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# ASSET PRICING WITH MEAN REVERSION: THE CASE OF SHIPS\*

IOANNIS C. MOUTZOURIS AND NIKOS K. NOMIKOS <sup>a</sup>

<sup>a</sup> Faculty of Finance, Cass Business School, City, University of London, 106 Bunhill Row, London EC1Y 8TZ, UK. Contact number: +44 (0)20 7040 0104. Email: [N.Nomikos@city.ac.uk](mailto:N.Nomikos@city.ac.uk).

We develop a heterogeneous-beliefs asset pricing model with microeconomic foundations that reproduces asset prices, cash flows and trading activity in a real asset economy. In contrast to the majority of financial markets' behavioural models, and in line with the nature of the shipping industry, in this model agents extrapolate fundamentals. Formal estimation of the model indicates that an economy where a small fraction of agents significantly extrapolates fundamentals can explain the positive relation between earnings, vessel prices, and trading activity.

*Keywords:* Behavioural Finance, Asset Pricing, Biased Beliefs, Cash Flow Extrapolation, Heterogeneous-Agents

*JEL Codes:* G02, G11, G12.

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## I. INTRODUCTION

It is well-established in the asset pricing literature that most rational expectations models fail to explain numerous empirical regularities related to asset prices. Among others, two prominent examples are the “excess volatility puzzle” (Leroy and Porter 1981) and the positive correlation between trading volume and asset prices (Barberis et al. 2018). To explain these findings, one of the tools that researchers have developed are heterogeneous beliefs economic models that incorporate behavioural biases, termed as heuristics (Barberis et al. 2015).

In this paper, we extend the application of heterogeneous beliefs models to real assets and specifically, ocean-going vessels. Shipping is a very important sector of the world economy since it facilitates international trade and the bulk transport of raw materials, affordable food, and manufactured goods (ICS, n.d.). Namely, over 80% of the total world trade by volume and more than 70% of its value is facilitated by sea (UNCTAD, 2017). Modern ships are highly sophisticated assets and the market value of a large ship can exceed 200 million USD, while the operation of merchant ships generates an estimated annual freight income of over half a trillion USD in freight rates (ICS, n.d.). It is not surprising therefore that dry-bulk freight rates (which are directly related to the types of vessels considered in this study) are considered a leading indicator of real economic activity (Kilian, 2009). Therefore, understanding the pricing and trading dynamics of this asset class is important in its own right.

We contribute to the asset pricing literature by demonstrating that in a real asset industry like shipping, high trading activity is associated with prosperous market conditions, reflected by high net earnings and vessel prices. To explain this finding, we develop an asset pricing model with microeconomic foundations where net earnings are mean-reverting but agents hold heterogeneous beliefs regarding the degree of mean reversion. This heterogeneity results in trading activity after strong shocks but especially following significant positive shocks due to short-sale constraints.

The contribution of this research to the existing shipping literature is twofold. First, it documents and aggregates several stylised facts related to trading activity in the second-hand market for vessels which had not been analysed before. Second, to the best of our knowledge, this is the first paper to provide a framework that can simultaneously reproduce the main empirical regularities corresponding to this market.

In broad terms, the shipping industry is divided into four markets: newbuilding, demolition, second-hand (or, equivalently, sale-and-purchase (S&P)), and freight (Stopford 2009). Although each sector is important in its own right, the dynamics of the S&P sector are particularly important as second-hand vessels are sold for dozens of million USD and total annual second-hand vessel sales amount to billions of USD (average annual vessel sales from 1995 to 2014 were \$18 billion). Thus, to have a complete view of the shipping industry and its asset pricing dynamics, it is necessary to examine the second-hand market for vessels and the associated trading dimension.

While there exist many asset pricing models for financial markets that incorporate trading activity, to the best of our knowledge, this paper is the first to formally account for this aspect in shipping. Namely, Beenstock and Vergottis (1989) and Kalouptsi (2014) propose rational expectations general equilibrium models which, however, do not examine trading activity in the second-hand market. Greenwood and Hanson (2015) develop a behavioural microeconomic model to explain stylised facts particularly related to the newbuilding, demolition, and freight markets and as such do not examine trading activity in the second-hand market. While the foundations of the behavioural mechanism proposed here are similar to that of Greenwood and Hanson (2015), we focus on the market for second-hand vessels instead of the new-building and demolition ones. In that respect, our model extends the recent shipping research to cover also the market for second-hand vessels and trading activity. Finally, while there have been numerous studies focusing on the volatility of vessel prices per se (Strandenes 1984; Adland et al. 2006; Adland and Koekebakker, 2007; Moutzouris and Nomikos 2019), our paper integrates the second-hand market by directly linking cash flows, asset prices, and trading activity through the means of a structural microeconomic model instead of reduced form estimation.

Our discrete time environment consists of two agent types: “conservatives” and “extrapolators”. Annual shipping net earnings are the sole state variable and to value the asset at each period agents

maximise recursively a constant absolute risk aversion (CARA) utility function defined over next period's wealth. While both types of agent value vessels based on the evolution of fundamentals, they are characterised by bounded rationality which is expressed in two forms.

First, agents form extrapolative expectations regarding the cash flow process; the conservative at a lesser degree compared to the extrapolator. From a psychological perspective, the extrapolation of fundamentals can be the result of several heuristic-driven biases, the most common being the “representativeness heuristic” according to which, individuals believe that small samples are representative of the entire population (Tversky and Kahneman 1974). Second, in the presence of incomplete information in the market, each agent type does not know the way in which his counterpart forms expectations about future market conditions and thus, how he derives his demand. Being boundedly rational though, each agent assumes that in all future periods, the other type will maintain his per-capita fraction of the asset.

Some key features of the Barberis et al. (2018) heterogeneous-beliefs extrapolative model are closely related to the one presented here; most crucially, we maintain the assumption of short-sale constraints which is consistent (much more than it is for equities) with the fact that vessels are physical assets and are not amenable to short sales. We also introduce important modifications which are required to capture stylised features of shipping markets (as analysed in Section II). For instance, a key modification is that in our model there is cash flow rather than returns extrapolation. Moreover, we examine an asset that generates a random cash flow at each period of its economic life, is subject to economic depreciation due to wear and tear and thus, age-dependent. In contrast, the asset in Barberis et al. (2018) generates a cash flow only at maturity and is age-independent. In their set-up, a series of positive cash flow shocks (and, in turn, a 3-stage displacement process) is a necessary requirement for overvaluation to occur while their model cannot accommodate cases when the asset is significantly undervalued since, as the authors argue, it was developed to explain asset bubbles per se. Undervaluation is an interesting feature of shipping markets observed in practice during market downturns (Greenwood and Hanson, 2015).

Due to the assumed form of extrapolation and the fact that the asset pays a cash flow at each period, in our model a single cash flow shock suffices to generate over- or, under-valuation of the asset or even

cause asset values to revert to their fundamental values within one period which, in turn, generates excess price volatility. This pattern is consistent with the nature of the shipping industry as large swings in earnings and asset values in short periods of time are quite common.

Finally, our model is flexible enough to allow agents to hold different degrees of extrapolative or, in general, distorted beliefs. Those modifications provide different challenges in the economic modelling of the market. Consequently, we provide a framework that can be adapted in other markets with similar characteristics, such as the commercial real estate sector.

Our simulation results suggest that even a small fraction of extrapolators, of 5%, can reproduce the observed stylised facts. Extrapolators in our model represent less experienced and/or less informed shipping investors and as such, they are expected to be relatively new entrants in the shipping industry. The fraction of conservative investors corresponds to the large number of established shipping companies that operate in the industry (Section II justifies this interpretation).

While there can be alternative explanations for the observed patterns in either trading activity or asset price behaviour (e.g. limits to arbitrage, productivity shocks, time-varying risk preferences, etc.), the proposed model has the advantage of simultaneously explaining, in a sufficient manner, numerous empirical regularities. In addition, as analysed in Section II, extrapolation of net earnings is particularly relevant to this market and is also supported by numerous academic studies.

The remainder of this paper is organised as follows. Section II presents the empirical regularities related to the second-hand market for ships that motivate the development of our model. Section III introduces the environment of our economy and the solution of the theoretical model. Section IV presents the dataset employed along with the simulation of the model. In addition, it provides an economic interpretation of the results. Section V examines several alternative hypotheses regarding the investor population composition. Section VI concludes.

## II. EMPIRICAL REGULARITIES

In this section, we summarise several stylised facts related to the market for second-hand vessels that motivate the development of the suggested framework. In addition, we provide a justification for the characteristics and the composition of the investor population in our economy.

To begin with, our framework explains the observed price behaviour of second-hand vessels. Vessel prices are highly volatile in annual and longer horizons (Moutzouris and Nomikos 2019) exhibiting modest positive autocorrelation (the autocorrelation coefficient in our sample is 0.49). Interestingly, Kavussanos and Alizadeh (2002), Alizadeh and Nomikos (2007), and Greenwood and Hanson (2015) document that market prices for ships are more volatile than theoretical, model-based prices. Namely, in the baseline simulation of our model, actual vessel prices appear to be 34% more volatile than the ones suggested by the rational expectations benchmark. Capturing this feature, defined as “excess volatility”, is one of the main aims of our model.

A well-analysed feature of the shipping industry is that vessel prices and net earnings are strongly correlated, consistent with second-hand vessel prices being responsive to changes in net earnings (Moutzouris and Nomikos 2019); we find a correlation coefficient of 0.76 in our sample. Thus, changes in net earnings result in changes in vessel prices in the same direction. Related to that, Greenwood and Hanson (2015) show that vessels are overvalued – compared to the price implied by rational expectations – during market peaks and undervalued during market troughs.

The most important contribution of our model though, is that it can reproduce and justify stylised facts related to trading activity. Alizadeh and Nomikos (2003) document a statistically significant relationship between vessel prices and trading activity. In line with this, the correlation coefficient between second-hand vessel prices and trading activity in our sample is equal to 0.71. Trading activity is also positively related to net earnings; the correlation coefficient in our data being 0.53. Thus, we can argue that trading activity is positively related to prevailing market conditions as reflected in both net earnings and vessel prices. In support of these findings, Papapostolou et al. (2014) illustrate that high sentiment periods – which coincide with prosperous market conditions – are related to increased activity

in the sale-and-purchase market for ships. In addition, we show that trading activity is positively related to absolute changes in net earnings (the correlation coefficient is 0.65); thus, a strong net earnings shock is followed by significant trading activity. Finally, our model also captures the relative illiquidity of the second-hand markets for vessels (Kalouptsidei 2014); in line with most markets for physical assets, the average annual sales turnover is relatively low, at 5.8% of the corresponding fleet size.

Furthermore, the proposed framework takes into account that net earnings yields are positively correlated with prevailing market conditions (the correlation coefficient in our sample being 0.88) and, in turn, strongly negatively forecast future net earnings growth. Our model accounts for the fact that in shipping, net earnings yield volatility is attributed to variability of net earnings growth, rather than variability in expected returns (Moutzouris and Nomikos, 2019). The latter stylised fact is important and is in line with market practice: shipping market participants characterise market conditions based on prevailing net earnings, rather than on realised returns, and as such it is much more plausible for investors to form biased expectations regarding fundamentals. Specifically, as Lof (2015) argues, biased beliefs models are very appealing when modelling boom-bust cycles as the ones characterising the shipping industry. Numerous academic studies support the argument that shipping market participants are affected by behavioural factors that may lead to extrapolation of fundamentals. Starting with Zannetos (1966), Metaxas (1971), and followed by Beenstock and Vergottis (1989), the argument of extrapolation of current market conditions has a long history in the shipping literature as well as in market practice.

This process is described with great clarity in Metaxas (1971): “The relatively brief periods of abnormally high freight rates have frequently led to undue optimism in the ordering of new tonnage. Thus, the industry has been characterised by an endemic tendency to overinvest.” Greenwood and Hanson (2015) build on those arguments and argue that firms may overestimate the persistence of exogenous demand shocks. As a result, there is over-investment in newbuilding vessels during booms (and excess scrapping of vessels in market troughs) because firms mistakenly believe that abnormally high (low) net earnings will persist into the future, consistent with the view of net earnings extrapolation.

The existence of earnings extrapolation and behavioural biases has also been supported by other studies. For instance, Sodal et al. (2009) state that shipping agents are in general slow in adjusting their expectations about future market conditions. Papapostolou et al. (2014) show that investor sentiment has a significant effect in the market for the sale-and-purchase of second-hand vessels. Moreover, Alizadeh and Nomikos (2007) and Alizadeh et al. (2017) use changes in net earnings to construct a momentum indicator for investment strategies consistent with the view that market participants adjust their market views with a lag. The aforementioned studies provide strong support for the extrapolation of net earnings in the dry bulk shipping industry thus justifying the key assumption of our model. In conclusion, since shipping market participants identify market conditions based on prevailing net earnings, it is highly possible that part of the shipping investor population myopically believes that short-term persistence of net earnings is also valid for longer horizons.<sup>1</sup>

We assume that net earnings follow an AR(1) process where extrapolators believe that the autocorrelation coefficient (or, equivalently, the perceived persistence of net earnings) is larger than the actual one. The AR(1) specification is in line with the characteristics of the shipping industry (Kalouptsi, 2014), is consistent with the actual net earnings process as suggested by the data (Moutzouris and Nomikos, 2019), and is also supported by the adopted extrapolation mechanisms in the shipping literature (Greenwood and Hanson, 2015). Unfortunately, there are no survey studies to document the views and expectations of shipping market participants (as it is the case in equity markets; e.g. Greenwood and Shleifer, 2014) and as such we do not have actual data regarding the perceived net earnings extrapolation process.<sup>2</sup>

Finally, our model assumes the existence of heterogeneous investors with extrapolative expectations which enables us to distinguish between more and less experienced shipping investors. In particular,

1. Note that while the 1-month autocorrelation of annual net earnings is equal to 0.98, the 1-year figure drops to 0.58 in our sample.

2. The perceived autocorrelations coefficients are calibrated under the objective (physical). This approach is in line with numerous recent articles in the field of behavioural empirical asset pricing (e.g. Barberis et al, 2015; Greenwood and Hanson, 2015; Barberis et al, 2018) that calibrate and simulate their models in the physical rather than the subjective measure.

extrapolators in our model represent relatively new entrants in this industry like private equity investors (such as the K/G and K/S partnerships in Germany and Norway, respectively). While those investors exhibit growing interest in shipping companies, as they are searching for new industries to invest and are backed by strong capital liquidity, they lack shipping market expertise compared to traditional owners (Syriopoulos, 2010).

Table I presents a summary of the stylised features captured by our model along with the corresponding references from the literature.

### III. ENVIRONMENT AND MODEL SOLUTION

Consider a discrete-time environment where the passage of time is denoted by  $t$ . The economy consists of two asset classes: a risk-free asset earning a constant rate of return equal to  $R_f$  and a risky asset class which consists of otherwise identical assets (i.e. ships) which are further categorised based on their age. All age classes have fixed per capita supply over time equal to  $Q$ .<sup>3</sup> In what follows, we restrict our attention to the modelling of the market for 5-year old vessels although, the same principles apply for the valuation of the other age classes. Following market practice, we assume that a newly-built vessel has an economic life of 25 years after which she is scrapped and exits the economy.<sup>4</sup> Accordingly, setting the time-step of the model,  $\Delta t$ , equal to one year implies that a 5-year old asset has  $T = 20$  periods of remaining economic activity.

In the context of our model, net earnings are the sole state variable. The owner of the ship is entitled to an exogenously determined stream of annual net earnings,  $\{\Pi_n\}_t^{t+T}$ . A feature of the shipping industry

3. This is justified by the fact that we are modelling a real asset with economic depreciation. Hence, the supply of the age-specific asset class cannot increase over time. Furthermore, scrapping rarely occurs before the 10<sup>th</sup> year of a vessel's life, thus, supply cannot be reduced either. Note that supply may differ across different age classes.

4. We consider the scrapping decision to be fixed at 25 years which is consistent with market practice. We should note though that occasionally shipping companies may scrap vessels of younger age due to technical obsolescence, poor market conditions or regulatory pressures, particularly when the cost of modifying a ship to comply with new regulations is prohibitive. In addition, companies may decide to scrap their vessels to raise cash and finance newbuilding orders to renew their fleet. These cases are not considered here as our primary focus is to model second-hand activity in the market and, thus, consider the scrapping decision to be exogenous.

is that a ship-owner at time  $t$  knows his net earnings for the period  $t \rightarrow t + 1$ .<sup>5</sup> In line with the nature of the industry and the existing literature (Greenwood and Hanson 2015), net earnings are assumed to follow a mean-reverting process:

$$(1) \quad \Pi_{t+1} = (1 - \rho_0)\bar{\Pi} + \rho_0\Pi_t + \varepsilon_{t+1},$$

where  $\bar{\Pi}$  is the long-run mean,  $\rho_0 \in [0,1)$ , and  $\varepsilon_{t+1} \sim N(0, \sigma_\varepsilon^2)$ , *i.i.d.* over time. Importantly, in contrast to  $\bar{\Pi}$ , parameters  $\rho_0$  and  $\sigma_\varepsilon^2$  are not public information.

The economy consists of two investor types,  $i$ : “conservatives” and “extrapolators”, denoted by  $c$  and  $e$ , respectively. We normalise the investor population related to each asset age-class to a unit measure and we further assume that the fractions of conservatives,  $\mu^c$ , and extrapolators,  $\mu^e$ , are fixed, both across all age classes and through each specific asset’s life. In what follows, we set  $\mu^c = \mu$ ; hence,  $\mu^e = 1 - \mu$ . The difference between the two types lies in the alternative ways in which they form expectations about future cash flows. Specifically, compared to extrapolators, conservatives’ perception is closer—in principle, it might even be identical—to equation (1). We assume that in agent  $i$ ’s mind, net earnings related to the valuation of the 5-year old vessel evolve according to

$$(2) \quad \Pi_{t+1} = (1 - \rho_i)\bar{\Pi} + \rho_i\Pi_t + \varepsilon_{t+1}^i,$$

in which  $\rho_0 \leq \rho_c < \rho_e < 1$  and  $\varepsilon_{t+1}^i \sim N(0, \vartheta_5^i \sigma_\varepsilon^2)$ , *i.i.d.* over time, where  $0 < \vartheta_5^e < \vartheta_5^c < 1$ . The strictly positive parameter  $\vartheta_5^i$  adjusts the—true—variance of the cash flow shock according to agent’s  $i$  perspective, while the subscript denotes the current age-class of the vessel being valued.

The parameters for the conservative agent  $\{\mu, \rho_c, \vartheta_5^c\}$  characterise completely the information structure of our model. When  $\mu = 1$ ,  $\rho_c = \rho_0$ , and  $\vartheta_5^c = 1$ , all agents have perfect information about the economy. This case is defined as the benchmark “rational” economy and we term this agent type as

5. We assume that vessels of the same age group are homogeneous (thus, maintenance policy does not matter) and are engaged in consecutive and identical, for the same age group, 1-year time-charter contracts (thus, chartering efficiency does not matter either). In practice, ship-owners and charterers agree upon the time-charter (leasing) rate of the vessel for a specified period of time (e.g. one year), at the commencement of the corresponding period. Thus, assuming that there is no default by either party in the agreement, the freight rate to be received for the following period – i.e. from  $t \rightarrow t + 1$  – is known in advance.

fundamentalist,  $f$ ; hence,  $\rho_f = \rho_0$  and  $\vartheta_5^f = 1$ . When  $\mu = 1$ ,  $\rho_c \neq \rho_0$ , and  $\vartheta_5^c \neq 1$  or, equivalently,  $\mu = 0$ , all agents have imperfect information about the economy. However, in all cases above, there is no information asymmetry among agents and, in turn, no trading activity in the market. Finally, when  $\mu \in (0,1)$ , information is both imperfect and asymmetric (Wang 1993) and, as a result, there is trading activity in the economy.

The timeline of the model is as follows. At each point  $t$ ,  $\Pi_t$  is realised and observed by all market participants. Furthermore, the 25-year old age class is scrapped and replaced by newly built vessels. Accordingly, both agent types determine their time  $t$  demands for each age class with the aim of maximizing a constant absolute risk-aversion (CARA) utility function, defined over next period's wealth. For the 5-year old vessel, this corresponds to

$$(3) \quad \max_{N_{5,t}^i} E_t^i \left[ -e^{-\alpha^i w_{t+1}^i} \right],$$

where  $\alpha^i$  and  $N_{5,t}^i$  are investor  $i$ 's coefficient of absolute risk-aversion and time  $t$  per-capita demand for the 5-year old vessel, respectively. Agent  $i$ 's next period's wealth,  $w_{t+1}^i$ , is given by

$$(4) \quad w_{t+1}^i = (w_t^i - N_{5,t}^i P_{5,t})(1 + R_f) + N_{5,t}^i (\Pi_t + P_{6,t+1}),$$

in which  $P_{5,t}$  and  $P_{6,t+1}$  are the prices of the 5- and 6-year old vessel at  $t$  and  $t + 1$ , respectively.<sup>6</sup>

In what follows, we normalise the rate of return of the risk-free asset to zero (Wang 1993). Therefore, investor  $i$ 's objective function becomes

$$(5) \quad \max_{N_{5,t}^i} E_t^i \left[ -e^{-\alpha^i (w_t^i + N_{5,t}^i (\Pi_t + P_{6,t+1} - P_{5,t}))} \right].$$

Accordingly, the time  $t$  price of the 5-year old vessel is endogenously determined through the market clearing condition

$$(6) \quad \mu N_{5,t}^c + (1 - \mu) N_{5,t}^e = Q.$$

6. In principle, each agent could invest a fraction of his wealth in every age-class of the risky asset. However, to obtain closed-form solutions for the demand functions, we assume that at each  $t$ , a new unit mass of investors solely interested in 5-year old vessels enters the industry. In turn, at  $t + 1$ , the same investor population will be solely interested in the 6-year old class, while a new unit mass related to the 5-year old class, will enter the market.

Following the same principles, the time  $t$  per-capita demand of agent  $i$  for the 6-year old vessel,  $N_{6,t}^i$  and the corresponding 6-year old vessel price,  $P_{6,t}$ , are determined (Appendix A). Finally, trading activity corresponding to period  $t - 1 \rightarrow t$  takes place in the market. In shipping, this activity refers to the sale and purchase market for second-hand vessels. Since this is a discrete-time model, we impose the assumption that trading occurs instantaneously at each point  $t$  (Barberis et al. 2018). Note that because vessels are real assets with limited economic life, their values are affected by economic depreciation due to wear and tear. Thus, a 5-year old vessel acquired at time  $t - 1$  will become a 6-year old one, when sold at time  $t$ . Accordingly, we define as trading activity the agent-specific change in demand for the same asset between points  $t - 1$  and  $t$ , multiplied by the respective population fraction:

$$(7) \quad V_{t-1 \rightarrow t} \equiv V_t = \mu^i |N_{6,t}^i - N_{5,t-1}^i|.$$

Figure I summarises the timeline of the model.

Consistent with the nature of the industry, we also impose short-sale constraints. Appendix A shows that the time  $t$  per-capita demand of agent  $i$  for the 5-year old vessel is

$$(8a) \quad N_{5,t}^i = \max \left\{ \frac{\frac{1 - \rho_i^{21}}{1 - \rho_i} (\Pi_t - \bar{\Pi}) + 21\bar{\Pi} - X_5^i \sigma_\varepsilon^2 Q - P_{5,t}}{Y_5^i \sigma_\varepsilon^2}, 0 \right\}$$

with

$$(8b) \quad \begin{cases} X_5^i = \left[ \frac{20}{(1 - \rho_i)^2} - \frac{(1 - \rho_i^{20})(1 + 2\rho_i - \rho_i^{20})}{(1 + \rho_i)(1 - \rho_i)^3} \right] \alpha^i \vartheta_5^i \\ Y_5^i = \left( \frac{1 - \rho_i^{20}}{1 - \rho_i} \right)^2 \alpha^i \vartheta_5^i \end{cases},$$

where both  $X_5^i$  and  $Y_5^i$  are strictly positive constants. Equation (8a) along with the market clearing condition (6) determine the equilibrium 5-year old vessel price at each  $t$ . From an economic perspective, the fraction term in (8a) reflects the expected one-period net income for investor  $i$  scaled by the product of investor's risk aversion times the risk he is bearing, according to his perception of the cash flow process. Note that to derive the agent-specific demand functions, we have assumed that agent

$i$  makes the simplifying assumption that his counterpart,  $-i$ , will hold his fraction of the risky asset constant at  $\mu^{-i}Q$ , irrespective of the corresponding future net earnings variable. This is equivalent to assuming that each agent type does not know the way in which his counterpart derives his demand, that is how he forms expectations about future market conditions.<sup>7</sup>

Since extrapolators have more erroneous beliefs about the true net earnings process, it might be the case that in the long-run their wealth will be significantly reduced, if not depleted.<sup>8</sup> Notice though that the use of exponential utility implies that the demand function is independent of the respective wealth level. This, in turn, allows us to abstract from the “survival on prices” effect (Barberis et al. 2015) and focus solely on the pricing and trading implications of the heterogeneous-agent economy. In reality, even if extrapolators were not able to invest due to limited wealth, it is plausible to assume that they would be immediately replaced by a new mass of investors with exactly the same characteristics. In shipping, this cohort could correspond to diversified investors with substantial cash holdings, such as private equity firms, but little or no prior experience of the industry.

**PROPOSITION.** (Equilibrium price for 5-year old vessels) In the environment presented above, a market-clearing price for the 5-year old vessel,  $P_{5,t}^*$ , always exists. We denote the net earnings thresholds at which extrapolators and conservatives related to the 5-year old vessel class exit the market by  $\Pi_5^e$  and  $\Pi_5^c$ , respectively.

First, when

$$(9a) \quad \Pi_5^e = \bar{\Pi} + \frac{(X_5^e - X_5^c - \frac{Y_5^c}{\mu})\sigma_\varepsilon^2 Q}{\frac{1 - \rho_e^{21}}{1 - \rho_e} - \frac{1 - \rho_c^{21}}{1 - \rho_c}} < \Pi_t < \bar{\Pi} + \frac{(X_5^e - X_5^c + \frac{Y_5^e}{1 - \mu})\sigma_\varepsilon^2 Q}{\frac{1 - \rho_e^{21}}{1 - \rho_e} - \frac{1 - \rho_c^{21}}{1 - \rho_c}} = \Pi_5^c,$$

both agents are present in the market, and the market clearing price,  $P_{5,t}^{*c+e}$ , is equal to

7. As analysed in Appendix A, in principle, investors will gradually realise that their beliefs about either the cash flow process and/or their competitors’ strategy are inaccurate (Barberis et al. 2015). We do not incorporate an explicit learning process in the model as this would gradually eliminate both the excess volatility and the observed patterns related to sale and purchase activity in the market. Instead, we adopt a rather indirect learning mechanism: we assume that agents become more “suspicious” as the specific asset’s age grows, and they indirectly respond by increasing their perceived risk associated with their investment.

8. Note that while extrapolators’ one-period changes in wealth are more volatile than those of conservatives, both types of agent realise approximately the same mean change (Section IV.C). Thus, there is no formal indication that extrapolators “suffer” (on average) by limitations of wealth more than conservatives do.

$$(9b) \quad P_{5,t}^{*c+e} = 21\bar{\Pi} + \frac{\mu Y_5^e \frac{1-\rho_c^{21}}{1-\rho_c} + (1-\mu)Y_5^c \frac{1-\rho_e^{21}}{1-\rho_e}}{\mu Y_5^e + (1-\mu)Y_5^c} (\Pi_t - \bar{\Pi}) - \frac{\mu Y_5^e X_5^c + (1-\mu)Y_5^c X_5^e + Y_5^c Y_5^e}{\mu Y_5^e + (1-\mu)Y_5^c} \sigma_\varepsilon^2 Q.$$

Second, in the case where  $\Pi_t \leq \Pi_5^e$ , extrapolators exit the market and the market-clearing price,  $P_{5,t}^{*c}$ , is given by

$$(10) \quad P_{5,t}^{*c} = 21\bar{\Pi} + \frac{1-\rho_c^{21}}{1-\rho_c} (\Pi_t - \bar{\Pi}) - \left[ X_5^c + \frac{Y_5^c}{\mu} \right] \sigma_\varepsilon^2 Q.$$

Third, in the case where  $\Pi_5^c \leq \Pi_t$ , conservatives exit the market and the equilibrium price,  $P_{5,t}^{*e}$ , is given by

$$(11) \quad P_{5,t}^{*e} = 21\bar{\Pi} + \frac{1-\rho_e^{21}}{1-\rho_e} (\Pi_t - \bar{\Pi}) - \left[ X_5^e + \frac{Y_5^e}{1-\mu} \right] \sigma_\varepsilon^2 Q.$$

As the first term of equations (9b), (10), and (11) indicates, the price of the vessel depends on the long-run mean of the cash flow variable multiplied by the total number of payments to be received until the end of the asset's economic life. The second term corresponds to the effect of perceived persistence in net earnings times its deviation from its long-run mean, which is the main source of over-or, under-valuation in the price of the risky asset. Finally, the last term corresponds to the aggregate discounting of future cash flows in order for investors to be compensated for the risk they bear (Wang 1993).<sup>9</sup> Note that due to the assumed form of extrapolation and the structure of our economy, a single cash flow shock suffices to generate substantial over-or, under-valuation of the asset or even cause asset values to revert to their fundamental value within one period. This pattern is consistent with the nature of the shipping industry as large swings in earnings and asset values in short periods of time are quite common, like for instance in the last quarter of 2008 when asset values dropped by more than 80%. In contrast, in the model of Barberis et al. (2018), we need to have a series of positive cash flow shocks and, in turn, a 3-stage displacement process (in line with Kindleberger [1978]), for overvaluation to occur.

9. Extending the proof of this Proposition, it is straightforward to show that a vessel age-specific market-clearing price always exists (Appendices A and B).

It is also useful to examine the benchmark rational economy, denoted by  $f$ , in which the market consists solely of agents who know the actual stochastic process that governs the evolution of net earnings. The equilibrium price of the 5-year old vessel in this case is

$$(12) \quad P_{5,t}^f = 21\bar{\Pi} + \frac{1 - \rho_0^{21}}{1 - \rho_0} (\Pi_t - \bar{\Pi}) - [X_5^f + Y_5^f] \sigma_\varepsilon^2 Q.$$

As equations (8a) and (8b) indicate, fundamentalists' perception of risk is given by the product  $\left(\frac{1 - \rho_0^{20}}{1 - \rho_0}\right)^2 \sigma_\varepsilon^2$ . In this benchmark case, this perception is correct. In the presence of extrapolators, though, it is just an approximation since future asset prices will also depend on extrapolators' future demand responses and not just on the riskiness of cash flows.

Moreover, the unconditional volatility of the fundamental price is given by

$$(13) \quad \sigma(P_{5,t}^f) = \frac{1 - \rho_0^{21}}{1 - \rho_0} \sigma(\Pi_t).$$

Finally, taking unconditional expectations on both sides of equation (12) and setting the unconditional mean of the net earnings variable equal to its long-run mean,  $\bar{\Pi}$ , yields

$$(14) \quad E[P_{5,t}^f] = 21\bar{\Pi} - [X_5^f + Y_5^f] \sigma_\varepsilon^2 Q.$$

**CORROLARY 1.** (Steady state equilibrium) We define the “steady state” of our economy as the one

in which the net earnings variable is equal to its long-run mean,  $\bar{\Pi}$ . As equation (1) indicates, the economy reaches this state after a sequence of zero cash flow shocks. In the steady state, the price of the risky asset is equal to its respective fundamental value. Furthermore, both types of agent are present in the market and each type holds the risky asset in analogy to his fraction of the total population. Accordingly, the “steady state” equilibrium price of the 5-year old vessel,  $\bar{P}_5^*$ , is given by

$$(15a) \quad \bar{P}_5^* = 21\bar{\Pi} - [X_5^i + Y_5^i] \sigma_\varepsilon^2 Q,$$

under the restriction

$$(15b) \quad X_5^c + Y_5^c = X_5^e + Y_5^e = X_5^f + Y_5^f = \frac{\mu Y_5^e X_5^c + (1 - \mu) Y_5^c X_5^e + Y_5^c Y_5^e}{\mu Y_5^e + (1 - \mu) Y_5^c}.$$

In a similar manner, the “steady state” equilibrium price of the 6-year old vessel is

$$(16a) \quad \overline{P}_6^* = 20\overline{\Pi} - [X_6^i + Y_6^i]\sigma_\varepsilon^2 Q,$$

under the restriction

$$(16b) \quad X_6^c + Y_6^c = X_6^e + Y_6^e = X_6^f + Y_6^f.$$

Therefore, if in two consecutive periods the net earnings variable is equal to its long-run mean, the change in the price of the asset is

$$(17) \quad \overline{P}_6^* - \overline{P}_5^* = -\overline{\Pi} - [X_6^i + Y_6^i - (X_5^i + Y_5^i)]\sigma_\varepsilon^2 Q.$$

The right-hand side of (17) is negative and corresponds to the one-year economic depreciation in the value of the vessel. Finally, in this case, there is no activity in the second-hand market since the change in share demand of each agent is equal to zero.

**CORROLARY 2.** (Deviation from the fundamental value) Whenever the net earnings variable deviates from its long-run mean, the model-generated price of the 5-year old vessel deviates from its fundamental value. In the following, we denote by  $D_{5,t}$  the degree of deviation; namely, a positive (negative) value of  $D_{5,t}$  corresponds to over(under)valuation of the asset relative to its fundamental analogue,  $P_{5,t}^f$ . Note that, in the following, we define as strong (weak) market conditions the case where net earnings are above (below) their steady state value,  $\overline{\Pi}$ .

First, in the case where both agents are present in the market the deviation,  $D_t^{c+e}$ , is given by

$$(18) \quad D_{5,t}^{c+e} = \frac{\mu \left( \frac{1 - \rho_c^{21}}{1 - \rho_c} - \frac{1 - \rho_0^{21}}{1 - \rho_0} \right) Y_5^e + (1 - \mu) \left( \frac{1 - \rho_e^{21}}{1 - \rho_e} - \frac{1 - \rho_0^{21}}{1 - \rho_0} \right) Y_5^c}{\mu Y_5^e + (1 - \mu) Y_5^c} (\Pi_t - \overline{\Pi}).$$

Since the fraction term is always positive, the sign of price deviation solely depends on the sign of net earnings deviation. Thus, during strong market conditions the asset is overpriced and vice versa.

Second, when only conservatives exist in the market the deviation,  $D_{5,t}^c$ , is estimated through

$$(19) \quad D_{5,t}^c = \left( \frac{1 - \rho_c^{21}}{1 - \rho_c} - \frac{1 - \rho_0^{21}}{1 - \rho_0} \right) (\Pi_t - \bar{\Pi}) - \left[ X_5^c + \frac{Y_5^c}{\mu} - X_5^f - Y_5^f \right] \sigma_\varepsilon^2 Q,$$

which is always negative. Thus, during weak market conditions, the vessel is undervalued.

Third, when only extrapolators are present, the discrepancy,  $D_{5,t}^e$ , is

$$(20) \quad D_{5,t}^e = \left( \frac{1 - \rho_e^{21}}{1 - \rho_e} - \frac{1 - \rho_0^{21}}{1 - \rho_0} \right) (\Pi_t - \bar{\Pi}) - \frac{\mu Y_5^e}{1 - \mu} \sigma_\varepsilon^2 Q,$$

which is always positive<sup>10</sup> and implies that as market conditions improve, the degree of overvaluation increases.

**CORROLARY 3.** (Sensitivity of exit points to the fraction of conservatives) As condition (9a)

suggests, the difference between the agent-specific exit points is due to quantities  $-\frac{Y_5^c}{\mu}$  and

$\frac{Y_5^e}{1-\mu}$  which implies that whenever  $Y_5^c/\mu \neq Y_5^e/(1-\mu)$  the two exit points will not be symmetric

around  $\bar{\Pi}$  and will respond differently to positive and negative shocks in freight earnings. Taking the first partial derivative of the extrapolators' 5-year exit point with respect to the fraction of conservatives yields

$$(21) \quad \frac{\partial \Pi_5^e}{\partial \mu} = \frac{1}{\mu^2} \cdot \frac{Y_5^c \sigma_\varepsilon^2 Q}{\frac{1 - \rho_e^{21}}{1 - \rho_e} - \frac{1 - \rho_c^{21}}{1 - \rho_c}}$$

which is strictly positive. Hence, the higher the fraction of conservatives, the more prone extrapolators are to exit from the market as freight rates decrease below the steady-state rate.

Similarly, the first partial derivative of conservatives' exit point with respect to their relative fraction is equal to

$$(22) \quad \frac{\partial \Pi_5^c}{\partial \mu} = \frac{1}{(1 - \mu)^2} \cdot \frac{Y_5^e \sigma_\varepsilon^2 Q}{\frac{1 - \rho_e^{21}}{1 - \rho_e} - \frac{1 - \rho_c^{21}}{1 - \rho_c}}$$

10. It is straightforward to verify this by plugging in (20) equation (9a).

which is strictly positive. Thus, the higher the fraction of conservatives, the less prone they are to exit from the market as freight rates increase. Hence, the asymmetry between the exit points increases as  $\mu$  deviates from the midpoint 0.5. The same principles apply for the 6-year old vessel valuation.

Appendix B shows that trading activity is quantified through

$$(23) \quad V_t = \mu^i \left| \max \left\{ \frac{\frac{1 - \rho_i^{20}}{1 - \rho_i} (\Pi_t - \bar{\Pi}) + 20\bar{\Pi} - X_6^i \sigma_\varepsilon^2 Q - P_{6,t}}{Y_6^i \sigma_\varepsilon^2}, 0 \right\} - \max \left\{ \frac{\frac{1 - \rho_i^{21}}{1 - \rho_i} (\Pi_{t-1} - \bar{\Pi}) + 21\bar{\Pi} - X_5^i \sigma_\varepsilon^2 Q - P_{5,t-1}}{Y_5^i \sigma_\varepsilon^2}, 0 \right\} \right|.$$

Due to the short-sale constraints, the agent-specific demand functions are not strictly monotonic with respect to the net earnings variable in the entire  $\Pi_t$  domain. As a result, trading activity depends on the realisation of the net earnings variable during the two corresponding consecutive dates,  $t - 1$  and  $t$ . In Appendix B, we examine all possible scenarios. Note that in the absence of constraints, absolute net earnings changes would be almost perfectly correlated with trading activity. The existence of short-sale constraints, however, means that the correlation between the two variables is much lower.<sup>11</sup>

Moreover, Corollary 3 demonstrates that both exit points increase (decrease) with the fraction of conservatives (extrapolators) and the perceived persistence on behalf of extrapolators (conservatives). Hence, the higher the values of the exit points, the more the two types of agent coexist during prosperous market conditions and the less they interact during adverse ones. Thus, a high value of  $\mu$ , along with a large spread between  $\rho_c$  and  $\rho_e$ , result in positive correlation between current net earnings and trading activity, positive correlation between current vessel prices and net earnings and positive correlation

11. In order to illustrate this point, let's assume for simplicity that there is no depreciation in the value of the asset; namely, we set  $N_{6,t}^i = N_{5,t}^i$ . Equivalently, we substitute  $A_5^c$  for  $A_6^c$  in equation (64) in Appendix B. Thus, in the absence of short-sale constraints, trading activity,  $V_t$ , is always equal to  $\mu |A_5^c| |\Pi_t - \Pi_{t-1}|$  and, in turn,  $\text{corr}(|\Pi_t - \Pi_{t-1}|, V_t) = 1$ .

between absolute net earnings changes and trading activity. These theoretical predictions are analysed in the next section.

#### IV. DATA AND SIMULATION OF THE MODEL

The dataset consists of annual observations on second-hand vessel prices, 1-year time-charter rates, fleet capacity, and second-hand vessel transactions, for the Handysize dry bulk sector. Our main source of shipping data is Clarksons Shipping Intelligence Network 2010. In addition, data for the U.S. Consumer Price Index (CPI) are obtained from Thomson Reuters Datastream Professional.

We assume that vessels are chartered (leased) in consecutive 1-year time-charter contracts; hence, only operating and maintenance costs are borne by the ship-owner. Following discussions with industry participants, we arrived at a figure of \$5,500 per day for these costs, as of December 2014, which is also used as the base real value since these costs generally increase with inflation. In addition, we assume that vessels spend 10 days per annum off-hire for maintenance and repairs. During this period, ship-owners do not receive the corresponding time-charter rates but bear the operating and maintenance costs (Stopford 2009). We also consider the commission that the shipbroker receives for bringing the ship-owner and the charterer into an agreement, which is 2.5% of the daily time-charter rate. Finally, in line with shipping practice, tax expenses are not considered in our analysis.

Thus, the annual net earnings variable is calculated as

$$(24) \quad \Pi_t \equiv \Pi_{t \rightarrow t+1} = 355 \cdot 0.975 \cdot TC_{t \rightarrow t+1} - 365 \cdot OPEX_{t \rightarrow t+1},$$

where  $TC_{t \rightarrow t+1}$  and  $OPEX_{t \rightarrow t+1}$  are measured in US\$ per day and refer to the corresponding time-charter rates and total operating and maintenance costs, respectively. Moreover, the one-year horizon log return is given by

$$(25) \quad r_{t+1} = \ln \left( \frac{\Pi_t + P_{6,t+1}}{P_{5,t}} \right),$$

where  $P_{5,t}$  and  $P_{6,t+1}$  refer to the current and next period's price of the 5 and 6-year old vessel, respectively. Since prices for generic 6-year old vessels are not readily available, we set  $P_{6,t} = 0.95P_{5,t}$ .<sup>12</sup>

In order to construct the annual trading activity variable,  $V_t$ , we scale the total number of second-hand transactions taking place within a year by the size of the fleet in the beginning of the respective year. Table II summarises descriptive statistics related to annual net earnings, 5-year old vessel prices, and annual trading activity. Panels A and B of Figure II illustrate the relation between trading activity and net earnings and trading activity and absolute one-year changes in net earnings, respectively. Evidently, trading activity is positively correlated with both variables, the correlation coefficients being 0.53 and 0.65, respectively. This indicates that trading activity in the sales and purchase market for second-hand vessels increases when freight rates are high but also when there is large movement in the level of freight rates.

To conduct the simulations, we calibrate two sets of model parameters. The first set contains the asset-level parameters,  $\{\bar{\Pi}, \rho_0, \sigma_\varepsilon^2, Q, T, R^f\}$ , and remains the same irrespective of the population composition and its characteristics. We set the long-run mean of net earnings,  $\bar{\Pi}$ , and the coefficient of persistence,  $\rho_0$ , equal to their sample counterparts, in Table II. We set the standard deviation of the error term,  $\sigma_\varepsilon^2$ , equal to 1 to reduce the number of discarded paths but at the same time ensure a sufficient degree of net earnings volatility. We set the remaining economic life of the 5-year old vessel,  $T$ , equal to 20. Finally, we normalise the fixed per capita supply,  $Q$ , to one and the risk-free rate of return,  $R^f$ , to zero.

The second set includes the agent-specific parameters  $\{\mu, \rho_i, \vartheta_5^i, \vartheta_6^i, \alpha^i\}$  for  $i \in \{f, c, e\}$ . Regarding the parameter  $\mu$ , we choose values within the interval  $[0,1]$ . While fundamentalists' characteristics are fixed by definition, the ones related to conservatives and extrapolators are recalibrated each time depending on the scenario used. Since fundamentalists form expectations about future net earnings

12. We have estimated the average ratio of 10- to 5-year old vessel prices to be approximately equal to 0.75. Accordingly, adopting a straight-line depreciation scheme implies  $P_{6,t} = 0.95P_{5,t}$ .

based on the true stochastic process,  $\rho_f$ ,  $\vartheta_5^f$ , and  $\vartheta_6^f$  are assigned values of 0.58, 1, and 1, respectively. Finally, we note that the coefficient of absolute risk aversion,  $\alpha^f$ , and the steady state equilibrium prices of the 5- and 6-year old vessels,  $\overline{P}_5^*$  and  $\overline{P}_6^*$ , respectively, are nested, as shown in Appendix B. We thus set  $\overline{P}_5^*$  equal to its long-run mean in Table II which, in turn, yields  $\alpha^f = 0.42$  and  $\overline{P}_6^* = 22.14$ .

The agent-specific parameters for extrapolators are estimated in a similar manner. Since these parameters are nested, for any chosen value of the key parameter of interest  $\rho_i$ , the values of the products  $\alpha^i \vartheta_5^i$  and  $\alpha^i \vartheta_6^i$  are endogenously determined (Appendix B). Hence, it suffices to arbitrarily fix either parameter  $\vartheta_5^i$ ,  $\vartheta_6^i$  or  $\alpha^i$ . This choice does not have any qualitative or quantitative impact on the results since only the value of their product matters. We thus set conservatives' and extrapolators' coefficients of absolute risk aversion to 0.35 and 0.15, respectively and through the equilibrium conditions we calculate the corresponding values for  $\vartheta_5^i$  and  $\vartheta_6^i$ . Table III summarises the model parameters.

We obtain the empirical moments of interest for each scenario with numerical simulations, using equation (1) to generate 10,000 sample paths for our economy where each path corresponds to 100 periods. We then estimate the average for each statistic under consideration and compare the model-generated moments to the actual ones across all valid paths (Barberis et al. 2015).<sup>13</sup> Table IV presents our model's predictions for several combinations of the agent-specific parameters  $\{\mu, \rho_c, \rho_e\}$ . In addition, the right-most column illustrates the actual empirical values for the quantities of interest.

To begin with, average prices and average earnings yields are very close to their actual values, irrespective of the selected parameterisation. This is expected since the steady-state equilibrium price has been set equal to the sample mean of the actual vessel prices. In addition, net earnings are exogenously determined and thus, do not depend on the chosen parameterisation. Furthermore, equations (9b), (10), and (11) imply a high positive correlation between net earnings and vessel prices. As a result, the autocorrelations of net earnings and 5-year old prices are closely related, irrespective of

13. In the simulations, we discard the paths that lead to negative values either for net earnings or vessel prices. We impose this restriction to be able to perform the predictive regressions which use log quantities as variables. Even if we do not discard these paths, the remaining results remain essentially the same.

the chosen scenario (Alizadeh and Nomikos 2007). Taken together, these facts explain why the latter statistic has similar values across all scenarios and is also very close to the actual value.

Excess price volatility is defined as the ratio of the standard deviation of 5-year old vessel prices in the extrapolative heterogeneous-agent economy to the standard deviation of the fundamental value for a given sequence of net earnings shocks. When this ratio is greater than one, heterogeneous-agent model prices are more volatile than the ones in the benchmark rational economy (Barberis et al. 2015); hence, this is a measure of whether actual vessel prices are more volatile than those obtained by optimally forecasted net earnings.<sup>14</sup> Our results suggest that this statistic is positively related to the perceived autocorrelation coefficient of both types of agent and negatively related to the relative fraction of conservatives. Thus, the higher the degree of net earnings extrapolation in the market or, the higher the participation of extrapolators, the higher the volatility of vessel prices.

The results related to the net earnings yields regressions confirm a well-analysed argument in the recent shipping literature: while vessel prices and net earnings are highly correlated, they do not change proportionately (Greenwood and Hanson 2015). Consequently, net earnings yields fluctuate significantly over time and strongly and negatively forecast future net earnings growth. In addition, the bulk of net earnings yield volatility is attributed to variations in expected net earnings growth and not to time-varying expected returns (Moutzouris and Nomikos 2019). A plausible explanation for this is that the average investor appears to anticipate, up to a certain degree, the mean-reverting character of net earnings. Thus, vessels are moderately over- or, under-valued in equilibrium and, in turn, earnings yields are high when market conditions are good and vice versa.

In contrast, if the average degree of extrapolation in the market were higher, changes in vessel prices would be larger than changes in net earnings. As result, earnings yield and net earnings would be negatively correlated, and earnings yield would be strongly positively related with future net earnings

14. To assign a benchmark value to this statistic we consider the volatility of vessel prices in a counterfactual fully rational economy, given by equation (13). Substituting in this formula the actual volatility of net earnings from Table II, we estimate the volatility of fundamental prices as  $\sigma(P_{5,t}^f) = 5.71$  which is then compared to the actual volatility of vessel prices in the data,  $\sigma(P_{5,t}) = 7.65$ . This results in a price volatility ratio of 1.34.

growth. What is more, a substantial fraction of earnings yield volatility would be attributed to future returns.<sup>15</sup> This scenario, illustrated in column (1) of Table V, is in sharp contrast with reality. As more conservative participants enter the market, however, results from model-implied predictive regressions approach the empirical values.

Since trading is the result of heterogeneous beliefs in the market, one expects that trading activity will increase with the degree of heterogeneity and decrease with the difference in the population fractions. Our numerical results suggest that this is precisely the case; when both agent types have a strong presence in the market and a noticeable belief disagreement, trading activity is high (column [2]) and vice versa. As we illustrate in the following, the exit points of the agents, the correlation between net earnings and trading activity and the correlation between vessel prices and trading activity, are sensitive to the choice of parameter  $\mu$ . Keeping the values of  $\rho_c$  and  $\rho_e$  constant, we see that for  $\mu$  equal to 0.1, 0.5, and 0.95, the respective correlation coefficients are negative, approximately zero, and positive, (columns [1] to [3]). Similarly, for fixed  $\mu = 0.95$ , columns (3) to (6) suggest that the correlation coefficient is positively related to  $\rho_e$  and negatively related to  $\rho_c$ .

In conclusion, for the model to capture the key stylised features of the market, the fraction of conservative investors must be high, conservatives must hold slightly extrapolative beliefs, and there must exist significant heterogeneity of beliefs among the two types of investors. Hence, the parameterisation  $\{0.95, 0.65, 0.90\}$  appears to capture sufficiently almost all stylised facts under consideration.<sup>16</sup>

This finding also has a very intuitive interpretation for the shipping industry. As analysed in Section II, the fraction of extrapolators,  $\mu$ , captures the relative participation of less experienced and/or less informed shipping investors or, equivalently, new entrants to the shipping industry such as private

15. As the presence of extrapolative beliefs in the market increases, the volatility of earnings yield decreases since changes in vessel prices weaken the effect of net earnings changes on earnings yield. To illustrate this argument, in scenarios (1) and (3) of Table IV, earnings yield volatilities – scaled by the earnings yield volatility in the benchmark rational economy – are 0.32 and 0.97, respectively.

16. Table IV presents the results for 6 scenarios, however, by conducting numerous simulations using alternative parameterisations, we observe that the main statistics under consideration are strictly monotonic functions of the respective population parameter in the intervals between the examined cases, for a given net earnings sequence. For example, keeping the values of  $\rho_c$  and  $\rho_e$  equal to 0.58 and 0.9, respectively, vessel price volatility is a strictly decreasing function of  $\mu$  in the interval  $[0.1, 0.5]$  (columns [1] and [2]). Results from these tests are available from the authors.

equity investors. According to Marine Money (January 2019), since the late 1990's private equity has been on average providing approximately 5% of the total shipping funds raised from capital markets which provides further support for our result regarding the fraction of extrapolators in the empirical estimation. Similarly, conservative investors in our model correspond to the large number of established shipping companies that operate in the industry. By being present in the market for decades, these companies have substantial prior experience and expertise about the key supply and demand drivers of the industry. In addition, they have access to superior market intelligence, compared to new entrants. Shipping is an industry that is still very much based on personal relationships and information is not shared equally among market participants. Business deals in the freight market are executed through a network of brokers and are not publicly reported. The absence of a central price reporting mechanism reinforces this effect. Thus, one should expect that a long-standing owner has closer relationships with other market participants and brokers and can obtain private signals about the market and the investment decisions of other owners (which, in turn, will affect future shipping supply).

This, in turn, leads to a deeper understanding of the market mechanism and a better ability to forecast the market. Although in principle information about the world economy might be equally available to both traditional shipowners and private equity investors, it may not be used equally efficiently to forecast future demand for shipping services. Therefore, the superior knowledge of the market and the private information of traditional shipowners may translate into "more rational" and accurate forecasts about future market conditions compared to relatively new investors.

It is well-documented that during prosperous periods, new entrants, impressed by high prevailing earnings and short-term returns, are eager to buy vessels which, subsequently, are more than keen to sell as conditions begin to deteriorate. In contrast, there are many cases where established owners remain in the market for long periods of time and have realised significant returns by selling vessels at the peak of the market and buying at the trough—a strategy known as "playing the cycles" (Stopford 2009). Our model accounts for this fact through the two exit points; namely, extrapolators exit during weak market conditions while conservatives during extremely strong ones. Finally, while in principle conservatives could form expectations about future net earnings based on the true process, the moments

of interest are better matched when they also hold slightly extrapolative beliefs. This feature is consistent with reality since, no matter how experienced investors are, it is highly unlikely they can forecast precisely the evolution of cash flows in such a volatile industry.

#### *IV.A. Sensitivity Analysis*

To provide more intuition on the mechanism that creates the positive correlation between net earnings and trading activity, we examine the sensitivity of agents' exit points and trading activity to key model parameters, based on the parameterisation  $\{0.95, 0.65, 0.9\}$  (Column [5] in Table IV). As Corollary 3 suggests, both agents' exit points are strictly increasing functions of conservatives' fraction. Panel A of Figure III plots this relation for  $\rho_c = 0.65$ ,  $\rho_e = 0.9$ , and  $\mu \in [0.05, 0.95]$ . Evidently, as  $\mu$  deviates from the midpoint 0.5, the asymmetry between the two exit points increases. Specifically, when  $\mu$  is 0.95 a slight reduction in the level of freight rates from the steady state suffices for extrapolators to exit the market, while conservatives remain active in the market even for very high levels of net earnings. The opposite is observed when the fraction of conservatives in the market is low. Panels B and C plot the sensitivity of both agents' exit points to the perceived persistence of extrapolators and conservatives, respectively. Namely, Panel B indicates that conservatives' exit point is a strictly decreasing function of the perceived persistence of extrapolators while extrapolators' exit point is a strictly increasing one. Finally, the opposite is true for the sensitivity of exit points with respect to conservatives' perceived persistence, as Panel C illustrates.

Overall, a large fraction of conservatives combined with a high  $\rho_e$  and a low  $\rho_c$  result in an exit point for extrapolators that is very close to steady state equilibrium earnings,  $\bar{\Pi}$ , and an exit point for conservatives that is much higher than the steady state.<sup>17</sup> This suggests that for the set of parameters that best matches the stylised features of the market, conservatives are always present in the market while extrapolators' optimal investment policy is to exit the market even during moderately weak market conditions.

17. Specifically, for the parameter set  $\{0.95, 0.65, 0.9\}$ , the calculated exit points are  $\Pi_5^e = 2.44$  and  $\Pi_5^c = 15.75$ , while  $\bar{\Pi} = 3.1$ .

Turning next into trading activity, Panel A of Figure IV illustrates the relationship between trading volume and the fraction of conservatives in the market following positive and negative two standard-deviation shocks in net earnings. Notice that for  $\mu = 0$  and  $\mu = 1$  there is no heterogeneity among agents and thus, no trading activity. For  $\mu = 0.5$  trading activity is approximately the same following a positive and a negative shock while, for values of  $\mu$  higher than 0.5, trading activity is higher following a positive shock and vice versa. This is in line with Panel A of Figure III, since for large values of  $\mu$  extrapolators exit relatively quickly following a negative shock in net earnings. In this case, extrapolators' demand and, in turn, their holdings of the risky asset become zero. In addition, from Corollary 1, both agents hold the risky asset according to their population proportions in steady state. Therefore, following a negative shock, trading activity equals  $(1 - \mu)Q$  (Appendix B) which, for large values of  $\mu$ , will be small. In contrast, following a positive shock, both agents are present in the market and trading activity is much higher than in the previous case.

This point is also illustrated in Panel B which presents the relation between trading activity and extrapolators' perceived persistence. As the latter variable deviates from  $\rho_c$ , the heterogeneity of beliefs and, in turn, trading activity in the market will increase. Up to a limiting value of extrapolators' persistence, denoted by  $\rho_e^*$  in the graph, trading activity in the positive and negative shock cases is approximately the same. In the interval  $(\rho_e^*, 1)$ , however, trading activity following a positive shock is higher compared to the one following a negative shock. In line with Panel B of Figure III, this follows from the extrapolators' exit point being a strictly increasing function of  $\rho_e$ . Accordingly, trading activity following a negative shock is equal to  $(1 - \mu)Q$ , which for our chosen setting is 0.05. As  $\rho_e$  increases further, extrapolators' exit point increases as well, however, trading activity after the negative shock is bounded since it cannot be higher than 0.05.

Therefore, it follows that to replicate the observed stylised features, in particular trading activity, extrapolators must form a relatively small percentage of the population. From an economic perspective, as mentioned above, this result is in line with the actual composition of participants in the shipping industry, that comprises a very large number of established ship-owning firms which, in numerous cases, have been present in the market for many decades.

#### IV.B. Impulse Response Functions

To gain further insight into market dynamics, we perform model-implied impulse response functions and examine the effect on the economy of a one-time shock in net earnings for the parameterisation  $\{\mu, \rho_c, \rho_e\} = \{0.95, 0.65, 0.9\}$ . Figures V and VI illustrate the behaviour of net earnings, 5-year old vessel prices, vessel demand, and trading activity following two standard-deviations positive and negative shocks, respectively. In panel B of each figure, we present the model-generated price of a 5-year old vessel (i.e. the market clearing price) as well as the respective agent-specific valuations; the latter refer to the “fair” value of the asset from each agent’s perspective.

In the following, the negative shock case is analysed. At  $t = 0$ , net earnings are equal to their long-run mean,  $\bar{\Pi}$ , and the economy is in steady state (Panel A of Figure VI). Hence, all four valuations coincide (Panel B), agents have the same per capita demand (Panel C), and there is no trading activity (Panel D). At  $t = 1$ , we perturb the steady state by generating a negative 2 standard-deviation (i.e., minus \$2 million) shock. The immediate first order effect is the decrease of current net earnings by this amount. Due to the mean reverting property of net earnings, the shock is completely attenuated within approximately 10 years. However, extrapolators expect net earnings to revert to their steady state value after more than 20 periods while conservatives in about 12 (Panel A). As a result, extrapolators consider the asset to be overvalued while, conservatives consider it to be undervalued, relative to the prevailing market clearing price (Panel B). Essentially, agents compare their valuation of the asset to its equilibrium price and not the fundamental price which, by not being fundamentalists, they totally ignore.

Extrapolators’ (conservatives’) demand is lower (higher) compared to the steady state of the economy. In particular, extrapolators’ demand for 5-year old vessels is zero, as their exit point is higher than the corresponding net earnings variable at  $t = 1$  and thus, they exit the market.<sup>18</sup> Following this rapid change in demand, there is significant trading activity in the second-hand market (Panel D) with extrapolators reducing their relative fractions of the risky asset while conservatives increasing theirs.

18. Namely,  $\Pi_{t=1} \cong 1.1$ ,  $\Pi_5^e \cong 2.44$  and  $\Pi_6^e \cong 2.42$ .

However, since the short-sale constraint binds in this case, trading activity is much lower compared to the positive shock case (Panel D of Figure V).

In year 2, net earnings are at a higher level as they revert to their long-run mean, although, they are still below both exit thresholds,  $\Pi_5^e$  and  $\Pi_6^e$ . Therefore, extrapolators stay out of the market and there is no trading activity. In year 3, net earnings are slightly higher than  $\Pi_6^e$  but still below  $\Pi_5^e$ .<sup>19</sup> Accordingly, there is some, rather small, trading activity. In year 4, trading activity becomes noticeably higher since demand for the 6-year old vessel is much higher than demand for the 5-year old vessel in year 3 (Panel C of Figure VI). From this point on, both agents are present in the market and trading activity strictly decreases with time until it becomes zero, when net earnings converge to their long-run mean. The positive shock case can be interpreted along the same lines. The main difference is that in the positive shock case, both agents are always present in the market (Panel C of Figure V) and, as a result, trading activity is significantly higher.

#### IV.C. *Expectations of Returns and Realised Returns*

We also examine the agent-specific expectations of future returns and the corresponding realised returns. Agent  $i$ 's one-period expected return from operating the vessel between its fifth and sixth years of economic life is given by

$$(26) \quad R_t^i \equiv R_{t \rightarrow t+1}^i = \frac{E_t^i[P_{6,t+1}] + \Pi_t - P_{5,t}}{P_{5,t}} = \frac{\frac{1 - \rho_i^{21}}{1 - \rho_i}(\Pi_t - \bar{\Pi}) + 21\bar{\Pi} - X_5^i \sigma_\varepsilon^2 Q - P_{5,t}}{P_{5,t}}.$$

Since there is one market clearing price at each  $t$ , agent  $i$ 's expected return depends on his specific beliefs and the current realisation of the net earnings variable. Since the numerator is an increasing function of  $\rho_i$ , during prosperous market conditions extrapolators have higher expected returns compared to conservatives and, in turn, are more eager to invest compared to conservatives and vice versa.

19. Specifically,  $\Pi_{t=2} \cong 1.94$  and  $\Pi_{t=3} \cong 2.43$  while  $\Pi_5^e \cong 2.44$  and  $\Pi_6^e \cong 2.42$ .

To assess which investor type's expectations are on average closer to realised returns in the following period, we define the agent-specific prediction error,  $Z_t^i$ , as the absolute deviation between agent  $i$ 's expected return and realised (actual) return:

$$(27) \quad Z_t^i = |R_t^i - R_t^a|,$$

where the realised returns,  $R_t^a$ , are estimated through

$$(28) \quad R_t^a \equiv R_{t \rightarrow t+1}^a = \frac{P_{6,t+1} + \Pi_t - P_{5,t}}{P_{5,t}}.$$

Plugging (26) and (28) in equation (27) yields

$$(29) \quad Z_t^i = \frac{\left| \frac{1 - \rho_i^{21}}{1 - \rho_i} (\Pi_t - \bar{\Pi}) + 21\bar{\Pi} - X_5^i \sigma_\varepsilon^2 Q - \Pi_t - P_{6,t+1} \right|}{P_{5,t}}.$$

In the heterogeneous-agent economy, the prediction error,  $Z_t^i$ , depends on the stochasticity of the net earnings variable and the determination mechanism of the equilibrium market price. If conservatives were able to incorporate in their valuation the strategy of extrapolators, then they would have always formed more accurate returns expectations and they would have been able to exploit their “more correct” beliefs. Due to incomplete information and bounded rationality, however, the equilibrium price and realised returns depend on a complex weighted average of both agents' beliefs, where the weights correspond to their population fractions in the economy.

We further clarify this argument by examining the heterogeneous-agent economy for three different values of  $\mu$ . The statistics under consideration are the mean and standard deviation of each agent  $i$ 's expected return; the mean and standard deviation of realised returns; and the mean and standard deviation of the agent-specific prediction error. In addition, we estimate the expected returns,  $R_t^f$ , realised returns, and prediction errors,  $Z_t^f$ , in the counterfactual rational economy. By construction, if no shock occurs between two consecutive periods, the rationally expected return is equal to the realised one.

Table V summarises the statistics of interest for three parameterisations of the heterogeneous-agent economy (Panels A-C) and the benchmark rational economy (Panel D). Evidently, when the market is

dominated by conservatives (Panel A), their average prediction error is much smaller than that of extrapolators. In support to this, conservatives' average expected return is also closer to the average realised return while the standard deviations of both expected return and prediction error are among the lowest across the cases considered. In this case, conservatives' prediction error is mainly attributed to the stochasticity of the error term and their slightly extrapolative expectations and to a lesser extent to incomplete information and bounded rationality. The opposite is true for extrapolators, since they constitute a very small fraction of the population. In contrast, when extrapolators constitute the largest fraction of the population, agent-specific prediction errors are quite high (Panel C). For conservatives, this error is now mainly attributed to incomplete information and bounded rationality, while in the case of extrapolators, the opposite applies. The case where each agent-type constitutes half of the populations (Panel B) lies somewhere in the middle.

Finally, Panel D of Table V shows that while the expectations of rational investors converge to the average realised returns, there still exists a small average prediction error between the two values which is solely attributed to the volatility of the cash flow shock. In line with this argument, the standard deviation of expected returns is very small which supports the view that in the benchmark rational economy, investors have essentially constant required returns. Consistent with the previous analysis and the characteristics of the shipping industry, the combination that best matches the observed features implies that conservatives constitute a large fraction of the population, hold slightly extrapolative beliefs, and there is sufficient discrepancy between the views of conservatives and extrapolators.

## V. ROBUSTNESS

We now proceed to test the robustness of our model's predictions by examining five alternative hypotheses regarding the characteristics of the investor population. Namely, we allow our economy to consist of (i) contrarians and fundamentalists, (ii) contrarians and extrapolators, (iii) fundamentalists, (iv) extrapolators only and (v) contrarians only. Accordingly, we compare the findings to the empirical values and results from our benchmark setting.

We introduce contrarian investors, denoted by  $x$ , in a straightforward manner. Specifically, we assume that they hold irrational beliefs regarding the net earnings process in the opposite way to that of extrapolators; that is, they overestimate the mean reversion of net earnings. Accordingly, their perceived persistence of net earnings,  $\rho_x$ , lies in the interval  $[0, \rho_0)$ . Apart from this feature, contrarians behave exactly as other agent types. In particular, they also neglect the future demand responses of the other types and they upgrade the perceived riskiness of their investments as they grow older. Therefore, the Proposition and Corollaries 1-3 can be directly extended to capture this alternative specification.

Table VI summarises the results obtained from these alternative hypotheses for a variety of investor population characteristics,  $\{\mu_i, \rho_x, \rho_i\}$  for  $i \in \{f, e\}$ . The estimation procedure and the basic parameter values are as in Section III. For reasons of brevity, we present only the statistics related to the main quantities of interest. Evidently, the results suggest that these alternative hypotheses are not able to simultaneously match sufficiently the empirical values. To begin with, in the heterogeneous-agent cases (Panels B and C), we observe that the main effect of contrarians' presence in the market is the attenuation of vessel price volatility. It seems that contrarians have the opposite effect on price volatility compared to extrapolators; that is, they generate less volatility than in the case of a benchmark rational economy. This is expected since vessel price volatility is an increasing function of perceived persistence.

Extending the analysis of Section III, in an economy consisting of contrarians and fundamentalists (Panel B) or, contrarians and extrapolators (Panel C), the latter will exit from the market when market conditions deteriorate. Accordingly, it is straightforward to interpret the remaining results in Table VI. For instance, a very small fraction of extrapolators combined with a sufficient degree of heterogeneity in beliefs (third row in Panel C) result in low average trading activity and positive correlation between trading activity and net earnings. However, due to the large presence of contrarians, the volatility of vessel prices is much lower compared to the benchmark case. On the other hand, the specifications that generate excess volatility (first and second row in Panel C) cannot simultaneously match the positive correlation between trading activity and net earnings and between trading activity and vessel prices. Finally, in a homogeneous-agent economy (Panel D) there is no trading activity in the market since beliefs' heterogeneity is what motivates trading in our model.

Finally, we also considered the co-existence of contrarians, fundamentalists and extrapolators in the economy. The results obtained from those extensions lie somewhere between the ones illustrated in Tables IV and VI; thus, they are not able to either improve the fit of the model regarding the main quantities of interest or to alter the economic interpretation of the results. In conclusion, we have illustrated that none of those alternative hypotheses regarding the composition of investors can reproduce the stylised facts under consideration.

## VI. CONCLUSIONS

In this article, we develop a behavioural asset-pricing model with microeconomic foundations to simultaneously investigate the formation of vessel prices and trading activity in the Handysize sector of the dry bulk shipping market. Our discrete-time economy consists of two agent types, conservatives and extrapolators, who differ only in the way in which they form expectations about future net earnings. Specifically, agents form their demand for the asset by extrapolating – at a different degree – current net earnings and underestimating the future demand responses of their competitors. As a result, prices fluctuate significantly more than in the benchmark rational model. While a model with homogeneous extrapolative beliefs can capture sufficiently well the observed price behaviour, it cannot account for trading activity in the second-hand market. Accordingly, by incorporating a heterogeneous beliefs framework, we can capture the positive relation between trading activity and both current market conditions and absolute changes in market conditions but also the aggregate level of trading activity in the market.

Both theoretical predictions and model simulations suggest that to simultaneously capture these stylised facts, conservatives must constitute a very large fraction of the population while shipping investors of both types must hold extrapolative expectations. Moreover, our model’s predictions are consistent with additional empirical regularities in the shipping literature such as that net earnings yields are highly positively correlated with prevailing market conditions and, in turn, strongly negatively

forecast future net earnings growth, but also that the bulk of the yield's volatility is attributed to expected cash flow variation and not to time-varying expected returns. To the best of our knowledge, this is the first time that a heterogeneous-beliefs asset pricing model with microeconomic foundations is applied to the shipping industry.

Finally, the proposed partial equilibrium model provides a framework for modelling the joint behaviour of earnings, physical asset prices and trading activity which can also be empirically evaluated in other markets with similar characteristics, such as the commercial real estate industry.

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## TABLES

Table I

Main Empirical Regularities and Findings in the Existing Literature Captured by the Proposed Behavioural Model

Stylised feature	References
Excess volatility of vessel prices	Kavussanos and Alizadeh (2002); Alizadeh and Nomikos (2007); Greenwood and Hanson (2015).
Positive correlation between vessel prices and net earnings	Greenwood and Hanson (2015); Moutzouris and Nomikos (2019).
Overvaluation (undervaluation) of vessels during market peaks (troughs)	Greenwood and Hanson (2015).
Positive correlation between trading activity and vessel prices	Alizadeh and Nomikos (2003).
Positive correlation between trading activity and net earnings	<i>Current Paper Only.</i>
Positive correlation between trading activity and absolute changes in net earnings	<i>Current Paper Only.</i>
Low trading activity	Kalouptsidi (2014).
Positive correlation between earnings yield and prevailing market conditions	Moutzouris and Nomikos (2019).
Earnings yield strongly predicts future net earnings growth	Moutzouris and Nomikos (2019).
Earnings yield volatility is attributed to variability of net earnings growth rather than variability in expected returns.	Moutzouris and Nomikos (2019).
Extrapolation of fundamentals	Zannetos (1966); Metaxas (1971); Beenstock and Vergottis (1989); Alizadeh and Nomikos (2007); Sodal et al. (2009); Stopford (2009); Greenwood and Hanson (2015); Alizadeh et al. (2017).

Table II

## Descriptive Statistics for Vessel Prices, Net Earnings, and Trading Activity

Variable	$T$	Mean	SD	Median	Max	Min	$\rho_0$
$\Pi$ (\$m)	26	3.10	2.39	2.42	9.96	0.91	0.58
$P$ (\$m)	26	22.86	7.65	22.32	50.23	13.43	0.49
$V$	20	0.058	0.020	0.054	0.099	0.031	0.11

*Notes.* This table presents the number of observations ( $T$ ) mean, standard deviation, ( $SD$ ), median, maximum, minimum, and 1-year autocorrelation coefficient, ( $\rho_0$ ), for net earnings,  $\Pi$ , 5-year old vessel prices,  $P$ , and trading activity,  $V$ . Shipping data are provided by Clarksons Shipping Intelligence Network 2010. The sample is annual, covering the period from 1989 to 2014, apart from trading activity data for which are available from 1995. Net earnings and prices are expressed in December 2014 million dollars through the U.S. Consumer Price Index (CPI), obtained from Thomson Reuters Datastream Professional. Since the generic 5-year old vessel price refers to a 32,000 dead-weight tonnage (dwt) carrier while time-charter rate series to a 30,000 dwt one, we multiply the initial rate series by  $\frac{32}{30}$ . Trading activity is scaled by the respective size of the fleet at the beginning of the corresponding period.

Table III

Parameter Values

Parameter	Assigned Value
$\overline{P}_5^*$	22.86
$\overline{\Pi}$	3.1
$\rho_0$	0.58
$\sigma_\varepsilon^2$	1
$Q$	1
$T$	20
$R_f$	0
$\mu$	{0.1,0.5,0.95}
$\alpha^f$	0.42
$\rho_f$	0.58
$\vartheta_5^f$	1
$\vartheta_6^f$	1
$\alpha^c$	0.35
$\rho_c$	{0.58,0.65,0.75}
$\alpha^e$	0.15
$\rho_e$	{0.9,0.99}

*Notes.* The table summarises the assigned values used in the simulations for: the average price of a 5-year old vessel,  $\overline{P}_5^*$ ; the average net earnings variable,  $\overline{\Pi}$ ; the 1<sup>st</sup> degree autocorrelation of net earnings,  $\rho_0$ ; the variance of net earnings shock,  $\sigma_\varepsilon^2$ ; the vessel supply,  $Q$ ; the remaining economic life of the 5-year old vessel,  $T$ ; the risk-free rate,  $R_f$ ; the fraction of conservatives in the investor population,  $\mu$ ; the coefficient of absolute risk aversion of fundamentalists,  $\alpha^f$ ; the perceived persistence of fundamentalists,  $\rho_f$ ; the 5- and 6-year variance adjustment coefficients of fundamentalists,  $\vartheta_5^f$  and  $\vartheta_6^f$ , respectively; the coefficient of absolute risk aversion of conservatives,  $\alpha^c$ ; the perceived persistence of conservatives,  $\rho_c$ ; the coefficient of absolute risk aversion of extrapolators,  $\alpha^e$ ; and the perceived persistence of extrapolators,  $\rho_e$ . Note that we list parameters  $\vartheta_5^i$  and  $\vartheta_6^i$  only for the fundamentalist since in the cases of conservatives and extrapolators these depend solely on the choice of  $\rho_i$ .

Table IV

Model Predictions for the Quantities of Interest

Quantity		$\{\mu, \rho_c, \rho_e\}$						Actual Data
		(1)	(2)	(3)	(4)	(5)	(6)	
		$\{0.1, 0.58, 0.9\}$	$\{0.5, 0.58, 0.9\}$	$\{0.95, 0.58, 0.9\}$	$\{0.95, 0.58, 0.99\}$	$\{0.95, 0.65, 0.9\}$	$\{0.95, 0.75, 0.99\}$	
1	$\bar{P}_5^*$	23.94	24.09	23.05	23.13	23.08	23.22	22.86
2	$Corr(P_{5,t}^*, P_{5,t+1}^*)$	0.52	0.52	0.53	0.53	0.53	0.53	0.49
3	$\sigma(P_5^*) \div \sigma(P_5^f)$	3.48	2.30	1.06	1.10	1.26	1.78	1.34
4	$Corr(P_{5,t}^*, \Pi_t)$	1.00	0.97	1.00	1.00	1.00	1.00	0.76
5	$\Pi_t \div P_{5,t}^*$	0.14	0.13	0.13	0.13	0.13	0.13	0.13
6	$Corr(\Pi_t, \Pi_t \div P_{5,t}^*)$	-0.81	0.80	0.98	0.98	0.98	0.95	0.88
7	$\beta_{\Delta\pi}$	<b>3.39</b>	<b>-0.98</b>	<b>-0.73</b>	<b>-0.73</b>	<b>-0.78</b>	<b>-0.99</b>	<b>-0.61</b>
8	Growth $R^2$	0.26	0.26	0.27	0.27	0.27	0.28	0.27
9	$\beta_r$	<b>3.54</b>	<b>-0.19</b>	<b>-0.05</b>	-0.06	-0.10	<b>-0.25</b>	0.09
10	Returns $R^2$	0.27	0.05	0.04	0.04	0.07	0.11	0.03
11	$\bar{V}_t$	0.12	0.28	0.05	0.08	0.05	0.07	0.06
12	$Corr(V_t, \Pi_t)$	-0.44	-0.04	0.38	0.44	0.37	0.43	0.53
13	$Corr(V_t,  \Pi_t - \Pi_{t-1} )$	0.69	0.74	0.79	0.73	0.80	0.73	0.65
14	$Corr(V_t, \bar{P}_5^*)$	-0.44	-0.04	0.38	0.44	0.37	0.43	0.71

*Notes.* This table summarises the heterogeneous-agent model's predictions for the quantities of interest presented in the left column. The last column presents the actual empirical values for those quantities. Columns (1) to (6) report the average value of each quantity across 10,000 simulated paths, for a given parameterisation  $\{\mu, \rho_c, \rho_e\}$ . The basic model parameters are presented in Table III. Row (1) refers to the mean of the 5-year old vessel prices; row (2) to the 1-year autocorrelation of those prices; the row (3) to the ratio of the standard deviation of 5-year old ship prices in the extrapolative heterogeneous-agent economy to the standard deviation of these prices in the counterfactual rational economy (equation (13), for  $i = f$ ), under the same net earnings sequence; row (4) to the correlation between net earnings and 5-year old vessel prices. Row (5) refers to the mean of the net earnings yield, defined as the ratio of net earnings to the respective 5-year old vessel price; row (6) to the correlation between net earnings and net earnings yield. Rows (7) and (8) report the slope coefficient and R-squared, respectively, of a regression of the one-period log net earnings growth on the log net earnings yield:  $\ln(\Pi_{t+1}/\Pi_t) = \alpha_{\Delta\pi} + \beta_{\Delta\pi} \cdot \ln(\Pi_t/P_{5,t}) + \varepsilon_{\Delta\pi,t+1}$ . Rows (9) and (10) report the slope coefficient and the R-squared, respectively, of a regression of one-period log returns on the log net earnings yield:  $r_{t+1} = \alpha_r + \beta_r \cdot \ln(\Pi_t/P_{5,t}) + \varepsilon_{r,t+1}$ , where log returns are estimated through (25). Slope estimates in bold indicate that results are significant at the 5% level. Finally, rows (11) to (14) present the mean of the annual trading activity, estimated using (7), the correlation between trading activity and net earnings, the correlation between trading activity and absolute 1-year changes in net earnings and the correlation between trading activity and 5-year old vessel prices, respectively.

Table V

## Expected Returns, Realised Returns, and Prediction Error

Variables	Mean	Standard Deviation
Panel A: $\{\mu, \rho_c, \rho_e\} = \{0.95, 0.65, 0.9\}$		
Conservatives' Expected Return	0.1093	0.0241
Extrapolators' Expected Return	0.1375	0.2906
Realised Return	0.0922	0.1350
Conservatives' Prediction Error	0.1057	0.0796
Extrapolators' Prediction Error	0.2838	0.2133
Panel B: $\{\mu, \rho_c, \rho_e\} = \{0.5, 0.65, 0.9\}$		
Conservatives' Expected Return	0.1083	0.1472
Extrapolators' Expected Return	0.1020	0.1950
Realised Return	0.1079	0.2546
Conservatives' Prediction Error	0.1914	0.1511
Extrapolators' Prediction Error	0.2806	0.2279
Panel C: $\{\mu, \rho_c, \rho_e\} = \{0.1, 0.65, 0.9\}$		
Conservatives' Expected Return	0.3927	0.8369
Extrapolators' Expected Return	0.1773	0.0712
Realised Return	0.2237	0.7411
Conservatives' Prediction Error	0.3747	0.4769
Extrapolators' Prediction Error	0.3927	0.5599
Panel D: Benchmark Rational Economy		
Expected Return	0.1054	0.0131
Realised Return	0.1083	0.1052
Prediction Error	0.0828	0.0627

*Notes.* This table summarises the mean and standard deviation of the quantities of interest presented in the left column for three different populations compositions across 10,000 simulations. Panel A presents the case where conservatives constitute a very large fraction of the population. Panel B illustrates the case where each agent type constitutes half of the population. Panel C summarises the case where extrapolators constitute a very large fraction of the population. Finally, Panel D presents the corresponding results for the benchmark rational economy.

Table VI

Model Predictions for the Quantities of Interest under Alternative Hypotheses

(1)	(2)	(3)	(4)	(5)	(6)	(7)
$\{\mu_i, \rho_x, \rho_i\}$	$\overline{P}_5^*$	$\frac{\sigma(P_5^*)}{\sigma(P_5^f)}$	$\overline{V}_t$	$Corr(V_t, \Pi_t)$	$Corr(V_t,  \Pi_t - \Pi_{t-1} )$	$Corr(V_t, P_5^*)$
Panel A: Actual Data						
-	22.86	1.34	0.06	0.53	0.65	0.71
Panel B: Contrarians and Fundamentalists						
{0.1,0.1,0.58}	22.93	0.51	0.04	0.07	0.99	0.07
{0.5,0.1,0.58}	22.96	0.72	0.12	0.02	1.00	0.02
{0.95,0.1,0.58}	23.00	0.97	0.02	-0.12	0.95	-0.12
Panel C: Contrarians and Extrapolators						
{0.9,0.1,0.9}	23.97	3.42	0.16	-0.46	0.67	-0.46
{0.5,0.1,0.9}	24.55	2.09	0.29	-0.03	0.68	-0.03
{0.05,0.1,0.9}	22.99	0.53	0.06	0.41	0.77	0.41
{0.5,0.1,0.65}	22.98	0.81	0.16	-0.01	0.99	-0.01
Panel D: Homogeneous-Agent Economy						
$\rho_f = 0.58$	23.00	1.00	-	-	-	
$\rho_e = 0.9$	23.72	3.73	-	-	-	

$$\rho_x = 0.1$$

22.93

0.47

-

-

-

---

*Notes.* This table summarises the heterogeneous-agent model's predictions for the quantities of interest presented in the second row. Table III summarises the basic model parameters. In addition, we have set  $\alpha_x = 0.55$ . Panel A presents the empirical values of these quantities for the period 1989-2014 (1995 to 2014 for trading activity). Panels B-D report the average value of each quantity across 10,000 simulated paths, for a given parameterisation depending on the population characteristics,  $\{\mu_i, \rho_x, \rho_i\}$  for  $i \in \{f, e\}$ . The second column refers to the mean of the 5-year old vessel prices; the third column refers to the ratio of the standard deviation of 5-year old vessel prices in the extrapolative heterogeneous-agent economy to the standard deviation of the fundamental value of the 5-year old asset, under the same net earnings sequence. Columns four, five, six, and seven present the mean annual trading activity, estimated using (23), the correlation between trading activity and net earnings, the correlation between trading activity and absolute 1-year changes in net earnings and the correlation between trading activity and 5-year old vessel prices, respectively.

## FIGURES

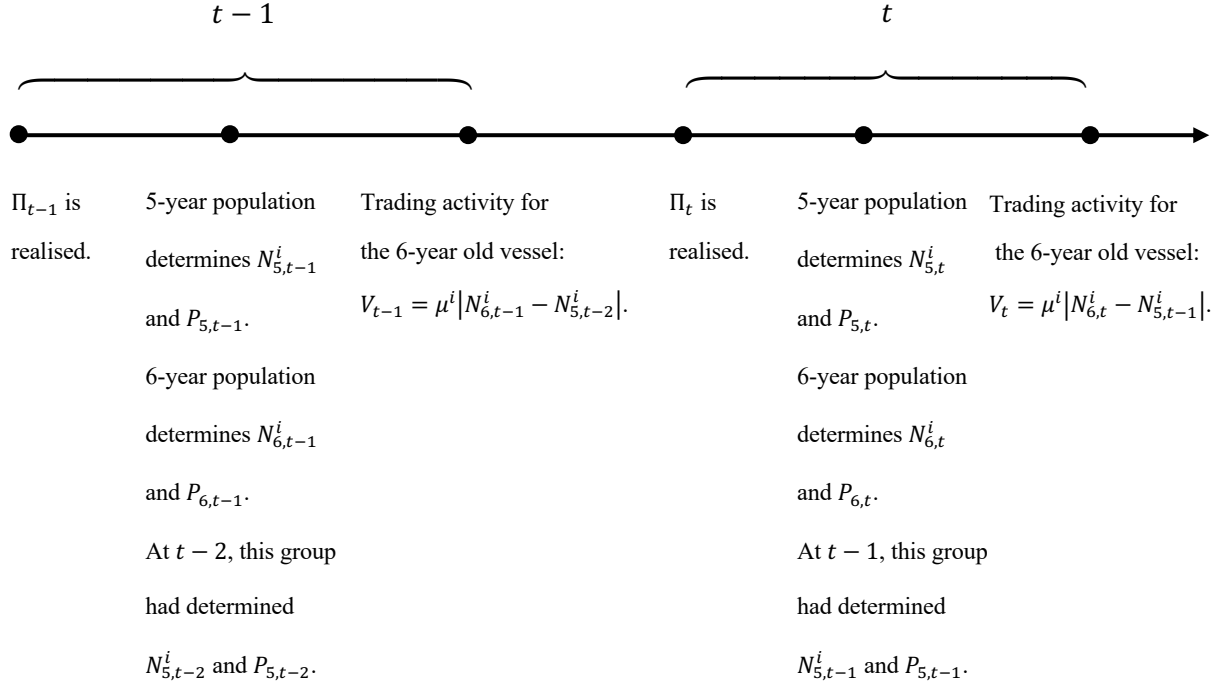
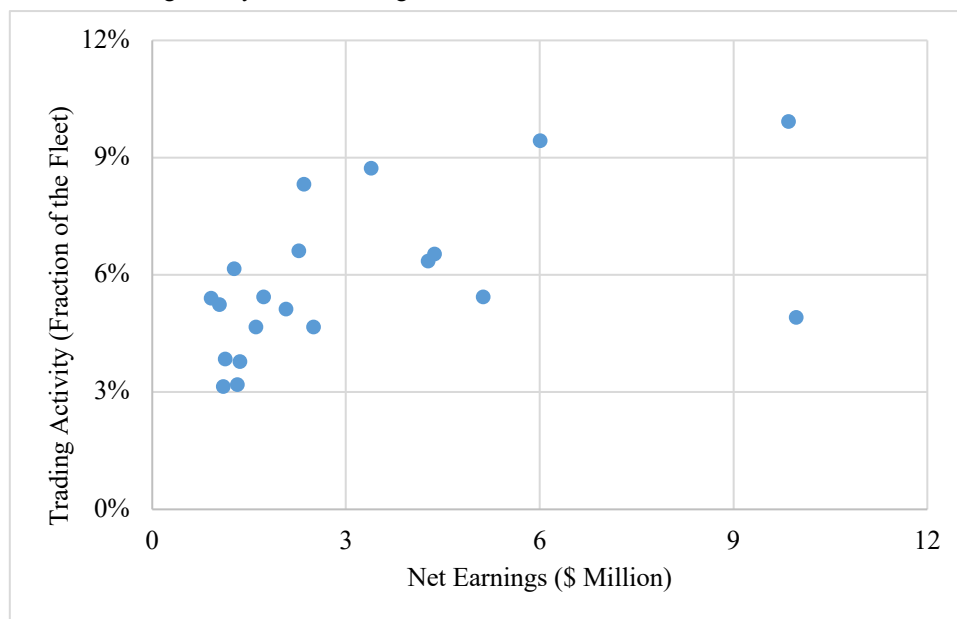


Figure I

Timeline of the Model

Panel A: Trading activity and net earnings



Panel B: Trading activity and absolute changes in net earnings

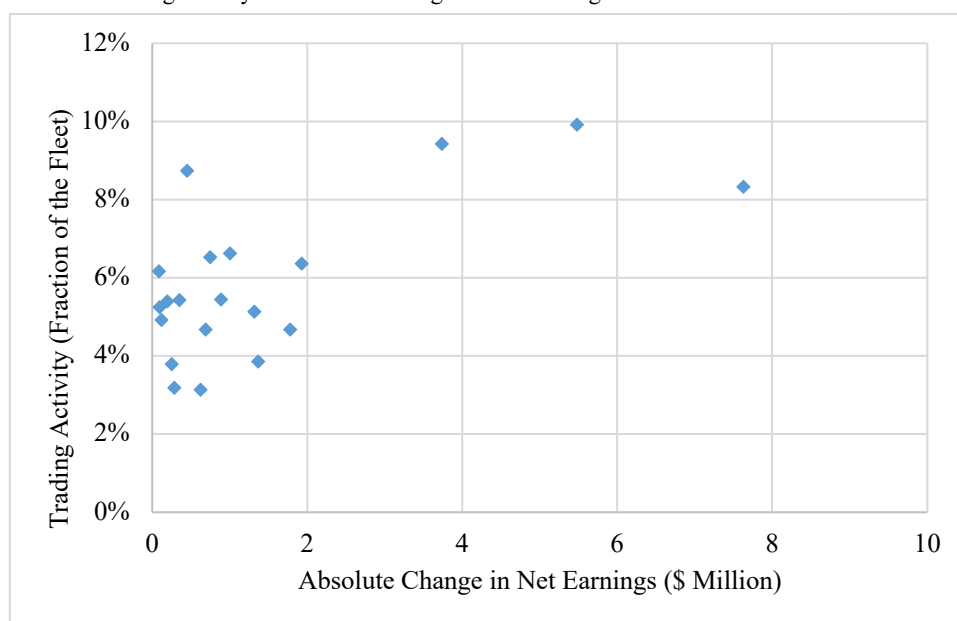
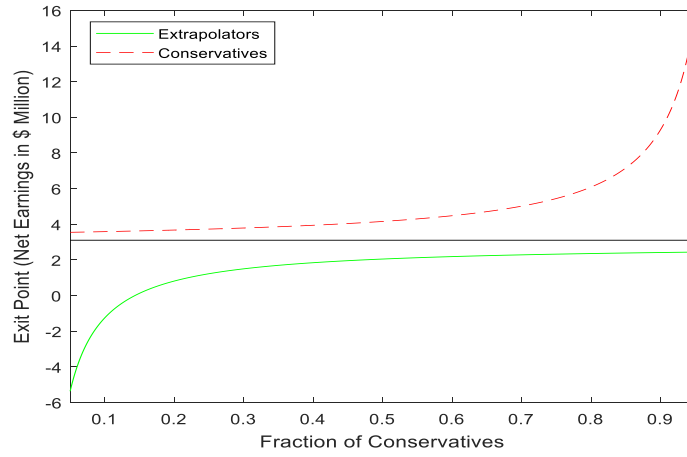


Figure II

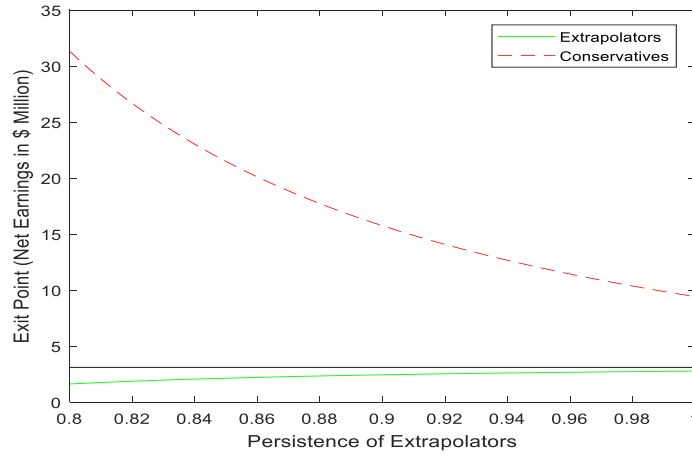
### Net Earnings and Trading Activity

Panel A depicts the relation between annual trading activity and annual net earnings. Panel B depicts the relation between annual trading activity and absolute changes in annual net earnings. The sample period is from 1995 to 2014. Annual trading activity is expressed as a percentage of the fleet at the beginning of the corresponding period. Prices and net earnings are expressed in December 2014 million dollars.

Panel A: Sensitivity of exit points to the fraction of conservatives



Panel B: Sensitivity of exit points to extrapolators' persistence



Panel C: Sensitivity of exit points to conservatives' persistence

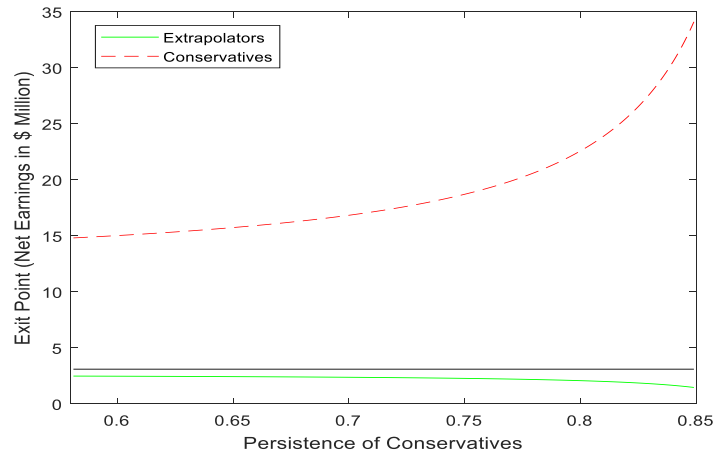
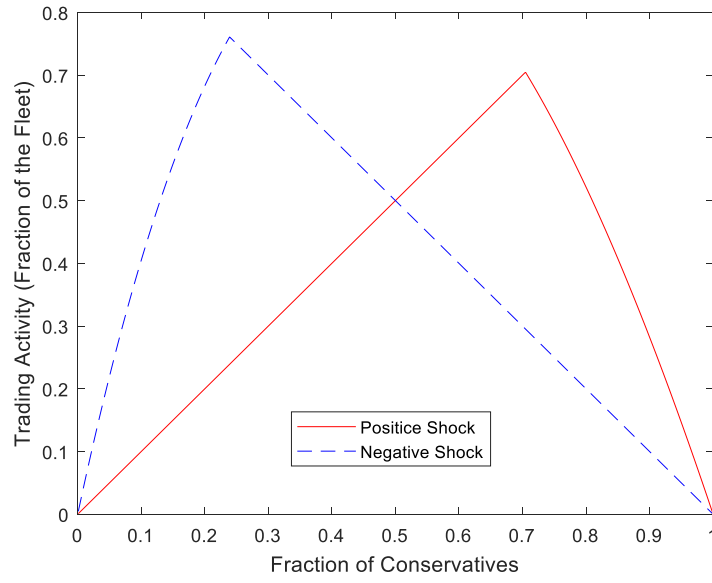


Figure III

### Sensitivity of Exit Points to Parameter Values

Figure III depicts the relation between agents' exit points and the key parameters of the model. Panel A illustrates the sensitivity to the fraction of conservatives for  $\rho_c = 0.65$  and  $\rho_e = 0.9$ . Panel B shows the sensitivity to extrapolators' perceived persistence for  $\mu = 0.95$  and  $\rho_c = 0.65$ . Panel C demonstrates the sensitivity to conservatives' perceived persistence for  $\mu = 0.95$  and  $\rho_e = 0.9$ . The horizontal line in each panel shows the steady state value of the net earnings variable.

Panel A: Sensitivity of trading activity to the fraction of conservatives



Panel B: Sensitivity of trading activity ratio to extrapolators' persistence.

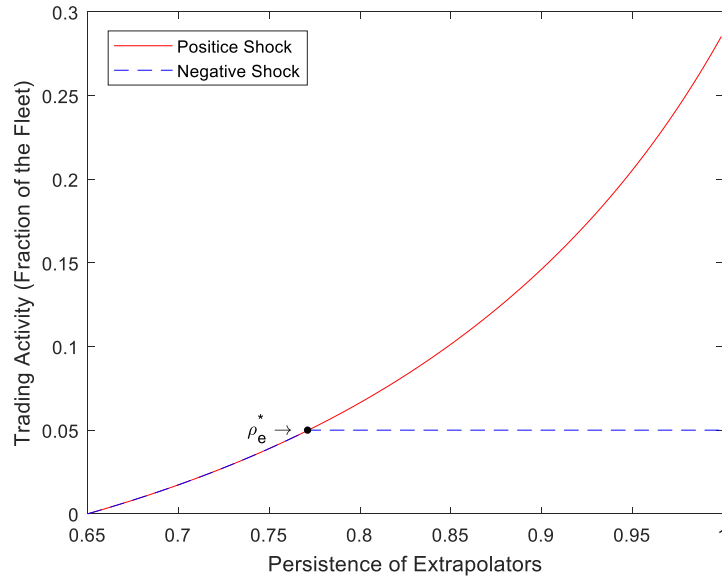


Figure IV

#### Sensitivity of Trading Activity to Key Model Parameters

Panel A presents the relation between the fraction of conservatives and trading activity following positive and negative two standard-deviation shocks for  $\rho_c = 0.65$  and  $\rho_e = 0.9$ . Panel B presents the relation between extrapolators' persistence and trading activity following positive and negative two standard-deviation shocks for  $\mu = 0.95$  and  $\rho_c = 0.65$ . The arrow indicates the limiting value of extrapolators' perceived persistence,  $\rho_e^*$ .

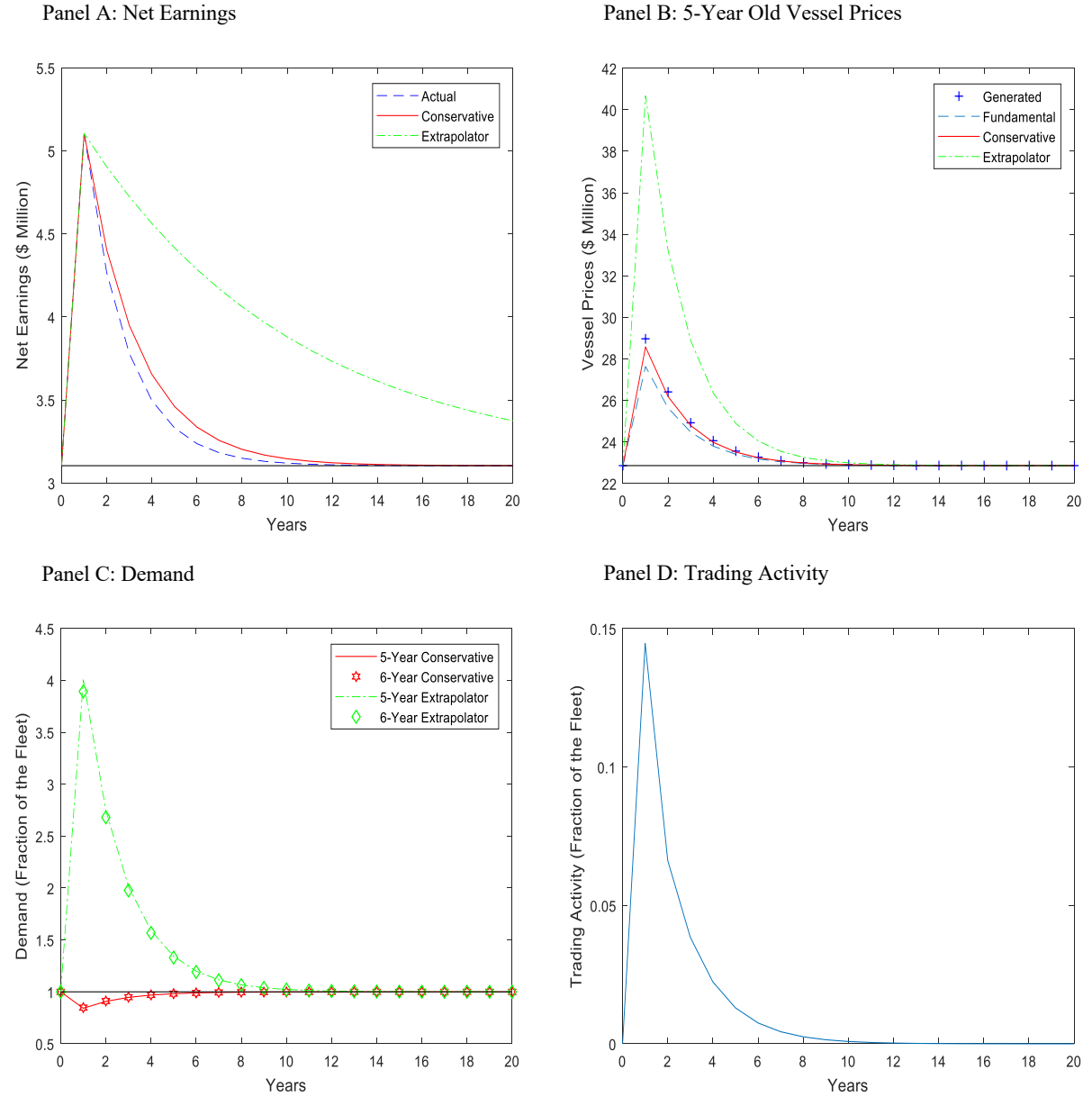


Figure V

### Model-Implied Impulse Response Functions Following a Positive Shock

Figure V displays model-implied impulse response functions following a positive two standard-deviation (\$2 million) shock to net earnings, for the parameterisation  $\{\mu, \rho_c, \rho_e\} = \{0.95, 0.65, 0.9\}$ . Panel A illustrates the actual evolution of net earnings and the evolution perceived by each type of agent. Panel B shows the model-generated 5-year old vessel prices and the agent-specific valuations. Panel C shows the agent-specific demand for the 5- and 6-year old vessels. Finally, Panel D plots the trading activity in the market. The horizontal line in each panel shows the steady state value of the corresponding variable.

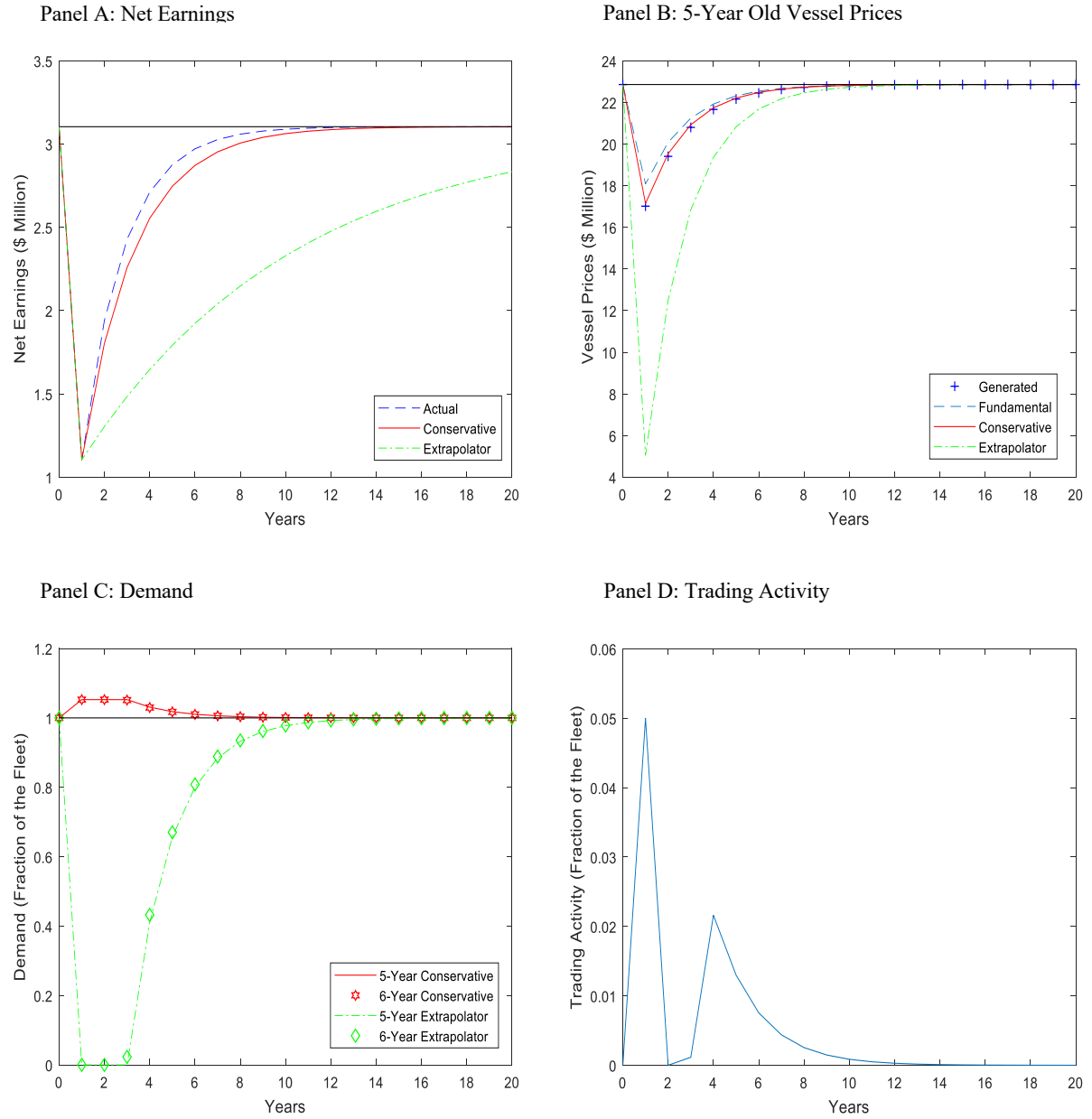


Figure VI

### Model-Implied Impulse Response Functions Following a Negative Shock

Figure VI displays model-implied impulse response functions following a negative two standard-deviation (\$2 million) shock to net earnings, for the parameterisation  $\{\mu, \rho_c, \rho_e\} = \{0.95, 0.65, 0.9\}$ . Panel A illustrates the actual evolution of net earnings and the evolution perceived by each type of agent. Panel B shows the model-generated 5-year old vessel prices and the agent-specific valuations. Panel C shows the agent-specific demand for the 5- and 6-year old vessels. Finally, Panel D plots the trading activity in the market. The horizontal line in each panel shows the steady state value of the corresponding variable.

## APPENDIX

### *A. Derivation of the Demand Functions for the Age-Specific Vessel*

*5-Year Old Vessel.* We begin by estimating the time  $t$  demand function for the 5-year old vessel for each agent type,  $N_{5,t}^i$ . Notice that, for notational simplicity, the age index corresponding to the 5-year old vessel is dropped in those derivations. Therefore,  $N_t^i$  and  $P_t$  refer to the time  $t$  agent  $i$ 's demand for and price of the 5-year old vessel, respectively. Accordingly, we have omitted the age index from all parameters. Since vessels are real assets with limited economic lives, we can estimate this demand recursively. Specifically, assuming that a newly-built vessel has an economic life of 25 years, at the terminal date,  $T$ , the residual price of the 25-year old asset must be equal to its scrap price. However, since scrap price is correlated with net earnings (Stopford 2009), we impose the simplifying assumption that it is equal to the net earnings variable corresponding to period  $T$ ; that is,  $P_T = \Pi_T$ .<sup>20</sup> From equation (5) of the main text, agent  $i$ 's objective at time  $T - 1$  is

$$(30) \quad \max_{N_{T-1}^i} E_{T-1}^i \left[ -e^{-\alpha^i (w_{T-1}^i + N_{T-1}^i (\Pi_{T-1} + P_T - P_{T-1}))} \right],$$

Using the fact that  $P_T = \Pi_T$  and, accordingly, incorporating (2) of the main text, results in

$$(31) \quad \max_{N_{T-1}^i} -e^{-\alpha^i (w_{T-1}^i + N_{T-1}^i ((1+\rho_i)\Pi_{T-1} + (1-\rho_i)\bar{\Pi} - P_{T-1})) + \frac{(\alpha^i N_{T-1}^i)^2}{2} \vartheta^i \sigma_\varepsilon^2}.$$

Hence, agent  $i$ 's first order condition yields

$$(32) \quad N_{T-1}^i = \frac{(1 + \rho_i)\Pi_{T-1} + (1 - \rho_i)\bar{\Pi} - P_{T-1}}{\alpha^i \vartheta^i \sigma_\varepsilon^2}.$$

The market clearing condition at  $T - 1$ , along with (32), suggest that

18. It is straightforward to assume a scrap value given by an AR(1) process where the long-run mean is equal to the average scrap value in our sample and the random term is highly correlated with the error term in (1) and (2). Alternatively, we could also assume a zero-terminal value of the asset.

$$(33) \quad \Rightarrow P_{T-1} = (1 + \rho_i)\Pi_{T-1} + (1 - \rho_i)\bar{\Pi} - \frac{\alpha^i \vartheta^i \sigma_\varepsilon^2}{\mu^i} [Q - (1 - \mu^i) N_{T-1}^i].$$

In a similar manner, at time  $T - 2$ , trader  $i$ 's objective is

$$(34) \quad \max_{N_{T-2}^i} \left\{ -e^{-\alpha^i (w_{T-2}^i + N_{T-2}^i (\Pi_{T-2} - P_{T-2}))} E_{T-2}^i \left[ e^{-\alpha^i N_{T-2}^i P_{T-1}} \right] \right\}.$$

Incorporating equation (33), the expectation in (34) can be expressed as

$$(35) \quad e^{-\alpha^i N_{T-2}^i \left[ (1 - \rho_i) \bar{\Pi} - \frac{\alpha^i \vartheta^i \sigma_\varepsilon^2}{\mu^i} Q \right]} E_{T-2}^i \left[ e^{-\alpha^i N_{T-2}^i \left\{ (1 + \rho_i) \Pi_{T-1} + \frac{\alpha^i \vartheta^i \sigma_\varepsilon^2}{\mu^i} (1 - \mu^i) N_{T-1}^i \right\}} \right].$$

At this point, we assume that each agent is characterised by an additional form of bounded rationality: agent  $i$  makes the simplifying assumption that in all future periods his counterpart, agent  $-i$ , will hold his per-capita fraction of the risky asset supply constant at  $\mu^{-i} Q$  (Barberis et al. 2018). Accordingly, using equation (2) of the main text, the objective function (34) is simplified to

$$(36) \quad \max_{N_{T-2}^i} \left\{ -e^{-\alpha^i [w_{T-2}^i + N_{T-2}^i ((1 + \rho_i + \rho_i^2) \Pi_{T-2} + (2 + \rho_i)(1 - \rho_i) \bar{\Pi} - \alpha^i \vartheta^i \sigma_\varepsilon^2 Q - P_{T-2})] + \frac{[\alpha^i (1 + \rho_i)]^2}{2} \vartheta^i \sigma_\varepsilon^2} \right\}.$$

Therefore, agent  $i$ 's first order condition implies

$$N_{T-2}^i = \frac{(1 + \rho_i + \rho_i^2) \Pi_{T-2} + (2 + \rho_i)(1 - \rho_i) \bar{\Pi} - \alpha^i \vartheta^i \sigma_\varepsilon^2 Q - P_{T-2}}{\alpha^i \vartheta^i (1 + \rho_i)^2 \sigma_\varepsilon^2}.$$

In a similar manner, agent  $i$ 's first order condition at time  $T - 3$  yields

$$N_{T-3}^i = \frac{(1 + \rho_i + \rho_i^2 + \rho_i^3) \Pi_{T-3} + (3 + 2\rho_i + \rho_i^2)(1 - \rho_i) \bar{\Pi} - \alpha^i \vartheta^i \sigma_\varepsilon^2 [1 + (1 + \rho_i)^2] Q - P_{T-3}}{\alpha^i (1 + \rho_i + \rho_i^2)^2 \vartheta^i \sigma_\varepsilon^2}.$$

Extending the above pattern up to 20 periods before the end of the vessels' economic activity and applying basic properties of geometric series, we obtain:

$$(37) \quad N_t^i = \frac{\frac{1 - \rho_i^{21}}{1 - \rho_i} (\Pi_t - \bar{\Pi}) + 21 \bar{\Pi} - X^i \sigma_\varepsilon^2 Q - P_t}{Y^i \sigma_\varepsilon^2}.$$

where:

$$(38) \quad \begin{cases} X^i = \left[ \frac{20}{(1 - \rho_i)^2} - \frac{(1 - \rho_i^{20})(1 + 2\rho_i - \rho_i^{20})}{(1 + \rho_i)(1 - \rho_i)^3} \right] \alpha^i \vartheta^i \\ Y^i = \left( \frac{1 - \rho_i^{20}}{1 - \rho_i} \right)^2 \alpha^i \vartheta^i \end{cases}.$$

Finally, to be consistent with the nature of the industry, we impose short-sale constraints for each investor type. Following Barberis et al. (2018) and adding the age subscript, equation (37) becomes

$$(39) \quad N_{5,t}^i = \max \left\{ \frac{\left( \frac{1 - \rho_i^{21}}{1 - \rho_i} (\Pi_t - \bar{\Pi}) + 21\bar{\Pi} - X_5^i \sigma_\varepsilon^2 Q - P_{5,t} \right)}{Y_5^i \sigma_\varepsilon^2}, 0 \right\}.$$

This corresponds to equation (8a) of the main text.

*6-Year Old Vessel.* Following the same procedure, it is straightforward to derive the demand functions for the 6-year old vessel,  $N_{6,t}^i$ . At this point, note that investors gradually realise that their beliefs about either the cash flow process and/or their competitors' strategy are inaccurate; that is, they learn from their misperception and, accordingly, try to correct it. In the context of this framework, however, we do not incorporate an explicit learning process for the following reason: if investors could directly correct and update their beliefs, the main observed regularities would not be reproduced by this environment as the value of the asset would be fast approaching its fundamental value in the benchmark rational case. As a result, there would be neither excess volatility nor the observed patterns related to second-hand activity in the market.

Accordingly, we adopt a rather indirect learning mechanism. Specifically, we assume that agents become more "suspicious" – or, equivalently, more risk averse – as the specific asset's age grows. This "suspicion" stems from the fact that they realise that the evolution of net earnings (and prices) does not evolve precisely in the way they expected in the previous period. As a result, agents indirectly respond by increasing the perceived risk associated with their investment. We model the update in agents' beliefs in a straightforward manner and assume that agent  $i$  at  $t$  increases the value of the perceived cash flow shock variance corresponding to the valuation of the 6-year old vessel,  $\vartheta_6^i \sigma_\varepsilon^2$ , compared to the one for the valuation of the 5-year old vessel at  $t - 1$ ,  $\vartheta_5^i \sigma_\varepsilon^2$ ; therefore,  $\vartheta_5^i < \vartheta_6^i$ . As a result, for a given  $t$ ,

investors related to different vessel-age classes have different beliefs about the variance of the error term.<sup>21</sup>

Thus, according to agent  $i$ , net earnings related to the valuation of the 6-year old vessel evolve as

$$\Pi_{t+1} = (1 - \rho_i)\bar{\Pi} + \rho_i\Pi_t + \varepsilon_{t+1}^i,$$

in which  $\rho_0 \leq \rho_c < \rho_e < 1$  and  $\varepsilon_{t+1}^i \sim N(0, \vartheta_6^i \sigma_\varepsilon^2)$ , *i.i.d.* over time, where  $0 < \vartheta_6^e < \vartheta_6^c$ . Despite their increased “suspicion”, however, agents remain irrational, since they still form biased forecasts of either the cash flow process or their competitors’ demand responses. Following the same procedure as for the 5-year old asset, agent  $i$ ’s time  $t$  demand for the 6-year old asset is

$$(40) \quad N_{6,t}^i = \max \left\{ \frac{\left( \frac{1 - \rho_i^{20}}{1 - \rho_i} (\Pi_t - \bar{\Pi}) + 20\bar{\Pi} - X_6^i \sigma_\varepsilon^2 Q - P_{6,t} \right)}{Y_6^i \sigma_\varepsilon^2}, 0 \right\},$$

where  $P_{6,t}$  refers to the time  $t$  price of the 6-year old vessel and

$$(41) \quad \begin{cases} X_6^i = \left[ \frac{19}{(1 - \rho_i)^2} - \frac{(1 - \rho_i^{19})(1 + 2\rho_i - \rho_i^{19})}{(1 + \rho_i)(1 - \rho_i)^3} \right] \alpha^i \vartheta_6^i \\ Y_6^i = \left( \frac{1 - \rho_i^{19}}{1 - \rho_i} \right)^2 \alpha^i \vartheta_6^i \end{cases}.$$

Note that agents adjust upwardly the perceived riskiness of the cash flow shock which implies that  $Y_6^i \sigma_\varepsilon^2 > Y_5^i \sigma_\varepsilon^2$ . Thus, the expected one-period net income for the 6-year old investment is scaled by a higher quantity compared to the respective 5-year old one.

### **B. Proposition and Corollaries**

*Proof of the Proposition.* This proof is based on Barberis et al. (2018). In order to prove the Proposition, it is convenient to define the aggregate demand at time  $t$  as  $N_{5,t} = \mu N_{5,t}^c + (1 - \mu) N_{5,t}^e$ , where the agent-specific demands are given by equation (8a). To begin with, we can directly observe

19. Apart from the economic intuition, this result is also an indirect implication of the model solution. Alternatively, we could have assumed that agent  $i$  becomes more risk averse, which would imply an increase of the CARA coefficient from period  $t - 1$  to  $t$ . Both methods yield exactly the same results. We impose this condition in order for the steady state equilibrium of our economy to be well-defined from a mathematical perspective. However, even if we did not impose this assumption, the steady state equilibrium restrictions would hold approximately, and our results would be essentially the same.

that the lower the price of the vessel, the higher the value of aggregate demand. On the other hand, demand can be equal to zero for a sufficiently high value of the vessel. Formally, aggregate demand is a continuous function of the vessel price,  $P_{5,t}$ .<sup>22</sup> Moreover, it is a strictly decreasing function of  $P_{5,t}$  (as a sum of strictly decreasing functions) with a minimum value of zero. Accordingly, since the market supply of vessels cannot be negative, there always exists a vessel price at which the aggregate demand for the risky asset at time  $t$  is equal to the aggregate supply of the vessel,  $Q$ . Due to monotonicity of the aggregate demand function, this price is unique. We call this value “market clearing price” or “equilibrium price” of the 5-year old vessel at each  $t$  and we denote it by  $P_{5,t}^*$ .

Accordingly, we define the price at which investor  $i$ 's short-sale constraint binds at time  $t$

$$(42) \quad \tilde{P}_{5,t}^i = \frac{1 - \rho_i^{21}}{1 - \rho_i} (\Pi_t - \bar{\Pi}) + 21\bar{\Pi} - X_5^i \sigma_\varepsilon^2 Q.$$

Since  $\frac{1 - \rho_i^{21}}{1 - \rho_i}$  is an increasing function of the perceived net earnings' persistence,  $\rho_i$ , there exists a net earnings threshold, denoted by  $\hat{\Pi}_5$  and given by

$$(43) \quad \hat{\Pi}_5 = \bar{\Pi} + \frac{(X_5^e - X_5^c) \sigma_\varepsilon^2 Q}{\frac{1 - \rho_e^{21}}{1 - \rho_e} - \frac{1 - \rho_c^{21}}{1 - \rho_c}},$$

such that

$$(44) \quad \Pi_t \leq \hat{\Pi}_5 \Leftrightarrow \tilde{P}_{5,t}^e \leq \tilde{P}_{5,t}^c.$$

Namely, when shipping net earnings are equal to or below this threshold, the cut-off price for extrapolators is equal to or lower to the one for conservatives and vice versa.

In order to simplify the illustration, we denote the highest and lowest cut-off prices at time  $t$  by  $\tilde{P}_{5,t}^1$  and  $\tilde{P}_{5,t}^0$ , respectively, so that  $\tilde{P}_{5,t}^1 \geq \tilde{P}_{5,t}^0$ . Furthermore, we define the aggregate demand when the price is equal to  $\tilde{P}_{5,t}^i$  as  $N_{\tilde{P}_{5,t}^i}$ . The fact that demand is strictly decreasing in vessel price implies

$$\tilde{P}_{5,t}^1 \geq \tilde{P}_{5,t}^0 \Leftrightarrow 0 = N_{\tilde{P}_{5,t}^1} \leq N_{\tilde{P}_{5,t}^0}.$$

20. As a sum of continuous functions. Notice that  $\max(f(x), 0)$  is continuous for all continuous  $f$  and in our case,  $f(P_{5,t})$  – which is given by plugging (8a) in equation (6) – is a continuous function of  $P_{5,t}$ .

Accordingly, we distinguish between two scenarios. First, assume that  $N_{\bar{P}_{5,t}^0} < Q$ , that is, aggregate demand at the lowest cut-off price at time  $t$  is lower than the market supply of vessels. Due to market clearing, however, total demand will adjust to be equal to total supply at each point in time. Therefore, aggregate demand at time  $t$ ,  $N_{5,t}$ , will increase and, accordingly, will become higher than  $N_{\bar{P}_{5,t}^0}$ . For demand to increase though, price must decrease beyond  $\bar{P}_{5,t}^0$  which is the lowest cut-off price at this point. In turn, this implies that the demand of the trader with the lowest cut-off price becomes positive as well. Hence, in this scenario, all traders in the market have strictly positive demand and both types of agents are active. Accordingly, substituting equation (8a) in the market clearing condition, (6) and rearranging for  $P_{5,t}$ , we obtain the equilibrium price of the vessel:

$$(45) \quad P_{5,t}^{*c+e} = 21\bar{\Pi} + \frac{\mu Y_5^e \frac{1-\rho_c^{21}}{1-\rho_c} + (1-\mu)Y_5^c \frac{1-\rho_e^{21}}{1-\rho_e}}{\mu Y_5^e + (1-\mu)Y_5^c} (\Pi_t - \bar{\Pi}) - \frac{\mu Y_5^e X_5^c + (1-\mu)Y_5^c X_5^e + Y_5^c Y_5^e}{\mu Y_5^e + (1-\mu)Y_5^c} \sigma_\varepsilon^2 Q.$$

This corresponds to equation (9b) of the main text.

Second, assume that  $N_{\bar{P}_{5,t}^1} \leq Q \leq N_{\bar{P}_{5,t}^0}$ .<sup>23</sup> It follows that the equilibrium price belongs in the interval defined by the lowest and the highest cut-off prices; that is,  $\bar{P}_{5,t}^0 \leq P_{5,t}^* \leq \bar{P}_{5,t}^1$ . Accordingly, in equilibrium, only the agents with the highest cut-off price will have strictly positive demand for the vessel. Intuitively, due to the market clearing condition, aggregate demand for the risky asset must equal aggregate supply. When the equilibrium price is lower than the highest cut-off price, the corresponding agents' demand becomes positive; thus, they are in the market. At the same time, the equilibrium price remains higher than the lowest cut-off price; hence, the corresponding agent type has zero demand and, in turn, stays out of the market. In conclusion, in this second scenario, only one type of agent is active in the market. Which type is this and, thus, the determination of the equilibrium price, depends on the prevailing market conditions.

21. The aggregate supply of the risky asset cannot be negative; thus, we cannot observe the scenario where  $Q < N_{\bar{P}_{5,t}^1}$  – since, by definition,  $N_{\bar{P}_{5,t}^1} = 0$ .

Specifically, when net earnings are below the threshold  $\widehat{\Pi}_5$ , then

$$\tilde{P}_{5,t}^0 = \tilde{P}_{5,t}^e \geq \tilde{P}_{5,t}^c = \tilde{P}_{5,t}^1 \Leftrightarrow N_{\tilde{P}_{5,t}^e} \leq N_{\tilde{P}_{5,t}^c}.$$

Namely, when market conditions are weak, and earnings are sufficiently low, the demand of extrapolators becomes zero and only conservatives have strictly positive demand. Therefore, for  $N_{5t}^e = 0$ , from equation (8a) along with the market clearing condition (6), we obtain the equilibrium price of the vessel in the scenario where only conservatives hold the risky asset

$$(46) \quad P_{5,t}^{*c} = 21\bar{\Pi} + \frac{1 - \rho_c^{21}}{1 - \rho_c} (\Pi_t - \bar{\Pi}) - \left[ X_5^c + \frac{Y_5^c}{\mu} \right] \sigma_\varepsilon^2 Q.$$

This equation corresponds to (10) of the main text. Furthermore, it is straightforward to find the critical point at which extrapolators exit the market. Namely, at this point, the short-sale constraint of extrapolators is binding; hence the equilibrium price of the market is given also by equation (42). Since, the equilibrium price at each  $t$  is unique, by equating (42) to (46), we can obtain the value of the net earnings variable at which extrapolators exit from the market,  $\Pi_5^e$ . Accordingly,

$$(47) \quad \Pi_5^e = \bar{\Pi} + \frac{(X_5^e - X_5^c - \frac{Y_5^c}{\mu}) \sigma_\varepsilon^2 Q}{\frac{1 - \rho_e^{21}}{1 - \rho_e} - \frac{1 - \rho_c^{21}}{1 - \rho_c}}.$$

As expected, since  $Y_5^c$  is positive,  $\Pi_5^e$  is lower than the threshold  $\widehat{\Pi}_5$ . This suggests that—depending on the sign of the fraction in condition (47)—even during unfavourable market conditions extrapolators can be present in the market. In other words, the higher the fraction of extrapolators, the more tolerant they are to low net earnings. From an economic point of view, this result is intuitive; the higher the fraction of extrapolators, the more difficult it becomes to be entirely driven out of the market, that is, to trade their aggregate holdings with the other part of the investor population.

In a similar manner, when net earnings are above the threshold  $\widehat{\Pi}_5$ , then

$$\tilde{P}_{5,t}^0 = \tilde{P}_{5,t}^c \geq \tilde{P}_{5,t}^e = \tilde{P}_{5,t}^1 \Leftrightarrow N_{\tilde{P}_{5,t}^c} \leq N_{\tilde{P}_{5,t}^e}.$$

Specifically, when market conditions are significantly favourable, the demand of conservatives becomes zero and only extrapolators have strictly positive demand. Therefore, for  $N_{5,t}^c = 0$ , from

equation 8a along with the market clearing condition (6), we obtain the equilibrium price of the vessel in the case where only extrapolators hold the risky asset

$$(48) \quad P_{5,t}^{*e} = 21\bar{\Pi} + \frac{1 - \rho_e^{21}}{1 - \rho_e} (\Pi_t - \bar{\Pi}) - \left[ X_5^e + \frac{Y_5^e}{1 - \mu} \right] \sigma_\varepsilon^2 Q.$$

This equation corresponds to (11) of the main text.

Following the same line of reasoning, i.e. equating (42) with (48), the value of the net earnings variable at which conservatives exit from the market,  $\Pi_t^c$ , is

$$(49) \quad \Pi_5^c = \bar{\Pi} + \frac{(X_5^e - X_5^c + \frac{Y_5^e}{1 - \mu}) \sigma_\varepsilon^2 Q}{\frac{1 - \rho_e^{21}}{1 - \rho_e} - \frac{1 - \rho_c^{21}}{1 - \rho_c}}.$$

Since  $Y_5^e$  is positive,  $\Pi_5^c$  is higher than  $\hat{\Pi}_5$ .

In conclusion, the necessary and sufficient conditions for agents to coexist in the market is

$$(50) \quad \Pi_5^e = \bar{\Pi} + \frac{(X_5^e - X_5^c - \frac{Y_5^c}{\mu}) \sigma_\varepsilon^2 Q}{\frac{1 - \rho_e^{21}}{1 - \rho_e} - \frac{1 - \rho_c^{21}}{1 - \rho_c}} < \Pi_t < \bar{\Pi} + \frac{(X_5^e - X_5^c + \frac{Y_5^e}{1 - \mu}) \sigma_\varepsilon^2 Q}{\frac{1 - \rho_e^{21}}{1 - \rho_e} - \frac{1 - \rho_c^{21}}{1 - \rho_c}} = \Pi_5^c.$$

Condition (50) corresponds to (9a) of the main text. Furthermore, (50) implies that when  $\Pi_t = \bar{\Pi}$  both agents are present in the market. ■

*Equilibrium Price for the 6-Year Old Vessel.* Extending the arguments illustrated above, it is straightforward to prove that a vessel age-specific market-clearing price always exists. Below, we state the equilibrium price conditions for the 6-year old vessel.

First, in the case where both agents are present in the market, that is, when

$$(51) \quad \Pi_6^e = \bar{\Pi} + \frac{(X_6^e - X_6^c - \frac{Y_6^c}{\mu}) \sigma_\varepsilon^2 Q}{\frac{1 - \rho_e^{20}}{1 - \rho_e} - \frac{1 - \rho_c^{20}}{1 - \rho_c}} < \Pi_t < \bar{\Pi} + \frac{(X_6^e - X_6^c + \frac{Y_6^e}{1 - \mu}) \sigma_\varepsilon^2 Q}{\frac{1 - \rho_e^{20}}{1 - \rho_e} - \frac{1 - \rho_c^{20}}{1 - \rho_c}} = \Pi_6^c,$$

the price is given by

$$(52) \quad P_{6,t}^{*c+e} = 20\bar{\Pi} + \frac{\mu Y_6^e \frac{1 - \rho_c^{20}}{1 - \rho_c} + (1 - \mu) Y_6^c \frac{1 - \rho_e^{20}}{1 - \rho_e}}{\mu Y_6^e + (1 - \mu) Y_6^c} (\Pi_t - \bar{\Pi}) - \frac{\mu Y_6^e X_6^c + (1 - \mu) Y_6^c X_6^e + Y_6^c Y_6^e}{\mu Y_6^e + (1 - \mu) Y_6^c} \sigma_\varepsilon^2 Q.$$

Second, when only conservatives hold the vessel, that is, when

$$(53) \quad \Pi_t \leq \bar{\Pi} + \frac{(X_6^e - X_6^c - \frac{Y_6^c}{\mu})\sigma_\varepsilon^2 Q}{\frac{1-\rho_e^{20}}{1-\rho_e} - \frac{1-\rho_c^{20}}{1-\rho_c}} = \Pi_6^e,$$

the price is given by

$$(54) \quad P_{6,t}^{*c} = 20\bar{\Pi} + \frac{1-\rho_c^{20}}{1-\rho_c} (\Pi_t - \bar{\Pi}) - \left[ X_6^c + \frac{Y_6^c}{\mu} \right] \sigma_\varepsilon^2 Q.$$

Third, in the scenario where only extrapolators hold the risky asset, that is, when

$$(55) \quad \Pi_6^c = \bar{\Pi} + \frac{(X_6^e - X_6^c + \frac{Y_6^e}{1-\mu})\sigma_\varepsilon^2 Q}{\frac{1-\rho_e^{20}}{1-\rho_e} - \frac{1-\rho_c^{20}}{1-\rho_c}} \leq \Pi_t,$$

the price equals

$$(56) \quad P_{6,t}^{*e} = 20\bar{\Pi} + \frac{1-\rho_e^{20}}{1-\rho_e} (\Pi_t - \bar{\Pi}) - \left[ X_6^e + \frac{Y_6^e}{1-\mu} \right] \sigma_\varepsilon^2 Q.$$

*Proof of Corollary 1.* From the Proposition and the definition of the “steady state” equilibrium, it is straightforward to derive equations (15a) and (15b) of the main text. Specifically, equation (8a) combined with the fact that in steady state  $\Pi_t = \bar{\Pi}$  result in

$$(57) \quad \bar{N}_5^i = \max \left\{ \frac{21\bar{\Pi} - X_5^i \sigma_\varepsilon^2 Q - \bar{P}_5^*}{Y_5^i \sigma_\varepsilon^2}, 0 \right\}.$$

Since, in the steady state, both agents coexist in the market, type  $i$ 's time  $t$  demand becomes

$$(58) \quad \bar{N}_5^i = \frac{21\bar{\Pi} - X_5^i \sigma_\varepsilon^2 Q - \bar{P}_5^*}{Y_5^i \sigma_\varepsilon^2}.$$

Substituting (58) in the market clearing condition (6), we obtain

$$(59) \quad \bar{P}_5^* = 21\bar{\Pi} - \frac{\mu Y_5^e X_5^c + (1-\mu) Y_5^c X_5^e + Y_5^c Y_5^e}{\mu Y_5^e + (1-\mu) Y_5^c} \sigma_\varepsilon^2 Q.$$

Moreover, both types hold the risky asset in analogy to their fraction of the total population if and only if

$$\bar{N}_5^i = \frac{21\bar{\Pi} - X_5^i \sigma_\varepsilon^2 Q - \bar{P}_5^*}{Y_5^i \sigma_\varepsilon^2} = Q \Leftrightarrow \bar{P}_5^* = 21\bar{\Pi} - (X_5^i + Y_5^i) \sigma_\varepsilon^2 Q.$$

However, since the steady state equilibrium price is unique

$$(60) \quad X_5^c + Y_5^c = X_5^e + Y_5^e = X_5^f + Y_5^f = \frac{\mu Y_5^e X_5^c + (1 - \mu) Y_5^c X_5^e + Y_5^c Y_5^e}{\mu Y_5^e + (1 - \mu) Y_5^c},$$

which corresponds to condition (15b) of the main text.

Similarly, condition (60) ensures that in steady state both agents are present in the market. Namely, the second terms of the right-hand side of conditions (47) and (49) are negative and positive, respectively; hence,  $\Pi_5^e < \bar{\Pi} < \Pi_5^c$ . Following the same procedure, we obtain the steady state equilibrium conditions for the 6-year old case. ■

*Agent and Age-Specific Parameters.* The steady state equilibrium conditions (15a) and (16a) imply that the model's parameters for the 5- and 6-year old vessels are nested. This interrelationship can be illustrated through the following system of equations

$$(61) \quad \alpha^i = \frac{21\bar{\Pi} - \bar{P}_5^*}{\left[ \frac{20 + (1 - \rho_i^{20})^2}{(1 - \rho_i)^2} - \frac{(1 - \rho_i^{20})(1 + 2\rho_i - \rho_i^{20})}{(1 + \rho_i)(1 - \rho_i)^3} \right] \vartheta_5^i \sigma_\varepsilon^2 Q},$$

and

$$\alpha^i = \frac{20\bar{\Pi} - \bar{P}_6^*}{\left[ \frac{19 + (1 - \rho_i^{19})^2}{(1 - \rho_i)^2} - \frac{(1 - \rho_i^{19})(1 + 2\rho_i - \rho_i^{19})}{(1 + \rho_i)(1 - \rho_i)^3} \right] \vartheta_6^i \sigma_\varepsilon^2 Q}. \quad (62)$$

The implications of this fact are analysed in the empirical estimation of the model.

*Trading Volume and Net Earnings.* Trading activity is measured as

$$(63) \quad V_t = \mu^i \left| \max \left\{ \frac{\frac{1 - \rho_i^{20}}{1 - \rho_i} (\Pi_t - \bar{\Pi}) + 20\bar{\Pi} - X_6^i \sigma_\varepsilon^2 Q - P_{6,t}}{Y_6^i \sigma_\varepsilon^2}, 0 \right\} - \max \left\{ \frac{\frac{1 - \rho_i^{21}}{1 - \rho_i} (\Pi_{t-1} - \bar{\Pi}) + 21\bar{\Pi} - X_5^i \sigma_\varepsilon^2 Q - P_{5,t-1}}{Y_5^i \sigma_\varepsilon^2}, 0 \right\} \right|.$$

Due to short-sale constraints, agent-specific demand functions are not strictly monotonic with respect to the net earnings variable in the entire  $\Pi_t$  domain; namely, strict monotonicity disappears whenever

the constraint binds. As a result, trading activity depends on the realisation of the net earnings variable between two consecutive dates,  $t - 1$  and  $t$ . In the following, we examine all possible scenarios.

In the first scenario, both agents are present in the market for two consecutive periods. Equivalently, conservative agents' demands for 5 and 6-year old vessels are positive. Incorporating the equilibrium prices from (45) at  $t - 1$  and (52) at  $t$  in equation (63) results in

$$(64) \quad V_t = \mu^i |A_6^i \Pi_t - A_5^i \Pi_{t-1} + (A_6^i - A_5^i) \bar{\Pi}|,$$

where

$$(65) \quad A_6^i \Pi_t - A_5^i \Pi_{t-1} + (A_6^i - A_5^i) \bar{\Pi} = N_{6,t}^i - N_{5,t-1}^i,$$

is agent  $i$ 's change in demand for the asset between periods  $t - 1$  and  $t$ . The agent-specific constants are given by

$$(66) \quad \begin{cases} A_5^c = \frac{(1 - \mu) \left( \frac{1 - \rho_c^{21}}{1 - \rho_c} - \frac{1 - \rho_e^{21}}{1 - \rho_e} \right)}{[\mu Y_5^e + (1 - \mu) Y_5^c] \sigma_\varepsilon^2} < 0 \\ A_6^c = \frac{(1 - \mu) \left( \frac{1 - \rho_c^{20}}{1 - \rho_c} - \frac{1 - \rho_e^{20}}{1 - \rho_e} \right)}{[\mu Y_6^e + (1 - \mu) Y_6^c] \sigma_\varepsilon^2} < 0 \end{cases}$$

and

$$(67) \quad \begin{cases} A_5^e = \frac{\mu \left( \frac{1 - \rho_e^{21}}{1 - \rho_e} - \frac{1 - \rho_c^{21}}{1 - \rho_c} \right)}{[\mu Y_5^e + (1 - \mu) Y_5^c] \sigma_\varepsilon^2} > 0 \\ A_6^e = \frac{\mu \left( \frac{1 - \rho_e^{20}}{1 - \rho_e} - \frac{1 - \rho_c^{20}}{1 - \rho_c} \right)}{[\mu Y_6^e + (1 - \mu) Y_6^c] \sigma_\varepsilon^2} > 0 \end{cases}.$$

Since trading volume in the market is the same irrespective of the agent type, in the following we examine this variable from the conservative agent's perspective. Accordingly, equation (64) becomes

$$(68) \quad V_t = \mu |A_6^c \Pi_t - A_5^c \Pi_{t-1} + (A_6^c - A_5^c) \bar{\Pi}|.$$

The second scenario is when both agents are present at time  $t - 1$  but conservatives exit at  $t$ , that is,  $N_{6,t}^c$  equals zero. Incorporating the equilibrium prices from (45) at  $t - 1$  and (56) at  $t$  in equation (63) yields

$$(69) \quad V_t = \mu |A_5^c(\Pi_{t-1} - \bar{\Pi}) + Q|.$$

In the third scenario, conservatives are not present in the market at time  $t - 1$  but both agent types are active at  $t$ . Proceeding in a similar fashion to before, equation (63) becomes

$$(70) \quad V_t = \mu |A_6^c(\Pi_t - \bar{\Pi}) + Q|.$$

The fourth scenario refers to the case where both agents are present in the market at time  $t - 1$  but extrapolators exit at  $t$ . In this case, trading activity is given by

$$(71) \quad V_t = \mu \left| A_5^c(\Pi_{t-1} - \bar{\Pi}) - \frac{(1 - \mu)}{\mu} Q \right|.$$

The fifth scenario is when only conservatives are present in the market at time  $t - 1$  but both types at  $t$ . Therefore, equation (63) becomes

$$(72) \quad V_t = \mu \left| A_6^c(\Pi_t - \bar{\Pi}) - \frac{(1 - \mu)}{\mu} Q \right|.$$

In the sixth (seventh) scenario, only agents of type  $i$  ( $-i$ ) are present in the market at time  $t - 1$  and only of type  $-i$  ( $i$ ) at  $t$ . In this case, (63) simplifies to

$$(73) \quad V_t = Q.$$

Furthermore, if in two consecutive dates agents  $i$  are out of the market, there is no trading activity.

Finally, if  $\mu = 0$ , or  $\rho_c = \rho_e$ , then there is no heterogeneity and the market clearing condition along with equations (8a) and (40) suggest that there is no activity in the second-hand market.

■