# Effects of tank sloshing on submerged oil leakage from damaged tankers

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## Abstract

In order to gain advanced understanding of the mechanism of the submerged oil leakage from damaged tankers subjected to wave-excited motions, a three-dimensional (3D) full scale numerical model is developed in this paper. The model is based on the three-phase Navier-Stokes equation and the continuity equation, which are solved by using the finite volume method (FVM). The volume of fluid (VOF) method is implemented to identify the interfaces between different phases and the k-ε turbulence model is employed to approximate the turbulence effects. The prototype of the oil tanker is taken as the side tank of VLCC, which is subjected to a periodically forced motions yielding a liquid sloshing inside the tank. After being validated by comparing its prediction with experimental data, the present model is utilized to a systematic investigation with wide range of applications including different motion amplitudes and periods. The dynamic characteristics of both the macroscopic parameters, e.g. the volume of the oil/water, and the microscopic parameters, e.g. the velocity distributions, are analyzed. The results fill the gap in the existing numerical and experimental work, in which the tank is assumed to be stationary, and produce a more reliable prediction on the dynamic process of the oil leakage and the stability of the damaged oil tankers subjected to wave actions.

Keywords: Liquid sloshing；Submerged oil leakage；Numerical simulation; damaged oil tanker; VOF

## 1. Introduction

The submerged crude oil leakage from damaged tanker in is considered as a major potential hazard to the ocean environment. A great effort has been devoted to both experimental and numerical investigations.

In terms of experimental studies, Yamaguchi (1992) has applied 1/50 side tank of the VLCC to explore the behavior of crude oil leakage in grounding condition (bottom rupture) and confirmed the importance of geometry and dynamic similarity; Debra et al. (2001) investigated the effect of oil density on the characteristics of oil leakage . Tavakoli et al. (2008, 2009, 2010, 2011, 2012) have carried out a systematical experiment with wide range of considerations including the draft factor, characteristics of oil leakage and capability of ballast tank for single hull tankers (SHT); Lu et al. (2014, 2015 and 2016) have carried out similar experiments but using double hull tankers (DHT), in which different positions of the rupture holes, initial water layers in the ballast tank (to reflect the time difference between the damage on the external hull and that on the internal hull) These experimental studies have explored the dynamic characterizes of the oil leakage for ideal conditions, i.e. fixed oil tankers originally placed in still water.

On the other hand, the analytical and empirical models for predicting the oil leakage were mainly established based on the theory of orifice flow subjected to hydrostatic conditions in the earlier stage(e.g. Dodge and Bowles, 1982; Fthenakis and Rohatgi, 1999; Fay, 2003). Such researches brought benefits on estimating the ultimate volume of spilled oil in an ideal condition. In order to reveal the hydrodynamic feature of the oil leakage process, which has been confirmed in the above-mentioned experimental studies, the computational fluid dynamics (CFD) has also been attempted (e.g. Chang III and Lin, 1994; Cheng and Gomes, 2010; Tavakoli et al., 2011, 2012; Yang et al., 2014, 2016, 2017). Similar to the experimental studies, the majority of the numerical work did not take into account of the effects of the tank motion on the oil leakage.

In fact, during the oil leakage process, the tanker is often subjected to the action of water wave, current and wind, yielding translational and/or rotational motion, which influences the oil leakage process. On the other hand, the oil leakage poses loading on the oil tanker and may significantly affect the motion of the damaged tankers. Consequently, the motion of the tanker, the oil leakage and the ocean environment need to be modelled simultaneously as an integrated system in both the numerical and experimental studies. However, in most of the existing researches, the tankers are assumed to be stationary and, therefore, the effects of the tanker motion on the oil leakage, as well as the external environmental fluid conditions, are ignored. Theoretically, the CFD models established for modelling oil leakages from fixed oil tankers can be extended to simulate the scenario where the oil tankers are subjected to a motion, its complexity emerges when comprehensively considering the integrated system combining the external environment (tide, current and wave), the ship response (damaged ship motion and sloshing) and the oil leakage (Zhang and Suzuki, 2006). To the best of our knowledge, only Yang et al. (2016) have preliminarily attempted to numerical simulate the oil leakage from a two-dimensional (2D) SHT subjected to pre-specified periodic motions without considering the action of wave and current. They have demonstrated that the tank motion does not only cause a periodic oscillation of the oil/water flow through the broken hole, but also results in a second long-duration stage of spilling after a quasi-hydrostatic-equilibrium condition occurs, leading to more significant amount of spilled oil. Nevertheless, their work was restricted by 2D assumption and is difficult to be applied to the reality.

In this paper, a three-dimensional numerical model is established and couples the oil leakage and the motion of the DHT to investigate the effect of tanker motion on both the oil leakage and the liquid sloshing inside the tank. Only the grounding scenario, in which the oil spills from the bottom of the tank, is considered. Dynamic characteristic of the oil leakage, the free surface deformation and the flow field inside the tank are obtained to analyze the mechanism of oil leakage from a grounded tank in motion. Similar to Yang et al. (2016), only pre-specified periodic tank motion with different amplitudes and periods of motion, are taken into account. Although it is understood that the oil leakage and external fluid motion in turn influence the motion of the tank, one may agree that the present research leads to a better understanding on the hydrodynamics associated with the oil leakage.

## 2. Mathematical formulation and Numerical approach

### 2.1 Governing equations

The submerged oil leakage from a damaged tanker is considered as a viscous multiphase transient flow. In general practices, the crude oil in cargo tank needs to be heated and insulated. During the oil spilling process, the oil may be mixed with water. Under such a condition, the crude oil, air, water and their mixtures are assumed as an incompressible Newtonian fluid. The surface tension is not taken into account. These assumptions have been proven to be acceptable in our previous numerical practices (Yang et al., 2017). Based on these assumptions, three-dimensional mass and momentum conservation equations can be described as follow, respectively.

$$\frac{\partial \rho}{\partial t} + \nabla \rho \vec{v} = 0 \qquad (1)$$

$$\frac{\partial}{\partial t}(\rho \vec{v}) + \nabla \cdot (\rho \vec{v}\vec{v}) = -\nabla P + \nabla[\mu(\nabla \vec{v} + \nabla \vec{v}^T - \frac{2}{3}\nabla \cdot \vec{v} I)] + \rho \vec{g} + \vec{F} \qquad (2)$$

Where, $\vec{v}$ is the velocity vector, $\rho$ is the density and $P$ is the pressure, $I$ is the unit tensor; $\vec{F}$ is the external rotational inertia force induced by sloshing motion of tank, $g$ is the gravitational body force, $\mu$ represents the molecular viscosity.

### 2.2 Interface tracking and transport

As indicated, three phase of fluids, i.e. water, air and oil, need to be taken into account in the model, the volume of fluid (VOF) technique is applied to track their interface. The VOF method identifies and tracks the interface by the volume fraction of each phase. By solving the corresponding transportation equation of $q^{th}$ phase,

$$\frac{\partial \alpha_q}{\partial t} + v_q \cdot \nabla \alpha_q = 0 \quad (3)$$

One can obtain the temporal-spatial distribution of the volume fractions, i.e. $\alpha_{air}$, $\alpha_{water}$ and $\alpha_{oil}$ for air, water and oil respectively, which satisfies,

$$\alpha_{air} + \alpha_{water} + \alpha_{oil} = 1 \qquad (4)$$

Once the volume fractions are obtained, the density and viscosity are approximated by the following form:

$$\rho = \alpha_{air}\rho_{air} + \alpha_{water}\rho_{water} + \alpha_{oil}\rho_{oil} \quad (5)$$

$$\mu = \alpha_{air}\mu_{air} + \alpha_{water}\mu_{water} + \alpha_{oil}\mu_{oil} \quad (6)$$

Where the subscripts, air, water and oil, refer to the air phase, water phase and oil phase, respectively. It is important to express that such assumption may not be acceptable for emulsive oil, which may be observed in the oil/water mixture. In this study, the emulsification is ignored. The previous numerical works (e.g. Yang et al., 2017) have confirmed that such assumption is acceptable in the numerical practices.

**2.3 Turbulence model**

Yang et al (2017) has discussed the role of the turbulence when modeling oil leakage from damaged oil tanker. Nevertheless, their discussions are based on the simulation in which a scaled model, instead of full-scale prototype, is applied. It is well-known that smaller-scale model may over-estimate the turbulence effects. In this paper, the full scale simulation is carried out and the standard *k*-*ε* turbulence model for multiphase flow is applied,

$$\frac{\partial \alpha_q}{\partial t}(\rho k)+\nabla(\rho k v)=\nabla(\frac{\mu_t}{\sigma_k}\nabla k)+\mu_t S^2-\rho\varepsilon \quad (7)$$

$$\frac{\partial \alpha_q}{\partial t}(\rho \varepsilon)+\nabla(\rho \varepsilon v)=\nabla(\frac{\mu_t}{\sigma_k}\nabla \varepsilon)+C_{1\varepsilon}\mu_t S^2-C_{2\varepsilon}\rho\frac{\varepsilon^2}{k} \quad (8)$$

In which, $\mu_t$ represents the turbulent viscosity, $S$ is the modulus of mean rate of strain tensor, $\mu_t=\rho C_\mu k^2/\varepsilon$, $S=\sqrt{2S_{ij}\square S_{ij}}$ ;

$C_\mu=0.09$ , $\sigma_k=1.00$ $\sigma_\varepsilon=1.30$ , $C_{1\varepsilon}=1.44$ , $C_{2\varepsilon}=1.92$ . $\rho=\sum_{i=1}^{N}\alpha_i\rho_i$ , $\vec{v}=\frac{\sum_{i=1}^{N}\alpha_i\rho_i v_i}{\sum_{i=1}^{N}\alpha_i\rho_i}$

**2.4 Model of tank sloshing motion**

Although, the tank motion is rarely considered in the studies of oil leakages. A massive research has been carried out for the liquid motion within a cargo tank without liquid leakage. The tank filling rate, the interaction of the liquid and the tank wall, the effect of excitation on liquid sloshing have been focused (Akyildız et al., 2006).. Different numerical methods have been applied in engineering practices and researches, including the potential theory, which assumes the fluid is inviscid, incompressible and irrotational (e.g. Kolaei et al.,2015; Vaziri et al., 2015) and the general viscous flow theory as what is applied in this paper. Limited examples of the latter are given here for completeness.

To explore the interaction between the liquid and tank wall, Hu et al .(2010) and Lee et al.(2011) adopted the CIP (Constrained Interpolation Profile) method to simulate the violent sloshing flow and estimate the impact pressure on the tank wall; The SPH (smoothed particle hydrodynamics) method was employed to analyze the effect of excitation frequency and baffle on the impact pressure and the liquid sloshing in swaying and surging tank (Zhang et al., 2014); the MPS ( Zhao et al., 2014) and the material point method (MPM, Lin et al., 2015) have been employed to model the breakups of the free surface and estimate the pressure field. With regard to the effect of excitation on the liquid sloshing, the VOF has been developed to investigate the liquid sloshing in container with pre-specified linear rotational accelerations (Elahi et al.,2015); To consider the coupling effect of the ship motion and the liquid sloshing flow, two coupled programs were adopted to solve the flow equation of liquid sloshing in a container and the motion equation of container, respectively(Kim et al., 2007); the coupled model of linear ship motion and nonlinear sloshing flows has been applied to estimate the dynamic behavior of liquid tank motion with regular wave(Li et al., 2014).

From the researches, it is commonly agreed that (1) the liquid sloshing is dominated by the motion amplitude and frequency of the tank. Typically, for a specific motion frequency, larger motion amplitude leads to more significant nonlinearity (Panigrahy et al., 2009); the sloshing is more violent when the motion frequency is closer to the natural frequency of the tank; (2) the filling rate influence the natural frequency of the tank and, consequently, that motion responses of liquid is related to filling depth in the moving tank and the forces and torque induced by liquid sloshing are not linear with motion amplitude (Liu and Lin, 2008). However, once the tank is damaged and the liquid leakages from the tank. It results in continuously loss of liquid inside the tank. Therefore, the filling rate of the tank is decreasing and consequently the natural frequency of the tank changes. This has been confirmed by Yang et al. (2016). This means that the hydrodynamics associated with damaged tank may be considerably different from that observed in the cases with sealed tank without liquid leakage.

Typically, the rolling motion of a damaged tanker is more likely. Especially, when subjected to a high sea state, rolling is significantly nonlinear due to the fact that the damping and the restoring torque change nonlinearly. To simplify the problem, the motion of oil tanker is described by a harmonic motion,

$$\theta = \theta_m \sin(wt + \theta_0) \qquad (9)$$

In which, $\omega$ is the angular frequency of the tank motion, $\theta$ is the instantaneous roll angle, $\theta_m$ is the motion amplitude and $\theta_0$ is the initial position. In this study, $\theta_0 = 0$ is taken. The rotational center is located at gravity center of the filling tank.

Due to the motion of the tank, an Eulerian mesh, which are used in our previous numerical practices for modelling oil leakage from fixed tank, is not suitable unless the immersed boundary method, in which the solid tank wall is also considered as an additional phase, is applied. The problem is overcome by utilizing the dynamic mesh technique, which moves the computational grid covering the fluid domain only to conform to the motion of the tank. This technique includes the spring analogy smoothing method and the local remeshing methods. The details can be found in Lin et al. (2014).

## 3. Model configuration and verification

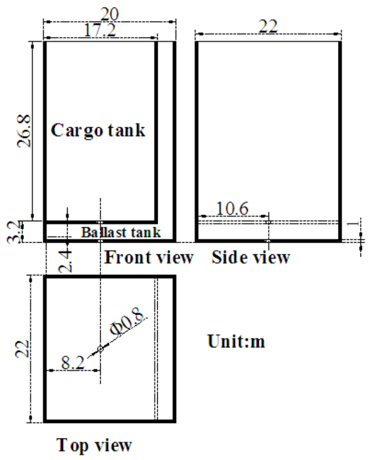
### 3.1 The model of submerged oil from moving tank

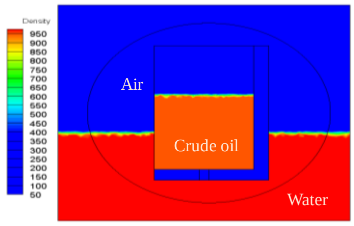
The full scale L-shape side tank of a VLCC (sketched in Fig. 1) is applied to investigate the mechanism and dynamic characteristics of crude oil leakage from a tank subjected to pre-specified harmonic motion. The length, width and height of the prototype are 20m, 22m, 30m, respectively. The height of the ballast tank is 2.4m. The oil leaks from the rupture hole placed on the bottom of the tank and the diameter of the rupture hole is 0.8m. The initial height of crude oil in cargo tank is 16.8m and the draft of the tank is 10.8m.

The typical properties of numerical crude oil are selected as real crude oil. The density and dynamic viscosity of the crude oil are 915kg/m$^3$ and 8.09×10$^{-2}$Pa·s, respectively; the density and dynamic viscosity of the water are 998.5kg/m$^3$ and 1.01×10$^{-3}$Pa·s, respectively; the density and the viscosity of air are

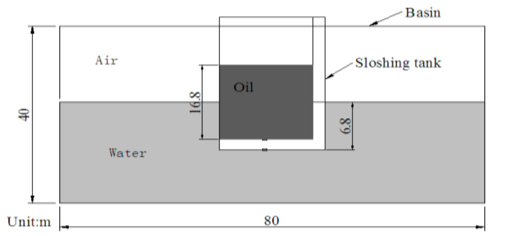
1.225kg/m3 and 17.9×10$^{-6}$Pa·s respectively.

In the numerical simulation, the cargo tank is initially placed in a still water basin, as illustrated in Fig. 2. An unstructured mesh is generated and demonstrated in Fig. 3. The size of computational domain is 48m*21.2m*48m, the minimun mesh size, which is applied near the rupture hole, is 0.0027m$^3$, the maximun aspect ratio of the grid is 4:1, and equisize skew of 98%, mesh is 0~0.6. To maintain a good quality of the computational mesh, the cell in a cylinderical region near the cargo tank (circuled in Fig. 2(a)) are allowed to be rotated following the rotaiton of the cargo tank.




a) Model of crude oil leakage




b) The cross-sectional diagram

Fig.1 Sketch of the L-shape tank

Fig.2 Model of crude oil leakage from moving damaged tank

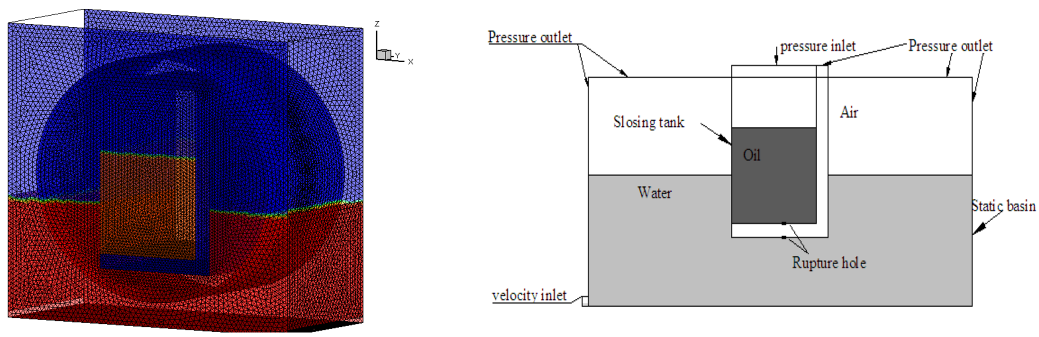



Fig.3Mesh arrangement and boundary conditions

The others are remain stationarity using the sliding mesh technique. It shall be noted that the motion of the tank result in a radiation wave near the cargo tank, such wave will be reflected by the side wall of the water basin and consequently influence the behaviour of the spilled oil outside the cargo basin. A damping zone may be applied to minimise such effects (Yang et al., 2016). In this study, the pressure outlet applied on the side wall and sufficient long (i.e. 80m) of the water basin can minimise the impact of the reflected waves from the basin wall in the duration of the simulation

### 3.2Verification of numerical methods

The validation of the numerical method were carried out by model tank sloshing and oil leakage from model tank at prior. Considering the fact that the experimental data for oil leakages from damaged tank in motion is not available. We consider the liquid sloshing in the un-damaged tank and the oil leakage from fixed damaged tank seperately.

The physical model of liquid sloshing without liquid leakge is taken from the exeperiments done by Akyildız et al.(2006) . The present numerical model is applied to estimate the pressure on tank wall. In this case, the geometry of the cargo tank is presented in Fig.4, the filling rate is 75%, different motion amplitudes ranges from 4° to 8° are used, whilest the motion period is 3.14s. the pressure sensor is located on the side wall and is 0.31m below the mean water surface.

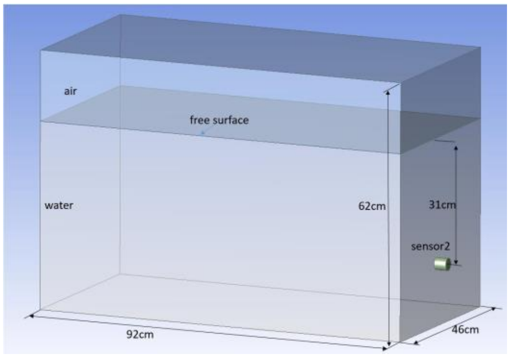
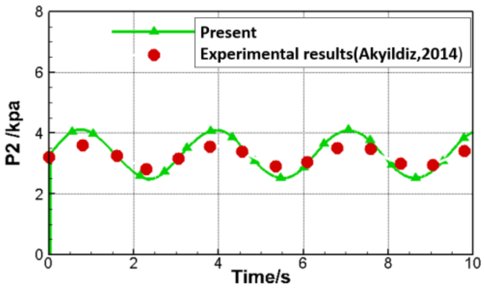
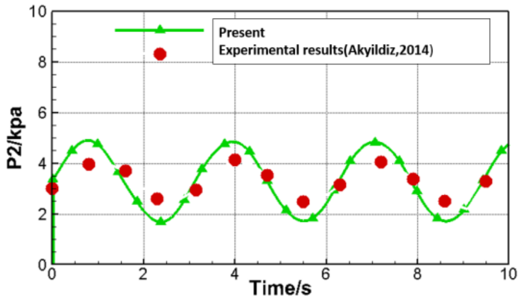



Fig.4 The geometry of the cargo tank applied to validate sloshing method.

Fig.5 compares the present numerical results and the experimental data. This shows a satisfactory agreement on predicting the pressure caused by liquid sloshing in the cargo tank.




a. T=3.14s, A=4°




b. T=3.14s, A=8°

Fig.5 Comparison of impact pressure with the experimental results of Akyildız

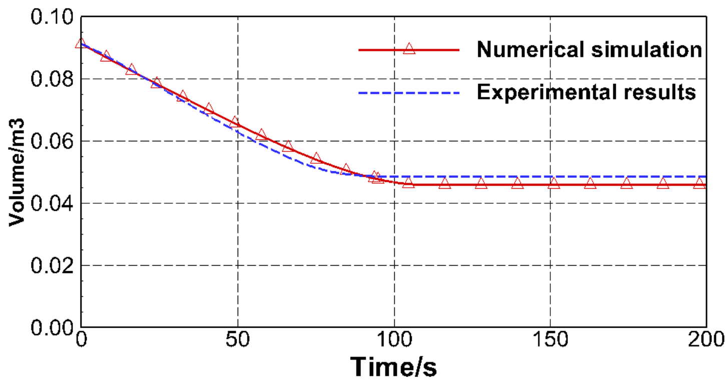



Fig.6 Comparison of crude oil leakage from cargo tank between experimental and numerical results

We have carried out the model test for the oil leakage from fixed damaged DHT (Lu et al.,2014,2015,2016). The corrsponding results are used here for validation purpose. Some results are shown in Fig.6. In this case, the gemetry of the tanker is the same as that used in this study but 1/40 scale is used in the experiment. The initial oil height in the cargo tank is 0.42m and the draft of the tank is 0.27m. As observed that the volume of the oil remaining in the cargo tank predicted by the present model agree with the experimental data. The relative error of is 4.1% which is regared to be acceptable in engineering.

From the comparion between the numerical predictions and the corresponding experimental results, one may agree that the present model is acceptable for numerically investigating the oil leakage from damaged tank subjected to harmonic motions.

### 3.3 Conditions of damaged tank motion

In the ocean environment, the most commonly seen wave period ranges from 4s to 9s. The period of tanker motion is mainly determined by sea waves and tanker’s structure. In accordance with requirements of tanker design, the natural period of ship should keep off effective period of wave to avoid violent resonance oscillation. According to safe requirements of tanker operation, the requirements of transverse stability of sloshing ship have to be achieved to tolerate the transverse torques caused by waves and winds. The stability of ship can be divided into initial stability (less than 10°~15°) and the larger angle stability (larger than 15°).Considering the engineering practices, the amplitudes and the periods of the tank motions used in the experiments are listed in Tab.1.

Tab.1 Conditions of damaged tank motion

| Case | amplitude(°) | period(s) |
|---|---|---|
| 1 | 2 | 10 |
| 2 | 5 | 10 |
| 3 | 10 | 10 |
| 4 | 15 | 10 |
| 5 | 20 | 10 |
| 6 | 10 | 4 |
| 7 | 10 | 6 |
| 8 | 10 | 8 |

The natural frequency of the tank without considering the oil leakage is less 4s. Since the main purpose of this study is to explore the mechanical feature of the oil leakage, the motion frequency selected in this study is away from the natural frequency, avoiding violent sloshing occurrence inside the cargo tank. The effect of the violent sloshing and wave breakup will be discussed in the future study.

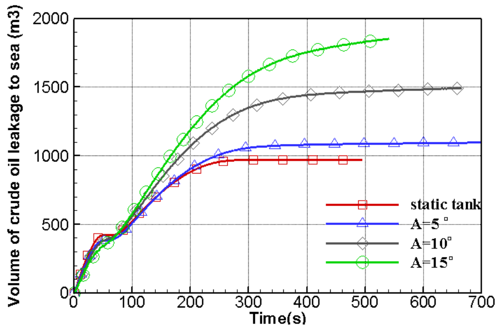



Fig.7 comparison between numerical results of moving tank and static tank (Period 10s)

## 4. Results and discussions

Based on the oil leakage for stationary damaged tanks, relevant experimental and numerical investigations have been carried out to explore the mechanism of the oil leakage. In this study, the full-scale numerical results of the crude oil leakage from tank in motion are compared with the corresponding results in the cases with fixed tank to quantify the effects of the tank motions on the oil leakage. Fig.7 compares the time histories of the volume of the spilled oil into the external environment in the cases with different motion amplitudes where the motion period is 10s. As observed, in the early stage of the leakage ($t<100s$), the macroscopic volume of the spilled oil into the external are close regarding the motion amplitude. However, in the later stage ($t>100s$), the effect of the tank motion on the spilled oil volume become significant, the larger motion amplitude, the more volume of oil leakage.

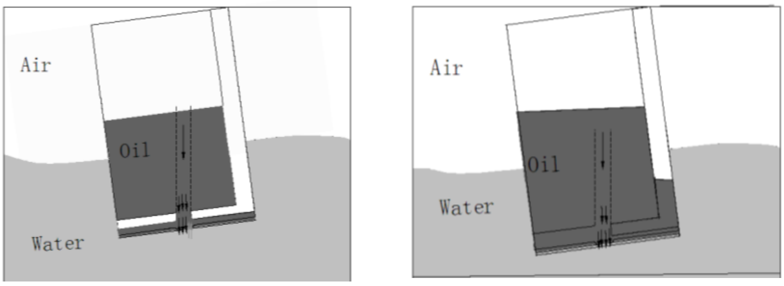
### 4.1 Dominated forces analysis of crude oil leakage

The crude oil leakage from moving tank is complex and different from the cases with stationary tank. During the process of crude oil leakage, the dominate forces driving the crude oil motion and the characteristics of oil-water flow near the broken hole are significantly different at different stages of the leakage. Following the analysis on the oil leakage from stationary tank in our previous work (e.g. Yang et al, 2014), it is convenient to split the entire process into three stages by considering the total heads at (1) the oil surface in the cargo tank; (2) the oil/water mixture surface in the ballast tank and (3) the free surface of the external environment. Without tank motions (Yang et al, 2014), the total head at (1) and (3) are dominated by the potential head. With tank motions, which do not only lead to the liquid sloshing in the cargo tank and ballast tank, but also excite radiation wave in the sea environments, amplify the velocity head at three positions.

In the initial stage, the oil in the cargo tank leakages into the ballast tank through the internal hole by the head difference between (1) and (2); the water flows into the ballast tank due to the head difference between (2) and (3). The oil jet out of the cargo tank and the water jet towards the cargo tank interacts with each other. Consequently, not all of the crude oil from the cargo tank flows into the sea, some remains in the ballast tank. The oil-water mixture sloshes in the ballast tank initiated by the tank motion, leading to a fluctuating discharge of the crude oil leakage into the sea. This stage is referred to as the gravity flow stage, which is featured by a convective crude oil and water flows near the broken hole, as shown in Fig.8. During this stage, the potential head difference is more significant than the velocity head difference. Thus, the dominate factor is the potential head difference. The liquid sloshing plays insignificant role as demonstrated by the time histories before t $t \approx 100s$ in Fig. 7.

As more fluids (both oil and water) are accumulated in the ballast tank, especially after the bottom space of the ballast tank is fully filled and the liquid level in the ballast tank increases more rapid in the vertical section of the ballast space, the potential head difference between (1) and (2), which drives the oil moving into the ballast tank, and that between (2) and (3), which drives the water moves from the sea into the ballast tank, become less significant and the velocity head difference caused mainly by the tank motions play more important role. This stage is referred to as the transition stage, which is featured by a unidirectional water/oil mixture flows into the sea through the external hole, as shown in Fig.8. It is expected that more significant sloshing motion (e.g. larger motion amplitude or motion frequency closer to the natural frequency) leads to more significant velocity head difference and thus drive more oil leakage. This is confirmed by the numerical results shown in Fig. 7.

This stage finishes when a quasi-static pressure equilibrium is established, in which the potential head difference at positions (1), (2) and (3) approaches zero. In the case with stationary tank, no further oil leakage, however, in the cases with tank motions, the velocity head difference, especially at (1) and (3), exists. This leads to a fluctuation of the total head (mainly the velocity head caused by the liquid sloshing inside the tank or the radiation waves outside the tank) at (1) and (3). If the total head at (1) is higher than at (3), the crude oil and water in ballast tank are driven into the sea. Otherwise, the external water/oil mixture near the hole is ejected into ballast tank. Therefore, the flow direction at the external hole is bi-directional (moves in or out), as illustrated in Fig. 10. Overall, the oil in the cargo tank continues leakage into the sea. As expected, more significant tank motion leads to higher velocity head difference at (1) and (3), leading to more volume of oil leakage as shown in Fig.7.




a) before internal hole is submerged    b) after internal hole is submerged

Fig.8 Characteristics of crude oil and water in gravity flow stage

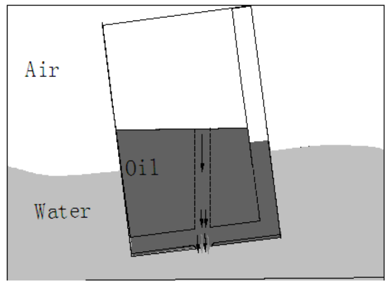
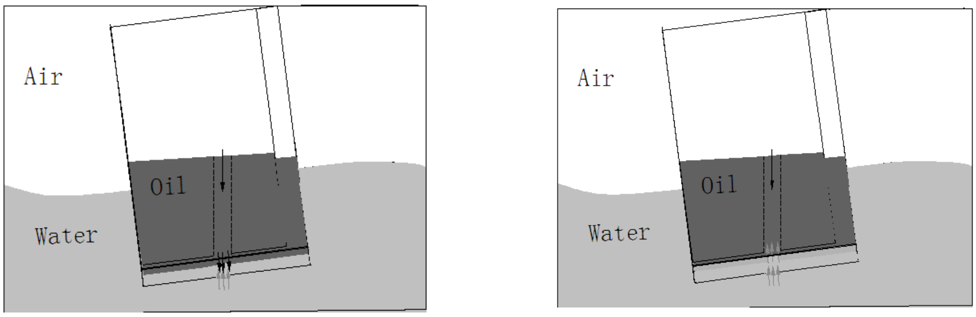



Fig.9 Characteristics of crude oil and water in transition flow stage




a) water flow into ballast tank    b）water flow into cargo tank

Fig.10 Characteristics of crude oil and water in sloshing flow stage

#### 4.2Mechanism of crude oil leakage from tanks in motion

Based on the analysis in the previous Section, theoretical models are established here in order to predict the oil leakage from the damaged oil tank in motions. Following the discussion in the previous section, Bernoulli's equation is applied when establish the theoretical model. It is worth noting that the streamline in the cargo tank, near the broken hole inside the ballast tank and in the region outside of the oil tanker are not continuous in reality, in particular near the interface between the oil and water, due to the involvement of the fluid phase changes in space from crude oil in the cargo tank to the oil/water mixture in the ballast tank and external environment. This invalid the application of the Bernoulli's equation. However, considering small density ratio between the oil and the water, one may assume that the effect of the phase changing can be ignored and the streamline is continuous. It is also assumed that the effect due to the acceleration of the fluid in the Bernoulli's equation may be ignored within a short duration, and therefore, the oil leakage process is regarded as a quasi-steady procedure. This means a steady Bernoulli's equation is only applied to each time step. Similar assumptions have also been used by Tavakoli et al. (2008) and been justified by our previous correlation analysis (Yang et al, 2014) in the cases with fixed oil tanker.

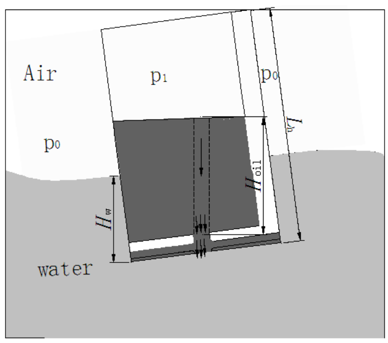



Fig.11 The schematic diagram of crude oil and water flow subjected to damaged tanks in

motion

By using this assumptions, it is not difficult to apply the steady Bernoulli's equation at the oil surface in the cargo tank (marked as 1 in Fig. 11) and the external hole on the external hull surface (marked as 2), leading to

$$H_{oil} + d + \frac{p_1}{\rho_{oi1} g} + \delta \frac{v_s^2}{2g} = \frac{p_2}{\rho_{oi1} g} + \frac{v_d^2}{2g} + h_w + h_f \tag{10}$$

Similarly, one can also establish

$$p_2 = \rho_w g H_w + P_0 \tag{11}$$

By considering the energy equilibrium between position 2 and the free surface of the external environment. In Eq. (10) and (11), $H_{oil}$ is the height of the crude oil in cargo tank; $d$ is the vertical distance between the inner hull and the outer hull; $v_s$ is the fluctuating velocity of the oil surface in the cargo tank; $\delta$ is a correction coefficient of additional kinetic energy caused by the tanker motion; $P_0$, $P_1$ and $P_2$ are the static pressure at three positions described above and $P_1=P_2=0$; $\rho_{oil}$ and $\rho_w$ are the density of crude oil and the water, respectively; $v_d$ is the velocity of crude oil passing through external hole at Position 2; $h_w$ is the viscous head loss and $h_w \approx \zeta \frac{v_d^2}{2g}$ where $\zeta$ is the loss coefficient; $h_f$ is the head loss caused by the collision/dilution of the crude oil and the water in the ballast tank, $h_f \approx \eta \frac{v_d^2}{2g}$ where $\eta$ is the loss coefficient.

In this study, the motion of tanker is described by Eq. (9), so the rotational speed of the tanker can be obtained as followed:

$$\frac{d\theta}{dt} = A\frac{2\pi}{T}\cos(\frac{2\pi}{T}t) \tag{12}$$

Subjected to such motion, the oil surface in the cargo tank and the water surface in the external environment are temporally-spatially varying due to the liquid sloshing and/or radiation waves. This means that $H_{oil}$ and $H_w$ are unsteady and non-uniform. For the same reason, the velocity $v_s$ fluctuates spatially due to the liquid sloshing in the cargo tank. It is difficult to quantity their spatial distribution when the tank is subjected to motions with different amplitudes and frequencies. However, the purpose of Eqs. (10-11) are to establish a macroscopic equilibrium of the energy at different positions. One may give the characteristic values of $H_{oil}$, $H_w$ and $v_s$, by callibrating the coefficient $\delta$, the total head at the oil surface in the cargo tank can be approximately with a minimising error. In this practice, $H_{oil}$ and $H_w$ are taken as the values corresponding to the mean oil surface and water surface, respectively. The characteristic value of $v_s$ is defined as

$$v_s(t) = (H_{oil} - \frac{l_h}{2})\frac{d\theta}{dt} = \pi(2H_{oil} - l_h)\frac{A}{T}\cos(\frac{2\pi}{T}t) \tag{13}$$

Where, $l_h$ is the height of damaged tank. One may notice that the value of $v_s$ defined in Eq. (13) is simplied the translational velocity of the tank motion at the centre of the mean oil surface in the cargo tank. Since the main focus of the present investigation is the effect of tanker motion on crude oil leakage, the maximum velocity ($v_{s_{max}}$) may also be an alternative to the characteristic value, i.e.

$$v_{s_{max}} = \pi(2H_{oil} - l_h)\frac{A}{T} \quad (14)$$

Considering Eqs. (10-13), the velocity ($v_d$) and volume ($V_d$) of the crude oil leakage subjected to a tanker motion can be described as followed,

$$v_d = \frac{1}{\sqrt{1+\zeta+\eta}}\sqrt{2g(H_{oil} + d - \frac{\rho_w}{\rho_h}H_w) + \delta\pi^2(2H_{oil} - l_h)^2(\frac{A}{T})^2} \quad (15)$$

$$V_d = \int v_d dt \quad (16)$$

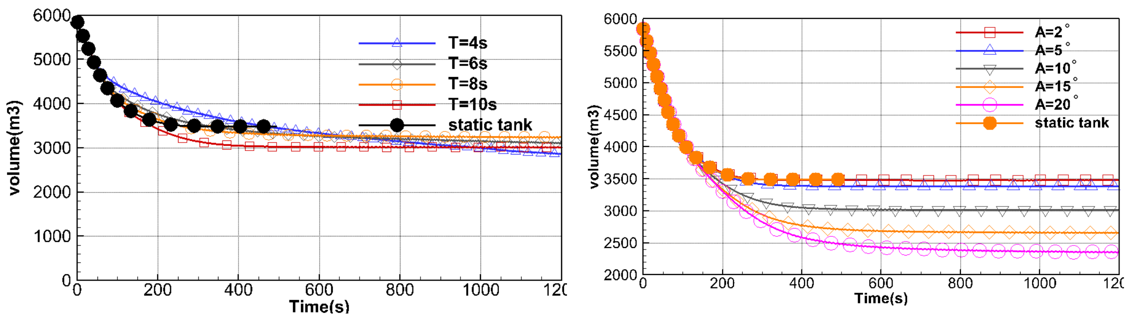
Where coefficients $\delta$, $\zeta$ and $\eta$ can be calibrated using a systematic numerical investigation by the numerical model described above. Obviously, the first term $H_{oil} + d - \frac{\rho_w}{\rho_h}H_w$ represents the potential head difference and the 2nd term $(2H_{oil} - l_h)^2 \delta\pi^2(\frac{A}{T})^2$ indicates the significance of the velocity head difference. As indicated in the previous section, the entire leakage procedure can be divided into three different phases by the relevant significance of the potential head and the velocity head.

In the gravity flow stage, the effect of tanker sloshing, i.e. the 2nd term is significantly less than the first term and therefore $\delta = 0$. The coefficient $\eta \neq 0$ before the inner hole is submerged by the fluid in the ballast tank (i.e. the oil jet from the ballast tank impacts on the surface of the oil/water mixture in the ballast tank), whereas $\eta = 0$ afterwards. In the sloshing flow stage (the third stage), the first term $H_{oil} + d - \frac{\rho_w}{\rho_h}H_w \approx 0$ and $\eta = 0$. In the transition flow stage, both terms need to be considered. Consequently, the equation describing the oil leakage velocities at different stages are summarized in Tab.2.

Tab.2 The prediction model of crude oil leakage from moving damaged tank

| Leakage stage | Dominate forces | Velocity of crude oil leakage to sea |
|---|---|---|
| Gravity flow | Potential head difference (Liquid level; Density) | $v_d = \frac{1}{\sqrt{1+\zeta+\eta}}\sqrt{2g(H_{oil} + d - \frac{\rho_w}{\rho_h}H_w)}$ (after the inner hull is fully submerged, $\eta$=0) |
| Transition flow | Liquid level; density; Fluctuation pressure | $v_d = \frac{1}{\sqrt{1+\zeta+\eta}}\sqrt{2g(H_{oil} + d - \frac{\rho_w}{\rho_h}H_w) + \delta\pi^2(2H_{oil} - l_h)^2(\frac{A}{T})^2}$ |

| | | |
|---|---|---|
| Sloshing flow | Velocity head at the oil tank(Fluctuation pressure) | $v_d = \frac{\pi}{\sqrt{1+\zeta}}\sqrt{\frac{\delta}{1+\zeta}(2H_{oil}-l_h)}\frac{A}{T}$ |




a) T=10s b) A=10°

Fig.12 Time histories of volume of crude oil in the cargo tank

### 4.3 Macroscopic characteristics of crude oil leakage

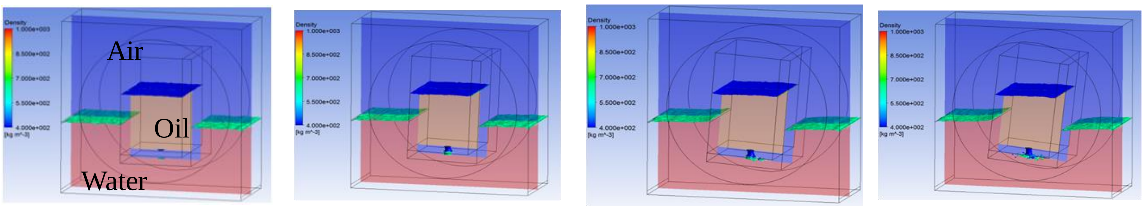
Fig. 12 compares the time histories of the volume of the crude oil in the cargo tank in the cases with different motion conditions of the tank. As expected that in the early stage of the leakage, i.e. in the gravity flow stage (~100s), the motions of the tank do not influence the volume of the crude oil in the cargo tank. After a certain duration, the volume of the oil in the cargo tank approaches a steady value, expect the case with T = 4s and A = 10°. For a specific motion period (T=10s , Fig. 11(a)), larger motion amplitude leads to lower volume of crude oil in the tank and longer duration before the volume becomes steady; For a specific motion amplitude (A=10°, Fig. 11(b)), a longer period results in a shorter duration before the volume becomes steady when T is longer than 4s. For T = 4s and A = 10°, the volume of crude oil does not reach a steady state before the simulation stops. In this case, a significant volume of water is observed to flow into the cargo tank to replace the oil, leading to the continuous loss of oil from the cargo tank, although the total volume of the fluid in the cargo tank does not seem to be considerably different from other cases with longer motion period.

During the gravity flow stage, where the interfaces are captured by using the VOF method with volume of fraction of 0.5 being

1） Dynamic characteristics of the gravity flow stage

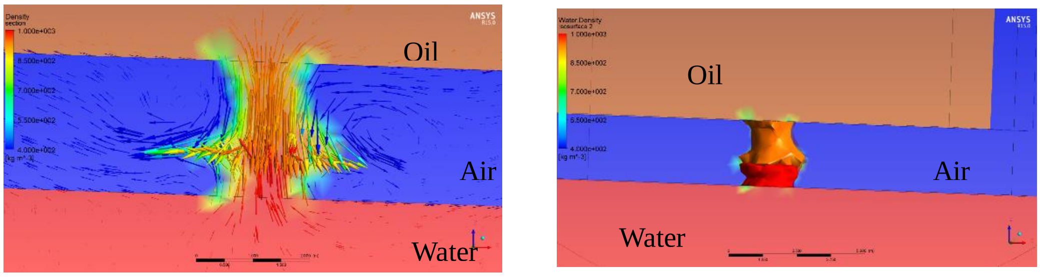
Similar to the case with fixed tank presented in Yang et al (2014, 2017), in the beginning of the oil leakage (referred to as the transient phase), an oil jet is formed at the internal hole and a water jet is formed at the external hole by corresponding potential head difference. They impact on each other, as illustrated later in Fig. 13 illustrates the oil-air interface (in dark blue), water-air interface (in green) and the water-oil interface (in red).

the interface. In this case, T=6s and A=10°. As the crude oil jet has a larger momentum (larger velocity and density) compared to the water jet. Consequently, the water jet is forced to keep out of the ballast tank and the crude oil jet primarily occupied the external hole. Snapshots of the velocity field near the broken hole and the interfaces between different phases are given in Fig. 14.

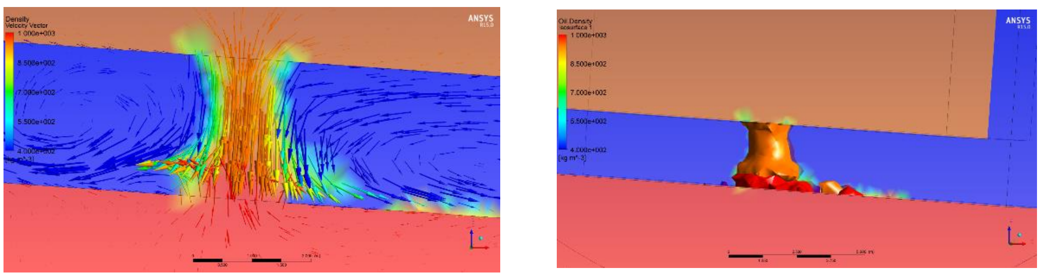



a) t=0s b) t=0.2s c) t=0.4s d) t=0.6s

Fig.13 Characteristics of free surface in transient stage of grounding accident

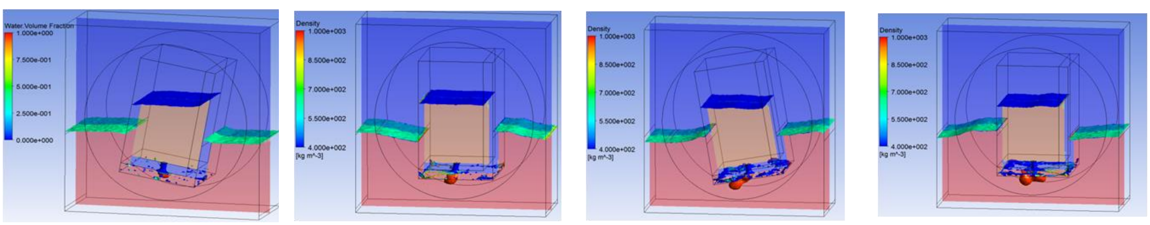



a) t=0.2s

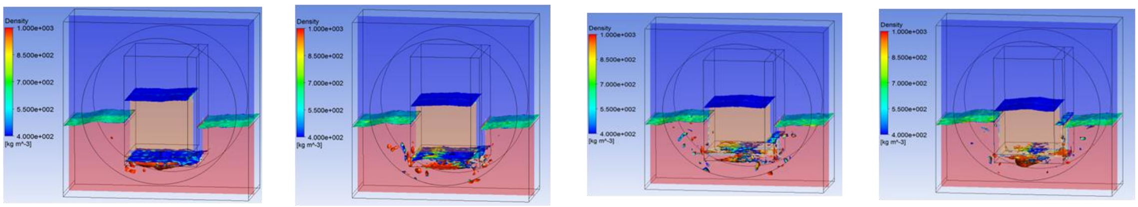


b) t=0.4s

Fig.14 The velocity vectors and phase interface in transient stage of grounding accident

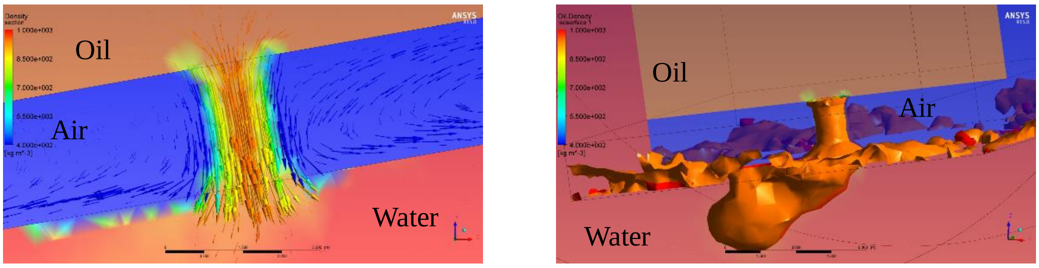


e) t=1.6s f) t=3s g) t=4.6s h) t=6s

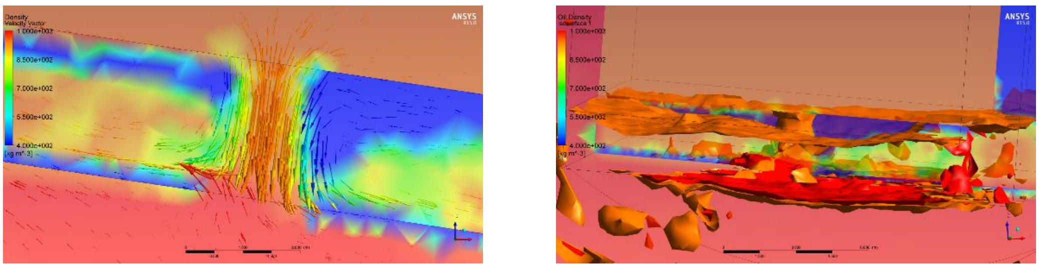


i) t=24s j) t=42s k) t=54s l) t=78s

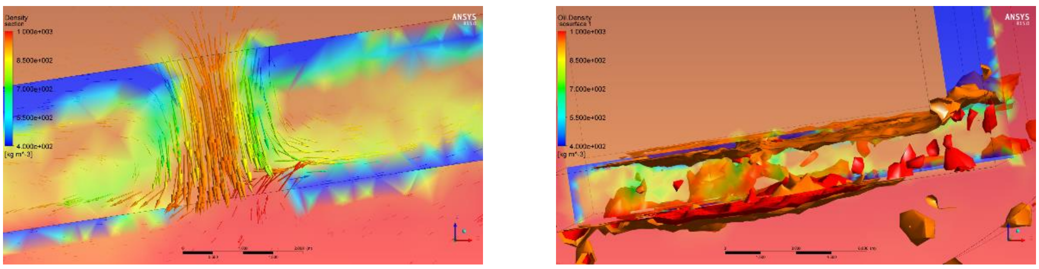
Fig.15 Characteristics of free surface in gravity flow stage




a) t=4.6s



b) t=42.9989s



c) t=45.9989s

Fig.27 the velocity vectors and phase interface in gravity flow stage

After the transient stage, the ballast tank starts being filled by the oil/water mixture. Snapshots corresponding to Fig. 13 and Fig. 14 are shown in Fig. 15 and Fig. 16, respectively for other time steps. Since the horizontal cross-section of the ballast tank near the bottom is approximately the same as the cargo tank, but the liquid height in this stage is small than the thickness of the ballast tank, the liquid inside the cargo tank is subjected to a violent liquid sloshing as demonstrated in Fig. 15. Oil bubbles entrapped by water and mixtures of oil-air-water are captured in the ballast tank (Fig. 16). Due to the block of the water outside the external hole, the velocity and so the momentum of the oil jet significantly reduced near the external hole. Therefore, water outside the tank may also move into the ballast tank. As a result, near the external hole, a convective oil and water flow is expected. Such convective flow may be significantly influenced by the tank motion. Larger motion amplitudes may amplify the convection and yields a greater volume of the water flowing into the ballast tank.

In the latter stage of gravity flow stage (t>24s), the bottom space of the ballast tank is full filled by the oil/water mixture, consequently, the liquid surface inside the ballast tank become significant smaller. The liquid sloshing in the ballast tank becomes insignificant. Simultaneously, the oil leakage from the cargo tank into the ballast tank no longer behaves as an oil jet but subjected to more significant viscous shearing force from the surrounding fluid. The energy carried by the spilled oil from the cargo tank quickly consumed by the viscous effects, reducing the amount of the oil leaked into the external sea. During this period, the velocity and pressure of the fluid near the external hole may suffer from fluctuating caused by the liquid sloshing in the cargo tank.

During this stage, for a specific motion amplitude (A=10°), a larger period typically leads to a smaller volume of water inflow and volume of the crude oil accumulated in ballast tank, as shown in Fig. 17 and Fig. 18. A larger motion period seems to produce a less momentum loss of the crude oil jet, which can prevent water outside the tank flowing into ballast tank at 1<t<40s. The crude oil in ballast tank almost increases linearly and water accumulates nonlinearly in ballast tank during this period.

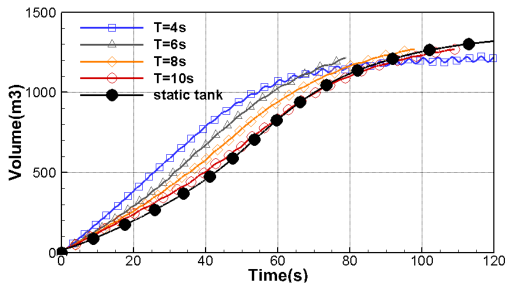



Fig.17 Time histories of volume of crude oil in ballast tank period (A=10°)

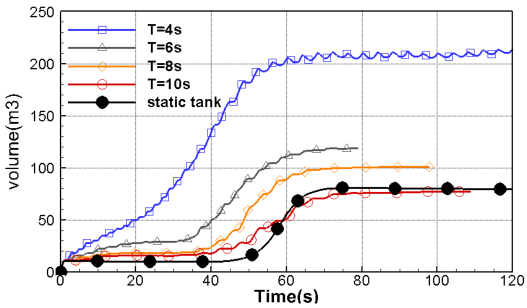



Fig.18 Time histories of volume of water in ballast tank (A=10°)

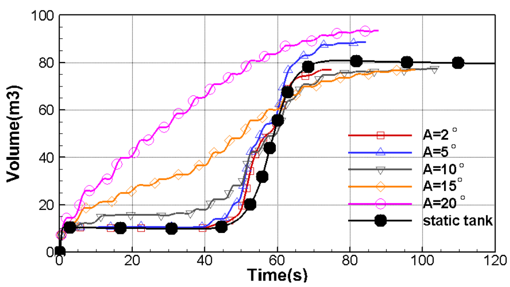



Fig.19 Time histories of volume of water in ballast tank (T=10s)

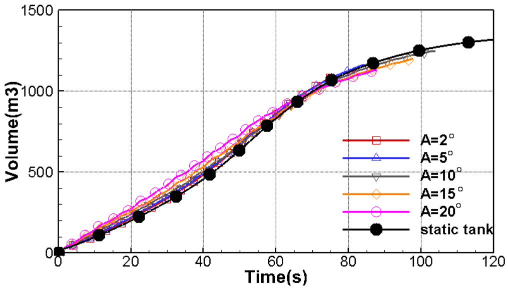
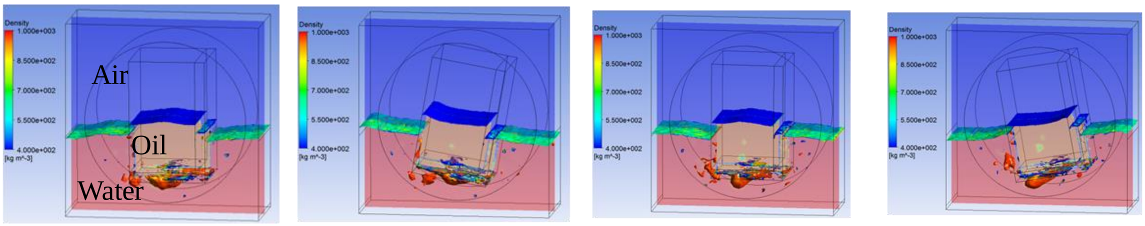


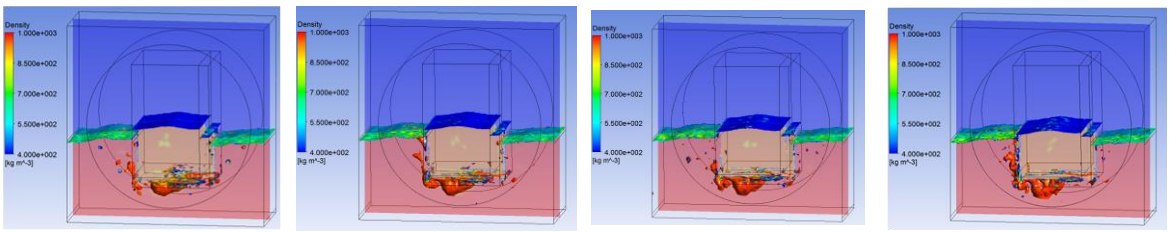

Fig.20 Time histories of volume of crude oil in ballast tank (T=10s)

For a specific motion period (T=10s), a larger motion amplitude leads to a greater volume of water inflow as shown in Fig. 19; however, the volume of the accumulated crude oil in ballast tank seems to be not sensitive to the motion amplitude as demonstrated by Fig.20. It is also interested to find that, when the motion amplitude is small, i.e. A<5°, at 1s<t<40s, the volume of water inflow into the ballast tank is almost constant partially due to the fact that the crude oil jet in such condition has more insignificant momentum loss, preventing water flowing into the ballast tank as shown in Fig.19.

2） Dynamic characteristics of transition stage

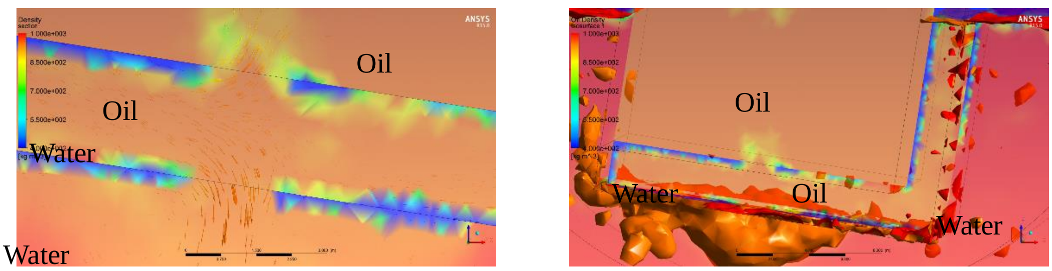



a) t=96s　　b) t=97.6s　　c) t=99s　　d) t=100.6s

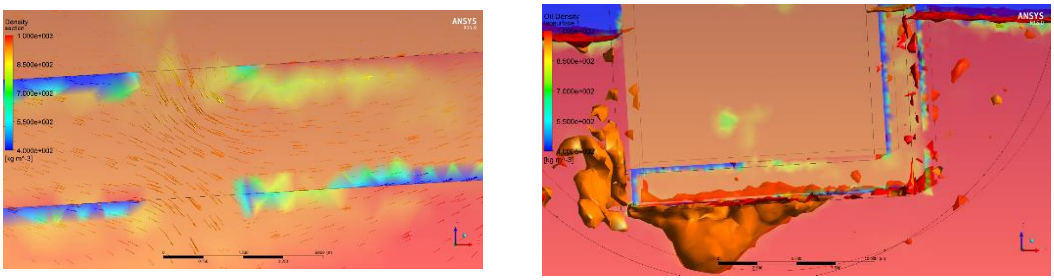


e) t=102s　　f) t=126s　　g) t=150s　　h) t=186s

Fig.21 Characteristics of free surface in transition stage




a) t=151.996s



b)  t=153.395s

Fig.22 The velocity vectors and phase interface in transient stage

Fig. 21 and Fig. 22 illustrate the flow field and the phase distributions at different instants in the transition stage corresponding to Fig. 13 and Fig. 14. In the transition stage, the height of crude oil in cargo tank is decreased considerably and the height of crude oil/water mixture in the ballast tank is increased dramatically. However, the head difference between the oil surface at the cargo tank ant that at the ballast tank at the hole continues driving the crude oil move into the ballast tank and leak into the sea. The total head of crude oil-water mixture in the ballast tank may be larger than that of the water outside the tank, and therefore, the mixture in the bottom space of the ballast tank is driven to flow into the sea by due to effects of liquid sloshing in the ballast space and the potential head difference. The overall effect causes the reduction of the water in the ballast tank. But, since the crude oil continues leaking from the cargo tank into the ballast tank, the volume of crude oil accumulated in the ballast tank may be greater than that lost from the ballast tank. Overall, the liquid height in ballast tank increased slowly.

For a specific motion amplitude (A=10°), a lower motion period does not only excite a larger sloshing energy, resulting in a greater fluctuating pressure, which make the crude oil/water mixture in the ballast tank flow into the sea quickly; it may also lead to a less duration time of the crude oil-water outflow and the less volume of the crude oil in the ballast tank at this stage as evidenced by Fig.23 and Fig.24, which compares the time histories of the volume of water and crude oil in the ballast tank during the transition period.

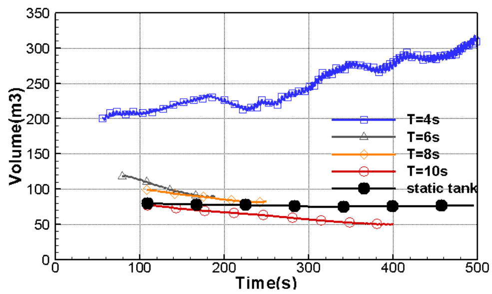



Fig.23 Time histories of volume of water in ballast tank (A=10°)

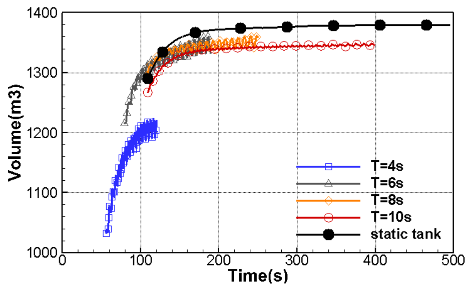



Fig.24 Time histories of volume of crude oil in ballast tank（A=10°）

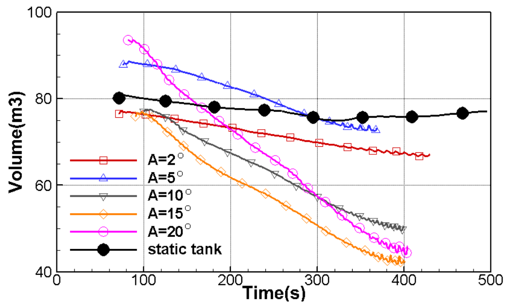



Fig.25 Time histories of volume of water in ballast tank (T=10s)

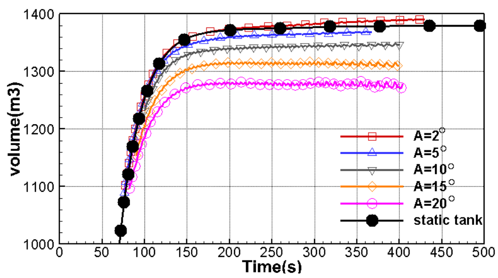
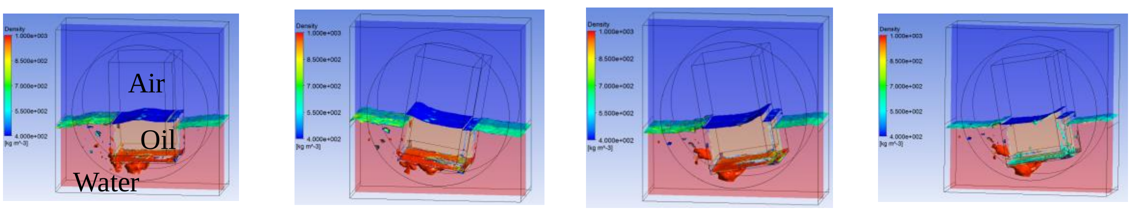



Fig.26 Time histories of volume of crude oil in ballast tank (T=10s)

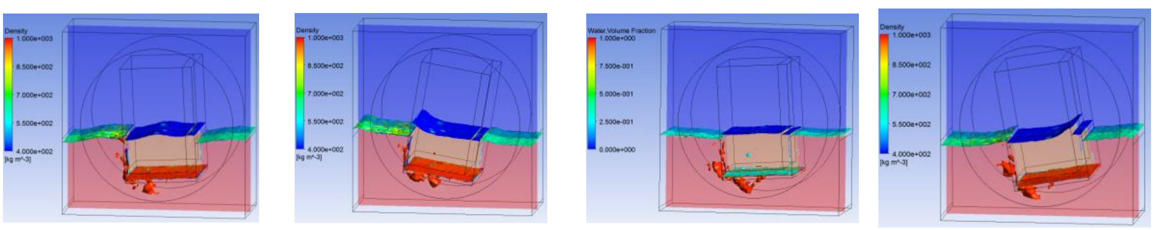
For a specific motion period (T=10s), a lager motion amplitude leads to a larger sloshing energy and fluctuating pressure, yielding a greater rate of water outflow and a less rate of crude oil accumulated in the ballast tank as illustrated in Fig.25 and 26. Furthermore, the duration, in which the volume of the crude oil in the ballast tank become steady, is not sensitive to the motion amplitude. In the latter stage, the volume of crude oil in ballast tank increases slowly and tends to be constant gradually ( Fig.26), but the water continues being lost from the ballast tank (Fig. 26).

3） Dynamic characteristics in sloshing flow stage

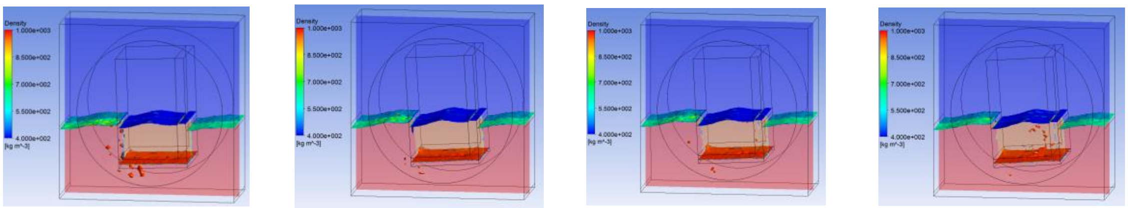
During the sloshing flow stage (i.e. t>360s for the cases shown in Fig. 13 and Fig. 27, the corresponding snapshots have been illustrated in Fig. 27 and Fig. 28), the volume of crude oil in the ballast tank has been decreased dramatically and the height of crude oil/water mixture in the ballast tank has reached its quasi-steady value. The potential head difference nearly disappears.




a) t=360s　b) t=361.84s　c) t=363.04s　d) t=364.64s

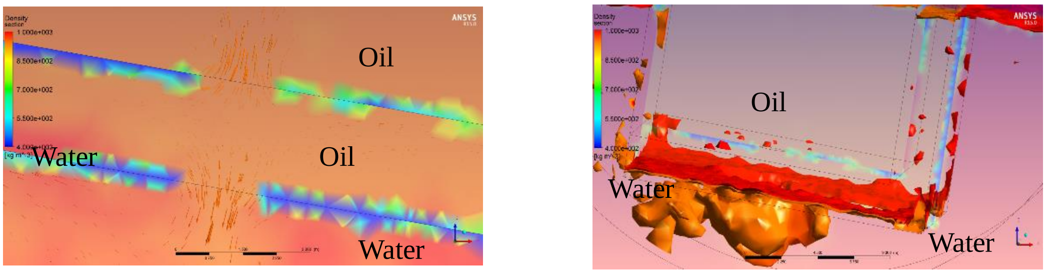


e) t=540.016s　f) t=541.617s　g) t=543.019s　h) t=544.62s

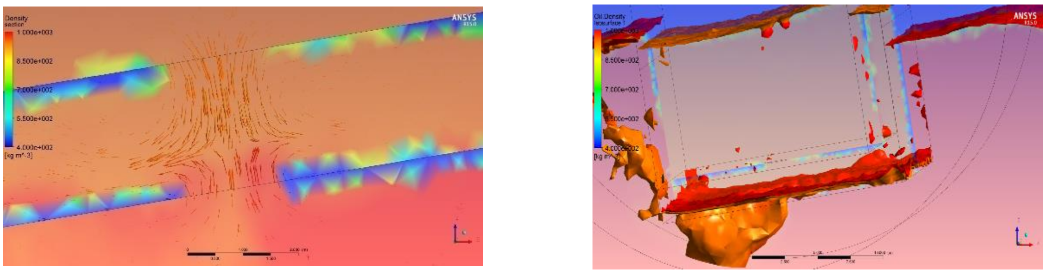


i) t=702.17s j) t=798.06s k) t=900.167s l) t=1062.13s

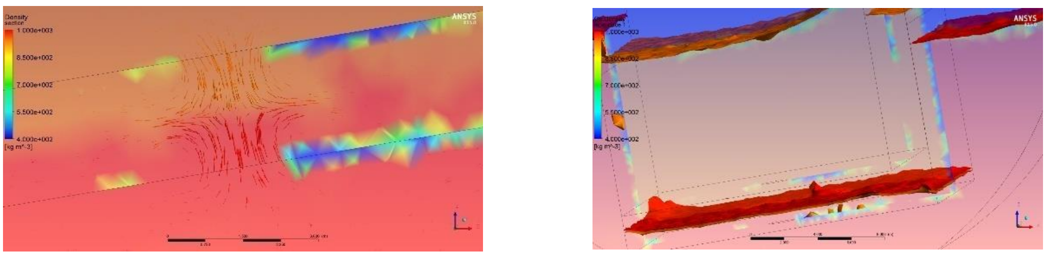
Fig.27 Characteristics of free surface in the sloshing flow stage.



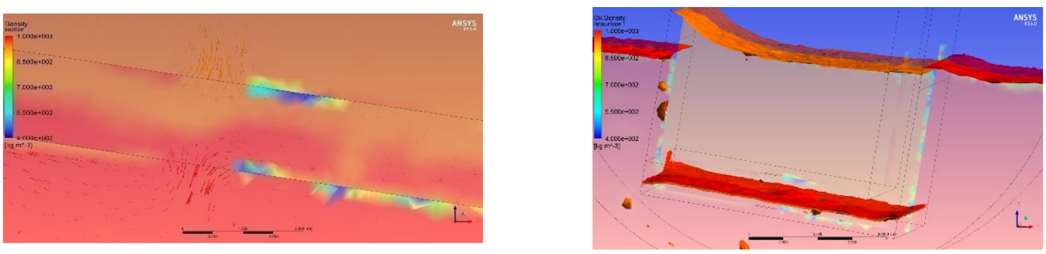

a) t=361.642s



b) t=365.045s



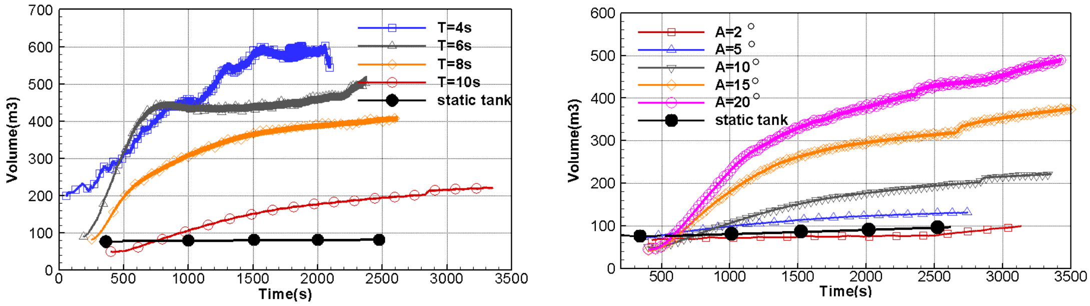
c) t=809.078s



d) t=811.681s

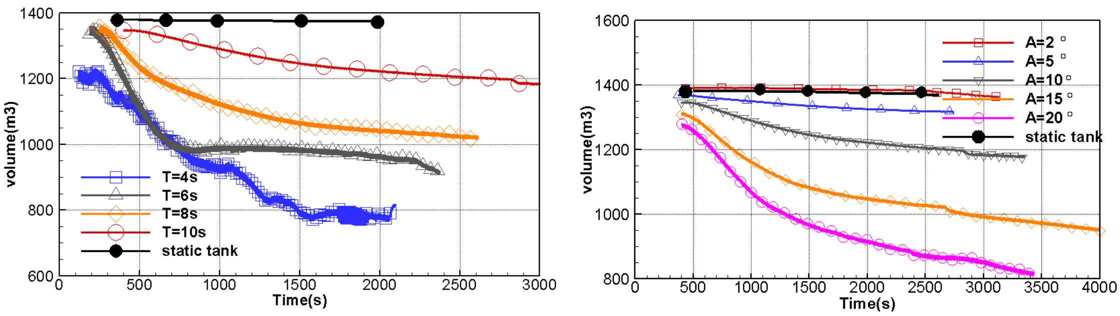
Fig.28 The velocity vectors and phase interface in sloshing flow stage

However, the velocity head difference fluctuates by the liquid sloshing, yielding a periodic fluid exchange between the ballast tank and the cargo tank through the internal hole. When the internal hole is submerged by oil in the ballast tank, the fluid exchange at through the internal hole does not contribute to the loss of the crude oil in the cargo tank. Nevertheless, near the bottom space of the ballast tank, the tank motion and so the liquid sloshing in the ballast tank breaks the potential head equilibrium near the external hole. A larger fluctuating velocity head in the surface of the ballast tank (and the oil surface in the cargo tank) can drive the liquid in the ballast tank to flow into the sea. The effect becomes more significant when the liquid sloshing in the tank become more significant. Nevertheless, the gravity effect may make the majority of the water (higher density than the oil) distribute on the bottom of the ballast tank. Consequently, the external hole is occupied by the water and the internal hole is occupied by the crude oil. This implies that the volume of the crude oil in cargo tank and volume of crude oil-water in ballast tank achieve a quasi-steady value. These are demonstrated by Figs. 29-30.




a) A=10° b) T=10s

Fig.29 Time histories of volume of water in ballast tank

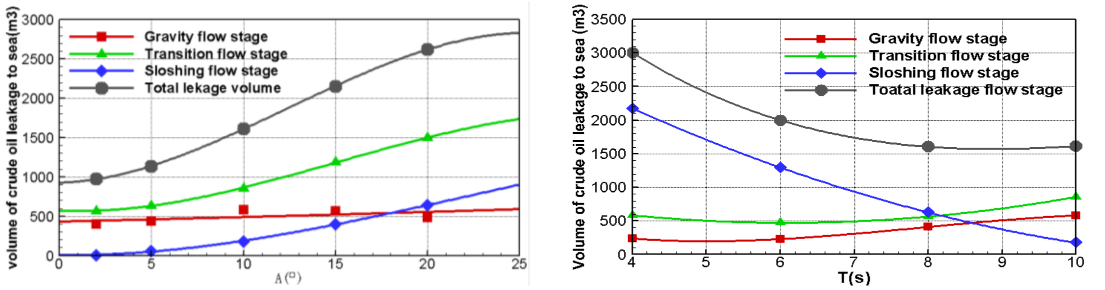



a) A=10° b) T=10s

Fig.30 Time histories of volume of crude oil in ballast tank

4.4 **Effect of period and amplitude of tank motion on submerged oil leakage**

In engineering practices, the volume of the crude oil leakage is the most important factor to be considered. These have been taken into account in the previous research with fixed oil tankers (Lu et al., 2014.2015, 2016). Herein, the effects of the tank motions on the ultimate volume of the oil leakage into the sea is investigated. This is taken as the ultimate volume of the oil leakage. The corresponding data are compared in Fig. 31.




a) T=10s b) A=10°

Fig.31 Effect of tank sloshing factors on volume of crude oil leakage to sea

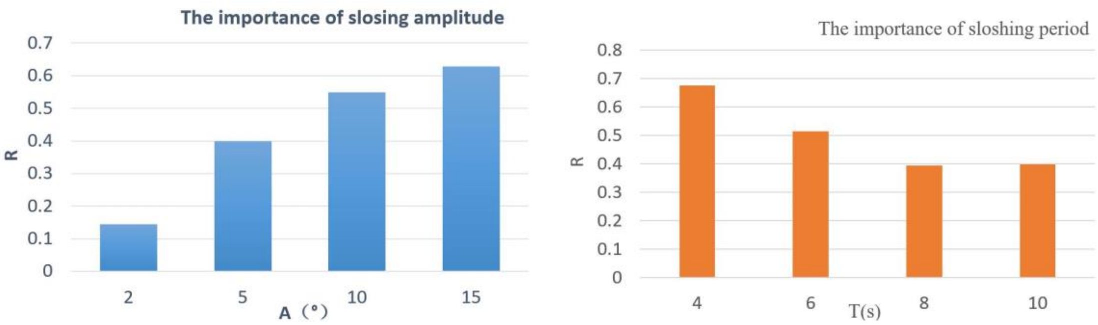



Fig.32 The importance of tank sloshing in the process of crude oil leakage to sea

For the convenience of exploring the importance of the tank motion in the oil leakage, the corresponding volumes of the oil leakages in three stages are also duplicated and plotted in Fig. 31. It is understood that the tank motion may be significantly affect the volume of the oil in the 2nd and 3rd stages where the effects of the velocity head in Eq. (10) brought by the tank motions are significant. As confirmed in Fig. 31 that the volumes of the oil leakage in the gravity flow stage are not sensitive to the motion of the tank. For specific motion period, (T=10s), the volumes of the oil leakage in the 2nd and 3rd stages, as well as the overall volume, increase as the motion amplitude increases, attributing to the fact that larger motion amplitude leads to more violent liquid sloshing in the tank (and so higher velocity head or energy in Eq. (10)). For a specific motion amplitude ( A=10°), the motion period seems to play less important role on the volume of the oil leakage during the transition and the gravity flow stages; however, the volume of the leaked oil in the sloshing stage decrease as the increase of the motion period. This may be due to the fact that as the natural period of the cargo tank is close to or shorter than 4s, as the motion period increases from 4s to 10s, the liquid sloshing in the tank become less violent.

According to above analysis, one may agree that the ratio R of the volume of the oil leakage in sloshing stage against the total oil leakage can be used to reflect the effects of the tank motion on the oil leakage. The corresponding results are shown in Fig. 32. As confirmed that the more severe sloshing (larger amplitude or periods closer to the natural period of the tank) can amplify the oil leakage.

## 5. **Conclusions**

The 3D numerical model is established to numerically model the crude oil leakage from damaged tanks subjected to pre-described motion. The effect of tank motion on submerged crude oil leakage is numerically investigated. In this model, the VOF method, dynamic mesh technique and the k-ε turbulence model are applied. The present model can lead to satisfactory results based on the numerical validation by comparing its predictions with exiting experimental data. After being validated, the model is applied to investigate the oil leakage from moving tanks with different motion conditions. It is concluded that the ultimate volume of the oil leakage into the sea is dramatically influenced by the motion of the tank. Typically, a larger motion amplitude and a period closer to the natural period of the tank result in more significant oil leakage.

By analyzing the numerical results obtained in this paper, a quasi-steady Bernoulli's equation is developed and utilized to analyze the dominate factors driving the oil leakage into the sea. This is the first semi-analytical model to describe the oil leakage from an oil tanker subjected to a pre-described motion. Although the most important benefit of this semi-analytical model is to directly predict the oil leakage, it is only used to assist the analysis of the numerical results in this paper. A further work will be done soon to calibrate the coefficients in the model leading to a practical tool. By using this model, the entire leakage process is suggested to be divided into the gravity flow stage, the transition stage and the sloshing stage. The latter two stages only exist in the cases with moving tanks, whereas the gravity flow stage is dominated by the potential head differences at the oil surface in the cargo tank, the liquid surface in the ballast tank and the water surface outside of the tank. The characteristics of the flow, including macroscopic values, e.g. the volume of the fluids, and the microscopic parameters, e.g. the fluid velocity and phase distributions, are explored using the numerical results at different stages. These conform to the analysis by using the semi-analytical model.

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## **References**

Akyildız H., Ünal, N.E., 2006. Sloshing in a three-dimensional rectangular tank: numerical simulation and experimental validation. Ocean Eng. 33(16), 2135-2149.

Chang, P.A., Lin, C.W., 1994. Hydrodynamic analysis of oil outflow from double hull tankers (CD-ROM). NAVAL SURFACE WARFARE

CENTER CARDEROCK DIV BETHESDA MD.

Cheng, L.Y., Gomes, D.V., Nishimoto, K., 2010. A numerical study on oil leakage and damaged stability of oil carrier. In: ASME 29th International Conference on Ocean, Offshore and Arctic Engineering. American Society of Mechanical Engineers. 829-836.

Dodge, F.T., Bowles, E.B., 1982.Draining of tanks with submerged outlets or without vacuum-relief. Journal of Fluids Engineering.104 (1), 67-71.

Debra, S.B., William, J.L., Jeffrey, L., 2001. Leaking tank experiments for heavy oils. J 2001 International oil spill conference .127-131.

Elahi, R., Passandideh, M., Javanshir, A., 2015.Simulation of liquid sloshing in 2D containers using the volume of fluid method. Ocean Engineering.96, 226-244.

Fthenakis, V.M., Rohatgi, U.S., 1999.A model of liquid releases from a submerged vessel. Journal of Loss prevention in the Process industries. 12(5), 437-449.

Fay, J.A., 2003. Model of spills and fires from LNG and oil tankers. Journal of Hazardous Materials.96 (2), 171-188.

Hu, C., Yang, K., K., Kim, Y., 2010. 3-D numerical simulations of violent sloshing by CIP-based method. Journal of Hydrodynamics, Ser. B. 22(5), 253-258.

Kolaei, A., Rakheja, S., Richard, M.J., 2015. Three-dimensional dynamic liquid slosh in partially-filled horizontal tanks subject to simultaneous longitudinal and lateral excitations. European Journal of Mechanics-B/Fluids.53, 251-263.

Kim, Y., Nam, B.W., Kim, D.W., Kim, Y.S., 2007.Study on coupling effects of ship motion and sloshing. Ocean Engineering. 34(16), 2176-2187.

Lin, T.J., Guan, Z.Q., Chang, J.H., Lo, S.H., and 2014.Vertex-Ball Spring Smoothing: an efficient method for unstructured dynamic hybrid meshes. Computers & Structures.136, 24-33.

Lu, J.S., Gong, X.W., Yan, S.Q., Wen, X.F., Liu, Z.C., Wu W.Q., 2010. Experimental and numerical study on leakage of underwater hole on an oil tanker. In: The Twentieth International Offshore and Polar Engineering Conference. International Society of Offshore and Polar Engineers.

Lu, J.S., Liu, F.C., Zhu, Z.Y., 2014. Effects of initial water layer thickness on oil leakage from damaged DHTs. In: the Twenty-fourth International Ocean and Polar Engineering Conference. International Society of Offshore and Polar Engineers.

Lu, J.S., Yang, Z.B., Wu, H.X., Wu, W.Q., 2015.Viscous effects for submerged oil leakage from damaged tank of Double-Hull Tankers. In: Proceeding of the Twenty-fifth International Offshore and Polar Engineering Conference. 1047-1053.

Lu, J.S., Yang, Z.B., Wu, H.X., 2016. Model experiment on the dynamic process of oil leakage from the double hull tanker. Journal of Loss Prevention in the Process Industries.43, 174-180.

Liu, D, Lin, P., 2008. A numerical study of three-dimensional liquid sloshing in tanks. Journal of Computational Physics. 227(8), 3921-3939.

Lee, S.H., Lee, Y.G., Jeong, K.L., 2011. Numerical simulation of three-dimensional sloshing phenomena using a finite difference method with marker-density scheme. Ocean Engineering.38 (1), 206-225.

Li, J.G., Hamamoto, Y., Liu, Y., Zhang, X., 2014.Sloshing impact simulation with material point method and its experimental validations. Computers & Fluids. 103, 86-99.

Lin, G., Liu, J., Li, J., Hu, Z., 2015. A scaled boundary finite element approach for sloshing analysis of liquid storage tanks. Engineering Analysis with Boundary Elements.56, 70-80.

Panigrahy, P.K., Saha, U.K., Maity, D.,2009.Experimental studies on sloshing behavior due to horizontal movement of liquids in baffled tanks. Ocean Engineering. 36(3), 213-222.

Tavakoli, M.T., Amdahl, J., Ashrafian, A., Leira, B.J., 2008. Analytical predictions of oil spill from grounded cargo tankers. In: Proceedings of 27th International Conference on Offshore Mechanics and Arctic Engineering. 911-920.

Tavakoli, M.T., Amdahl, J., Shrafian, A., Leira, B.J., 2009. Investigation of interaction between Oil spills and hydrostatic changes. In: Proceedings of 28th International Conference on Offshore Mechanics and Arctic Engineering. 803-811.

Tavakoli, M.T., Amdahl, J., Shrafian, A., Leira, B.J., 2010.Analytical and numerical modeling of oil spill from side damaged tanker. In: 5th International Conference on collision and grounding of ships. 88-96.

Tavakoli, M.T., Amdahl, J., Shrafian, A., Leira, B.J., 2011.Experimental investigation of oil leakage from damaged ships due to collision and grounding. Ocean Engineering.38, 17-18.

Tavakoli, M.T., Amdahl, J., Leira, B.J., 2012.Analytical and numerical modeling of oil spill from a side tank with collision damage. Ship and Offshore Structures. 7, 73-86.

Turner, M.R., Ardakani, H.A., Bridges, T.J., 2015. Instability of sloshing motion in a vessel undergoing pivoted oscillations. Journal of Fluids and Structures.52, 166-180.

Vaziri, N., Chern, M.J., Borthwick, A.G.L., 2015. Effects of base aspect ratio on transient resonant fluid sloshing in a rectangular tank: A numerical study. Ocean Engineering.105, 112-124.

YAMAGUCHI, K., YAMANOUCHI, H., 1992. Oil spills from the double hull model tanks. The Report of Ship Technology Research Station. 29, 1-38.

Yang, H., Lu,J.S., Yan, S.Q., 2014. Preliminary numerical study on oil spilling from a DHT. In: the Twenty-fourth International Ocean and Polar Engineering Conference. International Society of Offshore and Polar Engineers, Busan, Korea.

Yang, H., Yan, S., Ma, Q., Lu, J. and Zhou, Y., 2017. Turbulence modelling and role of compressibility on oil spilling from a damaged double hull tank. International Journal for Numerical Methods in Fluids, 83(11), pp. 841–865

Yang H., Yan, S., Ma, Q.W., 2016, Effects of tank motions on oil spilling from damaged oil tanker, in 26$^{th}$ International Ocean and Polar Engineering Conference, Rhodes, Greece.

Zhang, Y., Wan, D., Takanori, H., 2014. Comparative study of MPS method and level-set method for sloshing flows. Journal of Hydrodynamics, Ser. B. 26(4), 577-585.

Zhao, W., Yang, J., Hu, Z., Tao, L., 2014.Coupled analysis of nonlinear sloshing and ship motions. Applied Ocean Research. 47, 85-97.