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FORM FINDING AND DIMENSIONING OF REINFORNCED CONCRETE SHELL ROOF FOR AKROTIRI (SANTORINI)

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ABSTRACT

Shells are long-span and inherently light-weight structures used for both their aesthetic and structural benefits. This paper develops a robust methodology for the reinforced concrete (RC) roof shell design of Akrotiri, an archaeological site in Santorini island, Greece. The methodology uses Oasys GSA and finite element package Abaqus which allow both form finding analysis and dimensioning to be conducted. Through the step-by-step application of this method, a protective shell cover is designed and its applicability demonstrated.

Keywords: *design optimisation, form-finding, dimensioning, structural design, roof shell, reinforced concrete shell*

1. INTRODUCTION

Shell structures provide efficient solutions for situations requiring long spanning, light-weight systems. The shape of a shell determines its efficiency by influencing its stiffness, capacity and structural behaviour. In current research, shell design optimisation is widely explored with wide-ranging objectives being investigated, such as shape, size, topology, environmental, and buckling optimisation. Multiple methods of optimisation are available, however, there is no standardised methodology for the simple production of a reinforced concrete (RC) shell design.

Through material development shell designs became increasingly viable; progressing from the masonry dome of St Paul's Cathedral in 1708, to the reinforced concrete roof of the Kresge Auditorium in Boston built in 1955 [1]. Later, modern material development enabled structures such as the grid shell at the Japan Pavilion, 2000 Expo in Hannover, Germany [1] where recyclable paper tubes were used to create a grid shell spanning over 25m. Today, with the creation of more daring architectural designs, steel is often chosen. Its high strength to weight ratio enables thinner, more aesthetically pleasing designs which are more cost effective than concrete. However, recent developments such as the use of textile formwork and composite material technology, mean that concrete shells could be made more economically and thus become more appealing and viable structures [2].

The material chosen to create a shell dictates its design in many ways. For example, concrete shells are dependent on compression being the focal stress in the system whereas structures formed using fabric membranes rely on tensile forces [1,3,4,5]. These different material requirements affect the type of method that can be used to design and optimise each shell.

Although timber and steel are some of the most predominately used materials for grid shells, concrete also has its benefits. With both a high strength and elastic modulus, concrete can provide its own advantages to shell design. This paper focusses on RC shells, which are created from grids (small intersecting beams) and have the same structural make-up as a basic kitchen sieve [1].

horizontal and vertical forces in the structure being in equilibrium. It ignores the effect of the material's



Figure 1: Developed Standard Methodology for Shell Design and Optimisation

One of the most well-known form finding methods is the 'hanging chain method', used by Otto and Isler [6], where hanging chains were employed to determine an optimal shell form. A modern-day version of this method, the Particle Spring system, naturally generates appropriate load paths for the structure where the "forces are in equilibrium" [7], and thereby all developed models are structurally sound. Although this method addresses the structural validity of a shell, it can create peculiar shell topologies, therefore making it hard to further optimise the designs [7]. Another method, known as the Force Density method, developed in 1971 by Linkwitz and Schek, allows the form finding problem to be linear instead of nonlinear, allowing significant simplification [1][8]. The use of linear analysis can be a benefit in terms of simplifying the problem, however, this can also result in unrealistic models which do not provide real-world practical applications. More recently researchers have practiced form-finding while accounting for horizontal seismic loading in addition to the vertical gravity loading in order to understand how formfound shells perform [9,10].

Thrust Network Analysis, another form finding method, aims to develop a funicular structure, i.e., a system that only experiences either tension or compression forces. This method focusses on the properties would have on the structure, which could be considered a negative aspect of this optimisation method [11]. Normal Property method, unlike Thrust Network Analysis, depends on the material properties of the structure to develop the form. Through the application of negative vertical loads to a grid outline and the material's response to them the method creates an optimal shape of the shell. As the material properties can greatly affect the structural behaviour and since RC is deliberately being analysed in this paper, the Normal Property method was chosen for developing this new design methodology.

2.0 METHODOLOGY

This paper intends to develop a standard, robust methodology for shell design, looking at both form finding and dimensioning, and by doing so aims to create a firm step-by-step basis for other researchers and designers. A visual interpretation of the developed methodology is shown in Figure 1, highlighting each stage and giving a brief description of the necessary steps.

Shell structures are dependent on their form, therefore, it is essential to begin with the design optimisation performed by the form finding process. One of the signature aspects of shells is their lightweight characteristic, which is made possible by

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both their optimal shape and thickness. Appropriate dimensioning is therefore also a key factor to consider. Through the thickness optimisation of a shell's structure, an efficient and aesthetic final design is aimed for.

This paper follows a problem formulation of minimum compliance design, which works to a minimum thickness of a shape optimised shell. Dimensioning, or more precisely, the optimisation of the shell thickness is integrated into the structural design optimisation procedure in a manual way. It is introduced by completing multiple iterations with different shell thicknesses during the static analysis stage (Figure 1, Stage 7). The shell thicknesses are user-defined and rely on the engineering judgement of the user. It is worth to note that the effect of imperfections is not considered in the analysis.

An additional step not usually found in shell design is included in this methodology - a critical modal analysis. It evaluates the serviceability limit state performance of the optimised shell and assesses the shell's reaction to vibrations. Due to the long span nature of shells they are susceptible to excitation from vibration caused by seismic or wind loadings. As this is a potential failure mode for shell structures, and especially those located in Greece, this is an important step to include in the design.

This methodology aims to be an updated step-bystep design optimisation process, with important considerations made on the choice of software used. A combination of software packages Oasys GSA and Abaqus were employed for the form finding and dimensioning stages, respectively.

The methodology has been tested with a case study, allowing the user to determine whether it is a feasible design. Special care must be taken by the user as the software may not produce the most optimum outcomes, particularly when designs are combined. Moreover, the application of the proposed methodology assessed the feasibility of using RC in long-span shell systems.

3.0 APPLCATION OF THE METHODOLOGY

3.1 The Site

The site of the case study is the Akrotiri, an Archaeological Site in the island of Santorini,

Greece. Now buried under volcanic rock, the site is a deep excavation with all the artefacts sitting below ground level. The site is exposed to moisture and large temperature changes, therefore, it is necessary to provide a protective cover as artefacts are likely to deteriorate with continued exposure [12]. Figure 2 shows the Akrotiri Archaeological Site, with its current protective cover since 2012, after a catastrophic collapse of the previous roof killing a tourist and injuring six more people in 2005.



Figure 2: Akrotiri Archaeological Site in Santorini

Archaeological sites contain evidence of historical occurrences and can provide vital information to an archaeologist's research. Hence, the need for shelters such as shells. These covers aim to prevent external environmental disturbances from causing deterioration to the uncovered artefacts [12] and their bespoke design is in demand.

Multiple considerations must be taken into account when designing protective covers for archaeological sites. Firstly, preventing the degradation of artefacts due to elemental effects such as sun, wind, and rain is key. Secondly, it is important for the protective cover not to interfere with the site and its artefacts. Long spanning shells have the capability to allow for minimal interference with the site, with their continuous nature also providing appropriate sun and rain protection. Simultaneously, minimum number of supports is required while they should provide the necessary capacity to withstand uplifting wind and other accidental loads. Such supports must be shallow, as less intrusive as possible to avoid damaging the artefacts. As archaeological sites rarely being regular shapes, shells are also the ideal system of protection because they can easily adapt to the required shape.

3.2 Boundary Conditions

The artefacts all sit below the ground level on the

Akrotiri Site therefore there was reasonable freedom with regards to vertical boundary conditions of the shell. However, due to the precious nature of the artefacts and the required accessibility to the site the shell was prevented from sitting directly above ground level. The horizontal boundary conditions were defined by the existing plan of the site and determined accordingly. The plan of the site, shown in Figure 3, is an irregular shape, was defined as a maximum of 160m (N-S) by 128m (E-W).



Figure 3: Plan View of Akrotiri Archaeological Site

3.3 Form Finding Application

When employing the Normal Property method to find a shell's optimal form, the material's stiffness is the main influencing factor. Using Oasys GSA, negative vertical loads were applied to the shell's grid (both beam and area elements). The loading conditions were different for the beam (grid) and area elements, defined as -0.3kN/m and -0.6kN/m², respectively. Multiple iterations were completed with changes applied to the model, such as a different number of cycles, pin placement and grid segments. The final form contained 4m x 4m grid sections, pins at obvious corners of the shell (leaving unsupported lengths of up to 117m) and a 100mm thick shell. Figure 4 and 5 show the optimised form created.



Figure 4: Plan View of Optimised Form produced from Oasys GSA



Figure 5: Left-Hand Side View of Optimised Form produced from Oasys GSA, showing the 3 apexes

3.4 Dimensioning Application

During the form finding analysis the application of appropriate loads, boundary conditions, and materials properties allowed the structural behaviour of the shell to be understood. Abaqus was chosen to perform the static analysis and design optimisation.

The magnitude of the load applied to the structure was determined using Eurocodes. Equation 1, shown below, was used as it is a standard load case used for ultimate limit state design in the UK and would enable both an efficient and feasible shell design.

$$Total \ Load = 1.35G_k + 1.5Q_{k,wind} + (1.5)(0.7)Q_{k,variable}$$

The permanent and variable actions on the shell were unfavourable, therefore, the largest factors were applied to these loads (1.35 and 1.5 respectively). The psi factor (0.7) is applied to the variable action depending on the use of the structure. Usually structures where crowds could gather use this factor, but so as not to preclude potential future use of the shell it was included in the load calculations. (2)

As discovered from the research conducted by Tomas and Marti on the optimisation of concrete shells, the contribution of the steel reinforcement did not need to be considered during the modelling [13]. Using the Eurocodes, a set of values defining the material properties of C30/37 concrete were determined. The below equation was used to determine strain and stress values so a stress against strain curve could be plotted [14].

EC2 - BS EN 1992-1-1 (3.1.5):

$$\frac{\sigma_c}{f_{cm}} = \frac{k\eta - \eta^2}{1 + (k - 2)\eta}; where \eta = \frac{\varepsilon c}{\varepsilon c 1};$$
$$k = 1.05 \text{Ecm} \times \frac{|\varepsilon c 1|}{\text{fcm}}$$

Where:

 $f_{cm} = mean \ value \ of \ concrete \ cylinder \ compressive \\ strength \ (MPa) = 30 \ MPa$

 E_{cm} = Secant modulus of elasticity of concrete = 33GPa

 $\varepsilon_{c1} = 2.2$ (‰); $\varepsilon_{cu1} = 3.5$ (‰); n = 2.0 (from Table 1, EC2)

Linear material properties were used, as due to the size of the model, it would have taken too long to run a suitable nonlinear analysis during the time constraints of the research.

To determine the validity of the shell several load cases were separately applied to the model. Each of the cases were based on certain severe loading conditions for such structures. The loads were applied normal to the shell's surface, instead of in line with an x, y, or z plane making the model more realistic.

The first load case was a UDL applied over the whole shell surface. Secondly, a line load was applied along the left-hand edge of the shell to mimic wind loading. The third load case applied a UDL over the left-hand half of the whole surface and finally the fourth load case applied UDL only over the top third of the shell surface. Each of these conditions were created to produce different reactions from the shell, both in terms of deflection and stress.

Mesh convergence was a key consideration in this methodology. The larger the mesh used when analysing the structure, the more generalised and possibly inaccurate the data produced could be. However, there is a limit to how detailed the mesh must be; a very fine mesh can considerably slow down the analysis without producing more accurate results. The appropriate seed size for the mesh was determined to be 325. If a seed size of 1000 was compared to that of 325, their maximum values respectively were, in terms of stress 70.97MPa and 212.5MPa and in terms of deflection 243.60mm and 278.20mm. This demonstrates the importance of a mesh convergence test in this situation, especially with respect to stress values. The table below (Table 1) shows the values determined from the mesh convergence.

Seed Size	Stress Max. (MPa)	Deflect. Min. (mm)
1000	70.97	243.60
500	179.60	256.50
400	172.90	277.40
350	193.1	278.20
300	225.2	265.00
325	212.5	278.20

 Table 1: Mesh Convergence Table

3.5 Modal Analysis Application

The site of Akrotiri in Santorini is prone to seismic disturbances as the island of Santorini is characterised as an active volcano, with small eruptions occurring reasonably often [15]. Vibrations can be induced in a shell by even light seismic actions but also by wind loading. As a long span system, a shell is inherently susceptible to excitations due to such dynamic loading, therefore modal analysis is an important analysis to perform during design [16].

Any structure has a limitless number of modal shapes and it would not have been feasible to analyse them all, therefore, only five different modes were examined. The modal analysis of a structure uses only the mass and stiffness (shape) to determine the shell's reaction to the vibrations. The modal analysis models were studied by examining the deflections, the mode shapes of the shell and the frequencies - the latter was used to evaluate the likelihood of resonance occurring.

4.0 RESULTS

To analyse the form finding and dimensioning processes of the shell, the deflections and stresses of the models, each with differing thicknesses, were compared. A variation of thicknesses, 100, 200, 300, 400, and 500mm, were applied to every load case. Each load case highlighted different areas of weakness in the shell.

4.1 Design Optimisation Results

4.1.1 Deflections

The application of the first load case (UDL on the entire shell surface) produced some important results as this is the most likely loading case to occur. The maximum deflections, of a negative magnitude (shown in Figure 6 in blue), occurred along the right-hand edge of the shell. This was the longest unsupported length in the shell, therefore was likely to attract large deflections.



Figure 6: Load Case 1, Model 3 (300mm) Overall Vertical Deflection, Range: 35.55 to -118.94mm

The second load case mimicked wind loading, applying a horizontal line load to the shell's left-hand edge. It showed the bottom edge of the shell to be the main area susceptible to large deflections under this load. This edge was a similar length to the right-hand edge, therefore, it made sense for some large deflections to be concentrated here. Similarly, the third loading case, which consisted of the left-hand half of the shell being loaded with a vertical UDL, produced the largest deflections around the bottom edge of the shell. This load case created both large positive and negative deflections, shown in Figure 7. It also produced the most severe failure mode of the shell structure, thus it was considered the governing case for the overall shell design.



Figure 7: Load Case 3, Model 4 (400mm) Overall Vertical Deflection, Range: 365.36 to -336.75mm

The fourth and final load case focussed on the upper half of the shell. This load case was included to understand what areas would be susceptible to the largest deflections. The maximum deflections, similarly to the first load case, passed along the righthand edge of the shell, shown in Figure 8 (the negative deflections shown in blue and the positive in red). The final load case created such large deflections in the shell that some of the thicknesses failed the deflection requirement.



Figure 8: Load Case 4, Model 3 (300mm) Overall Vertical Deflection, Range: 367.39 to -387.83mm

In summary, the deflections governed the shell thicknesses due to the serviceability limit state being surpassed. The shell was susceptible to large deflections due to its long spanning, unsupported lengths along the bottom and right-hand edges.

4.1.2 Stresses

The stresses experienced in the initial loading case had a large range, with a maximum of 300MPa in the 200mm thick shell. The stresses above 30MPa (material stress maximum) were at the corners of the shell. Figure 9 shows the stress distribution limited to a maximum of 30MPa (shown in red). It shows how the stresses passed through the creases of the shell and concentrated at the corners.



Figure 9: Load Case 1, Model 1 (100mm) Stress Diagram up to 30MPa, (Red = 30MPa)

Similar stress distributions, concentrating around the corners, were found for most of the models. It was demonstrated that these corner areas would require additional cross-sectional concrete area and reinforcement. The second model also illustrated that the movement of the stresses through the structure followed the form of the shell and mainly passed along the ridges between its apexes. For the third and governing load case, large stress ranges were produced even in the thicker shells. The 500mm thick shell reached a maximum of about 200MPa at the corners. However, even in this shell the stresses produced in its main, central body did not surpass 30MPa showing that the form of the shell was working well. It was concluded that the weight of the shell was causing these large stresses at the corners.

A comparison between the stress distributions, in Figures 10 and 11, illustrate how the different load cases resulted in contrasting stress concentration zones in the shell. The stresses in Load Case 2 (Figure 10) concentrated around the left-hand side of the structure while for Load Case 3 (Figure 11) concentrated around the bottom of the structure, even though this case applied a UDL to the left-hand side of the shell.



Figure 10: Load Case 2, Model 1 (100mm) Stress Diagram up to 0.5MPa (Red = 0.5MPa)



Figure 11: Load Case 3, Model 2 (200mm) Stress Diagram up to 30MPa (Red = 30MPa)

The stresses in the final load case were not as high, reaching at maximum around 90MPa in the thickest shell (500mm). The stress distribution for this design

concentrated along the right-hand edge of the shell as shown in Figure 12.



Figure 12: Load Case 4, Model 2 (200mm) Stress Diagram up to 30MPa, (Red = 30MPa)

Overall, all the load cases produced stress distributions which focused on different areas of the shell, and therefore, they all highlighted different areas of weakness in the shell.

The distinguishing feature of all the stress patterns of this RC shell structure was that throughout the central of the body of the shell the stresses were not large unfeasible magnitudes, demonstrating that the optimised form satisfactorily works. The highest stresses only occurred at the corners of the shell, due to the high weight that these areas were carrying to the supports. The locations of the supports were determined by the user prior to optimising the shell through the software; it is considered that if a greater number of pins were introduced, it could more evenly distribute the load, hence reduce the stresses around the corners of the shell. This could be applied in an additional model to validate the methodology.

4.2 Modal Analysis Results

The deflections produced by the five different modes were analysed with respect to serviceability limit state design. The results showed that the main axis of deflection was the z-axis (vertical direction) and the largest magnitude of deflection only reached 1mm. This is not necessarily a realistic value, however, it illustrates that looking at the structure purely in terms of modal failure, failure due to deflection is not going to occur.

The frequency values could be compared to frequencies exerted from earthquakes in certain areas to determine whether resonance was likely to occur. None of the frequency values were found to match up, thus, destructive resonance would not be likely to occur and no structural changes needed to be made to the shell.

The modal shapes produced by the shell have been also investigated during the analysis. The deflected shapes created by normal loading patterns were superimposed over the modal shapes. This highlighted areas of the shell which were susceptible to large deflections, and thus further reinforcement could be provided to these areas. Similar to the load patterns the areas experiencing higher deflections were around either the bottom or the right-hand edge of the shell.

5.0 CONSTRUCTABILITY

In design optimisation processes, it is crucial to keep in mind the constructability of the project. With shells in particular, forms are usually designed by a form finding process, thus, the shapes cannot usually be constructed using standard methods [17]. Due to such unique shapes, the formwork to create a concrete shell can be difficult and expensive. For a way around this issue, Felix Candela created only hyperbolic paraboloid shell structures, as their formwork could be built out of purely straight lines [17]. Recently a new approach has been developed to create flexible formwork for concrete shells. Mele and Block created a fabric surface that, once sprayed with wet concrete, uses the weight of the concrete to create the desired shape of the shell [18]. Moreover, similar projects like the one presented here have employed the use of shell and flat slab hybrids [19].

This concept could be applied to the resulted RC shell. Due to the size of this particular shell, it is suggested to be divided up into its three apexes (shown in Figure 13) and each one formed separately. By dividing up the shell, its manufacture is easier and it might allow the concreting to be completed inside which would lessen the chance of irregularities in the pouring and curing of the concrete.



Figure 13: Shape of the shell from the LHS, with the ridges highlighted in red

For instance, steel rebar can provide the necessary reinforcement to the shell form. It should be placed through the top and bottom of the concrete shell to protect against both gravity and uplift forces. Areas shown to be susceptible to large deflections should be given additional reinforcement to prevent large cracks from occurring. Cracking of the concrete is what would make the shell most susceptible to deterioration, therefore, it should be avoided alike other long span RC structures.

As the shell will initially be created as three separate sections the connection of these sections is also critical. At the connecting edge of each section tension couplers can be used. Using couplers, the forces experienced in one piece of shell can be shared with the connecting section and the shell can act in the way it was designed for. Further considerations for the dynamic (seismic) performance of these connecting parts should take place.

Temporary propping should enable the shell to be constructed. Large cranes should be used to hold the pieces of shell while the propping was put in place. The locations from which the crane will hold the shell, should have to be determined through analysis and design; this should prevent unexpected failure occurring during construction, especially since inside access for cranes and propping is somewhat limited due to the archaeological site.

A preliminary reinforcement design was completed using the maximum stress values induced in the shell. It was assumed that the compressive stress was distributed evenly over the depth of the section, therefore, the equation below was used to determine the appropriate required reinforcement in a 1m width of shell [20].

$$N_{Ed} \le 0.43 f_{ck} bh + 0.67 f_{yk} A_s \tag{3}$$

Through producing several iterations an appropriate amount of rebar was defined. It was a feasible area of reinforcement in terms of incorporating it into the concrete thickness, however, it did highlight how much strength the shell would require which brought into question whether such a large and long span shell (160 by 128m) was a feasible design.

6.0 CONCLUSIONS

This study demonstrates that through the application of the proposed design optimisation, involving form finding and dimensioning, a unique fully optimised RC shell roof was created despite limitations. The RC shell developed using this new design methodology has an optimal form which evenly distributes stresses through the main shell body. This design, due to its long unsupported spans, was mainly susceptible to serviceability limit state criteria through large deflections at its edges. It is the shell's strength and stiffness that is generated by its optimised shape which would aid in withstanding the forces and stresses large deflections would induce. Additional design work may be considered to reduce the excessive use of material (high thickness) in certain areas of the shell, as well as to examine the used of lightweight and green concrete.

The combination of the software and method employed allowed a shell to be studied in detail, while the designer judged the shell's suitability. The inclusion of a level of engineering judgement allowed efficient options not to be ignored. This could also be seen as a limitation of the methodology since the reliability of user judgement can vary.

The design of this long span shell roof highlighted a basic and obvious fact. That the longer span of a free edge the more susceptible it is to the negative effects of loading and vibrations, even if intensive optimisation methods have been employed. Consequently, a key consideration for this methodology was introduced, that distances between pins should be limited to prevent unnecessarily large forces being produced.

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