Recovery rates, default probabilities, and the credit cycle

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Abstract

In recessions, the number of defaulting firms rises. On top of this, the average amount recovered on the bonds of defaulting firms tends to decrease. This paper proposes an econometric model in which this joint time-variation in default rates and recovery rate distributions is driven by an unobserved Markov chain, which we interpret as the "credit cycle". This model is shown to fit better than models in which this joint time-variation is driven by observed macroeconomic variables. We use the model to quantitatively assess the importance of allowing for systematic time-variation in recovery rates, which is often ignored in risk management and pricing models.

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1. Introduction

It has been noted that default probabilities and default rates (i.e. the fraction of defaulting firms in the economy) and average recovery rates are negatively correlated (see e.g. Altman et al. (2005); Acharya et al. (2007)). Both variables also seem to be driven by the same common factor that is persistent over time and clearly related to the business cycle: in recessions or industry downturns, default rates are high and recovery rates are low (Figure 1).

[Figure 1 about here.]

This paper attempts to empirically characterize the time-series behaviour of default probabilities and recovery rates distributions using an econometric model in which both depend on an unobserved two-state Markov chain.[1] The estimated states correspond to bad times, in which default probabilities are high and recovery rates are low, and good times, in which the reverse is true, so that we can interpret this Markov chain as a "credit cycle".

We can use the estimated model to characterize systematic credit risk. For example, we can explore the effect of allowing for time-variation in recovery rate distributions on estimates of credit risk. Whereas time-variation in default probabilities is almost always taken into account when calculating loss distributions or pricing credit-risk sensitive instruments, it is often assumed that recovery rates are either constant, or that recovery rates are independent of default probabilities. Given the negative relationship between default probabilities and recovery rates, this seems likely to be a bad idea. From the point of view of a holder of a diversified portfolio of corporate bonds the fact that recovery rates are low precisely in situations in which many companies default is important because the negative relationship between recoveries and default probabilities amplifies the risk of the portfolio. Moreover, it appears that these amplified losses tend to occur in recessions, i.e. in situations in which the marginal utility of the representative investor is high (or the return on the market portfolio is low); the behaviour of recovery rates therefore amplifies *systematic* risk. It is clear that this will not only have implications for risk measurement, but also for pricing, although we will not discuss pricing in depth in this paper.[2] Here, we ask the following question: By how much

[1] Since the Markov chain is unobserved it creates (unconditional) dependence between defaults across firms, and between defaults and recovery rates, and therefore also plays a similar role to a dynamic frailty variable. Dynamic frailty variables have been used in the context of defaults e.g. by Duffie et al. (2006).

[2] One can conjecture, for instance, that model-implied prices of instruments that depend heavily on the systematic component of credit risk such as senior tranches of CDOs would be greatly affected by whether or not the model allows for systematic variation in recovery rates.

do you underestimate risk if you ignore the negative relationship between recovery rates and default probabilities?

Other issues that can be addressed in the context of our model are in what way time-variations in default probabilities and recovery rate distributions relate to the business cycle, and whether recoveries on bonds of different seniorities vary in the same way over the cycle.

Related questions have been addressed in the literature. The relationship between default probabilities and the business cycle, for example, has already been documented by e.g. Nickell et al. (2000), who estimate different rating transition matrices for periods of high, medium and low GDP growth and find that default probabilities especially seem to be affected, or Bangia et al. (2002), who estimate separate rating transition matrices for NBER recessions and expansions, and find that economic capital of banks should be about 30% higher in recessions. Using firm-level data, Bonfim (2009) and Carling et al. (2007), also report that when looking for default risk, taking into account macroeconomic conditions substantially improves the results. However, none of these papers consider recovery rates.

Altman et al. (2005) regress average recovery rates on default rates and macroeconomic variables, and find that recovery rates and default rates are closely linked, and that macroeconomic variables become insignificant once default rates are included as explanatory variables.[3] They suggest that this might be due to inelastic demand for defaulted securities; their hypothesis is that typical investors in defaulted securities (vulture funds) have limited capacity, and as a result, the price of defaulted securities falls by a large amount when many defaults occur. In related work Altman et al. (2001) also try to calculate by how much the 99% VaR increases when taking into account the negative relation between recovery rates and default rates. On the basis of a static model which is unrelated to their regressions, into which they substitute guessed parameter values, they claim that the 99% VaR for a representative portfolio might increase from a percentage loss of about 3.8% to 4.9% when moving from a model in which recovery rates are constant to one in which there is negative dependence between recovery rates and default probabilities.

Acharya et al. (2007) take an argument by Shleifer and Vishny (1992) as a starting point: Suppose an industry is in distress and firms in this industry default. If the assets of the defaulting firms consist of industry-specific assets, the firms best able to put these assets to good use might also be experiencing problems, and hence might be unable to buy the assets.

[3] Grunert and Weber (2009) obtain similar results using data on loans in Germany.

This could lead to low prices of these assets, and hence low recovery values. In a sense, this argument is similar to that proposed by Altman et al. (2005), except that here, the resale price of *real* assets is depressed whereas in the case of Altman et al. (2005), the resale price of financial assets (the defaulted bonds) is depressed. Acharya et al. (2007) find that especially in industries in which assets are very industry-specific, industry distress is related to low recoveries, providing evidence for the hypothesis of Shleifer and Vishny (1992). They argue that their results should have implications for pricing; which will of course only be the case if industry distress is associated with aggregate fluctuations such that industry credit risk has a systematic (and hence priced) component. Unfortunately, they do not investigate implications of their results for systematic risk.

Chava et al. (2008) propose a combined model of default probability and recovery rates to model expected losses. In their specification, default arrivals are governed by a continuous hazard that depends on observable firm-specific and economy-wide covariates and an unobserved industry-specific and constant frailty variable. Probit-transformed recovery rates are modelled as a linear function of possible covariates and an error. In this approach, the dynamics of recovery rate distributions and their relation to default rates are driven by time-variation in observed covariates, whereas in our approach, this time-variation is driven by the unobserved Markov chain, i.e. we essentially exploit the persistence in the factor that seems to be driving both recovery rate distributions and default rates.

Using our approach, we find that a latent credit cycle describes default rates and recovery rates relatively well. For instance, on the basis of a model that only uses bond seniority and our latent cycle as explanatory variables, we calculate the out-of-sample rolling root mean square error (RMSE) for predicted recovery rates in a similar manner to Chava et al. (2008) (we use only past information to predict, whereas they use contemporaneous information on their observed covariates). We obtain a value of 22.62%, which is similar to the lowest RMSE of 22.88% that they achieve. This seems to indicate that an approach that utilizes the persistence over time in aggregate recovery rates and default rates holds up well against one based on observable covariates, although note that the datasets are not directly comparable.

We examine to which extent the latent "credit cycle" simply picks up macroeconomic fluctuations by estimating versions of the model in which default probabilities and recovery rate distributions also depend on macroeconomic and other economy-wide variables. We find that macroeconomic variables in general are significant determinants of default probabilities but not so for recovery rate distributions. However, the unobserved credit cycle variable is highly significant, whether or not macroeconomic variables are included, and the fit of the model depends mainly on whether or not the unobserved credit cycle is included, and not on whether or not macroeconomic variables are included. This indicates that recovery rates and default rates are more tightly related to each other than to macroeconomic variables. Also, in our model the beginning of the state with high default probabilities and low recoveries that we interpret as a credit downturn precedes the start of a recession, and it can continue until after the end of a recession. The average duration of a credit downturn is in the range of 3.8 years. In contrast, a typical estimate of the average duration of a recession would be in the range of 4.1 to 4.7 *quarters* (Hamilton (1989)). It appears that the cyclicality in credit variables is related to, but somewhat distinct from the macroeconomic cycle.

Given our estimated models, we can show that credit risk is much higher in a dynamic model in which both default probabilities and recovery rates are allowed to vary, than in a static model. For a well-diversified representative portfolio, the 99% VaR is a percentage loss of about 3.4% in the dynamic case, as opposed to 2.4% in the static case. This increase is due mainly to time-variation in default probabilities, however: In a model in which only default probabilities vary over time, the 99% VaR is already about 3.3% (in contrast, in a model in which only recovery rate distributions are allowed to vary over time, the 99% VaR is about 2.3%, i.e. about equal to the VaR in the static case). This suggests that although variation in recovery rate distributions over time does have an impact on systematic risk, this impact is small relative to the importance of the time variation in default probabilities. Also, in relative terms, it is much smaller than the impact suggested by the calculations of Altman et al. (2001).

The rest of this paper is structured as follows: The model is presented in section 2. In section 3 we describe the data set used. Section 4 discusses the various different versions of the models we estimated and the estimation results, and section 5 explores the implications for credit risk management. Finally, section 6 concludes.

2. The model

In terms of the data generating process, the model can be thought of as follows: Time is discrete. In each period, the state of the credit cycle is determined by the evolution of a two-state Markov chain. This implies that the dynamics of the state of the credit cycle are parameterized by two probabilities, i.e the economy can either be in either state 1 or state 0, and if the economy is in state 1 it will remain in this state with a probability p

and move into state 0 in the next period with probability $1-p$. If it is in state 0, it will remain in this state with probability q and move into state 1 with probability $1-q$. Given the state of the credit cycle, the number of defaulting firms is then drawn using a state-dependent default probability.[4] For each defaulting firm, recovery rates are drawn from a state-dependent recovery rate distribution.[5]

We assume that conditional on observable information and the unobserved state of the credit cycle, defaults are independent, recoveries in different default events are independent and the number of defaulting firms and recoveries are also independent. As a consequence, unconditional dependence between default events (i.e. dependence when not conditioning on the unobserved credit cycle) is driven entirely by the unobserved state of the credit cycle. Intuitively, when the state of the credit cycle is not known, observing the default of company A would make it more likely that the credit cycle is in a downturn, especially if we were to observe a low recovery. This would increase the probability that we would attribute to seeing company B default with a low recovery. The events of company A and company B defaulting with low recovery are therefore not independent if we do not know the state of the credit cycle. The credit cycle here plays a role very similar to a dynamic frailty variable; in the context of defaults, dynamic frailty variables have been used previously e.g. by Duffie et al. (2006).

In our case, we will be able to estimate the probability of being in either state of the credit cycle by using either data on default rates, or data on recovery rates, or both, since both default probabilities as well as recovery rate distributions vary with the cycle.

We make the following assumptions with respect to specific functional forms of default probabilities and recovery rate distributions. Firstly, conditional on the state of the credit cycle (and possibly other explanatory variables), the arrival of defaults is described by discrete hazards of the form

$$\lambda_t = (1 + \exp\{\gamma_0 + \gamma_1 c_t + \gamma_2 X_t\})^{-1}, \tag{1}$$

where c_t is the state of the cycle, a binary variable that is 0 or 1, and X_t represent other

[4]Note that our model of default probabilities is very similar to the model of Giampieri et al. (2005). They do not model recovery rates, however.

[5]We do not impose the negative relationship between default rates and average recovery rates that is apparent in the data as part of the model. This negative relationship does, however, emerge once the model is estimated, which will allow us to label the state with high default rates and low recoveries, or low default rates and high recoveries as a credit 'downturn' and 'upturn' respectively.

variables that might or might not be included in various versions of the model - for example, we might let hazards depend on log GDP growth.[6,7] This functional form of the discrete hazard has been used by Shumway (2001).

Secondly, we assume that the recovery rate y_{ti} for a default at time t of firm i is drawn from a beta distribution. This distribution is well suited to modelling recoveries as it has support $[0, 1]$, is relatively flexible and requires only two parameters (which we call α and β). It is in fact often used by rating agencies for this purpose (see e.g. Gupton and Stein (2002)). We let the parameters of this beta distribution depend on the state of the cycle (c_t) and macroeconomic variables (X_t) as in the case of hazard rates, as well as the industry sector of the firm (Z_i) and the seniority of the instrument on which recovery is observed (S). In order to ensure positivity of α, β, we choose an exponential specification:

$$\alpha_{ti} = \exp\{\delta_0 + \delta_1 c_t + \delta_2' Z_i + \delta_3' Z_i c_t + \delta_4' S + \delta_5' S c_t + \delta_6 X_t\} \tag{2}$$

$$\beta_{ti} = \exp\{\zeta_0 + \zeta_1 c_t + \zeta_2' Z_i + \zeta_3' Z_i c_t + \zeta_4' S + \zeta_5' S c_t + \zeta_6 X_t\} \tag{3}$$

We obtain a likelihood function of the model as described in Appendix A by employing a slightly modified version of the method proposed by Hamilton (1989).

3. Data

The recovery data and the information on the defaulting firms is extracted from the Altman-NYU Salomon Center Corporate Bond Default Master Database. This data set consists of more than 2,000 defaulted bonds of US firms from 1974 to 2005. Each entry in the database lists the name of the issuer of the bond (this means that we can determine its industry as described by its SIC code), the date of default, the price of this bond per 100 dollars of face value one month after the default event, and a ‘bond category’.

The bond category is one of either Senior Secured, Senior Unsecured, Senior Subordinated, Subordinated or Discount. Altman and Kishore (1996) and Altman et al. (2005) label this bond category as ‘seniority’, and we will follow this convention here. It should be pointed

[6]Since we do not want to specify laws of motion for macroeconomic variables, we interpret the likelihood estimates as partial likelihood estimates whenever these variables are included in the different versions of the model.

[7]Note that we do not include firm-specific determinants of default probabilities, although it would be possible to do so in this framework. This is due to limitations of our data, which we discuss in the next section.

out, though, that it also appears to contain information on the presence or absence of collateral (Senior Secured versus the other categories), and the type of coupon (Discount versus the other categories), which we will take into account when interpreting results.

Typically, for a given firm and date, the database contains the prices of many bonds for a given seniority class, so we need to aggregate. We do this by taking averages (weighted by issue size).[8]

To abstract from having to model the dependence between recoveries on different seniorities for a single default event, we retain only the most senior recovery rate observation for each default event.[9]

We also aggregate data across time into periods corresponding to calendar years.[10] We assume that a default of the same firm within twelve months of an initial default event (e.g. in December and then March of the following year) represent a single default event.

We calculate the recovery rate as the post-default price divided by the face value. Some of the recovery rates calculated in this way are larger than 1. This probably reflects the value of coupons. We scale the recovery rates by a factor of .9 to ensure that our observations lie in the support of the beta distribution. We could achieve the same effect by defining our distribution to be a beta distribution over the support that equals the range of our actual data.

The data set does not contain information on the non-defaulting issuers that are implicitly tracked. To calculate the likelihood of observing a given number of defaults in a given period (cf. equation [4]), we need at least the total size of the population of firms from which the defaults are drawn (or equivalently, the number of non-defaulting firms). Since the Altman data provides comprehensive coverage of defaults of US corporate bond issuers, we use US corporate bond issuer default rates provided by Moody's to obtain this number.[11] Dividing the number of defaulting firms in each year in the Altman data by Moody's (issuer-weighted) default rate, we can obtain a number for the total population of firms under the assumption

[8]This is something commonly done in the literature. See for example Varma and Cantor (2005).

[9]In previous versions of this paper, we explored several ways of modelling this dependence. Different assumptions on this dependence do not affect the conclusions of the paper, so for the sake of simplicity we do not treat this here. Results are available from the authors on request.

[10]We have to aggregate to calendar years rather than quarters because our default rate data is annual. In a lot of applications, e.g. the Basel II regulatory framework, the risk measures of interest are annual and not quarterly.

[11]Note that Altman's definition of default and Moody's new definition of default coincide. In both cases, default is one of the following events: a failure to pay (except if cured in a grace period), bankruptcy, or a distressed exchange. The default rates that we have are based on Moody's new definition of default.

that both data sets track the same set of firms, or that both data sets exhibit the same default rate.[12] The available default rate data ranges from 1981 to 2005, so we lose observations from periods 1974-1980. However, these years cover only 9 observations in the Altman data.

After all adjustments, we obtain a combined data set which is structured as follows: We have 25 years of data. In each year, we know the size of the population of firms, and know the subset of the firms which default. For each of our 898 default events, we observe one recovery.[13]

In some of our specifications we let default probabilities and recovery rate distributions depend on macroeconomic and economy-wide variables. We use GDP, investment growth, and unemployment; the first two variables are extracted from NIPA tables, provided by the Bureau of Economic Analysis of the United States, while unemployment data comes from the Bureau of Labor Statistics of the U.S. Department of Labor. We also use annual returns of the Standard & Poor's 500 index, annual realized volatility of the Standard & Poor's 500 index, the VIX volatility index (all obtained from Yahoo Finance), the slope of the term structure (the 10-year yield minus 2-year yield, obtained from the Federal Reserve website), corporate bond spreads (Baa rate minus Aaa rate, obtained from the Federal Reserve website), and NBER recessions as published by the NBER.

An important caveat of our analysis is related to the fact that we do not have firm-specific information on the non-defaulting firms, which implies that we cannot estimate default probabilities that depend on firm-specific information. This means that we can only make statements about aggregate default risk. Since we have firm- and security-specific information for defaulting firms, we can make some statements about specific recovery risk. The lack of firm-specific information on non-defaulting firms, does, however, limit the extent to which we can decompose the combined aggregate risk into its constituents.

4. Estimation results

In this section, we compare various estimated versions of the model across several dimensions.

[12] Previous versions of this paper used Standard & Poor's global issuer-weighted default rates, which are calculated on the basis of defaults of both bond and loan issuers. The results on the basis of this data are similar.

[13] Tables with summary statistics are available at http://fmg.lse.ac.uk/∼max/recoveries.html

Macroeconomic variables versus the cycle. We address the question of whether our "credit cycle" is more or less useful than macroeconomic variables in explaining default rates and recovery rates by comparing a completely static version of the model (Model 1) in which λ_t, α_{ti} and β_{ti} do not vary (cf. equations [1],[2],[3]) with versions in which these parameters are either functions of the unobserved credit cycle (Model 2) or macroeconomic variables (Model 3), or both (Model 4). In order to check the fit of the different versions, we report Akaike information criteria and the Bayesian information criteria in addition to the parameter estimates in Table 1.

[Table 1 about here.]

It can be seen that introducing an unobserved credit cycle (Model 2) leads to a great improvement in fit over a completely static model (Model 1) as measured in terms of the information criteria. The coefficients on the cycle are highly significant. We also check the recovery rate densities implied by the models using a method proposed by Diebold et al. (1998). The basic idea is that applying the probability integral transform based on the predicted (filtered) distributions to the actual observations (the recovery rates in our case) should yield an i.i.d.-uniform series (the PIT series) under the null hypothesis that the density forecasts are correct. The correlograms and QQ-plots of PIT series indicate that the model with the unobserved credit cycle does a much better job at matching the time-variation in recovery rate distributions.[14]

Including a macroeconomic variable instead of the cycle, in this case log GDP growth (Model 3), also produces an improvement over the fit of the static model (Model 1), but it can be seen that this improvement is much more modest. Although log GDP growth is a significant determinant of default probabilities, it is not a significant determinant of recovery rates. In terms of the BIC, which strongly favours more parsimonious models, the model with constant and cycle (Model 2) fits even better than the model with constant, cycle and log GDP growth (Model 4).

We also ask whether the cycle is necessary at all, and compare a model in which we have only log GDP growth as an explanatory variable (Model 3) to one in which we have log GDP growth and the cycle as explanatory variables (Model 4). Like Duffie et al. (2006) we can

[14] A more detailed description of how we apply this test as well as the corresponding plots are available on request from the authors or at http://fmg.lse.ac.uk/$\sim$max/recoveries.html

calculate the Bayes factor, which is twice the difference in the log likelihood of the model with the latent variable (Model 4 in this case) and the log likelihood of the model without the latent variable (Model 3 in this case). This statistic is on the same scale as the likelihood ratio test statistic, but has a Bayesian interpretation. Duffie et al. (2006) cite literature that suggests that a Bayes Factor between 2 and 6 provides positive evidence for the model with the latent variable, and a Bayes factor between 6 and 10 suggest strong evidence. Our Bayes Factor is 267.81, which indicates very strong evidence in favor of the model that includes the latent cycle.

We also compare a version of Model 3 against a version of Model 4 with a frequentist test. Note that under the null that the cycle is not necessary, some coefficients, and the transition probabilities are not identified, which means that standard asymptotic results do not apply. We therefore use the test proposed by Hansen (1992) (cf. also Hansen, 1996), where in both models we drop some interactions terms for reasons of computational feasibility.[15] The p-value of the test statistic is 0.0029, which given the conservative nature of this test is very strong evidence against the null that the cycle does not matter.

The picture looks very similar for the other macroeconomic and economy-wide variables that we considered (unemployment, investment, the slope of the nondefaultable term-structure, the S&P 500 return, the realized volatility of the S&P 500 return, the VIX, an NBER indicator, corporate bond spreads, and lagged default rates and lagged recovery rates in the specification of default rates and recovery rate distributions respectively).[16]

We interpret this as saying that the default rate and recovery rates are much more tightly related to each other than to macroeconomic variables; or that there is more information in recovery rates about default rates, and vice versa, than there is information in e.g. log GDP growth about default rates. These results are consistent with those of Altman et al. (2005).

We also calculate the out-of-sample RMSE of our recovery rate predictions in a similar manner to the calculation performed by Chava et al. (2008), using the expected recovery rate of Model 2 (we use only past information to predict, whereas they use contemporaneous information on their observed covariates). We estimate the model on the data up until 1996,

[15]The test requires restricted estimation at each point of a grid whose dimension depends on the number of unidentified parameters. Through dropping interactions of the cycle with other variables in Model 4, we reduce the dimensionality of the grid from 29 to 5. Following Hansen, for each transition probability we consider 10 values, and for each coefficient 20 values. We also drop interactions of log GDP growth with other variables in Model 3 and Model 4 to speed up estimation for each point on the grid.

[16]We only report results for log GDP growth here, other results are available from the authors on request.

predict the state of the credit cycle for 1997 given the information up until 1996, and calculate the expected recovery rate under the model for each recovery rate observation in 1997, and calculate the corresponding prediction error. We then roll the estimation forward, estimate using data up until 1997, and predict recovery rates for 1998, and so on. The RMSE for these out-of-sample predictions is 22.62%, which is similar to the lowest RMSE of 22.88% obtained by Chava et al. (2008). Although these numbers are probably not directly comparable since they are based on different datasets, it is still interesting that the relatively simple Model 2 seems to do as well on the basis of past information as the more complicated models proposed by Chava et al. (2008) based on contemporaneous information on many interesting observed firm-specific, security-specific and economy-wide covariates.

To qualify how the dynamics of macroeconomic variables differ from the dynamics of credit variables, we plot the smoothed (i.e. based on the full dataset) probabilities of being in the state of the Markov chain which we label as a credit downturn over time and compare these to NBER recession dates in Figure 2 (this state is associated with a higher default probability and a lower mean recovery).[17]

As can be seen, we do not pick up the recession of 1981, probably because although average recovery rates are very low for 1981, default rates are also very low in our data, and we only have a single recovery observation for this year. It can be seen that the credit cycle as measured by our Markov chain had two major downturns, around the recession of the early 90s and around 2001, but in each case, the credit downturn started before the recession, and in the latter case ended after it.

[Figure 2 about here.]

To further investigate the difference in timings of credit and business cycle downturns, we check whether one of the credit variables leads the business cycle by running bivariate Granger causality test of default rates and log GDP growth, as well as average recovery rates and log GDP growth. While there appears to be no Granger causality in either direction between default rates and log GDP growth for our data, we find that average recovery rates at lags of 1 year, or in terms of quarterly data at lags of 2 and 4 quarters strongly Granger-cause log GDP growth.

[17]The procedure for calculating the smoothed probabilities is described in Appendix A.

In the following, we drop macroeconomic explanatory variables from our specifications and focus on models that contain only the credit cycle as a time-varying variable. This produces relatively parsimonious models that still manage to capture the dynamics of our credit-related variables.[18]

Default Rates. In the static model (Model 1), the model-implied default probability is constant at 2.10%. In one version of a model including a cycle (Model 2), the default probability in downturns is 3.36% and 1.20% in upturns (these are similar in all models that include a cycle).

To check the goodness of fit of the cycle-implied default rates, we regress observed default rates on a constant and estimated default probabilities of Model 2. These estimated default probabilities are obtained by mixing, for each period, the default probabilities of both states where the weightings are the (smoothed) probabilities of being in one state or the other. Given the coefficients of this regression we can test the default rate component of both the static and dynamic model. The null hypothesis for the dynamic model (that Model 2 correctly captures time-variation in default rates) is that the constant and the coefficient of the estimated default probability are equal to zero and one respectively. The p-value associated with this restriction is 0.9540, indicating that this null is not rejected.[19] We also explicitly consider restricting the coefficient of the estimated default probability to zero, and letting the constant be unrestricted. This restriction would correspond to the static model (Model 1). The p-value associated with this restriction is less than 10^{-5}, indicating that the static model (Model 1) does not describe the data well.

Seniority (bond category). We can examine the behaviour of recovery rate distributions for different seniorities over the cycle on the basis of Model 2, where here we use the rather broad definition of seniority used by Altman and Kishore (1996) and Altman et al. (2005), i.e. their 'bond category' which also includes information on the presence of collateral and coupons (see Section 3 for details).

We can distinguish two types of default events in our data, one for which we observe

[18]Note that this also has the effect of simplifying forecasting. In order to forecast from models that use macroeconomic variables as covariates we would need to posit an appropriate law of motion for these variables, an issue which we avoid here.

[19]The p-values are those of standard t-tests. In order to verify that these are appropriate, we bootstrapped the t-statistics of the regression to confirm that they do indeed follow a t-distribution.

only a single recovery associated with a given default event, and one for which we observe multiple recoveries of different seniority classes associated with the default event. As we have stated before, we only use the most senior recovery for estimating the model when we observe more than one recovery for a given default event. However, we create a dummy which is one in case we observe multiple recoveries for a default event, and interact this dummy with the seniority dummies. The idea here is that seniority can mean different things depending on the debt structure of the firm. For example, the recovery on a Senior Unsecured bond might be higher in situations where we also observe recovery on a junior bond, since in this case the first losses are taken by the holders of the junior bond; the junior bond performs the role of a "junior debt cushion".

[Table 2 about here.]

We calculate the average recoveries implied by our estimates for different seniority classes for all types of events in Table 2. We can see for example that in default events where recovery is observed only on a single bond, senior unsecured bonds have a higher recovery in upturns on average (47.3%), they have very similar (low) recoveries to bonds of all other seniorities in downturns (31.6%). Similarly to e.g. Acharya et al. (2007), we also find that this effect is more pronounced for senior securities. Acharya et al. (2007) also provide evidence that in their data, the effect is not present if the debt is secured. Comparing our estimated recoveries on Senior Secured bonds and Senior Unsecured bonds in both states of the cycle, we find that in our data, senior debt is more strongly affected by a credit downturn if it is secured. The difference in results might be due to differences in the definition of secured debt, or due to differences between industry distress as defined by Acharya et al. (2007) and the credit cycle as defined here.

Lastly, one would expect that a "junior debt cushion" in a given default event matters especially for Senior Unsecured bonds, since in this case, all bonds share the same collateral and any losses are first borne by the junior bond(s). Conversely, for a Senior Secured bond the existence of a "junior debt cushion" should matter less, since in this case, the Senior Secured bond has bond-specific collateral. Surprisingly, the existence of a "junior debt cushion" seems to matters (in upturns) for secured debt, but not for unsecured debt; we can see that Senior Secured bonds recover more in upturns in situations in which a junior bond exists (74.1% versus 55.9% in upturns), whereas the recoveries on Senior Unsecured bonds are not really affected by whether or not a junior bond exists. To check that this is a feature of the data and not an artifact of any modeling assumption, we also regress recoveries of Senior Secured bonds on a constant and a dummy that is one if a cushion exists (a recovery is observed on another bond). For our data the estimated coefficients are 39.1% and 11.6% on the constant and on the dummy respectively, implying that recoveries on Senior Secured bonds are 39.1% on average if no cushion exists, and 50.7% on average when some type of cushion does exist. The coefficient on the dummy is also significant at 1%. This does appear to be a feature of the data.

Industry. Lastly, we also consider the industry of the issuer as an additional explanatory variable for α_{ti} and β_{ti}. Since for some industries, we have a very low number of recovery and default observations in upturns, we aggregate industries into three groups: Group A: Financials, Leisure, Transportation, Utilities; Group B: Consumer, Energy, Manufacturing, Others; and Group C: Construction, Mining, Services, Telecoms. For our data, these groupings roughly correspond to industries with high, medium and low mean recovery rates respectively.[20] All industry groups show recovery rate distributions that vary over the cycle. In terms of the AIC, the model with industry groupings is slightly better than the model that only relies on the credit cycle (Model 2) or the model that relies on the credit cycle and macroeconomic explanatory variables (Model 4). In terms of the BIC, which strongly favours more parsimonious models, the model that only relies on the credit cycle (Model 2) still is the best model.

5. Implications for risk management

Altman et al. (2005) cite previous work (Altman et al., 2001) that states that allowing for dependence between default rates and recovery rates produces a 99% VaR of a percentage loss of about 4.9% for a representative portfolio, whereas the VaR which is calculated on the basis of a model that assumes no dependence between default rates and recovery rates is around 3.8%. In relative terms, this is an increase of around 29% in the VaR. These number are based on a very simple static model into which they substitute guessed parameter values. We are in a position to compare the VaR calculated when assuming recovery rate distributions are either static or time varying, but on the basis of an estimated model.

[20]The groupings were chosen to minimize the LR statistic of a test that restricts the coefficients on industry dummies to be the same within these groupings, i.e. to choose the most likely grouping given the data. The p-value of the likelihood ratio test for the given groupings is 10%.

We calculate (by simulation of 50,000 paths) the one-year loss distribution of a hypothetical portfolio of 500 senior unsecured bonds, issued by 500 separate issuers that are representative of the firms in our dataset. We compare the 99% VaR assuming that the bonds in the portfolio are either well described by a static model (Model 1) or by dynamic model which includes the credit cycle as an explanatory variable, where we look at a version of the model in which both default probabilities and recovery rates are functions of the credit cycle (Model 2), and a version in which only default probabilities are a function of the credit cycle and recovery rate distributions are static (Model 2a), and only recovery rate distributions are a function of the credit cycle and default probabilities are static (Model 2b). This allows us to examine the impact on risk calculations of ignoring the time-variation either in recovery rate distributions or in default probabilities.

Note that when estimating the models, the identification of the state can come either through recovery rates, or through the default rate, or through both. If we assume that recovery rate distributions vary, but default probabilities are constant over the cycle, the state is identified only through the time-variation in recovery rate distributions. Intuitively, this is the case because the number of defaults (or the default rate) in this case does not contain information about the state of the cycle (for a mathematical argument, see Appendix B). Conversely, if we assume that default probabilities vary with the cycle, but recoveries do not, then the state is identified only via the variation in the number of defaults (the default rate). When we estimate Models 2, 2a and 2b, the smoothed probabilities of being in either state as well as the transition probabilities vary very little across the models, even though identification is coming through different subsets of the variables. This reflects the close link between recovery rates and default rates evident e.g. in Figure 1.

Once the models are estimated, we can simulate from them. For this we have to choose the probability that we attach to being in a credit downturn today. We examine cases with this probability being equal to one (we know that we are in a downturn today), zero (we know that we are in an upturn today) and 33.5%, which corresponds to the unconditional probability of being in a downturn, given our estimated transition probabilities, i.e. under the assumption that we have no information on the current state of the cycle. The 99% VaRs calculated in this fashion are in Table 3.

A representative loss density, the one implied by the dynamic model (Model 2) based on the unconditional probability of being in a downturn is compared to the loss density implied by the static model (Model 1) in Figure 3. The dynamic model loss density is bimodal, reflecting the possibility of ending up either in an upturn with low default probabilities and high recoveries, or ending up in a downturn, with high default probabilities and low recoveries. As can be seen, the tail of the loss distribution implied by the dynamic model is much larger.

[Figure 3 about here.]

Table 3 shows that the 99% VaR ranges from a loss of 3.2% to a loss of 3.7% of the portfolio assuming the world is dynamic as described in Model 2, versus a 2.4% loss on the portfolio if we had assumed that the world is static as in Model 1. Even supposing that we are in an upturn today, in which case losses over one year are likely to be smaller, the dynamic model (Model 2) still produces a 99% VaR of 3.2%, which is larger than the VaR based on the static model (Model 1). This is because in the dynamic model, even though we are in an upturn today, we might go into a credit downturn tomorrow, with the associated higher default rates and lower recoveries.

[Table 3 about here.]

How much of this extra risk in the dynamic model is due to the variation in recovery rates, though? Assuming that recovery rates are drawn independently from distributions that do not vary over time, or that recovery rates are static parameters is very common in pricing and risk management applications. We therefore also calculate the loss distribution on the basis of Model 2a, which assumes that recovery rate distributions are constant, but that default probabilities vary with the credit cycle. The 99% VaR produced by this Model 2a ranges from 3.0% to 3.4%, which is only slightly lower than the VaR produced by the fully dynamic model (Model 2). Suppose we focus on the case in which we assume that the probability of being in an upturn initially is equal to the unconditional probability of being in an upturn. Going from a model in which both default probabilities and recovery rate distributions are static to one in which default probabilities and recovery rate distributions vary with the cycle increases the VaR by a factor of 1.42 or by 42%. We can decompose this increase by comparing VaRs of Model 1, 2 and 2a. We can see that of these 42%, 37% are due to letting the default probabilities vary, and only about 5% are due to the additional amplification effect of letting recovery rates vary over the cycle as well. Although not negligible, it is smaller than the 29% suggested by the calculations of Altman et al. (2001).[21]

[21] If we focus on the case in which we assume that that we are in a downturn, the amplification effect is

Conversely, we can estimate a model in which default probabilities are constant, but recovery rate distributions vary over the cycle (Model 2b). The VaR that this model produces is essentially the same VaR of the static model, indicating again that the variation in default probabilities over the cycle has a larger impact on losses than the variation in recovery rate distributions.

For different seniorities, or looking at the model that includes industry group dummies the picture looks very similar, so we do not report the corresponding VaRs here. Essentially, it appears that although the time-variation in recovery rate distributions does amplify losses, quantitatively speaking, having about 3-4 times the number of defaulting firms in downturns is more important than losing an extra 15% or so per default in downturns.

We can also phrase this point in the context of the expected loss on a portfolio (for this, we do not even need our estimated models). Altman et al. (2001) suggest the following numerical example: Suppose that the default probability in upturns is $r_U = 2\%$ and the default probability in downturns is $r_D = 10\%$, and the loss given default (1 minus the recovery rate) is a constant $L_U = 30\%$ in upturns and a constant $L_D = 70\%$ in downturns. Assume the probability of being in an upturn is $1/2$.

The expected loss for a large portfolio, calculated using the assumption that there is no relationship between loss given default and default probability would be

$$\text{expected loss} = E[L] \times E[r] = 6\% \times 50\% = 3\%.$$

Of course, expected loss and default probability are related here through the state of the cycle. Taking this into account, we have

$$\begin{aligned} \text{expected loss} &= E[L \times r] = L_U \times r_U \times \frac{1}{2} + L_D \times r_D \times \frac{1}{2} \\ &= \frac{1}{2} \times 30\% \times 0.02 + \times 70\% \times 0.1 = 3.8\% \end{aligned}$$

The difference is $E[L \times r] - E[L] \times E[r] = 80$bp. We could calculate this on the basis of our estimated models, but note that this difference is just the covariance of L and r, $E[Lr] - E[L]E[r] = \mathrm{Cov}(L, r)$. If the annual default rate is a reasonable estimator of the annual default probability, and the annual average recovery rate is a reasonable estimator of 1 minus the annual loss given default, then the sample covariance between these two quantities in our

larger, but still not as large as the one suggested by Altman et al. (2001). The VaR increases by 54.2% in going from Model 1 to Model 2, of which 12.5% are due to letting recovery rates vary.

data can be interpreted as an estimator of this difference in expected losses. This covariance is 6bp (rather than 80bp, as suggested by the above calculation) in our data. This indicates that the amplification effect of recovery rates on expected loss is relatively small.

6. Conclusions

This paper proposes and estimates an econometric model of the systematic time-variation in recovery rate distributions and default probabilities. On the basis of our estimated model, we can state that the time-variation in recovery rate distributions does amplify risk, but that this effect is much smaller than the contribution of the time variation in default probabilities to systematic risk. We also present evidence that indicates that default rates and recovery rates are more tightly related to each other than to macroeconomic variables, and that credit downturns seem to be only imperfectly aligned with recessions; they start before recessions and last longer. The different phases of the business and the credit cycle seem to be particularly evident in recovery rates - average recovery rates actually Granger-cause log GDP growth in our data set.

The results here also suggest some interesting avenues for future research. For example, a closer examination of the quantitative importance of the effect of recovery rates on prices would appear to be important. Also, a drawback of our dataset is that it contains no firm-specific information on non-defaulting firms, which limits the extent to which we can decompose the time-variation in default probabilities. The methodology proposed here, however, could easily be extended to such datasets. This would allow examining not only the systematic time-variation in credit risk, but also any possible relationship between recovery risk and firm-specific default probabilities over time, which could yield important insights. In addition, we have to limit ourselves to market recoveries of bonds. It would probably be of particular interest to banks to repeat the exercise with data that can say something about losses on loan portfolios, i.e. with data on ultimate recoveries of loans. Finally, the different phases of the credit cycle and the business cycle which are particularly apparent in the lead-lag relationship between recovery rates and GDP growth warrant further theoretical and empirical attention.

Appendix

A. The likelihood function

Let c_t be the unobservable state of the cycle (our Markov chain). Also, let p be the probability of remaining in state $c_t = 1$, and q the probability of remaining in state $c_t = 0$.[22]

Let d_t be the number of defaulted firms observed in period t. Firms are indexed by i. We denote the recovery observed for a default event at t of firm i as y_{ti}. We collect the recoveries on all default events at t into a vector which we call Y_t.

It is easy to see that the number of defaulting firms d_t conditional on the state c_t of the cycle in any period (and the variables that influence hazards, X_t) will be binomially distributed, given that we have assumed that firms default independently conditional on this information. The conditional default probability here is the discrete hazard described above. We can write

$$\Pr(d_t|c_t) = \binom{N_t}{d_t} \lambda_t^{d_t} (1-\lambda_t)^{N_t - d_t}, \tag{4}$$

where N_t is the number of firms in the population at time t, and

$$\lambda_t = \left(1 + \exp\left\{\gamma_0 + \gamma_1 c_t + \gamma_2 X_t\right\}\right)^{-1}. \tag{5}$$

This probability is also to be interpreted as being conditional on X_t and N_t, which is not explicit in the above notation.

For each default at t associated with firm i, we observe a y_{ti}. The density of y_{ti} conditional on the state is

$$g(y_{ti}|c_t) = \frac{1}{B(\alpha_{ti}, \beta_{ti})} (y_{ti})^{\alpha_{ti}-1} (1-y_{ti})^{\beta_{ti}-1}, \tag{6}$$

where

$$\alpha_{ti} = \exp\left\{\delta_0 + \delta_1 c_t + \delta_2' Z_i + \delta_3' Z_i c_t + \delta_4' S + \delta_5' S c_t + \delta_6 X_t\right\} \tag{7}$$

$$\beta_{ti} = \exp\left\{\zeta_0 + \zeta_1 c_t + \zeta_2' Z_i + \zeta_3' Z_i c_t + \zeta_4' S + \zeta_5' S c_t + \zeta_6 X_t\right\} \tag{8}$$

Putting recoveries and the number of defaults together, it is then easy to see that conditional on the state, the density associated with observing a given number of defaulting firms

[22] In our estimated models, $c_t = 0$ will correspond to a state with high default probabilities and low recoveries which we label as a 'credit downturn' and $c_t = 1$ to a state with low default probabilities and high recoveries which we label a 'credit upturn', but we do not impose this in the structure of the model.

d_t and associated recoveries Y_t in time period t is given by

$$f(Y_t, d_t|c_t) = \binom{N_t}{d_t} \lambda_t^{N_t}(1-\lambda_t)^{N_t-d_t} \prod_{i=1}^{d_t} g(y_{it}|c_t), \tag{9}$$

where the understanding is that the last product is 1 if $d_t = 0$.

The density of the data observed in period t, not conditional on the state, but conditioned on past information Ω_{t-1} can be written as

$$\begin{aligned} f(Y_t, d_t|\Omega_{t-1}) = & f(Y_t, d_t|c_t = 1) \Pr(c_t = 1|\Omega_{t-1}) + \\ & + f(Y_t, d_t|c_t = 0) \Pr(c_t = 0|\Omega_{t-1}). \end{aligned} \tag{10}$$

Note that given our assumptions, the past information Ω_{t-1} is not relevant for the conditional density of Y_t, d_t. Since the state is not observed, we need a way of deriving the filtered (and smoothed) probabilities of being in either state. Obviously,

$$\Pr(c_t = 1|\Omega_{t-1}) = p \cdot \Pr(c_{t-1} = 1|\Omega_{t-1}) + (1-q) \cdot \Pr(c_{t-1} = 0|\Omega_{t-1}), \tag{11}$$

where we used the notation defined previously, $p = \Pr(c_t = 1|c_{t-1} = 1, \Omega_{t-1})$, the probability of remaining in state 1, and $(1-q) = \Pr(c_t = 1|c_{t-1} = 0, \Omega_{t-1})$, the probability of moving from state 0 to state 1. We can rearrange [11] to obtain

$$\Pr(c_t = 1|\Omega_{t-1}) = (1-q) + (p+q-1) \Pr(c_{t-1} = 1|\Omega_{t-1}) \tag{12}$$

The probability $\Pr(c_{t-1} = 1|\Omega_{t-1})$ can be obtained via a recursive application of Bayes' rule:

$$\Pr(c_{t-1} = 1|\Omega_{t-1}) = \frac{f(Y_{t-1}, d_{t-1}|c_{t-1} = 1) \Pr(c_{t-1} = 1|\Omega_{t-2})}{f(Y_{t-1}, d_{t-1}|\Omega_{t-2})}. \tag{13}$$

Note that there is information in recoveries Y_t and d_t, about the state, i.e. identification of the state is obtained through both the number of defaults and recoveries.

We now have all elements to maximize our log likelihood function

$$L = \sum_{t=1}^{T} \log f(Y_t, d_t|\Omega_{t-1}), \tag{14}$$

which is a partial likelihood since we omit specifying the law of motion of X_t and N_t.

For a given parameter vector, the likelihood can therefore be calculated recursively, given some suitable initial conditions, e.g. the unconditional probabilities of being in each state implied by the transition matrix.

Once we have our estimated parameters, we can also calculate smoothed probabilities of being in each state at any period in time. Smoothed probabilities are those probabilities obtained from the whole sample of data. The expression for these probabilities is as follows:

$$\begin{aligned}\Pr(c_t = 1|\Omega_T) = \Pr(c_t = 1|\Omega_t) \times \frac{f(Y_{t+1}, d_{t+1}|c_t = 1)}{f(Y_{t+1}, d_{t+1}|\Omega_t)} \times \\ \times \frac{f(Y_{t+2}, d_{t+2}|c_t = 1)}{f(Y_{t+2}, d_{t+2}|\Omega_{t+1})} \times \cdots \times \frac{f(Y_T, d_T|c_t = 1)}{f(Y_T, d_T|\Omega_{T-1})}\end{aligned} \tag{15}$$

B. Identification of the state of the cycle

The state of the cycle is identified only via variables that depend on it, in the following sense:

1. When default probabilities are independent of the cycle and recovery rates are not, the filtered and smoothed probabilities of being in either state only depend directly on the recoveries.
2. When recoveries are independent of the cycle and default probabilities are not, the filtered and smoothed probabilities of being in either state only depend directly on the number of defaults.

To see that this is the case, suppose first that all values of recoveries y_{it} do not depend on the cycle, i.e. $g(y_{it}|c_t) = g(y_{it})$, but that the number of defaults d_t does. We need to show that in the recursive calculation of the probabilities of the state of the cycle, y_{it} does not enter. The prediction step, i.e. calculating $\Pr(c_t = 1|\Omega_{t-1})$ from $\Pr(c_{t-1} = 1|\Omega_{t-1})$ works as before and does not involve any data (see [12]). The filtering step, i.e. the step involving the calculation of $\Pr(c_{t-1} = 1|\Omega_{t-1})$ is now slightly simpler:

As before, we define $Y_t = \{y_{it}\}$. First note that

$$f(Y_{t-1}, d_{t-1}|c_{t-1}) = f(Y_{t-1}|d_{t-1}, c_{t-1})f(d_{t-1}|c_{t-1}) = f(Y_{t-1}|c_{t-1})f(d_{t-1}|c_{t-1})$$

because the value of recoveries and the number of defaults are independent conditional on the cycle as before. Additionally, if the y_{it} now do not depend on the cycle, we know that $f(Y_t|c_t) = f(Y_t)$, and we can write

$$f(Y_{t-1}, d_{t-1}|c_{t-1}) = f(Y_{t-1})f(d_{t-1}|c_{t-1}). \tag{16}$$

and

$$f(Y_{t-1}, d_{t-1}|c_{t-1} = 1) = f(Y_{t-1})f(d_{t-1}|c_{t-1} = 1). \tag{17}$$

Combining the previous equations with [10] indicates that for the given case, we can write $f(Y_{t-1}, d_{t-1}|\Omega_{t-2})$ as

$$\begin{aligned} f(Y_{t-1}, d_{t-1}|\Omega_{t-2}) & \\ =& f(Y_{t-1}) \{ f(d_{t-1}|c_{t-1} = 1) \Pr(c_{t-1} = 1|\Omega_{t-2}) \\ & \quad + f(d_{t-1}|c_{t-1} = 0) \Pr(c_{t-1} = 0|\Omega_{t-2}) \} . \\ =& f(Y_{t-1})f(d_{t-1}|\Omega_{t-2}). \end{aligned} \tag{18}$$

Inserting both [17] and [18] into our original equation for the filtering step, i.e. [13], we obtain

$$
\begin{aligned}
\Pr(c_{t-1} = 1|\Omega_{t-1}) = & \\
&= \frac{f(Y_{t-1}, d_{t-1}|c_{t-1} = 1)\Pr(c_{t-1} = 1|\Omega_{t-2})}{f(Y_{t-1}, d_{t-1}|\Omega_{t-2})} \\
&= \frac{f(Y_{t-1})f(d_{t-1}|c_{t-1} = 1)\Pr(c_{t-1} = 1|\Omega_{t-2})}{f(Y_{t-1})f(d_{t-1}|\Omega_{t-2})} \\
&= \frac{f(d_{t-1}|c_{t-1} = 1)\Pr(c_{t-1} = 1|\Omega_{t-2})}{f(d_{t-1}|\Omega_{t-2})}.
\end{aligned} \tag{19}
$$

Neither the calculation of $\Pr(c_{t-1}|\Omega_{t-1})$, nor the calculation of $\Pr(c_t|\Omega_{t-1})$ involves recovery rates directly.

In the converse case where default rates are not a function of the credit cycle, but recovery rates are, an equivalent argument will show that the filtering algorithm does not use the default rates.

Intuitively, if either recovery rates or default rates are independent of the cycle, they do not contain information about it - and they are therefore not useful in deducing the probability of being in either state of the credit cycle.

[Table 4 about here.]

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List of Figures

Figure 1. Historical default rates and average recovery rates
Historical default rates (fraction of defaulting firms) against time (from Moody's) and average recovery rates from the Altman data (see Section 3).

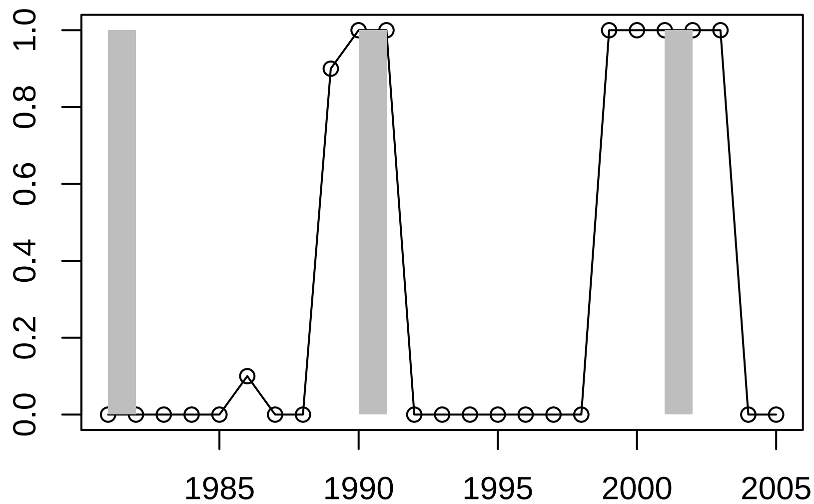

Figure 2. Probabilities of being in the credit downturn versus NBER recessions (annual)

Annual smoothed probability of being in a credit downturn as estimated on the basis of the version of the model that only uses the unobserved credit cycle as an explanatory variable. This is contrasted with NBER recessions (grey areas).

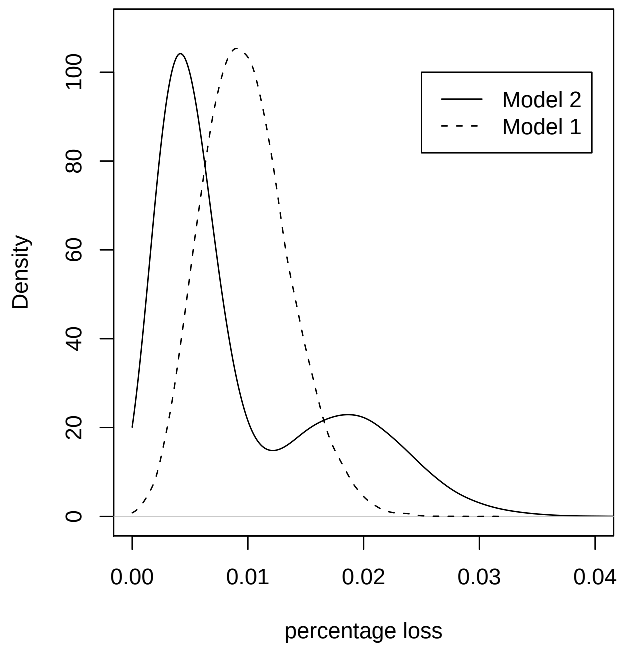

Figure 3. Loss Densities

One-year Loss densities implied by a static model (Model 1) and a dynamic model (Model 2) for a portfolio of 500 senior unsecured bonds issued by 500 different issuers. The initial probability of being in an upturn is assumed to be equal to the unconditional probability.

List of Tables

Table 1
Parameter estimates

Parameter estimates and measures of fit for various models that differ in the combination of variables that influence the hazard rate λ and the two parameters of the beta distribution α and β. Note that with the specification of Shumway (2001), a positive coefficient on a explanatory variable for λ implies that λ falls when the variable rises. sen2, sen3, sen4, sen5 are seniority dummies for Senior Unsecured, Senior Subordinated, Subordinated and Discount respectively, mult is a dummy that is one for observations corresponding to default events for which we observe multiple recoveries, cycle is the unobserved credit cycle, and macro is log GDP growth (versions with other macroeconomic variables are omitted). * denotes individual significance at 5%.

	M1		M2		M3		M4	
DEFAULT RATES	λ		λ		λ		λ	
constant	3.84*		3.36*		3.45*		3.11*	
cycle			1.05*				0.87*	
macro					13.42*		10.37*	
RECOVERY RATES	α	β	α	β	α	β	α	β
constant	0.40*	1.00*	0.47*	1.40*	0.33*	1.13*	0.39*	1.38*
sen2	0.04	0.15	-0.06	-0.06	-0.04	0.06	0.05	0.11
sen3	-0.18	-0.00	-0.26	-0.28	-0.30*	-0.13	-0.27	-0.25
sen4	0.33	0.38*	0.00	-0.09	0.26	0.31	-0.11	-0.48
sen5	-0.32	0.46	-0.25	0.43	-0.48	0.31	-0.31	0.38
mult	-0.22	-0.56*	-0.27	-0.53*	-0.41	-0.72*	-0.39	-0.68*
mult×sen2	-0.21	-0.26	-0.24	-0.39	-0.24	-0.29	-0.46	-0.50
mult×sen3	-0.14	-0.07	-0.26	-0.14	-0.21	-0.08	-0.49	-0.26
cycle			0.48*	-0.46			0.52*	-0.41
cycle×sen2			-0.07	0.24			-0.34	0.16
cycle×sen3			-0.29	0.20			-0.35	0.10
cycle×sen4			0.11	0.71			0.11	1.13*
cycle×sen5			-0.63	-0.46			-0.77	-0.59
cycle×mult			0.69	0.27			0.49	0.23
cycle×mult×sen2			0.03	0.16			0.33	0.36
cycle×mult×sen3			0.47	0.37			0.79	0.60
macro					6.40	-1.39	3.80	1.69
macro×mult					3.52	1.65	3.15	1.44
TRANSITION PROB.								
p			0.8699				0.8060	
q			0.7338				0.6411	
MEASURES OF FIT								
Log Likelihood	27.722		191.588		65.225		199.132	
AIC	-20.43		-309.18		-86.45		-314.26	
BIC	0.0681		-0.1465		0.0213		-0.1255	

Table 2
Implied Mean Recoveries

Implied mean recoveries in percent for Model 2, by state of the credit cycle (upturn or downturn), bond category (ranging from Senior Secured to Discount), and by number of recovery observations per default event. "Single rec." means events for which recovery is observed only on a single bond, "Multiple rec." means recoveries on the senior bond in default events for which several recoveries are observed. Note that here, we follow the convention of Altman and Kishore (1996) in labeling the bond category as 'seniority'.

		Upturn	Downturn
Single rec.	Sen. Sec.	55.9	31.6
	Sen. Unsec.	47.3	31.6
	Sen. Sub.	43.1	32.0
	Sub.	42.2	33.6
	Discount	33.6	18.6
Multiple rec.	Sen. Sec.	74.1	37.7
	Sen. Unsec.	66.6	41.2
	Sen. Sub.	61.2	35.3
	Sub.*	-	-

*We have no observations for which we observe several recoveries, and the recovery on a Subordinated bond is the recovery on the most 'senior' bond.

Table 3
Value-at-Risk

One-year 99% VaRs for hypothetical portfolios of 500 senior unsecured bonds (issued by 500 different issuers) based on a static and two dynamic models. In the dynamic models, default probabilities depend on the cycle, and recovery rate distributions may or may not depend on the cycle. For these models, we calculate loss distributions assuming that the probability of being in a downturn initially are 0 ("upturn"), equal to the unconditional probability ("uncond."), or equal to 1 ("downturn").

		MODEL SPECIFICATIONS		
	Model 1	Model 2	Model 2a	Model 2b
Explanatory variables for λ (default probability)				
	constant	constant cycle	constant cycle	constant
Explanatory variables for α,β (recovery rates)				
	constant seniority	constant cycle seniority	constant seniority	constant cycle seniority

Init. state		ANNUAL 99%-VAR		
upturn	2.4%	3.2%	3.0%	2.2%
uncond.	2.4%	3.4%	3.3%	2.3%
downturn	2.4%	3.7%	3.4%	2.6%

Table 4
Parameter estimates (2)

Parameter estimates and measures of fit for various models that differ in the combination of variables that influence the hazard rate λ and the two parameters of the beta distribution α and β. Note that with the specification of Shumway (2001), a positive coefficient on a explanatory variable for λ implies that λ falls when the variable rises. sen2, sen3, sen4, sen5 are seniority dummies for Senior Unsecured, Senior Subordinated, Subordinated and Discount respectively, mult is a dummy that is one for observations corresponding to default events for which we observe multiple recoveries, cycle is the unobserved credit cycle, and lagged def. rate and rec. rates are the previous annual default rate and mean recovery rate respectively. indB and indC are dummies corresponding to industry groups B and C. * denotes individual significance at 5%.

	M2a		M2b		M5		M6	
DEFAULT RATES	λ		λ		λ		λ	
constant	3.36*		3.85*		3.36*		3.41*	
cycle	1.04*				1.05*		1.03*	
lagged def. rate							-1.20	
RECOVERY RATES	α	β	α	β	α	β	α	β
constant	0.40*	1.00*	0.52*	1.48*	0.34*	1.15*	0.38	1.53*
sen2	0.04	0.15	-0.02	0.10	-0.06	-0.09	-0.07	-0.07
sen3	-0.18	0.00	-0.16	-0.02	-0.35*	-0.40*	-0.30	-0.32
sen4	0.34	0.38*	0.54	1.11*	0.06	0.01	-0.04	-0.11
sen5	-0.32	0.47	-0.23	0.32	-0.32	0.19	-0.23	0.45
mult	-0.22	-0.56*	-0.39	-0.54	-0.29	-0.49	-0.26	-0.56
mult×sen2	-0.21	-0.26	0.00	0.03	-0.18	-0.35	-0.25	-0.40
mult×sen3	-0.14	-0.08	-0.72	-0.57	-0.28	-0.17	-0.22	-0.05
lagged rec. rate							0.37	-0.40
cycle			0.18	-0.65*	0.48*	-0.44	0.55	-0.63
cycle×sen2			0.08	0.15	-0.15	0.13	-0.01	0.29
cycle×sen3			-0.25	-0.01	-0.26	0.25	-0.20	0.26
cycle×sen4			-0.46	-0.56	-0.09	0.41	0.20	0.79
cycle×sen5			-0.5	-0.16	-0.46	-0.14	-0.72	-0.61
cycle×mult			0.67	0.31	0.65	0.20	0.73	0.35
cycle×mult×sen2			-0.49	-0.42	0.06	0.22	-0.03	0.10
cycle×mult×sen3			0.76	0.80	0.52	0.42	0.34	0.20
cycle×lagged rec. rate							-0.39	0.44
indB					0.29*	0.43*		
indC					0.09	0.40*		
TRANSITION PROB.								
p	0.8487		0.9523		0.8742		0.8232	
q	0.7872		0.7634		0.7432		0.6443	
MEASURES OF FIT								
Log Likelihood	131.726		91.214		202.032		193.003	
AIC	-221.45		-110.43		-322.06		-302.01	
BIC	-0.1344		0.0695		-0.1395		-0.1118	