Investigation of cavitation and air entrainment during pilot injection in real-size multi-hole diesel nozzles

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Abstract

This paper investigates the complex multiphase flow developing inside the micro-orifices of diesel injector nozzles during pilot injection. High speed micro-visualisations of a transparent serial production nozzle tip replica are used to record the multiphase flow inside the flow orifices as well as near-nozzle spray development. The physical processes taking place are explained with the aid of a three-phase homogeneous mixture model utilized in the context of Large Eddy Simulations. Phase-change due to cavitation is modelled with a Rayleigh-Plesset equation based model, while compressibility of all the phases is considered. Numerical simulations shed light on the interaction between the vortex flow, liquid inertia and cavitation formation that take place simultaneously with air entrainment from the surrounding environment into the injector's sac volume during the injection and the dwell

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time between successive injections. The experimentally observed flow phenomena are well captured by the simulation during all injection phases. In particular the compression of pre-existing air bubbles inside the injector's sac volume during the injector opening, cavitation vapor condensation and air suction after the needle closure are well reproduced.
Keywords: LES, Multiphase flow, Cavitation, Fuel Injection, Realistic nozzle tip visualisation

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1. Introduction

New European Real Driving Emission (RDE) driving cycle legislations require significant research efforts to develop emission compliant and efficient passenger car engines [1]. In this context, the so-called digital injection schemes, used to split the fuel injection into multiple small injections with close separation among them, are widely applied in modern diesel engines in order to obtain simultaneous reductions in noise and emissions without compromising engine performance and fuel consumption [2, 3].Although the nozzle flow for static needle lift conditions has been extensively investigated (see selectively [4, 5, 6, 7]), not much work is available for the flow development during the dynamic operation of the injector, which plays a key influence on emissions [8, 9].

The digital injection schemes are often operated with fast injector needle opening and closing and with very small separation between injections; with typical dwell time of the order of $50\mu s$. This results in highly transient flow

Nomenclature

α_{air}	air volume fraction	[-]
α_{liq}	liquid fuel volume fraction	[-]
α_{nuc}	nuclei content	[-]
α_{vap}	vapor fuel volume fraction	[-]
$\boldsymbol{v}$	velocity field	[m/s]
λ_g	Taylor length scale	[m]
μ	viscosity	[Pa s]
μ_t	turbulent viscosity	[Pa s]
ρ	density	$[kg/m^3]$
ρ_{vap},ρ_{air}	vapour/air density	$[kg/m^3]$
σ	viscous stress tensor	[Pa]
τ_t	turbulent stresses	[Pa]
D	injection hole diameter	[m]
E	total energy	[J/kg]
F_{vap}, F_{cond}	empirical constants	$[m^{-1}]$
p	pressure field	[Pa]
R	gas constant	[J/kg/K]
R_b	bubble radius	[m]
R_e, R_c	evaporation/condensation rate	$[kg/m^3/s]$
Re	Reynolds number	[-]
T	temperature	[K]
y^+	non-dimensional wall disatance	[-]

and formation of massive cavitation inside the injection nozzle. In addition, modern diesel engines are operated under high injection pressure ($> 2500bar$) and utilise injectors with small injection hole diameters ($90 - 120\mu m$); these conditions pose significant difficulties in measuring and/or optically visualising the processes occurring in both the injector nozzle and within the high temperature combustion chamber. The majority of transparent real-size nozzle investigations featuring simplified single-hole geometries that generally confirm the presence of geometric-induced cavitation [10, 11, 12]. The work of [13, 14, 15], and the relevant early modelling work [16] were the first to substitute one of the holes of a production nozzle with a quartz window of identical geometric characteristics and was an experimental breakthrough that provided valuable information on flow and cavitation structures inside such micro-channels under realistic operating conditions; further studies were reported in [17]. A step forward was realised in [18], where a 3-hole, real-size, fully transparent nozzle allowed for unobstructed optical access inside the sac volume. Vortex cavitation is dramatically enhanced by vapour or air already present inside the nozzle volume [19]. Moreover, [20] showed that the structure of a vortex core is significantly affected by entrained vapour bubbles. Similarly, [21] demonstrated possible fragmentation of the vortex core so as to increase the vorticity at the core centre. Finally, the strong interaction observed between vortex properties and bubble dynamics[22], the coupling of radial and axial growth of bubbles trapped in vortices [23] and the interaction between shear (or normal strain) flow and bubble volume change [24] form a tremendously complex flow field inside an injector nozzle, where dynamic changes in the behaviour of vortices and vapour bubbles strongly affect the emerging fuel spray. Highly transient flow phenomena caused by the fast needle response times, give rise to formation of vortical structures and therefore, to string cavitation [25]. Transient effects have also been correlated to increased probability of surface erosion damage, which is attributed to both, geometric and string cavitation [26]. Cavitation in simplified nozzle replicas has been visualized even at pressures as high as $2000bar$, as shown in [27, 28]. Remarkably, in very recent studies, sonoluminescence from cavitation collapse observed in a simplified nozzle replica has been observed for the first time[29] and a neutron imaging technique has been developed overcoming the disadvantages of using materials transparent to visible light[30]. All aforementioned studies report data from one or just a few injection events. The group of the authors has reported in [31, 32, 33]] for the first time ensemble averaged images of cavitation developing in a real-size 6-hole transparent tip nozzle for single and pilot-main split injections up to $400bar$. Data from these investigations are further reported here and utilized for validation of the newly developed models. Only the very recent work of [34] has extended the range of operating conditions (injection pressures up to $1000bar$ and back pressures up to $30bar$) and geometrical features studied (hydro erosively ground inlet orifice) for long injections.

Given the limited information around the flow structure inside diesel injectors, fuel injection equipment manufacturers require robust predictive Computational Fluid Dynamics (CFD) tools, in order to understand the physical mechanisms taking place during injection. From a physical viewpoint, modelling of such flow conditions requires the fluid compressibility [35], mass transfer (cavitation, flash boiling, evaporation etc.) and heat transfer [36, 37, 38] to be taken into account, which increase the complexity as well as the computational cost of the simulations. Additionally, the fluid dynamic processes occur at high Reynolds number and therefore accounting for the effect of turbulence structures and vortex dynamics, is key in explaining how the spray is formed [39, 40, 41, 42]; this can only be resolved using very fine computational grids and scale resolving simulations such as Large Eddy Simulation (LES).

Recent LES including dynamic needle movement for the in-nozzle flow includes the work of Battistoni et al. [43] who simulated the start and end of injection for single hole nozzle using the cut cell cartesian method for modelling the boundary movement and a homogeneous relaxation model for cavitation phenomena. The work concludes that URANS predictions for the residual liquid back flow occur without fragmentation, while in LES liquid breaks up generating complex three dimensional structures. The URANS approach predicted at the end of the injection an annular void region stemming from the needle seat, which then re-condenses as the pressure is recovered. This was not observed in LES, where regions of low pressure are produced even in areas detached from the needle seat. The predicted near spray region was also different as no ligaments were formed in URANS; instead diffusion disperses the liquid in the surrounding air even if integral values like sac pressure and liquid volume fraction were not greatly affected. Ligament formation and gas ingestion into the nozzle at the end of injection are predicted, as observed experimentally in Phase Contrast X-ray images (for additional Phase Contrast X-ray studies see for example [9, 44]). The start-of-injection simulation shows how gas is ejected first, and liquid fuel starts being injected with a delay. The main result of these analyses is that if the sac is initially filled with gas, the liquid exit is delayed several tens of μs after the start of needle movement, which is in good agreement with the experimental evidence. This delay is of the order of $100\mu s$, and it is compatible with the duration of the first slow rising part of the needle movement. Orley et al. [45] used the cut cell cartesian method to simulate with implicit LES, a barotropic homogeneous equilibrium model for cavitation and a fully compressible 3-phase flow model a complete 9-hole diesel injector. The focus of the work was on the vortical development of the flow and the assessment of erosion sensitive areas during the operation of the injector. After the injector closing, strong collapse events of vapor structures in the needle seat and the sac hole cause the formation of violent shock waves. The authors highlighted that a fully compressible description of the flow is essential to capture such phenomena. It was also concluded that despite steady needle simulations capturing the main flow features reasonably well, vapor creation during the closing phase of the needle valve requires information on the previously developed flow; thus, reliable prediction of erosion-sensitive areas due to collapse events during and after the closing of the needle can only be predicted accurately by including the unsteady needle motion. Finally the work of Koukouvinis et al. [35] used a 2-phase dynamic needle approach based on a combination of layering and stretching algorithms together with a Rayleigh-Plesset based cavitation model with increased mass transfer, to compute the opening phase of two different injector designs; the findings have correlated the pressure peaks in the domain with areas that suffer from erosion. Whichever the chosen modelling approach may have been, previous studies have lacked validation [45], had indirect validation [35] or were not of direct relevance to modern applications [43], since a single hole nozzle lacks the complex sac recirculation flow present in modern diesel injectors.

The current work, to the best of the authors knowledge, presents for the first time a successful 3-phase LES of a diesel pilot injection including the compressibility of the phases, cavitation effects and the needle valve movement of a real size 6 hole nozzle for which validation is performed against transparent nozzle tip visualisations. The need to employ LES derives from the need to predict the complex vortical flow and liquid structures inside the sac during and after injection; moreover, and as it is shown, replicating the observed phenomena requires the inclusion of compressibility effects.

The present paper is structured in the following way: first an overview on the experimental results is given for a diesel pilot injection visualization of a transparent nozzle tip. Then the numerical methodology employed is described in detail, followed by the comparison of the CFD results with the transparent nozzle visualisations for which good agreement is obtained and interpretation of the observed phenomena is provided.

2. Experimentally observed multiphase phenomena

The development of the 3-phase simulation methodology has been validated against high speed visualisations of a transparent Delphi Technologies Diesel 6-hole nozzle tip manufactured by City, University of London. The metallic injector nozzle tip was substituted with a transparent acrylic tip. The design is a standard serial production geometry, i.e not just a multi-hole nozzle, but a fully operational, serial production type. The detailed results and findings of that experimental campaign as well as the setup details were reported in [31, 32, 33] and will not be repeated here. The 6-hole transparent tip has holes with no taper (zero conicity) and a nominal diameter (D) of $160\mu m$. The electrical pulse activation width for a pilot injection was $0.5ms$. High speed cameras recorded the events at a frame rate of $30000fps$. An example of a pilot injection for a rail pressure of 300bar into atmospheric conditions can be found in Figure 1. Given the image acquisition rate, the pilot injection including all major events after closing lasts for 24 frames. As discussed in [32, 33] air trapped in the sac after the end of the injection aggregates forming bubbles in the sac and occupying part of the hole. Prior to $233.33\mu s$ after the electrical trigger, no change is observed and therefore images are not shown. Then the trapped bubble shows slight expansion due to the initial volume created by the needle as it starts lifting ($300\mu s$ after the trigger) and subsequent compression ($400\mu s$ after the trigger) highlighting the need to model air compressibility. This is followed by void coming from the seat passage and its advection into the hole ($500\mu s$ after the trigger). Then, due to flow acceleration at the hole entrance, void structures are seen in the hole during the opening phase ($600\mu s$ after the trigger). During the needle closing phase, vapour increases substantially in the hole and void coming from the seat reappears ($633.33\mu s$ after trigger). At the end of the injection, the sac gets full with bubbles and the spray greatly weakens ($733.33\mu s$ after the trigger), followed by what seems to be air suction ($766.66\mu s$ after the

Figure 1: Experimental results. Time sequence of a pilot injection transparent nozzle tip visualisation.

trigger). Finally, a bubbly mixture is observed floating in the sac as well as an oscillatory movement of the air in the hole (1000μs after the trigger). An important input for nozzle flow moving needle simulations is the needle lift profile which was extracted from the images [31, 32, 33].

3. Modelling approach

The simulations are computed using the commercial CFD code ANSYS Fluent [46]. The nozzle flow is solved using a homogeneous, three-phase mixture model (liquid fuel, vapour fuel and air) where all phases share same velocity, pressure and temperature. The code is supplemented with user defined functions (UDFs) for implementation of the thermo-hydraulic properties of diesel and the needle movement.

3.1. Multiphase model

The properties appearing in the transport equations are determined by the presence of the component phases in each control volume. Defining α_{liq}, α_{vap}, α_{air} as the volume fraction of liquid fuel, air and vapour fuel in a cell, respectively, the density in each cell is given by: $\rho = \alpha_{liq}\rho_{liq} + \alpha_{vap}\rho_{vap} + \alpha_{air}\rho_{air}$.

All other properties (e.g. viscosity) are computed in this manner. Obviously, the volume constraint $\alpha_{liq} + \alpha_{air} + \alpha_{vap} = 1$, in each cell must be respected. The solved equations consist of the continuity, momentum and energy of the mixture, and the mass conservation equations for the vapor and the air:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}) = 0 \quad (1)$$

$$\frac{\partial \rho \boldsymbol{v}}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}) = -\nabla p + \nabla \cdot \sigma \quad (2)$$

$$\frac{\partial \rho E}{\partial t} + \nabla \cdot (\boldsymbol{v}(\rho E + p)) = \nabla \cdot (k_{eff} \nabla T) + \sigma \cdot \boldsymbol{v} \tag{3}$$

$$\frac{\partial \alpha_{vap} \rho_{vap}}{\partial t} + \nabla \cdot (\alpha_{vap} \rho_{vap} \boldsymbol{v}) = R_e - R_c \tag{4}$$

$$\frac{\partial \alpha_{air} \rho_{air}}{\partial t} + \nabla \cdot (\alpha_{air} \rho_{air} \boldsymbol{v}) = 0 \tag{5}$$

The source terms R_e and R_c represent the mass transfer between liquid and vapour phase due to cavitation. The effective viscous stress tensor is defined as $\sigma = \tau + \tau_t = \mu(\nabla v + (\nabla v)^T) + \tau_t$,

where μ is the viscosity of the mixture and τ_t are the turbulent stresses defined per the turbulence model being used. The energy is computed as the mass average for each phase and the internal energy of each phase is based on the local thermodynamic conditions of that phase [37].

The source terms appearing in the vapour volume fraction transport equation ($R_e - R_c$) represent the mass transfer between fuel liquid and vapour phases due to cavitation bubble expansion and collapse respectively. The calculation of these values is based on the Rayleigh-Plesset equation describing bubble expansion and collapse [47], and its magnitude is based on the Zwart-Gerber-Belamri cavitation model [48] which reads as:

$$R_e = F_{vap} \frac{(3\alpha_{nuc}(1 - \alpha_{vap})\rho_{vap})}{R_b} \sqrt{\frac{2}{3} \frac{max((p_{vap} - p), 0)}{\rho_{liq}}} \tag{6}$$

$$R_c = F_{cond}\frac{(3\alpha_{vap}\rho_{vap})}{R_b}\sqrt{\frac{2}{3}\frac{max((p-p_{vap}),0)}{\rho_{liq}}} \tag{7}$$

F_{vap} and F_{cond} are empirical calibration coefficients, α_{nuc} is the volume fraction associated with the nuclei contained in the liquid and R_b the assumed bubble radius and p_{vap} is the vapour pressure. According to [48], values of $R_b = 10^{-6}m$, $\alpha_{nuc} = 5\times10^{-4}$,$F_{vap} = 50$, $F_{cond} = 0.01$ give reasonable results in a wide range of flows. Nevertheless, as discussed in [49] the mass transfer magnitude for these values could be insufficient creating areas of unrealistic liquid tension and not reproducing correctly the Rayleigh-Plesset bubble collapse, the suggested solution is to increase the empirical calibration coefficients several orders of magnitude to approximate the model to a Homogeneous Equilibrium Model (HEM). However, within this work the original coefficients published in[48]were used.

3.2. Turbulence model

The target when using LES is to capture the large scales that are dependent of the physical domain simulated while modelling the sub-grid turbulent scales. This is achieved by filtering of the Navier-Stokes equations using a spatial low-pass filter determined by the cell size of the computational domain used. This operation leaves the flow equations unchanged, but transforms the equations into equations for the filtered magnitudes [50]. During this operation terms in the equations appear representing the sub grid scale contributions to the equations of motions and have to be modelled. The closure of the model requires calculating a suitable sub grid turbulent dissipation (viscosity) μ_t. For such purpose, the Wall-Adapting Local Eddy-Viscosity (WALE) model is chosen [51]. This model is capable of correctly reproducing the correct turbulence wall behaviour ($\mu_t \sim o(y^3)$) and becomes 0 at $y = 0$, being y the normal distance to the wall. Another advantage is that it returns a zero turbulent viscosity for laminar shear flows which allows the correct treatment of laminar zones in the domain, this is necessary for modelling the start of injection when flow velocities are low.

3.3. Fluid properties

High injection pressures and low lifts cause high injection speed velocities and important transient heating effects making an incompressible approach unjustifiable [36, 37, 35]. Even if for the transparent nozzle tip testing conditions the pressure is lower than engine conditions, the diesel liquid phase is modelled as a compressible liquid based on the measurements made for the calibration oil Normafluid ISO4113. This is the usual fuel for testing and calibrating diesel fuel injection systems in laboratory at an industrial level. All diesel properties that follow are taken from [52, 53], where details on how the measurement methodology, range of validity, method for fitting the coefficients and their values can be found. (see Figure 2 for plots of the density and viscosity values for different pressures and temperatures) These properties were implemented into ANSYS Fluent following the available User-Defined-Real-Gas-Model (UDRGM) functionality as in [37]. As

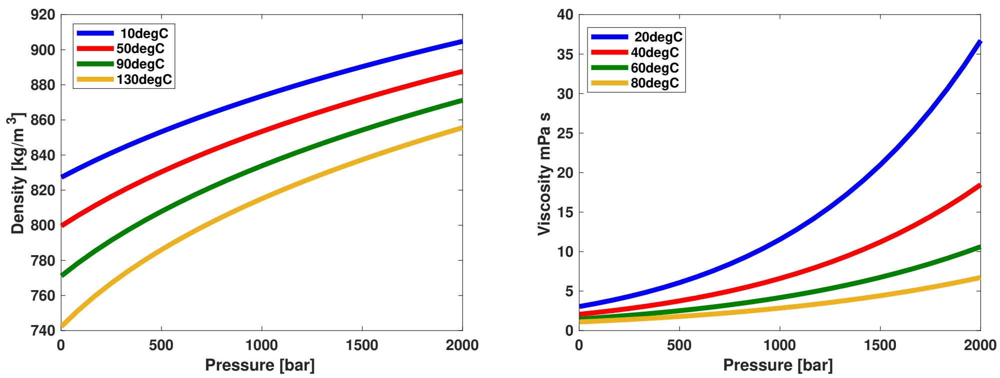

Figure 2: Diesel fuel properties implemented. Density (left) and viscosity (right) diesel fuel properties used.

mentioned in the experimental results section, air compressibility effects are observed during the sac filling event and therefore the air density is modelled as an ideal gas with equation of state $p = \rho RT$.

3.4. Moving mesh methodology. Mesh generation and boundary conditions.

Modelling the dynamic movement of the needle is inherently difficult. At low lifts the cells in the seat are squeezed into very small gaps deteriorating their quality, which can have an impact on the robustness and accuracy of the simulation. Moreover, the contact between walls is not trivial to model since the continuity of the mesh is broken. Recent advances have been reported in [54] where the immersed boundary method has allowed simulations to be performed even at zero needle lift; however, this method has not been adopted here and as a compromise, the closed needle is modelled using the seat surface as a wall when the needle lift is below $1\mu m$.

The approach followed is based on an interpolation approach between two topologically identical meshes (key-grids) with the same number of cells and was already employed by the authors in [55]. The initial mesh has a $1\mu m$ lift and the high lift mesh is based on the maximum lift reached for the pilot injection $36\mu m$. Based on the node position of this two meshes any intermediate lift is achieved by linear interpolation between the node position of the two key-grids. Another difficulty associated is the loss of resolution in the seat passage as the needle reaches high lifts, this requires interpolating the results into another pair of key-grids such as in [37]. For the pilot injection cases considered here, this was not needed due to the relatively low lift attained ($36\mu m$). Moreover, in order to save computational resources, just a 60^o sector is model (one hole) based on the nominal (target) geometry. Figure 3 (left) shows the computational domain, consisting of different surfaces; the hole, housing, needle, seat inlet and side surfaces. Additionally, a 2mm long conical discharge volume is added in order to move away the outlet boundary condition from the areas of interest. The computational mesh used for the LES flow simulation is a fully hexahedral mesh.

The LES settings are adapted from the basis of the previous successful studies on diesel [39, 40, 41, 42] and gasoline [55, 56] direct injection and primary breakup simulations. In order to choose the appropriate filter/mesh size for the LES, the Taylor micro-scales (λ_g) have been estimated. This length scale is the intermediate length scale at which fluid viscosity significantly affects the dynamics of turbulent eddies in the flow [57]. For the flow inside the transparent tip, the Reynolds number based on the nozzle hole diameter, outlet pressure and inlet temperature can be estimated to be $Re = \frac{(\rho V D)}{\mu} \sim 13000$. The Taylor micro-scales can then be approximated by [50]: $\lambda_g = \sqrt{\frac{10D}{Re}} = 4.4\mu m$. However, in order to resolve the smallest eddies close to the wall, the non-dimensional wall distance based on the friction velocity has to be of the order of 1 ($y^+ \sim 1$) [50]. Therefore, additional refinement close to the walls is needed. An estimate of this value based on the turbulent boundary layer theory yields a cell wall distance of $\sim 0.2\mu m$. In order to reach a value of $\sim 5\mu m$ in the bulk flow without increasing in excess the number of cells, a cell growth ratio of 1.1 was applied in the wall. Under these constraints, a $\sim 5M$ element mesh was produced, with a volume change between neighbouring cells below 3, minimum cell angle of 27^o and $3D$ determinant (normalized triple product of the vectors starting from each cell node) above 0.6 for both key-grids. Special care was taken to refine the needle seat area in the stream-wise direction in order not to exceed for low lifts aspect ratios of 100 in the direction of the bulk flow. Figure 3 (right) depicts the two meshes needed for the interpolation method, and a front view of the mesh showing the additional refinement in the seat area. A pressure boundary conditions was applied to the inlet of the domain. The pressure at the injector entrance in the high-pressure pipe was taken from the experimentally recorded values for every individual injection event. During the opening phase, pressure decreases at the injector entrance due to the increasing flow through it. At the end of the injection an over pressure is observed due to the water hammer effect after needle closing. The pressure

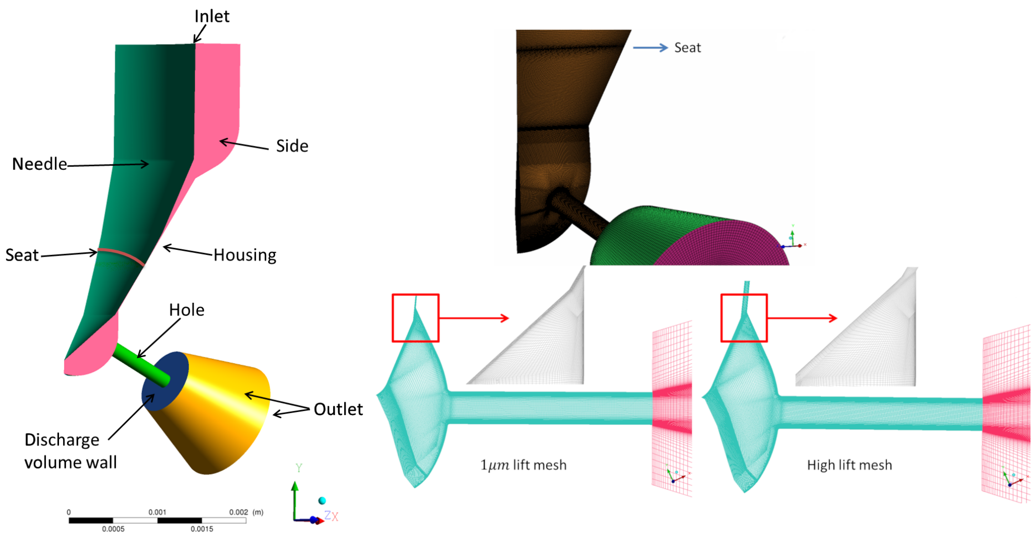

Figure 3: Geometrical model and mesh. Domain simulated and boundary conditions (left). Mesh showing seat refinement (right-top) and mesh cross section for both high and low lift meshes (right-bottom).

at the entrance of the injector was provided in [31]. A temperature of $300K$ was chosen for the flow entering the domain and an air mass fraction value of 2×10^{-5} was imposed to take into account the possible dissolved air since it is a typical value for fuel or water exposed to ambient pressure [58]. The non-slip boundary conditions was applied to the non-moving wall (housing, hole, discharge volume wall, and, seat surface below $0.1\mu m$) as well as to the needle according to the motion profile resulting from the needle lift profile extracted from the images [31]. Periodic boundary condition have been applied to the side surfaces. And fixed pressure outlet was applied to the outlet surfaces, with pressure 1bar and $300K$ and air volume fraction prescribed as 1 in the case of back-flow.

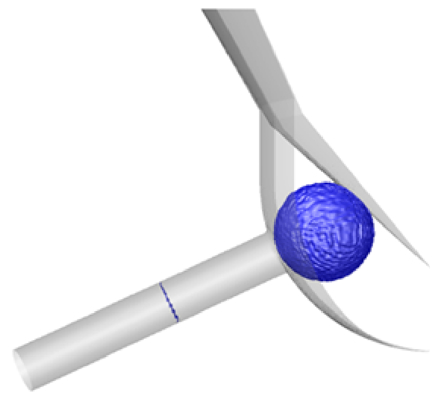

Figure 4: Initial simulation instant. Iso-surface of 0.5 liquid volume fraction and a mid-plane for the initial instant.

The experimental images of the transparent nozzle show trapped air bubbles inside the injector before the start of injection. The mechanism behind the appearance of this bubble is not straight forward to derive from the experimental images. Regardless, the LES nozzle flow simulation is initialised in qualitatively similar way; half of the hole is filled with air and an air spherical bubble is included in the sac (see Figure 4).

The computational domain above the seat surface is initialised at the pressure corresponding to that instant. Below the seat, the simulation is initialised at a pressure of $1bar$. All the domain is initialised at a temperature of $300K$ and with zero velocity. For the closing phase the movement of the needle is stopped when it reaches $1\mu m$ however the seat surface is not switched from interior to wall until the needle lift profile reaches $0.1\mu m$.

The solver used is segregated and pressure-based. The pressure-velocity coupling is achieved using the SIMPLEC algorithm [59]. The continuity equation was discretised using a second order upwind scheme [60] while for the momentum equation a bounded central differencing scheme based on the normalized variable diagram (NVD) approach together with the convection boundedness criterion (CBC) [61] was used. The bounded central differencing scheme is a composite NVD-scheme that consists of a pure central differencing, a blended scheme of the central differencing and the second-order upwind scheme, and the first-order upwind scheme. The first-order scheme is used only when the CBC is violated. This scheme has small numerical dissipation and sufficient numerical stability for industrial LES simulations [62]. Discretisation of the volume fraction equations was done with the quadratic upstream interpolation for convective kinetics (QUICK) scheme (in order to capture the high density ratios) [60], pressure interpolation with the body force weighted scheme [46] and the temperature equation was discretised with a first order upwind scheme. Finally the calculation of gradients was done using the Least Squares Cell-Based method.

The used solver is pressure-based and therefore the simulation stability is not limited by the acoustic wave propagation time scale. However, temporal resolution for LES requires minimum diffusion for the advection of the turbulent eddies. Therefore, an adaptive time step method is employed to ensure the advection CFL number stays below 1 throughout the computational domain.

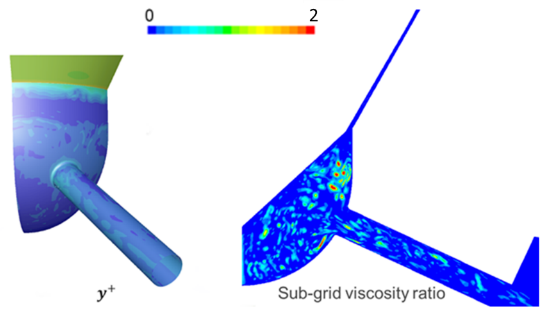

Figure 5: Mesh resolution evaluation. y^+ contours on the nozzle wall (left) and sub-grid viscosity ratio (right) for highest needle lift during the pilot injection.

3.5. LES mesh quality evaluation

The instantaneous fields of sub-grid viscosity ratio and y^+ for a characteristic moment at the highest lift ($t = 0.608ms$) are shown in Figure 5. Based on the y^+ the boundary layer resolution can be assessed; this value only exceeded 1 in areas above the seat and gradually transitions to values well under 1 ensuring a good wall shear resolution for the small eddies near the walls. Spatial resolution can be evaluated from the sub-grid viscosity ratio, which is defined as the sub-grid scale viscosity introduced by the WALE model divided by the molecular viscosity. Its value is mostly under 1 throughout the domain peaking at values of around 2 in the separation region that occurs at the entrance of the sac due, confirming the suitability of the mesh.

4. Results and discussion

The evolution of the volume fraction inside the nozzle for the different phases is shown in Figure 6. Additionally, the imposed needle lift extracted

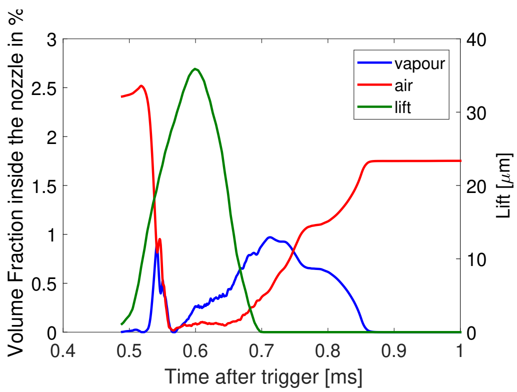

Figure 6: Integral results. Volume of vapour and air inside the nozzle and needle lift against time.

from the image sequence shown in Figure 1 is shown as well. The simulation is started at the physical time $0.4874ms$ coincident with a lift of $1\mu m$ for the imposed profile. During the opening phase it follows from this plot that initially there is air present inside the nozzle. This air is evacuated out of the nozzle while cavitation is generated showing a peak between $0.5ms$ and $0.6ms$ and decreases. As the injection transitions towards the closing phase the amount of vapour increases, showing a peak just after the needle closes, while the amount of air continually increases by a process of air suction as it will be shown in the following section.

A comparison between the transparent nozzle tip images and the simulation results at the start of the injection is shown in Figure 7. In particular, a snapshot of the predicted liquid volume iso-surface of 50 at $t = 0.532ms$ is shown. At the early stages of the injection the simulation reproduces the compression of the air bubble inside the sac volume. The compression is caused by the pressure build up in the sac, showing the need for modelling

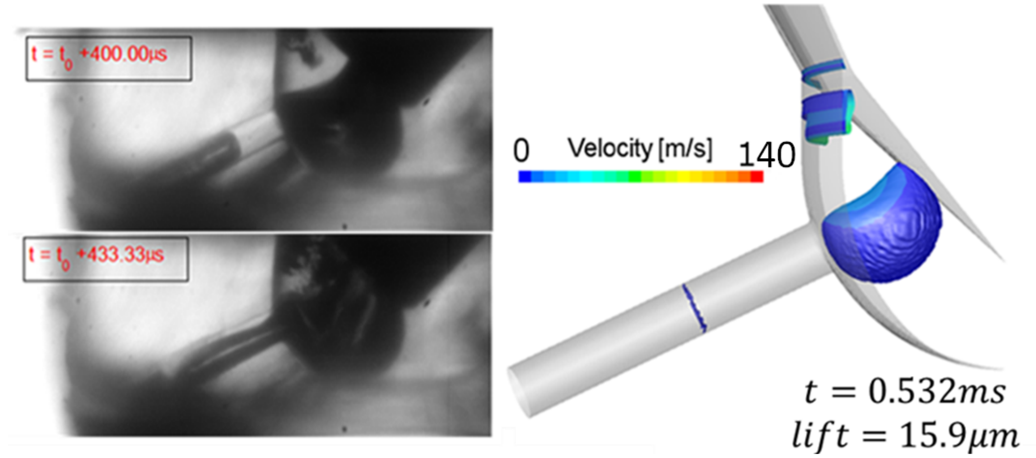

Figure 7: Start of injection results. Experimental visualisations (left), 50% liquid volume fraction iso-surface coloured by velocity magnitude (right).

the compressibility of the air. This is quickly followed by cavitation coming at the needle seat passage, due to flow separation and shear in this area.

Sample simulation results and the transparent nozzle tip images for the needle opening phase are shown in Figure 8. The CFD results indicate that cavitation produced at the sac entrance is transported directly into the hole. Simultaneously, the air bubble is further compressed and is pushed to recirculate parallel to the needle in the direction of the needle motion. Similarly to the experimental images, the air bubble is seen breaking down and mixing with any remaining cavitation into a fine bubbly mixture which is then advected into the hole.

As the needle lift increases and the flow further develops, the simulation indicates that air disappears from the sac volume, as seen in Figure 9. This is attributed to a combination of two effects. Firstly, the sac pressure build

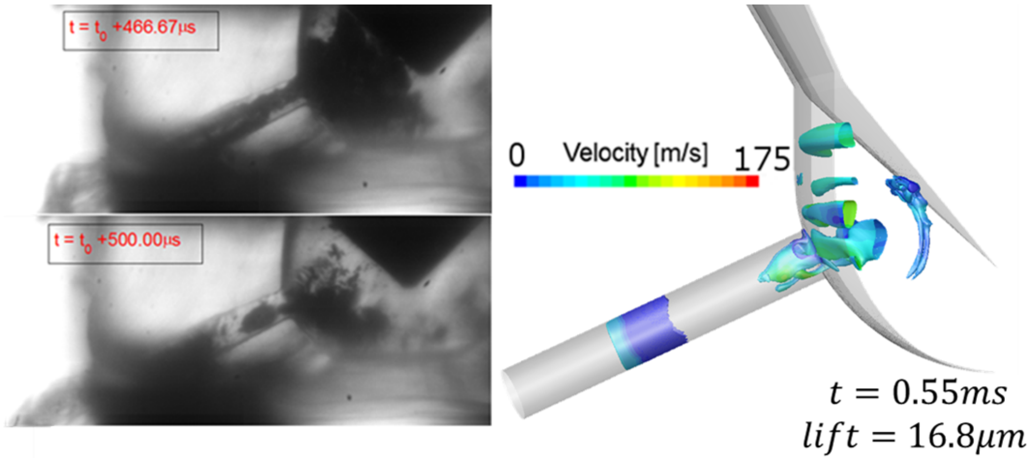

Figure 8: Needle opening phase results. Experimental visualisations (left), 50% liquid volume fraction iso-surface coloured by velocity magnitude (right).

up causes the air to be compressed, reducing its volume fraction. Secondly, as the air is trapped within the recirculation zone developing inside the sac volume, it enters into the injection hole, where it expands due to the local pressure drop at its entrance. This contributes to the void areas observed and suggests that the void observed experimentally is a combination of air and fuel vapour. In addition, part of the void visible in the simulation can be attributed to geometrical cavitation developed at the hole inlet upper lip which can also be seen from the experimental images.

The only two experimental frames available for the needle closing phase together with the simulation results are shown in Figure 10 (top). As the needle valve moves into the closing phase, the amount of void in the hole increases. This is in agreement with the simulation results from Figure 6, where volume content as a percentage of the injector volume of both air

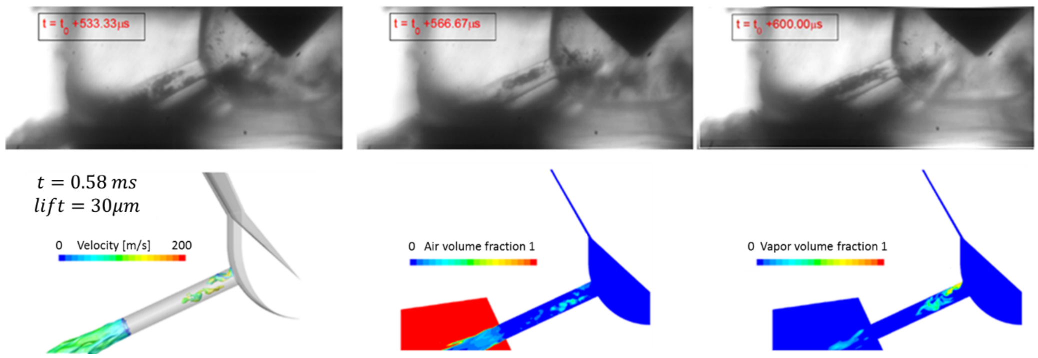

Figure 9: Results as flow further develops during the opening phase. Experimental visualisations for three time instances (top), 50% liquid volume fraction iso-surface coloured by velocity magnitude (bottom-left), air volume fraction contours (bottom-centre) and vapour volume fraction contours (bottom-right).

and vapour are plotted against time, it follows that these quantities increase during the needle closing phase. This void in the simulation has two sources, one from the unstable vortical flow from the sac coming into the hole and another due to geometrical cavitation in the hole inlet corner. Regarding the experimental results at very low lifts ($lift = 6\mu m$), a bubbly mixture appears in the sac and bubbles of size of the order of the hole diameter appear in the hole. At very low lifts ($lift = 6\mu m$), the simulation model predicts high velocities in the hole; however, since the flow coming from the seat is throttled, pressure loss in the sac is occurring and a void structure appears in front of the hole. The bubbly mixture in the sac volume correlates to the void structure created in front of the hole, which is predicted to be composed of a mixture of fuel vapour and expanded air. On the other hand, the visualised bubbles inside the hole correlate to the big amount of cavitation computed in the hole.

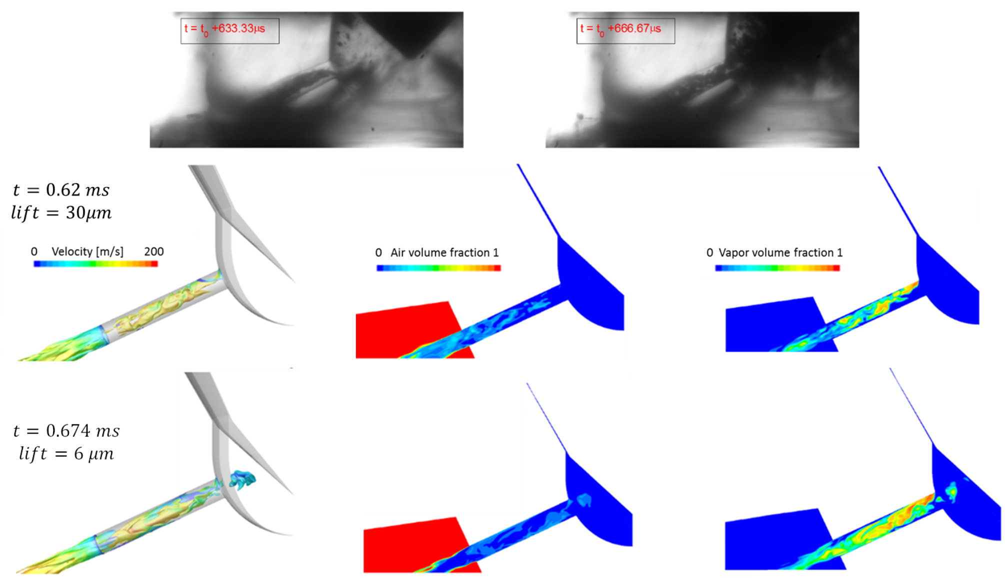

Figure 10: Needle closing results. Experimental visualisations for two time instants (top). Simulation results (center and bottom). For the simulation results 50% liquid volume fraction iso-surface coloured by velocity magnitude (left), air volume fraction contours (center) and vapour volume fraction contours (right) are presented.

A time sequence of the pressure field is presented in Figure 11. Before the needle valve closes, the predicted sac pressure is still higher than the ambient pressure ($t = 0.674ms$), but immediately after the needle valve closing ($t = 0.698ms$) a pressure wave is generated that travels towards the sac volume; this leaves the sac pressure below the ambient pressure ($t = 0.77ms$). In agreement with Figure 6, where air volume fraction inside the nozzle is seen to increase after needle closing, this induces the spray to weaken and air to be sucked back from the ambient into the nozzle until the sac pressure is balanced with the exterior pressure ($t = 1ms$).

Evidence is also provided in Figure 12, which shows a time sequence of air and vapour volume fraction fields. It clearly depicts the weakening flow momentum in the injection hole ($t = 0.698ms$) leading to air suction ($t = 77ms$). Finally, due to the pressure balancing with the ambient pressure, vapour completely disappears ($t = 1ms$), indicating that shortly after the needle closing only liquid and air remain inside the sac volume.

5. Conclusions

This paper presents an investigation of cavitation and air interaction during a diesel pilot injection of a standard serial production six-hole geometry. The focus was to understand the complex interaction between the needle motion, cavitation formation and development, and gas suction. The strategy followed has been to use high speed visualisations of a transparent nozzle tip to record the multiphase phenomena and to use CFD to explain the physics

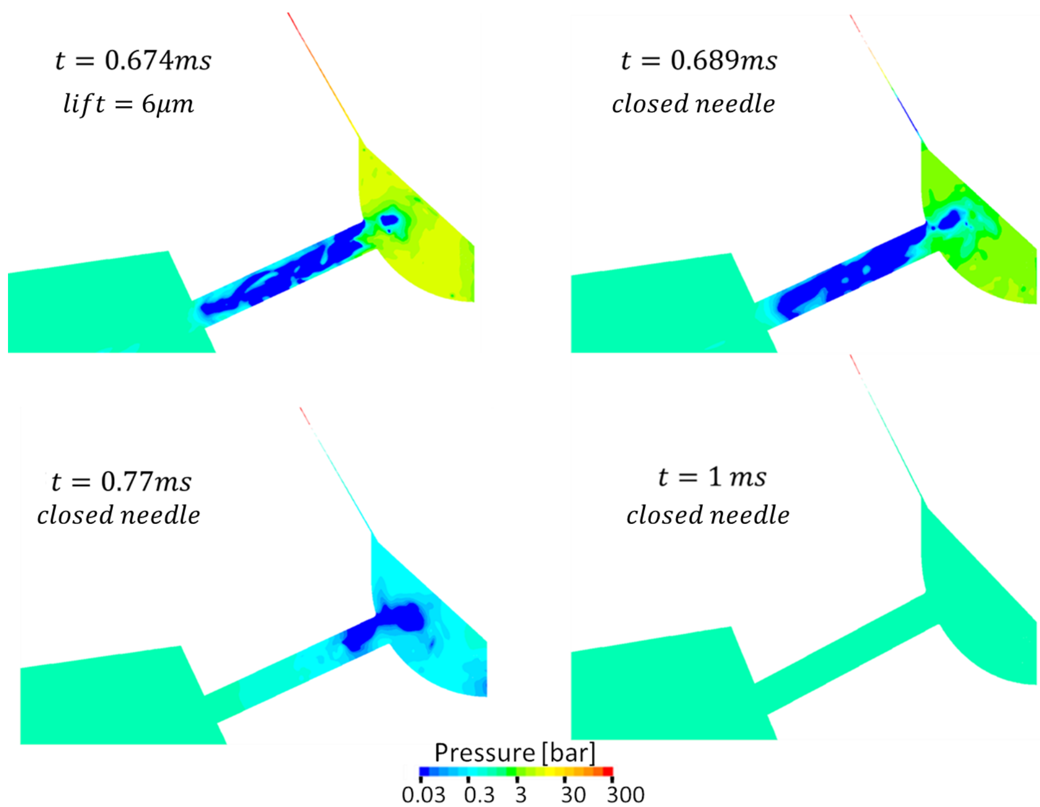

Figure 11: Pressure field time sequence. Notice that logarithmic scale has been used.

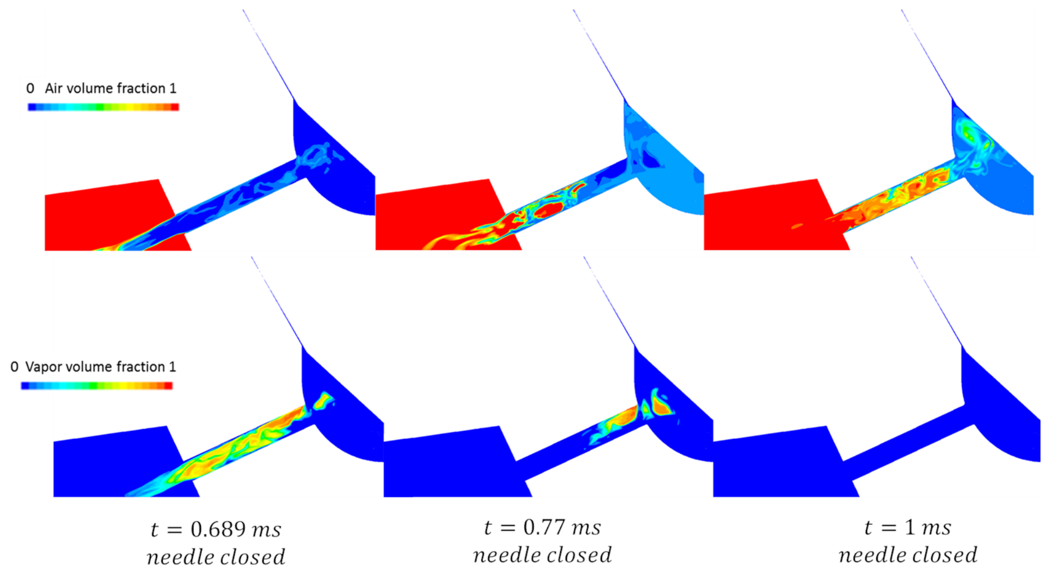

Figure 12: After needle closing results. Time sequence for air (top) and vapour (bottom) volume fraction fields.

behind the observations. The CFD methodology includes LES turbulence modelling, the needle valve boundary movement, cavitation effects through a Rayleigh-Plesset based cavitation model, and the compressibility of air and fuel. Starting from a flow field initialised according to the experimental observations (with an air bubble in the sac and a big portion of the hole filled with air), the main flow features observed are replicated by the simulations. In particular the following phenomena experimentally noticed have been explained and reproduced:

- The compression of the initial air bubble due to sac pressure build up. The inclusion of air compressibility in the simulation can be very relevant even for modest injection pressures in order to replicate the air compression in the sac at the start of the injection as well as the air expansion in the injection hole and sac.

- The appearance of cavitation stemming from the sac entry at the start of the injection, due to flow separation and shear.

- The sac flow recirculation in the sac and flow patterns inside the hole. One part of the void observed in the simulation can be attributed to cavitation both geometrical (developed at the hole inlet upper lip) and vortical (due to complex flow structure coming from the sac). Furthermore, the initial air inside the nozzle expands in the hole contributing to the void areas observed. This shows that the void observed experimentally is a combination of both air and fuel vapour.

- An increase of void inside the hole and in the sac during the needle valve closing. The underlying reason being the flow throttling, since liquid momentum is still high but flow passage very restricted.

- The air suction after the needle closing. The closure of the valve creates an expansion wave that leaves the sac pressure below the ambient. This induces vapour creation and air expansion in the sac and consequently air is sucked from the ambient into the nozzle. When the pressure in the sac is recovered, all vapour collapses. Therefore it is shown that the remaining foam at the end of the injection consists of a liquid and air mixture.

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