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Using inventory to mitigate the Ripple effect

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Abstract: A single disruption at a single location within a supply chain may effect many other locations of a the supply chain through the Ripple effect (Ivanov et al., 2014). In this article we focus on the role of inventory to mitigate the Ripple effect. Our main finding states that operational decisions (such as safety inventory level or service level) are highly interrelated with decisions that mitigate the Ripple effect (such as a high Resilience of the supply chain). Specifically, we find: 1) An increase in demand volatility may lead a firm to invest more in operational safety stock as well as in dedicated Risk Mitigation Inventory. 2) A high service level and high resilience of a supply chain can be conflicting objectives. Copyright © 2019 IFAC

1. INTRODUCTION

A single disruption at a single location within a supply chain may effect many other locations through the Ripple effect (Ivanov et al., 2014). In this article we want to discuss various levers that mitigate the Ripple effect. Broadly speaking there are two kind of operational risk mitigation levers available to a firm: (i) to hold additional capacity at a reliable source, so called reserve capacity (Lücker et al., 2018b) and (ii) to hold inventory that can be used to meet demand in the event of a disruption. In this article we want to further specify inventory as a risk mitigation lever. A broad stream of literature assumes that firms hold so-called Risk Mitigation Inventory (RMI). RMI is designed to be used to meet customer demand in the event of a supply disruption (Lücker et al., 2018a). As it is not designed to be used to mitigate demand uncertainty, RMI is different from safety inventory and is often held in addition to safety inventory. However, it is important to recognize that any available inventory will be used in the event of a disruption to supply the disruption demand. In particular, safety inventory and cycle inventory (if available) will be used as risk mitigation levers even though they are not designed a priori to serve in the event of a disruption.

Pharmaceutical firms often produce large batch seizes of the active pharmaceutical ingredient (API) of a drug. Such batches may supply more than a year of demand for the API. Clearly, in the event of a disruption, the pharmaceutical firm would use all available cycle inventory to mitigate the disruption.

With this explorative research we want to better understand the roles of safety and cycle inventory on the optimal risk mitigation strategy.

To illustrate our insights, this analysis focuses on a single echelon where inventory is replenished according to the popular (Q, R) policy. We conduct extensive numerical ex-periments to find that operational decisions (such as safety inventory level or service level) are highly interrelated with decisions that mitigate the Ripple effect (such as a high Resilience of the supply chain). Specifically, we find:

(1) An increase in demand volatility may lead the firm to invest more in operational safety stock as well as in RMI.
(2) A high service level and high resilience of a supply chain can be conflicting objectives.

Finding 1 reveals the counter-intuitive result that the operational safety stock and RMI can sometimes be complementary as RMI and operational safety stock may both be increasing in demand volatility. Finding 2 underlines that a resilient supply chain may not necessarily operate at a high service level. This is interesting because such a supply chain may be capable to deal with severe supply disruptions, but not necessarily with demand uncertainty.

2. RELATED WORK

The research topic of supply chain risk management has recently been explored by Ivanov and Dolgui (2018) who suggest to emphasize uncertainty in the supply chain rather than certain assumptions. In this line of research the authors identify key characteristics of these low-certainty-need supply chains.

The ripple effect was introduced by Ivanov et al. (2014). This effect suggests that the impact of a disruption propagates across an entire supply chain, affecting performance metrics along various dimensions. In (Ivanov et al., 2019) the authors combine findings on the ripple effect with supply chain digitization.

The work of Scheibe and Blackhurst (2018) studies disruption propagation in multi-tier supply chains. Based on grounded theory, the authors identify key themes that are relevant for decision-makers when being exposed to supply disruption risk. Further related literature focuses on

resilience in the procurement process. Based on developing resilience-based supplier selection criteria, Hosseini and Barker (2016) proposes a new Bayesian network paradigm.

3. METHODS

Let us briefly restate key elements of the (Q, R) policy: This continuous-review policy is based on ordering Q units when on-hand inventory hits a reorder point R. Key assumptions include that demand is stochastic and stationary and that the lead time T for placing an order is positive and constant. Although exact solutions are known, often heuristics are applied. Hadley and Whitin (1963) find an iterative heuristic solution, where the inventory holding costs h are approximated and where they assume that the lead time demand does not exceed the order quantity Q. This heuristic is based on the expected average annual cost given by

$$G(Q,R) = h\left(\frac{Q}{2} + R - \lambda T\right) + K\frac{\lambda}{Q} + \left(h + \frac{p\lambda}{Q}\right)n(R), \quad (1)$$

where λ is the demand rate, K the setup cost for placing an order, and p the backlog cost per unit of unsatisfied demand. The expected number of shortages that occur in one cycle is

$$n(R) = \int_{R}^{\infty} (x - R)f_T(x)dx, \quad (2)$$

where $f_T(x)$ describes the probability density function of the lead time demand. The operational safety stock SS is

$$SS = R - \lambda T. \quad (3)$$

Let us extend this policy to include disruptions by combining the total cost function for a disruption with the total cost function for the (Q, R) policy:

$$\hat{G}(Q,R,a) = G(Q,R)\left(1 - \omega_\tau\right)$$
$$+ \omega_\tau \int_{R}^{Q+R} \left(p \int_{I+a\tau}^{\infty} (x - I - a\tau)f_\tau(x)dx \right.$$
$$+ h \int_{0}^{I} (I - x)f_\tau(x)dx$$
$$+ c_A \int_{I}^{I+a\tau} (x - I)f_\tau(x)dx$$
$$\left. + c_A a\tau\left(1 - F_\tau(I + a\tau)\right) \right)\xi(I)dI$$
$$+ \hat{c}_A a. \quad (4)$$

In above total cost function, the first term $G(Q,R)\left(1 - \omega_\tau\right)$ represents the expected cost function if no disruption occurs (costs for the undisrupted (Q, R) policy). No disruption occurs with probability $(1 - \omega_\tau)$. The second term represents the the expected costs if a disruption occurs. The disruption probability is ω_τ. We refer to the literature (Lücker et al., 2018b) for a detailed discussion of the expected costs in the event of a disruption. We note

that the first of these terms represents the penalty cost, the second the RMI holding costs, the third and fourth the reserve capacity production costs. The last term represents the reserve capacity reservation cost and is always incurred regardless of whether a disruption takes place. $\xi(I)$ represents the probability distribution density for the inventory level I (e.g. on-hand inventory minus stockouts).

Using the truncated normal distribution we can rewrite the terms as follows:

$$\hat{G}(Q,R,a) = G(Q,R)\left(1 - \omega_\tau\right)$$
$$+ \omega_\tau \frac{1}{Q} \int_{R}^{Q+R} \left(p\left(\mu_\tau + \sigma_\tau \frac{f_\tau\left(\frac{I+a\tau-\mu_\tau}{\sigma_\tau}\right)}{1 - F_\tau\left(\frac{I+a\tau-\mu_\tau}{\sigma_\tau}\right)}\right) \right.$$
$$+ h\left(\mu_\tau + \sigma_\tau \frac{f_\tau\left(\frac{-\mu_\tau}{\sigma_\tau}\right) - f_\tau\left(\frac{I-\mu_\tau}{\sigma_\tau}\right)}{F_\tau\left(\frac{I-\mu_\tau}{\sigma_\tau}\right) - F_\tau\left(\frac{-\mu_\tau}{\sigma_\tau}\right)}\right)$$
$$+ c_A\left(\mu_\tau + \sigma_\tau \frac{f_\tau\left(\frac{I-\mu_\tau}{\sigma_\tau}\right) - f_\tau\left(\frac{I+a\tau-\mu_\tau}{\sigma_\tau}\right)}{F_\tau\left(\frac{I+a\tau-\mu_\tau}{\sigma_\tau}\right) - F_\tau\left(\frac{I-\mu_\tau}{\sigma_\tau}\right)}\right)$$
$$\left. + c_A a\tau\left(1 - F_\tau(I + a\tau)\right) \right)dI + \hat{c}_A a. \quad (5)$$

To further evaluate this objective function, we perform a numerical analysis of key parameters with MATLAB (fmincon optimizer).

We measure supply chain resilience with the risk measure conditional value at risk $CVaR_\alpha$ with threshold α. We consider two types of uncertainty in our supply chain: demand uncertainty and inventory uncertainty (only an uncertain amount of the total cycle inventory can be used in the event of a disruption). $CVaR$ is then given by the expected shortages beyond a threshold. A low expected shortfall ($CVaR$) corresponds to a high supply chain resilience, whereas a high expected shortfall ($CVaR$) corresponds to a low supply chain resilience.

4. EXPERIMENTAL RESULTS

In the following we provide numerical results for the extended $(\hat{Q}, \hat{R})$ policy. We focus on realistic scenarios with parameters that were aligned with the pharmaceutical company: $K = 100$, $\lambda = 1$, $p = 40$, $h = 1$, $T = 1$, $c_A = 20$, $\hat{c}_A = 2$, $\tau = 10$, $\omega_\tau = 0.05$, $\alpha = 0.95$ and a normally distributed demand with $\mu = 1$ and $\sigma = 0.3$. Throughout the experiments we find consistent results along a wide range of parameter choices that support the observations below. We analyze how the decision variables $\hat{Q}$, $\hat{R}$ and a^* as well as $CVaR_\alpha$ depend on key model parameters.

First, we describe the behavior of our decision variables and $CVaR_\alpha$ in various cost parameters:

Proposition 1. Sensitivity in various cost parameters:

- $\hat{Q}$ increases in c_A, $\hat{c}_A$ and may decrease or increase in h, p.
- $\hat{R}$ decreases in h and increases in c_A, $\hat{c}_A$, p.
- a^* decreases in c_A, $\hat{c}_A$ and increases in h, p.
- $CVaR_\alpha$ increases in h, c_A and $\hat{c}_A$, but may increase or decrease in p.

	Coefficient of variation CV:				
	$CV = 0.1$	$CV = 0.15$	$CV = 0.2$	$CV = 0.25$	$CV = 0.3$
R	1.08	1.13	1.17	1.21	1.25
$\hat{R}$	1.19	1.28	1.37	1.45	1.54
Q	13.57	13.52	13.47	13.43	13.40
$\hat{Q}$	10.04	10.07	10.09	10.11	10.14

Table 1. Sensitivity of R and $\hat{R}$ in the coefficient of variation of demand CV

These observations are in line with the Hadley-Whitin (Q, R) policy.

Next, we elaborate on the role of the coefficient of variation of demand CV:

Proposition 2. Sensitivity in the coefficient of variation of demand CV:

- $\hat{Q}$ decreases in CV.
- $\hat{R}$ and $CVaR_\alpha$ increase in CV.
- a^* may decrease or increase in CV.

Table 1 shows how $\hat{R}$ increases in CV. We also display the increase in the reorder point R from the Hadley-Whitin (Q, R) policy. Interestingly, the reorder point increases even more strongly in the case of a disruption ($\hat{R}$ increases by 29.8% over the range $CV = 0.1$ to $CV = 0.3$) than without disruptions (R increases by 15.3% over the range $CV = 0.1$ to $CV = 0.3$). Since the lead time is constant, the increase in R entails an increase in the safety stock. At the same time, in the case of a disruption, $\hat{R}$ increases more strongly, indicating that safety stock and RMI can be complements rather than substitutes.

Table 1 also shows that $\hat{Q}$ increases in CV, whereas Q is approximately constant in CV.

In contrast to previous analytical results (Lücker et al., 2018b), the reserve capacity production rate a^* may decrease in the coefficient of variation of demand CV. As demand volatility increases, more safety stock is built up. Hence, a lower reserve capacity production rate is optimal.

Next, we elaborate on the role of disruption probability ω_τ:

Proposition 3. Sensitivity in the disruption probability ω_τ:

- $\hat{Q}$, a^* and $CVaR_\alpha$ may increase or decrease in ω_τ.
- $\hat{R}$ increases in ω_τ.

In order to provide intuition behind this Proposition, we plot how the batch size (Figure 1), the reorder point (Figure 2), the reserve capacity production rate (Figure 3), and $CVaR_\alpha$ (Figure 4) depend on the disruption probability ω_τ. In Figure 1 we observe that $\hat{Q}$ varies strongly with the disruption probability ω_τ. Clearly, as the disruption probability increases, it is helpful to hold more inventory in the supply chain. As long as the disruption probability remains low (below 6%) it is cheaper to substantially increase the batch size rather than substantially increasing safety inventory. However, once we pass a threshold (6% in the numerical example), it becomes cost-efficient to substantially increase the reorder point (see Figure 2) because RMI and safety inventory provide the best buffer against the very likely disruptions (rather than the highly variable
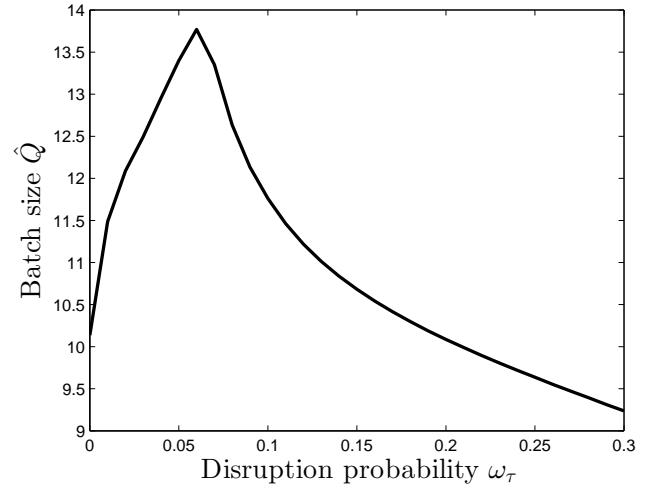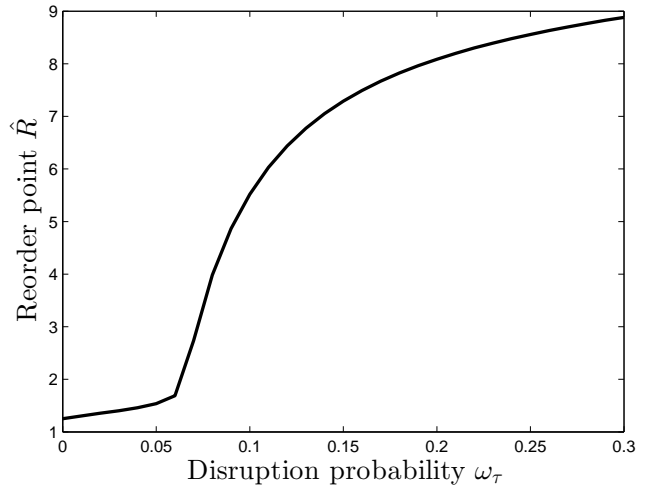

Fig. 1. Batch size $\hat{Q}$

Fig. 2. Reorder point $\hat{R}$

cycle inventory). However, since it is optimal to hold large amount of RMI and safety inventory, it is optimal to reduce the batch size for high disruption probabilities. This discussion shows that supply chain resilience can effectively be improved by not only holding RMI and safety inventory, but also by setting optimal batch sizes. These results contrast with the practice of some companies, which focus exclusively on stocking RMI without taking the batch size into account.

Regarding the reserve capacity, we observe that the reserve capacity is switched on only for disruption probabilities of $\omega_\tau \geq 0.2$. Then a^* increases in ω_τ until it reaches its maximum at $\omega_\tau \approx 0.06$ because low probability disruptions are better mitigated with reserve capacity than inventory. Once we pass the threshold of $\omega_\tau \approx 0.06$, it is optimal to reduce the reserve capacity because it is cheaper to mitigate disruptions with inventory rather than reserve capacity.

The dependency of $CVaR_\alpha$ on the disruption probability ω_τ can be explained as follows. $CVaR_\alpha$ increases in the disruption probability ω_τ until $\omega_\tau \approx 0.02$ as the
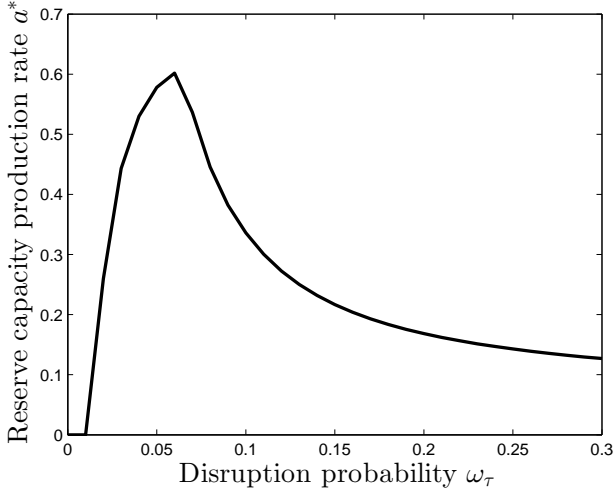
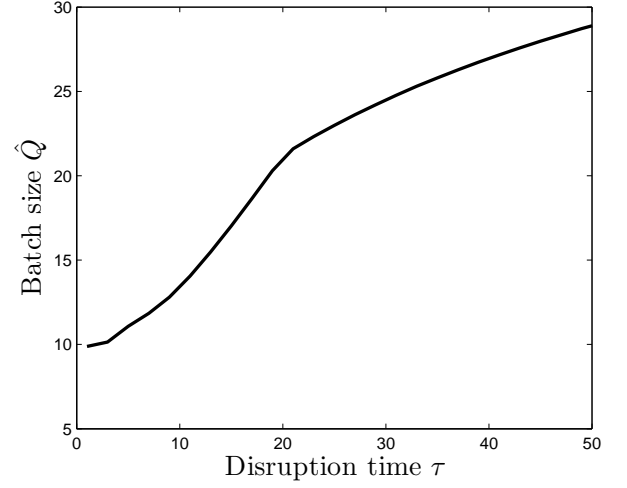Fig. 3. Reserve capacity production rate a^*
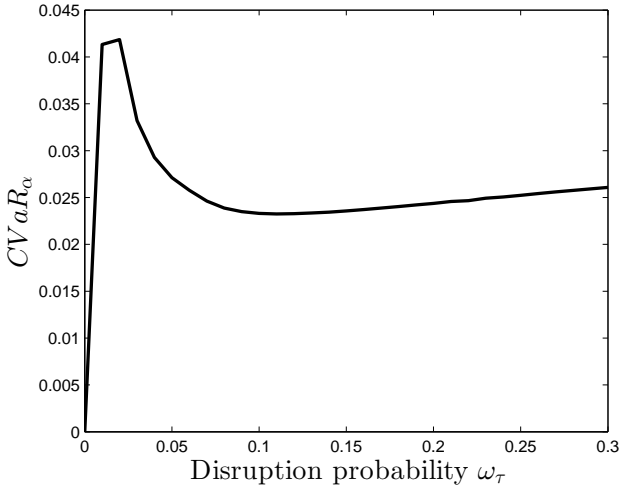

Fig. 5. Batch size $\hat{Q}$

Fig. 4. $CVaR_\alpha$ for $\alpha = 0.95$

Fig. 6. Reorder point $\hat{R}$

likelihood of not being able to supply in the case of a disruption increases. We have the maximum $CVaR_\alpha \approx 0.042$. Afterwards $CVaR_\alpha$ decreases in ω_τ until $\omega_\tau = 0.1$ as more RMI and reserve capacity are built up. For $\omega_\tau > 0.1$ $CVaR_\alpha$ increases slightly in ω_τ.

Next, we elaborate on the role of the disruption time τ:

Proposition 4. Sensitivity in the disruption time τ:

- $\hat{Q}$ and $\hat{R}$ increase in τ.
- a^* and $CVaR_\alpha$ may increase or decrease in τ.

In order to provide intuition behind this Proposition, we plot how the batch size (Figure 5), the reorder point (Figure 6), the reserve capacity production rate (Figure 7), and $CVaR_\alpha$ (Figure 8) depend on the disruption time τ. We find that the batch size $\hat{Q}$ and reorder point $\hat{R}$ increase in τ throughout the parameter range. It is very intuitive that more inventory of any kind is needed when disruptions tend to last longer. Note that when disruptions tend to be short ($\tau < 20$), it is optimal to increase the reserve capacity substantially, whereas the reorder point increases only slightly because it is more cost-efficient
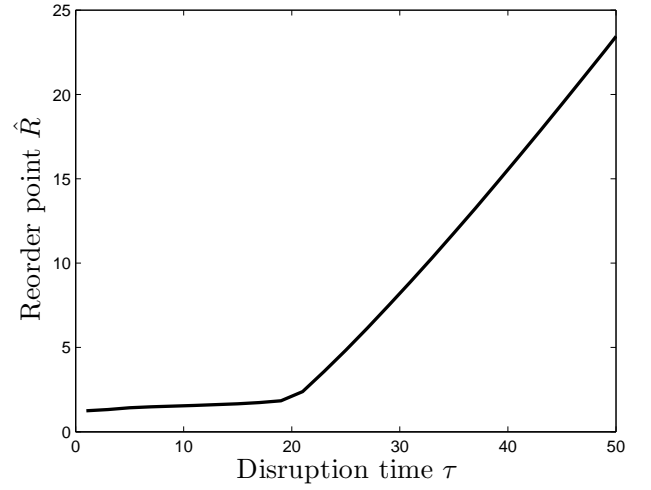
mitigating short disruptions using reserve capacity rather than inventory. In contrast, when disruptions tend to be longer ($\tau > 20$), the reorder point increases whereas reserve capacity decreases. Note that these results are in line with the analytical results of Lücker et al. (2018b).

$CVaR_\alpha$ increases in τ until $\tau = 5$ as the potential loss increases'. Afterwards $CVaR_\alpha$ decreases in τ because the potential total penalty cost increases, and it therefore pays off for the firm to invest in RMI and reserve capacity.

Finally, we elaborate on the role of the fixed costs K:

Proposition 5. Sensitivity in the fixed costs K:

- $\hat{Q}$ increases in K.
- $\hat{R}$ and a^* decrease in K.
- $CVaR_\alpha$ may increase or decrease in K.

As with the Hadley-Whitin (Q, R) policy, the batch size increases in K and the reorder point decreases in K. The decrease in $\hat{R}$ also entails a decrease in the service level as the lead time is kept constant. Given that $CVaR_\alpha$ may increase or decrease, keeping a high service level and high supply chain resilience as measured by $CVaR_\alpha$ can be
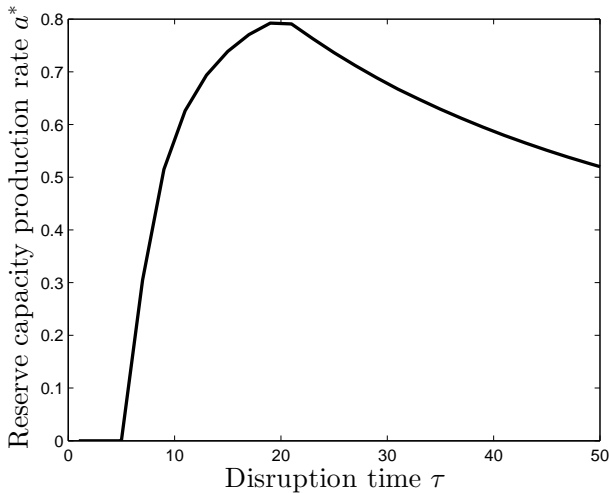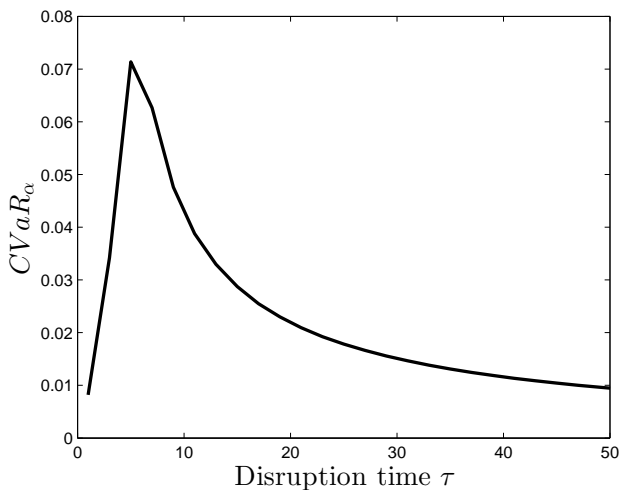
Fig. 7. Reserve capacity production rate a^*

Fig. 8. $CVaR_\alpha$ for $\alpha = 0.95$

conflicting objectives. In particular, keeping a high service level does not necessarily entail high supply chain resilience as measured by $CVaR_\alpha$.

5. CONCLUSIONS

This research suggests that operational decisions such as batch size or safety inventory levels are highly interrelated with decisions that deal with disruption risk (RMI and reserve capacity).

As avenues for future research we suggest to develop further insights on how these dependencies affect each other in more complex settings (multi-echelon, multi-product supply chains) and to provide a more rigorous modeling foundation.

REFERENCES

Hadley, G. and Whitin, T.M. (1963). *Analysis of inventory systems*. Prentice-Hall, Englewood Cliffs, NJ.

Hosseini, S. and Barker, K. (2016). A bayesian network model for resilience-based supplier selection. *International Journal of Production Economics*, 180, 68 – 87.

Ivanov, D., Sokolov, B., and Dolgui, A. (2014). The ripple effect in supply chains: trade-off 'efficiency-flexibility-resilience' in disruption management. *International Journal of Production Research*, 52, 2154–2172.

Ivanov, D. and Dolgui, A. (2018). Low-certainty-need (lcn) supply chains: a new perspective in managing disruption risks and resilience. *International Journal of Production Research*, 0(0), 1–18. doi:10.1080/00207543.2018.1521025. URL https://doi.org/10.1080/00207543.2018.1521025.

Ivanov, D., Dolgui, A., and Sokolov, B. (2019). The impact of digital technology and industry 4.0 on the ripple effect and supply chain risk analytics. *International Journal of Production Research*, 57(3), 829–846. doi:10.1080/00207543.2018.1488086. URL https://doi.org/10.1080/00207543.2018.1488086.

Lücker, F., Chopra, S., and Seifert, R. (2018a). Mitigating product shortages due to disruptions in multi-stage supply chains. *Working paper, Swiss Federal Institute of Technology EPFL, Lausanne*.

Lücker, F., Seifert, R., and Biçer, I. (2018b). Roles of inventory and reserve capacity in mitigating supply disruption risk. *International Journal of Production Research*.

Scheibe, K.P. and Blackhurst, J. (2018). Supply chain disruption propagation: a systemic risk and normal accident theory perspective. *International Journal of Production Research*, 56(1-2), 43–59. doi:10.1080/00207543.2017.1355123. URL https://doi.org/10.1080/00207543.2017.1355123.