CALCULATION OF STATIC CHARACTERISTICS OF LINEAR STEP MOTORS FOR CONTROL ROD DRIVES OF NUCLEAR REACTORS - AN APPROXIMATE APPROACH

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ABSTRACT

This paper describes an approximate method for calculating the static characteristics of linear step motors (LSM), being developed for control rod drives (CRD) in large nuclear reactors. The static characteristic of such a LSM which is given by the variation of electromagnetic force with armature displacement determines the motor performance in its standing and dynamic modes. The approximate method of calculation of these characteristics is based on the permeance analysis method applied to the phase magnetic circuit of LSM. This is a simple, fast and efficient analytical approach which gives satisfactory results for small stator currents and weak iron saturation, typical to the standing mode of operation of LSM. The method is validated by comparing theoretical results with experimental ones.

INTRODUCTION

Despite the Chernobyl accident, the contribution of atomic power plants in the total output of electric energy in many countries around the world is still rising. This intensive development of atomic energy and the tendency to increase the unit power output of reactors set the complex task of ensuring their safe, reliable and economic exploitation. This is accomplished by ensuring controllability of energy output locally as well as globally over the entire volume of the reactor core by suitably designing the CRD of nuclear reactors. Basically, it consists of movable control rods, made of neutron absorbing materials in the form of individual rods or group of rods (cassettes, clusters, etc.) and a driving mechanism to move them inside the reactor core [1-3]. The key element of this driving mechanism is the electric motor, upon the rational selection and reliable functioning of which, to a great extent depend the safety and reliability of the entire power plant. In recent years countries like Russia, USA, France, Germany and Italy are developing linear and discrete electromagnetic driving mechanisms for CRD with passive armature linear step motors [1-3]. These CRD with LSM are fast, highly reliable due to the simplicity of kinematics and accurate in the fixation of control rods.

One of the vital performance characteristics of LSM is the static characteristic which gives the variation of electromagnetic force produced by the motor with armature displacement. It determines motor performance in its standing (when the armature with controls rods attached to it is held at a fixed position by electromagnetic force) and dynamic (when the armature is moved by sequential excitation of stator windings) modes of operation. In order to design reliable and economically viable CRD with LSM it is important to be able to calculate and evaluate their static characteristics. Although rotating step motors are well covered in literature, there are a few published papers which concern linear step motors [4, 5]. Two methods, approximate and accurate have been developed [6, 7] for the calculation of static characteristics of LSM. The approximate method, described in this paper is based on the permeance analysis approach [5, 8] and gives satisfactory results for small stator currents and low iron saturation. This is quite useful for the fast evaluation of motor performance at the earlier stages of their CAD without needing the computationally intensive modelling of magnetic fields by the finite element method on which the accurate method is based. The approximate method was used to calculate the static characteristics of LSM designed by the researchers at "Ijorcki Javod" (St. Petersburg, Russia) for CRD (namely, linear synchronous electromagnetic drive, LSED) of large pressurised water reactors with electrical power output of 1000 MW (VVER-1000) and more [9, 10]. Some of the results are compared with experimental ones to establish the validity of the adopted approximate approach.

CONSTRUCTIVE FEATURES AND THE GEOMETRIC PARAMETERS OF A LSM

Figure 1 shows the longitudinal section of one of the phases of the 4-phase LSM designed at "Ijorcki Javod". It consists of the stator with cylindrical dc winding (1), poles (2) and ring elements in the form of
alternately arranged magnetic (3) and nonmagnetic (4) sleeves. The hermetically sealed cylinder (5) of the stator is made of nonmagnetic material (except the magnetic shunts under the poles) and withstands the high pressure inside the reactor vessel. The armature, in the form of a thin-wall hollow cylinder is made up of alternately arranged magnetic (6) and nonmagnetic (7) sleeves. These nonmagnetic sleeves are made up of the inner nonmagnetic material and acts as slide bearings. The height (length) of end (interphase) magnetic sleeves (8) of stator is higher than the height of those situated in between them (for example, 3). The outer casing of the motor (9) is made of magnetic material and acts as the outer magnetic circuit. The hollow cylindrical duct (10) in between the hermetically sealed cylinder of stator is chosen in such a way as to create a misalignment between stator and armature magnetic sleeves of adjacent phases. That is $l_{mi} = l_{mi} + t_4 + k_{ta} = l_m + t_4 (k = 1/m)$, where $k = 0, 1, 2$.

**TABLE 1 - Geometric parameters of LSM used in linear synchronous electromagnetic drives**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>LSED</th>
<th>LSED-4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Internal diameter of the armature cylinder Dci, mm</td>
<td>23</td>
<td>23</td>
</tr>
<tr>
<td>2. Internal diameter of armature magnetic sleeves Dai, mm</td>
<td>30</td>
<td>30.1</td>
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<tr>
<td>3. External diameter of armature magnetic sleeves Dae, mm</td>
<td>49</td>
<td>54.7</td>
</tr>
<tr>
<td>4. Internal diameter of stator magnetic sleeves Dai, mm</td>
<td>50</td>
<td>56.2</td>
</tr>
<tr>
<td>5. Internal diameter of stator magnetic sleeves Dsi, mm</td>
<td>76</td>
<td>76</td>
</tr>
<tr>
<td>6. Internal diameter of the no magnetic sleeves Dsi, mm</td>
<td>94</td>
<td>94</td>
</tr>
<tr>
<td>7. Internal diameter of stator poles Dp, mm</td>
<td>98</td>
<td>110</td>
</tr>
<tr>
<td>8. Width of stator poles bp, mm</td>
<td>16</td>
<td>16</td>
</tr>
<tr>
<td>9. Airgap between stator and armature Dg, mm</td>
<td>0.5</td>
<td>0.75</td>
</tr>
<tr>
<td>10. Width of the cooling duct dc, mm</td>
<td>16</td>
<td>16</td>
</tr>
<tr>
<td>11. Thickness of the hermetically sealed cylinder of stator $d_{si}$, mm</td>
<td>11</td>
<td>9</td>
</tr>
<tr>
<td>12. Height (length) of magnetic sleeves $l_{m}$, mm</td>
<td>29</td>
<td>25</td>
</tr>
<tr>
<td>13. Height (length) of nonmagnetic sleeves $l_{n}$, mm</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>14. Height (length) of end magnetic sleeves of stator $l_{mi}$, mm</td>
<td>92</td>
<td>81</td>
</tr>
<tr>
<td>15. Number of phases in stator $m$</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>16. Number of nonmagnetic sleeves of stator per phase $a$</td>
<td>8</td>
<td>9</td>
</tr>
</tbody>
</table>

**WORKING PRINCIPLE AND THE STATIC CHARACTERISTIC OF THE LSM**

With each voltage pulse current in the stator winding produces mmf $F$ that gives rise to the magnetic flux $\Phi = F/R(x) = F P(x)$, where $R(x)$ and $P(x)$ are phase reluctance and permeance respectively. Armature movement changes the mutual alignments of stator and armature magnetic sleeves resulting in the change of reluctance $R(x)$ and magnetic field energy $W(x) = F^2/2R(x) = F^2 P(x)/2$. These changes in energy $W(x)$ give rise to electromagnetic force $F(x)$ acting on the armature

$$F(x) = \frac{dW(x)}{dx} = \frac{F^2}{2} \frac{d}{dx} \left( \frac{1}{R(x)} \right) = \frac{F^2}{2} \frac{dP(x)}{dx}$$

(1)

where $x$ is the coordinate of armature displacement which characterises mutual alignments of stator and armature magnetic sleeves. Force $F(x)$ acts in such a direction as to increase field energy $W(x)$ and hence permeance $P(x)$. With armature movement, $P(x)$ and so the force $F(x)$ change periodically with a period of $t_4$ so that $F(x)$ changes with coordinate $x$ for a given phase. This is the static characteristic of the LSM shown in Figure 1. $x=0$ in Figure 3 corresponds to a certain armature position when the central line through stator nonmagnetic sleeve aligns with that of the armature magnetic sleeve. $F(x)$ attains its maximum for $x=x_m$ when the armature and stator magnetic sleeve corners
align opposite to each other and \( \frac{dW}{dx} \) becomes maximum. \( x=0 \) corresponds to maximum values of permeance \( P(x) \) and field energy \( W(x) \) and hence \( F(x)=\frac{dW(x)}{dx}=0; F(x)=0 \) also for \( x=\pm \alpha_2/2 \) when the magnetic sleeves of stator and armature are positioned opposite to each other and \( P(x) \) and \( W(x) \) have minimum values. Figure 4 shows static characteristics of all 4 phases of the LSM which are shifted in space by discrete step \( t_a \). This is because the height of end magnetic sleeves of stator are different from the height of those situated in between them. For a given armature position this creates different alignments of magnetic sleeves for different phases. \( F_1, F_2, F_3 \) and \( F_4 \) in Figure 4 correspond to electromagnetic forces produced by phases 1, 2, 3 and 4 respectively. To move the armature, phase windings are sequentially excited from pulse voltage source. If, for example only phase 1 is excited due to the total force \( P_1 + Q \) (\( Q \) is the force due to the weights of armature and control rods) the armature will take the equilibrium position corresponding to \( x=x_1 \) (Figure 4) for which \( P_1 + Q = 0 \). Now, if phase 1 is switched off and phase 2 is excited due to the force \( P_2 + Q \) armature will move to the new position for which \( x=x_2 = x_1 + t_a \). In this way armature is moved, each time by a distance of the discrete step \( t_a \) by sequentially exciting the phase windings in the order 1-2-3-4-1-2-... By reversing this order the armature can be made to move in the opposite direction.

**Calculation of the Static Characteristic of the LSM**

**Magnetic Circuit of a Phase of the LSM**

As can be seen from equation (1) to calculate the electromagnetic force \( F(x) \) it is necessary to determine the equivalent phase reluctance \( R(x) \) or permeance \( P(x) \). For approximate calculation of \( F(x) \) this is done by analysing the complex magnetic circuit of the LSM shown in Figure 5 which shows the probable flux paths through various parts of stator and armature. The total flux \( \Phi \) can be divided into stator and armature components \( \Phi_s, \Phi_a \) and the effective flux \( \Phi_e \) which crosses the airgap. Thus \( \Phi = \Phi_s + \Phi_a + \Phi_e \). Apart from these fluxes there can be, in principle leakage flux the lines of which take path entirely or partially through the air or nonmagnetic parts of the stator. The amount of such flux is small compared with the main flux \( \Phi \) and can, therefore be neglected. Figure 5 also shows the reluctances due to the flux paths \( R_s, R_a \) and \( R_e \) which constitute the phase reluctance \( R \). It consists of the reluctances of the outer casing \( R_o \), poles \( R_p \), nonmagnetic path between the poles and end magnetic sleeves of stator \( R_a \) and inner magnetic circuit \( R_{ic} \): \( R = R_c + 2R_p + 2R_a + R_{ic} \). Due to the large cross sectional areas of the outer casing and the poles their reluctances are considered negligibly small and \( R_c = R_a = 0 \). Reluctance \( R_p \) consists of the reluctances of the hermetically sealed cylinder \( R_{sc} \) and the cooling duct \( R_{cw} \) and can be approximately calculated using \( R_{pc} = R_{cw} + R_{sc} = \mu / \mu_0 S_c + \mu / \mu_0 S_s \) where \( S_c \) and \( S_s \) are the effective areas which the total flux crosses in the cooling duct and hermetically sealed cylinder respectively. Where \( \mu \) is the permeability.

Effective airgap areas \( S_c = \pi (D_p + D_se)bpe/2 \) where the effective pole width \( bpe = b + \delta_c + \delta_e \) in the presence of magnetic shunts and \( bpe = b + \delta_e \) in the absence of magnetic shunts in the hermetically sealed cylinder. The main difficulty is associated with the calculation of the reluctance \( R_{ic} \) which consists of the airgap reluctances \( R_1, R_2 \), reluctances due to end magnetic sleeves of stator \( R_{es} \), \( R_{es} \) and the reluctances of the magnetic \( R_{ms}, R_{ma} \) and nonmagnetic \( R_{ns}, R_{na} \) sleeves of stator and armature respectively. All these reluctances are functions of armature displacement \( x \) and need to be determined beforehand in order to calculate the phase reluctance \( R \) (or \( P \)). In the approximate method described in this paper given the above reluctances, \( R \) or \( P \) is calculated from the electric circuit analogue of the phase magnetic circuit shown in Figure 6. It takes into account complex flux distribution in the inner magnetic circuit of the LSM.

**Approximate Calculation of Permeances of the Airgap and Nonmagnetic Sleeves of Stator and Armature**

The approximate calculation of permeances of the airgap \( P_1, P_2 \) and nonmagnetic sleeves of stator and armature \( P_{ns}, P_{na} \) is based on the plotting of flux paths graphically or numerically in the airgap and
nonmagnetic sleeves. Alternatively, this may be done by assuming probable flux paths for various armature positions x. For these, it is assumed that the flux distribution in the above regions of the inner magnetic circuit does not depend on the saturation of stator and armature magnetic sleeves. After plotting the respective flux paths permeances are calculated analytically for various x. The results of such calculations for the 4-phase LSM used in LSED are shown in Figures 7 and 8. Due to the symmetry in flux distribution in the inner magnetic circuit it is sufficient to consider 0 ≤ x ≤ π/2. Figure 7 shows the variation of permeances P_1 and P_2 with armature displacement x which reflect the complex nature of flux distribution in the airgap for various positions of the armature. The constant redistribution of flux components \( \Phi_s, \Phi_a, \Phi_e \) with armature displacement x in the nonmagnetic sleeves is evident from the variation of permeances Pns and Pna shown in Figure 8.

### Calculation of Permeances Between the Armature and End Magnetic Sleeves of Stator

As said earlier the height of end magnetic sleeves of stator \( l_{mi} \) is different from that of other magnetic sleeves of stator and armature. This causes different flux distributions in the regions of end magnetic sleeves (shown schematically in Figure 9) for various armature positions x. The total flux \( \Phi \) in the end magnetic sleeve m branches out into components \( \Phi_s \) which shunts through the stator sleeves, and \( \Phi_s + \Phi_e = \Phi_{m1} + \Phi_{m2} + \Phi_{m3} \) which crosses the airgap taking three parallel paths before entering the armature sleeves. From Figures 5 and 6 permeance \( P_{c1} = 1/R_{c1} \) is defined as the permeance between the end magnetic sleeve m and the armature magnetic sleeve closest to it (marked 1 in Figure 9) through which the flux \( \Phi_s + \Phi_e \) flows. As shown in Figure 9 armature magnetic sleeves marked 2 and 3 also take part in conducting this flux. Considering all these the magnetic circuit shown in Figure 9 may be represented by the equivalent circuit shown in Figure 10 which is used to calculate the permeance \( P_{c1} \). For this it is assumed that the reluctances of magnetic sleeves 2 and 3 are negligibly small since a very small amount of flux passes through them and, therefore their flux density is considerably smaller than the saturation density. The following symbols are used in Figure 10: \( R_{m1}, R_{m2}, R_{m3} \) - airgap reluctances between m and 1, 2, 3 respectively (Figure 9); \( R_{nm1}, R_{nm2}, R_{nm3} \) - reluctances of magnetic sleeves between sleeves 1, 2, and 3 respectively. It may be assumed that \( R_{nm1} = R_{nm2} = R_{nm3} = 1/P_{na} \) and \( R_{m1} = R_{m1} = 1/P_{1} \). Reluctances \( R_{m2} \) and \( R_{m3} \) are calculated from the following:

\[
R_{m2} = \frac{\mu_0 l_{m1}(D_{ae} + D_{si})}{2}, \quad 0 \leq x \leq \frac{\pi}{2}
\]

\[
R_{m3} = \frac{\mu_0 l_{m1}(l_{m1} - x)(D_{ae} + D_{si})}{2}, \quad \frac{\pi}{2} \leq x \leq \pi
\]

The variation of \( R_{m3} \) with armature displacement x is taken into account in equation (3). Knowing the above reluctances \( P_{c1} \) is calculated from the equivalent circuit in Figure 10 using:

\[
P_{c1} = P_1 + \frac{1}{1 + \frac{1}{R_{m2} + R_{m3} + R_{na}}} \quad (4)
\]

Permeance due to the second end magnetic sleeve \( P_{c2} \) is calculated by considering the flux distribution in that region (see Figure 11). From the above this gives:

\[
P_{c2} = P_2 + \frac{1}{1 + \frac{1}{R_{m2} + R_{m3} + R_{na}}} \quad (5)
\]
In equation (5) the reluctance $R_{m3}$ is different from that given by equation (3) and is determined by taking

$$R_{m3} = \frac{\delta}{\mu_{0}(\ln(x-a)-x)(D_e+D_i)/2}$$

**Iterative Calculation of the Reluctances of Magnetic Sleeves, Fluxes and Equivalent Phase Permeance**

Reluctances of magnetic sleeves of stator and armature $R_{ms}$ and $R_{ma}$ are determined iteratively which lies in the basis of the approximate method of calculation of phase reluctance $R$ (or permeance $P$) and the static characteristics of LSM. For this, it is assumed that flux distributions in magnetic sleeves are uniform and, for a given alignment of stator and armature magnetic sleeves their permeabilities $\mu_s$ and $\mu_a$ depend only on the total flux passing through them and not on coordinate $x$. It is assumed that the variation of total flux in the magnetic sleeves with $x$ determined by airgap flux $\Phi_a$ is linear. From this the average calculated flux components in stator and armature magnetic sleeves $\Phi_{ac}$ and $\Phi_{ec}$ are given by $\Phi_{ac} = \Phi_{e} + \Phi_{a}/2$ and $\Phi_{ec} = \Phi_{e} + \Phi_{a}/2$. The effective permeabilities $\mu_{es}$ and $\mu_{ea}$ are calculated from the flux densities $B_s$ and $B_a$ determined by these flux components: $B_s = \Phi_{ac}/S_{ms}$, $B_a = \Phi_{ec}/S_{ma}$ where $S_{ms}$ and $S_{ma}$ are respectively the cross sectional areas of stator and armature magnetic sleeves. Now, from respective magnetisation curves $\mu_{es} = B_s/H_s$ and $\mu_{ea} = B_a/H_a$ where $H_s$, $H_a$ are field intensities corresponding to $B_s$ and $B_a$. Knowing $\mu_{es}$ and $\mu_{ea}$ magnetic sleeve reluctances $R_{ms}$ and $R_{ma}$ are calculated: $R_{ms} = \mu_{es}(\mu_{0}S_{ms})$ and $R_{ma} = \mu_{ea}(\mu_{0}S_{ma})$.

Knowing $R_{ms}$, $R_{ma}$ and other reluctances phase reluctance $R$, total flux $\Phi$ and its components $\Phi_s$, $\Phi_a$ and $\Phi_e$ are determined by solving the following set of equations of mmf balance for these flux paths (Figure 6):

$$F = \Phi R_{pe} + (\Phi_a + \Phi_0)(R_{1} + R_{2}) + + n(\Phi_e + \Phi_0)/2)R_{ms}$$

$$F = \Phi R_{pe} + (\Phi_a + \Phi_0)(R_{1} + R_{2}) + + n(\Phi_e + \Phi_0)/2)R_{ma}$$

$$F = \Phi R_{pe} + (\Phi_a + \Phi_0)(R_{1} + R_{2}) + + n(\Phi_e + \Phi_0)/2)R_{ms}$$

For the iterative solution of (7) it is convenient to rewrite the equations in terms of flux components in the $k$-th iteration:

$$\Phi_s(k) = \Phi_{e(k)}/R_s$$

$$\Phi_a(k) = \Phi_{e(k)}/R_a$$

$$\Phi_e(k) = \Phi_{e(k)}/R_e$$

The total flux $\Phi_{e(k)}$ in the $k$-th iteration is calculated from its components determined in the $(k-1)$-th iteration:

$$\Phi_{e(k)} = \Phi_s(k) + \Phi_a(k-1) + \Phi_e(k-1)$$

In each iteration, starting from the 2nd before the calculation of reluctances $R_{s(k)}$, $R_{a(k)}$ and $R_{e(k)}$ in the $k$-th iteration magnetic sleeve reluctances $R_{ms(k)}$ and $R_{ma(k)}$ are calculated using the flux components $\Phi_{e(k)}$, $\Phi_{s(k)}$ and $\Phi_{a(k)}$ determined in the $(k-1)$-th iteration. For this the procedure described at the beginning of this section for calculating $R_{ms}$ and $R_{ma}$ from $\mu_{es}$ and $\mu_{ea}$ using the flux components $\Phi_{s(k)}$ and $\Phi_{a(k)}$ is used. However, theoretical experiments show that the above method for updating the permeabilities can diverge the iterative process. To avoid this and to accelerate convergence the permeabilities of magnetic sleeves are corrected in each $k$-th iteration using

$$\mu_{es(k)} = q(\Phi_{s(k)}/H_s) + (1-q)\mu_{es(k-1)}$$

$$\mu_{ea(k)} = q(\Phi_{a(k)}/H_a) + (1-q)\mu_{ea(k-1)}$$
where relaxation factor $0.5 \leq q \leq 0.8$ and initially $\mu_i(0)=\mu_i(1)=100\mu_0$. The above iterative process is ended when the flux components in two successive iterations become very close. For this the following end condition can be used:

$$|\Phi_p^{(k)} - \Phi_p^{(k-1)}| + |\Phi_x^{(k)} - \Phi_x^{(k-1)}| + |\Phi_y^{(k)} - \Phi_y^{(k-1)}| \leq \epsilon$$

where $\epsilon$ is the end factor determined by accuracy requirements. Thus, using the method described above the total flux $\Phi(x)$ is calculated for various armature displacements $x$ from which phase permeance $P(x) = \Phi(x)/F$ is calculated. Finally, electromagnetic force $F(x)$ is calculated as a function of $x$ by differentiating $P(x) = f(x)$ using equation (1).

RESULTS AND DISCUSSIONS

Some of the results of above calculations for the 4-phase LSM used in LSED are presented in Figures 12 and 13. Figure 12 shows the variation of phase permeance with armature displacement for a small stator current $I=5$ A with $(1 - P_s(x))$ and without $(2 - P_{su}(x))$ saturation of magnetic sleeves taken into account. For $P_{su}(x)$ it is assumed that $\mu_s = \mu_m = \infty$ and hence, $R_{sm} = R_{ma} = 0$. As can be seen from Figure 12 the effect of saturation on phase permeance for small stator current is not significant. Figure 13 shows the static characteristic of the LSM calculated by the approximate method (1) for $I=5$ A and its comparison with the experimental characteristic (2) obtained from [10]. It shows, in general satisfactory agreement which validates the above described method. Maximum error of about 15% can be seen in the region of highest electromagnetic force for $x=11$ mm. This is due to the approximate incorporation of iron saturation, especially the saturation of magnetic sleeve corners which takes place (even for small currents) for certain armature positions for which stator and armature magnetic sleeve corners align opposite to each other.

CONCLUSIONS

A method has been developed for the approximate calculation of static characteristics of linear step motors which gives satisfactory results for small currents and weak iron saturation. This is a simple and fast analytical technique which can be readily used at the early stages of CAD of these motors to generate, verify and evaluate initial designs.

REFERENCES


[3]. Patent, France no. 2526241, Int. Cl. H02K 4/03, 1982


