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Modelling and Valuation of Guarantees in With-Profit and Unitised With-Profit Life Insurance Contracts

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Modelling and Valuation of Guarantees in With-Profit and Unitised With Profit Life Insurance Contracts

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Abstract

The purpose of this paper is to develop suitable valuation techniques for the broad category of participating life insurance policies. The nature of the liability implied by these contracts allows treating them as options written on the reference portfolio backing the policy. Consequently, our valuation approach is based on the classical contingent claim theory; in particular, Monte Carlo techniques are used to compute the values of the so called "policy reserve", that is the guaranteed payoff and the reversionary bonus, and the terminal bonus. The numerical results obtained are used to investigate the sensitivity of the policy reserve and the terminal bonus to changes in the model parameters. The paper also addresses the issue of a fair contract design for with-profit life insurance policies. Bearing in mind that the parameters characterizing the financial market are in general not under the control of the life insurance office, the implemented valuation procedure is used to determine the feasible set of design parameters that would lead to a fair contract.

1 Introduction

The modelling, valuation and pricing of participating life insurance contracts are important subjects for consideration because of the need for internal financial risk management of a life insurer, the need to demonstrate solvency and hence the ability to pay benefits, the need to measure profitability and be a desirable feature from the viewpoint of the policyholder. Thirdly, the benefits payable on early death (or surrender) may mean that time path as well as the terminal value are important to the policyholder. The participating contract may then be regarded as a product that is not directly available from the financial market and (in absence, for example, of a real risk free asset) may be contributing to a more complete market (Briys and de Varenne, 1994).

Following the pioneering work of Brennan and Schwartz (1976), most of the life insurance modelling literature has focused on unit-linked contracts, with minimum survival guarantees. Despite their historic and ongoing importance, participating contracts have been ignored because of their complexity and because the implicit guarantees seemed to be of minor significance in term of high interest rates and rising equity markets. As noted above, the economics environment has changed in recent years. The literature developed with single period models, which ignored the periodic build-up of the guarantees (Briys and de Varenne, 1994, 1997) but now focuses on multi-period models. Thus we would cite Bacinello (2002), Grosen and Jørgensen (2000, 2002), Hansen and Miltersen (2002), Jensen et al (2001), Miltersen and Persson (1999), Persson and Aase (1997) who have used market-based methodology, involving arbitrage free models, to investigate a range of different policy designs. Similarly, Wilkie (1987) and Hare et al (2000) have focused on UK designs but using a simulation-based asset model with arbitrage present.

Our approach is to consider and model the most common policy designs used in the UK for unitised with profits contracts and use a market-based methodology. These designs are common in many other European countries and Japan, where interest rate guarantees are offered.

The paper is organised as follows. Section 2 describes the valuation framework and Section 3 analyses in detail the numerical simulation-based results, with particular emphasis on a comparative statics sensitivity analysis in section 3.1 and consideration of the parameter choices consistent with the "fair value" principle in section 3.2.

2 Participating contracts and valuation framework

Participating life-insurance contracts are designed so that, in return for the payment of a fixed single or annual premia, they entitle the policyholder to a certain guaranteed benefit plus a regular, periodic reversionary bonus

(2001), we consider three smoothing schemes commonly used by insurance companies in the UK for the building up of the benefit and the reversionary bonus in relation to the reference portfolio A. Let r_G be the annual guaranteed rate. Then $r_P(t)$ is determined as follows.

<u>Scheme I</u> The rate credited on the policyholder account is the greater of the guaranteed rate r_G and the arithmetic average of the last τ period returns on the reference portfolio, so that

$$r_P(t) = \max\left\{r_G, \frac{\beta}{n}\left(\frac{A(t)}{A(t-1)} + \dots + \frac{A(t-n+1)}{A(t-n)} - n\right)\right\},$$

where $\beta \in (0,1)$ denotes the participating rate and n is the length of the smoothing period chosen as

$$n = \min(t, \tau).$$

<u>Scheme II</u> The policy rate is now based on the geometric average of the last τ period returns on the reference portfolio. In other words

$$r_P(t) = \max\left\{r_G, \beta\left(\sqrt[n]{\frac{A(t)}{A(t-n)}} - 1\right)\right\},$$

where β and n are defined as before.

<u>Scheme III</u> The last scheme considered in our analysis is based on the concept of a smoothed asset share. Let P^1 denote the unsmoothed asset share such that

$$\begin{aligned} P^1(t) &= P^1(t-1)(1 + r_P(t)) \\ r_P(t) &= \max\left\{r_G, \beta\frac{A(t) - A(t-1)}{A(t)}\right\}; \end{aligned}$$

then the policy reserve is defined as the average of the value at time (t) of the unsmoothed asset share with weight α, and the value at time $(t-1)$ of the smoothed asset share, i.e. the policy reserve itself, with weight $(1-\alpha)$. In other words

$$P(t) = \alpha P^1(t) + (1-\alpha) P(t-1)$$

with $\alpha \in (0,1)$ playing the role of the smoothing parameter (and hence replacing n).

for contracts expiring in 20 years and monitored on annual basis, i.e. the time step in each iteration is 1 year. The antithetic variable technique is implemented to increase the accuracy of the estimates.

3.1 Pricing and comparative statics

In this section, we consider the results obtained for the arithmetic crediting scheme (scheme I), and the smoothed asset share crediting scheme (scheme III) only. The results and analysis concerning the geometric scheme (scheme II) are similar to those obtained for scheme I: further details are available from the authors. Unless otherwise stated, the benchmark set of parameters is as follows:

$$A_0 = P_0 = 100; \quad r = 6\%; \quad r_G = 4\%; \quad T = 20.$$

$V_P(0)$: **Scheme I & II** In Figure 1, we consider the effect on the value of the policy reserve of different lengths of the smoothing period. As intuition suggests, V_P, is a decreasing function of n, the parameter governing this length. In fact, extending the averaging period reduces the volatility of the rate of return credited to the policy reserve, which in our model specification plays the role of the underlying asset. Consequently, as standard option theory shows, the option premium reduces.

Figure 2 shows the sensitivity of the policy reserve to the volatility parameter, σ, for different values of β ranging from 0.1 to 0.9 in steps of 0.1. From the plot, we observe that the value of the policy reserve, V_P, is an increasing function of the participation rate, β, at any level of σ. This is due to the fact that, as participation rate, the parameter β controls how much of the asset return is credited to the policy. Also, we observe that, as for any fixed strike option, the policy reserve is an increasing function of the underlying asset volatility. However, the policy reserve appears not to be very sensitive to σ when the participation rate, β, is low. In fact, as previously observed, β controls how much of the asset return feeds into the policy, and in this sense it acts as a "rescaling factor" of the asset volatility parameter. In other words, if β is small, little of the asset return volatility is transferred from the reference portfolio to the policy; however, as β increases, the policy reserve "inherits" more and more of the volatility risk affecting the reference portfolio. Different profiles of the policy reserve as a function of the guaranteed rate r_G are represented in Figure 3, for different levels of σ, from 0.1 to 0.5. As intuition suggests, the value of the policy reserve is increasing as the minimum guaranteed rate of return is raised. We note that the profile is approximately linear.

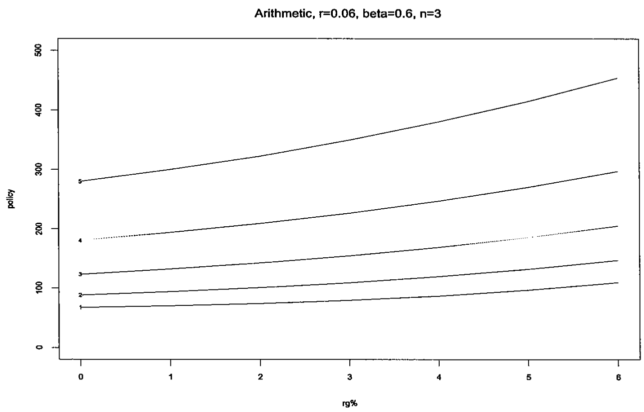

Figure 3: Arithmetic Scheme: Policy reserve vs the minimum guaranteed rate.

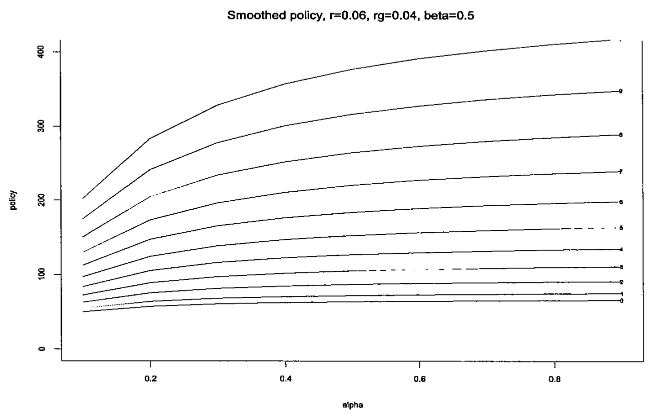

Figure 4: Smoothed Asset Share Scheme: Sensitivity of the policy reserve to the degree of smoothing.

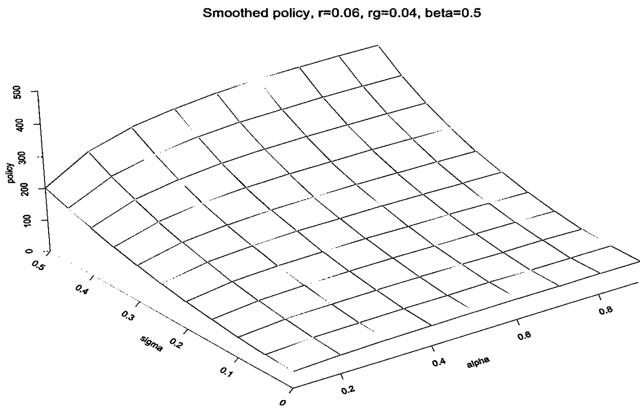

Figure 6: Smoothed Asset Share Scheme: The sensitivity of the policy reserve to changes in the volatility parameter and the degree of smoothing.

Consequently, the sensitivity of V_R to these parameters is opposite to that of V_P, although the shape is more complex since $R(T)$ is a convex function itself of $P(T)$. The return volatility, σ, however, affects both the reference portfolio (directly) and the policy reserve; and, as a result, the profiles that the value of terminal bonus exhibits are distinctive and particularly interesting.

Scheme I & II: Figure 7 shows that the value of the terminal bonus is a decreasing function of the participation rate, β. This is consistent with what has been previously observed: as seen in Figure 2, the policy reserve is more valuable as the proportion of the asset return which is credited to the policy is increased; at the same time the market value of the reference portfolio is independent of the design parameter β. However, the profiles we obtain for different levels of σ suggest a cross-over or inversion feature, which is particularly outlined in Figure 8. Here we have three corresponding graphs (for the same values of r, n, and r_G), but we show the value of the terminal bonus, V_R, plotted against σ for different values of β (ranging from 0.1 to 0.9, with $\beta = 0.1$ at the top and $\beta = 0.9$ at the bottom of each panel). As we can observe, the terminal bonus presents a different pattern depending on the value of the participation rate. For small values of β, V_R is an increasing function of

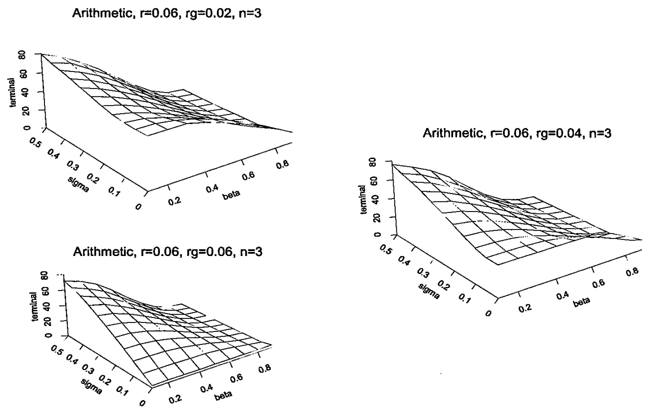

Figure 9: Arithmetic Scheme: the terminal bonus sensitivity to changes in the volatility and the participation rate.

σ, but as β is increased, the pattern of V_R shows an inversion of trend. We note that the effect almost disappears when the guarantee, r_G, equals the market interest rate (panel bottom left in Figure 8).

When the participation rate is low, the policy reserve is almost insensitive to σ, as we have seen before in Figure 2. This means that $P(T)$ is approximately constant. On the other hand, $A(T)$, the market value of the equity fund, is fully sensitive to changes in the volatility σ. Because of the downside protection offered by the guarantee, the terminal bonus $R(T)$ behaves like a conventional vanilla option and is an increasing function of σ. However, the higher the participation rate, β, the more of the volatility risk is transferred from the reference portfolio to the policy reserve. Consequently, as β increases, $A(T)$ and $P(T)$ both react similarly to changes in σ and the chance of exercising the terminal bonus option becomes smaller. V_R is effectively a premium for the probability mass in the tail of the distribution of $A(T)$, where the tail is defined by $P(T)$.

This phenomenon is attenuated for higher levels of the guarantee, as is shown in Figure 9. This Figure contains 3-dimensional pictures of V_R as function of σ and β for different choices of r_G. These

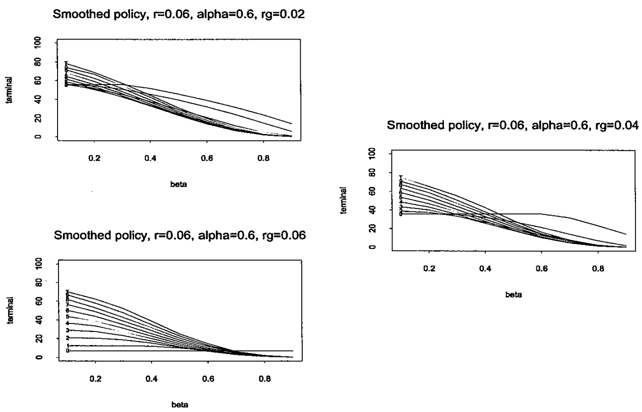

Figure 10: Smoothed Asset Share Scheme: the terminal bonus vs β profile.

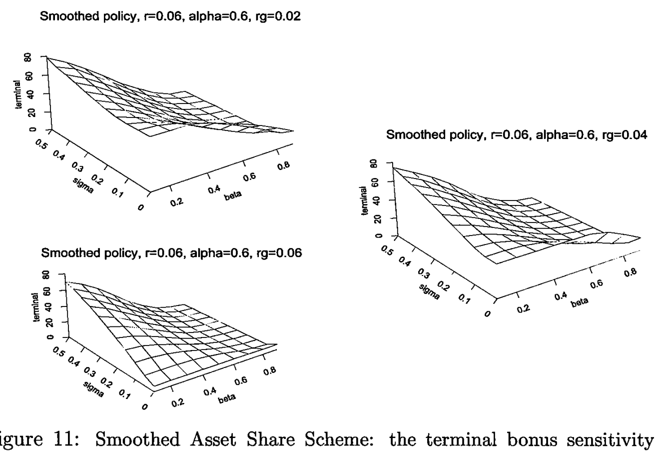

Figure 11: Smoothed Asset Share Scheme: the terminal bonus sensitivity to changes in the volatility and the participation rate.

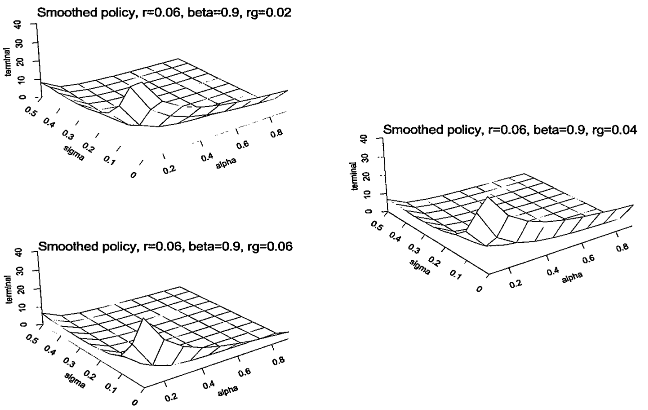

Figure 14: Smoothed Asset Share Scheme: the terminal bonus sensitivity to changes in the volatility and the degree of smoothing for maximum participation rate ($\beta = 0.9$).

3.2 Pricing and parameter selection: the "fair value" principle

So far we have considered the behaviour of the values of the policy reserve, V_P, and the terminal bonus, V_R, as the underlying model parameters are changed. In this section, we address the issue of a fair design for unitised with-profit life insurance contracts, i.e. the set of design parameters such that the value of the contract, as computed via arbitrage principles (see section 2), equals the initial premium paid by the policyholder. As seen in the previous sections, these contracts can be treated as financial derivative securities written on the reference portfolio. As such, their values depend on the specification of the contract design parameters: the level of the guaranteed return, r_G, the participation coefficients, β and γ, the smoothing parameters, n or α (according to which smoothing scheme is adopted for the reversionary bonus rate), and the term of the contract, T. The market parameters, like the reference portfolio volatility or the risk-free rate of interest, are also essential to complete the description of the contract. However, not every choice of these parameters determines an initially fair contract. Bearing in mind that the financial parameters are in general not under the control of the life insurance office, a possible guideline for the design of fair contracts may be obtained from an inspection of the insurer's balance sheet, here schematically

3.2.1 Guarantees and participation rates: scheme I

In this section, we explore possible combinations of (r_G, β, γ) such that the equilibrium condition (1) is satisfied. We focus in particular on the arithmetic crediting scheme, as the results obtained for the geometric scheme and the smoothed asset share scheme are similar.

In Figure 15, we plot the set of feasible combinations of the minimum guarantee, r_G, and the participation rate, β, for different levels of the terminal bonus rate, γ, and of the market volatility, σ. As the four panels show, there is a trade-off between r_G and β: in fact, if the contract offers a high guarantee, the policyholder is in a sense less willing to ask for a high participation rate as compensation for the "equity risk", that is for the risk of low returns from the reference portfolio. We also observe that, as the market conditions become more and more uncertain (i.e. when σ increases), the range of feasible choices for both r_G and β becomes smaller. In other words, the insurance company needs to reduce the benefits paid to the policyholder in order to contain the risk exposure implied by the contract. (This trade-off between the participation rate, β, and σ has also been observed by Briys and de Varenne, 1994).

In Figure 16, we consider the possible combinations of the minimum guaranteed rate of return, r_G, and the terminal bonus rate, γ, for different levels of the participation rate, β, in three market volatility scenarios. As in the previous case, we observe a trade-off between r_G and γ. In fact, when r_G is low, we expect the policyholder to require a larger percentage of the insurance final surplus in return for the initial premium. However, as the participation rate β increases, i.e. as more of the asset return is credited to the policy reserve, the insurer has to reduce both the guaranteed rate and the terminal bonus rate, especially when the market is very volatile, to the extent that, for a participation rate β as high as 70%, there are no feasible contracts when the market volatility is higher that 15% per annum.

Similar trends can be observed in Figure 17, in which we analyze the feasible combinations of the two participation rates, β and γ, for different levels of r_G and in different market volatility scenarios. Again, a trade-off between the parameters β and γ is observed. This trend suggests that in order to maintain the initial premium fixed, the insurance company has to lower the terminal bonus rate, γ, when the participation rate in the company profits, β, is high. The bottom-left panel shows the case in which the rate r_G equals the market interest rate; as the plot shows, the largest feasible rates are about 20% for the terminal bonus rate, γ, and 45% for the participation rate β, both in corresponding of the lowest volatility scenario ($\sigma = 10\%$) considered here.

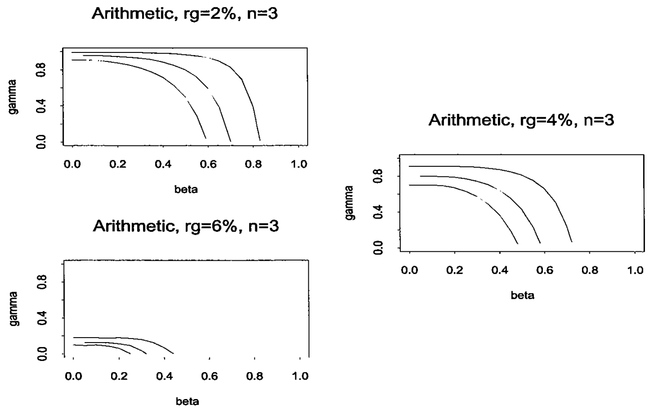

Figure 17: Feasible combinations of participation rates and terminal bonus rate vs the participation rate in the insurer profits.

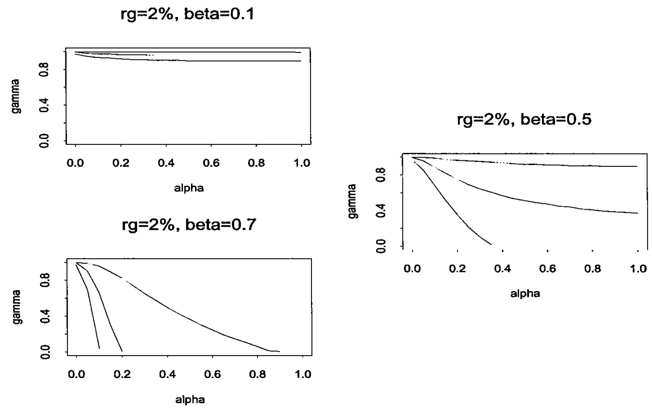

Figure 18: Smoothed Asset Share Scheme: the smoothing effect vs the terminal bonus rate. The case of a low guarantee.

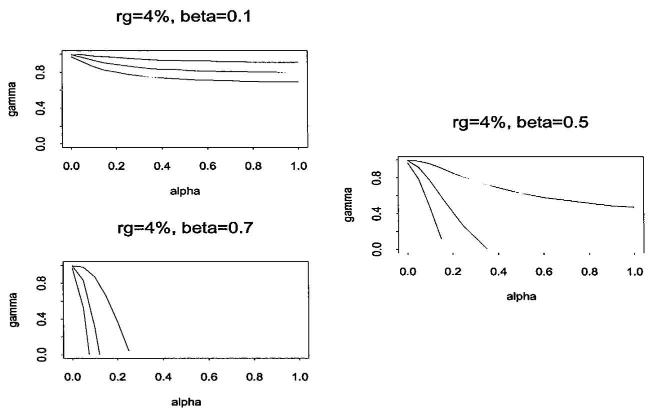

Figure 19: Smoothed Asset Share Scheme: the smoothing effect vs the terminal bonus rate. The case of a medium guarantee.

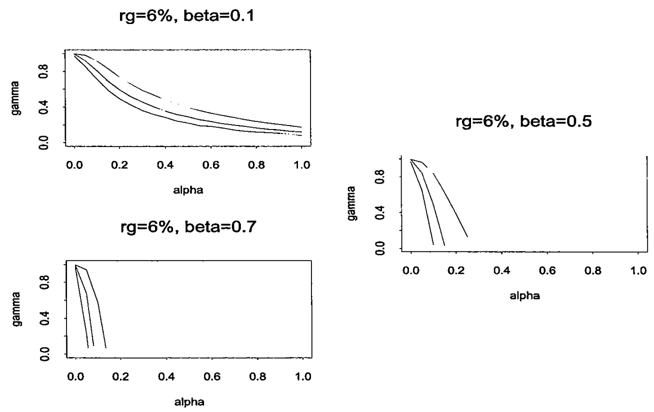

Figure 20: Smoothed Asset Share Scheme: the smoothing effect vs the terminal bonus rate. The case of guarantees equal to the market interest rate.

reducing the overall weight, α, of the policy reserve. For a fixed initial premium, the reduction allowed by the equilibrium condition (1) is larger for contracts offering a higher terminal bonus rate.

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