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OPAQUE SELLING*

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Abstract

We study "opaque" selling in multiproduct environments – a marketing practice in which sellers strategically withhold product information by keeping important characteristics of their products hidden until after purchase. We show that a monopolist will always use opaque selling, but it is not first-best optimal to do so. However, opaque selling might be used at the constrained optimum (with the monopolist's pricing behavior taken as given). For linear disutility costs, it is optimal for a monopolist to offer a single opaque product.

Keywords: Opaque products; product line design; product differentiation; price discrimination.
JEL classification: L12; L13; L15.

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1 Introduction

We study "opaque" selling in multiproduct environments – a marketing practice in which sellers strategically withhold product information by keeping important characteristics of their products hidden until after purchase. Opaque selling is particularly prevalent and growing in the travel/tourism industry.[1] Online intermediaries such as Hotwire.com and Priceline.com engage in opaque selling by concealing hotel names and locations or airlines and departure/arrival times. Economycarrentals.com, an online car rental intermediary, reveals the name of the car rental company only after the customer pays for the service. Other venues where opaque selling is employed include Japanese "fukubukuro" or "omakase", subscription beer or wine boxes, etc.[2]

Focusing on market segmentation (and thereby price discrimination) as a motive for withholding information, we investigate in this paper the equilibrium and welfare properties of opaque selling. We consider a standard Hotelling model with a continuum of consumers who differ with respect to their ideal tastes and a monopoly seller. In the baseline model, we assume that the seller is equipped with two base products that are located at the two end-points of the unit line $[0, 1]$. We then extend the analysis to the case of many products. Besides offering each base product individually for sale, the firm can also design and sell any number of lotteries that award one of the base products as the final prize, but the consumer cannot observe the outcome until after purchase. The questions we address are: When can the seller profit from selling opaque products? How are base product prices affected? How many opaque products does the seller offer concurrently? Does opaque selling improve social welfare?

The literature on opaque selling is quite recent. The price discrimination motive for a monopolist is addressed in Jiang (2007) and Fay and Xie (2008) in a symmetric two-product Hotelling framework. They find the conditions under which offering a given opaque

[1] Online spending on travel products in the US alone totalled \$103 billion in 2012, which constituted roughly 40% of all US online spending on retail products (excluding auctions). Source: www.comscore.com.

[2] "Fukubukuro" is a Japanese New Year custom in which merchants make grab bags filled with unknown random contents and sell them for a substantial discount. "Omakase" is a form of Japanese dining in which guests leave themselves in the hands of a chef in choosing their meals.

product improves profits. In a similar setting, Balestrieri, Izmalkov and Leao (2017) solve the optimal selling mechanism allowing non-uniform pricing and an endogenous number of lotteries. They show that, depending on the shape of the transportation cost function, the monopolist may offer a single lottery, a continuum, or lotteries with positive probabilities of no sale. Thanassoulis (2004) and Pavlov (2010) reach similar results in a random utility setting.[3]

The main elements of the above models are similar to ours. We contribute to this literature by offering a tractable framework that relies heavily on graphical tools and economic reasoning. Building on the methodology we developed in Anderson and Celik (2015, henceforth AC), we provide a simple graphical characterization of a monopolist's optimal strategy using the elementary tools of virtual valuation curves. Given a set of base products, a monopolist will offer only those opaque products that extend the upper envelope of virtual valuations. When the transportation costs are linear, it is optimal to offer a single opaque product that offers the same expected utility to all consumers. This approach greatly simplifies the standard mechanism design approach used in Balestrieri et al. (2017) while allowing for possibly asymmetric base products and non-uniform consumer distributions.[4] Moreover, we are able to show that this result extends to multiple base products when the monopolist is free to offer any number of lotteries. To the best of our knowledge, none of the above papers consider lotteries of more than two products. Finally, our methodology enables us to show that opaque selling is never first-best optimal, but might improve welfare in a second-best sense. We are not aware of any earlier papers that offer any welfare analysis of opaque products.

[3]Besides market segmentation, firms may use opaque selling to: 1) expand market size by offering a larger product line, 2) dispose left-over capacity through an intermediary without damaging brand name, and 3) secure against fluctuations in demand. Fay (2004), Shapiro and Shi (2008) and Tappata (2012) focus on motive 1 along with market segmentation, whereas Jerath et al. (2010) address a combination of motives 2 and 3 in a two-period model with two single-product capacity-constrained firms.

[4]In this respect, our contribution can be viewed similar in spirit to that of Bulow and Roberts (1989) who revisited the mechanism design approach of Myerson (1981) by applying the analysis of standard monopoly third-degree price discrimination.

2 Baseline model

Consider a market with a unit mass of consumers and a single firm (M) equipped with two horizontally-differentiated base products, $i = 1, 2$. Besides offering each product individually for sale, M can also sell lotteries that award one of the two products as the final prize. In the latter case, consumers do not observe the outcome until after purchase. Each consumer demands a single unit of the product yielding the highest expected utility, provided this is non-negative. M's problem is to design lotteries and choose prices of its products.

We describe each consumer by a unidimensional taste parameter θ, distributed over $[0, 1]$ according to a twice differentiable c.d.f. $F(\theta)$. Assume the corresponding density $f(\theta)$ is log-concave. This also ensures that $F(\theta)$ and $1 - F(\theta)$ are log-concave. The valuations are in the standard linear-cost Hotelling form: $u_1(\theta) = R_1 - t_1\theta$ and $u_2(\theta) = R_2 - t_2(1 - \theta)$, where $R_i, t_i >> 0$ for $i = 1, 2$. We allow R_i and t_i to differ across the two products to allow for asymmetric configurations. In particular, $R_1 \neq R_2$ captures any inherent quality differences across products. We assume identical constant marginal costs of production, which we normalize to zero. Hence, each $u_i(\theta)$ measures cost-normalized net valuation.

In all derivations below, we assume that $\max\{u_1(\theta), u_2(\theta)\} > 0$ for all θ. This means that it is socially optimal to serve all consumers with one of the products. A necessary and sufficient condition to ensure this holds is $\frac{R_1}{t_1} > 1 - \frac{R_2}{t_2}$. We also assume that, in the absence of lotteries, the monopolist offers both base products in strictly positive quantities. This requires that R_1 and R_2 are not too apart from each other. The precise restriction will be specified in the next subsection.

2.1 Equilibrium analysis - no lotteries

We first solve the optimal product selection, pricing and welfare properties without opaque selling. These results extend AC, who only considered vertical differentiation. Let p_i denote the price of product i. Suppose for now that the market is fully covered. There will then be a unique marginal consumer $\hat{\theta} \in [0, 1]$ indifferent between the two products, and M's profits are

$$\pi = p_1 F(\hat{\theta}) + p_2(1 - F(\hat{\theta})). \tag{1}$$

The choice of prices must obey:

$$u_1(\hat{\theta}) - p_1 = u_2(\hat{\theta}) - p_2 = 0. \tag{2}$$

Hence, consumer $\hat{\theta}$ gets zero utility. Otherwise M could increase both prices and still serve all consumers. Plugging $p_1 = u_1(\hat{\theta})$ and $p_2 = u_2(\hat{\theta})$ into (1):

$$\pi = u_1(\hat{\theta})F(\hat{\theta}) + u_2(\hat{\theta})(1 - F(\hat{\theta})).$$

Differentiating π with respect to $\hat{\theta}$, we get in any interior solution

$$u_1(\hat{\theta}) + u_1'(\hat{\theta})\frac{F(\hat{\theta})}{f(\hat{\theta})} = u_2(\hat{\theta}) - u_2'(\hat{\theta})\frac{1 - F(\hat{\theta})}{f(\hat{\theta})}. \tag{3}$$

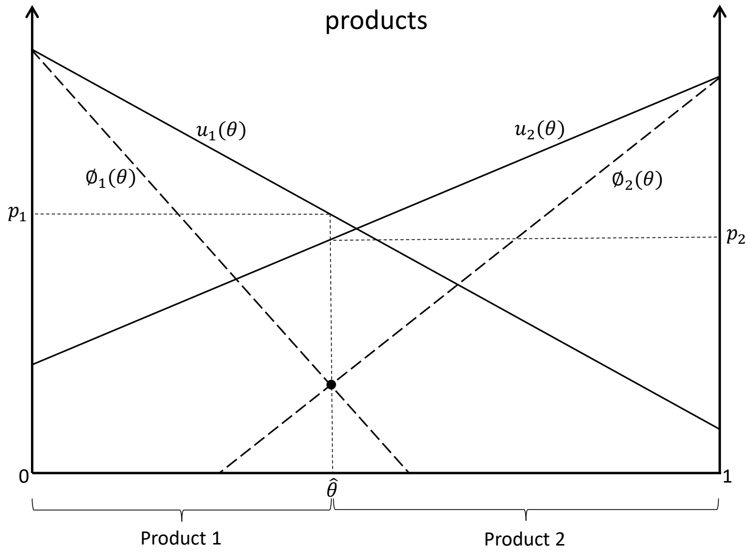
Each side of this equality measures the underlying "virtual" valuation for product 1 and 2, respectively, evaluated at $\theta = \hat{\theta}$. This is no coincidence; our analysis in AC was also based on conditional stand-alone inverse demands and the corresponding marginal revenue curves. Here, because of horizontal differentiation, we use virtual valuations. Define by $\phi_i(\theta)$ consumer θ's virtual valuation for product i:

$$\phi_i(\theta) = \begin{cases} u_i(\theta) + \frac{F(\theta)}{f(\theta)} u_i'(\theta) & , u_i'(\theta) < 0 \\ u_i(\theta) - \frac{1-F(\theta)}{f(\theta)} u_i'(\theta) & , u_i'(\theta) > 0 \end{cases}. \tag{4}$$

As in the theory of auctions, $\phi_i(\theta)$ here measures the highest surplus M can extract from a θ-type consumer. Log-concavity of $f(\theta)$ implies that $\frac{F}{f}$ is increasing and $\frac{1-F}{f}$ decreasing in θ. Under linear transportation costs, this ensures that each $\phi_i'(\theta)$ has the same sign as the corresponding $u_i'(\theta)$.

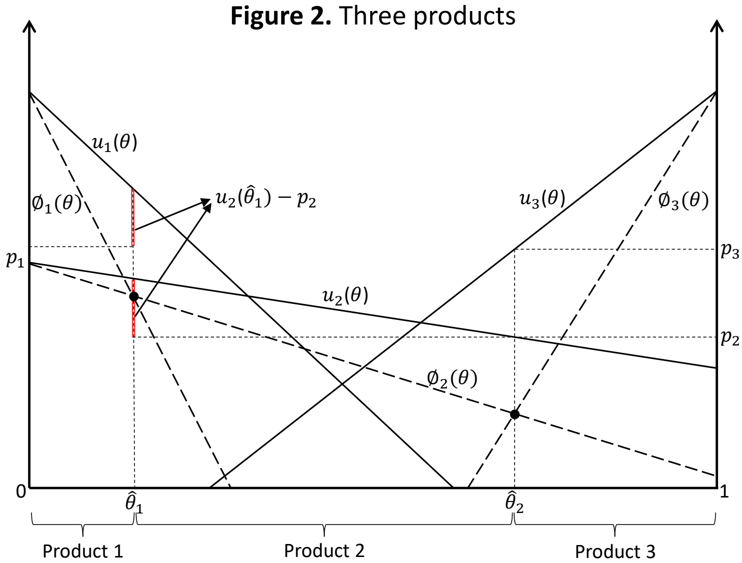
Graphically, it suffices to draw $\phi_1(\theta)$ and $\phi_2(\theta)$, and find the point $\hat{\theta}$ where they intersect. The corresponding prices to support this cutoff are then given by the constraints in (2), $p_1 = u_1(\hat{\theta})$ and $p_2 = u_2(\hat{\theta})$. This is graphically illustrated in Figure 1 below. To ensure that both products are sold in strictly positive quantities, we need $\phi_1(0) > \phi_2(0)$ and $\phi_1(1) < \phi_2(1)$. This requires $R_1 > R_2 - t_2\left(1 + \frac{1}{f(0)}\right)$ and $R_2 > R_1 - t_1\left(1 + \frac{1}{f(0)}\right)$, respectively. So, if $R_2 \geq R_1$ for instance, then we need that $\frac{R_2 - R_1}{t_2} < 1 + \frac{1}{f(0)}$.

Figure 1. Equilibrium with no opaque products



As in AC, this result can be generalized to any number of products: simply draw the virtual valuations for all products and find the upper envelope. M chooses its product line and the corresponding prices according to this envelope.[5] Figure 2 illustrates.

Figure 2. Three products



The above analysis also goes through when base utilities are not sufficiently high. In this case, $\phi_1(\theta)$ and $\phi_2(\theta)$ will not have an interior intersection and M will find it optimal

[5]Specifically, for a given set of available products $N = \{1, ..., n\}$, M will include product i in its product line if and only if $\phi_i(\theta) > \max\{\max_{j \neq i} \phi_j(\theta), 0\}$ for some $\theta \in (0, 1)$.

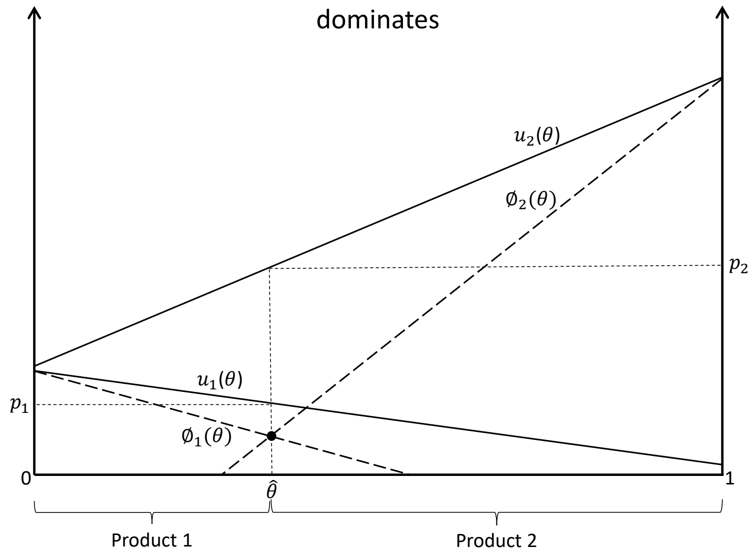
to serve a strict subset of consumers. Then, there will be two cutoff consumer locations, $\hat{\theta}_1 < \hat{\theta}_2$, such that consumers to the left of $\hat{\theta}_1$ purchase product 1, consumers to the right of $\hat{\theta}_2$ purchase product 2 and those in between stay out of the market. Equilibrium cutoff points in this case are given by $\phi_1(\hat{\theta}_1) = \phi_2(\hat{\theta}_2) = 0$ with the resulting prices $p_1 = u_1(\hat{\theta}_1)$ and $p_2 = u_2(\hat{\theta}_2)$.

First-best efficiency:

A product is socially optimal to offer for sale if, under marginal cost pricing, it generates the highest positive surplus for some consumers. In our framework, the set of socially optimal products corresponds to the upper envelope of valuation functions. If $u_i(\theta)$ belongs to this upper envelope, then it is socially optimal to consume product i. However, M's equilibrium behavior is governed fully by the upper envelope of virtual valuations. As a result, first-best product selection might differ from what M offers in equilibrium.

Consider the example depicted in Figure 3. Only product 2 should be consumed at the social optimum since $u_2(\theta) > u_1(\theta)$ for all θ. However, since $\phi_1(\theta)$ and $\phi_2(\theta)$ intersect at an intermediate point, M offers both products for sale. In other words, there is a market failure in terms of product selection.

Figure 3. When one product dominates



Even when M offers the socially optimal set of products, M's pricing might distort welfare via consumption inefficiencies. In Figure 1, for instance, while it is socially optimal that all consumers to the left of the intersection of $u_1(\theta)$ and $u_2(\theta)$ consume product 1, only those to the left of $\hat{\theta}$ consume it in equilibrium. As we will see next, opaque selling can restore some of this welfare loss by improving product match.

2.2 Opaque selling

We now allow M to offer opaque products along with the two base options. First, we consider a single lottery L_α that delivers products 1 and 2 with probabilities $\alpha \in (0,1)$ and $1-\alpha$, respectively. Consumers know α and are expected utility maximizers. M sets prices p_1, p_2 and p_{L_α} for the three products and consumers self-select. The expected valuation of lottery L_α for a consumer located at θ is

$$\begin{aligned} u_{L_\alpha}(\theta) &= \alpha u_1(\theta) + (1-\alpha)\, u_2(\theta) \\ &= \alpha R_1 + (1-\alpha)(R_2 - t_2) - (\alpha t_1 - (1-\alpha)\, t_2)\, \theta. \end{aligned}$$

In a fully-covered market configuration in which M sells all three products in positive quantities, there will be two threshold consumers, $\hat{\theta}_1 < \hat{\theta}_2$, such that consumers with $\theta < \hat{\theta}_1$ purchase product 1, consumers with $\theta > \hat{\theta}_2$ purchase product 2 and those in between purchase the lottery. Thus,

$$\pi = p_1 F(\hat{\theta}_1) + p_{L_\alpha}(F(\hat{\theta}_2) - F(\hat{\theta}_1)) + p_2(1 - F(\hat{\theta}_2)),$$

where prices satisfy

$$\begin{aligned} u_{L_\alpha}(\theta) - p_{L_\alpha} \geq 0 \text{ for all } \theta \in [\hat{\theta}_1, \hat{\theta}_2], \\ u_1(\hat{\theta}_1) - p_1 = u_L(\hat{\theta}_1) - p_{L_\alpha}, \\ u_2(\hat{\theta}_2) - p_2 = u_L(\hat{\theta}_2) - p_{L_\alpha}. \end{aligned}$$

Given that $u_{L_\alpha}(\theta) < \max\{u_1(\theta), u_2(\theta)\}$ for any $\alpha \in (0,1)$, **the lottery is never the most preferred product for any consumer**. In other words, a social planner would never use opaque selling in a first-best allocation. However, we show below that $\phi_{L_\alpha}(\theta)$ is **always** part of the upper envelope of virtual valuation functions as long as $\max\{u_1(\theta), u_2(\theta)\} > 0$ for all θ. Hence, it will always be offered in market equilibrium.

Proposition 1 *If* $\max\{u_1(\theta), u_2(\theta)\} > 0$ *for all* θ*, then, for any* $\alpha \in (0,1)$*, it is always optimal to offer lottery* L_α *for sale.*

Proof. Suppose first that $\phi_1(\theta)$ and $\phi_2(\theta)$ have an interior intersection, so $\phi_1(\hat{\theta}) = \phi_2(\hat{\theta}) > 0$. Assume, without loss of any generality, that $\alpha t_1 \leq (1-\alpha) t_2$ so that $u'_{L_\alpha}(\theta) \geq 0$ (the opposite case is symmetric). Then,

$$\begin{aligned}
\phi_{L_\alpha}(\theta) &= u_{L_\alpha}(\theta) - \frac{1-F(\theta)}{f(\theta)} u'_{L_\alpha}(\theta) \\
&= \alpha R_1 + (1-\alpha)(R_2 - t_2) - (\alpha t_1 - (1-\alpha) t_2)\left(\theta - \frac{1-F(\theta)}{f(\theta)}\right) \\
&= \alpha \phi_1(\theta) + (1-\alpha)\phi_2(\theta) + \alpha t_1 \frac{1}{f(\theta)}.
\end{aligned}$$

Evaluated at $\theta = \hat{\theta}$, it follows that $\phi_{L_\alpha}(\hat{\theta}) = \phi_1(\hat{\theta}) + \alpha t_1 \frac{1}{f(\hat{\theta})} > \phi_1(\hat{\theta})$. This means that the opaque product is part of the upper envelope of virtual valuations. Thus, from the analysis in the previous section, it is optimal to sell the opaque product. It is also easy to see that $\phi_1(0) = R_1 > \phi_{L_\alpha}(0)$ and $\phi_2(1) = R_2 > \phi_{L_\alpha}(1)$, so all three products will be offered in equilibrium.

Next, suppose that $\phi_1(\theta)$ and $\phi_2(\theta)$ do not have an interior intersection. In this case, there will be two cutoff consumer locations, $\hat{\theta}_1 < \hat{\theta}_2$, given by $\phi_1(\hat{\theta}_1) = \phi_2(\hat{\theta}_2) = 0$. Assume again that $\alpha t_1 \leq (1-\alpha) t_2$ so that $u'_{L_\alpha}(\theta) \geq 0$. Evaluated at $\theta = \hat{\theta}_2$, we have $\phi_2(\hat{\theta}_2) = R_2 - t_2(1-\hat{\theta}_2) - \frac{1-F(\hat{\theta}_2)}{f(\hat{\theta}_2)} t_2 = 0$. This implies that $\hat{\theta}_2 - \frac{1-F(\hat{\theta}_2)}{f(\hat{\theta}_2)} = 1 - \frac{R_2}{t_2}$. Then,

$$\begin{aligned}
\phi_{L_\alpha}(\hat{\theta}_2) &= \alpha R_1 + (1-\alpha)(R_2 - t_2) - (\alpha t_1 - (1-\alpha) t_2)\left(\hat{\theta}_2 - \frac{1-F(\hat{\theta}_2)}{f(\hat{\theta}_2)}\right) \\
& \alpha R_1 + (1-\alpha)(R_2 - t_2) - (\alpha t_1 - (1-\alpha) t_2)\left(1 - \frac{R_2}{t_2}\right) \\
&= \alpha t_1 \left[\frac{R_1}{t_1} - \left(1 - \frac{R_2}{t_2}\right)\right] > 0,
\end{aligned}$$

where the strict inequality follows from $\max\{u_1(\theta), u_2(\theta)\} > 0$. Thus, $\phi_{L_\alpha}(\hat{\theta}_2) > \phi_2(\hat{\theta}_2) = 0$, which implies that the opaque product is part of the upper envelope of virtual valuations. ■

By offering an opaque product, the monopolist is able to increase its profits via better market segmentation. The consumers in the middle do not have strong preferences for either product, so it is optimal for M to offer a different product to these consumers. This, in turn, enables M to charge a higher price to those consumers with a stronger preference for either product.[6]

Next, using a nice graphical property that we first explored and utilized in AC, we show that even when M can offer any number of opaque products with differing probabilities, it will choose to offer only a single one: the one with probabilities such that the resulting expected valuation is independent of θ.

Proposition 2 *It is optimal for M to offer only a single opaque product that delivers product* 1 *with probability* $\alpha = \frac{t_2}{t_1+t_2}$ *and product* 2 *with probability* $1-\alpha$.

Proof. The proof follows from the striking property that two valuation functions with slopes of same sign cross each other at a height of $\eta > 0$ if and only if their corresponding virtual valuation functions also cross at a height of η. To see this, take a lottery L_α with $\alpha t_1 \leq (1-\alpha) t_2$ so that $u'_{L_\alpha}(\theta) \geq 0$. Suppose $u_2(\theta) = u_{L_\alpha}(\theta) = \eta > 0$ for some θ. Then, at such θ,

$$R_2 - t_2(1-\theta) = \alpha(R_1 - t_1\theta) + (1-\alpha)(R_2 - t_2(1-\theta))$$

$$\Leftrightarrow R_1 - t_1\theta = R_2 - t_2(1-\theta)$$

$$\Leftrightarrow \theta = \frac{R_1 - R_2 + t_2}{t_1 + t_2}.$$

Hence,

$$\eta = R_1 - t_1\frac{R_1 - R_2 + t_2}{t_1 + t_2} = \frac{t_2 R_1 + t_1 R_2 - t_1 t_2}{t_1 + t_2}.$$

[6] Having two base products is crucial for this result. In their seminal paper, Riley and Zeckhauser (1983) show that a single-product monopolist cannot profit from using lotteries and that the optimal mechanism is a take-it-or-leave-it offer.

Similarly, if $\phi_2(\tilde{\theta}) = \phi_{L_\alpha}(\tilde{\theta})$ for some $\tilde{\theta}$, then it must be that

$$R_2 - t_2 + t_2\left(\tilde{\theta} - \frac{1-F(\tilde{\theta})}{f(\tilde{\theta})}\right) = \alpha R_1 + (1-\alpha)(R_2 - t_2) - (\alpha t_1 - (1-\alpha)t_2)\left(\tilde{\theta} - \frac{1-F(\tilde{\theta})}{f(\tilde{\theta})}\right)$$

$$\Leftrightarrow \tilde{\theta} - \frac{1-F\left(\tilde{\theta}\right)}{f\left(\tilde{\theta}\right)} = \frac{\alpha R_1 - \alpha(R_2 - t_2)}{\alpha t_1 + \alpha t_2}.$$

This, then, implies that at any such crossing,

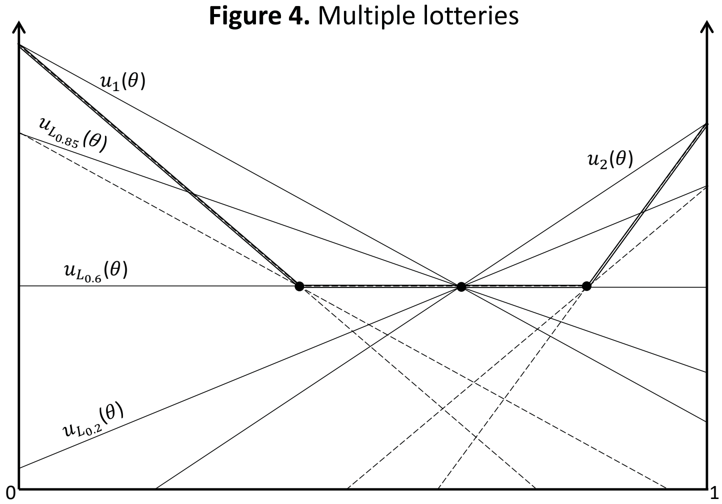
$$\phi_2(\tilde{\theta}) = \phi_{L_\alpha}(\tilde{\theta}) = R_2 - t_2 + t_2\left(\tilde{\theta} - \frac{1-F(\tilde{\theta})}{f(\tilde{\theta})}\right) = \eta.$$

Thus, virtual valuations cross at the same height as the corresponding valuations.

With this property in hand, it is easy to prove the result. First note that, all $u_{L_\alpha}(\theta)$ cross through the intersection point of $u_1(\theta)$ and $u_2(\theta)$ since $u_{L_\alpha}(\theta) = \alpha u_1(\theta) + (1-\alpha)u_2(\theta)$. Suppose that $u_1(\theta)$ and $u_2(\theta)$ intersect at $\theta = \bar{\theta} \in (0,1)$. Take any two lotteries L_{α_1} and $L_{\alpha_2}$with $\alpha_1 t_1 \leq (1-\alpha_1)t_2$ and $\alpha_2 t_1 \leq (1-\alpha_2)t_2$ so that so that $u'_{L_{\alpha_1}}(\theta), u'_{L_{\alpha_2}}(\theta) \geq 0$. Since $u_2(\theta)$, $u_{L_{\alpha_1}}(\theta)$ and $u_{L_{\alpha_2}}(\theta)$ cross each other at $\bar{\theta}$, their virtual valuations $\phi_2(\theta)$, $\phi_{L_{\alpha_1}}(\theta)$ and $\phi_{L_{\alpha_2}}(\theta)$ must cross each other at a single point where their height is $u_2\left(\bar{\theta}\right)$. In other words, there is a common pivotal point to the right of $\bar{\theta}$ with a height of $u_2\left(\bar{\theta}\right)$ where $\phi_2(\theta)$ and all $\phi_{L_\alpha}(\theta)$ with $\phi'_{L_\alpha}(\theta) \geq 0$ cross each other. The upper envelope of these virtual valuations will then contain only $\phi_2(\theta)$ and $\phi_{L_{\tilde{\alpha}}}(\theta)$ where $\tilde{\alpha} = \frac{t_2}{t_1+t_2}$. The latter is the virtual valuation for the lottery $u_{L_{\tilde{\alpha}}}(\theta) = \frac{t_2 R_1 + t_1(R_2 - t_2)}{t_1+t_2}$, which offers the same gross utility to all consumers ($u'_{L_{\tilde{\alpha}}}(\theta) = 0$). The argument is symmetric for $\phi_1(\theta)$ and all $\phi_{L_\alpha}(\theta)$ with $\phi'_{L_\alpha}(\theta) \leq 0$. Hence, besides the two base products, it is optimal to offer only the lottery with $\alpha = \frac{t_2}{t_1+t_2}$.

If, on the other hand, $u_2(\theta) > u_1(\theta)$ for all θ while $\phi_1(0) > \phi_2(0)$, then offering only the lottery with $\alpha = \frac{t_2}{t_1+t_2}$ is still optimal, but the monopolist will not sell the first base product at all since its virtual valuation will be dominated with that of the lottery. Hence, the possibility of opaque selling can induce the seller to stop offering a particular base product for sale, which can have important welfare consequences (see section 4). An example is illustrated in Figure 7.■

Figure 4. Multiple lotteries



Proposition 2 is illustrated in Figure 4. Tracing through the upper envelope of all virtual valuations, we get only the flat part besides ϕ_1 and ϕ_2 – the part associated with the lottery L_α where $\alpha = \frac{t_2}{t_1+t_2}$. In other words, the optimal lottery equates the virtual valuations – and thus marginal profits – of all three products M sells. By offering a single opaque product which is independent of θ, M makes sure it leaves no (expected) surplus to its consumers. Since this enables a higher markup on products 1 and 2, M has no incentive to offer any other lotteries.

Our graphical solution method is valid only when transportation costs are linear. Using a mechanism design approach, Balestrieri et al. (2017) also show that offering a single opaque product is optimal when the transportation costs are linear; otherwise, depending on the shape of the transportation costs, the monopolist offers a continuum of opaque products, or lotteries with positive probabilities of no sale. However, they restrict their analysis to two symmetric base products (i.e., $R_1 = R_2$ and $t_1 = t_2$) and uniform consumer distribution. Our approach greatly simplifies their analysis for linear transportation costs and extends it to asymmetric base products as well as non-uniform consumer distributions. Moreover, as we show in the next subsection, we can extend our results to the case of multiple base products.

3 Many products

Suppose now that M has many products and can design and sell any number of lotteries involving two or more products. We restrict attention to products that are located either at 0 or 1. Consider, for instance, the example depicted in Figure 2, where products 1 and 2 are located at 0 and product 3 at 1. Products 1 and 2 differ by their base utilities as well as transportation costs ($R_1 > R_2$ and $t_1 > t_2$), which can be thought of as product 1 having higher durability but offering more niche properties.[7] In this setting, would M offer multiple lotteries in equilibrium? Would it offer lotteries of three products along with lotteries of two products?

In Figure 2, $u_2(\theta)$ and $u_3(\theta)$ cross each other at a higher point than where $u_1(\theta)$ and $u_3(\theta)$ cross. We show below that, under linear transportation costs, it is optimal for M to offer only a single opaque product that delivers products 2 and 3 with the underlying probabilities $\frac{t_3}{t_2+t_3}$ and $\frac{t_2}{t_2+t_3}$ such that the resulting valuation function is flat. This configuration and the resulting market segmentation are depicted in Figure 5 below. The double-lined boundary is the upper envelope of all virtual valuations.

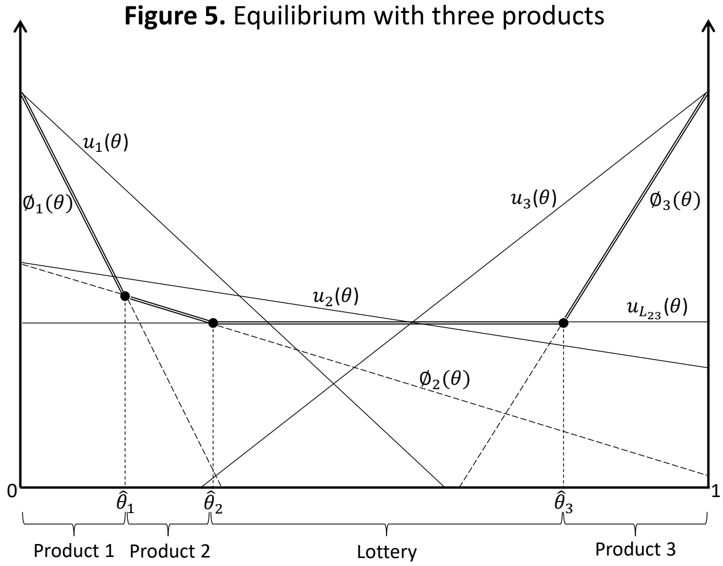
Proposition 3 *When M has three or more products, each located at either* 0 *or* 1*, it is optimal to offer only a single opaque product that delivers products j and k with probabilities $\alpha = \frac{t_k}{t_j+t_k}$ and $1-\alpha$, where products j and k are the two products whose intersection point is the highest among all product pairs with opposite slopes.*

[7] We can also construct such products from the standard Lancasterian characteristics model. Suppose each product characteristic is a 2-tuple $(\alpha_j, \beta_j) \in \mathbb{R}^2$ and each product x is a collection of a subset of given characteristics. If a θ-type consumer purchases product x, her gross utility is $u(\theta; x) = \sum_{j\in x} \alpha_j + \sum_{j\in x} \beta_j \theta$, where $\theta \in [0,1]$. Hence, each characteristic consists of two vertical attributes. Suppose there are only three characteristics, $(\alpha_1, \beta_1) = (R, -2t)$, $(\alpha_2, \beta_2) = (-t, t)$ and $(\alpha_3, \beta_3) = (0, 3t)$, and let product 1 have characteristic 1 only, product 2 has characteristics 1 and 2, and product 3 has characteristics 1 and 3. Then,

$$\begin{aligned}
u_1(\theta) &= R - 2t\theta, \\
u_2(\theta) &= (R-t) - t\theta, \\
u_3(\theta) &= (R+t) - t(1-\theta),
\end{aligned}$$

which corresponds to the example depicted in Figure 2. See Anderson and Celik (2018) for further details of this approach.

Figure 5. Equilibrium with three products

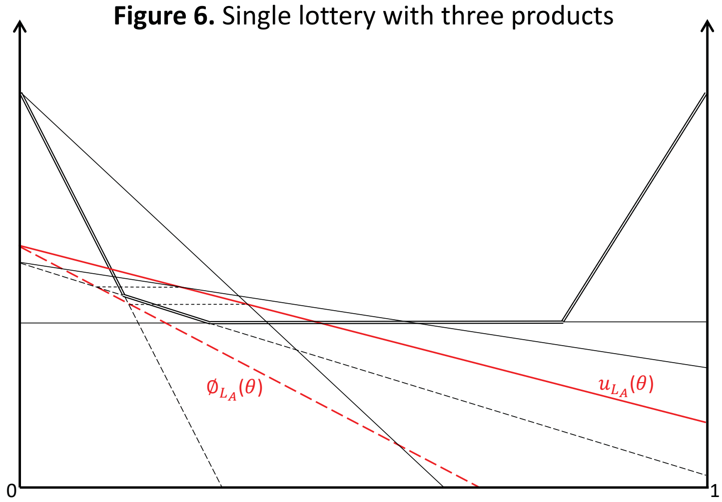


Proof. The proof directly follows from the property that was utilized in AC and in Proposition 2 above (i.e., the property that two valuation functions with slopes of same sign cross each other at the same height as their corresponding virtual valuation functions do). Consider the product configuration depicted in Figure 5. First, we know from Proposition 2 that any other lottery of products 2 and 3 will be strictly dominated by lottery L_{23} that delivers products 2 and 3 with probabilities $\alpha = \frac{t_3}{t_2+t_3}$ and $1-\alpha$. Second, any lottery of products 1 and 3 will be strictly dominated by lottery L_{23} since $u_2(\theta)$ and $u_3(\theta)$ cross each other at a higher point than where $u_1(\theta)$ and $u_3(\theta)$ cross. Next, observe that any lottery of products 1 and 2 will be unprofitable to offer because the virtual valuation of any such lottery will be strictly bracketed between $\phi_1(\theta)$ and $\phi_2(\theta)$, thus lying below the upper envelope $\max\{\phi_1(\theta), \phi_2(\theta)\}$.[8] Finally, any lottery L_A that delivers all three products with strictly positive probabilities will also be strictly dominated because, regardless of its slope, $u_{L_A}(\theta) < u_2(\theta)$ at the point where $u_1(\theta) = u_2(\theta)$ as well as where $u_2(\theta) = u_3(\theta)$. So, if $u'_{L_A}(\theta) \leq 0$, then it must be that $u_{L_A}(\theta) < \max\{u_1(\theta), u_2(\theta), u_{L_{23}}(\theta)\}$. Similarly, if $u'_{L_A}(\theta) \geq 0$, then it must be that $u_{L_A}(\theta) < \max\{u_{L_{23}}(\theta), u_3(\theta)\}$. Then, by the property of AC, $\phi_{L_A}(\theta)$ will be strictly below the upper envelope $\max\{\phi_1(\theta), \phi_2(\theta), \phi_{L_{23}}(\theta), \phi_3(\theta)\}$.

[8] This observation is more generally valid in the setting of AC with multiple products. Hence, under linear valuations, opaque selling with vertically differentiated base products is suboptimal.

An example is depicted in Figure 6 below. These arguments apply equally to any number of products that are located either at 0 and 1.■

Figure 6. Single lottery with three products



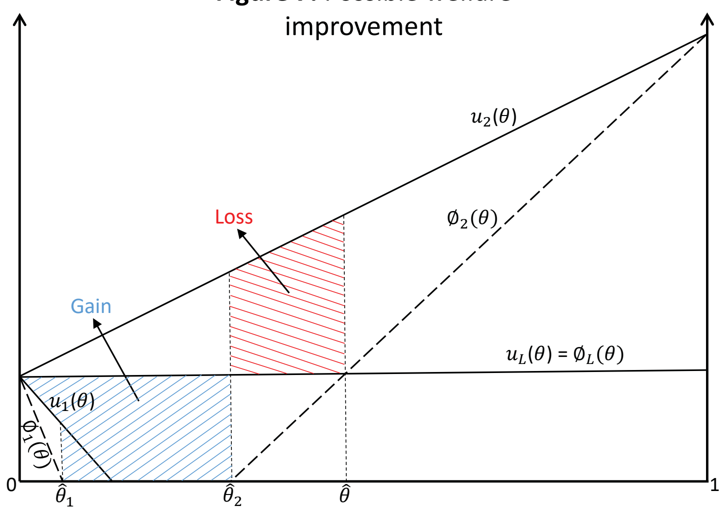
We are not aware of any earlier analysis of opaque selling with multiple base products. Whether a monopolist would offer multiple lotteries or any lotteries of more than two products in this setting are important questions and could generally be difficult to solve in a mechanism design framework. Our tractable graphical approach provides a simple and intuitive answer.

4 Welfare properties of opaque selling

We have argued that a social planner would never use opaque selling in a first-best allocation with marginal-cost pricing. However, first-best is rarely achievable. Here, we address the welfare implications of opaque selling in the second-best solution where the planner takes the seller's pricing behavior as given. Consider Figure 7 below. Without opaque selling, the monopolist sells product 1 to consumers with $\theta \leq \hat{\theta}_1$ and product 2 to consumers with $\theta \geq \hat{\theta}_2$, where $\hat{\theta}_2 > \hat{\theta}_1$. Thus, the market is not fully covered, but each product is consumed in strictly positive quantities (thus satisfying our restrictions that $\max\{u_1(\theta), u_2(\theta)\} > 0$, $\phi_1(0) > \phi_2(0)$ and $\phi_1(1) < \phi_2(1)$).

From a first-best perspective, product 1 should not be offered at all. But M has incentives to sell it because of price discrimination motives. Suppose now that M also offers an opaque product. By Proposition 2, the optimal opaque product to offer will be the one with $\alpha = \frac{t_1}{t_1+t_2}$, which corresponds to the flat valuation indicated as $u_L(\theta) = \phi_L(\theta)$ in the figure. Since this is above $\phi_1(\theta)$ for all θ, M will no longer offer product 1 for sale. $\phi_L(\theta)$ and $\phi_2(\theta)$ intersect at $\hat{\theta}$, so M will now serve the whole market, selling the opaque product to consumers with $\theta < \hat{\theta}$ at a price of $p_L = u_L(\hat{\theta})$ and product 2 to consumers with $\theta \geq \hat{\theta}$ at a price of $p_2 = u_2(\hat{\theta})$.

Figure 7. Possible welfare improvement



At the extensive margin, this will bring welfare gains because of the increased market coverage. In contrast, since M will increase p_2, there will be welfare losses at the intensive margin. To be more precise, all consumers who previously pruchased product 1 will switch to the opaque product, implying a welfare gain of $\int_0^{\hat{\theta}_1} (u_L(\theta) - u_1(\theta))\, dF(\theta)$ thanks to better expected product match. Those consumers who initially stayed out of the market are now willing to purchase the opaque product, which brings expected welfare gains of $\int_{\hat{\theta}_1}^{\hat{\theta}_2} u_L(\theta)\, dF(\theta)$. Thus, the total welfare gain is $\int_0^{\hat{\theta}_2} u_L(\theta)\, dF(\theta) - \int_0^{\hat{\theta}_1} u_1(\theta)\, dF(\theta)$. At the intensive margin, those consumers with $\theta \in (\hat{\theta}_2, \hat{\theta})$ switch from product 2 to the opaque product because of the new prices under opaque selling, which, due to decreased product match, causes a welfare loss of $\int_{\hat{\theta}_2}^{\hat{\theta}} (u_2(\theta) - u_L(\theta))\, dF(\theta)$. Overall, each of these might dominate, so the effect is unclear. In Figure 7, we depict an example with $F(\theta) = \theta$, where the gains (blue area) dominate the losses (green area). Hence, it is quite viable that opaque selling raises social welfare by increasing market participation and improving product match.[9]

As a specific example, take $u_1 = R - \theta$ and $u_2 = R + t\theta$, where $R < \frac{t}{1+t}$ to ensure that $\hat{\theta}_2 > \hat{\theta}_1$. Equlibrium values of $\hat{\theta}_1$ and $\hat{\theta}_2$ are easily found by $\phi_1 = R - 2\hat{\theta}_1 = 0$ and $\phi_2 = R - t + 2t\hat{\theta}_2 = 0$, which give $\hat{\theta}_1 = \frac{R}{2}$ and $\hat{\theta}_2 = \frac{t-R}{2t}$. The optimal opaque product to offer is the one with $\alpha = \frac{t}{1+t}$ and $1 - \alpha = \frac{1}{1+t}$, which implies $u_L = \frac{t}{1+t}(R - \theta) + \frac{1}{1+t}(R + t\theta) = R$, and hence $\phi_L = u_L$. The equlibrium value of $\hat{\theta}$ is then given by $\phi_2 = \phi_L$, implying $\hat{\theta} = \frac{1}{2}$. Now, looking at Figure 7, we can calculate the welfare gains and losses as

$$\text{Gains} \equiv \hat{\theta}_2 R - \frac{\hat{\theta}_1}{2}(R + u_1(\hat{\theta}_1)) = \frac{(t-R)\,R}{2t} - \frac{3R^2}{8} = \frac{(4t-7R)\,R}{8t},$$

$$\text{Losses} \equiv \frac{\hat{\theta} - \hat{\theta}_2}{2}(u_2(\hat{\theta}_2) - R + u_2(\hat{\theta}) - R) = \frac{R}{4t}\left(\frac{t-R}{2} + \frac{t}{2}\right) = \frac{(2t-R)\,R}{8t}.$$

Hence, welfare gains dominates the losses if $4t - 7R > 2t - R$, or equivalently if $R < \frac{t}{3}$. An example that satisfies these restrictions is $R = 0.3$ and $t = 1$.

5 Conclusion

We study opaque selling in a Hotelling setting using graphical tools to find the optimal solution to a multi-product monopolist's problem. We show that it is always profitable to offer an opaque product as long as it is socially optimal to serve all consumers with a product. For linear disutility costs, a monopolist offers a single opaque product even when it could offer many. Opaque selling is socially suboptimal, but might improve welfare in a second-best sense taking the monopolist's pricing behavior as given.

We can generalize some of these results. For instance, profitability of opaque selling for the monopolist (Proposition 1) extends to more general (non-linear) transportation costs. However, characterization of equilibrium with opaque products becomes less tractable. Nor-

[9] Varian (1985) showed that for social welfare to increase with price discrimination, output must increase too. This is also true in our model. If, without opaque selling, the monopolist finds it optimal to serve the whole market, then offering an opaque product will lower the social welfare.

mative results also go through in a more general setting: opaque selling is always socially suboptimal, but might improve welfare in a second-best sense.

We have assumed identical marginal costs. If they are different, the monopolist might have incentives to deliver the less-costly product to those consumers who purchased the opaque product, thus dishonoring the underlying lottery. Since consumers would anticipate this from the beginning, opaque selling will then fail. However, credible commitment power (e.g., announcing the lottery in the beginning and sticking to it thereafter) or binding capacity constraints will restore the result.

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