MENTAL TIME TRAVEL AND THE VALUATION OF FINANCIAL INVESTMENTS

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ABSTRACT

Purpose

To portray the valuation of financial investments as mental time travel.

Design/methodology/approach

In a series of thought investments, $1 invested in an investment fund is mentally projected forward in time and then discounted back to the present – with no objective time passing. The thought investments feature symmetric valuation (in which discount rates exactly match projection rates) and asymmetric valuation (in which discount rates and projection rates happen to differ). They show how asymmetric valuation can result in differences between the current personal value and market value of an investment and, by way of real-world illustration, between a closed-end investment fund's net asset value and its market value. We explore possible reasons for asymmetric valuation.

Findings

Thought investments illustrating mental time travel can be used to help understand both financial investment valuation generally and, more specifically, established explanations of the closed-end investment fund puzzle. We show how different expectations, different perceptions of time and risk and different risk and time preferences might help determine value.

Originality

There are vast literatures on prospection, discounting and future-orientated or intertemporal decision-making. Our innovation is to illustrate how these mental activities might combine to facilitate financial investment valuation. In particular, we show that a low personal discount rate could be a consequence of a shortened perception of future time and vice versa.

Keywords: mental time travel; time discounting; expectations; perceptions; preferences; closed-end fund puzzle; market valuation; personal valuation.

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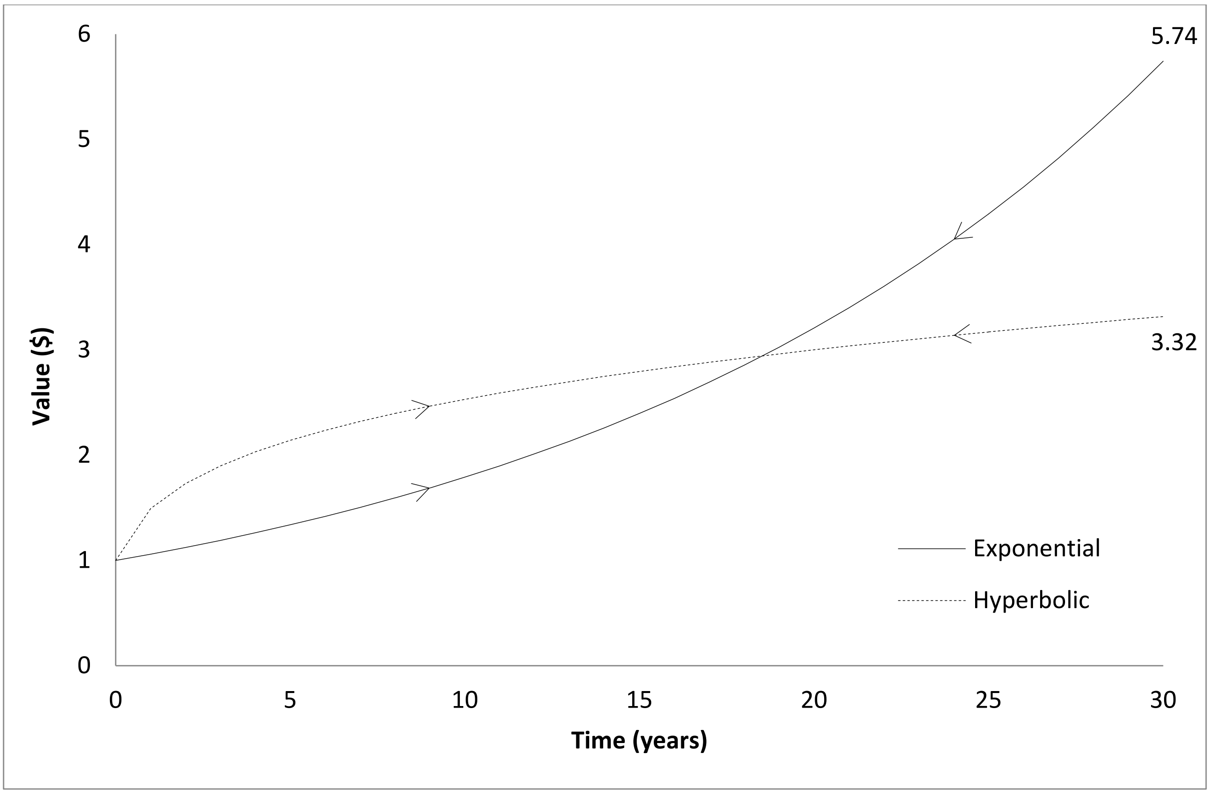
1. **INTRODUCTION**

This paper was motivated by a discussion at a conference that one of the authors attended some years ago about whether $1 invested in equities is worth more than, less than or the same as $1 invested in bonds. Valuing financial investments, such as equities and bonds, involves mental time travel – forwards and backwards. In the paper, we explore some of the different paths such travel might take. In a series of "thought investments": an invested $1 is projected forward; and the projected value is discounted back to the present. To keep things simple, we assume a positive expected return – in the form of capital appreciation rather than income – on all thought investments. We use the term "thought investment" in the same sense as "thought experiment". This is mentally simulated value transformation; and it is important to note that no objective time passes during the investment. Because future time and risk are subjective (i.e., they are "perceived" and they reflect preferences), so too are projected and discounted values.

Projections are integral to financial investment valuation. Indeed, all financial decisions can be viewed as being based on forecasts of future value (Utkus, 2011). Chart 1 is what we call a "value-time chart". It is derived from Mitchell and Utkus, 2004 and illustrates two projection methods. An invested $1 is projected forward 30 years: "exponentially" at 6% p.a. to $5.74 [1] and "hyperbolically" to $3.32 [2].

CHART 1: EXPONENTIAL AND HYPERBOLIC PROJECTION AND DISCOUNTING – SYMMETRIC VALUATION



Exponential projection is a common assumption in time value of money calculations with rates being postulated not to vary today, tomorrow or, indeed, many years from now. Exponential investors foresee future values increasing at a constant rate, thanks to the benefits of compounding. Not all investors project exponentially, however. Hyperbolic investors, by contrast, expect an invested $1 to grow more rapidly in the short term than in the long term. So, they perceive decreasing benefits to long-term investment – rewards are expected to accelerate quickly and then taper off (Mitchell and Utkus, 2004). The hyperbolic curve in Chart 1 reflects this: it rises more rapidly in the near future (the first five years) than in the far future (the last five years).

Chart 1 can also be read as illustrating two discounting methods. A future payment of $5.74 is discounted exponentially at 6% p.a. [3], while a payment of $3.32 is discounted hyperbolically [4], both to a current value of $1. Practitioners typically assume that discount functions are exponential (Angeletos *et al*., 2001), whereas a growing body of experimental evidence suggests that hyperbolic discounting – in which value declines at a more rapid rate in the short term than the long term – better describes how many individuals value delayed rewards (Thaler, 1981, Frederick *et al*., 2002, Laibson, 2003, Ainslie, 2005, and Berns *et al*., 2007). The hyperbolic curve in Chart 1 reflects this: it falls more rapidly in the near future (the first five years) than in the far future (the last five years). (Exponential and hyperbolic discounting are the two methods that feature most prominently in the time discounting research literature, but there are, of course, others: Doyle, 2013, for example, identifies over twenty methods.)

Chart 1 can thus be read as containing: a pair of exactly matching exponential projection and discount curves extending from year-0 to year-30; and a pair of exactly matching hyperbolic curves covering the same period. So read, each pair of curves traces a mental time travel journey, or thought investment, in which an initial value is instantaneously mentally simulated forward 30 years and then back to a current value.

These curves record investor expectations of future value. These expectations, in turn, reflect investor perceptions and preferences. Where such curves record market expectations, they reflect market perceptions and preferences. Where they record personal expectations, such curves reflect personal perceptions and preferences. Market expectations are aggregations of the personal expectations of all market participants, weighted by the size of their transactions. It follows that individual personal expectations, perceptions and preferences may or not match those of the market – and will generally only do so if the individual is a "typical" or "average" Investor.

The thought investments that follow envisage a $1, invested in an investment fund – a publicly traded closed-end mutual fund whose assets are publicly traded securities – at current market value, being projected forward and then discounted back to a personal value. Sometimes, personal value matches market value; sometimes, it does not. They further imagine $1, invested in an investment fund at net asset value (NAV), being projected forward and then discounted back to current market value. Sometimes, market value matches NAV; sometimes, it does not. (As we discuss later, differences between market value and NAV are a well-known puzzle in financial markets.) As with Chart 1, the specific numbers and the investment horizons are arbitrary being chosen for illustrative purposes.

In the next section, we look at personal symmetric and asymmetric valuation, together with explanations for the latter. In section 3, we do the same for market valuation. In section 4, we examine the connection between personal and market valuations, while section 5 concludes.

2. **PERSONAL VALUATION**

2.1 Symmetric personal valuation

Imagine a $1, invested in an investment fund at current market value, being projected forward 30 years and then discounted back to a personal value. We can see from Chart 1 that, when discount rates exactly track projection rates, $1, so invested, has a current personal value of $1. We might label this exact matching of discount and projection rates "symmetric valuation". So valued, $1 invested in an equity fund has the same personal value as $1 invested in a bond fund – both will be valued at $1. This is so whether personal valuation is exponential, hyperbolic or any other method [5].

Symmetric personal valuation does not require personal expectations to match market expectations. Think of the two projection curves in Chart 1 as recording the exponential market expectations and the hyperbolic personal expectations of the future value of $1 investment in an investment fund. Assume that personal expectations are based on the belief that this investment will outperform market expectations in the short term but fail to match the returns expected by the market over the 30-year period. Even if these personal expectations are irrational, $1 will still be personally valued at $1 if valuation is symmetric.

2.2 Asymmetric personal valuation

Personal valuation is not always symmetric. Sometimes, as we now elaborate, it is asymmetric – discount rates do not exactly match projection rates. This can arise both when the methods of projecting and discounting are the same and when they differ. Asymmetric valuation does not preclude the possibility of an invested $1 having a personal value of $1, but it does reduce its likelihood. In Chart 1, for example, we can identify an exponential projection–hyperbolic discounting combination and a hyperbolic projection–exponential discounting combination which both project a year-19 value of $2.96 and which both result in a discounted value of $1. We might term this "asymmetric matching". In the remainder of this section, we consider possible causes and consequences of asymmetric valuation.

2.2.1 *Psychological asymmetry*

Projecting and discounting are mental time travel exercises. Mental time travel allows us to mentally visit the future ("prospection") or the past ("retrospection") (Suddendorf and Corballis, 2007). Mental time travel theory has traditionally assumed symmetry between prospection and retrospection, viewing them as identical save for the direction of travel (Van Boven *et al*., 2008). Recent research, however, reveals that, for many of us, the ways in which we mentally represent the future and the past are not the same (Kane *et al*., 2012). Prospection differs, in context and experience, from retrospection (Van Boven *et al*., 2008). Future events, for example, are psychologically closer than past events of equivalent objective distance – a "temporal Doppler effect" (Caruso *et al*., 2013).

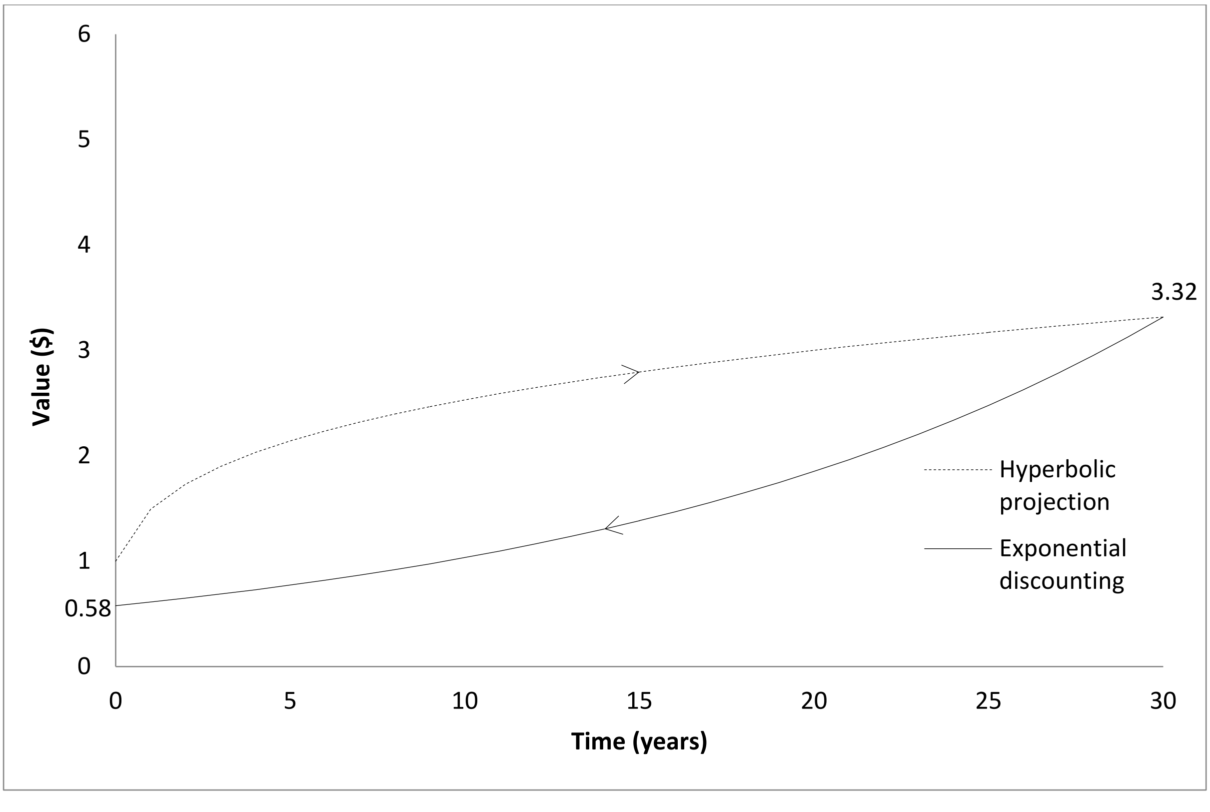
Discounting, as in Chart 1, is not retrospection (visiting the past). Rather, it is returning to the present from the future ("retrospective prospection", if you like). Like prospection and retrospection, projecting and discounting are sometimes viewed as being identical (Mitchell and Utkus, 2004, for example, entitle their diagram, reproduced here as Chart 1, "Exponential versus Hyperbolic Discounters: Growth of $1 Over Time"), but some researchers suspect that the two are psychologically distinct. Doyle, 2013, for example, suggests that, while their underlying formulae might be closely related mathematically – one being the inverse of the other – their mental operations might differ. Loewenstein, 1988, and Weber *et al*., 2007, for example, find that subjects value delayed and accelerated rewards at different rates. And, indeed, projection and discounting are likely to be treated differently from each other if they are motivated by different influences. We now look at the influence of expectations, perceptions and preferences on valuation.

2.2.2 *Expectations*

We noted, in the Introduction, that the temporal value curves in Chart 1 record investor expectations which, in turn, reflect investor perceptions and preferences. Expectations can be rational or irrational. Irrational investors are sometimes referred to as "noise traders". Noise traders are prone to "investor sentiment" – expectations not justified by the facts (Lee *et al*., 1991, Baker and Wurgler, 2007). Sometimes, noise traders project returns higher than those expected by sophisticated investors; sometimes, the opposite.

We also noted earlier that symmetric valuation does not require rational expectations. On the other hand, personal valuation will be asymmetric if personal and market projection rates differ, but personal discount rates match those of the market. Such might be the case if market discount rates, somehow, influence or dictate personal discount rates. This would require market discount rates to be common knowledge. If, as illustrated in Chart 2, the personal year-30 hyperbolic projected value of $3.32 was discounted at the market exponential rate of 6%, its current personal value would be $0.58 [6].

CHART 2: HYPERBOLIC PROJECTION WITH EXPONENTIAL DISCOUNTING – ASYMMETRIC VALUATION



If we compare the 30-year hyperbolic projection-exponential discounting combination in Chart 2 with Chart 1, we can see the influence of time on value. The curves in Chart 1 cross over at year-19. The hyperbolic curve in Chart 1 is steeper than the exponential curve for investments with durations below 19 years, but less steep for longer durations. The implication is that a $1 investment with a duration less than 19 years will have a personal value greater than $1 in the case of hyperbolic projection and exponential discounting; and an identical investment with a duration greater than 19 years will have a personal value less than $1 (as Chart 2 also confirms). They are equal for durations of 19 years. We can also see from Chart 1 that the opposite holds when exponential projection and hyperbolic discounting combine.

Personal valuation will also be asymmetric if personal expectations match those of the market, but personal and market discount rates differ. In the rest of this section, we assume that personal expectations do match market expectations and we consider how different perceptions and preferences might cause personal and market discount rates to differ.

2.2.3 *Perceptions – of risk and time*

We noted, earlier, that the temporal value curves in Chart 1 reflect investor perceptions and that, if personal perceptions differ from market perceptions, personal valuations might differ from market valuations. Two important perceptions relate to risk and time.

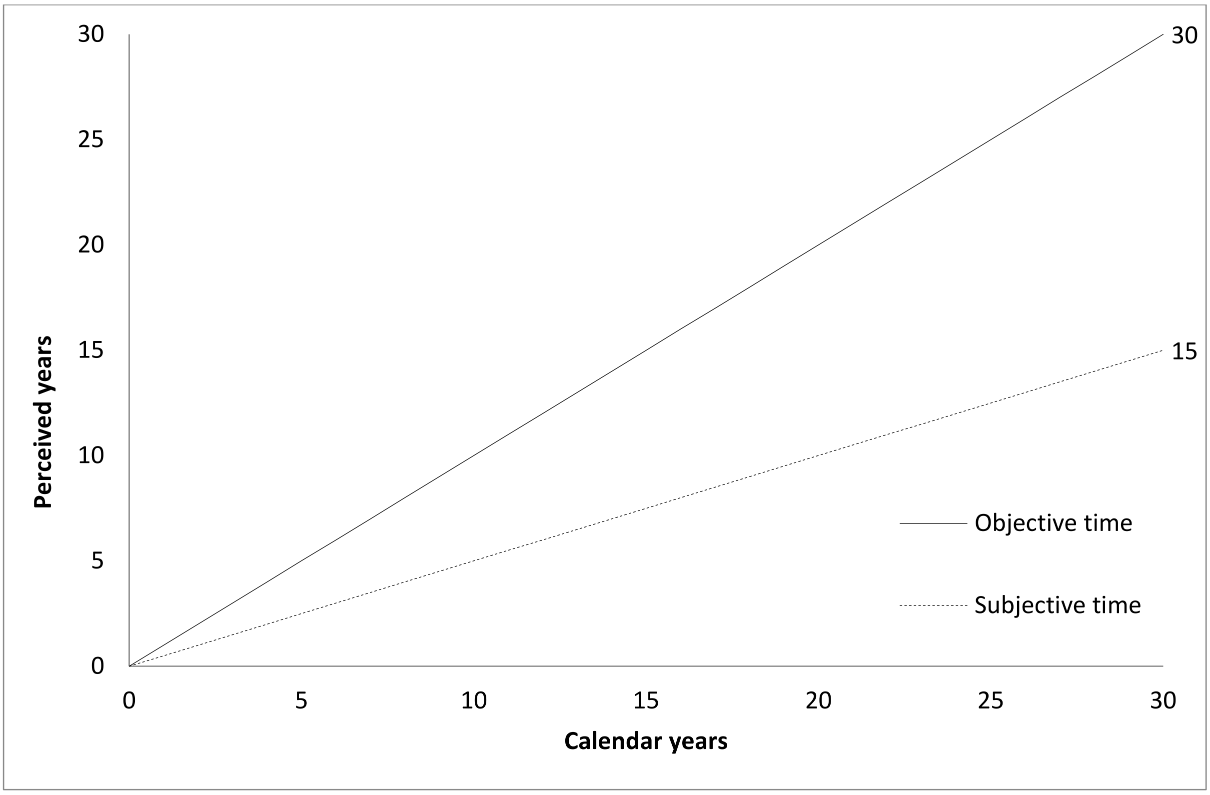
Think of the two curves in Chart 1 as recording the market's expectations of the future values of two investment funds (call them "Exponential" and "Hyperbolic"). Such expectations reflect the market's perceptions of these two funds. Assume that the market perceives that Hyperbolic is riskier than Exponential in the short term but less risky in the long term. Assume, too, that personal expectations of the future values of the two funds match those of the market. Individuals who perceive the two funds as being equally risky might be inclined to discount their projected values at the same rates and hence value one fund symmetrically and the other asymmetrically. Chart 2 can thus, alternatively, be read as illustrating the exponential discounting of Hyperbolic's projected 30-year value. So valued, Hyperbolic has a current personal value of $0.58.

Just as personal risk perceptions can differ from those of the market, so, too, can personal perceptions of prospective time duration (Zauberman *et al*., 2009). Specifically, some individuals' understanding of time differs from objective (or calendar) time. They might perceive, for example, that two years is less than twice as far ahead as one year (Bradford *et al*., 2014). Chart 3 (which is derived from Figure 3 in Zauberman *et al*., 2009) illustrates one way in which subjective time might differ from objective time.

In Chart 3, we assume (purely for ease of illustration) that biased time perception is such that two years of future calendar time is equivalent to just one year of perceived time. An individual who perceives future time in this way will underestimate a calendar period of 30 years' duration by 15 years and might consequently value a 30-year investment as illustrated in Chart 4.

Consider two individuals. The first, without time bias, values a $1 investment symmetrically and projects and discounts exponentially as in Chart 1. So, $1 investment over 30 years has a personal (present) value of $1. The second individual has a time bias when discounting but not when projecting, say, because they are aware that the year-30 investment value is predetermined. In Chart 4, the $1 investment is therefore projected forward exponentially, as in Chart 1. Assume for this second individual that, as in Chart 3, every two calendar-years' time is perceived subjectively as one year. This means that when discounting, the individual mentally jumps from A to B in Chart 4 and begins to discount 15 (rather than 30) years from the current date. So, the projected terminal investment value of $5.74 is discounted at the same exponential 6% rate as the projection rate but only for 15 perceived years. A $1 investment so valued has a personal value of $2.40 and therefore is attractive.

CHART 3: OBJECTIVE AND SUBJECTIVE TIME

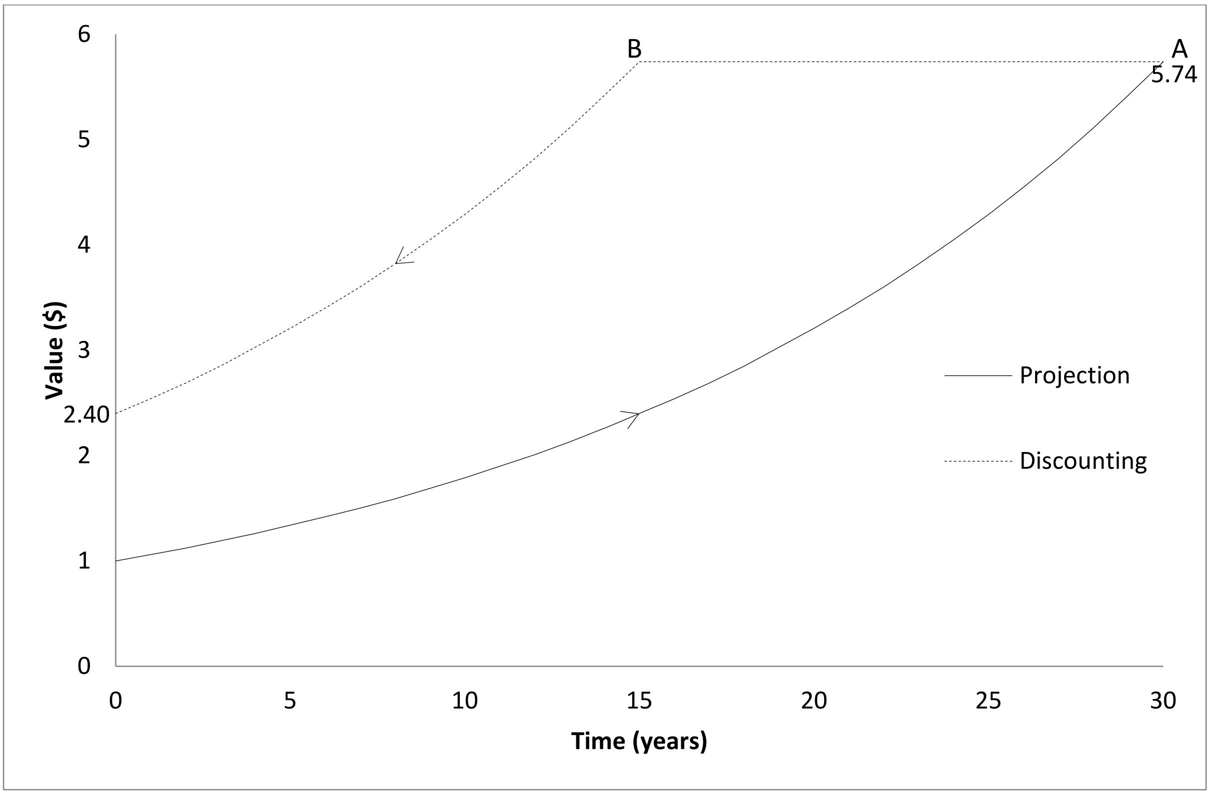


It is equally clear that had this second individual's time perception been biased in the opposite direction, so that one year of future calendar time was equivalent to more than one year of perceived time, then the personal value of the $1 investment would be worth less than $1 even though projection and discount rates are the same.

This illustrates how time perception can influence personal discount rates. A $2.40 personal value for the $1 investment in Chart 4 can result from either discounting the terminal value of $5.74 at the market discount rate of 6% over 15 years of perceived time or discounting it at a personal discount rate of 2.95% over 30 years of objective time. This implies that future time is much nearer mentally – or equivalently that perceived time passes more quickly than objective time – for individuals with low personal discount rates than for those with high personal discount rates relative to projection rates.

It is also possible – and indeed likely – that an individual's perception of time differs over their lifecycle. For young children, perceived time tends to pass more slowly than objective time: their next birthday party feels as though it is at least 10 years away! This is equivalent to young children discounting the future heavily. For old people, time speeds up – a whole year appears to pass in a few months. This is equivalent to old people having low personal discount rates.

CHART 4: EFFECT OF TIME BIAS ON VALUATION



All this is, of course, very bad for optimal lifecycle consumption and savings: young people as a whole prefer to consume rather than save, while old people find it difficult to dissave and consume their accumulated assets, since perceived time appears to pass so quickly. This behaviour can be readily explained in terms of Chart 3 and 4.

2.2.4 *Preferences – for time and risk*

We noted, earlier, that the temporal value curves in Chart 1 reflect investor preferences and that, if personal preferences differ from market preferences, personal valuations might differ from market valuations. Two important preferences relate to time and risk.

Frederick *et al*., 2002, and Hayden, 2016, distinguish *time preference* (discounting driven solely by a preference for the timing of a reward) from discounting generally (devaluation of that reward for any reason, including uncertainty). (For a comprehensive discussion, see Loewenstein, 1992, and Frederick *et al*., 2002.) Valuing a future reward less only because it is further away in time signifies positive time preference; a higher valuation indicates negative time preference; and equal valuation signals temporal neutrality.

Positive time preference is a synonym for impatience, implying a preference for today over the future (Van Liederkerke, 2004). Impatience can be due to a lack of foresight, to a weakness of will or self-control, to habit or to fashion (Van Liederkerke, 2004). Some of us experience psychological discomfort when asked to defer immediate gratification, even for a short period (Loewenstein, 1992, Olson and Bailey, 1981). Patience, on the other hand, is a characteristic of those who have self-control and can resist, or abstain from, immediate gratification. Patience is consistent with far-sightedness and with an extended or infinite time horizon. Patience is a requisite for low or negative discount rates.

Negative time preference has been linked to a desire for improving sequences of outcomes which, in turn, can be attributed partly to anticipation (Loewenstein and Prelec, 1991). Pleasure from anticipation is enhanced if the best comes last. Pleasure from anticipating the future and its rewards includes a "bequest motive" (Loewenstein, 1992) – a spirit of altruism which can mean enjoying giving for its own sake (Cowen and Parfit, 1992) or which can be personally motivated, as by the desire to build a family dynasty, or socially motivated, as by a concern for inter-generational welfare and for the protection of the environment.

Time preference has been linked with the degree to which a person feels connected to his or her future self (Herschfield, 2011). The concept of a life as a series of psychologically connected temporal selves is a notion developed by philosophers, such as Parfit (1971, 1982 and 1984), and explored by economists and behavioural scientists, such as Strotz (1955) and Frederick (2003 and 2006). The psychological connections at the core of this concept include our memories, personal characteristics and interests, all of which contribute to the make-up of our personal identity. Research suggests that strong psychological connections between the temporal selves facilitate saving – individuals who anticipate that their future personal identity will overlap considerably with their current identity tend to accumulate more financial assets than do those who sense little such overlap (Hershfield *et al*., 2009). On the other hand, the more a person's future self feels like a stranger, the more heavily that person might discount that stranger's assets.

Of course, the "brevity and uncertainty of human life" (Loewenstein, 1992) means that we can never be certain that our future selves will exist. Uncertainty is, however, "double-edged": it can cause the future to be valued more than the present, or less (Olson and Bailey, 1981). Existential uncertainty, for example, can incentivize financial provision for an unexpectedly long life.

The degree to which uncertainty influences personal discount rates depends, to a large extent, upon personal risk preference. Many of us are risk averse (Andersen *et al*., 2008). We prefer a certain reward to an uncertain reward of equal expected value (Kahneman and Tversky, 1984). Some of us, in contrast, are risk seeking. We prefer the gamble. Just as personal time preferences can differ from those of the market, so too can personal risk preferences.

Personal risk preferences can influence the valuation of leveraged investments. To the extent that they are willing and able to, individuals can borrow to fund investment. As with unleveraged investments, if investors discount the projected value of an investment funded with borrowings at its expected rate of return, valuation will be symmetric – a $1 investment will be valued today at a $1.

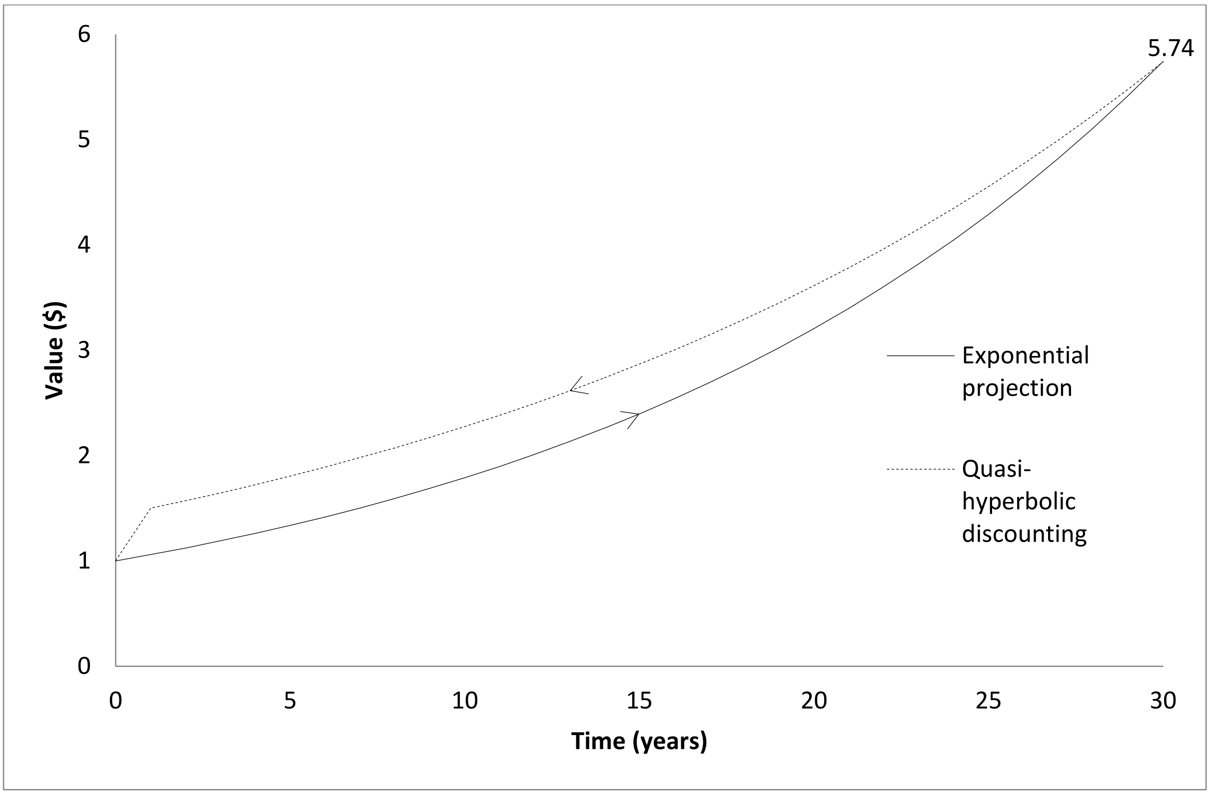
Personal borrowing can enable investments that might not otherwise be affordable but leveraged investment is riskier than unleveraged investment. A risk averse individual might be expected to avoid highly leveraged investments (discounting their projected values at more than their expected rates of return), whereas risk seekers might personally value leveraged investments more highly than does the market (using a lower discount rate than the expected rates of return).

Investor profiles can be constructed based on personal time and risk preferences. For example, those who are: patient and risk averse might be classified as "conservative" investors; impatient but risk averse as the "short-sighted, prudent"; impatient and risk seeking as "hotheads"; and patient and risk tolerant as "enterprising" investors (Arrondel and Masson, 2013). We might want to define these profiles relative to that of the market. For example, we might define conservative investors as being more patient and more risk averse than is the market.

Hyperbolic discounting provides a challenge to the stability of investor profiles over time. Discounting at a more rapid rate in the short term than the long term is typical of those of us who are impatient in the short term (hotheads), but patient over the long term (conservatives).

A feature of hyperbolic discounting is "time inconsistency" or "preference reversal" – we value one apple today over two tomorrow but prefer two apples in 51 days over one apple in 50 days (Thaler, 1981). We can see this if we study Chart 5, which combines exponential projection (assuming a 6% annual projection rate) with "quasi-hyperbolic" discounting [7] in another example of asymmetric valuation, and we think of value in terms of apples rather than dollars. (The quasi-hyperbolic model, also referred to as the "quasi-exponential" model (Sáez-Marti and Weibull, 2005), was designed to preserve exponential discounting for most periods, and all the useful mathematics that goes with it, yet capture the steep initial-period discount rate associated with hyperbolic discounting (Doyle, 2013)).

CHART 5: EXPONENTIAL PROJECTION WITH QUASI-HYPERBOLIC DISCOUNTING



In Chart 5, the quasi-hyperbolic discount curve is steeper than the projection curve at year-1 but less steep at year-30. This means that today we discount more heavily than at 6% the projected 1.06 apples at year 1 and so value them less than our 1 apple today. In contrast, we discount year-30 apples at a lower rate than that at which we discount year-29 apples – so the year-30 apples have a higher personal value now than have the year-29 apples. Of course, if we were to repeat our valuation exercise at year 29, we would then heavily discount the year-30 apples. This is a consequence of objective time passing. However, our thought investments, being mental time travel exercises, do not involve objective time passing, so we do not experience any preference reversals – and hence time inconsistency – in our framework.

3. MARKET VALUATION

3.1 Symmetric market valuation

When initial value is current market value, market valuation is symmetric – \$1 invested today at current market value has a current market value of \$1. But what if initial value is something other than current market value? What if it is net asset value? (The NAV of a fund is the market value of the fund's portfolio net of fund liabilities.)

Think of an investment fund with a current NAV of \$1 per share. Imagine this value being projected forward and discounted back by the market to current market value. Here, when symmetrically valued, a share in the fund has a current market value of \$1, i.e., the share price trades at NAV. This is so, regardless of the duration and the nature of the underlying investments.

When projecting NAV and discounting projected NAV back to current market value, market valuation is not always symmetric – market value does not always equal NAV. We consider how and why they might differ in the next sub-section. We do so by portraying as mental time travel four of the most important explanations of the "closed-end fund puzzle".

Investment funds can be open-end (with a variable issued share capital or number of units) or closed-end (with a fixed number of shares). Open-end fund shares always trade at NAV (they are symmetrically valued) because they can be issued or redeemed at NAV at any time. But this is not the case for closed-end funds. A closed-end fund occasionally trades at NAV, sometimes at a premium, but most typically at a discount, to NAV. Differences between the two measures (share price and NAV) are a well-known puzzle in financial markets. The closed-end fund puzzle is an exhaustive subject and we only scratch the surface here. (For comprehensive descriptions of the puzzle and its many explanations, see, for example, Lee *et al*., 1991, Deaves and Krinsky, 1994, Chay and Trzcinka, 1999, Ferguson and Leistikow, 2001, Dimson and Minio-Paluello, 2002, Berk and Stanton, 2007, and Bleaney and Smith, 2008.)

3.2 Asymmetric market valuation

3.2.1 *Expectations - management expenses hypothesis*

We noted, in the preceding section, that personal valuation will be asymmetric if personal and market expectations differ but personal discount rates match those of the market. Similarly, closed-end funds will trade at other than market price if expectations for the fund and for its underlying portfolio of assets differ, but the rates at which fund and portfolio projected values are discounted are the same.

One reason why the expected return on a fund might differ from that of its underlying portfolio is that the fund incurs management expenses. The discount to NAV at which a closed-end fund's shares trade, it is hypothesized, reflects the discounted value of future management fees (Gemmill and Thomas, 2002, and Ross, 2004).

Assume that $1 invested directly in the portfolio is projected to grow exponentially at 6% a year to $5.74 at the end of 30 years. Now assume that the fund accrues management expenses (equal to 1% p.a. of the value of the portfolio at the beginning of each year) whose value totals $0.79 per $1 invested by the end of year-30, thereby reducing the net return on the invested $1 from $5.74 to $4.95. Fund investors expect a compound annual return of 6% on their investment (the expected return on the portfolio) so they discount the projected $4.95 at this rate – the $1 investment has a current market value of $0.86, a 14% discount to current NAV; and the present value of the accrued expenses is $0.14. In this thought investment, the fund's share price differs from its NAV because market expectations of the return on its shares differ from those of the return on its underlying portfolio, but the fund discount rate is the expected portfolio return.

We can see that the discount to NAV at which a fund's share price trades will be a function of both the level of management fees and the life expectancy of the fund (the future period over which cash flows are discounted) [8].

3.2.2 *Expectations – management performance hypothesis*

Management expenses can explain why a fund's share price might trade at a discount to NAV, but not why it might trade at a premium nor why discounts should fluctuate. To explain these, we need to consider, additionally, management performance expectations which can mitigate or magnify the effect of management expenses. A share price premium to NAV, it is hypothesized, reflects investors' perception of management's ability to generate returns in excess of the expected return on the underlying portfolio, net of costs, whereas a discount greater than that of the present value of management expenses reflects anticipated managerial underperformance (Deaves and Krinsky, 1994, Berk and Stanton, 2004).

3.2.3 *Expectations – investment services hypothesis*

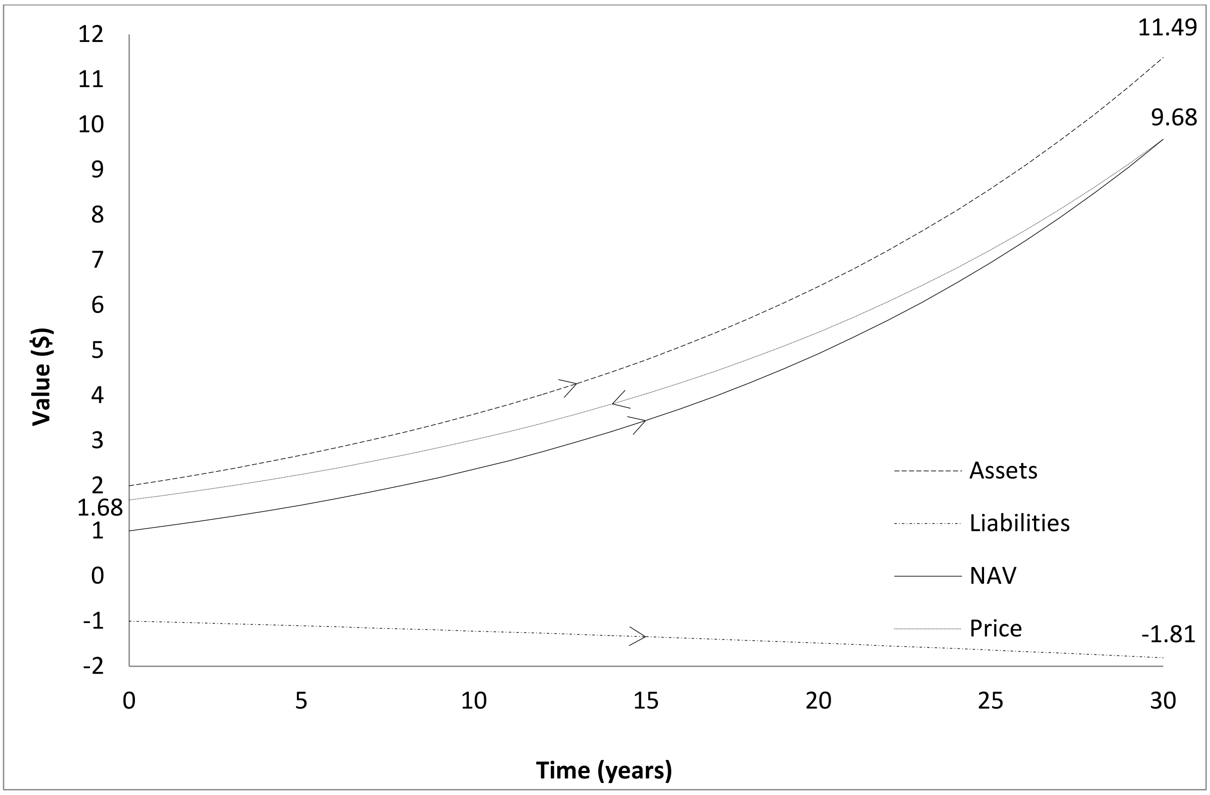
Closed-end funds incur management expenses in return for management services and, in addition to management expertise, investors receive, or expect to receive, other benefits which Cherkes, 2012, labels "investment services". These include: access to leverage; liquidity; and market-segmented investment. As with the management performance hypothesis, the investment services hypothesis envisages a trade-off between the value of these investment services and the cost of management services.

Leverage

Closed-end funds have the ability to borrow and they may do so when the expected investment returns from borrowing exceed borrowing costs. Borrowing increases the projected return on an equity investment in a fund, but it also increases its risk. The expected return on the investment should reflect this risk. As with investment in an unleveraged fund, if fund investors discount the projected future value of their investment at its expected rate of return, valuation will be symmetric – a $1 investment will be valued at a $1. As with personal borrowings, however, individuals who are more risk averse than the market might be expected to heavily discount the projected value of a highly leveraged investment fund, whereas individuals who are more risk seeking than is the market might be expected to value such funds more highly than does the market.

Research by Elton *et al*., 2013, finds that closed-end US bond funds can borrow at a much lower cost than can individual investors. Investment in such funds thus offers these investors borrowing facilities at rates not personally available. These investors, it is hypothesized, reduce the rate at which they discount such a fund's projected value so as to reflect the perceived value of cheap borrowings. Chart 6 illustrates the effect of leverage on the valuation of an investment fund. It assumes that a fund, with no previous liabilities and a current NAV of $1 per share, borrows $1 for each $1 of equity, at a compound annual interest rate of 2%. So, the fund has $2 of portfolio assets per share and $1 of borrowings per share. Chart 6 assumes that the portfolio assets grow at a compound annual rate of 6% (and that management performance expectations exactly match the cost of management expenses, so the net return on the fund equals the gross return of 6% on the underlying portfolio of assets). They have a projected value at the end of year 30 of $11.49. The $1 per share borrowing has a projected value of $1.81 at that time, implying that NAV per share has a projected value of $9.68 (i.e., $11.49 – $1.81, compared to the projected value of $5.74 for the unleveraged $1 investment). If that value is discounted at the expected annual return on portfolio assets of 6%, reflecting the perceived benefit of fund borrowings, the fund will have a current share price of $1.68, a premium to NAV. (Note that the growth rate of the net investment of $1 is not constant. It falls over time as the level of gearing falls: assets grow over time at a higher rate (6% p.a.) than liabilities (2% p.a.))

CHART 6: EXPONENTIAL LEVERAGED MARKET VALUATION



Liquidity

Closed-end funds can also offer investors what Cherkes, 2012, labels “liquidity transformation”. They can transform illiquid assets (their portfolios) that are costly for small investors to acquire into liquid securities (their shares or units) that are cheaper to acquire and easier to liquidate. As with management performance expectations, the hypothesis here is that the premium, or discount to, NAV at which a fund’s share price trades reflects the extent to which the perceived value of liquidity exceeds, or is exceeded by, the cost of the provision of management services. In the absence of management fees, a fund will trade at a premium to NAV because investors require a liquidity premium on the expected return of its underlying portfolio but not on that of the fund itself.

Market segmentation

A further service offered to closed-end fund investors is access to foreign investments through so-called “country funds”. Again, it is hypothesized, if investors value such a service, because it offers investment opportunities not otherwise available to them, they may be prepared to reduce the rate at which they discount a country fund’s projected value. Grullon and Wang, 2001, for example, report that U.S. closed-end funds investing in foreign securities tend to trade at lower discounts than those investing in domestic securities because it is easier for investors to diversify on their own in their home market.

3.2.4 *Perceptions – investor sentiment hypothesis*

We noted, in Section 3.2.1, that personal valuation will be asymmetric if personal expectations match those of the market but personal and market discount rates differ. Similarly, closed-end funds will trade at other than market price if expectations for the fund match those of its underlying portfolio, but the rates at which fund and portfolio projected values are discounted differ.

An alternative explanation for the puzzle focuses on the fund discount rate which, it is hypothesized, exceeds the expected portfolio return. We noted, in Section 2.2.2, that investors can be categorized as rational or as irrational (“noise traders”) and that noise traders are prone to “investor sentiment” – expectations about future cash flows and risk not justified by the information available to investors. The hypothesis here is that if there is a higher concentration of irrational investors in the ownership of a fund than in the ownership of the fund’s underlying portfolio of assets, and if there are limits to arbitrage, investing in the fund will be riskier than holding the same portfolio directly. As a consequence, the rate of return required on the fund (the discount rate used) by investors will be higher than that of its portfolio (the projected growth rate). Assuming the expected fund return matches that of its portfolio, the market value of the fund will be lower than its NAV. Movements in the level of the share price discount to NAV over time, it is hypothesized, reflect fluctuating noise trader expectations (Lee *et al*., 1991).

Assume that $1 invested in the fund at NAV is projected to grow at a compound annual rate of 6%, net of expenses, to $5.74 at the end of year 30. Assume also that, because of noise trader risk, rational investors in the fund require an annual rate of return of 7%. Discounting the projected $5.74 at a 7% discount rate gives a current market value of $0.75, a 25% discount to current NAV. (Note that the investor sentiment hypothesis does not imply that asymmetric valuation is necessarily irrational.)

Grullon and Wang, 2001, highlight a further difference in risk perception between fund and underlying portfolio investors. They suggest that closed-end fund discounts can arise when the quality of private information (such as security analysts' reports) possessed by (mainly institutional) investors in the underlying portfolio is better than that possessed by (mainly individual) fund investors. Because the fund is perceived as being riskier than its portfolio, its projected NAV is discounted at a rate higher than the expected return on its portfolio – so the fund trades at a discount to NAV.

4. THE CONNECTION BETWEEN PERSONAL AND MARKET VALUATIONS

In Section 2, we explained how personal valuations can differ from market valuations when personal expectations, perceptions and preferences differ from those of the market. The investor sentiment hypothesis assumes that the investors in a fund are distinct from those who invest directly in its portfolio of assets (Lee *et al*., 1991). We might think of the market valuation of a fund as being a personal valuation – the weighted average of all fund investors. If we do so, we might not be too puzzled that the market value of the fund should differ from its NAV. Indeed, we might attribute it to differences between the expectations, perceptions and preferences of those who invest in the fund and those who invest directly in its portfolio of assets.

We might expect, then, time and risk preferences to play a role in the valuation of closed-end funds. As in Section 2.2.2, think of the two temporal value curves in Chart 1 as recording the market's expectations of the NAVs of two investment funds. These expectations reflect the market's time and risk preferences. As before, assume that the market perceives that Hyperbolic is more risky than Exponential in the short term but less risky in the long term. Assume, too, that personal expectations of the future values of the two funds match those of the market as do personal risk perceptions. We might expect, for example, that:

- conservative investors (being both more patient and more risk averse than the market) will value Hyperbolic more highly than Exponential;
- short-sighted, prudent investors (being less patient and more risk averse than the market) will discount both investments more heavily than does the market, thereby personally valuing them at less than market value;
- hotheads (who are impatient gamblers) will favour speculation over investment in either fund; and
- enterprising investors (who are patient and risk seeking) might personally value a $1 invested in Exponential more highly than a $1 invested in Hyperbolic.

The actual market value of the fund will reflect the relative weight of these different investor types in the fund.

We might expect similar valuation differences if individual funds have distinct shareholder bases. If, for example, individual investors suffer more existential unease, have more loosely connected temporal selves and are more impatient and more risk averse than institutional investors, those investment funds which are owned mainly by retail investors might be expected to trade at wider share price discounts to NAV than do those with predominantly institutional shareholder bases.

In section 3, we highlighted some of the explanations offered for the closed-end fund puzzle. If we think of the market valuation of a fund as being a personal valuation, we might expect that these explanations would provide insights into how differing expectations, perceptions and preferences can lead to differing personal valuations. We might expect investor sentiment, for example, to be reflected not just in irrational expectations, but also in investor misperceptions and in extreme time and risk preferences. We noted that, if small investors are otherwise excluded from the investment services' opportunities offered by a closed-end fund, they might value such services more highly than do large investors in the fund's underlying portfolio. Accordingly, we might add leverage preference, liquidity preference and market segmentation preference to risk preference and time preference as rational explanations for asymmetric personal discounting.

5. CONCLUSION

In this paper, we have portrayed the valuation of financial investments as mental time travel. We began with exponential and hyperbolic valuation. We explained how the temporal value curves in Chart 1 can be read both as projecting forward an initial value and as discounting that projected value back to the present. So read, we have two pairs of curves (one pair exponential, one pair hyperbolic) each, in turn, comprising two curves identical save for the direction of travel. Each pair of curves traces a mental time travel journey, or thought investment, in which an initial value is instantaneously mentally simulated forward 30 years and then back to a current value. It is important to note that no objective time passes in these thought investments.

We called the exact matching of projection and discount curves in Chart 1 "symmetric valuation". We showed how, when valued symmetrically: personal value will be the same as market value; and an investment fund's share price will trade at NAV. This is so whether valuation is exponential, hyperbolic or any other method and regardless of the duration of the investment.

These curves record expectations and reflect perceptions and preferences. Personal expectations, perceptions and preferences will either match or differ from those of the market – where they match, personal valuations will match market valuations; where they do not match, personal valuations may differ from market valuations. Likewise, the expectations, perceptions and preferences of an investment fund's investors will either match or differ from those of its underlying portfolio of assets – where they match, the fund's shares will trade at NAV; where they do not match, share price and NAV may differ.

We called the non-exact matching of projection and discount curves "asymmetric valuation". Asymmetry can arise whether valuation is exponential, hyperbolic or any other method. We noted that there might be some combinations of method (such as exponential projection and hyperbolic discounting) where asymmetric valuation does not preclude the possibility that personal value will be the same as market value and an investment fund's share price will trade at NAV. We termed this "asymmetric matching" and we explained how the duration of the investment might be a determining factor of asymmetric matching.

We then explored possible reasons for asymmetric personal valuation which, we suggested, might not be unexpected if projecting and discounting are psychologically distinct. We looked at how personal expectations, perceptions and preferences might differ from those of the market. We suggested that it might be possible to construct investor profiles defined by personal preferences relative to those of the market.

To illustrate our theorizing, we portrayed as mental time travel four of the main hypotheses offered for the closed-end fund puzzle: management expenses; management performance; investment services; and investor sentiment. We showed how asymmetrically valuing a fund results in a fund share price discount, or premium, to NAV.

The management expenses and performance hypotheses propose differing fund and portfolio expectations with a common discount rate (the expected return on the portfolio). Here, for example, a fund might be valued at a discount to its NAV because it is expected to return less than its portfolio. The investor sentiment hypothesis, on the other hand, allows for common expected returns but with different discount rates – the fund discount rate being the expected portfolio return adjusted for noise trader risk. Here, the fund discount rate is greater than its expected return. The investment services hypothesis allows for both fund expectations and discount rates to differ from those of the portfolio.

The investor sentiment hypothesis rests, in part, on the dual assumptions of a fund investor base distinct from that of its portfolio and of a higher concentration of irrational investors in the ownership of a fund than in the ownership of its portfolio. It may be that the former assumption, alone, is sufficient to explain the closed-end fund puzzle. If the investors in a fund differ from those who invest directly in its portfolio, we might expect that the expectations, perceptions and preferences of fund investors will differ from those of direct investors. So, we should not be too surprised that the market value of a fund often differs from its NAV.

Indeed, it may be that even the assumption of a distinct fund investor base is unnecessary to explain the puzzle. It may be sufficient that investors perceive the fund as being an investment distinct from that of its underlying portfolio of assets. If investors' expectations and perceptions of the fund differ from those of its portfolio (as, for example, Grullon and Wang, 2001, discussed above, suggest), we might expect that the valuations of the fund and its underlying portfolio of assets will differ.

The key message, then, of this paper is that financial investment valuation is personal, with market value being no more than a weighted average of personal valuations. Value, therefore, lies in the mind of the beholders that make up the market place. This suggests that market value can differ from "fundamental" (or "intrinsic") value and can do so for extended periods.

If valuation is personal, the key to successful financial valuation lies in understanding how the personal expectations and preferences (of an individual investor or an individual fund's shareholder base) differ from those of the wider market. For example, if the market projects exponentially (at a constant rate r over time), those who discount hyperbolically (at a higher rate than r in the short term and at a lower rate in the long term), might find illiquid, long term investments (so called "golden eggs" such as pensions, real estate, savings bonds and certificates of deposit) more attractive than publicly traded investment funds (see Laibson, 1997). On the other hand, if markets offer higher rates of return for short investment periods than for long ones, those who discount the future at a constant rate will personally value short term securities over long term securities of a similar nature (Lowenstein and Prelec, 1992). (It also has powerful implications for savings behaviour and inducing changes to savings behaviour. For example, people who project hyperbolically but discount exponentially do not value long-term savings and need to be induced (or "nudged") into joining long-term savings plans along lines proposed by Thaler and Benartzi (2004) with their "save more tomorrow" programmes.)

We mentioned in the Introduction that this paper was motivated by a discussion about the relative values of a $1 invested in equities and another $1 invested in bonds. As we have seen, when valued symmetrically, $1 invested in equities has the same value as $1 invested in bonds. But this is rarely so when valuation is asymmetric. As we have explained, differing personal expectations, perceptions and preferences mean that some of us value equities over bonds, some of us prefer bonds, while yet others are happier with a trip down to the betting shop.

As a final point, we would argue that mental time travel is the starting point of all decision-making involving the future. It certainly applies to humans. It might even apply to other animals. Is the lion observing a wildebeest projecting forward – to estimate the benefits – and discounting backwards – to reflect the effort and risks involved – in order to determine the value of an attack?

NOTES

1. We use the formula $(1+r)^T$ to define the exponential projection function for a T-year investment horizon. Chart 1 assumes that r = 6%.
2. We use the formula $(1+\alpha T)^{\gamma/\alpha}$, where γ and α are constants, to define the T-year hyperbolic projection function (Loewenstein and Prelec, 1992, Laibson, 1998). Chart 1 assumes γ = 1 and α = 4.
3. We use the formula $(1+r)^{-T}$ to define the T-year exponential discount function. Chart 1 assumes that r =6%.
4. We use the formula $(1+\alpha T)^{-\gamma/\alpha}$ to define the T-year hyperbolic discount function (Loewenstein and Prelec, 1992, Laibson, 1998). Chart 1 assumes that γ = 1 and α = 4.
5. So long as the discount function associated with a method exists and can be defined as the inverse function of the projection function.
6. As in Chart 1, we use the formula $(1+\alpha T)^{\gamma/\alpha}$, where γ and α are constants, to define the T-year hyperbolic projection function. As in Chart 1, γ = 1 and α = 4. As in Chart 1, we use the formula $(1+r)^{-T}$ to define the T-year exponential discount function. As in Chart 1, r =6%.
7. The quasi-hyperbolic discount function is $\{1, \delta, \delta^2, \delta^3, \ldots \delta^T\}$ for t = 0, 1, 2, 3,…,T, respectively, with δ < 1. Here β = 0.7 and δ = 0.95 approximately, which approximately equates to a year 1 r of 50% and r thereafter of 3% per objective year (equivalent to 6% per subjective year). Derived from Angeletos *et al*., 2001.
8. Gemmill and Thomas, 2017, show that, for a non-income paying fund, the discount to NAV formula is

$$\left[\frac{(1+r-\gamma)}{(1+r-\theta)}\right]^T - 1$$

where γ is the expenses ratio, Θ is the benefit rate received from incurring management services and T is the expected life of the fund. Note that this formula assumes exponential growth and discounting valuation.

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