Product Line Design*

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December 24, 2014

Abstract

We characterize the product line choice and pricing of a monopolist from the upper envelope of net marginal revenue curves to the individual product demand functions. The equilibrium product line constitutes those varieties yielding the highest upper envelope. In a generalized vertical differentiation framework, the equilibrium line is exactly the same as the first-best socially optimal line. These upper envelope and first-best optimal line findings extend to symmetric Cournot oligopoly.

Keywords: Product line design, product differentiation, second-degree price discrimination, product line pricing, Cournot multi-product competition.
JEL classification: L12, L13, L15.

*We thank participants at the Berlin IO Day (March, 2014) for comments. In particular, thanks to Mark Armstrong for alerting us to the work of Itoh (1983) and to the recent work of Johnson and Myatt (2014). We also express gratitude to David Myatt, Justin Johnson, an Associate Editor and three anonymous referees at the Journal of Economic Theory for extremely helpful comments and suggestions. Fang Guo provided excellent research assistance. Simon Anderson thanks the NSF for support. Levent Celik thanks the Czech Science Foundation for support (GACR #15-22540S). All errors are our own.

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1 Introduction

Economic analysis typically describes single-product firms even though almost all actual firms offer many varieties in the same product class. Our contribution is two-fold. First, we provide a surprisingly simple characterization of the pricing of a monopolist's product line using marginal revenue curves. The monopolist prices its product line (and segments consumers) following the upper envelope of marginal revenue curves to the individual product demand functions. This pricing characterization determines the varieties to include in a product line as those in the upper envelope of marginal revenue.

Second, we show that, in the general Mussa-Rosen (1978) vertical differentiation framework, the equilibrium product range is the same as the first-best socially optimal range. This is because the upper envelope of demands corresponds exactly with the upper envelope of marginal revenues, even though the sets of consumers assigned to each variant can be quite different at the two solutions. Our main monopoly results extend cleanly to a symmetric Cournot oligopoly.

Our analysis builds on Itoh (1983) and Johnson and Myatt (2003, 2006), and it complements Johnson and Myatt (2014).[1] Itoh (1983) analyzes the effects on product prices of introducing a new variety in a standard Mussa-Rosen framework. We provide a simpler derivation of his results while extending them to more general preferences. To do so, we deploy an "upgrades" approach (which was implicit in the work of Itoh), whereby a product variety can be seen as a base product plus a series of upgrades corresponding to each additional higher quality variant in the range. Then, each upgrade can be associated to a price premium. Such an upgrades approach was pioneered by Johnson and Myatt (2003), who emphasized its usefulness in analyzing product line and pricing choices of multi-product firms in monopoly and Cournot duopoly (incumbent-entrant) contexts. Johnson and Myatt (2006) used the same approach to characterize product line choices for an n-firm multi-product Cournot oligopoly.

Mussa and Rosen (1978) recognized that the monopolist sets the net marginal revenue of each upgrade to zero: "the optimal assignment equates the marginal cost and marginal revenue of increments of quality" (1978, p.311).[2] We implement this insight in our graphical treatment of the monopolist's solution by noting that setting the net marginal revenue of a quality increment

[1] We also contribute to the resurgent research interest in price discrimination (e.g., Aguirre, Cowan and Vickers, 2010, Anderson and Dana, 2009, and Armstrong, 2013). See Armstrong (2006) for a survey of recent studies.

[2] We thank a referee for this quote and for a very perceptive take on the literature, on which we have drawn heavily.

to zero is equivalent to setting the net marginal revenue of adjacent qualities equal. This means that equilibrium prices can be identified from the intersections of the net marginal revenue curves, which implies that the equilibrium product line is determined from the upper envelope of the net marginal revenue curves: a variant with a dominated net marginal revenue will not be used.

Johnson and Myatt (2014) clarify some of Itoh's results and extend them to Cournot oligopoly, engaging the set-up in Johnson and Myatt (2003, 2006). In particular, they highlight Itoh's finding that the price of each variety is set equal to its stand-alone profit-maximizing price when the individual demand curves are ρ-linear. They then show that this property extends to Cournot oligopoly. They also show that equilibrium prices are often close to prices in stand-alone single product markets under other reasonable specifications. We draw heavily on their insights on the "upgrades" approach and on how to carry over the monopoly results to symmetric oligopoly. We also deliver, for both monopoly and Cournot oligopoly, a welfare equivalence between market and socially optimal provision of varieties for a general class of Mussa-Rosen preferences.

2 Model

Consider a single firm (M) with a set $N = \{1, 2, ..., n\}$ of possible variants to offer. There is a unit mass of consumers. Each will buy at most one unit of the product, choosing the variant that yields the greatest positive utility (otherwise, she does not purchase). M's problem is to determine the variants to offer and their prices, under the constraint that consumers self-select. We rule out weakly dominated variants: those with no sales are excluded from the product line.

We describe each consumer by a taste θ, distributed over $[0, 1]$ according to a twice differentiable c.d.f. $F(\cdot)$, and a corresponding willingness-to-pay $u_i(\theta)$ for variety $i \in N$. We order consumers such that $u_i'(\theta) > 0$ for all $i \in N$; i.e., if consumer θ has a higher willingness-to-pay for variety i than consumer θ', then she has a higher willingness-to-pay for all other varieties as well.

We call $P_i(q) = u_i\left(F^{-1}(1-q)\right)$ the conditional stand-alone inverse demand functions. Here, $P_i(q)$ is the price that leaves the consumer with a taste $\theta = F^{-1}(1-q)$ indifferent between buying and not. We label potential varieties so that a lower index implies a higher $P_i(0)$ (this means that the consumer $\theta = 1$ likes 1 best and n the least, etc.) Thus the inverse demand curves are ranked and labeled by their vertical intercepts.

We will impose the following single-crossing condition: for any two varieties i and j, $P_i(q) -$

$P_j(q)$ is monotonic in q. Hence, if $P_i(q)$ crosses $P_j(q)$ from above, and both varieties are commonly priced below the intersection price, then all consumers right of the intersection would buy j over i, and consumers left of the intersection would buy i over j. Define by $MR_i(q)$ the marginal revenue curve to the inverse demand curve $P_i(q)$:

$$MR_i(q) = P_i(q) + qP_i'(q).$$

Assume that each inverse demand curve $P_i(q)$ is strictly (-1)-concave to ensure that the corresponding $MR_i(q)$ is strictly downward sloping when $MR_i(q) \geq 0$. We also assume that any two MR curves can intersect at most once, and that $MR_i(1) \leq 0$ for all $i \in N$ (so that M does not serve the whole market). We assume zero production costs: the analysis readily extends to asymmetric constant marginal costs, rephrased in terms of net demand and net marginal revenue curves (see the CEPR Working Paper for details).

This setup covers a wide range of preferences, including Mussa-Rosen and many others. For instance, Itoh (1983) and Johnson and Myatt (2014) use a multiplicative specification $u_i(\theta) = s_i\theta$, where s_i is naturally interpreted as the quality of variety i as per Mussa and Rosen (1978). Then M faces a (stand-alone) demand $1 - F(p/s_i)$ at a price p. Inverting this for p gives the maximum price M can charge to sell q units of variety i as $P_i(q) = s_i F^{-1}(1-q)$, so inverse demands are multiplicative shifts. Our base specification does not require multiplicative preferences.

We now turn to determination of the equilibrium product line. We define quantities cumulatively and denote by q_j the cumulative amount of varieties $1, ..., j$ sold. In other words, q_1 is the quantity sold of variety 1 only, q_2 is q_1 plus the quantity sold of variety 2, q_3 is q_2 plus the quantity sold of variety 3, and so on. Thus, when M charges p_i for variety i and all varieties are consumed in strictly positive quantities, M's profit is

$$\pi = p_1 q_1 + p_2(q_2 - q_1) + ... + p_n(q_n - q_{n-1}).$$

We can alternatively rewrite π as

$$\pi = (p_1 - p_2) q_1 + (p_2 - p_3) q_2 + ... + p_n q_n.$$

Here we can think of variety n as the base product that all served consumers buy at p_n. Variety $n-1$ is an upgrade purchased at a premium $p_{n-1} - p_n$ by all but the last bracket of consumers, etc. Implicit in Itoh (1983), this "upgrades" approach to product line determination was first crystallized by Johnson and Myatt (2003).

Prices that support the cumulative quantities must obey n incentive compatibility constraints, which define the switch-points $q_1, q_2, ..., q_n$. Namely, $q_1, ..., q_{n-1}$ must satisfy the surplus equality:

$$P_i(q_i) - p_i = P_{i+1}(q_i) - p_{i+1}, \text{ for } i = 1, ..., n-1. \tag{1}$$

Hence, the q_1 highest consumer types get more surplus from variety 1 than variety 2 while the consumer corresponding to q_1 is indifferent between the two, and so on up to variety $n-1$. The last constraint says that the final $q_n - q_{n-1}$ consumers should prefer variety n to non-purchase, with the consumer at q_n being indifferent:

$$P_n(q_n) - p_n = 0. \tag{2}$$

These constraints enable us to uncover the key structure of the problem and envisage its simple solution. Incorporating them into the profit function, we are back to quantities:

$$\pi = (P_1(q_1) - P_2(q_1)) q_1 + (P_2(q_2) - P_3(q_2)) q_2 + \cdots + P_n(q_n) q_n. \tag{3}$$

Thus, quantities enter the profit function additively in separate terms. The choice of q_n maximizes base profits earned on all consumers served, the choice of q_{n-1} maximizes the incremental profits earned on those who purchase the first upgrade, and so on. At any interior solution, using the marginal revenue notation, we reach the following n first-order conditions:

$$MR_i(q_i) = MR_{i+1}(q_i), \text{ for } i = 1, ..., n-1, \tag{4}$$

$$MR_n(q_n) = 0. \tag{5}$$

This is a remarkably simple and intuitive characterization of a monopolist's pricing problem of its product line. Graphically, it suffices to find where the marginal revenue curves cross to determine the quantities, with the last crossing at zero determining the total number of consumers served. The prices that support these quantities are given by the incentive compatibility constraints. First, given q_n, constraint (2) determines the price on variety n, $p_n = P_n(q_n)$: this price depends on no other variety. The knowledge of p_n together with q_{n-1} then gives, via constraint (1), the price premium $p_{n-1} - p_n = P_{n-1}(q_{n-1}) - P_n(q_n)$. This process is repeated for the remaining prices. Figure 1 illustrates for $n = 3$.

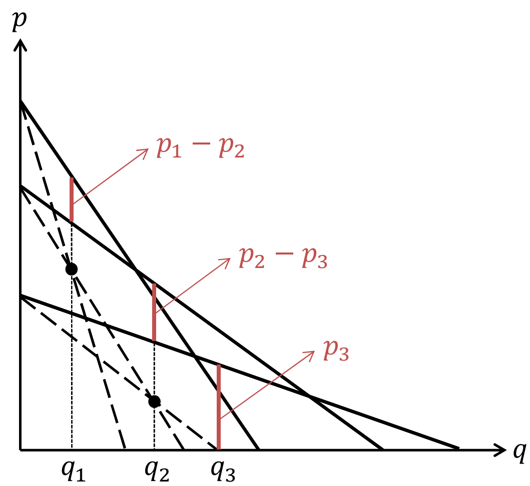

Figure 1
Equilibrium as the intersection of MR curves and recursive determination of prices (n=3)

If two successive equations in the above system (say, i^{th} and $(i+1)^{st}$) produce an outcome $q_i \geq q_{i+1}$, then variety $i+1$ will not be offered in equilibrium. Graphically, this happens when MR_{i+1} is not part of the upper envelope. Thus, for all n varieties to be offered in strictly positive quantities in equilibrium, each MR_i must be the highest MR curve for some interior quantity. Put differently, M will pick its product line according to the upper envelope of the marginal revenue curves. We summarize these in the following proposition.

Proposition 1 *M will include variety $i \in N$ in its product line if and only if*

$$MR_i(q) > \max_{j \neq i} MR_j(q) \text{ for some } q \in \left(0, MR_i^{-1}(0)\right).$$

Relabeling the set of varieties included in the product line as $N^* = \{1, 2, ..., n^*\}$, M will choose quantities by $MR_{n^*}(q_{n^*}) = 0$, and $MR_i(q_i) = MR_{i+1}(q_i)$, for $i = 1, ..., n^* - 1$. The corresponding net prices will be given by the following recursive system:

$$p_{n^*} = P_{n^*}(q_{n^*}), \text{ and } p_i = p_{i+1} + P_i(q_i) - P_{i+1}(q_i), \text{ for } i = 1, ..., n^* - 1.$$

Notice that the first-order condition captured by the marginal revenue equality is interpreted as a switch of a marginal consumer from variety $i+1$ to variety i. The choice of q_i holds constant all other switch-points so that a one unit increase in q_i means a one unit increase in production of variety i and a corresponding one unit decrease in production of variety $i+1$ (in order to keep the marginal consumer q_{i+1} fixed). With this in mind, the intuition for the marginal revenue equality at the switch-point is the following. The value to M of getting a marginal consumer to switch up from variety $i+1$ to variety i is the premium $p_i - p_{i+1}$. But to induce this switch, M must raise the consumer's surplus on variety i by enough. The derivative of the willingness-to-pay difference, $P_i'(q_i) - P_{i+1}'(q_i)$, indicates how much the premium must be reduced, and this premium reduction is suffered on M's demand base, q_i, of consumers buying varieties better than i. Pulling this together, the first-order profit derivative is $p_i - p_{i+1} + \left(P_i'(q_i) - P_{i+1}'(q_i)\right) q_i$, which rearranges to the difference in net marginal revenues at q_i.

A key property of equilibrium is that price and the quantity sold of an included variety $i \in N^*$ depend only on the varieties that come after it on the product line. Itoh (1983) showed this in a Mussa-Rosen setting with multiplicative preferences, which Johnson and Myatt (2014) later extended for a Cournot oligopoly. Using a graphical analysis, we are able to replicate most of Itoh's results. Moreover, our approach is more general as we represent each variety by a general stand-alone inverse demand function, which admits the Mussa-Rosen model as a special case.

Suppose M initially includes each one of the n varieties in its product line, and that there is a new variety called "variety $j.5$" whose MR curve expands the upper envelope in the proximity where MR_j and MR_{j+1} intersect. Using Proposition 1, we can summarize the effects of introducing variety $j.5$ into the product line as follows: (i) quantities $q_1, ..., q_{j-1}, q_{j+1}, ..., q_n$ remain unaffected since these are determined by local MR curves that do not involve $MR_{j.5}$. The quantities sold of varieties j and $j+1$ ($q_{j.5} - q_j$ and $q_{j+1} - q_{j.5}$ respectively) go down; (ii) prices $p_{j+1}, ..., p_n$ also remain unaffected; (iii) Suppose p_j changes by an amount Δ. Then $p_1, ..., p_{j-1}$ change in the same direction, also by Δ. These effects are illustrated in Figure 2 where M starts with three existing varieties, and the new variety comes in between varieties 2 and 3. Note that the area under the upper envelope of the MR curves measures M's profits. Therefore, the shaded area indicates the increase in M's profit as a result of introducing variety 2.5.

Itoh (1983) finds that addition of a new variety will improve consumer surplus for ρ-linear and ρ-concave demand curves. Since profits must be higher if the firm has chosen to introduce a new variety, aggregate welfare must improve as well. In the next section, we consider quite a different welfare link between optimum and equilibrium, by looking at the first-best optimum product line. This involves quite a different allocation of consumers to varieties than the equilibrium with its

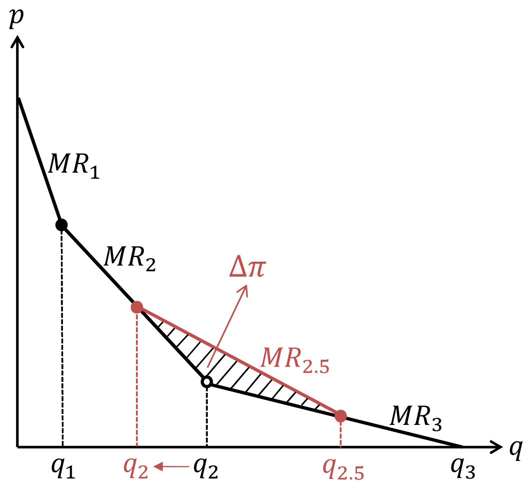

Figure 2
Extra profit and output effects of a new variety

monopoly pricing. Nonetheless, we show an equivalence result for the whole class of general Mussa-Rosen preferences (and therefore for the Itoh, 1983, and Johnson-Myatt, 2014, models).

3 Socially optimal product line

Given our assumption of zero production costs, the socially optimal matching of consumers to varieties follows the upper envelope of the inverse demand curves. Since this will typically differ from the way M will segment the market, the equilibrium outcome will be associated with consumption inefficiencies. For instance, while it is socially optimal that all consumers left of the intersection of $P_1(q)$ and $P_2(q)$ consume variety 1, only those left of the intersection of $MR_1(q)$ and $MR_2(q)$ consume it in equilibrium.

We will here focus on a particular class of inverse demand curves:

$$P_i(q) = \alpha_i - \beta_i \eta(q), \tag{6}$$

where each variety $i \in N$ is described by a pair $(\alpha_i, \beta_i) >> 0$, and a function $\eta(q)$ with $\eta'(q) > 0$. Without any loss of generality, set $\eta(0) = 0$. This class of demand curves is widely used in economics. For example, $\eta(q) = q^\rho$ means that the (direct) demand curve is ρ-linear. It also represents a general class of Mussa-Rosen preferences. Take, for instance, $u_i = s_i v(\theta)$ where $v'(\theta) > 0$. This translates into a net inverse demand curve $P_i(q) = s_i v\left(F^{-1}(1-q)\right)$, which can be rewritten as $P_i(q) = s_i v(1) - s_i\left[v(1) - v\left(F^{-1}(1-q)\right)\right]$. With this formulation, $\alpha_i = s_i v(1) > 0$,

$\beta_i = s_i > 0$, and $\eta(q) = v(1) - v\left(F^{-1}(1-q)\right)$ with $\eta'(q) > 0$.

Lemma 1 shows an important property of this class of inverse demand curves.

Lemma 1 *Assume $P_i(q) = \alpha_i - \beta_i \eta(q)$, $\forall i \in N$, where $(\alpha_i, \beta_i) >> 0$, $\eta(0) = 0$ and $\eta'(q) > 0$. For any two varieties $i, j \in N$, if $P_i(q) = P_j(q) = \phi > 0$ at some $q > 0$, then $MR_i(\tilde{q}) = MR_j(\tilde{q}) = \phi$ for some $\tilde{q} \in (0, q)$. Similarly, if $MR_i(\tilde{q}) = MR_j(\tilde{q}) = \phi > 0$ at some $\tilde{q} > 0$, then $P_i(q) = P_j(q) = \phi$ for some $q > \tilde{q}$.*

Lemma 1 says that two inverse demand curves cross if and only if their corresponding marginal revenue curves cross. Moreover, both of these crossings occur at the same height. This has an important implication: if a particular variety is part of the upper envelope of the marginal revenue curves, then it is also part of the upper envelope of the inverse demand curves (and vice versa). Thus, Lemma 1 immediately implies:

Proposition 2 *When each variety $i \in N$ has an inverse demand function $P_i(q) = \alpha_i - \beta_i \eta(q)$, as implied by the general Mussa-Rosen framework, the equilibrium product line M chooses is the same as the first-best socially optimal product line.*

This situation is graphically illustrated in Figure 3 below. Proposition 2 does not mean that consumers will purchase their first-best varieties in equilibrium. Since M will optimally segment the market, it will have to leave some informational surplus to high-valuation consumers. Therefore, not all consumers will end up consuming the varieties that are socially best for them.

4 Cournot oligopoly

The framework we have built for monopoly above extends very easily to a symmetric Cournot oligopoly. Suppose there are m identical firms. Similar to the monopoly notation, denote by q_i^j the cumulative amount of varieties $1, ..., i$ that firm j sells. We now introduce a new notation Q_i, which aggregates q_i^j across firms; i.e., $Q_i = \Sigma_{j=1}^m q_i^j$. Also, denote the symmetric Cournot residual marginal revenue by $SMR_i(Q; m)$. We present the main result in Proposition 3.

Proposition 3 *For a given set of available varieties $N = \{1, 2, ..., n\}$, m symmetric Cournot oligopolists will include variety i in their product lines if and only if*

$$SMR_i(Q; m) > \max_{j \neq i} SMR_j(Q; m) \text{ for some } Q \in \left(0, SMR_i^{-1}(0)\right),$$

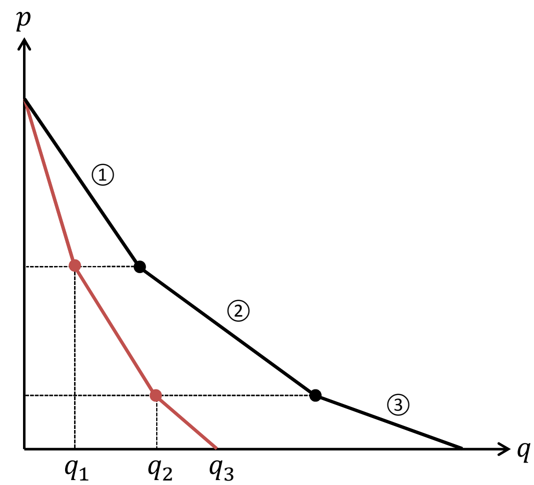

Figure 3
Upper envelope of demand curves vs. MR curves

where $SMR_i(Q;m) = P_i(Q) + QP_i'(Q)/m$. Quantities and prices will be given by analogy to Proposition 1. When each variety $i \in N$ has an inverse demand function in the form $P_i(q) = \alpha_i - \beta_i \eta(q)$, as implied by the general Mussa-Rosen framework, the equilibrium product line under symmetric Cournot oligopoly is the same as the first-best socially optimal product line.

Once again, equilibrium product line and consumer allocation will be determined by the upper envelope of the symmetric Cournot marginal revenue curves, with the corresponding prices given by the incentive compatibility conditions. These are precisely the same steps we followed to obtain the monopoly outcome, just adjusted for the number of firms. Moreover, the inverse demand form (6) again implies that the equilibrium and socially optimal product lines are the same (see the proof of Lemma 1 in the Appendix, which shows that SMR curves intersect at the same height as do the demand curves for an m-firm Cournot oligopoly).

The effects of introducing a new variety can also be tracked as above and as in Johnson and Myatt (2014). Consider a new variety (variety $j.5$) with a Cournot marginal revenue curve in the proximity where $SMR_j(Q;m)$ and $SMR_{j+1}(Q;m)$ intersect. Then: (i) quantities $Q_1, ..., Q_{j-1}, Q_{j+1}, ..., Q_n$ will stay unaffected; (ii) prices $p_{j+1}, ..., p_n$ will stay unaffected; (iii) Suppose p_j changes by an amount Δ. Then $p_1, ..., p_{j-1}$ change in the same direction, also by Δ.

5 Conclusion

In this paper, we analyze a monopolist's choice of its product line. Even though this is generally a complex problem, we are able to reach very clean results. In particular, we show that the monopolist's product line choice problem reduces to including those varieties that are part of the upper envelope of the net marginal revenue curves. The equilibrium quantities of the included varieties are then determined by finding where the associated marginal revenue curves cross. We also show that, for an important class of preferences, the monopolist offers only those product designs that are (first-best) socially desirable. However, since the monopolist will optimally segment the market to maximize its profits, there will be distortions in consumption, so consumers will not always get the variety that is best for them. We also show that these results smoothly extend to a symmetric Cournot oligopoly framework.

Appendix: Proofs

Proof of Lemma 1. Suppose $P_i(q) = P_j(q) = \phi$ for some q. Then $\alpha_i - \beta_i \eta(q) = \alpha_j - \beta_j \eta(q)$ at such q, and thus $\eta(q) = \frac{\alpha_i - \alpha_j}{\beta_i - \beta_j}$. Plugging this back into $P_i(q)$, we have

$$\phi = \alpha_i - \beta_i \frac{\alpha_i - \alpha_j}{\beta_i - \beta_j}.$$

Marginal revenue curve $MR_i(q)$ is given by $\alpha_i - \beta_i(\eta(q) + q\eta'(q))$. If $MR_i(\tilde{q}) = MR_j(\tilde{q})$ for some $\tilde{q}$, then it must be that

$$\eta(\tilde{q}) + \tilde{q}\eta'(\tilde{q}) = \frac{\alpha_i - \alpha_j}{\beta_i - \beta_j}.$$

Plugging this back into $MR_i(q)$, we get

$$MR_i(q) = \alpha_i - \beta_i \frac{\alpha_i - \alpha_j}{\beta_i - \beta_j} = \phi.$$

Thus marginal revenue curves always cross at the same height as the inverse demand curves.

Note that the same arguments also apply to a Cournot oligopoly with m firms, where we define

$$SMR_i(Q; m) = P_i(Q) + QP_i'(Q)/m = \alpha_i - \beta_i \left(\eta(Q) + Q\eta'(Q)/m\right).$$

If $SMR_i(\tilde{Q}, m) = SMR_j(\tilde{Q}, m)$ for some $\tilde{Q}$, it must be that $\eta(\tilde{Q}) + \frac{\tilde{Q}\eta'(\tilde{Q})}{m} = \frac{\alpha_i - \alpha_j}{\beta_i - \beta_j}$,implying

$$SMR_i(\tilde{Q}, m) = SMR_j(\tilde{Q}, m) = \alpha_i - \beta_i \frac{\alpha_i - \alpha_j}{\beta_i - \beta_j}.$$

Hence, the same property extends to a symmetric m-firm oligopoly. ■

Proof of Proposition 3. Take each firm's behavior as given and analyze firm k in isolation. For given prices $(p_1, ..., p_n)$, we can express firm k's profits as

$$\pi_k = (p_1 - p_2)\, q_1^k + \cdots + p_n q_n^k.$$

The incentive compatibility constraints are now given by:

$$P_i\,(Q_i) - p_i = P_{i+1}\,(Q_i) - p_{i+1}, \text{ for } i = 1, ..., n-1,$$

$$P_n\,(Q_n) - p_n = 0.$$

Incorporating these into the profit function, we can express profits as a function of quantities only:

$$\pi_k = (P_1\,(Q_1) - P_2\,(Q_1))\, q_1^k + (P_2\,(Q_2) - P_3\,(Q_2))\, q_2^k + \cdots + P_n\,(Q_n)\, q_n^k.$$

Let $SMR_i\,(Q;m) = P_i\,(Q) + QP_i'\,(Q)\,/m$ denote the symmetric Cournot residual marginal revenue. Then, in an interior equilibrium with $q_i^k = Q_i/m$, we reach the following first-order conditions:

$$SMR_i\,(Q_i;m) = SMR_{i+1}\,(Q_i;m)\,, \text{ for } i = 1, ..., n-1,$$

$$SMR_n\,(Q_n;m) = 0.$$

We now argue that any variety in the upper envelope of the SMR curves will be produced in equilibrium. To see this, suppose that varieties 1 and 2 are produced, with switch-point Q_1, but that variety 1.5 has a higher SMR at Q_1 (the argument applies for any variety). Without variety 1.5, profit is as above. Suppose now that firm k produces Δq_1^k units less of variety 1 and substitutes with Δq_1^k units of variety 1.5. So then,

$$\begin{aligned}\pi_k = &\left(P_1\left(Q_1 - \Delta q_1^k\right) - P_{1.5}\left(Q_1 - \Delta q_1^k\right)\right)\left(q_1^k - \Delta q_1^k\right) + (P_{1.5}\,(Q_1) - P_2\,(Q_1))\, q_1^k \\ &+ (P_2\,(Q_2) - P_3\,(Q_2))\, q_2^k + \cdots + P_n\,(Q_n)\, q_n^k.\end{aligned}$$

The change in profit is the difference

$$\Delta\pi_k = \left\{\left(P_1\left(Q_1 - \Delta q_1^k\right) - P_{1.5}\left(Q_1 - \Delta q_1^k\right)\right)\left(q_1^k - \Delta q_1^k\right) + P_{1.5}\,(Q_1)\, q_1^k\right\} - P_1\,(Q_1)\, q_1^k.$$

We can rewrite this expression as

$$\frac{\Delta\pi_k}{\Delta q_1^k} = \frac{P_1\left(Q_1 - \Delta q_1^k\right) - P_1(Q_1)}{\Delta q_1^k} q_1^k - P_1\left(Q_1 - \Delta q_1^k\right) - \frac{P_{1.5}\left(Q_1 - \Delta q_1^k\right) - P_{1.5}(Q_1)}{\Delta q_1^k} q_1^k + P_{1.5}\left(Q_1 - \Delta q_1^k\right).$$

In the limit as $\Delta q_1^k \to 0$, this becomes

$$\begin{aligned}\pi_k' &= -P_1'(Q_1)\, q_1^k - P_1(Q_1) + P_{1.5}'(Q_1)\, q_1^k + P_{1.5}(Q_1)\\ &= SMR_{1.5}(Q_1; m) - SMR_1(Q_1; m),\end{aligned}$$

where the last line follows from evaluating at $q_1^k = Q_1/m$. Thus, profit is higher if the putatively excluded variety is produced. A similar argument shows that any variety whose SMR lies below the upper envelope of the SMR curves will not be produced by any firm because it will not be profitable even if the rivals produce none of it. ■

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