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A novel nonlinear isolated rooftop tuned mass damper-inerter (IR-TMDI) system for seismic response mitigation of buildings

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Abstract

This paper conceptualizes a novel passive vibration control system comprising a tuned mass damper inerter (TMDI) contained within a seismically isolated rooftop and investigates numerically its effectiveness for seismic response mitigation of building structures. The working principle of the proposed isolated rooftop tuned mass damper inerter (IR-TMDI) system relies on the yielding of typical elastomeric isolators (e.g. lead rubber bearings) under severe earthquake ground motions to create a flexible rooftop which, in turn, increases the efficacy of the TMDI for seismic vibrations suppression. Herein, a nonlinear mechanical model is considered to explore the potential of IR-TMDI whereby the primary building structure is taken as linear damped single-mode system while the Bouc-Wen model is used to capture the nonlinear/hysteretic behavior of the rooftop isolators. An equivalent linear system (ELS), derived through statistical linearization, is used to expedite the optimal IR-TMDI tuning for different isolated rooftop properties, inertance, and primary structure natural periods under white noise excitations with different intensities as well as Kanai-Tajimi excitations for different soil conditions. It is found that tuning for maximizing TMDI seismic energy dissipation is more advantageous than tuning for minimizing primary structure displacement or acceleration response since it lowers deflection and force demands to the isolators and to the inerter. Further, significant primary structure displacement and acceleration reductions are achieved as the effective rooftop flexibility increases through reduction of the nominal strength of the isolators, which verifies the intended working principle of the IR-TMDI. This is also confirmed through response history analyses to the nonlinear model under four benchmark recorded ground motions. Moreover, for IR-TMDI with sufficiently flexible isolators, improved seismic structural performance with concurrent reduced deflection and force demands at the isolators is shown for all considered stationary excitations as the inertance scales-up, which is readily achievable technologically. Thus, it is concluded that the IR-TMDI mitigates effectively structural seismic response without requiring the inerter to span several floors, as suggested in previous studies, thus extending the TMDI applicability to both existing and low-rise new-built structures.

Keywords

Tuned mass damper inerter, floor isolation, Bouc-Wen model, statistical linearization, optimal energy design

33 1 Introduction and motivation

34 Over the past decades, various passive vibration control approaches for seismic structural response
35 modification and energy dissipation have been considered in the scientific literature and in practice for improving
36 the performance of buildings under earthquakes (e.g. Freddi et al. 2021). Common approaches involve the
37 insertion of a laterally flexible (isolation) layer comprising elastomeric and/or sliding bearings at the basement
38 of buildings (Naeim & Kelly 1999, Warn & Ryan 2012) or at an intermediate floor (Ryan & Earl 2010, Faiella
39 & Mele 2020). The isolation layer elongates the fundamental structural natural period while enabling local
40 seismic energy dissipation through nonlinear (hysteretic) behavior of the isolators. Both of these considerations
41 reduce the total lateral seismic loads. Further, the isolation layer changes the shape of the first (dominant) mode
42 of vibration which reduce interstorey drifts and floor accelerations throughout the building by localizing lateral
43 seismic deflection demands at the isolators. Other approaches consider various damping devices such as viscous
44 dampers (Berquist et al. 2019) and hysteretic dampers (Xie 2005) diffused throughout buildings as standalone
45 struts to resist earthquake loads through increased seismic energy dissipation (Whittle et al. 2012), or placed at
46 the isolation layer to increase energy dissipation in isolated buildings (Wolff et al. 2015), or incorporated within
47 tuned mass dampers (TMDs) (Soto & Adeli 2013, Elias & Matsagar 2017). The latter approach, of particular
48 interest to this work, relies on attaching a free-to-oscillate (secondary) mass to the top floor of the building
49 (primary) structure via viscous dampers and stiffeners which are designed/tuned to minimize the seismic
50 response of the building primary structure, dominated by the first mode of vibration (e.g. Rana & Soong 1998).
51 TMDs may be implemented either as a hanging heavy-weight pendulum from the top floor (Zemp et al. 2011,
52 Li et al. 2011) or by isolating the top floor (Villaverde 2002, Matta & De Stefano 2009) whereby the mass of the
53 rooftop slab becomes the TMD secondary mass while the isolators are appropriately dimensioned such that the
54 effective period of the isolated rooftop is tuned (i.e. matches closely) the fundamental frequency of the remaining
55 building. They may have linear viscoelastic, non-linear elastic, or even hysteretic behavior in which case they
56 oftentimes termed nonlinear energy sinks (Vakakis & Gendelman 2001, Tsiatas et al 2020).

57 Out of the different approaches discussed above, TMDs are the least considered in practice for the seismic
58 protection of buildings, although they are widely used for mitigating wind-borne oscillations in slender/tall
59 structures (Elias & Matsagar 2017, Colherinhas et al 2021). This is because TMDs require excessively large
60 secondary mass for the effective mitigation of earthquake-induced oscillations corresponding to more than 15%
61 of the building/structural mass (see e.g. De Angelis et al 2011 and therein references). To this end, in recent
62 years, the tuned mass damper inerter (TMDI) configuration proposed by Marian and Giaralis (2013, 2014) has
63 been widely considered in the literature for the seismic response mitigation of non-isolated buildings (Giaralis
64 & Taflanidis 2018, Ruiz et al. 2018, Taflanidis et al. 2019, Kaveh et al. 2020, Patsialis et al. 2021, Djerouni et
65 al. 2022) as it significantly relaxes the requirement for a large secondary mass. This is achieved by coupling a
66 conventional top-floor TMD with an inerter device which links/supports the secondary mass to an intermediate
67 floor, different from the top floor, thus resulting in a device assembly attached to two different building floors.
68 The inerter is a mechanical element that produces a relative acceleration- dependent force proportional to a so-
69 called inertance property measured in mass units (kg) (Smith 2002). Importantly, inertance scales-up practically
70 independently from the inerter physical mass using mechanical gearing or fluid mechanics principles (e.g.
71 Brzeski et al. 2017, Smith 2020, Pietrosanti et al. 2021) which enables large-scale inerter device prototypes,

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72 developed for earthquake engineering applications, to reach inertance of 10000 tons or more (e.g. Nakamura et
73 al. 2014, Nakaminami et al. 2017). Hence, in the TMDI configuration, the inerter contributes inertia (but not
74 weight) through large inertance by leveraging the relative kinematics (motion) during a seismic event between
75 the secondary mass and an intermediate floor. Notably, the beneficial effect of the inerter in the TMDI
76 configuration has been extended to the case of nonlinear energy sinks (Javidialesaadi & Wierschem 2019).

77 In this respect, Marian and Giaralis (2016) demonstrated numerically that the required secondary mass can
78 be significantly reduced by trading it to inertance for fixed structural seismic performance measured in terms of
79 top floor displacement. Further, Pietrosanti et al. (2020) and Wang & Giaralis (2021) proved that the TMDI
80 vibration mitigation performance improves for fixed inertance as the inerter spans more floors (i.e. is connected
81 to more than one floor below the top floor), by demonstrating analytically that the difference of the modal
82 coordinates between the two TMDI attachment floors amplifies the inertance property. In this regard, Giaralis
83 and Taflanidis (2018) established that the TMDI offers increased robustness to uncertainties in structural
84 properties and seismic excitation compared to TMD, as the inertance increases, especially when the inerter
85 connects the secondary mass two floors below the top floor. Moreover, Ruiz et al. (2018) and Taflanidis et al.
86 (2019) showed that by judicial tuning supported by bi-objective optimal design formulations, the TMDI achieves
87 improved structural seismic performance in terms of storey drifts and floor accelerations under stochastic ground
88 excitations when the inerter is let to span more than one floors. Additionally, Kaveh et al. (2020) and Djerouni
89 et al. (2022) demonstrated significantly improved seismic performance of linear buildings with TMDI spanning
90 several floors by using large sets of non-pulse and pulse-like recorded ground motions, respectively, while
91 Patsialis et al. (2021) extended the above conclusion to the case of inelastic (yielding) building structures.

92 Nevertheless, whilst TMDI configurations spanning several floors may be practically feasible in
93 tall/landmark buildings under wind excitations by considering an internal large opening in consecutive floor
94 slabs (see e.g. Giaralis and Petrini 2017, Dai et al. 2019), such configurations may have limited practical
95 applicability for existing structures as well as for low-to-mid-rise buildings in high seismically active areas.
96 Importantly, these practical limitations can be by-passed by introducing a local structural modification to the
97 primary structure such that the difference of the modal coordinates between the floors connected by the TMDI
98 increase, whereby exploiting the theoretical studies of Pietrosanti et al. (2020) and Wang & Giaralis (2021). To
99 this end, Wang and Giaralis (2021) considered a TMDI configuration spanning only one (the top) floor and
100 demonstrated that effective mitigation of wind induced oscillations in a 34-storey slender building are achieved
101 as the top floor becomes more flexible through a local reduction of cross-sectional columns and shear walls as
102 well as by increasing the top floor height. Still, the consideration of a soft large-height top floor may not be a
103 practically attractive structural modification in high seismicity areas.

104 To this end, recognizing that a locally flexible floor can be achieved through inter-storey (or partial) seismic
105 isolation of buildings (Villaverde 1998, Matta & De Stefano 2009, Ryan & Earl 2010, Faiella & Mele 2020),
106 this paper investigates numerically the potential of a seismically isolated top floor equipped with a TMDI for
107 enhanced structural seismic performance, schematically shown in Fig.1(a). In the proposed configuration termed
108 isolated rooftop tuned mass damper inerter (IR-TMDI), the hypothesis is that the isolators yield under severe
109 ground motion, creating a flexible rooftop which, in turn, increases the effectiveness of the TMDI for seismic
110 energy dissipation. In this regard, the isolators of the IR-TMDI are modelled as nonlinear hysteretic elements

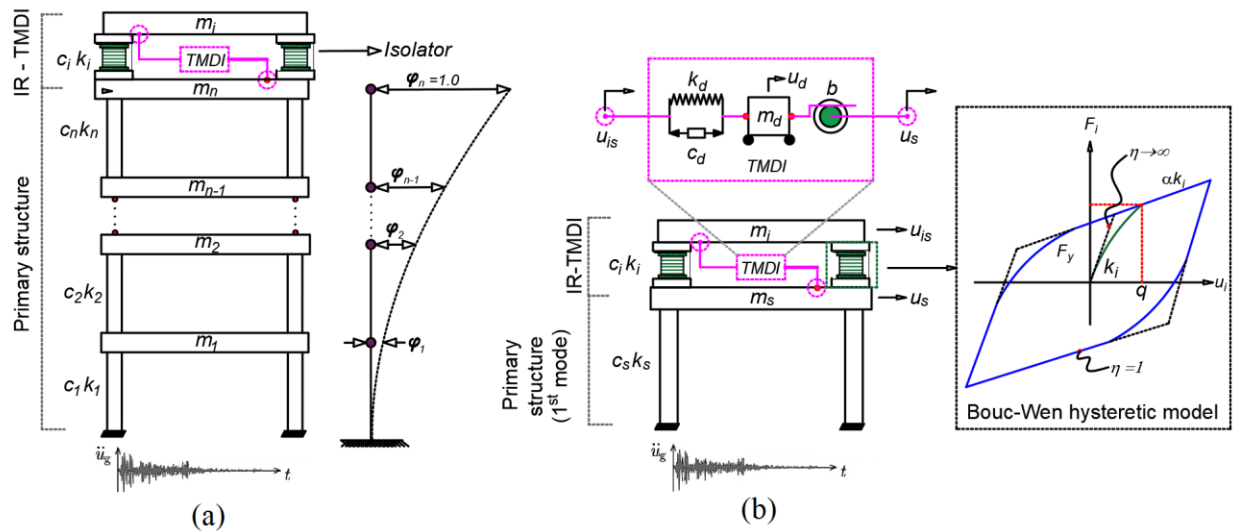
111 tracing a Bouc Wen hysteretic model (Wen 1976) which is widely adopted to capture the response of elastomeric
112 isolators (e.g. Nagarajaiah & Xiaohong 2000). From a practical viewpoint, the IR-TMDI is more advantageous
113 than a TMDI spanning several floors as it can be readily added atop of existing building structures or included
114 in the design of new structures as a service-only fake floor (Fig.1(a)). Indeed, the IR-TMDI resembles the rooftop
115 isolated TMD pioneered by Villaverde (1998) (see also Villaverde and Mosqueda 1999, and Matta & De Stefano
116 2009), but employs a TMDI for seismic energy dissipation, rather than tuning the isolators such that the added
117 floor plays the role of a TMD. Component-wise, the IR-TMDI is related to TMDI-equipped base isolated
118 structures studied by De Domenico and Ricciardi (2018) and De Angelis et al. (2019), among several other
119 researchers, in which the TMDI is housed in the basement and links the isolated layer to the ground to minimize
120 the seismic deflection demands of the isolators. However, the IR-TMDI develops distinct dynamics compared
121 to the systems discussed in the last publications as the inerter element is not grounded while the isolation bearings
122 are components of the vibration absorber, rather than a part of the primary structure to be seismically protected.
123 To this end, this paper first explores different criteria for the optimal IR-TMDI tuning by examining the primary
124 structure response separately from the isolated rooftop deflection. Then, a comprehensive parametric study is
125 undertaken to identify the influence of the isolated rooftop, seismic excitation, and primary structure properties
126 to the IR-TMDI potential for seismic response mitigation. Both the optimal IR-TMDI tuning and the parametric
127 study are facilitated by adopting a linear damped single degree of freedom (SDOF) oscillator as a proxy of the
128 primary structure representing the dynamics of its first/dominant mode of vibration. Further, statistical
129 linearization is employed to treat the nonlinear isolators under stationary stochastic ground excitation, thus
130 further expediting the optimal IR-TMDI tuning and appraisal of its potential for seismic response mitigation
131 through linear random vibrations analysis. Lastly, numerical data from nonlinear response history analyses using
132 the recorded ground motions from the benchmark seismic vibrations control problem in Ohtori et al. (2004) are
133 considered to verify the main trends of the seismic response of optimally tuned IR-TMDI equipped structures,
134 focusing on the flexibility of the isolation layer. The presentation begins by describing the IR-TMDI and
135 discussing some practical considerations before dwelling on the derivation of the nonlinear equations of motion
136 of an IR-TMDI equipped primary structure.

137 **2 Proposed nonlinear isolated rooftop tuned mass damper inerter (IR-TMDI) system**

138 *2.1 System description and practical considerations*

139 Consider a generic n -storey planar building structure whose lateral response to horizontal seismic ground
140 acceleration excitation, α_g , can be faithfully captured by a linear damped n degree-of-freedom (n -DOF)
141 dynamical model with lumped floor masses m_k ; $k=1,2,\dots,n$ as shown in Fig. 1(a). Aiming to suppress the seismic
142 response of the considered building (primary structure), the novel passive energy dissipation IR-TMDI system is
143 herein considered. The system comprises an isolated slab with mass m_i placed atop of the building, as discussed
144 by Villaverde (1998), and a linear TMDI (Marian and Giaralis 2013), placed in between the isolated slab and the
145 topmost building slab as depicted in Fig. 1(a). With reference to the inlet in Fig.1(b), the TMDI consists of a
146 secondary vibrating mass, m_d , which is connected to the isolated slab through a properly designed (or tuned)
147 viscoelastic connection, modelled by a linear spring with stiffness k_d in parallel with a dashpot with damping

148 coefficient c_d . Further, the secondary mass is linked to the topmost building slab by an inerter with inertance b
 149 (Smith 2002).



150 (a) 151 **Fig. 1:** Conceptualization and mechanical modelling of the IR-TMDI system for multi-storey buildings
 152 represented by (a) lumped mass dynamic model, and (b) Simplified single-mode dynamic model

153 As noted in the introduction, the proposed IR-TMDI system may be equally applicable for the seismic
 154 retrofitting of existing buildings as well as for achieving high-performing new buildings. This is because the
 155 isolated slab of the IR-TMDI can serve as a rooftop garden, as conceptualized by Matta and De Stefano (2009),
 156 or it may be part of a specially designed last floor, to house various services apart from the TMDI. In this regard,
 157 mass m_i may be taken to be comparable to the structural mass of a typical building floor, and, in every case, it
 158 should be interpreted as the top floor mass which is part of the primary/host building structure mass. Nevertheless,
 159 for the purpose of the optimal IR-TMDI design, the mass m_i needs to be treated as an integral IR-TMDI component
 160 since its value affect significantly the IR-TMDI tuning and performance as will become evident in later Sections..
 161 Further, any standard type of bearing (isolator) for base isolated buildings (Naeim & Kelly 1999) may be used to
 162 support the IR-TMDI slab. In this work, elastomeric isolators with nonlinear hysteretic behavior are assumed, as
 163 their implementation for rooftop isolation was shown to be practically feasible through experimental work
 164 (Villaverde & Mosqueda 1999) as well as through detailed design using commercially available bearings
 165 (Villaverde 2002, Ryan & Earl 2010).

166 Moreover, the TMDI can be made sufficiently compact (see e.g. Pietrosanti et al 2021) and thus can be
 167 readily integrated within the isolation layer. Specifically, TMDI compactness can be facilitated by using
 168 commercial fluid viscous dampers developed for seismic applications (e.g. Berquist et al. 2019) in place of the
 169 TMDI dashpot element (see also Villaverde 2002 and Rajana et al. 2022), as well as by adopting inerter devices
 170 with large inertance and small physical mass as the one prototyped by Nakaminami et al. (2017) reaching inertance
 171 $b > 10000t$. In this setting, the inertance b can endow exclusively the required inertia attribute to the TMDI for
 172 effective seismic response mitigation, which relaxes the need for a large secondary mass m_d . To this end, the
 173 secondary mass m_d of the IR-TMDI does not correspond to any block of steel or concrete with substantial dead
 174 weight as in conventional TMD implementations for building structures (e.g. Zemp et al. 2011, Li et al. 2011).

175 Instead, it is merely used to capture the influence of the relatively small self-weight (compared to the building
176 structural weight) of the TMDI damping and inerter devices and their end connections.

177 As a final remark on the practicalities of the proposed IR-TMDI, note that although a TMDI may behave in
178 a nonlinear fashion (e.g. due to the non-ideal/nonlinear inerter device behavior and/or fluid viscous damper
179 behavior), it has been established experimentally (e.g. Gonzales-Buelga et al. 2017, Pietrosanti et al. 2021) and
180 numerically (e.g. Brzeski & Perlikowski 2017, De Domenico et al. 2020, Rajana et al. 2022) that such nonlinear
181 effects do not compromise the vibration mitigation potential of the TMDI configuration. If anything, such
182 nonlinear effects are actually beneficial in earthquake engineering applications (De Domenico et al. 2020, Rajana
183 et al. 2022). Therefore the herein adopted assumption of a linear TMDI suffices for the purpose of appraising the
184 potential of the IR-TMDI for the structural seismic response mitigation.

185

186 2.2 Simplified single-mode structural model equipped with IR-TMDI

187 To support a meaningful tuning of the IR-TMDI while facilitating a comprehensive parametric investigation
188 of the IR-TMDI seismic response performance, the simplified dynamical model in Fig. 1(b) is considered
189 throughout this work in which the primary structure is represented by a linear damped 1-DOF modal oscillator
190 corresponding to the fundamental mode shape vector $\boldsymbol{\phi}$ of the n -DOF model. The latter is assumed to dominate
191 the seismic response of the uncontrolled (no IR-TMDI installed) building and therefore IR-TMDI tuning is taken
192 to target this first mode. In this context, the modal primary structure mass, m_s , inherent damping coefficient, c_s ,
193 stiffness, k_s , and seismic excitation \ddot{x}_g are defined as

$$194 \quad m_s = \boldsymbol{\phi}^T \mathbf{M}_s \boldsymbol{\phi}, \quad c_s = \boldsymbol{\phi}^T \mathbf{C}_s \boldsymbol{\phi}, \quad k_s = \boldsymbol{\phi}^T \mathbf{K}_s \boldsymbol{\phi}, \quad \text{and} \quad \ddot{x}_g = \frac{\boldsymbol{\phi}^T \mathbf{M}_s \mathbf{r}}{\boldsymbol{\phi}^T \mathbf{M}_s \boldsymbol{\phi}} a_g, \quad (1)$$

195 respectively, where \mathbf{M}_s , \mathbf{C}_s , and \mathbf{K}_s , are the mass, damping, and stiffness matrices, respectively, of the n -DOF
196 primary structure model, the superscript ‘‘T’’ denotes vector transposition and \mathbf{r} is the unitary vector. By
197 normalizing $\boldsymbol{\phi}$ in Eq.(1) such that the modal coordinate at the n -th floor is equal to 1 (see Fig 1(a)), the structural
198 modal displacement coordinate of the simplified model, u_s , relative to the ground motion coincides with the
199 relative displacement of the n -th floor, expected to be the maximum across all floors for regular buildings with
200 dominant first mode. Therefore, the modal 1-DOF oscillator with properties defined in Eq.(1) can be used as a
201 surrogate model of the MDOF to support single-mode optimal design/tuning of inertial absorbers (Rana & Soong
202 1998).

203 Focusing on the IR-TMDI modelling, with reference to the simplified system in Fig. 2(b), the inerter is
204 modelled as an ideal linear mechanical element developing a force given as (Smith 2002)

$$205 \quad F_b = b(\ddot{u}_d - \ddot{u}_s) = b(\ddot{u}_i + \ddot{u}_k), \quad (2)$$

206 where $u_i = u_{is} - u_s$ and $u_k = u_d - u_{is}$ with u_d and u_{is} being the relative to the ground displacements of the TMDI
207 secondary mass and of the isolated slab, respectively, as indicated in Fig. 1(b). In Eq.(2) and henceforth a dot over

208 a symbol denotes time differentiation. Further, the force developing at the viscoelastic connection of the TMDI
209 with the isolated slab is given as

$$210 \quad F_v = c_d (\dot{u}_d - \dot{u}_{is}) + k_d (u_d - u_{is}) = c_d \dot{u}_k + k_d u_k \quad (3)$$

211 Lastly, the nonlinear restoring force of the elastomeric isolation layer is modelled as a sum of a viscoelastic
212 and a hysteretic contribution given as

$$213 \quad F_i = c_i \dot{u}_i + \alpha k_i u_i + (1 - \alpha) F_y z \quad (4)$$

214 in which z is an auxiliary hysteretic state variable related to the isolator deflection u_i through the versatile Bouc-
215 Wen hysteretic model expressed as (Wen 1976)

$$216 \quad u_y \dot{z} = A \dot{u}_i - \gamma |\dot{u}_i| |z|^{\eta-1} - \delta \dot{u}_i |z|^\eta \quad (5)$$

217 Note that the Bouc-Wen model has been widely used in the scientific literature to model the behavior of laminated
218 rubber and lead-rubber bearings (e.g. Nagarajaiah & Xiaohong 2000), as well as sliding-friction elements (Tsiatas
219 and Karatzia 2020). Herein, a generic type of lead rubber bearing is assumed (e.g. De Domenico & Ricciardi
220 2018). In this context, in Eq.(4), c_i is a viscous damping coefficient modelling the energy dissipation attributed to
221 the rubber phase of the isolation layer, k_i is the initial stiffness prior to the yielding of the lead cores, αk_i is the
222 post-yielding stiffness, and F_y is the yielding strength of the isolators (see also Fig.1(b)). Further, in Eq.(5), u_y is
223 the yielding displacement, while δ , γ , η and A are the Bouc-Wen model parameters which control the shape and
224 inclination of the hysteretic $F_i - u_i$ loop (Wen 1976, Ikhouane et al 2006). In the numerical part of this work, the
225 Bouc-Wen parameters are taken as $\delta = \gamma = 0.5$, $A = \eta = 1$ and $\alpha = 0.10$, which model a standard smooth hysteretic
226 behaviour, as shown in the inset of Fig.1(b), widely adopted in the literature to model lead rubber isolators (e.g.
227 De Domenico & Ricciardi 2018).

228 Moreover, the stiffness of the isolated rooftop is quantified by the post-yielding effective period of the
229 isolation layer given as (Jangid 2010)

$$230 \quad T_i = 2\pi \sqrt{\frac{m_i}{\alpha k_i}} = 2\pi \sqrt{\left(\frac{u_y}{F_o g \alpha} \right)} \quad (6)$$

231 where $g = 9.81 \text{m/s}^2$ and F_o is the normalized yielding strength defined as

$$232 \quad F_o = \frac{F_y}{m_i g} \quad (7)$$

233 2.3 Nonlinear equations of motion of the simplified model

234 Using the modelling assumptions discussed above, the equations of motion of the simplified model in Fig.1(b)
235 are written as

$$236 \quad \begin{aligned} m_s \ddot{u}_s + c_s \dot{u}_s + k_s u_s - F_i - F_b &= -m_s \ddot{x}_g \\ m_i (\ddot{u}_i + \ddot{u}_s) + F_i - F_v &= -m_i \ddot{x}_g \\ m_d (\ddot{u}_k + \ddot{u}_i + \ddot{u}_s) + F_v + F_b &= -m_d \ddot{x}_g \end{aligned} \quad (8)$$

237 By introducing the dimensionless parameters: natural frequency of the primary structure, ω_s , and of the TMDI,
238 ω_d , defined as

$$239 \quad \omega_s = \sqrt{\frac{k_s}{m_s}} \text{ and } \omega_d = \sqrt{\frac{k_d}{b+m_d}}, \quad (9)$$

240 critical damping ratio of the primary structure, ξ_s , of the isolation layer, ξ_i , and of the TMDI, ξ_d , defined as

$$241 \quad \xi_s = \frac{c_s}{2m_s\omega_s}, \quad \xi_i = \frac{c_i}{2\omega_i m_i} \text{ and } \xi_d = \frac{c_d}{2(m_d+b)\omega_d}, \quad (10)$$

242 mass ratios of the isolated slab, μ_i , and of the TMDI, μ_d , and inertance ratio, β , defined as

$$243 \quad \mu_i = \frac{m_i}{m_s}, \quad \mu_d = \frac{m_d}{m_s} \text{ and } \beta = \frac{b}{m_s}, \quad (11)$$

244 the equations of motion in Eq.(8) are further written in matrix form as

$$245 \quad \mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} + \mathbf{p}z = -\mathbf{d}\ddot{x}_g, \quad (12)$$

246 where

$$247 \quad \mathbf{M} = \begin{bmatrix} 1 & -\beta & -\beta \\ \mu_i & \mu_i & 0 \\ \mu_d & \mu_d + \beta & \mu_d + \beta \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 2\xi_s\omega_s & -2\xi_i\omega_i & 0 \\ 0 & 2\xi_i\omega_i & -2\xi_d\omega_d(\mu_d + \beta) \\ 0 & 0 & 2\xi_d\omega_d(\mu_d + \beta) \end{bmatrix}, \quad (13)$$

$$\mathbf{K} = \begin{bmatrix} \omega_s^2 & -\alpha\mu_i\omega_i^2 & 0 \\ 0 & \alpha\mu_i\omega_i^2 & -\omega_d^2(\mu_d + \beta) \\ 0 & 0 & \omega_d^2(\mu_d + \beta) \end{bmatrix}, \quad \mathbf{p} = \begin{bmatrix} -\mu_i(1-\alpha)F_0g \\ \mu_i(1-\alpha)F_0g \\ 0 \end{bmatrix}, \quad \mathbf{d} = \begin{bmatrix} 1 \\ \mu_i \\ \mu_d \end{bmatrix} \text{ and } \mathbf{u} = \begin{bmatrix} u_s \\ u_i \\ u_k \end{bmatrix}.$$

248 It is important to recognize that the system of equations in Eq.(12) is nonlinear due to the nonlinear
249 (hysteretic) relationship between the coordinates z and u_i in Eq.(5). This nonlinearity significantly impedes the
250 analysis and, therefore, the optimal design (or tuning) of IR-TMDI by requiring computationally expensive
251 nonlinear response history analyses. To this end, Eq.(5) is treated via stochastic linearization in the next section
252 to derive an equivalent linear system with the view of expediting the IR-TMDI tuning and seismic response
253 assessment under stochastic seismic excitation.

254 3 Stochastic Linearization and random vibration analysis

255 3.1 Equivalent linear system

256 Commonly, stochastic ground excitation models are used in the optimal tuning of inertial dampers to account
257 for the uncertain nature of the earthquake ground motion (e.g. Marian and Giaralis 2014, Taflanidis et al. 2019).
258 For linear structural systems and absorbers, this is significantly facilitated by straightforward application of linear
259 random vibration analysis. In the presence of nonlinearities, computationally efficient stochastic linearization (SL)
260 approaches can be employed for the task (e.g. Sgobba and Marano 2010, Mitseas et al. 2018) which approximate

261 the nonlinear stochastic seismic response of structures by the response of an underlying equivalent linear dynamic
 262 system (ELS) under the same stochastic ground excitation without the need for nonlinear response history analysis
 263 (Spanos & Giaralis 2013). To this end, the SL approach introduced by Wen (1980) for the Bouc Wen model is
 264 herein applied to substitute Eq. (5) with an equivalent linear equation given as

$$265 \quad u_y \dot{y} + c_{eq} \dot{x}_i + k_{eq} y = 0. \quad (14)$$

266 In the last expression, y and \dot{x}_i are response processes of the ELS corresponding to the ordinates z and u_i of the
 267 nonlinear system. Further, c_{eq} and k_{eq} are deterministic parameters obtained by minimising the mean square error
 268 between the nonlinear system defined by Eqs.(5) and (12) and the ELS (Eqs.(14) and (12)), under the same \ddot{x}_g
 269 stationary Gaussian stochastic excitation process. Using the fact that y and \dot{x}_i are jointly Gaussian, the equivalent
 270 linear parameters in Eq.(14) are given as (Wen 1980)

$$271 \quad c_{eq} = \sqrt{\frac{2}{\pi}} \left(\gamma \frac{E[\dot{x}_i y]}{\sigma_{\dot{x}_i}} + \delta \sigma_y \right) - A \quad \text{and} \quad k_{eq} = \sqrt{\frac{2}{\pi}} \left(\gamma \sigma_{\dot{x}_i} + \delta \frac{E[\dot{x}_i y]}{\sigma_y} \right), \quad (15)$$

272 for $\eta = 1$, where $E[\cdot]$ is the mathematical expectation operator and σ_x denotes the standard deviation of the
 273 process x . Importantly, the equivalent linear parameters c_{eq} and k_{eq} are unknown as they depend on the unknown
 274 response statistics $\sigma_{\dot{x}_i}$ and σ_y of the ELS. Therefore, iterative linear random vibration analyses is required to
 275 determine the parameters in Eq.(15) together with the response statistics of the ELS. Such analyses can be
 276 efficiently undertaken for different seismic stochastic excitations in state space as discussed in the following
 277 section.

278 *3.2 Random vibration analysis and determination of the equivalent linear parameters*

279 In this work, efficient linear random vibration analysis is performed in state-space to obtain response
 280 statistics of the ELS, together with the equivalent linear parameters in Eq.(15) in the context of stochastic
 281 linearization, under different stationary stochastic excitations. This is achieved by first expressing the equations
 282 of motion of the ELS in state-space as

$$283 \quad \dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}w \quad (16)$$

284 where \mathbf{A} is the state matrix, \mathbf{B} is the excitation vector, \mathbf{x} is the vector of the unknown state variables, and w is
 285 Gaussian white noise excitation. Then, the covariance matrix $\mathbf{\Gamma}$ collecting all second-order response statistics of
 286 the processes in vector \mathbf{x} can be determined by solving the Lyapunov equation

$$287 \quad \mathbf{A}\mathbf{\Gamma} + \mathbf{\Gamma}\mathbf{A}^T + 2\pi S_0 \mathbf{B}\mathbf{B}^T = 0 \quad (17)$$

288 where S_0 is the spectral intensity of the white noise w . In the computational part of this study, the built-in
 289 MATLAB function *lyap* is used to obtain numerically the covariance matrix $\mathbf{\Gamma}$. The diagonal elements of the latter

290 matrix are the variances of the states in vector \mathbf{x} while the off-diagonal elements are the cross-variances of all the
291 states.

292 Notably, Eqs. (15) and (16) can be used to determine the response statistics of the ELS, for given equivalent
293 linear parameters in Eq.(14), under stationary white noise as well as colored noise (filtered Kanai-Tajimi)
294 excitations. For stationary white noise excitation, the state matrix, excitation vector, and state variable vector are
295 written as

$$296 \quad \mathbf{A} = \begin{bmatrix} \mathbf{0}_{(3,3)} & \mathbf{I}_{(3)} & \mathbf{0}_{(3,1)} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} & -\mathbf{M}^{-1}\mathbf{P} \\ \mathbf{0}_{(1,3)} & \left\{ 0 \quad \frac{c_{eq}}{q} \quad 0 \right\} & \frac{k_{eq}}{q} \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} \mathbf{0}_{(3,1)} \\ -\mathbf{M}^{-1}\mathbf{d} \\ 0 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} x_s \\ x_i \\ x_k \\ \dot{x}_s \\ \dot{x}_i \\ \dot{x}_k \\ y \end{bmatrix}, \quad (18)$$

297 respectively, where the response coordinates of the ELS are denoted by a different symbol from those of the
298 nonlinear system in Eqs. (5) and (12) (i.e. x is used in place of u , and y in place of z as discussed before) to
299 emphasize that the ELS response is different (i.e. an approximation) of the response of the nonlinear system. In
300 Eq.(18), $\mathbf{0}_{(m,n)}$ is the m -by- n zero matrix, $\mathbf{I}_{(m)}$ is the m -by- m identity matrix, and the exponent “-1” demotes matrix
301 inversion. Further, the spectral intensity S_o in Eq. (17) is related to the peak ground acceleration (PGA) for the
302 case of clipped white noise excitation with double-sided power spectral density function using the expression

$$303 \quad S_o = \frac{PGA^2}{18\omega_c} \quad (19)$$

304 where ω_c is the cut-off frequency of the excitation, derived under the approximate “ 3σ ” rule.

305 In case it is deemed important to account for site soil conditions, the ground excitation can be
306 modelled by a Gaussian stationary colored noise, represented in the domain of frequencies ω by the
307 widely-used for the purpose filtered Kanai-Tajimi (K-T) spectrum given as (Clough and Penzien 2003)

$$308 \quad S(\omega) = S_o \frac{\omega_g^4 + 4\xi_g^2 \omega_g^2 \omega^2}{(\omega_g^2 - \omega^2)^2 + 4\xi_g^2 \omega_g^2 \omega^2} \frac{\omega^4}{(\omega_f^2 - \omega^2)^2 + 4\xi_f^2 \omega_f^2 \omega^2}, \quad (20)$$

309 where ω_g and ξ_g are the natural frequency and damping ratio of the soil, respectively, which is
310 modelled as a white noise excited linear 1-DOF system with spectral intensity S_o . Moreover, ω_f and
311 ξ_f are parameters of a high-pass filter incorporated in Eq.(20) to eliminate spurious low-frequency
312 content. In subsequent sections, different soil conditions are accounted for by using the parameters in Table 1
313 derived in Giaralis and Spanos (2012).

314

315

316 **Table 1:** Filtered Kanai-Tajimi parameters for different soil conditions (Giaralis and Spanos 2012)

Soil type	ω_g (rad/s)	ζ_g	ω_f (rad/s)	ζ_f	Dominant period $2\pi/\omega_g$ (s)
firm	10.73	0.78	2.33	0.90	0.59
soft	5.34	0.88	2.12	1.17	1.18

317

318 Conveniently, the spectrum in Eq.(20) can be readily incorporated in the state-space formulation in Eq.(16) as a
 319 pre-filter excited by white noise (e.g. Taflanidis and Giaralis 2018) by augmenting the expressions in Eq.(18) as
 320 follows

$$\mathbf{A} = \begin{bmatrix}
 \mathbf{0}_{(3,3)} & \mathbf{I}_{(3)} & \mathbf{0}_{(3,1)} & \mathbf{0}_{(3,1)} & \mathbf{0}_{(3,1)} & \mathbf{0}_{(3,1)} & \mathbf{0}_{(3,1)} \\
 -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} & -\mathbf{M}^{-1}\mathbf{p} & \omega_f^2\mathbf{M}^{-1}\mathbf{d} & -\omega_g^2\mathbf{M}^{-1}\mathbf{d} & 2\xi_f\omega_f\mathbf{M}^{-1}\mathbf{d} & -2\xi_g\omega_g\mathbf{M}^{-1}\mathbf{d} \\
 \mathbf{0}_{(1,3)} & \left\{ \begin{array}{c} 0 \\ -\frac{c_{eq}}{q} \\ 0 \end{array} \right\} & -\frac{k_{eq}}{q} & 0 & 0 & 0 & 0 \\
 \mathbf{0}_{(1,3)} & \mathbf{0}_{(1,3)} & 0 & 0 & 0 & 1 & 0 \\
 \mathbf{0}_{(1,3)} & \mathbf{0}_{(1,3)} & 0 & 0 & 0 & 0 & 1 \\
 \mathbf{0}_{(1,3)} & \mathbf{0}_{(1,3)} & 0 & -\omega_f^2 & \omega_g^2 & -2\xi_f\omega_f & 2\xi_g\omega_g \\
 \mathbf{0}_{(1,3)} & \mathbf{0}_{(1,3)} & 0 & 0 & -\omega_g^2 & 0 & -2\xi_g\omega_g
 \end{bmatrix},$$

321 $\mathbf{B} = \{0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ -1\}^T,$ (21)

$$\mathbf{x} = \{x_s \ x_i \ x_k \ \dot{x}_s \ \dot{x}_i \ \dot{x}_k \ y \ x_f \ x_g \ \dot{x}_f \ \dot{x}_g\}^T$$

322 Thus, Eq.(17) can be used in conjunction with Eq.(21) to determine all the response statistics of the ELS for
 323 filtered Kanai-Tajimi excitation. In this case, the spectral intensity S_o is related to the PGA by (Sgobba and Marano
 324 2010)

325 $S_o = \left(\frac{4}{9\pi}\right) \frac{\xi_g \text{PGA}^2}{\omega_g \sqrt{1+4\xi_g^2}}.$ (22)

326 As previously discussed, the response statistics of the ELS in matrix $\mathbf{\Gamma}$ need to be determined together with
 327 the ELS parameters c_{eq} and k_{eq} by using iteratively Eqs. (15) and (17) until convergence (Roberts & Spanos 2003).
 328 For numerical implementation, seed values $c_{eq} = -1$ and $k_{eq} = 0$ are assumed in the computational part of this work
 329 and iterations are performed until consecutive values of both of the ELS parameters differ by less than 10^{-6}
 330 (stoppage criterion).

331 4 Optimal IR-TMDI tuning

332 4.1 Optimal design problem formulation and alternative tuning criteria (objective functions)

333 In this section, optimal tuning of the IR-TMDI in Fig.1 is sought by considering the response of the ELS
 334 defined in Section 3 with primary structure properties ω_s and ζ_s under stationary stochastic seismic excitation. For
 335 this purpose, an optimal IR-TMDI design problem is formulated involving 7 independent design parameters.

336 These are the TMDI damping and frequency ratios, ζ_d and $\lambda = \omega_d/\omega_s$, collected in vector \mathbf{v} and treated as primary
 337 design variables, as well as the isolation layer damping and effective natural period, ζ_i and T_i , and the IR-TMDI
 338 inertial ratios μ_i , μ_d , and β collected in vector \mathbf{v} and treated as secondary design variables. In this context, the
 339 proposed optimal IR-TMDI design/tuning problem seeks to determine the primary parameters in \mathbf{v} , within a
 340 prespecified search range $[\mathbf{v}_{\min}, \mathbf{v}_{\max}]$, to minimize an objective function J (tuning criterion), dependent on the
 341 ELS stationary response statistics, for a set of fixed secondary parameters in \mathbf{v} . This distinction between primary
 342 and secondary IR-TMDI design parameters is purposely made to facilitate comprehensive parametric
 343 investigations, presented in a subsequent section, aiming to quantify the influence of different IR-TMDI properties
 344 to the seismic response mitigation and performance of IR-TMDI for various primary structure properties and
 345 stochastic seismic excitations. Mathematically, the considered optimal IR-TMDI tuning problem can be written
 346 as

$$347 \quad \min_{\mathbf{v}} \{J(\mathbf{v}|\mathbf{v})\} \quad \text{with} \quad \mathbf{v}_{\min} \leq \mathbf{v} \leq \mathbf{v}_{\max} \quad \text{where} \quad \mathbf{v} = \left\{ \zeta_d, \lambda = \frac{\omega_d}{\omega_s} \right\}, \quad \mathbf{v} = \{\zeta_i, T_i, \mu_i, \mu_d, \beta\} \quad (23)$$

348 The optimal tuning problem in Eq.(23) can accommodate different tuning criteria, targeting the minimization
 349 of specific response quantities based on the choice of the objective function J . In this respect, one viable tuning
 350 criterion is the maximization of the TMDI energy dissipation index (EDI), proposed by Pietrosanti et al. (2017).
 351 This tuning criterion was shown to achieve efficient optimal TMDI designs for the seismic response mitigation of
 352 base isolated structures, striking a good balance between reducing seismic displacement and acceleration demands
 353 (De Domenico & Ricciardi 2018, De Angelis et al. 2019). In this work, the EDI is defined as the portion of the
 354 input stationary stochastic seismic energy dissipated by the TMDI damping element with coefficient c_d over the
 355 total energy dissipated by the ELS. The latter is the sum of the energy dissipated by the primary structure (through
 356 the inherent damping coefficient c_s), the TMDI, and the isolators (through the viscous damping coefficient c_i and
 357 the hysteretic behavior), as detailed in the energy balance analysis undertaken in the Appendix A. The EDI is
 358 computed as (see Appendix A for derivation)

$$359 \quad 0 \leq \text{EDI} = \frac{c_d \sigma_{\dot{x}_k}^2}{c_s \sigma_{\dot{x}_s}^2 + c_d \sigma_{\dot{x}_k}^2 + c_i \sigma_{\dot{x}_i}^2 + (1-\alpha) F_y E[\dot{x}_i y]} \leq 1 \quad (24)$$

360 and the associated objective function supporting a minimization problem in Eq.(23) is expressed as (De Domenico
 361 & Ricciardi 2018)

$$362 \quad J_i = \text{FEI} = 1 - \text{EDI}, \quad (25)$$

363 where FEI stands for the filtered energy index. Notably, FEI signifies the portion of the total energy that is not
 364 dissipated by the TMDI. Thus, the minimization of FEI leads to IR-TMDI designs which maximize energy
 365 dissipation to take place by the TMDI rather than by the isolation layer for given isolation layer properties and
 366 IR-TMDI inertial (mass and inertance) properties.

367 Further to the FEI, two alternative tuning criteria are also considered since they have been widely used in
 368 the optimal TMDI tuning of non-isolated structures under stochastic seismic excitations (e.g. Marian and Giaralis

369 2014, Pietrosanti et al. 2017, Rajana et al. 2022). One criterion seeks to minimize the relative displacement
 370 (deflection) variance of the primary structure and is introduced in Eq.(23) by the normalized objective function

$$371 \quad J_2 = \frac{\sigma_{x_s}^2}{\sigma_{x_o}^2}, \quad (26)$$

372 where x_o is the relative displacement of the uncontrolled primary structure (i.e. without IR-TMDI). This criterion
 373 is well-related to the minimization of the structural damage in the primary structure. The other criterion seeks to
 374 minimize the total acceleration variance of the primary structure and is introduced in Eq.(23) by the normalized
 375 objective function

$$376 \quad J_3 = \frac{\sigma_{x_s,abs}^2}{\sigma_{x_o,abs}^2} \quad (27)$$

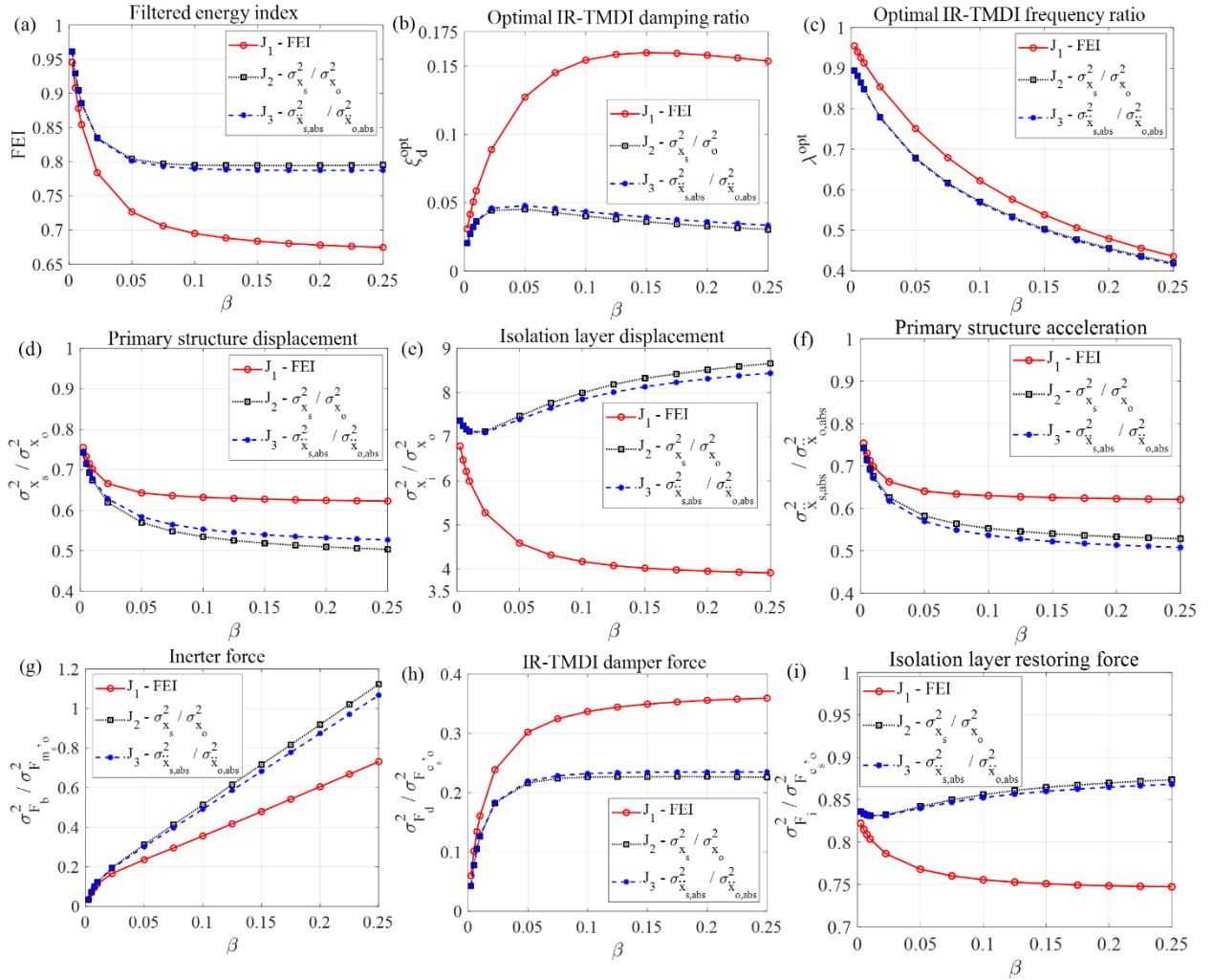
377 where $x_{s,abs}$ and $x_{o,abs}$ are the total acceleration of the controlled and the uncontrolled primary structure for the same
 378 stochastic excitation. The latter criterion is well-related to the minimization of the damage of non-structural
 379 components and equipment housed in the primary structure.

380 For all the adopted objective functions in Eqs.(25-27), the optimization problem in Eq.(23) is numerically
 381 solved using standard pattern search (Charles and Dennis, 2008) implemented in the built-in MATLAB®
 382 command `fminsearch`. In all the subsequent numerical work, the boundaries of the search range is set as $\mathbf{v}_{min} =$
 383 $\{0,0\}$ and $\mathbf{v}_{max} = \{2,2\}$. This search range suffices to determine a global optimum of the problem in Eq.(23) which
 384 was found to be convex for all sets of IR-TMDI parameters, primary structure properties, objective functions, and
 385 stochastic excitations investigated in the following Sections.

386 4.2 Numerical investigation of alternative tuning criteria

387 A numerical investigation is herein undertaken to appraise the potential of the three previously discussed
 388 tuning criteria for yielding advantageous IR-TMDI designs. This is pursued by solving the optimization problem
 389 in Eq.(23) for the objective functions in Eqs.(25)-(27) for different inertance ratios varying within the range [0,
 390 25%] and for white noise excited ELS with structural period $T_s = 2\pi/\omega_s = 1s$, structural inherent damping ratio ξ_s
 391 $=5\%$, isolation mass ratio $\mu_i = 10\%$, isolation period $T_i = 3.5s$ (assuming yielding displacement $u_y = 2cm$ and
 392 normalized yielding strength $F_o = 0.065$ in Eq.(7)), isolation damping ratio $\xi_i = 5\%$, and secondary mass ratio $\mu_d =$
 393 0.5% . The white noise intensity is assumed to correspond to PGA=0.3g with cut-off frequency $\omega_c = 15\pi$ in
 394 Eq.(19). Numerical results obtained from the optimal tuning and from the ELS response with optimal IR-TMDIs
 395 are presented in Fig. 2. Specifically, 9 different quantities are examined, described in the titles of each of the 9
 396 panels of Fig.2, against the inertance ratio for each of the three different tuning criteria. The top row of panels
 397 pertain to results from optimal tuning: the FEI objective function in Eq.(25) is plotted in Fig.2(a), while the optimal
 398 values of IR-TMDI primary design parameters in vector \mathbf{v} of Eq.(23) are plotted in Figs.2(b) and 2(c). The middle
 399 row of panels in Fig.2 present ELS response variance data from optimally tuned IR-TDMI normalized by response
 400 variances of the uncontrolled primary structure (with no IR-TMDI): Fig.2(d) plots the normalized relative
 401 displacement variance of the primary structure J_2 in Eq.(26), Fig.2(e) plots the deflection variance of the isolated
 402 layer normalized as in the J_2 index, and Fig.2(f) plots the normalized absolute acceleration variance of the primary

403 structure J_3 in Eq.(27). Lastly, the bottom row of panels in Fig.2 present data from forces developing in the ELS
 404 with optimally tuned IR-TMDI: Fig.2(g) plots the variance of the inerter force in Eq.(2) using ELS response
 405 acceleration variances normalized by the variance of the inertia force developed in the uncontrolled structure,
 406 Fig.2(i) plots the variance of the isolators' force in Eq.(4) using ELS response statistics normalized by the variance
 407 of the inherent damping force developed in the uncontrolled structure, and Fig.2(h) plots the variance of the force
 408 developed at the TMDI damping element (e.g. $F_d = c_d \dot{\sigma}_{\dot{x}_k}^2$) using the same normalization as in Fig.2(i).



409

410 **Fig. 2:** Optimal IR-TMDI properties and normalized system response variances plotted against the inertance
 411 ratio for different tuning criteria (objective functions) and for $T_s=1s$, $\xi_s=5\%$, $\mu_i=10\%$, $T_i=3.5s$, $\xi_i=5\%$,
 412 $\mu_d=0.5\%$, white noise excitation with $PGA=0.3g$ and $\omega_c=15\pi$ (Base case).

413 It is important to note that the ordinates of the plots in the middle and the bottom row of the panels are herein
 414 used solely to establish relative seismic response trends and, thus, are interpreted in a relative rather than in an
 415 absolute sense. This is because they correspond to linear systems (ELs) under stationary base excitation which
 416 have limited capability for dependable prediction of the expected seismic response of IR-TMDI structures. This
 417 is due to the fact that actual seismic excitation is non-stationary, as reflected on the time-evolving amplitude and
 418 frequency content of recorded earthquake strong ground motions (e.g. Spanos et al. 2007), while the potential of

419 the ELS to estimate the nonlinear behaviour of the hysteretic isolators is restricted by the well-known limitations
420 of statistical linearization (e.g. Roberts & Spanos 2003, Spanos & Giaralis 2013).

421 From the top row of panels in Fig.2, it is seen that IR-TMDI optimal tuning based on the FEI criterion
422 achieves significantly higher TMDI energy dissipation for any given value of inertance ratio compared to tunings
423 aiming at structural displacement minimization, J_2 , or acceleration minimization, J_3 , as readily deduced from
424 Fig.2(a). This is due to a significantly higher optimal TMDI damping ratio obtained by the FEI-based tuning
425 compared to the other two tuning criteria as seen in Fig.2(b), (e.g. more than three times higher for $\beta > 7\%$).
426 Notably, differences in the optimal TMDI frequency ratio among the three tuning criteria are non-negligible as
427 seen in Fig.2(c), but are much less prominent compared to differences in the optimal TMDI damping ratio. Still,
428 the system response kinematics data shown in the middle row of panels in Fig.2, evidence that FEI-based tuning
429 results in lower primary response reductions in terms of displacement variance, Fig. 2(d), and acceleration
430 variance, Fig. 2(f), compared to the other two examined tuning criteria, for any given inertance ratio. Further, it
431 is seen in Figs.2(d) and 2(f) that, as expected, J_2 -based tuning achieves the best (i.e. lowest) structural
432 displacement performance across the competing tuning criteria while J_3 -based tuning achieves the best structural
433 acceleration performance, respectively.

434 Nevertheless, FEI-based tuning poses significantly lower deflection demands to the isolation layer (i.e. to
435 the LRB bearings) compared to the tunings using J_2 and J_3 criteria as shown in Fig.2(e). For example, the required
436 isolation layer deflection variances for both J_2 - and J_3 -based tuning, for $\beta \geq 10\%$, are more than twice that of FEI-
437 based tuning. Furthermore, for FEI-based tuning, isolation deflection demands decrease with inertance, while
438 other competing tuning criteria necessitate higher isolation layer deflection with increasing inertance. At the same
439 time, FEI-based tuning reduces significantly the forces exerted by the IR-TMDI to the primary structure compared
440 to the J_2 and J_3 based tunings, that is the inerter force and the isolators' force as evidenced in Figs.2(g) and 2(i),
441 respectively. Further, the reduction of these forces become increasingly higher as the inertance ratio increases.
442 Specifically, it is seen in Fig.2(g) that for any given inertance value, the FEI-based normalized inerter force
443 variance is much lower compared to the values pertaining to the other tunings. More importantly, the reduction
444 of the inerter forces increases monotonically with the inertance ratio as the rates of increase of the inerter force
445 variance with inertance (slope of the curves in Fig.2(g)) is appreciably lower for the FEI-based tuning compared
446 to the J_2 and J_3 based tunings. Additionally, as with the inerter force variance, it is observed from Fig.2(i) that the
447 variance of the isolators' forces for FEI-based tuning vis-à-vis J_2 and J_3 based tuning is always lower while the
448 benefit (difference) increases with inertance. This is because the isolators' force reduces appreciably with
449 inertance for FEI-based tuning, while it slightly increases with inertance for the alternative tunings. In this regard,
450 although FEI-based tuning results in lower displacement and acceleration reductions compared to the J_2 and J_3
451 based tunings, it is overall more advantageous from a practical viewpoint since it yields reduced requirements for
452 isolation layer deflection and for isolators' and inerter force transmission which, collectively, become more
453 significant as the inertance increases. Notably, similar trends are found for different values of T_i and T_s than those
454 assumed in Fig.2, though results are not herein reported for the sake of brevity. To this end, it is recommended
455 that IR-TMDI is tuned for TMDI energy dissipation maximization (i.e. FEI tuning criterion in Eq.(25)) which
456 echoes previous recommendations in the literature on optimal tuning of TMDI with grounded inerter for the
457 seismic protection of base isolated structures (De Domenico & Ricciardi 2018, De Angelis et al. 2019).

458 As a closing remark to this section, it is worth noting that the recommended FEI-based tuning results in
459 higher TMDI damping forces compared to J_2 and J_3 based tunings as seen in Fig.2(h). The higher values in the
460 TMDI damping force variance are a direct result of the higher optimal TMDI damping coefficients coming from
461 the FEI-based tuning in Fig.2(b) for any arbitrary inertance ratio. In this respect, it is concluded that the FEI-based
462 tuning supports an IR-TMDI design whereby the TMDI is the main actor in mitigating primary structure seismic
463 response, as opposed to the bearings of the seismic isolation layer. In this setting, the significance of the TMDI
464 over the isolators for the seismic response mitigation increases as the inertance property increases. Indeed, for
465 FEI-based tuning, TMDI damping and inerter forces increase with inertance (Figs.2(g) and 2(h), respectively)
466 while the isolators' deflection and force demands reduce with inertance (Figs.2(e) and 2(i), respectively). The
467 latter consideration is of paramount practical importance since inertance can readily and economically scale-up in
468 real-life inerter devices (e.g. Nakamura et al. 2014, Nakaminami 2017, Pietrosanti et al. 2021), thus yielding an
469 increasingly efficient IR-TMDI for the seismic protection of primary building structures.

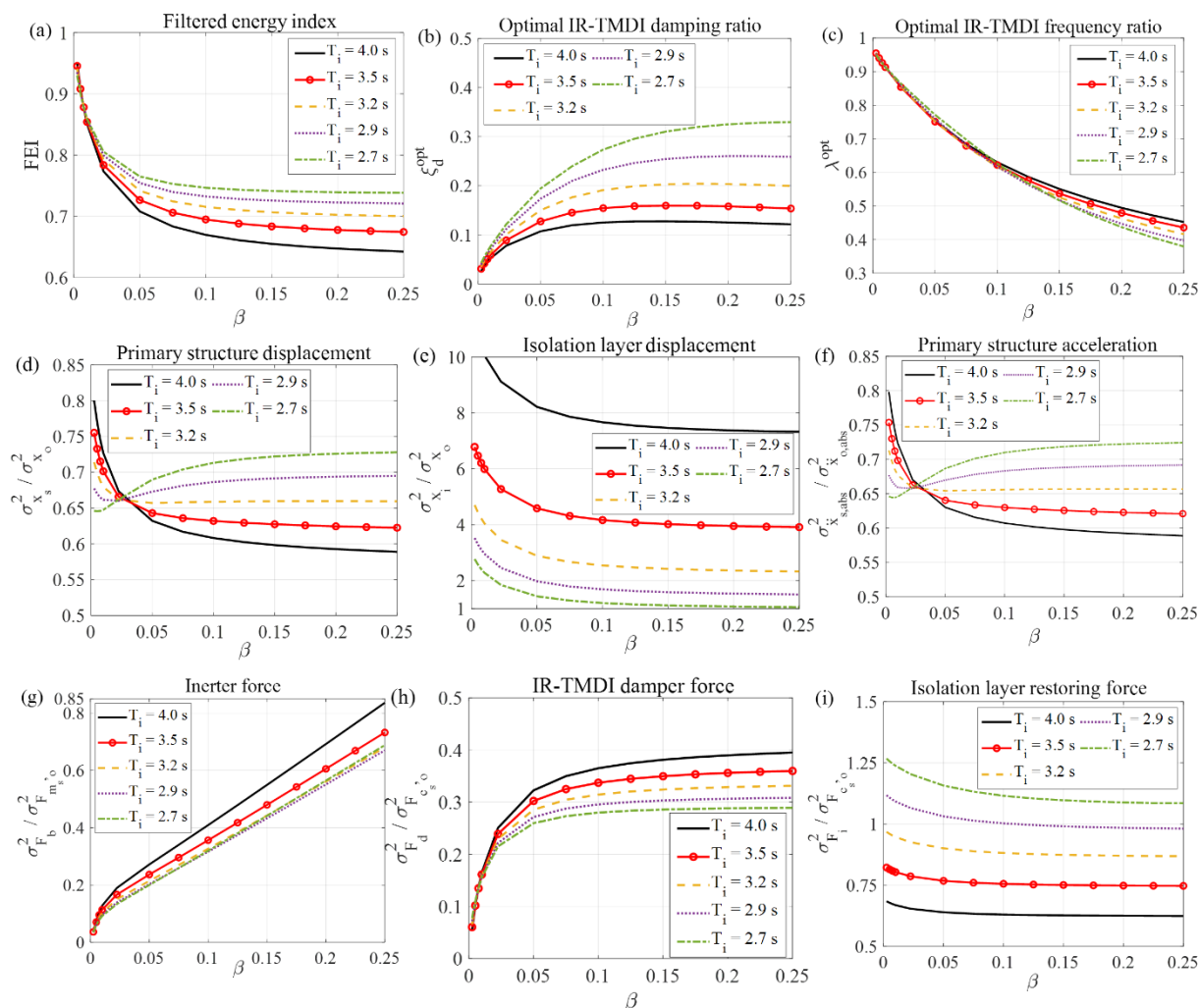
470 **5 Parametric investigation and assessment of optimally tuned IR-TMDI**

471 Having established a practically meritorious optimal tuning criterion for the proposed IR-TMDI (i.e. FEI in
472 Eq.(25)), this section presents and discusses numerical data from a comprehensive parametric investigation aiming
473 to assess the influence of the IR-TMDI properties (i.e. secondary design parameters in vector \mathbf{v} in Eq.(23)), of the
474 primary structure properties, and of the stationary excitation properties to the effectiveness of FEI-based optimally
475 tuned IR-TMDI. In doing so, the system properties and excitation considered in producing the results of Fig.2 are
476 taken as the base case whereas one parameter/property is let to vary each time to examine its influence/importance.
477 With one exception which will be pointed out later in the text, all numerical results in this section are presented
478 using the same plotting pattern and normalizations as in Fig.2 to facilitate comparisons across the different cases
479 and parameters considered vis-à-vis the base case. As a previously highlighted word of caution, the magnitude of
480 the reported kinematics (deflections) and forces in this section have only a relative significance and are herein
481 used to establish relative trends of the seismic response under varying system and excitation properties. This is
482 because they are derived from ELSs under stationary base excitation with limited capability for estimating the
483 expected seismic response of nonlinear IR-TMDI equipped structures. The presentation begins by examining the
484 influence of the isolation layer properties as they deviate from the base case in Fig.2.

485 *5.1 Influence of the isolated rooftop properties*

486 The stiffness k_i of the isolation bearings is expected to influence significantly the IR-TMDI seismic response
487 reduction potential, as it directly relates to the deflection of the isolated rooftop under seismic excitation and,
488 therefore, to the level of engagement of the TMDI. To quantify this influence, Fig.3 plots numerical data for 5
489 different values of the effective natural period of the isolation layer T_i in Eq.(6) obtained by varying the stiffness
490 k_i of the bearings, while all other system and excitation parameters are the same as in the base case of Fig.2. It is
491 found that the consideration of more flexible isolators or, equivalently, of longer period T_i , facilitates the seismic
492 energy dissipation by the TMDI as manifested by the reduced values of FEI with T_i for any given inertance ratio
493 in Fig.3(a). This effect results in improved primary structure performance as demonstrated by reductions to both
494 the structural displacement and acceleration variances in Figs.3(d) and (f), respectively, as the period T_i elongates

495 for any value of inertance ratio with $\beta > 2.5\%$. These improvements become more significant as the inertance ratio
 496 increases and are due to larger inerter and damper forces developed by the TMDI with increasing T_i as shown in
 497 Figs.3(g) and 3(h). It is worth noting that these higher TMDI forces are generated by the increased relative
 498 accelerations and velocities through the inerter and the damping elements. This is readily deduced for the inerter
 499 force in Fig.3(g) since the inerter force variance increases with T_i for any fixed inertance. Further, in Fig.3(h), the
 500 damping force variance increases with T_i for any given value of inertance, even though the optimal damping
 501 coefficient in Fig.3(b) reduces with T_i for the same inertance. Clearly, this is only possible if the relative velocity
 502 across the TMDI damping element increases with T_i .



503
 504 **Fig. 3:** Optimal IR-TMDI properties using the FEI tuning criterion and normalized system response variances
 505 plotted against the inertance ratio for different isolation layer periods T_i and for $T_s=1s$, $\xi_s=5\%$, $\mu_i=10\%$, $\xi_i=5\%$,
 506 $\mu_d=0.5\%$, white noise excitation with $PGA=0.3g$ and $\omega_c=15\pi$.

507 Nevertheless, the improved primary structure performance with T_i comes at the cost of increased deflections
 508 in the isolation layer as seen in Fig.3(e). In this regard, there is a trade-off between primary structure response and
 509 isolated layer response which is regulated by the stiffness of the isolation layer. At the same time, although
 510 improved primary structure performance is always achieved by the IR-TMDI compared to the uncontrolled
 511 structure (i.e. ordinates in Figs.3(d) and 3(f) are well below 1), the primary structure performance deteriorates as

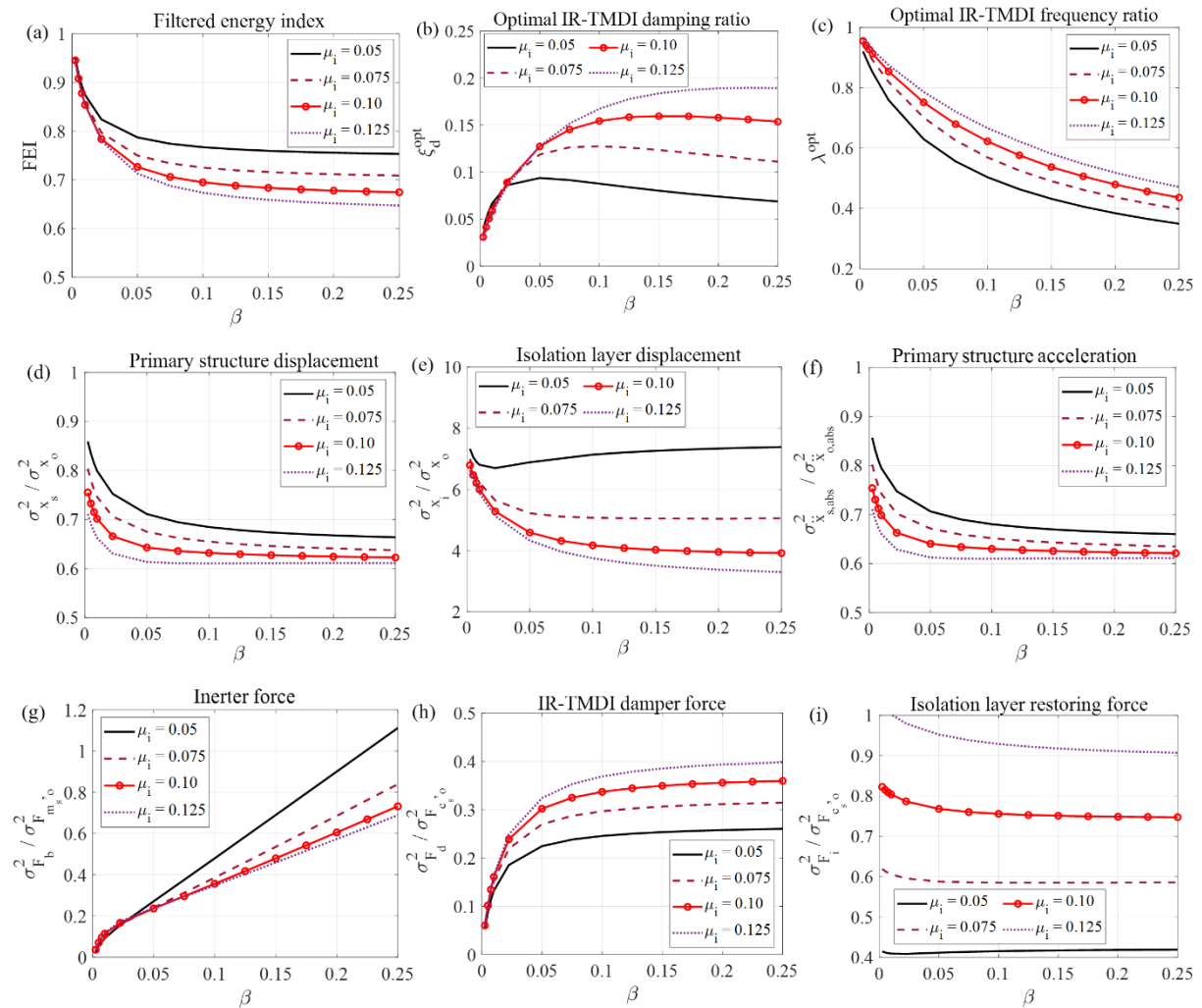
512 inertance increases when relatively stiff isolators are adopted (see cases of $T_i=2.7s$ and $2.9s$ in Figs. 3(d) and 3(f)).
513 To this end, a judicious selection of the isolators' stiffness k_i is required to promote higher values of T_i in conjunction
514 with high inertance, depending on the desired level of primary structure seismic response mitigation, while
515 ensuring the isolators' ability to perform large deflections without facing stability issues (see also Ryon & Earl
516 2010). This recommendation is further supported by the fact that the force demands of the isolators reduce as T_i
517 elongates for any inertance value as seen in Fig.3(i). Importantly, the latter observation confirms that the FEI-
518 based tuned IR-TMDI improves the primary structure performance through a better engagement of the TMDI
519 manifested by higher TMDI inerter and damping forces as the isolators become more flexible, provided there is
520 sufficient inertance (e.g. $\beta>2.5\%$ for the system considered), rather than through the action of the isolators (i.e.
521 through the force transmitted by the isolators to the primary structure).

522 Next, the influence of the isolated rooftop mass m_i is examined by plotting in Fig.4 the same type of
523 numerical data as in previous figures for 4 different values of the mass ratio μ_i including the value of the base
524 case, $\mu_i= 10\%$, while all other ELS and excitation parameters are the same as in the base case. To this effect, a
525 different isolators' stiffness k_i is chosen for each μ_i value such that the isolation layer period T_i is kept constant
526 and equal to the base case (i.e. $T_i= 3.5s$). This consideration establishes a meaningful comparison across systems
527 with different mass ratios μ_i and is widely employed in the literature to compare inter-storey isolated buildings
528 with different ratios of masses above and below the isolated layer (e.g. Faiella & Mele 2019). Parenthetically, it
529 is worth pointing out that if only μ_i is let to vary, then the isolation layer period T_i in Eq.(6) will change accordingly,
530 resulting in similar response trends as those in Fig.3.

531 It is seen in Fig.4(a) that the increase of the rooftop mass with constant T_i results in reductions to the FEI
532 index upon optimal IR-TMDI tuning (i.e. more energy is dissipated by the TMDI) for any given value of inertance.
533 These reductions in FEI yield improved primary structure performance for both displacement and accelerations
534 as shown in Figs.4(d) and 4(f), respectively. Here, the improved structural performance is due to significant
535 increase of the force exerted to the primary structure by the isolators with μ_i , as seen in Fig.4(i), while the
536 kinematics of the isolated rooftop reduce with μ_i , as evidenced by the reduced isolation layer deflections in
537 Fig.4(e). As a result, the level of engagement of the inerter (i.e. the relative acceleration between the TMDI
538 secondary mass and the top floor of the primary structure) reduces with μ_i , resulting in lower inerter force exerted
539 to the primary structure in Fig.4(g) as μ_i increases for fixed inertance. Still, the TMDI damping force increases
540 with μ_i due to the higher optimal damping ratios in Fig.4(b).

541 In this regard, it is found that the IR-TMDI achieves improved seismic primary structure performance either
542 by increasing the isolation rooftop period/flexibility, T_i , (for the same additive rooftop mass), or by increasing the
543 rooftop mass, μ_i (for the same period T_i). However, these improvements are achieved in significantly different
544 ways. On the one hand, increase of flexibility, T_i , leads to increased kinematics which engage more effectively
545 the TMDI inerter and the damper while reduce forces developing at the isolators at the expense of larger isolator
546 deflections. On the other hand, increase of rooftop mass reduce the deflection demands of the isolators, leading to
547 reduced inerter forces, at the expense of larger isolation layer forces. In view of the above, it becomes evident that
548 the stiffness and the mass properties of the isolated rooftop need to be carefully selected, accounting for practical
549 considerations concerning the additional (rooftop) mass that can be accommodated, especially in existing

550 buildings, and the required deformation demands that can be exhibited by the isolators without compromising
 551 their stability.

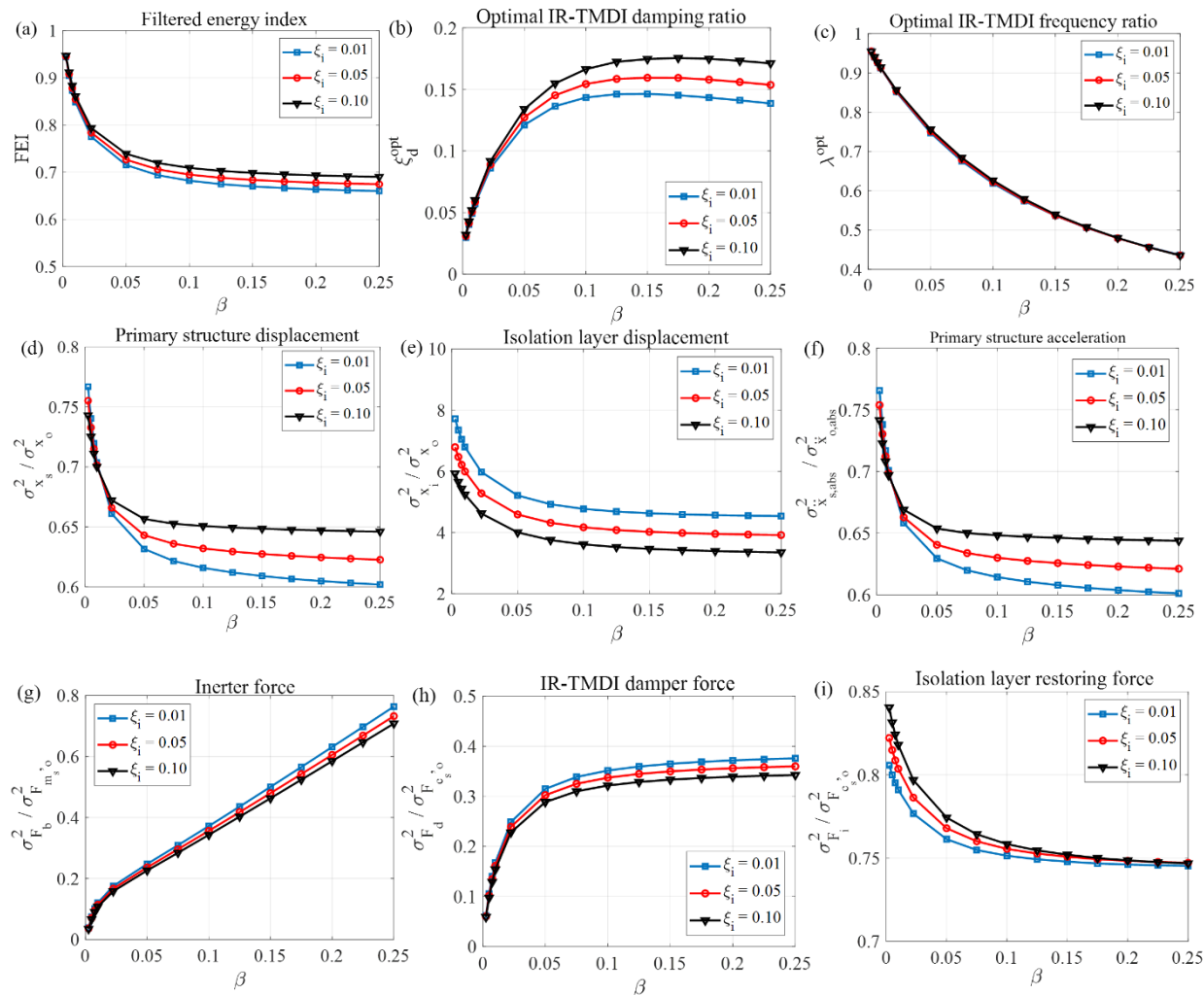


552

553 **Fig. 4:** Optimal IR-TMDI properties using the FEI tuning criterion and normalized system response variances
 554 plotted against the inertance ratio for different isolated slab mass ratios μ_i and for $T_s=1s$, $\zeta_s=5\%$, $T_i=3.5s$,
 555 $\zeta_i=5\%$, $\mu_d=0.5\%$, white noise excitation with $PGA=0.3g$ and $\omega_c=15\pi$.

556 Further to the isolators' stiffness and rooftop mass, the influence of the viscous damping ratio ζ_i of the
 557 isolation layer to the response of FEI-based tuned IR-TMDI is examined in Fig.5 whereby numerical data for
 558 three different ζ_i values are plotted while keeping ELS and excitation parameters the same as in the base case. It
 559 is found that by adopting low-damping bearings achieves lower FEI values (Fig.5(a)) and, consequently, better
 560 primary structure performance in terms of displacement (Fig.5(d)) and acceleration (Fig.5(f)). These
 561 improvements become consistently higher with inertance and are readily attributed to a better engagement of the
 562 TMDI through increased isolated rooftop kinematics/deflections (Fig.5(e)), similar to the case of reducing the
 563 stiffness of the isolators. In particular, lower isolation layer damping leads to higher TMDI inerter and damping
 564 forces and to lower shear force demands to the isolators. Interestingly, optimal TMDI frequency ratio is practically
 565 unaffected by changes to ζ_i , while optimal TMDI damping ratio reduces with ζ_i which further indicates that the
 566 increased damping forces and energy dissipation by the TMDI is due to larger kinematics (relative velocity) at the

567 ends of the damping element. In this respect, it is evident that the use of low-damping isolators (i.e. isolators
 568 which require a reduced damping coefficient c_i for modelling their resisting force in Eq.(4)) is preferable as it
 569 allows for more seismic energy dissipation to take place by the TMDI damper element and increases the amplitude
 570 of the TMDI damping and inerter forces over the isolators' force.



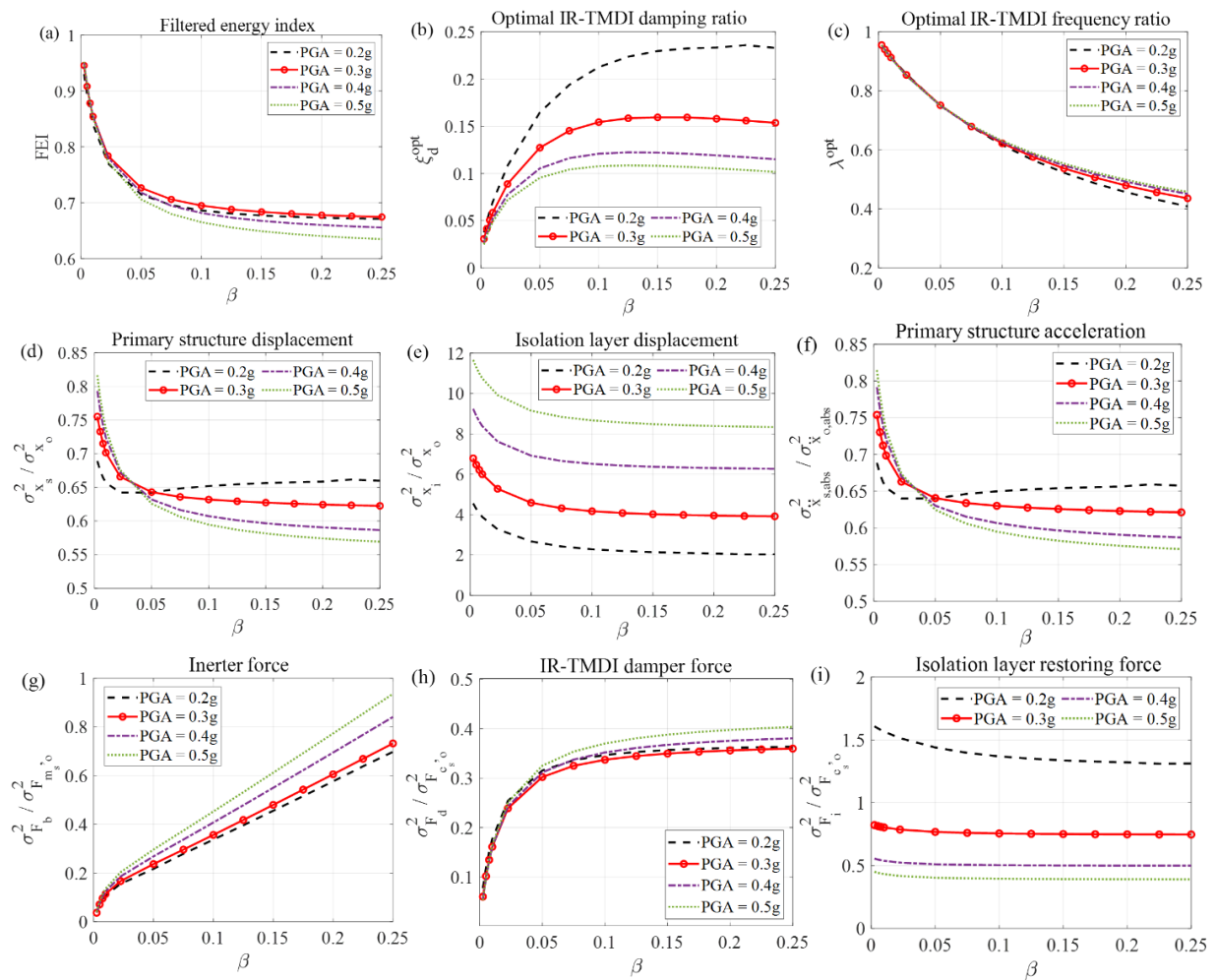
571
 572 **Fig. 5:** Optimal IR-TMDI properties using the FEI tuning criterion and normalized system response variances
 573 plotted against the inertance ratio for different isolated layer damping ratios ξ_i and for $T_s=1s$, $\zeta_s=5\%$, $\mu_i=10\%$,
 574 $T_i=3.5s$, $\zeta_i=5\%$, $\mu_d=0.5\%$, white noise excitation with $PGA=0.3g$ and $\omega_c=15\pi$.

575 **5.2 Influence of excitation properties**

576 In this section, attention is focused on assessing the influence of the amplitude and frequency content of the
 577 stationary base excitation adopted for the IR-TMDI tuning to the seismic response of the ELSs. For this purpose,
 578 Fig.6 furnishes numerical data pertaining to the base case properties with 4 different PGA values corresponding
 579 to different white noise excitation intensities as per Eq.(19). Further, in Fig.7 data for the base case system under
 580 the two colored noise (Filtered Kanai-Tajimi) excitations with different frequency content defined in Table 1 for
 581 $PGA=0.3g$ in Eq.(22) are compared to the data for white noise excitation with $PGA=0.3g$. These colored noise

582 excitations represent better the expected site-specific frequency content of seismic ground motions inasmuch as
 583 the local soil conditions are accurately captured by the Kanai-Tajimi filter parameters.

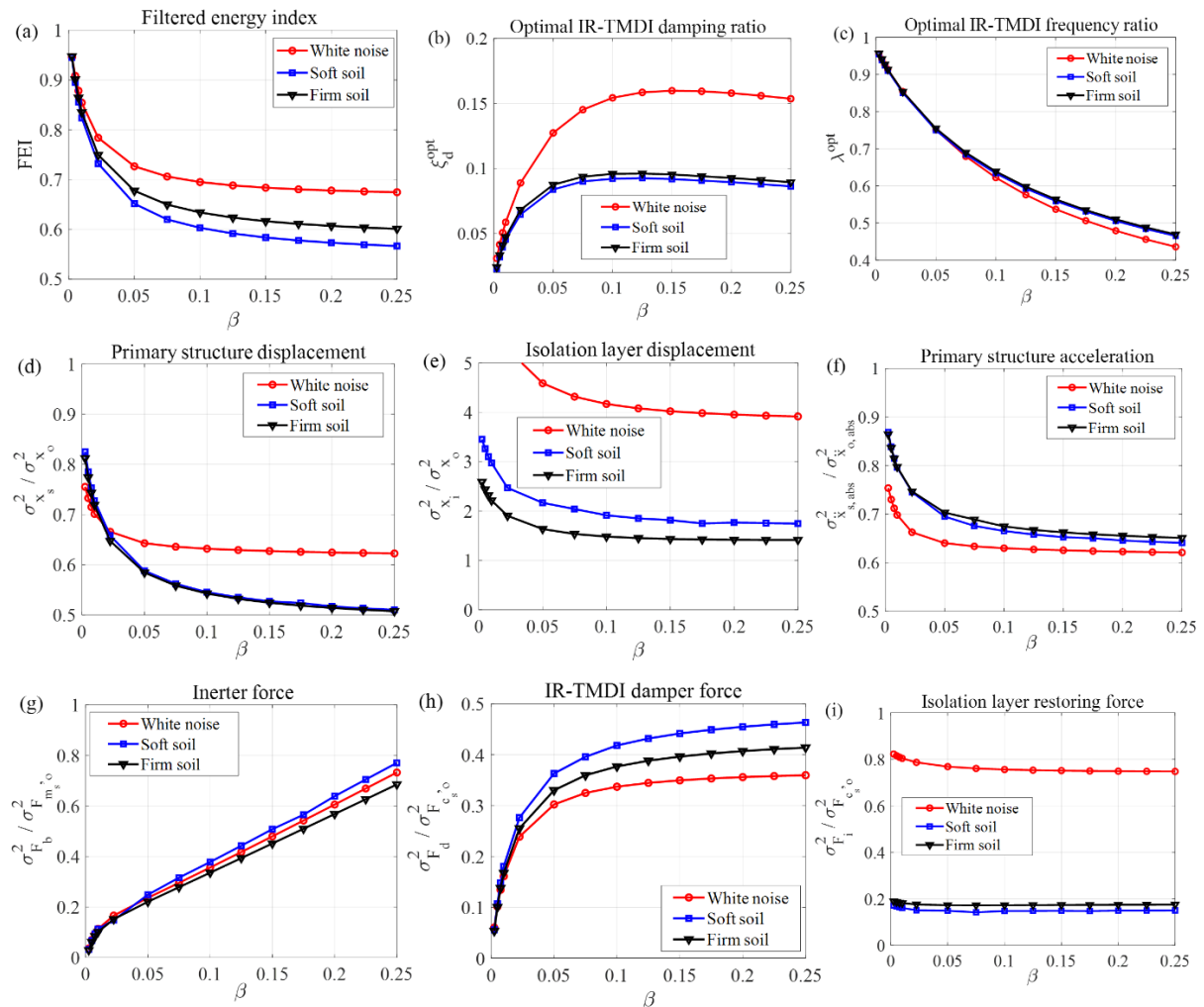
584 Examining first the variation of the excitation intensity in Fig.6, it is seen that the optimal TMDI damping
 585 ratio is rather sensitive to the assumed design PGA, which is not the case for the optimal TMDI frequency ratio
 586 in Fig.6(c). Importantly, the IR-TMDI becomes more efficient in reducing the primary structure response
 587 compared to the uncontrolled structure as PGA increases and for $\beta > 5\%$. This can be attributed to the larger
 588 isolation layer deflections induced by higher PGA excitation in Fig.6(e) which, in turn, engage better the TMDI
 589 as evidenced by the higher inerter and damping forces with PGA in Figs. 6(g) and 6(h), respectively. In this
 590 regime, the forces developed at the isolator reduce compared to the forces developed at the uncontrolled structure
 591 as PGA increases (Fig.6(i)) as a result of the increasingly important role of the TMDI in resisting seismic
 592 excitation with PGA.



593
 594 **Fig. 6:** Optimal IR-TMDI properties using the FEI tuning criterion and normalized system response variances
 595 plotted against the inertia ratio for white noise excitation with different PGA values and $\omega_c = 15\pi$, and for
 596 $T_s = 1s$, $\zeta_s = 5\%$, $\mu_i = 10\%$, $T_i = 3.5s$, $\zeta_i = 5\%$, $\mu_d = 0.5\%$.

597 Turning the attention to the data in Fig.7, it is seen that the frequency content of the seismic excitation does
 598 affect the optimal IR-TMDI tuning insofar as the optimal TMDI damping ratio in Fig.7(b) is significantly different
 599 for the colored noise excitations compared to the white noise excitation. Nevertheless, insignificant differences

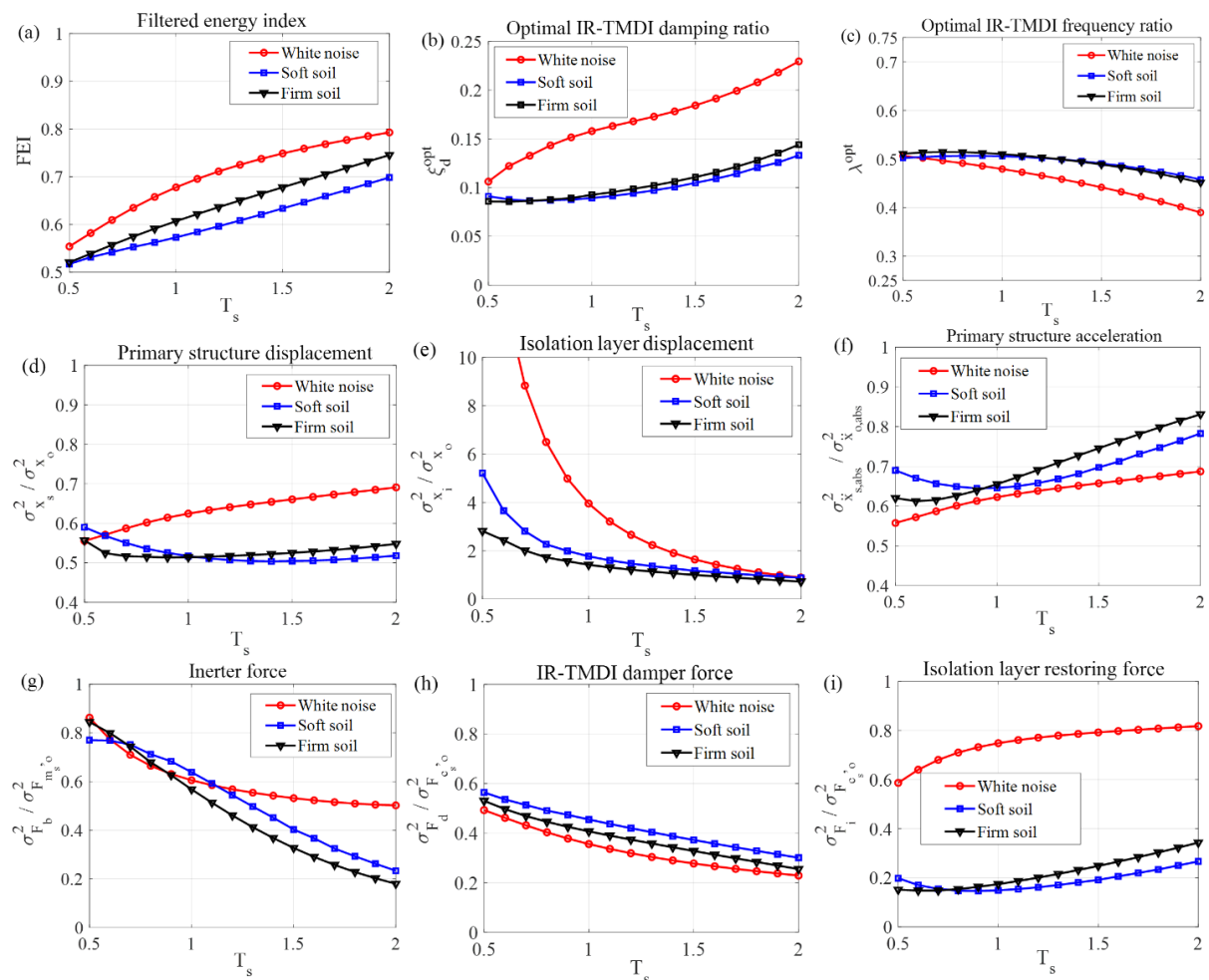
600 are observed in the TMDI frequency ratio and only for $\beta > 8\%$ in Fig.7(c). Notably, the IR-TMDI suppresses
 601 appreciably the primary structure response in terms of both displacement and acceleration in Figs.7(d) and 7(f),
 602 respectively, for all the considered excitations. In all cases, higher improvements to the structural seismic
 603 performance are noted with increasing inertance, though the level of the improvements does depend on the
 604 frequency content of the excitation. The structural response reductions are due to higher TMDI inerter and
 605 damping forces developed as inertance increases, for all different types of excitations, as seen in Figs.7(g) and
 606 7(h), respectively.



607 **Fig. 7:** Optimal IR-TMDI properties using the FEI tuning criterion and normalized system response variances
 608 plotted against the inertance ratio for stationary excitations with different frequency content and $PGA=0.3g$ and
 609 for $T_s=1s$, $\zeta_s=5\%$, $\mu_i=10\%$, $T_i=3.5s$, $\zeta_i=5\%$, $\mu_d=0.5\%$.
 610

611 Overall, the data in Fig.7 suggest that the frequency content of the excitation plays a major role to the system
 612 response. For instance, the colored noise excitations pose significantly lower deflection and resisting force
 613 demands to the isolators as seen in Figs.7(e) and 7(i). Further, the primary structure displacement response
 614 reductions in Fig.7(d) are consistently higher for the colored noise excitations compared to white noise excitation,
 615 while the opposite holds for the structural acceleration reductions in Fig.7(f). Nevertheless, the observed
 616 differences between colored and white noise excitations are dependent on the proximity of the natural period of
 617 the primary structure, T_s , with the dominant period of the colored excitations reported in Table 1. To investigate

618 the latter point, as well as to gauge the influence of the primary structure to the IR-TMDI tuning and structural
 619 response mitigation potential, Fig.8 furnishes “response spectra” by plotting all 9 quantities examined in Figs.2-
 620 7 for the three different excitations considered in Fig.7 and inertance ratio $\beta=20\%$ for different structural natural
 621 period T_s . It is seen that the optimal TMDI parameters in Figs.8(b) and (c) do not depend much on the dominant
 622 frequency of the colored noise excitation, though there are significant differences between white noise and colored
 623 noise excitations for $T_s > 0.6s$. Importantly, the trends and observations made in view of the data in Fig.7 on the
 624 differences between white and colored noise are valid for all different primary structures with $0.6s < T_s < 1.8s$.
 625 Specifically, higher primary structure displacement reductions are achieved for colored noise excitation vis-à-vis
 626 white noise excitation, except for the relatively stiff primary structures with $T_s < 0.6s$ (Fig. 8(d)), while the opposite
 627 holds for structural response acceleration (Fig. 8(f)). Further, higher deflection demands at the isolators are
 628 observed for white noise vis-à-vis colored noise which, though, reduce as the primary structure flexibility
 629 increases (Fig.8(e)). Moreover, the lower FEI ordinates in Fig.8(a) achieved by optimal tuning under different
 630 excitations reflect on consistently larger damping forces (Fig.8(g)) for the full range of primary structures
 631 considered. Further, the isolator force demands remain significantly lower for the colored noise vis-à-vis white
 632 noise, again for all primary structures (Fig.8(i)).

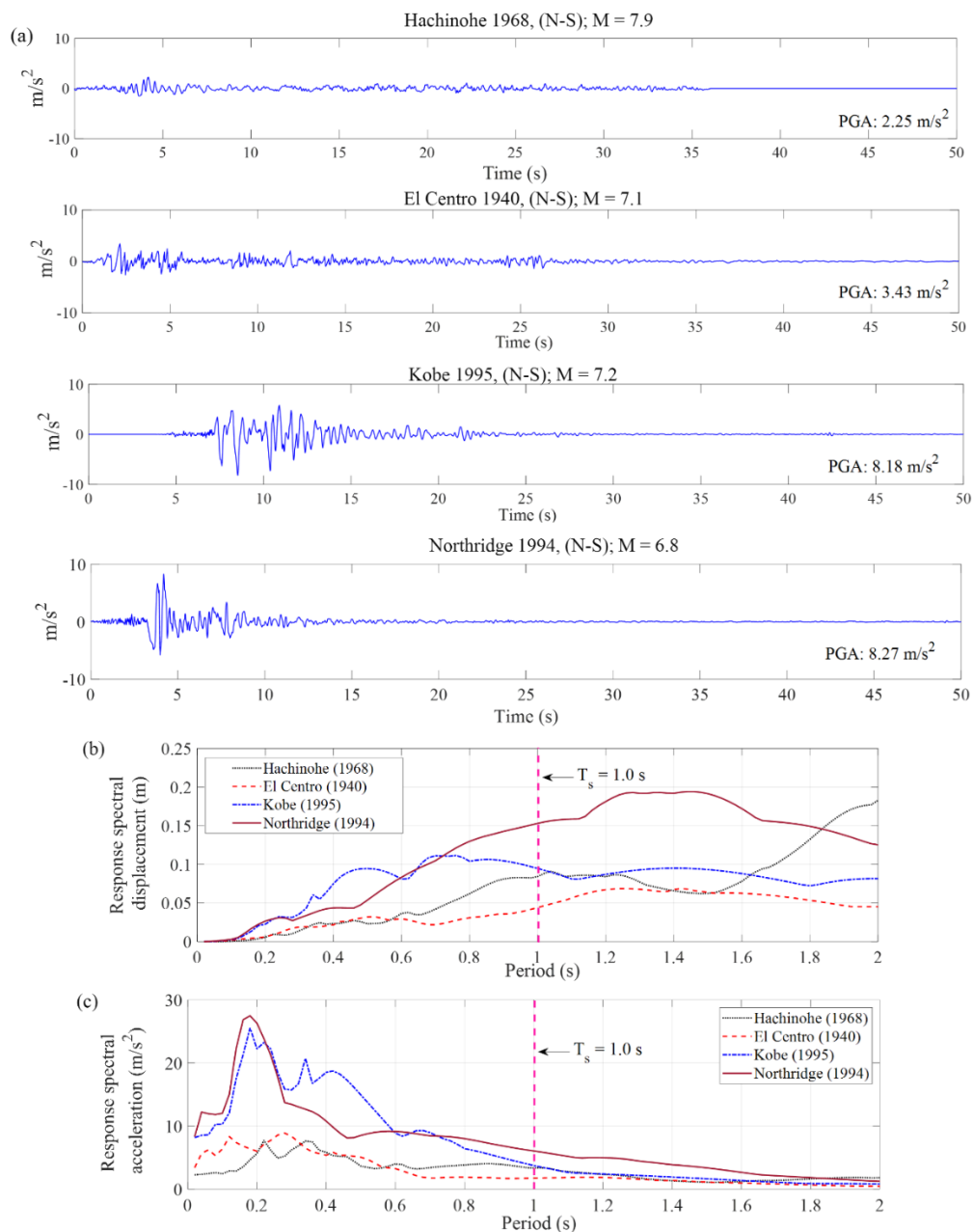


633

634 **Fig. 8:** Response spectra of optimal IR-TMDI properties using the FEI tuning criterion for stationary
 635 excitations with different frequency content and $PGA=0.3g$ and for $\xi_s=5\%$, $\mu_i=10\%$, $T_i=3.5s$, $\xi_i=5\%$, $\mu_d=0.5\%$
 636 and $\beta=20\%$.

637 **6 Verification of IR-TMDI effectiveness using nonlinear response history analysis**

638 In previous Sections, the response of the ELS in Eq.(4) under stationary excitations has been used as a proxy
 639 to appraise the effectiveness of the optimal IR-TMDI for primary structure seismic response mitigation. However,
 640 the response of IR-TMDI equipped structures to earthquake excitation is nonlinear and non-stationary due to the
 641 presence of the nonlinear isolators and the non-stationary amplitude and frequency content of naturally occurring
 642 seismic ground motions (GMs). To this end, it is herein deemed important to verify the IR-TMDI potential for
 643 structural seismic response mitigation by application of response history analyses to the nonlinear system
 644 equations in Eqs. (5) and (8) subject to recorded GMs. For this purpose, the base case system is considered
 645 equipped with the stiffest ($T_i= 2.7s$) and the most flexible ($T_i= 4s$) FEI-based optimally tuned IR-TMDIs
 646 previously studied in Fig.3.



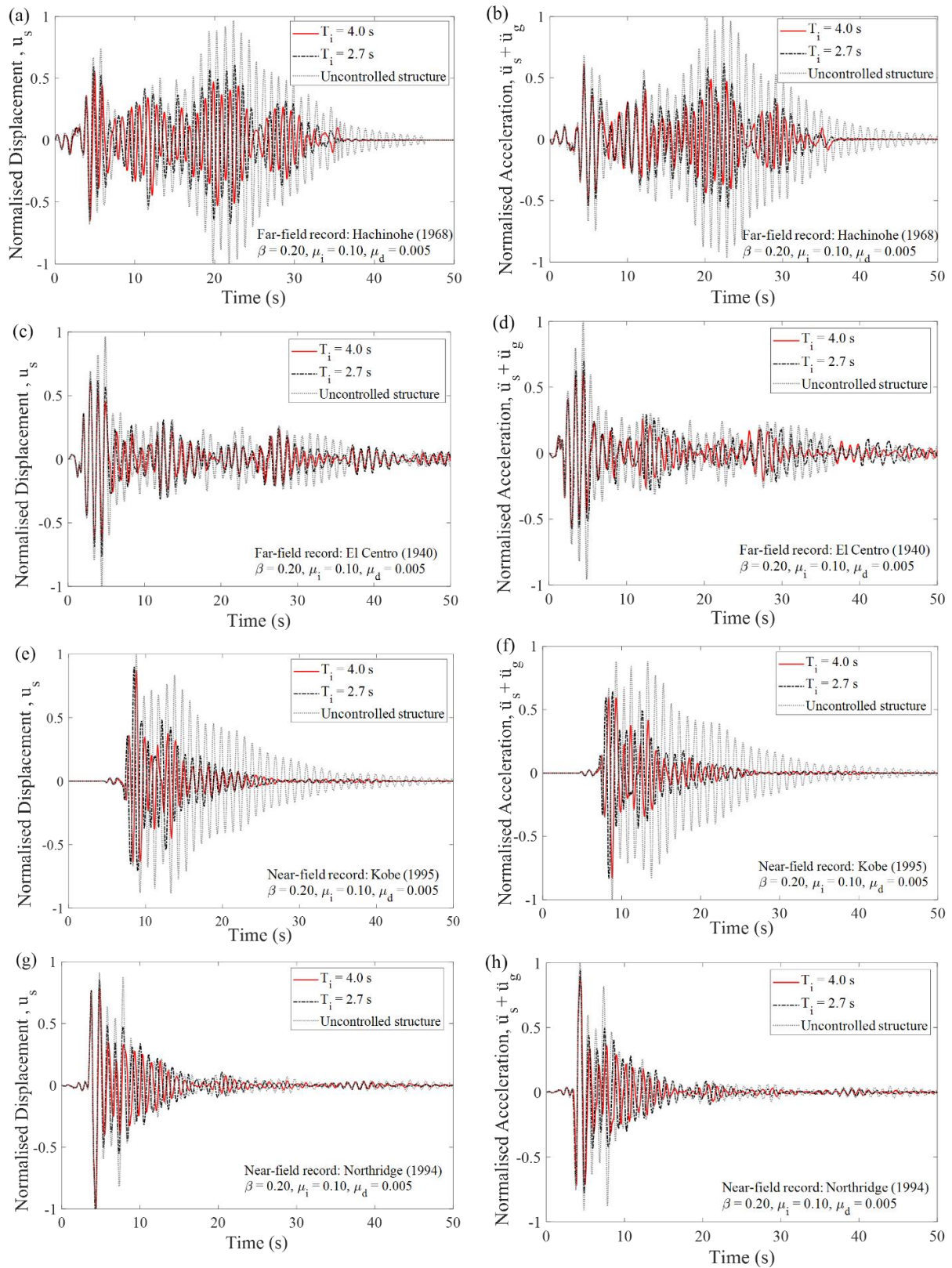
647
 648 **Fig. 9:** Considered recorded ground motions: (a) Time-histories, (b) Response spectral displacement for 5%
 649 damping ratio, (c) Response spectral acceleration for 5% damping ratio.

650 For numerical implementation, a common nominal yielding displacement $u_y=2\text{cm}$ is taken for the isolators
 651 while the normalized yielding strengths are set equal to 11% and 5% for the stiff and for the flexible isolators,
 652 respectively. Nonlinear response history analysis (NRHA) is undertaken for the 4 recorded GMs in Fig.9(a). These
 653 GMs are specified in Ohtori et al (2004) as part of a benchmark structural vibration control testbed problem. The
 654 El Centro and the Hachinohe records are far-field GMs, while the Northridge and the Kobe records are near-field
 655 GMs (see Ohtori et al 2004 for further details). Relative displacement and pseudo-acceleration response spectra
 656 of the GMs are shown in Figs.9(b) and 9(c) in which the T_s natural period of the primary structure is indicated.
 657 The as-recorded GMs in Fig.9(a) are herein scaled to $\text{PGA}=0.3\text{g}$ which has been assumed in the IR-TMDI optimal
 658 tuning. Time-domain numerical integration of the nonlinear equations of motion is performed in MATLAB using
 659 the standard `ode45` solver.

660 Structural response time-histories in terms of relative displacement and absolute acceleration are plotted in
 661 Fig. 10 for the two considered IR-TMDI equipped structures subject to the four GMs in Fig.9. To facilitate the
 662 appraisal of the IR-TMDI structural response mitigation potential with different T_i , the time-histories in each panel
 663 are normalized with respect to the highest absolute response of the uncontrolled primary structure whose response
 664 time-history is also plotted. Moreover, Table 2 provides the peak absolute and root-mean-square (RMS)
 665 displacement and acceleration response reductions achieved by the two different IR-TMDIs for each GM as well
 666 as their average value across the four GMs. Data in Fig.10 and Table 2 verify the trends observed and discussed
 667 in Section 5 in terms of the RMS values: despite the record-to-record variability, the IR-TMDI reduces appreciably
 668 RMS structural displacement and acceleration for each GM individually and, therefore, on the average.
 669 Reductions in terms of RMS acceleration is slightly by consistently higher than reduction in RMS displacement
 670 for all GMs and systems, while the IR-TMDI with the more flexible isolators performing significantly better for
 671 each GM. More importantly, similar observations are made for the peak response values, with the exception of
 672 the Northridge GM for which the peak displacement response of primary structure is 4% higher from the
 673 uncontrolled structure when equipped with the IR-TMDI with stiff isolators ($T_i= 2.7\text{s}$), while shows no
 674 improvement when equipped with the IR-TMDI with flexible isolators ($T_i= 4\text{s}$). This inability of the IR-TMDI to
 675 suppress the peak structural displacement for the Northridge GM is because the record begins with one early large
 676 pulse with long period under which the primary structure attains each peak response before the IR-TMDI is
 677 activated kinematically (i.e. moves with respect to the primary structure) to produce resisting inerter and damping
 678 forces and to dissipate energy. This is a well-reported in the literature disadvantage common to all inertial
 679 vibrations absorbers, including the TMDI (De Angelis et al. 2019). Still, the IR-TMDI does reduce the peak
 680 structural acceleration even for the Northridge GM, while the IR-TMDI with flexible isolators reduces the average
 681 peak structural displacement response from all the considered GMs by 20%.

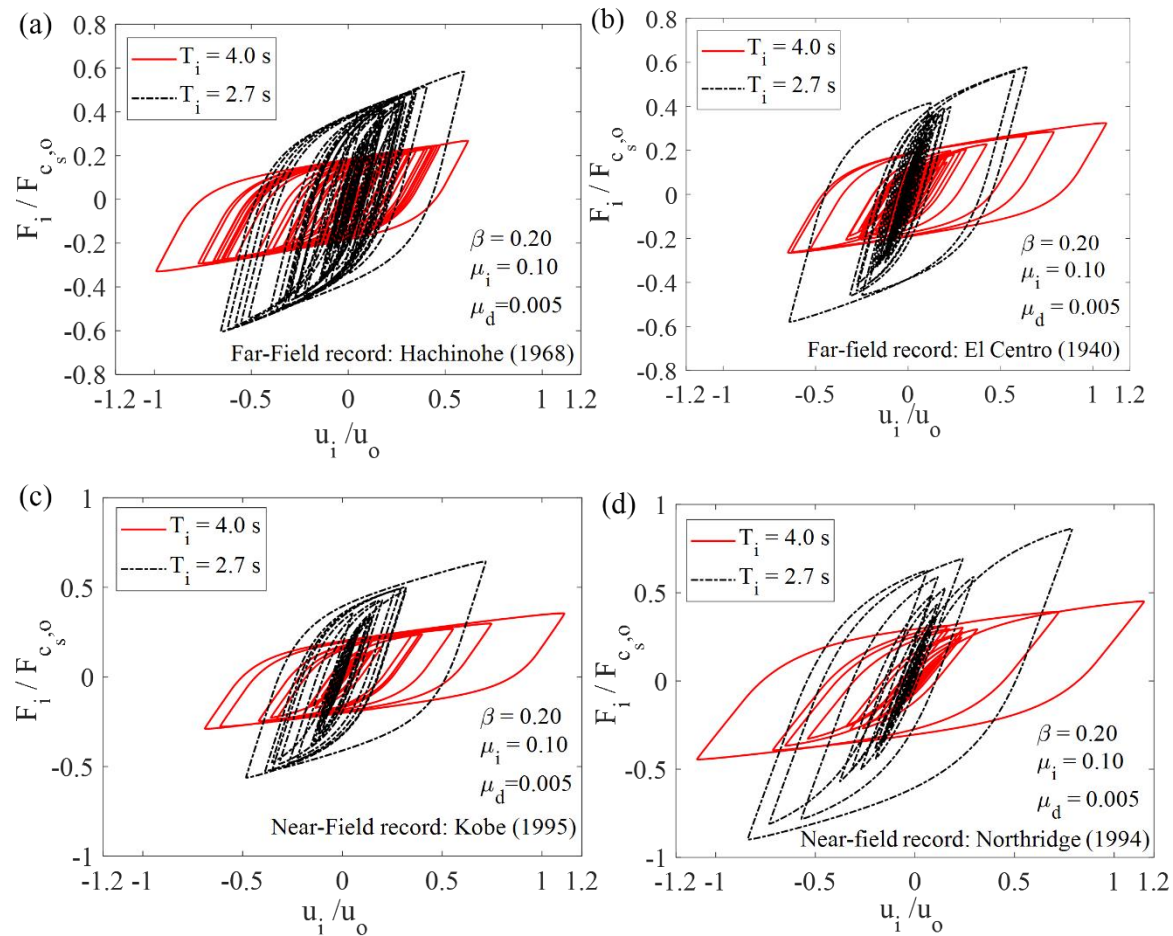
682 **Table 2:** Peak and RMS values of the normalized response time-histories of Fig.10

	Peak absolute response				RMS response			
	$T_i = 2.7\text{s}$		$T_i = 4.0\text{s}$		$T_i = 2.7\text{s}$		$T_i = 4.0\text{s}$	
	displ.	acc.	displ.	acc.	displ.	acc.	displ.	acc.
Hachinohe	0.71	0.66	0.70	0.67	0.64	0.58	0.52	0.48
El Centro	0.76	0.70	0.63	0.59	0.78	0.71	0.68	0.64
Northridge	1.04	0.95	1.00	0.90	0.91	0.83	0.83	0.75
Kobe	0.89	0.83	0.87	0.83	0.61	0.56	0.54	0.51
Mean values	0.85	0.78	0.80	0.75	0.73	0.67	0.64	0.60



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Fig. 10: Primary structure response time-histories normalized to the peak absolute response of the uncontrolled structure for the GMs in Fig.9.



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Fig. 11: Isolators' force-deformation plots for the GMs in Fig.9.

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Finally, Fig.11 plots the force-deformation curves (hysteretic loops) of the isolators for both the IR-TMDI equipped structures considered under the GMs in Fig.9. The plots are normalized with respect to the peak damping force and structural displacement of the uncontrolled primary structure. These data confirm that the IR-TMDI with more flexible isolators exhibits higher deflections than the stiffer isolators which, in turn, activate more effectively the TMDI, resulting in higher structural response reductions (Table 1). Evidently, the higher resisting forces developed by the stiffer isolators are not as effective as the action of the FEI-based optimally tuned TMDI in mitigating the seismic response of the primary structure. Clearly, this observation establishes the potential of the IR-TMDI for efficient seismic protection of structures, provided that a judicious selection of the isolators' stiffness in conjunction with FEI-based IR-TMDI tuning are adopted.

698 7 Concluding remarks

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Motivated by the insight that the TMDI vibration suppression capability improves by increasing the structural flexibility between the floors it is attached to, a novel passive energy dissipation system, IR-TMDI, comprising a TMDI contained within a seismically isolated rooftop, has been herein proposed and its potential for the seismic protection of buildings has been numerically demonstrated. The intended working principle of the IR-TMDI is to improve the structural seismic response mitigation effectiveness of the TMDI through the creation of a phenomenologically flexible floor achieved upon yielding of the isolation layer under severe ground motions.

705 This intended functionality of the IR-TMDI has been numerically verified by considering a nonlinear mechanical
706 model whereby a low-fidelity damped single mode representation was adopted for the primary structure, while
707 the yielding isolators were represented by the Bouc-Wen hysteretic model. The verification has been supported
708 by an equivalent linear system (ELS), derived through statistical linearization, which expedited the optimal IR-
709 TMDI tuning for various isolated rooftop properties, inertance, and primary structure natural periods under white
710 noise excitations with different intensities as well as Kanai-Tajimi excitations with different frequency content
711 corresponding to soft and firm soil conditions.

712 It was established that tuning for maximizing TMDI seismic energy dissipation (FEI-based tuning) is more
713 advantageous than tuning for minimizing primary structure displacement or acceleration response, as it lowers
714 deflection and force demands to the isolators as well as the inerter force. Moreover, significant primary structure
715 displacement and acceleration reductions are achieved as the effective rooftop flexibility increases, through
716 reduction of the nominal strength of the isolators, which verifies the intended working principle of the IR-TMDI.
717 This was further confirmed from numerical data pertaining to nonlinear response history analyses under four
718 benchmark recorded ground motions. Furthermore, for IR-TMDI with sufficiently flexible isolators (i.e. three
719 times more flexible than the primary structure), improved seismic structural performance with concurrent reduced
720 deflection and force demands at the isolators is shown for all considered stationary excitations as the inertance
721 scales-up, which is readily achievable technologically. Additional conclusions drawn from the parametric study
722 are that the use of higher damping isolators is detrimental to the primary structure response, larger relative
723 structural response improvements are achieved for higher seismic intensity (PGA), and that the relative
724 improvement in structural accelerations reduce as well as the deflection demands at the isolators for stiffer primary
725 structures.

726 Lastly, it was noted that improved structural seismic performance concurrently with reduced the deflection
727 demands at the isolators is also achieved by increasing the rooftop mass for fixed inertance and effective isolation
728 period. Nevertheless, these improvements come at the cost of significantly higher isolator forces to be transferred
729 to the structure, further to the increased weight that the isolators and the primary structure need to accommodate.
730 In this regard, whilst a detailed consideration of practical technological aspects and implementation of the
731 proposed IR-TMDI falls outside the scope of this work, the increase of rooftop mass is likely to be a least attractive
732 approach to improve IR-TMDI performance compared to reducing isolation natural period in practice, especially
733 for existing structures. Still, it is noted that the combination of relatively lightweight rooftops with large
734 deformation demands and low vertical loads may become detrimental to the stability of elastomeric bearings.
735 However, applications of lightweight base isolated residential houses and timber buildings as discussed by Ryan
736 and Earl (2010) as well as partial isolation in multi-storey buildings (Faiella and Mele 2020) support the practical
737 feasibility of IR-TMDI. To this aim, further research is warranted involving the detailed design and assessment
738 of IR-TMDI for benchmark/case-study multistorey buildings which is left for future work. Ultimately, this
739 consideration will further reinforce and quantify the herein established advantages of the IR-TMDI over TMDI
740 configurations spanning several floors, widely considered in the recent literature, rendering the IR-TMDI
741 particularly applicable to the bulk of buildings in high seismicity areas, that is, low-to-mid-rise new-built and
742 existing structures.

743 Acknowledgements

744 The first author gratefully acknowledges the financial support from the School of Science and Technology at
745 City, University of London through a fully funded PhD studentship.

746 Appendix A: Energy dissipation index derivation

747 In this Appendix, the energy-based performance criterion in Eq.(24) is derived. To this aim, the equations of
748 motion of the ELS in Eq.(16) are first written as

$$749 \quad m_s \ddot{x}_s + c_s \dot{x}_s + k_s x_s - (c_i (\dot{x}_{is} - \dot{x}_s) + \alpha k_i (x_{is} - x_s) + (1 - \alpha) F_y y) - b(\ddot{x}_d - \ddot{x}_s) = -m_s \ddot{x}_g \quad (\text{A.1a})$$

$$750 \quad m_i \ddot{x}_{is} + c_i (\dot{x}_{is} - \dot{x}_s) + \alpha k_i (x_{is} - x_s) + (1 - \alpha) F_y z - (c_d (\dot{x}_d - \dot{x}_{is}) + k_d (x_d - x_{is})) = -m_i \ddot{x}_g \quad (\text{A.1b})$$

$$751 \quad m_d \ddot{x}_d + c_d (\dot{x}_d - \dot{x}_{is}) + k_d (x_d - x_{is}) + b(\ddot{x}_d - \ddot{x}_s) = -m_d \ddot{x}_g \quad (\text{A.1c})$$

752 Next, the so-called equations of relative energy balance are derived by multiplying Eq. (A1a) by \dot{x}_s , Eq. (A1b)
753 by \dot{x}_{is} and Eq. (A1c) by \dot{x}_d and integrating over time to yield (e.g. Uang & Bertero 1988)

$$754 \quad E_{m_s}(t) + E_{c_s}(t) + E_{k_s}(t) - E_{c_{i,s}}(t) - E_{k_{i,s}}(t) - E_{h_{i,s}}(t) - E_{b,s}(t) = E_{g_s}(t) \quad (\text{A.2a})$$

$$755 \quad E_{m_i}(t) + E_{c_i}(t) + E_{h_i}(t) + E_{k_i}(t) - E_{c_{d,is}}(t) - E_{k_{d,is}}(t) = E_{g_i}(t) \quad (\text{A.2b})$$

$$756 \quad E_{m_d}(t) + E_{c_d}(t) + E_{k_d}(t) + E_b(t) = E_{g_d}(t) \quad (\text{A.2c})$$

757 In Eq.(A.2a), E_{m_s} , E_{c_s} and E_{k_s} are the kinetic energy, viscous damping energy, and elastic strain energy of the
758 primary structure, respectively, given by

$$759 \quad E_{m_s}(t) = m_s \int_0^t \dot{x}_s \dot{x}_s dt, \quad E_{c_s}(t) = c_s \int_0^t \dot{x}_s^2 dt, \quad \text{and} \quad E_{k_s}(t) = k_s \int_0^t x_s \dot{x}_s dt, \quad (\text{A.3})$$

760 $E_{c_{i,s}}$, $E_{k_{i,s}}$ and $E_{h_{i,s}}$ are the viscous damping energy, strain energy, and hysteretic dissipated energy transferred
761 from the isolation system to the primary structure, respectively, given by

$$762 \quad E_{c_{i,s}}(t) = -c_i \int_0^t \dot{x}_s^2 dt + c_i \int_0^t \dot{x}_{is} \dot{x}_s dt, \quad E_{k_{i,s}}(t) = -\alpha k_i \int_0^t x_s \dot{x}_s dt + \alpha k_i \int_0^t x_{is} \dot{x}_s dt, \quad \text{and} \quad (\text{A.4})$$

$$E_{h_{i,s}}(t) = (1 - \alpha) k_i u_y \int_0^t y \dot{x}_s dt,$$

763 $E_{b,s}$ is the energy transferred from the inerter to the primary structure, given by

$$764 \quad E_{b,s}(t) = -b \int_0^t \ddot{x}_s \dot{x}_s dt + b \int_0^t \ddot{x}_d \dot{x}_s dt, \quad (\text{A.5})$$

765 and E_{g_s} is the seismic excitation energy entering the primary structure, given by

$$766 \quad E_{g_s}(t) = -m_s \int_0^t \ddot{x}_g \dot{x}_s dt \quad (\text{A.6})$$

767 Further, in Eq.(A.2b), E_{m_i} , E_{c_i} , E_{k_i} and E_{h_i} are the kinetic energy, viscous damping energy, elastic strain energy
768 and hysteretic energy dissipation of the isolated rooftop, respectively, given by

$$769 \quad E_{m_i}(t) = m_i \int_0^t \dot{x}_{is} \dot{x}_{is} dt, \quad E_{c_i}(t) = c_i \int_0^t \dot{x}_{is}^2 dt - c_i \int_0^t \dot{x}_s \dot{x}_{is} dt, \quad (A.7)$$

$$E_{k_i}(t) = \alpha k_i \int_0^t x_{is} \dot{x}_{is} dt - \alpha k_i \int_0^t x_s \dot{x}_{is} dt \quad \text{and} \quad E_{h_i}(t) = (1-\alpha) k_i u_y \int_0^t y \dot{x}_{is} dt,$$

770 $E_{c_{d,js}}$ and $E_{k_{d,js}}$ are the damping element energy and spring energy transferred from the TMDI to the isolated
771 rooftop, respectively, given by

$$772 \quad E_{c_{d,js}}(t) = -c_d \int_0^t \dot{x}_{is}^2 dt + c_d \int_0^t \dot{x}_d \dot{x}_{is} dt \quad \text{and} \quad E_{k_{d,js}}(t) = -k_d \int_0^t x_{is} \dot{x}_{is} dt + k_d \int_0^t x_d \dot{x}_{is} dt, \quad (A.8)$$

773 and E_{g_i} is the seismic excitation energy entering the isolated floor system, given by

$$774 \quad E_{g_i}(t) = -m_i \int_0^t \ddot{x}_g \dot{x}_{is} dt \quad (A.9)$$

775 In addition, in Eq.(A.2b), E_{m_d} , E_{c_d} , E_{k_d} and E_b are the kinetic energy, viscous damping energy, elastic strain
776 energy, and inerter energy of the TMDI, respectively, given by

$$777 \quad E_{m_d}(t) = m_d \int_0^t \dot{x}_d \dot{x}_d dt, \quad E_{c_d}(t) = c_d \int_0^t \dot{x}_d^2 dt - c_d \int_0^t \dot{x}_{is} \dot{x}_d dt, \quad (A.10)$$

$$E_{k_d}(t) = k_d \int_0^t x_d \dot{x}_d dt - k_d \int_0^t x_{is} \dot{x}_d dt, \quad \text{and} \quad E_b(t) = b \int_0^t \dot{x}_d \dot{x}_d dt - b \int_0^t \ddot{x}_s \dot{x}_d dt,$$

778 and E_{g_d} is the seismic excitation energy entering the TMDI, given by

$$779 \quad E_{g_d}(t) = -\int_0^t m_d \ddot{x}_g \dot{x}_d dt. \quad (A.11)$$

780 Assuming Gaussian stationary stochastic seismic excitation and taking the mathematical expectation in both
781 sides of Eqs. (A.2), the following set of equations are derived in a small increment of time Δt under ergodic
782 conditions (see also Pietrosanti et al. 2017)

$$783 \quad E[\Delta E_{g_s}] = E[\Delta E_{m_s}] + E[\Delta E_{c_s}] + E[\Delta E_{k_s}] - E[\Delta E_{c_{i,s}}] - E[\Delta E_{k_{i,s}}] - E[\Delta E_{h_{i,s}}] - E[\Delta E_{b,s}]$$

$$E[\Delta E_{g_i}] = E[\Delta E_{m_i}] + E[\Delta E_{c_i}] + E[\Delta E_{h_i}] + E[\Delta E_{k_i}] - E[\Delta E_{c_{d,js}}] - E[\Delta E_{k_{d,js}}] \quad (A.12)$$

$$E[\Delta E_{g_d}] = E[\Delta E_{m_d}] + E[\Delta E_{c_d}] + E[\Delta E_{k_d}] + E[\Delta E_b]$$

784 In Eq.(A.12) the following incremental energy terms vanish $E[\Delta E_{m_s}] = E[\Delta E_{m_i}] = E[\Delta E_{m_d}] = E[\Delta E_{k_s}] = 0$ due
785 to the property $E[u\dot{u}] = 0$ which holds for any Gaussian temporal stochastic process u (e.g. Roberts & Spanos
786 2003). Further, it can be shown that $E[\Delta E_{k_i}] - E[\Delta E_{k_{i,s}}] + E[\Delta E_{k_d}] - E[\Delta E_{k_{d,js}}] + E[\Delta E_b] - E[\Delta E_{b,s}] = 0$ by
787 making use of the previous property together with the coordinate transformations $x_i = x_{is} - x_s$ and $x_k = x_d - x_{is}$ and
788 some algebraic manipulation. To this end, by summing the three expressions in Eq.(A.12), the total expected
789 incremental seismic input energy in the ELS system $E[\Delta E_{Total}]$ is found as

$$E[\Delta E_{Total}] = E[\Delta E_{g_s}] + E[\Delta E_{g_i}] + E[\Delta E_{g_d}] = c_s \sigma_{\dot{x}_s}^2 \Delta t + c_d \sigma_{\dot{x}_k}^2 \Delta t + c_i \sigma_{\dot{x}_i}^2 \Delta t + (1-\alpha) k_i u_y \sigma_{y_{x_i}}^2 \Delta t, \quad (A.13)$$

791 by noting that

$$E[\Delta E_{c_s}] = c_s E[\dot{x}_s^2] \Delta t = c_s \sigma_{\dot{x}_s}^2 \Delta t, \quad E[\Delta E_{c_i}] - E[\Delta E_{c_{i,s}}] = c_i E[(\dot{x}_s - \dot{x}_{is})^2] \Delta t = c_i \sigma_{\dot{x}_i}^2 \Delta t, \\ E[\Delta E_{c_d}] - E[\Delta E_{c_{d,js}}] = c_d E[(\dot{x}_d - \dot{x}_{is})^2] \Delta t = c_d \sigma_{\dot{x}_k}^2 \Delta t, \quad \text{and} \quad (A.14) \\ E[\Delta E_{h_i}] - E[\Delta E_{h_{i,s}}] = (1-\alpha) k_i u_y E[y(\dot{x}_{is} - \dot{x}_s)] \Delta t = (1-\alpha) k_i u_y \sigma_{y_{x_i}}^2 \Delta t$$

793 Finally, the energy dissipation index (EDI) defined by Pietrosanti et al. (2017) as the ratio of the energy
794 dissipated by the TMDI damping element over the total input energy is given as

$$EDI = \frac{E[\Delta E_{c_d}] - E[\Delta E_{c_{d,js}}]}{E[\Delta E_{Total}]} \quad (A.15)$$

796 for the herein considered ELS. Then, Eq.(24) follows from Eqs.(A.13)-(A.15).

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