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Form Finding and structural Optimization of Tensile cable dome using Parametric Modelling Tools

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Abstract

In this paper, a new framework of form finding and structural optimizations for tensile domes was developed using a cutting-edge parametric modelling tool Grasshopper in Rhino. The detailed exploration of this new techniques is presented. It is found that the use of this parametric tool allows a more intuitive, rapid and flexible design. Structural optimisation of the member sizes, topology and surface can be explored easily at an initial design stage in a project. Therefore, the proposed new framework provides a more effective and efficient way for form finding and structural optimization. Based on the new method, a prototype Tensile dome which is to replicate the existing Tensile Dome Georgia dome is designed and analyzed. The structural behavior of the cable domes is investigated. Using this new framework, two ellipse shape Tensile domes with new geometrical configuration are developed. They exhibit enhanced load bearing capacity, therefore can be used the future long span structure projects.

Keywords

Form finding, parametric design, force density Method, dynamic relaxation method, Grasshopper

1. Introduction

Lightweight tensile structures, such as tensile cable domes using membrane as a roof cover, offer aesthetic richness and material efficiency [1]. The lightweight membrane structures originated from Frei Otto's soap film in 1950. Tensile structures offer a wide range of design choices and innovation such as building envelope , facades. [2] [3] The initial geometrical shape of tensile structures depends on the equilibrium of internal force the structure. Any discontinuity in the membranes will lead to wrinkling, deformation and therefore, reduce life expectancy of the structure. Hence, for

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this type of structure, to resist applied loads, an adequate level of prestress is fundamental. The stress-strain relationship is geometrical non-linear which requires specific design software for the analysis. [4]

For long span structures, self-weight is critical in its design. Therefore, lightweight tensile structures such as cable domes are widely used. A cable dome consists of ridge cables, diagonal cables, hoop cables, vertical struts, an inner tension ring and an outer compression ring. The cables are in tension and the struts are in compression. Initial pre-stresses should be applied to cables at its self-equilibrium state, which significantly influence the behaviour of a loaded structure and greatly contribute to its stiffness and stability. The ridge cables are anchored into the compression ring, a large concrete ring beams for most of the cases, to resist the huge tensile force from the cables. The cable dome is a geometrical non-linear system, the structural analysis can be divided into two phases: the first phase is the initial equilibrium (form finding); the second phase is static analysis.

The design of cable domes requires a balance between minimize steel usage sufficient stiffness through the judicious use of tension cables. The key task of optimization is to find the minimal amount of material for a given structure with enough

stiffness. The geometry of tensile structures can be created by free-form mathematical methods, such as hyperbolic catenary hanging shapes related to the funicular structures [5]. The philosophy of form finding by self-forming processes developed into the concept of ‘minimal surfaces’, the smallest surface areas requiring the least potential energy.

1.1 Form finding

In Tensile cable dome design, form finding is the first important step. Form finding is the process of determining an initial equilibrium shape of a structure. For tensile structures, the form finding process adheres to the principle of ‘form follows force’ where the geometry depends on the relationship between topology and forces.

In the past, the form finding was made through experimentation using physical models, [6]. Physical form finding method is time consuming. New technological advanced physical form finding in computational methods Different numerical methods have been introduced over the years. In 1975, Argyris developed a form finding method based on Finite Element Method.Barnes introduced a Dynamic Relaxation Method (DRM). The Dynamic Relaxation Method is defined by a system of springs and particles. The applied load causes the particles to move until they eventually stabilise in a state of equilibrium. In the 1970's, Schek introduced the Force Density Method that remains the most used methods. The Stiffness Matrix Method comes from structural analysis and uses the elastic and geometric stiffness matrices. This method includes unnecessary material properties that increase cost and have difficulty in controlling a stable convergence. [7]. The force density method can solve the equilibrium equation of a tensile structure that is transformed into a discrete cable network, nodes and lines. [20] In a cable network, pin-jointed network structures reach the state of equilibrium when the internal and external forces are balanced. [21] The Force Density Method is material independent. It produces results that may serve only as preliminary, and additional iterations may be necessary.

1.2 Structural optimization

The 'optimal topology' of a structure aims to minimize material and weight, maximize stiffness, enhance load bearing capacity. The design of free-form surface and envelopes has three main aspects: sizing, topology and shape, related to the spatial configuration and geometry. [22]

Structural optimization can be categorized into:

1. reduce size of individual members
2. Optimize the topology to decrease numbers of steel members, therefore decrease the weight and excessive stresses.

1.3 Parametric modelling

According to [23], parametric modelling is a new modelling process with the ability to change the shape of model geometry through modification of the parameters It is implemented through computer programming code such as a script to define the dimension and the shape of the model. The model can be visualized in 3D draughting programs to resemble the attributes of the original project. Parametric modelling was first invented by 3D draughting software Rhino. The key advantage of parametric modelling is, when setting up a 3D geometric model, the shape of model geometry can be changed as soon as the parameters such as the dimensions or curvatures are modified, therefore no need to redraw the model whenever it needs a change. This greatly saved the time for engineers, especially, in the scheme design stage. Before the advent of parametric modelling, the scheme design was not an easy task for designers, as the model is prone to be changed frequently requiring great amount of work in modification of analysis model., Conventional methods to track changes are more complex and keeping the quality of the design in complex structures with a tight deadline can be specially challenging. It requires more time, and usually several physical models would have to be built, to test the design options. Parametric modelling allows the designer to modify the entire shapes of the model, not just individual members. It can simulate the behaviour of the structure under loading and make initial calculations to study the feasibility of the design options. This can be visualised and changes to the geometry can be performed so we can instantly get a sense of what ‘s happening to the global structural system when we modify the geometry. We can see the effect in the whole system when modifications are performed only partially. therefore, greatly saves engineers’ time [21,22].

1.4 The use of the parametric tools for form finding and Structural optimization

From above literature review, it can be seen that form finding and design optimization for tensile structure is a complicated task. Particularly, in the current design project, the geometry of the structure become increasing complicated [21,22]. The traditional form finding techniques and structural optimization are steel need significant improvement [24]. As they are not capable to cope the increasing complex structural geometry. They are prone to deal simple form of structures such as a truss. It is also difficult for them to consider the effect of buckling and the width to height ratio etc. The current complex structural forms require a better solution that integrates all these factors. [25]

Therefore, in this paper, a new digital tool Grasshopper in Rhinocerous 3D [26] is used for parametric modelling and design optimization of Tensile domes which. The new method enables simulation, evaluation and pre-rationalization of design outcomes for this type of structure efficiently. The use of digital tools for topology optimization and form finding enables us a more integrated and fully cooperative design [6],. The use of digital tools provides engineers and architects a faster way of rationalizing forms. Using the new tools, form finding is able to develop an initial design scheme and choosing member sizes while taking into consideration aesthetics and vision of the designer. [27] By optimization of shape and topology, significant cost savings can be made by reducing the amount of material required for a structure.. Form finding and structural optimization method using the parametric tool Grasshopper

In this paper, a new script was developed by the authors using parametric tool Grasshopper in Rhinoceros 3D. This script is used to create a geometry of cable domes and perform subsequent form finding and structural optimization. This script was also used for exporting the final geometry to Oasys GSA for non-linear analysis. Several iterative process between Rhinoceros and Oasys GSA were made with the objective of minimising material usage and reactions on foundations while achieve allowable deflection by providing sufficient stiffness from the cables .

2.1 Workflow of the new digital tool framework

The Grasshopper which is a graphical algorithm editor integrated within Rhino3D's modelling tools are used as the digital tool in this paper. Grasshopper allows users to automate tasks in Rhino 3D or other programs that can interoperate within Rhino interface. Grasshopper provides a multi-dimensional data structure algorithm, called a definition, that allows data matching and a program workflow that organizes building blocks of pockets of algorithms in a logical, user friendly way while offering a highly efficient open source, customisable software to create, organise and manipulate complex mathematical formulas and geometries.

Grasshopper performs mathematical operations and can evaluate conditions and manipulate large sets of information. The workflow of Grasshopper allows users to create a visual representation of code, in the form of blocks connected by wires. These blocks are mainly parameters that store data and components that perform actions resulting in new data.

In the framework, input parameters can be modified dynamically and interactively. For example, three-dimensional grid-based interface and value list of input parameters allow exchange of data from external software such as Excel or Oasys GSA. The development of the interoperability of Grasshopper with other programs such as Autodesk Revit and SAP 2000 make Grasshopper a powerful tool for teamwork of structural engineers, and architects.

An interface for Grasshopper and Oasys GSA is provided using Geometry Gym. The use of parametric tools in form finding of tensile structures allows the user to freely model and make changes at any stage of the design process. A parametric model created with grasshopper allows automatic generation of different versions of a tensile dome by just change a set of input or variables. The parametric model is fully integrated and responds to changes in real time.

The structural analysis result such as cable deflections produced by Oasys GSA can be used to control parameters and set limits under which the length of the components can be shortened, simulating a pre-stress condition of the cables and setting constraints in the design. This allows early optimisation tools that affect design decisions.

Design engineers and architects define how a set of parameters will best influence design through writing scripts pipeline.

Parametric design is the interface in between architects and designers where they can bring the problems to the canvas and discuss about geometry and structure. Cooperation is facilitated from the early concept design stage throughout the whole project duration. This wasn’t a possible before with the traditional methods, such as physical methods or conventional software package.

Structural engineers are now designing the algorithms to perform structural analysis more efficiently and they are also participating more of the design, facilitating more collaboration and accommodation changes along the pipeline in a more efficient way.

2.1 Geometry set up

Figure 1 shows the code pipeline diagram using Grasshopper in Rhino, which is used to generate the initial geometry of a tensile cable dome.

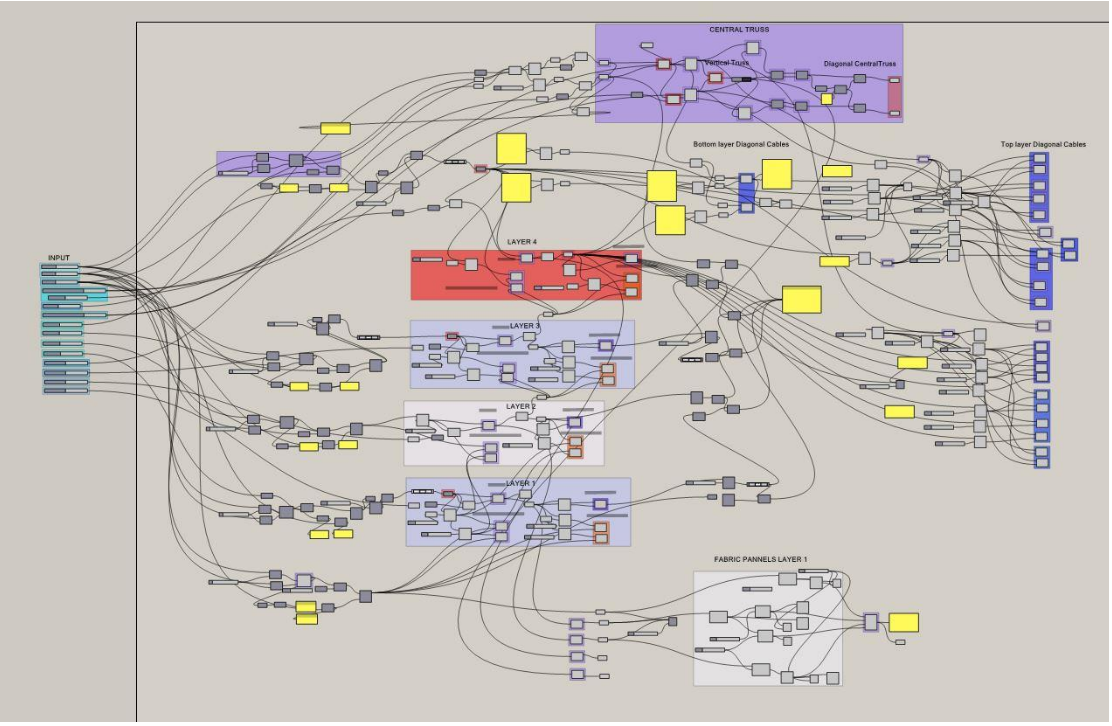
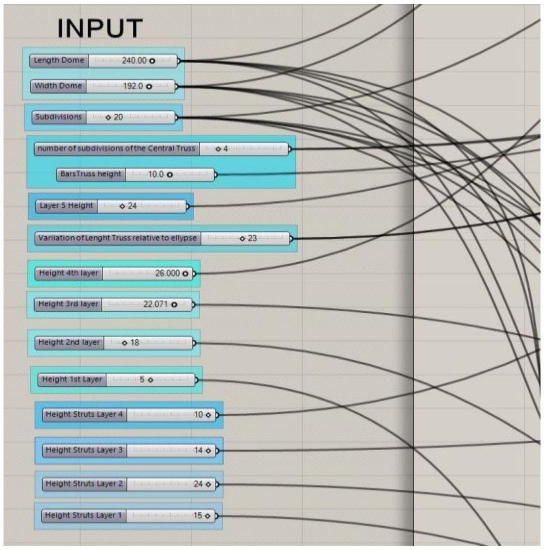


Fig. 1: Code design diagram of using Grasshopper to generate the tensile structure

Figure 2 shows the INPUT section which is used for changing the design parameters by simply using sliders to control the number and member sizes of cables and struts in a tensile dome.



Length Dome	240.00
Width Dome	192.0
Subdivisions	20
number of subdivisions of the Central Truss	4
BarsTruss height	10.0
Layer 5 Height	24
Variation of Lenght Truss relative to ellypse	23
Height 4th layer	26.000
Height 3rd layer	22.071
Height 2nd layer	18
Height 1st Layer	5
Height Struts Layer 4	10
Height Struts Layer 3	14
Height Struts Layer 2	24
Height Struts Layer 1	15

Fig. 2: INPUT control of parameters

Figure 3 and 4 show the pipeline code for generating the first ring of cables and struts in tensile dome.

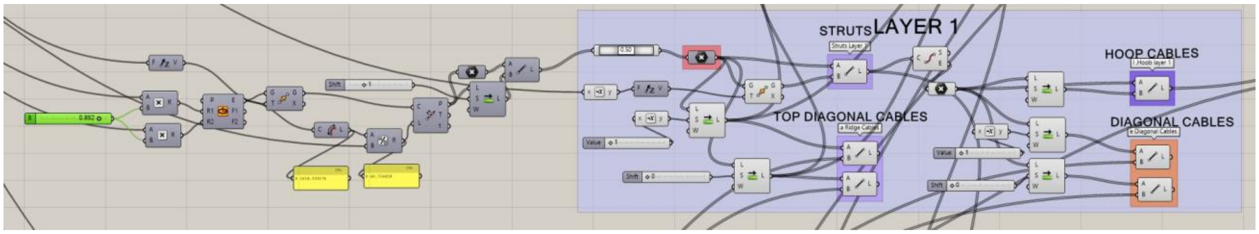


Fig. 3: The code for generating the first ring of cables and struts in tensile dome.

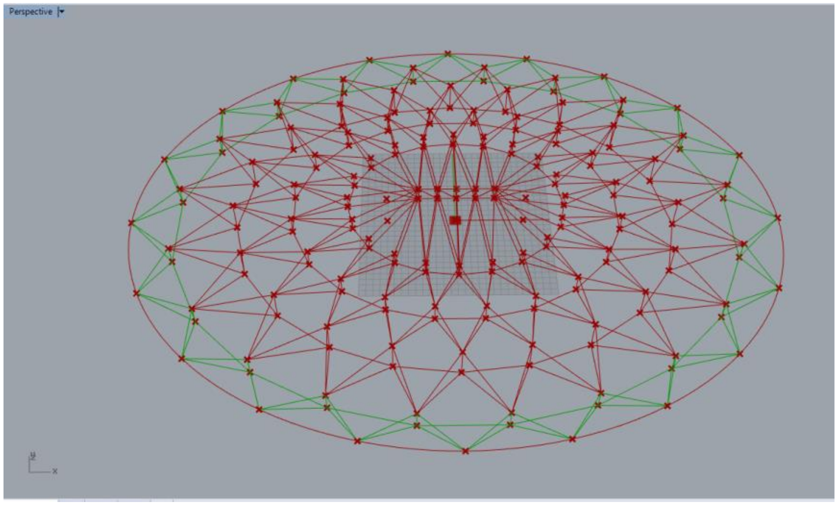


Fig.4: first ring of cables and struts generated by Grasshopper (in Green colour).

Figure 5 and 6 show the pipeline code for generating the second ring of cables and struts in tensile dome. Similarly, all other ring of cables and structures are defined using Grasshopper.

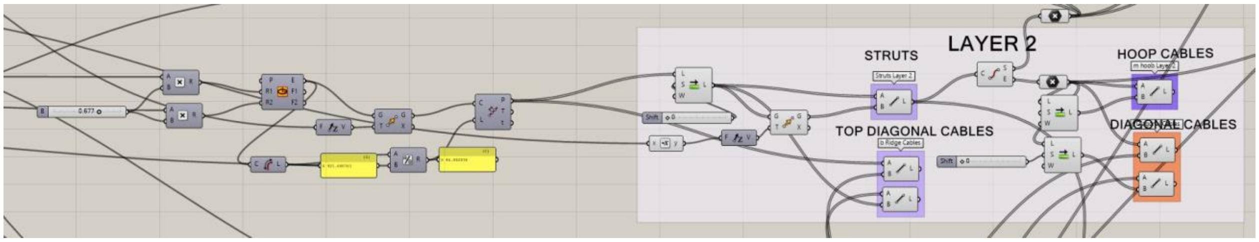


Fig. 5: The code for generating the second ring of cables and struts in tensile dome.

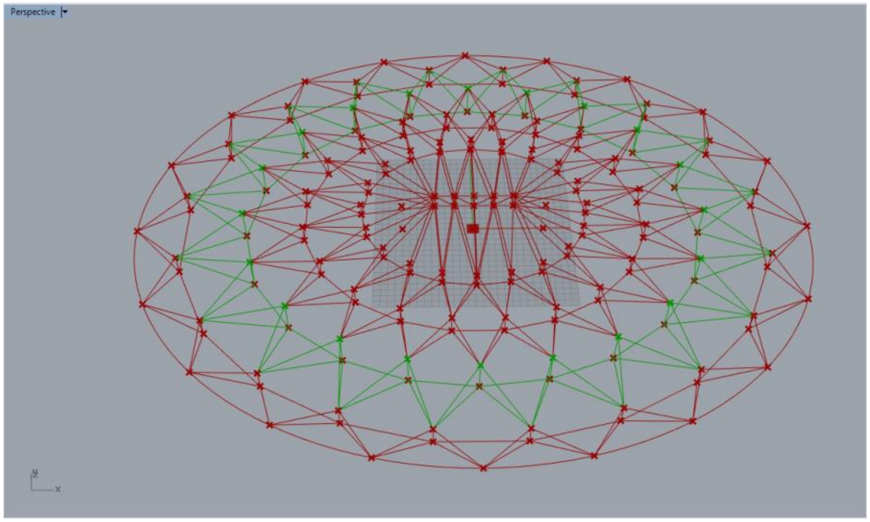


Fig 6: Second ring of cables and struts in tensile dome by Grasshopper (in Green colour).

Figure 7 and 8 show the pipeline code for generating the CENTRAL TRUSS in the tensile dome using Grasshopper.

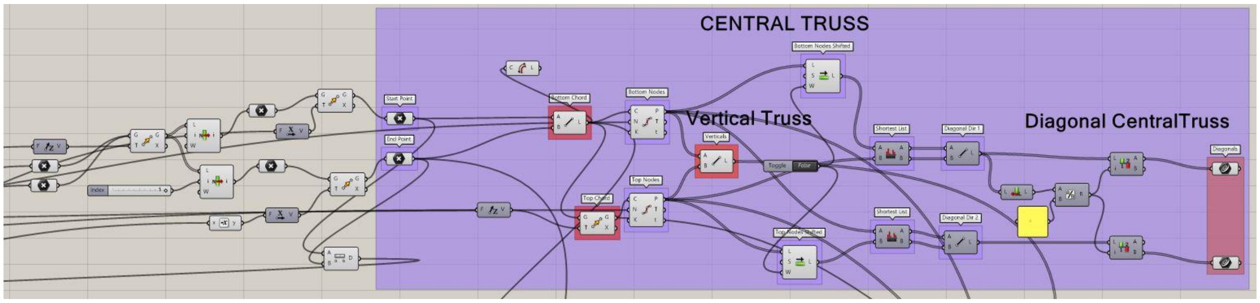


Fig 7: CENTRAL TRUSS definition.

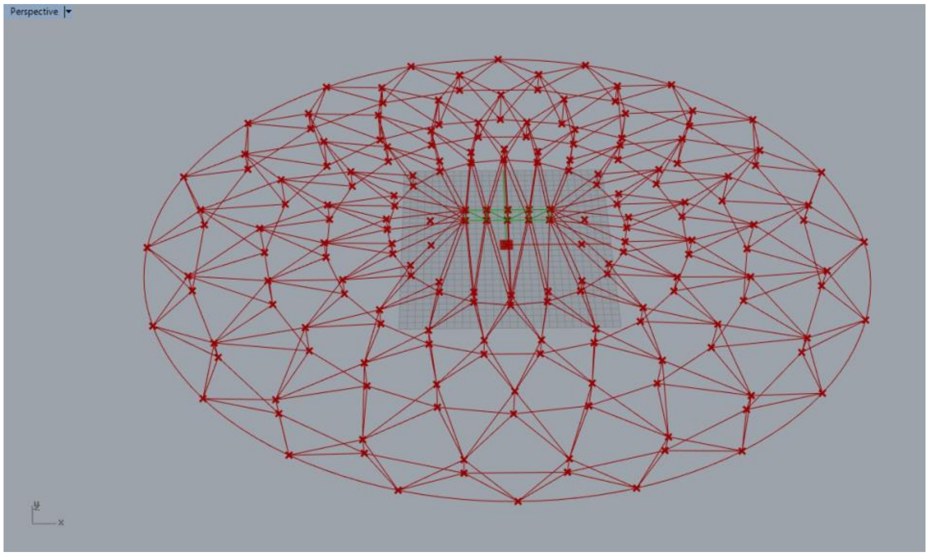


Fig. 8: CENTRAL TRUSS generated by Grasshopper definition (in Green colour).

It can be seen that, as the whole structure is generated using the parametric tools, so it become easier to perform form finding and structure optimization by just changing parameters.

2.2 Form finding and structural optimization

The strength of Grasshopper is that it can analyse a given geometry and extracted data from it that can be further used for making new geometry. With the plugin Kangaroo for grasshopper boundary conditions loads can be defined. Kangaroo [26] is a numerical simulation plugin for Grasshopper, that perform geometrically non-linear structural analysis of the digital models using a dynamic procedure. This plugin uses Dynamic Relaxation Method (DR) for form finding to find a static equilibrium solution. After form finding,a structural analysis can be further explored by exporting it to a commercial software such as GSA. Geometry Gym for Grasshopper is a interface between Rhinoceros and GSA. These tools provide efficient means for manipulating and generating a 3d model with form and shape as fundamental factors in its performance. Using above process; the form finding, and structural Optimization of a cable dome can be achieved.

2.3 Comparison of the new method to the existing methods

2.3.1 New method

The interdisciplinary design process of parametric tools in the way we can automate some processes to facilitate flexibility and quality in the design process.

Initial sizes of members can be calculated to assess initial design. Changes are adjusted automatically saving time along the design pipeline. As a result, the quality of the design is enhanced. In the case of tensile cable domes this is a great advantage because this allows determining a set of parameters to assess feasibility and optimise the individual members and the structural system allowing changes at any time. Form finding optimizes the right level of pre-stress force by the way loads are applied to the system.

Parametric modelling can simulate the behaviour of the structure under loading and make initial calculations to study the feasibility of the design options. This can be visualised and changes to the geometry can be performed so engineers can instantly get a sense of force and deflection change in the global structural system when we modify the geometry. The efficiency of the whole system under modifications can be checked. For an example, the form finding method could be thought of as a memory foam mattress that will take the shape of your body when you lie on it. In the case of tensile structures, the pre-stress force will define the shape of the net, so to understand how the forces are distributed is key in the design. Form finding with the means of parametric design will allow this conversation in between forces and shape. The benefits of implementing parametric design in the early stages of construction are that creative thinking and rationalisation are involved early in the design process. It requires thinking and analysing the system and its parts - computer programming offers a set of tools to create an algorithm that will assist on those changes. So, when coding the script, we are addressing the body of the design and the assumptions that we are doing, identifying the variables. With the movement towards prefabrication, off site construction and 3D printing, a circular economy will need an organization of the construction pieces that allows rationalization but also flexibility of the design. Compatible with additive manufacturing processes, parametric modelling will facilitate this transition”.

1.3.2 Conventional methods

For conventional method, tracking changes are more complex and keeping the quality of the design in complex structures with a tight deadline can be specially challenging. It requires more time, and usually several models would have to be built, to test the design options.

2. Case studies of Form finding and design optimization of the tensile dome using parametric tool

A parametric model of a tensile elliptical dome was created in Rhinoceros 3D Grasshopper as it shown in Figure 9. It replicates the existing Tensile structure Georgia Dome (the dome was demolished). Different parameters were set out allowing variations in the size of the elements and their connectivity. The resulting free-form geometry was then rationalised by changing parameters through the pipeline code in Grasshopper. The form finding and optimization results can be changed freely and quickly in each iteration through varying the parameters. An initial form finding of different options was done with the plugin Kangaroo by using the dynamic relaxation method. The results were compared and were then exported to GSA for further form finding which allows an initial sizing of the elements with the force density method and further non-linear analysis was carried out and results compared. Therefore, an iterative process of form finding was carried out with the objective of minimising the sizes of the cross-sections areas of cables by distributing the force density ratios. The rigidity and stiffness of the dome comes are provided through the equilibrium of internal force between cables and struts. To avoid slackening of the cables, a high level of prestress is required. It will determine the sizes of the cables, thus the cost of the materials and the construction process.

Meanwhile, different forms of cable domes model 2 and 3 are proposed and compared. The topology of the different models created was associated with different force density ratios and a non-linear analysis was performed and the results were compared.

For comparative study purpose, the dimensions of all the models have the same height 30 m, width 192 m, length 240 m and 20 sub-divisions at each layer. As the dome has an elliptical shape, the cables on the same layer have different lengths. The dome has a triangulated geometry similar with the Levy's Georgia Dome and has a span of 240 m.

The following 3 models have different numbers of concentric elliptical layers and the process of form finding contributes to find the optimal pre-stresses of the cables and the distribution of the forces in the network. The cross sections areas of the ridge cables from the outer layer to the centre are 85.4 cm^2, 46 cm^2, 22 cm^2, 22 cm^2. The diagonal cables are 48 cm, 56 cm, 24 cm and 24 cm respectively. The cross-section areas of the hoop cables are 190 cm^2, 268 cm^2 and 75 cm^2. The yield strength of the cables is 1670 N/mm^2, and the tensile strength of the cables is f_t = 1860 N/mm^2.

Using these specifications, if a ridge cable on the outer layer has a CSA of 85.4 cm^2, the maximum stress reached before failure will correspond to 158,844 KN. The struts are S275 circular tubes in compression.

It was determined that an initial prestress for of averaging 30% of cable tensile capacity was needed. Thus, a starting value of 30% was taking to define the initial force densities value and the iterative process of form finding [29]. For a ridge cable with a cross sectional area (CSA) of 85.4 cm^2 , the initial force density value $q = f_y * CSA * 0.3 = 4279$ KN was taken.

A live load of 0.6 KN/m^2 and is applied on the roof and a projected wind load of 1 KN/m$^{2.}$ The non-linear analysis is defined for a combination of the internal load due to pre-stresses, the dead load of 1 KN/ m^2. A combination of internal loads due to pre-stress, dead load and snow is also compared.

Figure 9 to 12 illustrate how Grasshopper was used to first generate a tensile dome; this model was then exported into Oasys GSA using Geometry Gym.

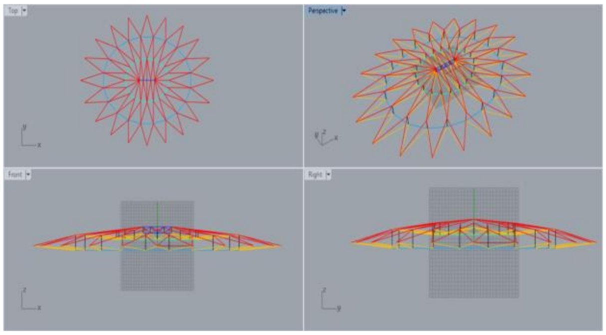


Figure 9 Model 1in Rhinoceros 3D

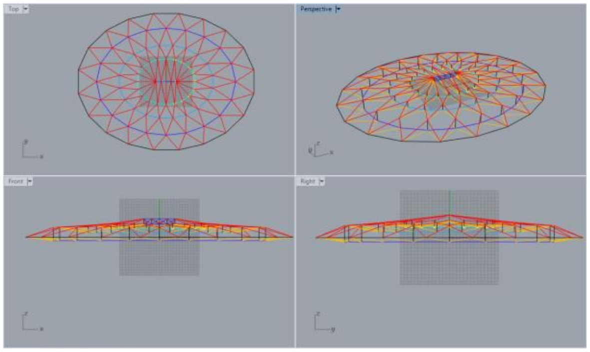


Figure 10 Model 2 in Rhinoceros

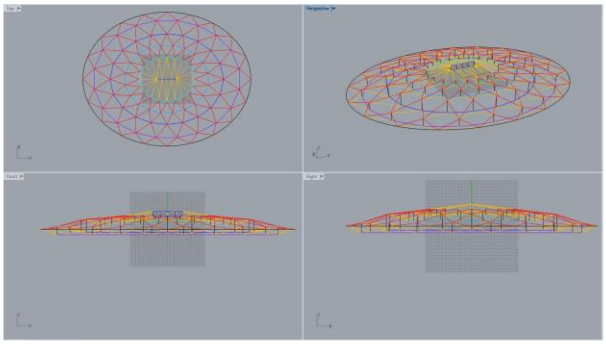


Figure. 11 Model 3

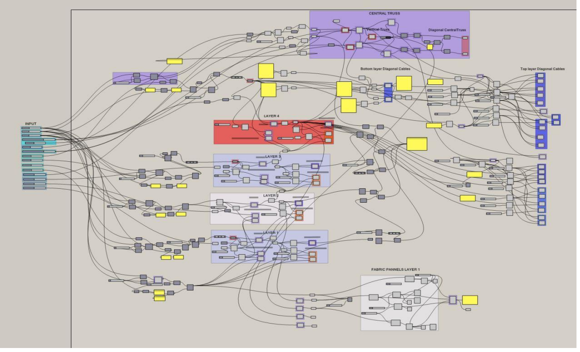


Figure 12 Grasshopper pipeline coding

3.1 Model 1

It can be seen that, after loading, the forces on the ridge cables decrease, especially those on the top and inner layers. The forces on the outer and lower layer of diagonal cables increase, while for the inner and upper layer the forces decrease. The forces on the hoop cable increase more rapidly on the bottom layer.

The results show in Figure 13 that Model 1 presents considerable vertical displacement and deformation. The distribution of the stresses and dimensions of the cables and struts need to be further adjusted. The necessary changes are performed in the parametric model in Grasshopper. Longer struts are added, and higher pre-stresses are considered. The central zone of the tensile dome needs to be reinforced with vertical struts and adding another concentric ring of ridge cables and diagonal cables seems to be a good idea since the horizontal distances in between the struts are too large and the cables need to be highly pre-stressed to maintain the stresses.

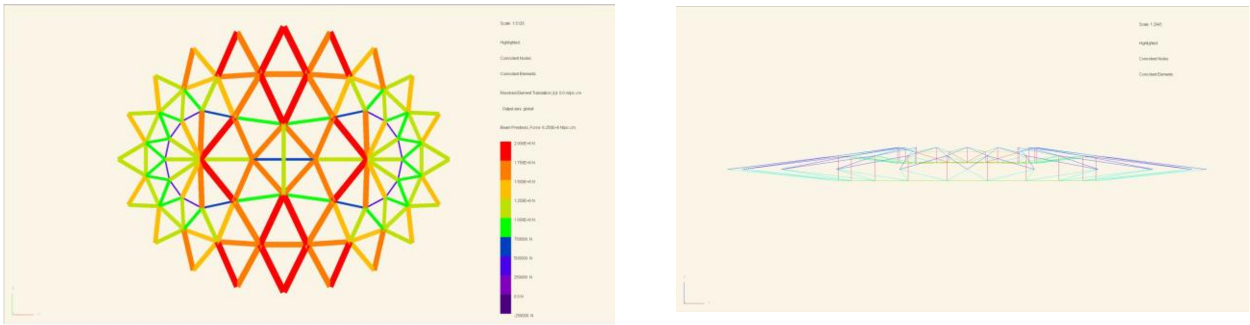


Figure 13 Form finding result of Model 1.

3.2 Model 2

The new Model 2 is an improvement of the Model 1. An extra layer of struts and ridge and diagonal cables is added. There are 12 different types of elements with different rations of pre-stresses. The pre-stress of the cables is gradually increased during the form finding with the force density method in Oasys GSA. During the form finding the same dimensions of height, width and length are kept and an efficient use of the materials explored through the intelligent distribution of the stresses in the network. Figure 14 to 16 show the result of the analysis.

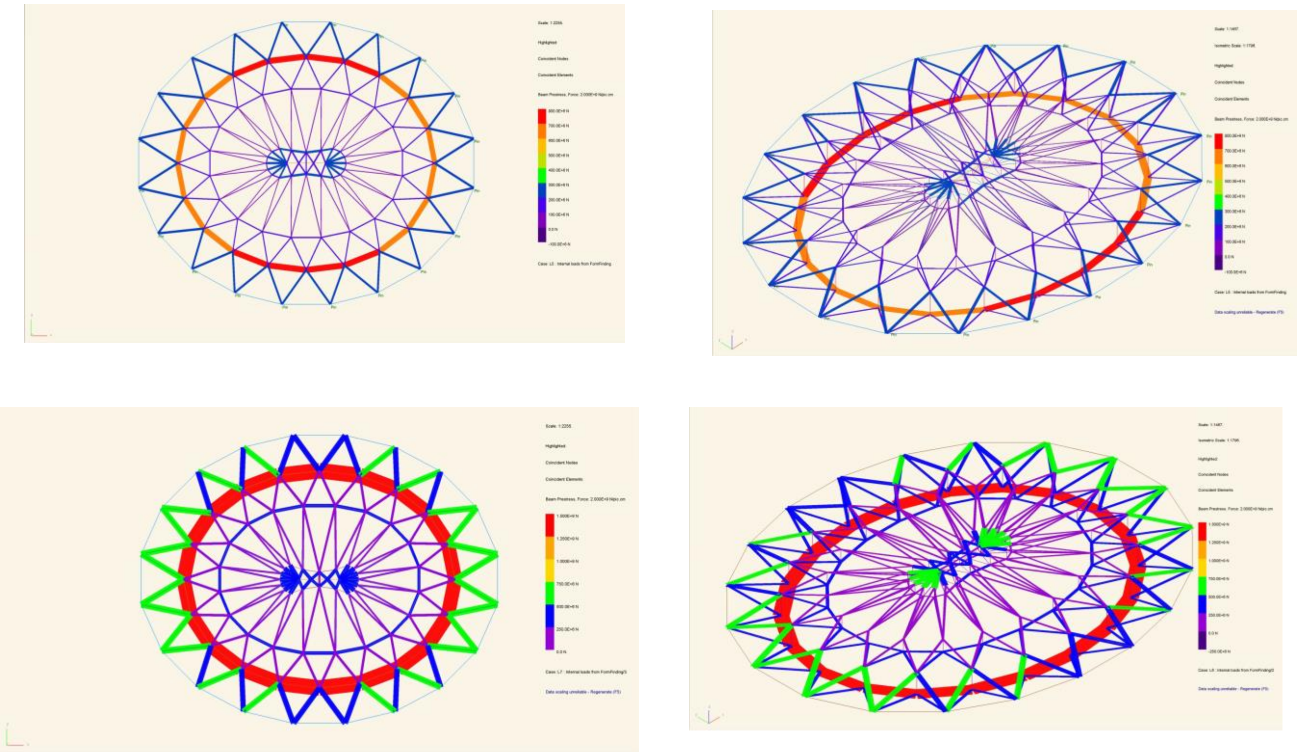


Figure 14 Form finding result of Model 2.

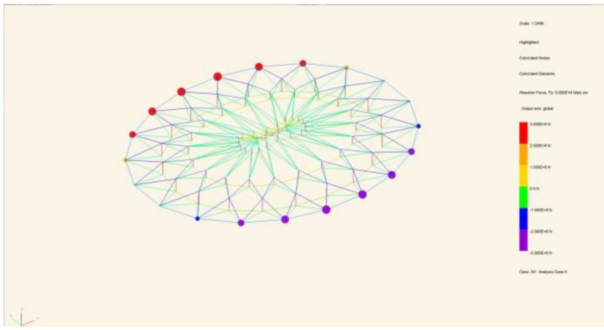


Figure 15 Static analysis Result (Reactions) of Model 2.

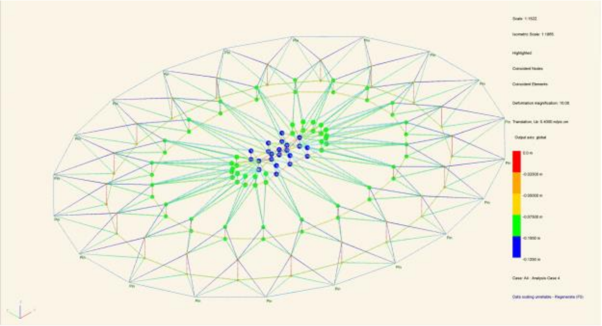


Figure 16 Static analysis Result (Vertical deflection Uz) Model 2

Model 2 requires more cables and struts than the model 1. However, it performs well with an allowable maximum vertical displacement of 506 mm. The results of form finding of the model 2 are geometrically more satisfactory.

3.3 Model 3

Model 3 is the further improvement of Model 2 where more cables were added.. The central zone is again the weakest part of the structure. The design process of has the main objective of strengthening the central part of the dome.

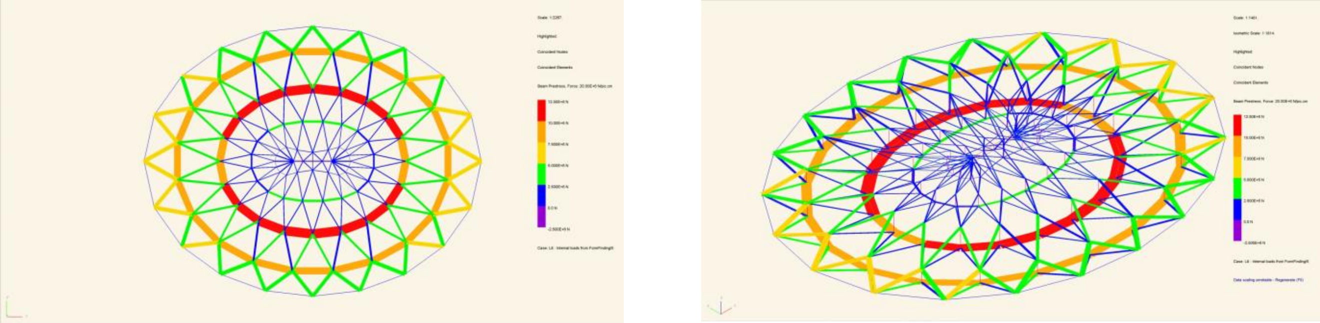
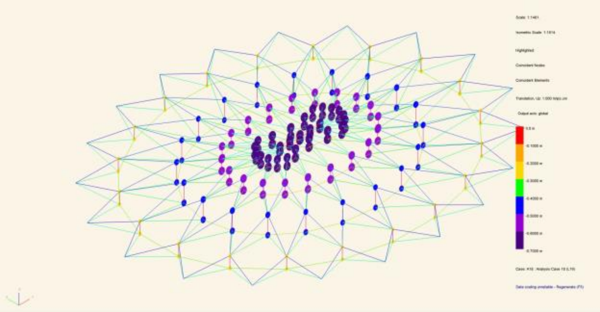


Figure 17 Form finding result Model 3



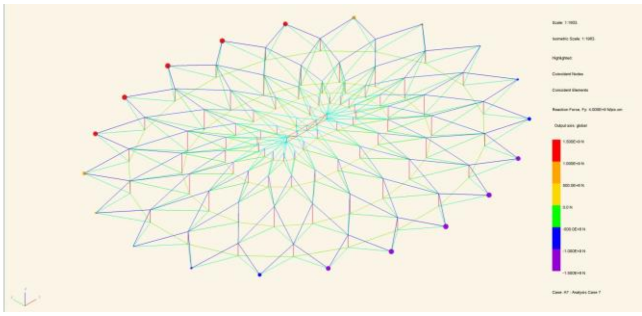


Figure 18 Static analysis result of Model 3(Displacements Uz)   Figure 19 Static analysis result of Tensile Dome Model 3. Reactions Fx

Figure 17 to 19 show the analysis result of Model 3. The initial force densities result is in the range of 4278 KN to 1092 KN for the ridge and diagonal cables, and 13416 KN to3787 KN for the tension hoop cables. After a form finding in Oasys GSA, it has then been increased up to the 60%, with a maximum of 26822 KN, compared to the initial 30% of the prestress level of a cable.   he stresses are distributed thought a more homogeneous network. The maximum vertical displacements have decreased with the new geometry and pre-stress distribution. By comparison, under the same load conditions the Model 3 performed better than Case 1 and Case 2. mainly due to the higher pre-stresses of the cables. The weakest part of the dome is the central zone. To strengthen the central section of the dome has been necessary, especially the ridge and diagonal cables at the central section.

3.4 Analysis results discussion

In the elliptical domes, the weakest part of the structure is the central region between the centre point and the first hoop cable, the slackening of the cables may cause failure. This applies particularly to the ridge and diagonal cables. The failure occurs mainly by breaking of the hoops and diagonal cables, the buckling of the struts or both.

The configuration of the cable net can be either in a wedge shape or in a triangular shape A combination of both will improve the stability and performance of a tensile dome.

The benefits of implementing parametric design in the early stages of construction are that creative thinking and rationalisation are involved early in the design process. It requires thinking and analysing the system and computer programming offers a set of tools to create an algorithm that will assist on those changes.

3. Conclusions

In this paper, a new parametric tool for form finding and design optimization of Tensile structure is presented. Using the new tool, a prototype model which replicate the Georgia dome is designed and analyzed. Based on the parametric form finding and optimization results, this dome is further improved, two new geometrical configuration of ellipse shape Tensile domes were designed.
Following conclusion can be made:

- Computational design with the means of parametric tools allows an initial design evaluation of multiple form options. It facilitates an intuitive design exploration of topology. It drives structural optimization and exploration of design in a more efficient way.
- It is found that any improvement on the design of the tensile dome will focus on strengthening the ridge and diagonal cables of the inner and upper part of the central section of the dome while minimizing the use of steel on the structure

- The two new geometrical configurations of ellipse shape Tensile domes were developed, which is primarily to strengthen the central part of the dome. The new types of Tensile domes exhibit better load bearing feature.

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