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Rotorcraft Fuselage/Main Rotor Coupling Dynamics Modelling and Analysis

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Abstract. This work considers the fuselage/main rotor coupling nonlinear dynamics under a modal analysis study. The authors provide an advanced application and study of a simulation model that captures the complex fuselage/main rotor dynamics and validate it with previous existing theories. The model has been built up by using VehicleSim, software specialized in modelling mechanical systems composed by rigid bodies. The rotors are articulated, the main rotor considers flap, lag and feather degrees of freedom for each of the equispaced blades and their dynamic couplings. Once the validation is made, the main rotor angular speed is defined as time varying function such that heterodyning behaviour can be detected in the fuselage. The detection of this behaviour is not a simple task and this helicopter model provides an accurate system for its analysis using a short time Fourier transform processing.

Keywords. Fuselage, Rotor, Modal Analysis, Heterodyning.

1 Introduction

The helicopter structure is undergoes to dynamic loads from aerodynamic forces and vibrations induced by the engine and the rotors. As modifications are made to a helicopter, these can be critical in the structural behaviour of the rotocraft. The tests are fundamental to validate and to improve the design, and these should provide information about internal performance under certain conditions. The corresponding results are needed to assess the impact and to reduce the maintenance time between flights [1].

The vibration measurement of a helicopter is significantly relevant for structural inspection and performance evaluation. Overall, the conventional modal analysis via vibration measurement uses transducer as contact accelerometer to measure dynamic response and inertia shakers to excite the structure, among others. However, the traditional measurement system presents several practical limitations as (a) mass-loading effect, due to the transducer mass interferes with the real response of structure. (b) Difficulty to select an appropriate artificial exciter as in the case of a complex structure and (c) low spatial information, as a contact transducer is able to cover the response only from one measurement point [2].

The proneness of the systems to vibrations is a design issue in mechanics, especially in aeronautic, heavy equipment industries, etc. The experimental verification of design values is essential to model the validation, in order to assure the safety and the serviceability of the structures. In particular, modal parameters such as eigenfrequencies and mode shapes among others, ease these tasks. This can be also employed for structural health monitoring purposes and quality control. The boundary conditions and damping depend on the vibration amplitude, and if the structure moves as a whole or contains rotating parts, the corresponding modal parameters are determined by the rotation speed of the parts, as a helicopter [3]. Zaharia [4] derived the characteristic frequencies for a helicopter carbon fiber blade to study the occurrence of the resonance phenomena. The modal analysis allowed an accurate studio of the behaviour of the blade taking into account aspects as the geometry of the blade, the coupling between the blade and the rotor hub as well as the blade material. The results of the modal analysis of the blade showed that their frequencies were superior to those functional of a blade. Garinis et al. [5] studied a modified Gazelle helicopter blade using modal analysis. A new modelling approach was presented for obtaining natural frequencies and normal modes of rotor blade. Several methods of eigenvalue extraction were analysed to find optimal a method, which was able to be used in dynamic analysis of composite structures containing honeycomb cores. It was found that the most effective method for matrix decomposition and eigenvalue extraction was Lanczos method. Nour et al. [6] simulated using finite elements method the behaviour of a blade of different materials under an aerodynamic load. This work was carried out to assess the aerodynamic loads applied and it was evaluated by a numerical simulation the frequencies and the eigenmodes.

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The stresses acting on the structure for different modes were also calculated. As a result, the vibration responses were obtained for different excitation modes and unbalance.

Simulation models as well as their development and validation processes play a main role in the helicopter research field. Different simulation techniques and software packages have been used in the modelling process, see [7]. Manimala et al. [8] derived and validated a simulation model of the Bell-412 in the FLIGHTLAB software for handling qualities and flight control research. The following main steps were carried out, rotor hub design, modelling of the hub and blade retention, blade aerodynamics description, rotor inflow, tail rotor and fuselage modelling, and finally model validation. Shevtsov et al. [9] used Simulink/SimMechanics/Matlab environments to develop a scale five-bladed articulated rotor where each blade was taken into account as a solid body. The model considered important aspects such as the geometrical dimensions, mass, hub and blades inertia tensor, etc. The blades had flap, lag and feather hinges. Hover and forward flight simulations showed that even for a simplified aerodynamic environment of the blade air flow interaction, the helicopter rotor dynamic model allowed to determine the vibration sources and provided an estimation of the frequencies vibration levels introduced by the control actions on the flight modes. Qi-You et al. [10] have recently used the software MSC.ADMAS and MSC.NASTRAM to establish the dynamic analysis model of a coupled rotor/fuselage for vibration reduction studies. The equation of coupled system has been set up on the theory of multibody dynamics. The results of this work were compared with those of uncoupled modelling method, and these displayed that the dynamic features derived by the model of coupled rotor/fuselage were some different.

Over the past decades, helicopter simulations and vibration reduction have received the attention of a large scientific community. Rotor and fuselage interaction is an important issue when trying to study helicopter vibrations, in order to reduce the cost of the experimental tests and to derive more accurate outcomes. In Yeo and Chopra [11], vibration analysis of a coupled rotor/fuselage system was done using a detailed 3-D finite element model of the AH-1G airframe from the DAMVIBS program. The blade was taken to be an elastic beam undergoing flap bending, lag bending, axial deformation and elastic twist. For the coupled rotor/fuselage vibration analysis was considered the three-dimensional NASTRAN finite element models of the AH-1G helicopter. Airframe modal data were calculated using NASTRAN and used as input to the coupled rotor/fuselage vibration analysis program. Aerodynamic loads acting on the fuselage were not considered in the study. Parametric studies were carried out to test the prediction of vibration of a rotorcraft. The predicted vibration results were contrasted to load survey flight test data of the AH-1G helicopter. Modelling of difficult components such as engine, doors, panels, transmission etc., was necessary in predicting airframe natural frequencies and it was important for the accurate prediction of airframe natural frequencies. An estimation of these frequencies was required around the resonance, due to a small change of these ones could induce to large error in the calculation of the vibration. In order to figure out the dynamic instabilities of a rotorcraft coupled rotor/fuselage system, Guo-Cai et at. [12] determined the responses of the rotor blade flap and lag displacements as well as the fuselage motion using nonlinear differential equations. The time-domain methods and eigenanalyses were used to calculate dynamic instabilities. The results were contrasted with wind tunnel test data of reference and the corresponding outcomes displayed a good agreement between analytical and experimental data in the ground resonance cases. There are exist some nonlinear descriptions of helicopter dynamics, but only for reduced order models [13] or without taking into consideration the interaction of the main rotor with the fuselage [14].

It is fundamental to understand the internal performance of the helicopter to predict its dynamic response to external excitations. The addition and subtraction of frequencies are presented in aeronautical systems. In fact, the high frequency vibration has been behind the trigger of helicopter structural failure. As two signals of frequencies are combined, the result is the addition and the subtraction of frequencies. This is called sidebands and the deliberate used of this process is known as heterodyning [15]. On the other hand, the variable speed rotor is a research field for the rotorcraft operations improvement. Chen et al. [16] proposed an analytical expression for the varying angular velocity, the authors analysed blade's nonlinear vibrations under the action of this velocity. Thin-walled beams were considered in this study as they had been used in a broad range of fields including aerospace and mechanical industry. The model of rotating thin-walled beams could be used in the design of turbine blades, helicopter blades, etc. Vibration analysis plays a central role in reliability of rotating blades. Vibrations and harmonics generation have been tackled in existing literature, in fact, rotor slot harmonics are mostly utilized for condition monitoring. Keysan and Ertan [17] showed that appearing harmonics from a rotor slotting machine can be used for control or diagnostic purposes. This study illustrated a novel approach to the identification of rotor slot harmonics which consumed very little computation time.

The study of internal performance under the action of rotor varying angular speed can also be a convenient tool to find heterodyning behaviour on helicopters. The corresponding detection in a helicopter was done by Boetz [18], an experiment was carried out with a rotorcraft to quantify the performance of a multifunctional CO2 laser heterodyne detection system. In order to evaluate vibration signatures, Kranz [19] described a method to classify targets due to different types of these have different vibration spectra. The study was focused especially in helicopters. The signals showed the characteristic vibration signatures, which were evaluated by mathematical algorithms. Hence, the classification of targets was feasible. In addition to this, vibrometer systems are commercially available, the frequency of oscillation in heterodyne systems can be measured for instance, using two-frequency laser among other methods [20]. Siringoringo and Fujino [2] developed a model that represented the connection properties in terms of additional damping and stiffness. The relevance of this, for the structural damage detection was studied. The work was an extension of the previous study and it was formulated a modal-based damage detection method using laser Doppler vibrometer (LDV). Two types of LDV were used, the scanning laser (SLDV) and the reference laser (RLDV) according to the data acquisition procedure displays in Figure 1. The results showed that the presence of simulated damage in a steel connection was able to be detected by quantifying the additional stiffness and damping as well as by tracking the modal phase difference.

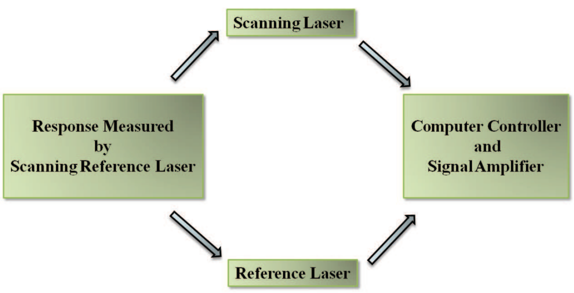

Figure 1. Data acquisition method [2].

In the context of the literature here mentioned, in this work, the dynamical coupling between the main rotor and the fuselage should be tackled as well as the main rotor varying angular speed according to the expression proposed by Chen et al. [16]. This one should be simulated with the objective of identifying heterodyning phenomena and the corresponding harmonics, because the frequency of the main rotor is a function of the angular speed at which it rotates. The vibration frequencies are equal to the main rotor rotating frequency or multiple of it and this is the frequency that the main rotor drives, in the fuselage [21]. In addition, according to Boetz [18] and Kranz [19], the vibration detection in a rotorcraft and the evaluation of their signatures are cumbersome tasks. In the same line, modal-based damage detection method has shown that the data acquisition procedure is also complex [2]. In view of these considerations, a new simulation tool should be implemented to ease the detection and the evaluation of internal performance as well as to shed light on data collection at development rotorcraft platform. The new approach should provide a suitable environment to simulate the helicopter model behaviour as well as to design experimental tests, using two tools such as the modal analysis and the heterodyne operation. In order to facilitate the operation the portability should also be tackled.

Taking this into consideration, it is undertaken the task to establish a first approach, which is able to attend the requirements previously determined. For this purpose, a multibody software specialized in modelling mechanical systems composed by rigid bodies is used, which derives a realistic nonlinear model of the fundamental dynamics [22]. The helicopter nonlinear behaviour and the nonlinear equations of motions are generated. The state-spaces matrices are also obtained and these are an advantage with respect other software package that are able to be found in the specialist literature. For example, VehicleSim has allowed to study the dynamic couplings existing on the blades through the appeared nonlinear terms in the equations of motion, being these the key to understand the energy exchange process between the flap and lag degrees of freedom, see [23].

As the portability is required, a single code should be built up, which should provide modal analysis results and to simulate heterodyning behaviour for a helicopter model. As a consequence, the rotorcraft model presents the following characteristics, the blades are rigid and the flexibility is not considered. Due to it is not required to do in a first approach of VehicleSim as a modelling tool in such case. The VehicleSim methodology and the rigid bodies are sufficient to test the VehicleSim features, this would require to use an external software, however, it is searching the implementation in a single code, without needing the action of external commands to VehicleSim. The state-space generated by VehicleSim can be a simulation alternative tool to the modal analysis proposed by other authors. In fact, operational modal analysis of the rotating helicopter blade is able to find a support tool, because of it can facilitate the studies carried out in this field, as for example the works presented by Zaharia [4] and Nour et al. [6]. On the other hand, the study of the modified performances in this helicopter model could be used to predict experimental heterodyning phenomena, they may be used to design control methods if that is required, and also for diagnostic purposes as the harmonics are associated to the distortion on the system [17].

The main contributions of this paper are: (a) To study the coupling between the fuselage and the main rotor by using modal analysis, this has been done using VehicleSim. It is a new approach implemented to search a simulation tool, which provides information relative to helicopter behaviour under expected conditions. Being this, an alternative to the works proposed by Shevtsov et al. [9] and Qi-You et al. [10]. (b) To set out and to discuss the VehicleSim main features as a modelling tool in the rotorcraft field. This is fundamental to understand the VehicleSim performance, if contributions such as Qi-You et al. [10], Verdult et al.[14], Guo-Cai et at. [12], Siringoringo and Fujino [2] can be complemented or extended. (c) To demonstrate the impact of the main rotor varying angular speed on the system. This is carried out to study of rotorcraft performance under complex mechanisms and finally (d) to analysis the heterodyning behaviour and harmonics on the fuselage, as a consequence of the main rotor varying angular speed. In order to assess the helicopter model capability for the detection as well as the evaluation of different behaviour. This could open a door to works as Boetz [18] and Kranz [19]. Additionally, future studies could be carried out, if a connection is established between the simulation and the experimental heterodyne detection. Nevertheless, the present work aims to be a test platform to the helicopters that require mechanical assesses during their corresponding procedure of manufacturing.

The outline of the paper is as follows; Section 2 presents the helicopter modal analysis. Section 3 provides a description of the modelling aspects carried out in this work and describes the modelling software used: VehicleSim. Section 4 studies the fuselage/main rotor coupling nonlinear dynamics under a modal analysis, and the heterodyning behaviour detected in the fuselage is also dealt with. Finally, Section 5 summarizes the main conclusions.

2 Helicopter Modal Analysis

Control of dynamic systems has been historically the first motive for developing system identification algorithms. The interaction between the different research fields of system identification and the modal testing was kept limited, as the main concern in modal testing was computational performance. Due to the increasing computational power, a structure is able to be tested in operational rather than in laboratory conditions. The modal analysis has been became an active field of research [3]. In the rotorcraft field is needed to analyse the vibration to predict the natural frequencies as well as the response to the expected excitation. These allow to assess dynamic properties of the system and to predict its behaviour [5].

2.1 Modal Analysis and Simulations

The structural dynamic problem of a helicopter involves three topics such as measurement of dynamic features of the system, calculation of vibrational excitation, formulation of helicopter structural dynamic model as well as estimation of its corresponding parameters [24]. As a consequence of this, the design process of helicopter structures needs an accurate analysis of their dynamic properties. The modal analysis is a computational stage for the spectral analysis and the harmonic or transitory analysis through effects overlapping. This is considered linear although the problem with vectors and associated values, that requires to be solved, involves numerical iterative methods [4]. The most important stages that are needed for the helicopter vibratory phenomena are provided in Figure 2. The difficulty degree changes for every single case, and this depends on the complexity of the mathematical model selected as well as the manner in which the stability solution is described.

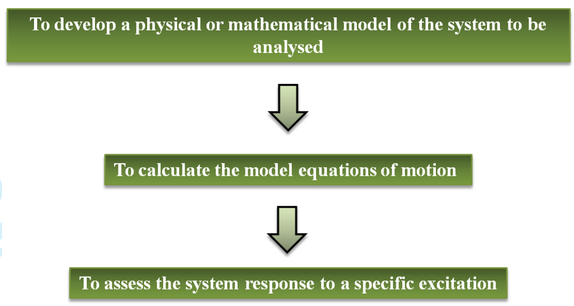

Figure 2. The phases of a internal performance analysis [5].

It has been observed that recent studies the helicopter internal phenomena raised significantly, due to the new specific software packages that ease the access to faster results from the development stage of the product. The main programs for finite element analysis and modal analysis are Hypermesh, Abaqus, Ansys, Ansa, Nastran. It should be noted that there is a special interface that accounts for all the software specific elements for each of these programs [4].

3 Modelling Aspects

The main goal of this section is to describe the modelling process for implementing the helicopter model. The rotating systems have been studied and analysed by formulations and software tools that consider the reference rotation motion of the system. Such applications are able to be efficient and effective, however these can present lack of generality. It has to be taken into account that the rotating systems are distinguished by the non negligible angular motion which they are subjected to.

3.1 Helicopter Model Description

The rotorcraft modelled consists of three subsystems: fuselage, main rotor and tail rotor, see Figure 3. A rotocraft is able to be modelled in different manners; one of them is as multibody system with combination of several interacting subsystems and constraints between the different degrees of freedom. For example, the coupling between the flap-feather degrees of freedom on the tail rotor can be carried out using the restrictions and constraints that VehicleSim allows to implement. The rotorcraft's main body presents six degrees of freedom. Three three rotations around the (X, Y, Z) axes and three translations along the same axes. The aerodynamic model is not considered in here because this work is focused to set up a type of pure varying angular speed on the main rotor to study its impact on the system.

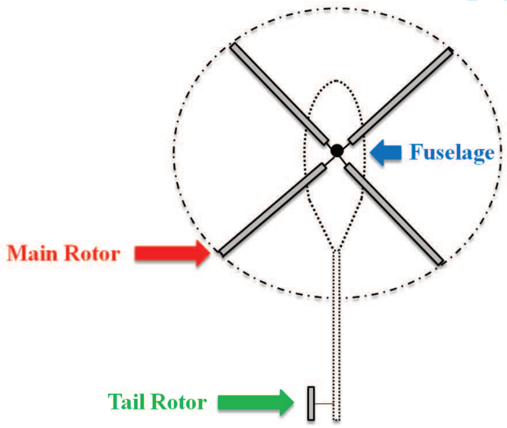

Figure 3. Helicopter model configuration.

As it can be seen in Figure 3, the rotorcraft is on conventional configuration i.e., the main rotor is in perpendicular to the tail rotor. The main and the tail rotor consist of four and two equally spaced blades, respectively. The first one was joined to a central hub and the second one was linked to a secondary shaft. The main rotor hinges have three degrees of freedom the flap, the lag and the feather, see Figure 4. The flap hinge that allows the blade to move in a plane containing the blade and the shaft about either the flap hinge. The flap hinge is more often designed to be a short distance from the centre line, this is called an "offset" (eR) [25, 26]. A blade that is free to flap is undergoing large Coriolis moments in the plane of rotation, a lag hinge is introduced to reduce these moments. This degree of freedom allows the blade motion to be parallel to the disc plane [27]. The blade is able to feather around an axis parallel to the blade span. Blade feather control is performed through a linkage of the blade to a swashplate [28]. The tail rotor hinges have two degrees of freedom the flap and the feather. Furthermore, the feather-flap coupling is also considered in this rotor. The blades are rigid in both rotors, the case of flexible blades is a natural extension of the work here presented. However, the authors objectives can be achieved without taking into account this. There is a proportional ratio between the rotors' angular speeds, which are constant. Some helicopter model parameters are displayed in Table 1.

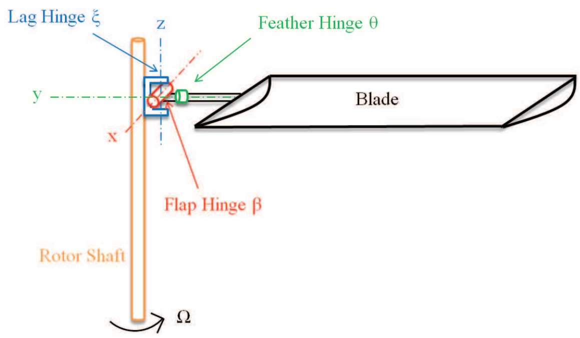

Figure 4. Schematic diagram of main rotor's hub and hinges system.

3.2 Heterodyning Performance Modelling

In order to validate the helicopter model, the heterodyning behaviour is studied using the main rotor's varying angular speed. This is implemented via the angular velocity defined by Chen et al. [16] (see Equation 1) and it is included in the main rotor model using a PD controller to regulate the speed, this is carried out according to time dependent.

$$\Omega(t) = \Omega_1 + \Omega_2 cos(\Omega_3 t) \tag{1}$$

Ω_1 is the main rotor angular speed. Ω_2 and Ω_3 are additional angular speeds. These are introduced with the objective of testing heterodyning behaviour appearing on the main rotor. Furthermore, it can provide information about other conditions on structure components of the system. Equation (1) allows to change the main rotor varying angular speed and its corresponding impact can be studied on the fuselage.

The controller gains have been tuned manually. VehicleSim enables from the browser to modify the numerical values of the parameters, if these have been established in symbolic form on the code. Thereby, the tuning is simplified due to every execution from the browser generates swiftly the simulation results, allowing to assess the response to the error between the corresponding actual rotor angular speed and the prescribed rotational speed. The analysis of this error and the user decision making are fundamental to obtain the needed calibration. VehicleSim does not produce by itself any controller' gains data. The user is who determines the method to follow, this is, if the program generates any information in this subject matter, it is because the user must set up the suitable work environment. In order to tune the numerical values of the controller' gains, external knowledge from other software platform or rotorcraft model has not been taken into account. The modelling process and the calibrations done by the authors, have led to a proper tuning of the controller as well as a stable response into VehicleSim and the corresponding gain numerical values have been figured out using this software package only.

3.3 Helicopter and Control

The helicopter trim control is a essential aspect that has to be dealt with particular attention in the rotorcraft model. The coordinates in the inertial frame are (x, y, z), Euler angles (θ, ϕ, ψ) (roll, pitch, yaw), linear velocities ($\dot{x}$, $\dot{y}$, $\dot{z}$) and angular velocities ($\dot{\theta}$, $\dot{\phi}$, $\dot{\psi}$) in the body coordinate frame. Several controllers on the corresponding translational and rotational degrees of freedom must be added. Thereby, the helicopter's translational position control will be restored and the angular position.

For stability aims, the error between the reference and the actual states must be measured at each time step of the simulation. The outputs of these controllers need to be applied between the fuselage and the inertial frame to derive a suitable action control. For example, in order to control the rotorcraft's position and displacement on the X axis, the longitudinal position prescription for the fuselage's position has to determine in order to represent this. It provides the reference value for the longitudinal position; it allows to describe the origin of the reference system in the space. The output of this controller represents an applied force at the fuselage's centre of mass. Thus, the demanded longitudinal position control is reached. Both lateral and vertical position controls were also modelled equally to the longitudinal position control. Furthermore, in order to control the helicopter's pitch degree of freedom, a PID controller is added. The input is the difference between the actual and desired fuselage's pitch position. A reference value is defined at the beginning of model script. The output is a torque applied between the fuselage and the inertial frame. Roll and yaw control are equally modelled. As it can be seen, the helicopter dynamic model is implemented using PID controllers, because their features allow to design a code in VehicleSim without needs additional software. In addition, this control approach is appropriate to achieve the authors objectives.

3.4 Rotorcraft Dynamic Modelling and VehicleSim

The helicopter model is implemented using VehicleSim multibody software [22]. This one consists of two separated tools as VehicleSim Lisp and VehicleSim Browser. The first is the tool used to derive the equations of motions from a multibody description of any dynamical system. It is able to be set up to generate the nonlinear equations of motion or the linearized equations of motion. VehicleSim commands are used to describe the components of a multibody system in a parent/child relationship in accordance with their joints and physical constraints. Both type of equations are symbolically reported as functions of all the parameters defining the model and a Matlab file of the linearised state-space model in symbolic form is also generated, see [23, 32]. Second, VehicleSim Browser provides a graphical interface from which the nonlinear simulations are able to be executed and the different databases are able handled.

In order to implement the helicopter model, this one is subdivided into its constituent bodies for the aim of building up the VehicleSim code [22]. The model is set up by adding bodies according to the structure shown in Figure 5. The first body to be added is the fuselage which is implemented as the child of the inertial frame. At the same time, the fuselage is the parent of the rotors. The corresponding fuselage translation and rotation degrees of freedom are enabled, as shown in Figure 5. The main rotor subsystem has been modelled in the same parent/child structure and it is implemented as the child of the fuselage, rotating around Z axis. The rotor is the parent of the flap hinges that rotates around X axis and this one is the parent of the lag hinges, which rotates around Z axis. The feather hinges are the child of the lag hinges and each blade is added to the program structure as the child of each feather hinge that rotates around Y axis. Similar procedure is done to model the tail rotor, taking into account the number of blades as well as their corresponding rotation axes. Additional data parameters such as masses, inertia matrices, coordinates of the centre of masses, the allowed translations and rotations of each body, which conform the dynamical system, are able to be described in a sequential manner along with the rigid bodies [31].

4 Results: Rotorcraft Model Performance

In this section is studied the fuselage's behaviour and its coupling with the main rotor and vice versa. In order to shed light on these performances, various simulations are done and modal analysis is used, due to the most software

Parameters	Magnitude	Units
Helicopter mass	2200	kg
Main rotor blade mass	31.06	kg
Tail rotor blade mass	6.21	kg
Fuselage-tail rotor longitudinal distance	6.00	m
Fuselage-main rotor vertical distance	1.48	m
Fuselage-tail rotor vertical distance	1.72	m
Main rotor blade length	4.91	m
Tail rotor blade length	0.98	m
Tail rotor gearing	5.25	–

Table 1. Rotorcraft model parameters [29, 30].

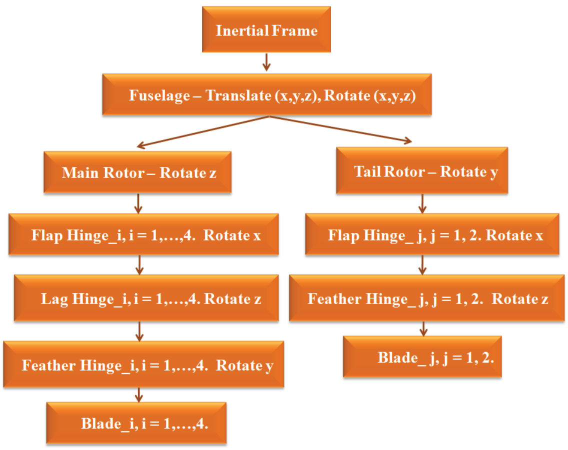

Figure 5. Main structure of the rotocraft model.

destined to solve problems in modern control, must ensure that the rotorcraft equations of motion are successfully assembled, before a solution is determined using these tools [14]. The estimation of the structural response and the modal frequencies are essential to design rotating components [33, 34]. There is a further consequence of the use of a rotating frame of reference that affects vibration frequencies created in the rotor and transmitted to the fuselage, the frequencies generated in the rotor may contain the rotational frequency of the rotor and the external perturbation frequency; both added and subtracted [15]. As a consequence, the heterodyning behaviour is studied on the rotorcraft model using short time Fourier transform.

4.1 Nonlinear Equations for Flap/Lag Coupled Motion

In a dynamic model, the Coriolis force acting on the blade for the flap motion corresponds to the term $2m\Omega\dot{\xi}$. In the case of the lag motion, Coriolis inertia forces are represented by the term $2B\Omega\dot{\beta}sin\beta$, where B is the moment of the inertia [27,28]. These terms can be identified in the nonlinear equations for the flap and lag coupled motion. Using the corresponding nonlinear set up of VehicleSim the complex expressions of both flap and lag degrees of freedom are obtained. The modelling capabilities of VehicleSim start becoming evident now, as obtaining these equations by hand could be a cumbersome process.

- Flap equation of motion

$$\begin{aligned}
&\ddot{\beta}\left(I_{blx}cos^2\xi + I_{bly}sin^2\xi + m_{bl}y_{bl}^2cos^2\xi\right) - 2m_{bl}sin\xi y_{bl}^2 \\
&cos\xi\dot{\beta}\dot{\xi} = -k_{fj}\beta cos\xi + \Omega cos^2\xi sin\beta\left(I_{bly} - I_{blz}\right) \\
&\left(\dot{\xi} + \Omega cos\beta\right) - \dot{\beta}cos\xi sin\xi\left(I_{bly} - I_{blz}\right)\left(\dot{\xi} + \Omega cos\beta\right) \\
&-I_{blx}cos^2\xi\Omega\dot{\xi}sin\beta + I_{blx}cos\xi\dot{\beta}sin\xi\left(\dot{\xi} + \Omega cos\beta\right) \\
&+\Omega sin\beta sin^2\xi\left(I_{blx} - I_{blz}\right)\left(\dot{\xi} + \Omega cos\beta\right) + \dot{\beta}cos\xi sin\xi \\
&\left(I_{blx} - I_{blz}\right)\left(\dot{\xi} + \Omega cos\beta\right) + I_{bly}sin\xi\Omega\dot{\beta}cos\beta cos\xi \\
&-I_{bly}\dot{\xi}sin\xi\dot{\beta}cos\xi - I_{bly}\dot{\xi}sin\xi\Omega sin\beta sin\xi - m_{bl}cos\xi y_{bl} \\
&eR\Omega^2 sin\beta - m_{bl}cos\beta\Omega^2cos^2\xi y_{bl}^2 sin\beta \\
&-\underbrace{2m_{bl}\Omega\dot{\xi}sin\beta y_{bl}^2cos^2\xi}_{(a)}
\end{aligned} \tag{2}$$

where β is the flap angle, ξ is the lag angle, m_{bl} is the main rotor blade mass, y_{bl} is the blade centre of mass, k_{fj} is the main rotor flap hinge spring stiffness, Ω is the main rotor angular speed, eR is the offset. I_{blx}, I_{bly}, I_{blz} are the blade moments of inertia around their corresponding axis.

- Lag equation of motion

$$\begin{aligned}&\ddot{\xi}\left(I_{blz}+m_{bl}y_{bl}^2\right)+m_{bl}sin\xi cos\xi y_{bl}^2\dot{\beta}^2=-k_{lj}\xi-d_{lj}\dot{\xi}\\&+I_{blx}\Omega\dot{\beta}cos^2\xi sin\beta-\left(I_{bly}-I_{blx}\right)sin\xi cos\xi\Omega^2 sin^2\beta\\&+\Omega\dot{\beta}\left(I_{blz}-I_{bly}cos^2\xi\right)sin\beta-\dot{\beta}sin\xi\left(I_{blx}-I_{bly}\right)\\&\left(\dot{\beta}cos\xi+\Omega sin\beta sin\xi\right)+m_{bl}sin\xi cos\xi y_{bl}^2\Omega^2 sin^2\beta\\&-m_{bl}y_{bl}sin\xi eR\Omega^2 cos\beta+m_{bl}\Omega sin\beta y_{bl}^2\dot{\beta}\\&-m_{bl}\Omega\dot{\beta}sin\beta y_{bl}^2 cos^2\xi+m_{bl}\Omega\dot{\beta}sin\beta y_{bl}^2 sin^2\xi\end{aligned} \tag{3}$$

k_{lj} is the main rotor lag hinge spring stiffness and d_{lj} main rotor lag hinge damping coefficient. The last three terms in equation (3), can be rearranged taking into account that $sin\xi=\sqrt{1-cos^2\xi}$ and this yields:

$$\begin{aligned}&m_{bl}\Omega sin\beta y_{bl}^2\dot{\beta}-m_{bl}\Omega\dot{\beta}sin\beta y_{bl}^2 cos^2\xi\\&+m_{bl}\Omega\dot{\beta}sin\beta y_{bl}^2 sin^2\xi=m_{bl}\Omega sin\beta y_{bl}^2\\&\dot{\beta}\left(1-cos^2\xi\right)+m_{bl}\Omega\dot{\beta}sin\beta y_{bl}^2 sin^2\xi\end{aligned} \tag{4}$$

And this is:

$$\begin{aligned}&m_{bl}\Omega sin\beta y_{bl}^2\dot{\beta}\left(1-cos^2\xi\right)+m_{bl}\Omega\dot{\beta}sin\beta y_{bl}^2 sin^2\xi=\\&2\Omega\dot{\beta}sin\beta m_{bl}y_{bl}^2 sin^2\xi\end{aligned} \tag{5}$$

which is the corresponding Coriolis term expected to appear in the lag motion equation.

The Coriolis terms have been mathematically described for flap and lag coupled motions as equations (2), (3) and (5) show. These equations have displayed to contain the terms that the theory predicts in terms of coupling of blade's flap and lag degrees of freedom, testing in this way, the correct and adequate blade motion modelling by using VehicleSim.

4.2 Fuselage Motion and Blade Flap Coupling

This subsection studies the coupling between the fuselage's motion and the blade flap dynamics, using modal analysis in order to search a simulation tool, which is able to show information about internal behaviour under expected conditions. As a consequence of this, if the rotor is rotating in the absence of a flap spring and in vacuum, the rotor remains aligned to the original position as there is no other mechanism to generate a turning moment on the blade. Otherwise, the model would show some static deflection on the flap hinge when a spring is added to it, the blade will develop a persistent oscillation about the shaft orientation with an inertial moment appearing due to out of the plane flap and the centrifugal moment being continually in balance, of course, this is assuming that the gravity force is not considered [29].

In order to study this, the fuselage is allowed vertical motion, the remaining of degrees of freedom are not considered in this case, and the rotor blades are mounted with flap and lag hinges. A simulation is carried out for the helicopter model operating at equilibrium conditions i.e., the main and tail rotors blades have initial flap angles $\beta_0=0$, thereby, the blades do not vary independently their displacements. In this manner, the rigid blades fluctuations are determined and the conditions of dynamic equilibrium are achieved. The main rotor angular speed is 44.4 rad/s, the tail rotor angular speed is 233.1 rad/s and gravity is not taken into account. Once the simulation conditions are defined, the nonlinear equations of motion are obtained in C, and these are used to generate the states' time histories from the nonlinear simulation carried out. The root locus diagram is derived by linearising the model, VehicleSim generates the symbolic state-space matrices A, B, C, D for linear analysis (in a Matlab form), see Figure 6. In order to obtain the eigenvalues/eigenvectors, the state's time histories are imported from the nonlinear model into the symbolic linearised model matrices at each timestep of the running interval. The result is shown as a root locus diagram in Figure 7 (a). Figure 7 (b) shows details of the root locus for the main rotor flap mode and coupled fuselage-main rotor flap mode, which are not shown clearly in Figure 7 (a). At it can be seen, this figure shows a well damped lag mode (d_{lj} = 349.58 Nms/rad) located into the left hand side of the plane, it is stable. The flap hinges do not have a damper in any rotor, therefore their corresponding flap modes follow a harmonic motion (they are located on the imaginary axis). These modes are marginally stable. The oscillation between the fuselage and the blades' flap motion on the main rotor is simulated for the case of four blades having initial flap angles $\beta_0 = 0.0175$ rad, flap spring, and gravity is not considered. The expected system behaviour is shown in Figure 8 (a), whilst the time history of the fuselage's and blade's centre of masses positions is shown in Figure 8 (b). Vertical oscillations appear on the fuselage as a consequence of the dynamic coupling with the blades' flap motion. The frequency of the oscillations that appear in the fuselage coincides with the blades' flap frequency, being the vertical displacement direction the opposite clearly, the system behaviour coincides with the expected one.

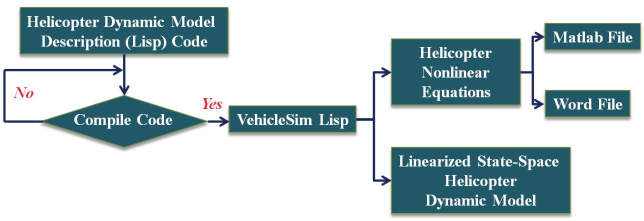

Figure 6. Flow chart of VehicleSim simulation method for the state-space.

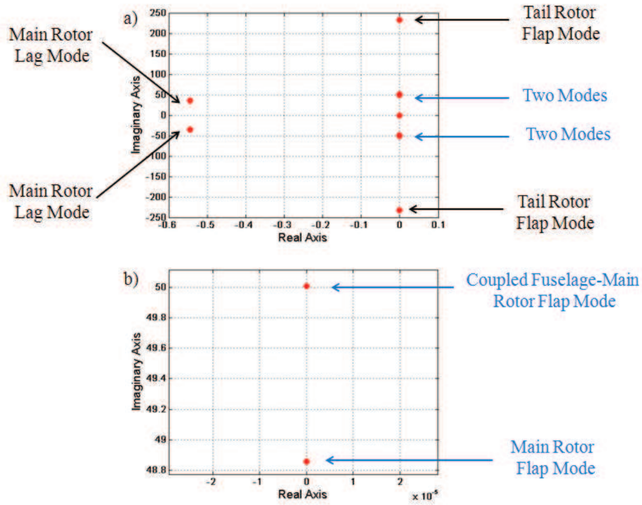

Figure 7. a) Root locus for the fuselage, main and tail rotors. b) Details of the root locus for flap modes.

4.2.1 Modal Analysis

Taking into account that the modal analysis is an active field of research, and due to it allows to evaluate dynamic properties of the system as well as to predict its behaviour. This section makes use of the linearised model obtained in VehicleSim and a modal analysis approach is done to study the fuselage's vertical motion and the blades' flap. In order to better understand the existing natural coupling between both subsystems a modal shape analysis is carried out.

It is well known that the eigenvalues λ, of a matrix A satisfy the equation:

$$(A - \lambda I)\,\phi = 0 \tag{6}$$

being ϕ an eigenvector. The solution of this algebraic equation is obtained by solving the characteristic equation:

$$det\,(A - \lambda I) = 0 \tag{7}$$

The corresponding eigenvectors, are found for each eigenvalue λ_i by:

$$A\phi_i = \lambda_i \phi_i \tag{8}$$

The eigenvectors components give a measure of the relative activity between the state variables and the eigenvalues when a particular mode is excited. In other words, the ith element of the eigenvector ϕ_i measures the activity of the state variable x_i in the ith eigenvalue [29,35].

The helicopter's model in this work has twenty four state variables, $q(t) = \{x_1, x_2, x_3, ..., x_{24}\}^T$, the first variable x_1 describes the fuselage's vertical translation, x_2 describes the main rotor rotation, x_3, x_5, x_7 and x_9

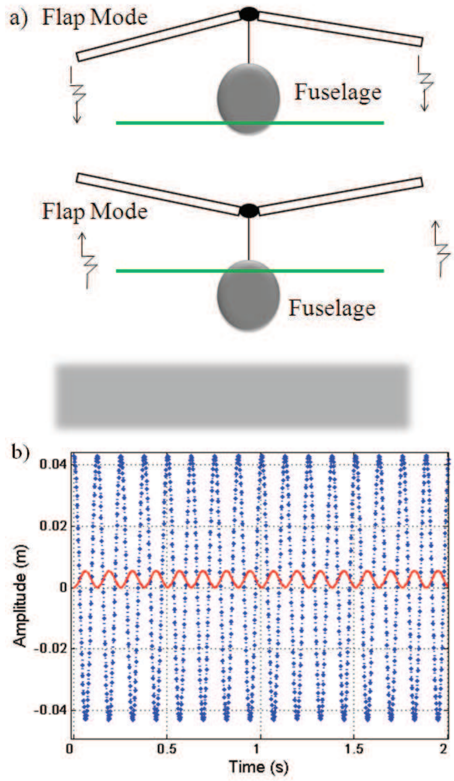

Figure 8. a) Expected fuselage/main rotor coupled vertical displacement. Green line, equilibrium position [29]. b) VehicleSim results, fuselage's centre of mass vertical displacement (solid red line). Blades' centre of mass (dotted blue).

describe the flap angles of the first, second, third and fourth main rotor blades, respectively. x_4, x_6, x_8 and x_{10} describe the lag angles of the first, second, third and fourth main rotor blades, respectively. x_{11} describes the tail rotor angular displacement, x_{12} and x_{13} describe the flap angles of the first and second tail rotor blades. x_{14} describes the fuselage vertical speed. x_{15}, x_{17}, x_{19} and x_{21} describe the flap angular speeds of the first, second, third and fourth main rotor blades. x_{16}, x_{18}, x_{20} and x_{22} describe the lag angular speed of the first, second, third and fourth main rotor blades. Finally, x_{23} and x_{24} describe the flap angular speeds for the first and second tail rotor blades. Table 2 shows the obtained eigenvalues associated to the root locus represented in Figure 7.

The transition matrix is obtained and each eigenvalue is associated to its corresponding eigenvector such that the various modes of oscillations can be identified. The interest in here is to study the coupled vertical dynamics between the fuselage and the main rotor, therefore, the mode under study would be the fuselage's vertical displacement and the main rotor blades' flap motions. The eigenvector associated to the eigenvalue λ_2 has been represented in Figure 9. It

Name	Eigenvalues
λ_1, λ_{13}	± 0.00
λ_2, λ_{14}	$0.00 \pm i50.01$
λ_3, λ_{15}	$0.00 \pm i48.86$
λ_4, λ_{16}	$0.00 \pm i48.86$
λ_5, λ_{17}	$0.00 \pm i48.86$
λ_6, λ_{18}	$0.00 \pm i232.97$
λ_7, λ_{19}	$0.00 \pm i232.97$
λ_8, λ_{20}	$-0.54 \pm i35.38$
λ_9, λ_{21}	$-0.54 \pm i35.38$
λ_{10},λ_{22}	$-0.54 \pm i35.38$
λ_{11},λ_{23}	$-0.54 \pm i35.38$
λ_{12},λ_{24}	± 0.00

Table 2. System eigenvalues corresponding to the simulation of a helicopter model with main, tail rotor and fuselage's vertical degree of freedom. The main rotor blades have flap/lag degrees of freedom and the tail rotor blades have flap degree of freedom.

shows the influence of the state variable associated to the fuselage vertical speed (x_{14}) and the flap angular speeds of the first, second, third and fourth main rotor blades (x_{15}, x_{17}, x_{19} and x_{21}), the rest of the state variables are zero. Figure 10 shows details of Figure 9, as it can be seen, the state variables associated to the fuselage's vertical translation (x_1) and the flap angles of the first, second, third and fourth main rotor blades (x_3, x_5, x_7 and x_9) are also activated. The eigenvector associated to the eigenvalue λ_2 is given by coupled motions between the fuselage vertical displacement and the main rotor blades' flap angles. The rest of the eigenvalues in Table 2 are associated to the blades' motions i.e., the eigenvalues (λ_3-λ_5, λ_{15}-λ_{17}) are associated to the main rotor blades' flap motion. (λ_6, λ_{18}) and (λ_7, λ_{19}) are related to the tail rotor blades' flap motion and (λ_8-λ_{11}, λ_{20}-λ_{23}) are associated to the main rotor blades' lag motion. The expected coupling motion between the fuselage displacement and the main rotor blades flap has been found as expected [29].

4.3 Fuselage Influence on the Flap Mode

Due to the design process of a rotocraft requires an accurate test of their dynamic properties. The coupling between the fuselage and the main rotor can also be studied when the main rotor shaft is rotated in pitch ϕ(t) and roll ζ(t). In this case, the rotor blades experience additional gyroscopic accelerations caused by mutually perpendicular angular velocities in pitch $\dot{\phi}$(t), roll $\dot{\zeta}$(t) and the main rotor angular speed (Ω). The flap dynamics on the steady state can be

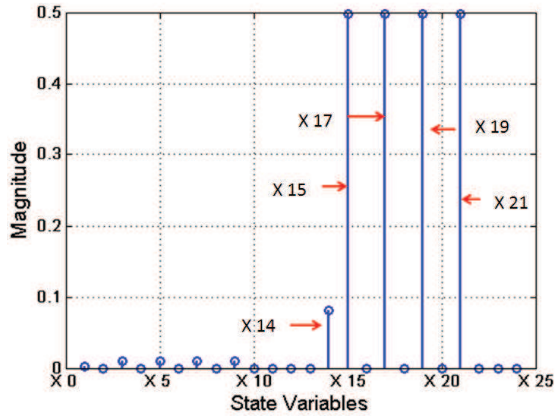

Figure 9. Magnitude of eigenvector components associated to $\lambda_2 = 0.0 + i50.01$.

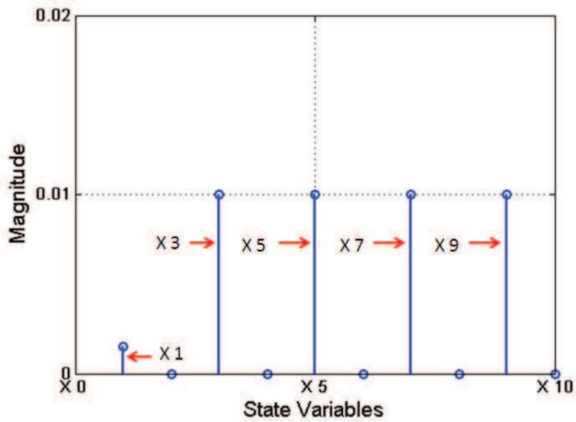

Figure 10. Details of the magnitude of eigenvector components associated to $\lambda_2 = 0.0 + i50.01$. The state variables associated to the fuselage's vertical displacement and the flap angles of the first, second, third and fourth main rotor blades show their influence in this mode.

represented as:

$$\beta = \beta_{1C} cos(\Omega t) + \beta_{1S} sin(\Omega t) \tag{9}$$

where:

$$\beta_{1C} = \frac{2}{\Omega\left(\lambda_\beta^2 - 1\right)} \omega_p \tag{10}$$

$$\beta_{1S} = \frac{-2}{\Omega\left(\lambda_\beta^2 - 1\right)} \omega_r \tag{11}$$

$\lambda_\beta^2 = 1.2$ is a standard value applicable to this model. β_{1C} is the longitudinal flap angle in response to the roll rate and β_{1S} the lateral flap angle in response to the pitch rate.

The flap angles due to the gyroscopic motions experienced can be calculated when any rotating mass is rotated out of plane [29].

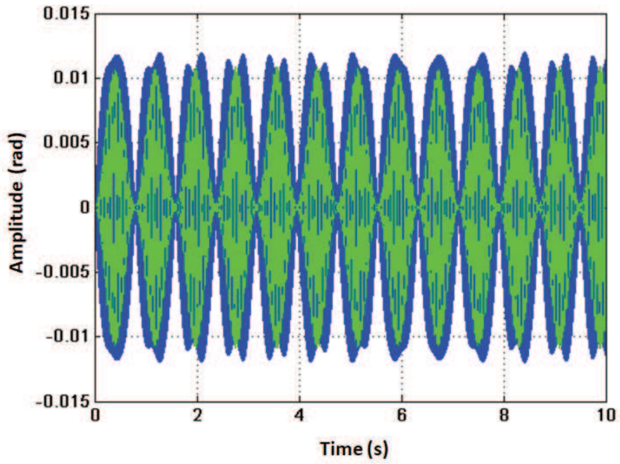

Figure 11. Presentation of the error between main rotor blades flap amplitude transmitted by the fuselage to the blades. Main rotor blades' flap amplitude measured from VehicleSim results (green). Theoretical flap amplitude (blue).

Based on the previous considerations, if now a torque is introduced on the fuselage's Y axis inducing a pitch rotation, and the initial flap angles on the blades are zero, this torque should be transmitted through the shaft to the flap hinges on each blade. In order to validate this behaviour in the system, a torque is included in the VehicleSim model as a constant value from the initial time, its magnitude being 1750 Nm. In here, flap hinges are included in the model to allow for the flap degree of freedom, but neither lag hinges nor tail rotor have been included in the model as this section is only focused in the analysis of the coupling between fuselage and flap degrees of freedom.

It is expected that the flap dynamics on the four main rotor blades, respond to variations on the fuselage's dynamics according to Equation (9). Figure 11 shows a presentation of the error between the flap angle amplitude obtained by Equation (9) (blue), and the blades flap angles when this is induced by a shaft torque that forces a fuselage's pitch and roll motions (green). It is clear to see that the simulation model follows the theoretical expectations in an accurate manner.

4.4 Study of the Heterodyning Performance

The purpose of this subsection is to model and to study the fuselage behaviour in order to identify their frequencies and the corresponding harmonics as a consequence of the variation of the main rotor angular speed. It is well known that the major source of vibration in a helicopter is the main rotor, whereas in fixed-wing aircraft, the vibrations are originated by the engines or are caused by atmospheric turbulences in between other reasons [26]. In the most general case, vibrations appearing due to the main rotor's action tend to be equal or multiple to the main rotor frequency, and this frequency is a function of the angular speed at which it rotates [21].

4.4.1 Main Rotor Angular Speed

The varying angular speed should generate additional spectral components and the combination of frequencies (heterodyning) should be detected on the fuselage [15]. Furthermore, harmonics should also appear when additional distortions are considered i.e., the varying rotating speed raises its value. In order to test this, several simulations are carried out introducing a mass unbalance in the main rotor's blades, this is, m_{bl1} = 31.06 kg, m_{bl2} = 31.26 kg, m_{bl3} = 29.96 kg and m_{bl4} = 31.16 kg. The simulations are carried out with a cyclic feather angle of 0.1 rad. The action of gravity is not considered, so, the system can be assumed to be in vacuum. As a consequence of the existing mass unbalance as well as the varying main rotor angular speed, the fuselage should show a frequency spectrum according to these, which should be studied.

The main rotor's equation of motion obtained in VehicleSim is:

$$\dot{\Omega}[I_{rtz} + I_{blz}cos^2\theta + I_{blx}sin^2\theta + m_{bl}(y_{bl}^2 + eR(eR+ 2y_{bl})) - k_d] = k_d((2\pi)^2 f_1 f_2 sin\,(2\pi f_3 t)) - k_p(2\pi(f_1+ f_2 cos(2\pi f_3 t)) - \Omega) - (2I_{blx} - 2I_{blz})\,\Omega\dot{\theta}cos\theta sin\theta \quad (12)$$

k_p and k_d are the proportional and derivative controller gains which allows to control the main rotor angular speed. θ is the feather angle. I_{rtz} is the main rotor moment of inertial around the Z axis. I_{blx} and I_{blz} are the blades' moments of inertia around the X and Z axes, and y_{bl} is the centre of mass of the blade along the Y axis. Equation (12) is the equation of motion for one main rotor blade that is mounted with flap, lag and feather hinges and it shows the contribution of the varying angular speed on the main rotor. This equation is analogous for all the blades in the main rotor.

To study the expected behaviour on the fuselage, several combinations of frequencies should be considered. In this work, f_1 = 7 Hz (according to the standard main rotor angular speed for this helicopter model, 44.4 rad/s). On the other hand, the frequency f_3 will adopt the values

$f_3 = f_1$ and $f_3 = f_1/2$, these cases are of special interest because the fuselage is sensitive to variations from the main rotor at frequencies equal or multiple of this frequency [28].

f_2 will be modified to regulate the amplitude of the varying main rotor angular speed, therefore, this additional perturbation will be considered to study its influence on the system. f_2 = 0.16 Hz and f_2 = 0.40 Hz, these values are chosen as low and high varying rotating speed respectively, other values could also be considered but these are given as examples allowing to illustrate the behaviour under study.

4.4.2 Heterodyning and Main Rotor Speed

Helicopters are subjected to periodic loads and internal phenomena that initiate and propagate in many components. A study of these at different frequency ranges can provide an indication of the nature of a fault and this gives some diagnostic capabilities. In fact, a change in these loads within a determined frequency range can be associated to a particular system component.

This subsection presents the study of simulated helicopter model under the action of main rotor varying angular speed with the aim to detect harmonics as well as heterodyne frequencies on the fuselage. The spectral analysis is used to study the particular behaviour on this system. In order to carry out the proposed analysis, two steps are needed: (a) to detect the fuselage performances appearing as a consequence of main rotor's varying speed and (b)identification and characterization of them. This will be done by analyzing spectrograms obtained when a Short Time Fourier Transform is applied (STFT), see [36].

The STFT of a signal $X(t)$ can be defined as:

$$S_x(t,\omega) = \int_{-\infty}^{\infty} X(\tau)h(\tau - t)e^{-j\omega\tau}d\tau \qquad (13)$$

X(t) is the signal under study and $h(t)$ is a finite support window function. The properties of the window function $h(t)$ have a significant effect on the STFT result and should be selected. The spectrograms of the signals have been derived with an overlapping of 99% to get a suitable compromise between time and frequency resolutions and a 44 seconds time window. The high and low frequency components were neglected, due to the size of the window, the spectrograms for the first and the last seconds are not able to be displayed. However, the corresponding frequencies and harmonics are able to be identified, properly [37].

If the main rotor angular speed varies and it is composed by two signals of different frequencies, the frequencies of each one must be added and subtracted on the fuselage [15]. As the main rotor angular speed is varying and the main rotor blades masses are unbalanced, the fuselage may experience modified behaviour. In order to analyse the additional frequencies on the fuselage, as a consequence of the main rotor only, the tail rotor angular speed degree of freedom is not considered. The impact will be studied in the roll and pitch degrees of freedom. In here, only two axes (X and Y) are studied as these provide enough information for analysis purposes.

A first simulation is carried out and the main rotor angular speed varies as a combination of frequencies. The harmonics should also appear as additional distortions are considered. The combination is given as $f_3 = f_1$ = 7 Hz and f_2 = 0.16 Hz. Note that the results of this simulation provide the time history of the fuselage's roll angle, this is a time dependent sequence processed in a STFT. The resulting spectrogram is displayed in Figure 12. As shown, there are two main frequencies appearing: 7 Hz = f_1 and 14 Hz = $f_1 + f_3$. Clearly, a combination of frequencies (heterodyning) has been detected on the fuselage according to the expected sidebands. In a similar manner, the spectrogram for the fuselage's pitch motion is represented in Figure 13, two main frequencies are clearly identified, their values are 7 Hz and 14 Hz and this coincides with the expected behaviour.

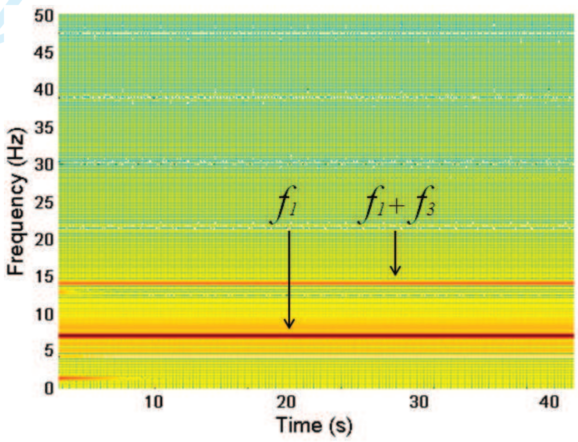

Figure 12. Fuselage's roll angle spectrogram with varying main rotor speed at frequencies f_1 = 7 Hz, f_2 = 0.16 Hz and $f_3 = f_1$ = 7 Hz.

The presence of frequency sidebands is a severe failure indicator in mechanical system. These can be described as a series of sinusoidal signals superimposed on a background of random noise. The authors test the possible appearance of harmonics as a consequence of an increasing varying main rotor angular speed. f_2 increases from 0.16 Hz to 0.40 Hz.

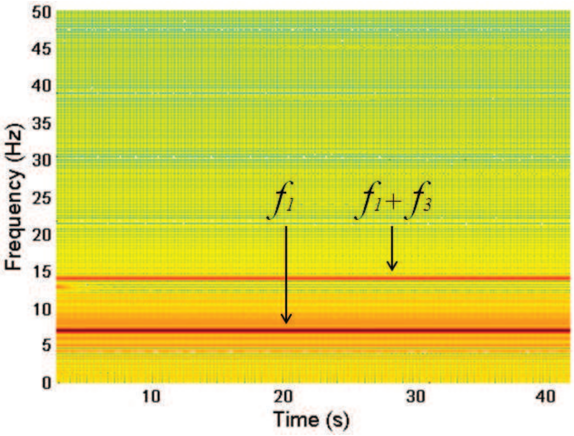

Figure 13. Fuselage's pitch angle spectrogram with varying main rotor speed at frequencies f_1 = 7 Hz, f_2 = 0.16 Hz and f_3 = f_1 = 7 Hz.

As $f_3 = f_1$ = 7 Hz and f_2 = 0.40 Hz. The corresponding fuselage's roll angle spectrogram is represented in Figure 14. A comparison can be done between figures 12 and 14, the frequencies 7 Hz and 14 Hz appear in Figure 12, and harmonics appear with frequencies 21 Hz = $f_1 + 2f_3$, 28 Hz = $f_1 + 3f_3$ and 42 Hz = $f_1 + 5f_3$ in Figure 14, as a consequence of the increase in f_2. As it can clearly be seen, the spectrum is given as a series of sinusoidal signals. The main source of these is attributable to the harmonics appearing on the main rotor as a result of the varying angular speed as well as an increase on its magnitude. Similarly, a spectrogram is obtained for the fuselage's pitch angle, Figure 15 displays similar sequence of harmonics as in the case of roll angle for this set of frequencies.

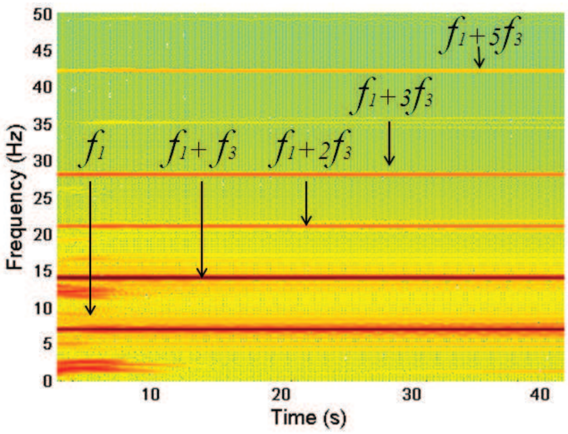

Figure 14. Fuselage's roll angle spectrogram with varying main rotor speed at frequencies f_1 = 7 Hz, f_2 = 0.40 Hz and f_3 = f_1 = 7 Hz.

A second combination of frequencies can be carried out, in order to show that heterodyning is equally obtained.

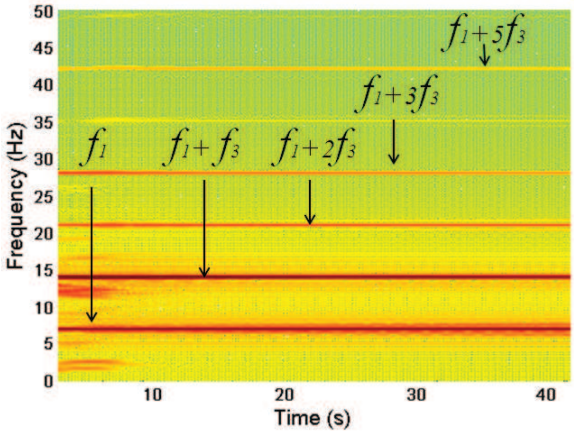

Figure 15. Fuselage's pitch spectrogram with varying main rotor speed at frequencies f_1 = 7 Hz, f_2 = 0.40 Hz and f_3 = f_1 = 7 Hz.

In this case, $f_3 = f_1/2$ = 3.5 Hz and f_2 = 0.16 Hz. The fuselage's roll angle spectrogram shows three main frequencies at 3.5 Hz = $f_1 - f_3$, 7 Hz = f_1 and 10.5 Hz = $f_1 + f_3$. If the spectrogram is analysed for the fuselage's pitch angle, similar heterodyning behaviour is obtained. In addition to this, if now f_2 = 0.40 Hz and f_3 remains as $f_3 = f_1/2$ = 3.5 Hz, the spectrograms of the roll and pitch motion signals show harmonics with frequencies at 14 Hz = $f_1 + 2f_3$, 17.5 Hz = $f_1 + 3f_3$, this is a consequence of the increase on f_2. Therefore, the heterodyning behaviour in this mode has been obtained under the combinations of frequencies, as it was expected. In the light of results shown, the helicopter model built up in VehicleSim is capable to reproduce theoretical behaviour predicted by the specialist literature.

5 Conclusions

The assessments are essential to enhance the design of the helicopters and these provide data relative to the internal performance under certain conditions. Furthermore, the structural analysis and its evaluation have to be continually tackled; however, these are not straightforward tasks. As a consequence, this paper presents an advanced application of a helicopter dynamic model to analysis the complex fuselage/main rotor dynamics and it is capable to transmit perturbations from the main rotor to the fuselage. The model has been set up by using VehicleSim, a program specialized in modelling mechanical systems composed by rigid bodies. It allows to define a system as a composition of several bodies and constrains by using a parental relationship structure. This software package has been employed due to the nonlinear dynamics couplings are taken into account, the system nonlinear equations and state-space matrices are derived. So, the main VehicleSim features have been dealt with.

The modal analysis has been used to study and to validate the main rotor and fuselage coupling. The modes shapes were studied and the corresponding outcomes showed the existence of a coupling between the fuselage vertical motion and the main rotor blade's flap mode. The gyroscopic displacement on the fuselage generated the impact of this one on the flap mode. The results displayed a good agreement with theoretical findings existing in the specialist literature.

This work studies the impact as a varying angular speed of the form $\Omega(t) = \Omega_1 + \Omega_2 cos(\Omega_3 t)$ is acting on the main rotor. Ω_1 is the main rotor angular speed, Ω_2 and Ω_3 are additional angular speeds. In fact, the main rotor angular speed changes by means of a PD controller. This allows to obtain different combinations of frequencies between the main rotor standard frequency and the additional frequencies. These combinations are important to study the fuselage behaviour. Various tests in absence of external forces were carried out to study the fuselage's roll and pitch axes under the action of only the main rotor varying angular speed. The pitch and roll axes have been sufficient to evaluate the capability of the rotorcraft model as a detection tool.

Simulations for heterodyning conditions were carried out for various cases i.e., when frequency f_3 changes its value as $f_3 = f_1$ and $f_3 = f_1/2$. On the other hand, sequences of harmonics on the spectrograms were observed as the main rotor varying angular speed increases its value. The obtained results match those predicted by theoretical approaches. This work opens the door for future helicopter studies, in order to establish a link between the simulation and the experimental heterodyne detection.

This model, in which couplings and nonlinearities are maintained and captured, enhances the accuracy of prospective research, thereby this approach responds to the demands determined for this purpose. However, the complexity of the dynamical model should be increased, as for example, including the blades flexibility. However, the main goal has been to set up the basis of a helicopter model code by using VehicleSim only, and to test its capability to carry out modal analysis and to simulate heterodyning behaviour. Being these, two required stages to establish a tool, which is able to link the helicopter simulation and the test platforms. In summary, the work here presented provides a simulation tool for the identification of heterodyne performance in a helicopter model through fuselage/main rotor coupling using a multibody software as well as modal analysis.

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