# IMPERFECTION SENSITIVITY OF NON-TRIANGULATED CYLINDRICAL SHELL CONFIGURATIONS

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**Abstract**: *Non-triangulated lattice shells are rarely adopted for large structures due to their lower resistance and stability compared to triangulated shells. As a result, only a few studies are available for the stability evaluation of non-triangulated shell configurations. However, these shell structures can be used for roofs with lower dimensions and lower resistance requirements. As studies related to the imperfection of non-triangulated shell structures are rare, an attempt has been made to study the effect of initial geometric imperfection on the overall resistance of cylindrical shells created using parameterisation principles. The influence of the different magnitude of initial geometric imperfection on the overall resistance of non-triangulated reticulated shells was studied under the uniform gravity load. The results illustrate the significant influence of initial geometric imperfection on the load capacity of non-triangulated shell configurations compared to the triangulated shell configurations. The effect of the initial geometric imperfection on the load capacity increases with the edge valency of faces and decreases with the edge valency of vertices. This study provides information to designers for designing non-triangulated shell structures by considering initial geometric imperfections encountered in the system.*

**Keywords**: Reticulated shell; Non-triangulation; Geometric imperfection; Degrees of freedom; Valency; Parameterisation.

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## 1 INTRODUCTION

Lattice cylindrical shells are the structural system used to cover large spaces. They are generated when the generatrix or directrix is a straight-line (Figure 1). The internal forces developed in the members due to external loads are primarily axial forces due to the geometry of these structures [1]. In addition to that, the external loads acting on the structures are wind load and snow load. As a result, cylindrical shells are generally lightweight and economical. They are typically adopted for railway station platforms, aircraft hangers, and storage yards. Single-layer cylindrical shells are economical when the span of the roof is not very large. However, multi-layer shell structures need to be adopted when the span is large.

As the shell structures are light and the members are subjected to axial force, stability investigations on single-layer reticulated shells are essential. The large span also makes them more vulnerable to instabilities. Lattice cylindrical shells are prone to geometric imperfections during fabrication and erection. Initial geometric imperfection arises due to members' lack of fit, connection deformation, etc. The structure with geometric imperfections will have lower load resistance than the ideal structure, which can cause instability behaviours with lower magnitude of the applied load. Hence, stability investigations of single-layer reticulated shells should consider both geometrical nonlinearities and initial geometrical imperfections. Linear analysis is insufficient, and geometrical nonlinear behaviour should be considered while analysing these structural systems [2].

There are several studies of single-layer reticulated shells with initial geometric imperfections. In general, sensitivity to geometric imperfections is found in the post-buckling behaviour of shell configurations. Sheik identified the most damaging areas due to geometric imperfections by analysing thirteen single-layer cylindrical shells [3]. The loading, span-to-rise ratio, and support conditions significantly impact the imperfection sensitivity of triangulated cylindrical shells [4]. They also found that shell structures designed for unsymmetrical loads are less sensitive to geometric imperfections. Cao et al. studied the imperfection sensitivity of single-layer cylindrical shells. They found that the geometric imperfection coefficient (the ratio of the critical load of the shell structure with imperfection to the critical load of the shell structure without imperfection) should be 0.8 when the initial geometric imperfection is of magnitude S/500 [5]. A plethora of investigations on imperfection sensitivity is conducted on single-layer cylindrical domes. Chen et al. suggested two methods for imperfection analysis: the random imperfection method and the conformable imperfection mode method [6]. The random imperfection method uses random values, and the conformable imperfection mode method uses the first buckling mode as the initial geometric imperfection. The imperfection sensitivity analysis conducted on the 50m Kiewitt-6 dome showed that a slight increase in the initial geometric imperfections could cause a sharp decrease in the buckling load [7]. However, the reduction in buckling load slows down with a further increase in the magnitude of geometric imperfections. The reduction in load capacity of the single-layer dome due to geometric imperfections can be explained with respect to member buckling. Fan et al. divided the stresses among the members as bending stress and axial stress. The member buckling was initiated due to the increase in the ratio of bending stress to axial stress of members. This increase was due to the initial geometric imperfections present in the structure. As a result, the buckling load was decreased [8]. The initial curvature of the members can change the plasticity development and the buckling mode of the single-layer reticulated domes [9]. An improved shape optimisation method, considering bending strain energy, resulted in low imperfection sensitivity and high buckling load [10]. A study on free form single-layer lattice structures showed that greater joint stiffness could increase structural imperfection sensitivity [11]. These investigations show the importance of imperfection sensitivity investigation on lattice structures.

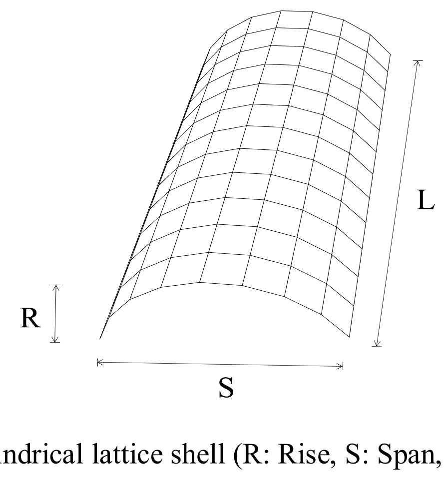



Figure 1: Cylindrical lattice shell (R: Rise, S: Span, and L: Length)

Non-triangulated shells are used when the span is small and the external load is minimal. Imperfection studies of non-triangulated shell configurations are required to optimise their design parameters. Various configurations can be generated with the help of Formex Algebra [12]. The structural behaviour of any configuration can be identified based on its geometric elements. In this paper, the imperfection sensitivity of two non-triangulated shell configurations with different parameters is compared with that of the triangulated shell configuration.

## 2 PARAMETERISED SHELL CONFIGURATIONS

All configurations, including shell structures, are created with basic elements: vertices, edges, faces, and cells. Hence, studying the relationship between these elements is significant for finding an efficient configuration. Different parameters are defined to identify the relation between the elements. The variation in these parameters is helpful for the creation of different non-triangulated shell configurations.

### 2.1 Basic parameters in a configuration

The basic parameters give the relation between the elements within a structure. Parameters are defined as dimensionality, valency, and extent [13]. The first parameter, dimensionality, is comparable to the degrees of freedom of an element. The dimensionality of a vertex is zero as it is fixed to a location. The dimensionality of an edge is one as it is one dimension in nature. Similarly, the dimensionality of the face and cell are two and three, respectively. The second parameter, valency, is the number of elements of a given dimensionality that connect to the elements of another dimensionality. For example, the vertex valency of an edge is two as an edge connects two vertices. Likewise, the vertex valency of an isolated cube is eight as the cube has eight vertices (Figure 2). A configuration can have twelve types of valencies. The edge valency of vertices and edge valency of faces were selected in this study to monitor the effect of parameters on the imperfection sensitivity of shell configurations. The third parameter, extent, is the measurable quantities for different elements. For example, the extent of an edge is length, and the extent of a face is area. Extent is useful when calculating the total length or weight of the members in a configuration.

### 2.2 Nomenclature

A nomenclature was derived from the parameters to differentiate between different shell configurations [14]. They are based on the variation in the edge valency. The standard configurations have vertices with identical edge valency or faces with identical edge valency. Commonly adopted structures have faces with identical edge valency. The edge valency of the vertices for a face is arranged in the clockwise direction to create the nomenclature for the configurations with identical edge valency of faces (Figure 3).

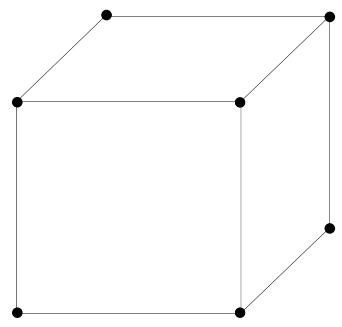


Figure 2: Cube explaining the vertex valency of a cell

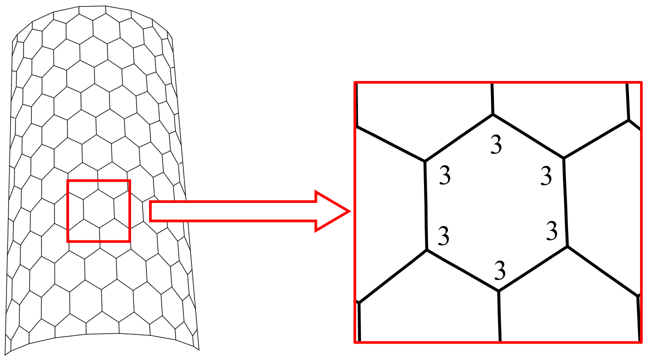
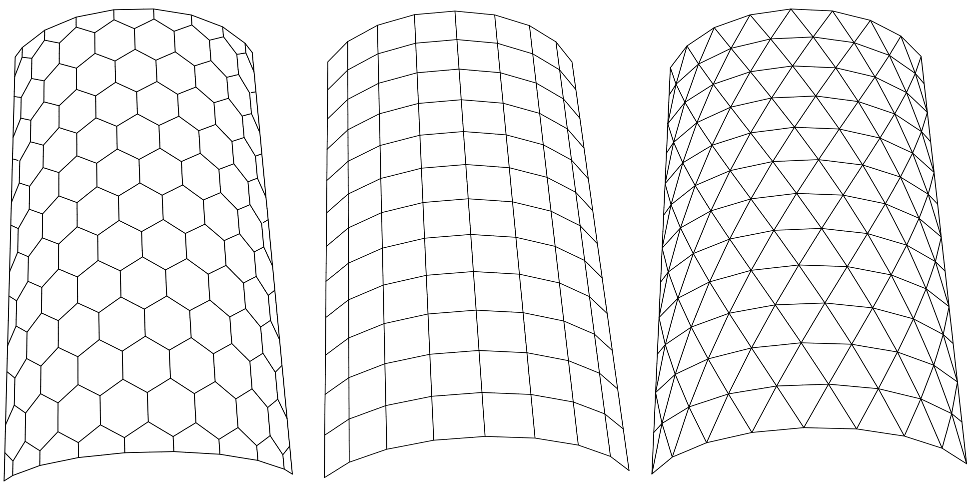



Figure 3: 'V.3.3.3.3.3.3' - Nomenclature for the configuration explained with edge valency of vertices

## 2.3 Non-triangulated shell configurations

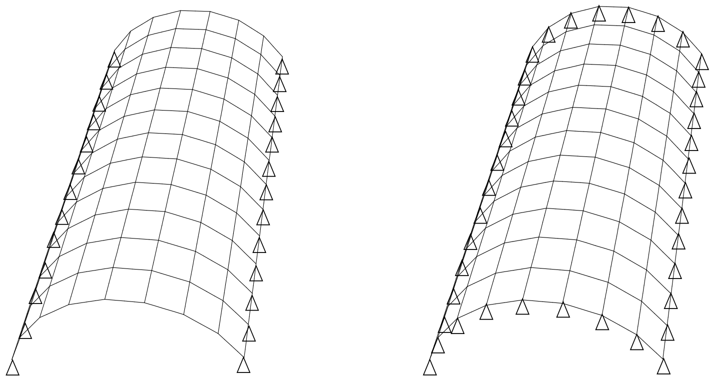
The parameterisation helps in creating different reticulated shell configurations with non-triangulated faces. Two non-triangulated cylindrical shells were selected to study the initial geometric imperfection sensitivity: configuration with edge valency of face is six (Figure 4(a)) and configuration with edge valency of face is four (Figure 4(b)). The studies are incomplete without comparing them with the triangulated cylindrical shell configuration. Triangulated shells (Figure 4(c)) are widely adopted due to their stability and the efficiency in the load-bearing capacity. Hence, the imperfection sensitivity of the selected non-triangulated shell structures was compared with the imperfection sensitivity of the triangulated shell structure.

Support conditions and span-to-rise ratios of the three cylindrical shells were varied to study the effect of edge valency on imperfection sensitivity. The length-to-span ratio (L/S) of two was adopted for all the cylindrical shells as large length is a common feature among the cylindrical shells and vaults. The span (S) of 10m was used for the cylindrical shells. Two support conditions were chosen to examine the effect of support conditions on the imperfection sensitivity of cylindrical shells (Figure 5). The longitudinal edges alone are provided with pin support in the first support condition (SC-1), and all the edges, including curved edges, are supported in the second support condition (SC-1). In addition to the two given support conditions, three span-to-rise ratios (S/R) were adopted for the cylindrical shells: three, five and seven. They are denoted as R3, R5, and R7, respectively.



(a) (b) (c)

Figure 4: The shell configurations considered in the study: (a) V.3.3.3.3.3.3 (b) V.4.4.4.4 (c) V.6.6.6



(a) SC-1 (b) SC-2

Figure 5: The support conditions considered in the study (SC-1: Longitudinal edges pin-supported and SC-2: All the edges pin-supported)

## 2.4 Average edge valency of the cylindrical shell configurations

The edge valency of vertices and the edge valency of faces are different at the edge of the configurations from the interior portion of the configuration. For example, the edge valency of the vertices near the support is three, whereas the edge valency of the vertices at the interior portion is four for configuration V.4.4.4.4 (Figure 4(b)). Hence, 'average edge valency' of vertices is defined to capture the difference in edge valency across the structure (Equation 1). $v$ is the average edge valency of the vertices and $N_0^r$ is the number of vertices for which the edge valency is 'r'. $N_0$ is the number of vertices. Similarly, the edge valency of faces is different in the support region of the shell configuration compared to that in the interior region for many shell structures. Hence, 'average edge valency' of the face is defined (Equation 2). $f$ is the average edge valency of the faces and $N_2^s$ is the number of faces for which the edge valency is 's'. $N_2$ is the number of faces. The average edge valency was calculated while comparing the imperfection sensitivity of shell structures.

$$v = \sum_{r=2}^{n} \frac{rN_0^r}{N_0} \tag{1}$$

$$f = \sum_{s=3}^{n} \frac{sN_2^s}{N_2} \tag{2}$$

## 2.5 Initial geometric imperfection

The investigation of cylindrical shell configurations with perfect geometry cannot capture the actual behaviour due to imperfections available in real structures. The position of members or connections in the erected structure will be different from that of the design due to fabrication and erection errors. Hence, the effect of the initial geometric imperfection should be studied to find out the performance deviation from the actual behaviour of structures. A reticulated shell is modelled with initial geometric imperfection in multiple ways. The popular methods adopted to model geometric imperfections are the random imperfection mode method and the eigenvalue buckling mode methods [6]. This study adopted geometric imperfections based on the first eigenvalue buckling mode. The first buckling mode is the most critical mode for a structure, and the resulting deformation based on the first buckling mode reduces the load capacity of the structure to the maximum amount. Therefore, imperfections were modelled as per the scaled value of deformation based on the first buckling mode (Figure 6). Mode shapes were identical to the first support condition (Figure 6(a)). Mode shapes were different for each shell with the second support condition (Figure 6(b)). Therefore, the effect of edge valency is higher on the cylindrical shells with the second support condition than on the cylindrical shells with the first support condition.

The magnitude of the imperfection varied from S/1000 to S/100 for the cylindrical shells based on the results from previous studies [4,9]. Here, S is the span of the cylindrical shells (kept as 10 metres in all cases). The imperfection sensitivity investigation examines the limit load of shell configurations under uniform gravity loading.

## 3 IMPERFECTION SENSITIVITY BASED ON NONLINEAR ANALYSIS

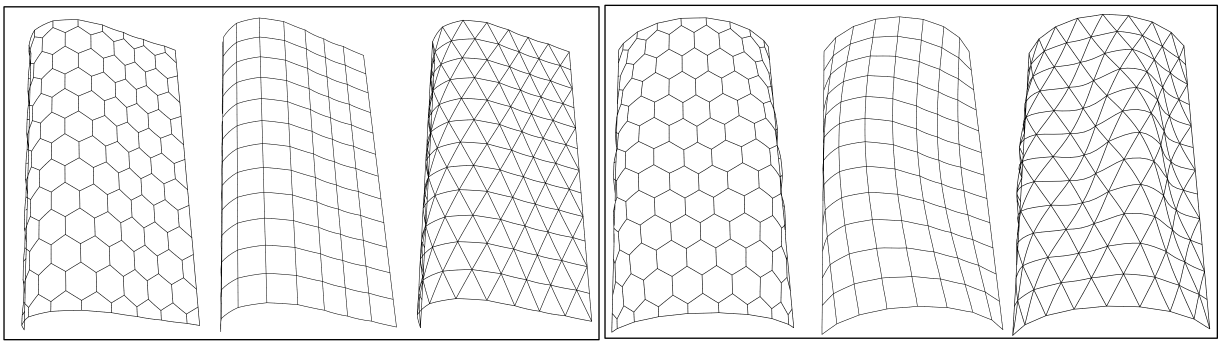
Three cylindrical shell configurations with different edge valency, two support conditions, and three span-to-rise ratios were subjected to Geometrically and Materially Nonlinear Imperfection Analysis (GMNIA) to study the effect of initial geometric imperfection on overall resistance. Different magnitudes of geometric imperfection were modelled in the study. The variation in the limit load with the change in magnitude of the initial geometric imperfection and the effect of average edge valency on the limit load was analysed.

### 3.1 Modelling

Three configurations were modelled using the finite element package, Abaqus [15]. The member cross-sections were calculated based on the IS 800 (2007) – Indian code of practice for general construction in steel [16]. The cross-section of members was arrived at by restricting the slenderness ratio to prevent member buckling. The study adopted a Circular Hollow Section (CHS 100) with an outer diameter of 114.3 mm and a thickness of 3.6 mm. Members modelled with Timoshenko beam element 'B32: A three-node quadratic beam in space' with two elements in each member. Connections were modelled as perfectly rigid joints. The circular hollow sections modelled with structural steel of Young's modulus 200 GPa and Yield stress 250 MPa with elasto-perfectly plastic constitutive behaviour. Initial geometric imperfections were assigned based on the first buckling mode (Figure 6).

### 3.2 Nonlinear analysis

The effect of the different magnitude of imperfections on three cylindrical shells was analysed during the initial stage. The influence of average edge valency on the limit load capacity of cylindrical shells was studied in the second stage. Uniform gravity loading was applied on the shell structures to examine the imperfection sensitivity during both stages.



(a) SC-1 (b) SC-2

Figure 6: The first buckling mode for cylindrical shells with two support conditions (R3)

3.2.1 Variation in the magnitude of imperfection

Initial geometric imperfection based on the first buckling mode was varied to compare the reduction in the overall resistance of selected shell configurations with a span-to-rise ratio of 3. The comparison of the results shows that the maximum reduction in limit load was observed for the configuration V.6.6.6 when an initial geometric imperfection of magnitude S/1000 was adopted (Figure 7). A similar trend was observed for cylindrical shells with both support conditions. The reduction in limit load capacity is nearly equal for all the shell structures when the initial geometric imperfection is very high (i=S/100) for shell structures with longitudinal edges are supported. Maximum reduction in load capacity was observed for configuration V.6.6.6 when an initial geometric imperfection of magnitude S/100 was provided. However, the reduction was significantly low for a lower magnitude of imperfection (i=S/1000 and i=S/750) for V.6.6.6. The increase in initial geometric imperfection amplified the reduction in load capacity. Among the three shell structures, V.4.4.4.4 exhibited a minor reduction in load capacity when all the sides were pin-supported.

The configuration V.3.3.3.3.3.3 exhibited more sensitiveness to the initial geometric imperfection when a smaller magnitude of imperfection was introduced. However, the reduction in limit load capacity of configuration V.6.6.6 was the highest for an increased magnitude of imperfection for the shell structures with all the edges supported.

3.2.2 Imperfection sensitivity with edge valency

The three configurations have different edge valencies of vertices and faces. The effect of edge valency on the limit load capacity of cylindrical shells with initial geometric imperfection was studied in the final stage of the study. Cylindrical shells with three span-to-rise ratios and two support conditions were monitored while the initial geometric imperfection varied from S/1000 to S/100.

As the weight of the three configurations is not identical, a parameter 'λ' is introduced to reduce the influence of weight while comparing the limit load (Equation 3). Here, P is the limit load capacity in kilonewtons, and W is the weight of the structure in kilonewtons. The limit load capacity based on the value λ was compared for shell structures with different average edge valency of vertices (Figure 8) and average edge valency of faces (Figure 9).

$$\lambda = \frac{P}{W} \tag{3}$$

The limit load factor λ increases and reaches the maximum value and decreases with the average edge valency of vertices for shell configurations with longitudinal edges supported (Figure 8). The value of λ increases with the average edge valency of vertices for shell structures with initial geometric imperfection and all the edges supported. For the configuration with low average edge valency of the vertices and low span-to-rise ratio, a slight variation in the limit load capacity was observed with the change in magnitude of the initial geometric imperfection. As the average edge valency of the shell structure increases, the variation in the value of λ is high with the change in magnitude of the initial geometric imperfection. Among the shell configurations, V.4.4.4.4 is found to have the maximum reduction in the value of λ when the magnitude of the initial geometric imperfection increases. The value of the parameter increases with the average edge valency of vertices for shell structures with initial geometric imperfection, and all edges are supported. For shell structures with a lower span-to-rise ratio and all the edges supported, the variation of λ with change in magnitude of initial geometric imperfection is lower for shell structures with lower average edge valency of vertices.

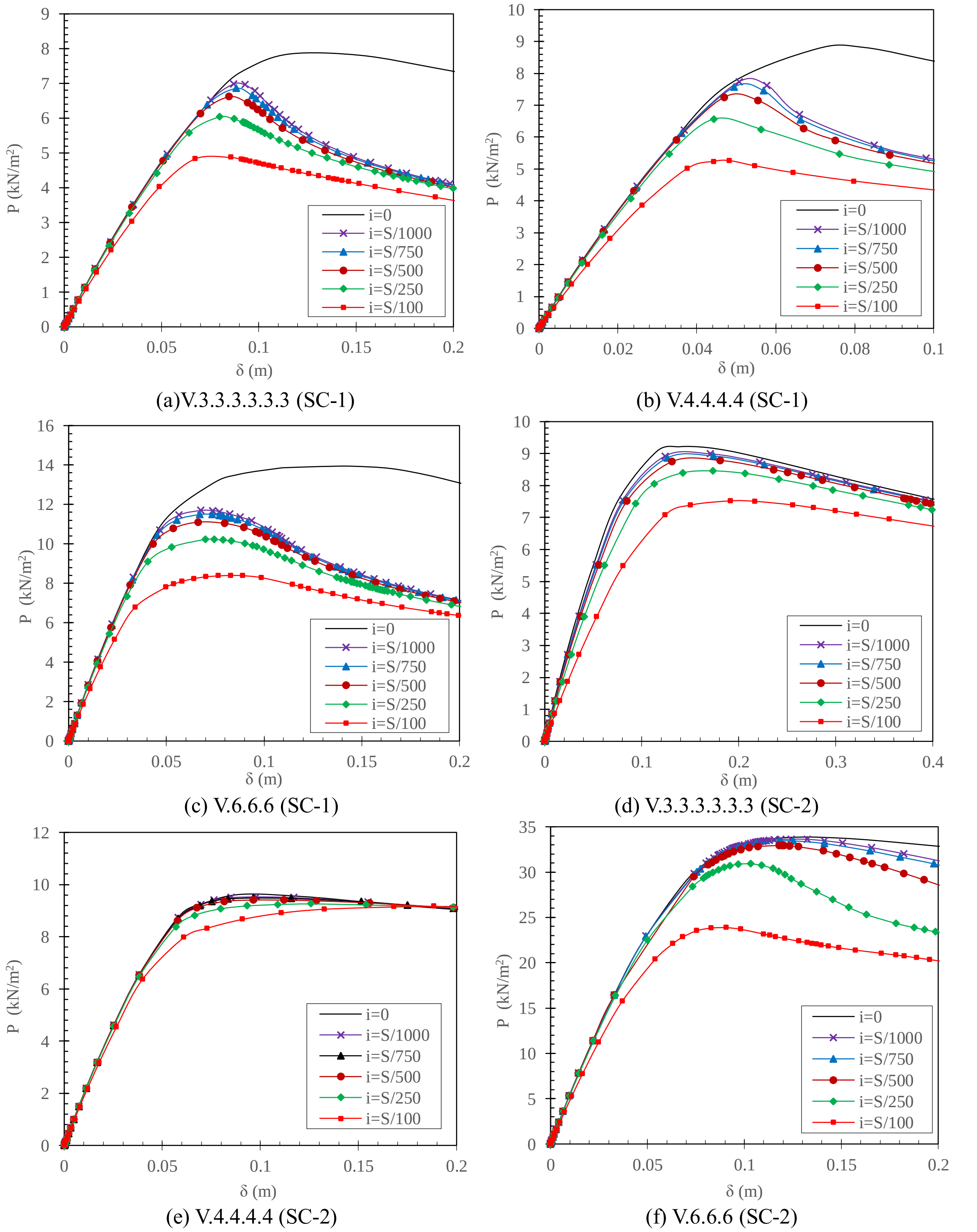



Figure 7: Load-deflection response for cylindrical shells with different magnitudes of initial geometric imperfection (R3)

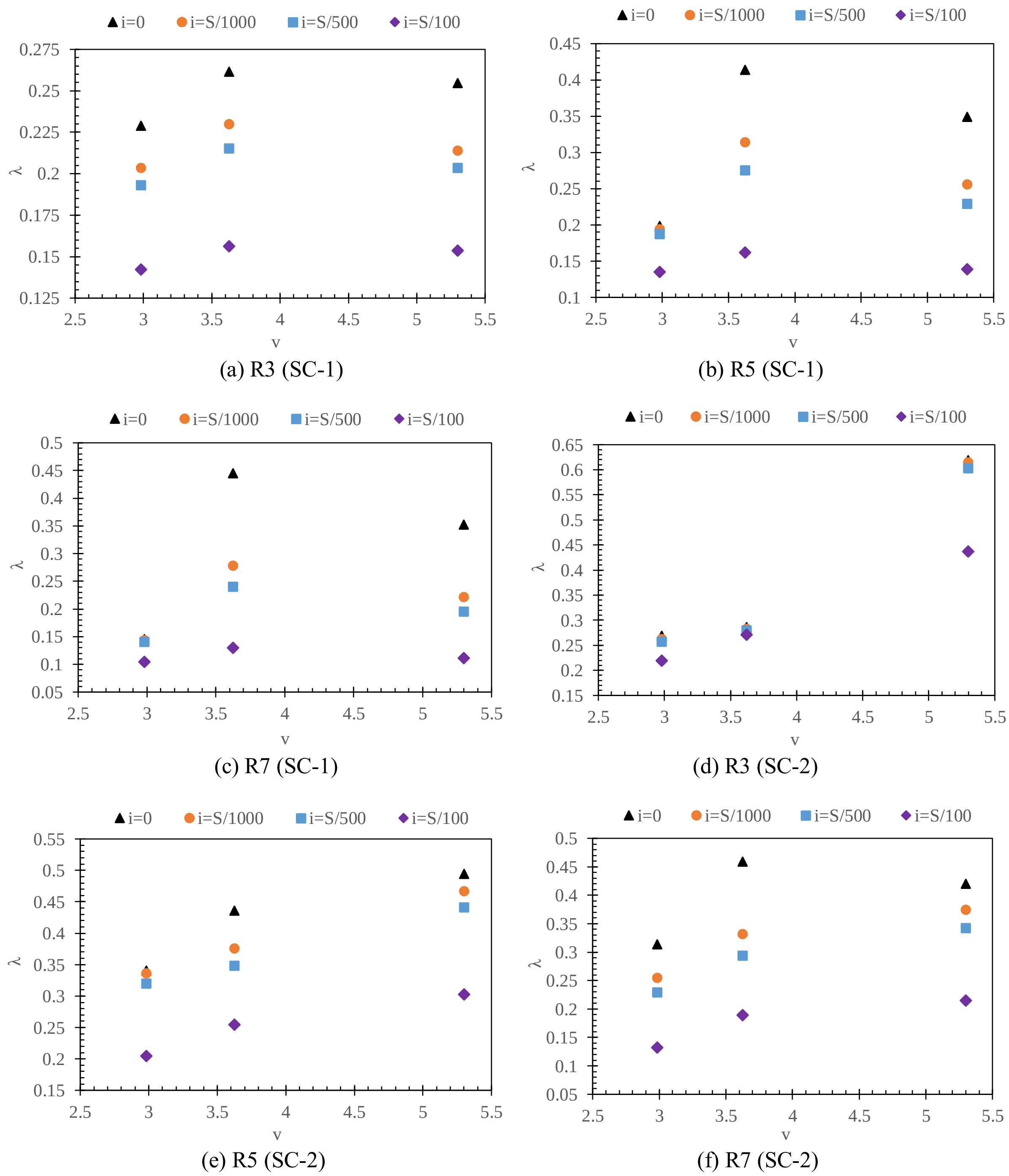



Figure 8: Variation of λ with average edge valency of vertices for different span-to-rise ratios

This variation of λ increases with the average edge valency of vertices and the span-to-rise ratio. In summary, the greater magnitude of the edge valency of vertices enables more load paths and reduces the effect of initial geometric imperfection. Further increase in the imperfection has more effect on these shell structures as further reduction in reserve capacity is possible. However, for shell structures with lower edge valency of vertices, the sudden decrease in load capacity with a small magnitude of imperfection is high, so a further reduction in the load capacity is negligible. Hence, the observation validates the effect of edge valency of vertices on the imperfection sensitivity of single-layer cylindrical shells.

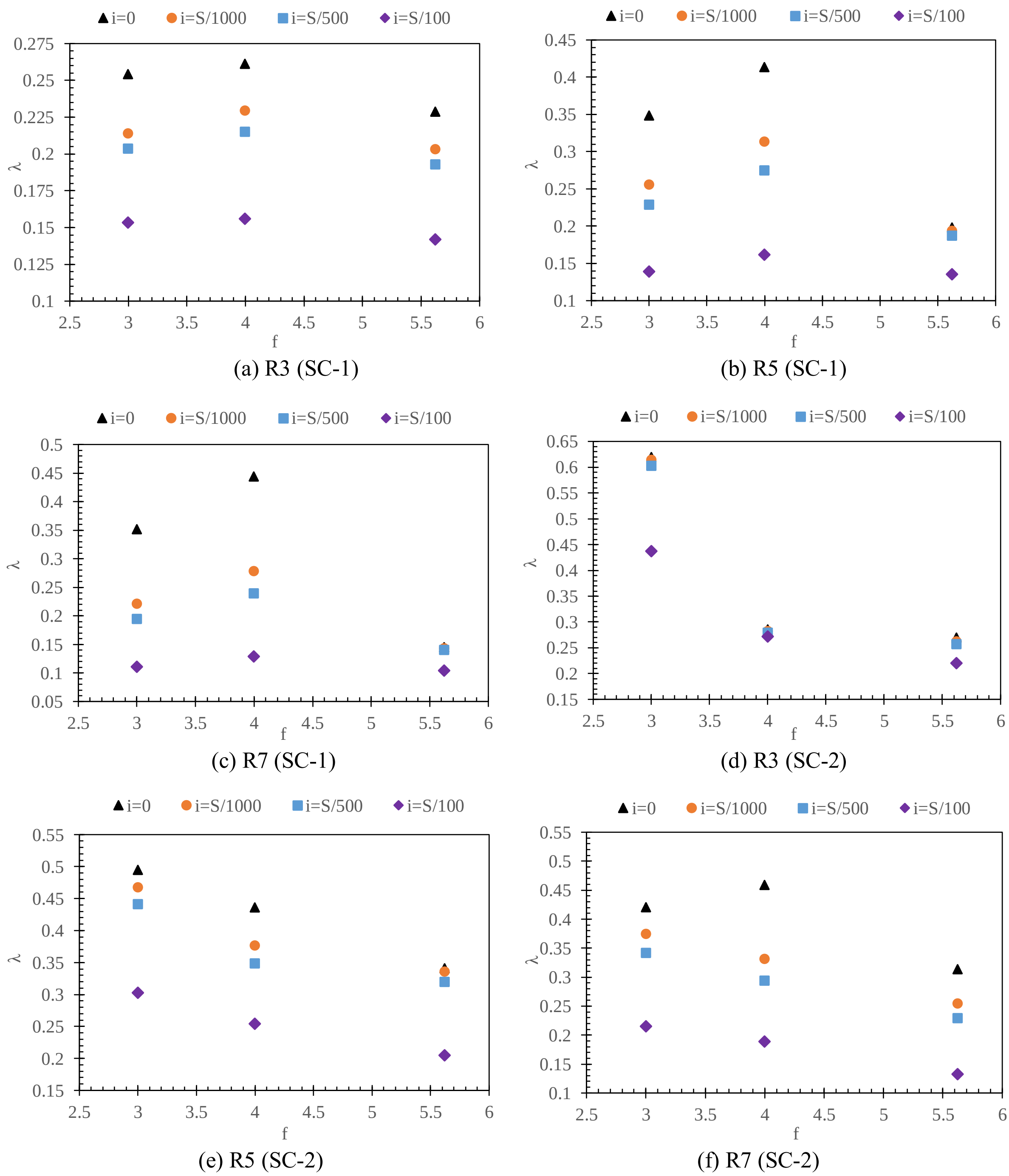



Figure 9: Variation of λ with average edge valency of faces for different span-to-rise ratios

The load capacity of cylindrical shells with imperfection increases to attain the maximum value and then decreases with the average edge valency of faces for shell structures with longitudinal edges supported (Figure 9(a)-9(c)). The load capacity decreases with the increase in the average edge valency of faces when all the edges are supported (Figure 9(d)-9(f)). The variation in the parameter λ is lower for higher average edge valency of faces among the shell structures with longitudinal edges supported. The behaviour is different from the comparisons made with respect to the average edge valency of vertices. The higher magnitude of edge valency of faces results in a lower magnitude of edge valency of vertices. As a result, the number of possible load paths decreases and imperfection sensitivity increases.

The study shows the impact of edge valency on the imperfection sensitivity of cylindrical shells. The variation was observed across different span-to-rise ratios and selected support conditions. In general, the increase in the edge valency of vertices reduces sensitivity against initial geometric imperfection.

## 4 DISCUSSION

The imperfection sensitivity analysis of three configurations with different edge valency of vertices and faces shows the impact of edge valency on the overall resistance of shell structures. Higher edge valency of vertices increases the possible load paths in a structure. The stability and the overall resistance of the shell structures will increase when the potential number of load paths increases. These results are in agreement with the findings by Sheik on single-layer shell configurations [3]. The effect of geometric imperfection decreases for the shell structure with higher edge valency of vertices and lower edge valency of faces. Hence, triangulated shell configurations are always efficient due to their higher edge valency of vertices.

The imperfections were introduced as per the first buckling mode of the cylindrical shell based on the assumption that the first buckling mode provides the least favourable coordinates for members with initial geometric imperfection. Even though it is true for many shell configurations, further studies are needed to validate the results for cylindrical shells with different support conditions. This study used only the three configurations with variable edge valency. Different configurations are possible within a single nomenclature itself. For example, the shell configuration with nomenclature "V.4.4.4.4" can have a different possible arrangement by changing the orientation of the members about the longitudinal axis and changing the angle between the neighbouring members. The effect of member orientation on imperfection sensitivity suggests future scope for the study to include 'inter-member angle' as an additional parameter. Furthermore, the 'extent' of edges (length of members) and dimensions of member cross-sections (outer radius and thickness) will affect the limit load of shell configurations as the slenderness ratio of individual members governs the local buckling of members. Local buckling of members can cause further member buckling and progressive collapse [2]. These factors further enhance the interest for supplementary investigation with additional parameters.

## 5 CONCLUSION

The influence of edge valency on the imperfection sensitivity of non-triangulated single-layer cylindrical shells was investigated with the help of two configurations with different edge valencies. The results were compared with the commonly adopted triangulated shell configuration. The geometrically and materially nonlinear imperfection analysis - by including the different magnitude of initial geometric imperfection, two support conditions, and three span-to-rise ratios - shows higher imperfection sensitivity of shell structures with lower edge valency of vertices and higher edge valency of faces. The higher edge valency of vertices enhances the possible load paths for the configuration, which enhances the stability and the overall resistance of shell structures with initial geometric imperfections. The load capacity evaluation based on parameter $\lambda$ suggests that the overall resistance of shell structures increases to reach the maximum value and decreases with edge valency of vertices when longitudinal edges are supported. Overall resistance increases with the average edge valency of vertices when all the edges are supported. The overall resistance of the shell structure initially increases to reach the maximum value and then decreases with the average edge valency of faces when longitudinal edges are supported. However, overall resistance decreases with the average edge valency of faces when all the edges are supported. It can be concluded that shell configurations with average edge valency of vertices and average edge valency of faces between 3.5 and 4.5

will exhibit the maximum resistance when the shell structure is supported on the longitudinal edges. However, shell configurations with the highest average edge valency of vertices and the lowest average edge valency of faces will demonstrate maximum resistance when all the edges are supported.

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