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Citation: Fusai, G., Longo, G. & Zanotti, G. (2022). Interest rate structured products: can they improve the risk–return profile?. *The European Journal of Finance*, 28(13-15), pp. 1481-1512. doi: 10.1080/1351847x.2021.1967180

This is the accepted version of the paper.

This version of the publication may differ from the final published version. To cite this item please consult the publisher's version.

Permanent repository link: <https://openaccess.city.ac.uk/id/eprint/26803/>

Link to published version: <https://doi.org/10.1080/1351847x.2021.1967180>

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Interest rate structured products: can they improve the risk-return profile?

Gianluca Fusai* Giovanni Longo† Giovanna Zanotti‡

Abstract

In this paper we investigate the contribution of interest rate structured bonds to portfolios of risk-averse retail investors. We conduct our analysis by simulating the term structure according to a multifactor no-arbitrage interest rate model and comparing the performance of a portfolio consisting of basic products (zero-coupon bonds, coupon bonds and floating rate notes) with a portfolio containing more sophisticated exotic products (like constant maturity swaps, collars, spread and volatility notes). Our analysis, performed under different market environments, as well as volatility and correlation levels, takes into account the combined effects of risk-premiums required by investors and fees that they have to pay. Our results show that capital protected interest rate structured products allow investors to improve risk-return trade-off if no fees are considered. With fees, our simulations show that structured products add value to the basic portfolio in a very limited number of cases. We believe our paper contributes to understanding the role of structured products in investors portfolios also in light of the current regulatory debate on the use of complex financial products by retail investors.

JEL: G11, G17, E47.

Keywords: Structured products, efficient frontier, portfolio diversification, interest rate derivatives, term structure model

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1 Introduction

Structured products are financial instruments with embedded derivatives designed to provide original payoffs and tailor risk/return profiles for investors. Their payoff is dependent on a formula based on some underlying securities (such as stocks indexes, basket of stocks, interest rates, commodities). They may go by various names (principal protected notes, accelerated return notes, range notes, barrier notes are just a few examples) and different wrappers (notes, certificates, for example). Structured products are built by manufacturers (investment banks) and offered by distributors (usually large commercial banks) as medium term notes with a term that can vary from a few months to several years.

According to ESMA annual report 2020 [21], the total outstanding stock of SRPs held by EU retail investors at the end of 2019 was around EUR 400bn. This is far less than holdings in UCITS (more than EUR 4.5tn) less than half of the holdings in AIFs sold to retail investors (EUR 1tn). Based on this data, the retail market for structured products made up around 2% of the financial net worth of EU households in 2019. According to EU-SIPA (European Structured Products Association) the market volume of investment and leverage products issued as securities stood at EUR 281 billion at the end of 2020 for Austria, Belgium, Germany, and Switzerland, was equal to 281 bilion EUR. The total number of exchange traded products was equal to 448.035 for investment products and 1237343 for leverage products. In Italy the amount issued at the end of 2020 was equal to almost 13 bilion EURO down from the pick of almost 17 billion in 2019 but in line with the constant growth in the market since 2016. Structured products are owned by a wide spectrum of clients ranging from institutional to retail individuals. The aim of this paper is to investigate the contribution of structured products to portfolios of retail investors. There are several reasons that could explain why structured products could add values to portfolios. The structuring process generates payoffs able to match any desired wealth distribution. In this regard, structured products increase the investment opportunities available. The literature has proved that stocks and bonds alone cannot provide exposure to all risk factors (such as volatility and price jumps). Derivatives use makes it possible to diversify across risk factors and to receive the associated risk premia. In spite of this consideration, derivatives, alone or combined in a wrapper as structured products, are often not considered by traditional asset allocation models. From a theoretical point of view this could be justified by the consideration that in a complete market, derivatives are redundant since they could be replicated by a dynamic trading strategy in stocks and bonds. But if markets are not complete, then excluding derivatives from asset allocation could lead to suboptimal results. On the other hand, structured products are perceived as costly, overly complex and lacking transparency. Investors may have difficulties in understanding all relevant characteristics of complex products. This should not be an issue for institutional users of structured products, but it is so for retail clients. Furthermore, financial institutions and retail clients have strong information asymmetries on pricing the products. This could allow financial institutions to charge higher fees, not fully displayed to investors, which significantly reduce the final performance of the instrument. The increased volumes of structured products issued in the market has drawn the attention of regulators. In July 2013, a report on "Retailisation in the EU" by ESMA highlighted that, "from a consumer protection perspective, retail investors may face difficulties in understanding the drivers of risks and returns of structured products". In the Opinion *Structured Retail Products - Good practices for product governance arrangements*,

46 (March 2014) ESMA writes that *it is good practice for manufacturers to ensure financial*
47 *products meet the financial needs, investment objectives, knowledge and experience of the*
48 *target market identified by the manufacturer.* This same idea is behind MIFID2 product
49 governance: financial products have to be designed by the manufacturer with reference to a
50 potential target client.

51 In this paper we aim to contribute to the current debate on the use of structured prod-
52 ucts by trying to measure the value added to retail investors by the inclusion of structured
53 products to the efficient frontier and to the risk - return profile. We focus on interest rate
54 linked products. We consider a base portfolio consisting of traditional interest rate products
55 (zero coupons, fixed and floating coupon bonds) and we add structured products (constant
56 maturity swaps, collared floating rate notes, spread notes and volatility notes). In spite of
57 its simplicity, our base portfolio is very significant for retail investors, who are traditionally
58 bondholders and allocate to the equity component only a small portion of their wealth. Then
59 we add to the basic portfolio the structured products mentioned above. They all have capital
60 protection at maturity and they generate coupons throughout their lives. But, compared to
61 traditional bonds, they allow to gain exposure to factors such as the change in the slope of
62 the term structure and the volatility of interest rates. To the best of our knowledge, there
63 are no previous studies on the contribution of interest rate-linked structured products to
64 portfolios of risk averse investors. We investigate how this convenience is robust to different
65 initial market environments like interest rate term structure shapes, as well as volatility and
66 correlation in its changes. Finally we examine how the combined effect of the risk-premium
67 required by investors and fees can change the portfolio allocation with respect to the one
68 consisting only of basic securities. The analysis is conducted by simulating the evolution
69 of the term structure of interest rates using the popular multifactor no-arbitrage Gaussian
70 (G2++) model, see Brigo and Mercurio ([9]). Simulations are used to price the different
71 bonds, using the risk-neutral probability measure, but also to generate real-world scenarios,
72 using the physical or natural measure, and incorporating in the model investors' risk-aversion
73 parameters. Simulations are performed under different scenarios concerning the shape of the
74 term structure of interest rates and investor's risk appetite.

75 2 Literature review

76 Most academic papers studying structured products have focused on pricing related issues
77 (Chen and Kesinger ([12]), Wasserfallen and Schenk ([45]), Burth et al. ([10]), Stoimenov
78 and Wilkens ([40], [41]) and Baule et al. ([3]), Wellmeier et al. ([44]), Bernard et al. ([5])
79 between others). These studies examine the difference between the quoted price and the
80 theoretical fair value of the SP, and they reach the conclusion that they are on average
81 mispriced. Other academic studies have examined the factors that could explain the growth
82 of SPs and what benefits they can offer. Fisher ([25]) finds that rational motives, such as
83 diversification and cost management, as well as irrational motives, such as betting, induce
84 retail investors to buy structured products. Branger and Bruer ([7]) analyze if retail in-
85 vestors with a buy and hold trading strategy can benefit from an investment in structured
86 products. They show that the benefit of investing in typical retail products is equivalent to
87 an annualized risk-free excess return of at most 35 basis points. Taking transaction costs
88 into consideration, benefits are reduced to 14 basis points. Their analysis is however limited

89 to SPs written on a single index. Hens and Rieger ([30]) analyze the benefits in term of util-
90 ity gains that can be achieved using structured products to deviate from a linear exposure.
91 They show that some of the most used structured products are not optimal for rational retail
92 investors if the utility function is concave. Using a different, non-concave utility function,
93 the gains become significant but still too small to compensate premium costs. They conclude
94 that behavioral factors such as loss-aversion or probability mis-estimation more than utility
95 gains explain the growing demand for structured products. The importance of behavioural
96 factors in explaining structured products demand was also found by Breuer and Perst ([8]).
97 Vanduffel ([43]) shows that structured products allow issuers to gain margins and investors
98 to gain returns. However investors should be very careful to analyse structured products,
99 since potential cash flows can turn into losses in the case of changes in market expectations.
100 Henderson and Pearson ([29]) investigate the dark side of financial innovation, concluding
101 that if investors misunderstand financial markets or suffer from cognitive biases and assign
102 incorrect probability weights to events, then financial institutions can exploit these biases by
103 creating products that pay off in the states that investors overweigh and do not pay off in the
104 states that investors underweigh. In this context, investors mis-price instruments and assign
105 a value that is greater than the fair value. Structured products allow financial institutions
106 to gain from the willingness of investors to overpay. Rieger ([38]) analyses the properties
107 that a product should have to maximize the utility function of an investor. Results show
108 that optimal products should follow the market, that is, they must be co-monotone with
109 the market portfolio (in the case of the CAPM) or with the inverted state price function
110 (in the general case). Chen ([13]) examines the role of derivatives on hedge fund portfolios,
111 showing that funds that use derivatives exhibits lower risk and are less likely to liquidate
112 in poor market condition. Jessen and Jorgensen ([32]) develop an optimal portfolio choice
113 model to describe the role of structured bonds in holdings of small retail investors. The set
114 of investment opportunities available to investors are a risky index, a bank account and a
115 structured product linked to another index. Investors are rational and maximize expected
116 utility at maturity. The results, based on different utility functions, show that structured
117 products have the highest relevance for investors with medium risk aversion. They also show
118 that the portion of structured products is very sensitive to change in the cost of construction.
119 Jessen and Jorgensen conclude that retail investors should include structured products in
120 their portfolios to reach higher diversification. The positive contribution of derivatives use
121 to the performance of pension funds (Cui, Oldenkamp, Vellekoop [14]) and to hedge funds
122 (Chen [13]) has been proved. Ofir and Wiener ([37]) show that behavioral biases explain the
123 investment in structured products among professional investors and claims the importance
124 of a specific regulation to increase investors' protection. Cui et al. ([14]) show that even
125 relatively small investments in derivatives allow pension funds to improve certainty equiva-
126 lent rates of return and other important performance measures. Derivatives enable pension
127 funds to capture diffusion risk, jump risk and volatility risk and to earn the associated risk
128 premia improving the risk return ratio. Deng et al. ([15]) show that ex-post returns of
129 structured products issued by 13 US brokerage firms since 2007 are highly correlated with
130 the returns of large capitalization equity markets in the aggregate, but individual struc-
131 tured products generally under-perform simple alternative allocations to stocks and bonds.
132 Cèlerier and Vallée (2017) investigate why banks design and offer structured products based
133 on two competing theories: risk sharing and catering investors. According to the first theory
134 (Allen and Gale 1994; Duffie and Rahi 1995), innovation in the design of financial products

135 improves risk sharing and contributes to complete the market offering to investors products
 136 that better match their risk return profile. According to the catering theory banks introduce
 137 innovation to cater to risk seeking investors. The paper, based on a study of the hurdle
 138 rate, the complexity and the risk of structured products, conclude that product complex-
 139 ity has increased from 2002 to 2010, that higher headline rates are associated with higher
 140 complexity and higher exposure to the risk of complete losses, that the spread between
 141 headline rates and interest rates increase when interest rates are low and finally, that higher
 142 markup are associated with higher complexity. Based on these findings they conclude that
 143 innovation in product design is coherent only with the catering theory. Maringer, Pohl and
 144 Vanini ([33]) analyse structured products with a focus on the Swiss market. They address
 145 three main questions: how structured products performed in the period 2008-2014, what the
 146 costs for investors at issuance are, how structured products can be used. The study shows
 147 that 80% of all issued products generated positive performance. The total expense ratio
 148 is estimated in a range between 0.3% and 1.7%. Finally, investors use structured products
 149 mainly to take rapid advantage of market opportunities. Entrop et al. ([19]) measure the
 150 risk adjusted performance achieved by investors buying structured products. They find that
 151 alphas are typically negative, even when transaction costs are ignored. Furthermore the
 152 under-performance increases with product complexity, since higher implicit price premiums
 153 are charged.

154 3 The products

155 Dybvig ([18]) has shown that, in a complete market the most efficient way to achieve a wealth
 156 distribution is by purchasing ‘simple’ structured products, whose payoffs only depend on the
 157 value of the underlying asset at maturity and not at intermediate times. Similar results have
 158 been obtained also in incomplete markets: see for example Vanduffel et al. ([43]). For this
 159 reason, in our analysis we do not consider path-dependent products. However, the products
 160 analysed cover a wide spectrum of interest rate products that over the years have been very
 161 popular among investors. Interest rate structured products have been little studied in the
 162 literature, which is more focused on equity based SPs. Moreover, we assume that: all the
 163 SPs considered are default free and have a maturity of five years and annual cash flows.
 164 Those are typical expires and payment frequencies for bonds offered to retail clients.

165 In what follows, we denote with $P(t, T)$ the time t discount factor for maturity T . This
 166 quantity is bootstrapped from market quotations of LIBOR and swap rates. LIBOR and
 167 swap rates are often also used as reference rates in the determination of the coupon payment.
 168 We recall that the (annualized) LIBOR rate $L(t, T)$ quoted at time t for maturity T is related
 169 to $P(t, T)$ by the relationship

$$L(t, T) = \frac{1}{\alpha_{t,T}} \left(\frac{1}{P(t, T)} - 1 \right), \quad (1)$$

170 where $\alpha_{t,T}$ is expressed in years and measures the time fraction (computed according to a
 171 given day count convention) between the two dates t and T .

172 The swap rate $S(t; \tau_n)$ quoted at time t with tenor $\tau_n = t_n - t$ is related to the term

173 structure of discount factors by the relationship

$$S(t; t_n) = \frac{1 - P(t, t_n)}{\sum_{i=1}^n \alpha_{i-1, i} P(t, t_i)}, \quad (2)$$

174 where $t_i, i = 1, \dots, n$ are the fixed leg swap payment dates.

175 In general, SPs cash flows consist in a periodic payment (fixed or variable) $C(t_i)$ at times
176 $t_i, i = 1, \dots, n$, and the payment of the notional N at maturity t_n

$$C(t_i) = \begin{cases} N \times \alpha_{i-1, i} \times c_i, & i = 1, \dots, n - 1, \\ N \times (1 + \alpha_{n-1, n} \times c_i), & i = n, \end{cases}$$

177 where c_i is the annualized coupon rate, which can be fixed or be floating according to some
178 formula as described below in Section 3.1. Here t_0 refers to the issue date of the bond and
179 corresponds also to the start of the first coupon payment. In particular, if c_i is constant, we
180 have a fixed rate bond. If it randomly changes according to some reference rate, we have a
181 floating rate bond.

182 A detailed description of the various coupon formulas is provided in the next section.

183 3.1 Plain Vanilla Bonds

184 We first consider plain vanilla products, such as zero-coupon bonds, and fixed and floating
185 rate coupon bonds. By construction, the issue price of the bonds considered is set equal to
186 the par value N .

187 **Zero-coupon bond (ZCB)** In this case, there are no intermediate cash-flows but a single
188 payment at maturity t_n given by N^{zcb} that is equal to

$$N^{zcb} = \frac{N}{P(t, t_n)}$$

189 so that the present value of the payoff is exactly N .

190 **Coupon bond (CB)** The fixed rate bond pays at times t_i a fixed coupon $c_i^{cb} = c, i =$
191 $1, \dots, n$, and the notional at expiry. The coupon c is chosen according to the formula

$$c^{cb} = \frac{1 - P(t, t_n)}{\sum_{i=1}^n \alpha_{i-1, i} P(t, t_i)},$$

192 so that the present value of all bond payments is equal to the face value N .

193 **Floating rate note (FRN)** This note has at the payment date t_i a variable coupon rate
194 determined according to the level of the LIBOR rate

$$c_i^{frn} = L(t_{i-1}, t_i).$$

195 Notice in particular, that we adopt the standard convention of fixing the rate at the beginning
196 of the coupon period, i.e. at time t_{i-1} , whilst the payment is due at t_i (this is the so-called
197 reset in advance pay in arrears convention). Moreover, in this case the coupon tenor is the
198 same as the reference rate tenor, and assuming no default risk, it is well known that the issue
199 price of the bond is exactly equal to the par value. Therefore, in the above coupon formula
200 we do not include a fixed spread component.

201 3.2 Structured products

202 The structured bonds considered here are floating rate notes in which the coupon is set
203 according to: a) a swap rate (constant maturity swap), b) a LIBOR or a swap rate, with
204 a collar structure, c) a difference of two swap rates (spread note), d) the absolute value of
205 the difference between a swap rate and a fixed rate (volatility note). Detailed descriptions
206 of these notes follow.

207 **Constant maturity swap (CMS)** This structure allows investors to take a position on
208 the long term part of the term structure. The coupon rate is determined according to the
209 formula

$$c_i^{cms} = m \times S(t_i; \tau),$$

210 where m is the participation factor. The main difference with respect to the floating rate
211 note is that coupon (annual) and reference rate (a long maturity rate) have different tenors
212 and the fact that the reset and pay in arrears convention applies, i.e. the reset and payment
213 dates coincide. Therefore, this bond at issue is not quoted at par. For this reason, the
214 participation factor m is introduced, so that the CMS fair price at inception is equal to the
215 par value.

216 **Floating rate note with a collar (FRNC)** This structure protects the buyer and the
217 seller against sudden up or down movements in short term rates. This is made possible by
218 inserting a cap (maximum rate) c , a floor (minimum rate) f and a spread component δ in
219 the coupon formula

$$c_i^{frnc} = \min(\max(L(t_{i-1}, t_i), f), c) + \delta, i = 1, \dots, n.$$

220 Here the three components are adjusted to ensure that the issue price is equal to the notional
221 N . Clearly, this can be achieved using different strikes combinations. Therefore, we set the
222 floor f and the cap c equal respectively to 90% and 110% of the average forward LIBOR
223 rate¹. Then we adjust the spread parameter δ to ensure that the bond is issued at par value.

¹Given the term structure of discount factors, we compute the simple forward rates with starting dates being the coupon reset dates and as final date the coupon payment date. Given we have different coupon

224 Here, the standard reset in advance rule applies.

225 **Constant maturity-swap with a collar (CMSC)** Similarly to the previous structure,
226 this one protects the seller and the buyer against large changes in the reference rate, here
227 taken to be a long term rate, i.e. a swap rate. As in the previous case, the coupon includes
228 a cap c , a floor f and a spread component

$$c_i^{cm\,sc}(t_i) = \min(\max(S(t_i; \tau), f), c) + \delta.$$

229 We set the floor (cap) equal to 90% (110%) of the forward swap rate and we adjust the
230 spread component so that the bond is issued at par value. Here the coupon resets in arrears.

231 **Spread note (SPREAD)** This SP pays a coupon related to the difference between a
232 long term swap rate and a short term one. Through this product, the investor bets on the
233 steepening of the swap curve. In general, the rate with shorter tenor is subtracted from the
234 rate with longer tenor. We consider as short term rate the 2-year swap rate and as long term
235 rate the 10-year swap rate. The coupon formula is given by

$$c_i^{spread} = \min(\max((S(t_i; \tau_1) - S(t_i; \tau_2)) \times m, f), c) + \delta,$$

236 Also in this case, the coupon benefits from a floor rate and a cap rate. In our simulations,
237 the floor f is set at half of the average value of the simulated reference variable, whilst the
238 cap is set at twice such average value.

239 **Volatility note (VOL)**. The payoff depends on the absolute value of the difference between
240 a swap rate and a fixed amount c , so that large deviations of the swap rate with respect to
241 a reference value c guarantee large coupons to the bond holder. Small deviations, will pay
242 small amounts. The coupon rate formula is

$$c_i^{vol} = m \times |S(t_i; \tau) - c|$$

243 In our simulations the maturity of the reference swap rate is taken to be $\tau = 10yrs$. The
244 reference value c is set equal to half the expected value of the reference rate in 5 years. The
245 participation factor m is chosen to guarantee that the bond quotes at par at inception.

246 3.3 Pricing of structured products

247 According to the no-arbitrage principle, the bond fair value $\pi(t)$ is set equal to the following
248 expected value under the risk-neutral measure

$$\pi(t) = \sum_{i=1}^n \alpha_{i-1,i} \times \tilde{E}_t \left(\frac{c_i}{B(t, t_i)} \right) \times N + P(t, t_n) \times N,$$

dates, we consider the average forward rate.

249 where $B(t, T)$ is the so called money market account, i.e. the T value of a unit initial
 250 investment in a risk-free account at time t

$$B(t, T) = \exp\left(\int_t^T r(s) ds\right).$$

251 In this expression r is the stochastic instantaneous rate whose dynamics is made explicit in
 252 section 4.

253 Given the pricing formula, the various parameters δ and m , in the coupon formula are
 254 adjusted to ensure that the fair price at issue is equal to the notional N . For example, in
 255 the CMS case the multiplicative factor m is chosen so that

$$m \times \sum_{i=1}^n \alpha_{i-1,i} \times \tilde{E}_t\left(\frac{S(t_i; \tau)}{B(t, t_i)}\right) + P(t, t_n) = 1.$$

256 The expectation in this expression is estimated via Monte Carlo simulation of the stochastic
 257 interest rate model presented in section 4. In some cases, we need to perform some numerical
 258 search routine. We proceed as follows. We simulate the state variables of the G2++ model
 259 and then we search for the contract parameters such that the contract price is equal to N .

260 4 The term structure model

261 For pricing and for conducting our simulations we have adopted the two-additive factors
 262 Gaussian G2++ Model, see Brigo and Mercurio [9]. The model is based on the general
 263 Heath, Jarrow and Morton [28] framework for the arbitrage-free modelling of the evolution
 264 of interest rate curves. In the G2++ specification, the short rate is assumed to be the sum
 265 of two mean-reverting correlated Gaussian factors plus a deterministic function, that allows
 266 the user to fit exactly the observed term structure of spot rates. The model provides closed
 267 form expression for discount bonds, European options on zero-coupon bond, caps and even
 268 swaptions via a simple univariate integration. This allows a fast parameter calibration to
 269 market quotations. Moreover, due to the factor structure, the model allows for a non perfect
 270 correlation between changes of rates of different tenors. This is a well known empirical
 271 feature that cannot be captured by one-factor term structure models. Given that different
 272 SPs should react differently to the movements of different parts of the interest rate curve,
 273 to have a model that captures a non-perfect correlation across different spot rates is of the
 274 foremost importance. In addition, the model turns out to be Markovian in the mean-reverting
 275 factors. Hence it allows for an efficient and fast Monte Carlo simulation with respect to other
 276 model specifications, like the LIBOR market model. Finally, the model allows for negative
 277 interest rates, a phenomenon registered in the Eurozone since August 2014.

278 In the G2++ short rate model, the instantaneous short rate $r(t)$ is given as the sum of
 279 two stochastic components, x and y and a deterministic function ϕ

$$r(t) = x(t) + y(t) + \phi(t), \tag{3}$$

280 with $x(0) = y(0) = 0$, and $r_0 = \phi(0)$. The deterministic function $\phi(t)$ is linked to the
 281 observed market forward curve. Indeed, its role is to guarantee that the model zero-coupon
 282 bond prices perfectly fit the market ones at the initial time. We discuss in section 6 how
 283 this function relates to the observed market discount curve.

284 The risk-neutral processes $\{x(t), t \geq 0\}$ and $\{y(t), t \geq 0\}$ follow Ornstein-Uhlenbeck
 285 dynamics

$$\begin{aligned} dx(t) &= -ax(t) dt + \sigma d\widetilde{W}_1(t), x(0) = 0, \\ dy(t) &= -by(t) dt + \eta d\widetilde{W}_2(t), y(0) = 0. \end{aligned}$$

286 Here $d\widetilde{W}_1$ and $d\widetilde{W}_2$ are the (risk-neutral) increments of two correlated Brownian motions

$$\widetilde{E}_t \left(d\widetilde{W}_1 d\widetilde{W}_2 \right) = \rho dt,$$

287 where ρ is the correlation coefficient. The parameter restrictions are

$$a, b, \sigma, \eta > 0, \rho \in [-1, 1].$$

288 Parameters a and b are interpreted as mean-reversion coefficients of the two stochastic
 289 factors $x(t)$ and $y(t)$. In order to avoid model identification issues, we must require that the
 290 mean reversion coefficients a and b are different. Moreover, if the mean-reversion coefficients
 291 a and b are positive, the latent factors, and therefore the short rate as well, have a long-run
 292 stationary distribution that is Gaussian. In particular the two stochastic factors revert to 0
 293 under the risk-neutral measure and the short rate reverts to the deterministic function $\phi(t)$.
 294 Moreover, the short rate has a stationary distribution given by

$$\lim_{t \rightarrow \infty} r(t) \sim \mathcal{N}(M, S_r^2),$$

295 5127390190442444 codice cc1145 where

$$M = \lim_{t \leftarrow \infty} \phi(t), \tag{4}$$

296 and

$$S_r = \sqrt{\frac{\sigma^2}{2a} + \frac{\eta^2}{2b} + 2\rho \frac{\sigma\eta}{a+b}}. \tag{5}$$

297 Stochastic discount bond prices $P(t, T)$ at a future date t are obtained as product of two
 298 quantities

$$P(t, T) = P^{mkt}(0, t, T) H(x, y, t, T)$$

where $P^{mkt}(0, t, T)$ is the forward zero-coupon price

$$P^{mkt}(0, t, T) = \frac{P^{mkt}(0, T)}{P^{mkt}(0, t)}$$

and $H(x, y, t, T)$ is an exponential affine function of the two stochastic factors

$$H(x, y, t, T) = \exp\left(-\frac{1-e^{-a(T-t)}}{a}x(t) - \frac{1-e^{-b(T-t)}}{b}y(t) + \frac{1}{2}(V(t, T) - V(0, T) + V(0, t))\right). \quad (6)$$

In the above formulas, $P^{mkt}(0, t)$ refers to the initial exogenously specified market term structure of discount factors whose construction is discussed in more detail in Section 6. The expression for the function $V(t, T)$ is given in formula (4.10) in Brigo and Mercurio [9] and represents the variance of $\int_0^T (x(u) + y(u)) du$.

Given the jointly Gaussian assumption on x and y , zero-coupon bond prices at any future date have a lognormal distribution. In addition, the risk-neutral dynamics of the zero-coupon bond price is

$$\frac{dP(t, T)}{P(t, T)} = r(t) dt - \sigma D(T - t; a) d\widetilde{W}_1 - \eta D(T - t; b) d\widetilde{W}_2,$$

where the function $D(\tau; \theta)$ is related to the (stochastic) duration of the zero-coupon bond price

$$D(\tau; \theta) = \frac{1 - e^{-\theta\tau}}{\theta}.$$

Therefore, $\sigma D(T - t; a)$ and $\eta D(T - t; b)$ represent the contribution to the bond price volatility of the volatility in the two factors, x and y .

Given the risk-neutral model specification that allows us to price SPs in a way that precludes arbitrage opportunities across all maturities, we now introduce the so called risk-adjusted dynamics that reflect market participants' risk preferences. Indeed, the above dynamics are relevant for pricing the structured products at the initial time. However, to compare the performance of the different products we also need the dynamics under the so called physical (or risk natural or real world) measure. This requires a specification of the risk premia required by the market for taking the risk given by the two Brownian motions. The literature on the specification of this risk-premium is very extensive. A discussion can be found in Singleton [39]. However, for the sake of simplicity and also for better understanding of our results we assume that the risk premia are constant, but then we perform the analysis under different risk aversion scenarios. In practice the specification of the risk-premium consists in introducing two parameters λ_1 and λ_2 and in replacing the risk neutral Brownian motions $\widetilde{W}_i(t)$ by a new Brownian motions $W_i(t)$ via the change of drift

$$\widetilde{W}_i(t) = -\lambda_i t + W_i(t), i = 1, 2.$$

324 The dynamics of zero-coupon prices under the true measure becomes²

$$\frac{dP(t,T)}{P(t,T)} = (r(t) + \lambda_1\sigma D(T-t;a) + \lambda_2\eta D(T-t;b)) dt + \sigma D(T-t;a) dW_1 + \eta D(T-t;b) dW_2.$$

325 where the quantity

$$\lambda_1\sigma D(T-t;a) + \lambda_2\eta D(T-t;b)$$

326 represents the the expected return, over $r(t)$, of holding a zcb bond with time to maturity
 327 $T-t$ for an instant. By exploiting the risk-neutral and risk-adjusted dynamics, it is now
 328 possible to decompose yields into expectations of future interest rates and term premia. In
 329 particular, over a period of length dt , the excess return from holding a zero-coupon bond
 330 expiring in T with respect to the instantaneous risk-less investment is given by

$$E_t(d\ln(P(t,T))) - r(t) dt = TP(t, t+dt, T) dt. \quad (7)$$

331 Here $TP(t, t+dt, T)$ is the so called (instantaneous) bond term premium and it represents
 332 the instantaneous excess return of a zero-coupon bond with time to maturity $T-t$ with
 333 respect to the riskless bank account return, see Duffee [17]. Indeed, in general, longer term
 334 notes are perceived as riskier and therefore require a premium to compensate for this extra
 335 risk. The term premium is then found aggregating temporally and averaging

$$\begin{aligned} TP(t,T) &= \frac{1}{T-t} \int_t^T TP(t,s,T) ds \\ &= \frac{1}{T-t} \int_t^T (\lambda_1\sigma D(T-s;a) + \lambda_2\eta D(T-s;b)) ds \\ &= \lambda_1 \frac{\sigma}{a} \left(1 + \frac{1}{T-t} \frac{e^{-a(T-t)} - 1}{a} \right) + \lambda_2 \frac{\eta}{b} \left(1 + \frac{1}{T-t} \frac{e^{-b(T-t)} - 1}{b} \right). \end{aligned}$$

336 The long-run term premium is then

$$\bar{TP} = \lim_{T-t \rightarrow \infty} TP(t,T) = \lambda_1 \frac{\sigma}{a} + \lambda_2 \frac{\eta}{b}.$$

337 In the specification (8), the term premium for a given time to maturity $T-t$ is assumed to
 338 be time homogeneous, i.e. it depends only on the bond time to maturity and its sign cannot

²For the sake of completeness, the dynamics of the two factors, under the new measure are

$$\begin{aligned} dx(t) &= a \left(\frac{-\lambda_1\sigma}{a} - x(t) \right) dt + \sigma dW_1(t), x(0) = 0, \\ dy(t) &= b \left(\frac{-\lambda_2\eta}{b} - y(t) \right) dt + \eta dW_2(t), y(0) = 0. \end{aligned}$$

Under the true measure, the two factors will now revert to $-\lambda_1\sigma/a$ and $-\lambda_2\eta/b$. Depending on the sign of λ_i , these long-run values can be negative, null or positive. In addition, the deterministic function $\phi(t)$ is no longer the unbiased forecast of the future instantaneous rate.

339 change over time, depending on the fluctuations of the interest rates investors' risk tolerance,
 340 see Fama and French [23]. However, we believe that this does not represent a problem for
 341 our setup. At first, we are considering bonds having the same maturity and therefore our
 342 results will not be affected by the values that the term premium takes at different maturities.
 343 The second reason is that our simulations are performed by assigning different values to the
 344 parameters λ_i , so that we can generate different shapes and changes of sign in the initial
 345 term premium structure.

346 We stress that portfolio allocation aims at modelling the probability distribution of the
 347 market prices at a given future investment horizon under the *true* probability distribution
 348 of the market prices, as opposed to the risk-neutral probability measure used for derivatives
 349 pricing, see Meucci [35] and Giordano and Siciliano [26]. Based on this distribution, the buy-
 350 side community takes decisions on which securities to purchase to improve the prospective
 351 payout profile of their position. In practice, the estimation of the true probability distri-
 352 bution, i.e. the estimation of the parameters λ_i (as opposed to the calibration procedure
 353 required to obtain the risk-neutral distribution), represents the main quantitative challenge
 354 in risk and portfolio management. This is discussed in section 6.

355 5 Optimal Investing in SPs and Performance Measures

356 The investor aims to build an optimal portfolio containing plain vanilla bonds (i.e. ZCB,
 357 CB and FRN) and structured products as well. The construction of the optimal portfolio is
 358 done as follows. Let us define $PV_j^{(k)}$ as the simulated present values of future cash flows of
 359 the products j , $j = 1, \dots, P$ in simulation k , $k = 1, \dots, K$

$$PV^{(k)} = \sum_{s=1}^n \frac{C^{(k)}(s\Delta)}{B^{(k)}(s\Delta)} + \frac{N}{B^{(k)}(n\Delta)}. \quad (8)$$

360 where $C^{(k)}(s\Delta)$ is the coupon cash flow paid at time $s\Delta$ in simulation k . The actual
 361 computation of the coupon is done via Monte Carlo simulation and is discussed in detail
 362 in appendix in section A (see in particular formula (16) therein). In the following it is
 363 convenient to work in return terms by defining the gross logarithmic return $R_j^{(k)}$

$$R_j^{(k)} = \ln \left(\frac{PV_j^{(k)}}{N} \right),$$

364 and the net log-return $R_{j,g_j}^{(k)}$

$$R_{j,g_j}^{(k)} = \ln \left(\frac{PV_j^{(k)}}{N(1+g_j)} \right) = R_j^{(k)} - \ln(1+g_j).$$

365 where the fee g_j is zero if $j = 1, 2, 3$ and non-negative if $j = 4, \dots, P$. We have decided to
 366 set the fee for the basic products to zero because in general they are very liquid instruments,
 367 being largely traded in many markets, typically issued by governments and largely available
 368 to the vast majority of retail investors. Instead, for the remaining products, the up-front
 369 payment required to the investor is $N \times (1 + g)$.

370 We assume the investor implements a buy and hold strategy and, among all admissible
 371 portfolios, choose the optimal portfolio in the subset of mean-variance efficient portfolios.
 372 Therefore, using the K simulated scenarios, the investor computes the expected return for
 373 each asset j

$$\mu_{j,g_j} = \frac{1}{K} \sum_{k=1}^K R_{j,g_j}^{(k)}, \quad (9)$$

374 and collects them in the mean vector μ_g

$$\mu'_g = [\mu_{1,g_1} \quad \mu_{j,g_j} \quad \mu_{P,g_P}]. \quad (10)$$

375 Similarly, we can estimate the covariances $V_{j,i}$ $j, i = 1, \dots, P$ between R_{j,g_j} and R_{i,g_i}

$$V_{j,i} = \frac{1}{K} \sum_{k=1}^K \left(R_{j,g_j}^{(k)} - \mu_{j,g_j} \right) \left(R_{i,g_i}^{(k)} - \mu_{i,g_i} \right), \quad (11)$$

376 and collect them in the covariance matrix \mathbf{V}^3 . Henceforth, the investor solves with respect to
 377 the vector of holdings \mathbf{w} , $\mathbf{w} \in R_+^n$, the following mean-variance problem with no-short selling
 378 constraint

$$\begin{aligned} & \min \frac{1}{2} \mathbf{w}' \mathbf{V} \mathbf{w} \\ & \text{sub} \\ & \mu'_g \mathbf{w} = m \\ & \mathbf{1}' \mathbf{w} = 1 \\ & \mathbf{w} \geq \mathbf{0}, \end{aligned} \quad (12)$$

379 where m is the target expected return required by the investor. The target expected re-
 380 turn m in (12) is taken considering twenty equally spaced points in the range $[m^{low}, m^{high}]$
 381 where, in order to avoid portfolios concentrated in a single product, we set m^{high} to be the
 382 gross average of the positive expected returns and m^{low} is the expected return of the global
 383 minimum variance (GMV) portfolio obtained solving the problem (12) excluding the budget
 384 constraint⁴.

385 Given that the fee amount does not affect the estimation of the covariance matrix, the

³Given that the number of simulations is very large, there is no significant difference in using the biased or the unbiased estimate of the covariance matrix. In addition, given the fact that we are using log-returns, notice that the fees do not affect the variances and the covariances. For this reason we omit the dependence of the covariance matrix on g .

⁴In practice, $m^{high} = \frac{\sum_{j=1}^P \mu_{j,g_j} \mathbf{1}_{\mu_{j,g_j} > 0}}{\sum_{j=1}^P \mathbf{1}_{\mu_{j,g_j} > 0}}$ and m^{low} is the expected return on the portfolio that solves $\min \frac{1}{2} \mathbf{w}' \mathbf{V} \mathbf{w}$, $\text{sub } \mathbf{1}' \mathbf{w} = 1$ and $\mathbf{w} \geq \mathbf{0}$ whilst $m^{high} = \max_{j=1, \dots, P} \mu_j$, i.e. the largest element in μ . If all the products have a negative expected return, we set m^{high} equal to the average expected returns of the best three products.

386 GMV composition turns out to be independent of the fee structure. In general, this portfolio
 387 will attract an individual with infinite risk-aversion. On the other side, as an investor
 388 considers to move away from this portfolio to get some additional return, the charged fee will
 389 reduce the expected return of SPs and therefore their attractiveness in the efficient portfolio.
 390 Therefore, as fees increase, a risk-neutral investor will tend to invest only in basic products,
 391 i.e. she will positionate herself on the other extreme of the efficient frontier.

392 The comparative analysis that we conduct is related to the composition of the optimal
 393 portfolios that are solutions of the problem (12) for different values of m as we vary the fee
 394 level and the risk-aversion of the investor.

395 To evaluate the investment strategy in SPs facing fees, we use the two-step mean variance
 396 approach in Meucci [34] and proceed as follows.

- 397 1. First, we compute the efficient frontier by solving the above optimization problem.
 398 We emphasize that to do so we do not need to assume normality of returns or mean-
 399 variance preferences. This step only reduces the dimension of the market to the family
 400 of efficient portfolios.
- 401 2. Second, for a given utility function u , we determine the portfolio belonging to the
 402 efficient frontier returning the maximum expected utility. For this portfolio, we deter-
 403 mine the amount invested in basic and in structured products and we also compute
 404 the expected utility *ExpUt* associated to it

$$ExpUt(s) = \frac{1}{K} \sum_{k=1}^K u \left(\sum_{j=1}^P w_j(s) R_{j,g_j}^{(k)} \right).$$

405 where $w_j(s)$ is the optimal weight of bond j in the efficient portfolio s . We adopt
 406 an exponential utility function $u(x) = -e^{-\lambda x}$, where λ is the (constant) risk-aversion
 407 parameter⁵.

408 6 Model calibration and implementation

409 The simulation of the G2++ model and the performance analysis of investing in SPs requires
 410 the following steps.

411 **Term structure of market discount factors** The exogeneously specified market dis-
 412 count curve $P^{mkt}(0, t)$, $t > 0$ is estimated, following standard industry practice, by a boot-
 413 strapping procedure of LIBOR money market deposits (with maturities of 1, 3, 6 and 9
 414 months) and interest rate swaps (with maturities of 1, 2, 3, 4, 7, 10, 15 and 20 years) ob-
 415 tained from Bloomberg. Feldhutter and Lando [24] show that swap rates are indeed the
 416 most parsimonious proxy for riskless rates. Then, in order to have the spot rate at each sim-
 417 ulation step, we have interpolated the derived term structure of continuously compounded

⁵Notice that the utility function is defined in terms of log-returns. This is equivalent to adopt a power utility $u(x) = \frac{x^{1-\gamma}-1}{1-\gamma}$ defined on the terminal wealth, i.e. $W_0 e_p^R$ and by setting $\lambda = -(1-\gamma)$.

418 spot rates using the parametric Nelson-Siegel functional form. Accordingly, the continuously
 419 compounded spot rate is given by

$$R^{NS}(0, \tau) = -\frac{\ln P^{NS}(0, \tau)}{\tau} = \beta_0 + \left(\beta_1 + \frac{\beta_2}{\kappa} \right) \frac{1 - e^{-\kappa\tau}}{\kappa\tau} - \frac{\beta_2}{\kappa} e^{-\kappa\tau}.$$

420 In particular, the parameter β_0 captures the long run level of the spot curve, whilst pa-
 421 rameters β_1 and β_2 enable to generate various shapes of the term structure. In detail, β_1
 422 measures the slope of the term structure: a positive (negative) β_1 represents an upward
 423 (downward) sloping term structure. β_2 can be positive or negative, and allows to generate a
 424 term structure with a hump or a trough, respectively. Finally, the parameter κ determines
 425 both the steepness of the slope factor and the location of the maximum. The Nelson-Siegel
 426 model parameters have been fitted by minimizing the sum of squared errors between market
 427 and model rates

$$\min_{\beta_0, \beta_1, \beta_2, \kappa} \sum_{i=1} (R^{NS}(0, \tau_i) - R^{mkt}(0, \tau_i))^2.$$

428 In particular, we have calibrated the model to five different term structure scenarios (labelled
 429 A, B, C, D, and E) representative of different shapes: negatively sloped, positively sloped,
 430 average level, near flat and negative rates. In particular, these curves were observed at the
 431 following dates: (A) June 6th, 2008; (B) September 28, 2007; (C) average level in the period
 432 1/1/2005 to 30/09/2010; (D) May 20th, 2009; (E) May 2th, 2021. The five different curves
 433 are represented in Figure 1. The calibrated parameters are then given in Table 1. The
 434 deterministic function $\phi(t)$ appearing in (3) is then related to the calibrated parameters
 435 through the following formula⁶

$$\phi(T) = \beta_0 + \beta_1 e^{-\kappa T} + \beta_2 \kappa T e^{-\kappa T} + \frac{1}{2} \frac{\partial V(0, T)}{\partial T}.$$

436 The sensitivity of our results to the choice of the deterministic function $\phi(t)$ is then captured
 437 by fitting the Nelson-Siegel model to the different term structure shapes given in Table 1
 438 and considering the different model parametrizations given in Table 2.

439 **Parameter calibration of the G2++ model.** The analysis by De Jong et al. [16]
 440 suggests that the volatility implied by interest rate options, such as caps and swaptions, is
 441 a poor predictor of future volatility, because it consistently overestimates realised volatility.
 442 For this reason, we calibrate the model using historical volatilities and correlations, estimated
 443 using the sample covariance matrix of changes in spot rates with maturities from 1 to 5 years
 444 and with reference to the period January 1st 2005 to September 30th, 2010. However, to give

⁶The perfect fit at initial time between model and market zero-coupon bonds is possible if the following restriction is satisfied

$$\int_t^T \phi(s) ds = -\ln \frac{P^{mkt}(0, T)}{P^{mkt}(0, t)} + \frac{1}{2} (V(0, T) - V(0, t)), \quad (13)$$

By taking the partial derivative with respect to T and assuming that the initial discount curve is given by the Nelson-Siegel parametric function, we obtain the expression in the main text.

445 robustness to our analysis, we consider a market implied calibration as well, choosing the
 446 parameters that best fit the implied volatility swaption surface adopting the same procedure
 447 as in Brigo and Mercurio [9], page 166. Historically calibrated parameters are reported in
 448 scenarios I-III of Table 2. Market implied calibrated parameters are given in the scenarios
 449 IV and V in the same Table. We also add two additional parameter settings, i.e. VI and VII
 450 that are very similar to the settings V. In the parameter settings VI we set the parameters in
 451 order to better reflect the historical volatility of changes in the 5 year Euro spot rate in the
 452 period 2010-2020 and generate a lower term premium than in settings V. Indeed, in setting
 453 V the long-run term premium is around 15.25% that appears to be too large. So in settings
 454 VI, we impose that the long-run term premium takes a more reasonable value of 7.45%. The
 455 parameter settings VII instead impose that market participants are risk-neutral because the
 456 risk-premium parameters λ_1 and λ_2 are zero.

457 **Risk-Premium Parameters.** As previously discussed the size and sign of the risk-premium
 458 parameters λ_1 and λ_2 determine the performance of a long term bond with respect to a
 459 rolling investment in a short-term zero-coupon bond. Given that all term premia estimates
 460 are model-dependent, and also subject to parameter uncertainty we have considered different
 461 parametrizations labelled from I to VII that are provided in Table 2. Figure 2 shows how
 462 the term premium given in (8) behaves for different time to maturities. In particular, the
 463 different parametrizations generate different shapes and signs of the term premium. Among
 464 these, we also consider the case where both parameters λ_1 and λ_2 are zero: that is, market
 465 participants are assumed to be risk-neutral (scenarios II and VII). The parameter values
 466 have been chosen consistently with the values reported in the literature. The evolution of
 467 term premia has been of particular interest since the Federal Reserve (FED) and the Eu-
 468 ropean Central Bank began large-scale asset purchases. Over this time, short-term interest
 469 rates have been close to zero, and the term premium has been compressed and has at times
 470 even been negative. The FED term premium estimates are obtained from a five-factor, no-
 471 arbitrage term structure model described in detail in Adrian, Crump and Moench [1]. They
 472 reports estimates⁷ that over the last twenty years average at 1.79%, with a maximum value
 473 of 3.45% and a minimum of -0.87%. EUTERPE⁸ proposes a model for the estimation of
 474 term premia in the Euro Area (EA) that relies on an affine term structure framework with
 475 interrelations between yields, volatility and macroeconomic factors. They report a 10 year
 476 term premium that varies from 0.9790% in January 2000 to -1.1320% in April 2021. In par-
 477 ticular, the maximum reported value is 1.08%, the minimum -1.74% and the average 1.08%.
 478 The values of λ_1 and λ_2 considered here allow us to approximately reflect these estimates. In
 479 particular, given the calibrated parameters of the G2++ model, we choose the parameters
 480 λ_1 and λ_2 so that the 5-year and the long-run term premium, proxied by the 10-year one,
 481 are matched. We match the maximum and the average US term premium and the minimum
 482 and the average EUR term premium. This last case allows us to deal with a change in the
 483 sign of the term premium. In addition, we also consider the case of zero-term premium. The
 484 combinations of risk-neutral parameters and term premium parameters are given in Table 2.

485 **Risk-neutral model simulation and product characteristics.** Given the initial term
 486 structure and the model parametrization, we have simulated the model using the risk-neutral

⁷See https://www.newyorkfed.org/research/data_indicators/term_premia.html

⁸See <https://www.unive.it/pag/39846>

487 specification. This allows us to price the different SPs and to fix the payoff parameters such
 488 as the floor, cap and participation ratio so that the bond is priced at par. In order to find
 489 a unique solution, we have set the floor and the cap equal respectively to half and twice the
 490 expected value of the reference variable, and then solving for the remaining parameter (i.e.
 491 the participation factor or the spread) so that the issue price is equal to the face value.

492 **Real world model simulation.** Then, we have resimulated the stochastic factors under
 493 the true probability measure, i.e. replacing the risk neutral Brownian motions \tilde{W}_i by the
 494 the new Brownian motions W_i , to generate the cash flows at each payment date. Then we
 495 have computed the returns gross returns $R_j^{(k)}$ and by translation the net returns $R_{j,g_j}^{(k)}$. In
 496 this way we have obtained an array of dimensions $K \times N$, where K is the number of Monte
 497 Carlo simulations and P is the number of SP'-s considered, $P = 8$. This array contains the
 498 simulated present values $PV_j^{(k)}$ of the random cash flows we can achieve in simulation k
 499 investing in the j th SP. By varying the fee amount, it is also immediate to generate the
 500 simulated present values net of fees.

501 **Fees.** From conversations with practitioners and according to the results of previous stud-
 502 ies, it appears that subscription fees (implicit and explicit) can have large variations depend-
 503 ing on the issuer, on the underlying (interest, index, equity, commodity, etc.), on the presence
 504 of exotic components and on the maturity of the contract. According to this discussion, and
 505 as said in section 5, we have set the fee for the basic products to zero, whilst, for SPs, we
 506 assume that the up-front payment required to the investor is $N \times (1 + g)$. By doing this, we
 507 are assuming that there are no further hidden costs due to product mispricing, creditworth-
 508 ness of the issuer or liquidity costs related to the difficulty of liquidating the holdings prior
 509 to maturity (indeed, our investor is using a buy-and-hold strategy).

510 Instead of specifying *a priori* the fee amount, we have decided to proceed as follows.
 511 Notice that the inclusion of fees only reduces the expected return of SPs. Therefore, as we
 512 increase the fee level, efficient portfolios, with the exception of the global minimum variance
 513 portfolio, will tend to increase the amount allocated to basic products. We call g_{nosp} the fee
 514 amount such that the portfolios belonging to the efficient frontier⁹ have invested a maximum
 515 amount of 3% in SPs, i.e. in practice the SP investment is very residual and the efficient
 516 frontier can be built using only basic products. We have considered a 3% threshold, because
 517 as illustrated in Table 8 in the next section, this is the minimum percentage invested in SPs
 518 included in GMV portfolios, across all possible scenarios: an infinitely risk-averse individual
 519 will invest at least this amount in SPs.

520 As described in section 5, a risk-averse individual, once has built the efficient frontier,
 521 picks the portfolio that maximizes her expected utility. Clearly, her choice should be affected
 522 by her risk-aversion coefficient and by the fee level. Therefore, we have (numerically)
 523 computed the fee amount g_{basic}^λ such that an investor with an exponential utility, picks an
 524 efficient portfolio that includes only basic securities. Notice also that in general g_{nosp} is
 525 different from g_{basic}^λ because in the first case we require that *all* efficient portfolios invest a
 526 maximum of 3% in SPs, while in the latter case we require that only the efficient portfolio
 527 that is optimal for an individual with exponential utility has a zero investment in SPs.

528 Additional information can also be obtained by comparing portfolios fully invested only

⁹In those portfolios we do not consider the global minimum variance portfolio

529 in basic securities (BASIC) or in SPs. We can compute the maximum fee level $g_{sp>b}$ such
 530 that a portfolio invested only in SPs dominates, i.e. has lower risk for given expected
 531 return or larger expected return for given risk, a portfolio invested only in BASIC securities.
 532 Similarly, we can compute the minimum fee level $g_{b>sp}$ that makes the investment only in
 533 BASIC securities more convenient. In both cases, the critical level is found by imposing
 534 that the GMV invested only in SPs (BASIC) stays above the efficient frontier made only of
 535 BASIC (SP) securities. If there is no dominance the fee level is set 0.

536 These quantities will provide us with synthetic measures of the maximum cost that
 537 makes investment in SPs not convenient. We can then compare these maximum cost with
 538 the empirical evidence. In addition, given the above critical levels, we can then investigate
 539 what should be the optimal portfolio for an investor that combines BASIC and SP products.

540 An illustration of the role of the different critical fee levels is given in figure 3 with
 541 reference to an hypothetical scenario. In this figure, the red curve is the efficient frontier
 542 built considering ALL products assuming that there are no fees. The cyan frontier is the
 543 efficient frontier built using only BASIC securities. If the fee amount is set at g_{nosp} , the
 544 ALL efficient becomes the circled red curve: along this curve efficient portfolios will have
 545 a maximum weight of 2.92% allocated to SPs. The circled curves are the efficient frontiers
 546 built including only SPs varying the fee level. The yellow curve assumes no fees. The purple
 547 and green curves are the SP efficient frontiers with a fee level at g_{SPB} and g_{BSP} . g_{SPB} is
 548 the maximum fee level such that a portfolio containing only SPs dominates a portfolio made
 549 of basic securities. $g_{sp>b}$ is the minimum fee level such that a portfolio of basic securities
 550 dominates the portfolio of SPs. Figure 3 also suggests that, if no fees are paid, there is a
 551 sizeable improvement in the expected return-risk trade-off: the red efficient frontier, that
 552 includes SP, dominates the ones built investing in basic securities or in SPs only. If the
 553 fee increases up to 3%, a portfolio of basic and SPs still dominates a portfolio made only
 554 of basic or SP securities. Indeed, the circled red frontier stays above the cyan and green
 555 curves. Investing only in SPs, the circled yellow curve, dominates the investment in basic
 556 securities. However, if fees are set at 1% level, the circled yellow curve becomes the purple
 557 green curve and the two curves start to cross. If fees increase to 3%, the investment in SPs
 558 is now dominated by the investment in basic securities.

559 7 Numerical Results

560 We have performed a preliminary analysis examining the mean vector, the standard devia-
 561 tions, the average correlations and the composition of the global minimum variance portfolio
 562 across all 35 scenarios, i.e. five initial curves and seven different G2++ parametrization¹⁰.
 563 The above quantities have been estimated by running $K = 500,000$ Monte Carlo simulations
 564 with antithetic variates.

565 Table 3 illustrates the expected return, before fees are paid, for each product in each
 566 scenario. We observe that the ZCB and the FRN have a very strong relationship between
 567 expected return and term premium but of opposite sign. Indeed, as shown in Appendix B
 568 and confirmed in Table 4, where we regress the expected return of the different products

¹⁰An additional parameters setting, assuming that the term premium is zero has been considered, but not reported here.

569 on the term premium (TP), the long-run volatility (S_r), the long run (β_2) and the short
 570 term interest rates (β_1), the relationship, is perfect and positive for the ZCB. The regression
 571 coefficient is approximately equal to the length of investment period (5 years)¹¹ and the R^2
 572 coefficient is 100%. The FRN contract has also a very strong relationship (R^2 of 96%) with
 573 the term premium, but of opposite sign. This makes sense in our setup. The FRN is paying
 574 a coupon that is related to the realized LIBOR rate that, on its turn, is determined by the
 575 two G2++ factors, whose mean is inversely related to the risk-premium parameters λ_1 and
 576 λ_2 and therefore to the term premium. The calculation of the present value of this cash flow
 577 depends on the money market account, whose dynamics is also inversely affected by a larger
 578 value of the risk premium parameters. In conclusion, a larger term premium reduces both
 579 the numerator (the future LIBOR rate) and the denominator (the money market account
 580 used as deflator). The net effect turns out to be a lower expected return of the FRN. The
 581 remaining interest rate products have still a significant relationship with the term premium,
 582 but it is less strong respect to the two previous cases, due to the presence of optionalities
 583 in the coupon calculation. In particular, we notice that the expected return of the VOL
 584 has a weak dependency on the term premium and a much stronger relationship with the
 585 interest rate volatility. Indeed, by construction, the expected cash flow of this product is
 586 greater, greater the interest rate volatility. The regression results also shows that there is no
 587 significant relationship between expected return and level and slope of the term structure,
 588 where these quantities are proxied by the Nelson-Siegel parameters β_0 and β_1 .

589 So the main insight of this Table is that the main driver of the sign of the expected return
 590 for the different bonds is the term premium. The ZCB, followed by the FRN, has also a
 591 large variability in the expected return across scenarios, given that its sign depends on the
 592 sign of the term premium. The performance of the FRNC tracks the performance of the
 593 FRN. An exception is represented by the VOL, whose expected return is mainly driven by
 594 the interest rate volatility, and by the SPREAD note. We notice that the expected return on
 595 this bond has the same sign as the term premium, but it also differs significantly across term
 596 structure scenarios. Indeed, in scenario B, i.e. a very steep term structure, the bond has
 597 an expected return, in absolute value, larger respect to the other term structure shapes. In
 598 particular, this bond, in average, has a large expected return if the term premium is positive
 599 and the term structure is very steep. However, a steep curve associated with a negative term
 600 premium, e.g. scenario II-B, is very penalising.

601 This suggests that a combination of plain vanilla coupon bond and floating rate note can
 602 be of some appeal to risk-adverse investors whenever there is uncertainty on the sign of the
 603 term premium. Products like the VOL note can attract investors with high risk appetite
 604 that have strong views on a possible increase of the interest rate volatility. The SPREAD
 605 note can appear attractive if there is a view towards a positive term premium and a very
 606 steep term structure.

607 Table 5 reports the standard deviation of each bond in the different scenarios over the five
 608 year investment period. In this table, we also report the term premium and the asymptotic
 609 volatility S_r of the short rate. In Table 6 we have the results of regressing the standard
 610 deviation of the different products on the 5-year Term Premium, the asymptotic volatility
 611 S_r , and the short and long term rate implicit in the initial spot curve (i.e. the parameters
 612 β_0 and β_1). We observe that S_r explains most of the volatility of the different products

¹¹Indeed, the term premium is a time average, whilst the expected return is computed over the full five years period.

613 (R^2 greater than 90%) and it fully explains the volatility of the ZCB¹² and of the FRN. An
614 exception is the SPREAD note whose volatility is mainly determined by the short and long
615 term rates.

616 The results of the two regressions therefore suggest that products like CMS, FRNC, CMS,
617 and CMSC have exposures to market and model variables similar to plain vanilla bonds. On
618 the other side, the SPREAD and the VOL could give some benefits to an investor because
619 they are paying off, i.e. they have a higher expected return or a lower volatility, in specific
620 market environments.

621 Scenarios V and VII differ for the value of the correlation coefficient ρ among the two
622 interest rate factors. In scenario V, we assume a negative correlation, as it is typically the
623 case in the calibration of the G2++ model. In scenario VII, we set it at 0. We see that the
624 most affected is the ZCB both in terms of expected return and variance, as it should be. A
625 negative correlation among the factors lowers the value of the variance term $V(t, T)$ and this
626 affects, as shown in Appendix B, both the expected return and variance of the ZCB return.
627 A change in the correlation of the factors also affects the remaining products, mainly the
628 expected return of the FRN and FRNC and the volatility of the VOL note.

629 It is also interesting to assess the securities in terms of their contribution to the portfolio
630 diversification, and this is mainly captured by the cross-correlation. In Table 7 we produce,
631 for each scenario, the average correlation of each product with the remainings. We observe
632 that the FRN, followed by the CMS, has in general a significant negative correlation with
633 all the other products. This suggests that these products can have an important role in
634 diversifying the interest rate risk of a portfolio.

635 Given these preliminary remarks, Table 8 reports the composition of the global minimum
636 variance (GMV henceforth) portfolio¹³, across the different scenarios. The last two columns
637 of this Table reports the amount that the GMV portfolio allocates respectively to BASIC and
638 SPs. As the preliminary analysis has just suggested, given the properties of the CB and of
639 the FRN in terms of volatility and correlation, it is not a surprise to see that large proportion
640 of the portfolio is invested in these products, that are also preferred to a ZCB investment.
641 Indeed, across all scenarios, the weight assigned to BASIC products is always greater than
642 90%. Large part of this investment is allocated to the CB (in the range 70%-85%) and then
643 to the FRN (range 8%-16%). The maximum weight assigned to SPs does not exceed 18%
644 (see last column of Table 8) and in general is below 8%. When this threshold is exceeded, a
645 significant weight is allocated to the SPREAD note (scenarios III-B/C and VI-D).

646 We have analyzed the determinants of the composition of this portfolio, by regressing
647 the amount invested in basic products on the same independent variables as before. The
648 regression results are illustrated in Table 9 at asset and at aggregate level. In particular, at
649 asset level, the main determinants of the weights of the different products in the GMV are
650 the long-run volatility, and the level of the term structure. At aggregate level, we observe
651 that higher the long-term rate β_0 and lower the short term rate, larger the amount invested
652 in BASIC products.

653 Then we have analyzed how the above results are affected by the introduction of fees,
654 by computing the critical fee levels g_{nosp} , g_{basic}^λ , $g_{sp>b}$ and $g_{b>sp}$. An illustration of their role
655 is given in Table 10, where we can read: the first two columns refer to the parameter and

¹²This is also proved in Appendix B.

¹³In this Table we do not report the scenario related to negative rates, because we will deal with this case separately later on, due to the particular care we have to use in designing the different products.

656 curve settings; then we have the fee level g_{nosp} , such that all efficient portfolios will have a
657 maximum amount of 3% invested in SPs (third column), and the actual amount invested in
658 SPs (fourth column); the maximum fee g_{basic}^λ such that a risk-averse investor will have no
659 investment in SPs (fifth column) and then the effective amount that this investor will invest
660 in SPs given the fee g_{basic}^λ (sixth column); the maximum fee level $g_{sp>b}$ such that there exists
661 a portfolio of SPs that dominates the BASIC portfolio and the weight allocated to SPs in
662 a combined SPs-BASIC portfolio for the given fee level; the minimum fee level $g_{B>SP}$ such
663 that there exists a portfolio of BASIC securities that dominates the SP portfolio and the
664 corresponding amount allocated to SPs in a combined portfolio; the last columns give the
665 optimal percentage invested in SPs in a portfolio combining basic and SPs given a fee level
666 of level $g_{B>SP}$ (column eight) and a fee level of 1%, 3% and 5% (last column).

667 A first remark can be made relative to g_{nosp} . In general, the exclusion of SPs from efficient
668 portfolios happens at fees as low as 2-3%. An exception is the parameter scenario II, charac-
669 terized by a quite exceptional negative term-premium that generates good performances of
670 the FRNC and the FRN, as seen earlier on. Therefore, there is a significant weight assigned
671 to these products. For example in the scenario II-A, SPs receive a weight lower than 5% if
672 the fee is above 6.9%.

673 Then, we can examine g_{basic}^λ , the fee level such that a risk-averse investor invests only
674 in basic securities. Recall, that, as described in section 5, a risk-averse individual, once has
675 built the efficient frontier, picks the portfolio that maximizes her expected utility. Clearly, the
676 investment decision should be affected by her risk-aversion coefficient and by the fee level.
677 Therefore, we should compute g_{basic}^λ for different values of the risk-aversion coefficient λ .
678 In practice, given the very small differences in the expected return and the volatility of the
679 different products it turns out that investors will pick the same efficient portfolio, whatever
680 the level of their risk-aversion so that it turns out that g_{basic}^λ is independent on λ . This
681 implies that when the critical fee level of column five is achieved, all risk-averse investors
682 will invest only in basic products. In general, g_{basic}^λ is not too different from g_{nosp} , and again
683 with the exception of Scenario II, it takes values in the range 0-4.5%. Correspondingly, the
684 weight allocated to SPs in the optimal portfolio is less than 1%, a very marginal amount.

685 Then, we see that in column seven $g_{sp>b}$ is always zero. This means that it never happens
686 that a portfolio made of SPs only dominates a portfolio invested in BASIC securities only.
687 However, column 6 says that, given a fee equal to $g_{sp>b}$, it can be convenient to hold SPs up
688 to 96% in a combined portfolio of BASIC and SPs. Similarly, in the ninth column we have
689 $g_{b>sp}$, i.e. the minimum fee such that a portfolio invested only in BASIC securities dominates
690 the SP portfolio. In agreement with the previous column, this is always the case, albeit it
691 is still optimal to hold BASIC and SPs together. This is illustrated in panel (a) of figure 4.
692 The solid red curve represents the efficient frontier made of BASIC and SPs products. The
693 circled red curve is the same efficient frontier when the fee is set at the maximum level of
694 6.9%. Given this fee, the maximum weight allocated to SPs along the efficient frontier is 5%.
695 The cyan curve is the efficient frontier made of BASIC securities only. Given that it does not
696 coincide with the circled red curve, it means that efficient portfolios contain SPs. The circled
697 yellow curve is invisible because it is covered by the circled purple curve, that represents the
698 efficient frontier made of SPs with zero fee. Then the circled green represents the efficient
699 frontier made of SPs when the fee is set at 1%. This picture clearly synthesizes that, in our
700 simulations, in general the efficient frontier made of BASIC products stays always above the
701 efficient frontier made of SPs products. However, a combined portfolio, generates an efficient

702 frontier that dominates the BASIC and the SPs frontiers. Panel (b) of figure 4 illustrates
703 instead the case where there is no-added value in SPs: here the efficient frontier contains
704 only BASIC securities.

705 Finally, the last three columns of the Table give the average amount invested in SPs
706 along the efficient frontier, given different fee levels (1%, 3% and 5%). In scenario III-B,
707 increasing the fee from 1% to 3%, reduces the average investment in SPs from 20% to a low
708 value as 0.1%. If the fee level is 5% (last column), efficient portfolios, with the exception of
709 cases V and VI and curve setting D, will invest no more than 6.74% in SPs. An 8% fee (not
710 reported in the Table) reduces the average amount invested in SPs to less than 2%.

711 7.1 Discussion

712 It is interesting to compare the fee critical levels of our simulations with the empirical
713 evidence. The literature suggests that the greater the complexity of the product, the higher
714 the overpricing, i.e. the (implicit) fees charged to the investor. For example, Henderson
715 and Pearson [29] investigate the overpricing of a popular type of structured products in the
716 U.S. and estimate that it amounts to about 8%, resulting in a negative expected return.
717 They conclude that it is difficult to rationalize purchases of structured products by informed
718 rational investors. Stoimenov and Wilkens [40] find a lack of transparency in the German
719 market of SPs, in the sense that these products appear to be overpriced and thus favor the
720 issuing institutions. Stoimenov and Wilkens [41] consider leverage products in the German
721 retail market and show that these products most guarantee risk-free profits for their issuers.
722 They find that, at issuance, structured products sell at an average of 3.89% above their
723 theoretical values and the overpricing can increase to 5.17% for more complex products.
724 Similarly, significant mispricing in favor of issuers has been found by Benet et al. [4] with
725 reference to reverse-exchangeable securities, which are traded on the AMEX (American Stock
726 Exchange). An analysis of the Italian retail market has been carried out by Billi and Fusai
727 [6]. They have considered around 500 fixed income products issued in the Italian retail
728 market in the year 2009. They estimate, in the primary market, an average premium over
729 theoretical values in the range of 2% to 6%. They also find that mispricing usually has a
730 positive relationship with product complexity. Also consider that this mispricing is on the
731 top of the explicit fees charged to the investor.

732 On this basis, the critical fee levels discussed in our simulations appear to be below the
733 typical level charged to investors. If we set the fee level at 8% as Stoimenov and Wilkens
734 [40] do, the amount invested in SPs is in average below 2%, again with the exception of
735 scenario II, where the weight averages around 3.5%. In conclusion, our simulation results
736 are quite disappointing regarding the convenience of SP when plausible fees are charged.
737 Indeed, without fees the investor can considerably improve its risk-return trade-off, but in
738 the presence of fees, the diversification benefits of investing in SP completely fades away
739 and the investment in SPs turns out to be negligible or null. The main reason is in large
740 part related to the very small differences in expected returns and standard deviations among
741 the different products. A second reason is due to the fact that basic products have very
742 low volatility and negative cross correlations. Therefore, they allow the investor to achieve
743 large diversification benefits and the additional contribution of other SPs turns out to be
744 completely marginal if fees are charged.

745 As discussed in our simulations, there are still scenarios where SPs appear to provide

746 significant results if fees are contained. This opens up also another interesting problem re-
747 lated to the role of the financial advisors, that should help their clients to identify which
748 are the most reliable products for given projected scenarios. Instead, Hoechle et al. [31]
749 show that structured products generate substantial profits also for distributors (and not
750 only for issuers) and the high profitability of these products induces financial advisors to
751 promote them strongly to their customers. The revised version of the European Markets in
752 Financial Instruments Directive (MiFID II) tries to cope with this by stating that indepen-
753 dent financial advisors must transfer all commissions and fees paid by third parties to their
754 clients. However, these new rules only apply to financial advisors declaring themselves to be
755 independent and leaving the business model of nonindependent advisors largely unchanged.

756 8 Sensitivity analysis and robustness to model assump- 757 tions

758 In this section we discuss the sensitivity of our results to model parameters and to the model
759 choice. In particular, we analyze the role of mean-reversion, the effect of negative rates and
760 then the choice of a non-Gaussian multifactor interest rate model.

761 **Role of mean-reversion and factor volatility** Important model parameters are the
762 mean-reversion coefficients a and b and the volatility of the two factors. According to equa-
763 tions 5 and 8, these parameters determine both the long-term volatility of the short rate and
764 through the function D the bond volatility and then the term premium and its long-run value
765 that, by letting $\tau \rightarrow \infty$ in equation 8, is given by $\lambda_1\sigma/a + \lambda_2\eta/b$. Therefore ceteris paribus,
766 a larger value of a and b will imply, due to a stronger mean-reversion, a lower long-term
767 volatility, and a lower long-run term-premium.

768 We have considered the scenario I and, we have increased a from to 1.7981 to 3.5836 and
769 b from 0.0517 to 0.1034, and left unchanged all the remaining parameters. This implies that
770 the long-run volatility decreases from 5.892% to 4.167% and the long-run term premium
771 decreases from 3.450% to 1.725%.

772 Based on the regression results we presented earlier on, given the reduction in the term
773 premium, we expect that the expected return of all products, with the exception of the
774 FRN and FRNC, will decrease. In particular, the most affected will be the ZCB. Similarly,
775 given the reduction in the long-run volatility of the short rate, we expect, according to
776 the regression results, a reduction also in the standard deviation of all products, with the
777 exception of the SPREAD note that has a regression coefficient estimate near zero.

778 This is indeed reflected in our simulations. In Table 11 we compare the characteristics
779 of the different products in the original Scenario I and in the new setting (scenario VIII),
780 both combined with the term structure A. We observe, as expected, that, given that the
781 mean-reversion coefficients double the mean return of the ZCB is near halved from 12.95%
782 to 6.90%. A similar reduction in the expected return is observed for all the other products,
783 with the exception of all remaining products, FRN and the FRNC. All products see the
784 volatility of their return to be significantly reduced, with the exception of the SPREAD.
785 The practical effect is that the GMV optimal portfolios will see a significant increment of

786 the weight allocated to the CB and a reduction of weight allocated to SPREAD. Additional
787 information can be found examining the critical fee levels. In particular, in the new setup
788 gives $g_{nosp} = 0.056\%$ versus a value of 2.1% in scenario IA. In addition, if the fee is set
789 at 1% , SPs are completely excluded from the efficient frontier. In conclusion, this analysis
790 shows that a reduction in the interest rate volatility implies a greater investment in BASIC
791 products.

792 **Negative rates** The recent experience has seen central banks, mainly in Europe and
793 Japan, imposing negative interest rates. This represents a challenge for sellers of SP. Indeed,
794 negative coupon “payments” are not feasible, therefore issuers have mainly two options.
795 The first option is to add a very large spread to the floating rate, but this will imply that
796 the bond has to be issued well above the par and this would discourage investors that
797 are used to buying around par. Another possibility is to add a floor to the FRN, but
798 like the previous option, the FRN would get expensive as the investor would be required
799 to pay for that protection. According to a report by PIMCO, the American investment
800 management firm that focuses on fixed income and manages more than \$1.92 trillion in assets,
801 not surprisingly, most governments and agencies have stopped issuing FRNs, whilst credit
802 FRNs are instead still being issued, given that the credit spreads are typically higher than
803 those on government bonds and high enough to provide a comfortable buffer against negative
804 rates. In our framework the issuer is assumed to be credit-free and the bonds are issued at
805 par, and therefore the only possibility is to assume that investors are not worried about the
806 possibility of receiving negative coupons, and in such a context they still try to maximize
807 their expected utility by investing in products returning a negative interest. Moreover, given
808 that in our setup, we do not consider a currency market that could potentially compensate
809 for the loss caused by the negative return in the domestic currency, and given that there are
810 no reasons linked to the solvency of the issuer, the only plausible reason for investing in SP
811 is that they believe that these rates would decrease further and therefore they would prefer
812 bond structures that protect against such events. This is confirmed in our simulations: also
813 with negative rates, very risk adverse investors still prefer fixed rate bonds to the investment
814 in SPs. Increasing the subscription fees progressively eliminates SPs from the composition
815 of optimal portfolios. As illustrative example, Figure 5 provides an illustration of this case.
816 A fee of 2.7% implies that the efficient frontier contains only BASIC products.

817 **Model assumption** In this section, we further analyze the robustness to model assump-
818 tions. As alternative to the G2++ model, among the several possibilities offered in the
819 literature, we have considered, for its generality, the flexible stochastic volatility multi-factor
820 model of the term structure introduced by Trolle and Schwartz [42]. As the G2++ model,
821 this model belongs to the Heath, Jarrow and Morton [28] class. The Trolle and Schwartz
822 model has multiple factors driving the forward rate curve with each factor exhibiting stochas-
823 tic volatility. The model allows for correlation between each factor and the corresponding
824 stochastic volatility. The dynamics of the forward curve can be described in terms of a
825 finite number of state variables which jointly follow an affine diffusion specification. For
826 the purpose of the present paper, we have considered the risk-neutral and real-world model
827 specification with one factor, one stochastic volatility and six state variables. The full model
828 identification under both the risk and real world measures requires the estimation of 11 pa-
829 rameters. In practice, we have considered the parameter estimates reported in column 1 in

830 Table 1 in [42]. These parameters have been estimated via maximum likelihood using both
 831 swaption and cap prices. The stochastic volatility has been simulated using the Andersen
 832 method [2], whilst for the remaining state variables we have adopted an Euler scheme with
 833 daily time steps (250 steps per year).

834 For the sake of generality we have considered an investor with preferences described
 835 by an utility function defined on the portfolio universe and not only to the ones that are
 836 mean-variance efficient. Hence, the problem that the investor is now facing is

$$\begin{aligned}
 & \max_{\mathbf{w}} \frac{1}{K} \sum_{k=1}^K u(\mathbf{w}'\mathbf{R}^{(k)}) \\
 & \text{sub} \\
 & \mathbf{w} \geq \mathbf{0} \\
 & \mathbf{1}'\mathbf{w} = 1
 \end{aligned} \tag{14}$$

837 where \mathbf{R} is the $K \times n$ array containing the K simulated returns of the n different fixed income
 838 products performed using the TS model. We have solved numerically the above problem by
 839 assuming an exponential utility function defined in terms of the portfolio return $R = \mathbf{w}'\mathbf{R}^{(k)}$.

840 The simulations confirm that, whatever the term structure scenario, the investment in
 841 basic securities dominates the investment in structured portfolios. For illustration, we pro-
 842 vide figure 6 where we present the simulated cumulative distribution function of the best
 843 portfolio invested in basic securities or only in structured portfolios. We consider a fee of
 844 0.5%. In figure 6 the simulated cumulative distribution function of basic products, being
 845 on the right-most part of the figure, dominates in terms of first order stochastic dominance,
 846 see Hadar and Russell [27] the distribution function relative to the portfolio of structured
 847 products. In addition, whenever the fees are positive, the optimal portfolio contains only
 848 basic securities.

849 In conclusion, even in the the Trolle and Schwartz model, the simulations confirm the
 850 findings we previously obtained with the G2++ model.

851 9 Conclusion

852 In this paper we have discussed the relative convenience of investing in a portfolio of fixed
 853 income structured products. We have shown that, without fees, structured products can
 854 improve the risk-return trade off for a retail investor. This result is in general not robust to
 855 the presence of fees: in this case the optimal portfolio consists only of basic products such
 856 as zero-coupon bonds, coupon bonds and floating rate notes, and the percentage invested
 857 in structured products appears to be marginal or even not significant. Only under very
 858 particular configurations of the term premium and shape of the current term structure,
 859 investment in SPs can still be convenient, mainly with reference to products like VOL and
 860 SPREAD notes.

861 Investment banks advertise that structured notes guarantee a portfolio diversification
 862 that better suits specific investment needs. According to the simulation results presented
 863 in this paper, instead fixed income SP are not always designed to be in the best interests
 864 of investors and do not always allow to achieve a better risk-reward tradeoff. Additional
 865 disadvantages of SP are the pricing of the implicit derivatives components that could lead to

866 potential mispricing. Furthermore investor should consider the credit risk in the event the
867 issuing investment bank forfeits its obligations, and the liquidity of secondary market. Most
868 structured products are nowadays traded in secondary market and market investors wishing
869 to liquidate their holding prior to maturity will need to find a buyer for their investment
870 in the secondary market. The secondary market trading price for a structured product will
871 be subject to a bid-offer spread whose determination depends on several factors and market
872 participants may be disadvantaged, for example, incurring high transaction costs for certain
873 types of trades.

874 In conclusion, the main point of the present paper is that in an hypothetical transparent
875 market where fees are small and explicit, SP's should be included in the investor portfolio.

876 However, in opaque markets where the investor is charged hidden fees due to mispricing
877 or credit or liquidity reasons, SPs should not be included.

878 After the financial crisis, retail clients protection and increased financial markets trans-
879 parency has been driving the agenda of regulators. In the United States, based on the 2010
880 Dodd-Frank Wall Street Reform and Consumer Protection Act, much higher attention has
881 been given to payments from product providers to financial advisors.

882 In Europe, MIFID 2, MIFIR, PRIIPS Regulation and Product governance rules, have
883 contributed to increase transparency on costs and returns as well as on the appropriateness
884 of financial products to real needs of clients. On an advice given on April 2020 by ESMA to
885 the European commission on "inducements and costs and charges disclosure", ESMA states
886 that MiFID II disclosure regime generally works well and helps investors to make informed
887 investment decisions while understanding of inducements by clients should be increased.
888 On March 29 2021 ESMA has clarified that inducement are justifies only in the presence
889 of an additional or higher-level service, to the relevant client, proportional to the level of
890 inducements received. Our paper shows that fees are critical to explain the role of SP in
891 portfolios and that the role of regulators to keep high standard on costs disclosure and to
892 limits inducements is crucial to drive financial markets toward higher efficiency.

References

- [1] Adrian, T, R Crump and E Moench (2013): Pricing the term structure with linear regressions, *Journal of Financial Economics*, vol 110, no 1, pp 110–38.
- [2] Andersen, Leif B.G., Efficient Simulation of the Heston Stochastic Volatility Model (January 23, 2007). Available at SSRN: <https://ssrn.com/abstract=946405> or <http://dx.doi.org/10.2139/ssrn.946405>
- [3] Baule, R. Entrop, O. Wilkens, M. (2008). Credit Risk and Bank Margins in Structured Financial Products: Evidence from the German Secondary Market for Discount Certificates. *Journal of Futures Markets* 28(4), 376–397.
- [4] Benet, A. Giannetti, A. Pissaris, S. (2006). Gains from Structured Products Markets: The Case of Reverse Exchangeable Securities (RES). *Journal of Banking and Finance* 30(1), 111-132.
- [5] Bernard, C., Boyle P. P. and Gornall (2011). Locally-Capped Investment Products and the Retail Investor. *The Journal of Derivatives*, 18(4), 72-88.
- [6] Billi, M. and G. Fusai (2010). Trasparenza Bancaria e commissioni implicite: le emissioni obbligazionarie private in Italia dopo Patti Chiari. w.p. Università L. Bocconi.
- [7] Branger, N. and Breuer, B. (2008). The Optimal Demand for Retail Derivatives. Available at SSRN: <http://ssrn.com/abstract=1101399> or <http://dx.doi.org/10.2139/ssrn.1101399>
- [8] Breuer, W. and Perst, A. (2007). Retail banking and behavioral financial engineering: The case of structured products. *Journal of Banking and Finance* 31(3): 827-844.
- [9] Brigo, D. and Mercurio, F. (2006). *Interest Rate Models - Theory and Practice. With Smile, Inflation and Credit*. Series: Springer Finance. 2nd ed.
- [10] Burth S., T. Kraus, H. P. Wohlwend (2001). The Pricing of Structured Products in the Swiss Market. *Journal of Derivatives*. 9(2): 30-40.
- [11] Carpenter, S. B. and Demiralp S. (2011). Volatility, Money Market Rates, and the Transmission of Monetary Policy, *Finance and Economics Discussion Series*, Board of the Governors of the Federal Reserve System.
- [12] Chen, A. H. and Kensinger J. W. (1990). An analysis of market-indexes certificates of deposit. *Journal of Financial Services Research*, 4(2): 93-110.
- [13] Chen Y. (2011). Derivatives Use and Risk Taking: Evidence from the Hedge Fund Industry. *Journal of Financial and Quantitative Analysis*. 46 (4): 1073-1106.
- [14] Cui, B. Oldenkamp, M. Vellekop (2013). When do Derivatives Add Value in Pension Fund Asset Allocation? *Rotman International Journal of Pension Management*, 6(1): 46-57.

- 928 [15] Deng, G. Dulaney, T. Husson, T. McCann, C. J. and Yan, M. (2015). Ex-Post Structured
929 Product Returns: Index Methodology and Analysis. *The Journal of Index Investing*,
930 24(2): 45-58.
- 931 [16] De Jong, F., Driessen, J., and Pelsser, A. (2001). Libor Market Models Versus Swap
932 Market Models for Pricing Interest Rate Derivatives: An Empirical Analysis. *European*
933 *Finance Review*, 5(3): 201-237.
- 934 [17] Duffee, G. (2002). Term Premia and Interest Rate Forecasts in Affine Models. *Journal*
935 *of Finance*, 57(1): 405-443.
- 936 [18] Dybvig, P. H. (1988). Inefficient dynamic portfolio strategies or how to throw away a
937 million dollars in the stock market. *The Review of Financial Studies*, 1(1): 67-88.
- 938 [19] Entrop, O. McKenzie, M. D. Wilkens, M. and Winkler, C. (2014). The Performance of
939 Individual Investors in Structured Financial Products. *Review of Quantitative Finance*
940 *and Accounting*, 36(3):569-604.
- 941 [20] ESMA (2014). Structured Retail Products - Good practices for product governance
942 arrangements, (consulted on October 2015).
- 943 [21] ESMA (2021). Performance and Costs of EU Retail Investment Products ESMA Annual
944 Statistical Report 2021
- 945 [22] ESMA (2014). MiFID practices for firms selling complex products, (consulted on Octo-
946 ber 2015).
- 947 [23] Fama, E. and French, K. R. (1993). Common risk factors in the returns on stocks and
948 bonds. *Journal of Financial Economics* 33(1): 3-56.
- 949 [24] Feldhutter, P. and Lando, D. (2008). Decomposing swap spreads. *Journal of Financial*
950 *Economics* 88(2) 375-405.
- 951 [25] Fischer, R. (2007). Do Investors in Structured Products Act Rationally? Available at
952 SSRN:<http://ssrn.com/abstract=1011008> or <http://dx.doi.org/10.2139/ssrn.1011008>.
- 953 [26] Giordano, L. and Siciliano, G. (2013). Real-World and Risk-Neutral Probabilities in the
954 Regulation on the Transparency of Structured Products. CONSOB Working Papers No.
955 74.
- 956 [27] Hadar, J.; Russell, W. (1969). Rules for Ordering Uncertain Prospects. *American Eco-*
957 *nomic Review*. 59 (1): 25-34.
- 958 [28] Heath, D., Jarrow, R. and Morton, A. (1992). Bond Pricing and the Term Structure of
959 Interest Rates: A New Methodology for Contingent Claims Valuation. *Econometrica*,
960 60(1):77-105.
- 961 [29] Henderson, B. J. and Pearson, N (2011). The dark side of financial innovation: A case
962 study of the pricing of a retail financial product. *Journal of Financial Economics* 100(2):
963 227-247.

- 964 [30] Hens, T. and Rieger, M. O. (2009). The Dark Side of the Moon: Structured Products
965 from the Customer's Perspective. EFA 2009 Bergen Meetings Paper. Available at SSRN:
966 <http://ssrn.com/abstract=134236>
- 967 [31] Hoechle, D., Ruenzi, S., Schaub, N. and Schmid, M. (2018) Financial Advice and Bank
968 Profits. *The Review of Financial Studies*, Volume 31, Issue 11, November 2018, Pages
969 4447–4492.
- 970 [32] Jessen, P. and Jørgensen, P. L., (2012). Optimal Investment in Structured Bonds. *Jour-*
971 *nal of Derivatives*, 19(4): 7-28.
- 972 [33] Maringer, D. Pohl, W. and Vanini, P. (2015). Structured Products: Perfor-
973 mance, Costs and Investments. Available at SSRN: <http://ssrn.com/abstract=2620300>
974 or <http://dx.doi.org/10.2139/ssrn.262030>
- 975 [34] Meucci, A. (2005). *Risk and Asset Allocation*, 1st ed. 2005. Springer Finance.
- 976 [35] Meucci, A.(2011). "P" versus "Q": Differences and Commonalities between the Two
977 Areas of Quantitative Finance, *The Quant Classroom* by Attilio Meucci, GARP Risk
978 Professional, February, p. 43-44.
- 979 [36] McCoy, E. (2019). A Calibration of the Term Premia to the Euro Area. *European*
980 *Economy Discussion Papers* 110, September.
- 981 [37] Ofir, M. and Wiener, Z. (2012). Investor Sophistication and the Effect
982 of Behavioral Biases in Structured Products Investment. Available at SSRN:
983 <http://ssrn.com/abstract=2193287> or <http://dx.doi.org/10.2139/ssrn.2193287>
- 984 [38] Rieger, M. O., (2011). Co-Monotonicity of Optimal Investments and the Design of
985 Structured Financial Products. *Finance and Stochastics*, 15(1): 27-55
- 986 [39] Singleton, K. (2006). *Empirical Dynamic Asset Pricing*, 2006. Princeton University
987 Press.
- 988 [40] Stoimenov P.A. and Wilkens A. (2005). Are structured products "fairly" priced? An
989 analysis of the German market for equity-linked instruments. *Journal of Banking and*
990 *Finance* 29(12): 2971-2993.
- 991 [41] Stoimenov, P.A. and Wilkens, S.(2007). The pricing of leverage products: An empirical
992 investigation of the German market for 'long' and 'short' stock index certificates. *Journal*
993 *of Banking & Finance* 31(3): 735-750.
- 994 [42] Trolle, A. B., and Schwartz, E. S. (2009). A General Stochastic Volatility Model for the
995 Pricing of Interest Rate Derivatives. *Review of Financial Studies*, 22(5), 2007-2057.
- 996 [43] Vanduffel, S. (2010). Thou Shalt Buy 'Simple' Structured Products Only. *Journal of*
997 *Financial Transformation*, 28: 12-14
- 998 [44] Wallmeier, M. and Diethelm, M. (2009). Market Pricing of Exotic Structured Prod-
999 ucts:The Case of Multi-Asset Barrier Reverse Convertibles in Switzerland. *Journal of*
1000 *Derivatives* 17(2): 59-72.

1001 [45] Wasserfallen W., Schenk, Ch. (1996). Portfolio Insurance for the Small Investor in
1002 Switzerland. *The Journal of Derivatives*, 3(3):37-43.

1003 A Monte Carlo Simulation

1004 The G2++ specification is a Markovian model in the two state variables $x(t)$ and $y(t)$,
 1005 which are jointly Gaussian distributed. This fact implies that Monte Carlo simulation can
 1006 be performed in a straightforward manner. We detail the steps.

- 1007 1. Given the model parameters, fix the time step Δ , the bond maturity $T = n\Delta$ and the
 1008 initial value of the money market account $B(0) = 1$. In the simulations we set $T = 5$
 1009 and $\Delta = 1/12$. Set also $x(0) = 0$ and $y(0) = 0$.
- 1010 2. Simulate the two stochastic factors according to the true probability measure from a
 1011 bivariate normal distribution

$$\begin{bmatrix} x^{(k)}(j\Delta) \\ y^{(k)}(j\Delta) \end{bmatrix} \Big| \mathcal{F}_{(j-1)\Delta} \sim \mathcal{N}(M^{(k)}(j\Delta), V(\Delta)), j = 1, \dots, n,$$

1012 where¹⁴

$$\begin{aligned} M^{(k)}(j\Delta) &= \begin{bmatrix} x^{(k)}((j-1)\Delta)e^{-a\Delta} - \lambda_1\sigma_1(1 - e^{-a\Delta}) \\ y^{(k)}((j-1)\Delta)e^{-b\Delta} - \lambda_2\eta(1 - e^{-b\Delta}) \end{bmatrix}, \\ V(\Delta) &= \begin{bmatrix} \frac{\sigma^2}{2a}(1 - e^{-2a\Delta}) & \rho\sigma\eta\frac{1 - e^{-(a+b)\Delta}}{a+b} \\ \rho\sigma\eta\frac{1 - e^{-(a+b)\Delta}}{a+b} & \frac{\eta^2}{2b}(1 - e^{-2b\Delta}) \end{bmatrix}. \end{aligned}$$

- 1013 3. Simulate the short rate according to

$$r^{(k)}(j\Delta) = x^{(k)}(j\Delta) + y^{(k)}(j\Delta) + \phi(j\Delta),$$

1014 and the discount curve according to

$$P^{(k)}(j\Delta, T) = \frac{P^{mkt}(0, T)}{P^{mkt}(0, j\Delta)} \exp(A^{(k)}(j\Delta, T)), \quad (15)$$

1015 where $A^{(k)}$ is given by the exponent in expression (6).

- 1016 4. Update the money market account. A possibility is to use the trapezium rule

$$B^{(k)}(j\Delta) = B^{(k)}((j-1)\Delta) e^{(r^{(k)}((j-1)\Delta) + r^{(k)}(j\Delta))\frac{\Delta}{2}}.$$

1017 Unfortunately, this step introduces a discretization error that can be avoided by sam-
 1018 pling from a trivariate normal distribution. For sake of clarity and space saving, we
 1019 do not detail the exact simulation step that we have implemented. The advantage of
 1020 using the exact simulation scheme rather the above trapezium approximation is that

¹⁴The formula for the mean vector and covariance matrix are given in Brigo and Mercurio [9].

1021 we can simulate the state variables only on the bond reset dates, with a significant
1022 time computational saving.

1023 5. If $j\Delta$ corresponds to a coupon date, given the values of the stochastic factors $x^{(k)}(j\Delta)$
1024 and $y^{(k)}(j\Delta)$ and the discount curve (15), compute the coupon $c^{(k)}(j\Delta)$ and discount it
1025 at the initial date using the simulated value of the money market account, i.e. compute

$$\frac{C^{(k)}(j\Delta)}{B^{(k)}(j\Delta)}.$$

1026 For example, if the coupon is tied to a reference rate once we use the simulated dis-
1027 counted curve at the reset date to compute the corresponding value of the reference
1028 rate according to formula 1 or 2.

1029 6. For each product, in the simulation k we compute the present value $PV^{(k)}$ of the
1030 cash-flows

$$PV^{(k)} = \sum_{j=1}^n \frac{C^{(k)}(j\Delta)}{B^{(k)}(j\Delta)} + \frac{N}{B^{(k)}(n\Delta)}. \quad (16)$$

1031 (In the above expression if $j\Delta$ is not a coupon date, we set the coupon to zero).

1032 7. We repeat steps 1-6 for $k = 1, \dots, K$, where K is the number of simulations.

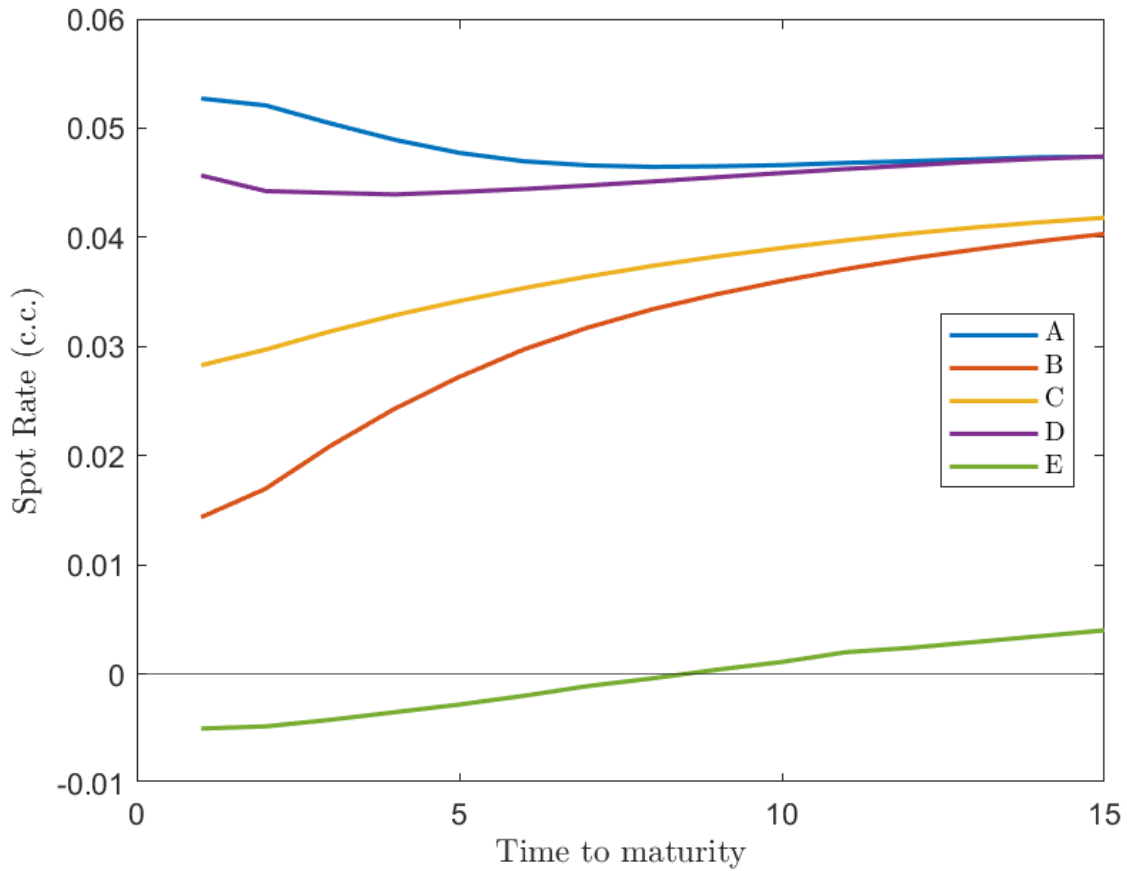


Figure 1: Initial term structures shapes. Four different initial shapes are considered: (A) negatively sloped on June 6th, 2008; (B) positively sloped on September 28th, 2007; (C) average level in the period 1/1/2005 to 30/09/2020; (D) flat on May 20th, 2009. (E) negative rates on May 2nd, 2021.

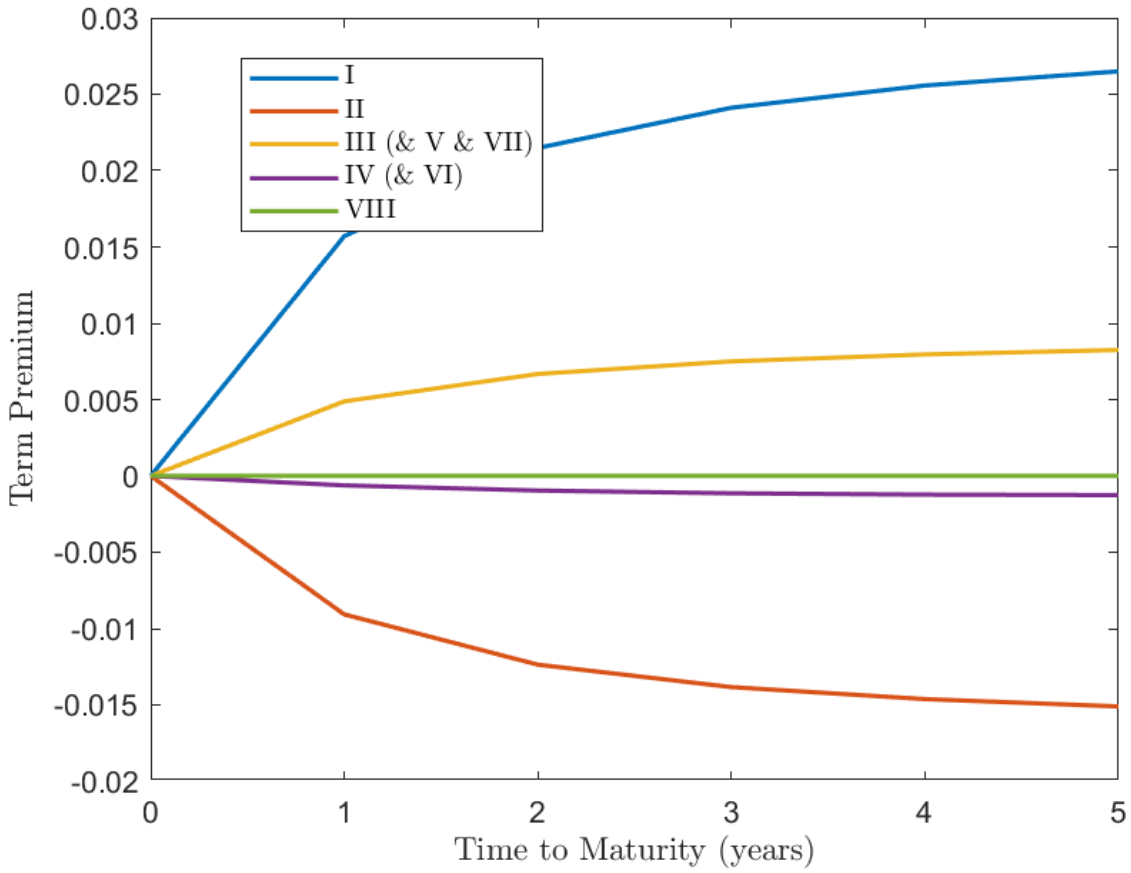


Figure 2: Behavior of the term premium under different G2++ parametrization. Model parameters according to the different scenarios are illustrated in Table 2.

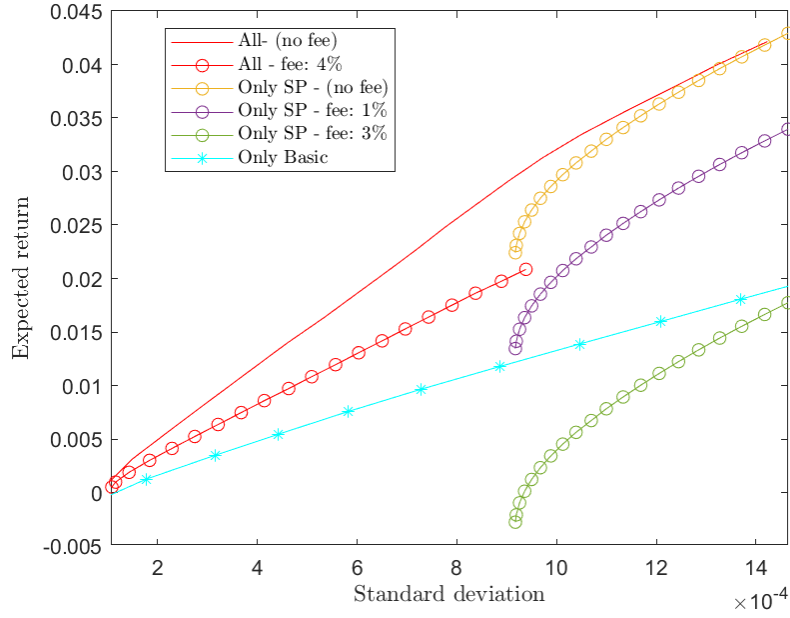
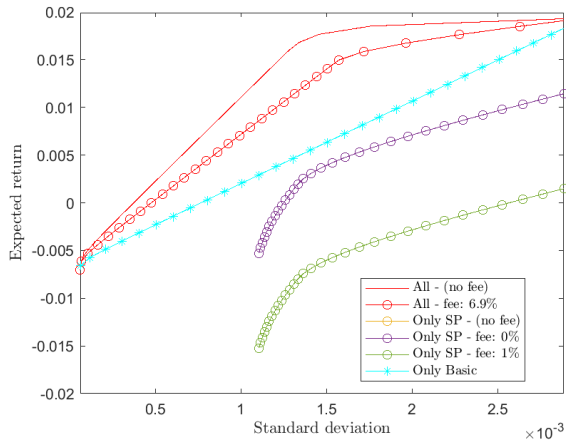
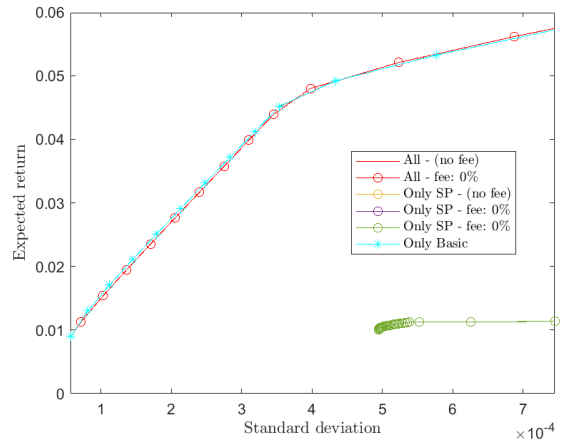


Figure 3: Hypothetical efficient frontiers. ALL: refers to the efficient frontier containing all products; only Basic is the efficient frontier made only of ZCB, CB and FRN; SP are the efficient frontiers containing only SPs varying the fee level (0%, 1% and 3%). The ALL - fee is the efficient frontier built assuming a fee level (4%) such that the maximum amount invested in SPs is 3%.



(a) Scenario II-A



(b) Scenario III-A

Figure 4: Efficient frontiers in two different scenarios varying the fee level.

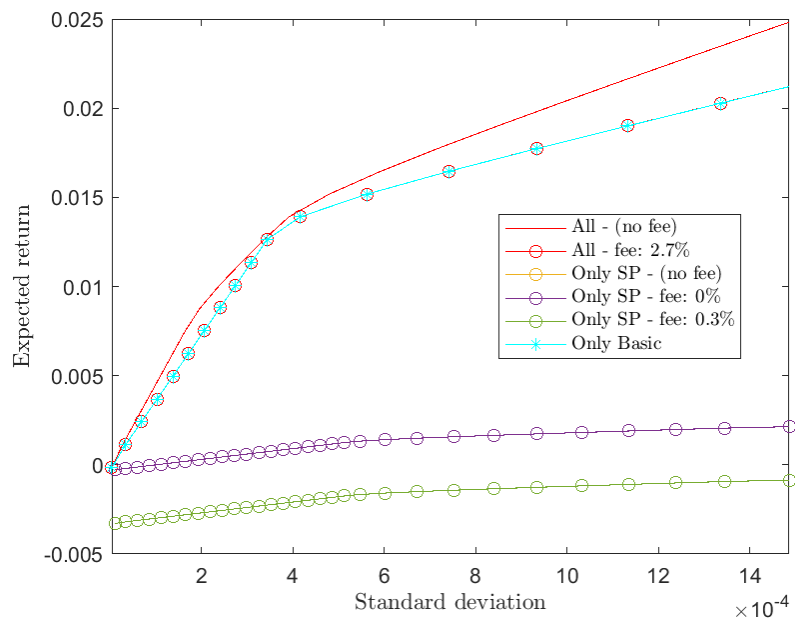


Figure 5: Efficient frontier in scenario III-E (negative rates)

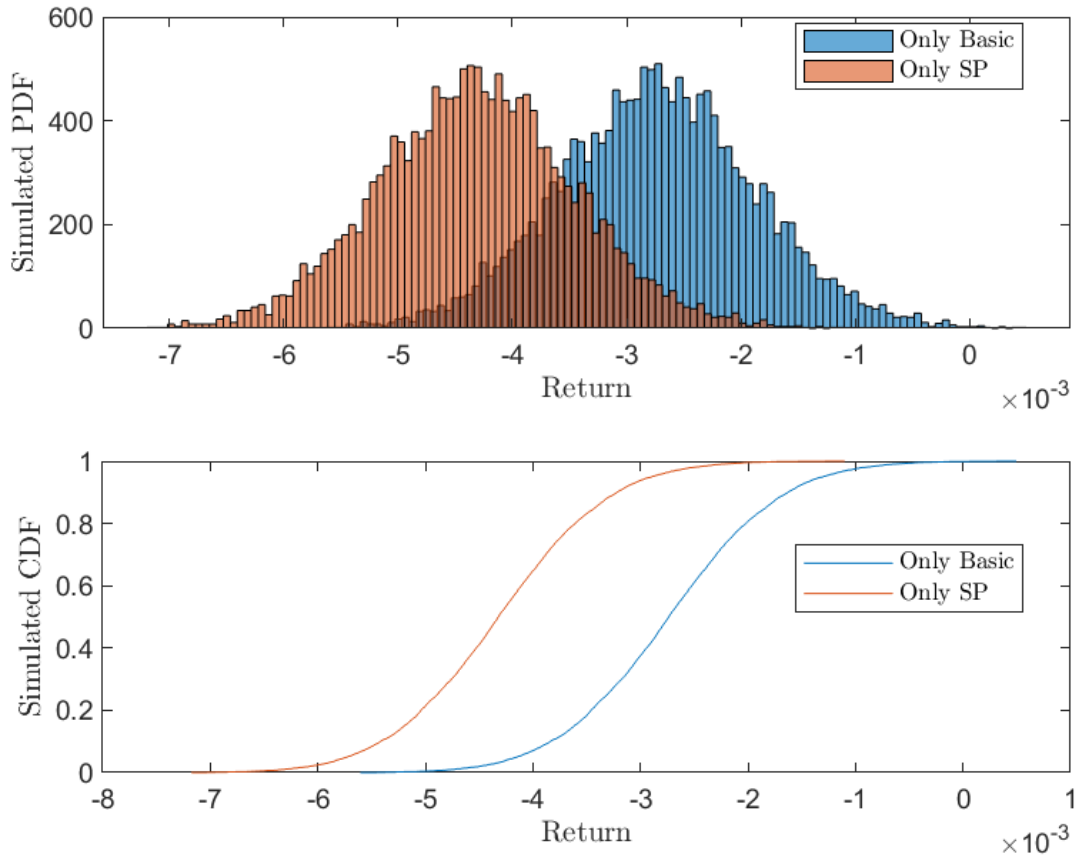


Figure 6: Simulated probability density (top figure) and simulated cumulative distribution (bottom part) of the optimal portfolio for an individual with logarithmic utility function under the term structure scenario A. Simulations are performed using the Trolle and Schwartz [42] model and assuming a 1% fee.

Scenario	A	B	C	D	E
β_0	5.7770%	4.8579%	4.8438%	5.1262%	1.2252%
β_1	-0.1721%	-3.6404%	-2.1799%	-0.3668%	-1.7045%
β_2	-0.6322%	-1.9082%	-0.4342%	-0.6751%	-0.5947%
κ	17.9718%	56.4758%	34.0464%	37.0178%	28.9778%

Table 1: Parameters of the Nelson-Siegel model in five different scenarios of the term structure of spot rates.

1033 B The expected return and variance on a ZCB

1034 In this section, we show how to compute analytically in the G2++ model the expected return
1035 on a ZCB expiring in T . For notational convenience we set $t = 0$. The amount that is paid
1036 at expiry is $(1 + c)N$, where $c = 1/P(0, T) - 1$, so that the ZCB is issued at par value. The
1037 logarithmic return is

$$R_{zcb} = \ln \frac{(1 + c)N}{B(T_n)N}$$

1038 where $B(T) = e^{\int_0^T r(s)ds}$. Therefore

$$R_{zcb} = -\ln(P(0, T)) - \int_0^T r(s)ds = -\ln(P(0, T)) - \int_0^T (x(s) + y(s))ds - \int_0^T \phi(s)ds$$

1039 and, using footnote 6 (with $t = 0$),

$$R_{zcb} = -\int_0^T (x(s) + y(s))ds - \frac{1}{2}V(0, T).$$

1040 We aim to compute the expected return and its variance. If the expectation is taken under
1041 the risk-neutral measure, given that the two factors have zero mean, we have

$$\tilde{E}_t(R_{zcb}) = -\frac{1}{2}V(0, T) \tag{17}$$

1042 This term, in the interest rate literature, is called convexity adjustment and is due to the
1043 non-linear relationship between price and return. If the expectation is computed under the
1044 real-world measure, the factor dynamics has to be adjusted for the risk-premium, i.e.

$$\begin{aligned} x(t) &= -\lambda_1 \frac{\sigma}{a} (1 - e^{-at}) + \sigma \int_0^t e^{-a(t-s)} dW(s), \\ y(t) &= -\lambda_2 \frac{\eta}{b} (1 - e^{-bt}) + \eta \int_0^t e^{-b(t-s)} dW(s). \end{aligned}$$

Scenario	I	II	III	IV	V	VI	VII	VIII
λ_1	34.771	-20.229	10.795	-0.086	0.385	-0.086	0.385	34.771
λ_2	0.015	-0.001	0.005	0.026	-0.001	0.026	-0.001	0.015
a	1.7918	1.7918	1.7918	0.7735	0.7735	0.7735	0.7735	3.5836
b	0.0517	0.0517	0.0517	0.082	0.082	0.082	0.082	0.1034
σ_1	0.0015	0.0015	0.0015	0.0223	0.0223	0.0223	0.0223	0.0015
η	0.019	0.019	0.019	0.0104	0.0104	0.0104	0.0104	0.019
ρ	-0.6441	-0.6441	-0.6441	-0.702	-0.702	-0.702	0	-0.6441
S_r	5.892%	5.892%	5.892%	2.450%	2.450%	2.450%	3.132%	4.167%
$TP(0, 5)$	2.650%	-1.510%	0.825%	-0.126%	0.825%	-0.126%	0.825%	1.435%
<i>Long Run TP</i>	3.438%	-1.738%	1.090%	0.075%	1.090%	0.075%	1.090%	1.725%

Table 2: Parameters for the G2++ model in eight different scenarios and assuming different values of the risk-premium parameters λ_1 and λ_2 . The first three scenarios refer to parameters calibrated using the sample covariance matrix and then assuming different values for the term premium at 5 years and in the long-run. The scenarios IV-VI are characterized by a lower asymptotic volatility of the short rate and by a reduced value of the 5 year term premium, i.e. near 1% and 0%. The scenario VII has been chosen to assess the effect of zero-correlation with respect to scenario V. Parameters in scenario VIII have been arbitrarily chosen. Concerning the choice of the risk premium: in scenario I, we use the maximum value of the term premium estimated in the US market in the period Jan. 2000 to April 2021, see [1]; in scenario II, we use the minimum value of the term premium estimated in the EURO market in the same period; in scenario III, V and VII, we use the average value of the term premium in the period 2000-2021 in the US market; in scenario IV and VI, the average value in the EUR market; in scenario VIII, we vary the mean-reversion coefficients of Scenario I, keeping constant all the other parameters. The following two scenarios refer to parameters calibrated using market quotations of swaptions and assuming a term premium that can take positive or negative values at different horizons. The row labelled S_r gives the asymptotic volatility of the short rate. The rows labelled $TP(0, 5)$, and *Long Run TP* give the 5 and long-run term premiums.

Param.	Curve	$TP(5)$	S_r	β_0	$\beta_0 + \beta_1$	zcb	cb	frn	cms	frnc	cmsc	spread	vol
I	A	2.65%	5.89%	5.78%	5.60%	12.95%	1.53%	-3.44%	0.43%	-2.04%	0.71%	1.95%	0.06%
I	B	2.65%	5.89%	4.86%	1.22%	12.95%	0.93%	-4.21%	0.27%	-1.65%	0.78%	3.26%	0.13%
I	C	2.65%	5.89%	4.84%	2.66%	12.95%	1.14%	-3.95%	0.31%	-1.91%	0.73%	1.13%	0.11%
I	D	2.65%	5.89%	5.13%	4.76%	12.95%	0.93%	-4.28%	0.17%	-1.69%	0.63%	0.80%	0.16%
I	E	2.65%	5.89%	1.23%	-0.48%	12.95%	-0.11%	-5.82%	0.24%	-0.10%	0.17%	0.40%	-0.11%
II	A	-1.51%	5.89%	5.78%	5.60%	-7.85%	-0.86%	1.94%	-0.37%	1.14%	-0.49%	-0.86%	-0.32%
II	B	-1.51%	5.89%	4.86%	1.22%	-7.85%	-0.52%	2.34%	-0.23%	0.90%	-0.54%	-2.69%	-0.24%
II	C	-1.51%	5.89%	4.84%	2.66%	-7.85%	-0.64%	2.21%	-0.27%	1.06%	-0.50%	-1.68%	-0.27%
II	D	-1.51%	5.89%	5.13%	4.76%	-7.85%	-0.52%	2.38%	-0.19%	0.94%	-0.45%	-1.44%	-0.26%
II	E	-1.51%	5.89%	1.23%	-0.48%	-7.85%	0.06%	3.18%	-0.08%	0.05%	-0.13%	-0.83%	0.05%
III	A	0.83%	5.89%	5.78%	5.60%	3.82%	0.45%	-0.82%	0.13%	-0.66%	0.21%	0.70%	-0.08%
III	B	0.83%	5.89%	4.86%	1.22%	3.82%	0.27%	-1.02%	0.08%	-0.53%	0.23%	1.32%	-0.01%
III	C	0.83%	5.89%	4.84%	2.66%	3.82%	0.33%	-0.95%	0.09%	-0.61%	0.21%	0.60%	-0.03%
III	D	0.83%	5.89%	5.13%	4.76%	3.81%	0.27%	-1.04%	0.05%	-0.55%	0.18%	0.42%	-0.04%
III	E	0.83%	5.89%	1.23%	-0.48%	3.82%	-0.03%	-1.45%	0.07%	-0.03%	0.05%	0.21%	-0.03%
IV	A	-0.13%	2.45%	5.78%	5.60%	-0.67%	-0.07%	0.24%	-0.15%	0.20%	-0.14%	-0.13%	-0.23%
IV	B	-0.13%	2.45%	4.86%	1.22%	-0.67%	-0.04%	0.29%	-0.10%	0.21%	-0.16%	-0.33%	-0.15%
IV	C	-0.13%	2.45%	4.84%	2.66%	-0.67%	-0.06%	0.27%	-0.12%	0.22%	-0.15%	-0.18%	-0.19%
IV	D	-0.13%	2.45%	5.13%	4.76%	-0.67%	-0.04%	0.29%	-0.11%	0.22%	-0.14%	-0.16%	-0.17%
IV	E	-0.13%	2.45%	1.23%	-0.48%	-0.67%	0.01%	0.36%	0.06%	0.01%	-0.05%	-0.09%	0.01%
V	A	0.83%	2.45%	5.78%	5.60%	4.09%	0.44%	-1.76%	-0.20%	-1.64%	-0.19%	0.75%	-0.85%
V	B	0.83%	2.45%	4.86%	1.22%	4.09%	0.26%	-2.03%	-0.13%	-1.67%	-0.20%	1.71%	-0.57%
V	C	0.83%	2.45%	4.84%	2.66%	4.09%	0.33%	-1.94%	-0.17%	-1.74%	-0.21%	0.84%	-0.69%
V	D	0.83%	2.45%	5.13%	4.76%	4.09%	0.26%	-2.04%	-0.19%	-1.66%	-0.24%	0.69%	-0.64%
V	E	0.83%	2.45%	1.23%	-0.48%	4.09%	-0.03%	-2.55%	0.27%	-0.06%	-0.14%	0.38%	-0.02%
VI	A	-0.13%	2.45%	5.78%	5.60%	-0.67%	-0.07%	0.24%	-0.15%	0.20%	-0.14%	-0.13%	-0.23%
VI	B	-0.13%	2.45%	4.86%	1.22%	-0.67%	-0.04%	0.29%	-0.10%	0.21%	-0.16%	-0.33%	-0.15%
VI	C	-0.13%	2.45%	4.84%	2.66%	-0.67%	-0.06%	0.27%	-0.12%	0.22%	-0.15%	-0.18%	-0.19%
VI	D	-0.13%	2.45%	5.13%	4.76%	-0.67%	-0.07%	0.25%	-0.14%	0.21%	-0.15%	-0.01%	-0.21%
VI	E	-0.13%	2.45%	1.23%	-0.48%	-0.67%	0.01%	0.36%	0.06%	0.01%	-0.05%	-0.09%	0.01%
VII	A	0.83%	3.13%	5.78%	5.60%	3.95%	0.42%	-1.64%	-0.20%	-1.42%	-0.18%	0.60%	-0.81%
VII	B	0.83%	3.13%	4.86%	1.22%	3.95%	0.26%	-1.89%	-0.13%	-1.27%	-0.19%	1.47%	-0.53%
VII	C	0.83%	3.13%	4.84%	2.66%	3.95%	0.32%	-1.80%	-0.16%	-1.41%	-0.20%	0.70%	-0.66%
VII	D	0.83%	3.13%	5.13%	4.76%	3.95%	0.39%	-1.68%	-0.17%	-1.44%	-0.17%	0.06%	-0.74%
VII	E	0.83%	3.13%	1.23%	-0.48%	3.95%	-0.03%	-2.39%	0.25%	-0.05%	-0.12%	0.34%	-0.02%
					min	-7.85%	-0.86%	-5.82%	-0.37%	-2.04%	-0.54%	-2.69%	-0.85%
					max	12.95%	1.53%	3.18%	0.43%	1.14%	0.78%	3.26%	0.16%

Table 3: Expected return of the different structured products in each term scenario (A, B, C, D, E) and parameters setting (I-VII) over the five year investment period. The expected returns have been estimated using the simulated G2++ interest rate model under the physical measure. In the third column, we have the 5-year term premium. In the fourth column the long-run interest volatility (S_r) and then the long-run rate (β_0) and in the fifth column the short term rate ($\beta_0 + \beta_1$).

Product	Intercept	$TP(0, 5)$	S_r	β_0	$\beta_0 + \beta_1$	R^2
zcb	0.001	4.997	-0.072	0.000	0.000	1
cb	-0.002	0.331	0.002	0.049	0.006	0.76
frn	-0.009	-1.689	0.100	0.084	0.006	0.96
cms	0.000	0.112	0.027	-0.048	0.001	0.76
frnc	0.001	-0.641	0.094	-0.157	0.018	0.76
cmsc	-0.004	0.218	0.058	0.002	-0.002	0.78
spread	-0.003	0.757	-0.093	0.196	-0.111	0.75
vol	-0.003	0.020	0.078	-0.053	-0.014	0.63

Table 4: Coefficient estimates of the regression of the expected return of the different bonds with respect to the 5-year Term Premium ($TP(0,5)$), asymptotic volatility S_r of the short rate, long-run level of the spot curve (β_0) and short term rate ($\beta_1 + \beta_0$). Bold estimates are significant at 1% level.

Parameters	Curve	$TP(5)$	S_r	zcb	cb	frn	cms	frnc	cmsc	spread	vol
I	A	2.65%	5.89%	0.43%	0.03%	0.39%	0.21%	1.86%	1.18%	0.19%	5.59%
I	B	2.65%	5.89%	0.43%	0.02%	0.44%	0.14%	1.40%	1.29%	0.35%	3.95%
I	C	2.65%	5.89%	0.43%	0.02%	0.42%	0.17%	1.73%	1.26%	0.07%	4.76%
I	D	2.65%	5.89%	0.43%	0.02%	0.44%	0.16%	1.65%	1.33%	0.04%	4.62%
I	E	2.65%	5.89%	0.43%	0.00%	0.52%	0.09%	0.00%	0.66%	0.02%	0.85%
II	A	-1.51%	5.89%	0.43%	0.02%	0.30%	0.17%	0.29%	0.78%	0.31%	4.34%
II	B	-1.51%	5.89%	0.43%	0.01%	0.33%	0.11%	0.78%	0.87%	0.43%	3.11%
II	C	-1.51%	5.89%	0.43%	0.02%	0.32%	0.14%	0.39%	0.85%	0.34%	3.74%
II	D	-1.51%	5.89%	0.43%	0.01%	0.34%	0.13%	0.48%	0.89%	0.19%	3.70%
II	E	-1.51%	5.89%	0.43%	0.00%	0.39%	0.07%	0.00%	0.43%	0.06%	0.74%
III	A	0.83%	5.89%	0.43%	0.03%	0.35%	0.19%	1.21%	0.99%	0.26%	4.97%
III	B	0.83%	5.89%	0.43%	0.02%	0.38%	0.13%	0.72%	1.08%	0.27%	3.54%
III	C	0.83%	5.89%	0.43%	0.02%	0.37%	0.16%	0.94%	1.06%	0.34%	4.26%
III	D	0.83%	5.89%	0.42%	0.02%	0.38%	0.15%	0.90%	1.11%	0.26%	4.14%
III	E	0.83%	5.89%	0.43%	0.00%	0.45%	0.08%	0.00%	0.54%	0.15%	0.80%
IV	A	-0.13%	2.45%	0.08%	0.00%	0.06%	0.05%	0.15%	0.11%	0.24%	0.48%
IV	B	-0.13%	2.45%	0.08%	0.00%	0.07%	0.03%	0.27%	0.12%	0.34%	0.39%
IV	C	-0.13%	2.45%	0.08%	0.00%	0.06%	0.04%	0.12%	0.12%	0.19%	0.48%
IV	D	-0.13%	2.45%	0.08%	0.00%	0.07%	0.04%	0.17%	0.18%	0.25%	0.71%
IV	E	-0.13%	2.45%	0.08%	0.00%	0.08%	0.03%	0.00%	0.30%	0.19%	0.76%
V	A	0.83%	2.45%	0.08%	0.00%	0.06%	0.05%	0.35%	0.13%	0.23%	0.59%
V	B	0.83%	2.45%	0.08%	0.00%	0.07%	0.03%	0.41%	0.14%	0.41%	0.48%
V	C	0.83%	2.45%	0.08%	0.00%	0.07%	0.04%	0.41%	0.15%	0.36%	0.58%
V	D	0.83%	2.45%	0.08%	0.00%	0.07%	0.04%	0.55%	0.22%	0.32%	0.85%
V	E	0.83%	2.45%	0.08%	0.00%	0.08%	0.03%	0.00%	0.39%	0.19%	0.79%
VI	A	-0.13%	2.45%	0.08%	0.00%	0.06%	0.05%	0.15%	0.11%	0.24%	0.49%
VI	B	-0.13%	2.45%	0.08%	0.00%	0.07%	0.03%	0.27%	0.12%	0.34%	0.39%
VI	C	-0.13%	2.45%	0.08%	0.00%	0.06%	0.04%	0.12%	0.12%	0.19%	0.48%
VI	D	-0.13%	2.45%	0.08%	0.00%	0.06%	0.04%	0.13%	0.10%	0.03%	0.40%
VI	E	-0.13%	2.45%	0.08%	0.00%	0.08%	0.03%	0.00%	0.30%	0.19%	0.76%
VII	A	0.83%	3.13%	0.15%	0.01%	0.12%	0.08%	0.82%	0.22%	0.23%	0.94%
VII	B	0.83%	3.13%	0.15%	0.01%	0.14%	0.05%	0.73%	0.25%	0.51%	0.77%
VII	C	0.83%	3.13%	0.15%	0.01%	0.13%	0.06%	0.85%	0.25%	0.40%	0.91%
VII	D	0.83%	3.13%	0.15%	0.01%	0.13%	0.07%	0.82%	0.20%	0.04%	0.80%
VII	E	0.83%	3.13%	0.15%	0.00%	0.16%	0.04%	0.00%	0.40%	0.23%	0.75%
			min	0.08%	0.00%	0.06%	0.03%	0.00%	0.10%	0.02%	0.39%
			max	0.43%	0.03%	0.52%	0.21%	1.86%	1.33%	0.51%	5.59%

Table 5: Standard deviation of the return of the different structured products under different scenarios. The standard deviations have been estimated using the simulated G2++ interest rate model under the physical measure. The column S_r gives the long-run volatility of the short rate.

Product	Intercept	$TP(0, 5)$	S_r	β_0	$\beta_0 + \beta_1$	R^2
zcb	-0.002	0.001	0.101	0.000	0.000	1
cb	0.000	0.001	0.004	<i>0.002</i>	0.000	0.92
frn	-0.001	0.023	0.090	-0.010	-0.001	1
cms	-0.001	0.006	0.030	0.005	0.006	0.98
frnc	-0.009	0.231	0.139	0.192	-0.025	0.9
cmsc	-0.005	0.084	0.221	0.036	-0.010	0.94
spread	0.000	-0.020	-0.008	0.090	-0.056	0.9
vol	-0.030	0.173	0.858	0.269	0.079	0.91

Table 6: Coefficient estimates of the regression of the standard deviation of the different bonds with respect to the 5-year Term Premium ($TP(0,5)$), asymptotic volatility S_r of the short rate, long-run level of the spot curve (β_0) and short term rate ($\beta_1 + \beta_0$). Bold numbers are significant at 1% level.

1045 Using now the definition of term premium, we have

$$E_0(R_{zcb}) = (T - 0) \times TP(0, T) - \frac{1}{2}V(0, T),$$

1046 Therefore $T \times TP(0, T)$ is the difference between the expected return under the real world
1047 measure and under the risk neutral measure.

1048 Finally, the variance of the return follows quite easily, indeed we have

$$V_0(R_{zcb}) = V(0, T).$$

1049 Notice that the expected return and the variance of the ZCB investment do not depend on
1050 the shape of the current term structure. This is also reflected in our simulations, see for
1051 example Tables 3 and 5: the estimated return and volatility of the ZCB is independent on
1052 the term structure scenario.

Parameters	Curve	zcb	cb	frn	cms	frnc	cmsc	spread	vol
I	A	12.78%	11.90%	-39.76%	-22.85%	18.74%	19.81%	-38.77%	18.16%
I	B	14.22%	15.47%	-40.89%	-25.11%	17.41%	16.83%	-36.53%	15.12%
I	C	17.00%	17.74%	-43.59%	-26.54%	23.53%	22.88%	-22.80%	19.43%
I	D	18.44%	19.23%	-45.05%	-28.44%	25.57%	26.11%	-13.80%	21.79%
I	E	-36.68%	7.57%	7.93%	-28.09%	7.57%	-21.92%	6.07%	6.87%
II	A	33.21%	31.52%	-60.06%	-30.38%	41.23%	44.88%	40.23%	42.18%
II	B	16.18%	16.51%	-43.35%	-23.52%	-28.53%	23.32%	16.16%	22.44%
II	C	15.18%	15.97%	-42.12%	-21.90%	-29.65%	22.22%	13.13%	20.30%
II	D	12.80%	13.21%	-39.92%	-22.18%	-32.21%	19.32%	6.89%	17.84%
II	E	-27.50%	-0.82%	-1.95%	-18.83%	-0.82%	-12.97%	-14.94%	-3.70%
III	A	18.90%	19.57%	-45.62%	-25.51%	20.52%	22.49%	-29.50%	21.28%
III	B	9.67%	11.05%	-36.61%	-21.45%	11.24%	13.55%	-42.02%	11.89%
III	C	13.05%	13.98%	-39.74%	-22.44%	16.17%	16.73%	-41.79%	15.18%
III	D	12.52%	13.88%	-39.13%	-22.83%	18.07%	16.87%	-38.04%	14.14%
III	E	-45.24%	15.81%	16.40%	-33.69%	15.81%	-25.56%	19.80%	17.17%
IV	A	5.80%	3.49%	-32.50%	-8.70%	20.32%	24.43%	7.06%	23.10%
IV	B	-3.23%	-2.69%	-24.06%	-8.36%	-11.47%	14.36%	-4.57%	12.11%
IV	C	5.72%	4.33%	-33.32%	-10.56%	11.29%	20.18%	-3.56%	18.92%
IV	D	10.27%	9.44%	-37.84%	-16.64%	7.96%	18.73%	-6.60%	18.01%
IV	E	-33.59%	5.32%	4.87%	-20.54%	5.32%	-22.11%	1.12%	0.93%
V	A	9.51%	7.22%	-37.08%	-14.24%	15.72%	22.35%	-4.22%	21.88%
V	B	9.60%	8.68%	-37.62%	-15.60%	8.18%	18.83%	-9.69%	18.51%
V	C	11.14%	9.24%	-39.09%	-16.70%	13.81%	20.97%	-9.13%	20.74%
V	D	14.80%	13.08%	-42.79%	-21.28%	14.09%	21.24%	-9.88%	21.20%
V	E	-33.69%	5.49%	4.94%	-20.00%	5.49%	-23.39%	2.44%	0.14%
VI	A	5.99%	3.65%	-32.76%	-8.92%	20.49%	24.51%	6.97%	23.21%
VI	B	-3.21%	-2.73%	-24.10%	-8.19%	-11.45%	14.35%	-4.56%	12.06%
VI	C	5.93%	4.50%	-33.57%	-10.65%	11.32%	20.28%	-3.51%	19.03%
VI	D	2.43%	0.31%	-29.95%	-7.60%	15.24%	22.35%	-3.13%	21.10%
VI	E	-33.52%	5.26%	4.78%	-20.47%	5.26%	-22.11%	1.05%	0.88%
VII	A	16.09%	15.77%	-38.02%	1.52%	23.98%	28.77%	-9.95%	27.30%
VII	B	12.33%	12.78%	-35.34%	-2.10%	10.77%	24.87%	-24.84%	23.81%
VII	C	13.50%	13.17%	-36.17%	-1.40%	16.63%	26.28%	-23.03%	25.20%
VII	D	12.11%	11.45%	-34.31%	1.70%	19.08%	27.18%	-20.95%	25.76%
VII	E	-39.13%	11.56%	8.31%	-14.61%	11.56%	-27.49%	10.42%	6.56%
	min	-45.24%	-2.73%	-60.06%	-33.69%	-32.21%	-27.49%	-42.02%	-3.70%
	max	33.21%	31.52%	16.40%	1.70%	41.23%	44.88%	40.23%	42.18%

Table 7: The first two columns identify the scenario setting. Then we have the average correlation of each product with the remainings.

Parameters	Curve	zcb	cb	frn	cms	frnc	cmsc	spread	vol	basic	SP
I	A	2.84%	77.94%	9.35%	0.75%	0.25%	0.33%	8.30%	0.23%	90%	10%
I	B	3.02%	82.80%	8.27%	1.09%	0.29%	0.43%	3.73%	0.36%	94%	6%
I	C	4.42%	76.02%	11.20%	1.12%	0.25%	0.34%	6.41%	0.25%	92%	8%
I	D	4.27%	82.56%	10.31%	1.30%	0.25%	0.30%	0.80%	0.22%	97%	3%
II	A	0.14%	86.61%	10.90%	0.28%	0.68%	0.22%	1.03%	0.14%	98%	2%
II	B	1.02%	87.74%	8.50%	0.99%	0.35%	0.33%	0.79%	0.28%	97%	3%
II	C	0.84%	87.24%	8.95%	0.60%	0.81%	0.25%	1.10%	0.22%	97%	3%
II	D	0.23%	90.23%	6.86%	0.42%	0.61%	0.19%	1.25%	0.22%	97%	3%
III	A	3.66%	79.89%	12.91%	0.49%	0.29%	0.28%	2.30%	0.18%	96%	4%
III	B	5.87%	72.10%	8.60%	1.07%	0.59%	0.57%	10.69%	0.50%	87%	13%
III	C	4.51%	72.33%	11.18%	1.01%	0.76%	0.90%	8.75%	0.56%	88%	12%
III	D	3.10%	83.00%	7.84%	0.61%	0.19%	0.26%	4.69%	0.32%	94%	6%
IV	A	2.30%	78.75%	15.06%	0.34%	1.29%	0.96%	0.74%	0.56%	96%	4%
IV	B	0.10%	85.27%	6.63%	2.14%	1.82%	1.49%	1.25%	1.30%	92%	8%
IV	C	2.00%	75.95%	15.42%	0.55%	2.40%	1.28%	1.56%	0.84%	93%	7%
IV	D	0.69%	81.25%	13.36%	0.61%	1.85%	0.72%	1.00%	0.53%	95%	5%
V	A	2.08%	78.35%	13.75%	2.26%	1.03%	0.85%	0.91%	0.76%	94%	6%
V	B	7.62%	69.75%	15.44%	3.61%	0.92%	0.86%	0.96%	0.83%	93%	7%
V	C	5.31%	72.16%	15.59%	3.26%	1.03%	0.87%	0.97%	0.81%	93%	7%
V	D	7.00%	70.90%	16.03%	3.51%	0.65%	0.56%	0.86%	0.48%	94%	6%
VI	A	1.62%	76.32%	16.89%	0.53%	1.72%	1.23%	0.97%	0.72%	95%	5%
VI	B	0.09%	85.33%	6.62%	2.13%	1.81%	1.48%	1.24%	1.29%	92%	8%
VI	C	1.93%	75.66%	15.48%	0.56%	2.49%	1.38%	1.65%	0.85%	93%	7%
VI	D	4.24%	58.14%	19.66%	1.75%	2.54%	2.02%	10.23%	1.42%	82%	18%
VII	A	0.00%	87.67%	8.29%	1.02%	0.57%	0.64%	1.26%	0.55%	96%	4%
VII	B	5.63%	74.67%	11.41%	3.57%	1.07%	1.22%	1.77%	0.65%	92%	8%
VII	C	1.41%	87.52%	7.56%	1.20%	0.42%	0.49%	1.06%	0.34%	96%	4%
VII	D	0.00%	82.20%	6.71%	0.70%	0.51%	0.51%	8.91%	0.47%	89%	11%
	min	0.00%	58.14%	6.62%	0.28%	0.19%	0.19%	0.74%	0.14%	82.04%	2.35%
	max	7.62%	90.23%	19.66%	3.61%	2.54%	2.02%	10.69%	1.42%	97.65%	17.96%

Table 8: Composition of the global minimum variance portfolio

Product	Intercept	$TP(0, 5)$	S_r	β_0	$\beta_0 + \beta_1$	R^2
zcb	0.074	0.773	-0.263	-0.678	-0.152	0.79
cb	0.189	-1.282	1.581	11.903	-1.919	0.99
frn	0.064	0.062	-0.977	1.515	0.300	0.93
cms	0.062	0.070	-0.309	-0.624	-0.129	0.88
frnc	0.577	0.084	-0.303	-12.136	2.007	0.98
cmsc	0.021	-0.044	-0.199	-0.060	-0.055	0.92
spread	-0.003	0.381	0.590	0.149	-0.004	0.54
vol	0.016	-0.045	-0.120	-0.070	-0.048	0.91
basic	0.327	-0.447	0.340	12.741	-1.771	0.99
SP	0.673	0.447	-0.340	-12.741	1.771	0.98

Table 9: Coefficient estimates of the regression of the composition of the GMV portfolio with respect to the 5-year Term Premium $TP(0, 5)$, asymptotic volatility S_r of the short rate, long-run level of the spot curve (β_0) and short term rate ($\beta_1 + \beta_0$). Bold estimates are significant at 1% level.

Param	Curve	g_{nosp}	%SP	g_{basic}^λ	%SP	$g_{sp>b}$	%SP	$g_{b>sp}$	%SP	%SP ($g = 1\%$)	%SP ($g = 3\%$)	%SP ($g = 5\%$)
1	1	2.1%	5.0%	1.1%	0.0%	0.0%	91.1%	0.0%	91.1%	2.3%	0.7%	0.2%
1	2	3.6%	4.7%	3.6%	0.9%	0.0%	77.0%	0.0%	77.0%	12.2%	5.2%	0.0%
1	3	0.0%	0.0%	0.0%	0.0%	0.0%	93.8%	0.0%	93.8%	0.0%	0.0%	0.0%
1	5	0.0%	0.0%	0.0%	0.0%	0.0%	96.0%	0.0%	96.0%	0.0%	0.0%	0.0%
2	1	6.9%	5.0%	32.7%	1.0%	0.0%	0.0%	1.0%	0.0%	6.6%	2.6%	2.6%
2	2	18.0%	5.0%	23.4%	1.0%	0.0%	5.7%	0.2%	5.7%	4.1%	4.0%	3.8%
2	3	17.4%	5.0%	28.5%	1.0%	0.0%	3.8%	1.0%	3.8%	3.3%	3.3%	3.2%
2	5	18.5%	5.0%	28.2%	1.0%	0.0%	3.4%	0.9%	3.4%	3.5%	3.4%	3.3%
3	1	0.2%	1.1%	0.2%	0.7%	0.0%	61.9%	0.0%	61.9%	0.0%	0.0%	0.0%
3	2	2.1%	5.0%	1.6%	0.6%	0.0%	86.1%	0.0%	86.1%	20.0%	0.1%	0.1%
3	3	1.1%	0.2%	1.1%	0.0%	0.0%	75.9%	0.0%	75.9%	0.0%	0.0%	0.0%
3	5	0.5%	0.4%	0.5%	0.3%	0.0%	90.0%	0.0%	90.0%	0.0%	0.0%	0.0%
4	1	0.6%	3.7%	0.7%	0.2%	0.0%	0.0%	0.0%	0.0%	0.1%	0.1%	0.2%
4	2	0.3%	4.4%	0.6%	1.0%	0.0%	6.7%	0.0%	6.7%	0.1%	0.1%	0.2%
4	3	0.5%	4.5%	0.7%	0.7%	0.0%	5.0%	0.0%	5.0%	0.1%	0.1%	0.2%
4	5	0.6%	4.8%	1.3%	1.0%	0.0%	8.7%	0.0%	8.7%	1.2%	0.1%	0.1%
5	1	0.7%	5.0%	0.5%	0.0%	0.0%	50.9%	0.0%	50.9%	0.2%	0.1%	0.1%
5	2	0.8%	2.9%	0.2%	0.8%	0.0%	50.2%	0.0%	50.2%	0.4%	0.1%	0.1%
5	3	0.8%	4.7%	0.3%	0.6%	0.0%	49.6%	0.0%	49.6%	0.2%	0.1%	0.1%
5	5	0.8%	5.0%	0.3%	0.7%	0.0%	50.4%	0.0%	50.4%	0.2%	0.1%	0.0%
6	1	0.6%	4.1%	0.7%	0.2%	0.0%	0.0%	0.0%	0.0%	0.1%	0.1%	0.2%
6	2	0.3%	4.4%	0.6%	1.0%	0.0%	6.7%	0.0%	6.7%	0.1%	0.1%	0.1%
6	3	0.5%	4.3%	0.7%	0.8%	0.0%	5.0%	0.0%	5.0%	0.1%	0.1%	0.2%
6	5	0.6%	4.9%	1.3%	1.0%	0.0%	8.9%	0.0%	8.9%	1.2%	0.1%	0.1%
7	1	0.0%	2.0%	0.3%	0.6%	0.0%	56.1%	0.0%	56.1%	0.0%	0.0%	0.0%
7	2	2.6%	4.8%	4.5%	0.9%	0.0%	61.9%	0.0%	61.9%	4.1%	2.2%	0.3%
7	3	1.4%	4.9%	2.6%	0.9%	0.0%	52.5%	0.0%	52.5%	3.3%	0.3%	0.0%
7	5	1.5%	4.7%	2.6%	0.8%	0.0%	54.2%	0.0%	54.2%	3.4%	0.3%	0.0%

Table 10: The first two columns identify the scenario setting. The third column gives the maximum fee g_{nosp} such that the amount invested in SP is no greater than 3% across all efficient portfolios. The fourth column gives the actual amount invested in SPs if the fee in column three is applied. The fifth column gives the maximum fee g_{basic}^λ such that a risk-averse investor will minimize the investment in SPs and the adjacent column the actual percentage allocated to SPs given g_{basic}^λ . Columns seven (nine) gives the maximum (minimum) fee such that an investment in SPs (BASIC) only dominates an investment in BASIC (SP) products only (the fee is set at 0 if dominance is not possible). The adjacent columns gives the percentage allocated to SPs in a portfolio containing BASIC and SPs if the fees $g_{sp>b}$ or $g_{b>sp}$ are charged. The last three columns give the average (across all efficient portfolios, excluding the GMV) amount invested in SPs given a 1%, 3% or 5% fee g_{basic}^λ .

Scenario I A	zcb	cb	frn	cms	frnc	cmsc	spread	vol
Ex. Return	12.95%	1.53%	-3.44%	0.43%	-2.04%	0.71%	1.95%	0.06%
Std. Deviation	0.43%	0.03%	0.39%	0.21%	1.86%	1.18%	0.19%	5.59%
GMV	2.84%	77.94%	9.35%	0.75%	0.25%	0.33%	8.30%	0.23%
Scenario VIII A								
Ex. Return	6.90%	0.84%	-0.61%	0.43%	-0.42%	0.49%	0.58%	0.14%
Std. Deviation	0.34%	0.02%	0.28%	0.13%	1.15%	0.65%	0.21%	3.05%
GMV	0.00%	92.59%	7.35%	0.00%	0.00%	0.00%	0.00%	0.06%

Table 11: Expected return and standard deviations of the different structured products in the parameter settings V-A and VIII-A.