Numerical evaluation on shell buckling of empty thin-walled steel tanks under wind load according to current American and European design codes

Chrysanthos Maraveas*[1], Georgios A. Balokas[2] and Konstantinos D. Tsavdaridis[3]

[1] School of Mechanical, Aerospace and Civil Engineering, University of Manchester, UK

[,2]C.Maraveas Partnership – Consulting Engineers, Athens, Greece

[3]Institute for Resilient Infrastructure, School of Civil Engineering, University of Leeds, UK

Abstract

Liquid storage steel tanks are vertical above-ground cylindrical shells and as typical thin-walled structures, they are very sensitive to buckling under wind load, especially when they are empty or at low liquid level. Previous studies revealed discrepancies in buckling resistance of empty tanks between the design method proposed by the American Standard API 650 and the analytical formulas recommended by the European Standard EN1993-1-6 and EN1993-4-2. This study presents a comparison between the provisions of current design codes by performing all types of numerical buckling analyses recommended by Eurocodes (i.e. LBA-linear elastic bifurcation analysis, GNA-geometrically nonlinear elastic analysis of the perfect tank and GNIA-geometrically nonlinear elastic analysis of the imperfect tank). Such analyses are performed in order to evaluate the buckling resistance of two existing thin-walled steel tanks, with large diameters and variable wall thickness. In addition, a discussion is unfolded about the differences between computational and analytical methods and the conservatism that the latter method imposes. A sensitivity study on the geometric imperfections and the boundary conditions is also conducted. Investigation on the boundary conditions at the foot of the tank highlights the sensitivity to the fixation of the vertical translational degree of freedom. Further, it is indicated that the imperfection magnitude recommended by the EN1993-1-6 is extremely unfavorable when applied to large diameter tanks. Comments and conclusions achieved could be helpful in order to evaluate the safety of the current design codes and shed more light towards the most accurate one.

Keywords: steel tanks; shell buckling; finite element modeling; nonlinear analysis; wind load

1. Introduction

Above-ground, vertical tanks of cylindrical shape are constructed in industrial and agricultural plants to store various fluids such as petroleum, oil, fuel etc. They are welded, thin-walled structures with large diameters, and hence buckling may occur when they are subjected to wind loads at their empty or partially filled state. Failure of such tanks results, in most cases, in a tremendous loss of financial and human resources, as well as composes a threat to public safety and an environmental hazard. Studies concerning wind-induced buckling of steel tanks have been increasing over the past few decades, since structural stability becomes critical for response and a major concern for the designer.

Early studies approached this matter based on analytical formulations of energy theory and tried to verify results with experiments [1]. Following, numerical approaches have been conducted extensively, inserting the imperfection sensitivity parameter [2,3]. Different tank variations have been investigated, like open-topped [4] and fixed-roof [5-7], combining computational methods and experimental results. Jaca and Godoy [8] indicated that buckling of tanks sometimes can occur under moderate wind load during their construction. Another subject of interest is the wind buckling behavior of grouped, arranged tanks [9,10,11]. The simulation of wind load distribution acting on the tank shell is an open research field [12,13,14]. Innovative ways of strengthening and improving buckling capacity have been proposed [15]. Sosa and Godoy [16] and Burgos et al. [17] have recently taken a turn towards analytical methods, in order to improve buckling evaluation by proposing new methodologies.

This study aims to appraise the efficiency of current design specifications in addressing structural stability of empty, large tanks when subjected to wind actions. Most recent codes (EN1993-1-6 and EN1993-4-2) have not yet seen many field applications and their results may raise doubts. This paper offers a comparison between API 650 and the Eurocodes, by performing three types of buckling analysis recommended by the EN1993-1-6 for numerical investigation and relating the results with previous studies [18] conducted with analytical methods proposed by the aforementioned codes. Thus, the stability of two existing large-diameter, steel tanks at empty state is evaluated.

The study is organized as follows: Section 2 describes the design philosophy of API 650 [19] and EN1993-1-6. In Section 3 the geometry of the two existing tanks is presented in detail and Section 4, presents the finite element models used for analyses and the wind pressures simulated for each code. Section 5 describes the linear bifurcation analysis (LBA) and in Section 6 and 7 geometrically nonlinear buckling behavior is investigated for perfect (GNA) and imperfect (GNIA) models respectively. In Section 8 comparison results are discussed and finally in Section 9, some helpful conclusions are reached.

2. Description of current code provisions

The most commonly used standards for assessing the structural stability of thin-walled structures are the API 650 and EN1993-1-6. The American Standard API 650 provides two empirical methods (the one-foot method and the variable design point method) for selecting the thickness of each shell course, depending on the geometry of the tank, the operational liquid level, the material used, the density of the contained fluid and the allowance for corrosion. The aforementioned methods are based on the concept of limiting the tensile stresses of the shell due to hydrostatic pressure while they do not consider for buckling. The buckling limit state is considered only indirectly, via an empirical design method that mandates stiffening of the shell (with circumferential girders at specific heights) depending on the thickness, height and wind velocity. The lack of mathematical formulation for evaluating the shell stability poses a major disadvantage.

On the contrary, the European standard EN1993-1-6 [20] contains the theoretical background and provides the state-of-the-art methodologies for evaluating explicitly the buckling resistance of shell structures. Provisions include analytical expressions for calculating the buckling capacity in terms of stresses and also propose several numerical methods, like linear bifurcation analysis for obtaining the critical elastic buckling load as well as analyses that include geometrical and material nonlinearities and imperfections. Even though its provisions are limited to axisymmetric geometries, the European Standard has a wide range of applications with regard to cylindrical tanks. It is of paramount importance that the code quantifies the buckling resistance in terms of critical stresses or critical loads. An analytical procedure for evaluating the buckling resistance of shells with variable wall thickness has also been developed. Most of the approaches recommended by the European Standard require the use of computational methods, such as the finite element method, for analyzing the shell. The use of simplified expressions, according to basic principles of mechanics, for determining the design stresses is permitted only in certain cases. However, it should be highlighted that the European Standard is still very recent, and its applicability to the field construction has not been adequately confirmed up to date.

3. Geometry of the tanks

The two existing, thin-walled and large diameter steel tanks under investigation (T-776 and T-761) are shown in **Fig. 1**. They are located at the refinery of Motor Oil Hellas S.A. (Korinthos, Greece). Both tanks are cylindrical, self-supported (not anchored to the foundation), with flat bottoms and are considered empty. Tank T-776 supports a conical roof with a slope equal to 1/6, while the other tank is open-topped. The conical roof is supported by a truss structure with three section groups (L125x75x8, HEM1000 and SHS_80x80x8). The geometrical data of both tanks, including distinct locations of the ring stiffeners (wind girders) along the circumference, are presented in **Table 1**. It can be seen that the aspect ratio of tanks (H/D) is quite low (0.43 for T-776 and 0.22 for T-761).

Fig. 1 On-site pictures of tank T-776 (left) and tank T-761 (right)

Both tanks have variable wall thicknesses and their cylindrical shell is divided in nine courses. The width and thickness of each shell course along with relevant information regarding the bottom and roof (where applicable) are summarized in **Table 2**. The choice of the case studies is based on the variability of the characteristics (aspect ratio, stiffeners, roof tops etc.) which cover a wide range of different structural behaviours for practical storage tanks.

Table 1 Geometrical characteristics of tanks T-776 and T-761

Tank ID	Shell Height (m)	Roof Height (m)	Inside Diameter (m)	1st Wind Girder Height* (m)	2nd Wind Girder Height* (m)
T-776	20.032	3.911	46.939	14.860	-
T-761	19.500	-	88.430	15.350	18.400

*Wind girder height is measured from the bottom of the tank

Table 2 Shell courses details for tanks T-776 and T-761

Course No.*	Course Thickness (mm)		Course Width (mm)	
	T-776	T-761	T-776	T-761
1	22.25	38.60	2438	2222
2	18.93	37.18	2438	2222
3	16.24	28.20	2438	2222
4	13.57	24.59	2438	2222
5	10.9	19.96	2438	2222
6	8.22	15.60	1940	2222
7	8.00	11.20	1940	2222
8	8.00	9.50	1940	2222
9-top	8.00	9.50	1940	1724
Bottom Shells	6.40	6.40	2102	Variable
Roof Shells	5.00	-	1502	-

*Courses are numbered from bottom to top (i.e. No1 refers to the bottom shell course etc.)

4. Computational models

A separate 3D finite element (FE) model was created for each tank. The commercial FE package ABAQUS [21] was used to simulate the tanks with geometric and material properties similar to the existing structures and to perform the required analyses. The S8R5 element type was used for the cylindrical shell and the bottom of each tank (but also for the roof shell for tank T-776). It is a rectangular, doubly curved, thin, continuum shell element with reduced integration and 8 nodes. Each node has 5 degrees of freedom: translations in each spatial coordinate and two rotations with respect to the in-plane axis.

Such characteristics satisfy the modeling requirements of EN1993-1-6 [20]. The remaining structural parts (wind girders, top curb angles, roof trusses etc.) were simulated with beam elements. Element B31 was used, which is a 3D Timoshenko beam element with linear interpolation, which allows for transverse shear deformation and is suitable for thick as well as slender beams. Discretization was selected to account for shell thickness gradual change and the location of stiffeners. Regarding the boundary conditions at the bottom, the most common assumption in similar studies is the fully fixed state. In this study tanks are simply supported (unanchored) so linear elastic, compression-only, translational springs were used to model the foundation of the tanks at the vertical (meridional) direction, in order to allow the wind-induced uplift of the tank. The constants for the support springs were determined from the soil factor. A direct comparison is made between the two assumptions for the boundary conditions (fully fixed and compression-only vertical springs) for all analyses performed. In all computational models, the material was modeled as elastic and isotropic for all structural members, with the modulus of elasticity equal to E=2.1x10^5 N/mm^2 and Poisson's ratio equal to v=0.3. The FE models of both tanks are presented in **Fig. 2**.

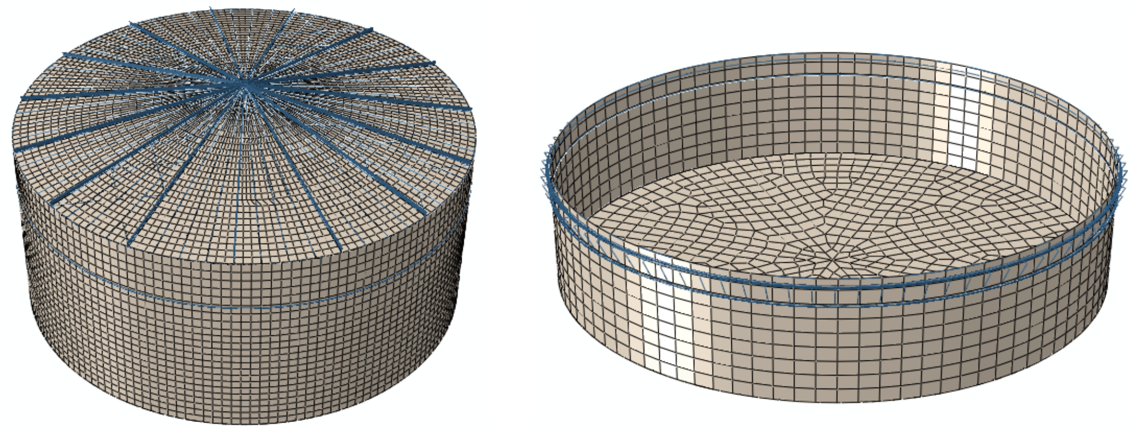


Fig. 2 Simulation of T-776 (left) and T-761 (right) with the FEM

The wind load is simulated as pressure distribution acting on the circumferential shell. According to current code provisions for cylindrical shell structures, this pressure varies along both height and circumference of the shell. The height variation is not significant for tanks [20], hence the pressure is assumed to be constant along the height, as opposed to silos. It has been experimentally observed how cosine families can represent circumferential pressures on shells, so most of the formulations established to define circumferential patterns of pressure employ Fourier cosine series. Wind pressure can generally be defined as:

$$q_w = c_p \bar{q} \tag{1}$$

Where c_p is the wind pressure coefficient and $\bar{q}$ is the pressure value at a specific height, on the incidence of the wind (windward). Wind pressure coefficient is specified using Fourier series decomposition:

$$c_p(\theta) = \sum_{i=0}^{m} a_i \cos(i\theta) \tag{2}$$

Where θ is the angle measured from the windward direction ($\theta=0°$ for windward and $\theta=180°$ for leeward) and a_i is the Fourier coefficients. Several proposals have been made for these coefficients [13]. EN1991-1-4 [22] includes formulations for calculating specific values of the distribution based on various parameters, but does not provide the Fourier coefficients used for obtaining these formulations. This study followed the proposal of Greiner [23] as it seems to more accurately approximate the shape of the EN1991-1-4 pressure distribution, using the expression:

$$c_p(\theta) = -0.55 + 0.25\cos(\theta) + 1\cos(2\theta) + 0.45\cos(3\theta) + 0.15\cos(4\theta) \tag{3}$$

EN1993-4-2 [24] allows wind simulation through an equivalent uniform pressure throughout the circumference of the tank, when several requirements are fulfilled. API 650 assumes uniform wind loading on any occasion. This assumption is investigated numerically and the results for both pressure distribution proposals are compared. **Fig. 3** demonstrates a representative schematic distribution of wind pressure based on the above formulations, as well as the uniform wind pressure.

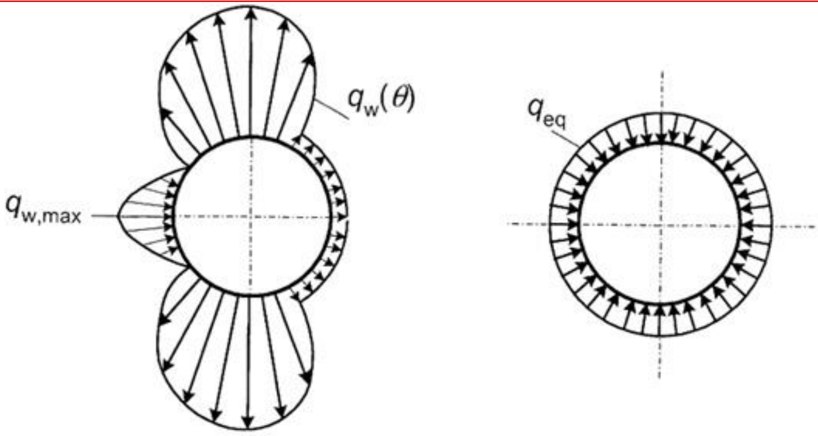
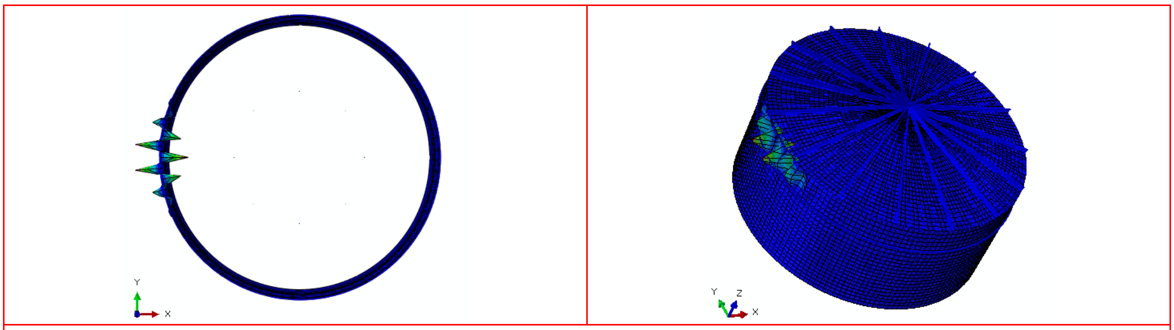


Fig. 3 Distribution of wind pressure on cylindrical tanks (left) and equivalent uniform pressure (right)

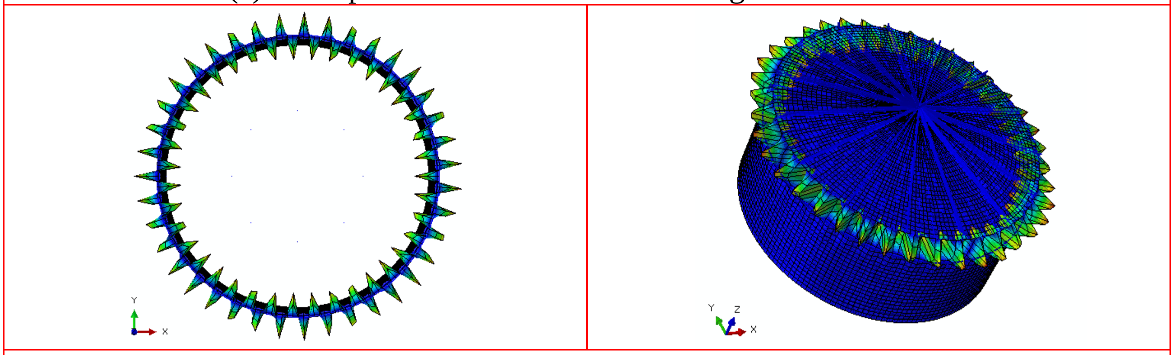
5. Linear Bifurcation Analysis (LBA)

EN1993-1-6 recommends linear elastic bifurcation (eigenvalue) analysis as a method which evaluates the linear bifurcation eigenvalue for a thin-walled shell structure on the basis of the small deflection linear elastic shell bending theory, related to the perfect geometry of the middle surface of the shell. It is a linear perturbation procedure that can be the first step in a buckling analysis, providing a preliminary evaluation of buckling behavior. It obtains the lowest eigenvalue at which the shell may buckle into a different deformation mode, assuming however no change of geometry, no change in the direction of action of the loads and no material degradation. Imperfections of all kind are ignored, but buckling mode results can be introduced as an initial geometric imperfection in the non-linear analysis.

The bifurcation buckling analysis for both tanks under wind load is presented herein. Wind pressure distribution, as proposed by EN1993-1-6, is applied and uniform pressure adopted by API 650 is investigated for comparison. Results concerning discrepancies between fully fixed boundary conditions and compression-only translational elastic springs are also provided. The buckling capacity is calculated through buckling load factors (λ) that multiply the reference wind pressure such that λ^c is a critical load of the tank, for a wind profile that is assumed constant during the load process. Reference pressures for both EN1993-1-6 and uniform distributions were set up to 1 kPa so that λ^c would directly represent the critical load. Results for critical buckling wind loads are summarized in **Table 3**. Buckling deformation modes corresponding to the critical load factors are presented in **Fig. 4** and **Fig. 5**, for tank T-776 and tank T-761 respectively. It is observed that buckling occurs at the windward region when EN1993-1-6 wind pressure distribution is applied, with a slightly greater critical load than uniform pressure for T-761. As Zhao and Lin [4] recommended, it might be acceptable for the structural designer to neglect the negative wind pressures, as the windward positive pressure govern the buckling capacity. At final, buckling initiated at the thinner shell courses for both tanks. The buckling mode of tank T-761 (**Fig. 5**) is located below both wind girders, as they are set at a small distance and offer great stiffness to the thin upper shell courses. It is also noted that fully fixed boundary conditions (only concerns EN19993-1-6 wind pressure) might overestimate buckling capacity for T-761.

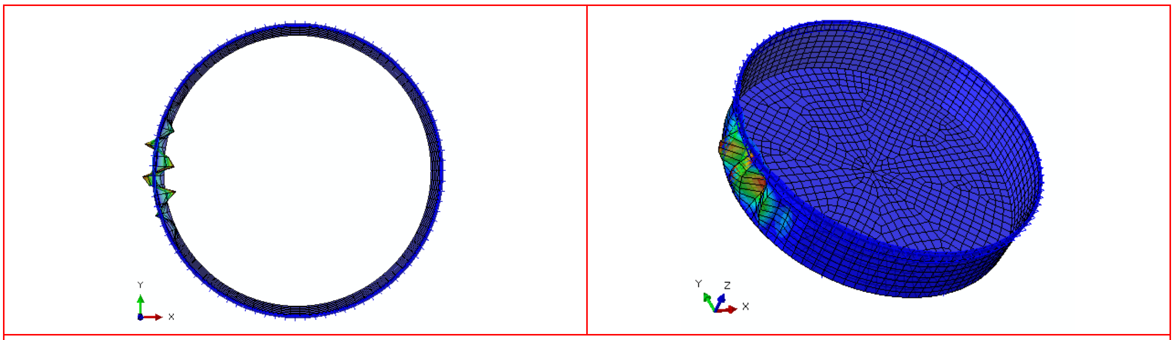


(a) Wind pressure distribution according to EN1993-1-6

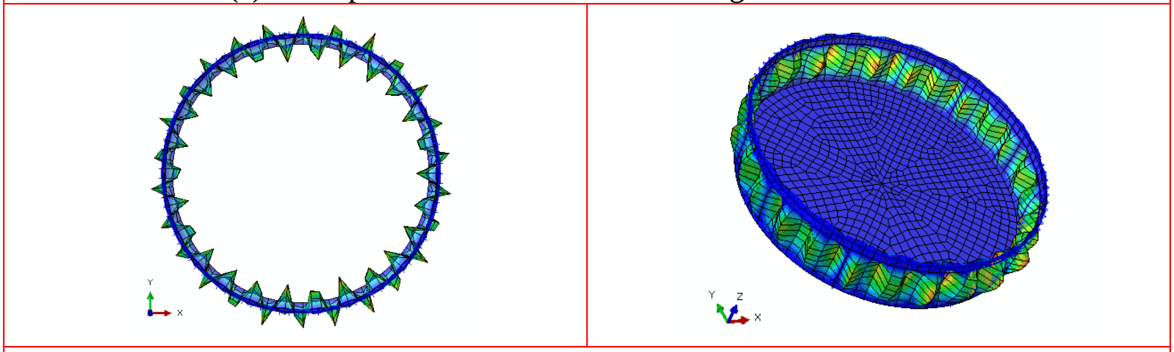


(b) Uniform wind pressure distribution

Fig. 4 Buckling deformation modes for tank T-776



(a) Wind pressure distribution according to EN1993-1-6



(b) Uniform wind pressure distribution

Fig. 5 Buckling deformation modes for tank T-761

Table 3 Critical buckling load factors* obtained from linear bifurcation (eigenvalue) analysis

Wind pressure type	T-776		T-761	
	Compression-only springs	Fully fixed	Compression-only springs	Fully fixed
EN1993-1-6	λ^c=5.5456	λ^c=5.9928	λ^c=9.4464	λ^c=10.339
uniform	λ^c=5.6518	λ^c=5.6518	λ^c=8.5665	λ^c=8.5665

*All buckling load factors coincide with critical loads measured in kPa.

6. Geometrically Nonlinear Analysis (GNA)

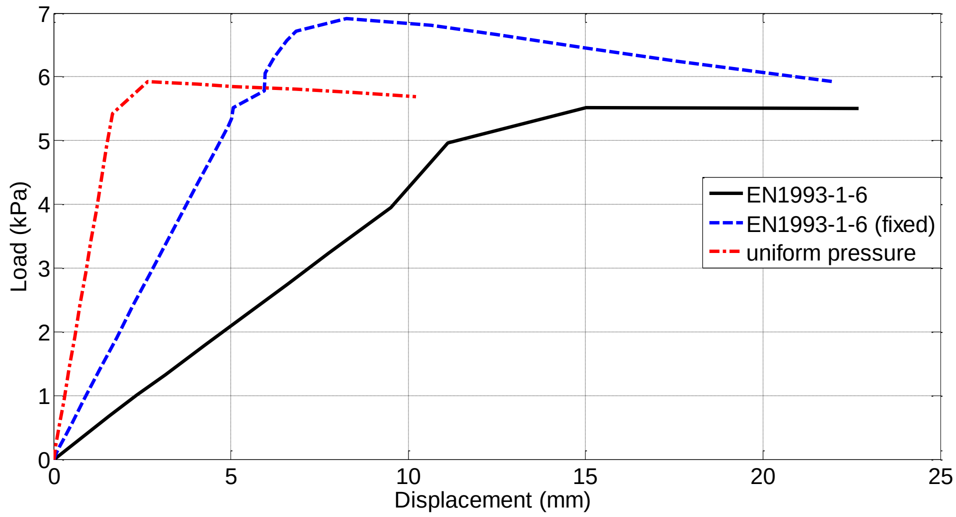
In order to obtain a more accurate buckling behavior, EN1993-1-6 proposes a geometrically nonlinear analysis. This is based on the principles of shell bending theory applied to the perfect structure, including the nonlinear large deflection theory for the displacements which accounts for any change in geometry due to the actions on the shell. The nonlinear analysis satisfies both equilibrium and compatibility of the deflections under conditions in which the change in the geometry of the structure caused by loading is included. The resulting field of stresses matches the definition of primary plus secondary stresses. This study did not account for material nonlinearity, since stresses acting on the shell of such thin walled structures are low while the buckling occurs before the material yielding. ABAQUS software uses the incremental and iterative Newton-Raphson method to obtain solutions for nonlinear problems, by applying the specified loads gradually and incrementally working towards the final solution. Riks algorithm was chosen for the different variations of Newton-Raphson method, which is an arc-length technique that can provide solutions even in cases of complex, unstable responses of the structure investigated [21].

The results of the geometrically nonlinear analysis are presented herein. The load-displacement curves (equilibrium paths) were computed by selecting a node on the windward meridian of the cylindrical shell and in the wind direction, representative of the maximum displacements experienced by the tank. This node was used as the degree of freedom to plot load-displacement curves that are presented in **Fig. 6** for tanks T-776 and T-761. Both results for fixed boundary conditions under EN1993-1-6 wind pressure and for uniform wind pressure adopted by API650 are displayed for comparison. The critical buckling loads obtained from GNA were slightly lower than those from LBA, as depicted in **Table 4**. It is observed that prebuckling equilibrium paths are linear, while buckling occurs suddenly for T-761 (across the maximum load), as opposed to T-776 that because of the roof shell and the wind girder, buckling occasionally develops a slightly unstable response near the critical point. The wind pressure distribution imposed by Eurocode leads to larger displacements. It should be highlighted that LBA should always precede nonlinear analysis to ensure none bifurcation point failed to be detected numerically in the equilibrium path [20]. Post buckling behavior should not concern the designer, as thin-walled structures do not develop any post buckling resistance.

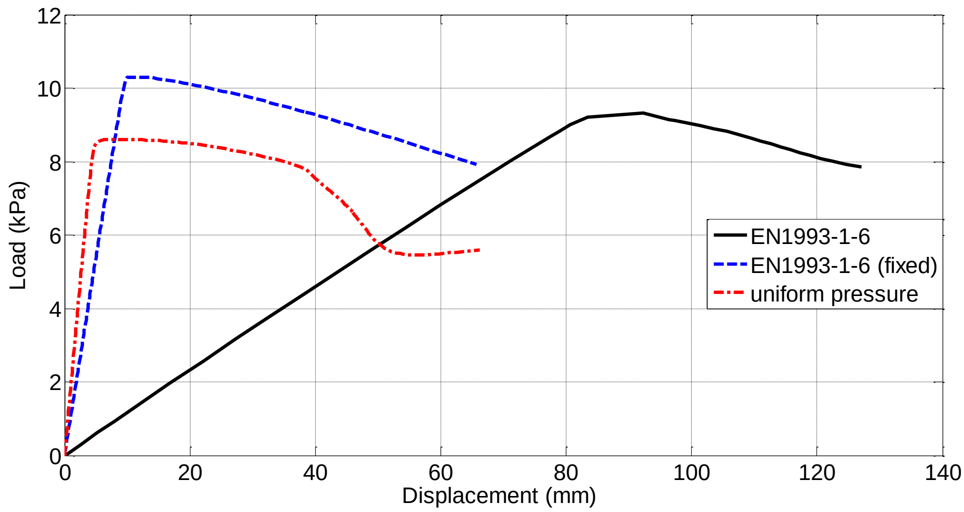
Table 4 Critical buckling load factors* obtained from GNA and comparison with LBA

Analysis type	T-776			T-761		
	EN1993-1-6	EN1993-1-6 (fully fixed)	Uniform pressure	EN1993-1-6	EN1993-1-6 (fully fixed)	Uniform pressure
GNA	λ^c=5.5191	λ^c=5.5827	λ^c=5.4296	λ^c=9.3286	λ^c=10.3049	λ^c=8.5594
GNA/LBA (%)	99.5	93.2	96.1	98.8	99.7	99.9

*All buckling load factors coincide with critical loads measured in kPa.



(a) T-776



(b) T-761

Fig. 6 GNA load-displacement curves

7. Geometrically Nonlinear Analysis with Imperfections (GNIA)

Previously considered analyses describe the ideal buckling behavior of tanks T-776 and T-761, starting with a "perfect" FE model. However tanks and structures in practice, contain features that cause changes in the geometry, such as minor deviations in the shape, eccentricities, local indentations etc., which can be error-induced or damage-induced.

These features known as imperfections can alter the buckling behavior radically and should always be considered in the design process.

In order to estimate the magnitude of geometrical imperfections, EN1993-1-6 [20] defines three fabrication tolerance quality classes (Class A: excellent, Class B: high and Class C: normal fabrication). Class selection is based on representative sample measurements conducted on the unloaded and completed structure. A clear distinction, based on the imperfection type being considered, is made from fabrication quality tolerance measurements. More specifically they are categorized as: a) out-of-roundness measurements, which are associated with the internal diameter of the shell, b) non-intended eccentricity measurements at the joints of the connected plates, c) dimple measurements, in the meridional direction and along the circumference of the shell, including measurements across the welds and d) flatness measurements at the interface of the shell and its bottom. The fabrication quality class is assessed separately for each measurement type, according to the tolerances specified in EN1993-1-6 [20]. The lowest quality class is then assigned to the shell structure.

It is clear that when the imperfection cannot be measured or otherwise be made known (e.g. the structure has not been build and the contractor does not have accurate tolerance data), the imperfect shape with the most unfavorable effect should be assumed and applied to the perfect model geometry. The eigenmode-affine pattern should be used unless a different unfavorable pattern can be justified [20]. The amplitude of the adopted equivalent geometric imperfection form should be taken as dependent on the fabrication tolerance quality class. The maximum deviation of the geometry of the equivalent imperfection from the perfect shape should be given by the above expression:

$$\Delta w_{0,eq} = \max\left\{\Delta w_{0,eq,1} ; \Delta w_{0,eq,2}\right\} \tag{4}$$

Where:

$$\Delta w_{0,eq,1} = l_g U_{n1} \tag{5}$$

$$\Delta w_{0,eq,2} = n_i \ t \ U_{n2} \tag{6}$$

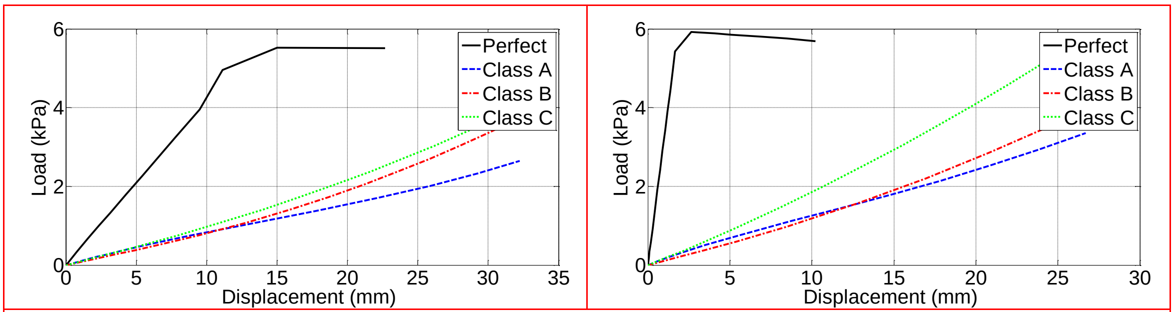
Where l_g is the corresponding gauge length, t is the local shell wall thickness, n_i is a multiplier to achieve an appropriate tolerance level (recommended value: 25) and U_{n1}, U_{n2} are the dimple imperfection amplitude parameters for the corresponding fabrication tolerance quality class. Recommended values for U_{n1} and U_{n2} are given in **Table 5**. The gauge length parameter included in Eq. (5) should be taken for both the meridional and the circumferential directions as the maximum of the following:

$$l_{g_x} = 4\sqrt{rt} \tag{7}$$

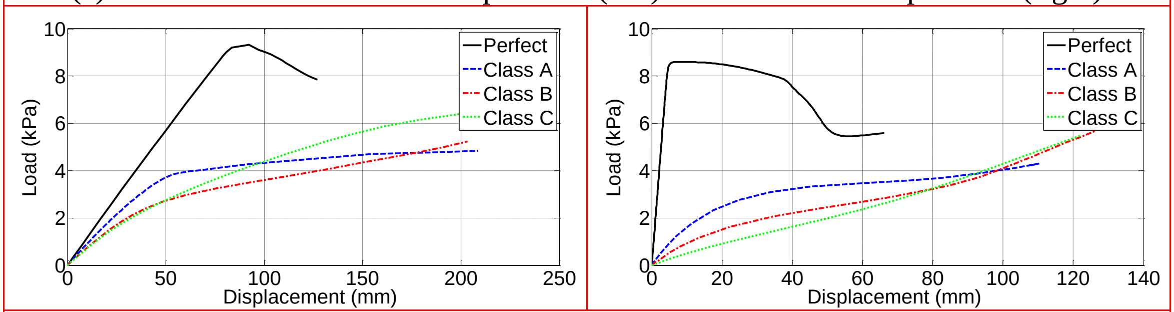
$$l_{g_\theta} = 2.3\left(l^2 rt\right)^{0.5} \quad , \quad \text{but } l_{g\theta} \le r \tag{8}$$

$$l_{g_w} = 25t \quad , \quad \text{but } l_{g_w} \le 500mm \tag{9}$$

Where r is the radius of the middle surface normal to the axis of revolution, t is the shell thickness (in case of variable wall thickness the minimum is selected) and l is the meridional length of the shell segment.



(a) T-776 for EN1993-1-6 wind pressure (left) and uniform wind pressure (right)



(b) T-761 for EN1993-1-6 wind pressure (left) and uniform wind pressure (right)

Fig. 7 GNIA load-displacement curves

Imperfection measurements have not been carried out for tanks T-776 and T-761 up to day, thus relevant results for all fabrication quality classes are presented. **Fig. 7** displays load-displacement curves for both tanks and for both wind pressures. Eigenmode results obtained from LBA were used as the imperfection shapes. Imperfection amplitudes for each quality class were calculated by implementing geometrical characteristics of the tanks (**Tables 1** and **2**) in the formulas presented in this section. The results of the amplitudes $\Delta w_{0,eq}$ are summarized in **Table 5**. It is safe to deduce that imperfections cause an impressive decrease in the critical buckling load and essentially change the shape of the equilibrium path, providing a highly nonlinear behavior. It is observed that as the imperfection amplitude increases, the curves become smoother and it is impossible to derive a critical load value, as there is no more a maximum point. Particularly for tank T-776 (**Fig. 7a**), the equilibrium paths for all quality classes indicate that the tank is actually stiffened. This is a result of the large values of imperfection amplitudes $\Delta w_{0,eq}$ calculated by the EN1993-1-6 formulas, for the large diameter tanks of this study ($\Delta w_{0,eq}$ normalized by thickness reaches 6.8*t*). Critical load estimations for equilibrium paths with such rapid increase of displacements are made through the largest tolerable deformation according to EN1993-1-6 [20], which occurs when the maximum local rotation of the shell surface attains the value of 0.1 radians. Another criterion is that the largest tolerable deformation can be taken as d/t_{min}=3 for practical tanks [4]. It is highlighted that the decrease in the buckling capacity due to imperfections can reach up to -60%, as demonstrated in **Fig. 7**.

Table 5 Recommended values for parameters U_{n1} and U_{n2} and imperfection amplitudes $\Delta w_{0,eq}$

Fabrication tolerance quality class	Description	Recommended value of U_{n1}	Recommended value of U_{n2}	$\Delta w_{0,eq}$ for T-776 (mm)	$\Delta w_{0,eq}$ for T-761 (mm)
Class A	Excellent	0.010	0.010	21.09	25.92
Class B	High	0.016	0.016	33.74	41.47
Class C	Normal	0.025	0.025	52.73	64.80

8. Comparison between the current design Standards

An attempt to compare and investigate discrepancies between the shell buckling evaluation methods offered by current code provisions is made. The results obtained numerically by linear (LBA) and nonlinear (GNA and GNIA) analyses are contrasted to the results emerged from the analytical formulations included in standards, and presented in a previous study [18]. Comparison is performed through von Mises equivalent stress, and constitutes a representation of the resulting two-dimensional field of primary stresses, thus accounts for in-plane stress interaction on the cylindrical shell of the tank [20]. The well-known mathematical expression for von Mises criterion is as follows:

$$\sigma_{eq} = \sqrt{\sigma_x^2 - \sigma_x \sigma_\theta + \sigma_\theta^2 + 3\tau_{x\theta}^2} \tag{10}$$

Where σ_x, σ_θ and $\tau_{x\theta}$ represent the meridional, circumferential and in-plane shear stresses respectively. A main feature of this rule's behavior of metal components in a multi-axial stress state is that the stress components with the same sign (e.g. biaxial compression) support each other while stresses with different signs or additional shear stress decrease the capacity in each direction.

As it was aforementioned, API 650 does not quantify buckling resistance by means of analytical expressions and it is not possible to evaluate the safety level provided through the equivalent von Mises stresses. Methods proposed are well-suited for design purposes due to their simplicity; however there is a lack of a specific procedure for determining the critical stress-state pertaining to buckling of the tank shell. Oppositely, EN1993-1-6 provides the “stress design” method that includes sophisticated mathematical formulas for calculating critical meridional, circumferential and shear stresses on every shell course of the tank and eventually attaining the design stresses by implementing limiting safety factors. The stresses of the perfect models, as well as the imperfect ones for every fabrication quality class, obtained from the study of Maraveas and Miamis [18] were implemented in Eq. (10) for every shell course. Results are displayed for comparison with the analogous equivalent stress obtained from numerical analysis at the shell segment that buckled (**Table 6**).

It is noted that von Mises stresses for the imperfect model of tank T-776 are not defined as the critical loads cannot be identified by the nonlinear force-displacement curves. Large imperfection amplitudes provide a progressively stiffening response to the shell and large displacements (**Fig. 7a**). The largest tolerable deformation criterion provided by EN1993-1-6 is rather arbitrary for T-776, since the stresses corresponding to 0.1 radians of local rotation are extremely high while the shell has already buckled. Consideration of maximum tolerable displacement as $d=3t_{min}$ is more realistic, but still specifies very high stresses. Instructions for the corresponding code for such cases are not satisfactory, hence further work deemed necessary in order to decide the methods to limit the structure’s deformation.

The results in **Table 6** exhibit satisfactory convergence between the stress design analytical method and the nonlinear numerical method for the perfect model under wind pressure distribution proposed by EN1993-1-6. Consequently, the proposed approach for variable thickness tanks in Annex D of the code is considered safe. Nevertheless, the safety factors implemented by the stress design method regarding imperfections found to be very conservative as opposed to the numerical results for the buckling capacity. It should also be highlighted that models under uniform wind pressure may not display significant discrepancies in terms of critical loads (**Fig. 7**) in comparison with actual wind pressure at the circumference of the shell proposed by EN1993-1-6; however they present a different behavior regarding displacements and unfavourable results regarding von Mises stresses.

Table 6 Comparison of equivalent von Mises stresses (MPa)

Tank ID	Method type	Perfect	Class A	Class B	Class C
T-776	Analytical stress design EN1993-1-6	48.78	21.21	18.07	13.84
	GNA & GNIA EN1993-1-6	52.36	-	-	-
	GNA & GNIA uniform pressure	18.44	-	-	-
T-761	Analytical stress design EN1993-1-6	51.95	22.50	18.58	14.04
	GNA & GNIA EN1993-1-6	51.11	49.44	40.92	36.09
	GNA & GNIA uniform pressure	28.80	26.59	24.42	23.07

Regarding API 650, due to the different problem approach a previous study [18] showed that both tanks satisfy its requirements optimally; hence this suggests the use of an earlier version of the code for their design. This study revealed that although the unfavourable uniform wind pressure is adopted while higher design wind velocity is used [19], the entire lack of imperfections in the process is dominating.

9. Concluding remarks

This study focuses on shell buckling evaluation of two empty and large diameter storage tanks under wind load as demonstrated by current design code provisions used in a great number of world-wide applications. The main conclusions of this research may be summarized as follows:

- Linear bifurcation analysis (LBA) is a good indicator for buckling capacity as the critical loads obtained converge with those of geometrically nonlinear analysis. It should always precede a geometrically nonlinear analysis as it offers the bifurcation points and buckling modes that can be used as imperfection shapes.
- Fully fixed boundary conditions may overestimate the buckling capacity of tanks. Unanchored structures' foundation could be simulated with linear compression-only springs in the vertical direction and thus display a more sensible behavior, allowing the tank to partly elevate due to horizontal wind load.
- Uniform wind pressure distribution is evaluated as the more unfavorable compared to the EN1993-1-6 distribution, which is experimentally confirmed. It also provides a different behavior to the shell, allowing smaller displacements and stresses.
- Imperfection amplitudes proposed by EN1993-1-6 decrease considerably the nonlinear buckling resistance of cylindrical tanks but also cause a progressively

stiffening response with rapidly growing displacements for such large diameter structures. The standard's "largest tolerable deformation" criterion for estimating the critical load is arbitrary and does not offer reasonable and satisfactory results.

- Analytical formulations of stress design method by EN1993-1-6 combined with the proposed method for variable thickness tanks offer satisfactory results for perfect tank models. However, the imperfection requirements of the aforementioned methods are very conservative and limit the buckling resistance remarkably.
- Empirical design methods proposed by API 650 are arbitrary, as they do not quantify the buckling critical state and do not account for imperfection sensitivity. Therefore the provided safety level is questionable; the theoretical background of the methods should be investigated, and improvements in future editions should be considered.

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