A Bayesian Generalised Linear Model for the Bornhuetter-Ferguson Method of Claims Reserving

by

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Abstract
This paper shows how a Bayesian model within the framework of generalised linear models can be applied to claims reserving. It is shown that this approach is closely related to the Bornhuetter-Ferguson technique. The Bornhuetter-Ferguson technique has previously been studied by Benktander (1976) and Mack (2000), who advocated using credibility models. The present paper uses a fully Bayesian parametric model, within the framework of generalised linear models.

Keywords
Bornhuetter-Ferguson method; Chain-ladder technique; Generalised linear models; Loss run-off triangles.

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1. Introduction

The Bornhuetter-Ferguson method (Bornhuetter and Ferguson, 1972) has proved useful for certain classes of general insurance business. In particular, when the data are very unstable, a method such as the chain-ladder technique can produce unsatisfactory results. In order to stabilise the results, the Bornhuetter-Ferguson method uses an external initial estimate of ultimate claims. This is then used with the development factors of the chain-ladder technique, or something similar, to estimate outstanding claims. It is sometimes stated that the Bornhuetter-Ferguson method is a Bayesian method, because of the initial estimate of ultimate claims which is supplied as prior information. This method has been investigated by a number of authors, and the recent paper by Mack (2000) provides an excellent summary of this work. Mack (2000) gives details of a similar approach to that advocated in this paper, using a credibility theory approach, first suggested by Benktander (1976). The present paper is based very much on Generalised Linear Models, and the theory in this paper is not applicable to all sets of data (in particular, it may break down for negative incremental claims).

This paper shows how the Bornhuetter-Ferguson method is related to the Generalised Linear Models approach to claims reserving, using a Bayesian approach. The advantages of this are as follows:
i) It provides further help for the actuary to understand what assumptions are made when the Bornhuetter-Ferguson method is used.
ii) It clarifies the connection between the Bornhuetter-Ferguson method and the chain-ladder technique, particularly for the case when Generalised Linear Models are used.
iii) It allows mean square prediction errors to be calculated and indicates how the Bornhuetter-Ferguson method could be incorporated into a DFA exercise.
iv) It provides a range of Bayesian generalised linear models which could be used in a claims-reserving exercise, of which the Bornhuetter-Ferguson method and the chain-ladder technique are special cases.

The approach taken in this paper to the Bornhuetter-Ferguson method is based on the approach of Verrall (2000), in which it was shown that the chain-ladder technique can be expressed in a number of different ways as stochastic models. This paper uses the same framework and examines the Bornhuetter-Ferguson method, giving a number of insights into this method. The stochastic model developed in section 3 encompasses the Bornhuetter-Ferguson method, as it is usually implemented by actuaries, as a special case.

Without loss of generality, we assume that the data consist of a triangle of incremental claims:

$$\{C_{ij} : j = 1,\ldots,n-i+1; i = 1,\ldots,n\}.$$

The cumulative claims are defined by:

$$D_{ij} = \sum_{k=1}^{j} C_{ik}$$

and the development factors of the chain-ladder technique are denoted by $\{\lambda_j : j = 2,\ldots,n\}$.

No tail factors are applied, and claims are only forecast up to the latest development year (n) so far observed. It would be possible to extend this to allow a tail factor, using the same methods, but no specific modelling is carried out of the shape of the run-off beyond the latest development year.

The paper is set out as follows. Section 2 describes briefly the Bornhuetter-Ferguson method, as currently used by actuaries. Section 3 defines a Bayesian generalized linear model which has the Bornhuetter-Ferguson method as a special case. Section 4 uses this model to examine the relationship between the Bornhuetter-Ferguson method and the chain-ladder technique, and section 5 contains some concluding comments.

2. The Bornhuetter-Ferguson method

The Bornhuetter-Ferguson method can be summarised as follows.

1. Obtain an initial estimate of ultimate claims for each accident year.
2. Estimate the proportion of ultimate claims that are outstanding for each accident year, using, for example, the chain-ladder technique.
3. Apply the proportion from 2 to the initial estimate of ultimate claims from 1, to obtain the estimate of outstanding claims.

The usual way of expressing this is as follows:

Let the initial estimate of ultimate claims for accident year i be M_i
The estimate of outstanding claims for accident year i is

$$M_i\left(1-\frac{1}{\lambda_{n-i+2}\lambda_{n-i+3}\ldots\lambda_n}\right) \quad = M_i\frac{1}{\lambda_{n-i+2}\lambda_{n-i+3}\cdots\lambda_n}(\lambda_{n-i+2}\lambda_{n-i+3}\cdots\lambda_n-1)$$

Thus, $M_i\frac{1}{\lambda_{n-i+2}\lambda_{n-i+3}\cdots\lambda_n}$ replaces the latest cumulative claims for accident year i, to which the usual chain-ladder parameters are applied to obtain the estimate of outstanding claims. For the chain-ladder technique, the estimate of outstanding claims is $D_{i,n-i+1}(\lambda_{n-i+2}\lambda_{n-i+3}\cdots\lambda_n-1)$. Thus, it can be seen that the difference between the Bornhuetter-Ferguson method and the chain-ladder technique is the factor that is used to multiply the development factors. For the chain-ladder technique, this is $D_{i,n-i+1}$ and for the Bornhuetter-Ferguson method, this is

$$M_i\frac{1}{\lambda_{n-i+2}\lambda_{n-i+3}\cdots\lambda_n}.$$

3. A Generalised Linear Model for the Bornhuetter-Ferguson Method

This section defines a Bayesian generalised linear model which has the Bornhuetter-Ferguson method as a special case. The stochastic model is based on the generalized linear model for the chain-ladder technique defined by Renshaw and Verrall (1998), the (over-dispersed) Poisson model. The difference between the Poisson and the over-dispersed Poisson is the variance. For the Poisson distribution, the variance is equal to the mean, while for the over-dispersed Poisson the variance is equal to $\varphi\times$ the mean. In other words, if Y has an over-dispersed Poisson distribution with mean μ, the variance of Y is $\varphi\mu$. This implies that $\frac{Y}{\varphi}$ has a Poisson distribution with mean (and variance) $\frac{\mu}{\varphi}$.

The (over-dispersed) Poisson model for the chain-ladder technique can be written as follows

$$C_{ij} \sim \text{independent over-dispersed Poisson, with } E\left[C_{ij}\right] = x_i y_j \text{, and } \sum_{k=1}^{n} y_k = 1.$$

$$\text{i.e. } \frac{C_{ij}}{\varphi} \sim \text{independent Poisson, with } E\left[C_{ij}\right] = \frac{x_i y_j}{\varphi} \text{ , and } \sum_{k=1}^{n} y_k = 1. \tag{3.1}$$

$x_i = E\left[D_{in}\right]$, expected ultimate cumulative claims (up to the latest development year so far observed). The column parameters, $\left\{ y_j : j = 1,\ldots,n; \sum_{j=1}^{n} y_j = 1 \right\}$, can be interpreted as the proportions of ultimate claims which emerge in each development year. This model can be reparameterised as follows:

$$\frac{C_{ij}}{\varphi} \sim \text{Poisson, with } E\left[C_{ij}\right] = \frac{z_i y_j}{\varphi \sum_{k=1}^{n-i+1} y_k} \qquad \text{and} \qquad \sum_{k=1}^{n} y_k = 1. \tag{3.2}$$

In this case, $z_i = E\left[D_{i,n-i+1}\right]$, which is the expected value of cumulative claims up to the latest development year observed in accident year i.
In the chain-ladder model, no prior assumptions are made about the row parameters, $\{x_i : i = 1,\ldots,n\}$. The key assumption of the Bornhuetter-Ferguson method is that there is prior knowledge about these parameters, and thus the Bornhuetter-Ferguson method uses a Bayesian approach. The prior information can be summarised as the following prior distributions for the row parameters:

$$x_i \sim \text{independent } \Gamma(\alpha_i, \beta_i)$$

This Bayesian model can be implemented (and indeed is probably best implemented) using an Markov chain Monte Carlo (MCMC) approach, through the software winBUGS (Spiegelhalter et al, 1996) . However, it is useful to look at the predictive distribution for the data, in order to compare the Bornhuetter-Ferguson method with the chain-ladder technique. As in Verrall (2000), we consider just one row of data, for simplicity of exposition:

$$C_{i1}, C_{i2}, \ldots, C_{i,n-i+1}.$$

As we are considering only one row of data, we drop the i suffix, and write the model for C_j given $z(j)$ as

$$\frac{C_j}{\varphi} \mid z(j) \sim \text{Poisson, with mean } \frac{z(j)y_j}{\varphi S_j} \tag{3.3}$$

where $S_m = \sum_{k=1}^{m} y_k$

and $z(j) = E[D_j]$ is the expected value of aggregate claims up to development year j.

The label j has been attached to z, since the definition of z is different for each C_j.

Now $z(j) = E[D_j] = E[D_{j-1}] + E[C_j] = z(j-1) + \dfrac{z(j)y_j}{S_j}$

and hence $z(j) = \dfrac{z(j-1)}{1 - \dfrac{y_j}{S_j}} = \dfrac{z(j-1)S_j}{S_{j-1}}$.

Thus, the conditional distribution of C_j given $z(j-1)$ is

$$\frac{C_j}{\varphi} \mid z(j-1) \sim \text{Poisson, with mean } \frac{z(j-1)y_j}{\varphi S_{j-1}}. \tag{3.4}$$

The following theorem gives the recursive distribution of $z(j)$, and is needed mainly for the corollary, which gives the predictive distribution of C_j.

Theorem

$$z(j) \mid C_1, C_2, \ldots, C_j \sim \Gamma\left(\alpha + \frac{D_j}{\varphi}, \frac{\beta\varphi + S_j}{\varphi S_j}\right), \text{ for } j = 1,2,\ldots n.$$

Proof

We prove this theorem by induction. Consider first the distribution of $z(1) \mid C_1$.

$$\frac{C_1}{\varphi} \mid z(1) \sim \text{Poisson, with mean } \frac{z(1)}{\varphi}, \text{ since } y_1 = S_1.$$

Note that $z(n) = x$, and that $z(j-1) = \dfrac{z(j)S_{j-1}}{S_j}$, and hence, $z(1) = xS_1 = xy_1$.

The prior distribution of x is $x \sim \Gamma(\alpha,\beta)$, and hence the prior distribution of $z(1)$ is $z(1) \sim \Gamma\left(\alpha,\frac{\beta}{y_1}\right)$. A standard Bayesian prior-posterior analysis gives the distribution of $z(1)$, given C_1:

$$f(z(1) \mid C_1) \propto \left(\frac{z(1)}{\varphi}\right)^{\frac{C_1}{\varphi}} e^{-\frac{z(1)}{\varphi}} z(1)^{\alpha} e^{-\frac{\beta}{y_1}z(1)}$$

from which it can be seen that

$$z(1) \mid C_1 \sim \Gamma\left(\alpha+\frac{C_1}{\varphi},\frac{\beta}{y_1}+\frac{1}{\varphi}\right)$$

i.e. $$z(1) \mid C_1 \sim \Gamma\left(\alpha+\frac{D_1}{\varphi},\frac{\beta\varphi+S_1}{\varphi S_1}\right)$$

Hence the theorem is true for $j = 1$. Suppose it is true for $j-1$:

$$z(j-1) \mid C_1,C_2,\ldots,C_{j-1} \sim \Gamma\left(\alpha+\frac{D_{j-1}}{\varphi},\frac{\beta\varphi+S_{j-1}}{\varphi S_{j-1}}\right)$$

Since $\frac{C_j}{\varphi} \mid z(j-1) \sim$ Poisson, with mean $\frac{z(j-1)y_j}{\varphi S_{j-1}}$, a standard Bayesian analysis gives the posterior distribution of $z(j-1)$ as

$$z(j-1) \mid C_1,C_2,\ldots,C_{j-1},C_j \sim \Gamma\left(\alpha+\frac{D_{j-1}}{\varphi}+\frac{C_j}{\varphi},\frac{\beta\varphi+S_{j-1}}{\varphi S_{j-1}}+\frac{y_j}{\varphi S_{j-1}}\right).$$

i.e. $$z(j-1) \mid C_1,C_2,\ldots,C_j \sim \Gamma\left(\alpha+\frac{D_j}{\varphi},\frac{\beta\varphi+S_j}{\varphi S_{j-1}}\right).$$

Since we have a relationship between $z(j)$ and $z(j-1)$, we can obtain the distribution of $z(j)$, conditional on the information received up to development year j by a straightforward transformation:

If $\quad z(j-1) \sim \Gamma(a,b) \quad$ then $\quad z(j) \sim \Gamma\left(a,\frac{bS_{j-1}}{S_j}\right)$.

Hence,

$$z(j) | C_1, C_2, \ldots, C_j \sim \Gamma\left(\alpha + \frac{D_j}{\varphi}, \frac{\beta\varphi + S_j}{\varphi S_{j-1}} \frac{S_{j-1}}{S_j}\right)$$

i.e. $$z(j) | C_1, C_2, \ldots, C_j \sim \Gamma\left(\alpha + \frac{D_j}{\varphi}, \frac{\beta\varphi + S_j}{\varphi S_j}\right)$$

which completes the recursive proof of the theorem.

Corollary

The predictive distribution of $\frac{C_j}{\varphi}$ is

$\frac{C_j}{\varphi} | C_1, C_2, \ldots, C_{j-1} \sim$ Negative binomial, with parameters

$k = \alpha + \frac{D_{j-1}}{\varphi}$ and $p = \frac{\beta\varphi + S_{j-1}}{\beta\varphi + S_j}$.

Proof

The predictive distribution of $\frac{C_j}{\varphi}$ is

$$f\left(\frac{C_j}{\varphi} | C_1, C_2, \ldots, C_{j-1}\right) = \int f\left(\frac{C_j}{\varphi} | z(j-1)\right) f\left(z(j-1) | C_1, C_2, \ldots, C_{j-1}\right) dz(j-1)$$

$$= \int \frac{\left(\frac{z(j-1)y_j}{\varphi S_{j-1}}\right)^{\frac{C_j}{\varphi}} e^{-\frac{z(j-1)y_j}{S_{j-1}}}}{\frac{C_j}{\varphi}!} \frac{\left(\frac{\beta\varphi + S_{j-1}}{\varphi S_{j-1}}\right)^{\alpha + \frac{D_{j-1}}{\varphi}}}{\Gamma\left(\alpha + \frac{D_{j-1}}{\varphi}\right)} z(j-1)^{\alpha + \frac{D_{j-1}}{\varphi} - 1} e^{-\left(\frac{\beta\varphi + S_{j-1}}{\varphi S_{j-1}}\right) z(j-1)} dz(j-1)$$

$$= \frac{\left(\frac{y_j}{\varphi S_{j-1}}\right)^{\frac{C_j}{\varphi}} \left(\frac{\beta + S_{j-1}}{S_{j-1}}\right)^{\alpha + D_{j-1}}}{\frac{C_j}{\varphi}! \Gamma\left(\alpha + \frac{D_{j-1}}{\varphi}\right)} \int z(j-1)^{\alpha + \frac{D_{j-1}}{\varphi} + \frac{C_j}{\varphi} - 1} e^{-\left(\frac{\beta\varphi + S_{j-1}}{\varphi S_{j-1}} + \frac{y_j}{S_{j-1}}\right) z(j-1)} dz(j-1)$$

$$=\frac{\left(\frac{y_j}{\varphi S_{j-1}}\right)^{\frac{C_j}{\varphi}}\left(\frac{\beta\varphi+S_{j-1}}{\varphi S_{j-1}}\right)^{\alpha+\frac{D_{j-1}}{\varphi}}}{\frac{C_j}{\varphi}!\Gamma\left(\alpha+\frac{D_{j-1}}{\varphi}\right)}\frac{\Gamma\left(\alpha+\frac{D_j}{\varphi}\right)}{\left(\frac{\beta\varphi+S_j}{\varphi S_{j-1}}\right)^{\alpha+\frac{D_j}{\varphi}}}$$

$$=\frac{\Gamma\left(\alpha+\frac{D_j}{\varphi}\right)}{\frac{C_j}{\varphi}!\Gamma\left(\alpha+\frac{D_{j-1}}{\varphi}\right)}\left(\frac{y_j}{\beta\varphi+S_j}\right)^{\frac{C_j}{\varphi}}\left(\frac{\beta\varphi+S_{j-1}}{\beta\varphi+S_j}\right)^{\alpha+\frac{D_{j-1}}{\varphi}}$$

which is a Negative binomial distribution, with parameters

$$k=\alpha+\frac{D_{j-1}}{\varphi} \quad \text{and} \quad p=\frac{\beta\varphi+S_{j-1}}{\beta\varphi+S_j}$$

4. The relationship between the Bornhuetter-Ferguson method and the chain-ladder technique

This section compares the predictive distribution for the chain-ladder technique from Verrall (2000) and the predictive distribution for the Bayesian model derived above. Since they are both negative binomial distributions, we can compare them by looking at the means and variances. In particular, the means show clearly the differences in the assumptions made by the two approaches. We restore the row suffix, *i*.

For the chain-ladder technique, the mean of the predictive distribution is $\frac{D_{i,j-1}y_j}{S_{j-1}}$. Note that, since $\lambda_j=\frac{S_j}{S_{j-1}}$, this can also be written as $(\lambda_j-1)D_{i,j-1}$. Also, since $D_{i,j}=D_{i,j-1}+C_{i,j}$, the mean of the predictive distribution for aggregate claims is $\lambda_j D_{i,j-1}$.

The mean of the predictive distribution for $\frac{C_j}{\varphi}$ for the Bornhuetter-Ferguson method is

$$\frac{\left(\alpha_i+\frac{D_{i,j-1}}{\varphi}\right)\frac{y_j}{\beta_i\varphi+S_j}}{\frac{\beta_i\varphi+S_{j-1}}{\beta_i\varphi+S_j}}=\frac{\left(\alpha_i+\frac{D_{i,j-1}}{\varphi}\right)y_j}{\beta_i\varphi+S_{j-1}}$$

and hence the mean of the predictive distribution for C_j is

$$\varphi\frac{\left(\alpha_i+\frac{D_{i,j-1}}{\varphi}\right)y_j}{\beta_i\varphi+S_{j-1}}=\left(\frac{S_{j-1}}{\beta_i\varphi+S_{j-1}}\frac{D_{i,j-1}}{S_{j-1}}+\frac{\beta\varphi_i}{\beta_i\varphi+S_{j-1}}\frac{\alpha_i}{\beta_i}\right)y_j$$

$$=\left(Z_{ij}\frac{D_{i,j-1}}{S_{j-1}}+(1-Z_{ij})\frac{\alpha_i}{\beta_i}\right)y_j$$

where $Z_{ij}=\dfrac{S_{j-1}}{\beta_i\varphi+S_{j-1}}$.

It can be seen that this is in the form of what is called by actuaries "a credibility formula". In modern statistical terms, it is a natural trade off between 2 competing estimates for the row parameter. Note that y_j is the proportion of ultimate claims that emerge in development year j.

This is then multiplied by the prior mean of the ultimate claims, $\dfrac{\alpha_i}{\beta_i}$, for the Bornhuetter-Ferguson method, or an estimate of ultimate claims from the data, $\dfrac{D_{i,j-1}}{S_{j-1}}$, for the chain-ladder technique. We have here a combination of these two, and thus the stochastic model in this paper has the chain-ladder as one extreme (no prior information about the row parameters), and the Bornhuetter-Ferguson method as the other extreme (perfect prior information about the row parameters).

It is interesting to note that the Bornhuetter-Ferguson method assumes that there is perfect prior information about the row parameters, and does not use the data at all for this part of the estimation.

The mean of the predictive distribution can also be written as

$$\left(Z_{ij}D_{i,j-1}+(1-Z_{ij})\frac{\alpha_i}{\beta_i}S_{j-1}\right)\frac{y_j}{S_{j-1}}.$$

Since $\lambda_j=\dfrac{S_j}{S_{j-1}}$ and $S_n=1$, $\lambda_j\lambda_{j+1}\cdots\lambda_n=\dfrac{S_j}{S_{j-1}}\dfrac{S_{j+1}}{S_j}\cdots\dfrac{S_n}{S_{n-1}}=\dfrac{1}{S_{j-1}}$. Hence, we may also write the mean as

$$\left(Z_{ij}D_{i,j-1}+(1-Z_{ij})\frac{\alpha_i}{\beta_i}\frac{1}{\lambda_j\lambda_{j+1}\ldots\lambda_n}\right)\frac{y_j}{S_{j-1}}=\left(Z_{ij}D_{i,j-1}+(1-Z_{ij})\frac{\alpha_i}{\beta_i}\frac{1}{\lambda_j\lambda_{j+1}\ldots\lambda_n}\right)(\lambda_j-1).$$

It can then be seen that the two values used for the row parameter are the equivalent of those in section 2:

$D_{i,j-1}$ and $\frac{\alpha_i}{\beta_i}\frac{1}{\lambda_j\lambda_{j+1}\ldots\lambda_n} = M_i\frac{1}{\lambda_j\lambda_{j+1}\ldots\lambda_n}$.

The credibility factor, $Z_{ij} = \frac{S_{j-1}}{\beta_i\varphi + S_{j-1}}$, governs the trade-off between the prior mean and the data. Notice that the further through the development we are, the larger S_{j-1} is, and the more weight is given to the chain-ladder estimate. The choice of β_i is governed by the prior precision of the initial estimate for ultimate claims, and this should be chosen with due regard given to the over-dispersion parameter (an initial estimate of which could be obtained from the over-dispersed Poisson model of Renshaw and Verrall, 1998).

5. Conclusions

This paper has shown that the Bornhuetter-Ferguson method can be written as a Bayesian model within the framework of generalised linear models. It has also shown that the method as currently implemented by actuaries can be regarded, within the framework of generalised linear models, as an extreme case of a Bayesian model, which assumes perfect prior information about the row parameters. It would perhaps be more sensible to use a slightly less exact prior distribution in practice, and thus apply a model somewhere between the Bornhuetter-Ferguson method and the chain-ladder technique. The theory derived in this paper shows how the approach to Bornhuetter-Ferguson method described in Mack (2000) can be applied when a generalised linear model is used. The Bayesian model derived in this paper may break down if there are negative incremental claims values, and is therefore probably only suitable for paid data.

References

Benktander, G. (1976) *An approach to Credibility in Calculating IBNR for Casualty Excess Reinsurance.* In The Actuarial Review.

Bornhuetter, R.L. and Ferguson, R.E. (1972) *The Actuary and IBNR* Proc. Cas. Act. Soc., LIX, pp 181-195

Mack, T. (2000) *Credible Claims Reserves: The Benktander Method.* ASTIN Bulletin, Vol. 30, pp 333-347

Renshaw, A.E. and Verrall, R.J. (1998) *A stochastic model underlying the chain ladder technique.* British Actuarial Journal 4, IV, pp 903-923

Spiegelhalter, D.J., Thomas, A., Best, N.G. and Gilks, W.R. (1996) *BUGS 0.5: Bayesian Inference using Gibbs Sampling*, MRC Biostatistics Unit, Cambridge, UK.

Verrall, R.J. (2000) *An Investigation into Stochastic Claims Reserving Models and the Chain-Ladder Technique.* Insurance: Mathematics and Economics, Volume 26, pp 91-99.

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