Distributed Optimal Power Flow for Unbalanced Radial Systems with Time-varying Communication

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***Abstract*—This paper proposes a distributed multi-period optimal power flow (OPF) formulation for unbalanced three-phase radial distribution systems over time-varying communication networks. To this end, we model the three-phase unbalanced network, distributed generators (DG), and electric vehicles' (EV) behaviour with inter-temporal constraints. Moreover, we represent the objectives of the distribution system operator and those of prosumers, e.g., who wish to minimise the cost of DG or the degradation cost of the EV batteries. We first formulate the centralised OPF that requires knowledge of DG costs; EV information in terms of desired energy, departure and arrival times that prosumers are reluctant in providing. Moreover, the computational effort required to solve the centralised OPF in cases of numerous DGs and EVs is very intensive. As such, we propose a distributed solution of the OPF over a time-varying communication network. We illustrate the proposed framework through a 33-bus distribution feeder.**

I. Introduction

Power systems are changing radically with the deepening penetration of distributed generation resources and flexible demand, e.g., electric vehicles (EVs). All these changes are leading towards the development of "fractal" grids that yield flexible, controllable, and interoperable systems that may be operated in an efficient and safe manner [1]. Fractal grids are more complex due to the numerous small scale devices that are connected to the system. As such centralised power system operation approaches are harder to implement due to the computational complexity as well as privacy concerns of the entities involved. Decentralised approaches for all timescales of power system operations, i.e., primary control occurring in real time to optimal power flow (OPF) occurring in an hour to days timescale, have been proposed in the literature to cope with the aforementioned challenges (see, e.g., [2], [3], [4]).

OPF is usually formulated as an optimisation problem with the objective of cost minimisation subject to network and variables' limit constraints. Historically OPF was implemented in transmission systems, since distribution systems were not very "active" with minimum presence of distributed generation and flexible demand. As such, recent efforts have been made in proposing distributed solutions for OPF in distribution systems. In [5] a semidefinite program is proposed with relaxations that are shown to be exact for radial networks to solve the OPF in a distributed manner. In [6], the authors proposed an alternating direction method of multipliers (ADMM) based algorithm to solve the second-order cone program (SOCP) relaxation of the OPF problem for balanced radial networks over a time-invariant communication network. In [7], the authors proposed a distributed algorithm to solve the SOCP OPF relaxation for radial distribution systems over time-varying networks. The authors in [8] developed a chordal conversion based convex iteration algorithm to solve the three-phase OPF problem that improves computational efficiency and guarantees optimality in some distribution feeders. Inter-temporal constraints and privacy concerns are not addressed in this paper. In [9] a distributed strategy for the optimal dispatch of islanded microgrids where units can communicate only with their neighbours is presented; the primary, secondary as well tertiary control is modelled, however, without taking network effects into account. The authors in [10] used a machine learning model to reconstruct the optimal set points to OPF problems; thus developing a decentralized OPF methodology.

In this paper, we propose a distributed multi-period OPF formulation for unbalanced three-phase distribution systems over time-varying communication networks. The entities that participate in the OPF are the distribution system operator (DSO), distributed generators (DGs) and EV owners. The objective of the DSO is to minimise cost subject to the physical limits of the network. A linear model is used for the modelling of three-phase unbalanced networks, which is based on a fixed-point interpretation of the AC power flow equations [11]. DGs wish to minimise their cost of operation while the generators' operation is within their limits. Last, EV owners wish to charge their vehicle at minimum cost without degrading their battery while operating within its charging rate limits. We use the aforementioned objectives and constraints to formulate the centralised OPF. Such a formulation requires knowledge of DG costs; EV information in terms of desired energy, departure and arrival times that prosumers are reluctant in providing. Moreover, the computational effort required to solve the centralised OPF in cases of numerous DGs and EVs is very intensive. In this regard, we propose a distributed solution of the multi-period OPF over a time-varying communication network using the algorithm presented in [12]. The contributions of the paper are as follows: development of a distributed OPF methodology that (i) allows for inter-temporal constraints, e.g., EV charging constraints; (ii) explicitly models unbalanced three-phase networks; (iii) guarantees convergence to the set of optimal primal solutions under a time-varying communication network; and (iv) maintains privacy concerns by using only local information.

II. PRELIMINAIRIES

A. Network Modelling

A linear model is used for the modelling of three phase unbalanced networks, as described in [11]. The authors have validated its accuracy compared to a full AC power flow. Let us assume that the system has N_{bus} three-phase buses denoted by the sets $\mathscr{N}_{\text{bus}} = \{1, \ldots, N_{\text{bus}}\}$ and the phases $\Phi = \{a, b, c\}$; and ℓ lines denoted by the set $\mathscr{L} = \{1, \ldots, \ell\}$. The study period is denoted by $\mathscr{T} = \{1, \ldots, T\}$ with T intervals of size Δt. For simplicity, we assume that the network has only phase to line connections. We denote by $Y \in \mathbb{C}^{3N_{\text{bus}} \times 3N_{\text{bus}}}$ the admittance matrix; by $s \in \mathbb{C}^{3N_{\text{bus}}}$ the complex power injections at each bus and by $v \in \mathbb{R}^{3N_{\text{bus}}}$ the magnitude of the bus complex voltages. We assume node 0 is the slack bus and partition the admittance matrix and the voltage magnitude vector as follows $Y = \begin{bmatrix} Y_{00} & Y_{0L} \\ Y_{L0} & Y_{LL} \end{bmatrix}$, where $Y_{00} \in \mathbb{C}^{3\times3}$, $Y_{L0} \in \mathbb{C}^{3(N_{\text{bus}}-1)\times 3}$, $Y_{0L} \in \mathbb{C}^{3\times 3(N_{\text{bus}}-1)}$, and $Y_{LL} \in \mathbb{C}^{3(N_{\text{bus}}-1)\times 3(N_{\text{bus}}-1)}$; and $v = [v_0, v_L]^\top$ where $v_0 \in \mathbb{R}^3$ is the slack bus voltage magnitude and $v_L \in \mathbb{R}^{3(N_{\text{bus}}-1)}$ the voltage magnitudes at remaining buses. Let us assume that the real (reactive) power injections are denoted by $p \in \mathbb{C}^{3(N_{\text{bus}}-1)}$ ($q \in \mathbb{C}^{3(N_{\text{bus}}-1)}$) and the real (reactive) power phase to line load is denoted by $p_d \in \mathbb{C}^{3(N_{\text{bus}}-1)}$ ($q_d \in \mathbb{C}^{3(N_{\text{bus}}-1)}$) for all buses than the slack bus, i.e., $\forall\ n \in \mathscr{N}_{\text{bus}}/\{0\}$. We consider D DG owners denoted by the set $\mathscr{D} = \{1, \ldots, D\}$. In order to determine the location of the DGs, we need to determine the node and the phase that they are connected to. To this end, each DG owner $r \in \mathscr{D}$ has a duplet $\mathscr{R}_r = \{n_r, \phi_r\}$, where $n_r \in \mathscr{N}_{\text{bus}}$ is the node that the DG is connected to and $\phi_r \in \Phi$ the phase. We also consider a collection of E EVs denoted by the set $\mathscr{E} = \{1, 2, \ldots, E\}$. Similarly, in order to determine the location of the EVs, we need to determine the node and the phase that they are connected to. To this end, we define for each EV $j \in \mathscr{E}$ the duplet $\mathscr{H}_j = \{n_j, \phi_j\}$, where $n_j \in \mathscr{N}_{\text{bus}}$ is the node that the EV is connected to and $\phi_j \in \Phi$ the phase. The set of all duplets for DGs and EVs are denoted by $\mathscr{H}$ and $\mathscr{R}$ respectively. We denote by $y_j(t)$ the charging power of EV j at time t; and $\tilde{y}(t) \in \mathbb{R}^{3N_{\text{bus}}}$ vector that has zero entries for buses and phases that do not have an EV, and is $y_j(t)$ for bus n_j and phase ϕ_j as determined by the duplet $\mathscr{H}_j = \{n_j, \phi_j\}$.

The fixed-point linearisation around a nominal point $(\hat{s}, \hat{v})$ renders the following relationships for the network representation:

$$v(t) = \quad K \begin{bmatrix} p(t) - p_d(t) - \tilde{y}(t) \\ q(t) - q_d(t) \end{bmatrix} + b, \forall t \in \mathscr{T}, \tag{1}$$

where $K = \text{diag}(h)\text{Re}(\text{diag}(h)^{-1}M)$, $b = |h|$, with $M = \begin{bmatrix} 0_{3\times3(N_{\text{bus}}-1)} & 0_{3\times3(N_{\text{bus}}-1)} \\ Y_{LL}^{-1}\text{diag}(\bar{\hat{v}}_L)^{-1} & -jY_{LL}^{-1}\text{diag}(\bar{\hat{v}}_L)^{-1} \end{bmatrix}$, and $h = \begin{bmatrix} \hat{v}_0 \\ -Y_{LL}^{-1}Y_{L0}\hat{v}_0 \end{bmatrix}$, where $\text{Re}(\cdot)$ denotes the real part of a complex number and $\overline{(\cdot)}$ its conjugate. The complex power at the substation denoted by $s_0 = p_0 + jq_0 \in \mathbb{C}^3$ is given by:

$$s_0(t) = \quad G \begin{bmatrix} p(t) - p_d(t) - \tilde{y}(t) \\ q(t) - q_d(t) \end{bmatrix} + c, \forall t \in \mathscr{T}, \tag{2}$$

where $G = \text{diag}(\hat{v}_0)\overline{Y}_{0L}\overline{M}$, and $c = \text{diag}(\hat{v}_0)\left(\overline{Y}_{00}\bar{\hat{v}}_0 - \overline{Y}_{0L}\overline{Y}_{LL}^{-1}\overline{Y}_{L0}\bar{\hat{v}}_0\right)$. Equation (2) represents two equations, one for the real and one for reactive component.

B. EV Modelling

We introduce the energy consumed by EV j for commuting at period $\mathscr{T}$ by e_j, $j \in \mathscr{E}$. We denote by $\pi_j(t)$ the availability of EV j at time t, i.e., if $\pi_j(t) = 1$ then EV j is available for charging at time t, if $\pi_j(t) = 0$ then EV j is not available. The charging constraints associated with the charging variables are the following:

$$\sum_{t\in\mathscr{T}} \pi_j(t) y_j(t) \Delta t = e_j, \forall j \in \mathscr{E}, \tag{3}$$

which ensures that each vehicle has received the right amount of energy at the end of the time horizon. The initial and final SOC are implicitly represented in (3) by appropriately defining e_j, for $j \in \mathscr{E}$. There are limits associated with each charging power which can be expressed as follows:

$$0 \leq y_j(t) \leq \pi_j(t) y_j^{\max}(t), \tag{4}$$

where $y_j^{\max}(t)$ is the maximum value, e.g., 3.7 kW for slow charging. Equation (4) ensures that at times when the EV j is not available for charging $y_j(t)$ will be zero. In this work, we consider one-directional charging; this can be easily extended to bi-directional charging. The degradation cost of the EV battery is taken into account by minimising the second order polynomial of the charging rates [13]:

$$\sum_{t\in\mathscr{T}} \sum_{j\in\mathscr{E}} y_j^2(t). \tag{5}$$

III. CENTRALISED OPTIMAL POWER FLOW FOR DISTRIBUTION SYSTEMS

A. Objectives

The objectives of the DSO refer to the minimisation of the cost of real power procured at the substation:

$$\sum_{t\in\mathscr{T}} \sum_{\phi\in\Phi} \lambda_0(t) p_0^{\phi}(t) \Delta t, \tag{6}$$

where $\lambda_0(t)$ is the locational marginal price (LMP) at the substation at time t and $p_0^{\phi}(t)$ is the injection at phase ϕ at time t at the substation and Δt is the time interval, e.g., 5 minutes. A byproduct of (6) is that each EV $j \in \mathscr{E}$ procures the desired energy e_j at minimum cost, as stated in Section II-B. The DSO objective also includes a term that ensures that voltage levels throughout the network are operating close to the reference voltage:

$$\sum_{t\in\mathscr{T}} \sum_{n\in\mathscr{N}_{\text{bus}}} \sum_{\phi\in\Phi} (v_n^{\phi}(t) - v_{\text{ref}})^2, \tag{7}$$

where v_{ref} is the reference voltage. The objective of DG owners $r \in \mathscr{D}$ is that the cost of DG is minimised, which may be defined as follows:

$$\sum_{t\in\mathscr{T}} \sum_{r\in\mathscr{D}} c_r(t) p_{n_r}^{\phi_r}(t) \Delta t, \tag{8}$$

where $c_r(t)$ is the cost of DG generation connected to node n_r and phase ϕ_r as defined by the duplet $\mathscr{R}_r = \{n_r, \phi_r\}$at time

t. The EV owners wish to minimise the effect of charging in the degradation of EV batteries, as given in (5).

B. Constraints

The network constraints are represented in (1), (2). The EV charging related constraints given in (3), (4) that describe the inter-temporal state of charge dynamics, non-negativity and charging rate constraints are also included. The voltage magnitude constraints are denoted by

$$v_n^{\phi,\min} \leq v_n^{\phi}(t) \leq v_n^{\phi,\max}, \forall n \in \mathscr{N}_{\text{bus}}, \phi \in \Phi, \forall t \in \mathscr{T}. \tag{9}$$

The real and reactive power injections by DG are:

$$p_n^{\phi,\min} \leq p_n^{\phi}(t) \leq p_n^{\phi,\max}, \forall \{n,\phi\} \in \mathscr{R}, \forall t \in \mathscr{T}, \tag{10}$$
$$q_n^{\phi,\min} \leq q_n^{\phi}(t) \leq q_n^{\phi,\max}, \forall \{n,\phi\} \in \mathscr{R}, \forall t \in \mathscr{T}. \tag{11}$$

For $\{n,\phi\} \notin \mathscr{R}$ we have $p_n^{\phi} = q_n^{\phi} = 0$.

C. OPF Formulation

The decision variables for each $n \in \mathscr{N}_{\text{bus}}$ and $\phi \in \Phi$ is the real power injection $p_n^{\phi}(t)$; the reactive power injection $q_n^{\phi}(t)$; the voltage magnitude $v_n^{\phi}(t)$; and for each EV $j \in \mathscr{E}$ is the charging schedule $y_j(t)$, for all $t \in \mathscr{T}$. The centralised OPF is formulated as follows:

$$\begin{aligned} \min_{\substack{\{p_n^{\phi}(t), q_n^{\phi}(t), v_n^{\phi}(t), y_j(t)\} \\ t \in \mathscr{T}, n \in \mathscr{N}_{\text{bus}}, \phi \in \Phi, j \in \mathscr{E}}} \quad & (5) + (6) + (7) + (8) \\ \text{subject to } & (1) - (4), (9) - (11). \end{aligned} \tag{12}$$

The proposed framework can directly capture the case where positive weights are attached to each term in the objective function; minimising the weighted sum of objective functions; and obtaining different points on the Pareto front of (12). This is constitutes a topic of current research. The coefficients of (5) and (7) may be interpreted as the cost (in pounds) of EV battery degradation and voltage deviation from the reference respectively. So then the objectives are expressed in the same units; thus comparable.

IV. Proposed Distributed Optimal Power Flow for Distribution Systems

We consider $K+1$ entities, i.e., agents, that participate in the OPF solution. These are K agents that are either DG or EV owners or both and one agent that is considered to be the DSO. We assume that the communication network that these agents use to exchange information is time-varying as is in reality. The optimisation problem given in (12) may be seen as an optimisation problem where each agent optimises a local objective subject to local constraints, but needs to agree with the other agents in the network on the value of some decision variables that refer to the usage of shared resources, i.e., the power at the substation and the network usage, which are represented by coupling constraints. More specifically, each agent i has its own vector $x_i \in \mathbb{R}^{n_i}$ of n_i decision variables, e.g., the voltage magnitude, the charging schedule; its local linear constraint set $A_i x_i = b_i$ and $D_i x_i \leq 0$, these include constraints such as (3), (4), (9)-(11); and its objective $f_i(x_i) : \mathbb{R}^{n_i} \to \mathbb{R}$, e.g., (5), (6)-(8). The coupling constraints refer to (1) and (2); (1) has $3N_{\text{bus}}T$ constraints and (2) has $6T$ (since (2) refers to two equality constraints per time step) thus in total the coupling constraints are $3T(N_{\text{bus}}+2)$. We denote the total number of coupling constraints by $w = 3T(N_{\text{bus}}+2)$ and the coupling constraints as $\sum_{i=1}^{K+1} Z_i x_i = \zeta$, where $Z_i \in \mathbb{R}^{w \times n_i}$ and $\zeta \in \mathbb{R}^{w}$. Each agent contributes to the coupling constraints with Z_i. Now we may rewrite (12) in compact form as

$$\begin{aligned} \min_{\{x_i\}_{i=1}^{K+1}} \quad & \sum_{i=1}^{K+1} f_i(x_i) \\ \text{subject to } & A_i x_i = b_i, i = 1, \ldots, K+1, \\ & D_i x_i \leq 0, i = 1, \ldots, K+1, \\ & \sum_{i=1}^{K+1} Z_i x_i = \zeta, \end{aligned} \tag{13}$$

where A_i, b_i, D_i for each agent i are defined as follows for the three different types of agents, i.e., DSO, DG owners and EV owners. The nodes/phases that do not contain an EV or DG are considered to be a responsibility of the DSO agent and are equal to $3N_{\text{bus}} - E - D$. The DSO agent, which without loss of generality, is indexed by 1, has a decision variable $x_1 \in \mathbb{R}^{n_1}$ with $n_1 = 3(3N_{\text{bus}} - E - D)T$. More specifically:

$$x_1 = [p_n^{\phi}(t), q_n^{\phi}(t), v_n^{\phi}(t) : \{n,\phi\} \notin \mathscr{H} \cup \mathscr{R}, n \in \mathscr{N}_{\text{bus}}, \phi \in \Phi, t \in \mathscr{T}]^{\top}.$$

Its objective is defined as $f_1 = \sum_{\substack{\{n,\phi\} \notin \{\mathscr{H} \cup \mathscr{R}\} \\ n \in \mathscr{N}_{\text{bus}}, \phi \in \Phi \\ t \in \mathscr{T}}} f_n^{\phi}(t)$, where

$$f_n^{\phi}(t) = \begin{cases} (v_n^{\phi}(t) - v_{\text{ref}})^2, \{n,\phi\} \notin \{\mathscr{H} \cup \mathscr{R}\}, n \in \mathscr{N}_{\text{bus}}, \phi \in \Phi, \\ \lambda_0(t) p_0^{\phi}(t) + (v_0^{\phi}(t) - v_{\text{ref}})^2, n = 0, \phi \in \Phi \end{cases}.$$

The limiting constraints for agent 1 given in (9)-(11) may be represented as the matrix $D_1 = [C_n^{\phi}(t) : \{n,\phi\} \notin \{\mathscr{H} \cup \mathscr{R}\}, n \in \mathscr{N}_{\text{bus}}, \phi \in \Phi, t \in \mathscr{T}]^{\top}$, where $C_n^{\phi} \in \mathbb{R}^{6n_1 \times n_1}$, i.e., one block row for the minimum and one for the maximum limit associated with each of the three variables. In this case A_i, b_i are a zero matrix and vector respectively since the DSO agent does not have any local equality constraints. For agent i that is an EV owner of EV j connected to node n_j and phase ϕ_j determined by the duplet $\{n_j, \phi_j\} \in \mathscr{H}_j$, we define the vector $x_i \in \mathbb{R}^{n_i}$ with $n_i = 4T$ by

$$x_i = [p_{n_j}^{\phi_j}(t), q_{n_j}^{\phi_j}(t), v_{n_j}^{\phi_j}(t), y_j(t) : t \in \mathscr{T}]^{\top},$$

and the objective function

$$f_i = \sum_{t \in \mathscr{T}} \left((v_{n_j}^{\phi_j}(t) - v_{\text{ref}})^2 + y_j^2(t) \right).$$

We rewrite (3) as $A_i x_i = b_i$, where $A_i \in \mathbb{R}^{1 \times n_i}$ and $b_i \in \mathbb{R}$ and the limiting constraints given in (4) and (9)-(11) as $D_i = [C_{n_j}^{\phi_j}(t) : t \in \mathscr{T}]^{\top}$, where $C_{n_j}^{\phi_j} \in \mathbb{R}^{8 \times n_i}$, i.e., one block row for the minimum and another one for the maximum limit associated with each of the four variables.

For agent i that is a DG owner $r \in \mathscr{D}$ connected to node n_r and phase ϕ_r determined by the duplet $\{n_r, \phi_r\} \in \mathscr{R}_r$, we define the vector $x_i \in \mathbb{R}^{n_i}$ with $n_i = 3T$ by

$$x_i = [p_{n_r}^{\phi_r}(t), q_{n_r}^{\phi_r}(t), v_{n_r}^{\phi_r}(t) : t \in \mathscr{T}]^{\top},$$

and the objective function

$$f_i = \sum_{t \in \mathscr{T}} \left((v_{n_r}^{\phi_{n_r}}(t) - v_{\text{ref}})^2 + c_r(t) p_{n_r}^{\phi_{n_r}}(t) \right).$$

We rewrite the limiting constraints given in (9)-(11) as $D_i = [C_{n_r}^{\phi_r}(t) : t \in \mathscr{T}]^\top$, where $C_{n_r}^{\phi_r} \in \mathbb{R}^{6 \times n_i}$, i.e., one block row for the minimum and one for the maximum limit associated with each of the four variables. In this case A_i, b_i are a zero matrix and vector respectively since agent i that owns a DG does not have any local equality constraints.

The problem given in (13) satisfies the convexity assumptions and connectivity properties of the communication network of [12]; thus the distributed algorithm below may be used to reach the optimal solution, without disclosing information about their local objective and constraint functions, nor about the function encoding their contribution to the coupling constraint. The proposed method can handle time-varying communication networks since it is based on an extension of dual decomposition based algorithms. Traditional dual decomposition techniques (see, e.g., [14]) or the ADMM (see, e.g., [15]) require time-invariant communication networks since they involve communication among all agents that are coupled via the constraints, which may not be possible in a time-varying connectivity set-up. To this end, notice that the adopted algorithm does not require the dual variable updates to be performed in a centralised manner, and each agent maintains a different estimate of the dual variables. This is in contrast with standard algorithms based on the alternating direction method of multipliers. Moreover, there is no Augmented Lagrangian term, as one would typically encounter in the alternating direction method of multipliers. Moreover, primal-dual subgradient based consensus algorithms (see, e.g. [16]) that could also be used in a time-varying setup assume that coupling constraints are known to all agents, thus violating privacy concerns. As such the proposed framework exhibits attractive features to reach the optimal solution in a time-varying communication network while preserving privacy since agents do not have to share their local information.

A distributed solution of (13) based on [12] is as follows:

Algorithm Distributed OPF

1: **Initialization**
2: $k = 0$.
3: Consider $\hat{x}_i(0)$ such that $A_i \hat{x}_i(0) = b_i$, $D_i \hat{x}_i(0) \leq 0$, for all $i = 1, \ldots, K+1$.
4: Consider $\kappa_i(0) \in \mathbb{R}^w$, for all $i = 1, \ldots, K+1$.
5: **For** $i = 1, \ldots, K+1$ **repeat until convergence**
6: $\ell_i(k) = \sum_{j=1}^{K+1} a_j^i(k) \kappa_j(k)$.
7: $X_i = \{x_i : A_i x_i = b_i,\, D_i x_i \leq 0\}$
$x_i(k+1) \in \arg\min_{x_i \in X_i} f_i(x_i) + \ell_i(k)^\top Z_i x_i$.
8: $\kappa_i(k+1) = \ell_i(k) + c(k)(Z_i x_i(k+1) - \frac{\zeta}{K+1})$
9: $\hat{x}_i(k+1) = \hat{x}_i(k) + \frac{c(k)}{\sum_{\gamma=0}^{k} c(\gamma)}(x_i(k+1) - \hat{x}_i(k))$.
10: $\tilde{x}_i(k+1) = \begin{cases} \hat{x}_i(k+1) & , k < k_{i,s} \\ \frac{\sum_{\gamma=k_{i,s}}^{k} c(\gamma) x_i(k+1)}{\sum_{\gamma=k_{i,s}}^{k} c(\gamma)} & , k \geq k_{i,s} \end{cases}$.
11: $k \leftarrow k+1$.

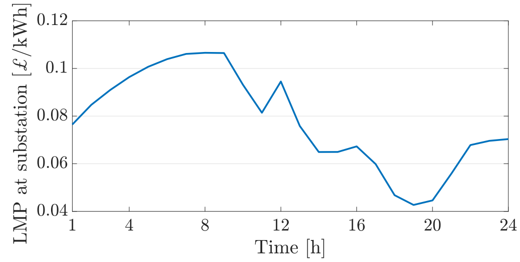

Fig. 1: LMP at the substation over a 24 hour period.

In the Algorithm above w is the row dimension of the Z_i matrices, i.e., the number of coupling constraints which are $w = 3T(N_{\text{bus}} + 2)$, $c(k)$ is the subgradient step-size usually set to $c(k) = \frac{\beta}{k+1}$ for some $\beta > 0$, $k_{s,i} \in \mathbb{N}_+$ is the iteration index related to a specific event, namely, the convergence of the Lagrange multipliers, as detected by agent i. The use of this algorithm ensures that no local information related to the primal problem is exchanged between the agents. In particular, only the estimates of the dual vector are communicated; thus addressing privacy concerns of the agents. The communication network between the agents may be time-varying and has to satisfy the following constraints: $a_j^i(k) \in [0,1)$, for all $k \geq 0$, $\sum_{j=1}^{K+1} a_j^i(k) = 1$, $\forall i = 1, \ldots, K+1$, $\sum_{i=1}^{K+1} a_j^i(k) = 1$, $\forall j = 1, \ldots, K+1$. The interpretation of these constraints is that each agent is mixing information received by other agents at a non-diminishing rate in time. Moreover, this mixing is a convex combination of the other agent estimates, assigning a non-zero weight to its local one. The communication graph is strongly connected, i.e., for any two agents there exists a path of directed edges that connects them. Step 9 of the algorithm is a running average of the primal iterates which are constructed as they are shown to exhibit superior convergence properties with respect to $x_i(k)$ while step 10 performs a reset of this average at a certain iteration index as this has been shown to speed up practical convergence. It has been shown that the dual iterates $\kappa_i(k)$ generated by the algorithm converge (by means of the gradient ascent computation of step 8) to an optimal dual vector which $\hat{x}_i(k)$ achieve asymptotically the optimal objective value. Notice that $\kappa_i(k)$ are mixed in step 6 to generate the weighted average $\ell_i(k)$. The stopping criterion of the algorithm is that the primal variables of the problem do not change (up to a numerical tolerance) across a number of iterations equal to the period of the graph for all agents. More details about the algorithm may be found in [12].

V. Numerical Results

In this section, we use the 141-bus distribution feeder to validate the proposed framework [17]. We consider a collection of 10 EVs and a study period of $\mathscr{T} = \{1, ..., 24\}$

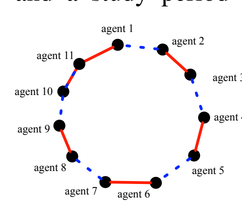

Fig. 2: Time-varying communication network.

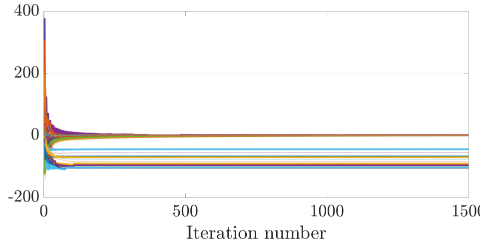

Fig. 3: Evolution of the agents estimates $\{\kappa_i(k)\}_{i=1}^{11}$.

with intervals of size $\Delta t = 1$ h. The minimum (maximum) allowed voltage level is 0.95 pu (1.06 pu). The maximum charging value is $y_j^{\max} = 10$ kW for all $j = 1, \dots, 10$. The LMP at the substation is depicted in Fig 1. In this example, we divide the participants into 11 agents, i.e., the EVs (10) and the DSO. The optimisation problem of the DSO has 9408 decision variables and local constraints set defined by 18816 inequalities. The optimisation problem of each EV has 96 decision variables and local constraints set defined by 1 equality and 192 inequalities. There are 3408 coupling equality constraints, and therefore we have 3408 Lagrange multipliers associated with them. The communication network is depicted in Fig. 2 where approximately only half the agents communicate with the other half at any time-step. More specifically, the communication network corresponds to a graph, whose edges are divided into two groups: the blue and the red ones, which are activated alternatively. We set $c(k) = \frac{1000}{k+1}$, $k_{i,s} = 600$ for $i = 1, \dots, 11$. We ran the proposed algorithm for 1500 iterations with $\kappa_i(0) = 0$, $i = 1, \dots, 11$ and the evolution of the Lagrange multipliers is depicted in Fig. 3. As we may see they converge to the optimal value from around 600 iterations. In Fig. 4 the evolution of the objective value is depicted. We may see a jump at iteration number $k_{i,s} = 600$ for $i = 1, \dots, 11$ since the Lagrangian multipliers have converged and we only use estimates for $\hat{x}_i(k)$ based on values after iteration $k_{i,s}$. According to step 10 of the proposed distributed algorithm the "jump" at iteration $k_{i,s}$ speeds up practical convergence by "resetting" the running average estimate. The run time for this specific case study is 3.7 minutes in a Macbook Pro with 3.1 GHz Dual-Core Intel Core i5 with 8 GB memory.

VI. Conclusions

In this paper, we developed a distributed multi-period OPF for radial distribution systems. More specifically, we represented the unbalanced three-phase network; we provided a detailed modelling of EVs, i.e., representing their times of arrival and departure, SOC, required energy, inter-temporal constraints and objectives. Next, we formulated the centralised multi-period OPF that incorporates the objectives and constraints of DGs and EVs; and has a detailed representation of the underlying three-phase power network. We proposed a distributed solution to the multi-period OPF that converges to the optimal solution under a time-varying communication network with no exchange of sensitive information. Through the numerical examples, we demonstrated that the proposed framework performs well and the optimal solution is achieved.

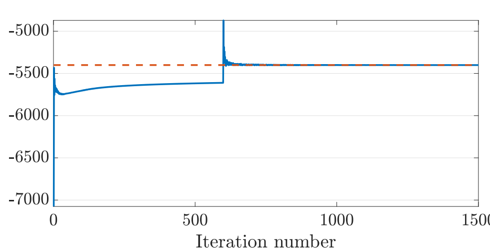

Fig. 4: Evolution of objective function until it reaches the optimal value (red line).

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