



## City Research Online

### City, University of London Institutional Repository

---

**Citation:** Listios, I., Pilbeam, K. & Asteriou, D. (2020). DSGE Modelling for the UK Economy 1974-2017. Bulletin of Economic Research, doi: 10.1111/boer.12255

This is the published version of the paper.

This version of the publication may differ from the final published version.

---

**Permanent repository link:** <https://openaccess.city.ac.uk/id/eprint/24199/>

**Link to published version:** <https://doi.org/10.1111/boer.12255>

**Copyright:** City Research Online aims to make research outputs of City, University of London available to a wider audience. Copyright and Moral Rights remain with the author(s) and/or copyright holders. URLs from City Research Online may be freely distributed and linked to.

**Reuse:** Copies of full items can be used for personal research or study, educational, or not-for-profit purposes without prior permission or charge. Provided that the authors, title and full bibliographic details are credited, a hyperlink and/or URL is given for the original metadata page and the content is not changed in any way.



## ARTICLE

# DSGE modelling for the UK economy 1974–2017

Ioannis Litsios<sup>1</sup> | Keith Pilbeam<sup>2</sup> | Dimitrios Asteriou<sup>3</sup>

<sup>1</sup> Plymouth Business School, Department of Accounting, Economics & Finance, University of Plymouth, Plymouth, UK

<sup>2</sup> Department of Economics City, University of London, London, UK

<sup>3</sup> Oxford Brookes Business School, Oxford Brookes University, Oxford, UK

**Correspondence**

Ioannis Litsios, Plymouth Business School, Department of Accounting, Economics & Finance, University of Plymouth, Plymouth, UK.

Email: [ioannis.litsios@plymouth.ac.uk](mailto:ioannis.litsios@plymouth.ac.uk)

**Abstract**

We build four different dynamic stochastic general equilibrium (DSGE) models for a small open economy reflecting both neoclassical and Keynesian specifications. A DSGE model with full price and wage flexibility is initially constructed and then modified through nominal wage and price rigidities. The ability of the models to replicate important features of the business cycle activity in the UK is explored through statistical and econometric analysis. Evidence suggests that a monetary shock under the Taylor model with price stickiness can replicate a significant portion of the business cycle activity in the UK economy.

**KEYWORDS**

DSGE, long run dynamics, monetary-fiscal-technology shocks, price rigidities, short run dynamics, simulated business cycles, small open economy, wage rigidities

**JEL CLASSIFICATION**

E52, E32, E24, E27, F41

## 1 | INTRODUCTION

This paper compares the predictions of several dynamic stochastic general equilibrium (DSGE) models with a view to developing an improved understanding of observed fluctuations in small open economies. An analytical framework, synthesising both neoclassical and Keynesian approaches, is proposed resulting in the construction of four different DSGE models, the predictions of which can be tested in the context of any small open economy. For definiteness, the current research focuses exclusively on the UK economy.

-----  
This is an open access article under the terms of the [Creative Commons Attribution](https://creativecommons.org/licenses/by/4.0/) License, which permits use, distribution and reproduction in any medium, provided the original work is properly cited.

© 2020 The Authors. *Bulletin of Economic Research* published by Trustees of the Bulletin of Economic Research and John Wiley & Sons Ltd

The benchmark model, as presented in section 2 reflects a neoclassical economy under the assumptions of perfect wage and price flexibility. Since the various theoretical predictions of the neoclassical models have triggered researchers to question their appropriateness to replicate data well, we examine the empirical implications of the neoclassical model in relation to the UK economy. We then proceed by modifying the neoclassical model with elements of nominal rigidities. Consequently, three Keynesian variants emerge: one with inflexible wages, and two others focusing explicitly on inflexible prices. The models are calibrated, simulated and evaluated in the presence of three different shocks: a domestic monetary shock, a domestic fiscal shock, and a domestic technology shock.

The ability of the models to replicate important features of the business cycle activity in the UK is explored after using both statistical and econometric analysis. As a result, the extent to which simulated business cycles can reproduce actual economic activity is primarily examined through moment comparisons between actual and simulated data. We also compare impulse responses generated by a VAR model for the UK with those predicted by the various models.

In order to test the empirical implications of our economic modelling in the long run, we employ a VECM model and we test for the coexistence of the purchasing power parity (PPP) and the uncovered interest rate parity (UIRP) in the long run. The empirical results are then compared to the long run predictions of the models related to these two international relationships.

In relation to our theoretical specification, the intertemporal asset-pricing model is assumed to reflect the representative agent's optimal allocation within a small open economy framework. The assumptions behind the agent's portfolio construction are crucial for the economic modelling. As distinct from other approaches in the literature, where the representative household is assumed to optimally allocate wealth between consumption and financial investment,<sup>1</sup> we assume that the domestic agent holds a portfolio of four different assets including domestic bonds, foreign bonds, domestic stocks, and real money balances. We explicitly introduce investment in foreign bonds as an additional source of openness apart from that of exports and imports.

In addition, as distinct from other literature on New Open-Economy Macroeconomics (NOEM), where microfoundations are explicitly embedded into a dynamic stochastic general equilibrium model (DSGE), the utility function employed in the current analysis, is a multiplicative iso-elastic Cobb Douglas utility function, which has been used by Finn, Hoffman, and Schlagenhauf (1990) and Poterba and Rotemberg (1986) in a closed economy framework.<sup>2</sup> The power Cobb Douglas utility function is tractable and maintains the scale independence property of a power utility<sup>3</sup>. Utility is assumed to be derived from consumption and real money balances. Following the seminal works of Kydland and Prescott (1982) and Prescott (1986), a major area of research has been oriented towards the role of the monetary sector in an otherwise conventional real business cycle (RBC) model.<sup>4</sup> The current research considers the microfoundations of the distinctive

<sup>1</sup> See for example Galí and Monacelli (2004) where the representative agent is assumed to invest only in shares of domestic firms. In a different specification, Amber et al (2004) allow investment in domestic and foreign bonds and real money balances with no reference to equity holdings.

<sup>2</sup> Given the framework of our economic modelling we embed Finn's et al (1990) utility function into an open economy environment. As in Finn et al. (1990) it is assumed for tractability that labour supply is provided inelastically. We leave inclusion of leisure in the utility function for later research recognizing the fact that leisure choice could provide further insights into the parametrization of our sticky price models.

<sup>3</sup> Other attractive utility functions, like the Epstein and Zin (1989, 1991) and Weil (1989) could have been employed in our modeling but they are omitted here for simplicity.

<sup>4</sup> See Galí (2008) for a discussion of the role of money within a classical monetary model. Cooley and Hansen (1989) is an early example of a closed economy under the assumptions of perfect competition and fully flexible prices and wages.

properties of money within a “New Open Economy Macroeconomic” (NOEM) structure. The substantive reason for including money in the utility function, is that we need to give to the asset called “money” some function to distinguish it from other assets, like interest bearing bonds or stocks. This in turn, reflects our ability to explicitly generate a demand for real money balances at the level of aggregation of our macroeconomy. In fact, real money balances are assumed to reflect the role of money for transaction purposes, in the absence of a formal transaction mechanism. Consequently, we do not focus on a cash in advance (CIA) model, like Svensson (1985) and Lucas (1982) or on a time shopping model.

In its neoclassical foundations, the model reflects a small open economy, which consists of an optimizing representative agent, who holds both domestic and foreign assets, together with the fiscal and monetary branches of the government, and production and foreign sectors. Domestic firms produce a homogeneous product and it is assumed that wages and prices are fully flexible and that full employment is achieved.

We then introduce nominal rigidities in order to emphasize the micro-economic details of wage inflexibility. The novelty of our approach is to introduce a dynamic stochastic general equilibrium model with rational expectations and overlapping labour contracts within the small open economy framework after modifying the benchmark neoclassical model. This is achieved after introducing a variant of Taylor’s (1979) overlapping contract mechanism into a dynamic stochastic modelling, assuming that a proportion of decision makers (firms-unions) negotiate a nominal contract wage growth, which last for two subsequent periods. Each contract is written relative to other contracts, where firms-unions must look both backward and forward in time.

Finally, we proceed by incorporating price inflexibility into the neoclassical model following two different approaches. The first innovative approach combines elements from both Calvo (1983) and Rotemberg, (1982a, b), whereas the second is a reflection of Taylor’s (1979) model under the assumption that prices are a constant mark-up over wage costs. An interesting aspect of the way that the two models with price stickiness are constructed, which has important implications for the implementation of the dynamic analysis, is the fact that an inflation adjustment equation appears explicitly. Given the innovative construction of these models, such an equation is not identical between the Calvo-Rotemberg specification and the Taylor model with price stickiness. This has further important implications for the calibration of the monetary policy rule in these models.

The paper is organized as follows: Section 2 provides the neoclassical DSGE model with full price and wage flexibility. Section 3 presents a Keynesian variant, after reformulating the baseline neoclassical model with a staggered wage setting mechanism, and two New Keynesian specifications with price rigidities. Section 4 presents the solution to the new Keynesian models and section 5 discusses the empirical implications of the models. Section 6 presents the short-run dynamic predictions of the models. Section 7 concludes and provides suggestions for further research.

## 2 | A NEOCLASSICAL DYNAMIC STOCHASTIC GENERAL EQUILIBRIUM MODEL FOR A SMALL OPEN ECONOMY

This section presents a model of a small open economy (referred to as the domestic economy) under the assumptions of perfect wage and price flexibility. The underlying neoclassical economy consists of identical, infinitely lived households who maximize the present value of their lifetime utility subject to a sequence of constraints. Agents are assumed to invest in both domestic and foreign assets and to consume a composite index of home and foreign goods. There is a monetary

and a fiscal branch of the government, a foreign sector, and an aggregate production technology for domestic firms.

## 2.1 | The households

The small open economy is assumed to be inhabited by a representative agent who derives utility from consumption and real money balances. The utility function employed is an iso-elastic Cobb Douglas utility function of the form<sup>5</sup>:

$$E_t \left[ \sum_{t=0}^{\infty} \beta^t \left[ C_t^\delta \left( \frac{M_t}{P_t} \right)^{1-\delta} \right]^\gamma \frac{1}{\gamma} \right] \gamma < 1, 0 < \beta < 1, 0 < \delta < 1 \quad (1)$$

where  $E_t$  denotes expectation at  $t$ ,  $C_t$  denotes real consumption,  $M_t$  nominal money balances,  $P_t$  the price index of a composite good consumed domestically,  $\delta$  is a preference parameter,  $\beta$  the subjective time discount factor and  $1 - \gamma$  the index of relative risk aversion. The budget constraint for the domestic investor is given by:

$$y_t + \frac{M_{t-1}}{P_t} + \frac{B_{t-1}^D (1 + i_{t-1}^D)}{P_t} + \frac{B_{t-1}^F (1 + i_{t-1}^F)}{e_t P_t} + \frac{S_{t-1} (p_t^s + d_{t-1})}{P_t} = C_t + \frac{M_t}{P_t} + \frac{B_t^D}{P_t} + \frac{B_t^F}{e_t P_t} + \frac{S_t p_t^s}{P_t} \quad (2)$$

The left hand side of equation (2) represents total real wealth derived from real income  $y_t$ , real money balances  $\frac{M_{t-1}}{P_t}$ , real return from investment in domestic bonds at  $t - 1$ <sup>6</sup>, real return from investment in foreign bonds at  $t - 1$ <sup>7</sup> and real return from investment in domestic stocks at  $t - 1$ <sup>8</sup>. The nominal exchange rate  $e_t$  is defined to be the amount of foreign currency per unit of domestic currency. Given the probability distribution of future shocks, the agent observes his total real wealth and then proceeds with an optimal consumption and portfolio allocation plan. Utility is assumed to be derived from the following constant elasticity of substitution (CES) composite:

$$C_t = \left[ \alpha^{\frac{1}{\Theta}} (C_t^h)^{\frac{\Theta-1}{\Theta}} + (1 - \alpha)^{\frac{1}{\Theta}} (C_t^f)^{\frac{\Theta-1}{\Theta}} \right]^{\frac{\Theta}{\Theta-1}} \quad (3)$$

where,  $C_t^h, C_t^f$  represent consumption of domestically produced goods and goods imported from the foreign country. The degree of home bias in preferences is given by  $\alpha \in [0, 1]$  and the substitutability between domestic and foreign goods by  $\Theta > 1$ . Defining  $P_t^h$  and  $P_t^f$  as the price indexes of domestically produced goods and goods produced in the foreign economy (all expressed in units

<sup>5</sup> The power Cobb Douglas utility function retains the attractive property of the scale independence by assuming non-separability in consumption and real money balances.

<sup>6</sup>  $B_{t-1}^D$  is the amount of domestic currency invested in domestic bonds at  $t - 1$  and  $i_{t-1}^D$  is the nominal rate of return on these domestic bonds.

<sup>7</sup>  $B_{t-1}^F$  is the amount of foreign currency invested in foreign bonds at  $t - 1$  and  $i_{t-1}^F$  is the foreign rate of return on these foreign bonds.

<sup>8</sup>  $p_{t-1}^s$  is the share price at  $t - 1$ , which is equal to  $P_{t-1}$  on the assumption that capital and consumption are a homogeneous good.  $S_{t-1}$  denotes the number of shares purchased at  $t - 1$  and  $d_{t-1}$  the value of the dividend earned.

of domestic currency) the utility-based CPI of the composite good consumed domestically is given by<sup>9</sup>:

$$P_t = \left[ \alpha (P_t^h)^{1-\Theta} + (1-\alpha) (P_t^f)^{1-\Theta} \right]^{\frac{1}{1-\Theta}} \quad (4)$$

The optimal allocation from this static optimization problem, leads to the following symmetric isoelastic demand functions for both domestic and foreign goods:<sup>10</sup>

$$C_t^h = \alpha \left( \frac{s_t}{q_t} \right)^\Theta C_t; C_t^f = (1-\alpha) (q_t)^{-\Theta} C_t \quad (5)$$

The real exchange rate is defined by  $q_t = \frac{P_t^*}{e_t P_t}$  and the terms of trade by  $s_t = \frac{P_t^f}{P_t^h}$ . The dynamic optimization plan yields the following necessary first order conditions for real money balances, domestic bonds, foreign bonds, and domestic stocks:

$$1 = E_t \left\{ \left[ \left( \frac{C_t}{C_{t+1}} \right)^{\gamma\delta-1} \left( \frac{m_t}{m_{t+1}} \right)^{\gamma(1-\delta)} \left( \frac{\beta}{1+\pi_{t+1}} \right)^{-1} \right] \right\} \quad (6)$$

$$\left( - \left[ \frac{C_t^{\gamma\delta}}{C_{t+1}^{\gamma\delta-1}} \frac{m_t^{[(1-\delta)\gamma]-1}}{m_{t+1}^{\gamma(1-\delta)}} \frac{(1-\delta)}{\delta} \left( \frac{\beta}{1+\pi_{t+1}} \right)^{-1} \right] \right) \quad (6)$$

$$1 = E_t \left\{ \beta \left( \frac{C_{t+1}}{C_t} \right)^{\gamma\delta-1} \left( \frac{m_{t+1}}{m_t} \right)^{\gamma(1-\delta)} R_{t+1}^D \right\} \quad (7)$$

$$1 = E_t \left\{ \beta \left( \frac{C_{t+1}}{C_t} \right)^{\gamma\delta-1} \left( \frac{m_{t+1}}{m_t} \right)^{\gamma(1-\delta)} R_{t+1}^F \right\} \quad (8)$$

$$1 = E_t \left\{ \beta \left( \frac{C_{t+1}}{C_t} \right)^{\gamma\delta-1} \left( \frac{m_{t+1}}{m_t} \right)^{\gamma(1-\delta)} R_{t+1}^S \right\} \quad (9)$$

<sup>9</sup> The domestic price equivalent of the foreign price index can be written as  $P_t^f = \frac{P_t^{f,*}}{e_t}$  and the foreign currency equivalent of the domestic price index as  $P_t^{h,*} = P_t^h e_t$ . Variables that correspond to the foreign economy are denoted with an asterisk “\*”.

<sup>10</sup>  $C_t^h$ , as given in equation (5), will not be used explicitly in the construction of the general equilibrium model but will be reflected in the economy wide resource constraint in subsection 2.6 as the difference between aggregate consumption  $C_t$  and imports. See Appendix I for full derivation of the isoelastic demand functions.

where  $C_t$  is real consumption,  $m_t = \frac{M_t}{P_t}$  are real money balances,  $\pi_t$  is domestic inflation,  $R_{t+1}^D$  is real return on domestic bonds,  $R_{t+1}^F$  is real return on foreign bond holdings (in terms of domestic consumption units) and  $R_{t+1}^S$  is real return on a unit of domestic stock. Both domestic and foreign bonds are assumed to be one period discount bonds paying off one unit of domestic currency at the beginning of next period. Real return at  $t + 1$  reflects the return of giving up a consumption unit at  $t$ . Therefore, real bond returns for domestic and foreign assets are given accordingly as:<sup>11</sup>

$$R_{t+1}^D = \frac{1 + i_t^D}{1 + \pi_{t+1}}; R_{t+1}^F = \frac{q_{t+1}(1 + i_t^F)}{q_t(1 + \pi_{t+1}^*)} \quad (10)$$

## 2.2 | Domestic firms

Firms in this neoclassical economy are identical and they produce according to a continuously differentiable, strictly increasing and strictly concave production function:

$$Y_t = A_t K_{t-1}^\xi \quad 0 < \xi < 1 \quad (11)$$

Firms produce in a competitive environment without any price adjustment mechanisms and wages are assumed to be fully flexible. Output  $Y_t$  is produced with capital  $K_{t-1}^\xi$  carried over from  $t - 1$ , where  $\xi$  is the share of capital in the production function and  $A_t$  is the level of technology. Labour supply is assumed to be fixed and normalized to be equal to 1. Given the production function, the real return on equities is equivalent to:

$$R_{t+1}^S = \xi K_t^{\xi-1} A_{t+1} + (1 - \phi) \quad (12)$$

where  $\phi$  the depreciation rate of capital.

As previously mentioned,  $R_{t+1}^S$  is the real return at  $t + 1$  on one unit of domestic stock. This can be written as  $[MPPK - \phi] + 1$  after assuming that capital and consumption are a homogeneous good.  $MPPK$  is the marginal physical product of capital and  $\phi$  is the depreciation rate of capital. It is assumed that domestic equities are claims on domestic firms' aggregate profits.

## 2.3 | The Government

The Treasury sets the government expenditure  $G_t$  and the monetary authority responds to the deficit by controlling both the money stock and the level of borrowing in the economy. The consolidated government identity is given by:

$$-\frac{B_{t-1}^D (1 + i_{t-1}^D)}{P_t} + \frac{B_t^D}{P_t} + \frac{M_t}{P_t} - \frac{M_{t-1}}{P_t} = G_t \quad (13)$$

<sup>11</sup>  $\pi_t^*$  denotes foreign inflation at  $t$  and is assumed to be exogenous to the model.

Equation (13) says that the government spending and its interest bearing liabilities, must be financed by borrowing from the private sector, and by changes in the stock of non-interest bearing debt held by the monetary authority.<sup>12</sup>

## 2.4 | The monetary authority

The monetary economics literature suggests that the exact formulation of the interest rate rule in a small open economy remains an open question. A number of studies, including Monacelli (2004) examine the role of exchange rates in the optimal monetary rule. In our neoclassical framework, we follow McCallum and Nelson (2000), and we assume that the monetary authority adjusts the nominal rate of interest after forecasting both inflation and output gap. Although deviations of the nominal exchange rate from its long run target are not introduced explicitly into the monetary policy rule, they are not totally ignored. Exchange rates could be one of the factors affecting the nominal rate of return, in as much as they influence inflation and output. As a result, the following Taylor (1993) rule is employed<sup>13</sup>:

$$R_{t+1}^D (1 + \pi_{t+1}) = (1 + \bar{i}^D) + \Theta_\pi [E_t (1 + \pi_{t+1}) - (1 + \bar{\pi})] + \Theta_y [E_t (Y_{t+1}) - (\bar{Y})] \quad (14)$$

Any deviation from this rule is perceived as a monetary policy shock.

## 2.5 | The foreign sector

Since we are analysing a small open economy, domestic exports  $C_t^{h,*}$  are taken as given. Imports are defined in equation (5) as  $C_t^f = (1 - \alpha)(q_t)^{-\Theta} C_t$ . Assuming that the balance of payments is zero, the trade balance is given by:

$$-C_t^{h,*} + [(1 - \alpha)q_t^{-\Theta} C_t] = -\frac{B_t^F}{e_t P_t} + \frac{B_{t-1}^F (1 + i_{t-1}^F)}{e_t P_t} \quad (15)$$

Equation (15) states that the trade deficit or surplus should be equal to the difference between receipts and payments from domestic investment in foreign bonds.

## 2.6 | The economy wide resource constraint

The economy wide resource constraint describing the goods market equilibrium in this neoclassical economy is given by:

$$Y_t = C_t + I_t + G_t + C_t^{h,*} - [(1 - \alpha)q_t^{-\Theta} C_t] \quad (16)$$

<sup>12</sup>  $G_t$  could be interpreted as a budget deficit in the presence of taxes, in as much these taxes are of the nature of lump sum taxes which do not affect optimizing behavior. Interest bearing liabilities are represented by debt held by the public.

<sup>13</sup> Variables symbolized with an upper bar correspond to steady state values.  $\Theta_\pi, \Theta_y$  are constant parameters.



where  $C_t$  is real consumption,  $I_t$  is investment,  $G_t$  government spending,  $C_t^{h,*}$  domestic exports and  $[(1 - \alpha)q_t^{-\Theta}C_t]$  domestic imports. Using the definition of output in equation (11), and rewriting equation (16) in terms of real consumption, we derive the final version of the economy's resource constraint, expressed as:

$$C_t = A_t K_{t-1}^{\xi} - [K_t - K_{t-1}(1 - \phi)] - G_t - C_t^{h,*} + [(1 - \alpha)q_t^{-\Theta}C_t] \quad (17)$$

Investment is characterized by the law of motion of capital as  $I_t = K_t - K_{t-1}(1 - \phi)$ .

In line of most of the literature in monetary economics (as explained in section 2.4), the monetary authority conducts monetary policy based entirely on a Taylor (1993) nominal interest rate rule. The nominal interest rate is adjusted after forecasting deviations of both inflation and output from their steady state levels. Consequently, the money supply adjusts endogenously and satisfies the demand for money in order to achieve equilibrium in the money market. An explicit demand equation for real money balances can be derived after combining the first order conditions for real money, domestic bonds and domestic stocks, along with the marginal utilities for consumption and real money balances.<sup>14</sup>

### 3 | NOMINAL RIGIDITIES

#### 3.1 | Model variation I: The Keynesian model with wage stickiness

The inclusion of the micro-economic details of nominal rigidities, as part of the whole macro expectations adjustment mechanism, has become a distinctive feature of recent theoretical models that seek to explain economic fluctuations. These monetary models seek to embed nominal rigidities within a fully specified DSGE model and to explain the implied real effects of monetary disturbances. The intention of this section is to depart from the assumption of perfect wage and price flexibility and illustrate how the benchmark neoclassical model illustrated in section 2 can be modified in order to capture the various dynamics in the presence of nominal wage rigidities.

As opposed to early literature on wage stickiness,<sup>15</sup> Taylor (1979, 1980) introduced explicitly a staggered wage setting mechanism for labour contracts in a model with rational expectations. Although earlier models were successful in explaining the persistent effects of monetary policy on real variables and to a further extent the specific role attributed to staggered contracts in generating those effects, it was apparent that the structural relations were not evolved from an explicit model of individual optimization as projected by the new vintage of macro models. In addition, there was a money supply rule for the monetary authority, as opposed to a specific policy instrument (short-term interest rate) which becomes part of the private sector's expectation process. Therefore, this section introduces a dynamic stochastic general equilibrium model with rational expectations and overlapping labour contracts within the small open economy framework constructed in section 2. In the current analysis the parameters of the Taylor mechanism are considered to be structural and not policy dependent.

<sup>14</sup> Deriving such an equation is beyond the scope of the current paper.

<sup>15</sup> Early literature based on Sargent and Wallace (1975) and Fischer (1977) introduced wage stickiness into a model with rational expectations by assuming long-term labour contracts as a source of a Keynesian-like element of temporary rigidity.

Under the staggered contract specification, wages are not determined in a synchronized manner but it is assumed that a proportion of decision makers (firms-unions) negotiate a nominal contract wage growth which lasts for two subsequent periods. During this process, unions should take under consideration both contracts that have already been written prior to current negotiations, as well as contracts that are going to be written by other firms in the future. In other words, each contract is written relative to other contracts, where firms must look both backward and forward in time. The growth in the aggregate nominal wage in the economy is then defined as an average of current and past wage growth contracts.

The reformulation of the neoclassical model follows several steps. First, new equations characterizing the staggered wage mechanism and the firms' optimization process are introduced and then several existing equations are expanded and modified. The two extra equations determining (i) the nominal wage growth when firms negotiate new contracts and (ii) the aggregate nominal wage growth are given by:

$$\tilde{X}_t - \lambda \tilde{X}_{t-1} - (1 - \lambda) E_t \tilde{X}_{t+1} - \omega \lambda \tilde{Y}_t - \omega (1 - \lambda) E_t \tilde{Y}_{t+1} \approx 0 \quad (18)$$

$$\tilde{W}_t - \lambda \tilde{X}_{t-1} - (1 - \lambda) \tilde{X}_t \approx 0 \quad (19)$$

For convenience, equations (18) and (19) are presented in their log-linear versions around a steady state, where  $\tilde{X}_t$  is the log deviation of the nominal contract wage growth ( $X_t = \frac{x_t}{x_{t-1}}$ ) set by unions at  $t$ , and  $\lambda$  is the degree of bias with respect to previous and forward wage contracts. The log deviation of output from its steady state  $\tilde{Y}_t$  is a proxy for the current labour condition.  $\tilde{W}_t$  denotes the log deviation of the aggregate nominal wage growth ( $W_t = \frac{w_t}{w_{t-1}}$ ) from its steady state level and  $\omega$  is a constant positive parameter.<sup>16</sup>

It is assumed that wage negotiators (unions) have expectations about inflation and set the nominal contract wage growth in order to secure a given real wage target in the future. This is reflected in equation (20) below:

$$E_t [\tilde{C}_t - \tilde{\pi}_{t+1}] \approx 0 \quad (20)$$

Given  $W_t$  firms will optimize and demand labour up to the point where the growth of the marginal physical product of labour is equal to the real product wage growth. This is reflected in the following equation, where  $z_t$  is the level of marginal physical product of labour and  $\pi^h$  the domestic product inflation.

$$\frac{W_t}{1 + \pi_t^h} = \frac{z_t}{z_{t-1}} \quad (21)$$

The production function under the neoclassical specification should now incorporate explicitly the labour choice  $L_t$ . Therefore, equations (11) and (12) are replaced by:

$$Y_t = A_t K_{t-1}^\xi L_t^{1-\xi}; R_t^S = A_t \xi L_t^{1-\xi} K_{t-1}^{\xi-1} + (1 - \phi) \quad (22)$$

<sup>16</sup>  $x_t$  is the level of nominal contract wage and  $w_t$  the level of nominal aggregate wage.

and the marginal physical product of labour will be given as:

$$z_t = A_t(1 - \xi)K_{t-1}^\xi L_t^{-\xi} \quad (23)$$

The intuition behind the Keynesian specification is that employment may deviate from its natural level i.e. its normalized value of 1<sup>17</sup>.

## 3.2 | New Keynesian models with price stickiness

In this section, the New Keynesian paradigm is introduced into the frictionless neoclassical model in order to explore the dynamic effects of the underlying shocks in the presence of price inflexibility. It is assumed that agents-firms have the ability to set prices in a monopolistically competitive environment. As distinct from the wage inflexibility, price stickiness allows for a more explicit modelling of the monopolistic seller's reaction to the economic environment. Being precise about the behaviour of the monopolistic seller could generate important theoretical and empirical insights.

This sub section introduces nominal price stickiness by following two different approaches. The first approach combines elements of both Calvo (1983) and Rotemberg, 1982a, 1982b) and is referred to as the "Calvo-Rotemberg" price setting mechanism. The second approach originates from Taylor's (1979, 1980) overlapping contract mechanism.

### 3.2.1 | Model variation II: Price stickiness á la Calvo-Rotemberg

In this specification, it is assumed that if a firm is allowed to adjust its price at time  $t$ , it will set its price  $p_{it}$  so as to minimize the following quadratic loss function subject to the random process of when it will be able to adjust again in the future:<sup>18</sup>

$$\min_{p_{it}} \sum_{\kappa=0}^{\infty} \omega_c^\kappa \rho^\kappa E_t (p_{it} - p_{t+\kappa}^*)^2 \quad (24)$$

where  $\omega_c^\kappa$  is the Calvo probability that the firm has not adjusted after  $\kappa$  periods,  $\rho$  is the firm specific discount factor,  $E_t$  denotes expectation at  $t$ ,  $p_{it}$  is the actual price at  $t$  and  $p_{t+\kappa}^*$  the optimal or target price. The solution to the above problem yields the following two equations which will be used to modify the neoclassical model. For analytical convenience, these equations are presented below in terms of log-linear deviation around the steady state.

$$\tilde{V}_t = (1 - \omega_c \rho) \tilde{p}_t + \omega_c \rho E_t \tilde{V}_{t+1} \quad (25)$$

<sup>17</sup> At steady state, it is assumed that the nominal contract wage growth and the aggregate nominal wage growth are both equal to the price growth. The steady state under the Keynesian specification corresponds to the values that a frictionless model (in this case the neoclassical model) generates.

<sup>18</sup> All variables are in logs.

$$\tilde{P}_t^h = (1 - \omega_c)\tilde{V}_t + \omega_c\tilde{P}_{t-1}^h \quad (26)$$

$V_t$  is the price set by all firms adjusting at  $t$  and is a weighted sum of current and expected future target prices.<sup>19</sup> The aggregate CPI of domestically produced goods at  $t$ , denoted by  $P_t^h$ , is given by the weighted average of the past domestic product consumer price index  $P_{t-1}^h$  weighted at  $\omega_c$ , and of the optimal price set by firms adjusting their prices,  $V_t$  weighted at  $1 - \omega_c$ .<sup>20</sup>  $p_t^*$  is assumed to reflect the optimal price without any restrictions associated with the price adjustment. In the current analysis, this price is approximated as a constant frictionless mark-up  $\nu$  over the nominal marginal cost<sup>21</sup>. In log deviation terms from its steady state, the target price is given by:

$$\tilde{p}_t^* = \tilde{P}_t^h + \tilde{m}c_t \quad (27)$$

where  $\tilde{m}c_t$  is the log deviation of real marginal cost from its steady state value and  $\tilde{P}_t^h$  is the log deviation of the price index of domestically produced goods from its steady state value.

Under the limiting case of no price rigidities ( $\omega_c = 0$ )  $\tilde{P}_t^h = \tilde{p}_t^*$ , and consequently, given equation (27) it follows that  $\tilde{m}c = 0$ . Output is chosen such that the real marginal cost is constant ( $mc = 1/\nu$ ). When  $\omega_c > 0$ ,  $P_t^h \neq p_t^*$  and the log deviation of real marginal cost from its steady state ( $\tilde{m}c_t$ ) equals to the difference between the log deviation of the real product wage ( $\tilde{w}_t$ ) from its steady state and the log deviation of the marginal product of labour ( $\tilde{z}_t$ ) from its steady state:<sup>22</sup>

$$\tilde{m}c = \tilde{W}_t - \tilde{P}_t^h - \tilde{z}_t = \tilde{w}_t - \tilde{z}_t \quad (28)$$

This deviation is approximated by the log deviation of the output gap from its steady state value. This is the difference between the log deviation of the sticky price equilibrium level of output from its steady state ( $\tilde{Y}_t^s$ ) and the log deviation of the flexible price equilibrium level of output from its steady state ( $\tilde{Y}_t$ ) defined as:  $\tilde{Y}_t = \tilde{A}_t + \xi\tilde{K}_{t-1}$ . In log deviation terms from its steady state value, the output gap is given by<sup>23</sup>:

$$\tilde{x}_t = \tilde{Y}_t^s - \tilde{Y}_t \quad (29)$$

<sup>19</sup> It is assumed that all firms face the same technology and that they have demand curves with constant and equal demand elasticities.

<sup>20</sup> Upper case  $P_t^h$  represents the aggregate price level of domestically produced goods, as opposed to lower case  $p_t$ , which represents prices set by individual firms.

<sup>21</sup>  $p_t^* = \nu p_t^h mc_t$ . This is a standard result in a model of monopolistic competition, under the assumption that firms that adjust their prices at time  $t$  will do so in order to maximize the expected discounted value of current and future profits. It is assumed that nominal marginal cost reflects wage costs and that  $\nu$  is the desired or frictionless mark-up.

<sup>22</sup>  $\tilde{w}_t = \tilde{W}_t - \tilde{P}_t^h$  is the log deviation of real product wage from its steady state at  $t$ , and  $\tilde{z}_t = \tilde{A}_t + \xi\tilde{K}_{t-1}$  the log deviation of the marginal product of labour from its steady state at  $t$ .

<sup>23</sup> In the absence of the price adjustment mechanism, the sticky price level of output and the flexible price level of output coincide and reflect the level of output at steady state. The marginal product of labour is constant, which implies that the nominal wage follows the change in the domestic product price level, given that the real marginal cost is constant. It is important to notice that the model is silent about the nominal wage mechanism out of steady state.

It follows from the above approximation, that  $\tilde{m}c = \frac{1}{\zeta}\tilde{x}_t$  so the output gap can be written as:

$$\tilde{x}_t = \zeta\tilde{w}_t - \zeta\tilde{A}_t - \zeta\zeta\tilde{K}_{t-1} \quad (30)$$

where  $\zeta > 0$ .

Given the above analysis, the inflation rate for the CPI of domestically produced goods (new-Keynesian Phillips curve) expressed in log deviation terms from its steady state, is given by:

$$\tilde{\pi}_t^h = \rho E_t \tilde{\pi}_{t+1}^h + \frac{\sigma}{\zeta} \tilde{x}_t \quad (31)$$

where  $\sigma = \frac{(1-\omega_c)(1-\omega_c\rho)}{\omega_c}$ .

In log deviation terms, domestic inflation  $\tilde{\pi}_t$  is given by equation (32) below and depends on inflation of domestically produced goods ( $\tilde{\pi}_t^h$ ) coming from equation (31) and from the rate of change in the real exchange rate ( $\tilde{q}_t - \tilde{q}_{t-1}$ ).

$$\tilde{\pi}_t = \tilde{\pi}_t^h + \left(\frac{1-\alpha}{\alpha}\right)\tilde{q}_t - \left(\frac{1-\alpha}{\alpha}\right)\tilde{q}_{t-1} \quad (32)$$

Combining (31) and (32) we get equation (33) which implies that domestic inflation at time  $t$ , depends on the expected domestic inflation, on the past, current and future real exchange rates and on the current output gap.<sup>24</sup>

$$\tilde{\pi}_t = \rho E_t \tilde{\pi}_{t+1} - \left(\frac{\rho(1-\alpha)}{\alpha}\right) E_t \tilde{q}_{t+1} + \left(\frac{(1-\alpha)(\rho+1)}{\alpha}\right)\tilde{q}_t - \left(\frac{(1-\alpha)}{\alpha}\right)\tilde{q}_{t-1} + \left(\frac{\sigma}{\zeta}\right)\tilde{x}_t \quad (33)$$

### 3.2.2 | Model variation III: The Taylor price specification

A second approach to price stickiness originates from Taylor's (1979, 1980) overlapping contract mechanism. As shown in sub-section 3.1, Taylor developed his model by introducing a contract wage-setting mechanism. However, under the assumption that prices are a constant mark-up over wage costs, Taylor's specification can generate an alternative model for price stickiness.

We develop a two period version of Taylor's (1979, 1980) overlapping contract mechanism, as an alternative way of introducing price inflexibility into the benchmark neoclassical model.<sup>25</sup> Given the assumption that prices are a constant mark-up over wage costs, the wage adjustment mechanism can motivate a delay mechanism for prices. With this assumption, the log of the price level is given by:

$$P_t^h = WG_t + \vartheta \quad (34)$$

where  $\vartheta$  is the log mark-up, normalized for convenience to be equal to zero ( $\vartheta = 0$ ) and  $WG_t$  the level of the aggregate nominal wage. It follows that the aggregate price level,  $P_t^h$  in log-terms can

<sup>24</sup> See Appendix II for full derivation of equations 31, 32 and 33.

<sup>25</sup> Earlier attempts to formulate a model of price stickiness appear in Grossman (1974), and Barro and Grossman (1976). McCallum (1994) investigates the properties of such a model, referring to it as the "P bar" model.

be written as:

$$P_t^h = \lambda X_{t-1} + (1 - \lambda)X_t \quad (35)$$

where  $(X_t)$  is now the nominal contract wage under the above specification.

Equation (34) has important implications for the construction of the Taylor price specification. In fact, the assumption behind this equation, that prices are a constant mark-up over wages, distinguishes this model from the one analysed in sub section 3.1 because it implies that the log of the average expected real wage over the life of the contract (assumed a two period contract) is equal to zero. This is not necessarily the case in the Taylor analysis of pure wage inflexibility. Given this implication, the nominal contract wage (in log terms) can be written as:

$$X_t = \frac{1}{2}P_t^h + \frac{1}{2}E_t P_{t+1}^h \quad (36)$$

Equation (36) suggests that in the current analysis contract wages follow a delay mechanism, which depends on the current and expected future price levels. Equations (35) and (36) reflect the Taylor price specification mechanism. For analytical convenience, they are given below in log-linear deviation terms around the steady state:

$$\tilde{P}_t^h = \lambda \tilde{X}_{t-1} + (1 - \lambda) \tilde{X}_t \quad (37)$$

$$\tilde{X}_t = \frac{1}{2} \tilde{P}_t^h + \frac{1}{2} E_t \tilde{P}_{t+1}^h \quad (38)$$

Given equations (37) and (38), we get equation (39) below, which shows that the value of the domestic product CPI depends on the previous period's domestic price level as well as on the expectations of future domestic prices:

$$\tilde{P}_t^h = \lambda \tilde{P}_{t-1}^h + (1 - \lambda) E_t \tilde{P}_{t+1}^h + \lambda \tilde{\eta}_t \quad (39)$$

where  $\tilde{\eta}_t = E_{t-1} \tilde{P}_t^h - \tilde{P}_t^h$  is an expectation error term<sup>26</sup>. Expressed in domestic product inflation, equation (39) implies that:

$$\tilde{\pi}_t^h = \left( \frac{1 - \lambda}{\lambda} \right) E_t \tilde{\pi}_{t+1}^h + \tilde{\eta}_t \quad (40)$$

Under the Taylor price specification domestic inflation at time  $t$  is given by equation (41) below and depends on expected domestic inflation, on the past, current, and future real exchange rates,

<sup>26</sup> In the absence of the price adjustment mechanism, equation (35) indicates that  $X_t = P_t^h$ . The average expected real wage will remain constant and at the same level as in the presence of price stickiness. However, the levels of the contract wage and the price level will now be the same. All firms will set  $P_t^h$  which could be interpreted as a contract price. At this stage, the analysis parallels the approach followed under the Calvo-Rotemberg specification in subsection 3.2.1 We approximate this price as a frictionless constant markup over the nominal marginal cost. This implies that  $\tilde{m}c = 0$ . The error term will be zero and the value of the relative price at steady state will be equal to 1.

and on the expectation error term  $\tilde{\eta}_t$ <sup>27</sup>.

$$\tilde{\pi}_t = \left( \frac{1-\lambda}{\lambda} \right) E_t \tilde{\pi}_{t+1} - \left( \frac{(1-\lambda)(1-\alpha)}{\lambda\alpha} \right) E_t \tilde{q}_{t+1} + \left( \frac{1-\alpha}{\lambda\alpha} \right) \tilde{q}_t - \left( \frac{1-\alpha}{\alpha} \right) \tilde{q}_{t-1} + \tilde{\eta}_t \quad (41)$$

As in the Calvo-Rotemberg model, we approximate the difference between the log deviation of real wage from its steady state and the log deviation of the marginal product of labour from its steady state value, with the log deviation of the output gap from its steady state. However, under the assumption that domestic product prices are a constant markup over wage costs, equation (30) becomes:

$$\tilde{x}_t = -\tau \tilde{A}_t - \tau \xi \tilde{K}_{t-1} \quad (42)$$

where  $\tau < 0$ .

#### 4 | THE SOLUTION TO THE NEW KEYNESIAN MODELS

This section shows how the benchmark neoclassical model is modified in order to incorporate price stickiness based on the two different price adjustment mechanisms analysed in section 3. Related to the set of expectational equations, the Euler equations are common in all models. After introducing price stickiness, the monetary policy rule depends on the expected deviations of domestic inflation and output gap from their steady state values. Although the monetary policy rule follows the same pattern in both new Keynesian models with price stickiness, an important implication of the Calvo-Rotemberg model is that expected domestic inflation is directly affected by the expected output gap. However, this is not the case under the Taylor price specification. We introduce the log deviation of the output gap, as the difference between the log deviation between the sticky and flexible price equilibrium levels of output and we modify accordingly the economy wide resource constraint.

An important implication is that under the new Keynesian approach, what enters in the economy wide resource constraint is the equilibrium output produced under the sticky price regime and not the flexible price level of output. However, the production function as defined in sub section 2.2 reflects the level of output produced under the flexible price equilibrium. In other words, there is not an explicit equation in the model for the production of the sticky price level of output. This is reflected from the fact that we use the output gap to approximate the deviation of real marginal cost from its steady state.

Another noteworthy point, is that the steady state level of output, denoted by  $\bar{Y}$ , is the steady state level of output of the flexible price model, which does not necessarily coincide with the steady state of output under the sticky price regime. Although the steady state in the two new Keynesian models corresponds to the values that a frictionless model would generate, such a steady state would heavily depend on the value of the desired or frictionless mark-up  $\nu$ . Assuming for simplicity that  $\nu = 1$ , the steady states under the neoclassical and the new Keynesian models coincide.

Finally, we make use of equations (30) and (42) that reflect the assumption that the difference between the log deviation of the real wage and the log deviation of the marginal product of labour

<sup>27</sup> See Appendix III for full derivation of equations 40 and 41.

from their steady states is approximated by the log deviation of the output gap. Price stickiness is introduced by the relevant equations for domestic inflation given by equations (33) and (41).

## 4.1 | The exogenous shock processes

The exogenous domestic shocks are assumed to evolve according to the following autoregressive processes:<sup>28</sup>

*Domestic monetary shock:*

$$\log v_t = T \log v_{t-1} + \varepsilon_{vt} \quad \varepsilon_{vt} \sim i.i.d.N(0; \sigma^2) \quad 0 \leq T \leq 1 \quad (43)$$

*Domestic fiscal shock:*

$$\log G_t = \Delta \log G_{t-1} + \varepsilon_{Gt} \quad \varepsilon_{Gt} \sim i.i.d.N(0; \sigma^2) \quad 0 \leq \Delta \leq 1 \quad (44)$$

*Domestic technology shock:*

$$\log A_t = \psi \log A_{t-1} + \varepsilon_{At} \quad \varepsilon_{At} \sim i.i.d.N(0; \sigma^2) \quad 0 \leq \psi \leq 1 \quad (45)$$

## 4.2 | Models' parameterization

The next step in the solution process is to choose parameter values for the specified models. Once the parameters are calibrated, Uhlig's (1999) algorithm is applied in order to simulate the models and analyse impulse response functions generated by the various shocks.<sup>29</sup>

### 4.2.1 | The Neoclassical model

Based on the business cycle literature for a small open economy, we choose values for the following parameters  $\{\delta, \gamma, \beta, \xi, \phi, \Theta_\pi, \Theta_y, \alpha, \Theta\}$ . Following standard values in the literature,<sup>30</sup> we calibrate

<sup>28</sup> This paper focuses on the analysis of domestic shocks. However, the models are suitable for analyzing foreign shocks like those generated from exogenous disturbances in the rate of return in foreign bonds, from foreign inflation, and from domestic exports. It is assumed that there is no contemporaneous correlation among the shocks. There is scope for analysis where the shocks are correlated but for tractability reasons is omitted from the current paper.

<sup>29</sup> We proceed by taking a log-linear approximation of all variables around their steady state values. The steady state equilibrium is defined as one in which output ( $\bar{Y}$ ), consumption ( $\bar{C}$ ), capital ( $\bar{K}$ ) and government spending ( $\bar{G}$ ) are constant through time. An implication of the steady state property is that real money holdings are constant (see equation 6 in section 2.1). The fact that real money balances are constant requires that prices should change at the same rate as the nominal stock of money (money neutrality). In addition, since the growth of nominal money supply does not affect real equilibrium, the model also exhibits the property of superneutrality. It can be proved that  $\bar{q} = \bar{s} = 1$  which implies that the Purchasing Power Parity (PPP) holds in steady state i.e.  $\bar{q} = \bar{P}_t^* / \bar{e}_t \bar{P}_t = 1 \Rightarrow \bar{P}_t = \bar{P}_t^* / \bar{e}_t$ . In addition, a property of the steady state is that the real return on assets is the same and equal to  $1/\beta$ . Because real interest rates are equal across the world, the international Fisher equation holds. Consequently, the Uncovered Interest Rate Parity (UIRP) also holds in steady state. The above relationships do not necessarily hold out of steady state.

<sup>30</sup> The values chosen are informed by , McCallum and Nelson(2000) , Monacelli (2004), Galí and Monacelli (2004) and Smets and Wouters (2003).



the discount factor  $\beta$  to be equal to 0.99. This implies an annual steady state real interest rate of 4 per cent. The depreciation rate of capital  $\phi$  is set equal to 0.025, which implies an annual depreciation of capital of 10 per cent. The share of capital in the production function  $\xi$  is set equal to 0.36. We set the preference parameter in the utility function  $\delta$  equal to 0.65. The coefficient  $\gamma$  is equal to  $-0.3$ , which assigns a value of less than unity to the elasticity of intertemporal substitution. This is close to most of the RBC literature that assumes an elasticity of substitution between 0.5 and 1.<sup>31</sup> Smets and Wouters (2003) report a similar value after estimating a dynamic stochastic general equilibrium model for the euro area.

The calibration of the monetary policy rule parameters plays an important role in the analysis. The benchmark value set by Taylor (1993) for the response coefficient on inflation is  $\Theta_\pi = 1.5$ . This value reflects the “Taylor principle” that the nominal rate is changed more than one for one with deviations of inflation from its steady state or target value. In the monetary literature, this active monetary policy is a necessary condition to ensure equilibrium determinacy. However, in the constructed neoclassical model it is assumed that prices are fully flexible. As a result, an inflation adjustment equation, which is present in the models with price rigidities, is absent from the current analysis. Given this characteristic of the neoclassical model, we experiment with different calibrated values for the monetary policy rule, and investigate how the dynamic responses of the economy can be affected under each parameterization. The conceptual experiment introduces elements of sensitivity analysis to the model solution.

Since we want to analyse the small open economy of the UK, part of our calibration experiment will include parameter estimates coming from estimated dynamic models using quarterly data from the UK. In a simplified version of a model previously constructed by Galí and Monacelli (2004), Lubik and Schorfheide (2007) estimate a structural small open economy model for a number of countries, including the UK. They report that the Central Bank performs a moderately anti-inflationary policy with an estimated value for  $\Theta_\pi = 1.30$  with little concern on output with a value of  $\Theta_Y = 0.23$ .

Given the above results, we calibrate the monetary policy rule in the neoclassical model with three different sets for the structural parameters. Initially, we assume a passive monetary policy with values of  $\Theta_\pi = 0.5$  and  $\Theta_Y = 1.5$ . Then we assume that the monetary authority reacts one for one to both inflationary and output movements i.e.  $\Theta_\pi = 1$  and  $\Theta_Y = 1$ , and finally that the monetary authority follows an active policy by mainly reacting to inflationary movements i.e.,  $\Theta_\pi = 1.30$  and  $\Theta_Y = 0.23$ . We consider the first assumption as the starting point in our experiment, and then we investigate how the dynamic behaviour of several key variables can change under the alternative specifications.

Finally, we follow Monacelli (2004) and set the substitutability between domestic and foreign goods  $\Theta$  equal to 1.5. The degree of home bias in preferences for the domestic consumers (degree of openness) is set equal to  $\alpha = 0.85$ .

## 4.2.2 | The Keynesian model with wage stickiness

The Keynesian model with wage stickiness follows the same parameterization with the one specified in sub section 4.2.1. However, parameters  $\lambda$  and  $\omega$  related to the nominal wage growth

<sup>31</sup> We acknowledge the fact that there is a great debate in the literature related to the coefficient of relative risk aversion. In particular, the equity premium puzzle can be explained on the grounds of a very high coefficient of relative risk aversion, which in turn implies a small value for the elasticity of intertemporal substitution.

determination in equation (18) must also be calibrated. As a result, we follow Taylor (1979) and set  $\lambda = 0.5$  and  $\omega = 0.2$ . Related to the monetary policy rule, we are going to perform the same experiments as with the neoclassical model and investigate the dynamic effects of the Keynesian model under the three different sets of parameters.

### 4.2.3 | The new Keynesian models with price stickiness

The new Keynesian models with price stickiness follow the same parameterization as the previous models with respect to the following parameters:  $\{\delta, \gamma, \beta, \xi, \varphi, \alpha, \Theta\}$ . However, several important aspects must be explored in relation to the endogenous interest rate rule that the monetary authority follows. More specifically, in both models with price stickiness domestic product inflation follows an adjustment mechanism. In the Calvo-Rotemberg model domestic inflation is directly affected by the output gap. This has important implications for the way that the monetary authority can affect economic activity. In particular, the monetary authority adjusts the nominal rate of interest based on the expected deviations of both domestic inflation and output gap from their steady state values. When the monetary authority expects inflation to rise, it increases the nominal interest rate enough in order to increase the real interest rate and generate a fall in the output gap (the monetary authority also targets the output gap directly). Given equation (31) the fall in the output gap should be sufficient to stop a self-fulfilling change in domestic product inflation, which is consistent with the Taylor principle. However, we must also stress that according to equation (32) domestic inflation is also related to the nominal exchange rate.<sup>32</sup> In other words, inflation can increase through a nominal depreciation. Consequently, when the monetary authority forms its expectations about future domestic inflation, it must predict fluctuations in both domestic product inflation and the nominal exchange rate. This is another indication that the exchange rate is not totally ignored in our modelling even though it does not directly appear in the monetary policy rule. In other words, when the monetary authority sets a nominal interest rate it seeks to control domestic inflation by affecting the domestic product inflation (directly) and the exchange rate (indirectly), through changes in the output gap.

Quite interestingly, the above scenario does not perfectly replicate the way that the monetary authority affects domestic inflation under the Taylor price specification. This is because in the Taylor price mechanism domestic product inflation is not directly affected by the output gap. However, by setting the nominal interest rate and affecting the output gap, it can affect only indirectly domestic inflation. From this analysis, it is apparent that under the Taylor price specification, the Taylor principle is not present. That is why in our calibration of the endogenous monetary policy rule in the Taylor sticky price model we give more emphasis on the parameter  $\Theta_Y$ .

In the Calvo-Rotemberg model we assume that  $\Theta_\pi > 1$ . Setting  $\Theta_\pi > 1$  is consistent with the Taylor principle and could exclude the possibility of multiple equilibria. However, we have to realize that although we can set parameter values in order to avoid stationary sunspot equilibria in a closed economy, that may not always guarantee determinacy in an open economy framework. Consequently, advocating an active monetary policy rule may not be sufficient to

<sup>32</sup>This is reflected from the fact that the change in the real exchange rate can be written as:  $\bar{q}_t - \bar{q}_{t-1} = \bar{\pi}_t^* - \bar{\varepsilon}_t - \bar{\pi}_t$ , where  $\varepsilon_t$  is the change in the nominal exchange rate. All variables are expressed in log percentage deviations from their corresponding steady state values.

prevent aggregate instability. This is due to the effects that nominal exchange rate can exert on domestic inflation.<sup>33</sup>

To calibrate the monetary policy rule we focus on estimated parameters coming for the UK economy. Related to the Calvo–Rotemberg specification we follow Lubik and Schorfheide (2007) and set  $\Theta_\pi = 1.30$  and  $\Theta_Y = 0.23$ . In addition, we follow Galí, Gertler, and Lopez-Salido (2001) and set  $\omega_c = 0.475$  and  $\rho = 0.837$ . Since the Calvo-Rotemberg model does not endogenously determine a direct relationship between the real marginal cost and the output gap<sup>34</sup>, we calibrate the parameter  $\zeta$  to be equal to 0.95. This is consistent with Lubik and Schorfheide (2007) that report a value of 0.7 for the parameter that links output gap with domestic inflation.

Related to the Taylor price specification, we set  $\lambda = \omega_c = 0.475$  in order to secure comparability between the two models. In fact, by setting the same value for  $\lambda$  and  $\omega_c$ , the two models with price stickiness can produce the same average frequency of price changes. We calibrate the parameter  $\tau$  to be equal to parameter  $\zeta$  recognizing the fact that the two parameters may not necessarily coincide. Finally, related to the endogenous monetary policy rule, we experiment with two different sets of parameters, assuming that the monetary authority targets mainly fluctuations in output. Initially we assume that  $\Theta_\pi = 0.5$  and  $\Theta_Y = 1.5$  and then we set  $\Theta_\pi = 1$  and  $\Theta_Y = 1$ . This parameterization reflects sensitivity experiments, since to the best of our knowledge, there are no estimated values in the literature for  $\Theta_\pi$  and  $\Theta_Y$  generated by a DSGE model where price rigidities are introduced through a similar specification as the one analysed in sub section 3.2.2. The models' parameterization is presented in Table 1 in Appendix IV.

## 5 | THE EMPIRICAL IMPLICATIONS OF THE MODELS

The purpose of the empirical specification is to test the dynamic predictions of the models in order to explore their empirical validity. The conceptual experiment follows several steps. Initially, statistical analysis is employed in order to investigate the ability of the theoretical models to replicate the actual business cycle behaviour of the UK economy. For this purpose, we collect quarterly data for several key macroeconomic variables from the period 1974: Q1 to 2017:Q4, using which we compare moments and time series generated by the simulated data from the different models with those coming from actual economic activity in the UK.<sup>35</sup>

The next step in the empirical evaluation is to employ a cointegrated VAR model in order to measure the degree to which the long-run predictions of the models are borne out in observed time series data. Consequently, we test the co-existence of two international relationships in the long run, namely the purchasing power parity (PPP) and the uncovered interest rate parity (UIRP) for the UK. Finally, impulse responses are generated through a VAR model in order to compare the short-term dynamics coming from real time series, with those predicted by the various theoretical models.

<sup>33</sup> The analysis behind the determinacy/indeterminacy regions in  $(\Theta_\pi, \Theta_Y)$  space under the baseline calibration, is left for future research.

<sup>34</sup> This endogenous relationship can be found in the literature in models that generate an optimal condition for labour-leisure choice. See Galí and Monacelli (2004) for such a representation.

<sup>35</sup> Data are collected from Datastream.

## 5.1 | Simulated business cycles for the British Economy

In order to investigate the predictions of the models, we compare moments that summarize actual economic activity in the UK economy with similar moments coming from the theoretical models. After taking the logarithm of each series, the HP filter is applied in order to extract the cyclical components.<sup>36</sup> In the construction of the real return on domestic bonds  $R_t^D = \frac{1+i_t^D}{1+\pi_t}$  we have used the 3-month nominal Treasury bill rate  $i_t^D$  and the quarterly inflation rate  $\pi_t$ . Private real consumption  $C_t$  has been deflated by using the Consumer Price Index (CPI), whereas the government expenditure on durable goods  $G_t$  has been deflated by the GDP deflator.  $Y_t$  is GDP volume and  $q_t$  is the real effective exchange rate.

The cyclical component of the GDP has been measured as a reference variable against which the relative volatility of other series is examined. Figure 1 in Appendix IV displays the business cycle components for the UK major macroeconomic aggregates. The cyclical components are expressed in percentage deviations from their long-run trends. This secures compatibility with the simulated data as generated by the various theoretical models. Business cycle statistics for the UK economy are provided in Table 2 in Appendix IV.

### *Volatility in the British economy*

- Consumption is slightly more volatile than real output with relative standard deviation of 1.19 (Table 2; Figure 1-panel 1);
- Real domestic bond return is almost as volatile as real output with relative standard deviation of 1.02 (Table 2; Figure 1-panel 2);
- Inflation is more volatile than real output with relative standard deviation of 1.37 (Table 2; Figure 1-panel 3);
- Government spending is more volatile than real output with relative standard deviation of 1.21 (Table 2; Figure 1-panel 4);
- The real effective exchange rate is substantially more volatile than real output with relative standard deviation of 3.57. (Table 2; Figure 1-panel 5);

**Comovement** Figure 1 exhibits that most macroeconomic aggregates for the UK economy are acyclical. From Table 2 we can see that only consumption is procyclical, in the sense that it has a strong positive contemporaneous correlation with output of 0.77. All other variables mostly exhibit low correlations, with government expenditure reaching the value of  $-0.30$ .

### *Persistence*

All macroeconomic variables, demonstrate significant persistence. From Table 2, we can see that the first order autocorrelation is within the range of 0.60 to 0.86. Given the stylized facts of aggregate activity for the UK economy the underlying experiment is based on moment comparison between actual economic experience, with moments that summarize the economic activity

<sup>36</sup> We employ the HP filter in order to secure compatibility with the simulated data. In the literature, there has been a controversy on the suitability of the various filters like the Hodrick and Prescott (1981) filter, the band pass filter and the Beveridge and Nelson (1981) decomposition in the analysis of the business cycles. Prescott (1986) argues that the HP filter is designed to eliminate stochastic components that have periodicities more than thirty-two quarters. On the contrary, the band-pass filter developed by Baxter and King (1999) passes through components with periodicities between six and 32 quarters. The benefit of the HP filter is that it can extract the same trend from a set of different variables.

predicted by the four models. The statistics for each model are generated from simulated data that reflect the different parameterizations of the monetary policy rule. The evaluation of each model follows several steps depending on the magnitude of the various disturbances that are allowed to affect the economy. Those disturbances are generated by domestic technology shocks, domestic monetary shocks and domestic fiscal shocks. The summary statistics are generated from simulated data where the standard deviation of one of the shocks is relatively larger than the standard deviation of the others.<sup>37</sup> Due to the way that our simulated data is generated, we do not actually expect the simulations to predict the values of the variables in actual data. That is why we proceed with comparing the relative variability of the series.<sup>38</sup>

We suggest that a monetary shock in the Taylor sticky price model under the parameterization of  $\Theta_{\pi} = 0.5$  and  $\Theta_Y = 1.5$  appears to replicate a significant portion of the behaviour of the UK economy. Looking at the first panel in Figure 2 it seems that the specific model gives a good account of the quarter-to-quarter variation in output for the UK economy. The correlation between the actual and simulated data is 0.53 and the model seems to work quite well in major recessions and expansions. Comparing the ratio of the model's to empirical standard deviation, we suggest that the model explains 80% of business cycle fluctuations in the UK economy.<sup>39</sup> Interestingly, the chosen parameterization is consistent with the monetary policy target of the Bank of England, particularly after the late 70's when the Government had begun to set targets for the growth of the money supply. It was not until the late 90's that an explicit inflation target was first adopted.

Turning to the individual components of output, the correlation in consumption is no more than 0.14. The fact that we observe a more volatile pattern for consumption could be attributed to the selected value of the coefficient of relative risk aversion in our modelling. A prediction of our economic modelling is that the variance of consumption varies inversely with the coefficient of relative risk aversion. The lower the coefficient of relative risk aversion, the higher the variance of consumption. As we assume a relatively low coefficient of relative risk aversion (following the literature) the model predicts a high variability of consumption, much higher than is observed. If the coefficient of relative risk aversion is chosen to be high, something that satisfies the equity premium puzzle, then more realistic variability i.e., lower variability in consumption is predicted. This can explain the low variability in the observed time series. We assume that with the iso-elastic formulation of our utility function, the equity premium puzzle and the risk free rate puzzle

---

<sup>37</sup> It is important to stress the primary role that the choice of the standard deviation of each shock plays in the creation of the simulated data. In fact, the standard deviations of the shocks are used to create an appropriate matrix with the residuals (epsilons) of the shocks. This matrix, along with the matrices estimated from the recursive equilibrium law of motion, generate simulated time series for the corresponding variables in the system of equations. We investigate which shock, in which theoretical model, and under which specific parameterization of the monetary policy rule, can reproduce a more realistic pattern for the business cycle activity in the UK.

<sup>38</sup> Due to the manifold of different sets of simulated data we present that particular set that most accurately replicates the actual economic activity in the UK. Overall, 27 different sets of simulated data, depending on the type of the shock in each theoretical model, are employed. The specific parameterization of the monetary policy rule, given the particular characteristics of each model, is based on the economic rationale given in section 4. Given that the current research is based on calibration, these values are informed from existing estimated dynamic models based on quarterly time series for the UK economy. Fruitful future research could further evaluate the ability of each model to capture the statistical properties of the UK data after estimating the structural parameters of the models through Bayesian techniques.

<sup>39</sup> The variance ratio is used by King and Rebelo (1999). Criticism for the interpretation of the ratio can be found in Eichenbaum (1991).

coincide.<sup>40</sup> Related to inflation, we report a correlation of 0.19, whereas it drops to 0.04 for the real domestic return and becomes negative for the government expenditure and the real exchange rate.

With respect to government spending, the simulated data are less volatile than actual data. This is probably due to the way that the consolidated government budget identity is constructed in our theoretical modelling. Introducing explicitly a level of taxation in the model, possibly in the form of distortionary taxes, will not only affect the individual's optimization plan but is also likely to result in a more volatile expression of the budget deficit. Such an implication could change the volatility pattern that the exogenously set government spending follows.

## 5.2 | Testing the long-run empirical validity of the economic modelling

In order to further explore the empirical validity of our economic modelling in the long run, we construct a “vector error correction model” (VECM). The purpose of using an error correction model representation is to investigate the extent to which a set of variables can generate a long-run equilibrium relationship, which can then be associated with an economically meaningful interpretation. We test for the Fisher relationship, which implies that both the purchasing power parity (PPP) and the uncovered interest rate parity (UIRP) hold in the long-run<sup>41</sup> and examine whether the results are consistent with the long-run predictions of the four dynamic stochastic general equilibrium models as constructed in sections 2 and 3 of the paper. According to the properties of the economic modelling the following equation holds at steady state:<sup>42</sup>

$$li_t^D = li_t^F + (l\pi_t - l\pi_t^*) \quad (46)$$

To empirically test the validity of the economic predictions implied by equation (46) in the long-run, a VECM of the following form is employed.<sup>43</sup>

$$\Delta x_t = \Gamma_1^m \Delta x_{t-1} + \Gamma_2^m \Delta x_{t-2} + \dots + \Gamma_{k-1}^m \Delta x_{t-k+1} + \Pi x_{t-m} + \varepsilon_t \quad (47)$$

where  $x_t = li_t^D, li_t^F, (l\pi_t - l\pi_t^*)$  a  $(3 \times 1)$  vector of variables,  $m$  denotes the lag placement of the ECM term<sup>44</sup>,  $\Delta$  denotes the difference operator, and  $\Pi = a\beta'$ ; where  $a$  and  $\beta$  are  $p \times r$  matrices of coefficients with  $r < p$  (here  $p$  is the number of variables and  $r$  denotes the number of stationary co-integrated relationships).

<sup>40</sup> For an analytic discussion of the equity premium puzzle and the risk free rate puzzle, see Campbell, Lo, and MacKinlay (1997).

<sup>41</sup> See Johansen and Juselius (1992) for a similar representation.

<sup>42</sup> All variable are expressed in logs.  $li_t^D = \log(1 + i_t^D)$  where  $i_t^D$  the nominal domestic interest rate,  $li_t^F = \log(1 + i_t^F)$  where  $i_t^F$  the foreign nominal interest rate,  $l\pi_t = \log(1 + \pi_t)$  where  $\pi_t$  the domestic inflation rate and  $l\pi_t^* = \log(1 + \pi_t^*)$  where  $\pi_t^*$  the foreign inflation rate.

<sup>43</sup> Some of the advantages of the VECM are that it reduces the multicollinearity effect in time series, that the estimated coefficients can be classified into short-run and long-run effects, and that the long-run relationships of the selected macroeconomic series are reflected in the level matrix  $\Pi$  and so can be used for further co-integration analysis. See Juselius (2006).

<sup>44</sup> For an  $I(1)$  analysis  $m$  should be equal to 1.



To test for co-integration among a set of integrated variables the Full Information Maximum Likelihood (FIML) approach is employed as proposed by Johansen (1988, 1991).<sup>45</sup> Having uniquely identified potential co-integrating vectors, stationarity among the variables can be tested, while imposing specific restrictions.

Without drawing any crucial distinction among the four theoretical models, we explore their long-run implications by using quarterly data over the period 1974:Q1 to 2017:Q4 for the UK economy.<sup>46</sup> Evidence from the Phillips Perron, the Augmented Dickey Fuller and the Kwiatkowski, Phillips, Schmidt, and Shin (KPSS) tests suggest that all variables are integrated of order 1 i.e., I(1). Given the evidence of non-stationarity, we seek to identify potential linear combinations among the I(1) variables that can generate a stationary process. Given the statistical evidence from the various lag order selection criteria we select 2 lags for the underlying empirical model and proceed by applying the Johansen procedures in order to test for the co-integration rank.<sup>47</sup> The foreign interest rate is treated as a weakly exogenous variable, thus long run forcing in the co-integrated space. This is economically justifiable, as the assumption of a small open economy reflects the fact that domestic policy decisions, or more generally the domestic economic activity, do not have a significant impact on the evolution of foreign variables.<sup>48</sup> The results suggest one co-integrating vector, which is presented below normalized with respect to the domestic nominal interest rate:<sup>49</sup>

$$li_t^D = 1.00 (li_t^F) + 0.32 (l\pi_t - l\pi_t^*)$$

(8.70)                      (2.15)

The above results indicate that the coefficient of the nominal domestic foreign interest rate is equal to 1 (as predicted by the Fisher relationship) and highly significant. Although the coefficient for the inflation differential is not equal to 1 it still comes with the correct sign and is significant. Consequently, although both estimated coefficients are not equal to unity there is still favourable evidence of some degree of coexistence of the purchasing power parity and the uncovered interest rate parity in the long-run as predicted by our theoretical setup.

## 6 | TEST THE SHORT RUN DYNAMIC PREDICTIONS OF THE MODELS

Applying the generalized impulse response analysis,<sup>50</sup> we evaluate the dynamic properties of the theoretical models analysed in sections 2 and 3 relative to the impulse responses generated from

<sup>45</sup> The main advantage of such an approach is that it is asymptotically efficient since the estimates of the parameters of the short-run and long-run relationships are carried out in a single estimation process. In addition, through the FIML procedure potential co-integrating relationships can be derived in an empirical model with more than two variables.

<sup>46</sup> Data from the United States are used as a proxy for foreign variables. Data are collected from Datastream.

<sup>47</sup> For the lag order selection we employ the AIC, SBC and HQ tests. See Johansen (1995) for co-integration rank testing.

<sup>48</sup> From an econometric point of view, imposing such a restriction on the appropriate adjustment coefficient is testable. The restricted co-integration test suggests that this is an acceptable description of the data. The stability of the VECM is also tested through the inverse roots of the AR characteristic polynomial. The analysis confirms that the VECM is stable since all the inverted roots lie inside the unit circle.

<sup>49</sup>  $t$ -statistics are reported in parentheses.

<sup>50</sup> As opposed to orthogonalized impulse responses, where shocks are orthogonalized using the Cholesky decomposition before estimating the impulse responses (Sims, 1980), we follow the generalized impulse response approach as proposed by

a VAR model for the UK economy. In order to secure comparability, impulse responses reflect the impact of a positive, once-and-for-all, monetary, fiscal and technology shock on nominal domestic bond return, GDP, inflation and real effective exchange rate. All variables are in logarithms and the sample covers quarterly data for UK from 1974:Q1 to 2017:Q4.<sup>51</sup>

## 6.1 | Dynamic responses to a monetary shock

Figure 3 in Appendix IV presents the estimated dynamic responses of the nominal domestic bond return, GDP, inflation and real effective exchange rate to an exogenous monetary tightening for the British economy and for the neoclassical model under the parameterization of  $\Theta_{\pi} = 0.5$ ;  $\Theta_Y = 1.5$ . The horizontal axis presents the number of quarters after the hit of the shock whereas the vertical axis measures the deviation from the initial value. For the UK economy (actual data) after the occurrence of the monetary shock, the nominal domestic bond return increases by 0.7 per cent, portraying a strong persistence for 2 quarters, declining gradually thereafter. In response to that tightening, GDP marginally increases by 0.07 per cent and then declines, reaching a trough after 6 quarters at a level of 0.31 per cent below its original value exhibiting a reverting trend thereafter.

Inflation increases on the impact of the shock by 0.26 per cent, reflecting the “price puzzle” often present in empirical studies.<sup>52</sup> It reaches a peak of 0.68 per cent after 3 quarters and gradually returns to its original level 7 quarters after the impact of the shock declining thereafter in a rather persistent manner. Combined with the interest rate reaction, this may be evidence of non-money neutrality of the monetary shock, which is typically a characteristic of the presence of nominal rigidities.

Related to the dynamic responses of the neoclassical model we can observe a somewhat similar pattern with the one generated from the VAR model especially on the impact of the shock. More specifically, inflation increases by 0.17 per cent upon the impact of the shock (reflecting the “price puzzle”) reaching its peak deviation of 0.61 per cent after 2 quarters. After the realization of the shock, the monetary authority forecasts fluctuations in both inflation and output. Evidence from the impulse responses suggest that the monetary authority forecasts an increase in inflation and output and reacts by increasing the nominal rate of interest in order to mitigate the effects of the shock. The price puzzle could be explained on the grounds that the monetary authority acts too late to prevent inflation from rising, or that it is unable to offset the factors that led it to predict higher inflation. The nominal domestic return increase by 0.64 per cent upon the impact of the shock (compared to 0.7 per cent for the UK economy) after which it decreases reaching a trough after 3 quarters. Output marginally increases by 0.04 per cent after which it begins to decline reaching a trough after 3 quarters. However, we should stress that although we can

---

Pesaran and Shin (1998) building on earlier work by Koop, Pesaran, and Potter (1996). Overcoming the difficulties behind identification, the generalized impulse responses are crucially invariant to the reordering of the variables (as opposed to the various orthogonalized approaches) by taking into account the historical patterns of correlations amongst the different shocks.

<sup>51</sup> Data are collected from Datastream.

<sup>52</sup> Commonly, the price puzzle is explained on the grounds that the variables included in a VAR model do not span the whole set of information available to Central Banks when they decide to take action. See Walsh (2003). Another explanation given by Barth and Ramey (2001) is that a tight monetary policy operates on both aggregate supply and aggregate demand. There is a negative supply effect (positive cost shock), which reflects the cost channel of monetary policy.



observe a similar initial reaction to the shock for the nominal domestic return, the output, and the inflation rate between the actual and the simulated data, the speed of adjustment following the shock does not exhibit a similar pattern. After the realization of the shock there is a nominal and real exchange rate appreciation to restore the balance of payments. The real exchange rate appreciates by 0.57 per cent in the UK economy and 0.87 per cent in the simulated model with  $\Theta_{\pi} = 0.5$ ;  $\Theta_Y = 1.5$ .

A similar dynamic behaviour of the economy can be observed under the alternative parameterization of  $\Theta_{\pi} = 1$ ;  $\Theta_Y = 1$  compared with the previous parameterization. The main characteristic is the change of the magnitude of the responses. More specifically, the nominal domestic return increases twice as much reaching the value of 1.34 per cent as compared to 0.64 per cent in the previous case. Inflation increases by 0.29 per cent (compared to 0.17 per cent in the previous case) whereas the output marginally increases again by 0.04 per cent after which it declines reaching a trough after 2 quarters. The real exchange rate appreciates by 0.21 per cent.

An active monetary policy rule, i.e.,  $\Theta_{\pi} = 1.30$ ;  $\Theta_Y = 0.23$  in the neoclassical model, exhibits an even higher reaction in the nominal domestic return, that increases by 2.58 per cent, and to the inflation rate that increases by 4.04 per cent on the realization of the shock. The real exchange rate appreciates by 6.75 per cent and output decreases after a minimal rise of 0.01 per cent. Overall, evidence suggests that a passive monetary policy under the parameterization of  $\Theta_{\pi} = 0.5$ ;  $\Theta_Y = 1.5$  can better replicate the way that key macroeconomic variables react to a monetary tightening in the UK economy particularly at the realization of the shock. The subsequent adjustment following the impact of the shock is less comparable.<sup>53</sup>

Related to the three different parameterizations of the monetary policy rule we can declare that the Taylor model with wage stickiness can exhibit non-money neutrality and mostly generate higher persistence compared to the neoclassical model. In addition, the magnitude of the responses following the hit of the shock is less comparable between the sticky wage model and the VAR model for the UK economy.<sup>54</sup>

Finally, related to the Taylor model with price stickiness under the parameterization of  $\Theta_{\pi} = 0.5$ ;  $\Theta_Y = 1.5$  we can observe from Figure 4 in Appendix IV that the impulse responses from the simulated data are much closer to the estimates from the VAR model for the UK economy as compared to any of the other theoretical models previously discussed. To facilitate comparison Table 3 in Appendix IV presents relevant statistics from both actual and simulated data.

Evidence suggests that the nominal domestic return increases by 1.08 per cent (0.7 per cent for actual data) fading out slowly returning to its initial value after 5 quarters (6 quarters for the UK VAR model). In addition, the correlation coefficient between actual and simulated data for the nominal domestic return is 0.91. Output increases marginally by 0.04 per cent (0.07 per cent for actual data) before reaching its trough after 4 quarters (6 quarters for actual data). The correlation between actual and simulated data for output is 0.60. In relation to the inflation rate we observe an initial increase of 0.52 per cent (0.26 per cent for actual data) before returning to its initial value after 6 quarters (7 quarters for actual data). The correlation coefficient between actual and simulated data for inflation is 0.76. Finally, evidence suggests that the real effective exchange rate appreciates by 0.62 per cent (0.57 per cent for actual UK data) with a rather lower correlation coefficient of 0.20 with the actual data. It is worth noting that according to the statistical analysis

<sup>53</sup> The graphs for the alternative parameterizations for the neoclassical model i.e.,  $\Theta_{\pi} = 1$ ;  $\Theta_Y = 1$  and  $\Theta_{\pi} = 1.30$ ;  $\Theta_Y = 0.23$  are available upon request.

<sup>54</sup> The graphs are available upon request.

in section 5.1, this model seems to replicate quite well the cyclical behaviour of the real GDP in the UK economy.<sup>55,56</sup>

## 6.2 | Dynamic responses to a fiscal shock

Figure 5 in Appendix IV presents the estimated dynamic responses of the nominal domestic bond return, GDP, inflation and real effective exchange rate to an exogenous fiscal expansion for the British economy and for the neoclassical model under the parameterization of  $\Theta_{\pi} = 1$ ;  $\Theta_Y = 1$ . Evidence coming from the UK economy suggest that after the realization of the shock the nominal domestic return slightly increases by 0.01 per cent. Following the fiscal shock output marginally increases by 0.04 per cent and inflation rises to by 0.09 per cent 2 quarters after the shock. The real exchange rate appreciates by 0.33 per cent and follows a rather persistence pattern thereafter. The simulated neoclassical model with  $\Theta_{\pi} = 1$ ;  $\Theta_Y = 1$  seems to replicate the fiscal shock reaction in the UK economy better than any other simulated model. Table 4 in Appendix IV presents the relative statistical properties between the VAR and the simulated model.<sup>57</sup>

## 6.3 | Dynamic responses to a technology shock

This section analyses the estimated impulse responses from the VAR model after the occurrence of a positive technology shock. It is important to mention that we express the technology shock in the VAR model as a positive shock to the level of output. This is due to the otherwise less reliable approach of generating time series data for the level of technology, as capital stock estimates are notoriously unreliable and error prone. As a result, following the technology shock, we can observe an increase in the nominal domestic return by 0.08 per cent reaching its peak deviation of 0.25 per cent, 5 quarters after the realization of the shock. Output increases by 0.7 per cent constantly declining thereafter and inflation falls initially by 0.12 per cent. There is a real exchange rate appreciation by 0.32 per cent.

Related to our economic modelling, it seems that the dynamic predictions of the Calvo-Rotemberg specification under the values  $\Theta_{\pi} = 1.30$ ;  $\Theta_Y = 0.23$  and the Taylor wage model with parameterization  $\Theta_{\pi} = 0.5$ ;  $\Theta_Y = 1.5$  can replicate the economic activity in the UK economy better than any other simulated model. Figures 6 and 7 in Appendix IV depict the relevant dynamic responses. More specifically, related to the Calvo-Rotemberg model, upon the impact of the shock the nominal domestic return increases by 0.28 per cent, output by 1.28 per cent, inflation by 0.38 per cent and there is a real exchange rate appreciation of 0.44 per cent. Table 5 in Appendix IV depicts the relevant statistical properties for the Calvo-Rotemberg model where it

<sup>55</sup> Related to the alternative parameterization, i.e.,  $\Theta_{\pi} = 1$ ;  $\Theta_Y = 1$  the Taylor model with sticky prices does not generate plausible results in terms of the magnitude of the responses and the statistical properties between the actual and simulated series. The results are available upon request.

<sup>56</sup> Related to the Calvo-Rotemberg specification model the calibrated values from the estimates reported by Lubik and Schorfheide (2007) i.e.  $\Theta_{\pi} = 1.30$ ;  $\Theta_Y = 0.23$  can replicate to some extent the responses from the UK economy. However, the magnitude of the responses and the statistical properties of the series, cannot outperform the behaviour of the Taylor sticky price model under the parameterization of  $\Theta_{\pi} = 0.5$ ;  $\Theta_Y = 1.5$ . Both graphical and statistical evidence for this model are available upon request.

<sup>57</sup> The outcome from the other simulated models does not provide better results. The results are available upon request.

can be observed that the standard deviations between the actual and the simulated data are very close, especially for the nominal domestic return, inflation and the real exchange rate, and that the correlation coefficient for output is 0.95.

In relation to the Taylor wage specification evidence suggests that the nominal domestic return increases by 0.13 per cent, output by 0.71 per cent (0.7 for the UK data) inflation by 0.14 per cent and there is a real exchange rate appreciation of 0.77 per cent. Table 6 in Appendix IV presents the relevant statistics where it can be observed that the standard deviations for domestic return, output and the real exchange rate are very close. The correlation coefficient for output is less comparable to the Calvo-Rotemberg model.

## 7 | CONCLUSION AND IMPLICATIONS

This paper has constructed and analysed four different dynamic stochastic general equilibrium models in order to investigate the dynamic effects of a small open economy within both neo-classical and Keynesian economic environments. The benchmark neoclassical model, analysed in section 2, reflects a small open economy under the assumptions of perfect wage and price flexibility. The neoclassical model is then modified by incorporating nominal wage rigidities through the Taylor's (1979) overlapping contract mechanism. The role that nominal rigidities play in new open economy macroeconomic (NOEM) models further highlights the desirability to investigate the dynamic effects of price stickiness. This issue is addressed after introducing price inflexibility through the Calvo-Rotemberg price setting mechanism and through the Taylor's price specification.

To test the empirical validity of our economic modelling we have employed both statistical and econometric analysis using time series from the UK economy. The ability of the theoretical models to replicate important features of business cycle activity in the UK has been examined through moment comparisons between actual and simulated data. The simulated data has been generated for each model separately, depending on the specific shock that is assumed to hit the economy and on the chosen calibrated values for the monetary policy rule. Although the simulated data from most of the models can generate statistics that are often compatible with those of actual data, we suggest that a monetary shock under the Taylor model with sticky prices and with parameter values of  $\Theta_{\pi} = 0.5$ ;  $\Theta_Y = 1.5$  can replicate a significant portion of the business cycle activity in the UK.

The particular specification is also supported by a VAR model constructed for the UK economy. Comparing the short run dynamics generated by the VAR model after a monetary tightening with impulse responses coming from the Taylor model with price stickiness under the parameterization of  $\Theta_{\pi} = 0.5$ ;  $\Theta_Y = 1.5$  we found that the specific theoretical model can replicate well the behaviour of output, inflation, and the nominal domestic return for the UK. The specific model is also consistent with the notion of non-money neutrality, which is a characteristic of nominal rigidities. The predictions of the model have important policy implications for the UK economy especially within the current Brexit uncertainty, which is also associated with a weak global economic environment. In the absence of any strong Taylor principle effects, it seems that the monetary authority in the UK should place particular emphasis on the expected output gap while setting the nominal rate of interest. Given the expectations for a weaker growth, mainly due to the Brexit uncertainty and to the expected global recession, the monetary authority would be more likely to cut interest rates for a long period of time, in order to stimulate demand rather than to increase rates in order to safeguard against inflationary pressures arising from the

expected pound depreciation. This is reflected in the theoretical setup of the Taylor price model where more emphasis has been placed on the  $\Theta_Y$  parameter.

The Calvo-Rotemberg model with parameters  $\Theta_\pi = 1.30$ ;  $\Theta_Y = 0.23$  can also replicate a significant portion of the UK activity however the results are less favourable compared to the Taylor price specification.

Although a monetary shock in the Taylor model with price stickiness as previously analysed can be perceived as a plausible representation of data in the UK we cannot declare that other specifications do not perform reasonably well. Evidence from the VAR model suggests that under a fiscal shock the neoclassical model under the parameterizations of  $\Theta_\pi = 1$ ;  $\Theta_Y = 1$  can generate plausible dynamics.

Related to the technology shock, it seems that the predictions of the Calvo-Rotemberg specification with values of  $\Theta_\pi = 1.30$ ;  $\Theta_Y = 0.23$  and the Taylor wage model with  $\Theta_\pi = 0.5$ ;  $\Theta_Y = 1.5$  are quite close to the estimated responses coming from the VAR model for the British economy.

To further explore the empirical implications of the models, we have also employed a VECM model in order to test for the long run coexistence of the purchasing power parity (PPP) and the uncovered interest rate parity (UIRP). In the context of the steady state properties of the models, we established some evidence for the long run behaviour of the UK economy, which is consistent with the steady state properties of our economic modelling.

In order to further investigate the empirical accuracy of the various models constructed in this paper, recent developed econometric techniques can also be employed. A further evaluation of the calibrated versions of the models would generate an interesting step forward. Although this was addressed in the current paper through moment comparison between actual and simulated data, the research can be extended by evaluating the calibrated versions of the singular log-linearized models with regard to their capacity to replicate observed data for the UK economy. Following Watson's (1993) methodology on comparing stochastically singular models with data, statistics can be generated in order to summarize the degree to which each of the models fit the UK data. Guided from the evaluation process, the theoretical models can then be augmented with a number of structural shocks in order to estimate the structural parameters through a Bayesian approach technique.<sup>58</sup> After estimating the structural parameters the models can be revaluated in order to investigate the extent to which their ability to capture the UK data has improved. If any of the models is able to match the properties of the data, then it can provide a useful tool for monetary policy analysis within an empirically plausible framework.

## ORCID

Ioannis Litsios  <https://orcid.org/0000-0003-0895-9817>

Keith Pilbeam  <https://orcid.org/0000-0002-5609-8620>

Dimitrios Asteriou  <https://orcid.org/0000-0001-7640-6507>

## REFERENCES

- Barro, R., & Grossman, H. (1976). *Money, employment and inflation*. Cambridge University Press.
- Barth, M., & Ramey, V. (2001). The cost channel of monetary transmission. *NBER Macroeconomics Annual*, 16, 199–240.
- Baxter, M., & King, R. (1999). Measuring business cycles: Approximate band-pass filters for economic time series. *Review of Economics and Statistics*, 81, 575–593.

<sup>58</sup> See Harrison and Oomen (2010).

- Beveridge, S., & Nelson, C. (1981). A new approach to decomposition of economic time series into permanent and transitory components with particular attention to measurement of the business cycle. *Journal of Monetary Economics*, 7, 151–174.
- Calvo, G. (1983). Staggered prices in a utility-maximizing framework. *Journal of Monetary Economics*, 12, 383–398.
- Campbell, J., Lo, A., & MacKinlay, A. (1997). *The econometrics of financial markets*. NJ: Princeton University Press.
- Cooley, T., & Hansen, G. (1989). Inflation tax in a real business cycle model. *American Economic Review*, 79(4), 733–748.
- Eichenbaum, M. (1991). Real business cycle theory: Wisdom or whimsy? *Journal of Economic Dynamics and Control*, 15, 607–626.
- Epstein, L., & Zin, S. (1989). Substitution, risk aversion and the temporal behavior of consumption and asset returns: A theoretical framework. *Econometrica*, 57, 937–968.
- Epstein, L., & Zin, S. (1991). Substitution, risk aversion and the temporal behavior of consumption and asset returns: An empirical investigation. *Journal of Political Economy*, 99, 263–286.
- Finn, M., Hoffman, D., & Schlagenhauf, D. (1990). Intertemporal asset pricing relationships in barter and monetary economies. *Journal of Monetary Economics*, 25, 431–451.
- Fischer, S. (1977). Long-term contracting, sticky prices, and monetary policy. *Journal of Monetary Economics*, 3, 317–323.
- Galí, J., Gertler, M., & Lopez-Salido, D. (2001). European inflation dynamics. *European Economic Review*, 45(7), 1121–1150.
- Galí, J., & Monacelli, T. (2004). Monetary policy and exchange rate volatility in a small open economy. *Review of Economic Studies*, 72, 704–734.
- Galí, J. (2008). *Monetary policy, inflation and the business cycle*. NJ: Princeton University Press.
- Grossman, H. (1974). The cyclical pattern of unemployment and wage inflation. *Econometrica*, 41, 403–413.
- Harrison, R., & Oomen, Ö (2010). Evaluating and estimating a DSGE model for the United Kingdom. Bank of England 2010; Working Paper No. 380.
- Hodrick, R., & Prescott, E. (1981). Post war business cycles: An empirical investigation. *Discussion Paper No. 451 (Northwestern University)*.
- Johansen, S. (1988). Statistical analysis in co-integrated vectors. *Journal of Economic Dynamics and Control*, 12, 231–254.
- Johansen, S. (1991). Estimation and hypothesis testing of co-integration vectors in gaussian vector autoregressive models. *Econometrica*, 52, 389–402.
- Johansen, S., & Juselius, K. (1992). Testing structural hypotheses in a multivariate cointegration analysis of the PPP and the UIP for UK. *Journal of Econometrics*, 53(1–3), 211–244.
- Johansen, S. (1995). *Likelihood – based inference in cointegrated vector autoregressive models*. Oxford: Oxford University Press.
- Juselius, K. (2006). *The cointegrated VAR model: Methodology and applications*. Oxford: Oxford University Press.
- King, R., & Rebelo, S. (1999). Resuscitating real business cycles. In J. Taylor & M. Woodford (Eds.), *Handbook of Macroeconomics* (Vol. 1, pp. 927–1007). Amsterdam: Elsevier.
- Koop, G., Pesaran, H., & Potter, S. (1996). Impulse response analysis in nonlinear multivariate models. *Journal of Econometrics*, 74, 119–147.
- Kydland, F., & Prescott, E. (1982). Time to build and aggregate fluctuations. *Econometrica*, 50(6), 1345–1370.
- Lucas, R. E. (1982). Interest rates and currency prices in a two country world. *Journal of Monetary Economics*, 10, 353–359.
- Lubik, A., Tomas, & Schorfheide, F. (2007). Do Central Banks respond to exchange rate movements? A structural investigation. *Journal of Monetary Economics*, 54(4), 1069–1087.
- McCallum, B., & Nelson, E. (2000). Monetary policy for an open economy: An alternative framework with optimizing agents and sticky prices. *Oxford Review of Economic Policy*, 16(4), 74–91.
- McCallum, B. (1994). A semi-classical model of price-level adjustment. *Carnegie-Rochester Conference Series on Public Policy*, 41, 251–284.
- Monacelli, T. (2004). Into the mussa puzzle: Monetary policy regimes and the real exchange rate in a small open economy. *Journal of International Economics*, 62, 191–217.
- Pesaran, H., & Shin, Y. (1998). Generalized impulse responses in linear multivariate models. *Economics Letters*, 58, 17–29.

- Poterba, J., & Rotemberg, J. (1986). Money in the utility function: An empirical implementation. National Bureau of economic Research, Working Paper Series (1796).
- Prescott, E. (1986). Theory ahead of business cycle measurements. *Quarterly Review Federal Reserve Bank of Minneapolis*, 10, 9–22.
- Rotemberg, J. (1982a). Monopolistic price adjustment and aggregate output. *Review of Economic Studies*, 49, 517–531.
- Rotemberg, J. (1982b). Sticky prices in the United States. *Journal of Political Economy*, 90, 1187–1211.
- Sargent, J., Thomas, & Wallace, N. (1975). Rational expectations, the optimal monetary instrument, and the optimal money supply rule. *J.P.E.*, 2, 241–254.
- Sims, C. (1980). Macroeconomics and reality. *Econometrica*, 48, 1–48.
- Smets, F., & Wouters, R. (2003). An estimated dynamic stochastic general equilibrium model of the euro area. *Journal of the European Economic Association*, 1(5), 1123–1175.
- Svenson, L. E. O. (1985). Money and asset prices in a Cash-in-Advance Economy. *Journal of Political Economy*, 919–944.
- Taylor, J. (1979). Staggered wage setting in a Macro model. *American Economic Association*, 69, 108–116.
- Taylor, J. (1980). Aggregate dynamics and staggered contracts. *Journal of Political Economy*, 88, 1–23.
- Taylor, J. (1993). Discretion versus policy rules in practice. *Carnegie-Rochester Series on Public Policy*, 39, 195–214.
- Uhlig, H. (1999). A toolkit for analyzing nonlinear dynamic stochastic models easily. In R. Marimon & A. Scott (Eds.), *Computational methods for the study of dynamic economies* (pp. 30–61). Oxford: Oxford University Press.
- Walsh, C. E. (2003). *Monetary theory and policy*. United States of America, Massachusetts; MIT Press; 2003.
- Weil, P. (1989). The equity premium puzzle and the risk free rate puzzle. *Journal of Monetary Economics*, 24, 401–421.

## SUPPORTING INFORMATION

Additional supporting information may be found online in the Supporting Information section at the end of the article.

**How to cite this article:** Litsios I, Pilbeam K, Asteriou D. DSGE modelling for the UK economy 1974–2017. *Bulletin of Economic Research*. 2020;1–29.  
<https://doi.org/10.1111/boer.12255>