Monte Carlo Simulation of the Effect of Human Skin Melanin in Light-Tissue Interactions

Raghda Al-Halawani¹, Subhasri Chatterjee¹, Panayiotis. A. Kyriacou¹, Senior Member, IEEE

Abstract— Recent reports have highlighted the potential challenges skin pigmentation can have in the accurate estimation of arterial oxygen saturation when using a pulse oximeter. Pulse oximeters work on the principle of photoplethysmography (PPG), an optical technique used for the assessment of volumetric changes in vascular tissue. The primary aim of this research is to investigate the effect of melanin on tissue when utilising the technique of PPG. To address this, a Monte Carlo (MC) light-tissue interaction model is presented to explore the behaviour of melanin in the visible range in the epidermis. A key novelty in this paper is the ability to model the Modified Beer Lambert Law (MBLL) through a fully functional three-dimensional (3D) model in reflective optical geometry. Maximum photon penetration depth was achieved by red light, however, limited bio-optical information was retrieved by moderately and darkly pigmented skin at source-detector separations of less than 3 mm. The current MC model can be modified to provide a more realistic representation of skin by assessing the impact of scatter in the dermis, as well as absorption of light by blood, water, and lipids.

I. INTRODUCTION

The field of tissue optics is continuously developing as it proves pertinent in medical diagnostics. As a result, there lies a huge focus on designing more advanced technologies to retrieve important information about light-tissue interactions through non-invasive techniques, such as photoplethysmography (PPG). When an optical source is placed on the surface of a volume of tissue, multiple interactions occur before light is transmitted and detected by a photodetector. The resulting waveforms represent the changes in light absorption from non-pulsatile (DC PPG) such as melanin, muscle, venous blood, etc., and pulsatile blood (AC PPG) between the systolic and diastolic phases of the heart.

Monte Carlo models (MCM) have been extensively used as a computational tool to study vital aspects of light-tissue interactions in PPG. It is a mathematical technique which involves modelling the probability of different outcomes with random variables. In comparison to other methods such as random walk theory, diffusion approximation etc., it accounts for the stochastic nature of turbid media, such as single or multi-layered biological tissue.

A. Absorption and Beer Lambert Law

Photon absorption can occur in two different forms; radiative and non-radiative. The latter involves the emission of heat after relaxation of the photon to the ground state, and is employed in the following MCM. All skin chromophores, such as melanin, water, etc., are capable of absorbing a certain number of photons per unit distance. This is characterised by the absorption coefficient ($\mu_a$), which is wavelength dependent. The intensity of light, which is assumed constant after transmitted across the tissue boundary, is given by [2]:

$$ I_t = I_0 e^{-\mu_a L} = I_0 e^{-\varepsilon C L}. \quad (1) $$

Where $\varepsilon$ is the molar extinction coefficient of a medium, $C$ is the concentration of the medium, and $L$ is the total path length of photons travelling through the medium. The absorbance ($A$), which is the amount of light absorbed by a medium, is given by the ratio of the intensity of transmitted and incident light [2]:

$$ A = \mu_a \cdot L = \varepsilon \cdot C \cdot L. \quad (2) $$

B. Scattering and the Modified Beer Lambert Law

The number of scattering events per unit distance is characterised by the scattering coefficient ($\mu_s$) which is also wavelength dependent. Similarly, scattering can take two different forms, namely elastic and inelastic scattering. The former causes a change in the direction of light propagation due to collisions between scattered particles while preserving

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kinetic energy [2], which is more relevant to the intended application. The type of scattering that occurs depends on the microphysical properties of the particles or molecules. The model is simulated such that it undergoes Mie scattering and hence, the Henyey-Greenstein phase function is used to present the relationship between the angle of deflection (θ) and the anisotropic factor (g), to determine photon directionality [3]:

\[ p(\theta) = \frac{1-g^2}{4\pi(1-2g\cos \theta + g^2)^2}. \]  

(3)

The amount of light lost due to scattering is denoted by a parameter, G [2]:

\[ A = \mu_a \ast L + G = \varepsilon \ast C \ast L + G. \]  

(4)

The aim of this paper is to explore light-tissue interactions underlying the PPG technique on a basic level, as they have shown to be inadequate in analysing the complex light-tissue interactions in biological tissue [1]. To one’s knowledge, the effect of melanin concentration on these interactions has not yet been fully explored, despite reports suggesting the existence of bias in oximetry measurement on subjects with different skin pigmentation since the 1980’s [4-8].

II. MATERIALS AND METHODS

A. Region of interest

The epidermal layer of a finger, with a thickness of 0.25 mm [1] is modelled for three different wavelengths. These include blue light (450 nm), green light (550 nm), and red light (650 nm). The epidermis contains only a specific concentration of melanin and water, however, the presence of water is disregarded for model simplification. The author in [9] provides the volume fractions of melanosomes (V_mel) in the human epidermis:

<table>
<thead>
<tr>
<th>Description</th>
<th>V_mel (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Light-skinned adults</td>
<td>1.3 – 6.3</td>
</tr>
<tr>
<td>Moderately pigmented adults</td>
<td>11 – 16</td>
</tr>
<tr>
<td>Darkly pigmented adults</td>
<td>18 – 43</td>
</tr>
</tbody>
</table>

Table 1: Volume fraction of melanosomes in the epidermis for different skin pigments.

The MCM was simulated for a V_mel range of 5% - 20%, increasing in increments of 5%, in an attempt to investigate light-tissue interactions across the entire range of human skin pigments.

Firstly, the absorption coefficients of melanin at different wavelengths were calculated [9]:

\[ \mu_{a_{mel}} (\text{mm}^{-1}) = 6.6 \ast 10^7 \ast \lambda^{-3.33}. \]  

(5)

Secondly, the skin baseline was assumed to be the same as bloodless rat skin, as the epidermis is an avascular skin layer. Therefore, the absorption coefficient was calculated [9]:

\[ \mu_{a_{skinbaseline}} = 7.84 \ast 10^7 \ast \lambda^{-3.255}. \]  

(6)

Combining (5) and (6), the absorption coefficient of the epidermis was computed [9]:

\[ \mu_{a_{epidermis}} = V_{mel} \ast \mu_{a_{mel}} + (1 - V_{mel}) \ast \mu_{a_{skinbaseline}}. \]  

(7)

The scattering coefficient of the epidermis alone (and not the combined sublayers of skin) was difficult to retrieve from secondary sources. In addition, the inability to obtain data experimentally imposed greater challenges regarding the simulation of a relatively accurate model. As a result, some assumptions were made to approximate \( \mu_{s_{epidermis}} \). Some experimental data [10] has shown that over the visible spectrum, the reduced-scattering coefficient (\( \mu_{rs_{epidermis}} \)) of the epidermis was always 2 – 3 mm\(^{-1}\) higher than the reduced-scattering coefficient of the dermis. To calculate \( \mu_{rs} \), the following equation is used:

\[ \mu_{rs} = \mu_s \ast (1 - g). \]  

(8)

Therefore, by assuming that \( \mu_{s_{epidermis}} \) is 3 mm\(^{-1}\) smaller than \( \mu_{s_{dermis}} \) (since absorption by melanin dominates in the epidermis), data for the scattering coefficients of the epidermis was estimated from [11]. Moreover, the simulation was limited to the reliable retrieval of data of the anisotropic factors of the epidermis at the selected wavelengths. Instead, the anisotropic factors of skin at similar wavelengths [12, 13] were used to estimate \( g \).

Table 2: Scattering coefficients and anisotropic factors of the epidermis for blue, green, and red light.

<table>
<thead>
<tr>
<th>Tissue</th>
<th>( \lambda ) (nm)</th>
<th>( \mu_s ) (mm(^{-1}))</th>
<th>( g )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Human epidermis</td>
<td>450</td>
<td>9.41</td>
<td>0.8</td>
</tr>
<tr>
<td></td>
<td>550</td>
<td>4.722</td>
<td>0.85</td>
</tr>
<tr>
<td></td>
<td>650</td>
<td>2.598</td>
<td>0.9</td>
</tr>
</tbody>
</table>

B. Monte Carlo simulation procedure

The monolayer model was simulated by adapting the Monte Carlo procedure described in [2, 14] using MATLAB. A set of conditions were implemented to govern the movement of 10\(^8\) photons in the tissue.

Fig. 1 Illustration of the Cartesian coordinate system and reflectance PPG adapted from [2].
To account for scattering, the spherical nature of the photons was represented using the Cartesian coordinates \( x, y, z \) (Fig. 1). In reflective optical geometry, the source-detector separation \( d \) was adjusted by the increasing the distance between the lateral \( (x) \) direction relative to a collimated beam from a point source i.e., located at \((0,0,0)\). Photons were launched perpendicular to the surface of the epidermis.

Figure 2 shows a vector \( \vec{r} \) which denotes the instantaneous direction of a photon, makes angles \( \alpha, \beta, \) and \( \gamma \) with the \( x, y, \) and \( z \) axes, respectively. Therefore, all the position vectors, which are defined by their direction cosines \( (u_x, u_y, u_z) \), were converted with respect to the Cartesian coordinate system to the spherical polar coordinate system:

\[
\begin{align*}
    u_x &= \sin \theta \cos \phi, \quad u_y = \sin \theta \sin \phi, \quad u_z = \cos \theta. \\
    \theta &= 2 \times \xi \times \sin^{-1}(NA), \quad \phi = 2 \times \xi \times \pi. 
\end{align*}
\]

\[ \text{Fig. 2 The relationship between the spherical polar coordinate and Cartesian coordinate system adapted from [2].} \]

A photon was launched with an initial weight equal to unity. The amount of light lost due to the air-tissue refractive index mismatch \( \left( R_I \right) \) was subtracted from the photon’s initial weight before it entered the epidermis. The photon moved a random distance \( (l) \), which combined the effects of absorption and scattering (the attenuation coefficient, \( \mu \)), to achieve the most realistic photon distribution within the tissue [14]:

\[
\frac{l}{\mu} = \frac{-\ln(\xi)}{\mu} = \frac{-\ln(\xi)}{\mu}. 
\]

This was used to calculate the new position coordinates \( (x_2, y_2, z_2) \) of the photon along with the direction cosines, respectively [2]:

\[
\begin{align*}
    x_2 &= x + l \times u_x, \quad y_2 = y + l \times u_y, \quad z_2 = z + l \times u_z. 
\end{align*}
\]

Once a photon was backscattered to the photodetector located on the tissue surface \( (z=0) \), a conditional if loop was used to check whether the photon was detected by the photodetector i.e., if the photon satisfied the survival criteria. The \( x \) and \( y \) coordinates of a transmitted photon were computed (13), and used to determine whether the photon was detected by calculating the size of the photon (14), and the angle of acceptance (15) [2]:

\[
\begin{align*}
    x_3 &= z \times \frac{u_x}{u_z} + x, \quad y_3 = z \times \frac{u_y}{u_z} + y. \\
    x_3^2 + y_3^2 &< r^2. \\
    \theta_{acc} &= \sin^{-1}(NA). 
\end{align*}
\]

To reduce statistical uncertainty, implicit photon capture, a variance reduction technique, was used to improve the efficiency of the MC simulation by considering a ‘photon cluster’ instead of a single photon entering the tissue [14]. If a photon cluster did not hit the boundary, a fraction of its weight was absorbed \( (wa) \), and the remainder continued to travel to the next interaction site [14]:

\[
wa = \left( \frac{u_z}{u_{\mu}} \right) \times w = \Lambda \times w. 
\]

A vector ‘\( \text{killw} \)’ was also created to record all the terminated photons if a) they were transmitted but not detected or b) the weight of the photon cluster did not exceed the pre-defined weight threshold \( (10^{-3}) \) i.e., intensity was negligible due to continuous absorption. If the remainder of a photon cluster, given by \( w = w - wa \) [2], was not transmitted, detected, or terminated, then it was scattered. The direction cosines of these photons were only updated if they were not extremely close to the \( z \) axis, using the following relationships [15]:

\[
\begin{align*}
    u_{x1} &= \frac{\sin \theta (u_y u_z \cos \phi - u_y \sin \phi)}{\sqrt{1 - u_z^2}} + u_x \cos \theta, \\
    u_{y1} &= \frac{\sin \theta (u_y u_z \cos \phi + u_x \sin \phi)}{\sqrt{1 - u_z^2}} + u_y \cos \theta, \\
    u_{z1} &= -\sin \theta \cos \phi \sqrt{1 - u_z^2} + u_z \cos \theta. 
\end{align*}
\]

III. RESULTS AND DISCUSSION

For all wavelengths, maximum photon penetration depth was achieved when \( V_{\text{med}} = 5\% \). The results suggested that for lightly pigmented individuals, one could see beyond the dermal sublayers when \( \lambda = 450 \text{ nm} \) \((0.98 \text{ mm})\), and possibly the first layer of fat when \( \lambda = 550 \text{ nm} \) \((2.06 \text{ mm})\), and even more so when \( \lambda = 650 \text{ nm} \) \((3.42 \text{ mm})\). As a result, retrieval of bio-optical information for moderately and darkly pigmented subjects was limited. As expected, blue light was mostly absorbed by melanin at all concentrations and hence, \( d \) did not exceed 1 mm.
It can be seen that light absorption increases as melanin concentration increases, resulting in a progressive reduction in photon detection (Fig. 3(a)). Adjusting the source-detector separation to 3 mm (Fig. 3(b)) leads to negligible outcomes when $V_{\text{mel}} = 10\% - 20\%$, proving impractical for reflectance PPG applications such as pulse oximetry. This may justify why calibration curves for such devices are generated on majority lightly pigmented subjects.

Moreover, MBLL is successfully demonstrated in Figure 4, as the linear relationship between absorbance and melanin concentration remains true for a scattering medium. The y-intercepts i.e., values of $G$ are less than zero. These values depend on the system parameters i.e. the optical properties of the epidermis and the sensor geometry. Therefore, negative G values may indicate the dominance of absorption in the epidermal layer of a human finger.

**Fig. 3** Verification of the modified Beer Lambert law (MBLL) - Linear relationship between absorbance and melanin concentration.

**Fig. 4** (a) Photon propagation of red light when $d = 1$ mm for different skin pigmentation. (b) Photon propagation of red light when $d = 3$ mm for different skin pigments.

**IV. CONCLUSION**

This paper has presented a Monte Carlo model of the human epidermis to analyse the effect of melanin concentration on light-tissue interactions in reflective optical geometry. It has successfully complied with the principles of tissue optics, namely the modified Beer Lambert law. The results suggest that light is highly absorbed by melanin, especially as the source-detector separation increases. It is clear that retrieval of bio-optical information from moderately and darkly pigmented subjects is limited.

The current MCM can be modified to provide a more realistic representation of skin by assessing the impact of scatter in the dermis, as well as absorption of light by blood, water, and lipids. Other modifications include removal of the point source to provide defined source dimensions for experimental purposes. Overall, such MCM may be utilised to predict the behaviour of photons under the influence of melanin in tissue.

**REFERENCES**