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Speculative Trade and the Value of Public Information

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Speculative Trade and the Value of Public Information*

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Abstract

In environments with expected utility, it has long been established that speculative trade cannot occur (Milgrom and Stokey [1982]), and that the value of public information is negative in economies with risk-sharing and no aggregate uncertainty (Hirshleifer [1971], Schlee [2001]). We show that these results are still true even if we relax expected utility, so that either Dynamic Consistency (DC) or Consequentialism is violated. We characterise no speculative trade in terms of a weakening of DC and find that Consequentialism is not required. Moreover, we show that a weakening of both DC and Consequentialism is sufficient for the value of public information to be negative. We therefore generalise these important results for convex preferences which contain several classes of ambiguity averse preferences.

JEL-Classifications: D81, D83, D91

Keywords: Updating, Ambiguity, Dynamic Consistency, Bayesian, Consequentialism, Value of Information, No Trade, Speculative Trade.

1 Introduction

In markets with subjective expected utility (SEU), it has long been established by Milgrom and Stokey [1982] that speculative trade cannot occur. This means that if the market is at an ex ante Pareto optimal allocation, differential information among traders in the interim stage can never imply that it is common knowledge that there is another allocation which Pareto dominates it. Moreover, in standard neoclassical economies with complete markets and symmetric information, it is shown with an example by Hirshleifer [1971] and more generally by Schlee [2001] that public information makes everyone weakly worse off, as it destroys opportunities for mutual insurance.

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By adopting the SEU model, both these results implicitly assume the following two properties. The first is *Dynamic Consistency*, which requires that an action plan is optimal when evaluated with the updated preferences of a later period, *if and only if* it is optimal when evaluated with the preferences of an earlier period. DC ensures that an ex ante optimal action plan will remain optimal at every period and irrespective of how information is updated. In the SEU model, DC is expressed through the Bayesian updating of the prior. The second property, *Consequentialism*, requires that conditional preferences do not depend on past actions, foregone payoffs or unrealized events.

These two properties are normatively appealing and form the basis of backward induction and dynamic programming. However, several decision theoretic models violate them. For instance, preferences which are ambiguity sensitive must either relax DC or Consequentialism (Siniscalchi [2009]), whereas Dominiak et al. [2012] show experimentally that subjects violate DC.

The purpose of this paper is to examine whether no speculative trade and the negative value of public information for mutual insurance are still valid in general models with convex preferences, where either DC or Consequentialism is violated. We find that they are. On the one hand, we relax Consequentialism and adopt the weaker Status Quo Bias, which has been proposed in axiomatic work by Masatlioglu and Ok [2005], Sagi [2006] and Ortoleva [2010], and identified experimentally by Samuelson and Zeckhauser [1988]. On the other hand, we adopt weak DC as formulated and motivated in Galanis [2019], in a single-agent environment with convex preferences.

First, we find that Consequentialism can be dropped completely and that the “if” part of weak DC is the minimum requirement which ensures that there is no speculative trade. This means that if every trader’s preferences satisfies this axiom, there can be no speculative trade, whereas if at least one trader violates it, there are examples with speculative trade. Galanis [2019] showed that the “if” part of weak DC characterises the Bayesian updating of subjective beliefs, which are identified by Rigotti et al. [2008] in the context of convex preferences and can be interpreted as the prices for Arrow-Debreu securities which characterize Pareto efficient allocations. Hence, an intuitive axiom for single-agent environments has natural implications for a multi-agent environment.

Second, under Status Quo Bias, we find that the “only if” part of DC implies that the value of public information is negative for mutual insurance, so traders would prefer not receiving more free information. Moreover, the “only if” part of weak DC implies that the value of public information is weakly negative, so that the traders would prefer to mix more with less information, instead of getting less information for sure. These results mirror the findings of Galanis [2019] for single-agent environments: the “only if” part of DC characterises an agent who always prefers receiving more to less information, whereas the “only if” part of weak DC characterises an agent who prefers to mix more with less information, instead of getting less information for sure. In other words, if information is (weakly) valuable for each agent, then public information is (weakly) not valuable for all agents.

1.1 Related literature

Rigotti et al. [2008] (RSS) identify the subjective beliefs generated by a large number

of models of ambiguity aversion, based on an idea of Yaari [1969], making our approach very general. These models are the convex Choquet model of Schmeidler [1989], the multiple priors model of Gilboa and Schmeidler [1989], the variational preferences model of Maccheroni et al. [2006], the multiplier model of Hansen and Sargent [2001], the smooth second-order prior models of Klibanoff et al. [2005] and Nau [2006], the confidence preferences model of Chateauneuf and Faro [2009] and the second-order expected utility model of Ergin and Gul [2009].¹

Ma [2001] proves that there is no speculative trade using DC and a weakening of Consequentialism, called piecewise monotonicity, whereas Galanis [2018] examines speculative trade under unawareness. Halevy [2004] shows that Consequentialism can be weakened, using Resolute Conditional preferences and Conditional Decomposition, but retains the full force of DC. In this paper, we prove the same result by weakening both DC and Consequentialism.

The paper proceeds as follows. Section 2 presents the model. In Section 3, we show that the “if” part of weak DC is the minimum requirement that precludes speculative trade. In Section 4, we show that if each agent individually considers information to be (weakly) valuable, then public information is not (weakly) valuable in competitive risk-sharing environments with no aggregate uncertainty. All proofs are contained in the Appendix.

2 Model

2.1 Preliminaries

Fix a finite set of payoff relevant states S , with typical element s . The set of consequences is \mathbb{R}_+ , interpreted as monetary payoffs. Let $\mathcal{F} = \mathbb{R}_+^S$ be the set of acts, with the natural topology. An act $f \in \mathcal{F}$ maps each state s to a monetary payoff. Given $x \in \mathbb{R}_+$, let $x \in \mathcal{F}$ be the constant act with payoff x at each state s . Let X be the set of constant acts. An act f is strictly positive if $f(s) > 0$ for all $s \in S$. Let \mathcal{F}_+ be the set of strictly positive acts.

For any two acts $f, g \in \mathcal{F}$ and event $E \subseteq S$, we denote by fEg the act h such that $h(s) = f(s)$ if $s \in E$ and $h(s) = g(s)$ if $s \notin E$. Define $f \geq_E g$ if $f(s) \geq g(s)$ for all $s \in E$, with strict inequality for some $s \in E$. Equality $f =_E g$ and strict inequality are similarly defined. Let E^c be the complement of E with respect to S .

Given events $E, F \subseteq S$ and probability measure $p \in \Delta E$, where $F \subseteq E$ and $p(F) > 0$, denote by $p_F \in \Delta F$ the measure obtained through Bayesian conditioning of p on F . Formally, for any event $G \subseteq S$, $p_F(G) = \frac{p(G \cap F)}{p(F)}$. We write $\mathbb{E}_p f := \sum_{s \in E} p(s) f(s)$

for the expectation of f given p .

Let \mathcal{E} be a collection of nonempty events $E \subseteq S$ which contains S . The decision maker is endowed with a collection of conditional preference relations, $\{\succsim_{E,h}\}_{E \in \mathcal{E}, h \in \mathcal{F}}$, one for each event $E \in \mathcal{E}$ and each act $h \in \mathcal{F}$. The interpretation is that in a previous period the agent had chosen act h and in the current period he learns that event E

¹Note that RSS adopts a domain of preferences over monetary acts, whereas these models allow for more general domains. Galanis [2019] discusses their differences.

has occurred. His updated preference relation is then $\succsim_{E,h}$. The ex ante preference relation $\succsim_{S,h}$ does not depend on the act h and is denoted by \succsim .

A partition Π of S is a collection of mutually disjoint events, whose union is S . It is finer than another partition Π' if, for each $E' \in \Pi'$, there exists $E \in \Pi$ with $E \subseteq E'$. We then say that Π' is coarser than Π .

2.2 Revealed preference

Given preference relation $\succsim_{E,h}$, we say that act f is *revealed preferred* to act g , written $f \succsim_{E,h}^* g$, if $f \succsim_{E,h} ag + (1-a)f$ for all $a \in [0,1]$, so that f is weakly preferred to all convex combinations of f and g . Preference relation $\succsim_{E,h}^*$ is transitive but not necessarily complete.

The interpretation of $f \succsim_{E,h}^* g$ is that f is weakly preferred to g under $\succsim_{E,h}$ and g is inside a “budget set”, which is constructed given f as the agent’s endowment and some prices for the Arrow-Debreu securities, one for each state. If these prices were to prevail and the agent chose f , it would be revealed that the agent prefers f over g .

2.3 Convex preferences

We consider the following axioms on preferences $\{\succsim_{E,h}\}_{E \in \mathcal{E}, h \in \mathcal{F}}$, for all events $E \in \mathcal{E}$ and acts $h \in \mathcal{F}$.

Axiom 1. (*Preference*). $\succsim_{E,h}$ is complete and transitive.

Axiom 2. (*Continuity*). For all $f \in \mathcal{F}$, the sets $\{g \in \mathcal{F} : g \succsim_{E,h} f\}$ and $\{g \in \mathcal{F} : f \succsim_{E,h} g\}$ are closed.

Axiom 3. (*Strong Monotonicity*). For all $f \neq_E g$, if $f \geq_E g$, then $f \succ_{E,h} g$.

Axiom 4. (*Convexity*). For all $f \in \mathcal{F}$, the set $\{g \in \mathcal{F} | g \succsim_{E,h} f\}$ is convex.

Axiom 5. (*Conditional Preference*) For all $f, g \in \mathcal{F}$, if $f =_E g$ then $f \sim_{E,h} g$.

These axioms are standard and the first four imply that each $\succsim_{E,h}$ is represented by a continuous, increasing and quasiconcave function $U_{E,h} : \mathcal{F} \rightarrow \mathbb{R}$. The fifth axiom specifies that if the agent knows that event E has occurred, his preferences depend only on what acts prescribe inside E . We say that preferences $\{\succsim_E\}_{E \in \mathcal{E}, h \in \mathcal{F}}$ are convex if they satisfy Axioms 1 through 5 and strictly convex if they additionally satisfy Axiom 6.

Axiom 6. (*Strict Convexity*). For all $f \neq_E g$ and $\alpha \in (0,1)$, if $f \succ_{E,h} g$, then $\alpha f + (1-\alpha)g \succ_{E,h} g$.

2.4 Consequentialism

Consequentialism requires that the agent’s preferences depend only on the received information and not on the act that was chosen in the previous period.²

²Some papers refer to Consequentialism as the conjunction of Axioms 5 and 7.

Axiom 7. (*Consequentialism*) For all $f, g \in \mathcal{F}$ and events $E \in \mathcal{E}$, $\succsim_{E,f} = \succsim_{E,g}$.

A weakening of Axiom 7 has been proposed in axiomatic work by [Masatlioglu and Ok \[2005\]](#), [Sagi \[2006\]](#) and [Ortoleva \[2010\]](#), where preference relation $\succsim_{E,h}$ depends on a “status quo” act (or frame) h . It specifies that if the agent ever prefers f over g (given some status quo h), then he would also prefer it if the status quo was f . In other words, the status quo exerts attraction towards itself.

Axiom 8. (*Status Quo Bias*) For all $f, g, h \in \mathcal{F}$ and events $E \in \mathcal{E}$, if $f \succsim_{E,h} g$ then $f \succsim_{E,f} g$.

As pointed by [Masatlioglu and Ok \[2005\]](#), Status Quo Bias is documented not only by experimental studies but also by empirical work in actual markets. For instance, [Madrian and Shea \[2001\]](#) examined how the default choice influenced participation in 401(k) saving plans, whereas [Samuelson and Zeckhauser \[1988\]](#) identified Status Quo Bias experimentally, in a study concerning portfolio choices.

2.5 Dynamic Consistency

DC provides restrictions on how two acts, which are identical outside of the conditioning event E , should be compared before and after E is known to have occurred. We break DC into two Axioms and adopt the names proposed by [Ghirardato \[2002\]](#).

Axiom 9. (*Consistency of Implementation*) For all acts $f, g \in \mathcal{F}$ and events $E \in \mathcal{E}$, if $f \succsim g$ and $f =_{E^c} g$ then $f \succsim_{E,f} g$.

Axiom 10. (*Information is Valuable*) For all acts $f, g \in \mathcal{F}$ and events $E \in \mathcal{E}$, if $f \succsim_{E,f} g$ and $f =_{E^c} g$ then $f \succsim g$.

Suppose that f and g specify the same payoff at each state not belonging to event E and that f is weakly preferred to g ex ante. Consistency of Implementation says that if the agent has chosen f ex ante and he is informed that event E has occurred (so that his preferences are $\succsim_{E,f}$), then in the interim stage f is still weakly preferred to g . Information is Valuable specifies the converse.

In a single-agent setting, [Galanis \[2019\]](#) provides an extensive discussion of DC and motivates the following version of weak DC, using the revealed preference relation, \succsim^* .

Axiom 11. (*Weak Consistency of Implementation*) For all acts $f \in \mathcal{F}_+$, $g \in \mathcal{F}$ and events $E \in \mathcal{E}$, if $f \succsim^* g$ and $f =_{E^c} g$ then $f \succsim_{E,f} g$.

Axiom 12. (*Weak Information is Valuable*) For all acts $f \in \mathcal{F}$, $g \in \mathcal{F}_+$ and events $E \in \mathcal{E}$, if $f \succsim_{E,f} g$, $f =_{E^c} g$ and $g \succ f$ then $g \not\succeq^* f$.

Weak Consistency of Implementation requires that if f is revealed preferred (but not necessarily weakly preferred) to g ex ante, then f is weakly preferred to g , conditional on E .³ Weak Information is Valuable specifies that if f is weakly preferred to g conditional on E and f but ex ante strictly preferred to f , then g is not revealed preferred to f ex ante.

³We also require that f is a strictly positive act.

2.6 Subjective beliefs

RSS define the *subjective beliefs at an act* f and preference relation $\succsim_{E,h}$ to be the set of all normals (normalized to be probabilities) of the supporting hyperplanes of f ,

$$\pi_{E,h}(f) = \{p \in \Delta S : \mathbb{E}_p g \geq \mathbb{E}_p f \text{ for all } g \succsim_{E,h} f\}.$$

RSS provide two alternative definitions for subjective beliefs and show that all three coincide for strictly positive acts. First, suppose that the agent's endowment is act f and we interpret a probability measure as a set of prices, one for each Arrow-Debreu security which pays 1 in a particular state and 0 otherwise. Given preference relation $\succsim_{E,h}$, the *subjective beliefs revealed by unwillingness to trade at* f contain the measures (prices) for which the agent would be unwilling to trade his endowment,

$$\pi_{E,h}^u(f) = \{p \in \Delta S : f \succsim_{E,h} g \text{ for all } g \text{ such that } \mathbb{E}_p g = \mathbb{E}_p f\}.$$

Second, let P be a set of measures (prices) such that whenever another act k is unaffordable for every $p \in P$, then there exists a mixture of k with endowment f that the agent would strictly prefer to his endowment. The smallest such P of measures contains the *subjective beliefs revealed by willingness to trade at* f . Formally, let $\mathcal{P}_{E,h}(f)$ denote the collection of all compact, convex sets $P \subseteq \Delta S$ such that if $\mathbb{E}_p g > \mathbb{E}_p f$ for all $p \in P$, then $\epsilon g + (1 - \epsilon)f \succ_{E,h} f$ for sufficiently small $\epsilon > 0$. Then, the subjective beliefs revealed by willingness to trade at f are denoted by $\pi_{E,h}^w(f) = \bigcap \mathcal{P}_{E,h}(f)$. RSS show that for strictly positive acts f , $\pi_{E,h}(f) = \pi_{E,h}^u(f) = \pi_{E,h}^w(f)$.

3 Speculative trade

In this and the next section, we show that weak DC has economic content in multi-agent settings, such as financial markets. First, we show that Axiom 11 is the minimum requirement which precludes speculative trade. In a single-agent setting, Galanis [2019] shows that Axiom 11 is equivalent to Bayesian updating of subjective beliefs.

Consider an economy consisting of I agents, with $|I| = m$ and typical element i . Each agent's consumption set is the set of acts \mathcal{F} . He is endowed with a collection of convex preferences $\{\succsim_{E,h}^i\}_{E \in \mathcal{E}, h \in \mathcal{F}}$.

An economy is a tuple $(\{\succsim_{E,h}^i\}_{i \in I, E \in \mathcal{E}, h \in \mathcal{F}}, e, \{\Pi^i\}_{i \in I})$, where $e \in \mathbb{R}_{++}^S$ is the aggregate endowment and $\{\Pi^i\}_{i \in I}$ denotes the information structure, where each $\Pi^i \subseteq \mathcal{E}$ is a partition of S . If in period 0 the resulting allocation is f , then in period 1 and at state s , agent i considers states in $\Pi^i(s)$ to be possible and has conditional preferences $\succsim_{\Pi^i(s), f^i}^i$.

An allocation is a tuple $f = (f^1, \dots, f^m) \in \mathcal{F}^m$. It is feasible if $\sum_{i=1}^m f^i = e$. It is interior if $f^i(s) > 0$ for all $s \in S$ and for all i . Given an event $E \subseteq S$, let $K^i(E) = \{s \in S : \Pi^i(s) \subseteq E\}$ be the set of states where i knows E . Event E is self evident if $E \subseteq K^i(E)$ for all $i \in I$. That is, an event is self evident if whenever it happens, everyone knows it. An event F is common knowledge at s if and only if there exists a self evident event E such that $s \in E \subseteq F$ (Aumann [1976]).

We say that there is speculative trade if an allocation is ex ante Pareto efficient (according to preferences $\{\succsim^i\}_{i \in I}$) but at some state $s \in S$ it is common knowledge that there exists a Pareto improvement.

Definition 1. *There is speculative trade in economy $\langle \{\succsim_{E,h}^i\}_{i \in I, E \in \mathcal{E}, h \in \mathcal{F}}, e, \{\Pi^i\}_{i \in I} \rangle$ at an ex ante Pareto efficient allocation f if there is agent $j \in I$, state s' and feasible allocation g such that event $H = \{s \in S : g^i \succsim_{\Pi^i(s), f^i}^i f^i \text{ for all } i \in I \text{ and } g^j \succ_{\Pi^j(s), f^j}^j f^j\}$ is common knowledge at s' .*

We now show Axiom 11 is necessary and sufficient for preventing speculative trade. In particular, if all agents satisfy Axiom 11 then there is no speculative trade, whereas if at least one fails it, there are economies with speculative trade. Let \mathbb{P} be the collection of convex preferences $\{\succsim_{E,h}\}_{E \in \mathcal{E}, h \in \mathcal{F}}$. Note that we do not require Consequentialism or Status Quo Bias.

Proposition 1. *If $\{\succsim_{E,h}^i\}_{i \in I, E \in \mathcal{E}, h \in \mathcal{F}} \in \mathbb{P}^I$ satisfy Axiom 11 then there is no speculative trade in any economy $\langle \{\succsim_{E,h}^i\}_{i \in I, E \in \mathcal{E}, h \in \mathcal{F}}, e, \{\Pi^i\}_{i \in I} \rangle$ and at any interior allocation f . Conversely, if $\{\succsim_{E,h}^k\}_{E \in \mathcal{E}, h \in \mathcal{F}} \in \mathbb{P}$ fails Axiom 11 then there exist preferences $\{\succsim_{E,h}^i\}_{i \in I, E \in \mathcal{E}, h \in \mathcal{F}} \in \mathbb{P}^I$ satisfying Axiom 11 and economy $\langle \{\succsim_{E,h}^i\}_{i \in I \cup \{k\}, E \in \mathcal{E}, h \in \mathcal{F}}, e, \{\Pi^i\}_{i \in I \cup \{k\}} \rangle$ such that there is speculative trade at an allocation f .*

To provide a sketch of the proof for one direction, suppose by contradiction that allocation $\{f^i\}_{i \in I}$ is ex ante Pareto efficient but in the interim it is common knowledge at some state s' that allocation $\{g^i\}_{i \in I}$ is a Pareto improvement. From Aumann [1976], there exists a self evident event F containing s' , where $s \in F$ implies $g^i \succ_{\Pi^i(s), f^i}^i f^i$ for all $i \in I$.⁴ Axiom 5 implies $g^i \Pi^i(s) f^i \succ_{\Pi^i(s), f^i} f^i$. Because each Π^i partitions F , Axiom 11 implies $\mathbb{E}_p(g^i F f^i) > \mathbb{E}_p f^i$ for all $p \in \pi^i(f^i)$. By convexity of preferences, $\epsilon g^i F f^i + (1 - \epsilon) f^i \succ^i f^i$ for small enough $\epsilon > 0$, which contradicts that $\{f^i\}_{i \in I}$ is ex ante Pareto efficient.

Following Kajii and Ui [2009], Martins-da-Rocha [2010] also shows that Bayesian updating of subjective beliefs precludes speculative trade. But both papers assume Consequentialism, which is not needed here. Halevy [2004] proves the absence of speculative trade by assuming DC but relaxing Consequentialism to the following two properties: Resolute Conditional preferences (which is similar to our Axiom 5) and Conditional Decomposability (which we do not require). Ma [2001] proves the result using DC and a weakening of Consequentialism, called piecewise monotonicity. Galanis [2018] examines speculative trade in an environment with unawareness, where DC is violated but Consequentialism is not.

4 The value of public information

In a single-agent setting, Galanis [2019] shows that information is (weakly) valuable if and only if Axiom 10 (Axiom 12) is satisfied. In this section, we explore whether public information is valued in multi-agent settings where each agent values information

⁴We can assume strict preference for everyone due to Strong Monotonicity.

individually. In the standard environment with expected utility, where each agent values information, [Hirshleifer \[1971\]](#) first argued with an example that if agents trade in order to mutually insure, then more public information could make everyone worse off. [Schlee \[2001\]](#) generalised this result, showing that the value of public information is negative in an expected utility model with a common prior, risk aversion and no aggregate uncertainty.⁵

For general convex preferences, we find that as long as Bayesian updating of subjective beliefs (Axiom 11) and Status Quo Bias are satisfied, if every agent (weakly) values information, then public information is (weakly) not valuable.⁶

In order to rule out pure indifference to betting, we assume strictly convex preferences. We also assume the following axiom, which is proposed by RSS.

Axiom 13. (*Translation Invariance at Certainty*). For all acts $h \in \mathcal{F}$ and events $E \in \mathcal{E}$, for all $g \in \mathcal{F}$ and all constant bundles $x, x' > 0$, if $x + \lambda g \succ_{E,h} x$ for some $\lambda > 0$, then there exists $\lambda' > 0$ such that $x' + \lambda' g \succ_{E,h} x'$.

RSS show that Axiom 13 is satisfied by most classes of ambiguity averse preferences and it implies that subjective beliefs do not change across constant acts: $\pi_{E,h}^i(x) = \pi_{E,h}^i(x')$ for all constant acts $x, x' > 0$. We henceforth write $\pi_{E,h}^i$ instead of $\pi_{E,h}^i(x)$ for all constant acts $x > 0$.

We also impose a slight variation of Axiom 11. First, it applies only to constant acts. Second, it should apply not only between the ex ante preference relation \succ^* and the interim $\succ_{E,x}$, but also between $\succ_{F,h}^*$ and $\succ_{E,x}$, where $E \subseteq F$ and $F \in \mathcal{E}$. In other words, it is as if we consider a multi period model where the agent first learns F and then E .

Axiom 14. (*Multi Period Weak Consistency of Implementation*) For all acts $x, g, h \in \mathcal{F}$, where $x > 0$ is constant, and events $F, E \in \mathcal{E}$ with $E \subseteq F$, if $x \succ_{F,h}^* g$ and $x =_{E^c} g$ then $x \succ_{E,x} g$.

There are two periods, 0 and 1. In period 0, the agents' common information structure about period 1 is represented by partition Π of S . Hence, there is symmetric information among all agents. The initial allocation is $\{e^i\}_{i \in I}$, where $e^i \in \mathcal{F}_+$. The aggregate endowment is $\sum_{i \in I} e^i = e \in \mathbb{R}_{++}^S$. We assume that there is no aggregate uncertainty, so e is constant across all states in S . The economy in period 0 is a tuple $\langle S, \succ^1, \dots, \succ^m, e \rangle$.

In period 1, all agents are informed that some event $E \in \Pi$ has occurred and trade, using their conditional preferences. Hence, information is symmetric. Trading at each $E \in \Pi$ generates an act for each agent, which is evaluated in period 0 using preference relation \succ^i .

⁵[Schlee \[2001\]](#) uses the [Blackwell \[1951\]](#) criterion of more information. Moreover, he proves this result in two other cases, that we do not examine. First, there are some risk neutral agents who fully insure the risk averse ones. Second, all agents are risk averse and the economy has a representative agent.

⁶Similar results are shown by [Galanis \[2015, 2016\]](#), in an environment with unawareness, where DC is violated but Consequentialism is not.

Given event $E \in \Pi$, an allocation for economy $\langle E, \succsim_{E,h^1}^1, \dots, \succsim_{E,h^m}^m, e \rangle$ is a tuple $f_E = (f_E^1, \dots, f_E^m) \in \mathcal{F}^m$.⁷ It is feasible if $\sum_{i=1}^m f_E^i = e$. It is interior if $f_E^i(s) > 0$ for all $s \in E$ and for all i . A feasible allocation f_E is full insurance if each f_E^i is constant across all states in E . It is Pareto optimal if there is no feasible allocation g_E such that $g_E^i \succ_{E,h^i}^i f_E^i$ for all $i \in I$ and $g_E^j \succ_{E,h^j}^j f_E^j$ for some $j \in I$.

Fix a collection of convex preferences $\{\succsim_{E,h}^i\}_{i \in I, E \in \mathcal{E}, h \in \mathcal{F}}$. Let $\mathcal{M} = \{\Pi, e\}$ be an aggregate decision problem, where $\Pi \subseteq \mathcal{E}$ is a partition of S and $e \in \mathbb{R}_{++}^S$ is the aggregate endowment, assumed constant across states.

Given event $E \in \Pi$ and economy $\langle E, \succsim_{E,h^1}^1, \dots, \succsim_{E,h^m}^m, e \rangle$, define $f_E = \{f_E^i\}_{i \in I} \in \mathcal{F}^m$ to be an equilibrium allocation if it is feasible and there are prices $p \in \mathbb{R}_+^S$, with $p(s) = 0$ if $s \notin E$, such that, for each $i \in I$, $\mathbb{E}_p f_E^i \leq \mathbb{E}_p e^i$ and $f_E^i \succ_{E,h^i}^i g$ for all g such that $\mathbb{E}_p g \leq \mathbb{E}_p e^i$.

We say that interior allocation $\{f^i\}_{i \in I}$ is admissible for aggregate decision problem $\mathcal{M} = \{\Pi, e\}$ if, for each agent $i \in I$, for each $E \in \Pi$, $f^i =_E f_E^i$, where $f_E = \{f_E^i\}_{i \in I}$ is an equilibrium allocation of economy $\langle E, \succsim_{E,f^1}^1, \dots, \succsim_{E,f^m}^m, e \rangle$. In words, $\{f^i\}_{i \in I}$ is admissible if, under some equilibrium, agent i receives f_E^i at each $E \in \Pi$.

We compare aggregate decision problems by evaluating the admissible acts they generate.

Definition 2. *Aggregate decision problem $\mathcal{M}_1 = \{\Pi_1, e\}$ is not more valuable than $\mathcal{M}_2 = \{\Pi_2, e\}$ if whenever $\{g^i\}_{i \in I}$ is admissible for \mathcal{M}_2 , there exists $\{f^i\}_{i \in I}$ which is admissible for \mathcal{M}_1 and $g^i \succ_{E,h^i}^i f^i$, for all $i \in I$. It is weakly not more valuable if $ag^i + (1-a)f^i \succ_{E,h^i}^i f^i$ for some $a \in (0, 1]$.*

We say that public information is (weakly) not valuable if an aggregate decision problem with a finer partition is always (weakly) not more valuable than a decision problem with a coarser partition.

Definition 3. *Public information is (weakly) not valuable for $\{\succsim_{E,h}^i\}_{i \in I, E \in \mathcal{E}, h \in \mathcal{F}}$ if, for all endowments e and partitions Π_1, Π_2 of S , Π_1 finer than Π_2 , aggregate decision problem $\mathcal{M}_1 = \{\Pi_1, e\}$ is (weakly) not more valuable than $\mathcal{M}_2 = \{\Pi_2, e\}$.*

The following Proposition shows that if information is (weakly) valuable for each agent, then public information is (weakly) not valuable.

Proposition 2. *Suppose strictly convex preferences $\{\succsim_{E,h}^i\}_{i \in I, E \in \mathcal{E}, h \in \mathcal{F}}$ satisfy Axioms 8, 13 and 14. Then, Axiom 12 implies that public information is weakly not valuable, whereas Axiom 10 implies that it is not valuable.*

A Appendix

Proof of Proposition 1. Fix $j \in I$. From Axioms 2 and 3 we have that $H = G \equiv \{s \in S : g_0^i \succ_{\Pi^i(s), f^i}^i f^i \text{ for all } i \in I\}$ for some feasible allocation g_0 , as we can always

⁷Note that we define the aggregate endowment of the economy as a map from S (rather than E) to \mathbb{R}_{++} . This is without loss of generality because, from Axiom 5, what the endowment prescribes outside of E is irrelevant. For consistency, we do the same for all subsequent acts.

distribute a small enough portion of j 's allocation to everyone else. We subsequently show that G cannot be common knowledge at any s , denoting g_0 by g .

Because f is an ex ante efficient allocation, there does not exist a feasible allocation h such that $h^i \succ^i f^i$ for all $i \in I$. Suppose that there exists feasible allocation g such that G is common knowledge at $s \in S$. Let F be a self evident event such that $s \in F \subseteq G$. Note that each Π^i partitions F . Then, we have that for each $i \in I$, for each $s' \in F$, $g^i \succ_{\Pi^i(s'), f^i}^i f^i$. From Axiom 5, $g^i \Pi^i(s') f^i \succ_{\Pi^i(s'), f^i}^i f^i$. Using Axiom 11 we have that $f^i \not\prec_{\Pi^i(s'), f^i}^{*i} g^i \Pi^i(s') f^i$. Noting that f^i is strictly positive, so that $\pi^{ui} = \pi^i$, and from the definition of π^{ui} , we have that $\mathbb{E}_p(g^i \Pi^i(s') f^i) > \mathbb{E}_p f^i$ for all $p \in \pi^i(f^i)$. Because this is true for all $s' \in S$ such that $\Pi^i(s') \subseteq F$ and Π^i partitions F , we have that $\mathbb{E}_p(g^i F f^i) > \mathbb{E}_p f^i$, for all $p \in \pi^i(f^i)$. Define $h^i = g^i F f^i$ and $h = \{h^i\}_{i \in I}$.

Allocation f is interior, hence $\pi^{ui}(f^i) = \pi^{wi}(f^i) = \pi^i(f^i)$. Because $\mathbb{E}_p h^i > \mathbb{E}_p f^i$ for all $p \in \pi^{wi}(f^i)$, we have that for small enough ϵ^i , $\epsilon^i h^i + (1 - \epsilon^i) f^i \succ^i f^i$. By taking $\epsilon < \epsilon^i$ for all $i \in I$, we have that $\epsilon h^i + (1 - \epsilon) f^i \succ^i f^i$ for all $i \in I$. Moreover, $\epsilon h + (1 - \epsilon) f$ is feasible because both f and h are feasible. Hence, f is not ex ante efficient, a contradiction.

Conversely, suppose that $\{\succ_{E,h}^1\}_{E \in \mathcal{E}, h \in \mathcal{F}}$ fails Axiom 11. This means that for some event $E \in \mathcal{E}$ and acts $f \in \mathcal{F}_+$, $g \in \mathcal{F}$, with $f =_{E^c} g$, we have $g \succ_{E,f}^1 f$ and $\mathbb{E}_{p_0} g \leq \mathbb{E}_{p_0} f$ for some $p_0 \in \pi^{1u}(f)$. Consider an economy with two agents, 1 and 2. Their information structure is identical, so that $\Pi^1 = \Pi^2 = \Pi = \{E, E^c\}$. Let $e = f + g$. Agent 2 has preferences represented by expected utility. In particular, $h \sim^2 h'$ if and only if $\mathbb{E}_{p_0} h = \mathbb{E}_{p_0} h'$. His conditional preferences given E or E^c are given by updating p_0 using Bayes' rule. This is well defined because Axiom 3 implies Axiom 8 in Galanis [2019], hence from Lemma 2 in that paper we have that $p(E), p(E^c) > 0$.

We next show that allocation $h = \{f, g\}$ is ex ante Pareto efficient. Suppose there exists allocation $\{x, y\}$ such that $x \succ^1 f$ and $y \succ^2 g$. Because $p_0 \in \pi^{1u}(f)$, we have that $\mathbb{E}_{p_0} x > \mathbb{E}_{p_0} f$. Moreover, $\mathbb{E}_{p_0} y \geq \mathbb{E}_{p_0} g$. These inequalities imply that $\mathbb{E}_{p_0}(x + y) > \mathbb{E}_{p_0}(f + g) = \mathbb{E}_{p_0} e$, which implies that $x + y \neq e$, hence $\{x, y\}$ is not feasible. A similar argument applies if $x \succ^1 f$ and $y \succ^2 g$.

Note that $f \succ^2 g$ because $\mathbb{E}_{p_0} g \leq \mathbb{E}_{p_0} f$. Given E and since $f =_{E^c} g$, we have that $\mathbb{E}_{p_0 E} g \leq \mathbb{E}_{p_0 E} f$, which implies $f \succ_{E,g}^2 g$. Because $g \succ_E^1 f$, at each $s \in E$ it is common knowledge that allocation $h' = \{g, f\}$ Pareto dominates $h = \{f, g\}$, hence there is speculative trade. □

Proof of Proposition 2. First note that because Axiom 6 implies Axiom 7 (No Flat Kinks) in Galanis [2019], Proposition 1 in that paper implies that if information is (weakly) valuable then $\{\succ_{E,h}^i\}_{i \in I, E \in \mathcal{E}, h \in \mathcal{F}}$ satisfy Axiom 10 (Axiom 12), for each $i \in I$. Let e be the endowment and suppose partition Π_1 is finer than partition Π_2 . Let $\{g^i\}_{i \in I}$ be admissible for $\mathcal{M}_2 = \{\Pi_2, e\}$, defined as follows. Let $\{g_{E_2}\}_{E_2 \in \Pi_2}$ be a tuple where, for each $E_2 \in \Pi_2$, g_{E_2} is an equilibrium allocation for economy $\langle E_2, \succ_{E_2, g^1}^1, \dots, \succ_{E_2, g^m}^m, e \rangle$ with (normalized) prices $p^{E_2} \in \Delta S$, such that $p(s) = 0$ if $s \notin E_2$. For each $E_2 \in \Pi_2$, let $g^i =_{E_2} g_{E_2}^i$. From the first welfare theorem, g_{E_2} is Pareto optimal. From Proposition 9 in RSS, g_{E_2} is a full insurance allocation. From Axiom 3, $p^{E_2} > 0$ for all $s \in E_2$ and $g_{E_2}^i(s) = \mathbb{E}_{p^{E_2}} e^i$, for all $s \in E_2$ and all $i \in I$. Hence, g_{E_2} is also an

interior allocation. Proposition 9 in RSS implies that $p^{E_2} \in \bigcap_i \pi_{E_2, g^i}^i$.

Proposition 1 in RSS shows that $\pi_{E, g^i}(f) = \pi_{E, g^i}^{ui}(f)$ for all strictly positive acts. Moreover, Axiom 3 implies Axiom 8 (Weak Full Support) in Galanis [2019], hence Lemma 2 in that paper implies $p^{E_2}(E_1) > 0$. It is straightforward that we can use the proof of Proposition 2 in Galanis [2019], but applying Axiom 14 instead of Axiom 11, to show that there is Bayesian updating of subjective beliefs at a constant act, between any events $F, E \in \mathcal{E}$, where $E \subseteq F$. We therefore have $p_{E_1}^{E_2} \in \bigcap_i \pi_{E_1}^i$, for each $E_1 \subseteq E_2$, where $E_1 \in \Pi_1$ and $p_{E_1}^{E_2}$ is the Bayesian update of p^{E_2} on E_1 .

Define allocation $\{f^i\}_{i \in I}$ as follows. If $E_1 \subseteq E_2$, where $E_1 \in \Pi_1$ and $E_2 \in \Pi_2$, then $f^i =_{E_1} f_{E_1}^i$, where $f_{E_1} = (f_{E_1}^1, \dots, f_{E_1}^m)$ is such that, for each $i \in I$, $f_{E_1}^i(s) = \mathbb{E}_{p_{E_1}^{E_2}} e^i$ for all $s \in E_1$. Hence, each f_{E_1} is a full insurance allocation. Because $p_{E_1}^{E_2} \in \pi_{E_1}^i$, $f_{E_1}^i$ is weakly preferred to each act h that is affordable given prices $p_{E_1}^{E_2}$. Because f_{E_1} is feasible, it is an equilibrium allocation of economy $(E_1, \succsim_{E_1, f^1}^1, \dots, \succsim_{E_1, f^m}^m, e)$.

By construction, $\mathbb{E}_{p^{E_2}} f^i = \mathbb{E}_{p^{E_2}} g_{E_2}^i$. Because $p^{E_2} \in \bigcap_i \pi_{E_2}^i$, we have that $g_{E_2}^i \succsim_{E_2, g^i}^i f^i$, for all $i \in I$ and all $E_2 \in \Pi_2$. Axiom 5 implies that $g^i \succsim_{E_2, g^i}^i f^i$ for each $E_2 \in \Pi_2$ and each $i \in I$.

Enumerate the partition cells of $\Pi_2 = \{E_1, \dots, E_n\}$. If $n = 1$ then $\Pi_2 = \{S\}$ is the uninformative partition and $g^i \succsim^i f^i$ for each $i \in I$, so we are done. Suppose that $n \geq 2$. For cell $1 \leq k \leq n$ define act h_k^i as follows. Let $h_k^i(s) = g^i(s)$ if $s \in E_j$, where $1 \leq j \leq k$, and $h_k^i(s) = f^i(s)$ otherwise. Note that $h_n^i = g^i$ and let $h_0^i = f^i$. For each $1 \leq k \leq n$, from Axiom 5, we have that $g^i \succsim_{E_k, g^i}^i f^i$ implies $h_k^i \succsim_{E_k, g^i}^i h_{k-1}^i$. Axiom 8 implies $h_k^i \succsim_{E_k, h_k^i}^i h_{k-1}^i$. Applying Axiom 10 we have $h_k^i \succsim^i h_{k-1}^i$. By Axiom 1, we have that $g^i \succsim^i f^i$, for each $i \in I$, which implies that \mathcal{M}_1 is not more valuable than \mathcal{M}_2 . Therefore, public information is not valuable.

For the second claim, for each $E_2 \in \Pi_2$ define $h_{E_2}^i = g_{E_2}^i E_2 f^i$. From Axiom 5, $g_{E_2}^i \succsim_{E_2, g^i}^i f^i$ implies $h_{E_2}^i \succsim_{E_2, g^i}^i f^i$. Axiom 8 implies $h_{E_2}^i \succsim_{E_2, h_{E_2}^i}^i f^i$. From Axiom 12, either $h_{E_2}^i \succsim^i f^i$ or $f^i \succ^i h_{E_2}^i$ and $f^i \not\succeq^{*i} h_{E_2}^i$. If $h_{E_2}^i \succsim^i f^i$, Axiom 6 implies that for all $a \in (0, 1)$, $ah_{E_2}^i + (1-a)f^i \succ^i f^i$. This means that $\mathbb{E}_p(ah_{E_2}^i + (1-a)f^i) > \mathbb{E}_p f^i$ for all $p \in \pi^i(f^i)$, or that $\mathbb{E}_p h_{E_2}^i > \mathbb{E}_p f^i$. Hence, in both cases we have that $\mathbb{E}_p h_{E_2}^i > \mathbb{E}_p f^i$ for all $p \in \pi^i(f^i)$. Repeating this argument for all $E_2 \in \Pi_2$, we have that $\mathbb{E}_p g^i > \mathbb{E}_p f^i$ for all $p \in \pi^i(f^i)$. By definition, for small enough $\epsilon^i > 0$, $\epsilon^i g^i + (1 - \epsilon^i) f^i \succ^i f^i$. By taking $\epsilon < \epsilon^i$ for all $i \in I$, we have that $\epsilon g^i + (1 - \epsilon) f^i \succ^i f^i$ for all $i \in I$, hence public information is weakly not valuable. \square

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