Distributed LQR design for identical dynamically coupled systems: Application to load frequency control of multi-area power grid

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***Abstract*— The paper proposes a distributed LQR method for the solution to regulator problems of networks composed of dynamically dependent agents. It is assumed that these dynamical couplings among agents can be expressed in a state-space form of a certain structure. Following a top-down approach we approximate a centralized LQR optimal controller by a distributed scheme the stability of which is guaranteed via a stability test applied to convex combination of Hurwitz matrices. The method is applied to *N*-identical-area power grid where a distributed state-feedback Load Frequency Controller (LFC) is proposed to achieve frequency regulation under power demand variations. An illustrative numerical example demonstrates the applicability of the method.**

I. INTRODUCTION

Networked systems are important in engineering and of special interest to control community due to their wide spectrum of applications. Vehicle platoons, groups of UAVs, supply chains and multi-area power systems are just a few paradigms. These schemes can be decomposed into a number of distinct subsystems (or just systems) often referred to as agents, each one having autonomous actuation capacity. Agents, despite their independent operation, have also the ability to cooperate with certain of their counterparts within the network towards a common objective [1], [2]. In some cases, the topology of the network may be imposed by structural links such as in power systems where agents correspond to power generators and the interconnections are represented by power transmission lines [3], [4].

Here, we focus on networks composed of identical dynamically coupled linear time-invariant (LTI) systems. We assume that couplings are expressed in state-space form of a certain structure. Each system representing an agent can produce actuation signals independently and is dynamically coupled with certain number of its peers referred to as neighboring agents with whom it can exchange state information. Effectively, we assume that the topology of physical couplings and the topology of information exchange among agents are described by the same graph. Network stabilization is one of the most challenging problems in multi-agent network control. In typical situations the mere complexity of the system makes centralized control schemes either impossible or undesirable.

In this paper, we propose a stabilizing distributed LQR-based controller. Via a top-down method we approximate a centralized LQR optimal controller by a distributed scheme. Overall network stability is guaranteed via a stability test applied to a convex combination of Hurwitz matrices. This condition is consistent with the stability of a class of network topologies which is identified. Sufficient condition for stability of convex combination of Hurwitz matrices can be found in [5]. Our approach is inspired by [6]. Therein, the subsystems constituting the network are dynamically decoupled and the stability of the distributed scheme relies on the stability margins of LQR control. Unfortunately, this elegant feature, which automatically guarantees stability of the distributed scheme in [6], does not hold in the presence of dynamical couplings between neighboring agents. Thus, the control design proposed in our work extends the results in [6] to a more general setting. This represents the main contribution of this paper.

A distributed LQR method has also been presented in [7]. This consists of a bottom-up approach in which optimal interactions between self-stabilizing agents are defined to minimize an upper bound of a global LQR criterion. It is still assumed that systems are open-loop decoupled. A thorough procedure for designing distributed controllers for a class of coupled systems based on a decomposition approach has been presented in [8]. The validity of this method relies on certain structural properties satisfied by the system matrices. Rigorous methods for cooperative control design for multi-agent systems with distributed or decentralized pattern have been established in [9]–[13].

Our definition of multi-agent networks with dynamical couplings is motivated by the structure of multi-area power systems. Thus, to illustrate the applicability of our control algorithm, we consider a large-scale power network formed of identical control areas interconnected through weak transmission lines referred to as tie-lines. In the study, we focus on Load Frequency Control (LFC) of a multi-area power system [14]. Textbooks providing an introduction to power system design and LFC can be found in [15], [16]. Analytical methods for designing decentralized and distributed load frequency control are presented in [4], [17], [18] while a set-theoretic method for LFC in the context of cyber-physical power systems can be found in [19]. Here, we formulate the LFC problem as a large-scale optimal control problem in the absence of state and input constraints. An arbitrary number of identical areas is considered. Our interest in distributed LFC

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arises from the need to avoid centralized schemes which have often high communication and processing costs and suffer from a single-point-of-failure drawback [4]. The proposed distributed LFC controller is stabilizing even if tie-line interconnections are added to or removed from the overall system, as long as this does not violate the stability condition stated in Section III. This powerful feature gives integrity to the control subsystem of each area and enhances the resilience of the system in the presence of interconnection variations.

From a power system perspective, the assumption that the dynamical models of each area are identical may be unrealistic in practice. However, it simplifies the design problem considerably, which is especially hard due to the coupling terms appearing in the model. Future work will attempt to eliminate or relax this assumption. Preliminary results can be found in [20], [21]. The model and the control strategy considered in this work have also been utilized in [22]. Therein, extensive simulation results under considerable perturbations in the model parameters of each area suggest that this hypothesis is valid and that our results can be extended to the general case.

The remaining of the paper is organized in six sections. In Section II preliminaries on graph theory are presented. The main results of our work appear in Section III and IV, where the distributed control algorithm is derived. Section V is devoted to multi-area power grid control design along with simulation results. The main conclusions of the paper and suggestions for further work are included in Section VI.

II. Preliminaries

A graph $\mathcal{G}$ is defined as the ordered pair $\mathcal{G} = (\mathcal{V},\mathcal{E})$, where $\mathcal{V}$ is the set of nodes (or vertices) $\mathcal{V} = \{1,\cdots,N\}$ and $\mathcal{E} \subseteq \mathcal{V}\times\mathcal{V}$ the set of edges (i,j) with $i \in \mathcal{V}$, $j \in \mathcal{V}$. The degree d_j of a graph vertex j is the number of edges which start from j. Let $d_{max}(\mathcal{G})$ denote the maximum vertex degree of the graph $\mathcal{G}$. We denote by $\mathcal{A}(\mathcal{G})$ the $(0,1)$ adjacency matrix of the graph $\mathcal{G}$. In particular, $A_{ij} = 1$ if $(i,j) \in \mathcal{E}\ \forall\ i,j = 1,\cdots,N$, $i \neq j$ and zero otherwise. Let $j \in \mathcal{N}_i$ if $(i,j) \in \mathcal{E}$ and $i \neq j$. We call $\mathcal{N}_i$ the neighborhood of node i. The adjacency matrix $\mathcal{A}(\mathcal{G})$ of undirected graphs is symmetric. We define the Laplacian matrix as $\mathcal{L}(\mathcal{G}) = \mathcal{D}(\mathcal{G}) - \mathcal{A}(\mathcal{G})$, where $\mathcal{D}(\mathcal{G})$ is the diagonal matrix of vertex degrees d_i. Let $S(\mathcal{L}(\mathcal{G})) = \{\lambda_1(\mathcal{L}(\mathcal{G})),\cdots,\lambda_N(\mathcal{L}(\mathcal{G}))\}$ be the spectrum of $\mathcal{L}$ associated with an undirected graph $\mathcal{G}$ arranged in nondecreasing semi-order. The following two results are standard [6], [23].

Proposition 1. *Let $\mathcal{G}$ be a complete graph (with all possible edges) with N_L vertices and $\mathcal{L}(\mathcal{G})$ be the corresponding Laplacian matrix. Then $S(\mathcal{L}(\mathcal{G})) = \{0,N_L,\cdots,N_L\}$.*

Proposition 2. *Let A, B be matrices of appropriate dimensions and $\mathcal{L}$ be Laplacian matrix with spectrum $S(\mathcal{L}) = \{\lambda_1,\cdots,\lambda_N\}$. Then the spectrum $S(I\otimes A + \mathcal{L}\otimes B)$ can be reduced to $\bigcup_{i\in[1:N]} S(A+\lambda_i B)$ with $\lambda_i \in S(\mathcal{L})$.*

III. Centralized LQR design for Dynamically Coupled Systems

Consider a network of N_L dynamically coupled LTI systems referred to as agents. At local level the dynamics of the i-th agent is represented in state-space form as:

$$\dot{x}_i = A_1 x_i + A_2 \sum_{j=1,j\neq i}^{N_L} (x_i - x_j) + B u_i,\ x_{0,i} = x_i(0) \tag{1}$$

where $x_i \in \mathbb{R}^n$, $u_i \in \mathbb{R}^m$ are states and inputs of the i-th system, respectively. A complete graph $\mathcal{G} = (\mathcal{V},\mathcal{E})$ with Laplacian matrix $\mathcal{L}_c$ models the topology of the physical links between agents. Node $i \in \mathcal{V}$ of $\mathcal{G}$ corresponds to local state x_i while edge $(i,j) \in \mathcal{E}$ corresponds to the $x_i - x_j$ term in (1). Now construct the aggregate state $\tilde{x} \in \mathbb{R}^{nN_L}$ and input vector $\tilde{u} \in \mathbb{R}^{mN_L}$ by stacking all state and input vectors, respectively, of all N_L systems taken in ascending order. The aggregate state-space of the network becomes:

$$\dot{\tilde{x}} = \tilde{A}\tilde{x} + \tilde{B}\tilde{u},\ \tilde{x}_0 = \tilde{x}(0) \tag{2}$$

with

$$\tilde{A} = I_{N_L}\otimes A_1 + \mathcal{L}_c \otimes A_2,\ \tilde{B} = I_{N_L}\otimes B \tag{3}$$

Consider now LQR control problem for the network of N_L coupled systems:

$$\min_{\tilde{u}}\ J(\tilde{u},\tilde{x}_0)\ \text{s.t.}\ \dot{\tilde{x}} = \tilde{A}\tilde{x} + \tilde{B}\tilde{u},\ \tilde{x}_0 = \tilde{x}(0) \tag{4}$$

where the cost function is

$$J(\tilde{u},\tilde{x}_0) = \int_0^\infty \tilde{x}'\tilde{Q}\tilde{x} + \tilde{u}'\tilde{R}\tilde{u}\ dt \tag{5}$$

with

$$\tilde{Q} = I_{N_L}\otimes Q_1 + \mathcal{L}_c\otimes Q_2 \text{ and } \tilde{R} = I_{N_L}\otimes R. \tag{6}$$

Here, the weighting matrices $Q_1 = Q_1' \geq 0$ and $R = R' > 0$ penalize local states and inputs of each node, respectively, while $Q_2 = Q_2' \geq 0$ weighs relative state differences between subsystems. The following assumptions guarantee a solution to LQR problem (4).

Assumption 1. *Let $C_1'C_1 = Q_1$. The pair (A_1,B) is stabilizable and (A_1,C_1) is observable.*

Assumption 2. *Let $C_{12}'C_{12} = Q_1 + N_L Q_2$. The pair $(A_1 + N_L A_2, B)$ is stabilizable and $(A_1 + N_L A_2, C_{12})$ is observable.*

With Assumption 1 and 2 in force, problem (4) admits a unique stabilizing solution $\tilde{u} = \tilde{K}\tilde{x}$, which gives minimum performance index (5) equal to $\tilde{x}_0'\tilde{P}\tilde{x}_0$ (see [24] and references therein). Then, the optimal LQR gain is $\tilde{K} = -\tilde{R}^{-1}\tilde{B}'\tilde{P}$, where $\tilde{P}$ is the symmetric positive definite (s.p.d.) solution to the (large-scale) Algebraic Riccati Equation (ARE):

$$\tilde{A}'\tilde{P} + \tilde{P}\tilde{A} - \tilde{P}\tilde{B}\tilde{R}^{-1}\tilde{B}'\tilde{P} + \tilde{Q} = 0. \tag{7}$$

Due to the specific formulation of (4), matrices $\tilde{K}$ and $\tilde{P}$ retain certain structure which will prove essential for designing stabilizing distributed controllers in the next section.

Theorem 1. *Assume $\tilde{P}$ is the s.p.d solution to (7) associated with the optimal solution to (4). Let $\tilde{P} \in \mathbb{R}^{nN_L\times nN_L}$ be decomposed into N_L^2 blocks of dimension $n\times n$, each denoted by $\tilde{P}_{ij}$. Then, the following hold true.*

I. *$\sum_{j=1}^{N_L}\tilde{P}_{ij} = P$ where $P = P' \geq 0$ is the stabilizing solution to single-node ARE:*

$$A_1'P + PA_1 - PBR^{-1}B'P + Q_1 = 0. \tag{8}$$

II. $\tilde{P}_{ij} = \tilde{P}_{kl} = \tilde{P}_2$ *for all* $j \neq i$, $l \neq k$ *where* $\tilde{P}_2$ *is symmetric matrix associated with the node-level ARE:*

$$(A_1 + N_L A_2)'(P - N_L \tilde{P}_2) + (P - N_L \tilde{P}_2)(A_1 + N_L A_2) \\ -(P - N_L \tilde{P}_2) B R^{-1} B' (P - N_L \tilde{P}_2) + Q_1 + N_L Q_2 = 0. \quad (9)$$

Detailed proof of Theorem 1 can be found in [22]. By assumption, matrices $\tilde{R}$ and $\tilde{B}$ are selected block diagonal. Consequently, the gain $\tilde{K} = -\tilde{R}^{-1}\tilde{B}'\tilde{P}$ retains the same structure with $\tilde{P}$. This leads to the following Corollary.

Corollary 1. *Assume* $\tilde{K} = -\tilde{R}^{-1}\tilde{B}'\tilde{P}$ *is the optimal state-feedback gain obtained from the solution to* (4) *with* $\tilde{P}$ *being the s.p.d solution to* (7). *Let* $\tilde{K} \in \mathbb{R}^{mN_L \times nN_L}$ *and* $\tilde{P} \in \mathbb{R}^{nN_L \times nN_L}$ *be decomposed into* N_L^2 *blocks of dimension* $m \times n$ *and* $n \times n$ *denoted by* $\tilde{K}_{ij}$ *and* $\tilde{P}_{ij}$, *respectively. Then, the following are true;*

I. $\tilde{P} = I_{N_L} \otimes P - \mathcal{L}_c \otimes \tilde{P}_2$.
II. $\sum_{j=1}^{N_L} \tilde{K}_{ij} = -R^{-1}B'P$ *for* $i = 1, \cdots, N_L$.
III. $\tilde{K}_{ii} = -R^{-1}B'P + (N_L - 1)R^{-1}B'\tilde{P}_2$ *for* $i = 1, \cdots, N_L$.
IV. $\tilde{K}_{ij} = -R^{-1}B'\tilde{P}_2$ *for* $i, j = 1, \cdots, N_L$ *and* $j \neq i$.
V. $\tilde{K} = I_{N_L} \otimes K - \mathcal{L}_c \otimes K_2$, *where* $K = -R^{-1}B'P$ *and* $K_2 = -R^{-1}B'\tilde{P}_2$.

In view of Corollary 1, the closed-loop matrix $\tilde{A} + \tilde{B}\tilde{K}$ can be written as:

$$A_{cl} = I_{N_L} \otimes (A_1 + BK) + \mathcal{L}_c \otimes (A_2 - BK_2). \quad (10)$$

Since $\tilde{u} = \tilde{K}\tilde{x}$ is stabilizing, the matrix A_{cl} is Hurwitz. Via Proposition 2 the spectrum of A_{cl} can be decomposed as

$$S(A_{cl}) = \bigcup_{i=1}^{N_L} S\left(A_1 + BK + \lambda_{c,i}(A_2 - BK_2)\right), \quad (11)$$

where $\lambda_{c,i} = \{0, N_L, \cdots, N_L\}$. The following remark follows straightforwardly from the special spectrum of the Laplacian matrix of a complete graph.

Remark 1. The matrix $A_1 + BK + \alpha N_L(A_2 - BK_2)$ is Hurwitz for $\alpha = 0$ and $\alpha = 1$.

In the sequel we require that:

Condition 1. The matrix $A_1 + BK + \alpha N_L(A_2 - BK_2)$ is Hurwitz for all $\alpha \in [0, 1]$.

In essence, Condition 1 states that all convex combinations of two Hurwitz matrices

$$\mu \bar{A}_1 + (1 - \mu)\bar{A}_2 \text{ with } \mu \in [0, 1] \quad (12)$$

are Hurwitz, where $\bar{A}_1 = A_1 + BK + N_L(A_2 - BK_2)$ and $\bar{A}_2 = A_1 + BK$. Sufficient conditions for Hurwitz stability of convex combination of Hurwitz matrices can be found in Theorem 2.2 in [5]. In general, the validity of the condition can be checked via a simple graphical test.

IV. Distributed Control Design for Dynamically Coupled Systems

Let a network be formed of N identical and dynamically coupled LTI systems. Each subsystem has the ability to exchange state-information with all counterparts it is coupled with. The two graphs defining energy and information exchanges are assumed to be identical. This is denoted by $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ with Laplacian matrix $\mathcal{L}$. Let the dynamics at local level of the i-th system be

$$\dot{x}_i = A_1 x_i + A_2 \sum_{j \in \mathcal{N}_i} (x_i - x_j) + Bu_i, \; x_{0,i} = x_i(0) \quad (13)$$

where $x_i \in \mathbb{R}^n$ and $u_i \in \mathbb{R}^m$. The aggregate state-space of the network becomes

$$\dot{\hat{x}} = \hat{A}\hat{x} + \hat{B}\hat{u}, \; \hat{x}_0 = \hat{x}(0) \quad (14)$$

where $\hat{x} \in \mathbb{R}^{nN}$, $\hat{u} \in \mathbb{R}^{mN}$ and

$$\hat{A} = I_N \otimes A_1 + \mathcal{L} \otimes A_2, \; \hat{B} = I_N \otimes B. \quad (15)$$

Note that $\mathcal{L}$ in (15) does not necessarily correspond to a complete graph in contrast to (3) and generically matrix $\hat{A}$ in (15) is sparse. A stabilizing distributed controller for (14) is constructed in the following Theorem.

Theorem 2. *Consider a network of* N *coupled systems with dynamics described in* (13) *modelled by graph* $\mathcal{G}_N$ *with Laplacian matrix* $\mathcal{L}_N$. *Let* λ_N *be the maximum eigenvalue of* $\mathcal{L}_N$ *and denote by* d_{max} *the smallest integer which is greater than or equal to* λ_N. *Consider problem* (4) *for* $N_L = d_{max}$, *define* P *and* $\tilde{P}_2$ *via* (8) *and* (9), *respectively, and assume Condition 1 holds. Define also:* $K = -R^{-1}B'P$, $K_2 = -R^{-1}B'\tilde{P}_2$ *and*

$$\hat{K} = I_N \otimes K - \mathcal{L}_N \otimes K_2. \quad (16)$$

Then, the closed-loop matrix

$$A_{cl} = I_N \otimes (A_1 + BK) + \mathcal{L}_N \otimes (A_2 - BK_2) \quad (17)$$

is Hurwitz.

Proof: Consider the spectrum $S(A_{cl}) = S(I_N \otimes (A_1 + BK) + \mathcal{L}_N \otimes (A_2 - BK_2))$. Let $V_N \otimes I_n$ be state-space transformation where $V_N \in \mathbb{R}^{N \times N}$ is an orthogonal matrix whose columns are the eigenvectors of $\mathcal{L}_N$. In the transformed coordinates, $\bar{A}_{cl} = I_N \otimes (A_1 + BK) + \Lambda_N \otimes (A_2 - BK_2)$ where $\Lambda_N = \text{diag}(0, \lambda_2, \cdots, \lambda_N)$ with $\lambda_N \leq d_{max}$. Then

$$S(\bar{A}_{cl}) = \bigcup_{i=1}^{N} S(A_1 + BK + \lambda_i(A_2 - BK_2)) \quad (18)$$

where λ_i, $i = 1, \cdots, N$ are the eigenvalues of $\mathcal{L}_N$. Condition 1 holds, hence $(A_1 + BK) + \alpha d_{max}(A_2 - BK_2)$ is Hurwitz for all $\alpha \in [0, 1]$. Consequently $\bar{A}_{cl}$ is also Hurwitz since $\lambda_i \in [0, d_{max}]$ for all $i = 1, \cdots, N$. This proves the Theorem. $\square$

Remark 2. For a time-varying graph $\mathcal{G}(t) = (\mathcal{V}, \mathcal{E}(t))$ with fixed number of vertices (N) and time-varying edges the maximum eigenvalue of the time-varying Laplacian matrix $\mathcal{L}(t)$ is bounded by $2N$. Consequently, solving (4) for $N_L = 2N$ and assuming Condition 1 is in force leads to a distributed controller $\hat{K}$ which stabilizes the network for all possible interconnection schemes among the N systems. Naturally, this does not imply stability of switching between stable network topologies.

V. Multi-Area Power Grid

We consider power system networks which can be decomposed into multiple distinct dynamical subsystems, referred to as areas, each one having two primary characteristics; (1) it comprises of either a single generator or a group of generators, and (2) it maintains a single frequency across its geographical expanse. The areas are responsible for meeting the demand of their own consumers and are interconnected with each other through tie-lines, over which they exchange power normally scheduled for a contracted value.

A. Modelling

Let a multi-area power system be formed of N areas the interconnection topology of which is modeled by an undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$. Each node $i \in \mathcal{V}$ represents an area and an edge $(i, j) \in \mathcal{E}$ between two nodes denotes interaction between the two nodes/areas. We note that the edge (i, j) of the graph determines coupling terms in the dynamics of area i and j and also indicates state-information exchange between node i and j. We denote by $\mathcal{N}_i$ the set of all the adjacent nodes to i. The open-loop linearized dynamics of the i-th interconnected area is represented by a model widely used in literature [15], the block diagram of which is shown in Figure 1.

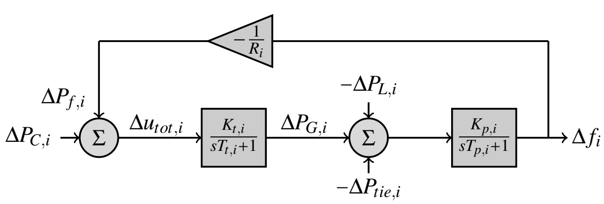


Fig. 1: Block representation of the i-th interconnected area.

The linearized dynamics of the total power inflow ($\Delta P_{tie,i}$) to the i-th area from all interconnected areas $j \in \mathcal{N}_i$ is

$$\Delta P_{tie,i} = \int \sum_{j \in \mathcal{N}_i} K_{tie,i,j} (\Delta f_i - \Delta f_j) \; dt. \tag{19}$$

Notation Δ indicates deviation from steady-state operation. The total control signal of the i-th area is the sum of two components: $\Delta u_{tot,i} = \Delta P_{f,i} + \Delta P_{C,i}$, namely the primary frequency control action, defined as $\Delta P_{f,i} = -\frac{1}{R_i} \Delta f_i$ and the automatic generation control (AGC) signal $\Delta P_{C,i}$. The first is a fixed static linear control law while the second (AGC) is the control signal to be designed (see [15], [16] for details). The disturbance signal $\Delta P_{L,i}$ denotes time-varying demand of the i-th area consumers. It is assumed to be unknown, piecewise constant power load deviations with known bounds. Here, we study the case where $\Delta P_{L,i,\min} \leq \Delta P_{L,i} \leq \Delta P_{L,i,\max}$ for $i = 1, \cdots, N$. All variables and parameters involved in the block representation of Fig. 1 are summarized in Table I.

A thorough description of the power system model considered in this paper can be found in [22]. The frequency regulation problem of a multi-area power grid of N identical areas is stated in the next paragraph.

TABLE I: Parameter Values for each Control Area.

Parameter	Symbol	Value	Units
Nominal Frequency	f^o	50	H_z
Power Base	$P_{B,i}$	2000	MW
Load Dependency Factor	D_i	16.66	MW/H_z
Speed Droop	R_i	1.2×10^{-3}	H_z/MW
Generator Inertia Gain	H_i	5	s
Turbine Static Gain	$K_{t,i}$	1	MW/MW
Turbine Time Constant	$T_{t,i}$	0.3	s
Area Static Gain	$K_{p,i}$	0.06	H_z/MW
Area Time Constant	$T_{p,i}$	24	s
Tie-line Coefficient	$K_{tie,i}$	1090	MW/H_z

B. Problem statement

Power load change in the i-th area of an interconnected power system causes the electrical frequency f_i to deviate from its nominal value. Due to interconnections among the areas through power transmission tie-lines and the dependence of the power exchange between the i-th and j-th area upon the respective difference $\Delta f_i - \Delta f_j$ (see eq. (19)), any power load deviation occurring in the i-th area will also affect the linked j-th area, causing transients in its frequency f_j. Here, we formulate the LFC of multi-area power systems as an optimal control problem in the absence of state and input constraints. The special case of N identical areas is considered. According to Fig. 1, the state-vector x_i of the i-th area is constructed as $x_i = [\Delta f_i \; \Delta P_{G,i} \; \Delta P_{tie,i} \; \int z_i]'$, where the signal $z_i = \beta \Delta f_i + \Delta P_{tie,i}$ is referred to as Area Control Error (ACE). A usual choice for β (bias factor) is $D_i + \frac{1}{R_i}$, [15]. Parameters D_i and R_i are defined in Table I. Note that the state augmentation by the integral of the ACE signal enforces integral action into the control scheme of each area. This is a standard technique for disturbance rejection problems (see [22] for more details). The aggregate dynamics of the network can now be represented by a state-space model of the form:

$$\begin{aligned} \dot{\hat{x}} &= (I_N \otimes A_1 + \mathcal{L} \otimes A_2)\hat{x} + (I_N \otimes B_u)\hat{u} + (I_N \otimes B_w)\hat{w} \\ \hat{x}_0 &= \hat{x}(0) \end{aligned} \tag{20}$$

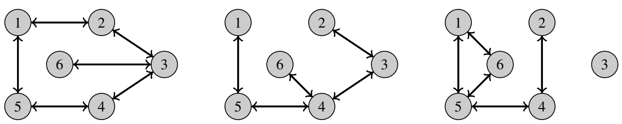
with $\hat{x} = \mathrm{col}(x_1, \cdots, x_N)$, $\hat{u} = \mathrm{col}(\Delta P_{C,1}, \cdots, \Delta P_{C,N})$, $\hat{w} = \mathrm{col}(\Delta P_{L,1}, \cdots, \Delta P_{L,N})$ and

$$\underbrace{\begin{bmatrix} -\frac{1}{T_p} & \frac{K_p}{T_p} & -\frac{K_p}{T_p} & 0 \\ -\frac{K_t}{RT_t} & -\frac{1}{T_t} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ B & 0 & 1 & 0 \end{bmatrix}}_{A_1}, \underbrace{\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ K_{tie} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}}_{A_2}, \underbrace{\begin{bmatrix} 0 \\ \frac{K_t}{T_t} \\ 0 \\ 0 \end{bmatrix}}_{B_u}, \underbrace{\begin{bmatrix} -\frac{K_p}{T_p} \\ 0 \\ 0 \\ 0 \end{bmatrix}}_{B_w}$$

where the subscript i has been neglected from all entries of A_1, A_2, B_u, and B_w since all areas are assumed to be identical. Next, the distributed LQR control design presented earlier is applied to a six-area power network subjected to power load deviations and switching topology.

C. Numerical example

Consider a power system network of six identical areas interconnected via tie-lines. Assume the network has switching topology where a single interconnection scheme is modelled by a time-invariant graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$. Three topology schemes are shown in Fig. 2 which are considered in the simulations. The corresponding Laplacian matrix for each case is given in (21). The switching topology considered is utilized to demonstrate stable network operation under the same control scheme for a class of network topologies. We stress that the stability of switched stable systems is beyond the scope of this work.



(a) Topology S_1, Laplacian $\mathcal{L}_1$. (b) Topology S_2, Laplacian $\mathcal{L}_2$. (c) Topology S_3, Laplacian $\mathcal{L}_3$.

Fig. 2: Tie-line switching topology of six interconnected control areas.

In the study, we consider one scenario where disturbances $\Delta P_{L,i}$ for $i = 1, \cdots, 6$ occur at different instants well-spread out over the simulation interval. During the simulation period the network interconnections are altered according to the order shown in Fig. 2. The distributed controller constructed in Section IV is used here to drive the AGC signal $(\Delta P_{C,i})$ of each area. The control objective of each area is to meet its load demand shown in Fig. 3 and regulate its frequency. The construction of the distributed controller is summarized next.

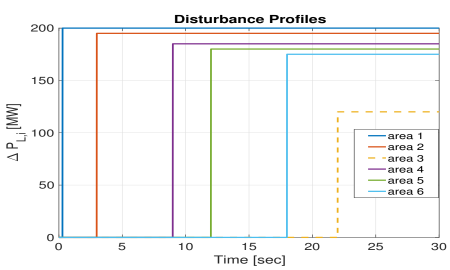


Fig. 3: Power demand deviation $\Delta P_{L,i}$, $i = 1 \cdots, 6$.

The maximum eigenvalue of each matrix $(\mathcal{L}_1, \mathcal{L}_2, \mathcal{L}_3)$ in (21) is 4.3028, 4.3028 and 4.1701, respectively. We take the smallest integer denoted by d_{max} which is greater than or equal to the maximum of these (4.3028), i.e., $d_{max} = 5$. We solve optimal problem (4) for $N_L = d_{max} = 5$ for two different selections of the weights $\tilde{Q}$, $\tilde{R}$. In the first, $\tilde{Q} = I_5 \otimes Q_1$, with $Q_1 = \mathrm{diag}(100, 10, 10, 5000)$ and $\tilde{R} = I_5 \otimes 100$ while in the second $\tilde{R}$ is kept the same and $\tilde{Q} = I_5 \otimes Q_1 + \mathcal{L}_5 \otimes Q_2$ where $Q_2 = 200 Q_1$ and $\mathcal{L}_5$ is Laplacian matrix corresponding to complete graph (all possible edges) with 5 nodes. The matrix Q_2 penalizes the relative state-difference $(x_i - x_j)$ between neighboring areas in (5). According to Theorem 2, we compute the respective K and K_2 state-feedback gains for each tuning. These are:

$$\begin{aligned} K &= \begin{bmatrix} -2502.857 & -1.203 & -1.757 & -7.071 \end{bmatrix} \\ K_2 &= \begin{bmatrix} 342.491 & 0.104 & -0.225 & 0.000 \end{bmatrix} \end{aligned} \tag{22}$$

for the first tuning where $Q_2 = 0$ and

$$\begin{aligned} K &= \begin{bmatrix} -2502.857 & -1.203 & -1.757 & -7.071 \end{bmatrix} \\ K_2 &= \begin{bmatrix} 12084.071 & 2.356 & 6.374 & 43.329 \end{bmatrix} \end{aligned} \tag{23}$$

for the case where $Q_2 = 200 Q_1$. Note that $K = -R^{-1} B_u' P$ is the same for both cases since P is the solution to a single node-level ARE with parameters (A_1, B_u, Q_1, R). We also test the validity of Condition 1 which can be seen to hold. Fig. 4 displays the real part of the maximum eigenvalue of $(A_1 + B_u K) + \alpha d_{max}(A_2 - B_u K_2)$ with $\alpha \in [0, 1]$ for both tunings. In essence, this implies stable operation of the network under both control schemes for all possible topologies corresponding to Laplacian matrices with maximum eigenvalue bounded by d_{max}.

At network level the distributed stabilizing controller $\hat{K}$ takes the form

$$\hat{K} = I_6 \otimes K - \mathcal{L}_s \otimes K_2 \tag{24}$$

where $\mathcal{L}_s$ is given in (21) according to the topology with $s = [1:3]$. Node-wise, the AGC signal at each area is

$$\Delta P_{C,i} = K x_i - K_2 \sum_{j \in \mathcal{N}_i} (x_i - x_j) \tag{25}$$

with $i = 1, \cdots, 6$ and $j \neq i$. In effect each area requires accessibility to the local and neighboring states in order to construct its control signal. Fig. 5 and 6 show the response of the frequency deviation and the total power-flow deviation of the tie-lines from the equilibrium operation of each area for the two control schemes given in (22) and (23). The time-intervals for the three topologies S_1, S_2 and S_3 are depicted in Fig. 5a. Stable operation is guaranteed for both control choices even for area-3 (denoted by yellow bold in the graphs) which is isolated over the third part of the simulation during which the network acquires the topology S_3 (Fig. 2c).

Note also, the magnitude of the total power flow over the tie-lines is significantly limited in the case where the controller is designed as in (23). This stems from the large weighting matrix Q_2 selection in the performance index (5). Since the relative state-difference between neighboring areas is highly penalized, the areas tend to acquire same frequency

$$\mathcal{L}_1 = \begin{bmatrix} 2 & -1 & 0 & 0 & -1 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & -1 & 3 & -1 & 0 & -1 \\ 0 & 0 & -1 & 2 & -1 & 0 \\ -1 & 0 & 0 & -1 & 2 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 \end{bmatrix}, \mathcal{L}_2 = \begin{bmatrix} 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & -1 & 3 & -1 & -1 \\ -1 & 0 & 0 & -1 & 2 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 \end{bmatrix} \text{ and } \mathcal{L}_3 = \begin{bmatrix} 2 & 0 & 0 & 0 & -1 & -1 \\ 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 2 & -1 & 0 \\ -1 & 0 & 0 & -1 & 3 & -1 \\ -1 & 0 & 0 & 0 & -1 & 2 \end{bmatrix} \tag{21}$$

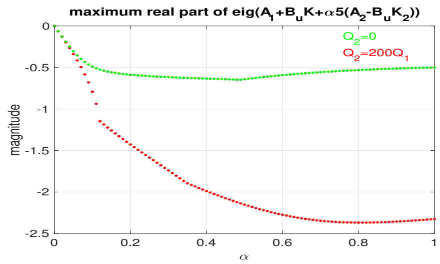
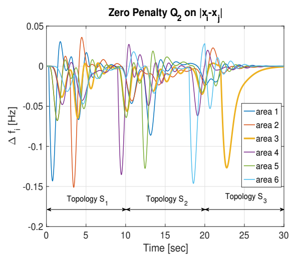
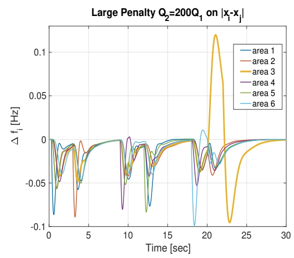


Fig. 4: Stability test and validity of Condition 1.



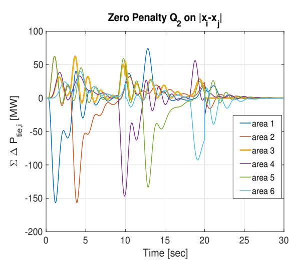
(a) $Q_2 = 0$.



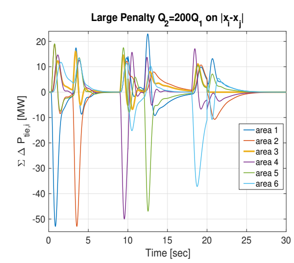
(b) $Q_2 = 200Q_1$.

Fig. 5: Frequency deviation for two different tunings.

deviations during the transients (see Fig. 5b) and thus the total power flow over the tie-lines given in (19) is kept low.



(a) $Q_2 = 0$.



(b) $Q_2 = 200Q_1$.

Fig. 6: Total power flow over tie-lines for two different tunings.

VI. CONCLUSIONS

Stabilizing distributed LQR-based controller for networks of identical dynamically coupled agents was proposed based on a large-scale LQR optimal problem. This method has originally been proposed in [6] for the decoupled case and was extended here to include couplings between the subsystems representing autonomous agents. First, a fully centralized controller was designed which was subsequently substituted by a distributed state-feedback gain with sparse structure. The control scheme was obtained by optimizing an LQR performance index with a tuning parameter utilized to emphasize/de-emphasize relative state difference between interconnected systems. Our approach enhances the multi-agent system modularity and leads to a simple and verifiable stabilizability condition for a class of network topologies. The control scheme was applied for two different tunings to a multi-area power system which was subjected to power load demand variations and switching topologies. The authors are currently working on the extension of the method to the case of non-identical dynamically coupled agents based on results proposed in [20], [21].

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