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Deposit Requirements in Auctions*

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Abstract

We examine optimal auction design when buyers may receive future outside offers. The winning bidder may choose to default upon observing her outside offer. Under the optimal mechanism, the bidder with the highest value wins if and only if her value is above a cutoff, and the winner never defaults. The optimal auction takes the form of a second-price auction with a reserve price and a deposit by the winning bidder. Under regularity conditions, both the optimal reserve price and the deposit increase when the distribution of outside offers worsens.

Keywords: Truthful direct mechanism; full compliance; outside offers; deposit requirements; reserve prices.
JEL codes: D44.

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1 Introduction

It often takes time for the winning buyer and the seller to complete the transaction with high-value auction items after the auction concludes. During this time, the winner may receive an alternative outside offer and renege on the original transaction if the outside offer is better.[1] In such a situation, it is a common practice for the seller to demand a non-refundable deposit from the winning bidder before settling the final transaction. If the winner defaults on the transaction, the deposit is forfeited. Deposit requirements (normally capped at a certain amount) have been widely adopted by sellers in practice. For instance, sellers in eBay auto auctions are allowed to set a deposit that is no more than $2,000$ US dollars. After paying the deposit, the winners can make final payments within a week or ten days. In UK real estate auctions, a buyer is required to pay a deposit of $6,000$ UK pounds immediately after winning; the buyer then has to make the final payment within twenty eight days.

It is, therefore, worthwhile to examine the impact of a deposit requirement on bidding strategy and the seller's revenue in the auction. A deposit has a direct effect that compensates the seller by making post-auction default costly. However, there is an indirect effect, as bidders who account for a possible future default will adjust their bids in response to the deposit requirement. It is not clear whether the seller benefits from requiring a deposit, and if so, at what level the deposit should be set. In addition, a large body of literature following Myerson (1981) has shown that a reserve price helps screen out low-value bidders and improve seller revenue. We explore the different roles of, and the links between the reserve price and the deposit requirement.

Our investigation starts by examining a class of truthful direct mechanisms, which can be described as follows: A seller would like to maximize revenue by allocating a single object among some bidders. Each bidder's private value is independently drawn from a common distribution. In the first stage, bidders report their values, and at most, one winner is selected to enter the second stage. All bidders discover their random outside offers in the second stage. In the second stage, the chosen winner reports her outside price, and the mechanism decides whether the winner completes the transaction or takes the outside offer. Relying on the necessary condition for first-stage incentive compatibility, we establish an upper bound for seller revenue under this class of mechanisms. Moreover, the upper bound of revenue entails the following allocation rules: In the

[1] A typical example is online auction marketplaces, i.e., eBay, where auctions are listed regularly and sequentially, and it is easy for buyers to conduct post-auction searches for better outside offers before the final transaction. In this case, sellers are faced with buyers' commitment issues. Resnick and Zeckhauser (2002) observe that the most common complaint by sellers in online auctions is that winning bidders do not follow through with the transactions. Dellarocas and Wood (2008) find that 81 percent of the negative feedback given to buyers in eBay auctions results from "bidders who backed out of their commitment to buy the items they won."

first stage, a bidder with the highest value is selected as the winner if and only if her private value is no less than a cutoff value, and in the second stage, the winner never defaults.

We next examine whether and how the upper bound of the seller's revenue can be achieved through a second-price auction with a winner-pay deposit and a reserve price. A winner-pay deposit increases default cost and therefore lowers the winner's incentive to exercise the outside offer. In the auction, the seller first announces the deposit and the reserve price. All bidders place their bids simultaneously, and the bidder with the highest bid wins (given that her bid is no less than the reserve price). The winning bidder then decides whether to pay the deposit before the random price of the outside offer is realized.[2] After the deposit is paid, the winning bidder can either complete the original transaction or take the outside offer and forfeit the deposit.[3] If the deposit is not paid, the winner only decides whether to take the outside offer. When the winner defaults on the original transaction after paying the deposit, she loses the deposit, and the seller keeps the object.

To better understand the impact of the deposit and the reserve price on equilibrium bidding strategies and seller revenue, we first study the optimal reserve price for each level of deposit and then examine the overall effect of a change in the deposit (with the optimal reserve price as a function of the deposit). We find that seller revenue strictly increases with the deposit until a specific cutoff value and then becomes flat. Maximal seller revenue, which equals the upper bound of revenue identified above, is achieved when the deposit is set sufficiently high such that the winner is deterred from defaulting. This result, in turn, indicates that a second-price auction with the identified deposit and reserve price is essentially the optimal design among the class of mechanisms considered.

The intuition of our result is as follows: bidders lower their bids when facing a higher deposit requirement in the auction. But setting a reserve price as the minimum bid excludes low-value bidders, which helps offset the negative effect of the deposit on seller revenue. Thus, it turns out that an auction with relatively low bids and full compliance is more profitable than one with high bids but partial compliance. Consequently, the seller raises the required deposit until the possibility of ex-post default decreases to zero.

To the best of our knowledge, our study is the first to examine the impact of deposit requirements on bidding strategies and seller revenue. Our paper is related to the literatures on auctions without buyer commitment and auctions with outside options, which we now review.

Auctions without buyer commitment. First, our paper is related to the growing literature on

[2] In the auction, the price of the outside offer can be private information for the winning bidder.

[3] In our model, we assume that if the final price is less than the deposit, the seller commits to pay back the difference when the winner chooses to complete the transaction.

auctions without buyer commitment. Asker (2000) studies an auction model in which bidders face uncertainty regarding their final valuations of the object, and this uncertainty can only be resolved after bidding has taken place. He shows that the inclusion of a withdrawal right (allowing the winner to default) raises the seller's expected revenue. Zheng (2001, 2009) considers the situation where bidders facing budget constraints can default on their bids. He shows that the default risk induced by financial constraints affects both equilibrium bidding strategies and seller revenues in auctions. Engelmann, Frank, Koch, and Valente (2015) study an auction model in which the seller can make a second-chance offer to the second-highest bidder if the auction winner fails to complete the transaction. Their analysis shows that the availability of such an offer reduces bidders' willingness to bid in the auction, thus lowering the seller's revenue, even when no default takes place.

Krähmer and Strausz (2015) investigate the effects of the withdrawal right on optimal sales contracts that involve only one buyer and one seller. In their contracting environment, the buyer, after having observed her private valuation, has the choice of either exercising her option as specified in the contract or withdrawing from it and choosing her outside option. Their findings show that the inclusion of default rights is equivalent to introducing ex-post participation constraints in the sequential screening model; even though sequential screening is still feasible with ex-post participation constraints, the seller no longer benefits from it. Instead, the optimal selling contract is static and coincides with the optimal posted price contract in the static screening model.[4]

Our study is closely related to that of Armstrong and Zhou (2016), who study optimal search deterrence in a one-seller-and-one-buyer setting. In their model, the buyer incurs a cost to search for an outside option. They primarily focus on the seller's choice between a buy-it-now discount offer and an exploding offer and study the optimal selling mechanism. Their analysis reveals that, at the optimum, the seller might charge a non-refundable deposit. Our paper is also linked to the literature on auction design with an optimal search.[5] One of the most important findings is from Crémer, Spiegel, and Zheng (2007, 2009), who consider the optimal auction design when the seller controls the set of participating bidders. They show that the optimal selling mechanism would feature fewer participants and more extended searches. Moreover, their papers assume the winner cannot recall the original transaction. On the contrary, our paper considers the case in which the winner can hold the object by paying the deposit and then compare it with the outside offer.

Although our study and the literature on auctions without buyer commitment share a common feature that the auction winner may renege on the original transaction, the focus of our paper differs. We provide a rationale for the seller's adoption of a deposit, which is commonly observed

[4]For example, see related studies by Ben-Shahar and Posner (2011) and Eidenmüller (2011).

[5]For example, see related studies from McAfee and McMillan (1988); Burguet (1996); Lee and Li (2020).

in real-world auctions, and characterize how the seller should set both the deposit and the reserve price in the auction.

Auctions with outside options. Our paper is also part of the literature on auctions with outside options. Cherry, Frykblom, Shogren, List, and Sullivan (2004) conduct a lab experiment to examine whether bidders consider the existence of outside options when formulating their bidding strategies in second-price auctions. Their results show that bidders reduce their bids whenever their resale values exceed the price of the outside option. Kirchkamp, Poen, and Reiss (2009) study the equilibrium bidding behavior of bidders in first-price and second-price auctions with outside options. They show that first-price auctions yield more revenue to sellers than second-price auctions, which may explain why first-price auctions are more common in practice.

Lauermann and Virág (2012) study how the presence of outside options influences whether an auctioneer prefers "opaque" or "transparent" auctions, which differ based on the information that bidders receive. They show that an auctioneer might choose opaque auctions to reduce the values of the bidders' outside options. Figueroa and Skreta (2007, 2009) examine revenue-maximizing auctions for multiple objects, where bidders' outside options depend on their private information and are endogenously chosen by the seller. They show that an optimal mechanism may or may not allocate the objects efficiently.

The existing models consider either only losing bidders having ex-post outside options or all bidders knowing their outside options before bidding. We instead consider a scenario where outside offers arrive after all bidders have submitted their bids, and they are available to all bidders, including the auction winner (i.e., the winner can choose to either complete the original transaction or default and take the outside offer). In addition, our analysis focuses on the role of a deposit requirement in such an environment.

The rest of the paper is organized as follows. In Section 2, we present the model setup and analyze the upper bound on seller revenue. Section 3 examines how a second-price auction with a deposit and a reserve price can achieve the revenue upper bound. Section 4 discusses distributions of the outside offers and the robustness of our results when relaxing some assumptions. Section 5 concludes the paper. The proofs of our main results are in the Appendix, and all the other non-essential proofs and computational details are included in Appendices S1 and S2, which are for online publication.

2 The Model and Analysis of General Mechanism

There are N potential risk-neutral bidders, where $1 < N < \infty$, who compete for an indivisible object. The seller's reservation value of the object is normalized to be zero. Bidders' private values, denoted by v_i, $i = 1, 2, ..., N$, are independent draws from a common atomless distribution $F(\cdot)$ with density $f(\cdot) > 0$ over the support of $[0, \bar{v}]$, where $\bar{v} > 0$. After the competition, each bidder i receives her own outside offer, which gives the same object (thus the same value v_i) but with a random price, denoted by p_i. Prices p_i, $i = 1, 2, ..., N$, are random draws from a common atomless distribution $\Phi(\cdot)$ with density $\varphi(\cdot) > 0$ over $[0, \bar{v}]$. Bidder i's v_i and p_i are independent and private information. We assume that F and Φ are common knowledge among the seller and bidders, and they are *regular* in the sense that the hazard rates $\frac{f(\cdot)}{1-F(\cdot)}$ and $\frac{\varphi(\cdot)}{1-\Phi(\cdot)}$ are increasing.

Instead of studying a specific auction format, we start by examining a class of direct mechanisms with two stages of bidder value reporting and arrival of future outside offers. As mentioned above, the purpose of doing this is to understand how to design the winner selection and the default deterrence, and more importantly, it allows us to explore the upper bound of seller revenue. In Section 3, we will show that the upper bound is indeed achievable by a second-price auction with properly set reserve price and winner-pay deposit. In this sense, focusing on the second-price auction has no loss of generality.

The class of direct mechanisms we consider can be described as follows: In the *first* stage, all bidders are asked to report their values. We use $\mathbf{v}' = (v'_1, v'_2, ..., v'_N)$ to denote the bidders' reports. Among those bidders, at most one winner is selected to participate in the second stage. The probability that bidder i is selected is denoted by $q_i^1(\mathbf{v}') \in [0, 1]$ with $\sum_i q_i^1(\mathbf{v}') \leq 1$ and bidder i's payment to the seller in the first stage is denoted by $m_i^1(\mathbf{v}') \in [0, +\infty)$.

In the *second* stage, the selected bidder observes the price p_i of her outside offer and makes a report, denoted by $p'_i \in [0, \bar{v}]$. The object is sold to the selected bidder if and only if $p'_i \geq \hat{p}_i(\mathbf{v}') \in [0, \bar{v}]$, and the associated payment to the seller in the second stage is denoted by $m_i^2(\mathbf{v}', p'_i) \in [0, +\infty)$. If $p'_i < \hat{p}_i(\mathbf{v}')$, the current transaction is not completed and the selected bidder takes the outside offer by paying p_i. The probability of completing the transaction, denoted by $q_2^i(v'_i, p'_i; \hat{p}_i)$, corresponding to $\hat{p}_i(v'_i)$, can therefore be written as follows:[6]

$$q_i^2(v'_i, p'_i; \hat{p}_i) \; = \; \begin{cases} 1 & \text{if } p'_i \; \geq \; \hat{p}_i(\mathbf{v}'); \\ 0 & \text{if } p'_i \; < \; \hat{p}_i(\mathbf{v}'). \end{cases} \tag{1}$$

[6]In fact, whether the object will be allocated to the selected bidder i in the second stage does not depend on other losing bidders' reports in the first stage, thereby implying that making the deposit a function of all the losing bids in the auction is unlikely to improve the seller's revenue.

We here look for the optimal truthful direct mechanism that maximizes the seller's expected revenue. In this design problem, the seller's choice variables are $\{q_i^1(\mathbf{v}'), m_i^1(\mathbf{v}'); \hat{p}_i(\mathbf{v}'), m_i^2(\mathbf{v}', p_i'); i = 1, 2, ...N\}$. Since we aim to establish an upper bound on seller revenue, in the following analysis, we relax the design problem by assuming that p_i in the second stage is public information. This relaxation would weakly increase the seller's revenue and moreover, by doing so, we can ignore the individual rationality (IR) and incentive compatibility (IC) constraints in the second stage but focus on the design problem in the first stage.

2.1 Implication of incentive compatibility in the first stage

Assuming that all other bidders tell the truth, let us now consider bidder i with value v_i and reporting v_i' in the first stage. In this case, the interim expected payoff of bidder i, denoted by π_i, is given by

$$\pi_i(v_i', v_i) = E_{\mathbf{v}_{-i},\, p_i} \left\{ \begin{array}{c} q_i^1(v_i', \mathbf{v}_{-i}) \Big[q_i^2(v_i', p_i; \hat{p}_i) v_i + (1 - q_i^2(v_i', p_i; \hat{p}_i)) \max\{v_i - p_i, 0\} \; - \; m_i^2(v_i', \mathbf{v}_{-i}, p_i) \Big] \\ + (1 - q_i^1(v_i', \mathbf{v}_{-i})) \max\{v_i - p_i, 0\} - m_i^1(v_i', \mathbf{v}_{-i}) \end{array} \right\}. \tag{2}$$

where the first term in the curly brackets is the expected payoff from being selected to enter the second stage (with probability of $q_i^1(v_i', \mathbf{v}_{-i})$) and the second term in the curly brackets is the expected payoff from not being selected to enter but facing the outside offer (with probability of $1 - q_i^1(v_i', \mathbf{v}_{-i})$).

To guarantee that no one has any incentive to lie about her value, incentive compatibility (IC) requires that, for all i, for all v_i and v_i',

$$\pi_i(v_i, v_i) \geq \pi_i(v_i', v_i),$$

indicating that any untruthful reporting cannot be beneficial for the bidder in the mechanism.

When bidder i with private value v_i chooses not to make a report (or always report zero) in the first stage but faces the outside offer directly, her expected payoff from the outside offer is given by $\int_0^{v_i}(v_i - p_i)\varphi(p_i)dp_i$, which can be simplified as $\int_0^{v_i} \Phi(p_i)dp_i$. Individual rationality (IR) requires that a bidder should not be better off by not participating, that is, for all i and v_i,

$$\pi_i(v_i, v_i) \geq \int_0^{v_i} \Phi(p_i)dp_i.$$

In particular, a bidder with value 0 will get zero payoff from her outside offer, i.e., we must have $\pi(0,0) \geq 0$ from IR.

We then have the following lemma (See Appendix for the proof).

Lemma 1. *If the mechanism is incentive compatible, then bidder i's expected payoff can be expressed as follows:*[7]

$$\pi_i(v_i, v_i) \;=\; \pi_i(0,0) + \int_0^{v_i} E_{\mathbf{v}_{-i},\, p_i} \left\{ \begin{array}{c} q_i^1(t, \mathbf{v}_{-i})\Big[q_i^2(t, p_i; \hat{p}_i) + (1 - q_i^2(t, p_i; \hat{p}_i))\mathbf{1}\{t - p_i \geq 0\}\Big] \\ + (1 - q_i^1(t, \mathbf{v}_{-i}))\mathbf{1}\{t - p_i \geq 0\} \end{array} \right\} dt. \tag{3}$$

2.2 An upper bound on seller revenue and full deterrence

Define virtual value as follows:

$$\lambda(v_i, p_i) \;=\; \begin{cases} p_i & \text{if } p_i \;\leq\; v_i; \\ J(v_i) & \text{if } p_i \;>\; v_i. \end{cases} \tag{4}$$

where $J(v_i) \equiv v_i - \frac{1-F(v_i)}{f(v_i)}$. Let us denote the seller's revenue function by R and given Lemma 1, we can then establish the following lemma (See Appendix for the proof).

Lemma 2. *The seller's revenue function R is given by*

$$R \;=\; E_{\mathbf{v}} \sum_i E_{p_i \leq v_i} \left\{ \left[v_i - \frac{1 - F(v_i)}{f(v_i)} \right] - p_i \right\} \;+\; E_{\mathbf{v}} \sum_i q_i^1(v_i, \mathbf{v}_{-i}) E_{p_i} \Big\{ q_i^2(v_i, p_i; \hat{p}_i) \lambda(v_i, p_i) \Big\}. \tag{5}$$

Now we are ready to establish an upper bound for seller revenue and characterize features of the associated allocation rules. Let $\check{v}^{so}$ and r^{so} be uniquely determined by

$$\begin{aligned} \frac{1 - F(\check{v}^{so})}{f(\check{v}^{so})}(1 - \Phi(\check{v}^{so})) \;&=\; \int_0^{\check{v}^{so}} [1 - \Phi(p_i)] dp_i, \;\text{ and} \\ r^{so} \;&=\; \int_0^{\check{v}^{so}} [1 - \Phi(p_i)] dp_i. \end{aligned} \tag{6}$$

Let us further use $v^{(1)} \equiv \max\{v_i, i = 1, 2, ..., N\}$ to denote the highest value among bidders and set

[7] This lemma can also be obtained by appealing to the version of the envelope theorem in Milgrom and Segal (2002).

the following allocation rules:

$$\hat{p}_i^{so}(v_i, \mathbf{v}_{-i}) \;=\; \begin{cases} 0 & \text{if } v_i \;\geq\; \check{v}^{so}; \\ \bar{v} & \text{if } v_i \;<\; \check{v}^{so}, \end{cases} \quad \text{and} \quad q_i^{so}(v_i, \mathbf{v}_{-i}) \;=\; \begin{cases} 1 & \text{if } v_i = v^{(1)} \text{ and } v_i \geq \check{v}^{so}; \\ 0 & \text{otherwise.} \end{cases} \tag{7}$$

In our analysis, $\check{v}^{so}$ is called the winning threshold, indicating that in the first stage, a bidder with the highest value is selected as the winner if and only if her value is no less than the threshold $\check{v}^{so}$.[8] We therefore have $q_i^1(\mathbf{v}') = 0$ and $m_i^1(\mathbf{v}') = 0$ if $v_i' < \check{v}^{so}$. We then have the following result (See Appendix for the proof).

Proposition 1. *An upper bound of seller revenue under the class of the mechanisms is given by*

$$R^* \;=\; N\big(1 - F(\check{v}^{so})\big)Q(\check{v}^{so})r^{so} \;+\; N\int_{\check{v}^{so}}^{\bar{v}} (1 - F(t))\big(t - \int_0^t \Phi(x)dx\big)dQ(t), \tag{8}$$

where $Q(\cdot) \equiv F^{N-1}(\cdot)$.

Our characterization above shows that to achieve the upper bound of revenue R^*, the seller needs to screen out bidders using threshold $\check{v}^{so}$ in the first stage and then induce full compliance, i.e., $\hat{p}^{so} = 0$, in the second stage. The winning bidder i's virtual value is p_i if p_i is lower than value v_i, and the virtual value is $J(v_i) = v_i - \frac{1-F(v_i)}{f(v_i)}$ when $p_i \geq v_i$. The seller sets a cutoff price $\hat{p}_i$ to maximize the expected virtual values, subject to the constraint that the winning bidder defaults if and only if the outside price is lower than threshold $\hat{p}_i$. Note that the virtual value function as a function of p_i starts from 0 and increases until $p_i = v_i$, and then it drops to a flat level of $J(v_i)$. Therefore, the seller must set $\hat{p}_i$ to either 0 or $\overline{v}$ depending on the level of $J(v_i)$. Since $J(v_i)$ increases with v_i, for $v_i \geq \check{v}^{so}$, the seller finds it optimal to set $\hat{p}_i$ to 0; and, for $v_i < \check{v}^{so}$, it is optimal to set $\hat{p}_i$ to $\overline{v}$. Note that, given that p_i is assumed to be known publicly, the identified maximum of seller revenue gives the upper bound the seller can achieve. In the next section, we will show that a second-price auction with properly designed deposits and reserve prices can still achieve the upper bound, even when p_i is private information.

3 Implementation by SPA with Deposit and Reserve Price

In this section, we consider a second-price auction game with deposit D and reserve price r.[9] Note that charging a deposit increases the default cost of the winning bidder and therefore, it affects

[8] In Proof of Proposition 1, we will provide the derivation of the threshold $\check{v}^{so}$.

[9] See the formal setup and timing of the auction game and characterizations of bidder strategies in online Appendix S1.

the cutoff $\hat{p}_i$ of whether to take the outside offer or complete the original transaction. Specifically, the auction game includes three stages: In stage $t = 0$, the seller sets up the deposit and reserve price. In stage $t = 1$, all the bidders submit simultaneous bids. Only those bids which are no less than the reserve price r are valid. In stage $t = 2$, the winning bidder decides whether to pay the deposit. In stage $t = 3$, an outside offer with price p_i arrives (which is the winning bidder's private information) and then the winner decides whether to default and take the outside offer or complete the current transaction, conditional on paying the deposit D.

Given the auction setup, the characterizations of bidder i's decisions on paying the deposit at $t = 2$ and taking the outside offer at $t = 3$ are straightforward. If the bidder does not win the auction, whether to pay the deposit becomes irrelevant and the only option she faces is the outside offer: taking the offer if $v_i \geq p_i$; otherwise, not to make the purchase. Conditional on winning, bidder i along the equilibrium path pays the deposit and faces two options between the original transaction and the outside offer:[10] the bidder completes the original transaction if the price for the outside offer is no less than the rest of the final payment; otherwise, the bidder takes the outside offer. Based on the strategies at $t = 2$ and $t = 3$, we employ backward induction to derive a bidder's equilibrium bidding strategy $b(v_i)$. Given D and r from the seller, define two cutoff values $\check{v} \in [0, \bar{v}]$ and $\hat{v} \in [0, \bar{v}]$ satisfying the following conditions:

$$r \;=\; \int_0^{\check{v}} [1 - \Phi(p_i)]dp_i, \;\; \text{and} \;\; D \;=\; \int_0^{\hat{v}} [1 - \Phi(p_i)]dp_i.$$

Note that if $b(\bar{v}) \leq r$, we then have $\check{v} \geq \bar{v}$, and no bidder participates. If $b(\bar{v}) \leq D$, we have $\hat{v} \geq \bar{v}$, and no bidder will bid more than the deposit D.

In Case (I) where $r \leq D$, we have $\hat{v} \geq \check{v}$. Bidder i's bidding strategy can be characterized as follows:

$$b(v_i) \;=\; \begin{cases} v_i \;-\; \displaystyle\int_{b(v_i)-D}^{v_i} \Phi(p_i)dp_i & \text{if } \; v_i \;>\; \hat{v}; \\ v_i \;-\; \displaystyle\int_0^{v_i} \Phi(p_i)dp_i & \text{if } \; v_i \in [\check{v}, \hat{v}]; \\ \varnothing & \text{if } \; v_i \;<\; \check{v}. \end{cases} \tag{9}$$

In Case (II) where $r \geq D$, we have $\hat{v} \leq \check{v}$; only the bidder with $v_i \in [\check{v}, \bar{v}]$ will submit a valid

[10]If the winning bidder chooses not to pay the deposit, this is an off-equilibrium behavior, and the seller will interpret the behavior as a default. Strategically, this off-equilibrium behavior is equivalent to the case where the bidder chooses not to bid in the auction but waits for the outside offer directly.

bid. The bidding strategy can be summarized as follows:

$$b(v_i) \;=\; \begin{cases} v_i \;-\; \displaystyle\int_{b(v_i)-D}^{v_i} \Phi(p_i)dp_i & \text{if } \; v_i \;\geq\; \check{v}; \\ \varnothing & \text{if } \; v_i \;<\; \check{v}. \end{cases} \tag{10}$$

Now let us examine the seller's optimal choices of reserve price r and deposit D at $t = 0$. We denote the seller's expected revenue by $R(r, D)$, and the expected revenue-maximizing problem with both instruments r and D can be written as follows:

$$\max_{(r,D)\in[0,\bar{v}]\times[0,\bar{v}]} R(r, D). \tag{11}$$

Depending on r and D, the bidders' equilibrium biding strategies are as characterized above. We thus rewrite (11) as follows:

$$\max_{D\in[0,\bar{v}]} \max\left\{ \max_{r:\, r\leq D} R(r, D); \;\; \max_{r:\, r\geq D} R(r, D) \right\},$$

where the sub-problem $\max_{r:\, r\leq D} R(r, D)$ corresponds to Case (I) where $r \leq D$ and the other sub-problem $\max_{r:\, r\geq D} R(r, D)$ corresponds to Case (II) where $r \geq D$. In each case, the seller's revenue equals the auction price if the winner does not default, the deposit if the winner does, and zero in the absence of any valid bids. For notation simplification, we write the seller's expected revenue in the former sub-problem as $\mathbb{E}_S^I[R(r, D)]$ and the latter as $\mathbb{E}_S^{II}[R(r, D)]$ in the following analysis.

In Case (I) where $r \leq D$, we write the bidding strategy in (9) as $b(v_i)$ and $\tilde{b}(v_i)$ for $v_i \in [\check{v}, \hat{v}]$ and $v_i \in (\hat{v}, \bar{v}]$, respectively. Then, the seller's expected revenue $\mathbb{E}_S^I[R(r, D)]$ is given by the following equation. According to Lemma 4 in online Appendix S1, $\check{v}$ is increasing in r but independent of D and $\hat{v}$ is increasing in D but independent of r in Case (I).

$$\begin{aligned} \mathbb{E}_S^I[R(r, D)] \;=\; & N\big(1 - F(\check{v})\big)Q(\check{v})r \;+\; N\int_{\check{v}}^{\hat{v}}\int_{\check{v}}^{v_i} b(x)dQ(x)dF(v_i) \;+\; N\int_{\hat{v}}^{\bar{v}}\int_{\check{v}}^{\hat{v}} b(x)dQ(x)dF(v_i) \\ & +\; N\int_{\hat{v}}^{\bar{v}}\int_{\hat{v}}^{v_i}\Big[\big(1 - \Phi(\tilde{b}(x) - D)\big)\tilde{b}(x) + \Phi(\tilde{b}(x) - D)D\Big]dQ(x)dF(v_i), \end{aligned} \tag{12}$$

where $Q(\cdot) \equiv F^{N-1}(\cdot)$. The first term occurs when only the highest value of all bidders is above $\check{v}$. The second term occurs when the highest and the second-highest values are between $\check{v}$ and $\hat{v}$. The third term occurs when the highest value is higher than $\hat{v}$ while the second-highest value is between $\check{v}$ and $\hat{v}$. The last term occurs when both the highest and the second-highest values are above $\hat{v}$.

In Case (II) where $r \geq D$, the bidding strategy $b(v_i)$ is given by (10), and the seller's expected revenue $\mathbb{E}_S^{II}[R(r,D)]$ is given by the following equation. According to Lemma 4 in online Appendix S1, $\check{v}$ is increasing in both r and D in Case (II).

$$
\begin{aligned}
\mathbb{E}_S^{II}[R(r,D)] \;=\; & N\big(1-F(\check{v})\big)Q(\check{v})\Big[\big(1-\Phi(r-D)\big)r \;+\; \Phi(r-D)D\Big] \\
& +\; N\int_{\check{v}}^{\bar{v}}\int_{\check{v}}^{v_i}\Big[\big(1-\Phi(b(x)-D)\big)b(x)+\Phi(b(x)-D)D\Big]dQ(x)dF(v_i),
\end{aligned} \tag{13}
$$

where the first term occurs when only the highest value of all bidders is above $\check{v}$, while the second term occurs when both the highest and the second-highest values are above $\check{v}$.

Our analysis will be carried out using the following two steps to examine the seller's problem with the instruments (r,D). *Step one*: for each given deposit level, we pin down the optimal reserve price. For this purpose, we analyze the seller's revenue maximization problems separately in Case (I) where $r \leq D$ and Case (II) where $r \geq D$. Let us denote the overall maximized expected seller revenue by $R^*(D)$ and the seller's optimal choice regarding the reserve price by $r^*(D)$. Then, combining the solutions of the two sub-problems gives $R^*(D)$ and $r^*(D)$, that is,

$$
\begin{aligned}
R^*(D) \;&:=\; \max\Big\{\max_{r:\, r\leq D} R(r,D);\; \max_{r:\, r\geq D} R(r,D)\Big\}, \;\text{ and} \\
r^*(D) \;&:=\; \arg\max\Big\{\arg\max_{r:\, r\leq D} R(r,D);\; \arg\max_{r:\, r\geq D} R(r,D)\Big\}.
\end{aligned}
$$

The characterization of $r^*(D)$ helps us identify the connection between the deposit and the optimal reserve price, which shows us how to set the reserve price optimally for any given deposit. *Step two*: we relax the condition so the seller is free to choose any level of deposit and examine the optimal deposit that maximizes $R^*(D)$ with the optimal reserve price $r^*(D)$, that is, $\max_{D\in[0,\bar{v}]} R^*(D)$. By doing so, we obtain the optimal deposit and the associated reserve price.

Following the steps mentioned above, we first separately characterize the seller's optimal choice of the reserve price for a given deposit under Case (I) and Case (II). Before proceeding further, we introduce notation of D^{so} as follows:

$$
D^{so} \;=\; \int_0^{\hat{v}^{so}}[1-\Phi(p_i)]dp_i, \tag{14}
$$

where $\hat{v}^{so} = \bar{v}$. Recall that $\check{v}^{so}$ and r^{so} are defined in (6). We then have $b(\hat{v}^{so}) = D^{so}$ and

$b(\check{v}^{so}) = r^{so}$. Clearly, D^{so} is strictly greater than r^{so}.

We then establish Lemmas 5 – 8 to help us characterize the seller's optimal choice of the reserve price r, given that D is set in the intervals of $[0, r^{so})$, $[r^{so}, D^{so}]$, and $(D^{so}, \bar{v}]$, respectively. Since these lemmas are rather technical, we instead put all the technical proofs for the seller's choice on reserve price for any given deposit in online Appendix S1. Let $r^I(D)$ and $r^{II}(D)$ denote the optimal reserve price maximizing $\mathbb{E}_S^I[R(r, D)]$ under Case (I) where $r \leq D$ and maximizing $\mathbb{E}_S^{II}[R(r, D)]$ in Case II where $r \geq D$, respectively. After obtaining the solutions for the sub-problems separately, we then compare the maximized seller revenues $\mathbb{E}_S^I[R(r^I(D), D)]$ and $\mathbb{E}_S^{II}[R(r^{II}(D), D)]$ across Cases (I) and (II) for a given $D \in [0, D^{so}]$. Note that with $D = r$ at the 45-degree line, $\check{v} = \hat{v}$, and revenue functions in both cases are the same, i.e., $\mathbb{E}_S^I[R(r^I(D), D)] = \mathbb{E}_S^{II}[R(r^{II}(D), D)]$. By doing so, we can establish the following result (See Appendix for the proof).

Proposition 2. *The optimal reserve price $r^*(D)$ can be characterized as follows:*

$$r^*(D) = \begin{cases} r^I(D) & \text{if } D \in [r^{so}, D^{so}]; \\ r^{II}(D) & \text{if } D \in [0, r^{so}). \end{cases} \tag{15}$$

From Proposition 2, it is easy to see that $r^*(D) = r^{so} \leq D$ when $D \in [r^{so}, D^{so}]$ and $r^*(D) \geq D$ when $D \in [0, r^{so})$.[11] In practice, due to (a) the time and/or budget constraints faced by the buyers (bidders), especially for the items with high values, and/or (b) regulations from the market regulators, like those real examples mentioned in the introduction, often an upper limit on the deposit that the seller can charge in the auction is imposed. Our result demonstrates how to set the corresponding optimal reserve price for a given deposit level. Let $\bar{D} \in [0, D^{so})$ denote the upper limit on the deposit, and then the reserve price is given by $r^*(\bar{D})$, as shown in Proposition 2. In this case, the cutoff $\hat{v}$ is lower than $\bar{v}$, and the possibility of default from the winner is not fully deterred.

Proposition 2 further allows us to define the optimal revenue function $R^*(D)$ as follows:

$$R^*(D) = \begin{cases} \mathbb{E}_S^I[R(r^*(D), D)] & \text{if } D \in [r^{so}, D^{so}]; \\ \mathbb{E}_S^{II}[R(r^*(D), D)] & \text{if } D \in [0, r^{so}). \end{cases} \tag{16}$$

[11] To provide some insights on the seller's choices of reserve price r and deposit D, we present numerical example 1 in online Appendix S2. Assume for simplicity that there are only two bidders ($N = 2$) and that both bidder valuations and outside offers are uniformly distributed on the unit interval ($v_i \sim U[0, 1]$ and $p_i \sim U[0, 1]$). Our computations show $r^{so} = 0.33333$ and $D^{so} = 0.5$. Note that if no post-auction outside offer exists, i.e., $\Phi(\cdot) = 0$, the optimal reserve price is 0.5. Moreover, when the seller sets $r \geq 0.5$, no valid bids will be submitted, and the seller's revenue decreases to zero. All details regarding the computations are in online Appendix S2.

Using (16), we move to step two to examine the optimal deposit and the associated reserve price. Suppose that the seller is free to set the level of the deposit in the auction; the following result can then be established (see Appendix for the proof).

Proposition 3. *$\frac{dR^*(D)}{dD} \geq 0$ for any $D \in [0, D^{so}]$. In particular, $\frac{dR^*(D)}{dD} = 0$ when $D = D^{so}$. As a result, the combination of D^{so} and r^{so} defined in (14) and (6) maximizes the seller's overall expected revenue.*

Proposition 3 describes how the overall seller revenue changes with the deposit requirement. A higher deposit requirement induces a higher overall expected revenue to the seller. The maximum is achieved at $D = D^{so}$, and the associated optimal reserve price is given by r^{so}. Then, $R^*(D^{so})$ is given by the following equation:

$$\begin{aligned} R^*(D^{so}) &= \mathbb{E}_S^I[R(r^{so}, D^{so})] \\ &= N\big(1 - F(\check{v}^{so})\big)Q(\check{v}^{so})r^{so} + N\int_{\check{v}^{so}}^{\hat{v}^{so}}\int_{\check{v}^{so}}^{v_i} b(x)dQ(x)dF(v_i) + N\int_{\hat{v}^{so}}^{\bar{v}}\int_{\check{v}^{so}}^{\hat{v}} b(x)dQ(x)dF(v_i) \\ &\quad + N\int_{\hat{v}^{so}}^{\bar{v}}\int_{\hat{v}^{so}}^{v_i}\Big[\big(1 - \Phi(\tilde{b}(x) - D)\big)\tilde{b}(x) + \Phi(\tilde{b}(x) - D)D\Big]dQ(x)dF(v_i). \end{aligned}$$

Given that $\hat{v}^{so} = \bar{v}$ and $b(x) = \int_0^x (1 - \Phi(p_i))dp_i$, the third and fourth terms become zero and the second term can be re-written as $\int_{\check{v}^{so}}^{\bar{v}}(1 - F(x))b(x)dQ(x)$ by changing order of the integration. The equation above can then be simplified as follows

$$\begin{aligned} R^*(D^{so}) &= N\big(1 - F(\check{v}^{so})\big)Q(\check{v}^{so})r^{so} + N\int_{\check{v}^{so}}^{\bar{v}}(1 - F(x))b(x)dQ(x), \\ &= N\big(1 - F(\check{v}^{so})\big)Q(\check{v}^{so})r^{so} + N\int_{\check{v}^{so}}^{\bar{v}}(1 - F(t))\big(t - \int_0^t \Phi(x)dx\big)dQ(t), \end{aligned}$$

showing that $R^*(D^{so})$ is identical to R^* in Proposition 1. Note that charging any deposit $D > D^{so}$ generates the same expected overall revenue as $D = D^{so}$ to the seller. It is clear by (14) that, at the optimum, $D = D^{so}$ implies that the cutoff $\hat{v}^{so}$ is equal to $\bar{v}$; the default possibility for the winning bidder is fully prevented, thereby resulting in full compliance in the auction, i.e., $\hat{p}^{so} = 0$. The analysis here suggests that

Theorem 1. *The second-price auction with D^{so} and r^{so} generates the upper bound of seller revenue R^*.*

Setting a high deposit, on the one hand, makes winning become less attractive and in turn gives bidders incentives to lower their bids in the auction; on the other hand, the deposit requirement increases the winner's default cost, which reduces the likelihood that she will withdraw from the original transaction. Our analysis indicates that the latter positive effect always dominates the former negative effect; the seller's revenue is maximized by setting a sufficiently high deposit to fully deter the possibility of default. In this case, the reserve price also plays an important role in excluding the possibility of selling the item to bidders with very low bids, thereby helping the seller offset the negative effect of the deposit requirement and increase revenue. Figure 4 in online Appendix S1 confirms that it is not optimal to use the deposit requirement alone; given any deposit requirement $D \in [0, D^{so}]$ in the auction, the overall seller revenue with a zero reserve price is strictly less than that of $r^*(D)$. However, if the reserve price is too high, it would (a) screen out bidders with reasonably high values and (b) induce a high ending price, which gives a higher incentive to the auction winner to default; both effects hurt the seller's expected revenue and thus limit the level of the reserve price the seller can set. As shown in Proposition 3, the optimal reserve price is set at r^{so}.

It is also of interest to compare the optimal reserve price r^{so} to that of Myerson (1981). We denote Myerson's optimal reserve price by r^m, which is given by $\frac{1-F(r^m)}{f(r^m)} = r^m$. From the characterization of r^{so} in (6), we have the following equation:

$$\begin{aligned} \frac{1-F(\check{v}^{so})}{f(\check{v}^{so})} &= \frac{\int_0^{\check{v}^{so}}[1-\Phi(p_i)]dp_i}{1-\Phi(\check{v}^{so})} \\ &\geq \frac{\check{v}^{so}(1-\Phi(\check{v}^{so}))}{1-\Phi(\check{v}^{so})} \\ &= \check{v}^{so}. \end{aligned}$$

This implies that $r^m \geq \check{v}^{so}$, as $\frac{1-F(v)}{f(v)}$ is decreasing in v. Furthermore, given that $r^{so} = \int_0^{\check{v}^{so}}[1 - \Phi(p_i)]dp_i \leq \check{v}^{so}$, we have $r^m \geq \check{v}^{so} \geq r^{so}$. Clearly, $r^m = \check{v}^{so} = r^{so}$ if and only if there exists no outside offer, i.e., $\Phi(\cdot) = 0$. Summarizing the comparison yields the following result.

Remark 1. *$r^m \geq \check{v}^{so} \geq r^{so}$. In particular, $r^m = \check{v}^{so} = r^{so}$ only when no outside offer exists, i.e., $\Phi(\cdot) = 0$.*

4 Discussion

We now discuss different distributions of the distribution of the outside offers and the robustness of our results when relaxing some important assumptions.[12]

[12] We also analyze the cases of percentage deposit and deposit proportionally deducted from full payment. See the related discussion in online Appendix S1.

4.1 Distribution of the outside offer

In this section, we study the impact of the outside offer in the auction. When the price of the outside offer is equal to 0 with probability 1, bidders will not enter the auction but choose to wait for the outside offer, regardless of r and D set by the seller. In this case, (14) implies that the optimal deposit D^{so} is 0. Another extreme case is when no outside offer exists for bidders after the auction, i.e., $\Phi(\cdot) = 0$. In this case, the bidding strategy and the seller revenue are the same as those in the standard second-price auction (with Myerson's reserve price). (14) states that the optimal deposit D^{so} in this case is equal to $\overline{v}$. It is clear in both cases that charging any deposit D will not affect the seller's revenue.

Next, we examine how different distributions of the outside offers affect the auction design. Given r and D, let $b(v_i, \Phi_1)$ and $b(v_i, \Phi_2)$ denote the equilibrium bidding functions corresponding to the distributions of the outside offer $\Phi_1(\cdot)$ and $\Phi_2(\cdot)$, respectively; and further denote the optimal reserve price, the optimal deposit, and the overall seller revenue by r_k^{so}, D_k^{so}, and $R^*(r_k^{so}, D_k^{so}, \Phi_k)$, respectively, corresponding to $\Phi_k(\cdot), k = 1, 2$. Our results are stated as follows (see Appendix for the proof).[13]

Proposition 4. *If the distribution of the outside offer worsens (in the sense of first-order stochastic dominance) from the bidders' perspective, i.e.,* $\Phi_1(\cdot) < \Phi_2(\cdot)$,

(i) *the equilibrium bid submitted by a bidder is higher for any given reserve price r and deposit D, that is,* $\check{v}_1 < \check{v}_2$, $\hat{v}_1 < \hat{v}_2$, *and* $b(v_i, \Phi_1) > b(v_i, \Phi_2)$;

(ii) *the seller sets a higher optimal deposit, that is,* $D_1^{so} > D_2^{so}$;

(iii) *the optimal seller revenue is higher, that is,* $R^*(r_1^{so}, D_1^{so}, \Phi_1) > R^*(r_2^{so}, D_2^{so}, \Phi_2)$.

Part (i) of Proposition 4 is intuitive; when the possibility for an attractive outside offer is smaller, the original auction becomes more attractive, which induces the bidders to submit higher bids. Interestingly, part (ii) shows that a worse distribution for the outside offer allows the seller to charge a higher deposit. This result can be explained as follows: A worse distribution induces bidders to bid more aggressively, thereby increasing the probability of post-auction default by the winner. As shown above, it is preferable for the seller to have full compliance in the auction, although it would lower bids from the bidders. Therefore, to deter the winner from defaulting, a

[13] To further illustrate the impacts of the distribution of an outside offer, we present numerical example 2, assuming that bidder i's valuation still follows a uniform distribution, but the distribution of the outside offer takes the form of $\Phi(p_i) = p_i^\alpha$ over $[0, 1]$, where $\alpha \geq 0$. We then compute how a change in α affects the sellers' choices regarding the optimal reserve price r^{so}, the optimal deposit D^{so}, and the optimal expected revenue R^*. The computational results are consistent with the predictions in Proposition 4, and all the details are presented in online Appendix S2.

higher deposit, combined with a higher reserve price (see Proposition 5), must be demanded in the auction. As a result, a worse distribution leads to a higher revenue to the seller, which gives part (iii).

We then turn to the comparisons of the threshold $\check{v}^{so}$ and the optimal reserve price r^{so}. Define function $\tau(v_i)$ as $\tau(v_i) \equiv \frac{\int_0^{v_i}[1-\Phi(p_i)]dp_i}{1-\Phi(v_i)}$. Differentiating $\tau(v_i)$ with respect to v_i yields

$$\begin{aligned}\tau'(v_i) &= 1 + \frac{\int_0^{v_i}(1-\Phi(p_i))dp_i}{1-\Phi(v_i)} \cdot \frac{\varphi(v_i)}{1-\Phi(v_i)} \\ &= 1 + \tau(v_i) \cdot \frac{\varphi(v_i)}{1-\Phi(v_i)} \\ &> 0.\end{aligned}$$

$\tau(v_i)$ is thus increasing in v_i, and $\tau(0) = 0$. If $\Phi_1(\cdot)$ dominates $\Phi_2(\cdot)$ in terms of the hazard rate, i.e., $\frac{\varphi_1(\cdot)}{1-\Phi_1(\cdot)} < \frac{\varphi_2(\cdot)}{1-\Phi_2(\cdot)}$, then $\tau_1(\cdot) < \tau_2(\cdot)$.[14] This indicates that, in order to satisfy $\frac{1-F(\check{v}_1^{so})}{f(\check{v}_1^{so})} = \tau_1(\check{v}_1^{so})$ and $\frac{1-F(\check{v}_2^{so})}{f(\check{v}_2^{so})} = \tau_2(\check{v}_2^{so})$, we must have $\check{v}_1^{so} > \check{v}_2^{so}$ and $r_1^{so} = \int_0^{\check{v}_1^{so}}[1-\Phi_1(p_i)]dp_i > \int_0^{\check{v}_2^{so}}[1-\Phi_2(p_i)]dp_i = r_2^{so}$. This gives us the following result.

Proposition 5. *$\check{v}_1^{so} > \check{v}_2^{so}$ and $r_1^{so} > r_2^{so}$, if $\frac{\varphi_1(\cdot)}{1-\Phi_1(\cdot)} < \frac{\varphi_2(\cdot)}{1-\Phi_2(\cdot)}$. The seller sets a higher optimal reserve price when the distribution of the outside offer worsens (in the sense of hazard rate dominance) from the bidders' perspective.*

4.2 Stochastic number of bidders

Rather than being fixed, the number of bidders may be stochastic. E.g., in internet auctions, an entering bidder does not know the number of rival bidders, but only the distribution of the number of potential bidders. One may then question whether the characterization of the optimal auction design with r and D still holds. First, regardless of whether the participation process is stochastic, conditional on winning, it is still optimal for bidder i to adopt the deposit strategy and the outside offer strategy characterized in (A4) and (A1) in online Appendix S1. Second, the equilibrium bidding strategy still follows the property of the second-price auction mechanism itself, that is, a bidder's bidding strategy does not depend on the number of entering bidders. Thus, if bidders' participation is stochastic, the combination of r^{so} and D^{so} is still optimal. Note that the optimal reserve price r^{so} and deposit D^{so} do not depend on the number of bidders.

[14] This also implies that $\Phi_1(\cdot)$ first-order stochastically dominates $\Phi_2(\cdot)$, that is, $\Phi_1(\cdot) \leq \Phi_2(\cdot)$; see more details in online Appendix B of Krishna (2002).

4.3 English ascending auction format

It is natural to examine whether our result still holds in the English ascending auction format instead of the simultaneous bidding format. We still solve the game by backward induction. Clearly, the optimality of the deposit strategy and the outside offer strategy in (A4) and (A1), respectively, will not change. The equilibrium bidding strategy we constructed is a weakly dominant strategy for bidders (see details in online Appendix S1); in the bidding stage of the English ascending auction, it cannot be optimal for the bidder to stay in after the current price exceeds the equilibrium bid or drop out before the current price reaches the equilibrium bid. Thus, the combination of r^{so} and D^{so} is still optimal if the auction takes the English ascending format.

4.4 Deposit requirement before bidding

Our analysis in the main text focused on the winner-pay deposit. In practice, another type of deposit exists where the seller requires all potential bidders to pay a certain amount as a deposit before submitting bids. After the auction ends, the seller refunds the deposits to all the bidders, except the winner. Then, the winner can choose to either default and lose the deposit or complete the current transaction by paying the final auction price minus the deposit. Such a deposit requirement is commonly used in art or antiques auctions. For instance, Sotheby's requires such a deposit for items. We call it *the all-pay deposit*.

It is then interesting to examine whether these two types of deposit requirements would result in different equilibrium strategies and seller revenues. Across the two deposit requirements, a bidder's strategy regarding the outside offer is the same conditional on winning, and the only difference is the timing of paying the deposit. Although a bidder needs to pay the deposit before bidding with the all-pay deposit requirement, this does not affect the expected surplus of the bidder and the construction of the equilibrium bid since the deposit will be refunded conditional on losing the auction. Thus, we can conclude that, in the auction game, the winner-pay deposit is strategically equivalent to the all-pay deposit requirement. However, this equivalence relies on the seller's commitment to refund the deposits to the losing bidders being credible. This may not always be true in reality; the seller's historical reputation and commitment issue may break down the equivalence.[15]

[15]In short, the ascending bidding procedure in Section 4.3 and the deposit requirement before bidding in Section 4.4 do not affect threshold $\check{v}^{so}$ and full compliance $\hat{p}^{so} = 0$ in the truthful direct mechanism. Therefore, it explains why they render the same optimal auction design.

5 Conclusion

In this paper, we study the role of a deposit requirement in auctions with future outside offers for all bidders and possibly defaulting from the winner. To examine the design problem and identify the upper bound on seller revenue, we start by examining a truthful direct mechanism, which includes two stages of bidder value reporting and arrival of outside offers (public information). Our characterization shows that it is optimal to set up threshold $\check{v}^{so}$ in the first stage and full compliance $\hat{p}^{so} = 0$ in the second stage. We next examine a second-price auction with a deposit and a reserve price, showing that the upper bound of revenue can be achieved when a sufficiently high deposit is charged to deter the winner from default fully. At the same time, a lower optimal reserve price (which is uniquely determined by $\check{v}^{so}$) than that of Myerson (1981) is required. Our study provides a rationale for the widely adopted deposit requirements in auctions where winner default is a recognized concern.

The environment we consider in the current study rules out some possible situations.[16] One relevant situation is that a bidder's private valuation may be positively or negatively correlated with the price of her outside offer. A positive correlation means that a higher outside price is more likely for a higher value. In this case, the auction winner who tends to value the object higher than the losers is less likely to default, and one can expect that a deposit requirement would play a less significant role helping the seller. When the correlation is negative, the winner tends to have a better outside offer, which means that the deposit requirement can play a more effective role in enhancing the seller's expected revenue. It would also be interesting to consider and examine the role of a deposit requirement in a common value or an affiliated private value setting. The mechanism design literature typically assumes independent private information across players due to the full surplus extraction result of Crémer and McLean (1988). With affiliated private values, their insight still applies in principle. As a result, the seller's revenue under the optimal mechanism is unlikely to be achievable by a second-price auction with deposit and reserve price. Another possible extension is to allow correlation in bidders' outside offers, which could arise since some common outside offers might be accessible to all of them. Our analysis should still apply, as bidders' payoffs only depend on their own outside offers. Moreover, the bidders' outside offers might also be correlated to the sellers' reservation values since the same market conditions can affect all of them. Our insight should also be able to accommodate this feature. These extensions are left for future research.

[16]We thank Robert Porter for raising this point.

Appendix: Proofs

In the Appendix, we provide proofs of Lemmas 1 and 2, and Propositions 1, 2, 3, and 4.

Proof of Lemma 1

Given (2), we can construct the following equation

$$\pi_i(v_i', v_i') \;=\; \pi_i(v_i', v_i) \;+\; E_{\mathbf{v}_{-i},\, p_i} \left\{ \begin{array}{c} q_i^1(v_i', \mathbf{v}_{-i}) \Big[q_i^2(v_i', p_i; \hat{p}_i)(v_i' - v_i) \\ +\, (1 - q_i^2(v_i', p_i; \hat{p}_i))(\max\{v_i' - p_i, 0\} - \max\{v_i - p_i, 0\}) \Big] \\ +\, (1 - q_i^1(v_i', \mathbf{v}_{-i}))(\max\{v_i' - p_i, 0\} - \max\{v_i - p_i, 0\}) \end{array} \right\}.$$

Therefore, for $v_i' < v_i$, $\pi_i(v_i', v_i) \leq \pi_i(v_i, v_i)$ and that yields

$$\pi_i(v_i', v_i') \;\leq\; \pi_i(v_i, v_i) \;+\; E_{\mathbf{v}_{-i},\, p_i} \left\{ \begin{array}{c} q_i^1(v_i', \mathbf{v}_{-i}) \Big[q_i^2(v_i', p_i; \hat{p}_i)(v_i' - v_i) \\ +\, (1 - q_i^2(v_i', p_i; \hat{p}_i))(\max\{v_i' - p_i, 0\} - \max\{v_i - p_i, 0\}) \Big] \\ +\, (1 - q_i^1(v_i', \mathbf{v}_{-i}))(\max\{v_i' - p_i, 0\} - \max\{v_i - p_i, 0\}) \end{array} \right\},$$

which we can re-write as follows

$$\begin{aligned} &\frac{\pi_i(v_i, v_i) \;-\; \pi_i(v_i', v_i')}{v_i - v_i'} \\ &\geq\; E_{\mathbf{v}_{-i},\, p_i} \left\{ \begin{array}{c} q_i^1(v_i', \mathbf{v}_{-i}) \Big[q_i^2(v_i', p_i; \hat{p}_i) \;+\; (1 - q_i^2(v_i', p_i; \hat{p}_i))(\max\{v_i - p_i, 0\} - \max\{v_i' - p_i, 0\})/(v_i - v_i') \Big] \\ +\, (1 - q_i^1(v_i', \mathbf{v}_{-i}))(\max\{v_i - p_i, 0\} - \max\{v_i' - p_i, 0\})/(v_i - v_i') \end{array} \right\}. \end{aligned} \tag{17}$$

Again, given (2), we can construct the following equation

$$\pi_i(v_i, v_i) \;=\; \pi_i(v_i, v_i') \;+\; E_{\mathbf{v}_{-i},\, p_i} \left\{ \begin{array}{c} q_i^1(v_i, \mathbf{v}_{-i}) \Big[q_i^2(v_i, p_i; \hat{p}_i)(v_i - v_i') \\ +\, (1 - q_i^2(v_i, p_i; \hat{p}_i))(\max\{v_i - p_i, 0\} - \max\{v_i' - p_i, 0\}) \Big] \\ +\, (1 - q_i^1(v_i, \mathbf{v}_{-i}))(\max\{v_i - p_i, 0\} - \max\{v_i' - p_i, 0\}) \end{array} \right\}.$$

Therefore, for $v_i' < v_i$, $\pi_i(v_i, v_i') \leq \pi_i(v_i', v_i')$ and that yields

$$\pi_i(v_i, v_i) \;\leq\; \pi_i(v_i', v_i') \;+\; E_{\mathbf{v}_{-i},\, p_i} \left\{ \begin{array}{c} q_i^1(v_i, \mathbf{v}_{-i}) \Big[q_i^2(v_i, p_i; \hat{p}_i)(v_i - v_i') \\ +\, (1 - q_i^2(v_i, p_i; \hat{p}_i))(\max\{v_i - p_i, 0\} - \max\{v_i' - p_i, 0\}) \Big] \\ +\, (1 - q_i^1(v_i, \mathbf{v}_{-i}))(\max\{v_i - p_i, 0\} - \max\{v_i' - p_i, 0\}) \end{array} \right\},$$

which we can re-write as follows

$$\frac{\pi_i(v_i, v_i) \ - \ \pi_i(v_i', v_i')}{v_i - v_i'}$$
$$\leq \ E_{\mathbf{v}_{-i},\, p_i} \left\{ \begin{array}{c} q_i^1(v_i, \mathbf{v}_{-i}) \Big[q_i^2(v_i, p_i; \hat{p}_i) \ + \ (1 - q_i^2(v_i, p_i; \hat{p}_i))(\max\{v_i - p_i, 0\} - \max\{v_i' - p_i, 0\})/(v_i - v_i') \Big] \\ + \, (1 - q_i^1(v_i, \mathbf{v}_{-i}))(\max\{v_i - p_i, 0\} - \max\{v_i' - p_i, 0\})/(v_i - v_i') \end{array} \right\}. \tag{18}$$

Combining (17) and (18) gives the left derivative of $\pi_i(v_i, v_i)$:

$$\frac{d\pi_i^-(v_i, v_i)}{dv_i} \quad = \quad E_{\mathbf{v}_{-i},\, p_i} \left\{ \begin{array}{c} q_i^1(v_i, \mathbf{v}_{-i}) \big[q_i^2(v_i, p_i; \hat{p}_i) + (1 - q_i^2(v_i, p_i; \hat{p}_i)) \mathbf{1}\{v_i - p_i \geq 0\} \big] \\ + \, (1 - q_i^1(v_i, \mathbf{v}_{-i})) \mathbf{1}\{v_i - p_i \geq 0\} \end{array} \right\}.$$

Then, let us consider the case where $v_i' > v_i$, we can construct the similar inequalities and obtain the right derivative of $\pi_i(v_i, v_i)$:

$$\frac{d\pi_i^+(v_i, v_i)}{dv_i} \quad = \quad E_{\mathbf{v}_{-i},\, p_i} \left\{ \begin{array}{c} q_i^1(v_i, \mathbf{v}_{-i}) \big[q_i^2(v_i, p_i; \hat{p}_i) + (1 - q_i^2(v_i, p_i; \hat{p}_i)) \mathbf{1}\{v_i - p_i \geq 0\} \big] \\ + \, (1 - q_i^1(v_i, \mathbf{v}_{-i})) \mathbf{1}\{v_i - p_i \geq 0\} \end{array} \right\}.$$

We can hence conclude that $\pi_i(v_i) = \pi_i(v_i, v_i)$ is differentiable everywhere and

$$\frac{d\pi_i(v_i, v_i)}{dv_i} \quad = \quad E_{\mathbf{v}_{-i},\, p_i} \left\{ \begin{array}{c} q_i^1(v_i, \mathbf{v}_{-i}) \big[q_i^2(v_i, p_i; \hat{p}_i) + (1 - q_i^2(v_i, p_i; \hat{p}_i)) \mathbf{1}\{v_i - p_i \geq 0\} \big] \\ + \, (1 - q_i^1(v_i, \mathbf{v}_{-i})) \mathbf{1}\{v_i - p_i \geq 0\} \end{array} \right\}.$$

Taking the integral of its derivative, we then have

$$\pi_i(v_i, v_i) \ = \ \pi_i(0, 0) \ + \ \int_0^{v_i} E_{\mathbf{v}_{-i},\, p_i} \left\{ \begin{array}{c} q_i^1(t, \mathbf{v}_{-i}) \big[q_i^2(t, p_i; \hat{p}_i) + (1 - q_i^2(t, p_i; \hat{p}_i)) \mathbf{1}\{t - p_i \geq 0\} \big] \\ + \, (1 - q_i^1(t, \mathbf{v}_{-i})) \mathbf{1}\{t - p_i \geq 0\} \end{array} \right\} dt.$$

We complete the proof. $\square$

Proof of Lemma 2

Given Lemma 1 and $\pi_i(0,0) = 0$, we can write the bidder i's expected payoff before learning type v_i as follows:

$$\begin{aligned} & \int_0^{\bar{v}} \pi_i(v_i, v_i) f(v_i) dv_i \\ = \quad & \int_0^{\bar{v}} \int_0^{v_i} E_{\mathbf{v}_{-i},\, p_i} \left\{ \begin{array}{c} q_i^1(t, \mathbf{v}_{-i}) \big[q_i^2(t, p_i; \hat{p}_i) + (1 - q_i^2(t, p_i; \hat{p}_i)) \mathbf{1}\{t - p_i \geq 0\} \big] \\ + \, (1 - q_i^1(t, \mathbf{v}_{-i})) \mathbf{1}\{t - p_i \geq 0\} \end{array} \right\} f(v_i) dt dv_i \\ = \quad & E_{\mathbf{v},\, \mathbf{p}} \left(\left\{ \begin{array}{c} q_i^1(v_i, \mathbf{v}_{-i}) \big[q_1^2 i(v_i, p_i; \hat{p}_i) + (1 - q_i^2(v_i, p_i; \hat{p}_i)) \mathbf{1}\{v_i - p_i \geq 0\} \big] \\ + \, (1 - q_i^1(v_i, \mathbf{v}_{-i})) \mathbf{1}\{v_i - p_i \geq 0\} \end{array} \right\} \cdot [\frac{1 - F(v_i)}{f(v_i)}] \right). \end{aligned}$$

Now let us look at the total expected surplus of the seller and buyers, denoted by TE, which is given by

$$TE \;=\; E_{\mathbf{v},\,\mathbf{p}} \sum_i \left[\begin{array}{c} q_i^1(v_i,\mathbf{v}_{-i})\big[q_i^2(v_i,p_i;\hat{p}_i)v_i + (1-q_i^2(v_i,p_i;\hat{p}_i))\max\{v_i-p_i,0\}\big] \\ +\,(1-q_i^1(v_i,\mathbf{v}_{-i}))\max\{v_i-p_i,0\}. \end{array} \right]$$

Note that the total surplus means the sum of payoffs of the seller and the bidders in our paper. Therefore, the expected surplus for the "third party" who offers the outside offer is not counted in. The seller's revenue R is the difference between the total expected surplus and the buyers' expected payoffs, that is,

$$\begin{aligned}
R \;&=\; TE - \sum_i \int_0^{\bar{v}} \pi_i(v_i,v_i) f(v_i) dv_i \\
&=\; E_{\mathbf{v}} \sum_i E_{p_i} \left[\begin{array}{c} q_i^1(v_i,\mathbf{v}_{-i})\big[q_i^2(v_i,p_i;\hat{p}_i)v_i + (1-q_i^2(v_i,p_i;\hat{p}_i))\max\{v_i-p_i,0\}\big] \\ +\,(1-q_i^1(v_i,\mathbf{v}_{-i}))\max\{v_i-p_i,0\} \end{array} \right] \\
&\quad - E_{\mathbf{v}} \sum_i E_{p_i} \left(\left\{ \begin{array}{c} q_i^1(v_i,\mathbf{v}_{-i})\big[q_i^2(v_i,p_i;\hat{p}_i) + (1-q_i^2(v_i,p_i;\hat{p}_i))\mathbf{1}\{v_i-p_i\geq 0\}\big] \\ +\,(1-q_i^1(v_i,\mathbf{v}_{-i}))\mathbf{1}\{v_i-p_i\geq 0\} \end{array} \right\} \cdot [\frac{1-F(v_i)}{f(v_i)}] \right) \\
&=\; E_{\mathbf{v}} \sum_i E_{p_i\leq v_i} \left\{ q_i^1(v_i,\mathbf{v}_{-i}) q_i^2(v_i,p_i;\hat{p}_i)p_i + [v_i - \frac{1-F(v_i)}{f(v_i)}] - p_i \right\} \\
&\quad + E_{\mathbf{v}} \sum_i E_{p_i>v_i} \left\{ q_i^1(v_i,\mathbf{v}_{-i}) q_i^2(v_i,p_i;\hat{p}_i)[v_i - \frac{1-F(v_i)}{f(v_i)}] \right\}.
\end{aligned}$$

Let us further define virtual value as follows:

$$\lambda(v_i,p_i) \;=\; \begin{cases} p_i & \text{if } p_i \;\leq\; v_i; \\ J(v_i) & \text{if } p_i \;>\; v_i. \end{cases}$$

where $J(v_i) \equiv v_i - \frac{1-F(v_i)}{f(v_i)}$. The seller's revenue function R can be re-written as follows:

$$R \;=\; E_{\mathbf{v}} \sum_i E_{p_i\leq v_i} \left\{ \left[v_i - \frac{1-F(v_i)}{f(v_i)} \right] - p_i \right\} \;+\; E_{\mathbf{v}} \sum_i q_i^1(v_i,\mathbf{v}_{-i}) E_{p_i} \left\{ q_i^2(v_i,p_i;\hat{p}_i)\lambda(v_i,p_i) \right\}.$$

We complete the proof. $\square$

Proof of Proposition 1

From (5), we can now identify the cutoff $\hat{p}_i^{so}(v_i, \mathbf{v}_{-i})$ such that the corresponding $q_i^2(v_i, p_i; \hat{p}_i)$ maximizes $E_{p_i}\left\{q_i^2(v_i, p_i; \hat{p}_i)\lambda(v_i, p_i)\right\} = E_{p_i \geq \hat{p}_i}\left\{\lambda(v_i, p_i)\right\}$, that is,

$$E_{p_i \geq \hat{p}_i}\left\{\lambda(v_i, p_i)\right\} = \begin{cases} \displaystyle\int_{\hat{p}_i}^{\bar{v}} J(v_i) d\Phi(p_i) & \text{if } \hat{p}_i \geq v_i; \\ \displaystyle\int_{\hat{p}_i}^{v_i} p_i d\Phi(p_i) + \int_{v_i}^{\bar{v}} J(v_i) d\Phi(p_i) & \text{if } \hat{p}_i < v_i. \end{cases}$$

Differentiating $E_{p_i \geq \hat{p}_i}\left\{\lambda(v_i, p_i)\right\}$ with respect to $\hat{p}_i$ yields

$$\frac{\partial E_{p_i \geq \hat{p}_i}\left\{\lambda(v_i, p_i)\right\}}{\partial \hat{p}_i} = \begin{cases} -J(v_i)\varphi(\hat{p}_i) & \text{if } \hat{p}_i \geq v_i; \\ -\hat{p}_i \varphi(\hat{p}_i) & \text{if } \hat{p}_i < v_i. \end{cases}$$

Clearly, if $J(v_i) \geq 0$, then $\frac{\partial E_{p_i \geq \hat{p}_i}\{\lambda(v_i, p_i)\}}{\partial \hat{p}_i} \leq 0, \forall \hat{p}_i$. Therefore, for $v_i < v_i^M$ where $J(v^M) = 0$, we should have

$$\hat{p}_i^{so}(v_i, \mathbf{v}_{-i}) = 0, \text{ if } v_i \geq v^M.$$

If $J(v_i) < 0$, then $\frac{\partial E_{p_i \geq \hat{p}_i}\{\lambda(v_i, p_i)\}}{\partial \hat{p}_i} \leq 0, \forall \hat{p}_i < v_i$; and $\frac{\partial E_{p_i \geq \hat{p}_i}\{\lambda(v_i, p_i)\}}{\partial \hat{p}_i} > 0, \forall \hat{p}_i > v_i$. Therefore, $\hat{p}_i^{so}(v_i, \mathbf{v}_{-i})$ is either 0 or $\bar{v}$. We thus compare $\varsigma(v_i) = \int_0^{v_i} p_i d\Phi(p_i) + \int_{v_i}^{\bar{v}} J(v_i) d\Phi(p_i)$ and 0 to pin down the optimal $\hat{p}_i^{so}(v_i, \mathbf{v}_{-i})$:

$$\hat{p}_i^{so}(v_i, \mathbf{v}_{-i}) = \begin{cases} 0 & \text{if } \varsigma(v_i) \geq 0; \\ \bar{v} & \text{if } \varsigma(v_i) < 0. \end{cases}$$

To this end, we study the property of $\varsigma(v_i)$ for $v_i < v^M$:

$$\varsigma'(v_i) = \frac{1 - F(v_i)}{f(v_i)} \Phi'(v_i) > 0.$$

Furthermore, $\varsigma(0) = \int_0^{\bar{v}} J(0) d\Phi(p_i) < 0$, and $\varsigma(v^M) = \int_0^{v^M} p_i d\Phi(p_i) > 0$. Thus, there must exist a unique cutoff $\check{v}^{so}$ such that $\varsigma(\check{v}^{so}) = 0$, i.e., $\varsigma(\check{v}^{so}) = \int_0^{\check{v}^{so}} p_i d\Phi(p_i) + \int_{\check{v}^{so}}^{\bar{v}} J(\check{v}^{so}) d\Phi(p_i) = 0$. This indicates that $\hat{p}_i^{so}(v_i, \mathbf{v}_{-i}) = 0$ if $v_i \in [\check{v}^{so}, v^M)$ and $\hat{p}_i^{so}(v_i, \mathbf{v}_{-i}) = \bar{v}$ if $v_i < \check{v}^{so}$. Furthermore, simplifying $\int_0^{\check{v}^{so}} p_i d\Phi(p_i) = -\int_{\check{v}^{so}}^{\bar{v}} J(\check{v}^{so}) d\Phi(p_i)$ gives

$$\frac{\int_0^{\check{v}^{so}} [1 - \Phi(p_i)] dp_i}{1 - \Phi(\check{v}^{so})} = \frac{1 - F(\check{v}^{so})}{f(\check{v}^{so})}. \tag{19}$$

Summarizing the discussion above yields

$$\hat{p}_i^{so}(v_i, \mathbf{v}_{-i}) = \begin{cases} 0 & \text{if } v_i \geq \check{v}^{so}; \\ \bar{v} & \text{if } v_i < \check{v}^{so}. \end{cases} \tag{20}$$

Under (20), the selected bidder i with $v_i \geq \check{v}^{so}$ will not choose to take the outside offer. We further have

$$R \;=\; E_{\mathbf{v}} \sum_i E_{p_i \leq v_i} \left\{ \left[v_i - \frac{1 - F(v_i)}{f(v_i)} \right] - p_i \right\} \;+\; E_{\mathbf{v}} \sum_i q_i^1(v_i, \mathbf{v}_{-i}) E_{p_i} \Big\{ q_i^2(v_i, p_i; \hat{p}_i^{so}(v_i, \mathbf{v}_{-i})) \lambda(v_i, p_i) \Big\}.$$

It is clear that $E_{p_i} \left\{ q_i^2(v_i, p_i; \hat{p}_i^{so}(v_i, \mathbf{v}_{-i})) \lambda(v_i, p_i) \right\} = 0$ for $v_i \leq \check{v}^{so}$, since $q_i^2(v_i, p_i; \hat{p}_i^{so}(v_i, \mathbf{v}_{-i})) = 0$ for $v_i \leq \check{v}^{so}$. When $v_i > \check{v}^{so}$, $q_i^2(v_i, p_i; \hat{p}_i^{so}(v_i, \mathbf{v}_{-i})) = 1$ and thus $E_{p_i} \left\{ q_i^2(v_i, p_i; \hat{p}_i^{so}(v_i, \mathbf{v}_{-i})) \lambda(v_i, p_i) \right\} = E_{p_i} \lambda(v_i, p_i)$. Note that $\lambda(v_i, p_i)$ strictly increases with v_i. To maximize R, it is clear to set

$$q_i^{so}(v_i, \mathbf{v}_{-i}) \;=\; \begin{cases} 1 & \text{if } v_i = v^{(1)} \text{ and } v_i \geq \check{v}^{so}; \\ 0 & \text{otherwise,} \end{cases} \tag{21}$$

where $v^{(1)} = \max\{v_i, i = 1, 2, ..., N\}$. In other words, only the bidder with the highest value is invited to enter the second stage, provided the bidder's value is no less than $\check{v}^{so}$. This is equivalent to setting the threshold at $\check{v}^{so}$ in the first stage.

In the mechanism, the total expected payment (the seller's revenue) R can be written as follows:

$$\begin{aligned}
R \;&=\; E_{\mathbf{v}} \sum_i E_{p_i \leq v_i} \left\{ \left[v_i - \frac{1 - F(v_i)}{f(v_i)} \right] - p_i \right\} \;+\; E_{\mathbf{v}} \sum_i q_i^1(v_i, \mathbf{v}_{-i}) E_{p_i} \Big\{ q_i^2(v_i, p_i; \hat{p}_i^{so}(v_i)) \lambda(v_i, p_i) \Big\} \\
&=\; N \int_0^{\bar{v}} \Big(\big[v_i - \frac{1 - F(v_i)}{f(v_i)} \big] \Phi(v_i) - \int_0^{v_i} p_i \varphi(p_i) dp_i \Big) dF(v_i) \\
&\quad +\; N \int_0^{\bar{v}} \Big(\int_0^{v_i} p_i \varphi(p_i) dp_i + \int_{v_i}^{\bar{v}} \big[v_i - \frac{1 - F(v_i)}{f(v_i)} \big] \varphi(p_i) dp_i \Big) Q(v_i) f(v_i) dv_i \\
&=\; N \int_0^{\bar{v}} \Big(v_i Q(v_i) + (1 - Q(v_i)) \int_0^{v_i} \Phi(p_i) dp_i - \frac{1 - F(v_i)}{f(v_i)} \big(Q(v_i) + (1 - Q(v_i)) \Phi(v_i) \big) \Big) f(v_i) dv_i \\
&=\; N \int_0^{\bar{v}} (1 - F(t)) \big(t - \int_0^t \Phi(x) dx \big) dQ(t).
\end{aligned}$$

where $Q(v_i) = F^{N-1}(v_i)$. When bidder i is selected, v_i should be no less than $\check{v}^{so}$. Let us further define $r^{so} = \int_0^{\check{v}^{so}} [1 - \Phi(p_i)] dp_i$, which gives the payment bidder i with value $\check{v}^{so}$ will pay conditional on being selected. Then, the expected payment from bidder i can be written as follows:

$$\begin{aligned}
&\int_{\check{v}^{so}}^{\bar{v}} \Big(Q(\check{v}^{so}) r^{so} + \int_{\check{v}^{so}}^{v_i} \big(t - \int_0^t \Phi(x) dx \big) Q'(t) dt \Big) f(v_i) dv_i \\
&=\; \big(1 - F(\check{v}^{so}) \big) Q(\check{v}^{so}) r^{so} + \int_{\check{v}^{so}}^{\bar{v}} (1 - F(t)) \big(t - \int_0^t \Phi(x) dx \big) dQ(t).
\end{aligned}$$

With N bidders in the mechanism, we then have

$$R^* \;=\; N \big(1 - F(\check{v}^{so}) \big) Q(\check{v}^{so}) r^{so} + N \int_{\check{v}^{so}}^{\bar{v}} (1 - F(t)) \big(t - \int_0^t \Phi(x) dx \big) dQ(t). \tag{22}$$

We complete the proof. $\square$

Proof of Proposition 2

Given the fact that $\mathbb{E}_S^I[R(r,D)] = \mathbb{E}_S^{II}[R(r,D)]$ when $r = D$ (implying that $\check{v} = \hat{v}$), Lemma 7 indicates that for any $D \in [r^{so}, D^{so}]$, $\mathbb{E}_S^I[R(r^I(D) = r^{so}, D)] > \mathbb{E}_S^I[R(r^I(D) = D, D)] = \mathbb{E}_S^{II}[R(r^{II}(D) = D, D)]$. Therefore, $r^*(D) = r^{so}$ for any $D \in [r^{so}, D^{so}]$; this proves part (i). Regarding part (ii), Lemma 8 states that for any $D \in [0, r^{so})$, $\mathbb{E}_S^I[R(r^I(D) = D, D)] = \mathbb{E}_S^{II}[R(r^{II}(D) = D, D)] < \mathbb{E}_S^{II}[R(r^{II}(D) = \tilde{r}^{II}(D), D)]$. Hence, $r^*(D) = \tilde{r}^{II}(D)$ for any $D \in [0, r^{so})$. Note that in this case, $r^*(D)$ is not necessary to be monotone in D when $D \in [0, r^{so})$. $\square$

Proof of Proposition 3

For $D \in [r^{so}, D^{so}]$, we have $R^*(D) = \mathbb{E}_S^I[R(r^*(D), D)]$ where $r^*(D) = r^{so}$. We have the following two steps to establish the result:

Step (i). Differentiating $\mathbb{E}_S^I[R(r^*(D), D)]$ with respect to $\hat{v}$, and evaluating it at $\hat{v} = \hat{v}(D)$ give the following equation:

$$\begin{aligned}
&\frac{\partial}{\partial \hat{v}} \frac{\mathbb{E}_S^I[R(r^*(D), D)]}{N}|_{\hat{v}=\hat{v}(D)} \\
&= \Bigg[\big(1 - F(\hat{v}(D))\big)b(\hat{v}(D)) \ - \ \big(1 - F(\hat{v}(D))\big)\Big[\big(1 - \Phi(\tilde{b}(\hat{v}(D)) - D)\big)\tilde{b}(\hat{v}(D)) + \Phi(\tilde{b}(\hat{v}(D)) - D)D\Big]\Bigg]q(\hat{v}(D)),
\end{aligned} \tag{23}$$

where $q(\cdot) = Q'(\cdot)$. Since $\tilde{b}(\hat{v}(D)) - D = b(\hat{v}(D)) - D = 0$, (23) can be re-written as follows:

$$\frac{\partial}{\partial \hat{v}} \frac{\mathbb{E}_S^I[R(r^*(D), D)]}{N}|_{\hat{v}=\hat{v}(D)} \ = \ \Big[\big(1 - F(\hat{v}(D))\big)D - \big(1 - F(\hat{v}(D))\big)D\Big]q(\hat{v}(D)) \ = \ 0. \tag{24}$$

This indicates that $\hat{v}(D)$ has no impact on $\mathbb{E}_S^I[R(r^*(D), D)]$.

Step (ii). Step (i) indicates that $\frac{\partial}{\partial \hat{v}} \frac{\mathbb{E}_S^I[R(r^*(D),D)]}{N}|_{\hat{v}=\hat{v}(D)} = 0$, and we then have that the impact of D through $r^*(D)$ is zero by envelope theorem, as $r^*(D) = r^{so}$ is an interior optimum for the given D. We thus have the following equation

$$
\begin{aligned}
\frac{d}{dD}\frac{\mathbb{E}_S^I[R(r^*(D),D)]}{N} \;&=\; \frac{\partial}{\partial D}\frac{\mathbb{E}_S^I[R(r,D)]}{N}\Big|_{r=r^*(D),\hat{v}=\hat{v}(D)} \\
&= \int_{\hat{v}(D)}^{\bar{v}} \big(1-F(x)\big)\Bigg[-\varphi(\tilde{b}(x)-D)\big(\Phi(\tilde{b}(x)-D)(\frac{\partial \tilde{b}(x)}{\partial D}-1)-1\big)\tilde{b}(x) \\
&\quad +\; \big(1-\Phi(\tilde{b}(x)-D)\big)\Phi(\tilde{b}(x)-D)(\frac{\partial \tilde{b}(x)}{\partial D}-1) \\
&\quad +\; \varphi(\tilde{b}(x)-D)\big(\Phi(\tilde{b}(x)-D)(\frac{\partial \tilde{b}(x)}{\partial D}-1)-1\big)D+\Phi(\tilde{b}(x)-D)\Bigg]dQ(x),
\end{aligned}
\tag{25}
$$

Note that $\frac{\partial}{\partial D}\frac{\mathbb{E}_S^I[R(r,D)]}{N}$ denotes the partial derivative of $\frac{\mathbb{E}_S^I[R(r,D)]}{N}$ with respect to D while fixing r and $\hat{v}$. Since $\frac{\partial \tilde{b}(x)}{\partial D} = \frac{-\Phi(\tilde{b}(x)-D)}{1-\Phi(\tilde{b}(x)-D)}$, we have

$$
\frac{\partial \tilde{b}(x)}{\partial D}-1 \;=\; \frac{-\Phi(\tilde{b}(x)-D)}{1-\Phi(\tilde{b}(x)-D)} - \frac{1-\Phi(\tilde{b}(x)-D)}{1-\Phi(\tilde{b}(x)-D)} = \frac{-1}{1-\Phi(\tilde{b}(x)-D)}. \tag{26}
$$

Plugging (26) into (25) shows

$$
\begin{aligned}
&\frac{d}{dD}\frac{\mathbb{E}_S^I[R(r^*(D),D)]}{N} \\
&= \int_{\hat{v}(D)}^{\bar{v}} (1-F(x))\Bigg[-\varphi(\tilde{b}(x)-D)\big(\Phi(\tilde{b}(x)-D)(\frac{\partial \tilde{b}(x)}{\partial D}-1)-1\big)\tilde{b}(x) \\
&\quad +\; \varphi(\tilde{b}(x)-D)\big(\Phi(\tilde{b}(x)-D)(\frac{\partial \tilde{b}(x)}{\partial D}-1)-1\big)D\Bigg]dQ(x) \\
&= \int_{\hat{v}(D)}^{\bar{v}} (1-F(x))\Bigg[-\varphi(\tilde{b}(x)-D)\big(\frac{-1}{1-\Phi(\tilde{b}(x)-D)}\big)\tilde{b}(x) \;+\; \varphi(\tilde{b}(x)-D)\big(\frac{-1}{1-\Phi(\tilde{b}(x)-D)}\big)D\Bigg]dQ(x) \\
&= \int_{\hat{v}(D)}^{\bar{v}} (1-F(x))\big(\frac{\varphi(\tilde{b}(x)-D)(\tilde{b}(x)-D)}{1-\Phi(\tilde{b}(x)-D)}\big)dQ(x),
\end{aligned}
\tag{27}
$$

Clearly, $\frac{d}{dD}\frac{\mathbb{E}_S^I[R(r^*(D),D)]}{N} \geq 0$ when D is in the interval of $[r^{so}, D^{so}]$. The equality holds if and only if $\hat{v}(D)=\bar{v}$, which implies that the seller charges $D=D^{so}$.

For any $D \in [0, r^{so})$, we have $R^*(D)=\mathbb{E}_S^{II}[R(r^*(D),D)]$ where $r^*(D)=\tilde{r}^{II}(D)>D$. The impact of r on seller revenue is solely through its impact on $\check{v}(r,D)$. At optimal $r^*(D)$, the marginal effect is zero, which means $\frac{\partial}{\partial \check{v}} \frac{\mathbb{E}_S^{II}[R(r^*(D),D)]}{N}|_{\check{v}=\check{v}(r^*(D),D)} = 0$. This result, together with envelop theorem, gives

$$\begin{aligned}
\frac{d}{dD}\frac{\mathbb{E}_S^{II}[R(r^*(D),D)]}{N} &= \frac{\partial}{\partial D}\frac{\mathbb{E}_S^{II}[R(r,D)]}{N}|_{r=r^*(D),\check{v}=\check{v}(r^*(D),D)} \\
&= \big(1-F(\check{v}(r^*(D),D))\big)Q(\check{v}(r^*(D),D))\big(\frac{\varphi(b(\check{v}(r^*(D),D))-D)(b(\check{v}(r^*(D),D))-D)}{1-\Phi(b(\check{v}(r^*(D),D))-D)}\big) \\
&\quad + \int_{\check{v}(r^*(D),D)}^{\bar{v}} (1-F(x))\big(\frac{\varphi(b(x)-D)(b(x)-D)}{1-\Phi(b(x)-D)}\big)dQ(x),
\end{aligned} \tag{28}$$

Note that $\frac{\partial}{\partial D}\frac{\mathbb{E}_S^{II}[R(r,D)]}{N}$ denotes the partial derivative of $\frac{\mathbb{E}_S^{II}[R(r,D)]}{N}$ with respect to D while fixing r and $\check{v}$. When the seller charges any D in the interval of $[0, r^{so})$, we have $\check{v}(r^*(D),D) < \bar{v}$ which indicates that $\frac{d}{dD}\frac{\mathbb{E}_S^{II}[R(r^*(D),D)]}{N} > 0$.

Summarizing the discussion above, we can conclude that charging $D = D^{so}$ is optimal. Any (D, r^{so}) with $D > D^{so}$ also maximizes seller's revenue, since seller revenue does not change in D by Lemma 5.□

Proof of Proposition 4

We separately prove parts (i), (ii), and (iii) as follow:

Part (i). Case (I) where $r - D \leq 0$. First, given that $\Phi_1(\cdot)$ first-order stochastically dominates $\Phi_2(\cdot)$, i.e. $\Phi_1(\cdot) < \Phi_2(\cdot)$, which means that $\Phi_1(\cdot)$ gives a worse outside offer to the buyers, we can easily establish the following facts: (a) $\int_0^{\check{v}_2}[1-\Phi_2(p_i)]dp_i = r = \int_0^{\check{v}_1}[1-\Phi_1(p_i)]dp_i$ implies $\check{v}_1 < \check{v}_2$; (b) $\int_0^{\hat{v}_2}[1-\Phi_2(p_i)]dp_i = D = \int_0^{\hat{v}_1}[1-\Phi_1(p_i)]dp_i$ gives $\hat{v}_1 < \hat{v}_2$.

Second, we compare equilibrium bidding strategies across $\Phi_1(\cdot)$ and $\Phi_2(\cdot)$, given $\Phi_1(\cdot) < \Phi_2(\cdot)$. For convenience, we write $b(v_i, \Phi_k)$ and $\tilde{b}(v_i, \Phi_k), k = 1, 2$ for $v_i \in [\check{v}_k, \hat{v}_k]$ and $v_i > \hat{v}_k$, respectively. We then have the following:

1. For $v_i \in [\check{v}_1, \check{v}_2)$, $b(v_i, \Phi_1) = \int_0^{v_i}[1-\Phi_1(p_i)]dp_i \geq r$ under $\Phi_1(.)$, but the bidder under $\Phi_2(.)$ does not submit a valid bid, equivalently, $b(v_i, \Phi_2) = 0$. Thus, $b(v_i, \Phi_1) > b(v_i, \Phi_2)$.

2. For $v_i = \check{v}_2$, given that the equilibrium bidding strategy is monotone and increasing, the following inequality must be true: $b(v_i, \Phi_1) > b(\check{v}_1, \Phi_1) = b(v_i, \Phi_2) = r$. Thus, $b(v_i, \Phi_1) > b(v_i, \Phi_2)$.

3. For $v_i \in (\check{v}_2, \hat{v}_1]$, we have $b(v_i, \Phi_1) = \int_0^{v_i}[1-\Phi_1(p_i)]dp_i > \int_0^{v_i}[1-\Phi_2(p_i)]dp_i = b(v_i, \Phi_2)$. Thus, $b(v_i, \Phi_1) > b(v_i, \Phi_2)$.

4. For $v_i \in (\hat{v}_1, \hat{v}_2)$, given that the equilibrium bidding strategy is monotone and increasing, we have $\tilde{b}(v_i, \Phi_1) > b(v_i, \Phi_1) = \int_0^{v_i}[1-\Phi_1(p_i)]dp_i > \int_0^{v_i}[1-\Phi_2(p_i)]dp_i = b(v_i, \Phi_2)$. Thus, $\tilde{b}(v_i, \Phi_1) > b(v_i, \Phi_2)$.

5. For $v_i = \hat{v}_2$, given that the equilibrium bidding strategy is monotone and increasing, the following inequality must be true: $\tilde{b}(v_i, \Phi_1) > b(\hat{v}_1, \Phi_1) = b(v_i, \Phi_2) = D$. Thus, $\tilde{b}(v_i, \Phi_1) > b(v_i, \Phi_2)$.

6. For $v_i \in (\hat{v}_2, \bar{v}]$, recall that we can re-write $\tilde{b}(v_i, \Phi)$ as $\int_{\tilde{b}(v_i,\Phi)-D}^{v_i}[1-\Phi(p_i)]dp_i = D$, therefore, we have $\int_{\tilde{b}(v_i,\Phi_1)-D}^{v_i}[1-\Phi_1(p_i)]dp_i = D = \int_{\tilde{b}(v_i,\Phi_2)-D}^{v_i}[1-\Phi_2(p_i)]dp_i$, which immediately indicates $\tilde{b}(v_i, \Phi_1) > \tilde{b}(v_i, \Phi_2)$.

Case (II) where $r - D \geq 0$. Recall that $\int_{r-D}^{\check{v}}[1-\Phi(p_i)]dp_i = D$. First, given that $\Phi_1(\cdot)$ first-order stochastically dominates $\Phi_2(\cdot)$, i.e. $\Phi_1(\cdot) < \Phi_2(\cdot)$, we can easily establish the following fact: $\int_{r-D}^{\check{v}_1}[1-\Phi_1(p_i)]dp_i = D = \int_{r-D}^{\check{v}_2}[1-\Phi_2(p_i)]dp_i$ implies $\check{v}_1 < \check{v}_2$.

Second, we compare equilibrium bidding strategies across $\Phi_1(\cdot)$ and $\Phi_2(\cdot)$, given $\Phi_1(\cdot) < \Phi_2(\cdot)$.

- For $v_i \in [\check{v}_1, \check{v}_2)$, $b(v_i, \Phi_1) \geq r$ under $\Phi_1(.)$, but the bidder under $\Phi_2(.)$ does not submit a valid bid, equivalently, $b(v_i, \Phi_2) = 0$. Thus, $b(v_i, \Phi_1) > b(v_i, \Phi_2)$.

- For $v_i = \check{v}_2$, given that the equilibrium bidding strategy is monotone and increasing, the following inequality must be true: $b(v_i, \Phi_1) > b(\check{v}_1, \Phi_1) = b(v_i, \Phi_2) = r$. Thus, $b(v_i, \Phi_1) > b(v_i, \Phi_2)$.

- For $v_i \in (\check{v}_2, \bar{v}]$, recall that we can re-write $b(v_i, \Phi)$ as $\int_{b(v_i,\Phi)-D}^{v_i}[1-\Phi(p_i)]dp_i = D$, therefore, we have $\int_{b(v_i,\Phi_1)-D}^{v_i}[1-\Phi_1(p_i)]dp_i = D = \int_{b(v_i,\Phi_2)-D}^{v_i}[1-\Phi_2(p_i)]dp_i$, which immediately indicates $b(v_i, \Phi_1) > b(v_i, \Phi_2)$.

Therefore, we can conclude that given reserve price r and deposit D, the equilibrium bid submitted by a bidder is higher when the distribution of the outside offer becomes worse in the sense of first-order stochastic dominance.

Part (ii). We examine how the optimal deposit D^{so} changes across different distributions of the outside offers. Recall that $\hat{v}_1 = \hat{v}_2 = \bar{v}$ corresponds to bids D_1^{so} and D_2^{so}. It is obvious that if $\Phi_1(\cdot) < \Phi_2(\cdot)$, then $D_1^{so} = \int_0^{\bar{v}}[1-\Phi_1(p_i)]dp_i > \int_0^{\bar{v}}[1-\Phi_2(p_i)]dp_i = D_2^{so}$.

Part (iii). Given that $\Phi_1(\cdot)$ first-order stochastically dominates $\Phi_2(\cdot)$, i.e., $\Phi_1(\cdot) < \Phi_2(\cdot)$, let us denote the optimal reserve price under $\Phi_i(\cdot)$ by $r_i^{so}, i = 1, 2$. Note that any sufficiently high D is optimal and fully deters the winner's default, and the optimal revenues under $\Phi_i(\cdot), i = 1, 2$ do not depend on D when it is optimally set. Taking such a D, under (r_2^{so}, D) the winner's default is fully deterred even with $\Phi_1(\cdot)$. Since $\Phi_1(\cdot) < \Phi_2(\cdot)$, under (r_2^{so}, D), we then have $\check{v}_1 < \check{v}_2$ and $b(v_i, \Phi_1) > b(v_i, \Phi_2)$ by Proposition 4 and its proof.

Under (r_2^{so}, D), seller revenue with $\Phi_1(\cdot)$ is given by

$$R(r_2^{so}, D, \Phi_1) \;=\; N\big(1-F(\check{v}_1)\big)Q(\check{v}_1)r_2^{so} + N\int_{\check{v}_1}^{\bar{v}}\big(1-F(x)\big)b(x, \Phi_1)dQ(x),$$

and the optimal seller revenue under $\Phi_2(\cdot)$ is given by

$$R^*(r_2^{so}, D, \Phi_2) \;=\; N\big(1-F(\check{v}_2)\big)Q(\check{v}_2)r_2^{so} + N\int_{\check{v}_2}^{\bar{v}} \big(1-F(x)\big)b(x,\Phi_2)dQ(x).$$

Define $R(v_1) = N(1-F(v_1))Q(v_1)r_2^{so} + N\int_{v_1}^{\bar{v}} \big(1-F(x)\big)b(x,\Phi_1)dQ(x)$. Differentiating $R(v_1)$ with respect to v_1 yields

$$\frac{dR(v_1)}{dv_1} \;=\; -Nf(v_1)Q(v_1)r_2^{so} + N\big(1-F(v_1)\big)Q'(v_1)\big(r_2^{so} - b(v_1,\Phi_1)\big) \;<\; 0,\ \forall v_1 > \check{v}_1.$$

Let $R^*(r_1^{so}, D, \Phi_1)$ denote the optimal seller revenue under Φ_1. Thus, it is clear that $R^*(r_1^{so}, D, \Phi_1) \geq R(r_2^{so}, D, \Phi_1) = R(\check{v}_1) > R(\check{v}_2) \geq R^*(r_2^{so}, D, \Phi_2)$. $\square$

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Online Appendices for "Deposit Requirement in Auctions" by X. Che, T. Li, J. Lu, and X. Zheng

1 Appendix S1

This online appendix covers the analyses of a second-price auction with deposit and reserve price (Section 1.1), percentage deposit (Section 1.2), and deposit proportionally deducted from full payment (Section 1.3).

1.1 Second-price auction format

Given the specific auction format (second-price auction) with deposit D and reserve price r, we modify the game in Section 2 as follows: A seller sells an indivisible object to N risk-neutral bidders through a sealed-bid, second-price auction, where $1 < N < \infty$. The seller's reservation value of the object is normalized to zero. Bidders' private values, denoted by v_i, $i = 1, 2, ..., N$, are independent draws from a common atomless distribution $F(\cdot)$ with density $f(\cdot) > 0$ over the support $[0, \overline{v}]$, where $\overline{v} > 0$. After the auction, each bidder i will receive an outside offer, which gives the same object (the same value v_i) but with a random price, denoted by p_i. Prices p_i, $i = 1, 2, ..., N$, are random draws from a common atomless distribution $\Phi(\cdot)$ with density $\varphi(\cdot) > 0$ over $[0, \overline{v}]$. F and Φ are common knowledge among the seller and bidders, and they are *regular* in the sense that the hazard rates $\frac{f(\cdot)}{1-F(\cdot)}$ and $\frac{\varphi(\cdot)}{1-\Phi(\cdot)}$ are increasing. Figure 1 depicts the timing of the game, which is comprised of four stages.

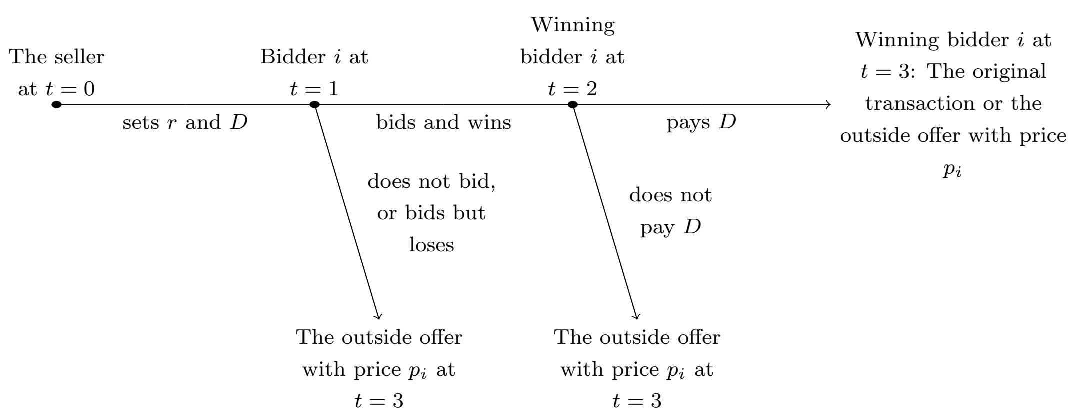


Figure 1: Timing

At stage $t = 0$, the seller announces a reserve price $r \in [0, \overline{v}]$ and a *winner-pay* deposit $D \in [0, +\infty)$. The seller commits to both the reserve price and the deposit requirement.

At $t = 1$, all N bidders observing r and D decide whether to enter and submit bids in the auction. If a bidder chooses not to enter, the bidder then simply waits for the outside offer with a random price at $t = 3$. For convenience, a nonparticipant's bid is denoted by "$\varnothing$." If bidder i enters the auction, we denote her bid by $b_i \in [r, +\infty)$. All the entering bidders simultaneously submit their bids. The highest-bid bidder wins the auction. If there is a tie, the object goes to each winning bidder with equal probability. The full payment (the auction price) is denoted by κ, which equals r if there is only one valid bid and equals the second-highest bid otherwise. Slightly abusing the notations, we have $\kappa \equiv \max\{r, b_{-i}^{(1)}\}$, where $b_{-i}^{(1)}$ stands for the losing bidders' highest bid. All the losing bidders will face their outside offers at $t = 3$. If there is no valid bid, the auction game ends, and the seller keeps the object.

At $t = 2$, the winning bidder i decides whether to pay D. Let us denote the winner's decision by $e_i \in \{0, 1\}$, with $e_i = 1$ for "paying D" and $e_i = 0$ for "not paying D." If the winning bidder i does not pay D, i.e., $e_i = 0$, the seller keeps the object, and the winning bidder is to face the outside offer or not make the purchase at $t = 3$. If $e_i = 1$, the winning bidder i has three options at $t = 3$: either completing the current transaction, or adopting the outside offer, or not purchasing at all. Note that we assume if $\kappa < D$ the seller, conditional on D being paid, pays $D - \kappa$ back to the winner when the final transaction goes through. Thus, the total payment to the seller is still the auction price κ.

At $t = 3$, the random price p_i for the outside offer is realized. Conditional on $e_i = 1$, the winning bidder i decides whether to complete the current transaction by paying the remaining payment $\kappa - D$, or to take the outside offer by paying price p_i, or to not make the purchase at all. If the winning bidder i does not complete the transaction, the deposit D is forfeited, and the seller keeps the object. We denote the winner's decision by o_i, where $o_i = 1$ for "taking the outside offer" and $o_i = 0$ for "completing the original transaction." For completeness, we write $o_i = NP$ for "no purchase."

For all other cases (a losing bidder or a winning bidder who does not pay the deposit D), we denote a bidder j's decision on whether to take the outside offer at $t = 3$ by $\breve{o}_j$, where $\breve{o}_j = 1$ for "taking the outside offer with price p_j" and $\breve{o}_j = NP$ for "no purchase."

The strategies of the seller and a bidder as well as the equilibrium concept are defined as follows: Before the bidding stage starts, the seller sets reserve price r and deposit D at $t = 0$. Given r and D, a bidder makes four decisions: first, how much to bid at the bidding stage at $t = 1$; second, conditional on winning the auction, whether to pay the deposit at $t = 2$; third, conditional on winning the auction and paying the deposit at $t = 2$, whether to take the outside offer with price p_i, or complete the original transaction by paying $\kappa - D$, or not make the purchase at all at $t = 3$; and fourth, conditional on not winning the auction at $t = 1$ or winning at $t = 1$ but not paying the deposit at $t = 2$, whether to take the outside offer with price p_i at $t = 3$.

The bidding strategy for bidder i at $t = 1$ is a mapping from private value v_i to bid b_i; that is, $b_i(v_i)$:

$[0, \infty) \rightarrow [r, +\infty) \cup \{\varnothing\}$. Note that b_i is valid if and only if $b_i \geq r$. Conditional on winning at the bidding stage, the strategy of the winning bidder i regarding whether to pay the deposit is a mapping from private value v_i and full payment κ to deposit payment decision e_i at $t = 2$: $e_i(v_i, \kappa) : [0, \overline{v}] \times [r, \infty) \rightarrow \{0, 1\}$.

Contingent on paying the deposit at $t = 2$, the strategy of the winning bidder i regarding whether to complete the original transaction or opt for the outside offer or not make the purchase is a mapping from private value v_i, full payment κ, and price p_i of the outside offer to decision o_i at $t = 3$: $o_i(v_i, \kappa, p_i) : [0, \overline{v}] \times [r, \infty) \times [0, \overline{v}] \rightarrow \{0, 1\} \cup \{NP\}$. Conditional on not winning the auction or winning but not paying the deposit, the strategy of bidder i regarding whether to purchase from the outside offer is a mapping from private value v_i and price p_i of the outside offer to decision $\breve{o}_i$ at $t = 3$: $\breve{o}_i(v_i, p_i) : [0, \overline{v}] \times [0, \overline{v}] \rightarrow \{0, 1\} \cup \{NP\}$.

The equilibrium concept in this paper is perfect Bayesian equilibrium (PBE). Given r and D, we first characterize bidder i's strategies concerning the outside offer at $t = 3$, i.e., $o_i(v_i, \kappa, p_i)$ and $\breve{o}_i(v_i, p_i)$, examine the winning bidder i's decision on paying the deposit at $t = 2$, i.e., $e_i(v_i, \kappa)$, and derive the equilibrium bidding strategy $b_i(v_i)$ at $t = 1$. Finally, we characterize the seller's optimal choices on r and D at $t = 0$.[1]

1.1.1 Decision on taking the outside offer at $t = 3$

If bidder i does not submit a valid bid, or submits a valid bid but does not win the auction, or does not pay D conditional on winning, the only choice left at $t = 3$ is the outside offer with random price p_i. The characterization of the bidder i's strategy $\breve{o}_i(\cdot, \cdot)$ is clear: takes the offer if $v_i \geq p_i$; otherwise, chooses not to make the purchase.

Next, we consider the bidder i's strategy $o_i(\cdot, \cdot, \cdot)$ conditional on winning and paying the deposit. The winning bidder i has three choices at $t = 3$: takes the outside offer with price p_i, or completes the original transaction with payment $\kappa - D$, or not make the purchase at all. The following is the optimal decision $o_i(v_i, \kappa, p_i)$ for the winning bidder i:

$$o_i(v_i, \kappa, p_i) = \begin{cases} 0 & \text{if } \big[e_i = 1 \text{ and } v_i \geq \kappa - D \text{ and } p_i \geq \kappa - D\big]; \\[2ex] 1 & \text{if } \big[e_i = 1 \text{ and } v_i \geq \kappa - D \text{ and } p_i < \kappa - D\big], \\ & \text{or } \big[e_i = 1 \text{ and } v_i < \kappa - D \text{ and } p_i \leq v_i\big]; \\[2ex] NP & \text{if } \big[e_i = 1 \text{ and } v_i < \kappa - D \text{ and } p_i > v_i\big]. \end{cases} \tag{A1}$$

When $v_i \geq \kappa - D$, the winning bidder i completes the original transaction if $p_i \geq \kappa - D$ and takes the

[1] In the following analysis, we restrict our attention to (weakly) undominated strategies. This helps rule out many uninteresting equilibria that generally arise in second-price auctions. For example, there may exist an equilibrium that a bidder submits $\bar{v}$ regardless of her true value and other bidders bid zero. Then, the winning bidder makes decisions on paying the deposit and taking the outside offer. However, this equilibrium is not admissible as bidding $\bar{v}$ is weakly dominated by submitting $b_i \leq v_i$, given bidding zero from other bidders. See discussion on all Nash equilibria of second-price auction by Blume and Heidhues (2004).

outside offer if $p_i < \kappa - D$. However, when $v_i < \kappa - D$,[2] the winning bidder takes the outside offer if $p_i \leq v_i$; otherwise, he chooses not to make the purchase.

1.1.2 Decision on paying the deposit at $t = 2$

Here we examine the winning bidder i's decision on paying deposit D at $t = 2$. If the winner pays the deposit D, i.e., $e_i = 1$, there are two possibilities: $\kappa - D \geq 0$ or $\kappa - D < 0$. In the case where $\kappa - D \geq 0$, depending on $p_i \lesseqgtr \kappa - D$, the winning bidder can choose to either complete the original transaction or take the outside offer $t = 3$, as described in (A1). However, if $\kappa - D < 0$, the price p_i of the outside offer will certainly be greater than $\kappa - D$, which implies that the winner will always complete the original transaction. Given the two cases above, we can construct the winning bidder i's payoff from paying D at $t = 2$, denoted by $\pi_D(v_i, \kappa)$, as follows:

$$\pi_D(v_i, \kappa) = \begin{cases} \int_0^{\kappa - D}(v_i - p_i)\varphi(p_i)dp_i + \int_{\kappa - D}^{\overline{v}}[v_i - (\kappa - D)]\varphi(p_i)dp_i - D & \text{if } \kappa - D \geq 0; \\ \int_0^{\overline{v}}[v_i - (\kappa - D)]\varphi(p_i)dp_i - D & \text{if } \kappa - D < 0. \end{cases} \tag{A2}$$

If the winning bidder i does not pay the deposit D, i.e., $e_i = 0$, her only choice at $t = 3$ is to face the outside offer with random price p_i. In this case, the expected payoff at $t = 2$, denoted by $\pi_{ND}(v_i)$, is given by

$$\pi_{ND}(v_i) = \int_0^{v_i}(v_i - p_i)\varphi(p_i)dp_i. \tag{A3}$$

It is clear that the winning bidder i is willing to pay the deposit D if and only if $\pi_D(v_i, \kappa) \geq \pi_{ND}(v_i)$. Defining $L = \pi_D(v_i, \kappa) - \pi_{ND}(v_i)$, our analysis includes the following two cases. In the case where $\kappa - D \geq 0$, $L(v_i, \kappa) = v_i - \kappa - \int_{\kappa - D}^{v_i}\Phi(p_i)dp_i$, which indicates that given κ and D, there should exist a unique threshold for bidder i's private value, denoted by $v_i'(\kappa) \in [0, \overline{v}]$, such that $L(v_i, \kappa) > 0$ if and only if $v_i > v_i'(\kappa)$. In the case where $\kappa - D < 0$, $L(v_i, \kappa) = v_i - \kappa - \int_0^{v_i}\Phi(p_i)dp_i$, implying that there should exist a unique threshold $v_i''(\kappa) \in [0, \overline{v}]$, such that $L(v_i, \kappa) > 0$ if and only if $v_i > v_i''(\kappa)$. Summarizing the discussion above gives the winning bidder's deposit strategy at $t = 2$:

$$e_i(v_i, \kappa) = \begin{cases} 1 & \text{if } \big[\kappa - D \geq 0 \text{ and } v_i > v_i'(\kappa)\big], \\ & \text{or } \big[\kappa - D < 0 \text{ and } v_i > v_i''(\kappa)\big]; \\ 0 & \text{otherwise.} \end{cases} \tag{A4}$$

1.1.3 Bidders' Equilibrium Strategies

Let us now examine the equilibrium bidding strategy at $t = 1$. We first present the following useful

[2] This is an out-of-equilibrium event, as it implies that the bidder submitted a bid greater than v_i.

lemma.

Lemma 3. *It is a dominated strategy for a bidder to submit any bid greater than her private value, i.e.,* $b_i > v_i$.

The proof of Lemma 3 is straightforward. Submitting $b_i > v_i$ makes the bidder win additionally when $b_{-i}^{(1)}$ is between v_i and b_i. Obviously, winning with such a price is weakly dominated by waiting for the outside offer with random price p_i. Lemma 3 allows us to focus on $b_i \leq v_i$ (which implies $\kappa \leq v_i$ and further $\kappa - D \leq v_i$, conditional on bidder i winning the auction) for the equilibrium characterization in the subsequent subgames.

Each bidder at $t = 1$ faces two choices: either bidding in the auction or waiting for the outside offer directly. In the equilibrium, the bid submitted by the bidder should make her indifferent between these two choices, specifically between "winning at her own bid and then paying the deposit" and "not submitting a bid but waiting for the outside offer directly." We call this insight the "payoff-indifference condition," which leads to a symmetric increasing bidding strategy, denoted by $b(v_i)$.[3] Furthermore, whether a bidder after winning has a chance to default and take the outside offer depends on both the deposit and the reserve price. Given the same r and D, a bidder with a relatively low private value would not choose to default after paying the deposit; however, this may not be true if the bidder is with a sufficiently high private value. Therefore, when we utilize the "payoff-indifference condition" to construct the equilibrium bidding strategy for bidders in the following, we need to separately consider two cases in which, after winning and paying, the bidder still has a chance to default. More specifically, we pin down the details of the equilibrium bidding strategy by considering Case (I) where $r \leq D$ and Case (II) where $r \geq D$.

Let us define two cutoff values $\check{v} \in [0, \bar{v}]$ and $\hat{v} \in [0, \bar{v}]$ satisfying the following conditions:

$$r = \int_0^{\check{v}} [1 - \Phi(p_i)] dp_i, \quad \text{and} \quad D = \int_0^{\hat{v}} [1 - \Phi(p_i)] dp_i.$$

Note that if $b(\bar{v}) \leq r$, we then have $\check{v} \geq \bar{v}$, and there is no participation from bidders. If $b(\bar{v}) \leq D$, we have $\hat{v} \geq \bar{v}$, and no bidder will bid more than the deposit D.

In Case (I) where $r \leq D$, we have $\hat{v} \geq \check{v}$. Let us consider bidding strategies for the bidders separately in the valuation intervals of $v_i \in [\check{v}, \hat{v}]$ and $v_i \in (\hat{v}, \bar{v}]$.

When $v_i \in [\check{v}, \hat{v}]$, the bidder, after winning, always chooses to complete the original transaction. The

[3] The bidding strategy $b(v_i)$ is a weakly dominant strategy. In the proof of Proposition B0, we will show that submitting a bid lower than $b(v_i)$ cannot be beneficial to the bidder. Bidding higher than $b(v_i)$ would increase the probability of winning but lower the bidder's expected payoff, which encourages the bidder to wait for the outside offer instead.

"payoff-indifference condition" gives

$$\int_0^{\overline{v}} \big[v_i - (b-D)\big]\varphi(p_i)dp_i \; - \; D \; = \; \int_0^{v_i} (v_i - p_i)\varphi(p_i)dp_i,$$

where the first term on the left hand side is the expected payoff from bidding and paying the deposit D (conditional on winning) in the auction, and the right hand side is the expected payoff from waiting for the outside offer directly. Rearranging the equation gives

$$b \; = \; v_i \; - \; \int_0^{v_i} \Phi(p_i)dp_i \iff b = \int_0^{v_i} [1 - \Phi(p_i)]dp_i. \tag{A5}$$

When $v_i \in (\hat{v}, \overline{v}]$, the bidder, after winning, has two options of either completing the original transaction or taking the outside offer, depending on $p_i \lesseqqgtr b - D$. The "payoff-indifference condition" gives

$$\int_0^{b-D} (v_i - p_i)\varphi(p_i)dp \; + \; \int_{b-D}^{\overline{v}} \big[v_i - (b-D)\big]\varphi(p_i)dp_i \; - \; D \; = \; \int_0^{v_i} (v_i - p_i)\varphi(p_i)dp_i,$$

where the first term and the second term on the left hand side present the expected payoffs from taking the outside offer if $p_i \leq b - D$ and completing the original transaction if $p_i > b - D$, respectively; the term on the right hand side is the expected payoff from waiting for the outside offer directly. Re-arranging the equation yields

$$b \; = \; v_i \; - \; \int_{b-D}^{v_i} \Phi(p_i)dp_i \iff D = \int_{b-D}^{v_i} [1 - \Phi(p_i)]dp_i. \tag{A6}$$

We can then summarize bidder i's bidding strategy at $t = 1$ in Case (I) as follows:

$$b(v_i) \; = \; \begin{cases} v_i \; - \; \displaystyle\int_{b(v_i)-D}^{v_i} \Phi(p_i)dp_i & \text{if } \; v_i \; > \; \hat{v}; \\ v_i \; - \; \displaystyle\int_0^{v_i} \Phi(p_i)dp_i & \text{if } \; v_i \in [\check{v}, \hat{v}]; \\ \varnothing & \text{if } \; v_i \; < \; \check{v}. \end{cases} \tag{A7}$$

It is easy to check that in (A5), $\frac{db}{dv_i} = 1 - \Phi(v_i) > 0$, and in (A6), $\frac{db}{dv_i} = \frac{1-\Phi(v_i)}{1-\Phi(b-D)} > 0$ and $\frac{db}{dD} = \frac{-\Phi(b-D)}{1-\Phi(b-D)} < 0$; $b(v_i)$ is increasing in v_i and decreasing in D. Clearly, (A6) coincides with (A5) when $v_i = \hat{v}$.

In Case (II) where $r \geq D$, we have $\hat{v} \leq \check{v}$; only the bidder with $v_i \in [\check{v}, \overline{v}]$ can submit a valid bid, and after winning, the bidder has two options of either completing the original transaction or taking the outside offer, depending on $p_i \lesseqqgtr b - D$. The construction of the equilibrium bidding strategy is similar to (A6) but with the cutoff value $\check{v}$, which can be summarized as follows:

$$b(v_i) \;=\; \begin{cases} v_i \;-\; \displaystyle\int_{b(v_i)-D}^{v_i} \Phi(p_i)dp_i & \text{if } \; v_i \;\geq\; \check{v}; \\ \\ \varnothing & \text{if } \; v_i \;<\; \check{v}. \end{cases} \tag{A8}$$

Again, it is easy to check that $b(v_i)$ is increasing in v_i and decreasing in D.

Based on the discussion above, we have the following result.

Proposition B0. *In the auction, bidder i adopts the weakly dominant bidding strategies* (A7) *or* (A8)*, depending on $r \lesseqgtr D$.*

Proof. Most of the result was already shown in the text in Section 1.1.3. The only task left is to establish the dominance of the equilibrium bidding strategy, in other words, check whether a bidder has an incentive to deviate from the bidding strategy characterized in (A7) and (A8). Given that bidder i submits $b(v_i)$ in the auction, and the payoffs are:

$$\Pi_i \;=\; \begin{cases} \pi_D(v_i, \kappa) & \text{if} \quad \text{winning ;} \\ \\ \pi_{ND}(v_i) & \text{if} \quad \text{not winning ,} \end{cases}$$

where $\pi_D(v_i, \kappa)$ and $\pi_{ND}(v_i)$ are given by (A2) and (A3), respectively, in Section 1.1.2 above. Since $b(v_i) \geq \kappa$, we have $\pi_D(v_i, \kappa) \geq \pi_{ND}(v_i)$ and the equality holds when $b(v_i) = \kappa$. Suppose now that bidder i does not follow the equilibrium bidding strategy $b(v_i)$ (in short, b) but submits z in the auction, then we consider the following two cases:

Case (a) where $z < b$. If $b > z > \kappa$, then bidder i still wins and that the surplus $\pi_D(v_i, \kappa)$ is not affected. If $\kappa > b > z$, bidder i still loses in the auction and the surplus $\pi_{ND}(v_i)$ is from facing the outside offer. If $b > \kappa > z$, the bidder will lose and obtain $\pi_{ND}(v_i)$, where, however, he would have won and got $\pi_D(v_i, \kappa)$ if he had bid b. Thus, there is no incentive to bid lower than b.

Case (b) where $z > b$. If $z > b > \kappa$, then bidder i still wins and that the surplus $\pi_D(v_i, \kappa)$ is not affected. If $\kappa > z > b$, bidder i still loses in the auction and obtains $\pi_{ND}(v_i)$ is from facing the outside offer. If $z > \kappa > b$, by doing so it increases the probability of winning, but it is more profitable for the bidder to simply wait for the outside offer directly, as $\pi_D(v_i, \kappa)$ is now strictly less than $\pi_{ND}(v_i)$. Thus, there is no incentive to bid higher than b.

Therefore, we can conclude that b is a weakly dominate strategy for bidder i in the auction game. $\square$

Proposition $B0$ characterizes how bidders respond in their bids when facing a deposit requirement in the auction; a higher deposit requirement induces a higher $\hat{v}$ which lowers the possibility of ex-post default (or the chance to take the outside offer). As a consequence, bidders submit lower bids in the auction as their best responses. If the seller chooses not to demand any deposit from the winner, i.e., $D = 0$, the only solution is $b(v_i) = v_i$.[4] We therefore have the following remark.

Remark 2. In the auction, bidders bid truthfully when the seller does not require a deposit.

This interesting observation indicates that the bidders submitting lower bids, i.e., lower than their private values, is not due to the existence of the post-auction outside offer but rather to the deposit requirement of the seller.

Furthermore, given the characterizations of the bidding strategies in Cases (I) and (II), the following lemma demonstrates how thresholds $\check{v}$ and $\hat{v}$ change with r and D.

Lemma 4. *(i) In Case (I) where $r \leq D$, $\check{v}$ is increasing in r but independent of D and $\hat{v}$ is increasing in D but independent of r. (ii) In Case (II) where $r \geq D$, $\check{v}$ is increasing in both r and D.*

Proof. Case (I) where $r - D \leq 0$. Given $r = \check{v} - \int_0^{\check{v}} \Phi(p_i)dp_i$, differentiating $\check{v}$ with respect to r yields

$$1 = \frac{\partial \check{v}}{\partial r} - \Phi(\check{v})\frac{\partial \check{v}}{\partial r} \quad \Leftrightarrow \quad \frac{\partial \check{v}}{\partial r} = \frac{1}{1 - \Phi(\check{v})} > 0. \tag{A9}$$

Thus, $\check{v}$ is increasing in r. Clearly, it is independent of D. $\hat{v}$ is given by $D = \hat{v} - \int_0^{\hat{v}} \Phi(p_i)dp_i = \int_0^{\hat{v}}[1 - \Phi(p_i)]dp_i$. Clearly, $\hat{v}$ increases with D but is independent of $r (\leq D)$.

Case (II) where $r - D \geq 0$. Given $r = \check{v} - \int_{r-D}^{\check{v}} \Phi(p_i)dp_i$, differentiating $\check{v}$ with respect to r yields

$$1 = \frac{\partial \check{v}}{\partial r} - \left(\Phi(\check{v})\frac{\partial \check{v}}{\partial r} - \Phi(r - D)\right) \quad \Leftrightarrow \quad \frac{\partial \check{v}}{\partial r} = \frac{1 - \Phi(r - D)}{1 - \Phi(\check{v})} > 0. \tag{A10}$$

Further, differentiating $\check{v}$ with respect to D shows

$$0 = \frac{\partial \check{v}}{\partial D} - \left(\Phi(\check{v})\frac{\partial \check{v}}{\partial d} + \Phi(r - D)\right) \quad \Leftrightarrow \quad \frac{\partial \check{v}}{\partial D} = \frac{\Phi(r - D)}{1 - \Phi(\check{v})} > 0. \tag{A11}$$

We then have that $\check{v}$ is increasing in both r and D. □

From Lemma 4, we derive the following corollary on how the equilibrium bidding strategy depends on the reserve price, which will be useful for the analysis of the optimal reserve price in the next section.

Corollary B0. *Reserve price r affects equilibrium bidding strategy $b(v_i)$ through $\check{v}$.*

[4] Given $r \geq 0$, if $D = 0$, then $b(\check{v}) \geq 0$, and the bidding strategy in (A8) can be written as $\int_b^{v_i}[1 - \Phi(p_i)]dp_i = 0$, which implies truthful bidding from bidders.

Proofs of Lemmas 5 - 8

The following four lemmas help us characterize the optimal reserve price maximizing the seller's revenue for a given deposit.

Lemma 5. $\mathbb{E}_S^I[R(r,D)]$ *is independent of* D *for any* $D \geq D^{so}$.

Proof. If the seller chooses any $D > D^{so}$, $\hat{v}$ takes the corner solution $\bar{v}$, i.e., $\hat{v} = \bar{v}$, and the seller's expected revenue can be re-written as follows:

$$\mathbb{E}_S^I[R(r,D)] \;=\; N\big(1-F(\check{v})\big)Q(\check{v})r \;+\; N\int_{\check{v}}^{\bar{v}}(1-F(x))b(x)dQ(x),$$

which does not change with D. Note that $b(x)$ does not depend on D when $D \geq D^{so}$. $\square$

Lemma 6. *(i) In Case (I) where* $r \leq D$*, given any* $D > D^{so}$, $\mathbb{E}_S^I[R(r,D)] = 0$ *for all* $r \in [D^{so}, D]$*. (ii) In Case (II) where* $r \geq D$, $\mathbb{E}_S^{II}[R(r,D)] = 0$ *for all* $r \geq D^{so}$.

Proof. Given the restrictions of $r \leq D$ in Case (I) and $r \geq D$ in Case (II), we show that the seller obtains zero revenue when $r \geq D^{so}$.

Case (I) where $r - D \leq 0$, seller revenue with any $D > D^{so}$ is given by

$$\mathbb{E}_S^I[R(r,D)] \;=\; N\big(1-F(\check{v})\big)Q(\check{v})r \;+\; N\int_{\check{v}}^{\bar{v}}(1-F(x))b(x)dQ(x).$$

Further, if the seller sets $r \geq D^{so}$ (but still less than D), then $\check{v}$ takes the corner solution $\bar{v}$, i.e., $\check{v} = \bar{v}$. As a result, no one can submit a valid bid and the seller's revenue decreases to zero.

Case (II) where $r - D \geq 0$. With $r \geq D^{so}$, $\check{v}$ takes the corner solution $\bar{v}$, i.e., $\check{v} = \bar{v}$. As a result, no one can submit a valid bid and the seller's revenue decreases to zero. $\square$

Lemmas 5 and 6 help us restrict our analysis to the area of $(r,D) \in [0,D^{so}] \times [0,D^{so}]$. Given any $D \in [0,D^{so}]$, let us use $r^I(D)$ to denote the optimal reserve price in Case (I) where $r \leq D$. We first maximize (12) without taking into account the restriction of $r \leq D$; in this case, we denote the unrestricted optimal reserve price by $\tilde{r}^I(D)$. We next check whether or not $r^I(D) = \tilde{r}^I(D)$, i.e., $\tilde{r}^I(D)$ can be implemented in Case (I). We follow the same procedure to examine (13) and denote the optimal reserve price in Case (II) by $r^{II}(D)$ and the unrestricted optimal reserve price without the constraint of $r \geq D$ by $\tilde{r}^{II}(D)$. The following lemmas present the optimal choices on the reserve price in Cases (I) and (II), given $D \in [0,D^{so}]$.

Lemma 7. *For any* $D \in [r^{so}, D^{so}]$*, (i) in Case (I) where* $r \leq D$, $r^I(D) = \tilde{r}^I(D) = r^{so}$*, and (ii) in Case (II) where* $r \geq D$, $r^{II}(D) = D \geq \tilde{r}^{II}(D)$.

Proof. Let us separately consider the following two cases. Part (i). We re-write $\mathbb{E}_S^I[R(r,D)]$ as follows:

$$\begin{aligned}\mathbb{E}_S^I[R(r,D)] \;&=\; N\big(1-F(\check{v})\big)Q(\check{v})b(\check{v}) \;+\; N\int_{\check{v}}^{\hat{v}}\big(1-F(x)\big)b(x)dQ(x)\\ &\quad+\; N\int_{\hat{v}}^{\bar{v}}(1-F(x))\Big[\big(1-\Phi(\tilde{b}(x)-D)\big)\tilde{b}(x) \;+\; \Phi(\tilde{b}(x)-D)D\Big]dQ(x).\end{aligned} \tag{A12}$$

Fix D, differentiating $\mathbb{E}_S^I[R(r,D)]$ with respect to r and plugging $\frac{\partial b}{\partial v_i}=1-\Phi(v_i)$ into the equation yield

$$\begin{aligned}\frac{\partial}{\partial r}\frac{\mathbb{E}_S^I[R(r,D)]}{N} \;&=\; \frac{\partial}{\partial \check{v}}\frac{\mathbb{E}_S^I[R(r,D)]}{N}\cdot\frac{\partial \check{v}}{\partial r}\\ &=\;\Big[-f(\check{v})Q(\check{v})b(\check{v}) \;+\; \big(1-F(\check{v})\big)q(\check{v})b(\check{v}) \;+\; \big(1-F(\check{v})\big)Q(\check{v})\frac{\partial b(\check{v})}{\partial \check{v}} \;-\; (1-F(\check{v}))q(\check{v})b(\check{v})\Big]\frac{\partial \check{v}}{\partial r}\\ &=\; Q(\check{v})\Big[-f(\check{v})b(\check{v}) \;+\; \big(1-F(\check{v})\big)\frac{\partial b(\check{v})}{\partial \check{v}}\Big]\frac{\partial \check{v}}{\partial r}\\ &=\; Q(\check{v})\Big[1-\Phi(\check{v}) \;-\; \frac{f(\check{v})}{(1-F(\check{v}))}\big(\check{v}-\int_0^{\check{v}}\Phi(p_i)dp_i\big)\Big]\frac{\partial \check{v}}{\partial r},\end{aligned} \tag{A13}$$

where $q(\cdot)=Q'(\cdot)$. Recall that $\frac{\partial \check{v}}{\partial r}=\frac{1}{1-\Phi(\check{v})}>0$ from (A9). When $\check{v}=0$, the derivative $\frac{\partial}{\partial r}\frac{\mathbb{E}_S^I[R(r,D)]}{N}$ is 0. But clearly, $\check{v}=0$ cannot be optimal, because $\frac{\partial}{\partial r}\frac{\mathbb{E}_S^I[R(r,D)]}{N}>0$ when $\check{v}$ is slightly above zero. As long as F is regular (increasing hazard rate), the optimum must occur when the term in the square brackets equals zero. We thus have at optimum

$$\begin{aligned}&\Big[1 \;-\; \Phi(\check{v}) \;-\; \frac{f(\check{v})}{(1-F(\check{v}))}\big(\check{v}-\int_0^{\check{v}}\Phi(p_i)dp_i\big)\Big] \;=\; 0\\ \Leftrightarrow\quad &\frac{\big(1-F(\check{v})\big)\big(1-\Phi(\check{v})\big)}{f(\check{v})} \;=\; \int_0^{\check{v}}[1-\Phi(p_i)]dp_i.\end{aligned} \tag{A14}$$

This gives $\check{v}=\check{v}^{so}$ and further, since $b(\check{v})=r$, we have $r=r^{so}=\check{v}^{so}-\int_0^{\check{v}^{so}}\Phi(p_i)dp_i=\int_0^{\check{v}^{so}}[1-\Phi(p_i)]dp_i$. The analysis above indicates that the unrestricted optimal reserve price $\tilde{r}^I(D)$ is exactly equal to r^{so} for any $D\geq 0$. Moreover, for any $D\in[r^{so},D^{so}]$, we have $r\leq D$ and the unrestricted optimal reserve price is feasible; therefore, $r^I(D)=\tilde{r}^I(D)=r^{so}$.

Part (ii). We re-write $\mathbb{E}_S^{II}[R(r,D)]$ as follows:

$$\begin{aligned}\mathbb{E}_S^{II}[R(r,D)] \;&=\; N\big(1-F(\check{v})\big)Q(\check{v})\Big[\big(1-\Phi(b(\check{v})-D)\big)b(\check{v}) \;+\; \Phi(b(\check{v})-D)D\Big]\\ &\quad+\; N\int_{\check{v}}^{\bar{v}}(1-F(x))\Big[\big(1-\Phi(b(x)-D)\big)b(x) \;+\; \Phi(b(x)-D)D\Big]dQ(x).\end{aligned} \tag{A15}$$

Fix D, differentiating $\mathbb{E}_S^{II}[R(r,D)]$ with respect to r yields

$$
\begin{aligned}
\frac{\partial}{\partial r}\frac{\mathbb{E}_S^{II}[R(r,D)]}{N} \;&=\; \frac{\partial}{\partial \check{v}}\frac{\mathbb{E}_S^{II}[R(r,D)]}{N}\cdot\frac{\partial \check{v}}{\partial r} \\
&= \Bigg[-f(\check{v})Q(\check{v})\Big[\big(1-\Phi(b(\check{v})-D)\big)b(\check{v})+\Phi(b(\check{v})-D)D\Big] \\
&\quad +\;\big(1-F(\check{v})\big)Q(\check{v})\frac{\partial b(\check{v})}{\partial \check{v}}\Big[\big(1-\Phi(b(\check{v})-D)\big)\;-\;\varphi(b(\check{v})-D)(b(\check{v})-D)\Big]\Bigg]\frac{\partial \check{v}}{\partial r} \\
&= \; Q(\check{v})\Bigg[-f(\check{v})\Big[\big(1-\Phi(b(\check{v})-D)\big)(b(\check{v})-D)+D\Big] \\
&\quad +\;\big(1-F(\check{v})\big)\frac{\partial b(\check{v})}{\partial \check{v}}\Big[\big(1-\Phi(b(\check{v})-D)\big)\;-\;\varphi(b(\check{v})-D)(b(\check{v})-D)\Big]\Bigg]\frac{\partial \check{v}}{\partial r}.
\end{aligned}
\tag{A16}
$$

Recall that $\frac{\partial \check{v}}{\partial r}=\frac{1-\Phi(r-D)}{1-\Phi(\check{v})}>0$ from (A10). When $\check{v}=0$, the derivative $\frac{\partial}{\partial r}\frac{\mathbb{E}_S^{II}[R(r,D)]}{N}$ is 0. But as long as F is regular (increasing hazard rates), $\frac{\partial}{\partial r}\frac{\mathbb{E}_S^{II}[R(r,D)]}{N}=0$ should occur when the term in the big square bracket is zero. Now let us establish the following property: For any $D\in[r^{so},D^{so}]$ and $r\geq D$, we have $\frac{\partial}{\partial r}\frac{\mathbb{E}_S^{II}[R(r,D)]}{N}\leq 0$, specially $\frac{\partial}{\partial r}\frac{\mathbb{E}_S^{II}[R(r,D)]}{N}=0$ if and only if $D=r^{so}$ and $r=D$. This property helps us to see under conditions which the term in the big square bracket is zero.

Given $b(\check{v})=r$, we re-write the term in the large square bracket of last line in (A16) and show that it has the same sign with the following Λ:

$$
\Lambda \;\equiv\; -\Big[\big(1-\Phi(r-D)\big)(r-D)+D\Big] \;+\; \frac{(1-F(\check{v}))}{f(\check{v})}\frac{\partial r}{\partial \check{v}}\Big[1-\Phi(r-D)\;-\;\varphi(r-D)(r-D)\Big]. \tag{A17}
$$

Plugging $\frac{\partial r}{\partial \check{v}}=\frac{1-\Phi(\check{v})}{1-\Phi(r-D)}$ into Λ, and simplifying it shows

$$
\Lambda \;=\; -\Big[\big(1-\Phi(r-D)\big)(r-D)+D\Big] \;+\; \frac{\big(1-F(\check{v})\big)\big(1-\Phi(\check{v})\big)}{f(\check{v})}\Big[1-\frac{\varphi(r-D)}{1-\Phi(r-D)}(r-D)\Big]. \tag{A18}
$$

Step (1). When $D=r^{so}$ and $r=D$, i.e., $\check{v}=\check{v}^{so}$, we have

$$
\Lambda \;=\; -r^{so} \;+\; \frac{\big(1-F(\check{v}^{so})\big)\big(1-\Phi(\check{v}^{so})\big)}{f(\check{v}^{so})} \;=\; 0. \tag{A19}
$$

Step (2). When $D=r^{so}$ and $r-r^{so}>0$, we can then write Λ as follows:

$$
\Lambda \;=\; -\Big[\big(1-\Phi(r-r^{so})\big)(r-r^{so})+r^{so}\Big] \;+\; \frac{\big(1-F(\check{v})\big)\big(1-\Phi(\check{v})\big)}{f(\check{v})}\Big[1-\frac{\varphi(r-r^{so})}{1-\Phi(r-r^{so})}(r-r^{so})\Big]. \tag{A20}
$$

Note that $-[(1-\Phi(r-r^{so}))(r-r^{so})+r^{so}] < -r^{so}$, and further, since $\check{v} > \check{v}^{so}$ and the hazard rates of F and Φ are increasing, we have $\frac{(1-F(\check{v}))(1-\Phi(\check{v}))}{f(\check{v})}$ decreases with $\check{v}$ and $\frac{1-\Phi(r-r^{so})}{\varphi(r-r^{so})}$ decreases with r, implying that the following inequality should hold

$$\begin{aligned}\frac{\big(1-F(\check{v})\big)\big(1-\Phi(\check{v})\big)}{f(\check{v})}\left[1-\frac{\varphi(r-r^{so})}{1-\Phi(r-r^{so})}(r-r^{so})\right] \;&<\; \frac{\big(1-F(\check{v})\big)\big(1-\Phi(\check{v})\big)}{f(\check{v})}\\ &<\; \frac{\big(1-F(\check{v}^{so})\big)\big(1-\Phi(\check{v}^{so})\big)}{f(\check{v}^{so})}.\end{aligned}$$

Therefore, we have $\Lambda < -r^{so} + \frac{(1-F(\check{v}^{so}))(1-\Phi(\check{v}^{so}))}{f(\check{v}^{so})} = 0$.

Step (3). Let us consider any $D \in (r^{so}, D^{so}]$. Define $\omega = r - D \geq 0$ and denote the inverse bidding function by $b^{-1}(v, D)$. (A18) can be re-written as follows:

$$\begin{aligned}\Lambda \;=\; &-\Big[\big(1-\Phi(\omega)\big)\omega + r^{so} + (D - r^{so})\Big]\\ &+\;\left[\frac{1-F(b^{-1}(\omega+r^{so}+(D-r^{so}),D))}{f(b^{-1}(\omega+r^{so}+(D-r^{so}),D))}\Big(1-\Phi(b^{-1}(\omega+r^{so}+(D-r^{so}),D))\Big)\right]\left[1-\frac{\varphi(\omega)}{1-\Phi(\omega)}\omega\right].\end{aligned} \tag{A21}$$

We now use D' to denote the deposit when $D > r^{so}$ and D'' to denote the deposit when $D = r^{so}$. Given the same v and $D' > D''$, we have $b(v, D') < b(v, D'')$, as $b(v, D)$ is decreasing in D. Further, $b(v, D)$ is increasing in v, indicating that given the same b, we should have $v' > v''$ for $b(v', D') = b(v'', D'')$. This property implies that with the same bid $\omega + r^{so}$, we should have $b^{-1}(\omega + r^{so}, D') > b^{-1}(\omega + r^{so}, D'')$. Thus, from equation (A21), we have

$$\begin{aligned}&\left[\frac{1-F(b^{-1}(\omega+r^{so}+(D'-r^{so}),D'))}{f(b^{-1}(\omega+r^{so}+(D'-r^{so}),D'))}\Big(1-\Phi(b^{-1}(\omega+r^{so}+(D'-r^{so}),D'))\Big)\right]\\ &<\;\left[\frac{1-F(b^{-1}(\omega+r^{so},D'))}{f(b^{-1}(\omega+r^{so},D'))}\Big(1-\Phi(b^{-1}(\omega+r^{so},D'))\Big)\right]\\ &<\;\left[\frac{1-F(b^{-1}(\omega+r^{so},D''))}{f(b^{-1}(\omega+r^{so},D''))}\Big(1-\Phi(b^{-1}(\omega+r^{so},D''))\Big)\right]\\ &<\;\left[\frac{1-F(b^{-1}(r^{so},D''))}{f(b^{-1}(r^{so},D''))}\Big(1-\Phi(b^{-1}(r^{so},D''))\Big)\right]\\ &=\;\frac{(1-F(\check{v}^{so}))(1-\Phi(\check{v}^{so}))}{f(\check{v}^{so})}.\end{aligned} \tag{A22}$$

Since $r \geq D > r^{so}$, we have $\Lambda < -r^{so} + \frac{(1-F(\check{v}^{so}))(1-\Phi(\check{v}^{so}))}{f(\check{v}^{so})} = 0$. We can conclude that for any $D \in [r^{so}, D^{so}]$ and $r \geq D$, $\frac{\partial}{\partial r}\frac{\mathbb{E}_S^{II}[R(r,D)]}{N} \leq 0$, in particular, $\frac{\partial}{\partial r}\frac{\mathbb{E}_S^{II}[R(r,D)]}{N} = 0$ when $D = r^{so}$ and $r = D$. This established property indicates that the unrestricted optimal reserve price $\tilde{r}^{II}(D)$ must be smaller than or equal to D, i.e., $\tilde{r}^{II}(D) \leq D$ for any $D \in [r^{so}, D^{so}]$ (the equality holds if and only if $D = r^{so}$ and $r = D$), and therefore, in Case (II) where $r \geq D$, $\tilde{r}^{II}(D)$ is not implementable and $\mathbb{E}_S^{II}[R(r,D)]$ is maximized at the boundary condition of $r^{II}(D) = D$.

Finally, Λ in (A18) is decreasing in r ($\leq D$), as $\check{v}$ increases with r. This indicates that the term in the big square bracket in (A16) single crosses zero when $r \leq D$. We thus have unconstrained optimum $\tilde{r}^{II}(D)$ is uniquely defined by $\Lambda = 0$. $\square$

Lemma 8. *For any $D \in [0, r^{so})$, (i) in Case (I) where $r \leq D$, $r^I(D) = D \leq \tilde{r}^I(D) = r^{so}$, and (ii) in Case (II) where $r \geq D$, $r^{II}(D) = \tilde{r}^{II}(D)$.*

Proof. We examine $r^I(D)$ and $r^{II}(D)$ for any $D \in [0, r^{so})$.

Part (i). From Lemma 7(i), we have shown that the unrestricted optimal reserve price $\tilde{r}^I(D)$ is equal to r^{so} for any $D \geq 0$. From (A14), since $\frac{(1-F(\check{v}))(1-\Phi(\check{v}))}{f(\check{v})}$ decreases with $\check{v}$ and $\int_0^{\check{v}}[1-\Phi(p_i)]dp_i$ increases with $\check{v}$, we have the following inequality for $D \in [0, r^{so})$ and $r \leq D$:

$$
\begin{aligned}
\frac{\big(1-F(\check{v})\big)\big(1-\Phi(\check{v})\big)}{f(\check{v})} - \int_0^{\check{v}}[1-\Phi(p_i)]dp_i \; &> \; \frac{\big(1-F(\check{v}^{so})\big)\big(1-\Phi(\check{v}^{so})\big)}{f(\check{v}^{so})} - r^{so} \\
&= \; 0.
\end{aligned} \tag{A23}
$$

The inequality above implies that $\frac{\partial}{\partial r}\frac{\mathbb{E}_S^I[R(r,D)]}{N} > 0$ for any $D \in [0, r^{so})$ and $r \leq D$. Therefore, under case (I) where $r \leq D$, $\tilde{r}^I(D)$ is not feasible for any $D \in [0, r^{so})$ and $\mathbb{E}_S^I[R(r,D)]$ is maximized at the boundary condition of $r^I(D) = D$.

Part (ii). Given the fact that $\frac{(1-F(\check{v}))(1-\Phi(\check{v}))}{f(\check{v})}$ decreases with $\check{v}$, (A18) gives the following inequality for any $D \in [0, r^{so})$ and $D = r$, that is,

$$
\begin{aligned}
\Lambda \; = \; \frac{\big(1-F(\check{v})\big)\big(1-\Phi(\check{v})\big)}{f(\check{v})} - D \; &> \; \frac{\big(1-F(\check{v}^{so})\big)\big(1-\Phi(\check{v}^{so})\big)}{f(\check{v}^{so})} - r^{so} \\
&= \; 0.
\end{aligned} \tag{A24}
$$

The inequality above implies that $\frac{\partial}{\partial r}\frac{\mathbb{E}_S^{II}[R(r,D)]}{N} > 0$ for any $D \in [0, r^{so})$ and $r = D$. Similar to proof of Lemma 7(ii), we can establish that Λ in (A18) is decreasing in r ($\leq D$). Therefore, we have that for any $D \in [0, r^{so})$ the unrestricted optimal reserve price $\tilde{r}^{II}(D)$ is greater than D and feasible in Case (II) where $r \geq D$, i.e., $r^{II}(D) = \tilde{r}^{II}(D)$. $\square$

1.1.4 Figures to illustrate the optimal D and r

We here use the following figures to illustrate the seller's choice on reserve price for any given deposit. Figure 2 presents a graphic illustration of how $r^I(D)$ varies when D changes in $[0, D^{so}]$. The X-axis is the reserve price, and the Y-axis is the deposit. The seller's choice of $r^I(D)$ is marked by bold red. For

$D \in [0, r^{so})$, $r^I(D)$ is equal to D, which is located on the 45-degree line, for $D \in [r^{so}, D^{so}]$, $r^I(D)$ is equal to r^{so}, and the seller's revenue becomes independent of D when $D > D^{so}$. Figure 3 graphically illustrates how $r^{II}(D)$ varies when D changes in $[0, D^{so}]$, which is marked by bold red. For $D \in [0, r^{so})$, $r^{II}(D)$ is greater than the deposit D (below the 45-degree line), for $D \in [r^{so}, D^{so}]$, $r^{II}(D)$ is equal to D (which is located along the 45-degree line), and for any $D > D^{so}$, the seller's revenue becomes independent of D. Figure 4 graphically depicts the optimal reserve price $r^*(D)$, which achieves the overall maximized seller revenue across Cases (I) and (II). In the figure, $r^*(D)$ is marked by bold blue. For $D \in [r^{so}, D^{so}]$, $r^*(D)$ is equal to r^{so} from Case (I), and for $D \in [0, r^{so})$, $r^*(D)$ is equal to $r^{II}(D)$ from Case (II).

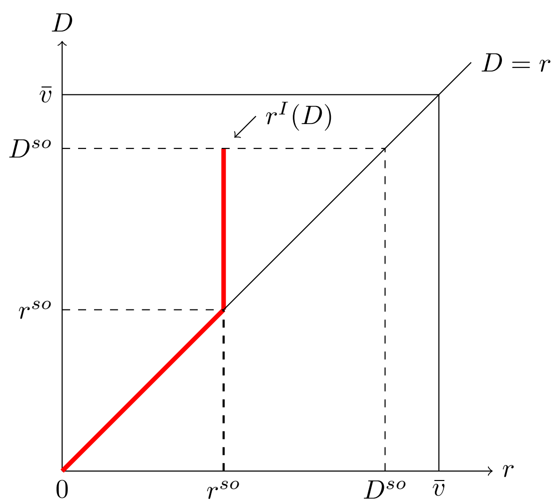


Figure 2: Case (I) where $r \leq D$

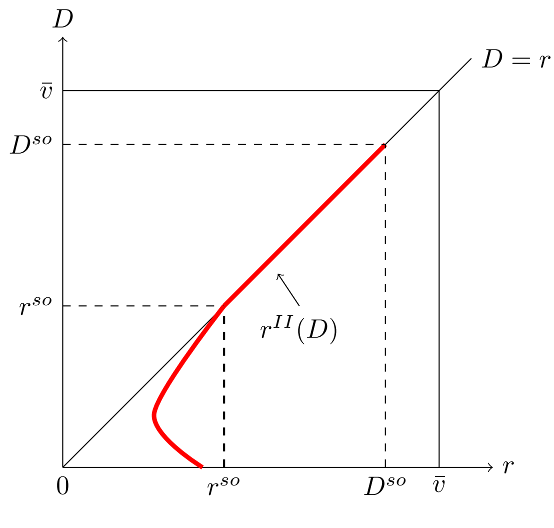


Figure 3: Case (II) where $r \geq D$

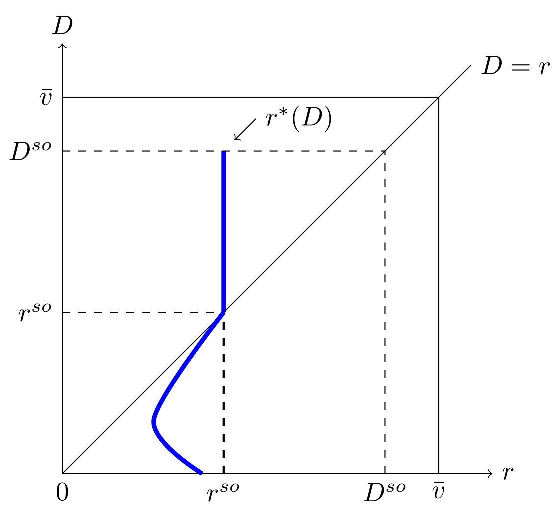


Figure 4: $r^*(D)$ for $D \in [0, D^{so}]$

1.2 Percentage deposit

Let $\gamma \in [0, 1]$ denote the percentage of the final price the seller requires as the deposit. Then, conditional on winning, the deposit amount bidder i as the winner needs to pay can be written as $D = \gamma\kappa$. Clearly, the analyses we have constructed for the outside option strategy in (A1) is not affected. We then look at the deposit strategy. Since the percentage deposit cannot be greater than the final price, the deposit strategy is the same as the case where $\kappa - D \geq 0$ in Section 1.1.2. Next let us examine the bidding strategy. Again, the construction of the equilibrium bid should make a bidder indifferent between "winning at her own bid and then paying the deposit" and "not submitting a bid but waiting for the outside offer directly," that is,

$$\int_0^{(1-\gamma)b}(v_i-p_i)\varphi(p)dp_i \;+\; \int_{(1-\gamma)b}^{\bar{v}}[v_i-(1-\gamma)b]\varphi(p_i)dp_i-\gamma b \;=\; \int_0^{v_i}(v_i-p_i)\varphi(p_i)dp_i$$
$$\Leftrightarrow \quad b \;=\; v_i-\int_{(1-\gamma)b}^{v_i}\Phi(p_i)dp_i.$$

Given r and γ, we write the bidding strategy as $b(v_i)$. It is easy to show that $b(v_i)$ is increasing in v_i and decreasing in γ, as $\frac{\partial b(v_i)}{\partial v_i}=\frac{1-\Phi(v_i)}{1-\Phi(b(v_i)(1-\gamma))(1-\gamma)}>0$ and $\frac{\partial b(v_i)}{\partial \gamma}=\frac{-b(v_i)\Phi(b(v_i)(1-\gamma))}{1-\Phi(b(v_i)(1-\gamma))(1-\gamma)}<0$. Given that $b(\check{v})=r$, the seller's expected revenue denoted by $\mathbb{E}_S[R(r,\gamma)]$ is given by

$$\begin{aligned}\mathbb{E}_S[R(r,\gamma)] \;&=\; N\big(1-F(\check{v})\big)Q(\check{v})\Big[\big(1-\Phi(b(\check{v})(1-\gamma))\big)b(\check{v})+\Phi(b(\check{v})(1-\gamma))\gamma b(\check{v})\Big]\\
&\quad+\; N\int_{\check{v}}^{\bar{v}}\int_{\check{v}}^{v_i}\Big[\big(1-\Phi(b(x)(1-\gamma))\big)b(x)+\Phi(b(x)(1-\gamma))\gamma b(x)\Big]dQ(x)dF(v_i)\\
&=\; N\big(1-F(\check{v})\big)Q(\check{v})\Big[\big(1-\Phi(b(\check{v})(1-\gamma))\big)b(\check{v})+\Phi(b(\check{v})(1-\gamma))\gamma b(\check{v})\Big]\\
&\quad+\; N\int_{\check{v}}^{\bar{v}}(1-F(x))\Big[\big(1-\Phi(b(x)(1-\gamma))\big)b(x)+\Phi(b(x)(1-\gamma))\gamma b(x)\Big]dQ(x).\end{aligned}$$

The seller chooses r and γ to maximize $\mathbb{E}_S[R(r,\gamma)]$. We introduce notations γ^{so} and r^{so} as follows:

$$\gamma^{so}=1 \text{ and } r^{so}=\int_0^{\check{v}^{so}}[1-\Phi(p_i)]dp_i,$$

where $\check{v}^{so}$ is determined by $\frac{(1-F(\check{v}^{so}))(1-\Phi(\check{v}^{so}))}{f(\check{v}^{so})}=\int_0^{\check{v}^{so}}[1-\Phi(p_i)]dp_i$. Let us write $r^*(\gamma)$ to denote the optimal reserve price, given any γ. We can then establish the following result.

Proposition B1. *$\frac{d\mathbb{E}_S[R(r^*(\gamma),\gamma)]}{d\gamma}\geq 0$ for any $\gamma\in[0,1]$. In particular, $\frac{d\mathbb{E}_S[R(r^*(\gamma),\gamma)]}{d\gamma}=0$ when $\gamma=\gamma^{so}$. As a result, the combination of γ^{so} and r^{so} maximizes the seller's revenue. Moreover, $\mathbb{E}_S[R(r^{so},\gamma^{so})]=R^*(D^{so})$.*

Proof. Our proof includes the following two steps:

Step (i). Differentiating $\mathbb{E}_S[R(r,\gamma)]$ with respect to r yields

$$\begin{aligned}&\frac{\partial}{\partial r}\frac{\mathbb{E}_S[R(r,\gamma)]}{N} \;=\; \frac{\partial}{\partial \check{v}}\frac{\mathbb{E}_S[R(r,\gamma)]}{N}\cdot\frac{\partial \check{v}}{\partial r}\\
&=\; Q(\check{v})\Bigg[-f(\check{v})\Big[\big(1-\Phi(b(\check{v})(1-\gamma))\big)b(\check{v})+\Phi(b(\check{v})(1-\gamma))\gamma b(\check{v})\Big] \;+\; (1-F(\check{v}))\frac{\partial b(\check{v})}{\partial \check{v}}\Big[\big(1-\Phi(b(\check{v})(1-\gamma))\big)\\
&\quad-\; \varphi(b(\check{v})(1-\gamma))(1-\gamma)^2 b(\check{v})+\Phi(b(\check{v})(1-\gamma))\gamma\Big]\Bigg]\cdot\frac{\partial \check{v}}{\partial r}.\end{aligned} \tag{A25}$$

When $\check{v} = 0$, the derivative is 0. But as long as F is regular (increasing hazard rate), then $\frac{\partial}{\partial \check{v}} \frac{\mathbb{E}_S[R(r,\gamma)]}{N} = 0$ should occur when the term in the large square bracket is zero. This gives the unrestricted $\check{v}(r^*(\gamma), \gamma)$ and further, the optimal reserve price $r^*(\gamma)$ is given by $b(\check{v}(r^*(\gamma), \gamma))$.

Step (ii). We then know that the impact of γ through $r^*(\gamma)$ is zero by envelope theorem. We thus have the following equation

$$\frac{d}{d\gamma} \frac{\mathbb{E}_S[R(r^*(\gamma), \gamma)]}{N} \;=\; \frac{\partial}{\partial \gamma} \frac{\mathbb{E}_S[R(r, \gamma)]}{N}\Big|_{r=r^*(\gamma), \check{v}=\check{v}(r^*(\gamma),\gamma)}. \tag{A26}$$

which is given by

$$\begin{aligned} & \frac{\partial}{\partial \gamma} \frac{\mathbb{E}_S[R(r, \gamma)]}{N}\Big|_{r=r^*(\gamma), \check{v}=\check{v}(r^*(\gamma),\gamma)} \\ =\; & \big(1 - F(\check{v}(r^*(\gamma), \gamma))\big)Q(\check{v}(r^*(\gamma), \gamma))\Big(\frac{-\varphi(b(\check{v}(r^*(\gamma), \gamma))(1-\gamma))(1-\gamma)}{\Phi(b(\check{v}(r^*(\gamma), \gamma))(1-\gamma))} \cdot \frac{\partial b(\check{v}(r^*(\gamma), \gamma))}{\partial \gamma}\Big) \\ & + \int_{\check{v}(r^*(\gamma),\gamma)}^{\bar{v}} (1 - F(x))\Big(\frac{-\varphi(b(x)(1-\gamma))(1-\gamma)}{\Phi(b(x)(1-\gamma))} \cdot \frac{\partial b(x)}{\partial \gamma}\Big) dQ(x). \end{aligned} \tag{A27}$$

Clearly, $\check{v}(r^*(\gamma), \gamma) < \bar{v}$ for any $\gamma \in [0, 1)$. Further, given $\frac{\partial b(v_i)}{\partial \gamma} < 0$, we therefore have $\frac{d}{d\gamma} \frac{\mathbb{E}_S[R(r^*(\gamma),\gamma)]}{N} > 0$, indicating that it is optimal for the seller to choose $\gamma^{so} = 1$. Plugging $\gamma^{so} = 1$ and $\frac{\partial b(v_i)}{\partial v_i} = \frac{1-\Phi(v_i)}{1-\Phi(b(v_i)(1-\gamma))(1-\gamma)}$ into the term in large square bracket of the last line of (A25) gives $\check{v}^{so}$ and r^{so}:

$$\begin{aligned} -f(\check{v}^{so})b(\check{v}^{so}) + (1 - F(\check{v}^{so}))\frac{\partial b(\check{v}^{so})}{\partial \check{v}^{so}} \;&=\; 0 \\ \Leftrightarrow \quad \frac{(1 - F(\check{v}^{so}))(1 - \Phi(\check{v}^{so}))}{f(\check{v}^{so})} \;&=\; \int_0^{\check{v}^{so}} [1 - \Phi(p_i)]dp_i, \end{aligned}$$

and r^{so} is determined by

$$r^{so} \;=\; \int_0^{\check{v}^{so}} [1 - \Phi(p_i)]dp_i. \tag{A28}$$

Note that the optimal reserve and bidding cutoff are exactly the same as in our main analysis.

Step (iii). Finally, plugging $\gamma^{so} = 1$ and r^{so} into $\mathbb{E}_S[R(r, \gamma)]$ yields

$$\begin{aligned} \mathbb{E}_S[R(r^{so}, \gamma^{so})] \;&=\; N\big(1 - F(\check{v}^{so})\big)Q(\check{v}^{so})b(\check{v}^{so}) \;+\; N\int_{\check{v}^{so}}^{\bar{v}} (1 - F(x))b(x)dQ(x) \\ &=\; \mathbb{E}_S^I[R(r^{so}, D^{so})] \\ &=\; R^*(D^{so}). \end{aligned} \tag{A29}$$

The intuition of the result above is consistent with what we had before. The seller's expected revenue is maximized when the default possibility from the winner is zero. To do this, the seller should set the percentage sufficiently high (equal to 100 percent) so that the winner's default possibility is fully deterred. Moreover, at the optimal level, the percentage deposit requirement is equivalent to the fixed amount deposit requirement, generating the same maximized seller revenue. □

1.3 Deposit proportionally deducted from full payment

So far, we have analyzed the deposit requirement in which the amount the winner paid will be completely deducted from the full payment. However, in practice sellers also commonly implement another type of deposit requirement where the amount paid by the winners is in addition to the full payment. For convenience, we call the former *the inclusive deposit requirement*, and the latter *the exclusive deposit requirement*. It is of interest to compare the two types of the deposit requirements, in particular, which one would benefit the seller more in terms of the expected revenue, or whether they would yield the same expected revenue to the seller. It seems that the exclusive deposit requirement is more profitable, as it is an additional fee, besides the final price, charged by the seller. However, because of this additional "cost" after winning, the bidders would strategically adjust their bids (intuitively, to submit lower bids) in the auction. Thus, the comparison is not trivial and obvious.

Instead of focusing only on these two particular deposit rules, we here provide a more general analysis. Denote the deduction proportion of the deposit D from the full payment by $\alpha \in [0, 1]$. Given α, the winner needs to pay $\kappa - \alpha D$ to complete the transaction. In other words, the winner's total payment is $\kappa + (1-\alpha)D$ for the object. Here, the inclusive and exclusive deposit requirements correspond to the two special cases with $\alpha = 1$ and $\alpha = 0$, respectively. In the second-price auction with $b(\check{v}) = r$ and $b(\hat{v}) = \alpha D$, bidders' bidding strategies at $t = 1$ can be re-written as follows:

In Case (I) where $r \leq \alpha D$, $b(v_i)$ is given by

$$b(v_i) = \begin{cases} v_i - \int_{b(v_i)-\alpha D}^{v_i} \Phi(p_i) dp_i - (1-\alpha)D & \text{if } v_i > \hat{v}; \\ v_i - \int_0^{v_i} \Phi(p_i) dp_i - (1-\alpha)D & \text{if } v_i \in [\check{v}, \hat{v}]; \\ \varnothing & \text{if } v_i < \check{v}, \end{cases}$$

and the associated seller revenue $\mathbb{E}_S^I[R(r, D, \alpha)]$ is given by

$$\begin{aligned} \frac{\mathbb{E}_S^I[R(r, D, \alpha)]}{N} = & \big(1 - F(\check{v})\big)Q(\check{v})\big(r + (1-\alpha)D\big) + \int_{\check{v}}^{\hat{v}} (1 - F(x))\big(b(x) + (1-\alpha)D\big) dQ(x) \\ & + \int_{\hat{v}}^{\bar{v}} (1 - F(x)) \Big[\big(1 - \Phi(\tilde{b}(x) - \alpha D)\big)\big(\tilde{b}(x) + (1-\alpha)D\big) + \Phi(\tilde{b}(x) - \alpha D) D\Big] dQ(x), \end{aligned}$$

where the bidding strategies are denoted by $b(v_i)$ and $\tilde{b}(v_i)$ for $v_i \in [\check{v}, \hat{v}]$ and $v_i \in (\hat{v}, \bar{v}]$, respectively.

In Case (II) where $r \geq \alpha D$, $b(v_i)$ is given by

$$b(v_i) \;=\; \begin{cases} v_i - \int_{b(v_i)-\alpha D}^{v_i} \Phi(p_i)dp_i - (1-\alpha)D & \text{if } \; v_i \geq \check{v}; \\ \varnothing & \text{if } \; v_i < \check{v}, \end{cases}$$

and the associated seller revenue $\mathbb{E}_S^{II}[R(r,D,\alpha)]$ is given by

$$\begin{aligned} \frac{\mathbb{E}_S^{II}[R(r,D,\alpha)]}{N} \;&=\; \big(1-F(\check{v})\big)Q(\check{v})\Big[\big(1-\Phi(r-\alpha D)\big)\big(r-\alpha D\big)+D\Big] \\ &+\; \int_{\check{v}}^{\bar{v}} (1-F(x))\Big[\big(1-\Phi(b(x)-\alpha D)\big)\big(b(x)-\alpha D\big)+D\Big]dQ(x). \end{aligned}$$

Define D^{so} and r^{so} as follows

$$D^{so} = \int_0^{\hat{v}^{so}} [1-\Phi(p_i)]dp_i \;\text{ and }\; r^{so} = \int_0^{\check{v}^{so}} [1-\Phi(p_i)]dp_i - (1-\alpha)D^{so},$$

where $\hat{v}^{so} = \bar{v}$ and $\check{v}^{so}$ is given by $\frac{(1-F(\check{v}^{so}))(1-\Phi(\check{v}^{so}))}{f(\check{v}^{so})} = \int_0^{\check{v}^{so}} [1-\Phi(p_i)]dp_i$. Interestingly, we see that although the thresholds of $\hat{v}^{so}$ and $\check{v}^{so}$ and the optimal deposit D^{so} are the same as before, the optimal reserve price r^{so} depends on the deduction proportion rule. Furthermore, r^{so} increases in α and therefore there must exist a threshold, called $\tilde{\alpha} > 0$, such that $r^{so} = 0$. This fact indicates that if the seller wants to implement deduction proportion rule $\alpha < \tilde{\alpha}$, the optimal reserve price should be a negative value, implying that submitting negative-valued bids from bidders should be allowed.

We next examine the impacts of the deduction proportion rule on seller's revenue. In particular, we are interested in comparing seller revenues across different proportion rules, answering the question of whether there exists an optimal proportion α to the seller. Let us denote the expected seller revenues and bidding strategies associated with two proportion rules α and α' by $\mathbb{E}_S^I[R(r,D,\alpha)]$ and $\mathbb{E}_S^I[R(r,D,\alpha')]$, and $b(v_i,\alpha)$ and $b(v_i,\alpha')$, respectively, where $\alpha,\alpha' \in [0,1]$ and $\alpha \neq \alpha'$. We can then establish the following result.

Proposition B2. *If submitting negative-valued bids is allowed, the seller's expected revenue with the optimal r^{so} and D^{so} is independent of the deduction proportion rule α.*

Proof. Our proof can be split into the following three steps: *Step* (i). We first assume that $\check{v}$, $\hat{v}$, and D are exactly the same across both deduction proportion rules α and α', then the equilibrium bidding strategies show:

If $v_i \in [\check{v}, \hat{v}]$, the equilibrium bidding strategy implies the following fact that

$$b(v_i,\alpha) + (1-\alpha)D = v_i - \int_0^{v_i} \Phi(p_i)dp_i = b(v_i,\alpha') + (1-\alpha')D. \tag{A30}$$

If $v_i \in (\hat{v}, \overline{v}]$, from the equilibrium bidding strategy, we have the fact that

$$\int_{\tilde{b}(v_i,\alpha)-\alpha D}^{v_i} [1-\Phi(p_i)]dp_i = D = \int_{\tilde{b}(v_i,\alpha')-\alpha' D}^{v_i} [1-\Phi(p_i)]dp_i. \tag{A31}$$

which implies

$$\tilde{b}(v_i,\alpha) - \alpha D = \tilde{b}(v_i,\alpha') - \alpha' D. \tag{A32}$$

Step (ii). Given what we have in step (i.), we can establish the equivalence between $\mathbb{E}_S^I[R(r,D,\alpha)]$ and $\mathbb{E}_S^I[R(r,D,\alpha')]$ as follows

$$\begin{aligned}
&\frac{\mathbb{E}_S^I[R(r,D,\alpha)]}{N} \\
=\ & \big(1-F(\check{v}(\alpha))\big)Q(\check{v}(\alpha))\big(r+(1-\alpha)D\big) + \int_{\check{v}(\alpha)}^{\hat{v}(\alpha)} \big(1-F(x)\big)\big(b(x,\alpha)+(1-\alpha)D\big)dQ(x) \\
&+ \int_{\hat{v}(\alpha)}^{\bar{v}} (1-F(x))\Big[\big(1-\Phi(\tilde{b}(x,\alpha)-\alpha D)\big)\big(\tilde{b}(x,\alpha)-\alpha D\big)+D\Big]dQ(x) \\
=\ & \big(1-F(\check{v}(\alpha'))\big)Q(\check{v}(\alpha'))\big(r+(1-\alpha')D\big) + \int_{\check{v}(\alpha')}^{\hat{v}(\alpha')} \big(1-F(x)\big)\big(b(x,\alpha')+(1-\alpha')D\big)dQ(x) \\
&+ \int_{\hat{v}(\alpha')}^{\bar{v}} \big(1-F(x)\big)\Big[\big(1-\Phi(\tilde{b}(x,\alpha')-\alpha' D)\big)\big(\tilde{b}(x,\alpha')-\alpha' D\big)+D\Big]dQ(x) \\
=\ & \frac{\mathbb{E}_S^I[R(r,D,\alpha')]}{N}.
\end{aligned} \tag{A33}$$

This implies that $\mathbb{E}[R(r,D,\alpha)]$ exactly equals $\mathbb{E}[R(r,D,\alpha')]$; both settings generate the same revenue to the seller.

Step (iii). The optimal $\check{v}^{so}$, $\hat{v}^{so}$, and D^{so} do not depend on the deduction proportion rule α. Therefore, if submitting negative-value bids from bidders is allowed, the maximized revenue of the seller is independent of the deduction proportion rule α. $\square$

A bidder will strategically adjust her bid as the best response to the deduction proportion rule implemented by the seller; a higher deduction proportion induces a higher bid from the bidder, that is, $b > b'$ if $\alpha > \alpha'$. Interestingly, we show that if negative-valued bids are allowed, the seller obtains exactly the same expected revenue for all α. We call this property the "deduction proportion independence." However, in practice, it is unlikely (if not impossible) to allow negative-valued bids. Obviously if the seller imposes a further constraint $r \geq 0$, implementing a lower α should generate a lower revenue to the seller.

Corollary B1. *If bidders can only submit non-negative bids, the maximized seller revenue is increasing in α.*

This result above indicates that the "deduction proportion independence" breaks down if a "non-negative-bid" restriction is imposed in the auction, and this implies that the exclusive deposit requirement is worse than the inclusive deposit requirement in terms of seller revenue. This provides an explanation of why the latter is relatively more popular in practice.

2 Appendix S2

This online appendix covers all the computation details in numerical examples 1 and 2.

2.1 Numerical example 1

Taking $N = 2$ and $v_i \sim U[0,1]$, $p_i \sim U[0,1]$, we have the following two parts for the computations.

Case (I) where $r \le D$. In the second-price auction, $\check{v}$ and $\hat{v}$ for any given r and D can be computed by the following two equations:

$$r = \check{v} - \frac{1}{2}\check{v}^2, \text{ and } D = \hat{v} - \frac{1}{2}\hat{v}^2.$$

Note that the optimal r^{so} and D^{so} can be computed by $\hat{v}^{so} = 1$ and $(1-\check{v}^{so})(1-\check{v}^{so}) = \int_0^{\check{v}^{so}} [1-p_i]dp_i$.

Bidder i's bidding strategy can be simplified as follows:

$$b(v_i, r, D) = \begin{cases} (D+1) - \left[-2v_i + v_i^2 + (D+1)^2 - D^2 \right]^{\frac{1}{2}} & \text{if } v_i > \hat{v}; \\ v_i - \frac{1}{2}v_i^2 & \text{if } v_i \in [\check{v}, \hat{v}]; \\ No & \text{if } v_i < \check{v}. \end{cases}$$

The associated seller's revenue function is given by

$$\begin{aligned} &\mathbb{E}_S^I[R(r,D)] \\ &= 2(1-\check{v})\check{v}r + 2\int_{\check{v}}^{\hat{v}} (1-v_i)b(v_i,r,D)dv_i \\ &\quad + 2\int_{\hat{v}}^{\bar{v}} (1-v_i)\left[\left(1 - (\tilde{b}(v_i,r,D) - D)\right)\tilde{b}(v_i,r,D) + (\tilde{b}(v_i,r,D) - D)D\right]dv_i \\ &= 2(1-\check{v})\check{v}r + \hat{v}^2 - \check{v}^2 - \hat{v}^3 + \check{v}^3 + \frac{\hat{v}^4}{4} - \frac{\check{v}^4}{4} \\ &\quad + 2\int_{\hat{v}}^{1} (1-v_i)\left[\sqrt{v_i^2 - 2v_i + 2D + 1} - v_i^2 + 2v_i - D - 1\right]dv_i, \end{aligned}$$

where $\check{v} = 1 - \sqrt{1-2r}$ and $\hat{v} = 1 - \sqrt{1-2D}$.

Case (II) where $r \ge D$. In a second-price auction, we can compute $\check{v}$ for any given r and D by the following equation:

$$r = \check{v} - \left[\frac{1}{2}\check{v}^2 - \frac{1}{2}(r-D)^2\right].$$

The equilibrium bidding strategy can be simplified as follows:

$$b(v_i, r, D) \;=\; \begin{cases} (D+1) - \left[(v_i - 1)^2 + 2D\right]^{\frac{1}{2}} & \text{if } v_i \geq \check{v}; \\ No & \text{if } v_i < \check{v}. \end{cases}$$

The associated seller's revenue function is given by

$$\begin{aligned} &\mathbb{E}_S^{II}[R(r, D)] \\ &= 2(1-\check{v})\check{v}\left[\Big(1-(r-D)\Big)r + (r-D)D\right] \\ &\quad + 2\int_{\check{v}}^{\bar{v}} (1-v_i)\left[\Big(1-(b(v_i,r,D)-D)\Big)b(v_i,r,D) + (b(v_i,r,D)-D)D\right]dv_i \\ &= 2(1-\check{v})\check{v}\left[\Big(1-(r-D)\Big)r + (r-D)D\right] \\ &\quad + 2\int_{\check{v}}^{1} (1-v_i)\left[\sqrt{2D+1-2v_i+v_i^2} - D - 1 + 2v_i - v_i^2\right]dv_i, \end{aligned}$$

where $\check{v} = 1 - \sqrt{1-4r+2D-(r-D)^2}$.

Our computations show $r^{so} = 0.33333$ and $D^{so} = 0.5$.[5] Moreover, when the seller sets $r \geq 0.5$, no valid bids can be submitted, and the seller's revenue decreases to zero. Given the deposit $D(= 0.2, 0.33333, 0.4)$, Figures 5 depicts how seller revenue functions in Cases (I) and (II) change in r. In each figure, the horizontal axes is reserve price r and the vertical axes is seller revenue $E_s(R)$. $E_s^I(R)$ is depicted by the blue curve and $E_s^{II}(R)$ by the red curve. Consistent with these properties established in Lemmas 7 and 8 and Proposition 2, our simulation results show that in Figures 5(a), given $D > r^{so}$, the maximized seller revenue is given by $E_s^I(R)$ with reserve price r^{so}. With $D = 0.33333$, both Cases (I) and (II) give the same maximized seller revenue with reserve price r^{so} in Figure 5(b). When $D = 0.2$, the maximized seller revenue is achieved under Case (II) with some reserve price less than r^{so}.

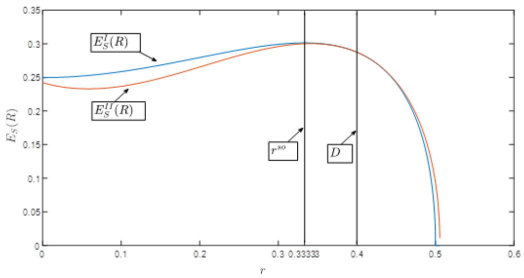
Figure 6 depicts the property characterized in Proposition 3; the seller's optimal revenue function $R^*(D)$ in (16) is increasing in D for $D \in [0, D^{so}]$ and becomes flat for any $D > D^{so}$. The global maximum seller revenue ($= 0.30157$) is achieved at $r = 0.33333$ and $D^{so} = 0.5$.

2.2 Numerical example 2

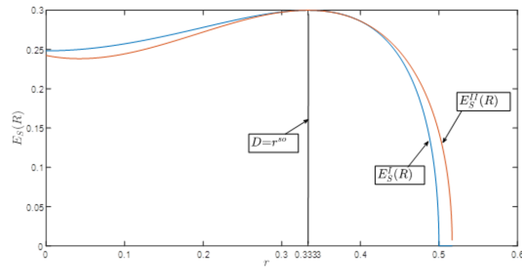
Taking $N = 2$ and $v_i \sim U[0,1]$, and $\Phi(p_i) = p_i^{\alpha}$ over $[0,1]$, we have the following two parts for the computations. First, the optimal D^{so} and r^{so} are given by

$$D^{so} = \hat{v}^{so} - \frac{(\hat{v}^{so})^{\alpha+1}}{\alpha+1} \quad \text{and} \quad r^{so} = \check{v}^{so} - \frac{(\check{v}^{so})^{\alpha+1}}{\alpha+1},$$

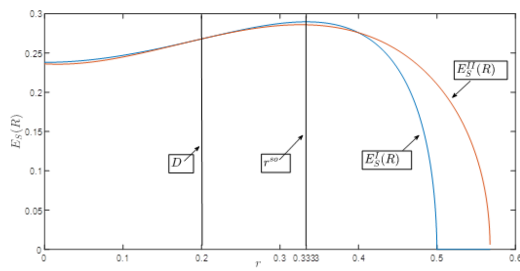
[5]Note that if no post-auction outside offer exists, i.e., $\Phi(\cdot) = 0$, the optimal reserve price is 0.5.



(a) $D = 0.4$



(b) $D = 0.33333$



(c) $D = 0.2$

Figure 5: $E_s^I(R)$ and $E_s^{II}(R)$ change in r

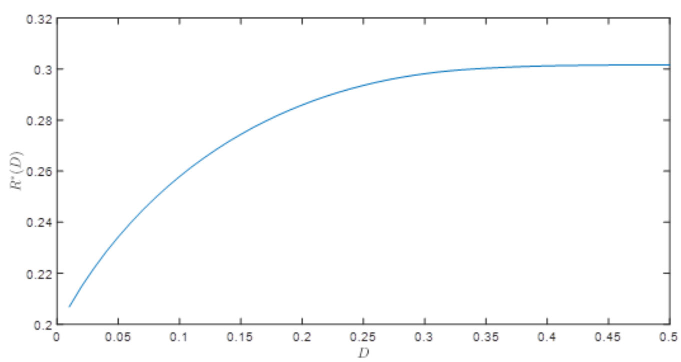


Figure 6: $R^*(D)$ changes in D

where $\hat{v}^{so} = \overline{v} = 1$ and $(1 - \check{v}^{so})(1 - (\check{v}^{so})^\alpha) = \check{v}^{so} - \frac{(\check{v}^{so})^{\alpha+1}}{\alpha+1}$. Then, we can compute the seller's optimal revenue with D^{so} and r^{so}, which is given by

$$\begin{aligned} R^* &= 2(1 - \check{v}^{so})\check{v}^{so} r^{so} + 2\int_{\check{v}^{so}}^{1} (1 - v_i) b(v_i, r^{so}, D^{so}) dv_i. \\ &= 2(1 - \check{v}^{so})\check{v}^{so} r^{so} + 2\int_{\check{v}^{so}}^{1} (1 - v_i) \left(v_i - \frac{v_i^{(\alpha+1)}}{\alpha + 1} \right) dv_i. \end{aligned}$$

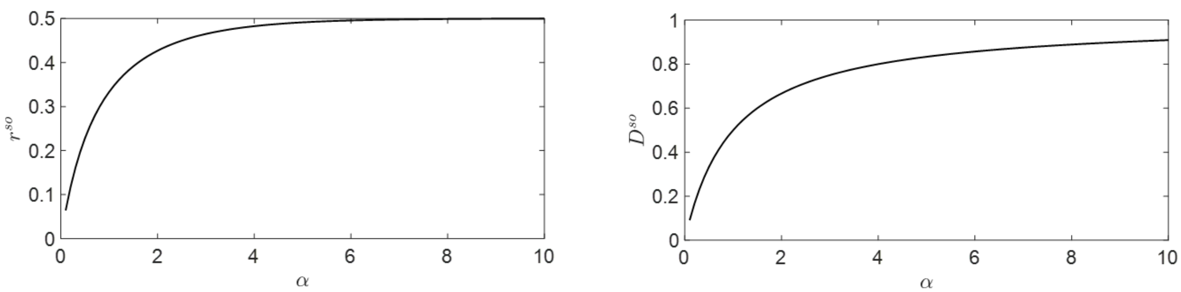
When $D = 0$, it is easy to check that the equilibrium bidding is $b(v_i) = v_i$, and the optimal $r^*(D = 0)$ is given by

$$(1 - r^*)\Big(1 - (1 + \alpha)(r^*)^\alpha\Big) - \Big(1 - (r^*)^\alpha\Big)r^* = 0.$$

The seller's revenue with $D = 0$ and $r^*(D = 0)$, which corresponds to the case of $r \geq D$, can be then written as follows:

$$\mathbb{E}_S^{II}[R(r^*)] = 2(1 - r^*)r^*\Big[r^* - (r^*)^{(\alpha+1)}\Big] + 2\int_{r^*}^{1}(1 - v_i)\left(v_i - v_i^{(\alpha+1)}\right)dv_i.$$

We then compute how a change in α affects the sellers' choices regarding the optimal reserve price r^{so}, the optimal deposit D^{so}, and the optimal expected revenue R^*. Figure 7 illustrates how r^{so} and D^{so} change with α. Consistent with our theoretical predictions, the figures show that when α increases, which corresponds to a worse chance for the outside option, both r^{so} and D^{so} increase.



(a) r^{so} and α

(b) D^{so} and α

Figure 7: The impact of α on r^{so} and D^{so}

Figure 8 depicts how R^* changes with α. Again, consistent with the theoretical prediction, an increase in α generates a higher R^*, which is captured by the black curve. To illustrate the role of a deposit in the auction with a post-auction outside offer, we compute and plot how seller revenue with $D = 0$ and the associated optimal reserve price, denoted by $r^*(D = 0)$, changes with α. As depicted by the red dotted curve, seller revenue increases in α as well. Interestingly, the difference between the two revenues is not monotonic. The improvement in seller revenue after charging a deposit is small when the likelihood of having an outside option, captured by α, is either too small or too large, but a relatively large improvement is achieved when the likelihood is in the medium range.

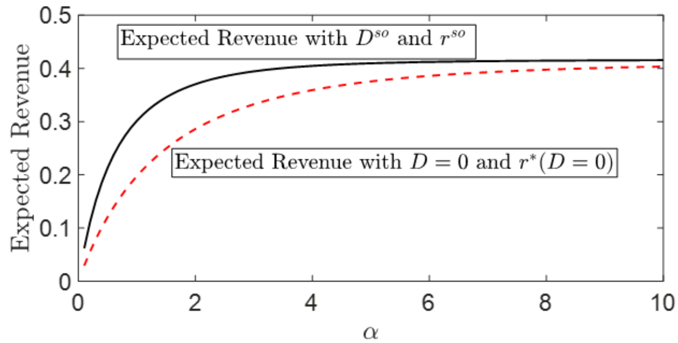


Figure 8: The relationship between R^{*} and α