Modelling Life Insurance New Business Risk

by

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Declaration

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Abstract

The development of new business plans is a very important responsibility affecting an ongoing life insurance company's profitability and solvency, and yet there is little life insurance literature in respect of the allocation of free assets to write new business. The uncertainty in fulfilling a company's future new business plans requires asset/liability models for both the market and the company to determine the effects of new business (strain) risk on a life insurance company in a competitive and stochastic environment. In this investigation, the decision concerning the allocation of free assets to write new business is taken as a proxy for the new business plan. Our research is based on non-profit non-linked insurance products.

The new business strategies (formulated with different pricing policies) and the market new business model (the relationship between the new business demanded and the price of the product / the policyholders' real income provide the framework for the simulated results. The market results form the base in modelling the company. The investigation makes a clear distinction between the quantity of new business demanded and that which is actually written.

We note the dominance of adequate free assets available and allocated to write new business in the new business plans in the course of the research. However, the sensitivity of the new business models can only be known if a company has sufficient free capital to write the quantity of new business demanded being generated by the models. Therefore, in producing a particular level of (relative) insolvency risk, a company's free assets allocation decision needs to be taken alongside the choice of new business and investment strategies. The results show that a company requires a relatively low constant proportion of free assets to finance new business over the long term.

It is shown that, in managing the new business (strain) risk with financial reinsurance including quota share basis of original terms reinsurance, a company can increase the proportion of free assets allocated to write new business only if the level of financing provided covers adequately the expenses related to the reinsured business.

As the research is based on non-profit non-linked products and on the assumption that there will be no future injection of capital to write new business over the projection year, we have suggested further research to consider the impact of future injection of capital by the shareholders and bonus distribution policy on the new business plans.

Chapter 1

Introduction

1.1 Background

The development of new business plans is a very important responsibility facing every company. For a new business plan to be successfully implemented there is the need to consider every relevant factor that would have considerable impact on the survival of the venture contemplated. These factors are both exogenous and endogenous to the company. This appears to be a lot more challenging when the business contemplated has to do with managing future uncertainties as is the case with the insurance business. An insurance contract generally is the acceptance of risk by the insurance company on behalf of a policyholder in respect of the occurrence of an insured event (Williams et al, 1998). Thus, the process of sales and underwriting of insurance contracts is crucial to the company's future solvency. And given the wide spectrum of events covered, the insurance business can be grouped into life insurance, health insurance, pension funds and general insurance. The focus of this research is on life insurance.

In developing new business plans, a life insurance company also requires an assessment of its business environment, both the internal factors which will determine the focus and thrust of the business plan and the external factors that would enhance or limit its actual successes, in order to ensure its long term sustainability and survival. New business plans ordinarily comprise among others the product design, availability of capital (free assets), pricing policy, channels of distribution, marketing strategy, underwriting, free assets allocation strategy to finance new business, the cost of sales, the expected new business volume, and other factors. Broadly speaking, free assets can be defined as "the excess of assets over supervisory liabilities and any required solvency margin" (Subject 302 Core Reading, Unit 4). In fact, the expected new business volume and the cost of sales are so relevant that they are also being recognised by the insurance regulators as some of the important factors that affect a life insurance company's future profitability and solvency (Luffrum, 1992). This is because writing an inappropriate volume of new business may drain the available free assets (due to new business strain on issue) to such an extent that it endangers the company's solvency or places a constraint on its investment policy.

Nonetheless, the level of capital (free assets) available and allocated to finance the new business is considered critical in assisting the development of an appropriate new business plan. This is because it determines the appropriate pricing policy, marketing strategy and the extent of underwriting to be adopted in achieving the objectives of sustainability and long term survival. In other words, the decision on premium rate adequacy may not only depend on the choice of new business strategy but also on the level of the free assets available to support the expected new business to be written. It needs to be noted that there has been little life insurance literature in respect of the allocation of free assets to finance new business. However, literature is replete with asset allocation strategies for the choice of investments, for example see Ross (1989), Hardy (1993) and Chadburn (1998). Since the focus of this study is to investigate new business risks, we have taken into account the relative importance of the free assets in developing new business plans that can ensure the solvency of the life insurance company.

One of the essential ingredients that can boost the future profitability and solvency of a life insurance business is the new business volume that an insurer would like to write (Ross 1989; Luffrum, 1992; Mehta, 1992). A life office's new business volume (quantity demanded and/or written) could be constrained by both internal factors (e.g. the pricing policy, product design, free assets) and external factors (e.g. the real disposable income, premiums charged by competitors and regulators' decisions) in a competitive environment (Taylor, 1986; Abbey and Brooks, 1998; Browne et al, 2001; McGaughey et al, 2001).

The new business sales are positively correlated to disposable income of prospective policyholders, see Browne et al (2001), if all other things remain the same (e.g. there are sufficient free assets available to finance new business). When the disposable income is relatively high, the cash flows to insurers are likely to increase. Therefore, we may expect disposable income to be positively related to an insurer's performance. "However, increasing the sales may result in a drain on surplus, leading to an alternative hypothesis that disposable income is negatively related to an insurer's performance", Browne et al (2001).

A good pricing policy could also enhance an insurer's new business volume and future profitability. Therefore the insurer is continually being faced with the problem of instituting an appropriate pricing policy that will make its products competitive in the market. Where the insurer's premium is considered relatively high compared to the prevailing market premium, this could result in low demand for insurance and hence low profitability which could lead to a high risk of insolvency, and vice versa. However, in some countries there is regulation of premium rates, for example, regulators may impose "restrictions on premium rates themselves (or certain elements of the premium basis, such as mortality and interest), or charges, that can be used for some types of contract. Such restrictions have been common in European countries and in some States in the United States", (Subject 302 Core Reading, Unit 4).

Thus, Derrig (1989) provides some highlights of various ways in which insurance regulators of some States in the United States have intervened in the pricing of property-liability insurance contracts in the past decades. The New York state Insurance Department promulgates regulation 72 (on 22nd January 1999) which prescribes rules for calculation of life insurance surrender cost index, http://www.ins.state.ny.us/re74f.htm.

A financially stressed life insurance company could be forced by the regulators to cease writing new business (i.e. close to new business), as a means of protecting the interests of existing policyholders. Regulators also have an indirect control of a life insurer's new business volume through regulation on solvency requirements as discussed below.

As many life insurance contracts are designed to meet the financial needs of the prospective policyholders, they could also incur significant valuation (new business) strains in their year of issue, and these have to be met from the office's free assets (Booth, et al 2005). "New business strain per policy arises when the premium paid at the start of a contract, less the initial expenses including commission payments, is not sufficient to cover the mathematical reserve that the company needs to set up" (Subject 302 Core Reading, Glossary). Thus, the new business strain is a function of the prudent supervisory valuation basis. While new business (valuation) strain risk could affect the financial profile of life insurance business, future new business has been recognised as one of the most significant factors that can determine the future financial condition of any life office (Booth, et al 2005). In other words, the future liabilities of a life office are affected by new business volume.

The risk of insolvency occurs where an insurer is not able to meet the statutory solvency requirements, and this could happen due to the new business (valuation) strains arising from writing an inappropriate volume of new business. It is not impossible for an insurer to become relatively insolvent (i.e. an insurer falling out of line with the rest of the market in terms of asset liability ratios), despite meeting the statutory solvency requirements (Hardy, 1996). The insolvency risk arising from the uncertainty in fulfilling the company's future new business plans can be described as the new business risk. It is the risk of having a higher or lower than expected volume of new business which may lead to the available resources becoming inadequate to meet statutory solvency requirements. The new business risks may arise from uncertainties including the following:

- The level of free assets available to write planned new business volume becoming inadequate to meet statutory solvency requirements (Luffrum, 1992);
- The premium rates charged may not be competitive to enable the company to achieve a desired level of new business volume (Subject 303 Core Reading, Unit 5);
- The inability to attract adequate volume of new business over which to spread expenses (i.e. fixed overheads), e.g. loss of business to competitors due to relative insolvency (Hardy, 1996; Subject 303 Core Reading, Unit 5); and
- Adverse selection by policyholders arising from inadequate risk classification methods as part of the pricing process (Cummins et al, 1991).

As a strategy of managing the uncertainties (which could lead to insolvency) the UK based insurance regulators, including the Financial Services Authority (FSA) recognise the need to place an indirect constraint on the amount of business that may be written through regulations and place minimum mathematical reserves and solvency margins to be held by the company, (Subject 302 Core Reading, Unit 4). These have the effect of limiting the free assets available within the company to write new business. It would also place a minimum requirement on the finances required to write a life insurance contract. The FSA recent rules and guidance require such mathematical reserves to be calculated on a realistic, market consistent basis using the gross premium valuation method, particularly for with profits business (Dullaway and Needleman, 2004). The professional guidance note (GN1) also places a specific requirement on the UK based appointed actuary to indicate to the directors the limits on the volume of business that the company may prudently accept. Life insurers can mitigate the new business (valuation) strains risk with measures such as proper product design to meet the policyholders' financial needs, mix of business and an appropriate reinsurance arrangement (preferably original terms arranged on quota shares basis) that can minimise the new business strain (Booth et al, 2005).

The foregoing discussion presupposes every new business to be self-financing. This is recognised by the UK actuarial profession, as stated in GN8, that “no explicit provision would need to be established in respect of an acquisition expense overrun if the new business is expected, on a prudent basis, to be self-supporting allowing for the repayment of any valuation strain”. However, in the unlikely event of an acquisition expense overrun where the new business is not self-financing, it is expected that an insurer would have made provision, using prudent estimates for all relevant assumptions, to accommodate such an overrun. For the purpose of simplicity, this investigation assumes that the sales cost equals the expense loadings in the pricing policy and the company under consideration is expected to be self-financing in future years based on an initial capital input at the projection date. Nonetheless, we have to assess and control the risks associated with new business plans so that an insurer would remain solvent to continue to attract future new business.

In evaluating the new business risk in non-linked life insurance, without loss of generality, we will therefore be confronted with providing answers to the following questions in this study.

First, will the company’s free assets be allowed to stay at a level considered adequate to continue its desired new business strategy over a given time horizon? This requires a company making decisions concerning the allocation of free assets to finance new business plans in order to maintain a sustainable growth rate from its own internal resources without creating solvency problems over the time horizon. This decision is taken alongside the choice of an appropriate investment strategy.

Second, how will the company's new business respond to changes in the level of solvency relative to the market? The management or regulator's timely decision to control a life company's new business volume in response to its relative insolvency would assist in preventing statutory insolvency.

Third, how will the company's new business respond to changes in the level of premiums being charged in a competitive market? This would require a company to operate alongside the market in aggregate so that the impact on new business of changes in the company's premium relative to the market premium can be assessed. The company could be allowed to respond to its worsening / improving solvency ratio through the premium being charged in order to remain competitive.

In evaluating new business risks in non-linked life insurance business we depend on the application of stochastic modelling of new business volume. This study (in contrast to previous works) has considered in detail some important assumptions that were overlooked in past studies. We note from previous studies [such as Cargill et al (1979), Taylor (1986), Daykin et al (1990), Browne et al (1993), Outreville (1996) and others] that the life insurance demand models proposed (some using econometric techniques) are calibrated with actual new business data (i.e. quantity of new business written). The model outputs are also taken as the actual new business volume written. This has the implicit assumption that the quantity demanded by prospective buyers of life insurance are the actual quantities written onto the books. We have, however, drawn a subtle distinction in our investigation between the quantity of new business demanded and that which is actually written. Ordinarily, the demand for insurance (i.e. the quantity of new business demanded) is affected by factors such as the price of the product and income of the prospective policyholder. The quantity of new business actually written onto the books is determined mainly by the level of free assets available to finance the new business. In practice, some prospective policyholders may not have been accepted for the purpose of insurance due to several reasons ranging from uninsurability, underwriting procedures, lack of free capital to write the business, and many other reasons. The above implicit assumption made in the previous studies also implies that there are sufficient free assets available to finance the quantity of new business demanded which may not always be true in practice.

Accordingly, this study recognises the importance to life insurance solvency of modelling the quantity of new business written as distinct from the quantity of business demanded since in practice the company has limited available capital to write the new business. This is because the both the quantity of new business demanded and that actually written can have an effect on the company's profitability, apart from the sales cost which is determined mainly by the quantity of new business demanded. Furthermore, the company's pricing policy assumes a particular level of new business volume to be written for it to be profitable, and where the quantity of new business volume expected under the pricing policy is not written due to lack of free assets, the company's expected profitability is likely to fall.

We also note that previous studies like Chadburn (1993) and Hardy (1993) have considered companies that are closed to new business at or a few years after the projection date. This is a situation that is not completely relevant in practice for a company that is a going concern, except for a company that is enmeshed in solvency problems or about to be taken over. This study is concerned with ongoing life insurance operations.

A number of previous studies [Hardy (1996), Booth et al (2005)] assume a deterministic approach to modelling new business volume (fixed level of new business growth) in life insurance but this study investigates new business risks on the basis of dynamic new business volumes (both quantity demanded and written).

Thus, this study adopts an integral approach that includes the elements of dynamic new business volumes in an on-going life insurance company in investigating the effects of free assets on price and new business taking into consideration the solvency requirements, and the feature of operating in a competitive market.

1.2 Related Literature

In the past few decades, the life insurance business worldwide has grown more rapidly than non-life business. A worldwide insurance market overview by the Association of British Insurers (ABI) reported approximately a 40% increase in life business premium income in contrast to approximately a 9% increase in non-life business premium income during the survey period 1988 to 2002. Although such increased new business volumes could have significant effects on a company's financial position, there appears to be little in the life insurance literature on modelling the volume of new business stochastically in a competitive market.

The volume of new business has been considered as one of the main factors defining a new business profile. Ross (1989) describes new business as one of the "four points of the actuarial compass in the with-profit world". He also notes that it is essential to examine the question "how much new business is it sensible for an office to take on?" even though its effects on "solvency may not be an issue but the investment policy could be adversely affected by too high a rate of expansion". However, it is generally noted that the free assets available to write new business remain a critical consideration to the survival of business plans. While the holding of capital is seen as an integral part of the overall business strategy, Ibeson et al (1999) argue that it "enhanced security, provides more valuable [non-life] insurance product for which a higher premium can be charged or more stringent terms imposed". Olivieri et al (2000) concur when they say that defining an appropriate level of assets in a life insurance business is essential in meeting the future obligations of an insurer.

Cummins et al (1991) explicitly assume that life insurance companies operating in the market "have adequate resources to offer any and all contracts considered likely to make an expected profit", although in practice the free assets available within a company to finance new business may be limited. Daykin and Hey (1990), Hardy (1996) and others not only ignore the possibility of an injection of new capital from external sources having given the initial assets at the projection date, but also the allocation of free assets to finance new business. This implies that the authors assume implicitly that there are sufficient free assets to write any quantity of new business generated from the new business models. In this investigation, we consider the effect of asset allocation strategies for financing both investments (which are studied as noted in section 1.1) and new business on a life insurance company's solvency.

The factors affecting the demand for life insurance have been extensively studied for many decades, using econometric models, and several papers [for example, Brown et al (1993), Outreville (1996) and many more] have been published which investigate the factors affecting the demand for life insurance. Some of these factors (for example real interest rates, anticipated inflation and disposable income) mainly relate to economic, financial and market performance.

Modelling the volume of new business, allowing for the price elasticity of demand of a product, in a competitive environment has also not attracted much attention in the life insurance literature, compared to the general insurance literature. In general insurance, this issue has been considered by Taylor (1986), Daykin et al (1990), and Emms et al (2004). In looking at the effect of price elasticity of demand, Daykin et al (1990) note that the increase or decrease in the volume of business written by a company is a function of whether premiums are lower or higher than the market rates. Hence, if the market as a whole begins to write business at a loss, any attempt by a particular insurer to maintain profitability will result in a reduction of its volume of business, as argued by Taylor (1986). He notes, therefore, that some market research on the shape of the demand function may assist an insurer seeking to determine a suitable underwriting strategy.

The price elasticity of demand for a life insurance product as a determinant for the product's price sensitivity appears to be more relevant to non-linked life insurance with protection-based products than savings-based products which have a relatively inelastic demand. Thus, Babbel (1985) uses a cost index to measure the price elasticity of demand for participating and non participating whole life insurance, and the results "of particular note is the finding of price elasticities for non participating insurance that were more than double the magnitude of those for participating insurance".

Ross (1989) proposes changing premium rates for with profit life insurance business gradually to reflect changing conditions, and "in practical terms it is probably necessary to set an extreme position beyond which premium rates would change, probably to support a minimum level of bonus and to allow bonus rates to alter over time". In this investigation, we allow premium rates to change as the solvency ratio changes in order to reflect the fact that we are modelling a non-linked non-profit life insurance market.

Chadburn (1993) investigates the effects of the expense ratio on the relative volumes of participating and non-participating new business. He examines the effects of the participating policyholders' returns on a mutual life company and observes that a company with a low expense ratio may require significant volumes of non-participating new business. This is because per participating policy returns depend on the extent to which non-proportionate expenses are covered by the profits from non-participating business. On the other hand, a company with a high expense ratio may control its per participating policy returns by controlling the volume of participating business and the level of non-proportionate expenses. McGaughey et al (2001) also agree that there could be possible effect on business volumes if expense allowances are taken into consideration. And the importance of the expense ratio (the firm's operating leverage) in the derivation of an operationally useful market model is further emphasised by Lester (1986).

Following the arguments presented by Lester (1986) on the effects of a firm's operating leverage on a market model, Taylor (1987) investigates how the optimal underwriting strategy will be affected by the introduction of components of fixed expenses. He notes that "the optimal premium rates taking expenses into account are precisely equal to the optimal premium rates ignoring expenses, increased by the marginal expense rate".

Emms et al (2004), based on Taylor (1986, 1987), consider two demand functions, the exponential demand function and a constant price elasticity of demand function, in investigating the 'optimal strategies for pricing general insurance'. Emms et al (2004) introduce stochasticity into the modelling of the market average premium, and a continuous time setting into the demand functions, in order not to assume any given scenario with certainty. The market average premium is modelled as a diffusion process, and the objective function is the maximisation, over a choice of premium strategies, of the expected utility of wealth over a finite time horizon. Using the same parameter value in both demand functions, the price elasticity under the exponential demand function is noted to be higher than the constant price elasticity of demand function if the insurer's premium is greater than the market average premium and vice versa. On the other hand, if the insurer's premium is greater than the market average premium, the volume of business lost is lower for the latter demand function than the former and vice versa. Simulated results are given for both demand functions for different parameter values and premium strategies. The results show that the maximised value of the objective function under the constant price elasticity of demand function is significantly higher than under the exponential demand function. This is possibly due to the inadequate penalty (in terms of loss of volume of business as explained above) under the constant price elasticity of demand function.

Chadburn (1993) does not consider the changes in non-participating business volume relative to changes in price (price/volume relationship) as considered by Taylor (1986, 1987) and Daykin & Hey (1990). However, Chadburn (1993) notes the relevance of price elasticity of demand for non-participating business. Also, we note that both Taylor (1986) and Emms et al (2004) treat the quantity of new business written as the quantity of business demanded which in most practical cases are distinct.

It needs to be noted that there has been empirical evidence relating the life insurer's insolvency to several exogenous and market factors (Browne et al 2001). They report that life insurer performance is positively related to portfolio returns on bonds and disposable personal income per capita and negatively related to unanticipated inflation. It becomes imperative therefore to evaluate the effects of some of these variables on new business volumes through modelling if the risk of insolvency is to be effectively managed.

In the work of Ng et al (2003) which relates to non life insurance, the authors warn that "by ignoring the link between profitability, credit rating and new business levels it is possible to ignore one of the most significant risks facing shareholders of an insurance company". They note that financial distress (relative insolvency) can occur when a company is experiencing low profitability, which can result in the indirect cost of customers being lost to its competitors as the financial position of the company worsens. Even though financial distress costs can be easily modelled according to Ng et al (2003), the assumption of fixed new business levels is seen as a major omission in the efforts to model it. Authors like Ross (1989) and Ng et al (2003) are all unanimous in appreciating the limitations of the deterministic approach in modelling new business volume.

Daykin et al (1990) in their work state that the impact of changes in premium rates relative to the non life insurance market can be assessed if a company is modelled alongside the market. The company's premiums are varied directly according to the solvency margin at the end of the previous year (for example, a worsening solvency margin is responded to by increasing the premium rates). In each simulation, items in the revenue account and the balance sheet are calculated in each projection year. In modelling the company's new business volume, the effect of the regulators' (or management) decisions is considered.

Hardy (1996) uses a stochastic model office to investigate the effect on relative solvency of different investment and bonus strategies. The author defines relative solvency as 'the probability that an individual insurer does not fall significantly out of line with the rest of the life insurance market, in terms of, for example, asset liability ratios or pay-outs to 'with-profits policyholders'. In other words a company that is relatively insolvent is unlikely to satisfy the obligation to meet policyholders' reasonable expectations or to be able to attract new business. This may lead to a breach of absolute solvency requirements at some point in the future that could invariably attract intervention from regulators. The author uses a model market office and four variant life offices, namely high equity, high reversionary bonus, high everything (e.g. higher equity exposure, higher reversionary bonus and others) and low initial assets to investigate two versions of the definition of probability of relative insolvency. In the first version, Hardy notes that a company is relatively insolvent if its solvency ratio is less than one and the market, as a whole, does not appear to be in difficulty (i.e. its solvency ratio is greater than one). In the second version, the author observes the office to be in difficulty for having solvency ratio significantly below the rest of the market, even if its solvency ratio is still greater than 1.0. The author obtains a new business volume by assuming a constant rate of growth only for the first five years of projection. The results show that the model market office has, as expected, the lowest risk in both versions of relative insolvency, while the office with low initial assets is clearly at more risk than the other types of office.

The results of Hardy (1996), using a constant new business growth rate for five years, may have been different if the author had adopted different new business policies (e.g. different rates of growth based on their initial assets) for each of the offices. The office with low initial assets would not have been at more risk than the other offices, as less new business volume would have been written, leading to little or no drain on capital. In this investigation, we examine the effect on relative solvency of different investment and new business strategies.

The work which has been done so far in the general insurance market may form a guide for modelling the non linked non profit life insurance market, since both markets are price sensitive as noted by Taylor (1986) and Babbel (1985) respectively. Generally, a prospective policyholder would like to assess the competitiveness of a life insurance product (with low investment content) by first comparing the price being charged before considering other factors affecting sales, as this will have a direct effect on his disposable income. Thus, in this investigation, the models proposed by Taylor (1986) and Daykin et al (1990) are adopted in modelling the price/volume relationship in a competitive non linked non-profit life insurance (for example, temporary insurance) market.

This research will take a step beyond the literature by considering not only the price of the product and regulators' decisions as the factors affecting the volume of new business, but also the disposable income of the prospective policyholders and the free capital available and allocated to finance new business. The sales cost is a key financial constraint on an insurer's marketing strategy and we assume that the cost of sales should not be greater than those in the product loading, as adopted in Ford et al (1998) for modelling a direct sales force. In other words, we do not consider the effect of additional sales cost on new business volume in this research.

From the related literature, we recognise the following factors as being capable of affecting a company's future new business plan in a life insurance market (hence could affect the new business model):

- Capital available to write new business,
- Capital required to write new business;
- Pricing policy relative to the market;
- Demand for the product, (as being affected by its price and disposable income of prospective policyholders); and
- Regulator's control of the company's solvency relative to the market.

1.3 Research Objectives

We have noted that a new business plan comprises amongst others the pricing policy and the expected new business volume, factors that are considered important by insurance regulators in determining a life insurance company's future profitability and solvency. But like every other business, an insurer faces some uncertainties in fulfilling its new business plan. Although, other studies have developed models for investigating these uncertainties, there are some assumptions that we believe are not practically feasible. For instance, some of the studies are based on the implicit assumption that the quantity of new business demanded by prospective buyers of life insurance policy is the actual quantity written, while others considered companies that are closed to new business at or some few years after projection date. Further, a "simplistic deterministic approach" [in the words of Ng et al (2003)] to modelling new business volumes in life insurance in evaluating risk has been adopted by others. We have therefore adopted a more encompassing approach that recognises these shortcomings in the literature in the application of stochastic modelling to evaluate the new business risks in non-linked insurance business. The overall purpose is to examine the effects of free assets (the excess of assets over supervisory liabilities and any required solvency margin) on price and new business volume taking into consideration the solvency requirements imposed by insurance regulators. Specifically, the study is designed to serve the following objectives.

- To examine the effects of new business risk on a life insurance company in a competitive and stochastic environment.
- To investigate the level of free assets to be allocated to finance new business over a given time horizon to produce a fixed level of insolvency risk under a desired new business strategy and investment strategy.
- To examine the effects of changes in the factors influencing new business volume on the measures of new business risk.
- To investigate the sensitivity of a company's future new business risk (i.e. measures of risk) to factors affecting new business.
- To consider risk control methods which enable an insurer to increase the proportion of free assets allocated to finance new business over a time horizon without facing the risk of insolvency.

1.4 Methodology

An asset-liability model for both the company and the life insurance market (level term (LTA) and/or non-profits endowment portfolios) is used to simulate the future liabilities and the assets stochastically in each projection year. On each simulated path, the model outputs (such as total value of assets backing liabilities, free assets, total value of statutory liabilities and solvency ratio) at annual intervals are computed. Thus, after sufficient simulations, relevant measures of risk (i.e. probability of insolvency, mean shortfall risk and probability of relative insolvency) for the company and/or the market based on the computed solvency ratio in each projection year may be calculated. The investigation also reviews the impact of the risk of changes on the factors affecting the new business and how the level of risk can be managed through underwriting, product design and mix of business and reinsurance. UK life insurance market data are analysed by using a regression analysis to obtain an initial set of model parameters (e.g. demand function parameters), and a sensitivity analysis to initial parameters can be used to determine the appropriate base values of the model parameters.

We model the quantity of new business demanded in a way that is similar to the approaches adopted by Taylor (1987), Daykin et al (1990) and Emms et al (2004). However, our new business model for quantity demanded allows for the effects of disposable income, regulator's (or management) control and any interactions between the company and the market under a given price elasticity of demand function. The different new business strategies considered allow the company to respond differently to its worsening (or improving) solvency ratio in the previous year by an adjustment to a profit-tested premium.

We assume that the company and the market would have two sets of new business models for the quantity of new business demanded. For the market model, while the first set of new business models test the price and new business volume relationship, the second set of new business models consider both the price and income effect on new business volume. For the company model, the first set of new business models test the effect of company's premium relative to the market premium and management decision on new business volume and the second set of new business models consider the effect of price and the regulator's control on new business volume. For both the company and market, the quantity of new business written is modelled as a function of free assets, new business strains and quantity of new business demanded.

For a given initial solvency ratio as an input at the projection date, we consider either a constant proportion of free asset allocation strategy or a dynamic allocation of free assets strategy to finance new business over the time horizon to produce a particular level of insolvency.

Initially, we consider a hypothetical portfolio (with cohort of lives aged 55 at entry) of 10-year level term assurance (LTA) contracts without options with which simulated results are obtained and analysed. Subsequently, we consider other hypothetical portfolios with different compositions by different entry ages and/or different product designs, and the simulated results obtained may be compared with those of the initial portfolio (where necessary) as part of the investigation of how the level of new business risk can be managed.

1.5 Overview/Structure of the Thesis

This thesis is organised as follows: in Chapter 2 we model the assets and liabilities of a company alongside the temporary life insurance market. In modelling the liabilities, we formulate new business strategies with different premium setting structures and adopt new business models for both the market and company to determine the quantity of new business demanded and written in a projection year. In modelling the assets, we propose investment strategies and use the Wilkie (1995) investment model to generate random investment returns. We estimate liability model parameters (e.g. the exponential and constant price elasticity of demand functions parameters) by carrying out a regression analysis on new business data from the Association of British Insurers (ABI) and financial data from the Department of Trade and Industry. We use the initial results from the regression analysis as a base in carrying out a sensitivity analysis in order to obtain a set of sensible base parameter values for the market and the company.

In Chapter 3 we consider measures of new business risk, and use the simulated results to carry out a sensitivity analysis (using the concept of hypothesis testing) of new business models and model parameters for the company and market to determine which model will be taken to represent the temporary life insurance market, and which company new business model will be used to interact with market. We examine the effect of factors affecting the new business volume on new business risk, and also how various types of life office (which differ by new business strategy and/or investment strategy) affect risk measures.

In Chapter 4 we consider tools such as underwriting, mix of new business, product design, constrained new business growth and reinsurance to manage the new business risk.

Chapter 5 concludes by summarising the main arguments, and findings, and suggesting possible areas for further research.

Chapter 2

Models and Assumptions

2.1 Introduction

In this chapter we consider the following:

- Model the assets and liabilities of a company as a going concern alongside the temporary insurance market;
- Estimate the initial model parameters using a regression analysis, and determine an appropriate set of base parameter values by sensitivity analysis;
- Provide a brief description of the structure of the model using the market; and
- Consider the model assumptions.

The market model results will form the base for modelling the company with the model assumptions given in the relevant sections. In addition, we make the following assumptions:

- The market and company are assumed to be mature (having ten years of existing business, of all possible durations) at the start of projection, and they write 10-year temporary insurance contracts to lives aged 55 at entry.
- Each cohort of new business actually written is assumed to occur at the start of year, and in the future they are identified and treated separately in each projection year.
- The past new business volumes (in policy numbers) for the market and company are assumed to grow at a fixed rate, in line with the past inflation rate (e.g. 3%).
- Both the market and the company have the same level of office premium (to be known as the profit-tested premium) for the existing business at the projection date, which is calculated using profit-testing techniques. This assumption is necessary in order not to allow the company's premium relative to the market premium affect its new business volume at the projection date.

- The profit tested premium attracts an assumed quantity of new business demanded and written (say 10,000 new polices) in year 0 in the market.
- An initial solvency ratio is chosen (arbitrarily) to determine the level of initial free assets having calculated the reserves for existing business at the projection date.
- The actual new business volume underwritten by the market and the company in a projection year depends on the capital available and therefore the less capital available the less business that is actually written. We assume that, if the capital required (new business strain) in writing a particular cohort of new business in a given year is more than the free assets available, then the company needs to write less new business. There is no allowance for a capital injection from external sources (e.g. from shareholders) other than the free assets generated internally (e.g. surplus arising from existing business) to finance future new business. This means that no new business is written in a year if there are no free assets.
- The future premium setting structure for the market is assumed to be a function of inflation, profit-tested premium and a measure of premium response to changes in a company's worsening / improving solvency ratio at the end of the previous year.
- The method of determination of an individual company's future office premium is similar to that of the market except that the price (premium) inflation rates for the company may be different.
- The expense loadings match the actual acquisition and management costs of a company operating in the market.
- The projection period of 20 years is considered, as the full effect of the impact of a substantial rise or fall in new business levels may become apparent only over a longer period.
- The initial assets at projection date are taken as a multiple (e.g. the initial solvency ratio) of the initial liabilities (e.g. reserves for the existing policies).

As the company is an integral part of the market, we assume that it interrelates with the market in the following ways:

- The company writes a proportion of the market volume of business in each projection year, the proportion is obtained though the demand function and management decision (or regulator's control). The company's price elasticity of demand function relates the company's office premium to that of the market office premium. This is used to determine the market share of the company when considering only the effect of the price elasticity of demand.
- The relationship between the solvency ratios for the company and market determines the relative solvency with which the management takes a decision on the company's proportion of new business demanded in the market.

The company differs from the market in the following respects:

- Different expense escalation rates are experienced by the company.
- Deaths and lapses in each future year are obtained stochastically. For the market they are obtained deterministically, as the variability in the deaths and lapses are likely to be more stable in the market than for the company due to the law of large numbers.

2.2 Market Liability Model

The notation for all the variables and constants in the market and company liability models is given in section 1.1 of appendix 1. However, they are also defined in the relevant sections as they are introduced.

2.2.1 Market New Business Models

We assume that the quantity of new business volume demanded in the temporary insurance market is dependent on two key variables: the economic conditions (i.e. the disposable income of a prospective policyholder which may be linked to real wage inflation) and the price of the product. "The investment conditions and financial position of a life office will impact on the likely volumes of certain types of business written. For example, in a recession the volume of new business may be expected to reduce. Also, if the office's free assets reduce to a low level, then it will be unable to write high volumes of new business, especially types of business with new business strain", Abbey and Brooks (1998)). In general terms, the quantity of new business demanded in year t can be expressed as a function of premium and/or real wage inflation and new business demanded in the previous year $t-1$

$$NM^j(t)=f\left(OP_m^j(t),OP_m^j(t-1),RWIR^j(t-1),\ NM^j(t-1)\right) \tag{1}$$

Where,

$NM^j(t)$ = Quantity of new business demanded (in terms of number of new policies) in the market in projection year t for simulation j.

$OP_m^j(t)$= Market office premium for new business issued in projection year t for simulation j.

$RWIR^j(t)$ = Real wage inflation rate for the projection year t for simulation j.

2.2.1.1 Effect of Real Wage Inflation on quantity of new business demanded

The disposable income of prospective policyholders will reduce during a period of economic downturns and this may affect the purchases of life insurance products and vice versa. We assume that the proportion of volume of exposure (measured in terms of quantity of new business demanded in year *t-1*) to be demanded in year *t* in the market (denoted by $RWP^j(t)$) is a linear function of the previous year's real wage inflation (denoted by $RWIR^j(t-1)$), which is simulated stochastically using Wilkie's model (1995). An increase in real wage inflation in year (t-1) would then lead to an increase in the quantity demanded in year t and vice versa. Thus, we propose

$$RWP^j(t)=a+b*RWIR^j(t-1) \tag{2}$$

$$for\quad 1\geq a>0 \quad and \quad b\ \geq 0.$$

2.2.1.2 Price Elasticity of Demand Functions (price effects)

We consider the two different demand functions in Taylor (1986, 1987) and Emms et al (2004), namely the exponential and constant price elasticity of demand functions. These demand functions are adopted and modified, with an adjustment to allow for price inflation, in determining the effect of changes in the current year's market price (denoted by $OP_m^j(t)$) relative to the preceding year's market price (denoted by $OP_m^j(t-1)$) on quantity of new business demanded in year t (denoted by $NM^j(t)$).

Constant Price-Elasticity Demand Function

$$CDF_m^j(t) \;=\; \left(\frac{OP_m^j(t)}{OP_m^j(t-1)*inf_p^j(t-1)}\right)^{-CK_m} \tag{3}$$

For some constant values, $CK_m > 0$, the price elasticity (see definition in the footnote)[1] of this function remains constant i.e. $E\left(OP_m(t)\right) = CK_m$

Where,

$CDF_m^j(t)$ = Constant price elasticity of demand function for the market in projection year t for simulation j.

$\inf_p^j(t)$ = Price inflation rate for the projection year (t-1, t) for simulation j.

Exponential Demand Function

$$EDF_m^j(t) = Exp\left\{\frac{-EK_m*(OP_m^j(t) - OP_m^j(t-1)*inf_p^j(t\text{-}1))}{OP_m^j(t-1)*inf_p^j(t\text{-}1)}\right\} \tag{4}$$

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[1] The price elasticity of demand is an economic measure of the sensitivity of the demand for a product to a small change in its price. It is defined as $E = -\frac{\text{Perecentage change in demand}}{\text{Percentage change in price}}$. In general, at any price (p) and demand (p, Q), where Q is the quantity, the price elasticity of demand is also defined as $E = -\frac{dQ}{dp}\frac{p}{Q}$.

For some constant values, $EK_m > 0$, the price elasticity (see definition in the footnote)[1] of this function increases linearly i.e. $E(OP_m(t)) = \frac{EK_m \times OP_m(t)}{OP_m(t-1)}$.

Where,

$EDF_m^j(t)$ = Exponential price elasticity of demand function for the market in projection year t for simulation j.

Interpretation of (3) and (4)

For the same parameter value $EK_m = CK_m$ the proportions of volume of exposure (measured in terms of quantity of new business demanded in year *t - 1*) to be demanded in year *t* under an exponential of demand function is lower than under a constant price elasticity of demand function, as shown in Emms et al (2004). For example, if $EK_m = CK_m = 2$, the proportion of the volume of exposure (quantity demanded in year *t-1*) to be demanded in year *t* in the market may arise by increasing the market premium by 1% over a year. The proportions in respect of the former and latter demand functions are in fact $\exp(-{}^{EKm}\!/_{100}) = 0.980198$ and $(1.01)^{-CKm} = 0.980296$ respectively, using equations (4) and (3) without the inflation adjustment. Clearly, these parameters will not have the same value if both equations are to produce the same proportion of exposure.

2.2.1.3 New Business Models for Quantity Demanded

In this investigation we assume that the market new business models defined below only generate the quantity of new business demanded and not the actual quantity of new business written. The quantity of new business demanded in the market in year t is proportional to the quantity of new business demanded in the preceding year (t – 1), and the proportion is assumed be the product of a real

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[1] The price elasticity of demand is an economic measure of the sensitivity of the demand for a product to a small change in its price. It is defined as $E = -\frac{\text{Perecentage change in demand}}{\text{Percentage change in price}}$. In general, at any price (p) and demand (p, Q), where Q is the quantity, the price elasticity of demand is also defined as $E = -\frac{dQ}{dp}\frac{p}{Q}$.

wage inflation effect (denoted by $RWP^j(t)$) and the demand function (denoted by $DF_m^j(t)$), see equation (5) below.

$$\frac{NM^j(t)}{NM^j(t-1)} = RWP^j(t) \times DF_m^j(t) \tag{5}$$

Where,

$DF_m^j(t)$ = Demand function for the market in projection year t for simulation j.

Thus, we propose four new business models for the market from equation (5) above.

Model M1: Exponential Demand Function (No Real wage Inflation Effect)

Let $DF_m^j(t) = EDF_m^j(t)$ and $RWP^j(t) = 1$ in equation (5)

Model M2: Exponential Demand Function and Real wage Inflation Effect

Let $DF_m^j(t) = EDF_m^j(t)$ in equation (5)

Model M3: Constant Price-elasticity of Demand Function (No Real wage Inflation Effect)

Let $DF_m^j(t) = CDF_m^j(t)$ and $RWP^j(t) = 1$ in equation (5)

Model M4: Constant Price-elasticity of Demand Function and Real wage Inflation Effect

Let $DF_m^j(t) = CDF_m^j(t)$ in equation (5)

2.2.1.4 New Business Model for Actual Quantity Written

The actual new business volume written (i.e. quantity written) is dependent not only on the quantity of new business demanded but also on the free assets being allocated to finance new business plans ($FAN_m^j(t)$) and the capital required to write the quantity of new business demanded (see calculation of $ENBS_m^j(t)$ in section 3A.10 of appendix 3).

$$FAN_m^j(t) \quad F'A_m^j(t \quad 1) \times \rho_m, \text{ where } 0 \leq \rho_m \leq 1 \tag{6}$$

$$\beta_m^j(t) = \frac{FAN_m^j(t)}{ENBS_m^j(t)}, \qquad 0 \le \beta_m^j(t) \tag{7}$$

$$ANM^j(t) = \begin{cases} \beta_m^j(t) \times NM^j(t) & if \quad 0 < \beta_m^j(t) < 1 \\ NM^j(t) & if \quad \beta_m^j(t) \ge 1 \\ 0 & if \quad \beta_m^j(t) = 0 \end{cases} \tag{8}$$

Where,

$ANM^j(t)$ = Quantity of actual of new business written (in terms of number of policies) in the Market in projection year t for simulation j.

$\beta_m^j(t)$ = The ratio of the allocated free assets to the total expected new business strain in the market in projection year t for simulation j.

$ENBS_m^j(t)$ = Total expected business strain for the market (i.e. capital required to write new business volume demanded) at start of year t for simulation j.

$FAN_m^j(t)$ = Free assets allocated to write new business in year t for simulation j.

$F'A_m^j(t)$ = Market's statutory free assets (including reserves released from expired policies) at the end of projection year t for simulation j.

ρ_m = Constant proportion of free assets allocated to write new business over a projection period.

Interpretation of (6), (7) and (8)

In equation (6), at the start of each projection year t a constant proportion of available free assets (including reserves which are released from expired policies) is allocated to write new business. Equation (7) gives the ratio of the allocated free assets to the total expected new business strain which determines the actual quantity of new business to be written. Equation (8) states that the overall quantity of new business demanded in the market is written if this ratio in equation (7) is equal to or greater than one (i.e. if the free assets used to write new business are sufficient to cover the new business strain of the quantity demanded). On the other hand, a fraction of the quantity of new business demanded is actually written if the ratio in equation (7) is less than one. Thus, no new business is written if there are no free assets available or allocated to write new business.

2.2.2 New Business Strategies

The aim of the new business strategies is to help in determining factors that measure the premium response to solvency in equation (11) below. The purpose of the office premium adjustment function (denoted by $AdjF_m^j(t)$) for a new business strategy in year t is to allow a company to respond to its worsening / improving solvency ratio at the end of the previous year through the premium being charged. The adjustment is based on the new business decision factor (denoted by NDF) which is chosen arbitrarily, and it is assumed that it is at least equal to one ($NDF \geq 1.0$). We assume that a profit-tested premium calculated (e.g. using the policy details in table 2.1 below) will be payable in projection year t when $SR_m(t-1) = NDF$. We propose four new business strategies in this investigation and they are discussed below.

2.2.2.1 New Business Strategy-1 (NBS-1)

A company will reduce (increase) its premium, relative to a profit-tested premium (denoted as $\overline{OP_m}$), to attract more (less) new business demanded in projection year t if its solvency ratio in year $(t-1)$ (denoted as $SR_m(t-1)$) is above (below) NDF. In this case the company's premium response to the solvency function (denoted by $PRF_m^j(t)$) is positive in equation (12) below (i.e. $l_m(+ve)$ in equation (11)). The above is illustrated in figure 2.1 showing the office premium charged in year t without the effect of price inflation under new business strategy 1.

In a competitive market, a company with free assets is likely to undertake risky ventures such as reducing its premium to attract more new business in order to increase its market share which would be at the expense of meeting statutory solvency requirements if an inappropriate volume of new business is written. In practice, this may be the most feasible strategy for large insurers that are risk tolerant, but would be a risky strategy in the sense that all available (or a greater percentage of) free assets may be used up to write new business thereby creating solvency problems.

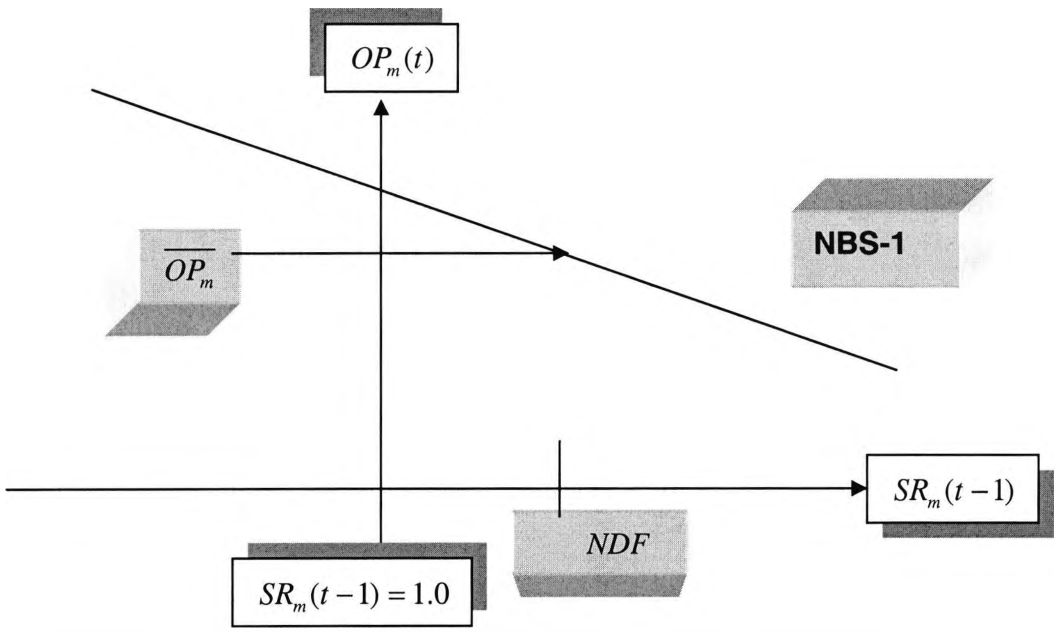

Figure 2.1: Office premium charged in year t under new business strategy 1.

However, the risk may be reduced if the allocation of free assets to finance new business expansion (which may be backed by investment in secure assets e.g. gilts) and the investment of the excess in volatile assets (e.g. equities) are explicitly allowed for, a decision to be investigated in chapter 3. In practice, the decision concerning the allocation of free assets to finance new business needs to be taken alongside the choice of an appropriate investment strategy.

2.2.2.2 New Business Strategy 2 (NBS-2)

A company will increase (reduce) its premium, relative to a profit-tested premium (denoted as $\overline{OP_m}$), to attract less (more) new business demanded in projection year t if its solvency ratio in year $(t-1)$ (denoted as $SR_m(t-1)$) is above (below) NDF. In this case the company's premium response to the solvency function $PRF_m^j(t)$ is negative in equation (12) below (i.e. $l_m(-ve)$ in equation (11)). The above is illustrated in figure 2.2 showing the office premium charged in year t without the effect of price inflation under new business strategy 2.

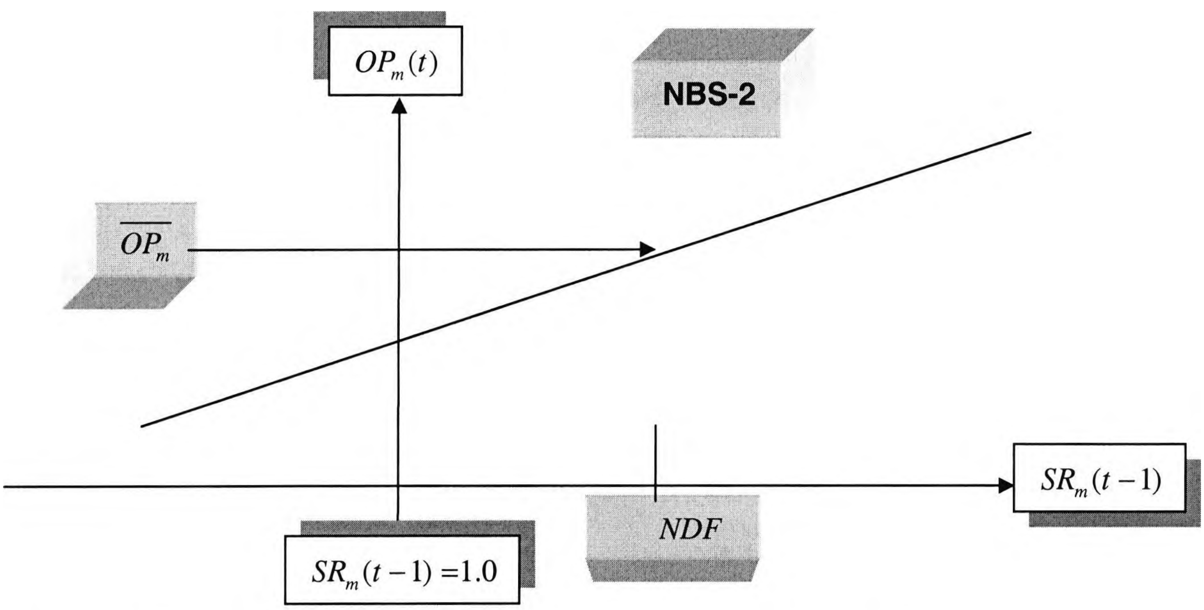

Figure 2.2: Office premium charged in year t under new business strategy 2.

In practice, a company that is risk averse is likely to hold back some capital for investment by writing less new business even though it has free assets. This strategy is unlikely to be adopted by a large insurer since its solvency position will improve to the detriment of its market share. However, it may be useful for insurers that are concerned with meeting the regulators' short-term statutory solvency requirements or phasing out a particular product line. The free assets allocation decision (in terms of proportion of free assets to finance new business) may be different for an insurer using this strategy (as it implicitly holds back capital for investment) when compared with new business strategy 1, an aspect also to be investigated in chapter 3.

The following two new business strategies (denoted as NBS-3 and NBS-4) which are discussed below are derived by combining components of strategies 1 and 2. For example, NBS-3 is similar to NBS-1 and NBS-2 when $SR_m(t \quad 1) > NDF$ and $SR_m(t-1) < NDF$ respectively, if NDF is the same for all new business strategies. The converse is true for NBS-4.

2.2.2.3 New Business Strategy 3 (NBS-3)

A company will reduce its premium, relative to a profit-tested premium (denoted as $\overline{OP_m}$), to attract more new business demanded in projection year t if its solvency ratio in year $(t-1)$ (denoted as $SR_m(t-1)$) is either above or below a new business decision factor (NDF). The extent of premium reduction depends on how far its solvency ratio in year t –1 is from the new business decision factor. The above is illustrated in figure 2.3 showing the office premium charged in year t without the effect of price inflation under new business strategy 3.

The maximum premium in any year (equal to the profit-tested premium $\overline{OP_m}$) is payable when $SR_m(t-1) = NDF$. In this case the company's premium response to the solvency function $PRF_m^j(t)$ is positive in equation (13) below (i.e. $l_m(+ve)$ in equation (11)).

An insurer intending to maintain a level of market share without unduly affecting its solvency requirements is likely to adopt strategy 3. The allocation proportion of free assets to finance new business is also considered differently for this strategy.

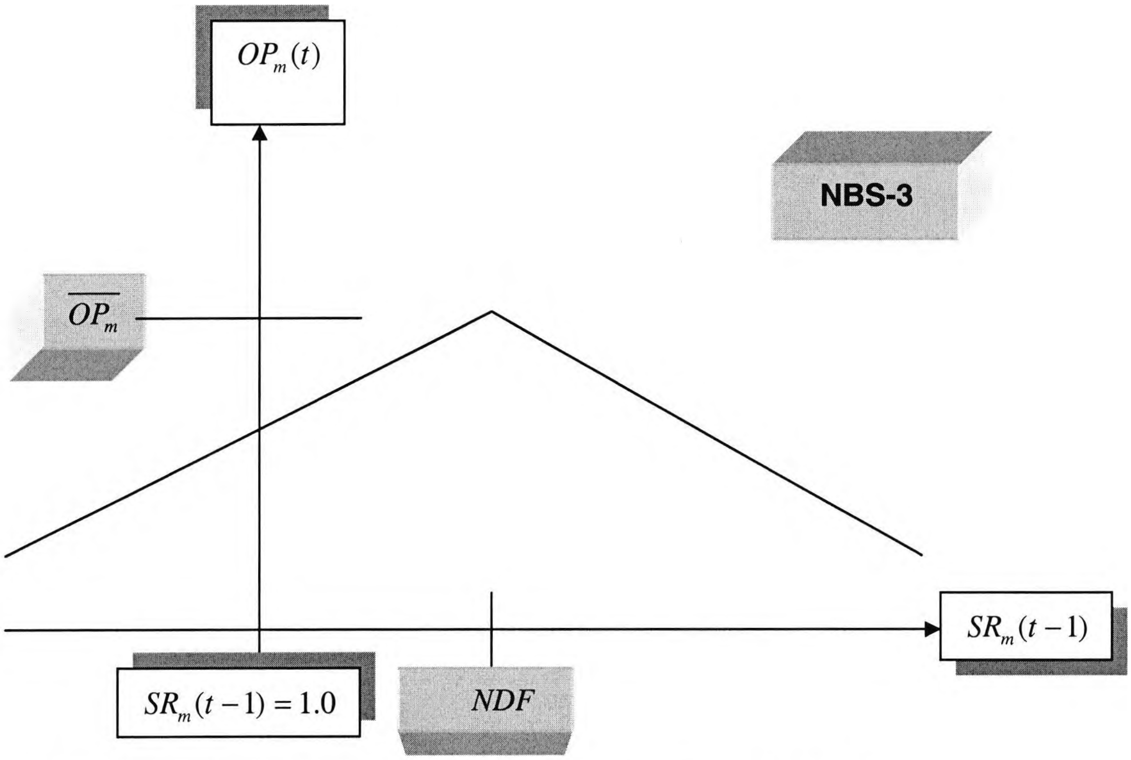

Figure 2.3: Office premium charged in year t under new business strategy 3

2.2.2.4 New Business Strategy 4 (NBS-4)

A company will increase its premium, relative to a profit-tested premium (denoted as $\overline{OP_m}$), to attract a reduced new business demanded in projection year t if its solvency ratio in year $(t-1)$ (denoted as $SR_m(t-1)$) is either above or below a new business decision factor (NDF). The extent of increase in premium depends on how far its solvency ratio in year t –1 is from the new business decision factor.

The minimum premium in any year (equal to the profit-tested premium $\overline{OP_m}$) is payable when $SR_m(t-1) = NDF$. In this case the company's premium response to the solvency function $PRF_m^j(t)$ is negative in equation (13) below (i.e. $l_m(-ve)$ in equation (11)). The above is illustrated in figure 2.4 showing the office premium charged in year t without the effect of price inflation under new business strategy 4.

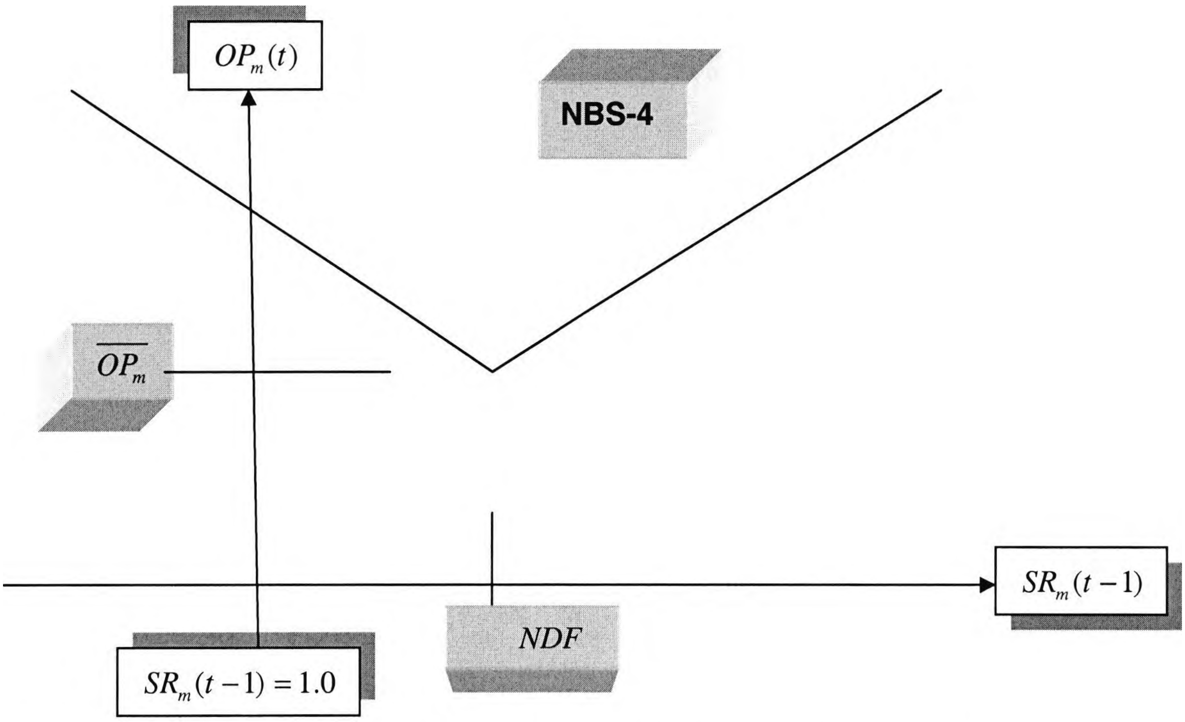

Figure 2.4: Office premium charged in year t under new business strategy 4

A company is less risk averse relative to a company adopting strategy 2 if the company's solvency ratio ($SR_m(t-1)$) is above *NDF* and vice versa. Thus the new business strategy 4 (NBS-4) is likely to hold less capital back for investment with the aim of increasing its market share slightly than new business strategy 2 NBS-2. Similarly, the decision on allocation of free assets to finance new business is considered differently for this strategy.

2.2.2.5 New Business Decision Factor

New business strategies may not necessarily have the same values for NDF. We assume that the new business decision factor for new business strategies 1 and 2 is slightly different from new business strategies 3 and 4.

We propose that for new business strategies 1 and 2, $NDF = 1.0$ whilst for new business strategies 3 and 4, $NDF > 1.0$. In this later case, the NDF is at least equal to the long-term mean solvency ratio derived deterministically in chapter 3 (i.e. $NDF \geq 1.05$). Thus, a company adopting new business strategies 3 and 4 is likely to have a lower and higher insolvency risk than a company with new business strategies 1 and 2 respectively. The reason for the above is explained below.

For instance, If $NDF = 1.0$ for all new business strategies, for a company adopting new business strategy 3, the premium is always reduced relative to a profit-tested premium. This means attracting more business than expected, thereby creating more solvency problems than a company adopting new business strategy 1. If $NDF > 1.0$, for new business strategy 3, the reduction in premium is likely to decrease and this will result in writing a relatively small volume of new business. Thus, the risk of insolvency is also likely to decrease. The degree of risk aversion relative to the desire for market share determines the level of NDF (i.e. the higher the degree of risk aversion the higher the NDF and vice versa for NBS-3). The insurer's risk aversion is greater than for an insurer adopting strategy 1 if the insurer's solvency ratio ($SR_m(t-1)$) is above NDF and vice versa. The converse of the above discussion is true for new business strategy 4 relative to new business strategy 2.

2.2.3 Annual Office Premium for New Business

We propose that the pricing policy under a given new business strategy is based on the assumption made in section 2.1, that is the office premium (denoted by $OP_m^j(t)$) for the coming year is a function of the profited-tested premium, price inflation and measure of premium response to changes in a company's worsening / improving solvency ratio at the end of the previous year.

$$OP_m^j(t) = \overline{OP_m} \times InfacF_p^j(t-1) \times AdjF_m^j(t) \tag{9}$$

$$InfacF_p^j(t) = \prod_{k=0}^{k=(t)} \left[1 + \inf_p^j(k)\right], \quad \inf_p^j(0) = \inf_p(0) = 3\% \, p.a. \tag{10}$$

$$PRF_m^j(t) = \frac{l_m(\pm ve)}{SR_m^j(t-1)} \tag{11}$$

We propose two alternative definitions for office premium adjustment function, $AdjF_m^j(t)$, with further explanation given below[2].

$$AdjF_m^j(t) = 1 - PRF_m^j(t) \times \left(SR_m(t-1) - NDF\right) \tag{12}$$

$$AdjF_m^j(t) = 1 - PRF_m^j(t) \times \left| SR_m^j(t-1) - NDF \right| \tag{13}$$

Where,

$AdjF_m^j(t)$ = Market office premium adjustment function for a new business strategy in year t for simulation j.

$InfacF_p^j(t)$ = Accumulation of price inflation up to the end of projection year t for simulation j.

$\inf_p^j(t)$ = Price inflation rate for the projection year (t-1, t) for simulation j.

$l_m(\pm ve)$ = A factor that measures the response of a company's premium to changes in its worsening (or improving) solvency ratio in the market.

$\overline{OP_m}$ = Profit-tested office premium for the market in section 2.2.5 (the same as premium for existing business at projection date i.e. $OP_m(0) = \overline{OP_m}$).

$PRF_m^j(t)$ = A company's premium response to solvency function in the market in projection year t for simulation j.

$SR_m^j(t)$ = Market's solvency ratio at the end of projection year t for simulation j.

[2] The modulus sign in equation (13) is used to reflect the shape of figures 2.3 and 2.4 for new business strategies 3 and 4 respectively, where there is a maximum or minimum turning point.

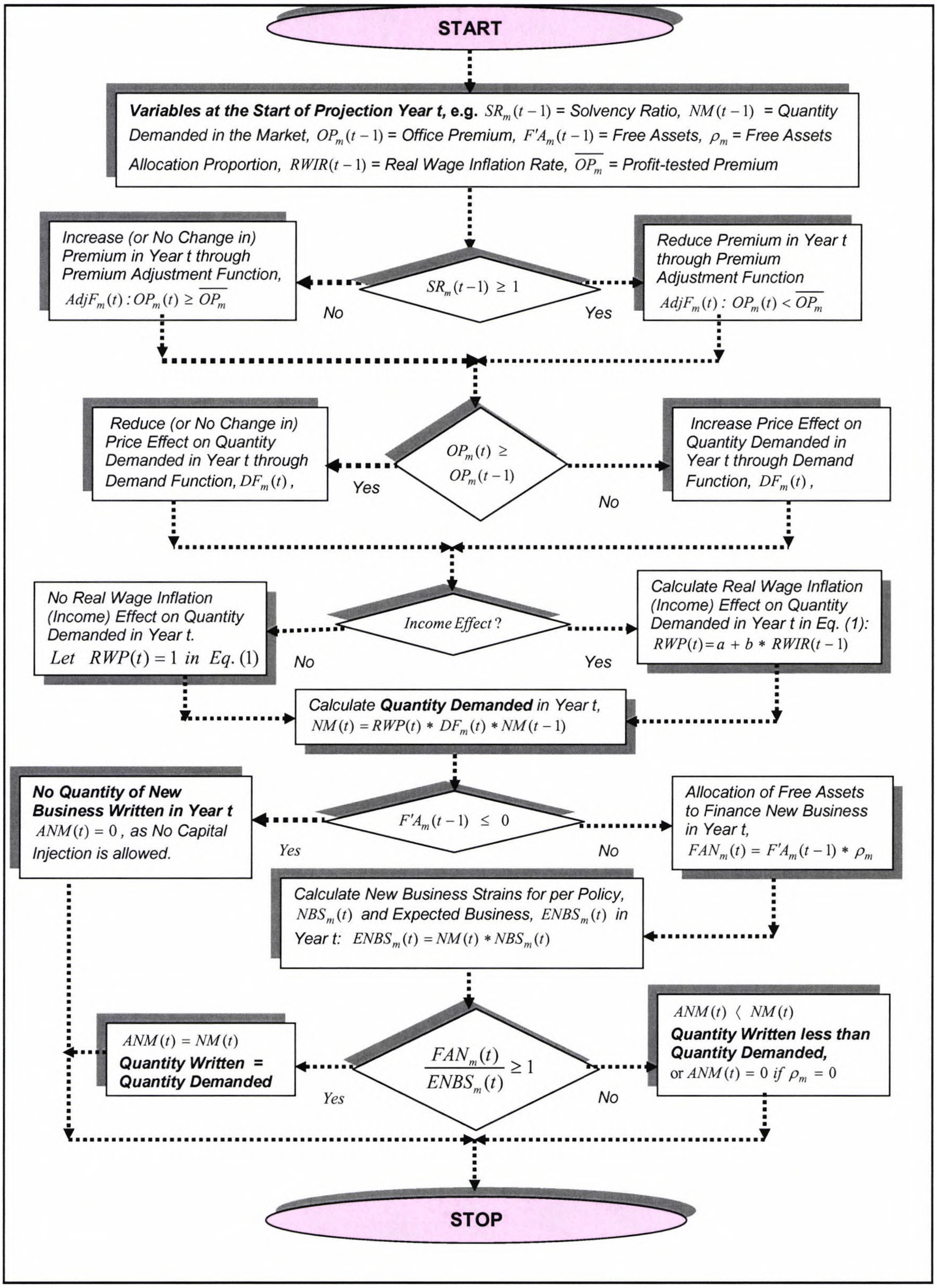

Figure 2.5: Schematic representation of market new business models under new business strategy 1.

Remarks

(1) The premium calculation in a future projection year, see equation (9), allows for both the effects of price inflation expectations and the size of the free assets (i.e. solvency ratio), but the former is not taken into account in the premium used in the demand function in generating the quantity of new business demanded under a given new business strategy. This is because a policyholder's ability to pay for insurance cover is dependent on the level of real disposable income and not on the nominal income. Diacon (1980) also concludes "that inflation has a marginally significant, negative effect on demand for both protection-based and savings-based life insurance".

(2) Equations (12) and (13) relate to the pricing policies for new business strategies (NBS-1 and NBS-2) and (NBS-3 and NBS-4) respectively without allowing for price inflation effect. The main difference between the two equations is that the profit-tested premium is assumed payable in year t when $SR_m(t-1) = NDF = 1.0$ and $SR_m(t-1) = NDF \geq 1.05$ under equation (12) and equation (13) respectively. Thus, for a given positive value of the measure of premium response to solvency (denoted by $l_m(\pm ve) \geq 0$) in equation (11)), equation (12) under new business strategy NBS-1 may produce a higher or lower premium than the profit-tested premium in a projection year while equation (13) under new business strategy NBS-3 is expected to produce a lower premium than the profit-tested premium in a projection year. The converse is true for comparing new business NBS-2 and NBS-4.

(3) Figure 2.5 above is a flow chart which summarises sections 2.2.1 and 2.2.2, the steps leading to writing the actual quantity of new business $ANM(t)$ in a projection year using new business strategy 1. Briefly, as a company's solvency ratio at the start of year t is greater than one it reduces its premium relative to a profit-tested premium in order to increase the quantity of new business demanded relative to the quantity demanded in the previous year through the

demand function (i.e. price effect only) under new business strategy 1 and vice versa. If the effect of income on demand is taken into account, the quantity of new business demanded in year *t* will increase further if the real wage inflation at the start of year is positive. The quantity of new business demanded that is actually written in year *t* is dependent on the extent to which the free assets available and allocated (denoted by $FAN_m^j(t)$) cover the expected new business strain at the start of year t (denoted by $ENBS_m^j(t)$). The decision concerning the constant proportion of free assets (denoted by ρ_m) allocated to finance new business over a projection period is taken at the projection date. The statutory free assets at start of year t (denoted by $F^{\prime}A_m^j(t)$) include reserves released from expired policies at the end of projection year t -1). No new business is written if no free assets are available or are allocated to finance new business ($\rho_m = 0$).

2.2.4 Withdrawal (i.e. lapses) Model

There are many factors affecting the rate of discontinuance of a policy as stated in Booth et al (2005) some of which are discussed below. Firstly, there is a correlation between adverse lapse experience and changes in economic factors such as economic downturns, which affect the disposable income of the policyholder. Secondly, the policyholder may have been sold a policy that was inappropriate to his (or her) needs from the outset, or one that subsequently become inappropriate, and he may become aware of it within a few years after the commencement of the policy and therefore discontinue the contract.

Thus, in this investigation we propose that the lapse experience in projection year t (denoted by ${}^{j}W_x^u(t)$) is dependent on the combined effect of disposable income of the policyholder which may be linked to the real wage inflation in year $t-1$ (denoted by $RWIR^j(t-1)$) and the duration of the policy (denoted by $C_x(u)$). We assume that the lapse rates before projection date are only affected by the duration of the policy. We also assume that the disposable income of a policyholder (real wage inflation) has a negative linear relationship with lapse rate, if the effect of duration is held constant or ignored, as shown in equation (14) below.

$$^{j}W_x^u(t) = C_x(u)\{1 - d \times RWIR^j(t-1)\} \tag{14}$$

where $0 \leq C_x(u) < 1$ *and* $0 \leq d$.

$^{j}W_x^u(t) =$ Withdrawal rate for a life aged x at entry with duration u years in the year projection t for simulation j.

$C_x(u)$ = Lapse rate at policy duration for a policy issued to a life aged x at entry.

The parameter value d will be chosen to ensure that $^{j}W_x^u(t) \geq 0$

2.2.5 Policy Design

The policy design follows that of Mc Gaughey et al (2001) but with modifications for the purposes of simplicity. The detailed assumptions represented in table 2.1 below are used to calculate the monthly office premiums (the profit tested premiums) applicable to all existing policies at entry ages 35, 45 and 55 before projection date, using the profit-testing technique adopted in Hylands & Gray (1990). However, we are only considering a market portfolio with temporary insurance policies issued to lives aged 55 at entry at this stage of the investigation. The effect of changing the entry age on simulated results for the policy design (i.e. Policy Type 1 in table 2.1), is considered later in chapter 4.

The guidelines for the profit testing calculations are given in section 2B of Appendix 2. We assume that the sum assured has remained constant for the period before the projection date but increases with inflation after the projection date and death benefits are paid at the end year of death. Both the existing and new policies will be affected by the expense assumptions in future projection years. We assume that expenses are paid at the start of the year and future expenses (i.e. per policy amount) will increase with inflation. Tax is not considered in future projection years for reasons of simplicity.

Table 2.1: Policy design for term assurance contract (Policy Type 1)

Age at Entry :		x	35, 45 and 55
Term of Contract		n	10 years
Annual Premium		OP(0)	£183.60 (£15.30 per month from profit testing) for age 35
			£342.96 (£28.58 per month from profit testing) for age 45
			£ 874.20 (£72.85 per month from profit testing) for age 55
		OP(t)	for policies issued in projection year t
Sum Assured:		SA(0)	£100,000 per policy for policies issued before projection date
		SA(t)	sum assured per policy for policies issued in projection year t
Expenses	Initial	Ie(0)	£115 per policy at start of first year
		Ier(0)	5% of annual premium at start of first year
	Renewal	Re(0)	£15 p.a (for Profit Testing -payable monthly from month 2,with inflation at the start of second month)
Commission	Initial	Icr(0)	125% of LAUTRO Commission for an initial period of of 48 months on non-indemnity terms (LAUTRO rules)
	Renewal	Rcr(0)	2.5% of premium outside the initial period
Inflation rates:	Expenses	$inf_e(0)$	4.7% p.a. for period before projection date
		$inf_e(t)$	expected inflation of expenses (per policy amount) in projection year t generated using Wilkie model
	Price	$inf_p(0)$	3% p.a (assumed initial rate at projection date)
		$inf_p(t)$	expected inflation of premium and sum assured in projection year t generated using Wilkie model
Tax rate:		Tax	31% (on a net/net basis without a deferral period- company will be taxed on investment income and obtain tax relief on expenses before projection date).. We ignore taxation in future projection periods.
Gross Investment Income:		r_t	7.75% p.a. (adjusted in line with Wilkie model)
Risk Discount Rate:		r_D	10% p.a. (adjusted in line with Wilkie model)
Mortality:		TM92 Table	70% of TM92, 5-year select period (male non-smoker)
Benefits:		SA(0)	Sum assured payable at the end of projection year of death for policies issued before projection date
		SA(t)	Sum assured payable at the end of projection year of death for policies issued in projection year t
Lapse Rates :			10% in first year and 5% thereafter.
Valuation Basis:			Gross premium prospective policy values Valuation rate of interest = 4.5% p.a. Mortality = 110% of Male non-smokers mortality. Expense Reserve = 20% of all renewal expenses held. Solvency capital = 4% of reserves plus 0.3% of capital at risk

2.2.6 Reserves

For statutory purposes, non-linked contracts are required to be valued using the net premium method or on some other basis (e.g. gross premium method) that produces a stronger reserve, Abbott (1984). "If a gross premium method is used, the 1994 Regulations [in UK] require that a check be made that it produces reserves at least as high as if a net premium method had been used. Hence there is a practical advantage in using a net premium method", (Subject 402 Core Reading, Unit 11).

However, the FSA has recently expressed a general dissatisfaction with the net premium valuation method for valuing liabilities particularly the with-profits business because of the implicit margins and allowance for future bonuses and expenses made within the valuation. "The FSA has adopted the pragmatic approach of allowing the non-profit business within a with-profits fund to be valued on an embedded value basis for the purposes of the realistic peak" Dullaway and Needleman (2004). Dullaway and Needleman (2004) note and discussed in details certain anomalies being introduced by the approach adopted by FSA.

We consider a net premium valuation method to determine the statutory reserves at the end of each projection year, as our investigation is only concerned with non-profit business. This method places a value on a life insurance company's liabilities by valuing the contractual liabilities to date allowing for mortality and interest and deducting the value of future net premiums. For simplicity, we assume that the valuation interest rate and mortality rate assumptions for valuing the future liabilities are the same as the assumptions for profit testing, which includes valuation as well as pricing, see table 2.1. The gross premium valuation method, used in the profit testing, values the future office premiums payable, future benefits and future expenses explicitly. Any resulting negative reserves (due to a Zillmer adjustment) early in the duration of the policy are eliminated so that the contract is not treated as an asset, for reasons of simplicity. The calculations do not include resilience reserves; see Luffrum (1992) for more details. The requirement that the net premium valued (including Zillmer adjustment) must be less than or equal to the office premium makes an implicit allowance for renewal expenses.

The capital required (new business strain) to write a new policy at the start of each projection year is also calculated using the net premium reserves, see section 3A.9 in appendix 3. New business strain arises when the premium paid at the start of a contract, less the initial expenses including commission payments, is not sufficient to cover the mathematical reserve that the company needs to set at that point. For a whole company, the capital requirement is the finance that it needs in order to be able to carry out its new business plans. This will enable the company to determine the actual new business volume written, see equation (8)

At the projection date, the total statutory reserves (including solvency margin) are determined for in-force policies from the different cohorts of existing business in order to derive the initial assets and hence the initial free assets in the market, having assumed a value for the initial solvency ratio.

2.2.7 Solvency ratio

We derive the initial market solvency ratio at projection date (denoted by $SR_m(0)$) by analysing the trend of solvency ratios (see table 2.2 below) of a set of companies in the UK market (based on their statutory DTI Returns, see section 2C of appendix 2) over the period 1994-2002. This may be a guide in estimating the base parameter values using sensitivity analysis later in this chapter. The market solvency ratio $SR_m^j(t)$ at the end of each projection year is calculated as the total statutory value of assets divided by the total statutory reserves.

2.3 Asset Model

2.3.1 Cash Flow

The total value of initial assets available at the projection date is the product of initial solvency ratio assumed and total initial statutory reserves calculated. At the end of each projection year the total assets in the market are determined with reference to the investment strategy. The market value of the assets at each future date is projected using the Wilkie stochastic asset model. Investment income arises from investing premiums less expenses and assets at the start of the year. The net cash flow is defined as the premium plus investment income less expenses and death benefits.

2.3.2 Investment Strategies

For simplicity, we assume that there are only two asset classes available in the market, namely long-dated fixed-interest gilts and equities and we consider four investment strategies in this investigation.

- **Asset-Liability Matching (ALM) Investment Strategy (i.e. Low Equities):** 100% of (free assets less capital required to write new business) invested in equities and 100% of all other assets invested in fixed interest securities (gilts) in order to back the liabilities. This is the investment strategy to be adopted by the market.
- **Static Investment Strategy (FPS) i.e. fixed proportion - High Equities:** 50% of assets invested in equities and 50% of assets invested in fixed interest securities (gilts).
- **Dynamic asset allocation Strategy (DS1):** This is similar to the approach described in Chadburn (1998) and Hardy (1993). A company is assumed to invest 80% of its assets in equities while its solvency ratio exceeds 1.35. If at the end of a year the solvency ratio falls below 1.35 the company moves progressively into gilts, reaching 100% in gilts if the solvency ratio is less than 1.05.
- **Dynamic asset allocation Strategy (DS2):** A company is assumed to invest 100% of its assets in gilts while its solvency ratio exceeds 1.35. If at the end of a year the solvency ratio falls below 1.35 the company moves progressively into equities, reaching 80% in equities if the solvency ratio is less than 1.05. In practice, this is a counter-intuitive strategy, where funds are switched into gilts

as the solvency position improves and into equities as solvency level becomes more adverse.

Remarks: Hardy (1993) states that the rationale for the dynamic asset allocation strategy (DS1) is "that the higher gilt investment allows the office to use a higher rate of interest in the valuation of liabilities, reducing the liability value and thereby improving the solvency position on the A/ L basis". Chadburn (1998) uses the strategy as a tool in managing insolvency risk in a with profit portfolio.

2.3.3 Inflation Rate and Investment Returns Model

In order to generate random investment returns, we use the Wilkie (1995) investment model for inflation, equity returns and returns for fixed interest government bonds, see Appendix 1. Future expenses (per unit of exposure), premiums and benefits levels are affected by volatility from random annual rates of inflation. We set different expected values taken arbitrarily for the price inflation time series in order to generate different inflation rates for expenses, premium and benefits. Thus, we assume a mean annual increase of 3% in premiums and benefits levels and a mean annual increase of 4.7% (which is the level suggested by Wilkie 1995) in per policy expenses, see section 1.3 in appendix 1. A similar approach has been adopted by Haberman et al (2004) for a study of income protection policies.

Wilkie's stochastic investment model is used because it is a model which is well structured, widely studied, [for example by Geoghegan et al (1992), Huber (1997), Booth et al (2005) and others], easily understood and commonly used in actuarial practice. However, there are other advantages and certain disadvantages of the Wilkie model as identified by the above authors. As the model has been developed for use in asset/liability modelling, it also makes it suitable for research which involves asset/liability modelling of a life insurance company (Booth et al, 2005).

2.4 Company Liability Model

2.4.1 Company New Business Model

As the company is an integral part of the market, the company can only write a proportion of the quantity demanded in the market. This proportion can change as a result of the operation of the company's price elasticity of demand relative to the market and the effect of either the company's management decision or the regulator's control (due to the company's level of solvency relative to the market leading to a loss of business to competitors). We will use the same demand function(s) as in the market for considering the effect of changes in company's price relative to the market price on new business volume demanded. Thus, the company's proportion of the quantity demanded in the market in projection year t for simulation j (denoted by $ft_c^j(t)$) is assumed to the product of the company's demand function (denoted by $DF_c^j(t)$) and the effect of either the company's management decision or the regulator's control (denoted by $PMS^j(t)$)

$$ft_c^j(t) = DF_c^j(t) \times PMS^j(t) \tag{15}$$

Where,

$ft_c^j(t)$= The company's proportion of the quantity demanded in the market in projection year t for simulation j, allowing for the effect of both the changes in a company's premium relative to the market premium and either the company's management decision or the regulator's control due to the company's level of relative solvency.

$DF_c^j(t)$ = Demand function for the company in projection year t for simulation j.

$PMS^j(t)$ = Company's proportion of the quantity demanded in the market in projection year t for simulation j due to the effect of either the company's management decision or the regulator's control.

2.4.1.1 Effect of a company's management decision or regulator's control

Equation (16) below has two parts (which we assume do not operate simultaneously for purposes of simplicity), namely the effect of regulatory control on a company's proportion of quantity of new business demanded in the market based on the company's solvency position and management decision. For simplicity, we assume that it is a management decision (not regulatory control) that the company's proportion (denoted by $PMS^{j}(t)$) of the quantity of new business demanded in the market (without the price effect) in a projection year t is the same proportion (denoted by $ft_{c}^{j}(t-1)$) of the quantity of new business demanded (allowing for both effect of changes in the company's premium relative to the market premium and either the management decision or regulator's control) in the previous year $t-1$. This assumption is necessary in order to make the company's proportion [$ft_{c}^{j}(t)$ in equation (15)] of the quantity demanded in the market in projection year t to be less than one when the effect of regulatory control in equation (16) is not considered.

On the other hand, we model the regulator's decision to control a company's proportion of the quantity demanded in the market in projection year t [denoted by $PMS^{j}(t)$] by considering the company's solvency ratio relative to the market solvency ratio in the previous year. We adopt the method used by Daykin et al (1990) in modelling the effect of a company's solvency margin (relative to a statutory level) on the proportion of the market written by the company.

$$PMS^{j}(t) = \begin{cases} \alpha_{R}^{j}(t) \quad ft_{c}^{j}(t \quad 1) & if \quad SR_{c}^{j}(t \quad 1) \quad 1 \\ [1 \quad F_{R}^{j}(t)] \quad ft_{c}^{j}(t \quad 2) + F_{R}^{j}(t) \quad ft_{c}^{j}(t \quad 1) & if \quad SR_{c}^{j}(t \quad 1) \geq 1 \\ ft_{c}^{j}(t \quad 1) & Management\ decision \end{cases} \qquad (16)$$

Where,

$\alpha_{R}^{j}(t)$ = A factor specified by the regulator(s) to reduce or increase the company's proportion of the quantity demanded in the market in projection year t for simulation j, if the solvency ratio of the company at the end of year t-1 is less than one.

$F_R^j(t)$ = Weighting factor specified by the regulator(s) to damp down the growth or fall in the company's proportion of the quantity demanded in the market in projection year t for simulation j, if the solvency ratio of the company at the end of year t-1 is greater than or equal to one.

$SR_c^j(t)$ = Company's solvency ratio at the end of projection year t for simulation j.

Equation (16) has two conditions under the regulator's control:

(a) If $SR_c^j(t-1) < 1$, the company's proportion of the quantity demanded in the market in projection year t (due to relative solvency), $PMS^j(t)$, is equal to the proportion of the quantity demanded in the market in projection year $t-1$ for simulation j, denoted by $ft_c^j(t-1)$, being reduced or increased by a factor specified by the regulator. We assume that the factor specified by the regulator denoted by $\alpha_R^j(t)$ is equal to the ratio of the company's solvency ratio to the market's solvency ratio at the end of projection year $t-1$.

(b) If $SR_c^j(t-1) \geq 1$, the company's proportion of the quantity demanded in the market in projection year t (due to relative solvency), $PMS^j(t)$, is equal to the weighted average of the company proportions of the market in projection year $t-1$, denoted $ft_c^j(t-1)$, and in projection year $t-2$, $ft_c^j(t-2)$. The weighting factor specified by the regulator (denoted by $F_R^j(t)$) is assumed to be equal to the ratio of the company's solvency ratio to the market's solvency ratio at the end of year $t-1$. However, if the market's solvency ratio is less than the company's solvency ratio, we assume that the company's proportion of the market in projection year t (due to relative solvency), $PMS^j(t)$ is equal to the company's proportion of the market in projection year $t-1$, $ft_c^j(t-1)$. In this case the weighting factor is assumed to be equal to one (i.e. $F_R^j(t)$ = 1). This choice is necessary in order to avoid $PMS^j(t)$ having negative values when $F_R^j(t)$ is greater than one.

Estimation of parameters in equation (16) under the regulator's control

Assumption: we assume that the two versions of relative solvency, as stated in Hardy (1996), will affect the company's proportion of the quantity demanded in the market (denoted by $PMS^j(t)$) in projection year t.

Version 1: An office is said to be relatively insolvent at time $t-1$ if $SR_c^j(t-1) < 1.0$ and either of these two conditions are met:

(i) $SR_m^j(t-1) \geq 1.0$

or

(ii) $SR_m^j(t-1) < 1.0$ and $SR_c^j(t-1) < SR_m^j(t-1)$;

then

$$\alpha_R^j(t) = {}^{SR_c^j(t-1)}\!\Big/_{SR_m^j(t-1)} \qquad 0 < \alpha_R^j(t) < 1$$

Version 2: An office is said to be relatively solvent at time $t-1$ if $SR_c^j(t-1) < 1.0$, $SR_m^j(t-1) < 1.0$ and $SR_c^j(t-1) \geq SR_m^j(t-1)$

then

$$\alpha_R^j(t) = {}^{SR_c^j(t-1)}\!\Big/_{SR_m^j(t-1)}; \qquad \alpha_R^j(t) \geq 1.$$

Version 3: An office is said to be relatively insolvent at time $(t-1)$ if $SR_c^j(t-1) \geq 1.0$, $SR_m^j(t-1) \geq 1.0$ and $SR_m^j(t-1) > SR_c^j(t-1) \geq 1.0$

then

$$F_R^j(t) = {}^{SR_c^j(t-1)}\!\Big/_{SR_m^j(t-1)}; \quad 0 < F_R^j(t) \leq 1$$

Version 4: An office is said to be relatively solvent at time $t-1$ if $SR_c^j(t-1) \geq 1.0$ and either of these two conditions are met:

(i) $SR_m^j(t-1) < 1.0$

or

(ii) $SR_m^j(t-1) \geq 1.0$ and $SR_m^j(t-1) < SR_c^j(t-1)$

then

$$F_R^j(t) = {}^{SR_c^j(t-1)}\Big/_{SR_m^j(t-1)}; \quad F_R^j(t) = 1 \;\; if \;\; F_R^j(t) > 1$$

2.4.1.2 Effect of a company's premium relative to the market premium

In this section, we consider the same demand functions as discussed in section 2.2.1.2, namely the exponential and constant price elasticity of demand functions, in determining the effect of a company's premium (denoted by $OP_c^j(t)$) relative to the market premium (denoted by $OP_m^j(t)$) on the company's proportion (denoted by $DF_c^j(t)$) of quantity of new business demanded in the market in projection year t, without allowing for the effect of management decision and regulator's control on new business (denoted by $PMS^j(t)$).

Exponential price elasticity of demand function (Price effect only)

$$EDF_c^j(t) = \left\{\frac{-EK_c * (OP_c^j(t) - OP_m^j(t))}{OP_m^j(t)}\right\} \qquad \text{For } EK_c > 0 \qquad (17)$$

Constant price elasticity of demand function (Price effect only)

$$CDF_c^j(t) = \left(\frac{OP_c^j(t)}{OP_m^j(t)}\right)^{-CK_c} \qquad \text{For } CK_c > 0 \qquad (18)$$

Where,

$CDF_c^j(t)$ = Constant price elasticity of demand function for the company in projection year t for simulation j.

$EDF_c^j(t)$ = Exponential price elasticity of demand function for the company in projection year t for simulation j.

$OP_c^j(t)$ = Company's office premium for new business issued in projection year t for simulation j.

2.4.1.3 Company's New Business Models

We propose four new business models for the company from equation (15) above.

<u>Model C1:</u> Exponential Demand Function and Management Decision

Let $DF_c^J(t) = EDF_c^j(t)$ and $PMS^j(t) = ft_c^j(t-1)$ in equation (15)

<u>Model C2:</u> Exponential Demand Function and Regulator's Control

Let $DF_c^J(t) = EDF_c^j(t)$ and $PMS^j(t) \quad ft_c^j(t-1)$ in equation (15), see equation (16)

<u>Model C3:</u> Constant Price-elasticity Demand Function and Management Decision

Let $DF_c^J(t) = CDF_c^j(t)$ and $PMS^j(t) \quad ft_c^j(t-1)$ in equation (15)

<u>Model C4:</u> Constant Price-elasticity Demand Function and Regulator's Control

Let $DF_c^J(t) = CDF_c^j(t)$ and $PMS^j(t) \quad ft_c^j(t-1)$ in equation (15), see equation (16)

We assume an insurer is not dominant in the market and thus, the company's proportion of quantity demanded in the market ($ft_c^j(t)$) is set not to exceed a certain level, $NBConF_c$ (e.g. 50% in year).

$ft_c^j(t) \leq NBConF$

Where,

$NBConF_c$ = The company assumed maximum proportion of the quantity demanded in the market in projection year t.

$ft_c(0)$ = The company's proportion of the quantity demanded and written in the market in year 0 (i.e. a year before projection date).

We also assume a fixed value for the company's proportion of the quantity demanded and written in the market before the projection date (i.e. $ft_c(0) = 0.2$).

Quantity of New Business Demanded

$$NC^{j}(t) \quad NM^{j}(t) * ft_{c}^{j}(t) \qquad (19)$$

Where,

$NC^{j}(t)$ = Company's quantity of new business demanded by prospective policyholders (in terms of number of new policies) in projection year t for simulation j.

Remarks:

(1) In equations (17) and (18), the demand functions consider the effect of changes in company's price relative to the market price on new business volume demanded. For simplicity, we assume that the demand function to be adopted by the company and the market models is the same. However, the estimated parameters in the company demand function may be different from the market, since the company's views about the market conditions may be different from other insurers.

(2) Figure 2.6 below is a flow chart which summarises section 2.4.1, the steps leading to the company's quantity of new business demanded in the market (denoted by $NC(t)$) in a projection year t using new business strategy 1. The company's premium in year t (denoted by $OP_{c}(t)$) relative to a profit-tested premium (denoted by $\overline{OP_{c}}$) is dependent on its solvency position in year t -1 (denoted by $SR_{c}(t-1)$ and price inflation factor in year t -1 [denoted by $InfacF(t-1)$] .

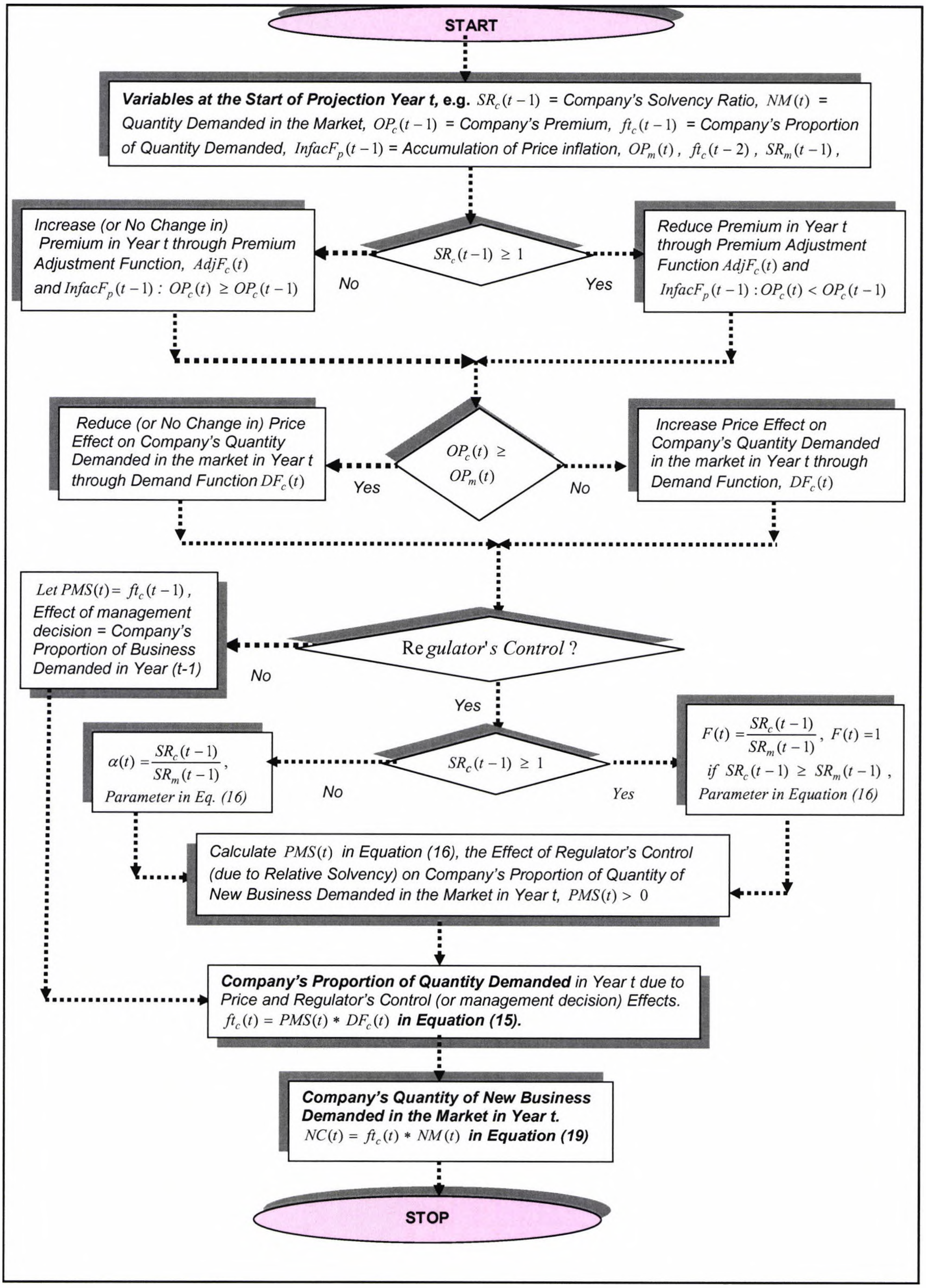

Figure 2.6: Schematic representation of interaction between the company and market models under new business strategy 1.

In a given year t, the company's proportion of quantity of the new business demanded in the market (denoted by $ft_c(t)$) is determined by the combined effect of changes in a company's premium relative to the market premium (denoted by $OP_m(t)$) in year t and either the regulator's decision based on the company's relative solvency position or the management decision [denoted by $PMS(t)$].

Firstly, consider the combined effect of price and management decision in a given projection year t. If the company's office premium exceeds the market premium (denoted by $OP_m(t)$), the company's proportion of quantity of new business demanded in the market in year t is reduced [i.e. $ft_c(t) \leq ft_c(t-1)$] relative to the previous year's proportion (that is, the effect of management decision is reduced by the demand function) and vice versa.

Secondly, consider the combined effect of price and regulator's decision in a given projection year t. The effect of changes in the company's premium relative to the market premium (the demand function) may be dampened by the regulator's decision which depends on the company's relative solvency ($PMS(t)$).

Thus, the company's quantity of new business demanded in year t (denoted by $NC(t)$) is a product of the company's proportion and the overall quantity of new business demanded in the market in year t (denoted by $NM(t)$).

2.4.2 Number of deaths and withdrawals in Projection Year t

The number of deaths and withdrawals from a particular cohort of lives (from new and existing business) are determined using the Monte Carlo's basic simulation technique by simulating two random numbers from uniform distributions $u_1 \sim U_1(0,1)$ and $u_2 \sim U_2(0,1)$. The dependent probabilities of death and withdrawal are defined as:

${}^{j}(aw)_x^u(t)$ = Dependent rate of withdrawal for a life aged x at entry with duration u in projection year t for simulation j

${}^{j}(aq)_x^u(t)$ = Dependent rate of death for a life aged $x+u-1$ at the start of projection year t, for simulation j.

The probability that a life aged x at entry with duration u at the start of projection year t will either die or withdraw over year t is $\left[{}^{j}(aq)_x^u(t) + {}^{j}(aw)_x^u(t)\right]$. We simulate the policyholder's mortality or withdrawal experience over year t by choosing a random fraction denoted by u_1 from the uniform distribution over the interval $(0,1)$ (denoted by $u_1 \sim U_1(0,1)$); if $u_1 < \left[{}^{j}(aq)_x^u(t) + {}^{j}(aw)_x^u(t)\right]$ we say the policyholder either dies or withdraws, otherwise he survives to the end of year t.

Similarly, for the policyholder who either dies or withdraws over year t, we simulate his actual experience over year t by choosing a random fraction denoted by u_2 from the second uniform distribution over the interval $(0,1)$ denoted by $u_2 \sim U_2(0,1)$); if $u_2 < \dfrac{{}^{j}(aq)_x^u(t)}{{}^{j}(aq)_x^u(t) + (aw)_x^u(t)}$ or $u_2 > \dfrac{{}^{j}(aw)_x^u(t)}{{}^{j}(aq)_x^u(t) + (aw)_x^u(t)}$ we say that the policyholder dies or withdraws over year t respectively.

Thus, we simulate the experience of a given cohort of assured lives (i.e. both new and inforce business) at the start of projection year t by simulating each policyholder's experience as stated above. The number of deaths and withdrawals over year t are counted, from which the number of policies inforce at the end of year t is obtained.

The above is equivalent to generating random variable $Y(t)$ from a multinomial (trinomial) distribution, $Y(t) \sim MULT\left(\eta S_m^j(t),\ {}^{j}(aq)_x^d(t),\ {}^{j}(aw)_x^u(t)\right)$

$\eta S_c^j(t) = \eta_c^j(t-1) + ANC^j(t)$

$\eta_c^j(t) = \eta S_c^j(t) - \eta d_c^j(t) - \eta w_c^j(t)$

Where, $s \leq t$, $1 \leq t \leq T$ and $-9 \leq s \leq t$

s = Policy issue year

t = Projection year

T = Projection period in years (e.g. 20 years)

$Y(t)$ = Random variable representing the number decrements (death and withdrawal).

$ANC^j(t)$ = Company's number of policies issued at the start of year t for simulation j.

$\eta S_c^j(t)$ = Company's number of policies in-force at start of year t for simulation j

$\eta w_c^j(t)$ = Company's number of withdrawals in year t for simulation j.

$\eta d_c^j(t)$ = Company's number of deaths in year t for simulation j.

$\eta_c^j(t)$ = Company's number of polices still in-force at the end of year t for simulation j.

Assumption: Deaths and withdrawals in the respective single decrement models are assumed to be uniformly distributed over each year of age.

2.4.3 Company's Annual Office premium for New Business i.e. $OP_c^j(t)$

We assume that the company's office premium at the projection date is the same profit-tested premium as the market. The revision of the premium rates is similar to the market (i.e. that the company's future office premium may increase with the same inflation rates being simulated using Wilkie's model).

2.4.4 Variant Company Models

We assume that there are individual life insurance companies operating alongside the market in the same business environment. For a given company new business model, the individual companies will only differ by the choice of new business strategies and investment strategies adopted, but they will all have the same liabilities and assets at the projection date. These variant life companies will interact with the market and they are ranked in terms of riskiness for a given risk measure.

Remarks

Methods and assumptions for obtaining all other elements in the market such as reserves, death benefits, future office premium, actual quantity of new business written, etc. are also relevant to the company model, except that the subscript m, representing the market is replaced by c for the company. We assume that the regulators will require a company to be wound-up if the company's solvency ratio in the future falls below a certain limit (denoted by $\mathrm{Re}\,gConF$), to be determined arbitrarily.

2.5 Estimation of Base Parameters

2.5.1 Initial Estimation of Market Model Parameters

A regression analysis is carried out on past market new business data (from the ABI) and inflation data for the period 1991 to 2002 in order to estimate the initial parameter values for the market new business models M2 and M4, see section 2A of appendix 2. Following the approach adopted in Taylor (1986) and Emms et al (2004), *we assume that if the market office premium in year t is 20%* (chosen arbitrarily) *higher than the market office premium in year* $t-1$, *this will produce only 85%* (also chosen arbitrarily) of the new business volume demanded in year $t-1$ through a demand function (e.g. $f\left(1.2\times OP_m^j(t), OP_m^j(t-1)\right)=0.85$).

Therefore the choice of the exponential demand function produces the constants ($EK_m = 0.81,\ a = 0.97 \text{ and } b = 4.81$), using market new business model M2, see table 2A.3 in appendix 2. However, the constant elasticity demand function produces the following parameter values, using new business model M4, $CK_m = 0.89,\ a = 0.96\ and\ b = 5.04$, see Table 2A.3. Table 2A.4 also shows a summary of initial parameter values for models M2 and M4 under different market conditions.

The market data produce results that are not reliable due to a lack of credibility in the level term insurance data obtained (e.g. the correlation coefficient – R is quite low and residual sums of squares appear to be high, see table 2A.4 and figures 2A.1 and 2A.2 in appendix 2). We note that the volume of data available is not only small but also the data relates to a mixture of different types of temporary insurance policy for the period under consideration (1991 to 2002).

Thus, the price and income effect on temporary life insurance policies cannot be easily revealed by the above classification of data, as there are many different non-mortgage term insurance policies in the market[3].

Table 2.2: Statistics of Solvency Ratios of a Set of UK Life Offices (Long Term Trends)

Year	*2002*	*2001*	*2000*	*1999*	*1998*	*1997*	*1996*	*1995*	*1994*
Mean	*1.060*	*1.079*	*1.129*	*1.152*	*1.111*	*1.137*	*1.148*	*1.205*	*1.141*
Median	*1.062*	*1.068*	*1.063*	*1.079*	*1.098*	*1.119*	*1.122*	*1.127*	*1.106*
Maximum	*1.147*	*1.292*	*2.915*	*2.703*	*1.326*	*1.414*	*1.473*	*2.564*	*1.453*
95th Percentile	*1.125*	*1.196*	*1.241*	*1.295*	*1.295*	*1.323*	*1.370*	*1.499*	*1.431*
5th Percentile	*1.015*	*1.009*	*1.010*	*1.004*	*1.009*	*1.009*	*1.015*	*1.015*	*1.019*
Minimum	*1.001*	*1.001*	*1.001*	*1.000*	*1.000*	*1.001*	*1.002*	*1.002*	*1.011*

Sources: DTI Returns (only the main company from each group is taken).
Solvency Ratio = Total assets / total liabilities

Data for UK life insurance companies (solvency and free assets ratios for long term business) for the period (1994–2002) have been extracted from DTI Returns (see appendix 2C). The data are not sub-divided into various classes of life insurance. Each company's solvency ratio and the average market solvency

[3] The key types of term insurance product defined in McGaughey et at (2001) include the followings: level term assurance (LTA), Decreasing term assurance (DTA) explicitly referred to as mortgage protection assurance, Family income benefit (FIB), Increasing term assurance (ITA), Business term assurance (BTA), Pension term assurance (PTA), Renewable term assurance (RTA), Convertible term assurance (CTA)

ratios are calculated for each year. The individual companies' solvency ratios for the period range from 1.00 to 2.92 while the average market solvency ratio ranges from 1.06 to 1.20 as shown in table 2.2 above. Thus, only five out of sixty companies considered have solvency ratios above 1.50 during the period, with AXA Sun Life plc having the highest solvency ratios in years 2000 and 2001. We may deduce from the above table that companies with solvency ratios less than one are not included.

We assume the following initial parameter values to enable us carry out a sensitive analysis of solvency ratio to changes in parameters in order to choose an appropriate set of base parameter values for both the market and company's new business models:

$$EK_m = CK_m = 0.85,\ \ a = 0.96,\ \ b = 4.0,\ \ d = 2.0,\ \ l_m(\pm ve) = 0.10,\ \ SR_m(0) = 1.50,\ \ \rho_m = 1.0$$

Some of these parameter values are obtained from the initial estimation above while others are chosen arbitrarily.

2.5.2 Choice of Market Parameters by Sensitivity Analysis

We are using a comparative statics approach (a sensitivity analysis) in order to determine more sensible base parameter values for the market and the company, having obtained the initial parameter values by regression analysis. Two types of sensitivity analysis are possible: the successive sensitivity analysis and the simultaneous sensitivity analysis, Darbellay (2001). In the successive sensitivity analysis, the value of a unique parameter is changed, and values of the other parameters do not change. The aim of this analysis is to determine the more influential parameter on the simulated results (e.g. the solvency ratios). In the simultaneous sensitivity analysis, the values of all the parameters are changed. "The aim of this analysis is to analyse the worst and best possible situations in taking respectively the pessimistic value of several parameters and in taking the optimistic value of the several parameters" Darbellay (2001). But to have a homogeneous comparison, we can take a constant percentage of variation for each parameter. The analysis (used in chapter three) may also take into account where necessary the interdependencies between the parameters.

In this section we consider the sensitivity of the simulated solvency ratios in year 20 to changes in each parameter value while the other initial parameters are held constant in order to estimate the base parameter values. We consider the new business strategy 1 and asset/liability matching (ALM) investment strategy with the appropriate new business model for each parameter in carrying out the sensitivity analysis. A projection over 20 years is chosen because the degree of sensitivity of the solvency ratio to a given parameter is more fully revealed over the longer term than in the early years' of projection, as we expect a higher volume of new business demanded over the long term. We also consider 1000 simulations in order to obtain reliable simulated results of the tails (e.g. 5^{th} percentile). Where there are no data with which to estimate initial values, we use different reasonable values taken arbitrarily for each parameter in the analysis.

2.5.2.1 Criteria for determining an appropriate set of market parameters

In practice, individual life insurance companies' solvency ratios normally do not exceed 2.0 (see table 2.2 and comments thereafter). We therefore assume that for each parameter, the individual market mean solvency ratio and the 95th percentile in projection year 20 should not exceed 1.20 and 1.60 respectively, based on table 2.2. The cumulative effect of a set of parameters should produce a market model solvency ratio in each projection year that does not exceed the maximum solvency ratios shown in table 2.2.

2.5.2.2 Sensitivity to changes in the market parameter a

This parameter is the constant (intercept) term in equation (2), $RWP^{j}(t) = a + b \times RWIR^{j}(t-1)$, and represents the underlying demand independent of real wage inflation. Table 2.3 shows the summary statistics of the distribution of solvency ratios in projection year 20 for different values of parameter a. The table shows that the solvency ratio is highly sensitive to a small increase in the parameter. This is because the quantity of new business demanded is also highly sensitive to the parameter a.

Table 2.3: Sensitivity of solvency ratio to changes in parameter a

Statistics for the distribution of solvency ratios in year 20 with new business model M2								
Parameter a	**0.84**	**0.88**	**0.92**	**0.96**	**0.97**	**0.98**	**0.99**	**1**
mean	*12.830*	*3.631*	*1.367*	*1.138*	*1.136*	*1.137*	*1.137*	*1.139*
Median	*7.817*	*1.767*	*1.178*	*1.135*	*1.135*	*1.139*	*1.143*	*1.147*
Std. Deviation	*14.849*	*4.632*	*0.877*	*0.232*	*0.213*	*0.204*	*0.197*	*0.194*
coeff. Of variation	*1.157*	*1.276*	*0.641*	*0.204*	*0.188*	*0.179*	*0.173*	*0.170*
Skewness	*3.122*	*3.759*	*5.108*	*2.404*	*1.035*	*0.280*	*-0.049*	*-0.097*
Maximum	*134.916*	*44.077*	*10.788*	*4.169*	*3.268*	*2.491*	*1.823*	*1.735*
95th Percentile	*39.437*	*12.296*	*2.578*	*1.485*	*1.469*	*1.455*	*1.451*	*1.443*
5th Percentile	*1.291*	*0.949*	*0.828*	*0.799*	*0.800*	*0.810*	*0.809*	*0.814*

As parameter a increases more new business is demanded and written (since a 100% allocation of free assets (i.e. $\rho_m = 1$) to finance new business is allowed). This will increase the new business strain leading to a decrease in the solvency ratio as the parameter increases. However, the table shows that there is marked change in the shape of the distributions, almost being symmetric (as the skewness tends to zero and coefficient of variation is less than 1) when the parameter is about 0.98. This is because as more new business is written the solvency ratio decreases until the free assets are used up. Subsequently there is little or no further new business being written (even if the quantity demanded increases with the parameter) but the surplus from existing business will improve the solvency position.

The table shows that the mean solvency ratio, 95th percentile and median in the range $0.96 < a < 0.99$ meet the criteria in section 2.5.2.1. So we choose $a = 0.98$.

2.5.2.3 Sensitivity to changes in the market parameter b

This is the rate of change in new business volume demanded with respect to disposable income (which may be linked to the real wage inflation) in equation (2). The quantity of new business demanded will increase with the parameter, provided that the real wage inflation is positive which reflects a buoyant economic climate, and this will lead to a drain on free capital if a substantial quantity of new business demanded is written.

The parameter has a maximum value for it to be realistic. The proportion of new business demanded in projection year t (i.e. $RWP^{j}(t)$ in equation (2)) due to the effect of real wage inflation will become negative if the real wage inflation in previous year (t-1) is negative and the parameter b also exceeds a maximum value. For any given value of b, if $RWIR^{j}(t-1) < -\frac{1}{b}$ then $RWP^{j}(t) \leq 0$. For example, if $RWIR^{j}(t-1) \leq -0.095$, then $RWP^{j}(t) > 0$ only when $b < 10$.

Table 2.4: Sensitivity of solvency ratio to changes in parameter b

Statistics for the distribution of solvency ratios in year 20 with new business model M2									
Parameter b	**0**	**1**	**2**	**3**	**4**	**6**	**8**	**10**	**12**
mean	*1.401*	*1.186*	*1.139*	*1.136*	*1.138*	*1.141*	*1.146*	*1.170*	*1.274*
Median	*1.213*	*1.139*	*1.132*	*1.133*	*1.135*	*1.136*	*1.140*	*1.138*	*1.146*
Std. Deviation	*0.852*	*0.410*	*0.249*	*0.228*	*0.232*	*0.228*	*0.247*	*0.451*	*2.112*
coeff. Of variation	*0.609*	*0.346*	*0.219*	*0.201*	*0.204*	*0.199*	*0.215*	*0.385*	*1.658*
Skewness	*7.556*	*7.611*	*2.057*	*1.204*	*2.404*	*2.056*	*4.150*	*11.362*	*3.140*
Maximum	*15.744*	*8.073*	*3.225*	*3.191*	*4.169*	*3.724*	*4.684*	*9.788*	*33.211*
95th Percentile	*2.637*	*1.657*	*1.500*	*1.484*	*1.485*	*1.466*	*1.478*	*1.473*	*1.566*
5th Percentile	*0.859*	*0.817*	*0.802*	*0.800*	*0.799*	*0.807*	*0.811*	*0.804*	*0.808*

Table 2.4 shows summary statistics of the distribution of solvency ratios in projection year 20 for different values of parameter b, $0 \leq b \leq 12$. It shows that the solvency ratio is sensitive to changes in the parameter, as reflected in the changes in the mean and the variance. As more quantity of new business is demanded and written when parameter b is increased, more free assets are used up and the solvency ratios decrease. The table also reveals that there is a

marked change in the shape of the distribution of solvency ratios (e.g. the coefficient of variation, 95th percentile, skewness are very low) when the parameter is about 6. This is because the increase in quantity demanded (as parameter b increases to a higher level) has no significant effect on the quantity of new business written if all the free assets are being used up, as explained in section 2.5.2.2 for the change in shape of the distribution. However, the distribution of solvency ratios becomes unstable and more sensitive to changes in parameter b particularly when $b \geq 10$, indicating that the parameter produces unrealistic results at higher values.

The distribution of solvency ratios becomes approximately symmetric when $3 \leq b < 8$. The criteria in section 2.5.2.1 is likely to be met within this range of values and so we choose b = 6.0.

2.5.2.4 Sensitivity to changes in the market parameter d

Policy lapses (or withdrawals) are negatively related to disposable income (which may be linked to the real wage inflation), and d in equation (14) represents the rate of change in policy lapses with respect to disposable income. This parameter does not affect the new business volume directly (either the quantity demanded or quantity written). As the parameter d increases there may be a significant change in the withdrawal rate depending on the level of the real wage inflation.

Table 2.5: Sensitivity of solvency ratios to changes in parameter d

Statistics for the distribution of solvency ratios in year 20 with model M2						
Parameter d	**0**	**2**	**4**	**6**	**8**	**10**
mean	*1.139*	*1.138*	*1.136*	*1.135*	*1.133*	*1.131*
Median	*1.135*	*1.135*	*1.132*	*1.129*	*1.126*	*1.124*
Std. Deviation	*0.232*	*0.232*	*0.232*	*0.230*	*0.230*	*0.231*
coeff. Of variation	*0.204*	*0.204*	*0.204*	*0.203*	*0.203*	*0.204*
Skewness	*2.231*	*2.404*	*2.518*	*2.540*	*2.661*	*2.742*
Maximum	*4.081*	*4.169*	*4.213*	*4.213*	*4.264*	*4.304*
95th Percentile	*1.488*	*1.485*	*1.482*	*1.476*	*1.476*	*1.470*
5th Percentile	*0.804*	*0.799*	*0.797*	*0.792*	*0.795*	*0.789*

The parameter *d* also has a maximum value for it to be realistic. The withdrawal rate in equation (14), ${}^{j}W_{x}^{u}(t) = C_{x}(u)\{1 - d * RWIR^{j}(t-1)\}$, will be negative if the real wage inflation rate and the value of the parameter *d* are above a given level, for example $RWIR^{j}(t-1) \geq 0.095$ and $d > 10$ respectively.

Table 2.5 shows the summary statistics of the distribution of solvency ratios in projection year 20 for different values of parameter d. The table shows that there is very little or no change in the distribution of the solvency ratios as the parameter d increases. This is an indication that lapses often have a largely neutral effect on the profitability of an insurance company transacting temporary insurance business. For such a company, there are two compensating effects. Firstly, an increase in lapses will reduce the assets since the initial cash-flow strains and future premiums from new and existing policies respectively cannot be recovered. Secondly, lapses from existing policies for which no benefits will be paid in respect of the premiums received may not only improve profitability in the long term but also lead to an early release of reserves.

As the results are not sensitive to the choice of the parameter d, we consider the base value (d = 2.0) to be an appropriate value.

2.5.2.5 Sensitivity to changes in the market parameters EK_m and CK_m

These parameters in equations (3) and (4) depend on the assumption regarding the elasticity of demand relative to the premium changes over the year. The parameters EK_m and CK_m indicate the extent of the price effect on the quantity of new business demanded in the exponential and constant demand functions respectively. The parameters directly affect the quantity of new business demanded which increases with the parameters, particularly for new business strategy 1 which reduces premium in order to attract more business , if there is no change in the current market premium.

Table 2.6: Sensitivity of solvency ratio to changes in parameter EK_m

Statistics for the distribution of solvency ratios in year 20 with new business model M2								
Parameter EK_m	**0.85**	**1**	**2**	**3**	**4**	**5**	**7**	**10**
mean	*1.138*	*1.138*	*1.138*	*1.139*	*1.142*	*1.144*	*1.145*	*1.146*
Median	*1.135*	*1.134*	*1.131*	*1.135*	*1.136*	*1.139*	*1.142*	*1.154*
Std. Deviation	*0.232*	*0.231*	*0.224*	*0.216*	*0.214*	*0.211*	*0.204*	*0.196*
coeff. Of variation	*0.204*	*0.203*	*0.196*	*0.190*	*0.187*	*0.185*	*0.178*	*0.171*
Skewness	*2.404*	*2.326*	*1.945*	*1.373*	*1.291*	*1.137*	*0.484*	*-0.001*
Maximum	*4.169*	*4.118*	*3.889*	*3.516*	*3.446*	*3.347*	*2.732*	*1.957*
95th Percentile	*1.485*	*1.484*	*1.480*	*1.476*	*1.462*	*1.459*	*1.459*	*1.456*
5th Percentile	*0.799*	*0.793*	*0.796*	*0.801*	*0.804*	*0.808*	*0.819*	*0.817*

Tables 2.6 and 2.7 show the summary statistics of the distribution of solvency ratios in projection year 20 for different values of parameters EK_m and CK_m. Comparing the two tables shows that the skewness and coefficient of variation are higher and stable in table 2.7 than those in table 2.6 as the parameter value ($EK_m = CK_m$) increases. However, table 2.6 shows that the skewness and coefficient of variation decrease rapidly to zero (implying a change in shape from positively skewed to symmetric) and a lower value respectively as the parameter value EK_m increases.

The above analysis reveals that there is a higher sensitivity to this parameter under the exponential elasticity demand function relative to the constant elasticity demand function. This may be due to the uncertainty arising from the high proportion of volume of exposure written with the former demand function. The tables show that the solvency ratios are not highly sensitive to changes in the parameter values. The overall insensitivity to this parameter is explained below.

As the parameters increase, a greater quantity of new business demanded will be generated under new business strategy 1, (and hence a higher quantity of new business is written if free assets are available) without a change in the premium rates. This will increase the premium income and the overall profitability of the company. On the other hand, the writing of more new business may also lead to a drain in the free assets, particularly in the short term. The overall effect of increasing the parameters may be a small increase in solvency ratio, if the premium income invested will offset the effect of new business strain on free assets.

Table 2.7: Sensitivity of solvency ratio to changes in the parameter CK_m

Statistics for the distribution of solvency ratios in year 20 with new business model M4								
Parameter CK_m	**0.85**	**1**	**2**	**3**	**4**	**5**	**7**	**10**
mean	*1.134*	*1.133*	*1.129*	*1.126*	*1.125*	*1.126*	*1.128*	*1.140*
Median	*1.129*	*1.127*	*1.118*	*1.118*	*1.109*	*1.107*	*1.101*	*1.118*
Std. Deviation	*0.234*	*0.233*	*0.236*	*0.237*	*0.242*	*0.250*	*0.263*	*0.289*
coeff. Of variation	*0.206*	*0.206*	*0.209*	*0.211*	*0.215*	*0.222*	*0.233*	*0.253*
Skewness	*1.717*	*1.704*	*1.704*	*1.792*	*1.967*	*2.277*	*2.667*	*3.251*
Maximum	*3.673*	*3.669*	*3.614*	*3.618*	*3.621*	*3.667*	*3.676*	*3.809*
95th Percentile	*1.486*	*1.483*	*1.478*	*1.473*	*1.465*	*1.479*	*1.500*	*1.523*
5th Percentile	*0.795*	*0.796*	*0.794*	*0.788*	*0.785*	*0.785*	*0.789*	*0.809*

Table 2.6 shows that there is just a little change in mean solvency ratios, percentiles and coefficients of variation as these parameters increase. The distribution of solvency ratios appears to be stable and symmetric when $EK_m = CK_m \geq 3$ and the criteria in section 2.5.2.1 are likely to be met within this range of values. We therefore choose $EK_m = 3.0$ and $CK_m = 3.1$ for the exponential price elasticity of demand function and constant price elasticity of demand function respectively. This means we have assumed that a 5% increase in the market premium will produce 86% of the volume of exposure to be written in the market (see table 2A.4 in appendix 2 as a guide). This shows that the initial assumption (i.e. a 20% increase in market premium over a year would produce only 85% of new business demanded) made in section 2.5.1 about market conditions may not reflect the price sensitivity of the term assurance market when compared with results of the sensitivity analysis for the market model.

2.5.2.6 Sensitivity to changes in the parameter $l_m(\pm ve)$

The parameter values $l_m(\pm ve)$ in equation (11) are the factors that measure a company's premium response to changes in its worsening (or improving) solvency in the market model under different new business strategies. The profit tested office premium (denoted by $\overline{OP_m}$) is charged independent of any new business strategy and therefore it is not affected by the company's solvency position (i.e. $l_m(+ve) = 0$).

Tables 2.8 shows the summary statistics of the distribution of solvency ratios in projection year 20 for different values of parameter $l_m(+ve)$. The table shows the distribution of solvency ratio tends towards symmetric as $l_m(+ve) > 0.3$. However, the table reveals that the solvency ratio is not highly sensitive to changes in the parameter and an explanation is given below.

Table 2.8: Sensitivity of solvency ratio to changes in parameter $l_m(+ve)$

Statistics for the distribution of solvency ratios in year 20 with model M2						
Parameter $l_m(+ve)$	**0**	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**
mean	*1.147*	*1.138*	*1.129*	*1.122*	*1.115*	*1.109*
Median	*1.142*	*1.135*	*1.126*	*1.121*	*1.115*	*1.112*
Std. Deviation	*0.244*	*0.232*	*0.218*	*0.209*	*0.204*	*0.200*
coeff. Of variation	*0.212*	*0.204*	*0.193*	*0.186*	*0.183*	*0.180*
Skewness	*2.195*	*2.404*	*1.660*	*1.154*	*0.964*	*0.862*
Maximum	*3.930*	*4.169*	*3.670*	*3.273*	*3.099*	*3.002*
95th Percentile	*1.498*	*1.485*	*1.461*	*1.447*	*1.431*	*1.418*
5th Percentile	*0.800*	*0.799*	*0.789*	*0.787*	*0.795*	*0.794*

Under new business strategy 1, an increase in the parameter will reduce the premium further if the company has free assets to write more new business, which in turn increases the volume of new business demanded. A particular premium rating structure may require a certain level of volume of business demanded (and quantity written) for it to be profitable. The solvency ratio may rise or fall, depending on the extent to which the premium decreases relative to the increase in volume of new business demanded (and quantity actually written). The rise in the volume of new business demanded (and hence an increase in quantity written) may also have an indirect effect on the solvency ratio. In this case the solvency ratio may fall because of the effect of a high new business strain which depends also on the extent of the premium reduction. A premium reduction strategy needs to be accompanied by an injection of more free capital to write the increased quantity of new business demanded. In this investigation, an injection of new capital from external sources is not allowed for and therefore the strategy may have its limitations. The premium income may fall if a lower quantity of new business is actually written due to insufficient free asset available (or allocated) to write the overall quantity of new business demanded. The above may reduce the sensitivity of the market solvency ratio to changes in the parameter.

In view of the above, the criteria in section 2.5.2.1 will be met if the likely range of values for the market parameter $l_m(+ve)$ will be $0.2 < l_m(+ve) \leq 0.4$. So we choose $l_m(+ve) = 0.3$.

2.5.2.7 Conclusion

The following set of base market model parameters will also produce a distribution of solvency ratios that do not exceed the maximum solvency ratios shown in table 2.2.

$EK_m = 3.0, \;\; CK_m = 3.1, a = 0.98, \;\; b = 6.0, \;\; d = 2.0, \;\; l_m(\pm ve) = 0.30$

The parameters $SR_m(0)$ and ρ_m, the market initial solvency ratio and free assets allocation proportion respectively, are considered as free variables.

2.5.3 Choice of Company Parameters by Sensitivity Analysis

The same initial parameter values ($EK_c = 0.85$, $l_c(+ve) = 0.10$, $SR_c(0) = 1.50$, $\rho_c = 1$) in section 2.5.1 will be used in addition to the following chosen parameter values $ft_c(0) = 0.2$, $NBConF = 0.50$ that are only relevant to the company to enable us carry out a sensitivity analysis of solvency ratio in order to estimate the appropriate base parameters. In this section, we consider only the effect of price (using models C1and C3) in estimating the appropriate parameter values.

2.5.3.1 Criteria for determining an appropriate set of company parameters:

The same criteria for the market in section 2.5.2.1 are used for the company (i.e. for each parameter the mean and the 95th percentile in projection year 20 should be about 1.20 and 1.60 respectively). Taking into account the main differences between the company and the market, particularly the size of the company relative to the market, randomness of deaths and lapses, the company can be expected to have more variability in the solvency ratio than the market. However, we still expect that the cumulative effect of a set of parameters should produce a company model solvency ratio in each projection year that does not exceed the maximum solvency ratios shown in table 2.2.

2.5.3.2 Sensitivity to changes in the company parameters EK_c and CK_c

These parameters in equations (17) and (20) also depend on the assumption regarding the company's price elasticity of demand relative to the market premium.

Tables 2.9 and 2.10 show the summary statistics of the distribution of solvency ratios in projection year 20 for different values of parameters EK_c and CK_c.

Table 2.9: Sensitivity of solvency ratio to changes in parameter EK_c

Statistics for the distribution of solvency ratios in year 20 with new business model C1								
Parameter EK_c	**0.85**	**1**	**2**	**3**	**4**	**5**	**7**	**10**
mean	1.229	1.177	1.152	1.172	1.193	1.193	1.233	1.289
Median	1.151	1.129	1.144	1.153	1.190	1.178	1.200	1.232
Std. Deviation	0.473	0.404	0.292	0.328	0.356	0.366	0.394	0.441
coeff. Of variation	0.385	0.343	0.253	0.280	0.299	0.307	0.319	0.342
Skewness	5.296	7.221	6.058	10.632	13.398	12.660	10.657	5.893
Maximum	7.367	7.844	6.097	8.215	9.572	9.588	9.693	8.694
95th Percentile	1.805	1.630	1.484	1.534	1.545	1.560	1.692	1.893
5th Percentile	0.817	0.795	0.771	0.823	0.816	0.830	0.828	0.811

They show that the distribution of solvency ratios appears to be unstable at lower parameter values. The mean and median solvency ratios increase as the parameter is above 2.0. The company is quite sensitive to the values of these parameters relative to the market parameter and the reasons are explained below.

Table 2.10: Sensitivity of solvency ratio to changes in parameter CK_c

Statistics for the distribution of solvency ratios in year 20 with new business model C3								
Parameter CK_c	**0.85**	**1**	**2**	**3**	**4**	**5**	**7**	**10**
mean	1.217	1.170	1.159	1.177	1.189	1.197	1.232	1.271
Median	1.149	1.129	1.150	1.167	1.168	1.179	1.202	1.212
Std. Deviation	0.461	0.403	0.308	0.311	0.349	0.380	0.359	0.426
coeff. Of variation	0.379	0.344	0.266	0.264	0.294	0.317	0.291	0.336
Skewness	5.697	7.898	7.186	9.405	12.735	12.948	7.808	5.370
Maximum	7.505	8.187	6.713	7.619	9.255	10.004	8.074	8.108
95th Percentile	1.782	1.591	1.522	1.542	1.529	1.576	1.677	1.879
5th Percentile	0.825	0.789	0.789	0.827	0.831	0.811	0.838	0.806

The company's volume of existing business is small and hence the positive net cash flows in each projection year are small. As the parameters increase, the proportion of the market, relating to the price effect, to be written by the company will increase under new business strategy 1 while the company's premium has not changed. A steady increase in volume of new business (without a change in premium rate) will increase the premium income. This may provide more funds for investments as expenses are likely to be covered, and thereby improving the company's solvency position.

The criteria in section 2.5.3.1 are likely to be met within the range of parameter values ($2 \leq EK_c, CK_c \leq 5$). Intuitively, we may expect the company model to have the same parameter values (i.e. $EK_m = 3.0$, $CK_m = 3.1$) as the market model since both of them will have same the demand function if they are to interact. However, the company's views about market conditions may be different from the rest of the market. We therefore choose $EK_c = 4.0$ and $CK_c = 4.1$ for the exponential and constant price elasticity cases respectively. This corresponds to an assumption that a 5% increase in the company's office premium relative to the market's office premium will produce an 82% of the market volume of exposure to be written by the company. This reflects the fact that an individual company is likely to suffer a greater loss of business than the overall market if it increases its premium by 5%.

2.5.3.3 Sensitivity to changes in the company parameter $l_c(\pm ve)$

Similar to the market model, under new business strategy 1, an increase in the parameter will reduce the premium further, which in turn increases the volume of new business demanded.

Table 2.11 shows the summary statistics of the distribution of solvency ratios in projection year 20 for different values of parameter $l_c(\pm ve)$. It reveals that the solvency ratio is highly sensitive to changes in the factor that measures a company's premium response to changes in its worsening (or improving) solvency. As the parameter increases, the distribution of solvency ratio is quite unstable, as reflected in the skewness and standard deviation. The table also shows that there is a marked difference in the distribution of the solvency ratios when the parameter is about 0.4. An explanation for the high sensitivity of the solvency ratio to the parameter is given below.

Table 2.11: Sensitivity of solvency ratio to changes in parameter $l_c(+ve)$

Statistics for the distribution of solvency ratios in year 20 with new business model C1						
Parameter $l_c(+ve)$	**0**	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**
mean	*2.953*	*1.229*	*1.128*	*1.102*	*1.073*	*1.078*
Median	*1.551*	*1.151*	*1.124*	*1.103*	*1.065*	*1.082*
Std. Deviation	*3.251*	*0.473*	*0.231*	*0.219*	*0.210*	*0.212*
coeff. Of variation	*1.101*	*0.385*	*0.205*	*0.199*	*0.196*	*0.197*
Skewness	*3.121*	*5.296*	*0.541*	*0.295*	*0.242*	*0.195*
Maximum	*12.270*	*7.367*	*2.421*	*2.346*	*1.930*	*2.060*
95th Percentile	*9.678*	*1.805*	*1.494*	*1.450*	*1.433*	*1.412*
5th Percentile	*0.923*	*0.817*	*0.765*	*0.761*	*0.742*	*0.744*

At $l_c(+ve) = 0$ the company's premium in any projection year is likely to be higher than the market premium since the parameter value in the market is $l_m(+ve) = 0.3$. This may lead to a significant reduction in new business volume over the projection period with the resultant effect of improving the company's solvency ratio. This is the assets and the surplus from existing business may grow faster than the liabilities since a high proportion of existing business in the portfolio will expire because without payment of survival benefit.

As the parameter $l_c(+ve)$ is increased, the company's premium is likely to be lower than the market premium with the expectation of increasing its market share, if all other things remain the same. However, if the effect of the company's relative solvency (regulator's control) or management decision on the quantity of new business demanded is considered, a decrease in premium does not necessarily attract a significant new business volume when the parameter is increased. This may adversely affect the company's premium income leading to a decrease in the solvency ratio as the parameter is increased. On the other hand, it may also improve the company's solvency position as it may avoid writing an inappropriate volume of business that will cause a significant drain on available free assets.

Table 2.11 shows that the mean solvency ratio and 95th percentile will meet the criteria in section 2.5.3.1 in the range $0.2 < l_c(+ve) < 0.5$. We choose $l_c(+ve) = 0.4$, as the company may prefer to have a different premium structure from the market in order likely to increase its market share and to reduce the overall sensitivity of the company's solvency ratio.

2.5.3.4 Sensitivity to changes in the company parameter *NBConF*

Table 2.12 shows the summary statistics of the distribution of solvency ratios in projection year 20 for different values of parameter *NBConF*. This parameter limits the company's proportion of new business demanded in the market (i.e. its market share) in the future projection year, having started with a 20% market share at the projection date. The company's market share is expected to increase with the parameter and it is likely to write more new business thereby draining its free assets. This is reflected in the mean solvency ratio in table below.

Table 2.12: Sensitivity of solvency ratio to changes in parameter *NBConF*

Statistics for the distribution of solvency ratios in year 20 with new business model C1						
Parameter NBConF	**0.25**	**0.3**	**0.35**	**0.4**	**0.45**	**0.5**
mean	*1.371*	*1.264*	*1.240*	*1.230*	*1.229*	*1.229*
Median	*1.163*	*1.156*	*1.152*	*1.151*	*1.151*	*1.151*
Std. Deviation	*0.887*	*0.587*	*0.506*	*0.479*	*0.473*	*0.473*
coeff. Of variation	*0.647*	*0.464*	*0.408*	*0.389*	*0.385*	*0.385*
Skewness	*4.346*	*4.826*	*5.064*	*5.239*	*5.297*	*5.296*
Maximum	*8.887*	*7.367*	*7.367*	*7.367*	*7.367*	*7.367*
95th Percentile	*2.869*	*2.025*	*1.870*	*1.805*	*1.805*	*1.805*
5th Percentile	*0.805*	*0.819*	*0.820*	*0.817*	*0.817*	*0.817*

The table also reveals that distribution of solvency ratios is more or less stable when $NBConF > 0.35$ as shown in the mean solvency ratio, standard deviation, median and the 95th percentile. This may be due to the effect of a limited amount of capital to write more new business.

In this investigation, we assume that an insurer is not dominant in the market. If, for example, the insurer currently underwrites 50% of the total premium income available in the market, any changes in his pricing policy will probably provoke responses in the pricing of other operators. In order to avoid such a scenario we choose *NBConF = 0.4.*

2.5.3.5 Sensitivity to changes in the company parameter $ft_c(0)$

This parameter is the company's proportion of new business demanded in the market (i.e. its initial market share) at projection date. We assume that the company's initial market share does not change before the projection date.

Table 2.13 shows the summary statistics of the distribution of solvency ratios in projection year 20 for different values of parameter $ft_c(0)$. It shows that an increase in the company's initial market share increases its future profitability and solvency. This is because the company's initial market share reflects its initial financial strength. This may determine the company's ability to finance new business and also to invest in risky assets (e.g. equities) in order to produce higher returns over the long term.

Table 2.13: Sensitivity of solvency ratio to changes in parameter $ft_c(0)$

Statistics for the distribution of solvency ratios in year 20 with new business model C1							
Parameter $ft_c(0)$	**0.1**	**0.15**	**0.2**	**0.25**	**0.3**	**0.35**	**0.4**
mean	*1.184*	*1.211*	*1.229*	*1.184*	*1.225*	*1.281*	*1.410*
Median	*1.098*	*1.151*	*1.151*	*1.119*	*1.133*	*1.152*	*1.173*
Std. Deviation	*0.497*	*0.462*	*0.473*	*0.500*	*0.612*	*0.700*	*0.994*
coeff. Of variation	*0.420*	*0.381*	*0.385*	*0.422*	*0.500*	*0.546*	*0.705*
Skewness	*4.696*	*6.097*	*5.296*	*7.271*	*7.328*	*6.561*	*5.438*
Maximum	*7.044*	*6.745*	*7.367*	*8.818*	*9.589*	*11.219*	*13.202*
95th Percentile	*1.816*	*1.681*	*1.805*	*1.664*	*1.796*	*2.192*	*3.043*
5th Percentile	*0.735*	*0.771*	*0.817*	*0.771*	*0.791*	*0.828*	*0.836*

It can be seen from the table that the shape of distribution of solvency ratio is sensitive to changes in the parameter as reflected in the skewness and 95th percentile. There is a marked difference in the distribution of solvency ratio when the parameter is about 0.25 (e.g. the mean, median and 95th percentile reaching their lowest value), which may be caused by random variation. The table shows that the mean solvency ratio and 95th percentile will meet the criteria in section 2.5.3.1 in the range $0.15 \leq ft_c(0) \leq 0.3$. We therefore choose $ft_c(0) = 0.25$.

2.5.3.6 Conclusion

We choose the following set of base parameters for the company new business models having considered the cumulative effect it will have on the company's solvency ratio in each projection year in order to meet the criteria in section 2.5.3.1.

$EK_c = 4.0$, $CK_c = 4.1$, $l_c(\pm ve) = 0.4$, $NBConF = 0.40$, $ft_c(0) = 0.25$

2.6 Structure of the Market Model

In this section, we show graphically and comment on the underlying structure of the market and/or company models using the simulated results (solvency ratios and net cash flows) over a 20-year projection period produced with the base set of parameter values given in sections 2.5.2.7 and 2.5.3.6 respectively. Thus, we consider the policy design in table 2.1, to be called (Policy Type 1), market new business model M1 and/or company new business model C1, new business strategy 1 and assets/Liability (ALM) investment strategy in order to illustrate the underlying structure of the models. The structure of the models is mainly affected by the policy design under consideration.

2.6.1 The underwriting pattern (net cash flows)

A non-profit product has a typical pattern of profitability over the policy term, a large negative profit (or strain) in year 1 and gradually rising small profits thereafter. The term assurance contract (Policy Type 1) considered under the market model has the above features but with a slight difference.

For (Policy Type 1) the initial expenses (including commission) are heavy so that it takes a number of years to recoup them before profitability can be attained. The net cash flow (excluding investment income) per policy (premiums – expenses – claims) over the policy term is negative in the first four years as a result of the high initial commission payable and positive thereafter. The fifth year has a large positive net cash flow and decreases in subsequent years, to a very small positive, negative or zero net cash flow in the last year. The jump in net cash flows from negative to a large positive between the 4^{th} and 5^{th} year of policy term is caused by the differences in commission terms in those years.

Figure 2.7 shows the box plots for net cash flows (excluding investment income) over the policy term for cohorts of new business issued a year before the projection date (a deterministic scenario) and issued in projection year 2 (a stochastic scenario) and using market new business model M1. Thus, figure 2.7 compares the effect of different cohorts of new business (with features such as premiums and sums assured which are obtained deterministically or stochastically) on a company's net cash flows.

Figure 2.7(a) shows that the policies issued a year before the projection date (the deterministic scenario) produce a large positive cash flow in projection year 4 (i.e. the fifth year of the policy) and a decreasing positive cash flow thereafter. The cash flow falls to negative, zero or positive values in projection year 9 (i.e. the last year of the policy).

Similarly, policies issued two years' before projection date produce a large positive net cash flow in projection year 3 (i.e. the fifth year of the policy). The combined net cash flows from all existing business in projection year 4 and after become positive because all the existing policies produce a positive net cash flow in each of these years (this is not shown in figure 2.7 due to space constraints). However, the above positive net cash flows are affected by negative net cash flows from new policies issued after the projection date. For instance, the policies issued in projection year 1 will produce a large positive net cash flow in projection year 5 and thereafter produce decreasing positive cash flows. Similarly, the policies issued in projection year 2 (the stochastic scenario) produce a large positive net cash flow in projection year 6 and there after produce decreasing positive net cash flows, as shown in figure 2.7(b).

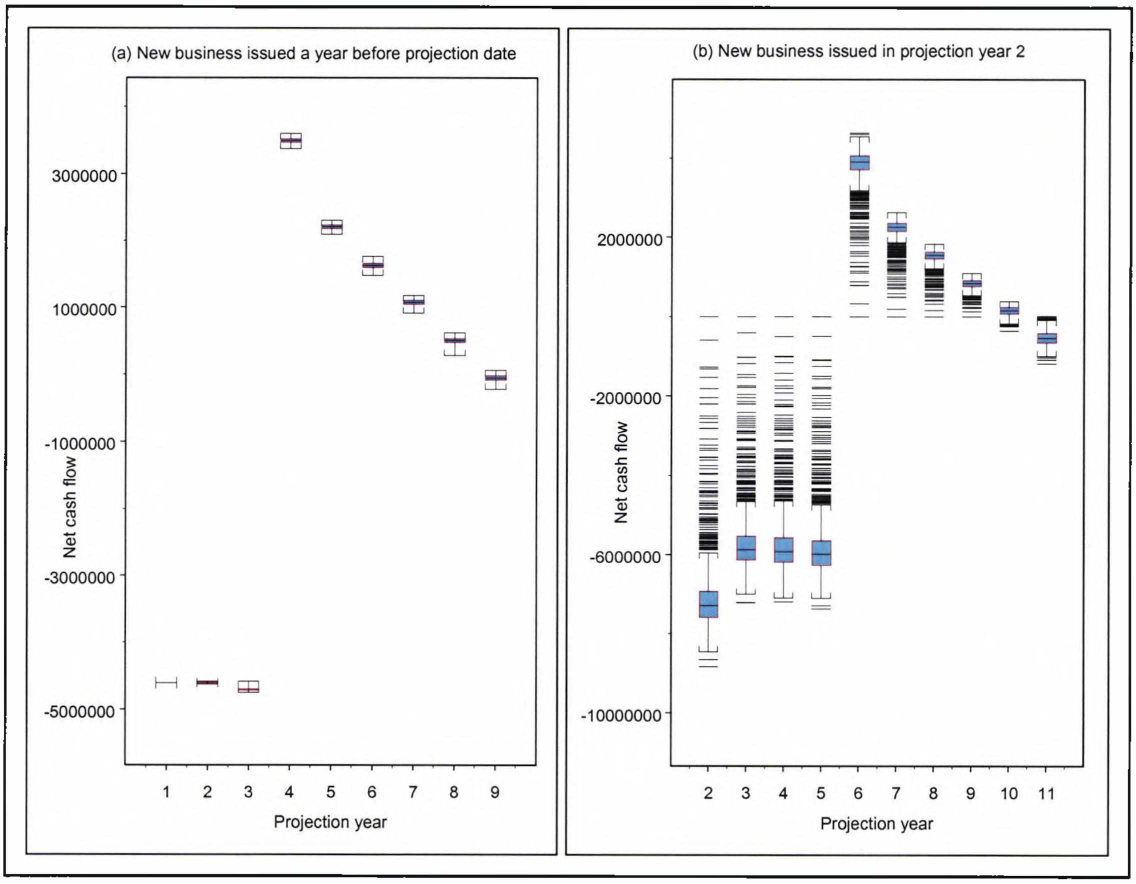

Figure 2.7: Distribution of net cash flows for market new business model M1

Figure 2.7 shows that the plots are much narrower in figure 2.7(a) than in figure 2.7(b). This reveals that there is less variability in net cash flows for policies issued a year before projection date, see figure 2.7(a), than those for policies issued in projection year 2, see figure 2.7(b). Figure 2.7(a) shows that there is little or no change in the shape of the distribution of net cash flows over the policy. Thus, the quartiles and the range of the distribution of net cash flows tend to remain the same over the policy term for policies issued a year before projection date. This is because the policies issued before the projection date have identical policy details such as such premium and sum assured (which are deterministically obtained) while the future premiums and sums assured for future new business issued in projection year 2 increase with inflation rates which are simulated stochastically. In other words, the size of past portfolios is known with certainty while the size of future portfolios depends on market conditions, the premium charged in a particular projection year and the free assets available.

Figure 2.7(b) shows that the median and quartiles of the distribution of net cash flows in the first year of the policy (i.e. in this case projection year 2) are lower than that in other policy years. This is because of the effect of initial new business strain in the year of issue arising from heavy initial commission. The figure also reveals that the shape of the distribution of net cash flows over the policy term changes (e.g. from positively skewed to negatively skewed) after the fourth policy year, and it becomes approximately symmetric at the end of policy term (projection year 11). In other words, the variability in net cash flows for policies issued in projection year 2 wears off over the policy term (e.g. the range of the distributions decreases over the policy term). This is probably due to the structure of the policy design under consideration, that is, the effect of heavy initial commission and the random effects which wear off over time.

2.6.2 The distributions of solvency ratios for the market and company

As the companies operating in the temporary insurance market are assumed to be self-financing, having received an initial free capital at the projection date, no new business will actually be written if there are no free assets in a projection year. Thus, the solvency ratio in each projection year is affected mainly by the pattern of net cash flows from both new business and existing business but also by the pattern of actual new business volume written. These features tend to produce a cyclical pattern of cash flows (peaks and troughs) for Policy Type 1.

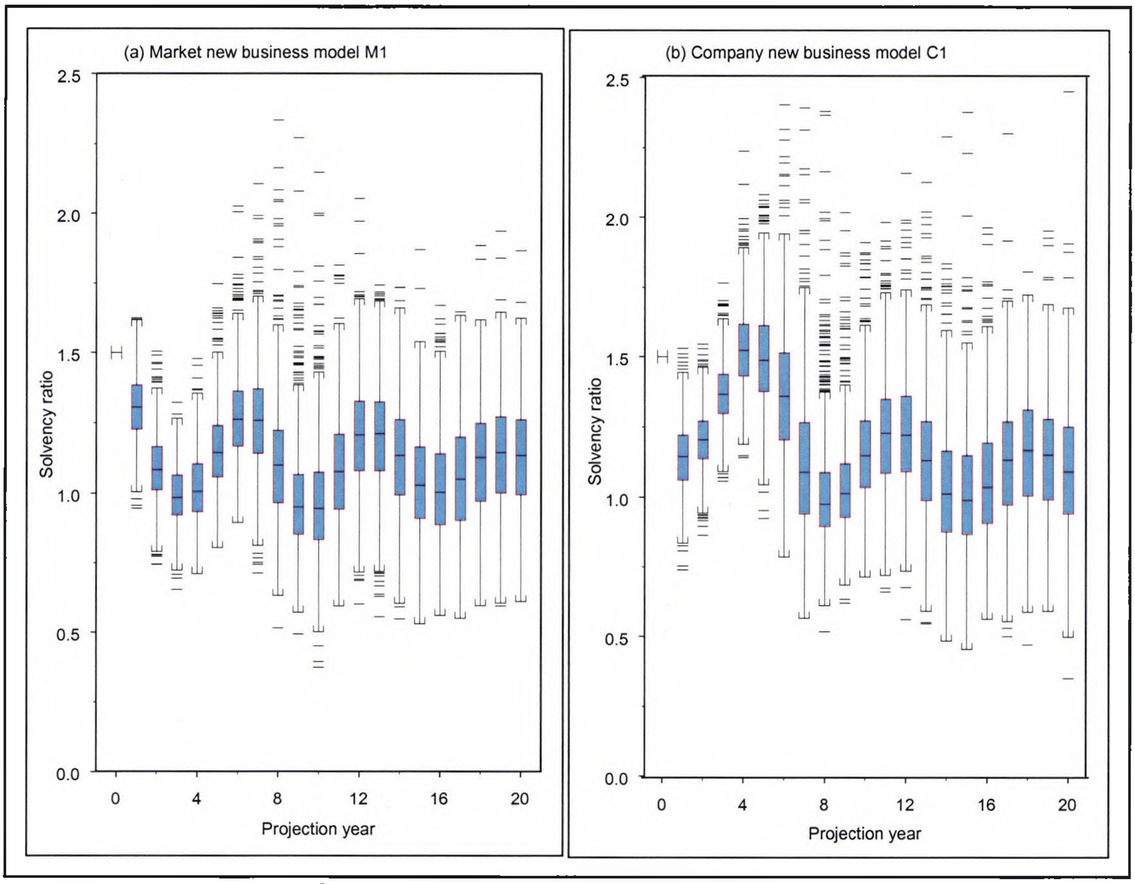

Figure 2.8: Distribution of solvency ratios for market and company models

Figure 2.8 shows the (box plots) distributions of solvency ratio in projection years for market new business model M1 and company new business model C1. Considering the median value of the distributions of the market and the company's solvency ratios in each projection year, the figure shows a pattern of underwriting results, peaks and trough of profitability. The above is due to the fact that different cohorts of new business will produce positive net cash flows at different times over the projection period. The figure shows that the company's solvency ratios, see figure 2.8(b), over the projection period are more sensitive than the market model, see figure 2.8(a), due to the greater variability in deaths and lapses.

Figure 2.8 also shows that, for the market and company having the same initial solvency ratio (e.g. 1.50), the median of the distribution of solvency ratios for the market tends to decrease while that for the company increases in the early projection years. Figure 2.8(a) shows that, for the market model, at least 75 % of the 1000 simulations have solvency ratios below the initial solvency ratio of 1.50 over the projection period. For the company model, see figure 2.8(b), the above holds only over the long term. The reason is explained below.

This is because the initial free assets for the company are sufficient to cover the new business strains in the early years of the projection as the company writes a relatively smaller volume of new business than the market in first few years of the projection. As all of the available free assets are used up in writing new business over the projection period, there is less investment freedom for producing higher returns from risky assets (e.g. equities) in the long term. This is more likely to hold for the market model than for the company model because of the large volume of new business written in the market which causes a drain on the free assets.

The simulated results in figure 2.8 show both the solvent and insolvent life companies whilst table 2.2 shows only the UK life insurance solvent companies with solvency ratios ranging from 1.00 t0 2.92. Thus, the simulated results appear to be robust.

2.6.3 Comments on measuring risk over a projection period

The emergence of peaks and troughs in profitability in projection years will make it difficult to consider some risk measures such as probability of insolvency (which measures the chance of insolvency occurring) in a particular year of projection. This is because the conclusions arising from such a risk measure in a projection year (e.g. with peaks in profitability) may be inconsistent and different from other projection years (e.g. with troughs in profitability). Therefore, in this case, it may be appropriate to consider the risk measure over a projection period, as it will not be affected by the emergence of peaks and troughs in profitability in a particular projection year. However, the above scenario will not create problems for risk measures such as the standard deviation of a distribution, the q-quantile or value-at-risk (VaR) [4]. In general terms, "the VaR is the amount of capital required to ensure, with a high degree of certainty (chosen arbitrary), that the enterprise doesn't become technically insolvent", Panjer (2002).

Thus, in this investigation, we propose that a simulation is deemed to have produced an insolvent outcome if the projected solvency ratio (or surplus) falls below one, (or zero) i.e. $SR_m^j(t) < 1$ at least once over the T-year projection period rather than in a projection year T. In other words a risk measure (e.g. probability of insolvency) can be defined as $P\{\min(SR_m^j(t) < 1 : \forall t, j)\}$ rather than $P\{(SR_m^j(T) < 1 : \forall j)\}$, for the j-th simulation, $SR_m^j(t)$, $t = 0, 1, 2, \ldots, T$, gives a stochastic development of solvency ratios over the projection period. Repeating this N times gives N simulated realisations of the solvency ratios.

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[4] Panjer (2002) defines a risk measure as "a mapping from the random variables representing the risks to the real line. A risk measure gives a single number that quantifies the risk exposure in a way that is meaningful for the problem at hand [e.g. the standard deviation of a distribution]. The q-quantile or VaR is defined as the q-quantile, x_q, is the smallest value satisfying $\Pr\{X > x_q\} = 1 - q$. As a risk measure, x_q is the Value-at-Risk, the size of loss for which there is a small (e.g. 1%) probability of exceedence".

Chapter 3

Measures of Risk and Simulated Results

3.1 Introduction

In this chapter we consider measures of new business risk, and use the simulated results from asset/liability framework for the market and company models as described in chapter 2 (see also section 3A in appendix 3) to:

- Investigate how a company with a particular level of initial solvency ratio can obtain a free assets allocation proportion (to finance new business) over a time horizon that produces a fixed level of insolvency risk under a desired new business strategy and investment strategy.
- Carry out a sensitivity analysis of risk measures for both market and company new business models in order to determine which of the models serves as a useful benchmark for representing the term assurance market, and which company new business model is a good choice allowing for the interaction of the company with the market.
- Carry out a sensitivity analysis of risk measures for both market and company model parameters in order to identify the most influential parameters affecting new business and insolvency risk.
- Illustrate the effectiveness of a company's new business plans (i.e. new business decision making process) to meet a given level of insolvency risk through the choice of values for these most influential parameters.
- Show the influence of the factors affecting new business plans on the risk measures.
- Demonstrate the effect on risk measures of various types of company model (which differ by new business strategy and/or investment strategy) interacting with the market model in a stochastic and competitive environment.

In this chapter, we consider a market and /or company portfolio with cohorts of business with entry age 55 to illustrate an adverse scenario (in terms of mortality risk and a potentially high probability of insolvency) which may face a company writing non participating business. We assume that 10, 000 such policies are issued in year 0, whose details are taken as input values for the new business at projection date. However, we will consider a portfolio of cohorts of business with mix of entry ages (e.g. 35, 45, and/or 55) to show the effect of a mix of business on solvency risk in chapter 4.

3.2 Measures of Risk

We consider three types of insolvency risk measures in this investigation, two of which (probability of insolvency and mean shortfall risk) are relevant to both the market and the company model, and the other (probability of relative insolvency) is only applicable to the company model. We consider the risk measures over a given time horizon (e.g. 20-year projection period). We have noted in section 2.6.3 of chapter 2 that it is more appropriate to consider a risk measure such as probability of insolvency over a projection period than in a particular projection year if the risk measure will be affected by the emergence of peaks and troughs in profitability in that year.

3.2.1 Probability of Insolvency

The probability of insolvency measures the chance of insolvency occurring over a period (or at a specified time horizon). Statutory insolvency is deemed to occur when the solvency ratio is less than one (e.g. $SR_m(t) < 1$) at least once in any simulation over a t-year projection period. The probability of insolvency is estimated by the proportion of outcomes with statutory insolvency from a predetermined number of simulations.

$$\hat{p}(t) = \frac{x(t)}{N} \qquad (20)$$

where $x(t)$ is the number of simulations with statutory insolvency over t-year projection period, N is the total number of simulations (e.g. 1000) and $\hat{p}(t)$ is the estimated probability of insolvency over a t-year projection period.

3.2.2 Mean Shortfall Risk

The mean shortfall risk measures the severity of insolvency if it occurs over a period (or at a specified time horizon). It may be defined as the product of probability of loss (i.e. insolvency) and expected value of loss should event occur (i.e. mean shortfall), see Haberman et al (2003). The mean shortfall (in terms of solvency ratio) may be defined as the conditional mean of the solvency ratios, conditional on the simulations whose solvency ratio is less than 1 over a t-year projection period. Where a simulation has two or more insolvent cases over a projection period, the worst case is taken. As stated in Haberman et al (2004), the mean shortfall is less sensitive to changes in the tail of the distribution being investigated and also satisfies the coherence requirement (proposed by Artzner (1999)) by being sub-additive. We define

$MS(t)$ = Mean shortfall over a t-year projection period, and

$MSR(t)$ = Mean shortfall risk over a t-year projection period.

$$MS(t) = \frac{1}{x(t)} \sum_{j=1}^{x(t)} SR_m^j(t) \quad \text{where } SR_m^j(t) < 1 \; for \; j = 1, 2, \ldots\ldots, x(t) \tag{21}$$

$$MSR(t) = \widehat{P}(t) \,.\, MS(t) = \frac{\sum_{j=1}^{x(t)} SR_m^j(t)}{N} \tag{22}$$

Remark: Haberman et al (2004) use a scaling factor (e.g. number of policies at entry) in the definition of mean shortfall (a conditional mean of the residual assets), for the purpose of facilitating comparison between cohorts of different sizes. However, no scaling is required in equation (21) as the mean shortfall is defined in terms of solvency ratios less than one rather than in amounts such as residual assets as in Haberman et al (2004)

3.2.3 Probability of Relative Insolvency

We consider two versions of relative insolvency as defined by Hardy (1996). It is defined as "the probability that an individual insurer does fall significantly out of line with the rest of the life insurance market, in terms of, for example, assets liability ratios, or pay-outs to with-profit policyholders".

3.2.3.1 Relative Insolvency: Version 1

This is the situation where an office is in difficulty through having a solvency ratio $SR_c^j(t)$ less than one and the market, as a whole, does not appear to be in difficulty. For some reasonable parameter value $RSV_1 > 1.0$ (chosen arbitrarily) an office is said to be relatively insolvent over time t if $SR_c^j(t) < 1.0$ *and* $SR_m^j(t) > RSV_1$.

The results for different values of RSV_1 may be considered (e.g. RSV_1 = 1.05, 1.15, 1.25).

We perform N simulations of the company model over time t. We set

$VX^j(t) = 1 \;\; when \;\; SR_c^j(t) < 1.0 \;\; and \;\; SR_m^j(t) > RSV_1$.

$VX^j(t) = 0 \;\; otherwise$.

The simulated frequency of relative insolvency occurring as shown below is assumed to be an estimate of probability (version 1) of the actual relative insolvency $P_{v1}(t)$.

$$\hat{P}_{v1}(t) = \frac{1}{N} * \sum_{j=1}^{N} VX^j(t) \tag{23}$$

3.2.3.2 Relative Insolvency: Version 2

This is the situation where an office is in difficulty through having a solvency ratio $SR_c^j(t)$ significantly below the rest of the market, even if the office's solvency ratio is still greater than 1.0. For some parameter value $RSV_2 < 1.0$ an office is said to be relatively insolvent over time t if: $\frac{SR_c^j(t)}{SR_m^j(t)} < RSV_2$. The results for different values of RSV_2, chosen arbitrarily, may be considered (e.g. RSV_2 = 0.6, 0.7, 0.8).

We perform N simulations of the model office over time t. We set

$$VX^j(t) = 1 \qquad when \ \frac{SR_c^j(t)}{SR_m^j(t)} < RSV_2$$

$$VX^j(t) = 0 \qquad otherwise$$

The simulated frequency of relative insolvency occurring as shown below is assumed to be an estimate of probability (version 2) of the actual relative insolvency $P_{v2}(t)$.

$$\hat{P}_{v2}(t) \quad \frac{1}{N} \quad \sum_{j=1}^{N} VX^j(t) \tag{24}$$

3.3 The New Business Plans of a Company

3.3.1 Introduction

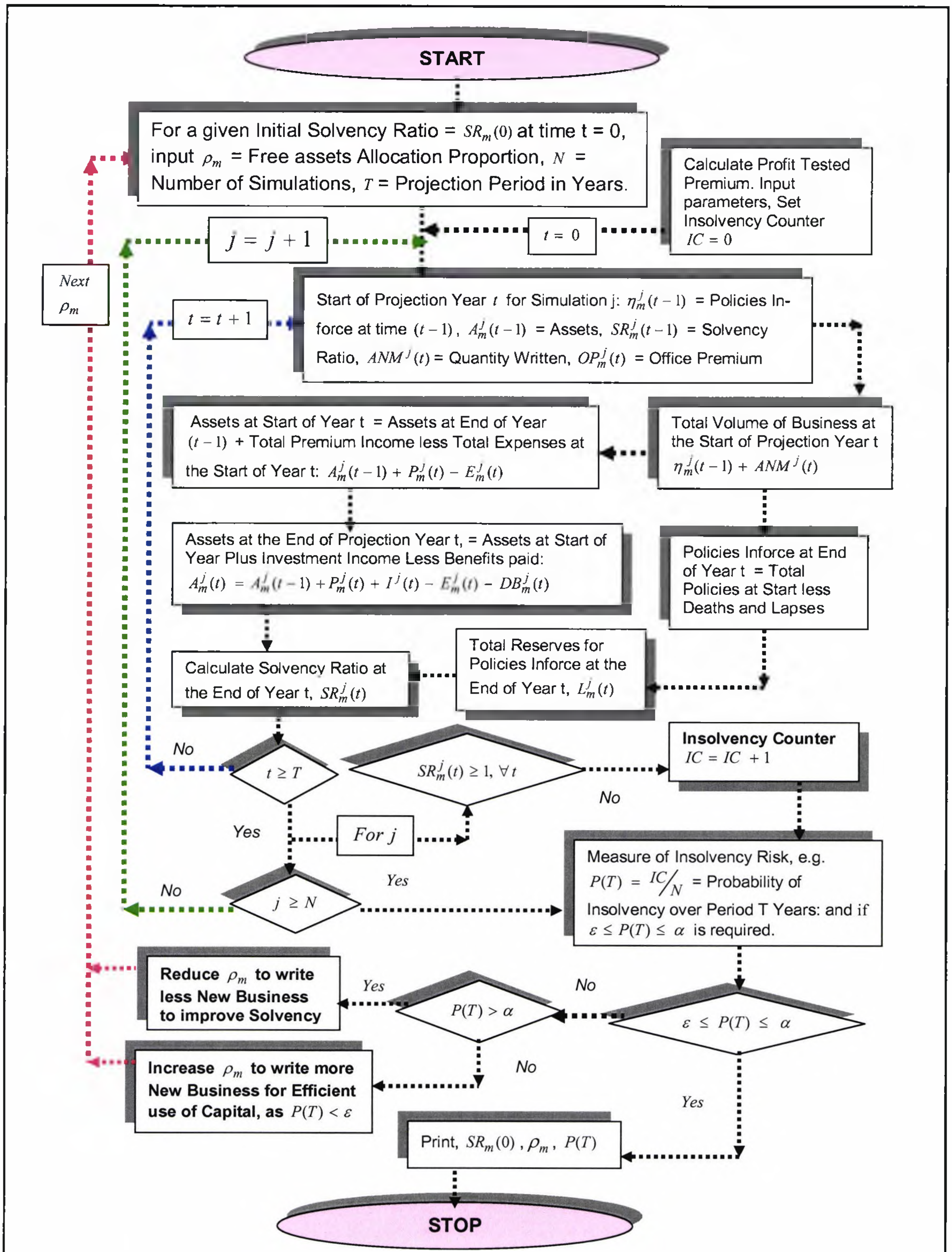
We note in section 1.1 that a company's new business plans are comprised of many factors such as product design, pricing policy, new business volume, channels of distribution, marketing strategy, free assets available and allocated to write new business. For simplicity, we assume that a company's decision concerning the allocation of free assets to write new business in order to produce a fixed level of insolvency risk for a given initial free capital and new business strategy together with its pricing policy (measure of a company's premium response to changes in its worsening (or improving) solvency position (denoted by $l_m(\pm ve)$) are taken as a proxy for the company's new business plans.

In other words, the decision concerning the free variables, namely the proportion of free assets allocated to finance new business (denoted by ρ_m) and/or the initial solvency ratio (denoted by $SR_m(0)$) constitutes the new business plans. This is because the free assets (measured in terms of solvency ratio) directly affect the quantity of new business written (see equation (8)) and new business strategy with its appropriate pricing policy, which in turn affects the quantity of new business demanded through the demand functions.

In this section, we consider two free assets allocation strategies to finance new business, namely a constant proportion of free assets allocation strategy and a dynamic strategy in allocation of free assets over a given projection period. We use the market new business model M1, new business strategy 1 (NBS_1), two insolvency risk measures (probability of insolvency and mean shortfall risk) and the ALM investment strategy. Further, we assume that a company's chosen pricing policy for any new business strategy (i.e. measure of premium response to changes in a company's solvency ratio, e.g. $l_m(+ve) = 0.30$ for NBS_1) remains the same over the projection period.

3.3.2 Constant Proportion of Free Assets allocation Strategy

Figure 3.1 below shows the investigation of how we can obtain a constant free assets allocation proportion to finance new business over a projection period for a given initial solvency ratio that produces an acceptable level of probability of insolvency. Briefly, figure 3.1 illustrates how the asset/liability framework for the market model (described in chapter 2 and section 3A in appendix 3) is used to produce simulated results (e.g. 1000 solvency ratios in each projection year) with which a level of risk (e.g. probability of insolvency over a projection period) is measured for a given combination of initial solvency ratio and constant free assets allocation proportion. The above process is repeated by changing only the free assets allocation proportion until an acceptance level of risk is produced for the given initial solvency ratio.



Schematic representation of investigation of a free assets allocation proportion to finance new business to produce a fixed level (or range) of insolvency risk over a period under a given investment strategy and new business strategy

Figure 3.1: Investigation of free assets allocation proportion to finance new business

The derivation of analytical expressions for the sequence of proportions of free assets allocated over a projection period for a given initial solvency ratio and required to produce an acceptable level of insolvency risk is, in general, very difficult, because of the complexity of the portfolio under investigation. Instead, we use contour lines (the constant insolvency risk curves) as a means of investigating and presenting the results in a simple manner. Section 3B in appendix 3 provides a general description of how contour lines (the constant insolvency risk curves) in figure 3.2 shown below are obtained. This is similar to the description in figure 3.1 except that an interpolation technique is carried out where necessary in order to simplify the calculations of the combinations along the curves in figure 3.2.

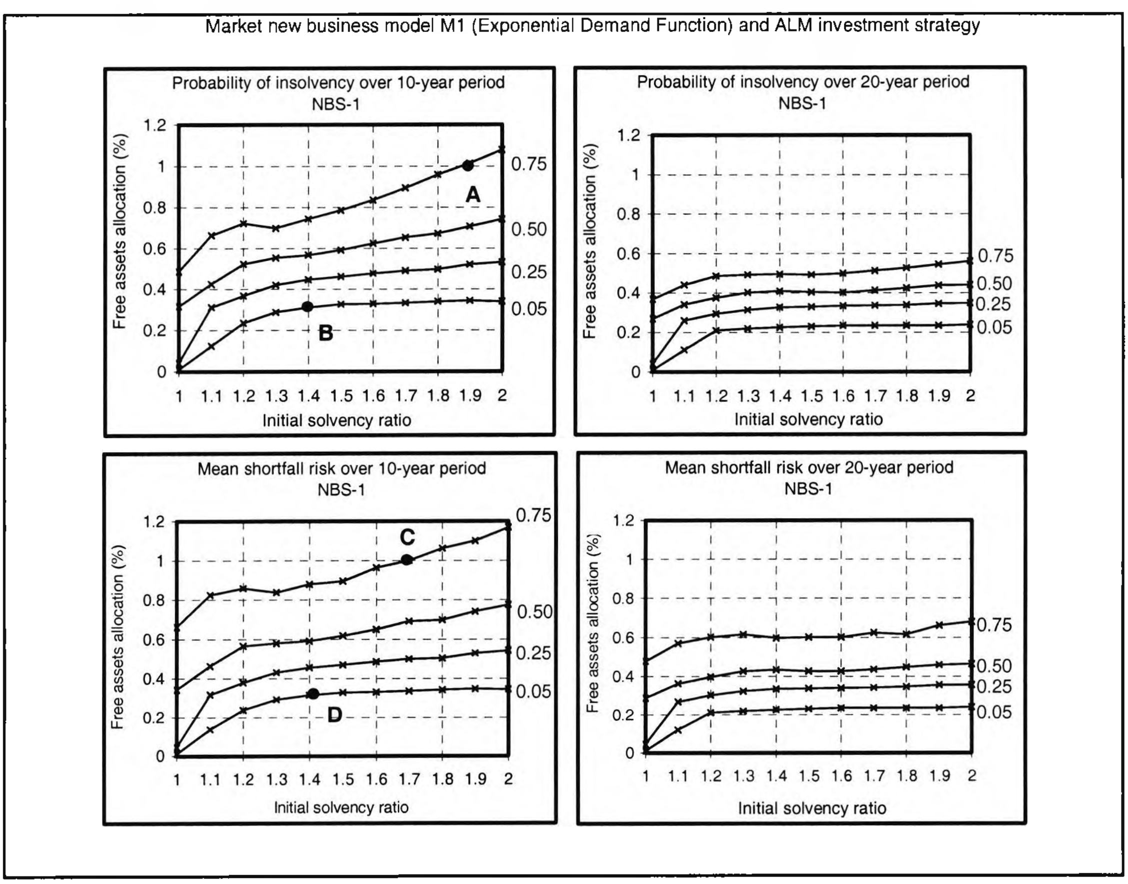


Figure 3.2: Constant proportion of free assets allocated to finance new business

Figure 3.2 shows a set of curves that will enable a company to choose freely a constant proportion of free assets to finance new business so that it has an adequate level of free assets to continue its desired new business strategy over a projection period. The curves are the constant insolvency risk curves (i.e. like contours) which allow a company to choose a sustainable growth rate from its own internal resources. Each curve gives the combination of a company's initial size (measured in terms of initial solvency ratio, $SR_m(0)$) and a choice of new business plan (e.g. the constant proportion (ρ_m) of free assets required to finance new business) over a projection period that produces a fixed level of insolvency risk. The probability of insolvency and the mean shortfall risk over a 10-year or 20-year projection period are illustrated in figure 3.2.

As expected, the insolvency risk associated with each curve increases (by moving upwards) as the company's desire for market share increases (i.e. requiring a higher proportion of free assets to finance new business) at the expense of meeting solvency requirements, all other things being equal. On the other hand, for a given contour line (a constant insolvency risk curve) the free assets allocation proportion increases with the initial solvency ratio. This reflects the fact that the financial strength of a company determines the proportion of free assets allocated to finance new business in order to increase its market share, if all other things remain the same. The shape of each curve reflects the extent to which the company's initial free assets cover the new business strain in the early projection years and the company still has sufficient assets for investment in equities with the intention of achieving higher returns. Each curve also shows a marked difference between a weak company (with a solvency ratio less than 1.30) and a strong company in terms of the proportion of free assets required to finance new business due to the above reason.

Figure 3.2 also shows that a company would require a lower constant proportion of free assets to finance new business over a longer projection period than a shorter projection period in order to maintain a given level of insolvency risk, except for a very weak company. The above reveals that a company may need more free assets to invest in equities (by writing a small volume of new business) over the longer term than over the short term in order to maintain the same level of insolvency risk over the different projection periods.

Figure 3.2 reveals a scenario whereby a company may require more than 100% of its available free assets to finance new business over a 10-year projection period in order to produce a fixed level of insolvency risk (e.g. at least 75%). This means that the company will have a need to demand for extra funds from external sources (which is not allowed for in this investigation) in order to finance its new business plans and achieve its objectives. Points A and C on the 75% contour lines (for probability of insolvency and mean shortfall risk) in figure 3.2, represent a combination of company's initial solvency ratio (about 1.9 and 1.7) respectively and 100% allocation of free assets over a 10-year projection period. For the given new business strategy 1, a company with an initial solvency ratio which exceeds the above limit will need funds from external sources to finance its future new business plans in order to achieve a 75% probability of insolvency over the projection period.

In practice, a company is more likely to target an insolvency risk of less than 10% over a time horizon like 10-20 years, and may also be required to do so by regulators. This objective may create the need to demand external funds in order to finance new business plans even if the free assets allocation proportion is below 100%, as long as the chosen allocation proportion produces an insolvency risk of more than 10%. Points B and D on the 5% contour lines (for probability of insolvency and mean shortfall risk) in figure 3.2 represent a combination of company's initial solvency ratio of 1.4 and free assets allocation proportions of about 31% and 32% respectively over a 10-year projection period. For the given new business strategy 1, a company with the corresponding initial solvency ratio will need funds from external sources if it needs to increase the allocation proportion above 31% to write more new business and it still wants to achieve a 5% probability of insolvency over the period.

The contour lines, for a given level of probability of insolvency in figure 3.2 are higher than the corresponding contour lines for mean shortfall risk. This reveals that, as expected, the two risk measures (probability of insolvency and the mean insolvency risk) affect the company's new business plans differently, in particular the constant proportion of free assets allocated to finance new business.

3.3.3 Dynamic Strategy for Allocation of Free Assets

For simplicity, we propose a dynamic proportion of free assets allocation strategy to finance new business similar to the dynamic investment strategies discussed in chapter 2. Thus, there may be many feasible dynamic structures to be considered, as a given level of initial solvency ratio may require different dynamic allocation strategies to produce the same level of insolvency risk. For purposes of illustration, we consider a hypothetical dynamic strategy for allocation of free assets to finance new business below by choosing some reasonable parameter values.

For example, a company is assumed to allocate 40% of its free assets to finance new business while its solvency ratio exceeds 1.4. If at the end of a year the solvency ratio falls below 1.4 the company gradually reduces the percentage allocation, reaching 5% if the solvency ratio is less than 1.05.

The dynamic allocation strategy will produce a corridor of free assets allocation proportions over a projection period. The dynamic strategy will produce a slightly lower insolvency risk over a projection period than the constant proportion of free assets allocation strategy if the maximum allocation percentage in the former strategy is the same as the constant proportion strategy (e.g. $\rho_m = 0.4$) for a given initial solvency ratio (e.g. $SR_m(0) = 1.50$). The above discussion is revealed in table 3.1.

Table 3.1: Free assets allocation strategies to finance new business

Risk measure	*Prob. of insolvency*		*Mean shortfall risk*	
Projection period	***10***	***20***	***10***	***20***
Constant Proportion	0.164	0.534	0.153	0.473
Dynamic allocation	0.141	0.501	0.130	0.436

For purposes of comparison, we allow only the maximum allocation percentage to change in the dynamic allocation strategy mentioned above. Thus, contour lines similar to figure 3.2 above are obtained, as shown in figure 3.3 for probability of insolvency and mean shortfall risk measures.

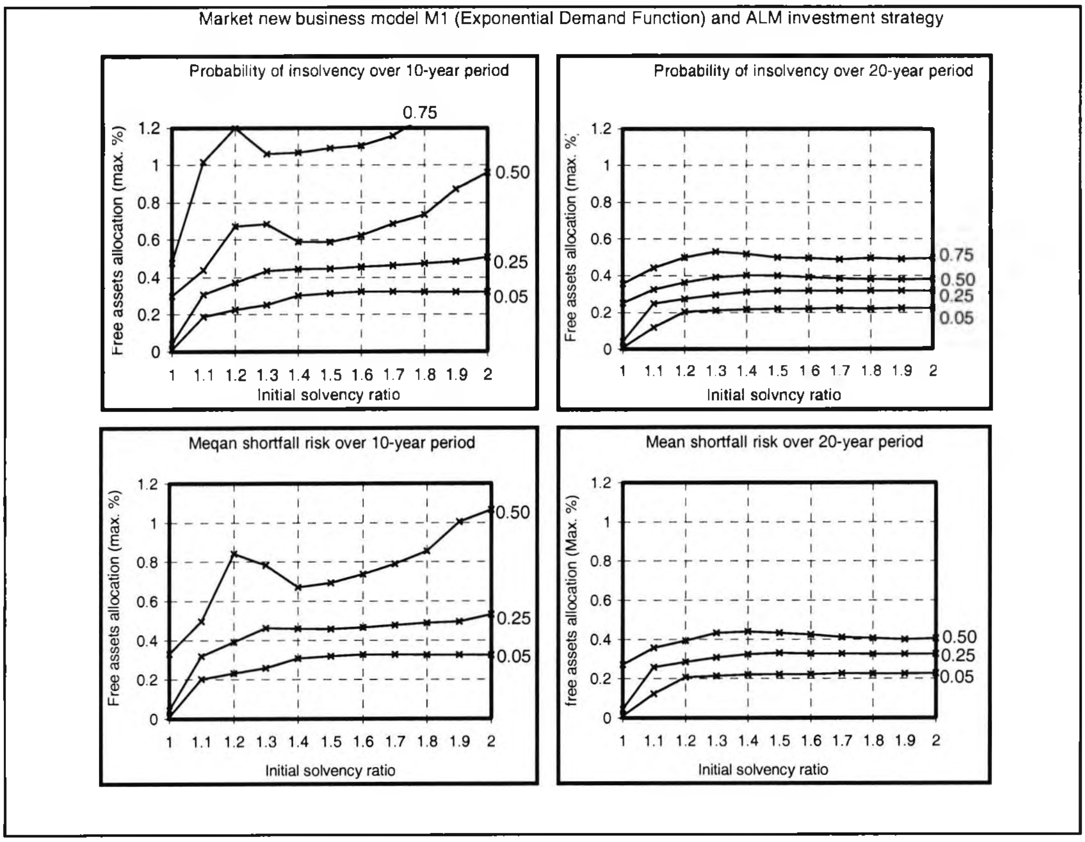


Figure 3.3: Dynamic allocation of free assets allocation to finance new business

Figure 3.3 shows that the contour lines for the dynamic strategy in allocation of free assets are quite unstable over a 10-year projection period than those of constant proportion of free assets allocation strategy, particularly at a high insolvency risk (e.g. above 50%). The above feature also holds when the maximum allocation percentage in the dynamic allocation strategy is higher than 40%. As the maximum allocation percentage increases above 40% whilst all other parameters in the dynamic allocation strategy remain the same, there is tendency for a company to write unstable volume of new business in the short term for a given level of initial free capital. Thus, the unstable contour lines may be a reflection of the unstable new business volume written.

For a 20-year projection period, the dynamic strategy in the allocation of free assets increases the insolvency risk for large companies (e.g. with an initial solvency ratio of above 1.50). On the other hand, it reduces the insolvency risk for the medium size companies (with initial solvency ratio of between 1.2 and 1.50) than the constant proportion of free assets allocation strategy. This is reflected in the 50% mean shortfall risk contour line for dynamic strategy being higher than that of constant proportion of free assets allocation strategy in respect of medium size companies. The above may be caused by the 1.4 solvency ratio upper bound specified in the dynamic strategy.

The above discussion may change if any of the values taken arbitrarily in the dynamic strategy is changed. Table 3.2 shows the effect on the risk measures of changes in solvency ratio upper bound in the hypothetical dynamic strategy for an initial solvency ratio of 1.50. As the solvency ratio upper bound increases while all other values are held constant, the insolvency risk decreases. This may arise from writing less new business. This is because a change in the solvency ratio upper bound in the dynamic strategy (for example from 1.4 to 1.6) will result in allocating less than 40% of a company's free assets to finance new business if its solvency ratio in a projection year is below 1.6.

Table 3.2: Effect of changes in maximum solvency ratio in dynamic strategy

Risk measure		*Prob. of insolvency*		*Mean shortfall risk*	
Projection period		***10***	***20***	***10***	***20***
maximum	***1.35***	0.145	0.513	0.134	0.448
solvency ratio	***1.4***	0.141	0.501	0.130	0.436
in dynamic	**1.5**	0.140	0.476	0.128	0.412
strategy	***1.6***	0.114	0.441	0.104	0.382

3.3.4 Conclusion

Browne et al (2001) state that insolvent life insurers tend to be smaller in size than solvent insurers and to change their product mix more often. The investigation in this section has also revealed that the level of insolvency risk for a given projection period decreases as the initial financial strength of the company increases.

For simplicity, we consider only the constant proportion of free assets allocation strategy in the subsequent sections in this investigation and therefore the dynamic allocation strategy is not considered further.

3.4 Sensitivity Analysis of New Business Models and Parameters

3.4.1 Introduction

In the following sections, we consider the sensitivity analysis of new business models and model parameters for both the company and market under the new business strategy 1 and the asset/liability matching investment strategy, as they are the most intuitive strategies in this investigation.

A model is a simplified version of reality. As models are not known with certainty they are usually only approximations to the real world (Haberman et al 2004). Therefore the choice of the model and assumptions involves risks, namely model risk (or specification error), parameter risk and process risk, and they are described below, see Daykin et al (1993), Hooker et al (1996) and Haberman et al (2004). Hooker et al (1996) noted that “typically, the process risk is the least significant, but most easily modelled and incorporated into a formula. The parameter uncertainty is measurable, but it is sometimes forgotten. The specification error is the least tractable of the three, but probably the most important”.

Model risk is the risk that the model chosen may not adequately represent the real world. For example, the selected price-elasticity of demand function and/or the real wage inflation model (e.g. market new business models M1 or M2) in this investigation may not reflect the price sensitivity of the term assurance market. Therefore there is the need to consider alternative model structures (e.g. market new business models M3 or M4), which may yield different results (e.g. different levels of insolvency risks over a given time horizon). The model sensitivity can be explored by statistical analysis of the differences in the results from the models.

Parameter risk is the risk that parameters used in the model are inappropriate for representing the future experience even if the underlying model is appropriate. The parameters that describe the structure of the models in this investigation are estimated mainly on the basis of the sensitivity analysis in chapter 2, because relevant term assurance market data are limited in quantity. Alternatively, the parameter values could also be simulated (e.g. using Bayesian approach) to allow for parameter risk, an aspect not considered in this investigation.

In section 2.5.2, we discuss both the successive and the simultaneous sensitivity analysis. For a successive sensitivity analysis, Darbellay (2001) uses the elasticity formula as defined below, as an estimate of the sensitivity of the valuation result $R(p)$ to changes in a parameter (p).

$$E_p(R) = \frac{\Delta R / R}{\Delta p / p}. \qquad (25)$$

where, $E_p(R)$ = The elasticity of the valuation result $R(p)$ in response to the parameter p, ΔR = change in valuation result, Δp = change in parameter (p). The parameter p is described as elastic if the result of the formula (25) is higher than 1 and inelastic if the result of the formula is lower than 1. This elasticity formula is used in the successive sensitivity analysis of a risk measure when we measure the response to a percentage change of a parameter value. For the simultaneous sensitivity, we will consider the effect on a risk measure of changing a few related parameters at the same time.

Process risk is the risk that "the outputs from the model will not adequately represent the range of possible outcomes for the actual future experience (i.e. the outcomes are subject to random fluctuations, even when the model and parameters are appropriate). Using Monte-Carlo simulation the risk can be reduced by increasing the number of simulations", Haberman et al (2003). The pooling of risk through writing more of the same business may also reduce the relative level of variability in the portfolio, i.e. risk per policy, as demonstrated by Cummins (1991) and Haberman et al (2004). This investigation will not consider the process risk further, as the risk is likely to be small given that the analysis is based on 1000 simulations.

We next investigate the sensitivity of the new business models and also consider the effects of choice of new business model on the quantity of new business demanded, the free asset allocation parameter and the insolvency risk.

3.4.2 Sensitivity of Market New Business Models

3.4.2.1 How the models affect the quantity of new business demanded

Tables 3.3 below shows the mean, standard deviation, coefficient of variation and median of the quantity demanded for market new business models M1, M2, M3 and M4 with an initial solvency ratio ($SR_m(0) = 1.50$) and free assets allocation proportion ($\rho_m = 0.4$) under NBS-1 over a 20-year projection period. The models differ mainly in respect of the quantity of new business demanded being produced, and therefore the true sensitivity of the models may only be known with certainty if a company has sufficient free capital to write any quantity of new business demanded. In practice, companies have limited amount of free capital (or constraints) to write new business.

Table 3.3: Quantity of new business demanded under NBS_1 $SR(0)=1.50$

Year	Model M1				Model M2			
t	μ_{NM}	Med_{NM}	σ_{NM}	CV_{NM}	μ_{NM}	Med_{NM}	σ_{NM}	CV_{NM}
1	11426.00	11426	0.00	0.0000	11609.00	11609	0.00	0.0000
2	11155.96	11174	311.80	0.0279	11504.73	11503	497.81	0.0433
5	10901.09	10920	391.12	0.0359	11789.18	11760	786.30	0.0667
10	11026.87	11066	556.46	0.0505	12897.12	12859	1190.32	0.0923
15	11011.98	11067	589.17	0.0535	14016.07	13946	1539.53	0.1098
20	10719.51	10783	782.76	0.0730	14765.64	14776	1970.73	0.1335
Year	Model M3				Model M4			
t	μ_{NM}	Med_{NM}	σ_{NM}	CV_{NM}	μ_{NM}	Med_{NM}	σ_{NM}	CV_{NM}
1	10230.00	10230	0.00	0.0000	10414.00	10414	0.00	0.0000
2	9956.86	9972	315.27	0.0317	10288.83	10248	981.05	0.0954
5	9731.75	9742	395.00	0.0406	10593.62	10494	1753.24	0.1655
10	9881.75	9897	565.21	0.0572	11720.85	11392	2765.84	0.2360
15	9885.04	9940	602.06	0.0609	12948.54	12289	3852.08	0.2975
20	9638.84	9694	775.92	0.0805	13778.34	12979	4702.46	0.3413

Symbols:
NM = Quantity of new business demanded
μ = Mean
σ = Standard deviation
CV = Coefficient of variation
NBS-1 = New business strategy 1
Med = Median

Comparing only the quantity of new business demanded in the table may give an insight to the differences between the models. The table shows that the models generate different quantities of new business demanded due to the underlying nature of the models. Models with both price and income effects (models M2 and M4) generate a higher quantity of new business demanded than models with only price effect (models M1 and M3).

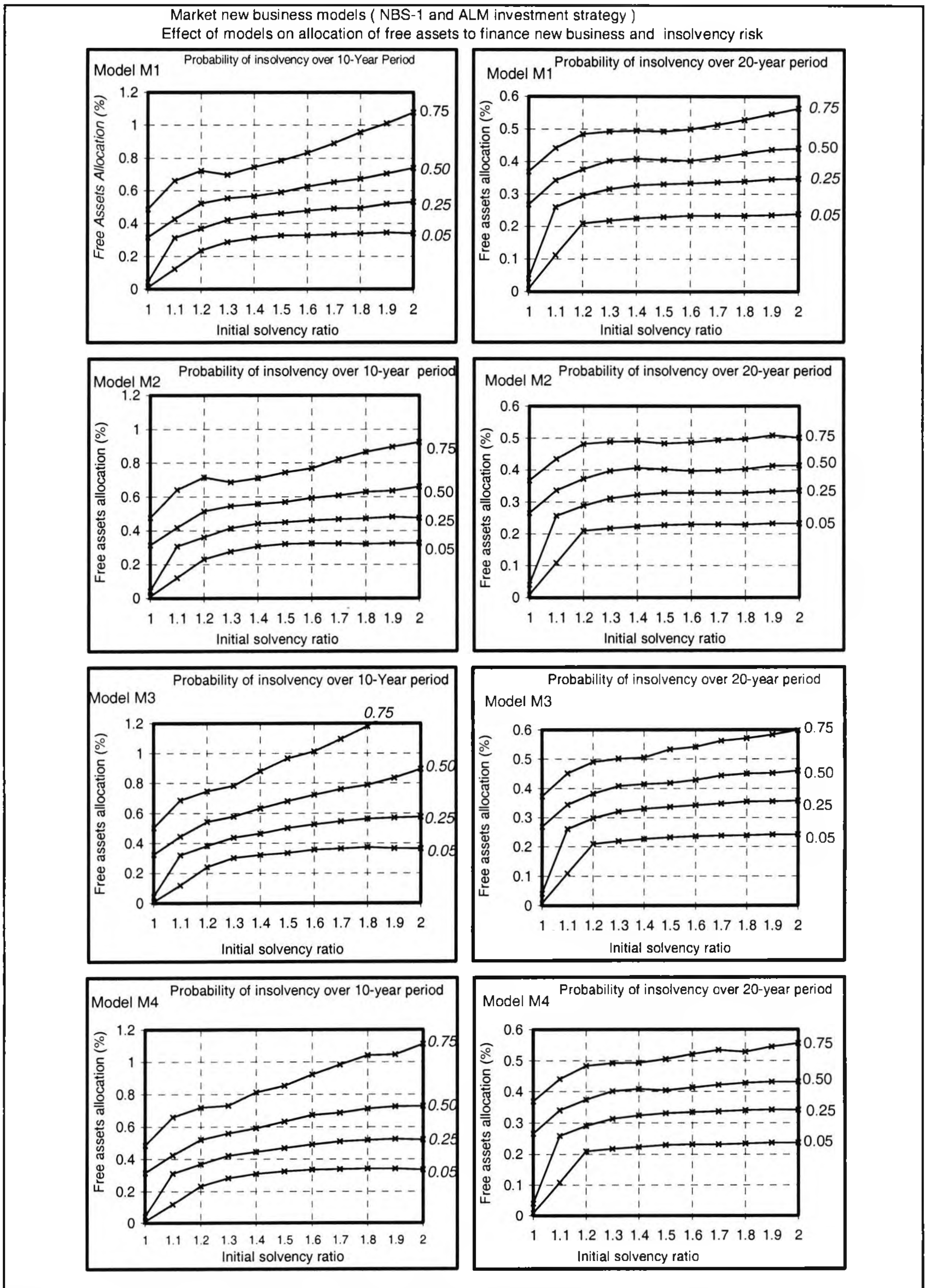


Figure 3.4: Effect of market new business models on insolvency risk.

Furthermore, models with exponential price elasticity of demand function (models M1 and M2) generate a higher quantity of new business demanded than models with constant price elasticity of demand function (models M3 and M4) respectively. Model M4 shows a much higher variability in quantity demanded than the other models, whilst model M1 has the least variability in quantity demanded in year 20. This is possibly due to the second order effect of the free assets allocation proportion on the quantity of new business demanded.

In this investigation, the free assets available and allocated (measured in terms of solvency ratio and free assets allocation proportion) have a direct (see equation (9)) and indirect effect on the premium respectively. The 40% allocation of free assets to finance new business creates more funds for improvement in a company's solvency position relative to a 100% allocation. This may in turn leads to a reduction in premium to attract more new business under new business strategy 1. The improvement in solvency may be through investments in risky assets which yield high returns. Thus, If the quantities of new business demanded were actually written, then the models that generate a higher new business volume are likely to be more sensitive to an insolvency risk (as this will drain the free assets) than the models that produce a smaller quantity of new business demanded.

3.4.2.2 How the new business models affect the insolvency risk

Figure 3.4 and figure 3C.1 in appendix 3 show the effect of the choice of the new business model on free asset allocation parameter ρ_m and the insolvency risk (probability of insolvency and mean shortfall risk respectively) under the new business strategy 1. As model M3 generates a much lower quantity of new business demanded than the other models (see table 3.3 above), its maximum volume of actual new business written is limited relative to other models even if there are enough free assets to write more new business. Thus, if the free assets allocation proportion for new business plan is high, it is unlikely for all allocated free assets to be used up, and this may enable a company to hold back free assets for investments (e.g. in equities). Therefore, for a given combination of company's initial solvency ratio and a constant proportion of free assets allocation to finance new business, figure 3.4 shows that model M3 will produce a lower insolvency risk (probability of insolvency) than the other models. This feature is reflected in the contour lines for M3 being higher than those of other models for a given initial solvency ratio, except for the very lower initial solvency ratios. This holds for both of the projection periods investigated and also for the mean shortfall risk shown in figure 3C.1.

Furthermore, model M2 produces a higher risk than model M4 which in turn produces a higher insolvency risk than model M1. The contour lines for model M2 are also lower than those of other models for a given initial solvency ratio. The ranking of riskiness of the model results from the different quantities of new business written under the models for a given combination of a company's initial solvency ratio and a constant proportion of free assets allocation to finance new business. Thus, the ranking is $M2 > M4 > M1 > M3$.

Considering the mean shortfall risk in figure 3C.1 in appendix 3 also leads to the same ranking of riskiness of the models except that the contour lines in figure 3C.1 are higher than the corresponding contour lines in figure 3.4.

3.4.2.3 The sensitivity of new business models

The sensitivity of one model relative to another can be investigated by using the concept of hypothesis testing on the difference between two model probabilities of insolvency over a projection period or by determining the 95% confidence interval for the difference between the models' true probabilities of insolvency over a projection period, for a given combination of initial solvency ratio and free assets allocation proportion. We expect that the sensitivity of the models may increase with an increasing free assets allocation proportion, as more new business volume will be written out of the quantity of new business demanded.

The description of the true (unknown) probability of insolvency given below follows that of Hardy (1993). We consider the probability that, statutory insolvency is deemed to occur when the solvency ratio is less than one (e.g. $SR_m(t) < 1$) at least once in any simulation over a t-year projection period. If the number of simulations is N, then at time t, the number of simulations, $X(t)$, for which the solvency ratio falls below 1.0 is a binomially distributed (since each simulation either does or does not do so) random variable. And $X(t) \sim B(N,\ p(t))$ where $p(t)$ is the true (unknown) probability of insolvency over time t on a given asset /liability model.

Then, it follows that:

$$\Rightarrow E\left[\frac{X(t)}{N}\right] = p(t)$$

$$V\left[\frac{X(t)}{N}\right] = \frac{p(t).[1-p(t)]}{N}$$

If we run N (e.g. 1000) simulations, and find that over time t the observed value of $X(t)$ is $x(t)$, so that the observed probability of insolvency over time t for the given asset / liability model is:

$$\hat{p}(t) = \frac{x(t)}{N}, \tag{26}$$

The absolute value observed for $\hat{p}(t)$ is likely to be highly dependent on the stochastic investment model used. However, since exactly the same 1000 investment scenarios will be used for each of the models, the effects of sampling error will be much reduced, as in Hardy (1993).

We can investigate the sensitivity of any two models by carrying out hypothesis testing of the difference in the sample probabilities of insolvency from any two models (say model 1 and model 2 with sample probabilities of insolvency denoted by $\hat{p}_1(t)$ *and* $\hat{p}_2(t)$ respectively). The procedure for hypothesis testing of the difference between two model probabilities of insolvency is as follow:

Step 1: The hypothesis

$H_0 : p_1(t) = p_2(t)$ (The null hypothesis is that there is no difference between the true probabilities of insolvency $p_1(t)$ *and* $p_2(t)$ over a period of t years for model 1 and model 2 respectively which are unknown)

$H_1 : p_1(t) \neq p_2(t)$ (The alternative hypothesis is that there is a difference between the true probabilities of insolvency $p_1(t)$ *and* $p_2(t)$ over a period of t years for model 1 and model 2 respectively which are unknown).

Step 2: Significance level and Critical Value

We set the significance level to 0.05 and corresponding critical value is $Z^* = 1.96$

Step 3: The test statistic: The distribution of $\hat{p}_1(t) - \hat{p}_2(t)$ is

$$\hat{p}_1(t) - \hat{p}_2(t) \sim N\left(p_1(t) - p_2(t),\ \frac{p_1(t)\left(1 - p_1(t)\right)}{N_1} + \frac{p_2(t)\left(1 - p_2(t)\right)}{N_2} \right),$$

so the test statistic is $z = \dfrac{\left(\hat{p}_1(t) - \hat{p}_2(t)\right) - \left(p_1(t) - p_2(t)\right)}{\sqrt{\dfrac{p_1(t)\left(1 - p_1(t)\right)}{N_1} + \dfrac{p_2(t)\left(1 - p_2(t)\right)}{N_2}}}$ (27)

However, $p_1(t)$ and $p_2(t)$ (the true probabilities of insolvency) in the denominator of equation (27) have to be replaced by an estimate of common value (denoted by $p(t)$) from the samples probabilities of insolvency.

Thus, $p(t) = \dfrac{N_1 \times \hat{p}_1(t) + N_2 \times \hat{p}_2(t)}{N_1 + N_2}$ and $p_1(t) = p_2(t) = p(t)$

We cannot simply replace $p_1(t)$ *and* $p_2(t)$ by $\hat{p}_1(t)$ *and* $\hat{p}_2(t)$ respectively because these (model probabilities of insolvency) are unequal, and to do so would contradict the null hypothesis that the true probabilities of insolvency are equal (Barrow, 2001).

As we run 1000 simulations, $N_1 = N_2 = 1000$, equations (26) and (27) can be used to obtain the probability of insolvency, denoted by $\hat{p}(20)$, for any particular model over a 20-year period, the p-value and z-value for testing the difference in probabilities of insolvency between two market new business models for a given combination of initial solvency ratio (e.g. $SR_m(0)$ = 1.50) and free asset allocation parameter (e.g. ρ_m = 0.4 or 0.6), and results are shown in tables 3.4 and 3.5.

Table 3.4: Testing on the differences between market new business models

Probability of insolvency over 20 years					*Difference of two probabilities of insolvency*			
Models	*M1*	*M2*	*M3*	*M4*	*between M1 and M2*		*between M1 and M3*	
ρ_m	$\hat{p}(20)$	$\hat{p}(20)$	$\hat{p}(20)$	$\hat{p}(20)$	*P-Value*	*Z-Value*	*P-Value*	*Z-Value*
0.4	*0.488*	*0.496*	*0.452*	*0.49*	*0.7205*	*-0.358*	*0.1068*	*1.613*
0.6	*0.898*	*0.915*	*0.841*	*0.876*	*0.1917*	*-1.306*	*0.0002*	*3.784*

Note:

ρ_m = *Propoortion of free assets allocated to finance new business*

$\hat{p}(20)$ = *Probability of insolvency over 20-year period from a given new businness model*

Table 3.5: Testing on the differences between market new business models

Probability of insolvency over 20 years					*Difference of two probabilities of insolvency*			
Models	*M1*	*M2*	*M3*	*M4*	*between M2 and M3*		*between M1 and M4*	
ρ_m	$\hat{p}(20)$	$\hat{p}(20)$	$\hat{p}(20)$	$\hat{p}(20)$	*P-Value*	*Z-Value*	*P-Value*	*Z-Value*
0.4	*0.488*	*0.496*	*0.452*	*0.49*	*0.0488*	*1.970*	*0.9287*	*-0.089*
0.6	*0.898*	*0.915*	*0.841*	*0.876*	*0.0000*	*5.056*	*0.1202*	*1.554*

Step 4: conclusion

Considering models M1 and M2, table 3.4 shows that for both free asset allocation proportions (ρ_m = 0.4 *and* $\rho_m = 0.6$), $Z-Value < Z^*$ (i.e. the observed Z-Values in the tables are less than the critical value of 1.96 at 5% level of significance). Thus, there is not sufficient evidence to suggest a significant difference between these models' probabilities of insolvency. An increase in the free assets allocation proportion also increases the actual new business volume written in both models thereby reducing the free assets available for investing in high income yielding assets. As the models generate a high quantity of new business demanded, any free assets allocated to finance new business are being used up.

Similarly, considering either models M1 and M3 or models M2 and M3, tables 3.4 and 3.5 show that there is sufficient evidence to suggest a significant difference in the probabilities of insolvency from these models only when the free assets allocation proportion to finance new business is high (e.g. $\rho_m = 0.6$), as $Z-Value > Z^*$ (i.e. the observed Z-Values in the tables are greater than the critical value of 1.96 at 5% level of significance). This is because not all of the allocated free assets under model M3 (If the allocation proportion is high) are used up in writing new business due to the small quantity of new business demanded being generated.

Tables 3.4 and 3.5 show that, for models M1 and M4, $Z-Value < Z^*$ for both free asset allocation proportions (i.e. $\rho_m = 0.4$ *and* $\rho_m = 0.6$). Thus, there is not sufficient evidence to suggest a difference between these models' probabilities of insolvency for both free allocation proportions, as both models produce a fairly high quantity of new business over the long term.

We also expect that testing of the difference in probabilities of insolvency between models M2 and M4 (results are not shown in the above tables) is likely to produce similar results as for models M1 and M3, since the underlying difference between Models M2 and M4 is the different demand functions used.

3.4.2.4 Conclusion of sensitivity analysis for market models

The above analysis shows that the model sensitivity increases with the free assets allocation proportion to finance new business plans. The model sensitivity analysis reveals that model M3 appears not to reflect adequately the price sensitivity nature of the term assurance market as it produces the lowest quantity of new business demanded. Model M2 also creates more insolvency problems in the market than models M1 and M4 because of the high quantity of new business generated and so may not be very appropriate to represent the market.

On the other hand, the analysis also shows that there is no significant difference between models M1 and M4. Nonetheless, Model M1 is more likely to represent the term assurance market than model M4, particularly if the price effect which is a prominent factor in this market is taken into account, and also due to fact that the exponential demand function (in model M1) is likely to be more sensitive to the probability of insolvency than the constant demand function (in model M4). Thus, when considering the interaction between the market and the company, we assume that new business Model M1 serves as the benchmark for representing the term assurance market.

3.4.3 Sensitivity of Market Model Parameters

In this section, we carry out a successive and a simultaneous sensitivity analysis of the probability of insolvency over a given projection period, with the aim of identifying the most influential factors that determine a company's solvency position. We use the elasticity formula, equation (25) in section 3.4.1, for the successive sensitivity analysis of probability of insolvency to model parameter. Whilst in the simultaneous sensitivity analysis, we compare the probability of insolvency produced by the base parameter values (reference scenario) with the probability of insolvency produced by changing some key parameter values at the same time. In carrying out the sensitivity analysis of the market model parameters, we consider the market new business model M2 (exponential demand function with real wage inflation effect) instead of new business model M1, as the former involves all the market parameters.

Parameter values for market new business model M2

$EK_m = 3.0$, $a = 0.98$, $b = 6.0$, $d = 2.0$, $l_m(\pm ve) = 0.30$, $SR_m(0) = 1.50$, $\rho_m = 0.4$

3.4.3.1 Successive sensitivity analysis of probability of insolvency

In table 3.6, the free assets allocation proportion to finance new business, ρ_m , is clearly the parameter to which the probability of insolvency is most sensitive, particularly over a longer term. The elasticity of free asset allocation proportion means that, for example, a variation of +25% in the free asset allocation proportion causes a variation of +55.75% (= 2.2 × 25%) in the probability of insolvency over a 20-year projection period. This parameter is the most important factor that determines a life company's level of solvency in the future, as it enables the company to meet its strategic aim to grow in size at a sustainable rate from its internal resources. Therefore, a substantial increase in the parameter may lead to writing an inappropriate volume of new business that will cause a drain in free assets, and thereby creating constraints on investment freedom and meeting statutory solvency requirements.

Table 3.6: Sensitivity of probability of insolvency to market parameters

Probbility of insolvency over projection period and elasticity	*P(10)*	*Elasticity*	*P(20)*	*Elasticity*
Reference Scenario (using base values for Model M2)	***0.167***		***0.538***	
+ 33% change in expo. demand function parameter, $EK_m = 4.0$	*0.168*	*0.018*	*0.537*	*-0.006*
- 33% change in expo. demand function parameter, $EK_m = 2.0$	*0.164*	*0.054*	*0.535*	*0.017*
+ 33% change in measure of premium response, $l_m(+ve) = 0.4$	*0.185*	*0.323*	*0.571*	*0.184*
- 33% change in measure of premium response, $l_m(+ve) = 0.2$	*0.152*	*0.269*	*0.505*	*0.091*
+1% change in constant term (intercept) in eq. (2), $a = 0.99$	*0.168*	*0.587*	*0.537*	*-0.182*
-1% change in constant term (intercept) in eq. (2), $a = 0.97$	*0.166*	*0.587*	*0.538*	*0.000*
+33% change in slope (rate of income) term in eq. (2), $b = 8$	*0.169*	*0.036*	*0.538*	*0.000*
-33% change in slope (rate of income) term in eq. (2), $b = 4$	*0.167*	*0.000*	*0.537*	*0.006*
+50% change in slope (rate of income) term in eq. (14), $d = 3$	*0.167*	*0.000*	*0.542*	*-0.004*
-50% change in slope (rate of income) term in eq. (14), $d = 1$	*0.171*	*-0.048*	*0.536*	*0.007*
+20% change in initial solvency ratio, $SR_m(0) = 1.8$	*0.250*	*2.485*	*0.654*	*1.078*
- 20% change in initial solvency ratio, $SR_m(0) = 1.2$	*0.355*	*-5.629*	*0.628*	*-0.836*
+25% change in free asset allocation proportion, $\rho_m = 0..5$	*0.438*	*6.491*	*0.838*	*2.230*
-25% change in free asset allocation proportion, $\rho_m = 0..3$	*0.039*	*3.066*	*0.198*	*2.528*

Note: *P(t) = Probability of insolvency over t-year period*

The table also reveals that the initial solvency ratio, $SR_m(0)$ (a measure of initial free capital) is considered the second most elastic parameter. For example, a variation of +20% in the initial solvency ratio causes a variation of +21.56% (= 1.078 $\times$ 20%) in the probability of insolvency over a 20-year projection period. Under the new business strategy 1, an increase in this parameter will lead to an increase in both quantity of new business demanded and written, particularly over the short term. The parameter also determines the long-term survival of a life insurance operation if an appropriate decision is taken on the asset allocation strategy and new business.

In considering the absolute values of the elasticity, the table above shows that the sensitivity of the probability of insolvency to all parameters reduces over time. This may be due to the effect of writing more new business volume in the short term than in the longer term under the new business strategy 1 under consideration.

The successive sensitivity analysis shows that the market model parameters other than the initial solvency ratio and the free assets allocation proportion to finance new business have a relatively insensitive effect on the probability of insolvency. This is because the quantity of new business demanded increases with these other parameters (except parameter d as discussed below), but only a proportion of the quantity demanded may actually be written due to the free assets allocation strategy being used to finance new business ($\rho_m = 0.4$). Therefore, the sensitivity of the probability of insolvency to these other parameters is constrained by the two key parameters (the market initial solvency ratio $SR_m(0)$ and free assets allocation proportion ρ_m).

The true sensitivity of the probability of insolvency to these other parameters may be revealed in a simultaneous sensitivity analysis where each of these other parameter values will change along with the initial solvency ratio and/or the free assets allocation proportion. However, in particular the parameter *d* (representing the rate of change in policy lapses with respect to disposable income in equation (14)) is highly inelastic relative to other parameters. For example, a variation of +50% in parameter *d* causes a variation of -0.2% (= -0.004 × 50%) in the probability of insolvency over a 20-year projection period. This indicates that the effect of lapses on profitability of an insurance company transacting temporary insurance business may be broadly neutral.

As discussed in section 3.4.2.2, the new business model M4 produces a slightly lower probability of insolvency than new business model M2 because the latter model generates more new business than the former model due to the underlying nature of the models (see table 3.3 in section 3.4.2.1). Thus, if a successive sensitivity analysis of probability of insolvency to model M4 parameters is carried out (results are not shown in detail), it is likely to reveal similar results (in terms of elasticity) as for the analysis with model M2 in table 3.6 since they differ only by nature of their demand functions.

3.4.3.2 Simultaneous sensitivity analysis of probability of insolvency

Sensitivity to changes in the parameters (Ek_m , ρ_m , *and* $SR_m(0)$)

The parameter Ek_m in equation (4) indicates the extent of the price effect on the quantity of new business demanded in the exponential demand function, and it affects the quantity of new business demanded directly. The quantity of new business demanded increases with the parameter and vice versa.

Tables 3.7 shows that for +25% variation in the free assets allocation proportion ρ_m , a change in the parameter Ek_m only produces a greater percentage change in the probability of insolvency over a long term if the initial solvency ratio is high (e.g. $SR_m(0) = 1.80$). Thus, a change in the parameter Ek_m (leading to a change in the quantity of new business demanded) will only have an effect on a company's solvency if it has sufficient free capital available and allocated a high proportion of free assets to finance new business.

Table 3.7: Sensitivity of probability of insolvency to parameters

Initial solvency ratio		$SR_m(0)$	*1.2*	*1.5*	*1.8*
Parameters	ρ_m	EK_m	*P(20)*	*P(20)*	*P(20)*
Base Values	***0.4***	***3***	***0.628***	***0.538***	***0.654***
% of Base Value	*- 25% (0.3)*	*- 33% (2)*	*0.31*	*0.197*	*0.194*
$\%\ \Delta\ in\ P(20)$			*-50.64*	*-63.38*	*-70.34*
% of Base Value	*+ 25% (0.5)*	*- 33% (2)*	*0.801*	*0.838*	*0.855*
$\%\ \Delta\ in\ P(20)$			*27.55*	*55.76*	*30.73*
% of Base Value	*- 25% (0.3)*	*+ 33% (4)*	*0.313*	*0.2*	*0.198*
$\%\ \Delta\ in\ P(20)$			*-50.16*	*-62.83*	*-69.72*
% of Base Value	*+ 25% (0.5)*	*+ 33% (4)*	*0.802*	*0.838*	*0.92*
$\%\ \Delta\ in\ P(20)$			*27.71*	*55.76*	*40.67*

Sensitivity to changes in the parameters in equation (2) a and b

Similarly to parameter Ek_m , the high sensitivity of the quantity of new business demanded to parameter a (the constant term representing the basic demand that is not linked to the real wage inflation) and b (the rate of change in new business demanded with respect to disposable income) is not fully reflected in the probability of insolvency if the free assets allocation proportion to finance new business only varies between 0.30 and 0.50 for a given initial solvency ratio. This is because there is little or no change in the quantity of new business written, as the parameters are changed, due to the low free assets allocation proportions.

Sensitivity to changes in the parameters ($l_m(+ve)$, ρ_m , and $SR_m(0)$)

Table 3.8: Sensitivity of probability of insolvency to parameters

Initial solvency ratio		$SR_m(0)$	*1.2*	*1.5*	*1.8*
Parameters	ρ_m	$l_m(+ve)$	*P(20)*	*P(20)*	*P(20)*
Base Values	***0.4***	***0.3***	***0.628***	***0.538***	***0.654***
% of Base Value	*- 25% (0.3)*	*- 33% (0.2)*	*0.282*	*0.158*	*0.165*
$\%\ \Delta\ in\ \ P(20)$			*-55.10*	*-70.63*	*-74.77*
% of Base Value	*+ 25% (0.5)*	*- 33% (0.2)*	*0.79*	*0.824*	*0.847*
$\%\ \Delta\ in\ \ P(20)$			*25.80*	*53.16*	*29.51*
% of Base Value	*- 25% (0.3)*	*+ 33% (0.4)*	*0.362*	*0.239*	*0.243*
$\%\ \Delta\ in\ \ P(20)$			*-42.36*	*-55.58*	*-62.84*
% of Base Value	*+ 25% (0.5)*	*+ 33% (0.4)*	*0.828*	*0.856*	*0.921*
$\%\ \Delta\ in\ \ P(20)$			*31.85*	*59.11*	*40.83*

The parameter values $l_m(\pm ve)$ in equation (11) are the measures of premium response to changes in a company's solvency ratio that affect the premium levels (i.e. premium setting structure) for different new business strategies. The parameter $l_m > 0$ relates to new business strategies 1 and 2 (denoted by NBS-1 and NBS-2 respectively) while the parameter $l_m < 0$ relates to new business strategies 3 and 4 (denoted by NBS-3 and NBS-4 respectively).

This parameter $l_m(+ve)$ under NBS-1 affects directly both the quantity demanded as well as the company's cash flows (e.g. premium and expenses). Table 3.8 reveals that the probability of insolvency is sensitive to this parameter for a given free assets allocation proportion and/or the initial solvency ratio. This is because the parameter $l_m(+ve)$ affects the premium income that also determines the profitability of a life insurance operation.

3.4.3.3 Conclusion of sensitivity analysis for market model parameters

The analysis shows that the parameter $l_m(+ve)$ is the next important factor other than the two key parameters (the free assets allocation proportion ρ_m and the initial solvency ratio $SR_m(0)$) affecting a company's profitability and solvency.

3.4.4 New Business Decision making

A collective decision on the choice of the important parameter values discussed above would determine the effectiveness of any new business plans of an insurer to meet its solvency requirements. This will enable a company to ensure that there are free assets available to support a volume of new business being attracted by charging an appropriate premium. For a given initial solvency ratio $SR_m(0)$, a company needs to decide on the combination of free assets allocation proportion ρ_m and the measure of premium response to changes in its solvency $l_m(\pm ve)$ under the chosen new business strategy in order to produce a given level of insolvency risk.

Table 3.9: Sensitivity of probability of insolvency to parameter $l_m(\pm ve)$ $SR_m(0)=1.50$

Probability of insolvency over 20-year projection period						*(NDF = 1.25 for NBS-3 and NBS-4)*			
New business strategy		*NBS-1*	$l_m(+ve)$	*NBS-2*	$l_m(-ve)$	*NBS-3*	$l_m(+ve)$	*NBS-4*	$l_m(-ve)$
Allocation proportion		$\rho_m=0.4$	$\rho_m=0.5$	$\rho_m=0.4$	$\rho_m=0.5$	$\rho_m=0.4$	$\rho_m=0.5$	$\rho_m=0.4$	$\rho_m=0.5$
Measure of premium response to solvency $l_m(\pm ve)$	*0*	*0.363*	*0.636*	*0.363*	*0.636*	*0.363*	*0.636*	*0.363*	*0.636*
	± 0.10	*0.432*	*0.734*	*0.248*	*0.51*	*0.403*	*0.685*	*0.296*	*0.583*
	± 0.20	*0.497*	*0.808*	*0.144*	*0.367*	*0.432*	*0.732*	*0.236*	*0.511*
	± 0.30	*0.534*	*0.834*	*0.064*	*0.205*	*0.459*	*0.768*	*0.169*	*0.42*
	± 0.40	*0.569*	*0.854*	*0.026*	*0.096*	*0.487*	*0.793*	*0.114*	*0.361*
	± 0.50	*0.591*	*0.865*	*0.009*	*0.035*	*0.501*	*0.814*	*0.073*	*0.266*

A prime responsibility of a life insurance company's actuary is to confirm that the premium rates being charged for new business are appropriate. Thus, the premium rates should be sufficient to enable the company in due course to meet its emerging liabilities, having regard to all aspects affecting the financial position of the company such as the new business expansion plans, investment policy and the extent of free assets. GN1 states that "should premium rates be such that business is expected to be written on terms which require support from the free assets,..the insurer's ability to continue to write business on such terms [needs to be assessed]". Therefore, the decision on premium rate adequacy may not only depend on the choice of new business strategy but also on the level of the free assets available to support the expected new business to be written.

We consider the new business model M1 (being taken to represent the temporary insurance market) and the probability of insolvency as a risk measure to investigate the new business decision making process. Table 3.9 shows that for NBS-1 and NBS-3, as the parameter $l_m(+ve)$ increases the probability of insolvency increases whilst as the parameter $l_m(-ve)$ decreases the probability of insolvency decreases under NBS-2 and NBS-4.

Table 3.9 reveals that NBS-1 and NBS-3 are the opposites of NBS-2 and NBS-4 respectively. When $l_m(\ ve) = 0$ the profit tested office premium (denoted by $\overline{OP_m}$) is charged independent of any new business strategy, as it is not affected by the company's solvency position, and therefore this will produce the same probability of insolvency for all new business strategies.

Figure 3.5 (a graphical presentation of table 3.9) is a tool that will enable a company to determine the level of free assets needed to support a new business volume that may be prudently accepted at a given level of premium under any new business strategy in order to produce a particular level of insolvency risk.

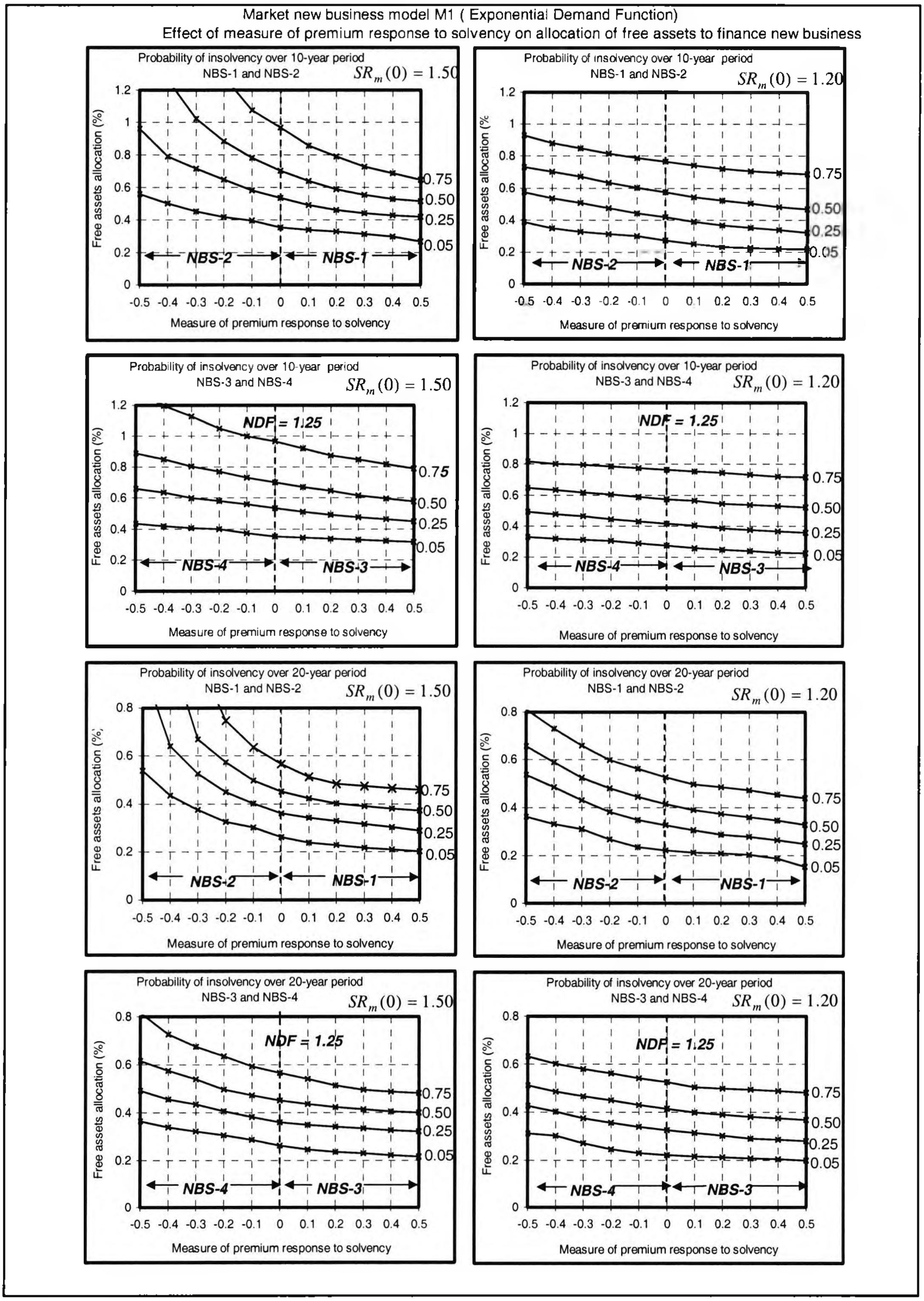


Figure 3.5: Effect of measure of premium response to solvency on free assets allocation proportion and probability of insolvency.

Figure 3.5 shows that, as the measure of premium response to change in a company's solvency increases, the company needs to decrease its allocation proportion in order to maintain the same probability of insolvency. As the parameter $l_m(\pm ve)$ increases in size (or becoming more positive) it leads to a reduction in the office premium needed to attract more business under any new business strategy. The more new business that is being written the higher is the drain on the free assets. Therefore, in order to maintain the same level of insolvency risk the company needs to reduce its allocation proportion.

The company's initial solvency ratio also determines the choice of combination of measure of premium response to solvency and the free assets allocation proportion to produce a given level of insolvency risk. The Initial solvency ratios 1.50 and 1.20 are also compared in figure 3.5, and the contour lines for an Initial solvency ratio of 1.50 lie above those of 1.20. This is true for any new business strategy and projection period considered. As expected, the contour lines for NBS-3 and NBS-4 are above and below the contours for NBS-1 and NBS-2 respectively. This reflects the riskiness of the new business strategies as discussed later.

Thus, the above discussion reveals that the company's new business plans (taken as the decision concerning the choice of premium setting structure and/or free assets allocation proportion to finance new business) of a company depend entirely on its financial strength, particularly the initial free assets if the company is self-financing.

3.4.5 Sensitivity of Company's New Business Models

The company being an integral part of the market, its volume of new business bears a functional relationship with the market new business volume generated by model 1. Thus the sensitivity analysis of the company's insolvency risk is affected by changes in the company's new business models.

3.4.5.1 How the new business models affect the insolvency risk

Figures 3.6 and 3C.2 in appendix 3 show the effect of the choice of new business model on free assets allocation parameter ρ_c and the insolvency risk (probability of insolvency and mean shortfall risk respectively). Figure 3.6 shows that there is a small difference between the model probabilities of insolvency for a given combination of initial solvency ratio and free assets allocation proportion. This is because the quantity of new business demanded under each model is only a small proportion of the market new business volume, and also the difference in quantity of new business written between the models is also very small. However, the contour lines show that models with constant price elasticity of demand function (new business models C3 and C4) prodsuce a slightly lower probability of insolvency than models with exponential price elasticity of demand function (new business models C1 and C2) only when the free assets allocation proportion is above 0.40.

A comparison of new business models C1 and C2 in figure 3C.2 shows that model C2 (with exponential demand function and regulator's control on new business) produces a slightly lower mean shortfall risk over a 10-year projection period than model C1 (with exponential demand function and management decision on new business). This is true particularly at a low initial solvency ratio (e.g. below 1.4) and a high free assets allocation proportion (e.g. above 0.40). This means that the regulator's control has more impact on a company with low free assets which is still allocating a high proportion of free assets to finance new business.

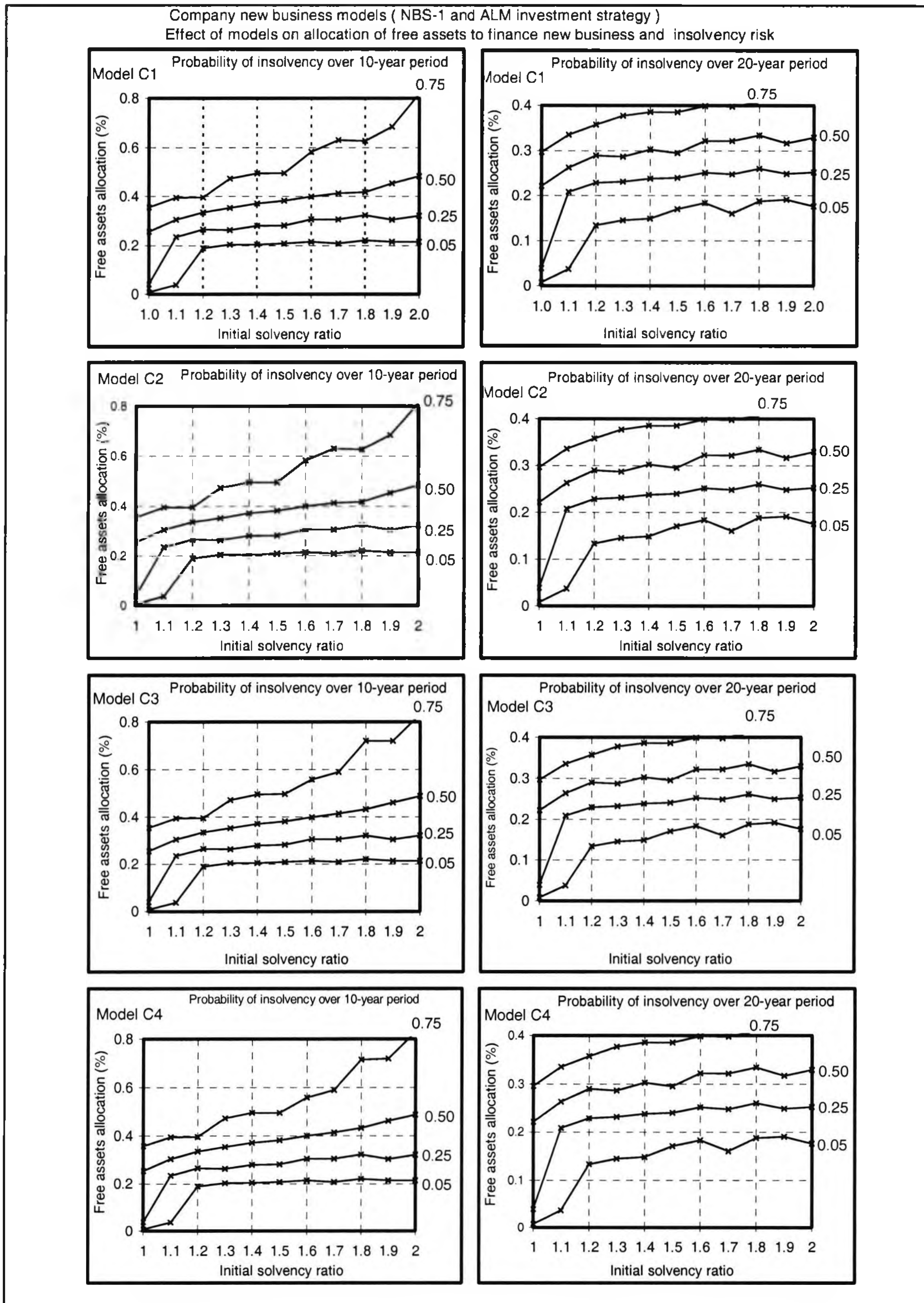


Figure 3.6: Effect of company new business models on insolvency risk

As the size of the company (in terms of free assets) is small relative to the market, a constant free asset allocation proportion of say 20% ($\rho_c = 0.2$) to finance new business over a 20-year projection period may produce more than a 5% level of insolvency risk. In contrast, a similar free assets allocation proportion in the market may produce less than a 5% level of insolvency risk, as revealed in comparing figures 3.4 and 3.6, except for the very low initial solvency ratios.

3.4.5.2 The sensitivity of new business models

Table 3.10: Testing on the differences between company new business models

Probability of insolvency over 20 years					*Difference of two probabilities of insolvency*			
Models	*C1*	*C2*	*C3*	*C4*	*between C1 and C2*		*between C1 and C3*	
ρ_c	$\hat{p}(20)$	$\hat{p}(20)$	$\hat{p}(20)$	$\hat{p}(20)$	*P-Value*	*Z-Value*	*P-Value*	*Z-Value*
0.4	0.79	0.79	0.789	0.79	*1.0000*	*0.000*	*0.9563*	*0.055*
0.6	0.973	0.963	0.967	0.962	*0.2039*	*1.270*	*0.4316*	*0.786*

Note:

ρ_c = *Propoortion of free assets allocated to finance new business*

$\hat{p}(20)$ = *Probability of insolvency over 20-year period from a given new businness model*

Similar to the market new business models, the sensitivity of the company new business models can also be investigated by hypothesis testing of the difference in probabilities of insolvency between any two models. Thus, tables 3.10 and 3.11 are produced following the procedure stated in section 3.4.2.3 using equations (26) and (27).

Table 3.11: Testing on the differences between company new business models

Probability of insolvency over 20 years					*Difference of two probabilities of insolvency*			
Models	*C1*	*C2*	*C3*	*C4*	*between C2 and C3*		*between C1 and C4*	
ρ_c	$\hat{p}(20)$	$\hat{p}(20)$	$\hat{p}(20)$	$\hat{p}(20)$	*P-Value*	*Z-Value*	*P-Value*	*Z-Value*
0.4	0.79	0.79	0.789	0.79	*0.9563*	*0.055*	*1.0000*	*0.000*
0.6	0.973	0.963	0.967	0.962	*0.6265*	*0.487*	*0.1654*	*1.387*

The tables 3.10 and 3.11 show that, for any pair of models (e.g. C1 and C2) and for both free asset allocation proportions ($\rho_m = 0.4 \; and \; \rho_m = 0.6$) with an initial solvency ratio of 1.50, $Z-Value < Z^*$ (the observed Z-Values in the tables are less than the critical value of 1.96 at 5% level of significance). Thus, there is not sufficient evidence to suggest a significant difference between any two new business models' probabilities of insolvency over a 20-year period. The explanation in section 3.4.5.1 for the small difference between the model probabilities of insolvency for a given combination of initial solvency ratio and free assets allocation proportion is also applicable here.

3.4.5.3 Conclusion of sensitivity analysis for company models

For simplicity, having chosen the market new business model M1 (with exponential demand function) for representing the term assurance market, we consider company new business model C1 (exponential demand function and management decision) in the subsequent analysis so that the demand function (exponential price elasticity of demand) to be adopted by the company model and the market model are the same. This choice appears reasonable, since the sensitivity analysis of company new business models shows that there is not significant difference between any two new business model probabilities of insolvency over a 20-year period.

3.4.6 Sensitivity of Company Model Parameters

We will consider company new business model C1 (exponential price elasticity of demand function) in the sensitivity analysis of probability of insolvency (or relative insolvency) to company model parameters.

<u>Parameter values for market new business model M1</u>

$EK_m = 3.0$, $d = 2.0$, $l_m(\pm ve) = 0.30$, $SR_m(0) = 1.50$, $\rho_m = 0.4$

<u>Parameter values for company new business model C1</u>

$EK_c = 4.0$, $l_c(\pm ve) = 0.40$, $ft_c(0) = 0.25$, $NBConF = 0.40$, $SR_c(0) = 1.50$, $\rho_c = 0.4$

3.4.6.1 Successive sensitivity analysis

<u>*Sensitivity of company's probability of insolvency to company parameters*</u>

As expected from the sensitivity analysis of the market model parameters, table 3.12 shows that the free assets allocation proportion ρ_c is the most sensitive parameter over the long term. However, over a 10-year projection period, the following parameters (the free assets allocation proportion ρ_c, initial solvency ratio $SR_c(0)$ and the company's maximum proportion of quantity of new business demanded in the market in a projection year $NBConF$) are elastic. The parameter $NBConF$ is elastic because the company's quantity of new business demanded increases with the parameter and this may have a second order effect on the quantity written and hence it affects the probability of insolvency. The level of sensitivity of probability of insolvency to the other parameters that are inelastic may be revealed in a simultaneous sensitivity analysis.

Table 3.12: Sensitivity of probability of insolvency to company model parameters

Probability of insolvency over projection period and elasticity		*P(10)*	*Elasticity*	*P(20)*	*Elasticity*
Reference scenario (using base values for Model C1)		***0.546***		***0.79***	
+25% Change in free asset allocation proportion,	$\rho_c = 0.5$	*0.760*	*1.568*	*0.940*	*0.759*
-25% Change in free asset allocation proportion,	$\rho_c = 0.3$	*0.299*	*1.810*	*0.521*	*1.362*
+20% change in initial solvency ratio,	$SR_c(0) = 1.8$	*0.473*	*-0.668*	*0.738*	*-0.329*
-20% change in initial solvency ratio,	$SR_c(0) = 1.2$	*0.772*	*-2.070*	*0.903*	*-0.715*
+25% change in expo. demand function parameter	$EK_c = 5.0$	*0.546*	*0.000*	*0.790*	*0.000*
-25% change in expo. demand function parameter	$EK_c = 3.0$	*0.546*	*0.000*	*0.790*	*0.000*
+25% change in measure of premium response,	$l_c(+ve) = 0.5$	*0.552*	*0.044*	*0.782*	*-0.041*
-25% change in measure of premium response,	$l_c(+ve) = 0.3$	*0.505*	*0.300*	*0.732*	*0.294*
+20% change in initial prop. of quantity demanded,	$ft_c(0) = 0.3$	*0.473*	*-0.668*	*0.761*	*-0.184*
+20% change in initial prop. of quantity demanded,	$ft_c(0) = 0.2$	*0.568*	*-0.201*	*0.786*	*0.025*
+25% change in max. Prop. of quantity demanded,	$NBConF = 0.5$	*0.567*	*0.154*	*0.797*	*0.035*
+25% change in max.prop. of quantity demanded,	$NBConF = 0.3$	*0.365*	*1.326*	*0.719*	*0.359*

Note: *P(t) = Probability of insolvency over t-year period*

Similarly to the market new business models, if a successive sensitivity analysis of probability of insolvency to company new business model C3 parameters is carried out (results are no shown in detail), it is likely to reveal similar results (in terms of elasticity) as for the analysis with company new business model C1 in table 3.12 since they differ only by nature of their demand functions.

Sensitivity of company's probability of insolvency to market parameters

Table 3.13: Sensitivity of company's probability of insolvency to market parameters

Probability of insolvency over projection period and elasticity	*P(10)*	*Elasticity*	*P(20)*	*Elasticity*
Reference scenario (using base values for Models M1 and C1)	***0.546***		***0.79***	
+25% Change in market free asset allocation proportion, $\rho_m = 0.5$	*0.524*	*-0.161*	*0.770*	*-0.101*
-25% Change in market free asset allocation proportion, $\rho_m = 0.3$	*0.555*	*-0.066*	*0.787*	*0.015*
+20% change in market initial solvency ratio, $SR_m = 1.8$	*0.519*	*-0.247*	*0.763*	*-0.171*
-20% change in market initial solvency ratio, $SR_m = 1.2$	*0.558*	*-0.110*	*0.796*	*-0.038*
+33% change in expo. demand function parameter, $EK_m = 4.0$	*0.551*	*0.027*	*0.787*	*-0.011*
-33% change in expo. demand function parameter, $EK_m = 2.0$	*0.507*	*0.214*	*0.751*	*0.148*
+33% change in measure of premium response, $l_m(+ve) = 0.4$	*0.556*	*0.055*	*0.802*	*0.046*
-33% change in measure of premium response, $l_m(+ve) = 0.2$	*0.514*	*0.176*	*0.761*	*0.110*

Table 3.13 shows that no market model parameter can be considered elastic. This reveals that changes in a particular market parameter have no significant effect on the company's probability of insolvency.

Sensitivity of probability of relative insolvency to company parameters

Table 3.14: Sensitivity of probability of relative insolvency to company parameters

Probability of relative insolvency over a period and elasticity	$P_{v1}(20)$	*Elasticity*	$P_{v2}(20)$	*Elasticity*
Reference scenario (using base values for Model C1)	***0.446***		***0.216***	
+25% Change in free asset allocation proportion, $\rho_c = 0.5$	*0.798*	*3.157*	*0.743*	*9.759*
-25% Change in free asset allocation proportion, $\rho_c = 0.3$	*0.163*	*2.538*	*0.060*	*2.889*
+20% change in initial solvency ratio, $SR_c(0) = 1.8$	*0.388*	*-0.650*	*0.189*	*-0.625*
-20% change in initial solvency ratio, $SR_c(0) = 1.2$	*0.703*	*-2.881*	*0.597*	*-8.819*
+25% change in expo. demand function parameter, $EK_c = 5.0$	*0.448*	*0.018*	*0.221*	*0.093*
-25% change in expo. demand function parameter, $EK_c = 3.0$	*0.445*	*0.009*	*0.215*	*0.019*
+25% change in measure of premium response, $l_c(+ve) = 0.5$	*0.463*	*0.152*	*0.236*	*0.370*
-25% change in measure of premium response, $l_c(+ve) = 0.3$	*0.426*	*0.179*	*0.215*	*0.019*
+20% change in initial prop. of quantity demanded, $ft_c(0) = 0.3$	*0.410*	*-0.404*	*0.221*	*0.116*
+20% change in initial prop. of quantity demanded, $ft_c(0) = 0.2$	*0.490*	*-0.493*	*0.320*	*-2.407*
+25% change in max. Prop. of quantity demanded, $NBConF = 0.5$	*0.475*	*0.260*	*0.243*	*0.500*
+25% change in max.prop. of quantity demanded, $NBConF = 0.3$	*0.329*	*1.049*	*0.162*	*1.000*

Note: $P_{v1}(t)$ = *Probability of relative insolvency (version 1) over t-year period*
$P_{v2}(t)$ = *Probability of relative insolvency (version 2) over t-year period*

In sections 3.2.3.1 and 3.2.3.2 of this chapter the probability of relative insolvency for both versions are defined, and we consider parameters $RSV_1 = 1.15$ and $RSV_2 = 0.70$ taken arbitrarily as the base parameters for determining the probability of insolvency versions 1 and 2 respectively in carrying out the sensitivity analysis.

Table 3.14 shows that for both versions of relative insolvency the free assets allocation proportion and the initial solvency ratio are elastic. Comparing table 3.12 with table 3.14 shows that the parameters have a more (elastic) influence on the probability of relative insolvency than the probability of insolvency. This reveals that a company operating with a given combination of initial free capital and free assets allocation proportion is likely to fall out of line with the rest of the market, despite meeting its statutory solvency requirements.

Sensitivity of probability of relative insolvency to market parameters

Table 3.15 shows that changes in the key market parameters (the free asset allocation proportion and the initial solvency) have considerable effect on the probability of relative insolvency. The table reveals that, where the market and company models have the same initial solvency ratio, the company's probability of relative insolvency will reduce if it allocates a lower proportion of its available free assets to finance new business over a 20-year period than the overall market and vice versa.

Table 3.15: Sensitivity of probability of relative insolvency to market parameters

Probability of relative insolvency over a period and elasticity		$P_{v1}(20)$	*Elasticity*	$P_{v2}(20)$	*Elasticity*
Reference scenario (using base values for Models M1 and C1)		***0.446***		***0.216***	
+25% Change in market free asset allocation proportion	$\rho_m = 0.5$	*0.292*	*-1.381*	*0.141*	*-1.389*
-25% Change in market free asset allocation proportion	$\rho_m = 0.3$	*0.674*	*-2.045*	*0.673*	*-8.463*
+20% change in market initial solvency ratio	$SR_m = 1.8$	*0.565*	*1.334*	*0.540*	*7.500*
-20% change in market initial solvency ratio	$SR_m = 1.2$	*0.146*	*3.363*	*0.025*	*4.421*
+33% change in market demand function parameter	$EK_m = 4.0$	*0.450*	*0.027*	*0.228*	*-2.653*
-33% change in market demand function parameter	$EK_m = 2.0$	*0.414*	*0.215*	*0.224*	*-1.667*
+33% change in measure of premium response	$l_m(+ve) = 0.4$	*0.469*	*0.155*	*0.246*	*0.417*
-33% change in measure of premium response	$l_m(+ve) = 0.2$	*0.445*	*0.007*	*0.232*	*-0.417*

On the other hand, where both the market and company models have the same free assets allocation proportion, the company's probability of relative insolvency will also reduce if it has a higher initial solvency ratio than the overall market's initial solvency ratio and vice versa.

The other market model parameters do not have a significant impact on the company's relative insolvency because they have only a second order effect on the assets and the liabilities of a life insurance company.

Sensitivity to relative insolvency parameters

Table 3.16 shows that the probability of relative insolvency increases (decreases) as the parameter RSV_2 (parameter RSV_1) increases, and therefore the probability of relative insolvency is sensitive to these parameters. For both parameters to produce approximately the same level of probability of relative insolvency (e.g. about 0.2), we consider parameters $RSV_1 = 1.25$ and $RSV_2 = 0.70$ to be the base relative insolvency parameter values to be used in subsequent analyses.

Table 3.16: Sensitivity of probability of relative insolvency to parameters, RSV_1 RSV_2

Relative insolvency version 1 over 20 years				*Relative insolvency version 2 over 20 years*			
RSV_1	*1.05*	*1.15*	*1.25*	RSV_2	*0.60*	*0.70*	*0.80*
$P_{v1}(20)$	*0.628*	*0.446*	*0.238*	$P_{v2}(20)$	*0.020*	*0.216*	*0.771*

3.4.6.2 Simultaneous sensitivity analysis

Sensitivity of probability of insolvency to parameters (Ek_c , ρ_c , *and* $SR_c(0)$)

The parameter EK_c in equation (17) indicates the extent of the price effect on the company's proportion of quantity of new business demanded in the market. The quantity of new business demanded increases with this parameter.

Table 3.17: Sensitivity of probability of insolvency to company parameters

Initial solvency ratio		$SR_c(0)$	*1.2*	*1.5*	*1.8*
Parameters	ρ_c	EK_c	*P(20)*	*P(20)*	*P(20)*
Base Values	***0.4***	***4***	***0.903***	***0.79***	***0.738***
% of Base Value	*- 25% (0.3)*	*- 25% (3)*	*0.544*	*0.521*	*0.376*
$\%\ \Delta\ in\ \ P(20)$			*-39.76*	*-34.05*	*-49.05*
% of Base Value	*+ 25% (0.5)*	*- 25% (3)*	*0.958*	*0.936*	*0.878*
$\%\ \Delta\ in\ \ P(20)$			*6.09*	*18.48*	*18.97*
% of Base Value	*- 25% (0.3)*	*+ 25% (5)*	*0.544*	*0.521*	*0.376*
$\%\ \Delta\ in\ \ P(20)$			*-39.76*	*-34.05*	*-49.05*
% of Base Value	*+ 25% (0.5)*	*+ 25% (5)*	*0.955*	*0.944*	*0.882*
$\%\ \Delta\ in\ \ P(20)$			*5.76*	*19.49*	*19.51*

For a given free asset allocation proportion, there is a small sensitivity of the probability of insolvency to changes in the parameter value EK_c, particularly for a company with a high initial solvency ratio, see table 3.17. This is because the quantity written does not change significantly with the quantity of new business demanded that arises from the changes in the parameter Ek_c. This leaves the company with enough available free assets for investment in risky assets.

<u>Sensitivity of probability of insolvency to parameters</u> $(l_c(+ve)$, ρ_c , *and* $SR_c(0))$

The parameter $l_c(+ve)$ is the measure of premium response to changes in a company's solvency position under a given new business strategy. As the company's solvency position improves, an increase in the parameter leads to a further reduction in premium relative to the market premium which in turn increases the company's proportion of quantity of new business demanded under the new business strategy 1.

Table 3.18 shows that there is a high sensitivity of the company's probability of insolvency to changes in the parameter $l_c(+ve)$ if there is a change in the free assets allocation proportion ρ_c and/or the initial solvency ratio $SR_c(0)$. This is because it affects both the quantity demanded and the cash flows of the company. For a given level of change in the free assets allocation proportion, the greater is the premium reduction arising from changes in the parameter value $l_c(+ve)$, the higher is the sensitivity of probability of insolvency. The reduction in premium may lead to an increase in quantity of new business demanded but not the quantity written, and therefore the overall premium income is likely to fall. This may reduce the company's solvency position.

Table 3.18: Sensitivity of probability of insolvency to company parameters

Initial solvency ratio		$SR_c(0)$	*1.2*	*1.5*	*1.8*
Parameters	ρ_c	$l_c(+ve)$	*P(20)*	*P(20)*	*P(20)*
Base Values	***0.4***	***0.4***	***0.903***	***0.79***	***0.738***
% of Base Value	*- 25% (0.3)*	*- 25% (0.30)*	*0.479*	*0.428*	*0.346*
$\%\ \Delta\ in\ P(20)$			*-46.955*	*-45.823*	*-53.117*
% of Base Value	*+ 25% (0.5)*	*- 25% (0.30)*	*0.970*	*0.914*	*0.868*
$\%\ \Delta\ in\ P(20)$			*7.420*	*15.696*	*17.615*
% of Base Value	*- 25% (0.3)*	*+ 25% (0.50)*	*0.651*	*0.488*	*0.498*
$\%\ \Delta\ in\ P(20)$			*-27.907*	*-38.228*	*-32.520*
% of Base Value	*+ 25% (0.5)*	*+ 25% (0.50)*	*0.960*	*0.927*	*0.918*
$\%\ \Delta\ in\ P(20)$			*6.312*	*17.342*	*24.390*

Sensitivity of probability of insolvency to parameters *($ft_c(0)$, ρ_c, and $SR_c(0)$)*

The parameter $ft_c(0)$ is the company's initial proportion of quantity of new business demanded and written in the market before the projection date. This parameter also determines the level of company's initial reserves and hence the initial assets (for a given initial solvency ratio) at the projection date.

For a given initial solvency ratio, the initial free assets increase with the parameter and this reduces the sensitivity of probability of insolvency, as shown in table 3.19 and vice versa, except for the low initial solvency ratio.

Table 3.19: Sensitivity of probability of insolvency to company parameters

Initial solvency ratio		$SR_c(0)$	*1.2*	*1.5*	*1.8*
Parameters	ρ_c	$ft_c(0)$	*P(20)*	*P(20)*	*P(20)*
Base Values	***0.4***	***0.25***	***0.903***	***0.79***	***0.738***
% of Base Value	*- 25% (0.3)*	*- 20% (0.20)*	*0.558*	*0.507*	*0.408*
$\% \; \Delta \; in \; P(20)$			*-38.21*	*-35.82*	*-44.72*
% of Base Value	*+ 25% (0.5)*	*- 20% (0.20)*	*0.984*	*0.93*	*0.917*
$\% \; \Delta \; in \; P(20)$			*8.97*	*17.72*	*24.25*
% of Base Value	*- 25% (0.3)*	*+ 20% (0.30)*	*0.527*	*0.421*	*0.361*
$\% \; \Delta \; in \; P(20)$			*-41.64*	*-46.71*	*-51.08*
% of Base Value	*+ 25% (0.5)*	*+ 20% (0.30)*	*0.962*	*0.887*	*0.815*
$\% \; \Delta \; in \; P(20)$			*6.53*	*12.28*	*10.43*

3.4.6.3 Conclusion of sensitivity analysis for company model parameters

The above shows that the sensitivity of a company's probability of insolvency (or relative insolvency) is strongly dependent on its initial capital (measured in terms of initial solvency ratio), the company's market share (company's maximum limit of proportion new business demanded in the market) and the chosen free assets allocation proportion to finance new business.

3.5 Effect of Factors Affecting New Business Plans on Risk

In this section and beyond, we consider no market new business models other than model M1 (which represents the term assurance market) except in section 3.5.5 where model M2 (exponential demand function and real wage inflation) is considered. In the subsequent sections, we consider the effects on new business plans and/or risk measures of changes in the following factors:

- Free assets available for financing new business and investments
- New business strategies (with different pricing policies)
- Investment strategies
- Valuation basis (capital required to write new business)
- Real wage inflation as a proxy for disposable income

3.5.1 Effect of Free Capital on New Business Strategies

3.5.1.1 Introduction

In practice, an ongoing life office does not write one block of business in year 1 and let it run off, with the profits emerging over the years. It will continue to write new business in year 2, year 3 and so on. Each block of new business creates its own financial strain (particularly the new business strain) and the surplus from business written in the previous years may not be enough to cover the total strain from the current new business written, while the growth in new business continues. This means that further transfers from the shareholders' fund (i.e. free assets) are necessary in the early years. However, there will be a time when the surplus from previous years will be enough to cover the strain from new business. At this time the demand for shareholders' fund will stop, and they will start to recover their capital slowly, see Goncalves (1999)[5].

"The strategic aim of most life insurance companies is to grow in size at a sustainable rate. However, writing new business usually involves new business strain and the company has to finance this from its internal resources, unless it is proprietary and its shareholders are willing to put in further capital" (Subject 402 Core Reading unit 13). In this investigation, we assume that further demand for capital from shareholders to write new business in future projection years is not allowed, but the reserves being released in respect of expired policies and the surplus arising from existing business would provide the capital needed internally to write future business.

The different new business strategies (with different pricing policies) discussed in chapter 2 will generate different levels of new business volume demanded by

[5] Goncalves (1999) determines the optimal retention level of a reinsurance treaty that maximises the utility of returns for a life insurance company. He assesses the utility of the extra return obtained by the shareholders, by utilising their capital in support of life insurance portfolios (with identical n-year term assurance policies) as opposed to investing it in a risk free asset.

prospective buyers but the actual new business volume (i.e. quantity actually sold and written on the books) will depend on the availability of free assets, see equation (8). We, therefore, assume that companies operating in the market are expected to be self-financing in future projection years based on an initial capital input at the projection date.

A company's self-financing capability in future projection years (arising from the reserves being released in respect of expired policies and the surplus from existing business) depends entirely on the cohorts (e.g. in terms of age at entry) of business attracted under a given new business strategy. A cohort of business with a low entry age is likely to have more surviving policies to the end of the term (arising from fewer deaths and so releasing more statutory capital at the end of the policy term) than a cohort of business with a high entry age[6]. Positive cash flows can also arise from early release of statutory capital and reserves from lapsed policies.

3.5.1.2 Effect of free assets on new business strategies

In this section, we consider the effect of initial free assets on the shape of sample paths under a deterministic scenario, as shown in figure 3.7. We also consider simulated sample paths in figure 3.8 for NBS-1 and NBS-2 to show how the free assets (i.e. solvency ratio) at the start of a projection year affect the premium and the new business volume (quantity demanded and quantity written).

[6] As mortality rate increases with age, it is more likely for a life aged 35 to survive the next n years than for a life aged 55. It is typical of a temporary insurance to have small policy reserves, and therefore it is mainly the capital at risk component of the statutory reserves of an expired policy that is released.

The level of initial free assets (measured in terms of initial solvency ratio, $SR_m(0)$) available to finance new business at the start of the projection year determines the shape of the sample path, particularly in the early projection years. If the company's starting position (measured in terms of initial solvency ratio) is above (below) the long-term mean solvency ratio (about 1.05) its solvency may decline (improve) over the first few years of the projection period. The reasons for the above are explained below.

Figure 3.7 presents deterministic projections which show the effect of initial capital on future solvency over a 40-year projection period (time horizon) for NBS-1, with the assumption that a company is allowed to use all its available free assets in writing new business (100% allocation of free assets, $\rho_m = 1$). As different levels of initial free assets will affect the early years' volume of actual new business written differently through the new business strategy, different sample paths may be produced.

The solvency position will improve in the early years of projection, if there are little or no initial free assets (i.e. initial solvency ratio less than 1.05) to write actual new business, particularly in years 1 and 2. The non existence of new business (or the presence of only a small volume of new business) in these years may improve the solvency position as the surplus arising from existing business is not being affected greatly by the heavy initial commission from new business. The surplus will also provide the capital required in writing future business. The actual new business growth in subsequent years (e.g. year 3 and beyond) may not be sustained over a longer term by the use of the surplus from business written in previous years as insufficient free assets are left to invest in equities to produce higher returns. This may lead to insolvency and consequently no new business being written in later projection years thereby generating peaks and troughs in the emergence of profits.

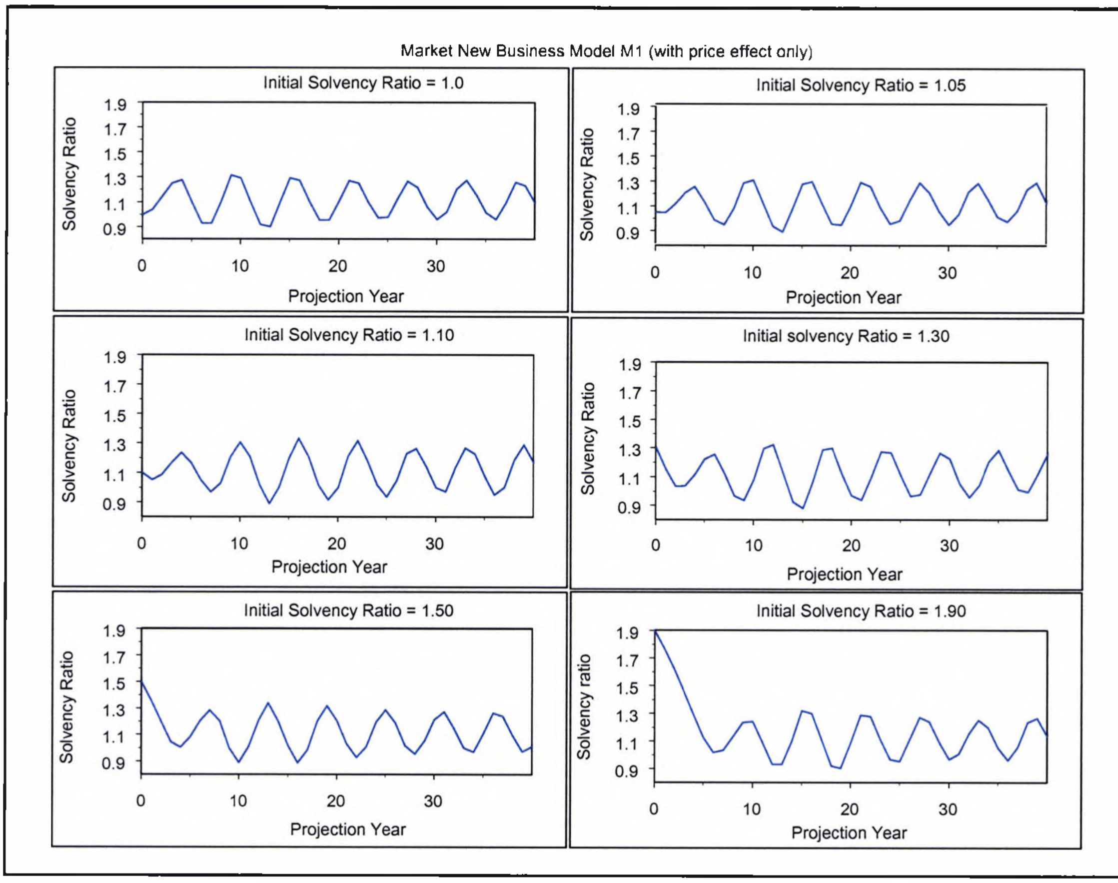


Figure 3.7: Deterministic sample paths for new business strategy 1 (NBS_1)

An increase in initial capital which is not sufficient to cover the new business strains in the early projection years will affect the future solvency of the company and this may also generate the peaks and troughs in profitability. As the increase in initial free capital covers the new business strains adequately in the early projection years, sufficient free assets may be left for investment in equities over a longer period. This is likely to improve the solvency position.

An increase in initial free assets (above 1.05) has no significant effect on a company's solvency over the very long term. This is because the more initial free assets are available, the more is the volume of new business written (with 100% free assets allocation strategy) over the first few years of the projection, and consequently the free capital is used up over time which may not necessarily improve the company's future long term profitability. The effect of the initial capital wears off over the extended projection period. The initial free assets essentially affect only the phase of the cycles as shown in figure 3.7.

However, the oscillations occurred mainly because the model does not allow for capital injections from external sources in future projection years other than the initial capital input at the projection date. Thus, figure 3.7 shows that a company's long term solvency position is insensitive to initial conditions if there is no future capital injection.

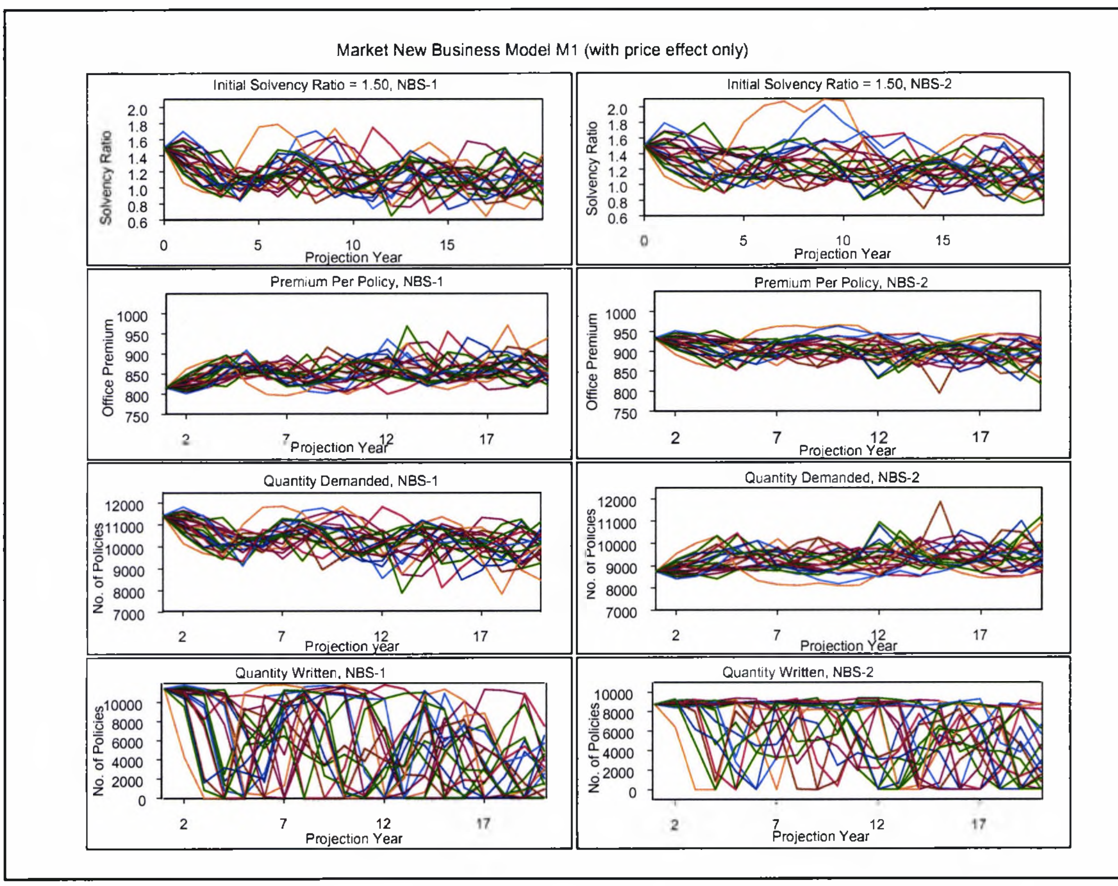


Figure 3.8: Effect of solvency ratio on new business strategies 1 and 2 (20 simulations)

Figure 3.8 illustrates (with 20 simulations) how changes in solvency ratio at the end of the previous year affect the current year's premium (without inflation effect) in adopting NBS-1 and NBS-2, and which in turn affects the quantity of new business demanded through the demand function. Under the new business strategy 1, as the free assets at the start of any projection year decrease, the premium increases and the quantity of new business demanded decreases. The converse is true for new business strategy 2.

For a given level of free assets the quantity of new business demanded is likely to be more under new business strategy 1 (NBS-1) than new business strategy 2 (NBS-2) due to the underlying nature of the strategies. The actual quantity of new business written is a function of the free assets at the start of the projection year and the quantity demanded. In the same vein, for a given level of free assets, the actual quantity written is likely to be more under new business strategy 1(NBS-1) than the new business strategy 2 (NBS-2), all other things being equal, as shown in figure 3.8. The effect of the above on insolvency risk is discussed in the next section.

3.5.1.3 Conclusion

We may expect a large volume of new business written to be positively related to an insurer's profitability, but the investigation in this section has revealed that writing more new business may result in a drain on free assets, leading to an increased probability of insurer insolvency. This is particularly true where no new capital injection is allowed. For a given level of initial free assets the new business strategy 1 may generate more new business volume than strategy 2, yet it has the highest probability of statutory insolvency relative to other strategies, if all available free assets are used for writing more new business.

3.5.2 Effect of New Business Strategies on New Business Plans and Risk

3.5.2.1 Introduction

Regulators do not allow companies to increase their market share without due consideration for solvency requirements. As the new business strategies are different due to the different underlying pricing policies, the free assets allocation decision needed to produce a given level of insolvency may also be different.

3.5.2.2 Effect of new business strategies

As the new business strategies with different pricing policies generate different quantities of new business demanded, they also create different levels of new business strain and the quantity of new business written for a given combination of initial solvency ratio and free assets allocation proportion. Figure 3.9 shows the means of the distributions of new business strain and new business volume written for the four new business strategies for a combination of initial solvency ratio (e.g. $SR_m(0) = 1.50$) and free assets allocation proportion (e.g. $\rho_m = 0.40$).

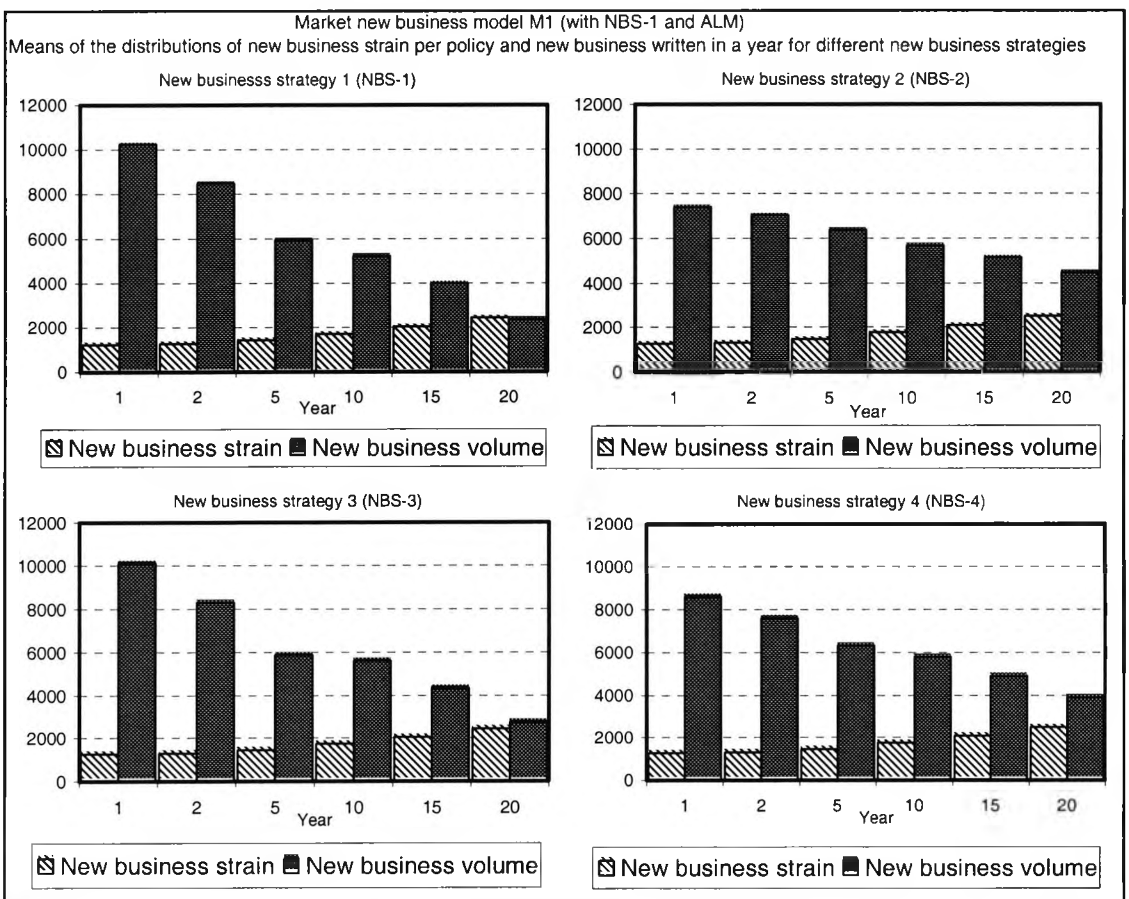


Figure 3.9: Distributions of new business strain and written for new business strategies

The pricing policy in new business strategy 2 allows a company to increase (reduce) its premium relative to a profit-tested premium in order to generate a small quantity of new business demanded if the company's solvency position improves (worsens). Figure 3.9 shows that the pricing policy in new business strategy 2 creates a slightly higher new business strain than other new business strategies over the projection period. This leads to writing a relatively lower volume of new business under new business strategy 2 than other new business strategies in the short term. Thus, new business strategy 2 holds back free assets for investment (e.g. in equities) in order to produce higher investment returns over the long term, thereby generating more free capital for financing more new business volume in the longer term than the other new business strategies, as shown in figure 3.9.

On the other hand, the pricing policy in new business strategy 1 allows a company to reduce (increase) its premium relative to a profit-tested premium in order to generate a high quantity of new business demanded if the company's solvency position improves (worsens). Figure 3.9 shows that new business strategy 1 has the lowest level of new business strain over the projection period and hence it has the highest volume of new business written in the short term than other strategies. A drain on the initial free assets may arise in the short term, leading to insufficient investment returns to support in the financing of future new business. Thus, new business strategy 1 leads to the lowest new business volume in the longer term than the other new business strategies, as shown in figure 3.9.

The pricing policy in new business strategy 3 (in new business strategy 4) allows a company to reduce (increase) its premium relative to a profit-tested premium in order to attract a high (low) quantity of new business demanded if the company's solvency position improves or worsens. The profit-tested premium is payable when the company's solvency ratio is equal to a new business decision factor (NDF = 1.25 which is chosen arbitrarily).

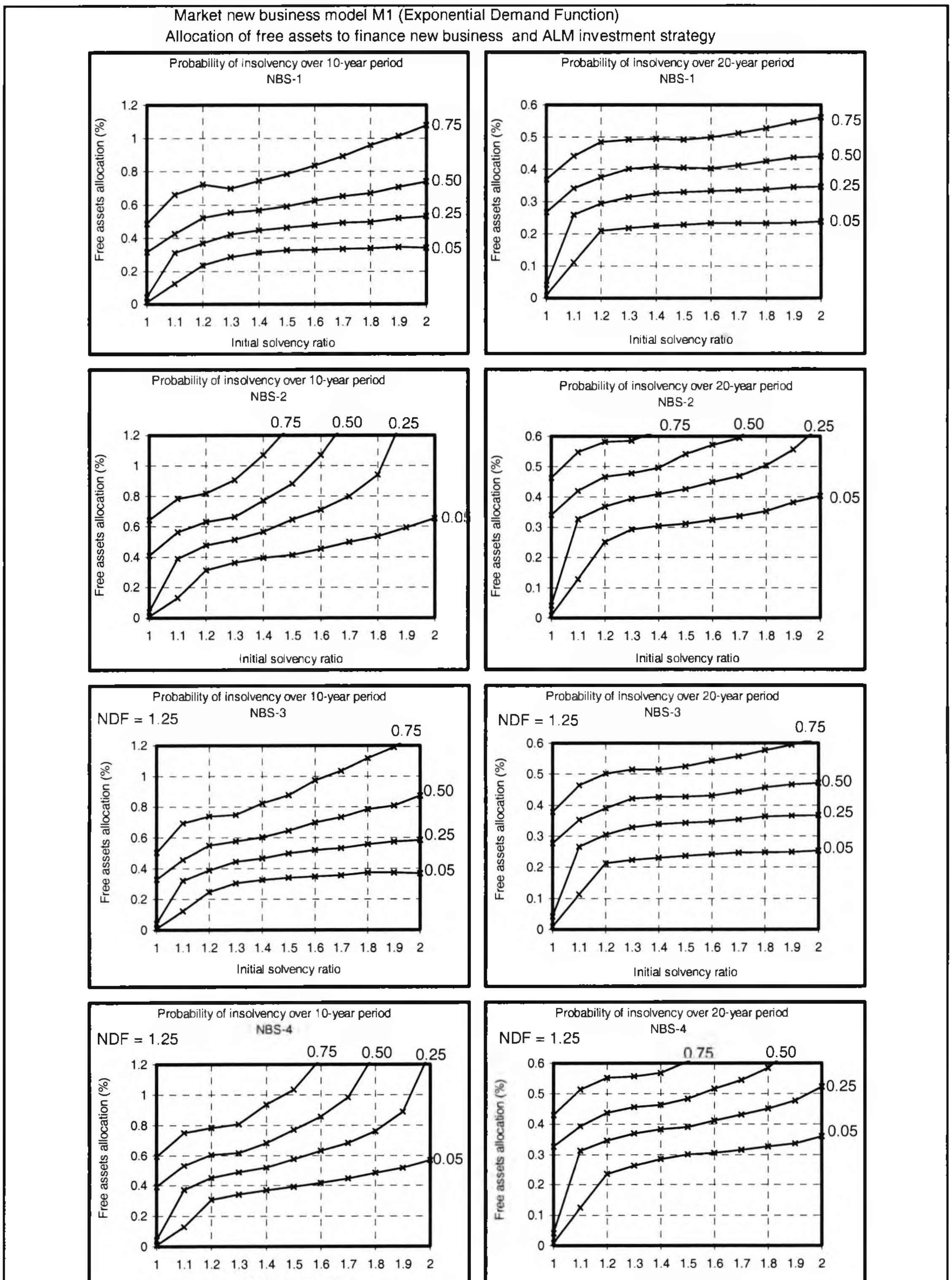


Figure 3.10: Effect of new business strategies on probability of insolvency

As we expect, new business strategies 3 and 4 have similar results to strategies 1 and 2 respectively, but new business strategy 3 writes more (lower) new business volume in the short term (long term) than new business strategy 4, as shown in figure 3.9.

Figure 3.10 shows how the new business strategies affect the allocation of a company's free assets to finance its new business to produce a given level of probability of insolvency. The effect of new business strategies can effectively be compared when there is sufficient free capital to implement them since the availability of free capital is a key factor that drives any new business strategy. Figure 3.10 shows that there is little or no difference in the contour lines (particularly at 5% probability of insolvency) for the different new business strategies when the initial free assets (initial solvency ratio) are small.

Figure 3.10 shows that the probability of insolvency risk is lowest under NBS-2 strategy and highest under NBS-1 strategy for a given combination of constant proportion of free assets to finance new business and initial size of a company. Furthermore, the NBS-3 has a lower and a higher probability of insolvency than NBS-1 and NBS-4 respectively. This holds for all projection periods (except for the very low values of initial size of a company) and also for mean shortfall risk measure as shown in figure 3C.3 of appendix 3. In other words, the constant proportion of free assets to finance new business over a projection period to produce a fixed level of insolvency risk (e.g. 5%) is highest under NBS-2 and lowest under NBS-1, for a given initial size of a company (see figure 3.10). The above is reflected in the ordinates of the contour lines for new business strategy 2 being higher than the other new business strategies.

The new business strategies (e.g. NBS-2 and NBS-4) that enable a company to write a relatively small new business volume in the short term improve the company's solvency position more than those strategies (e.g. NBS-1 and NBS-3) writing a substantial new business volume in the early projection years. Thus, the ranking of riskiness in new business strategies reflects the relative quantity of new business written in the short term.

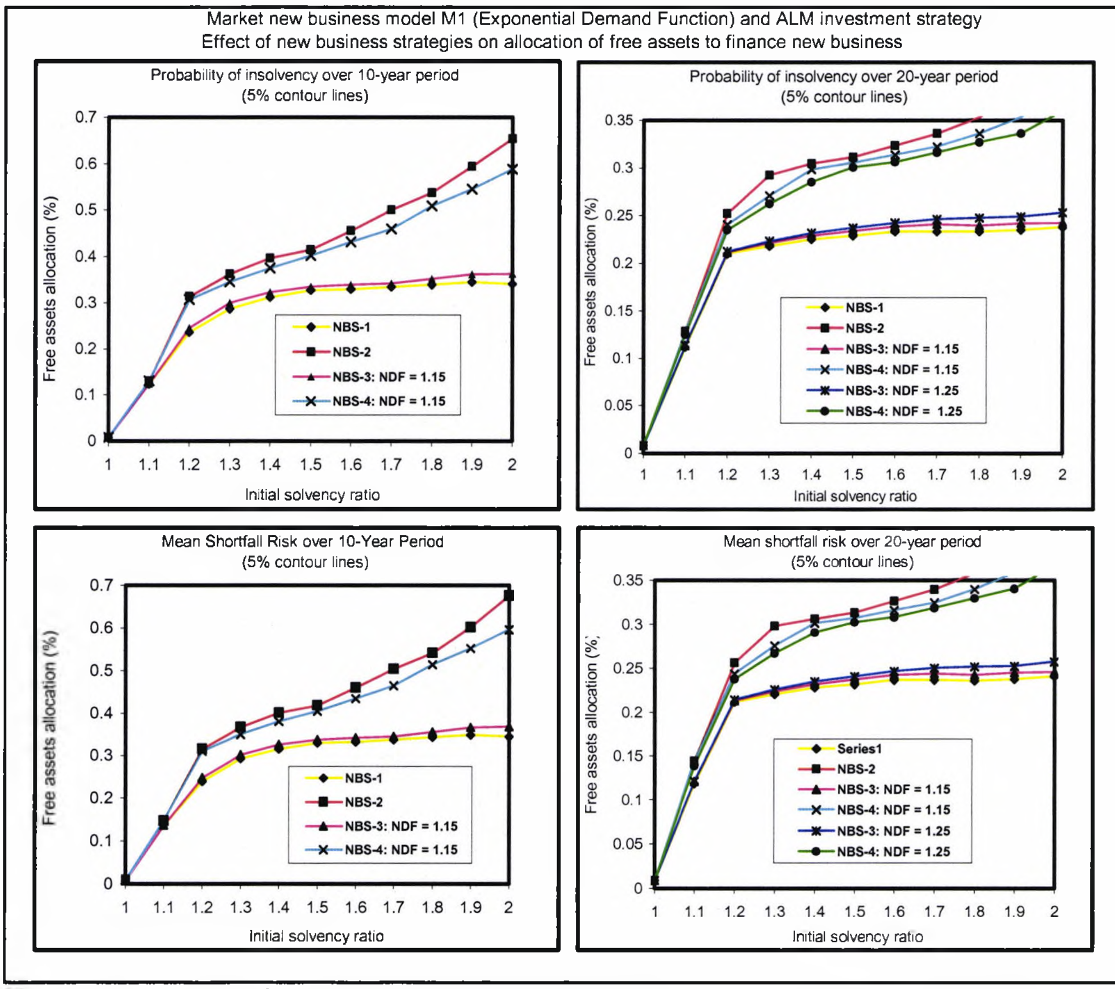


Figure 3.11: Effect of new business strategies on free assets allocation proportion

Figure 3.11 shows how the new business strategies affect the allocation of a company's free assets to finance its new business to produce a 5% insolvency risk and the effect of changes in new business decision factor (NDF) on new business strategies 3 and 4. Figure 3.11 also reveals that NBS-2 performs consistently better than the other new business strategies, in terms of proportion of free assets allocated to finance new business. Furthermore, NBS-4 performs better than NBS-3, which in turn performs better than NBS-1. NBS-4 holds back fewer free assets for investment relative to NBS-2, and it expects to generate more new business written than NBS-2 in the short term.

The chosen level of new business decision factor (NDF) for a given new business strategy may also affect the solvency position. Figure 3.11 also shows that, as the NDF increases from 1.15 to 1.25 in strategies 4 and 3, the probability of insolvency increases (and reduces) respectively. As NDF increases, a lower level of free assets is held back for investment in strategy 4 and hence more new business is expected to be written in the short tem which may result in insolvency.

3.5.2.3 Conclusion

A new business strategy that leads to writing more business (particularly in the short tem) is more likely to result in insolvency than a strategy with a lower volume of new business written, if all other things remain the same.

In practice, a company that has much concern for market share is likely to prefer new business strategy 1 relative to other strategies but at the cost of having solvency problems with the regulators in the long term, and hence there is need for a strategy on allocation of free assets. This is particularly true for a large insurer for whom, as noted by Young (2003), the level of risk aversion decreases as the wealth (free assets) increases. However, new business strategy 2 is most likely to be preferred by an insurer having or about to have problems with the regulators since its solvency position will improve in the short-term but to the detriment of its market share.

3.5.3 Effect of Investment Strategies on New Business Plans and Risk

3.5.3.1 Introduction

The choice of an appropriate investment strategy is relevant in making a decision on the level of free assets to be allocated in financing new business plans. An investment strategy that produces lower investment returns in the future is less likely to allow a company to allocate a high proportion of its available free assets to finance new business.

In this section, we use the four investment strategies proposed in section 2.3.2 namely, asset/liability matching (ALM) investment strategy, static (investing 50% in gilts and 50% in equities) allocation strategy, dynamic asset allocation strategies (DS1 and DS2) to investigate the effects of investment strategy on new business plans adopting new business strategy 1. Briefly, dynamic asset allocation strategy (DS1) allows a company to invest 80% of its assets in equities when its solvency ratio exceeds 1.35 and 100% in gilts if the solvency ratio is less than 1.05. In dynamic asset allocation strategy (DS2) the company invests 100% of its assets in gilts when its solvency ratio exceeds 1.35, and investing 80% in equities if the solvency ratio is less than 1.05.

3.5.3.2 Effect of investment strategies on new business plans and risk

Figure 3.12 and figure 3C.4 in appendix 3 show the constant insolvency risk contour lines over a given projection period for different investment strategies. For a given combination of constant proportion of free assets to finance new business and initial size of a company, the insolvency risk is lowest under the ALM strategy and highest under the DS1 strategy. This holds for all projection periods (except for the very low values of initial size of a company) and both risk measures. Figure 3C.4 shows that, at a high free assets allocation proportion, the dynamic assets allocation strategies produce a more unstable insolvency risk over the longer term than the ALM investment strategy and static strategy.

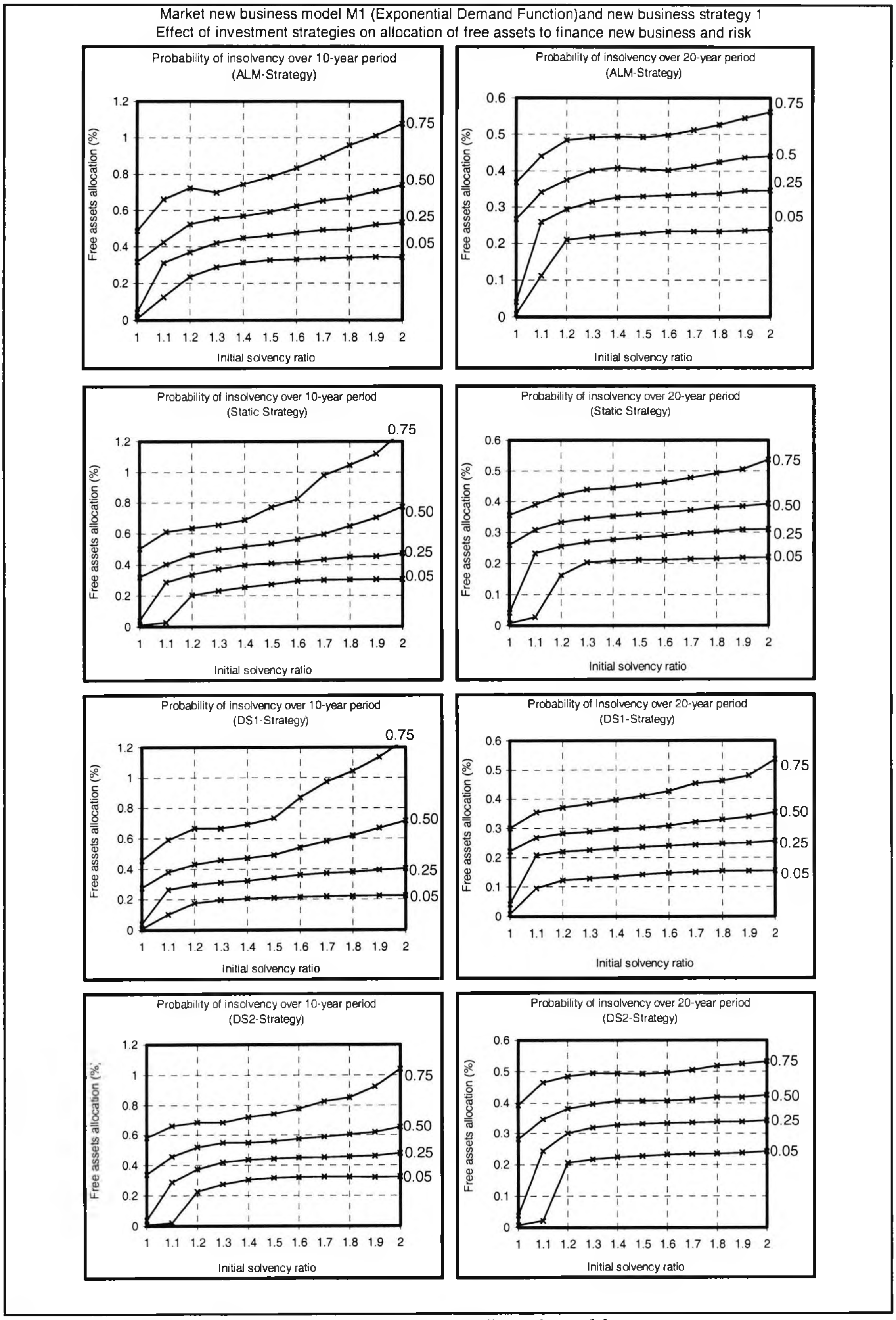


Figure 3.12: Effect of investment strategies on allocation of free assets

Figure 3.12 shows how a company's choice of investment strategy will affect its new business plans (i.e. the constant proportion of free assets required to finance new business over a projection period to produce a given insolvency risk) to meet its desired market share. Writing a high level of profitable non-profit business will produce a capital strain in the short term and reduce the statutory free assets but will increase the free assets in the long term. Figure 3.12 shows that a company with such a business plan may prefer the ALM investment strategy than the other investment strategies. This is because the ALM strategy will allow the company to use a higher asset allocation in less risky assets (e.g. gilts) than the other investment strategies. Thus, writing a profitable business and investing assets in a matched position (i.e. ALM strategy) may enable a company to write more new business (with higher free assets allocation proportion) over a longer term than the other investment strategies. Figure 3.12 shows that for an initial solvency ratio of 1.50, a company with an ALM strategy can allocate a higher proportion of its free assets to finance new business than a company adopting any of the other investment strategies in order to produce a 25% probability of insolvency over the long term.

Therefore, the ALM strategy may enable a company to meet its strategic aim (i.e. to grow in size at a sustainable rate) and still meet its statutory solvency requirements more effectively than the other strategies, particularly over the short term. However, the other investment strategies (e.g. static strategy) are likely to have more investment freedom but may produce more volatile and/or lower returns (leading to a higher insolvency risk) than the ALM strategy in the short term.

The reverse may be the case for a company writing non-profit business with a different pricing policy (i.e. it may prefer static strategy to ALM strategy). However, such a company may need to make a decision on the balance between its new business risk and investment risk so that it can continue its chosen new business plan (i.e. constant proportion of free assets to finance new business) without placing constraints on its investment freedom.

Considering the constant insolvency risk (e.g. 25%) contour lines over a given projection period for the different investment strategies in figure 3.12 shows that the ALM strategy performs better (in terms of free assets allocation proportion) than the other strategies. Further the DS2 Strategy performs better than the static strategy which in turn performs better than strategy DS1. This ranking also reflects the insolvency risk associated with each investment strategy, for a given combination of constant proportion of free assets to finance new business and initial size of a company.

The ranking of investment strategies reveals that if a company's new business plans imply an expected reduction in statutory free assets (due to significant new business strains), the company may need to move more investments into gilts (to have a more matched position) in order to maintain statutory solvency. However, this may in turn imply a lower expected return, leading to further reductions in the levels of free assets. This potentially vicious circle may lead the company to conclude that it needs to change its new business plans (subject 402 Core Reading, Unit 22).

Table 3.20: Probability of insolvency over 20-year period (under NBS-1) $\rho_m = 0.4$

	Static investment strategy			Dynamic investment strategy 1			Dynamic investment strategy 2			
$SR_m(0)$	30% Gilts	50% Gilts	70% Gilts	DS1(a)	DS1(b)	DS1(c)	DS2(a)	DS2(b)	DS2(c)	ALM
1	*0.903*	*0.852*	*0.791*	*0.898*	*0.881*	*0.853*	*0.765*	*0.752*	*0.75*	*0.829*
1.2	*0.812*	*0.715*	*0.597*	*0.825*	*0.767*	*0.665*	*0.557*	*0.522*	*0.51*	*0.578*
1.5	*0.741*	*0.654*	*0.48*	*0.737*	*0.68*	*0.571*	*0.481*	*0.443*	*0.431*	*0.488*
1.8	*0.679*	*0.559*	*0.438*	*0.668*	*0.581*	*0.487*	*0.451*	*0.42*	*0.409*	*0.433*

DS1(a) = Invest 80% in equities if SR(t) > 1.35 and 100% in gilts if SR(t) < 1.05
DS1(b) = Invest 60% in equities if SR(t) > 1.35 and 100% in gilts if SR(t) < 1.05
DS1(a) = Invest 40% in equities if SR(t) > 1.35 and 100% in gilts if SR(t) < 1.05

DS2(a) Invest 100% in gilts if SR(t) > 1.35 and 80% in equities if SR(t) < 1.05
DS2(b) Invest 100% in gilts if SR(t) > 1.35 and 60% in equities if SR(t) < 1.05
DS2(c) Invest 100% in gilts if SR(t) > 1.35 and 40% in equities if SR(t) < 1.05

The parameters used to describe the investment strategies in section 2.3.2 have been set at reasonable (although arbitrary) values and therefore the results (e.g. probability of insolvency) from the investment strategies in this section may change if these parameter values are altered. Thus, we expect that an increase in the proportion of assets invested in gilts under any investment strategy will reduce the insolvency risk. The difference between the results from the ALM strategy and the other investment strategies may reduce as the proportion of assets invested in gilts increases under these other investment strategies, see table 3.20.

3.5.3.3 Conclusion

Figures 3.12 is a tool that can enable a company writing non-participating business to meet its strategic aim to grow in size at a sustainable rate with its own internal resources being used to finance its new business plans. In the UK, GN1 states that "the Appointed Actuary must indicate any limits on the volume of business that may prudently be accepted" by a company, particularly those that are writing a business that involves significant new business strain. Figure 3.12 may assist an actuary in making decisions relating to new business volumes in order to meet such regulatory requirements under a given new business strategy.

The decision to allocate a constant proportion of free assets to finance new business to produce a particular level of insolvency risk over a given time horizon is dependent on the company's decision on the balance between its new business risk and investment risk.

3.5.4 Effect of Valuation Basis on New Business Plans and Risk

3.5.4.1 Introduction

The regulators recognise that one of the important factors that will affect the financial position of a life insurance company is its new business volumes. Thus, the regulations on the minimum mathematical reserves and solvency margins to be held by the company have the effect of limiting the free assets available within the company to write new business and of placing a minimum requirement on the finance required to write a life insurance contract.

3.5.4.2 Capital requirement to write new business

The company's capital requirement to write a cohort of new business is taken as the product of the expected volumes and the new business strain per contract together with any additional costs. For a suitable projection period, the company's expected volume of new business (i.e. quantity of new business demanded) in each year is obtained in accordance with its chosen new business strategy. The determination of new business strain per contract in each projection year will involve a set of assumptions (i.e. valuation basis which may be prescribed) in calculating the supervisory reserves (including solvency margin). A company's actual volume of new business (i.e. quantity of new business written) depends on the capital requirement and the free assets available. If a company writes too much new business it will run down its free assets to the extent that they no longer cover the required solvency margin thereby creating solvency problems. The above can be avoided by reducing the capital requirement and/ or increasing the free assets.

The ways in which the company may be able to reduce its capital requirement, as stated in Luffrum (1992), include the following:

- Redesign its contracts so as to reduce the new business strain to which they give rise.
- Weaken the reserving basis for new business in order to reduce the new business strain (e.g. add or increase a Zillmer adjustment, increase the valuation rate of interest or reduce the mortality rate).
- Cut back on the sales of those contracts that generate the highest new business strain or place limits on the amount that the company can safely write, in accordance with the regulators' requirement (GN1).

The finance for writing new business will come from the company's free assets, and below are some of the ways in which the free assets may be increased, see Luffrum (1992):

- Weaken the basis for valuing the liabilities in respect of the existing business.
- Change the investment policy so as to match more closely the liabilities
- Use of reinsurance.
- A proprietary company can seek further finance from owners (shareholders), an option not considered in this research.

The items listed above other than the valuation bases are considered in chapter 4, except that the effect of changes in investment policy has been considered in section 3.5.3. As a company's capital requirement and the free assets available to write new business are affected by the valuation basis, its new business plans (particularly the constant proportion of free assets required to finance new business over a projection period) may also be influenced by the choice of valuation basis. In the following sections, we consider the net premium valuation basis, and the effect of changes in valuation basis on a new business plan and the insolvency risk under new business strategy 1, as being the most intuitive strategy. The results for other new business strategies are not shown, as the conclusions arising from them are similar to those for new business strategy 1 under a given valuation basis.

3.5.4.3 Net premium valuation bases

The valuation basis adopted at any time depends on the purpose, methods of valuation and the type of contract considered in the valuation. For non-linked business in UK, "the format of the supervisory returns requires either a net premium or gross premium method. If a gross premium is used, the 1994 Regulations require that a check be made that it produces reserves at least as high as if a net premium method had been used. Hence there is a practical advantage in using a net premium method", (Subject 402 Core Reading unit 11). The net premium valuation method only takes into account future mortality and rate of interest. The regulations also allow the net premium to include a Zillmer adjustment (which takes account of the uneven incidence of the expenses) and the reserves produced are called Zillmerised net premium reserves. The maximum amount of initial expenses which can be incorporated into the Zillmer adjustment is the lower of 3.5% of sum assured or excess of the total initial expenses over one year's renewal expenses, see Luffrum (1992). The implied profit and expense loading is the difference between the actual office premium and the net premium.

However, FSA recent rules and guidance have favoured the use of gross premium valuation method in valuing a life insurance portfolio including non-profit business within a with-profits fund as discussed in section 2.2.6 of chapter 2.

If reserves are calculated for a particular contract on exactly the same basis as the premium basis, the profit (or loss) which will emerge will reflect the extent to which actual experience has improved or worsened compared to that initially expected when a contract was originally sold. As soon as the valuation basis is varied, part of the profit that will be earned in the future on account of the difference between the valuation basis and the premium basis is capitalised at the valuation date as a one-off profit due to the change in valuation basis. Thereafter, until the valuation basis is changed again, the emerging profit reflects the difference between the actual experience and that assumed in the valuation basis (see Booth et al 2005). For simplicity, we assume that the valuation basis (once chosen) does not change before and after projection date.

Interest Rate Assumption: The valuation rate of interest (according to UK regulations) must then not exceed the current yield obtainable on the assets backing the reserves for a particular contract. The net premium valuation reserves are insensitive to the changes in the future interest assumption due to the fact that all the valuation factors (i.e. annuity, assurance factors and the net premium itself) change with the interest assumption in compensating ways (the calculation details for these factors are not shown).

Mortality Assumption: The mortality assumption needs to be realistic and where there is an improvement in mortality rates over time, there may be a need to reduce the mortality rate in the valuation process. However, in this investigation, we assume a constant mortality rate over time (but the mortality rate still varies with age in the normal way) in order to create a contingency margin implicitly in the valuation process, because a reduction in mortality rate would produce smaller reserves and vice versa.

Table 3.21: Net premium valuation bases

Bases	Premium Basis	Optimistic Basis (Weak Basis)	Cautious Basis (Strong Basis)
Mortality	110% * 70% of TM92 (Ultimate)	99% * 70% of TM92 (Ultimate)	121% * 70% 0f TM92 (Ultimate)
Interest	*4.5%*	*5.4%*	*3.6%*
Zillmer	*113.65*	*125.01*	*102.28*

Zillmerised net premium reserves: The net premium valuation usually starts with zero at duration zero, but it is possible for the Zillmerised net premium reserve to be negative particularly at lower durations. In practice, negative reserves are eliminated since otherwise the contract would be treated as an asset.

Table 3.21 above shows the net premium valuation bases namely, premium, cautious and optimistic valuation basis. The premium valuation basis (i.e. the mortality and interest assumptions) is obtained from Mc Gaughey (2001) while the cautious and optimistic valuation bases are obtained by changing the premium basis arbitrarily (i.e. 10% change in the mortality assumption and Zillmer adjustment, and 20% change in the interest assumption). We consider both Zillmerised and non-Zillmerised reserves. For simplicity, the Zillmer adjustment is taken as 10% of the acquisition expenses, a small addition in order to keep the net premium less than the office premium.

3.5.4.4 Effect of valuation basis on new business plan and solvency risk

We expect that, for any new business strategy, the use of an optimistic basis valuation may result in a lower reserve for existing business, a lower capital required (new business strain) to write a new policy and hence higher free assets than adopting a premium or cautious basis valuation. In the same vein, the premium basis valuation results would be lower than the cautious basis valuation results and the free assets from the former would be higher than the latter valuation. Thus, we expect that the weakening of a company's reserving basis may increase its actual new business volume written particularly in the short term, as free assets are made available. The cost of the higher actual new business volume written is the drain on the free assets which increases the probability of insolvency.

The valuation bases may also have a second order effect on the actual volume of new business written. The free assets available at the start of a year are assumed to be the sum of the free assets and the reserves released in respect of the expired policies at the end of the previous year. The optimistic basis valuation may result in relatively smaller reserves being released to provide extra free assets in the long term for both investment and the financing of new business than other valuation bases.

Figure 3.13 shows the mean of the distribution of new business strain per policy and new business written in projection years for different valuation bases under new business strategy 1 for a combination of initial solvency ratio (e.g. $SR_m(0) = 1.50$) and free assets allocation proportion to finance new business (e.g. $\rho_m = 0.40$). The figure shows that, for non–Zillmerised valuation, an optimistic basis produces lowest new business strain per policy and quantity of new business written over the projection period than other valuation basis, whilst the cautious basis produces a higher new business strain per policy than the premium basis valuation. Figure 3.13 also shows that the premium basis (with Zillmer) valuation results in a lower new business strain per policy than the premium (with no Zillmer) basis valuation.

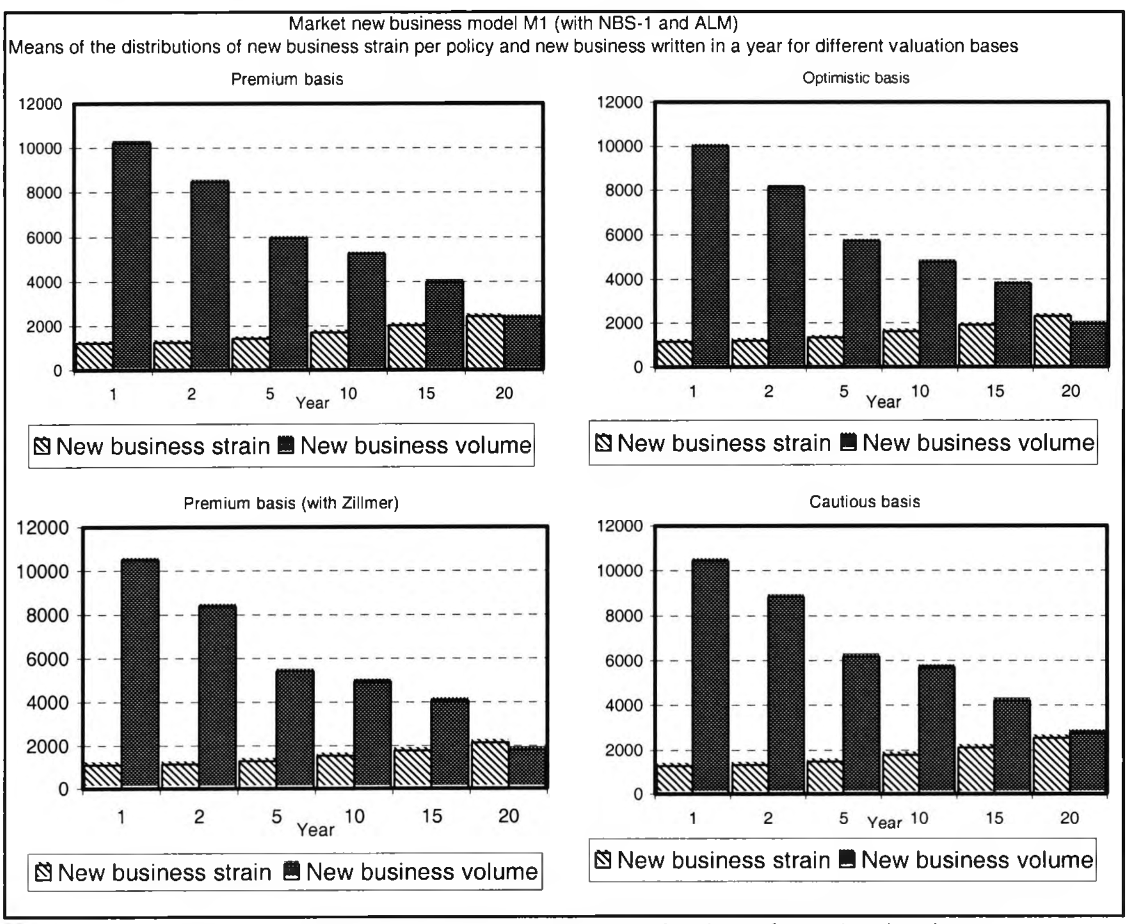


Figure 3.13: Distributions of new business strain and written for valuation bases

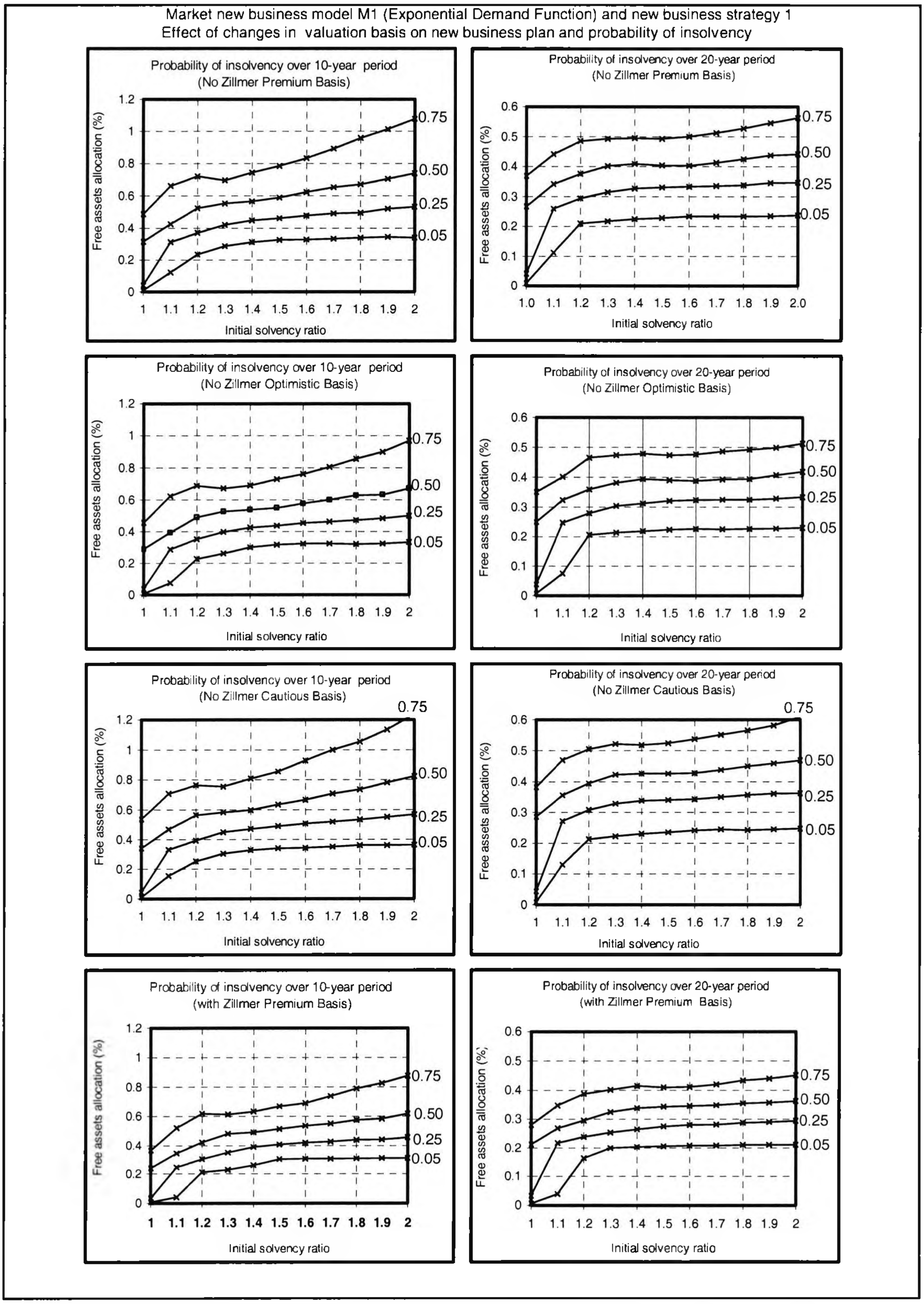


Figure 3.14: Effect of changes in valuation basis on new business plan and risk

In this investigation, we assume that the initial assets at projection date are taken as a multiple (e.g. the initial solvency ratio) of the initial liabilities (the reserves for the existing policies at the projection date). Similarly Hardy (1998) assumes that a mature model office has an initial Asset-Liability ratio (A-L) of 1.20 at the start of the projection and the initial assets are taken as 120% of the liabilities. As the optimistic basis results in lower reserves than the other basis, the initial free assets at the projection date and the reserves released to provide extra free assets in future years for a company adopting an optimistic basis are also lower than a company using either a premium basis or cautious basis valuation.

Thus, a company adopting cautious basis which results in higher reserves can afford to write more new business over a projection period than a company using either a premium or optimistic basis valuation, as shown in figure 3.13, and may still remain solvent. On the other hand, a company adopting premium basis (with no Zillmer) valuation is likely to write more new business over the projection period than a company using premium basis (with Zillmer), see figure 3.13.

Figure 3.14 shows how a company's choice of valuation basis may enable its future new business plans (i.e. the constant proportion of free assets required to finance new business over a projection period that produces a fixed level of insolvency risk) to meet its statutory solvency requirements. For a given combination of constant proportion of free assets to finance new business and initial size of a company, the probability of insolvency for any projection period is lowest under the cautious basis and highest under the optimistic basis, when considering the non-zillmerised net premium valuation. The zillmerised premium basis produces a higher solvency risk than the non-zillmerised premium basis. This may be as a result of the different levels of free assets and/or actual new business volume written under the different valuation bases.

The above analysis holds also for mean shortfall risk measure shown in figure 3C.5 of appendix 3. The contour lines (for a 5% solvency risk) in figures 3.14 and

3C.5 show that the cautious basis performs better (in terms of allocation proportion of free assets to finance a company's expansion plans) than the other valuation bases, whilst the premium basis performs better than optimistic basis.

Table 3.22 also reveals that the cautious mortality assumption helps a company to achieve its expansion plans better than the cautious interest assumption, as it produces the lowest probability of insolvency for any given free allocation proportion to finance new business. This indicates the importance of the mortality assumption in without profit non-linked temporary insurance business.

Table 3.22: Effect of changes in mortality and interest assumptions on risk

Probability of insolvency over 10-year projection period							
Initial solvency ratio		$SR_m(0) = 1.2$		$SR_m(0) = 1.5$		$SR_m(0) = 1.8$	
Allocation proportion		$\rho_m = 0.4$	$\rho_m = 0.6$	$\rho_m = 0.4$	$\rho_m = 0.6$	$\rho_m = 0.4$	$\rho_m = 0.6$
Valuation basis	*P-Basis*	*0.312*	*0.816*	*0.126*	*0.767*	*0.092*	*0.644*
	O-Mortality	*0.351*	*0.849*	*0.155*	*0.82*	*0.121*	*0.702*
	C-Mortality	*0.271*	*0.79*	*0.101*	*0.708*	*0.081*	*0.576*
	O-Interest	*0.319*	*0.819*	*0.129*	*0.769*	*0.098*	*0.651*
	C-Interest	*0.307*	*0.809*	*0.123*	*0.762*	*0.09*	*0.642*

Valuation Basis
P-Basis = Premium basis
O-Mortality = Premium basis with optimistic mortality assumption
C-Mortality = Premium basis with cautious mortality assumption
O-Interest = Premium basis with optimistic interest assumption
C-Interest = Premium basis with cautious interest assumption

3.5.4.5 Conclusion

Where the valuation basis and/or method are prescribed by regulations, a company cannot easily manipulate the valuation reserves to create free assets in order to write its desired level of new business when in fact it has not enough free assets to do so. The investigation reveals that the cost of changing the valuation basis (particularly to an optimistic valuation basis) for an on-going company in order to write more new business may be a drain on the free assets and this may lead to insolvency.

In practice, the regulations also allow a Zillmerised net premium valuation which may lead to a reduction in reserves relative to non-Zillmerised Valuation. All things being equal, this is likely to improve solvency if the free assets made available from the valuation are not used for writing an inappropriate volume of new business. The investigation has also revealed that a Zillmerised net premium valuation may create more solvency problems in the future for an on-going company writing more new business due to the increased free assets (if they are being used to invest in new business) than a non-Zillmerised net premium valuation.

3.5.5 Effect of Real Wage Inflation on New Business Plan and Risk

3.5.5.1 Introduction

An ongoing company's new business volume and its financial strength is not only affected by endogenous factors (such as price of the product, free capital etc. within the company's control) but also by exogenous factors (e.g. disposable income which may be linked to real wage inflation), Browne et al (2001). In practice, the impact of real wage inflation on an insurer's profitability and solvency (through increased demand for life insurance) depends on the insurer's decision to allocate more of its available free assets to finance new business. An increase in real wage inflation may directly affect the quantity of new business demanded but the quantity of new business actually written on the books depends on the available free assets being allocated to finance new business plans.

In this section, we compare the results of the market new business Models M1 (exponential demand function) and M2 (exponential demand function and real wage inflation effect) to show the impact of changes in real wage inflation (simulated using Wilkie's stochastic investment model) on an insurer's decision to allocate free assets to finance new business plans to achieve a particular level of solvency risk under different new business strategies. We note that, we have considered in previous sections the effect of rate of change in real wage inflation (parameter b in equation (2)) on the quantity of new business demanded when carrying out the sensitivity analysis.

3.5.5.2 Effect of real wage inflation on new business plan and risk

In practice, an increase in demand for insurance (through the wage inflation effect) may influence a risk tolerant insurer's desire to increase its market share, as the probability of a policy lapse (or surrender) is negatively related to disposable income of a policyholder, Browne et al (2001). The insurer is likely to increase its allocation proportion of free assets to finance new business in order to write more new business (i.e. to increase its market share if the insurer still has free assets available) but at the expense of increasing statutory solvency risk.

A comparison of figures 3.10 (in section 3.5.2.2) and 3.15 for market new business models M1 and M2 respectively shows that, for a given company's initial solvency ratio, the contour lines are higher in the grids under new business model M1 than under new business model M2. Therefore, the corresponding co-ordinates in figure 3.15 have a higher probability of insolvency than those in figure 3.10. The above reveals that, for a given free assets allocation proportion, new business model M2 (with combined effect of price and income) generates more new business and hence a higher drain on the free assets than new business model M1 (with only price effect). The above feature of the model is in line with the result that demand for life insurance is positively correlated with income of the prospective policyholders, [see Diacon (1980), Browne et al (1993)], and also an increase in sales may result in a drain on free assets leading to an insurer's insolvency as against the (ex ante) expectation that a large new business volume is positively related to an insurer's performance, (Browne et al, 2001).

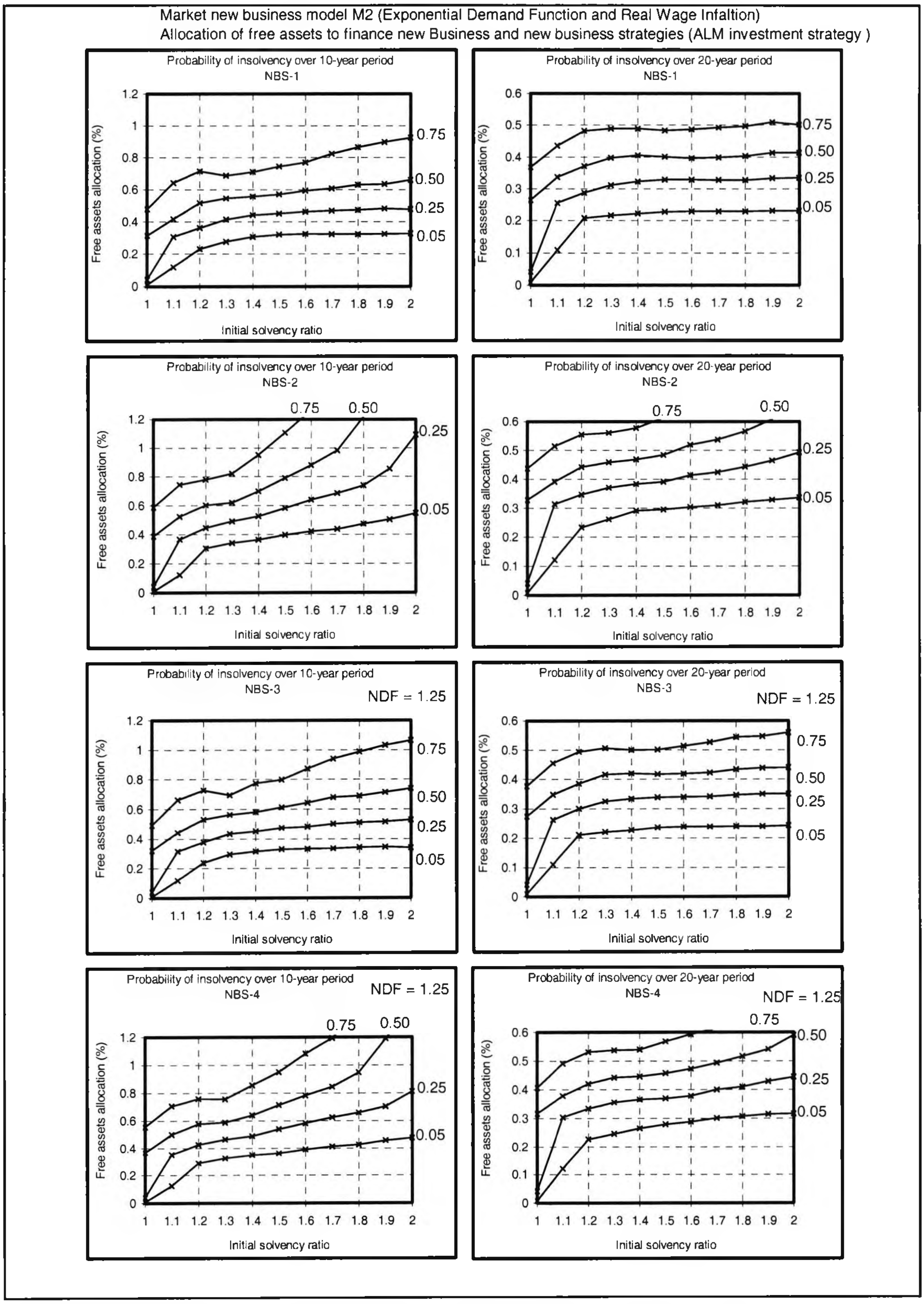


Figure 3.15: Effect of real wage inflation on new business plans and risk

The above analysis holds particularly for a company with high initial solvency ratio and constant proportion of free assets allocation (since the level of risk aversion decreases with increasing free assets), for all projection periods and also for all new business strategies. A comparison of figures 3C.3 and 3C.6 in appendix 3 reveals similar results as discussed above for comparing the figures 3.10 and 3.15, except that the position of the contours in figure 3C.3 is also higher than in figure 3C.6.

3.5.5.3 Sensitivity Analysis for Long Term Mean and Standard Deviation of Real wage Inflation

In this section, we investigate the sensitivity of probability of insolvency to the long-term mean rate and volatility of real wage inflation. We adopt Wilkie's (1995) notation and the long-term mean rate of real wage inflation parameter is $WMU = 0.016$ and the volatility (i.e. standard deviation) of real wage inflation parameter is $WSD = 0.0244$. In Wilkie's model, the estimation of the long-term mean rate and volatility of real wage inflation parameters has standard errors of 0.0029 and 0.0020 respectively. We consider different values of the long-term mean rate of real wage inflation in Wilkie's model. Wilkie's parameters (i.e. $WMU = 0.016$, $WSD = 0.0244$) are increased (or decreased) by 2 (or 5) times their standard error to obtain the new values as shown in table 3.23.

Table 3.23 shows the sensitivity of probability of insolvency over a 10-year projection period (under NBS-1) to the long term mean rate and the volatility of real wage inflation for a given combination of company's initial solvency ratio ($SR_m(0)$) and constant proportion of free assets (ρ_m). For different combinations of long term mean and volatility of real wage inflation, there is little or no change in the probability of insolvency. This holds for each combination of company's initial solvency ratio and constant proportion of free assets to finance new business plans, that we have investigated, and for all projection periods. This is mainly due to the small standard errors of the Wilkie model parameters, and the resultant effect on both the quantity of new business demanded and quantity of actual new business written of changes in the long-term mean rate and volatility of real wage inflation is also small. Therefore, we expect little or no effect on the solvency ratios in each projection year (and hence no significant change on probability of insolvency) of changes in the long-term mean rate and volatility of real wage inflation.

Table 3.23: Sensitivity to long term mean rate and std. dev. of real wage inflation

Probability of insolvency over 10-year projection period under NBS-1							
Initial solvency ratio		$SR_m(0) = 1.20$		$SR_m(0) = 1.50$		$SR_m(0) = 1.80$	
WMU	WSD	$\rho_m = 0.4$	$\rho_m = 0.6$	$\rho_m = 0.4$	$\rho_m = 0.6$	$\rho_m = 0.4$	$\rho_m = 0.6$
0.0014	0.0144	*0.303*	*0.811*	*0.129*	*0.769*	*0.108*	*0.651*
0.0014	0.0244	*0.302*	*0.812*	*0.126*	*0.771*	*0.105*	*0.651*
0.0160	0.0144	*0.322*	*0.828*	*0.145*	*0.814*	*0.127*	*0.712*
0.0160	0.0244	*0.325*	*0.828*	*0.143*	*0.81*	*0.127*	*0.704*
0.0160	0.0344	*0.325*	*0.826*	*0.141*	*0.797*	*0.128*	*0.711*
0.0304	0.0244	*0.333*	*0.831*	*0.15*	*0.838*	*0.146*	*0.756*
0.0304	0.0344	*0.337*	*0.837*	*0.151*	*0.834*	*0.148*	*0.754*

3.5.5.4 Conclusion

The above analysis shows that an increase in demand for insurance (through the real wage inflation effect) may only create solvency problems for a large risk tolerant insurer that allocates a high proportion of its free assets to write new business with the aim of increasing its market share. Thus, Browne et al (2001)'s hypothesis that "disposable income is negatively related to an insurer's performance" due to an increase in sales is likely to be more applicable to companies writing an inappropriate volume of new business without a proper free assets allocation strategy than companies with an appropriate free assets allocation strategy.

3.6 Effect of variant Life Offices (Company Models) on Relative Insolvency

In this section we examine how the company model interacts with the market model (leading to relative insolvency) in a competitive and stochastic environment.

3.6.1 Introduction

In practice, companies are more concerned about the regulator's intervention if a company, despite meeting the statutory solvency requirements, has fallen so seriously out of line with the rest of the market that it appears to be unlikely to meet policyholders' expectations, or be able to attract new business. The above scenario is likely to arise mainly due to the company's surplus distribution (for a with profit portfolio), investment and new business policies, as these affect the available free assets, (Luffrum, 1992). A company's decision to write an inappropriate volume of new business will run down its free assets, which in turn endangers the company's solvency position or places a constraint on its investment freedom and bonus distribution. On the other hand, (Luffrum, 1992), notes that by deferring the distribution of surplus, a company can release more free capital internally to ensure its solvency and enable it to support investment and new business policies, thereby improving its competitive position.

In this section, we use the stochastic model of a company offering temporary insurance contracts to investigate the effect on relative insolvency (as defined in section 3.2.3) of different investment and new business strategies. We expect a significant source of difference between life offices in UK to result from the relative effectiveness of their new business plans to meet solvency requirements. The life offices differ in making decisions on premium rate adequacy for a given new business strategy, which need to be taken alongside the assets allocation strategy and/or the new business strategy, as part of their overall business plans. Thus, in order to assess the risk that an office falls significantly out of line with the rest of the market, we consider variant life offices with different new business and investment strategies to interact with the market. The riskiness of variant life offices is examined on the basis of the probability of relative insolvency, using equations (23) and (24).

3.6.2 Variant Life Offices (Company Models)

For simplicity, all the variant offices (including the market model) are assumed to start the projection period with the same initial solvency ratio (e.g. 1.20 or 1.50). We assume that the variant life offices (using company new business model C1) interact with the market (using market new business model M1 which represents the term assurance market under new business strategy 1 and asset/liability matching ALM investment strategy). On the other hand, each variant life office has its own different combination of new business strategy (with its appropriate pricing policy i.e. the measure of premium response to solvency $l_c(\pm ve)$ and constant free assets allocation proportion to finance new business ρ_c) and investment policy.

As investment income is not an important factor affecting the price of a temporary insurance contract, we consider only two investment (ALM and static (e.g. 50% of assets invested in equities) strategies and the four new business strategies, as defined in sections 2.3.2 and 2.2.2 respectively, to form the variant life offices.

Firstly, we consider below the scenario where both the market and variant life offices have an initial solvency ratio of 1.20 at the projection date. Table 3.24 shows some possible combinations of constant free assets allocation proportion ρ_c and the measure of premium response to solvency $l_c(\pm ve)$ in order to produce a 10% or 25% probability of insolvency over a 20-year under a given new business strategy for the market model with an initial solvency ratio of 1.20. For simplicity, we assume that both the market model and the variant life offices (company models) choose from table 3.24 a desired new business plans in order produce a 25% probability of insolvency, if only operating under the market model assumptions.

Thus, we propose the following variant life offices with different combinations of new business plans (using table 3.24) and investment strategy:

Market: (NBS-1, ALM, $l_m(+ve) = 0.30$, $\rho_m = 0.28$), to produce 25% probability of statutory insolvency.

Variant Life offices:

LO-B: Same as market (NBS-1, ALM, $l_c(+ve) = 0.30$, $\rho_c = 0.28$)

LO-1: (NBS-1, and ALM, $l_c(+ve) = 0.40$, $\rho_c = 0.26$)

LO-2: (NBS-2, and ALM, $l_c(-ve) = -0.40$, $\rho_c = 0.48$)

LO-3: (NBS-3, and ALM, $l_c(+ve) = 0.40$, $\rho_c = 0.29$)

LO-4: (NBS-4, and ALM, $l_c(-ve) = -0.40$, $\rho_c = 0.40$)

LO-5: (NBS-1, and Static, $l_c(\ ve) = 0.40$, $\rho_c = 0.26$)

LO-6: (NBS-2, and Static, $l_c(-ve) = -0.40$, $\rho_c = 0.48$)

LO-7: (NBS-3, and Static, $l_c(\ ve) = 0.40$, $\rho_c = 0.29$)

LO-8: (NBS-4, and Static, $l_c(-ve) = -0.40$, $\rho_c = 0.40$)

Table 3.24: New business plans with market model to produce a level of insolvency risk

Initial solvency ratio $SR_m(0) = 1.20$		Asset/Liability Matching (ALM) strategy							
New business strategy		*NBS-1*		*NBS-2*		*NBS-3*		*NBS-4*	
Probability	P(20)	***0.10***	***0.25***	***0.10***	***0.25***	***0.10***	***0.25***	***0.10***	***0.25***
Over 20 years	$l_m(\pm ve)$	*0.40*	*0.40*	*-0.40*	*-0.40*	*0.40*	*0.40*	*-0.40*	*-0.40*
	ρ_m	*0.21*	*0.26*	*0.40*	*0.48*	*0.22*	*0.29*	*0.33*	*0.40*
	$l_m(\pm ve)$	*0.30*	*0.30*	*-0.30*	*-0.30*	*0.30*	*0.30*	*-0.30*	*-0.30*
	ρ_m	*0.22*	*0.28*	*0.35*	*0.43*	*0.23*	*0.29*	*0.31*	*0.37*

Table 3.25 shows the probability of statutory insolvency, probability of relative insolvency for both versions and also the probability that a variant office fails at least one of the relative tests over a 20-year projection period. The results for the base life office LO-B (which differs from the market mainly in the simulation of

deaths and withdrawals stochastically) indicate how much of the risk arises purely from the variation in deaths and withdrawals. The extra risk, 0.179 (0.429-0.250), the difference in probability of statutory insolvency between the base life office and the market arises from the simulation of deaths and withdrawals. On the other hand, the results for the variant life office LO-1 (which differs from the base life office only in the combination of free assets allocation proportion and the pricing policy i.e. the measure of premium response to solvency) indicate how much of risk (in term of probability of statutory insolvency) arises mainly from the variation in free assets allocation proportion. This is supported by the relative importance of the free assets allocation proportion as revealed in the sensitivity analysis of model parameters. The risk for the other variant offices arises as a result of the different new business and investment strategies adopted.

Table 3.25 shows that the base life office LO-B has the lowest risk in terms of relative insolvency tests (as expected), followed by LO-1, then the LO-3 and LO-5. For example, the probability (under version 2) that the base office LO-B solvency ratios falling below 70% of the market solvency ratios at least once over a 20-year period is about 1%, as compared to 45% for the variant office LO-2.

The Variant offices adopting new business strategy 2 (i.e. LO-2 and LO-6) are considerably more risky (irrespective of the investment strategy) than all the other offices, and followed by the variant offices adopting new business strategy 4 (i.e. LO-4 and LO-8). This is because the new business strategy 2 (which enables an office to hold back free assets for future investment in equities in order to produce higher returns over a longer term by writing less new business volume particularly in the short term) is only appropriate if there is sufficient initial capital at the projection date (e.g. the scenario with an initial solvency ratio of 1.50 is shown in table 3.27 below). New business strategy 4 also allows an office to hold back free assets for investment but to a lesser extent than new business strategy 2.

The table also shows that variant life offices adopting ALM investment strategy and a new business strategy other new business strategy 2 are clearly at less risk than the offices with a static investment strategy. Thus, where there is not sufficient initial free capital (e.g. an initial solvency ratio of less than or about 1.20), the ALM investment strategy is more appropriate for the offices than the static investment strategy, to maintain technical solvency.

Table 3.25: probability of statutory and relative insolvency over 20-year period

Life Office	Initial solvency ratio			$SR_c(0) = 1.20$
	Statutory insolvency	Relative insolvency version 1	Relative insolvency version 2	Relative insolvency versions 1 or 2
LO-B:(same as market)	*0.429*	*0.089*	*0.013*	*0.093*
LO-1: (NBS-1, ALM)	*0.331*	*0.082*	*0.071*	*0.123*
LO-2: (NBS-2, ALM)	*0.515*	*0.344*	*0.452*	*0.526*
LO-3: (NBS-3, ALM)	*0.451*	*0.132*	*0.058*	*0.158*
LO-4: (NBS-4, ALM)	*0.535*	*0.324*	*0.344*	*0.465*
LO-5: (NBS-1, Static)	*0.452*	*0.161*	*0.109*	*0.221*
LO-6: (NBS-2, Static)	*0.527*	*0.347*	*0.331*	*0.452*
LO-7: (NBS-3, Static)	*0.536*	*0.209*	*0.083*	*0.239*
LO-8: (NBS-4, Static)	*0.511*	*0.28*	*0.202*	*0.356*

The above reveals that, despite the fact that all the variant offices do not meet the statutory requirement (of say less than 25% probability of insolvency), regulators will be more concerned with the long term survival of the following variant life offices (LO-2, LO-4 and LO-6) if the overall market initial solvency ratio is about 1.20.

Secondly, we consider the same variant offices as discussed above but with an initial solvency ratio of 1.50 at the projection date. This requires a different set of values for the parameters (the constant free assets allocation proportion ρ_c and measure of premium response to solvency $l_c(\pm ve)$) under the different new business strategies.

Similarly, table 3.26 shows some possible choices of business plans in order produce a 25% probability of insolvency, if operating under the market model assumptions, available for both the market model and the variant life offices (company models) with an initial solvency ratio 1.50. Below are the new details for the market model and variant life offices with an initial solvency ratio of 1.50.

Market: (NBS-1, ALM, $l_m(+ve) = 0.30$, $\rho_m = 0.32$), to produce 25% probability of statutory insolvency.

Variant Life offices:

LO-B: Same as market (NBS-1, ALM, $l_c(+ve) = 0.30$, $\rho_m = 0.32$)
LO-1: (NBS-1, and ALM, $l_c(+ve) = 0.40$, $\rho_c = 0.30$)
LO-2: (NBS-2, and ALM, $l_c(-ve) = -0.40$, $\rho_c = 0.64$)
LO-3: (NBS-3, and ALM, $l_c(+ve) = 0.40$, $\rho_c = 0.33$)
LO-4: (NBS-4, and ALM, $l_c(-ve) = -0.40$, $\rho_c = 0.46$)
LO-5: (NBS-1, and Static, $l_c(+ve) = 0.40$, $\rho_c = 0.30$)
LO-6: (NBS-2, and Static, $l_c(-ve) = -0.40$, $\rho_c = 0.64$)
LO-7: (NBS-3, and Static, $l_c(+ve) = 0.40$, $\rho_c = 0.33$)
LO-8: (NBS-4, and Static, $l_c(-ve) = -0.40$, $\rho_c = 0.46$)

Table 3.26: New business plans with market model to produce a level of insolvency risk

Initial solvency ratio $SR_m(0) = 1.50$		Asset/Liability Matching (ALM) strategy							
New business strategy		*NBS-1*		*NBS-2*		*NBS-3*		*NBS-4*	
Probability	P(20)	***0.1***	***0.25***	***0.10***	***0.25***	***0.1***	***0.25***	***0.1***	***0.25***
Over 20 years	$l_m(\pm ve)$	*0.40*	*0.40*	*-0.40*	*-0.40*	*0.40*	*0.40*	*-0.40*	*-0.40*
	ρ_m	*0.23*	*0.30*	*0.50*	*0.64*	*0.26*	*0.33*	*0.39*	*0.46*
	$l_m(\pm ve)$	*0.30*	*0.30*	*-0.30*	*-0.30*	*0.30*	*0.30*	*-0.30*	*-0.30*
	ρ_m	*0.25*	*0.32*	*0.43*	*0.52*	*0.27*	*0.33*	*0.35*	*0.43*

Contrary to the discussions on table 3.25, table 3.27 shows that variant life office LO-8 has the lowest risk (in terms of all insolvency tests), followed by LO-6, and then by LO-4 and LO-2. This ranking of riskiness reveals that, with a high initial free capital at the projection date, offices adopting a static investment strategy and new business strategy 2 (or strategy 4) are likely to do better than other offices. The withholding of free assets for future investments (under new business strategy 2 or 4) looks reasonable with a static investment strategy since a high proportion of such assets are actually invested in equities over the longer term in order to produce higher returns.

The Variant offices adopting new business strategy 3 (i.e. LO-3 and LO-7) are considerably more risky (irrespective of the investment strategy) than the other offices, and followed by the variant offices adopting new business strategy 1 (i.e. LO-1 and LO-5) and then the base office. Table 3.27 reveals that the regulators will be more concerned with the future solvency of the variant life offices (LO-3 and LO-7) if the overall market initial solvency ratio is about 1.50, as they are not likely to attract new business in the long term.

Table 3.27: probability of statutory and relative insolvency over 20-year period

Life Office	Initial solvency ratio			$SR_c(0) = 1.50$
	Statutory insolvency	Relative insolvency version 1	Relative insolvency version 2	Relative insolvency versions 1 or 2
LO-B:(same as market)	*0.506*	*0.219*	*0.062*	*0.241*
LO-1: (NBS-1, ALM)	*0.490*	*0.183*	*0.046*	*0.195*
LO-2: (NBS-2, ALM)	*0.115*	*0.084*	*0.066*	*0.095*
LO-3: (NBS-3, ALM)	*0.530*	*0.254*	*0.100*	*0.303*
LO-4: (NBS-4, ALM)	*0.091*	*0.062*	*0.045*	*0.072*
LO-5: (NBS-1, Static)	*0.530*	*0.227*	*0.075*	*0.248*
LO-6: (NBS-2, Static)	*0.090*	*0.051*	*0.031*	*0.058*
LO-7: (NBS-3, Static)	*0.546*	*0.302*	*0.142*	*0.335*
LO-8: (NBS-4, Static)	*0.072*	*0.038*	*0.025*	*0.043*

3.6.3 Conclusion

A comparison of tables 3.25 and 3.27 shows that the solvency (either statutory or relative) position of the variant life offices adopting new business strategies 2 and 4 improves as the initial solvency ratio increases, irrespective of the investment strategy. The reverse is the case for the variant offices adopting new business strategies 1 and 3 as the initial solvency ratio increases. This is partly becuase of the underlying nature of the new business strategies and partly due to the higher constant proportion of free assets allocated to finance new business for variant life office with an initial solvency ratio 1.50. The new business strategies 1 and 3 enable offices to write more new business volume (which also causes more drain on free assets) than new business strategies 2 and 4.

The above discussion reveals that an appropriate free assets allocation strategy to finance new business is a more important aspect for the life offices adopting new business strategies 1 and 3 in order to remain solvent than those adopting new business strategies 2 and 4. In practice, life insurance companies solvency ratios rarely exceeds 1.50 (see table 2.2) and new business strategy 1, in particular, is the most intuitive strategy. Thus, decisions on assets allocation strategy need to be taken alongside a company's new business plans.

Chapter 4

Management of New Business Risk

4.1 Introduction

In the previous chapters, we consider a portfolio of n-year temporary insurance policies where all policyholders share the same characteristics: aged 55 at entry, sum assured (£100,000) and policy term (10 years). We also assume that 10, 000 such policies are issued in year 0, see market portfolio MP1 in table 4.1 below. The details of market portfolio MP1 are taken as input values for new business at projection date to obtain the simulated results (the measures of insolvency risk) over 10-year and 20-year periods.

In this chapter we consider, without loss of generality, different hypothetical market portfolios (as shown in table 4.1) with 10-year temporary insurance and /or non-profit endowment insurance policies. On the other hand, we also consider hypothetical company portfolios which interact with a given market portfolio, and they are assumed to write a proportion (not exceeding 25% in year 0) of the market portfolio. The hypothetical portfolios are using the most intuitive strategies, namely market new business model M1 and /or company new business model C1 with new business strategy 1 (NBS-1) and asset liability matching (ALM) investment strategy, to obtain simulated results over 20-year period. These results may be compared with the results from portfolio MP1 above where appropriate. Each hypothetical portfolio may be sub-divided into cohorts of different ages at entry with a pre-defined number of policies assumed to be issued in year 0. We assume that a market portfolio in table 4.1 may represent a set of companies in a target market with different sizes measured in terms of initial solvency ratio. The investigation in different sections in this chapter may use different combinations of the market portfolios in table 4.1.

Table 4.1: Distribution of new business by entry age in year 0 for market portfolios

Market portfolio	*Age at entry* x	Type of policy	*No. of new policies issued in year 0*	Annual office premium per policy in year 0	Sum assured in year 0
MP1	*55*	PT1	*10,000*	*£874.20*	*£100,000*
MP2	*45*	PT1	*10,000*	*£342.96*	*£100,000*
MP3	*35*	PT1	*5,000*	*£183.60*	*£100,000*
	55	PT1	*5,000*	*£874.20*	*£100,000*
MP4	*35*	PT1	*1,000*	*£528.90*	*£100,000*
	55	PT1	*5,000*	*£528.90*	*£100,000*
MP5	*35*	PT1	*2,000*	*£528.90*	*£100,000*
	55	PT1	*5,000*	*£528.90*	*£100,000*
MP6	*55*	PT1	*5,000*	£874.20	£100,000
	55	PT2	*5,000*	£874.20	£100,000
	55	PT3	*5,000*	£1,340.45	£10,000
	55	PT4	*5,000*	£890.75	£10,000
MP7	*35*	PT1	*3300*	*£183.60*	*£100,000*
	45	PT1	*3400*	*£342.96*	*£100,000*
	55	PT1	*3300*	*£874.20*	*£100,000*
MP8	*35*	*PT4*	*5,000*	*£859.25*	*£10,000*
	55	*PT4*	*5,000*	*£890.75*	*£10,000*
MP9	*35*	*PT1*	*2,500*	*£183.60*	*£100,000*
	55	*PT1*	*2,500*	*£874.20*	*£100,000*
	35	*PT4*	*2,500*	*£859.25*	*£10,000*
	55	*PT4*	*2,500*	*£890.75*	*£10,000*
MP10	*35*	*PT1*	*4,000*	*£183.60*	*£100,000*
	55	*PT1*	*4,000*	*£874.20*	*£100,000*
	35	*PT4*	*1,000*	*£859.25*	*£10,000*
	55	*PT4*	*1,000*	*£890.75*	*£10,000*

Note:
PT1- Policy Type 1 (10-year temporary insurance contract with initial commission payment period of 4 years)
PT2- Policy Type 2 (10-year temporary insurance contract with initial commission payment period of 1 year)
PT3- Policy Type 3 (10-year non profit endowment insurance contract with initial commission payment period of 4 years)
PT4- Policy Type 4 (10-year non profit endowment insurance contract with initial commission payment period of 1 year)
MP = Market Portfolio

As a reminder, we note that company new business model C1 and market new business model M1 both involve exponential demand functions. New Business Strategy 1 allows a company to reduce (increase) its premium relative to the profit-tested premium to attract more (less) quantity of new business demanded in a particular projection year if its solvency ratio is above (below) one in the previous year. In asset-liability matching (ALM) investment strategy, all liabilities are backed by fixed securities and only free assets are invested in equities. A brief description of new business models, new business strategies and investment strategies are also given in sections 1.1.2, 1.1.3, and 1.1.4 of appendix 1 respectively.

As the inter-relationship of all the different variables in this research is complex, we have inevitably employed graphical outputs of simulated results of the different hypothetical portfolios, in a form which can be easily understood, in investigating how a company's new business plans (proportion of free assets allocated to finance new business) and risk (through the risk measures as defined in chapter 3) over a 20-year period can be managed by:

- Underwriting
- Product design
- Mix of new business
- Constrain new business growth
- Reinsurance

4.2 Underwriting

4.2.1 Introduction

A life insurance company is likely to be exposed to two fundamental risks associated with pricing its products, namely that "the premium rates are not appropriate for the lives concerned, and that premium rates permit the prospective policyholders to select against the company", (Subject 302 Core Reading, Unit 14). In an attempt to counter the above risks some selection control of the insured lives (e.g. underwriting) must be exercised by the company. Luffrum (1990) defines underwriting as "the process by which an office decides whether to accept or decline or postpone a particular proposal made to it and, where it is decided to accept, on what terms". The ways in which underwriting can be used to manage the above risks, as stated in Subject 302 Core Reading Unit 14, include the following:

- To protect an insurer from adverse selection e.g. from the tendency for high risk lives to be more likely to insure or more likely to insure for higher coverage, (Cummins et al,1991).
- To identify lives with a substandard health risk for whom special terms would need to be quoted.
- Adequate risk classification within the underwriting process will help to ensure that all risks are priced fairly.
- To ensure that actual mortality experience does not depart too far from that assumed in the pricing of the contracts being sold.

The key aim of underwriting is accurate risk classification, so that the correct premium is charged and adverse selection is avoided.

In the following section, we consider how risk classification can be used to manage the new business risk, while the effect of the level of underwriting requirements on demand for insurance and hence on new business plans and risk are considered in section 4.5.

4.2.2 Risk Classification

A life insurance company's capital needs for financing new business and risk of insolvency can be managed by adequate risk classification methods as part of underwriting its new business. Cummins et al (1991) define risk classification as "the process of separating into groups (classifying) potential insureds (risks) by various pieces of information, such as age and occupation". Thus, risk classification will enable a company to identify high-and low-risk policyholders and offer life insurance coverage to each group at actuarially fair premium rates.

A life insurance company's inability to classify risks may arise if it is unable to measure loss probabilities accurately before issuing life insurance coverage or because regulation does not permit it to classify risks. In some countries, a life insurance company's ability to underwrite is restricted, for example, the use of the results of genetic testing in underwriting is prohibited. The anti-discrimination law in New Zealand prevents life insurers from refusing to insure anyone, whatever their state of health. However, there is no restriction on the insurers to charge an appropriate premium for the risk, (Subject 302 Core Reading, Unit 4).

An insurer's inability to classify risks may lead it to offer an average premium to everyone who applies for life insurance coverage. But this will result in fewer (more) low-risk (high-risk) consumers purchasing life insurance cover. Cummins et al (1991) note that adverse selection may arise because of an information asymmetry, that is, the prospective buyers of life insurance know their loss probabilities but the life insurance companies either do not know or are not permitted to use this knowledge.

As no risk group is subsidising any other under a well structured risk classification system, it is more likely for a life insurance company with appropriate risk classification methods to meet its statutory solvency requirements than a company without appropriate risk classification methods.

In the next section, we investigate (with different portfolios) how risk classification can assist a company in managing its capital requirement to finance its new business plans in order to meet a given level of statutory insolvency risk.

4.2.3 How can the new business risk be managed by risk classification?

In this section, we consider three simplified hypothetical market portfolios (MP3, MP4 and MP5 in table 4.1) based on the discussion and graphical illustration of adverse selection in (Cummins et al, 1991). We consider 10-year temporary life insurance policies, all with the same sum assured. Each portfolio has two types of policyholders, namely low-risk and high-risk policyholders aged 35 and 55 at entry respectively. Thus, the consumer's age represents the risk variable for risk classification in the pricing process.

In portfolio MP3 (with risk classification), the 10,000 new policies issued in year 0 in the temporary insurance market described in chapter 2 (portfolio MP1) are now subdivided into low risks and high risks, as shown in table 4.2 below. We assume that portfolio MP3 has an equal number of low and high risks (e.g. 5000 new policies in each risk group) in year 0, who are being charged the actuarially fair premiums.

Table 4.2: Market portfolios in year 0 for PT1 with (no) risk classification

Market Portfolios (MP)	Age at entry x	No. of new policies issued in year 0	Annual office premium per policy in year 0	Sum assured in year 0
MP3	*35*	*5,000*	*£183.60*	*£100,000*
(with risk classification)	*55*	*5,000*	*£874.20*	*£100,000*
MP4	*35*	*1,000*	*£528.90*	*£100,000*
(no risk classification)	*55*	*5,000*	*£528.90*	*£100,000*
MP5	*35*	*2,000*	*£528.90*	*£100,000*
(no risk classification)	*55*	*5,000*	*£528.90*	*£100,000*

Note:
PT1- Policy Type 1 (10-year temporary insurance contract with initial commission payment period of 4 years in table 2.1)

In response to the average premium $(£528.90=(£183.60+£874.20)\div 2$ for both ages 35 and 55) being charged under the no risk classification method of pricing, there are fewer low risks (as they are being overcharged) in portfolios MP4 and MP5 than in portfolio MP3. We assume that the low-risk group in portfolio MP5 are sufficiently risk averse to subsidise (to some extent) the high-risk group relative to the low risks in portfolio MP4, and hence there are also fewer low risks in portfolio MP4 than in portfolio MP5. In the market portfolios (without risk classification), the number of high risks purchasing life insurance remains the same since they are being undercharged.

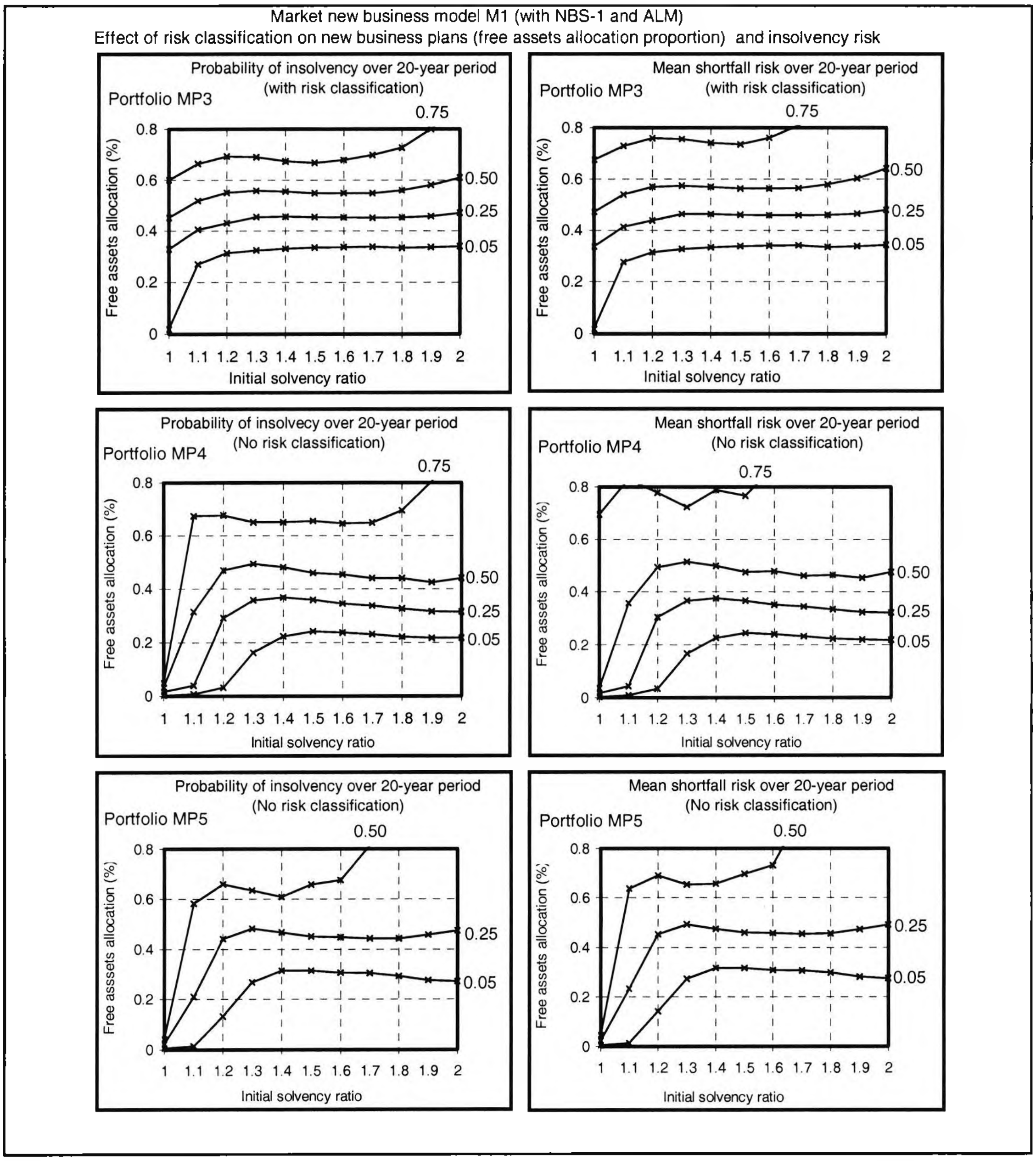


Figure 4.1: Effect of risk classification on new business plans and insolvency risk

We consider the details of each portfolio in table 4.2, taking as initial input values for new business issued in year 0, and the corresponding simulated results (the measures of insolvency risk) for each portfolio over a 20-year period are shown in figure 4.1. The effect of risk classification on the new business plans (proportion of free assets allocated to finance new business) and hence on the insolvency risk can be examined by comparing the simulated results in market portfolio MP3 (risk classification) with portfolio MP4 or portfolio MP5 (no risk classification).

Firstly, a comparison of portfolios MP3 and MP4 in figure 4.1 shows that all of the contour lines in portfolio MP4 (with no risk classification) are lower than those in portfolio MP3 (with risk classification). This reveals that, for a given initial solvency ratio, a company with risk classification will be able to allocate more free assets to finance new business to produce a given level of insolvency risk over a 20-year period than a company without risk classification. For a company with risk classification, the constant proportion of free assets allocated to finance new business increases with the company's initial size (measured in terms of initial solvency ratio) whilst the proportion of free assets allocated for a company without risk classification is unstable as its initial size increases. This is because a pricing method with risk classification, as expected, enables a company to produce a more stable level of profitability over a period in order to meet its statutory solvency requirements, than a pricing method without risk classification. The above is true for both measures of risk (probability of insolvency and mean shortfall risk).

Figure 4.1 shows that a pricing method without risk classification (e.g. in portfolio MP4) has a significant effect on small companies (e.g. with initial solvency ratio less than 1.2) than others. Companies with little or no free assets cannot afford the heavy losses that will arise from high-risk policyholders. On the other hand, the figure also shows that a pricing method without risk classification has a significant effect on large companies (e.g. with initial solvency ratio greater than 1.5) than medium size companies (e.g. with initial solvency ratio between 1.2 and 1.5). This is because a large company with no risk classification under new business strategy 1 is likely to reduce its premium substantially in order to attract more new business (i.e. more high-risk policyholders in this case as low-risks are not sufficiently risk averse in portfolio MP4) than a medium size company without risk classification. The large company's heavy death claims and low premium income may cause a substantial drain on its free assets leading to a higher level of insolvency risk relative to a medium size company.

Secondly, a comparison of portfolios MP3 and MP5 in figure 4.1 shows that, the contour lines in portfolio MP5 are likely to be higher than those in portfolio MP3 (particularly for a high level of insolvency risk) for a given size of a company except for the very small companies. This is caused by an increase in demand from the low risks, as they are more risk averse in portfolio MP5 relative to portfolio MP4 and subsidise (to some extent) the high risks. This reveals that, in this unlikely event that low risks become more risk averse, a company without risk classification may be able to allocate more free assets to finance new business in order to produce a given level of insolvency risk than a company with risk classification. This is because the company benefits from overcharging the low risk consumers which may not be always true in practice. This holds for both risk measures and for all companies (except the small companies) depending on the degree of risk aversion for low risks in the market.

In each portfolio, the contour lines (particularly above 50% level of risk) for mean shortfall risk are higher than those for probability of insolvency, but the discussions above also hold for this measure of risk.

4.2.4 Conclusion

A pricing method with risk classification will enable a company to have an efficient use of its free capital in writing more new business without having solvency problems. A pricing method with no risk classification has a more adverse effect (in terms of insolvency risk) on small companies than others, but even large companies that do not use risk classification are not fully secure.

The above results are not surprising because the failure to use risk classification is a type of systematic error which extra pooling of risks will not remove (i.e. as the size of the portfolio tends to infinity).

4.3 Product Design

4.3.1 Introduction

The factors which a company takes into consideration in designing or reviewing any product will need to reflect the nature of the existing product range and the distribution channels involved. These factors may include the following: profitability, marketability, competitiveness, financing requirement, and risk characteristics of the product, (Subject 302 Core Reading, Unit 8).

The combination of initial cash-flow (e.g. arising from marketing costs, underwriting costs, initial administration expenses and others), the need to hold prudent reserves and the need to establish a required solvency margin when writing a new business will lead to an initial capital strain, (Subject 302 Core Reading, Unit 8). Thus, the issue of a relatively large volume of new business with initial strains can seriously threaten statutory solvency, particularly for new or rapidly expanding life offices, (Booth et al, 2005). The initial capital may be recouped quickly or only very slowly depending on the contract design and the particular supervisory regulations. All other things being equal, a company is likely to prefer contract types and designs that lead it to recoup the invested capital quickly, so that the capital can be used to fund further profitable business.

For a traditional non-linked life insurance product (such as temporary insurance and endowment insurance) the initial capital strain can be reduced by reducing the initial acquisition expenses, initial administration expenses and valuation strain. The difficulty in reducing the strain is that these aspects may reflect the environment in which the product is sold rather than the product design per se.

However, “the most immediately controllable element in a product design context will be the commission scale”, (Subject 302 Core Reading, Unit 8).

In the next section, we consider how different contract designs may assist a company in reducing the capital needs for financing new business (initial new business strain) and its overall risk of insolvency in writing life insurance business. Thus, we consider the effect of changing the commission terms in the temporary insurance contract (policy type 1 (PT1) as shown in table 2.1 in chapter 2) which is being used for the analyses in previous chapters. We also compare the effect of a non-profit endowment insurance contract design (in terms of capital required to finance new business) with the temporary insurance contract design.

4.3.2 How product design can reduce a company’s capital needs and risk

In this section, we consider a simplified hypothetical market portfolio (MP6 in table 4.1), with four policy types (two temporary insurance and two non-profit endowment insurance policies), as shown in table 4.3. We have taken the sum assured for the temporary insurance policy (£100,000) to be different from that for the non-profit endowment insurance policy (£10,000) in order to make the difference in premium between the contract types not too large.

The contract details of each policy type are given in section 4A of appendix 4. Briefly, policy types 1 and 2 are temporary insurance policies. Policy type 2 is designed to have the same premium (£874.20) with policy type 1 but it differs only in terms of initial commission terms (e.g. 125% and 426% of LAUTRO commission for policy type 1 and policy type 2 respectively). On the other hand, policy types 3 and 4 are non-profit endowment insurance policies. Policy type 3 is similar to policy type 1 in terms of commission terms and expense loadings while it differs from policy type 4 in terms of commission payment period and expense loadings.

Table 4.3: Market portfolio with different policy types

Market Portfolio (MP)	Age at entry x	Type of Policy	Annual Office per policy at time 0	Sum Assured at time 0
MP6	55	PT1	£874.20	£100,000
	55	PT2	£874.20	£100,000
	55	PT3	£1,340.45	£10,000
	55	PT4	£890.75	£10,000

Note:

PT1- Policy Type 1 (10-year temporary insurance contract with initial commission payment period of 4 years)

PT2- Policy Type 2 (10-year temporary insurance contract with initial commission payment period of 1 year)

PT3- Policy Type 3 (10-year non profit endowment insurance contract with initial commission payment period of 4 years)

PT4- Policy Type 4 (10-year non profit endowment insurance contract with initial commission payment period of 1 year)

We assume that, for each policy type, portfolio MP6 issues 5,000 new policies to lives aged 55 at entry in year 0. The simulated results (probability of insolvency and mean shortfall risk) for individual policy types over a 20-year period are shown separately in figures 4.3 and 4.4 respectively. It is important to note that the overall results for market portfolio MP6 is not shown in this section and also in subsequent sections where it is considered.

Figure 4.2 shows the means of the distributions of new business strain per policy and new business volume written in projection years for a company writing the different policies stated in table 4.3, having an initial solvency ratio of 1.50 and allocating 40% of its free assets in writing new business over the projection period. The figure shows that there is a relatively larger new business strain arising from policy type 2 (with an initial commission of 426% of LAUTRO commission) than policy type 1 (with an initial commission of 125% of LAUTRO commission) in the first year of policy. We expect that the difference in capital strain between the two contracts (with the same level of premium) may decrease with increasing policy duration within the first four policy years. This is because the renewal commission (2.5% of premium) is payable from the start of second and fifth policy years under policy types 2 and 1 respectively.

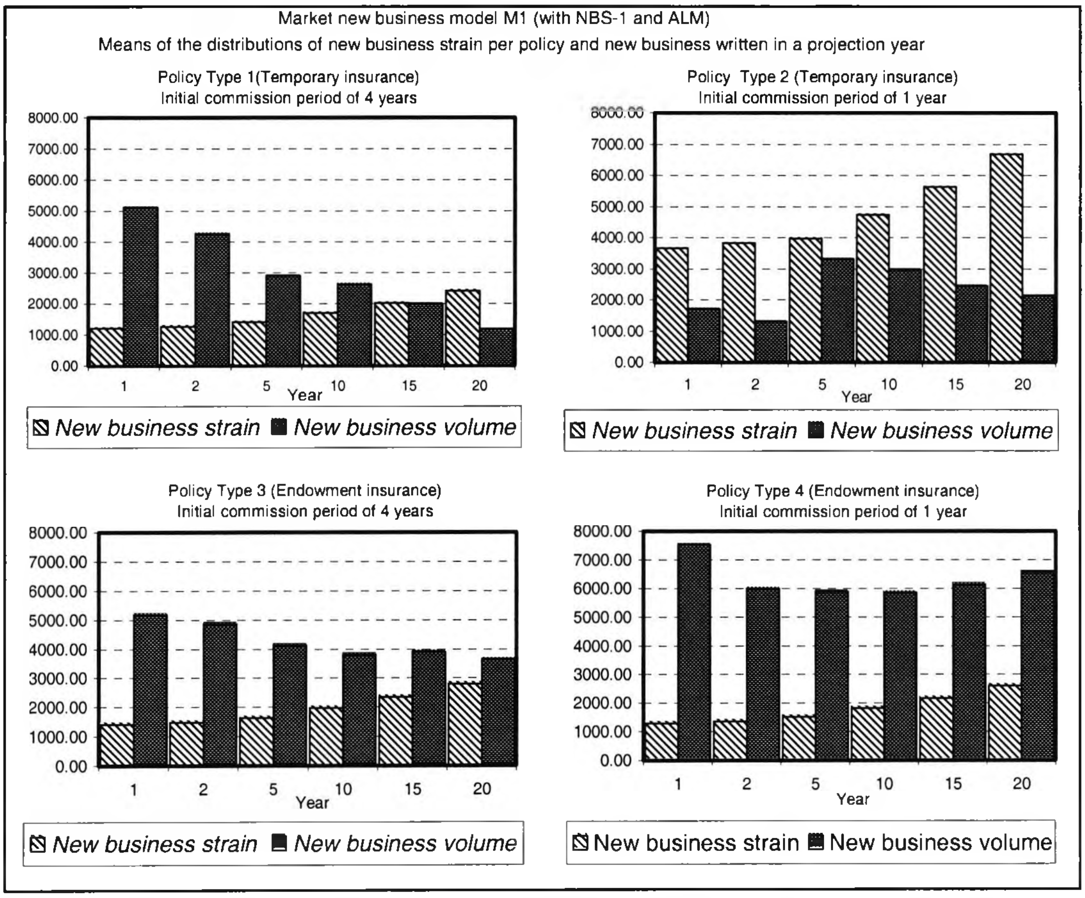


Figure 4.2: Distribution of new business strain and written for different policy types.

Thus, considering the new business strategy 1 and equation (8), a company with policy type 2 (having a relatively high new business strain) will reduce the quantity of new business written (proportionately in line with an increase in initial capital strain) more in the early years of projection than a company with policy type 1, see figure 4.2.

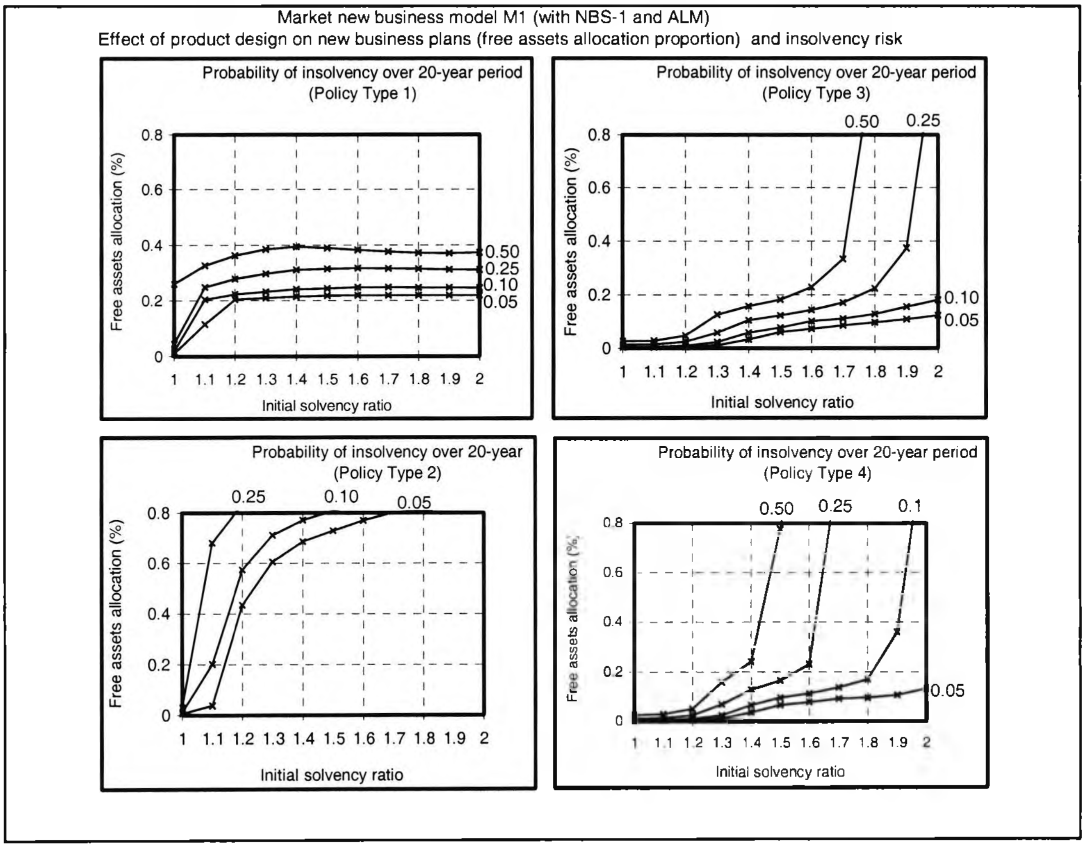


Figure 4.3: Effect of product design on new business plans and probability of insolvency.

Figure 4.3 shows the effect of contract types on the company's new business plans and probability of insolvency. The figure shows that, for a given combination of initial size of company (in term of initial solvency ratio) and free assets allocation proportion to finance new business, a company with policy type 2 is less risky than a company with policy type 1. This is reflected in the contour lines for policy type 2 which are higher than those for policy type 1, and the reasons are explained below.

Firstly, this arises because the company with policy type 2 is able to reduce its new business volume written (in line with an increase in initial capital strain) more than a company with policy type 1 in the early years of the projection. This leads to a more efficient use of free assets and improvement in the company's solvency position, since the company will not be writing inappropriate volume of new business which may cause a drain on its free assets.

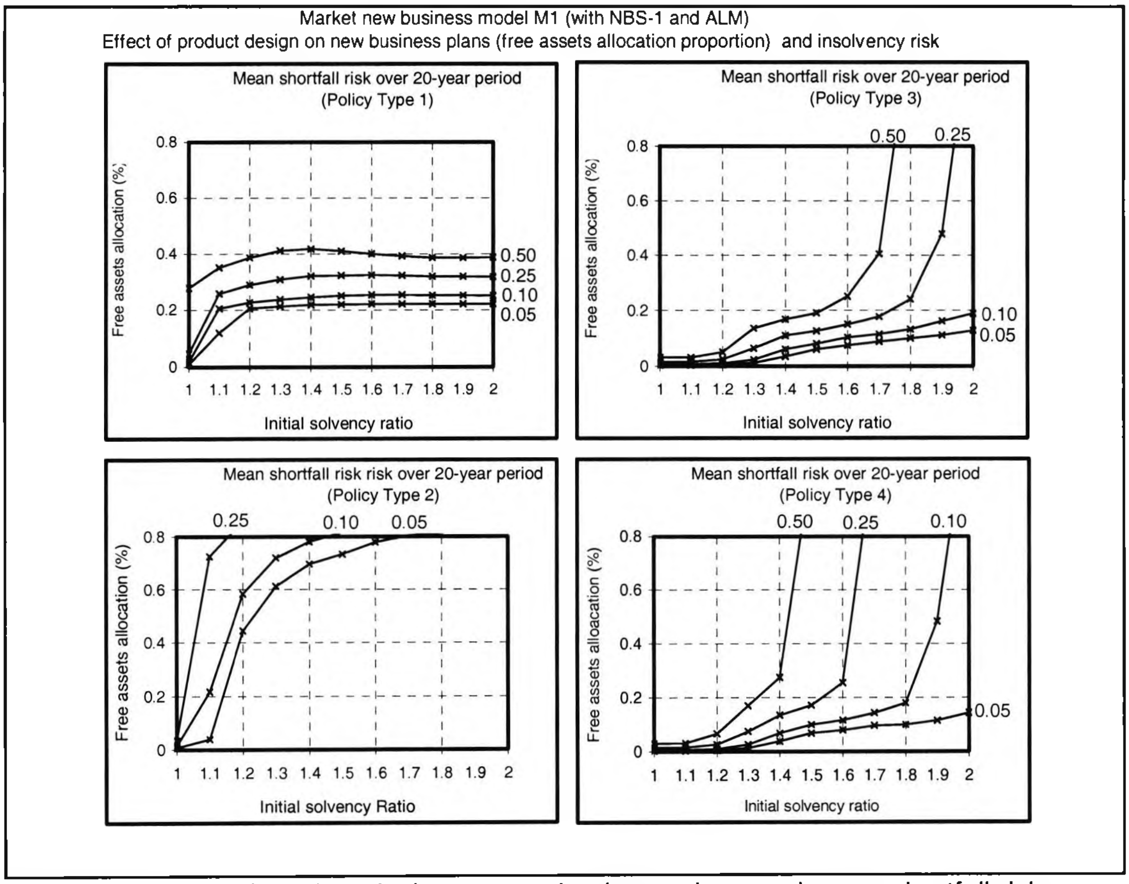


Figure 4.4: Effect of product design on new business plans and mean shortfall risk

Secondly, the shorter is the initial commission period in a product design (as with policy type 2) the more likely is it that the company with policy type 2 will meet its future solvency requirements. Thus, the company's initial capital invested in writing new business in the early years of a projection can be recouped quickly, which may enable it to write a relatively steady and higher volume of new business in the long term than a company with policy type 1, as shown in figure 4.2. The above holds for both measures of risk (probability of insolvency and mean shortfall risk) in figures 4.3 and 4.4 respectively, except for very small companies with little or no initial free assets.

Figures 4.2 and 4.3 also show that a company with policy type 2 is more likely to have an efficient use of its capital in writing new business in order to achieve a given level of probability of insolvency than a company with policy type 1. This is because the drain on the free assets, investment returns and positive net cash flows from the existing business may reduce, leading to an improvement in its future solvency position, as the company with policy type 2 is able to control the new business volume written (steady new business volume in figure 4.2). In other words, the company with policy type 1 writes a higher volume of new business in the early projection years than in the future years, which may result in placing a constraint on its investment freedom. Thus, a company with policy type 2, having a high initial solvency ratio of about 1.9, is likely to remain solvent irrespective of the proportion of free assets allocated to fund new business. We note that no contour lines are shown in figure 4.3 for a company with an initial solvency ratio of 1.90 and above. This means that the probability of insolvency is less than 5%.The above also holds for the mean shortfall risk measure (and the benchmark figure of 5%) as shown in figure 4.4.

For the purpose of comparison, we consider a non typical contract design in practice, a non profit endowment insurance contract (policy type 3 (PT3)), which has the same contract details (including the commission terms and expense loadings) as policy type 1 (temporary insurance contract), except for the surrender and maturity benefits payable in the former policy type.

The initial assets at the projection date are defined as a multiple (e.g. a given initial solvency ratio) of the liabilities (the reserves for in-force business calculated) at the projection date. Thus, the available free assets for financing future new business at the projection date for a temporary insurance portfolio are lower (as the reserves are lower) than for an endowment insurance portfolio (as the reserves are higher), for a given initial solvency ratio (e.g. $SR_m(0) = 1.50$). The relatively high initial free assets for policy type 3 allows a company to reduce its premium to attract more business under new business strategy 1 than a company with policy type 1 , particularly in the short term. The high initial free assets may enable a company with endowment insurance (e.g. policy type 3) to write a higher volume of new business over the projection period than a company with temporary insurance (e.g. policy type 1), as shown in figure 4.2. This is reflected in figure 4.2 which shows that the mean of the distribution of new business volume written for policy type 3 is higher than for policy type 1, despite the higher new business strain for policy type 3.

Comparing policy type 1 with policy type 3 in figure 4.3 shows that a company with little or no free assets is more at risk in writing a non profit endowment insurance (PT3) than a temporary insurance policy (PT1). This is because a company requires more initial free capital in writing a non-profit endowment insurance policy (PT3) than a temporary insurance policy (PT1) to produce a given level of insolvency risk. The relatively high volume of new business written by a company with policy type 3 in the early projection years may lead to a drain on the free assets thereby placing a constraint on investment in risky assets in the future and hence producing a higher level of insolvency risk.

"Broadly speaking, the lower the initial reserves, the lower the initial capital requirement, and the slower the increase in reserves over the contract's term, the faster any invested capital is released", (Subject 302 Core Reading, Unit 2). Thus, as it is typical for a temporary insurance policy to have relatively small reserves throughout the policy term, there is more likely to be a higher initial capital strain from a non-profit endowment insurance policy than from a temporary insurance, all other things being equal, as shown in figure 4.2.

As discussed above, policy type 4 (PT4) differs from policy type 3 (PT3) mainly in terms of initial commission payment periods and expense loadings, but both contracts are likely to have significant (not necessarily equal) initial capital strains. Figure 4.2 shows that policy type 4 produces a lower new business strain (and hence enables a company to write more new business) than policy type 3. This is because the policy details of type 4 produce an office premium lower than that for policy type 3, as shown in table 4.3. Furthermore, they have the same initial commission rate but different initial commission payment periods, and the initial per policy amounts for the policy types differ by just £35. Comparing policy type 4 with policy type 3 (in figure 4.3) shows that the contour lines for policy type 4 are higher than those for policy type 3, except for a company with insufficient initial free assets (i.e. initial solvency ratio of less 1.4). This reveals that that the period of initial commission payment has an effect on a company's free assets allocation proportion used to finance new business and on its overall level of insolvency risk.

Comparing the most typical contract types in practice (policy type 1 with policy type 4) in figure 4.3 also shows that the different contracts affect a company's new business plans (in terms of free assets allocation proportion) differently. The figure shows that only companies with sufficient initial free assets (e.g. initial solvency ratio of above 1.50) can afford to write substantial volume of non-profit endowment insurance contracts (PT4) without facing a higher risk of insolvency over a longer term relative to writing temporary insurance contracts (PT1). This is because of the relatively high initial capital requirement and the effect of short initial commission payment period for policy type 4. This also holds for the mean shortfall risk measure as shown in figure 4.4.

The 10% probability of insolvency contour lines in figure 4.3 show that policy type 2 has the lowest insolvency risk than other policy types, while policy type 3 produces the highest insolvency risk than others. Policy type 4 produces lower insolvency risk than policy type 1 when there are sufficient initial free assets, and vice versa. The above ranking of riskiness in policy types also holds for mean shortfall risk measure shown in figure 4.4.

4.3.3 Conclusion

The above analysis reveals that a company which is able to reduce its new business volume in line with an increase in new business strain arising from the contract design is likely to remain solvent and this may lead to an efficient use of free assets.

The shorter is the initial commission payment period in the product design, the more likely is the company to meet its solvency requirements, as it may enable the company to recoup quickly the initial capital invested than a longer initial commission period. Thus, this may also lead to more efficient use of free assets as reflected in the company's free assets allocation proportion to finance new business, particularly for the large companies.

Different contracts affect a company's new business plans (in terms of the free assets allocation proportion) differently. It is only companies with sufficient initial free assets that can afford to write non-profit endowment insurance contracts without facing a high risk of solvency over the longer term due to the relatively high initial capital requirement.

In the following sections, we consider policy types (PT1) and/or (PT4) temporary insurance and non-profit endowment insurance respectively in carrying out any further analysis, which are the product designs that are modified from UK insurance industry.

4.4 Mix of New Business

4.4.1 Introduction

A change in the mix of new business can lead to a change in the risk profile or capital needs of a company. The mix of new business can change by the nature and size of contract, and by source (distribution channels). The change in the mix of new business by nature of contract may involve a change in the mix by class of business (e.g. linked versus non linked business), type of contract (e.g. temporary insurance versus endowment insurance) and others.

In the following sections, we examine how the insolvency risk and the capital needs of a company to finance new business can be controlled by changing the mix of new business by source (distribution channels) and by type of contract.

4.4.2 Mix of new business by source (distribution channel)

The products sold by a life insurance company will have developed with regard to its target market which will have a bearing on the mortality assumptions used in its product pricing. Thus, an office may restrict its marketing effort to particular geographical areas or socio-economic groups giving rise to 'class selection'. The class of life may also be influenced by the distribution channels and the premium structure used by an office, (Luffrum, 1990).

In this section, we assume that the price for a life insurance contract varies by risk group (i.e. there is risk classification in the pricing process) and not by distribution channel. The different populations targeted by different channels of distribution can exhibit different experience in mortality (Subject 302 Core Reading, Unit 4). This is because different channels are likely to appeal to different people according to their level of income and financial awareness, which tend to correlate with their mortality experience (Subject 302 Core Reading, Unit 4). A change in distribution channel may also change the target market and this may have a resultant effect on the distribution of new business by age at entry in a portfolio. Thus, we may consider a change in distribution of new business by age at entry as a proxy for a change in the mix of new business by distribution channel.

The next subsection looks at the effect of changes in the distribution of new business by age at entry in the temporary insurance market.

4.4.2.1 How a change in mix of new business by entry age can reduce insolvency risk

In this section, we consider three simplified hypothetical market portfolios (MP2, MP3 and MP7 in table 4.1), each representing companies in a given target market, with the same contract type (Policy Type 1, PT1) but with a different distribution of new business by age at entry in year 0, as shown in table 4.4.

Table 4.4: Mix of new business by entry age in year 0 for PT1

Market portfolio (MP)	Age at entry x	No. of new poicies issued in year 0	Annual office premium per policy in year 0	Sum assured in year 0
MP2	*45*	*10,000*	*£342.96*	*£100,000*
MP3	*35*	*5,000*	*£183.60*	*£100,000*
	55	*5,000*	*£874.20*	*£100,000*
MP7	*35*	*3,300*	*£183.60*	*£100,000*
	45	*3,400*	*£342.96*	*£100,000*
	55	*3,300*	*£874.20*	*£100,000*

Note:
PT1- Policy Type 1 (10-year temporary insurance contract with initial commission payment period of 4 years in table 2.1)

We assume that each of these portfolios has issued 10,000 new policies in year 0, which are subdivided into risk groups (arbitrarily) with entry ages 35, 45 and/or 55, but with an average entry age of 45. The office premiums for these policies are calculated using the policy details in table 2.1 in chapter 2, except that the entry age is changed. The details of each portfolio in table 4.4 are taken as initial input values for the new business issued in year 0.

The market portfolios in table 4.4 differ from market portfolio MP1 (which represents companies operating in the temporary insurance market that target only 10,000 high-risk consumers aged 55 at entry in year 0), the scenario considered in previous chapters. Thus, these market portfolios are assumed to represent companies in different target markets having changed their distribution channels, leading to a change in the distribution of new business by age at entry which results in a lower average age at entry than that for market portfolio MP1.

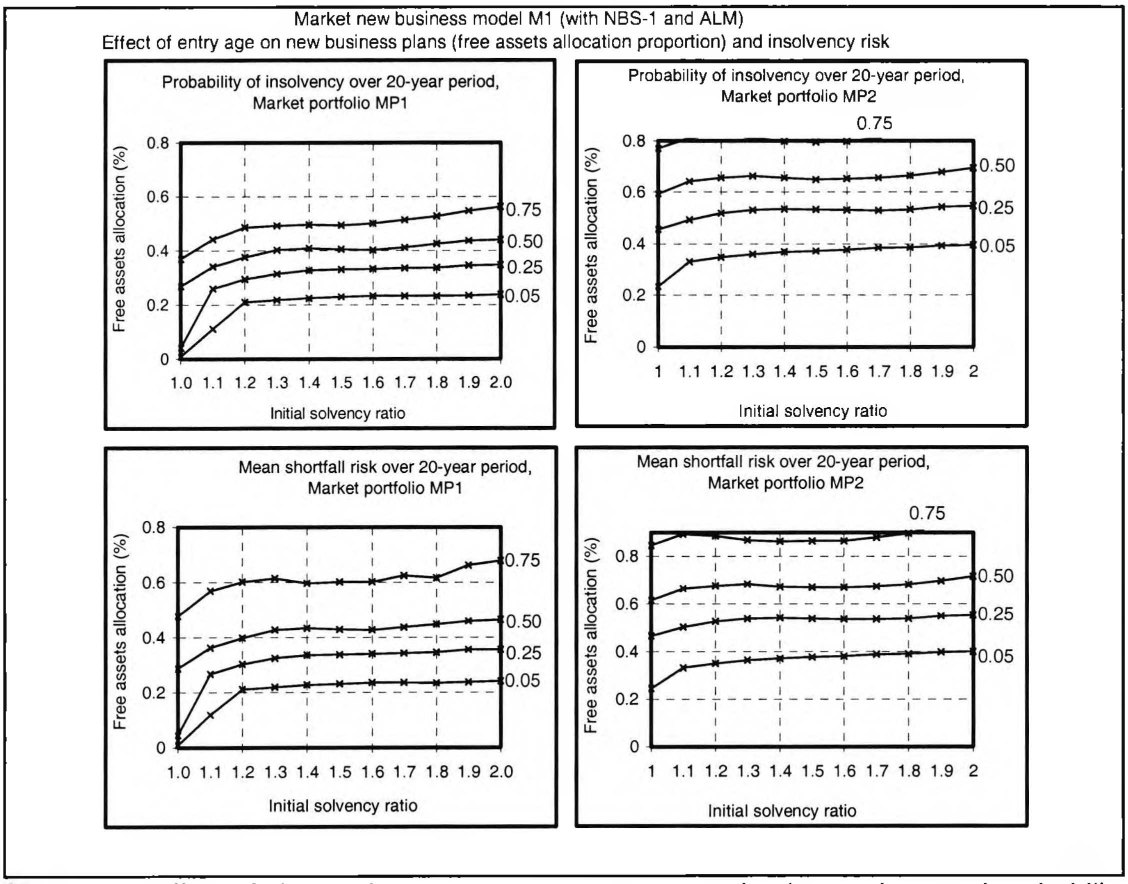


Figure 4.5: Effect of change in average entry age on new business plans and probability of insolvency.

However, market portfolio MP2 is similar to Market portfolio MP1 except for the lower entry age of 45. Thus, the analysis in the previous chapters may have changed if a different market portfolio (e.g. market portfolio MP2) is considered, as different average ages at entry may have different effects on the number of deaths and hence on a company's profitability and solvency. Figure 4.5 shows the simulated results (the probability of insolvency and mean shortfall risk over a 20-year period) for market portfolios MP1 and MP2.

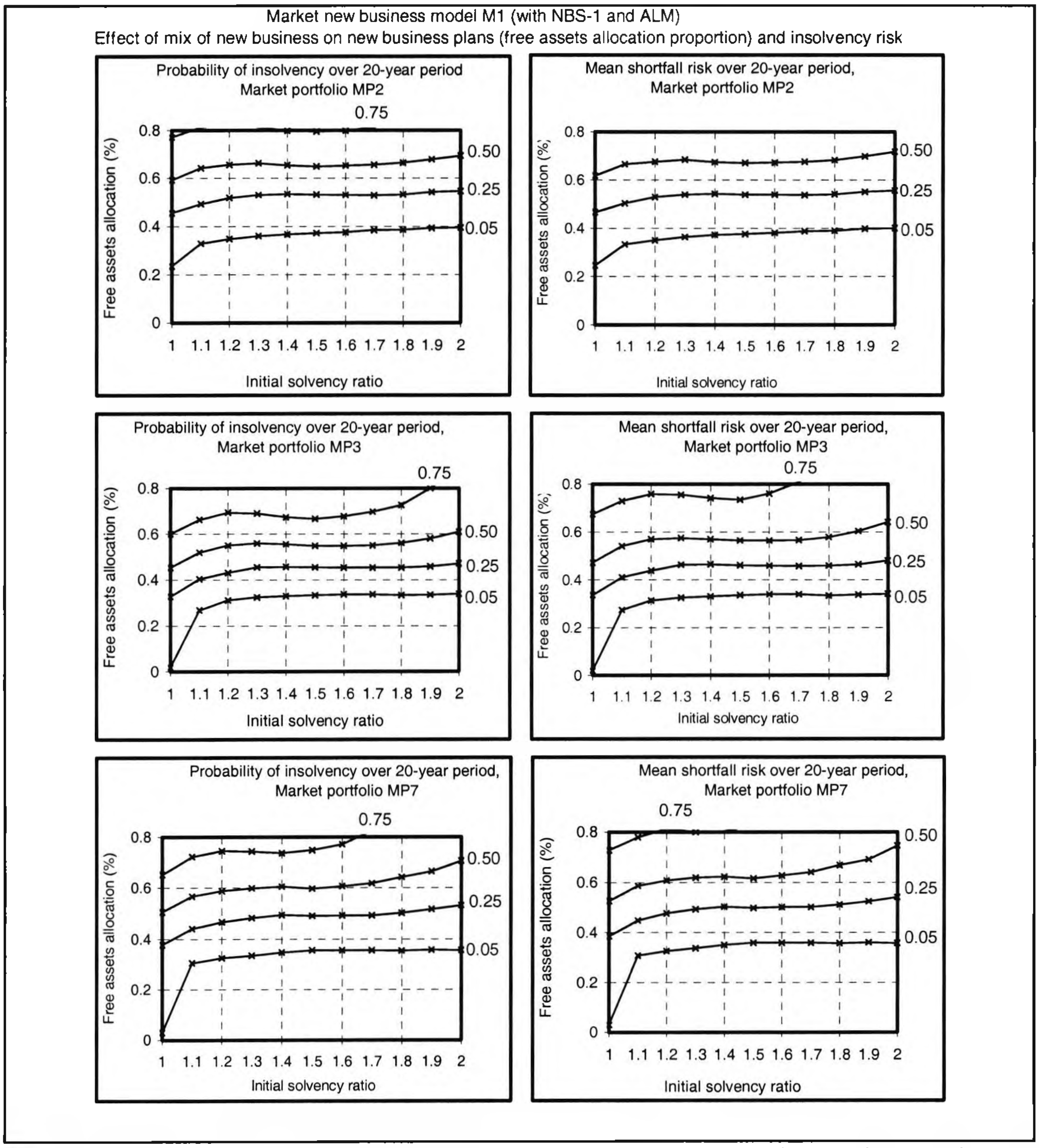


Figure 4.6: Effect of mix of new business by entry age on new business plans and mean shortfall risk.

Briefly, figure 4.5 shows that, for a given combination of constant proportion of free assets allocated to finance new business and a company's initial solvency ratio, the insolvency risk is lower under portfolio MP2 than under market portfolio MP1 (with only high-risk insureds aged 55 at entry). The contour lines for portfolio MP2 are higher than that those of portfolio MP1. This reflects the feature that more free assets are needed to write new business in portfolio MP1 in order to produce the same level of insolvency risk in portfolio MP2. As expected, portfolio MP1 is likely to pay more death claims than portfolio MP2, all other things being equal.

The above discussion reveals that different average ages at entry have different effects on a company's new business plans and insolvency risk. Similar results are also revealed by comparing market portfolio MP1 (shown in figure 4.5) with any other market portfolio in table 4.4 (shown in figure 4.6), except for the very low initial solvency ratios.

Below, we investigate the effect of changes in the distribution of new business by entry age on a company's new business plans and insolvency risk, without allowing for the effect of changes in average age at entry. Figure 4.6 shows the constant insolvency risk contour lines for both probability of insolvency and mean shortfall risk over a 20-year period for the market portfolios with an average entry age of 45. The figure shows that, for a given combination of constant proportion of free assets allocated to finance new business and a company's initial solvency ratio, the insolvency risk is lowest under portfolio MP2 and highest under portfolio MP3. Furthermore, the insolvency risk in market portfolio MP7 is higher (lower) than that in Portfolio MP2 (portfolio MP3). This ranking of riskiness in portfolios $MP3 > MP7 > MP2$ holds for both measures of risk.

The above ranking can be explained by the effect of pooling risk groups in a portfolio. The effect of pooling different risk groups, assumed to be independent in the market portfolios (via changes in distribution channel), may decrease the probability that an insurer will experience drastic financial fluctuations or ruin. "Pooling takes place when a number of independent, but not necessarily identical, risks are grouped together in order to share any losses that occur" Cummins et al (1991).

Table 4.5: Distribution of deaths in market portfolios over a 20-year period

Year	Market Portfolio MP2				
t	μ_d	Med_d	σ_d	CV_d	SK_d
1	192.000	192	0.000	0.000	-
2	190.896	191	1.859	0.010	0.257
5	171.004	171	6.442	0.038	0.117
10	98.325	98	12.961	0.132	0.394
15	84.498	83	19.138	0.226	0.653
20	69.117	67	20.166	0.292	0.914
Year	Market Portfolio MP3				
t	μ_d	Med_d	σ_d	CV_d	SK_d
1	329.000	329	0.000	0.000	-
2	324.559	325	3.428	0.011	-0.400
5	281.627	281	10.224	0.036	0.218
10	145.765	145	19.008	0.130	0.336
15	116.171	113	27.692	0.238	0.732
20	82.734	81	25.968	0.314	0.963
Year	Market Portfolio MP7				
t	μ_d	Med_d	σ_d	CV_d	SK_d
1	278.000	278	0.000	0.000	-
2	272.838	272	2.474	0.009	0.339
5	245.357	245	9.544	0.039	0.246
10	130.307	129	17.052	0.131	0.324
15	105.790	104	24.749	0.234	0.718
20	79.028	76	23.830	0.302	0.953

Symbols:

μ = *Mean* CV = *Coefficient of variation*

σ = *Standard deviation* Med = *Median*

SK = *Skewness* d = *Deaths*

Figure 4.6 shows that the probability of insolvency in a portfolio decreases as many different risk groups are pooled together through the sales effort (e.g. compare portfolio MP3 with portfolio MP7) or there are more low risks within the pooled portfolio (e.g. compare portfolio MP2 with portfolio MP3).

As mortality rate increases exponentially with age, the number of deaths in a portfolio with only low-risk groups (e.g. portfolio MP2) is expected to be lower than that in a portfolio with a mix of both low and high risks (e.g. portfolio MP3 and portfolio MP7), even though they all have the same average age at entry.

Thus, portfolio MP2 is expected to pay less in death claims over a period than portfolio MP3 and portfolio MP7. In other words, the expected death strain (i.e. the product of death rate and death strain at risk, summed over all policies in force at the start of a year) is likely to be lower in portfolio MP2 than in the other portfolios. The death strain at risk can be defined as the difference between the sum assured and reserve.

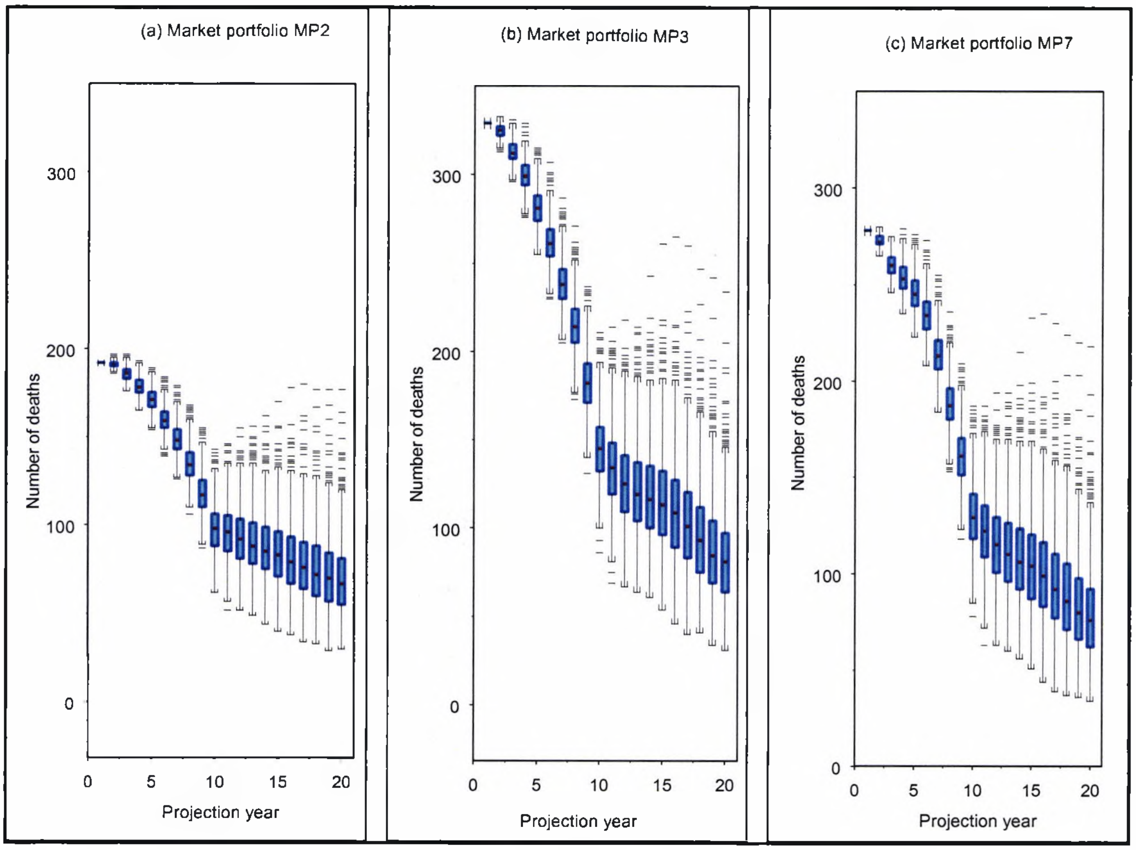


Figure 4.7: Distributions of deaths (Box plots) in market portfolios in projection years

The above analysis is confirmed in table 4.5 which shows the distribution of deaths in the different portfolios having an initial solvency ratio of 1.50 and allocating 40% of free assets to write new business over a 20-year period. The table shows that, over the short term, there is little or no difference in the coefficient of variation between the portfolios, see also figure 4.7. Figure 4.7 shows the distribution of deaths (box plots) in the market portfolios in projection years. However, over the longer term, portfolio MP2 has the lowest mean and coefficient of variation in deaths whilst portfolio MP3 has the highest mean and coefficient of variation in deaths. This is because of the differences in pooling low risk groups between portfolio MP2 and portfolio MP3.

Table 4.5 also shows that the shape of the distribution of deaths changes over the short term, particularly in portfolios MP3, where there is a high volume of high-risk groups. In portfolio MP3 the shape of the distribution changes from negatively skewed to positively skewed. Table 4.5 shows that the skewness of the distributions increases rapidly from about projection year 10 in all portfolios under consideration and this is also revealed in figure 4.7.

The constant insolvency risk (e.g. 5%) contour lines over a 20-year period for the different market portfolios in figure 4.6 show that portfolio MP2 performs better (in terms of free assets allocation proportion) than the other portfolios. Furthermore, portfolio MP7 performs better than portfolio MP3. This ranking also reflects the insolvency risk associated with each portfolio, for a given combination of constant proportion of free assets to finance new business and initial size of a company (measured in terms of initial solvency ratio).

The ranking of portfolios with the same average entry age reveals that the more different risk groups (e.g. portfolio MP7) or the more low risks (e.g. market portfolio MP2) are targeted by a company's distribution channels the more likely the company is to be able to use its free capital efficiently to finance more new business without having insolvency problems. Above all, it is worthy to note that we ignore any extra costs associated with using multiple distribution channels in this analysis.

A change in a company's mix of new business relative to the market portfolio may also have an impact on the company's probability of relative insolvency, and this aspect is considered in the next section.

4.4.2.2 How a change in mix of a company's new business can reduce its probability of relative insolvency

In this section, we consider four hypothetical company portfolios which interact with market portfolio MP7 in table 4.4 (assumed to represent the temporary insurance market for the purpose of analysis in this section). Thus, a company portfolio in year 0 does not exceed 25% of market portfolio MP7, as shown in table 4.6. We also assume that the market portfolio and the company portfolios have the same combination of initial solvency ratio ($SR_m(0) = SR_c(0) = 1.50$) and proportion of free assets allocated to finance new business ($\rho_m = \rho_c = 0.4$). The simulated results (using the details of each portfolio in the table as initial input values for new business issued in year 0) for each portfolio over a 20-year period are shown in table 4.7.

Table 4.6: mix of new business by entry age in company portfolio in year 0 for PT1

Company portfolio *(CP)*	Age at entry x	No. of new poicies issued in year 0	Annual office premium per policy in year 0	Sum assured in year 0
CP1	*35*	*825*	*£183.60*	*£100,000*
	45	*850*	*£342.96*	*£100,000*
	55	*825*	*£874.20*	*£100,000*
CP2	*35*	*495*	*£183.60*	*£100,000*
	45	*680*	*£342.96*	*£100,000*
	55	*825*	*£874.20*	*£100,000*
CP3	*35*	*660*	*£183.60*	*£100,000*
	45	*850*	*£342.96*	*£100,000*
	55	*495*	*£874.20*	*£100,000*
CP4	*35*	*825*	*£183.60*	*£100,000*
	45	*510*	*£342.96*	*£100,000*
	55	*660*	*£874.20*	*£100,000*

Note:
PT1- Policy Type 1 (10-year temporary insurance contract with initial commission payment period of 4 years in table 2.1)

Table 4.7 shows that the greater the number of low risks relative to high risks in a company's portfolio, the lower the company's probability of relative insolvency.

This is partly because of the low level of death benefits that are paid as explained in the previous section and partly due to the release of the capital component of statutory reserves in respect of policies surviving to the end of the policy term, as more low-risk policies are likely to be in-force at the end of the policy term than high risk policies.

On the other hand, as low-risk policyholders in a temporary insurance market are less risk averse, they are more likely to experience a high policy lapse rate than high-risk policyholders, all other things being equal. McGaughey et al (2001) note that, "ignoring selective lapsing, a longer term lapse will tend to improve profitability since no benefit paid in respect of the premiums received", and this will also result in an early release of regulatory capital and reserves. However, at short durations lapse will generally reduce profit. Thus, the release of reserves including the statutory capital component may provide the free capital needed for future investments and for attracting more new business. This holds for both versions of relative insolvency.

Table 4.7: Probability of insolvency (relative insolvency) in company portfolio

$SR_c(0) = 1.50$			$\rho_c = 0.4$	
Company portfolio *(CP)*	Statutory insolvency over 20-year	Relative insolvency version 1	Relative insolvency version 2	Relative insolvency versions 1 or 2
CP1	*0.376*	*0.207*	*0.028*	*0.218*
CP2	*0.517*	*0.365*	*0.108*	*0.402*
CP3	*0.269*	*0.108*	*0.016*	*0.114*
CP4	*0.298*	*0.127*	*0.023*	*0.138*

Thus, table 4.7 shows that portfolio CP3 performs better, in terms of either probability of statutory insolvency or probability of relative (both versions combined) insolvency, than the other portfolios. Furthermore, portfolio CP4 performs better than company portfolio CP1, which in turn performs better than portfolio CP2.

4.4.2.3 Conclusion

The above analysis reveals that the targeting of more low-risk and medium risk consumers in the life insurance sales process reduces a company's capital needs for financing new business and will also improve its statutory and relative solvency position in the market.

On the other hand, a company may decide to charge relatively higher premium for high risks so that there is a bigger safety loading in order to reduce the insolvency risk resulting from high risks, an aspect not considered in the research.

4.4.3 Mix of new business by type of contract

In section 4.3, we have already considered 4 homogeneous portfolios (under market portfolio MP6) made of distinct types of policy (policy designs). Furthermore, the effect of pooling different risk groups via distribution channel for portfolios with temporary insurance contract (policy type 1) is considered in section 4.4.2. Thus, in this section, we consider non homogeneous portfolios with mix of business by type of contract.

As different types of contract may have different risk characteristics, a significant unintended change in the mix of new business by contract may lead to a significant change in the risk profile or the capital needs of a company that are not within its internal resources. Thus, a change in a company's mix of new business which results in writing fewer policies of a contract type with a larger new business strain and more policies associated with a smaller new business strain may reduce its capital requirement and level of risk, (Subject 302 Core Reading, Unit 5). This is in line with the findings in section 4.3.2, when comparing the results for policy type 1 with policy type 2 in market portfolio MP6. In that section, we have shown that, as a company is able to reduce its new business volume in line with an increase in new business strain arising from the contract design, it is likely to remain solvent leading to an efficient use of free assets.

In the next section, we investigate how a company's capital needs to finance new business and its insolvency risk can be managed by changes in the mix of new business by contract type. We illustrate this using PT1 (temporary insurance with 4 years initial commission payment period) and PT4 (non-profit endowment insurance 1 year initial commission payment period).

4.4.3.1 How a change in mix of new business by contract can reduce the insolvency risk

Table 4.8: Mix of new business by type of contract in year 0 in market portfolio

Market Portfolio (MP)	Age at entry x	Type of Policy	No. of new poicies issued in year 0	Annual office premium per policy in year 0	Sum assured in year 0
MP3	*35*	*PT1*	*5,000*	*£183.60*	*£100,000*
	55	*PT1*	*5,000*	*£874.20*	*£100,000*
MP8	*35*	*PT4*	*5,000*	*£859.25*	*£10,000*
	55	*PT4*	*5,000*	*£890.75*	*£10,000*
MP9	*35*	*PT1*	*2,500*	*£183.60*	*£100,000*
	55	*PT1*	*2,500*	*£874.20*	*£100,000*
	35	*PT4*	*2,500*	*£859.25*	*£10,000*
	55	*PT4*	*2,500*	*£890.75*	*£10,000*
MP10	*35*	*PT1*	*4,000*	*£183.60*	*£100,000*
	55	*PT1*	*4,000*	*£874.20*	*£100,000*
	35	*PT4*	*1,000*	*£859.25*	*£10,000*
	55	*PT4*	*1,000*	*£890.75*	*£10,000*

Note:
PT1- Policy Type 1 (10-year temporary insurance contract with initial commission payment period of 4 years)
PT4- Policy Type 4 (10-year non profit endowment insurance contract with initial commission payment period of 1 year)

In this section, we consider four simplified hypothetical market portfolios (MP3, MP8, MP9 and MP10 in table 4.1), each involving contract types PT1 (temporary insurance) and/or PT4 (non-profit endowment insurance) issued to lives aged 35 and 55 at entry in different proportions. Table 4.8 shows the details of the portfolios in year 0, and the corresponding simulated results for probability of insolvency and mean shortfall risk in each portfolio over a 20-year period are shown in figures 4.8 and 4.9 respectively.

We note in section 4.3.2 that a company requires more initial capital to write a non-profit endowment insurance contract than a temporary insurance contract. In this section, we investigate the effect on a company of writing a balanced portfolio (the proportion of temporary insurance policies written relative to non-profit endowment policies) in terms of an efficient use of its free assets and the probability of remaining solvent over the longer term.

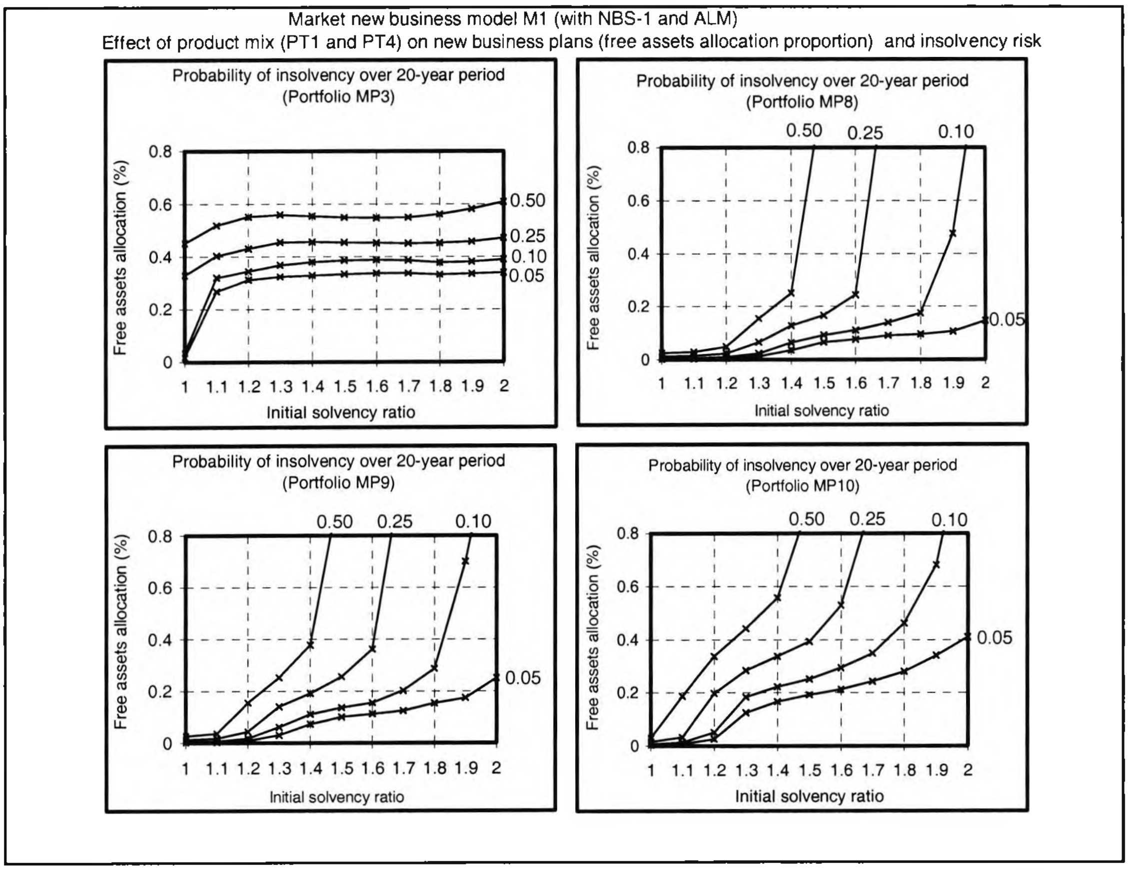


Figure 4.8: Effect of product mix on new business plans and probability of insolvency

Figure 4.8 shows the effect of product mix in market portfolios on new business plans and probability of insolvency. The figure shows that portfolio MP10 is less risky than the others (in terms of an efficient use of free assets allocated to finance new business in order to produce a given level of insolvency risk e.g. 5% probability of insolvency). Portfolio MP10 reduces its initial new business strain by writing relatively more temporary insurance policies than portfolios MP8 and MP9

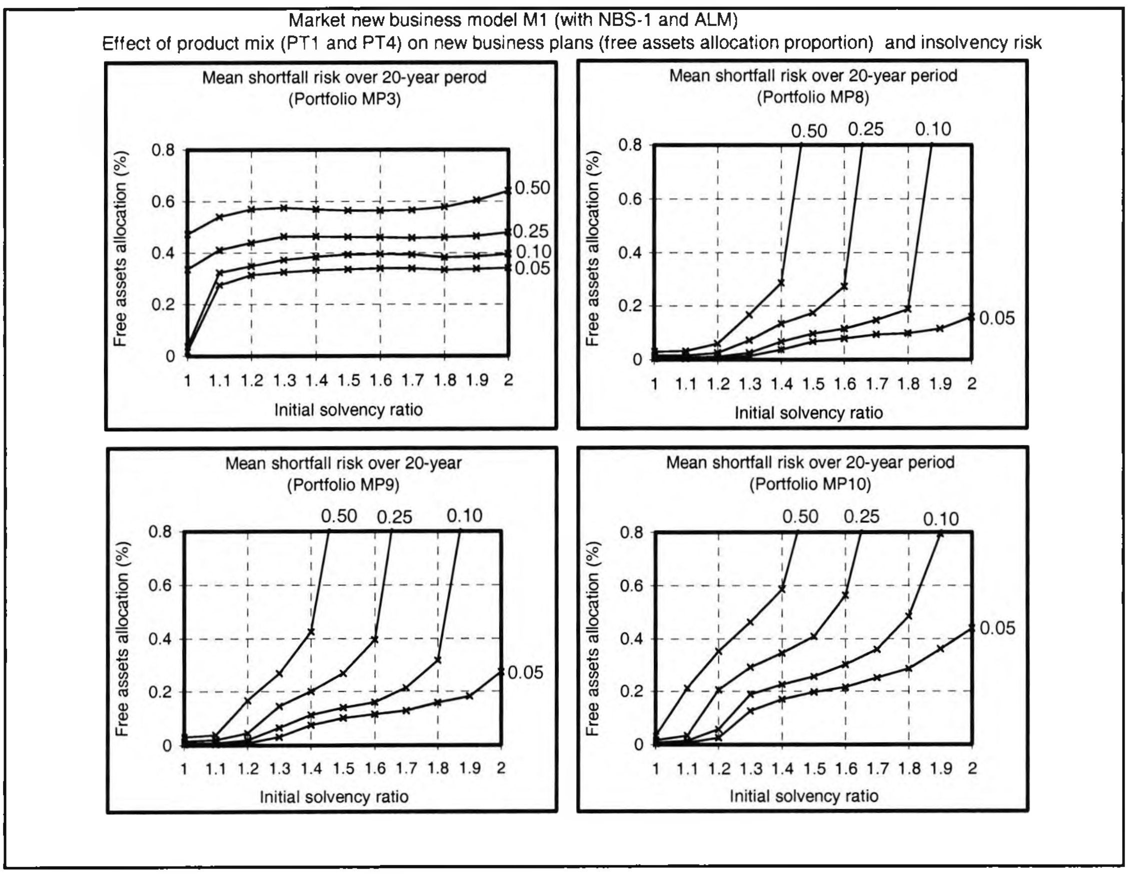


Figure 4.9: Effect of product mix on new business plans and mean shortfall risk.

Where there are sufficient initial free assets, portfolio MP10 still performs better than portfolio MP3 (which involves only temporary insurance policies of policy type 1). This is because portfolio MP10 is likely to pay a relatively smaller amount on death claims than portfolio MP3, which arises from the difference in sums assured between the two policy types PT1 and PT4 in portfolio MP10. Since the death benefit in policy type 1 (temporary insurance contract with sum assured of £100,000) is higher than that in policy type 4 (non-profit endowment insurance with sum assured of £10,000), the mortality risk (measured by the death strain at risk) in market portfolio MP10 is likely to be lower than that for market portfolio MP3.

Furthermore, the new business strain risk in market portfolio MP10 is also likely to be higher than that for market portfolio MP3, due to writing a relatively large volume of endowment insurance contracts with high reserves. Thus, with sufficient initial free assets, the new business strain risk in market portfolio MP10 can be eliminated or reduced considerably. On the other, where there are insufficient initial free assets, the relatively high new business strain risk in market portfolio MP10 cannot be eliminated or reduced considerably, thereby leading to a higher probability of insolvency than in market portfolio MP3.

Portfolio MP9 performs better than portfolio MP8, except for small companies (with low initial free assets). This is because portfolio MP9 reduces its initial new business strain risk by writing not only non-profit endowment insurance policies but also temporary insurance policies. Thus, the mix of contract types and the relative volume of business written in each contract type determine the balance between the new business strain risk and the mortality risk in a portfolio. The above analysis holds also for the mean shortfall risk measure as shown in figure 4.9.

Jacobson (1972) also indicates that "increased sales of term insurance and other low-premium plans of insurance" is one of the possible strategies an insurance company can take to offset adverse consequences of conditions such as less favourable combinations of termination rates (particularly at higher policy durations) and expense levels.

4.4.3.2 Effect of product mix on a company's probability of relative insolvency

Similarly to section 4.4.2.2, we consider 3 hypothetical company portfolios which interact with market portfolio MP10 in table 4.8 (assumed to represent the market comprising temporary and non-profit endowment insurance policies for the purpose of analysis in this section). Thus, a company portfolio in year 0 does not exceed 25% of market portfolio MP10, as shown in table 4.9. We also assume that the market portfolio and the company portfolios have the same combination of initial solvency ratio ($SR_m(0) = SR_c(0) = 1.50$) and proportion of free assets allocated to finance new business ($\rho_m = \rho_c = 0.4$). The simulated results (using the details of each portfolio in the table as initial input values for new business issued in year 0) for each portfolio over a 20-year period are shown in table 4.10.

Table 4.9: Mix of new business by contact type in company portfolio in year 0

Company portfolio (CP)	Age at entry x	Type of Policy	No. of new poicies issued in year 0	Annual office premium per policy in year 0	Sum assured in year 0
CP5	35	PT1	1,000	£183.60	£100,000
	55	PT1	1,000	£874.20	£100,000
	35	PT4	250	£859.25	£10,000
	55	PT4	250	£890.75	£10,000
CP6	35	PT1	1,000	£183.60	£100,000
	55	PT1	1,000	£874.20	£100,000
	35	PT4	200	£859.25	£10,000
	55	PT4	200	£890.75	£10,000
CP7	35	PT1	800	£183.60	£100,000
	55	PT1	800	£874.20	£100,000
	35	PT4	250	£859.25	£10,000
	55	PT4	250	£890.75	£10,000

Table 4.10 shows that the greater is the proportion of non-profit endowment policies in a company's portfolio, the lower is the company's probability of statutory (and relative) insolvency. The new business strain risk associated with writing policy type 4 (non-profit endowment insurance) relative to policy type 1 (temporary insurance) is very small in the company portfolios, for an initial

solvency ratio of 1.50. Thus, it is the mortality risk which determines the riskiness of a company portfolio.

Table 4.10 shows that company portfolio CP7 performs better, in terms of either probability of statutory insolvency or probability of relative (both versions combined) insolvency, than the other portfolios. Furthermore, company portfolio CP5 performs better than company portfolio CP6. The ranking reveals that a company's decision on the mix of business by type of contract needs to take into account of the mortality risk associated with contract types in order to be more relatively solvent than other companies.

Table 4.10: Effect of product mix on probability of relative insolvency

$SR_c(0) = 1.50$			$\rho_c = 0.4$	
Company portfolio (CP)	Statutory insolvency over 20-year	Relative insolvency version 1	Relative insolvency version 2	Relative insolvency versions 1 or 2
CP5	*0.242*	*0.038*	*0*	*0.038*
CP6	*0.292*	*0.08*	*0.002*	*0.082*
CP7	*0.201*	*0.017*	*0*	*0.017*

4.4.3.3 Conclusion

The above analysis reveals that the mix of new business by contract can significantly reduce a company's capital required to finance new business if the company writes more of a contract type with a relatively lower new business strain, particularly where there are insufficient free assets.

Following the discussions in sections 4.4.3.1 and 4.4.3.2, we note that the mix of business by type of contract is also affected by the mortality risk and new business strain risk associated with the contract types under consideration.

Thus, a company needs to seek a balance between the new business strain risk and the mortality risk associated with the contract types within its portfolio in order to remain solvent.

We assume that market portfolio MP10 represents the market proportions of temporary and endowment insurance policies issued in year 0 in subsequent sections where both contract types are considered. However, where only one policy type is considered, then either market portfolio MP3 or market portfolio MP6 is used.

4.5 Constrained New Business Growth

4.5.1 Introduction

A life insurance company's new business growth may be constrained by endogenous and exogenous factors. The endogenous factors may arise mainly from management decisions such as strict underwriting requirements, a change in new business strategy, placing a limit on the level of new business growth in a year in order to meet solvency requirements and others. The exogenous factors may include changes in economic conditions leading to a fall or rise in demand for insurance cover. The effect of changes in new business strategy has been investigated in section 3.5.2 and it is not considered further.

4.5.2 Control on new business growth

In reality, the total volume of exposure (quantity of new business demanded) in the life insurance market is finite and as such the rate of growth in new business in the insurance market in a year needs to be controlled. Thus, the quantity of new business actually written in the market in a year is also finite. Considering the U.K. market new business data for non-mortgage temporary insurance sales for the period 1991 to 2002 in table 2A.2 in appendix 2, the average rate of growth for the period is about 8%. For simplicity, we assume that the quantity of new business demanded in both the temporary insurance and endowment insurance market is expected to increase by not more than 10% in any projection year (i.e. $NBConF_m = 0.1$). Thus, we assume that there is a steady increase in demand for insurance over the projection period.

$$\frac{NM^j(t)}{NM^j(t-1)} \leq \left(1 + NBConF_M\right)$$

$NBConF_m$ = Market new business growth control factor.

$NM^j(t)$ = Quantity of new business demanded (in terms of number of new policies) in the market in projection year t for simulation j.

A life insurer may decide to control its new business growth (quantity of new business written) over a given period in order to remain solvent, irrespective of the market conditions and the availability of free assets to finance new business. Equation (8) specifies that companies operating in the market (represented by a market portfolio) write a proportion (denoted by $\beta_m^j(t)$, see equation (7)) of the quantity of new business demanded, and this proportion is the ratio of the available free assets that are allocated (denoted by $FAN_m^j(t)$) to the capital required (denoted by $ENBS_m^j(t)$) to write the new business in a year. For simplicity, we assume that the management of these companies in the market may decide to control the growth of quantity of new business written in a year by controlling the proportion (say $\beta_m^j(t) \leq 0.75$) of the quantity of new business demanded that needs to be written. Thus, $ANM^j(t) = \beta_m^j(t) \times NM^j(t)$, where $ANM^j(t)$ is the quantity of new business written (in terms of number of new policies) in the market in projection year t for simulation j.

In the next section, we consider the effect of controlling new business growth on new business plans and risk.

4.5.2.1 Effect of controlled new business growth on new business risk

We consider market portfolio MP10 in table 4.7 of section 4.4.3.1 which represents the market proportions of temporary and endowment insurance policies issued to lives aged 35 and 55 in year 0. The simulated results of market portfolio MP10 with or without control on the new business volume are shown in figures 4.10 and 4.11. Figure 4.10 shows the mean of the distributions of quantity of new business demanded and written over the projection period for portfolio MP10 with an initial solvency ratio of 1.50 and allocating 40% of free assets to write new business. Figure 4.11 shows the effect of constrained growth on new business plans and insolvency risk for portfolio MP10 (representing companies with different sizes measured in terms of initial solvency ratio).

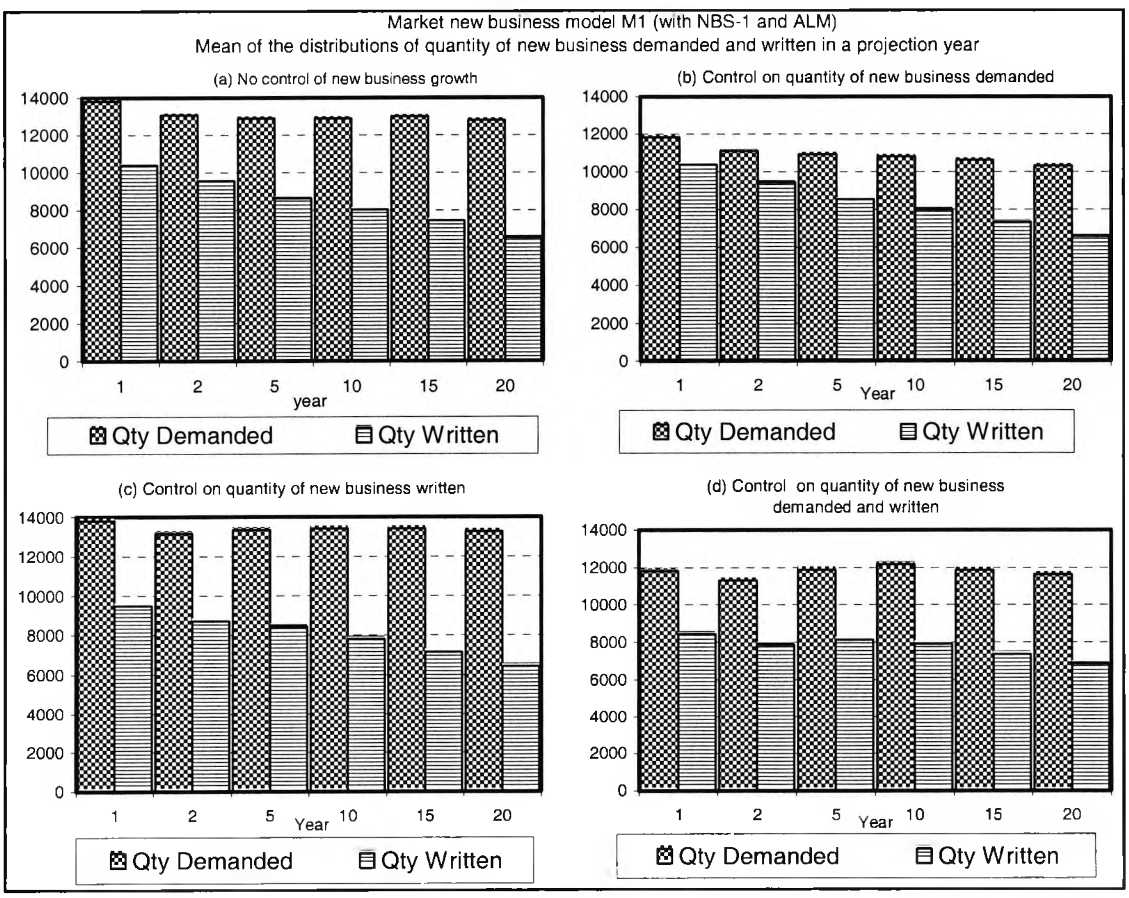


Figure 4.10: Distributions of quantity of new business demanded and written under with (no) new business control.

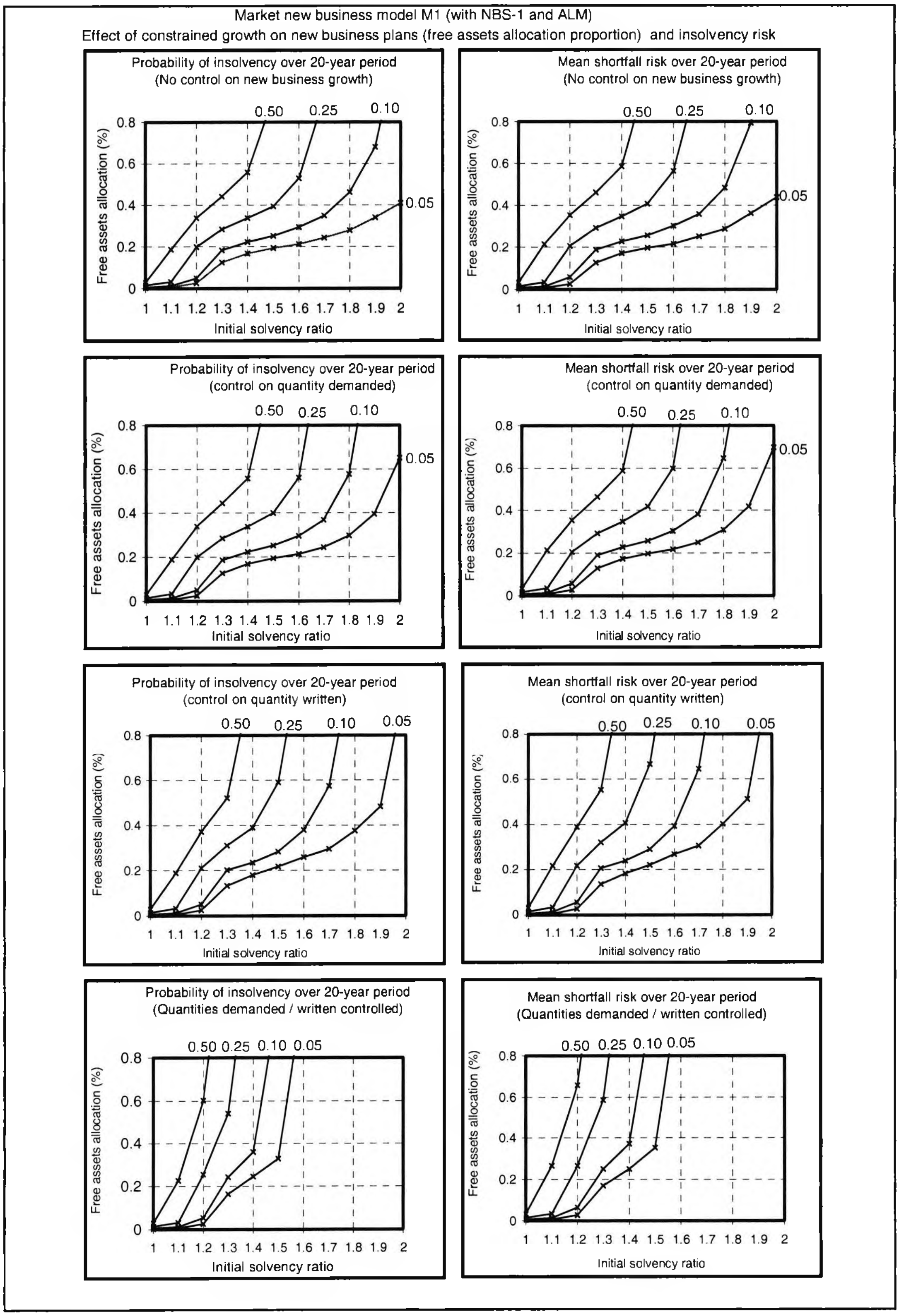


Figure 4.11: Effect of constrained growth on new business plans and insolvency risk

Firstly, we consider the effects of control of new business demanded on new business plans and insolvency risk. We assume that the rate of growth in quantity of new business demanded (control on quantity demanded) in a year does not exceed 10%. Thus, an increase in demand for temporary and non-profit endowment insurance contracts in the market is limited irrespective of changes in economic factors affecting the overall market demand for insurance. In other words, the growth of a company's quantities of new business demanded and written is indirectly constrained.

Comparing figure 4.10 (a) with figure 4.10 (b) shows that there is a significant decrease of about 15% and 20% in the quantity of new business demanded in years 1 and 20 respectively, as a result of the 10% growth limit on demand alone (i.e. control on quantity of new business demanded). Furthermore, the effect on the quantity of new business written of the 10% growth limit on demand is very small over the period. This is because the free assets available and allocated to finance new business which affect the quantity of new business written have not changed.

Figure 4.11 shows that for a given combination of initial solvency ratio and proportion of free assets allocated to finance new business in portfolio MP10, a steady rate of growth of 10% in market demand for insurance (control of quantity of new business demanded) may reduce its insolvency risk compared to the case of no control of its new business growth. Figure 4.11 shows that the contour lines for control of the quantity of new business demanded are higher than those without control of new business growth. This is because a steady growth in the quantity of new business demanded in the market places a cap on the maximum new business volume companies (market portfolio MP10) can write in a year, even though a company has sufficient free assets to write more new business. Thus, all free assets being allocated to finance new business may not be used up, thereby leading to a greater degree of investment freedom which may improve the company's solvency position over the long term. This is particularly true for medium and large companies, and also holds for both measures of risk.

Secondly, we consider the effect of control of quantity of new business written on new business plans and insolvency risk. We also assume that companies (market portfolio MP10) may decide to write not more than 75% of the quantity of new business demanded in the market (see control of quantity written in figures 4.10 and 4.11). For simplicity, we consider that the management (market portfolio MP10) decision to limit its new business growth (control of quantity written) is taken without allowing for the assumption of 10% steady growth in market demand for insurance (control of quantity demanded).

As we expect, comparing figure 4.10 (a) with figure 4.10(c) shows that there is a decrease in the quantity of new business written of about 8% and 2% in year 1 and year 20 respectively. Thus, the 75% control on growth of quantity of new business written has more effect in the early projection years when there are sufficient free assets in market portfolio MP10 than in the long term. Furthermore, there is only a small increase in the quantity of new business demanded of about 4% in the long term, since it is not directly controlled. However, there is a second order effect on the quantity of new business demanded when the quantity of new business written is controlled. As a smaller quantity of new business is written in the early years, more free assets become available and this allows the pricing policy under new business strategy 1 to reduce the premium in order to attract more new business through the demand functions.

Figure 4.11 shows that by writing not more than 75% of the quantity of new business demanded (control of quantity written) in market portfolio MP10 in a year leads to a greater improvement in its solvency position than the effect of a 10% steady growth in market demand for insurance. This is reflected in figure 4.11 which shows that, the contour lines for controlling the quantity of new business written are higher than those for controlling the quantity of new business demanded and for no control on new business volume. The 8% reduction in the quantity of new business written in the early years of projection leads to more free assets becoming available to finance investments in risky assets (equities). This may yield higher returns over a longer term which in turn improves a company's solvency position. The above is true for all companies except for companies with insufficient initial free assets, and also holds for both risk measures.

Thirdly, we consider the effect of controlling both the quantities of new business demanded and written on new business plans and insolvency risk. Figure 4.10 shows that the controlled quantity of new business written in year 1 is about 80% of the corresponding figure in year I without allowing for any form of control (comparing figure 4.10(a) with figure 4.10(d)). Both the quantities of new business demanded and written over the projection period shown in figure 4.10(d) are lower than the others in figure 4.10.

Figure 4.11 shows that controlling both the quantities of new business demanded and written further reduces a company's insolvency risk, as the contour lines become steeper compared to the case of controlling either the quantity of new business demanded or written. This is because the actual volume of new business written becomes so small that more than enough free assets are available to finance investments in risky assets (equities) which may yield higher returns over a longer term. This will in turn improve the solvency position of the company.

Generally, we note that even where there is no direct control of a company's new business volume (such as no control on new business growth in figure 4.11), the structure of the market model with the presence of in-force business and a substantial initial solvency ratio at the projection date may provide an indirect control of its insolvency risk.

4.5.2.2 Conclusion

The above analysis reveals that there is need for a company to control its new business growth, particularly where there are insufficient free assets, in order to remain solvent.

Generally speaking, even where there is no direct control of a company's new business, a company's existing block of business with a substantial initial solvency ratio at projection date (as provided in the market model) provides a powerful stabilising factor which enables the company to withstand adverse future conditions such as writing an inappropriate volume of new business.

4.5.3 The level of underwriting policy

Life insurance companies need to balance the benefits and costs of underwriting. The main benefit of increasing the level of underwriting will be an improved mortality experience in the insured population. This can be achieved through adequate risk classification methods as part of underwriting which helps to ensure that all risks are priced fairly as discussed in section 4.2.

In this section, we consider the effect of the costs of underwriting on a company's new business plans and insolvency risk. The costs of underwriting comprise the direct cost of obtaining medical evidence and the indirect cost of discouraging potential business (people who are healthy from applying for insurance) due to unnecessarily extensive underwriting. Prospective policyholders may be more inclined to take out life insurance contracts when the level of underwriting standards is lower. Also, the simpler is the underwriting process the quicker new business proposals can be processed. In addition, "the lower the level of underwriting the lower the expenses and hence the more competitive the product", (Subject 402 Core Reading, Unit 15) an aspect discussed below. Thus, a life insurance company's desire to maintain a high standard of future mortality experience (through stringent underwriting requirements) needs to be balanced against the desire to cater for as wide an insuring public as possible at normal rates, Luffrum (1990).

A life insurance contract that provides a benefit on death significantly in excess of the supervisory reserve being held (positive death strain) will be underwritten at entry. Thus, the extent of underwriting will be much greater for contracts that offer protection benefits (e.g. temporary insurance contract) than those that are fundamentally for savings (e.g. endowment insurance contracts), (Subject 402 Core Reading, Unit 15). A company may also have a claims management process, particularly for contracts that provide a benefit on sickness or disability, but this aspect is not considered in this investigation.

4.5.3.1 New business model allowing for underwriting effect

As the level of underwriting can have an indirect effect on sales irrespective of price effect, we assume that a reduction (an improvement) in the level of underwriting standards will increase (decrease) the price effect on sales. We also assume that an additional level of improvement (reduction) in underwriting standards beyond a certain level will have little or no additional effect on sales, all other things being equal. Given the above discussion and assumptions, we model the effect of indirect underwriting cost on sales using a logistic growth function as this is frequently used to model the growth in sales of a product for which saturation occurs in the market, see Pindyck et al (1998). In practice, a company's underwriting standards are reviewed regularly but not necessarily on a yearly basis, and for the purpose of simplicity, we assume that any underwriting decision taken does not change over the projection period.

Let UF = the effect of the level of underwriting standards on sales. Then we propose:

$$UF = \frac{2}{1 \quad e^{\frac{1}{4}z}} \qquad (28)$$

where z = the level of underwriting standards and figure 4.12 is a graphical presentation of equation (28).

Remarks on equation (28)

The form of the above logistic function is such that UF is a decreasing function of the level of underwriting standards (z) and $0 < UF < 2$ for all real values of z. In this investigation, UF represents the percentage of sales (quantity of new business demanded) of a life insurance product arising from an improvement (reduction) in the level of underwriting standards such that, at some level of underwriting standards, the additional percentage of sales will approach zero. Thus, $0 < UF < 2$ suggests that the percentage change in sales with respect to a change in underwriting standards from a company's normal underwriting standard ($z = 0$) is less than 100%. The values in the function UF are chosen to achieve the above objective.

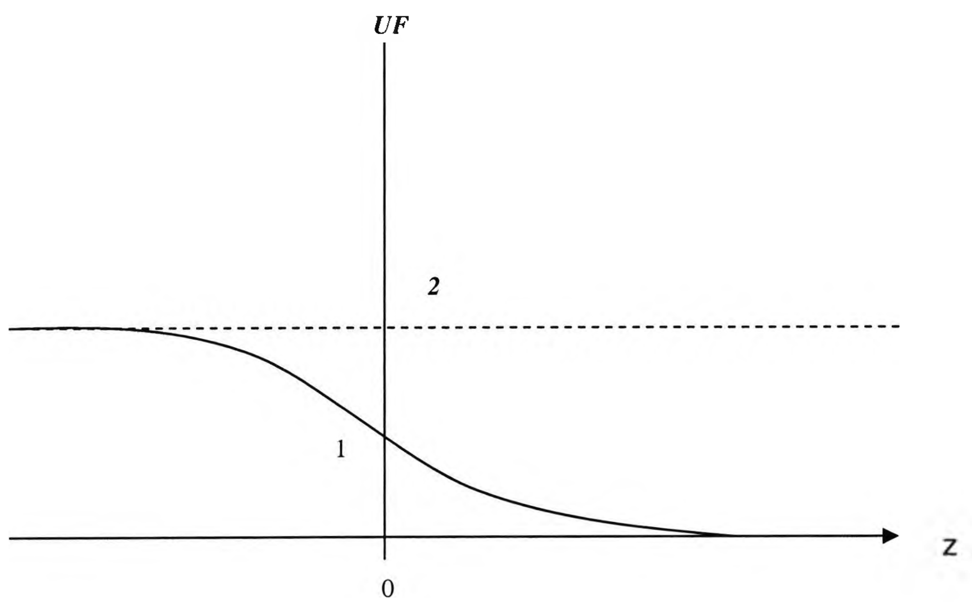


Figure 4.12: Effect of the level of underwriting standards on sales

It is worthy to note that it may be possible to use other methods than logistic function (e.g. by simulation) to model the indirect effect of underwriting standards on sales. However, our investigation adopts the logistic function since it has been used successfully to model the growth in sales for products in which saturation occurs as discussed above.

We assume that the improvement (reduction) in the level of underwriting denoted by $\gamma\%$ ($-\gamma\%$) does not exceed 20% (i.e. $-20 \leq \gamma \leq 20$).

$$z = \begin{cases} \gamma\% & \textit{level of improvement in underwriting} \\ 0\% & \textit{normal underwriting requirements} \\ -\gamma\% & \textit{level of reduction in underwriting} \end{cases}$$

If $z = \gamma\% = 0.1$ in equation (28) then $UF = 0.9875$ and thus, sales decrease by 1.25% which is approximately equal to $(\frac{1}{8}\gamma)\%$. On other hand, if $z = -\gamma\% = -0.1$ then $UF = 1.0125$ and sales increase by 1.25% which is also approximately equal to $(\frac{1}{8}\gamma)\%$. This is an illustration of a standard result which can be derived from equation (28) as follows:

$$UF = \frac{2}{1+e^{\frac{1}{4}z}} = \frac{2}{1+1+\frac{1}{4}z+\frac{1}{2}\frac{z^2}{16}+....} = \frac{2}{2+\frac{1}{4}z+\frac{z^2}{32}+...} = \frac{1}{1+\frac{z}{8}+\frac{z^2}{64}+....}$$

$UF \approx 1-\frac{1}{8}z+\mathrm{O}(z^2)$, and taking the first order approximation gives the standard result

$$UF \approx 1-\frac{1}{8}z \tag{29}$$

Thus, we propose a new market model for the quantity of new business demanded in order to allow for underwriting effect on sales.

$$\frac{NM^j(t)}{NM^j(t-1)} = RWP^j(t) \times DF_m^j(t) \times UF \tag{30}$$

Where,

$DF_m^j(t)$ = Demand function for the market in projection year t for simulation j.

$RWP^j(t)$ = Proportion of the previous year's new business volume demanded in year t in the market due to the effect of real wage inflation only, for simulation j.

$EDF_m^j(t)$ = The exponential price elasticity of demand function for the market in projection year t for simulation j, see equation (4) in chapter 2,

$NM^j(t)$ = The quantity of new business demanded (in terms of number of new policies) in the market in projection year t for simulation j.

Equation (30) above is a modification of equation (5) in chapter 2 and thus, the market new business model M1 (exponential demand function) is also modified as follows:

Let $DF_m^j(t) = EDF_m^j(t)$ and $RWP^j(t) = 1$ in equation (30)

For convenience, we assume three simplified underwriting policies (each having an indirect effect on sales) in the insurance market as defined and discussed below for temporary and endowment insurance contracts. The values for κ in the underwriting policies (represent a multiple of standard mortality rates) are taken from the premium basis for the above contract types while the values for γ are chosen arbitrarily. The published standard mortality rates are TM92 (5) select table and AM92 (2) select table for temporary insurance and non-profit endowment insurance contracts respectively.

- **Underwriting policy 1 (UP1)**: The premium basis mortality rate, $\kappa\%$ of a published standard mortality, is for lives in average good health (the normal mortality rates class) insured under the company's normal underwriting standards, and this has no indirect effect on sales, i.e. $z = 0, UF = 1$ in equation (29).
- **Underwriting policy 2 (UP2)**: An improvement in underwriting standards by $\gamma\%$ will improve the mortality experience by an equivalent of $(\kappa - \gamma)\%$ of a published standard mortality rate (e.g. for insured lives above average good

health), and this will reduce the price effect on sales by about $\left(\frac{1}{8}\gamma\right)\%$ in a year from equation (29).

- **Underwriting policy 3 (UP3)**: A reduction in underwriting standards by $\gamma\%$ will worsen the mortality experience by an equivalent of $(\kappa+\gamma)\%$ of a published standard mortality (e.g. for insured lives below average good health), and this will increase the price effect on sales by about $\left(\frac{1}{8}\gamma\right)\%$ in a year, i.e. $z=-\gamma\%,\ UF \square 1+(\frac{1}{8}\gamma)\%$ in equation (29).

$$\kappa\% = \begin{cases} 70\% & for\ non-profit\ temporary\ insurance\ policy \\ 90\% & for\ non-profit\ endowment\ insurance\ policy \end{cases}$$

As the extent of underwriting is much greater for contracts that offer protection benefits than those that are fundamentally for savings, (Subject 402 Core Reading, Unit 15), we assume different values of γ (taken arbitrary) for temporary insurance and non-profit endowment policies, and they are given below.

$$\gamma\% = \begin{cases} 20\% & for\ non-profit\ temporary\ insurance\ policy \\ 10\% & for\ non-profit\ endowment\ insurance\ policy \end{cases}$$

We assume that the effect of underwriting level on mortality experience for a life aged x (denoted by q_x') is expressed as a percentage of normal mortality: $q_x' = (\kappa \pm \gamma)\% \times q_x$, where q_x is the mortality rate of published standard mortality. This approach is used as it is computationally easier to make these adjustments to q_x rather than the theoretically correct μ_x.

In practice, occasions may arise from time to time when, (perhaps due to the combined effect of competition, increase in underwriting costs, improvements in mortality over time and others), an office deliberately extends the normal rates class to include certain impairments previously excluded. On the other hand, the office may decide to act more stringently in respect of impaired lives hitherto accepted at normal rates, Fisher and Young (1971). Thus, the normal rates class is not necessarily permanent, and for simplicity, we assume that changes in underwriting level from the normal underwriting requirements (Underwriting policy 1) are applicable to future new business.

4.5.3.2 Effect of the level of underwriting on sales and costs

A company which reduces its underwriting standards by seeking less underwriting evidence on grounds of cost may experience false economy which will not necessarily improve its solvency and competitive position, as illustrated by the example below based on Luffrum (1990). Thus, consider a male life aged 55 who proposes for a 10 year temporary insurance policy with a sum assured of £100,000. Suppose that a less (more) stringent underwriting could worsen (improve) the mortality experience by an equivalent of one-year addition to (deduction from) age. In practice, an addition to age is rarely used to specify the intensity of extra risk but a percentage of normal mortality is commonly used.

Then based on TM92 (5) select 4.5%, there will be a loss (saving) to the life insurance fund using equation (31) (equation (32)). The saving to the life insurance fund can pay for a substantial degree of underwriting.

Then in standard notation:

$$\left(P^{1}_{[x]:\overline{n|}} \times \ddot{a}_{[x+1]:\overline{n|}} - A^{\;1}_{[x+1]:\overline{n|}}\right) \times £100{,}000 \tag{31}$$

$$\begin{aligned}&\left(P^{1}_{[55]:\overline{10|}} \times \ddot{a}_{[56]:\overline{10|}} - A^{\;1}_{[56]:\overline{10|}}\right) \times £100{,}000\\ &= (0.08085 \times 8.047 - 0.65348) \times £100{,}000\\ &= -£288.0\end{aligned}$$

$$\left(P^{1}_{[x]:\overline{n|}} \times \ddot{a}_{[x-1]:\overline{n|}} - A^{\;1}_{[x-1]:\overline{n|}}\right) \times £100{,}000 \tag{32}$$

$$\left(P_{[55]:\overline{10|}}^{\;1} \times \ddot{a}_{[54]:\overline{10|}} - A_{[54]:\overline{10|}}^{\;1}\right) \times £100,000$$
$$= (0.08085 \times 8.092 - 0.65156) \times £100,000$$
$$= £267.82$$

The statement "the lower the level of underwriting the lower the expenses and hence the more competitive the product", (Subject 402 Core Reading, Unit 15) is true all other things being equal, but the above examples show that "if a bit more underwriting improved expected mortality experience enough to more than offset the cost of that underwriting, then the contract will be more competitive with more underwriting", (Subject 402 Core Reading, Unit 15). Thus, the more underwriting is justified to the extent that it pays for itself in improved experience.

Following the discussions above, we investigate the cost of changes in underwriting levels below, measured in terms of a multiple of an initial per policy expense (£115), see the premium basis in table 2.1 in chapter 2, using the temporary insurance policy type 1 (PT1). For simplicity, we assume that the per policy expense loading (£115) in the premium basis covers the normal underwriting costs, and any additional (reduction in) underwriting cost due to an improvement (a reduction) in underwriting levels is borne by the company, in order to make the contract competitive.

We consider market portfolio MP6 with an initial solvency ratio ($SR_m(0)=1.50$) and free assets allocation proportion ($\rho_m = 0.30$) which writes 5,000 temporary insurance policies (policy type 1) to lives age 55 at entry in year 0 on a normal underwriting basis (UP1). The simulated results produce a probability of insolvency and mean shortfall risk of about 20% over a 20-year period, point A in figure 4.13. In this case, there is no additional cost to the company in terms of extra direct underwriting cost (since the multiple of per policy expense is 1) and also no indirect effect on sales.

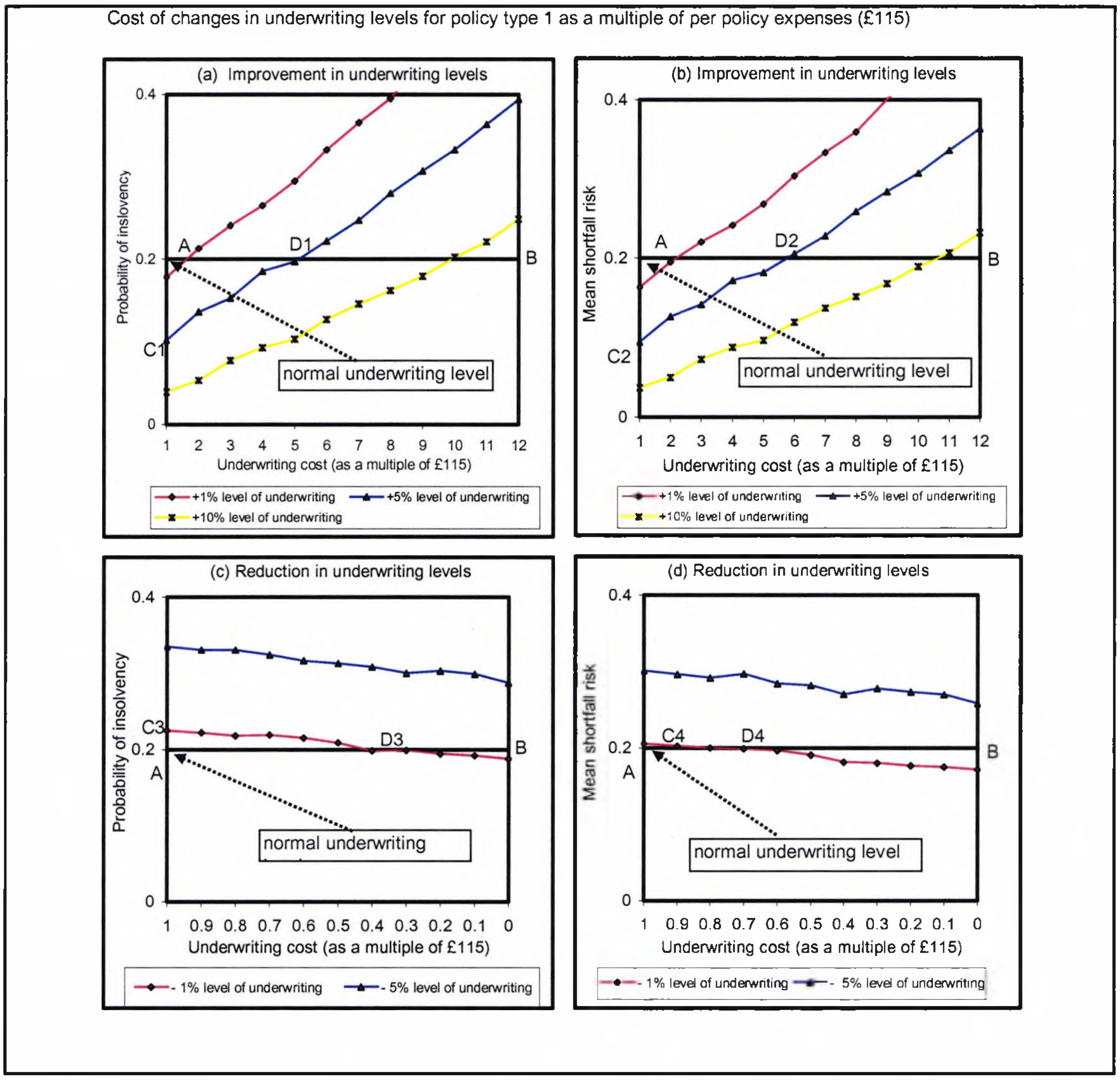


Figure 4.13: Cost of changes in underwriting levels for policy type 1.

If the company decides to improve its underwriting standard by +5%, there will be a fall in demand for insurance by 0.6% (i.e. $UF = 0.994$) without considering the price effect. The company's simulated results (probability of insolvency and mean shortfall risk) fall below the 20% level of normal underwriting basis (see points C1 and C2 in figure 4.13 (a) and (b) respectively) if the 5% increase in underwriting standards does not attract additional underwriting cost (i.e. if the multiple of per policy expense is still equal to 1). However, since an improvement in underwriting standards normally incurs additional underwriting costs, the company's level of insolvency risk increases with the level of additional underwriting costs.

Figure 4.13 (a) shows that the company's probability of insolvency is lower than the 20% level (below line AB), if the increase in underwriting cost is less than five times the per policy expense. Thus, the cost of improvement in underwriting standards by 5% on a company's probability of insolvency is about five times the per policy expense, see point D1 in figure 4.13 (a). Similarly, the cost of improvement in underwriting standards by 5% on a company's mean shortfall risk is about six times the per policy expense, see point D2 in figure 4.13(b).

The figure shows that the cost increases with the level of improvement in underwriting standards, but in practice, the underwriting cost is unlikely to exceed £575 $(= 5 \times £115)$ which means that an improvement in underwriting standard by +5% and above will reduce the company's insolvency risk.

On the other hand, If the company decides to reduce its underwriting standards by 1%, there will be an increase in demand for insurance by 0.125% (i.e. $UF = 1.00125$) without considering the price effect. The company's simulated results (probability of insolvency and mean shortfall risk) are above the 20% level of normal underwriting basis (see points C3 and C4 in figure 4.13 (c) and (d) respectively) if the 1% decrease in underwriting standards does not lead to a reduction in underwriting cost (i.e. if the multiple of per policy expense is still equal to 1).

Figure 4.13 (c) shows that the company's probability of insolvency is lower than the 20% level (below line AB), if the reduction in underwriting cost is more than 60% of per policy expense. Thus, the cost of reduction in underwriting standards by 1% on a company's probability of insolvency is about 40% of per policy expense, see point D3 in figure 4.13 (c). Similarly, the cost of reduction in underwriting standards by 1% on a company's mean shortfall risk is about 70% of per policy expense, see point D4. Figure 4.13(c) suggests that a small reduction in underwriting standards requires a very sharp reduction in expenses. We note also from 4.13(c) that a 5% and above reduction in underwriting standards will have an adverse effect on a company's solvency position if the initial solvency ratio of 1.50 and 30% of free assets allocated to write new business are maintained.

The above analysis may change if the company's initial solvency ratio and/or the free assets allocation proportion changes. For example, as the initial solvency ratio increases the probability of insolvency decreases, all other things being equal, and we expect the cost of underwriting as a multiple of per policy expense to decrease and the reverse is the case as the free assets allocation proportion increases. On the other hand, if there is a corresponding increase (decrease) in both the initial solvency ratio and the free assets allocation proportion at the same time the effect may be neutral. However, for purposes of simplicity, we assume that the cost of improvement (reduction) in underwriting levels does not exceed $£230 = 2 \times £115$ ($£57.5 = 0.5 \times £115$) in investigating the effect of underwriting levels on a company's capital needs and insolvency risk in the following section. We also consider a +10% (-10%) change in the level of improvement (reduction) in underwriting standards for temporary insurance contracts, and a +5% (-5%) change in the level of improvement (reduction) in underwriting standards for endowment insurance contracts in the investigation.

4.5.3.3 Effect of underwriting policy on new business plans and risk

In this section, we consider market portfolio MP10 in table 4.7 of section 4.4.3.1 which represents the market proportions of temporary and endowment insurance policies issued to lives aged 35 and 55 in year 0 as discussed in section 4.4.3.3. The simulated results of market portfolio MP10 with the three different underwriting policies described in section 4.5.3.1 are shown in figures 4.14 and 4.15 below.

In addition, we also consider a fourth underwriting procedure where normal and stringent underwriting standards are only applied to lives aged 35 and 55 at entry in the portfolio respectively. This differs from the other three underwriting policies in section 4.5.3.1 in the sense that each of these three underwriting policies is applied to lives in the portfolio irrespective of age at entry. In practice, "the conditions for dispensing with medical examination vary, but usually apply only to lives who are under a certain age (for example 45), with limits for the maximum sum assured and possibly on the class of assurance, and subject to medical examination if the office considers it desirable", Fisher and Young (1971).

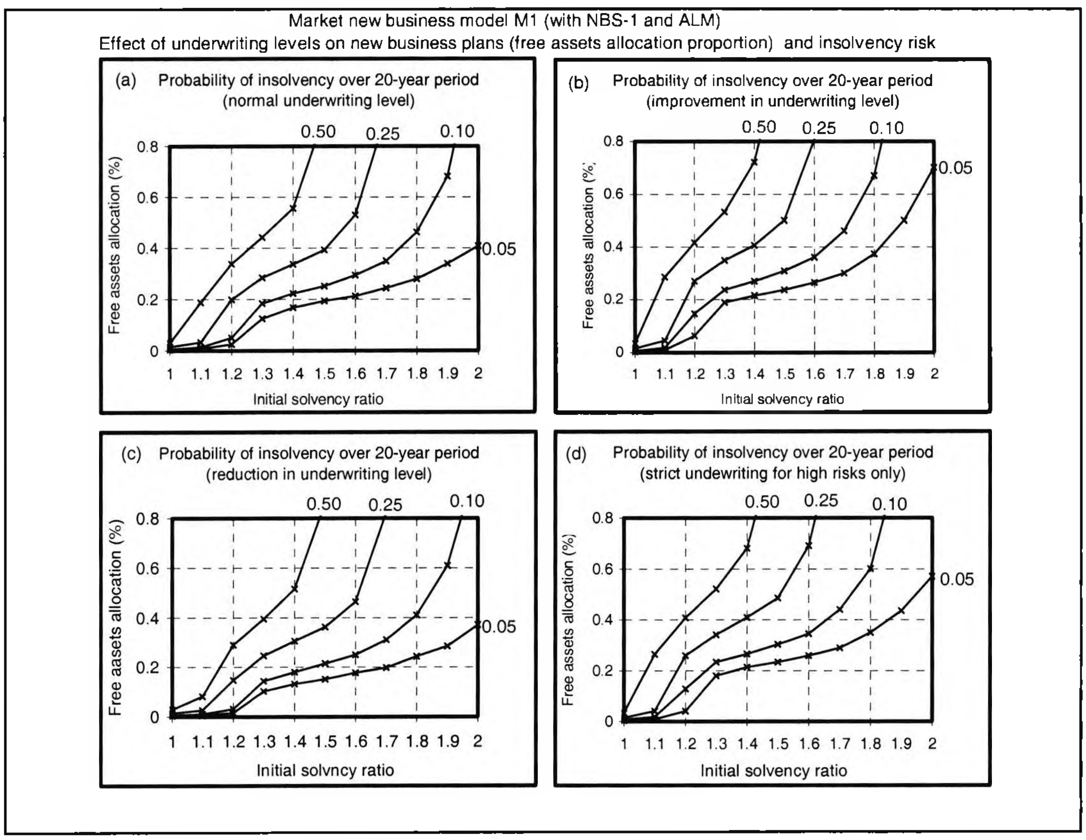


Figure 4.14: Effect of underwriting levels on new business plans and probability of insolvency.

As we expect, figure 4.14(b) shows that applying stringent underwriting standards to all insured lives in a portfolio enables a company to have better security in terms of lower probability of insolvency than applying a normal underwriting level, as shown in figure 4.14 (a) or lower underwriting level, as shown in figure 4.14(c). The contour lines (e.g. 5% probability of insolvency) for stringent underwriting are higher than those under other underwriting policies, and the above also holds for mean shortfall risk, as shown in figure 4.15. A stringent underwriting policy may have two effects on a life insurance portfolio, namely it reduces the company's initial capital strain as a relatively small volume of new business is written, and also it improves the mortality experience of the insured population. In other words, as the company's market share (in terms of volume of business written) reduces due to stringent underwriting standards, more free assets will become available for financing investments in risky assets (e.g. equities) over the long term in order to produce higher returns. However, the company's sustainable growth rate may not be achieved over the long term by applying stringent underwriting procedure in respect of every proposer.

On the other hand, figure 4.14 (c) shows that applying lower underwriting standards to all insured lives in a portfolio increases the company's probability of insolvency (e.g. 5% contour line for lower underwriting standards is lower than those of other underwriting policies). This is because the lower underwriting standards increase the company's market share (by writing an inappropriate volume of new business with relatively large initial capital strains) which may lead to a significant drain on free assets and hence placing a constraint in its investment freedom. The lower underwriting standards are likely also to worsen the mortality experience of the insured population leading to a higher insolvency risk than under normal or stringent underwriting standards. The above discussion is also true for mean shortfall risk, as shown in figure 4.15.

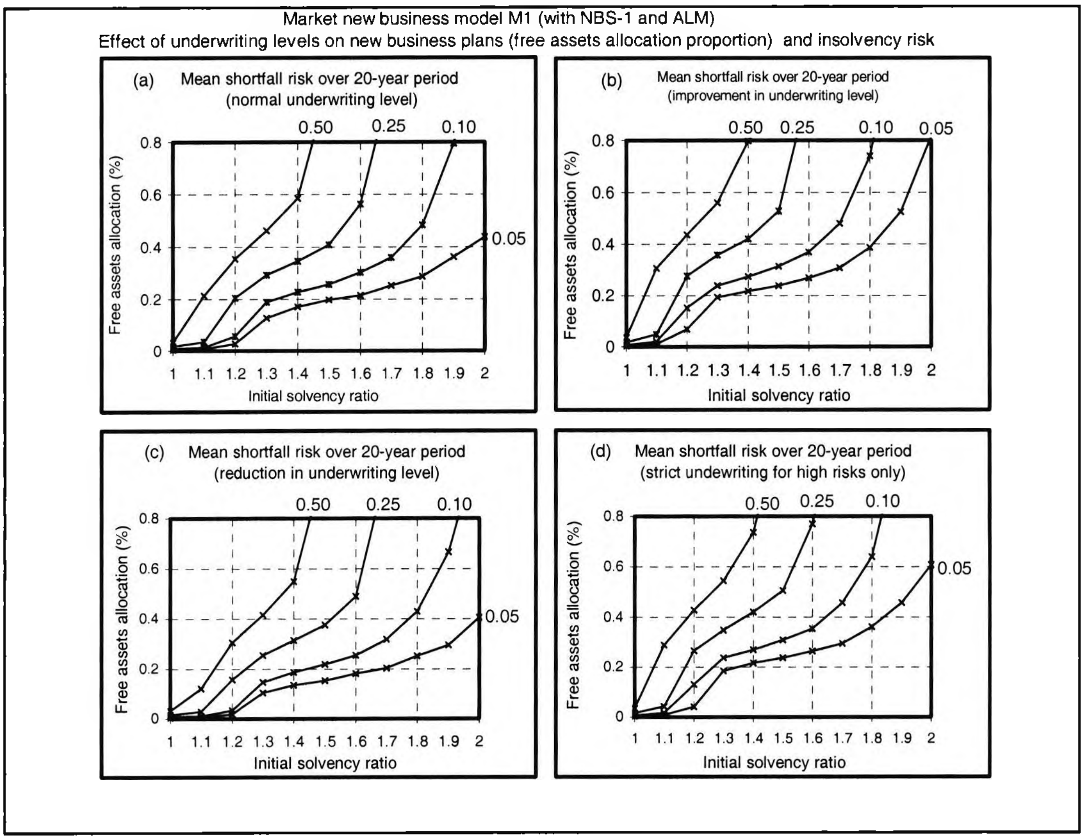


Figure 4.15: Effect of underwriting levels on new business plans and mean shortfall risk.

Comparing figure 4.14(a) with figure 4.14(d) shows that a company's probability of insolvency is lower under the fourth underwriting policy (most intuitive underwriting policy) where normal and stringent underwriting standards are applied to lives aged 35 and 55 respectively. This means that a company's insolvency risk is likely to increase by applying normal underwriting standards to all of an insured population if the portfolio consists of a high proportion of high risks (lives aged 55). Thus, the fourth underwriting policy may enable a company to achieve its strategic aim of an efficient use of capital by writing an appropriate volume of new business, and still provide sufficient free assets for long term investments in risky assets. This policy also improves the mortality experience of the insured lives and thereby reduces the company's overall probability of insolvency. The figure shows that the contour lines for the fourth underwriting policy in figure 4.14 (d) are higher than those under both normal underwriting policy in figure 4.14(a) and reduction in underwriting policy in figure 4.14(c).

However, the contour lines for the fourth underwriting policy are lower than those under the stringent underwriting standards in figure 4.14(b). This is also true for mean shortfall risk in figure 4.15.

4.5.3.4 Conclusion

The above analysis reveals that a company can maintain a growth rate over the long term by applying more stringent underwriting procedure to high-risk proposers than low-risk proposers.

However, applying stringent underwriting standards to all insured lives in a portfolio enables a company to have better security in terms of insolvency risk than lower underwriting standards. This is to the detriment of a company's future market share. On the hand, lower underwriting standards may increase the company's market share (by writing an inappropriate volume of new business) which may lead to a significant drain on free assets.

The figures above show that changes in the level of underwriting standards have more impact on companies with sufficient free assets than small companies. This is because the effectiveness of any underwriting policy also depends on the resourses available to the company, and thus a company needs to find a balance between no underwriting and rigorous underwriting in order to have an efficient use of its resources.

In this investigation, we assume that the underwriting expenses are borne by the company, but in practice, they are normally passed to the policyholder in form of increased premium. This is an aspect that may affect the company's sales volume and insolvency risk which is beyond the scope of this research.

4.6 Reinsurance

4.6.1 Introduction

An established office which begins to expand rapidly or a newly established life office (particularly with limited capital resources) can use reinsurance to reduce its new business strain and/or control its exposure to mortality risks (i.e. reduce claim payment fluctuation) to acceptable levels. "A direct writing company can directly reduce its new business strain by means of [financial reinsurance using] original terms or risk premium with financing commission reinsurance" (Subject 402 Core Reading, Unit 15). Thus, a life company's new business risk can be reduced by financial reinsurance (being sought for new business), either through an increase in its available capital or through a reduction of its financing requirement, the aspect (new business financing) considered only in this investigation.

Brett and Cowley (1993) define financial reinsurance (Fin Re) as "reinsurance that is motivated by financial as well as risk transfer goals. Fin Re employs the future profits contained in a block of new business or in-force business to achieve a financial objective for the direct writer". In practice, there are various forms of financial reinsurance that are available to life offices such as original terms reinsurance, deficit account financing, surplus relief and others which are described in detail by Spedding (1989), Brett and Cowley (1993), Gemmell et al (2000), McGauhey et al (2001) and others.

For simplicity, we consider only the first two forms of financial reinsurance stated above as they relate to new business financing, namely original terms and deficit (loan) account financing (i.e. risk premium reinsurance with financing commission) in this investigation. Under new business financing, there is a specific need for cash (in form of reinsurance commission, cash advance (loan) or premium rebate) from the re-insurer to cover the payment of the initial expenses incurred when writing the reinsured new business. Thus, new business financial reinsurance can reduce some of the initial cash-flow strain (arising from agent's commission and initial administration expenses) and/or reserving strain (due to the conservative nature of the statutory reserving basis) for a ceding company writing a regular premium product.

Briefly, Gemmell et al (2000) note that surplus relief (which takes future profits within a block of new or existing non-profit business and turns them into capital) is similar to new business financing, as they both aim to generate capital. However, surplus relief differs from a new business financing arrangement because the latter involves reinsuring blocks of new business as they are written (with a cash advance being made as each block of business written is being reinsured).

Original terms and deficit (loan) account financing are discussed in the next two sections.

4.6.2 Original terms reinsurance

Original terms reinsurance involves a sharing of all aspects of the original contract (such as the premiums, claims and the risks of investment and lapse) between the ceding company and the reinsurance company in a fixed proportion. The new business financing under the original terms treaty may take the form of a rebate, (e.g. a percentage of premium or a reinsurance commission) payable by the re-insurer to the ceding company, which is independent of the age/term combination of the policies falling within the treaty, (McGaughey et al 2001). The reinsurance commission is normally in two forms, namely an initial reinsurance commission to finance the agent's initial commission and the direct writer's initial expenses, and a renewal reinsurance commission to cover the ceding company's renewal expenses (Brett and Cowley, 1993).

Booth et al (2005) note that original terms reinsurance is usually "restricted to term assurance contracts, in which reserves are relatively small and, hence, the loss of investment profit is relatively minor". The original terms reinsurance can be arranged on either a quota share basis (a specified percentage of each policy is reinsured) or an individual surplus basis (the original sum assured in excess of the ceding company's retention on any individual life is reinsured, and so a larger proportion of bigger policies is reinsured) or a combination of the two methods, (Spedding, 1989).

Quota share reinsurance is most appropriate for reducing new business strain, as it gives an insurer the greatest control over the amount of financing it will receive in relation to the volume of new business it underwrites. Whilst surplus reinsurance is much more efficient at targeting the mortality risk as it enables a company to avoid high risk concentration by reinsuring a high proportion of its very large risks and none of its small cases, (Subject 302 Core Reading, Unit 14). Surplus reinsurance may not be appropriate in this investigation as the policies under consideration have the same sum assured.

Booth et al (2005) also note that original terms reinsurance arranged on a quota share basis (is often effected to obtain financial assistance) will reduce a company's capital strain, particularly in the first of the policy, in two main ways. These are a reduction in statutory reserves and solvency margins and payment of reinsurance commission (or a premium rebate) to the ceding company to cover (in respect of the reinsured portion of the policy) the commission that has been paid by the ceding company and part or all of the ceding company's other expenses. The reinsurer will also take its proportion of the reserving strain, and thus the ceding company does not hold reserves in respect of the reinsured portion of the business. In U.K "the solvency margin calculation however is based on the gross (before reinsurance) reserves, reduced on an aggregate basis (not per policy) by a maximum of 15% on account of reinsurances, and on the gross sums at risk, reduced by a maximum of 50%", (Spedding, 1989).

In this investigation, we assume that the direct offices in the market have a $\alpha_{QS}\%$ quota share treaty with a re-insurer to cover new business portfolio over the projection period. Thus, a ceding company needs to hold $(100-\alpha_{QS})\%$ statutory reserves under any policy at any time.

For simplicity, we also assume that the proportion reinsured by the ceding company does not exceed 50% (i.e. $0 \le \alpha_{QS} \le 50$). And in return the ceding company receives an initial and a renewal reinsurance commission equal to $\alpha_{QS}\%$ of its total initial and renewal expenses (including commission) respectively, which will cover in full the agent's commission and direct writer's expenses relating to the reinsured business.

As an alternative, the ceding company may receive a premium rebate (e.g. 90% of the annual reinsurance premium $(90\% \times \alpha_{QS}\% \times OP_m^j(t))$ payable only over the initial commission period) from the re-insurer instead of a reinsurance commission. The reinsurance commission terms may appear to be more generous than the premium rebate, as it is unlikely that the premium rebate covers in full both the initial and renewal expenses (including commission) of the reinsured portion of the policy. Thus, the effect of the reinsurance commission and the premium rebate on a company's new business plan and insolvency risk will depend on the extent which they reduce the company's capital strain.

4.6.3 Deficit account (loan) financing with risk premium reinsurance

In practice, an insurer may obtain a cash advance (which is usually represented as a reinsurance commission related to the volume of business reinsured) from a reinsurer which can be financed by means of deficit account financing with risk premium reinsurance. The business under the deficit account financing is split into tranches (e.g. monthly, quarterly or yearly new business) and all tranches are covered by a single treaty which is subdivided into two distinct sections, namely risk premium reinsurance and a cash advance (loan) typically related to the ceding company's commission payments (Brett and Cowley, 1993).

Under the risk premium reinsurance method, the amount reinsured is a proportion of the sum at risk (the excess of the sum assured over the reserve), and it is reinsured on a yearly renewable risk premium basis (that is, the reinsurance premiums which increase annually in line with the age of the lives are charged on a year-by-year basis), (Subject 302 Core Reading, Unit 14). Generally, the reinsured sum at risk may be determined on either the reducing retention method (a specified percentage of the sum at risk of each policy is reinsured) or the constant retention method (the sum at risk in excess of the ceding company's retention on any individual life is reinsured), (Spedding, 1989).

For new business financing arrangement, the same block of business is reinsured under both sections of the deficit account financing treaty. As the risk premium reinsurance is typically on a quota share basis, the reinsurance financing premium (i.e. a percentage of ceding company office premium) for the cash advance is also based on the same proportion (Brett and Cowley, 1993).

When the cash advance for each tranche of business is paid to the ceding company, it is also debited to a deficit account established for that tranche. "Each deficit account is credited periodically with a reinsurance financing premium, which is an agreed percentage of the reinsured portion of the office premium", (Brett and Cowley, 1993). Whist interest at the agreed rate is added, the outstanding cash advance is reduced each year as reinsurance financing premiums continue to be paid until the account reaches zero over the term of the policy (Spedding, 1989).

Once the deficit account balance is zero the financing reinsurance arrangement is recaptured by the ceding company, whist the risk premium reinsurance runs until the expiry of the business. The reinsurer makes a loss if the cash advance is not repaid by the reinsurance financing premium. Thus, the repayment terms for deficit account are set by the reinsurer, having considered the expected lapse experience of the portfolio of reinsurances, (Subject 302 Core Reading, Unit 14).

The above will improve the ceding company's balance sheet, particularly in the short term, as the liability will not show in the balance sheet. The ceding company's investment and lapse risk is not passed to the reinsurer.

In this investigation, we assume that the proportion reinsured (denoted by α_{RP}) by a ceding company under both sections of the deficit account financing treaty does not exceed 50% (i.e. $0 \leq \alpha_{RP} \leq 0.5$). This means that re-insurer pays α_{RP} of the sum at risk for death claims in return for a separately calculated premium (see section 4B in appendix 4) and also receives α_{RP} of the annual office premium in each projection year t (denoted by $\alpha_{RP} \times OP_m^j(t)$) from the ceding company to offset the deficit account balance of the cash advance made for the tranche of business issued in year s, where $s \leq t$. We assume that the maximum cash advance per policy (denoted by $CD^j(s) \quad 1.5 \times IC_m^j(s)$) is equal to one and half times the ceding company's initial commission (denoted by $IC_m^j(s) = 1.25 \times 0.35 \times OP_m^j(s)$) to cover the initial commission in the first year. However, a ceding company may receive only a proportion of the maximum cash advance (denoted by $LR^j(s) = \alpha_{RP} \times CD^j(s)$) as a loan depending on the proportion of business ceded. Thus, as the loan is small, it is likely that it is repaid before the tranche of business expires. A recursive formula for the deficit account balance in a projection year t for a tranche of new policies issued in year s is given in section 4B of appendix 4.

In the following sections, we consider the level of financing provided by quota share and deficit account financing which relates to new business financing, and the effect of financial reinsurance on a company's new business plan (the allocation of free assets to write new business) to produce a given level insolvency risk. We assume that the business reinsured is that of the future new business portfolio over the projection period. It does not include the in-force portfolio at the start of the projection period. The proportion of business ceded remains constant over the projection period.

4.6.4 Level of new business financing

The level of new business financing provided by a financial reinsurance can be defined as the difference between the new business strain before and after reinsurance (Brett and Cowley, 1993). The new business financing under a quota share treaty may improve the capital efficiency of the re-insured product by reducing the capital requirement, particularly in the first year, through sharing the reserving strain and expenses leading to a reduction in new business strain. The foregoing paragraphs will clarify the above.

In this section, we compare the level of financing being provided by the different new business financing methods described above. We consider market portfolio MP6 which issues 5,000 temporary insurance policies (policy type 1) to lives aged 55 at entry in year 0, having an initial solvency ratio of 1.50 and allocating 40% of its free assets in writing new business over the projection period.

Figure 4.16 shows the mean of the distributions of new business strain per policy and new business volume written in projection years for different Fin Re methods. On the other hand, figure 4.17 shows the level of financing provided by different Fin Re methods. Figure 4.17 is produced by taking the difference in the new business strain between before and after reinsurance in figure 4.16, for example, the difference in new business strain between no reinsurance in figure 4.16(a) and quota share reinsurance commission in figure 4.16(b).

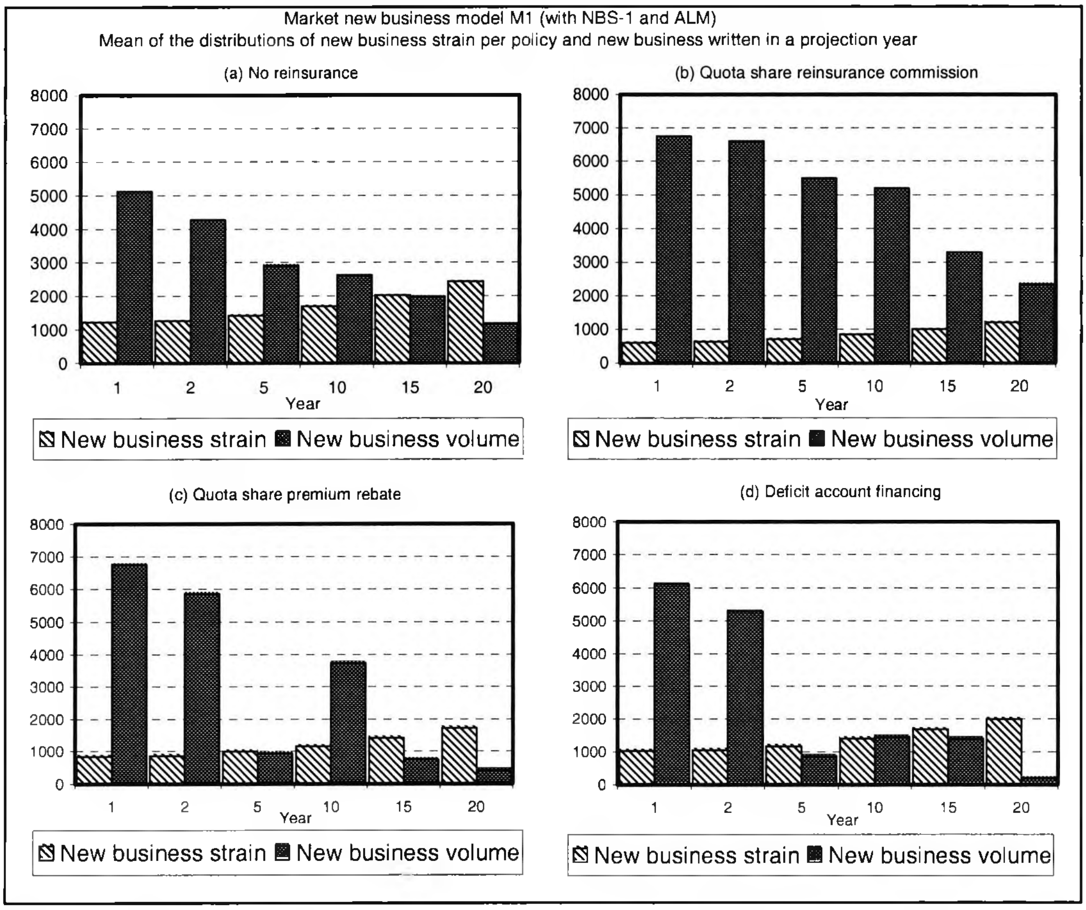
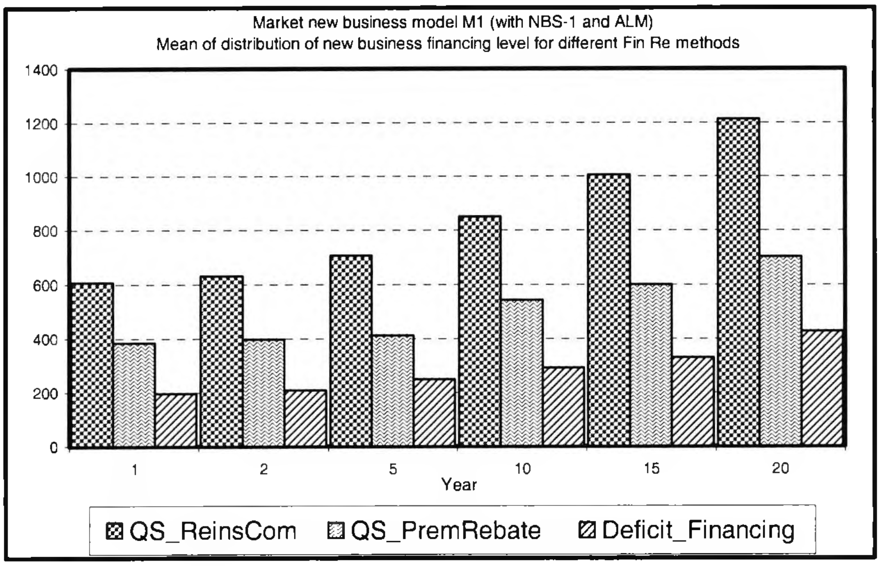


Figure 4.16: Distribution of new business strain and written for different financial reinsurance methods.

Figure 4.16 shows that quota share reinsurance commission reduces a company's new business strain more than the other methods, and hence it enables a company to write a greater new business volume with the same level of initial free assets than the other methods. Figure 4.16(b) shows that the quota share reinsurance commission enables a company to increase its new business volume by about 100% of the new business written for the no reinsurance case in year 20 as shown in figure 4.16(a). On the other hand, the quota share premium rebate in figure 4.16(c) and deficit financing in figure 4.16(d) reduce the company's new business volume written by about 60% and 80% of the new business written for the no reinsurance case in figure 4.16(a) respectively.



Note:
QS_ReinsCom = Quota share reinsurance (with reinsurance commission)
QS_PremRebate = Quota share reinsurance (with premium rebate)
Deficit_Financing = Risk premium reinsurance (with deficit account financing)

Figure 4.17: Distribution of new business financing level for different financial reinsurance methods.

Thus, quota share reinsurance commission provides the highest level of financing than other methods, as shown in figure 4.17. This is because reinsurance commission covers in full the initial and renewal expenses (including commission) relating to the reinsured business while the others may only cover adequately the initial commission. Figure 4.17 also shows that the quota share reinsurance premium rebate provides a higher level of financing than deficit account financing.

Figure 4.16 also shows that, as different levels of new business financing are produced by different financing methods, the volume of new business written also takes a different shape over the projection period depending on the financing methods. Financing methods that do not cover in full the expenses of the reinsured business produce troughs and peaks in the volume of new business written over the period. This is because free assets are required to offset the shortfall in difference between the financing provided and the expenses relating to the reinsured business.

4.6.5 Effect of Fin Re on company's new business plans and risk

As Fin Re treaty will "allow the same value of the free assets to support higher volumes of new business" McGaughey et al (2001), the decision concerning the allocation of free assets to finance new business needs to be taken not only alongside the choice of an appropriate investment strategy, but also the choice of an appropriate volume of business to be ceded. Since the original terms reinsurance is considered to be the most appropriate for term assurance contracts (Booth et al, 2005), we consider market portfolio MP3 in table 4.7 in section 4.4.3.1 (or see table 4.1) with 10,000 temporary insurance policies (policy type 1 with an initial commission period of 4 years) issued to lives aged 35 and 55 at entry (5000 policies in each age group) in year 0.

Firstly, we consider the effect of the level of new business financing through quota share reinsurance commission on a ceding company's new business plans and its insolvency risk. The simulated results (probability of insolvency and mean shortfall risk) of the temporary insurance portfolio allowing for quota share reinsurance commission without controlling the company's future new business growth is shown in figure 4.18.

Figure 4.18 shows a set of curves (contour lines) that, for a given initial solvency ratio (e.g. $SR_m(0)=1.50$), will enable a company to choose freely both of the following variables: the constant proportion of free assets allocated to finance new business (ρ_m) and the constant proportion of business ceded ($\alpha_{QS}\%$) under a quota share reinsurance treaty so that it has an adequate level of free assets to continue its desired new business strategy over a 20-year projection period. Each curve gives the combination of a company's percentage of business ceded and a choice of new business plan (the constant proportion of free assets required to write new business) over a 20-year projection period that produces a fixed level of insolvency risk (the probability of insolvency and the mean shortfall risk).

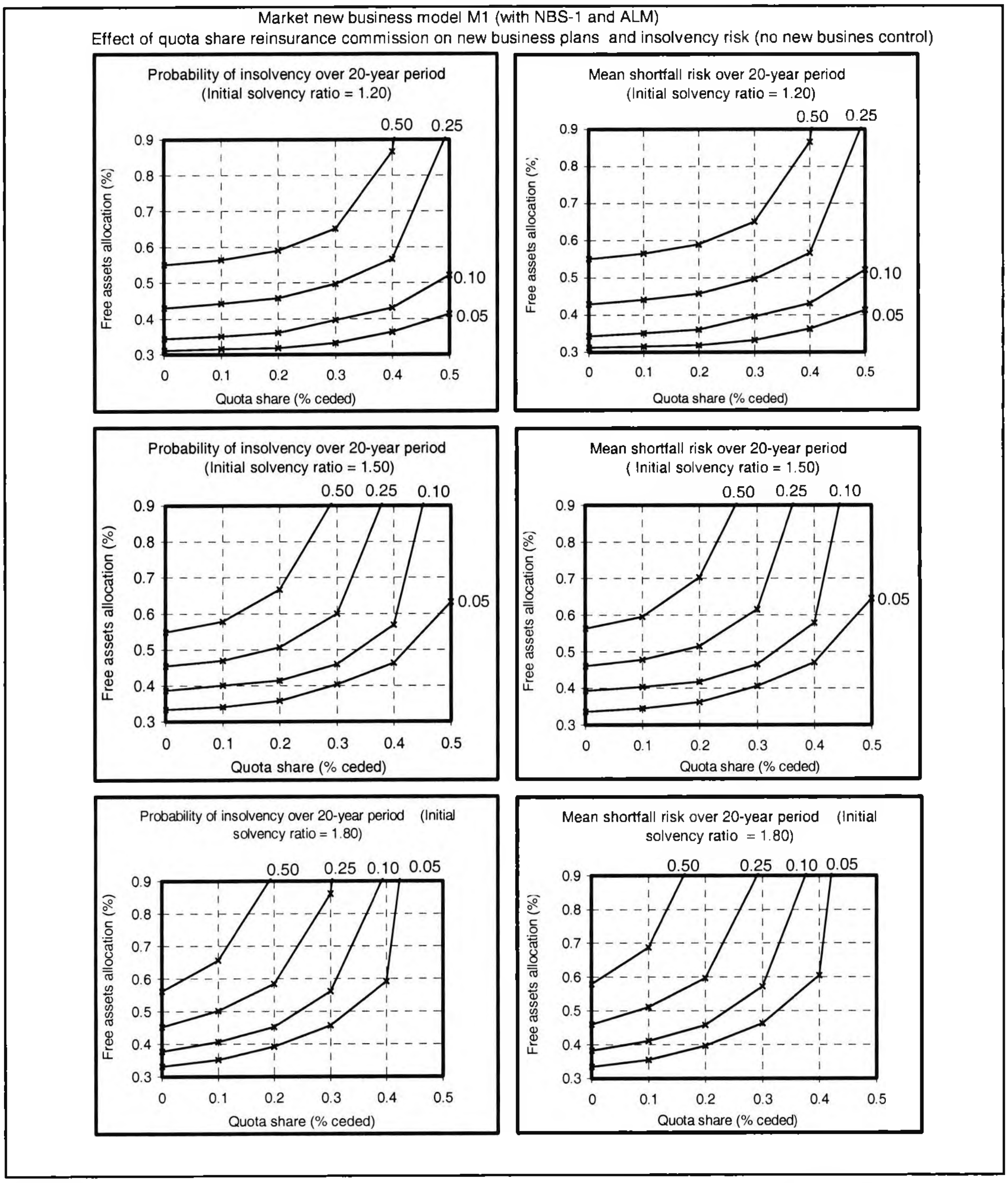


Figure 4.18: Effect of quota share reinsurance commission on new business plans and insolvency risk under no new business control.

As we expect, figure 4.18 shows that a company's need for new business financing through quota share reinsurance treaty, in order to increase its market share, decreases as the company's initial size (measured in terms of initial solvency ratio) increases. Consider a 5% probability of statutory insolvency requirement (that is a 5% constant insolvency risk curves in figure 4.18), a company with an initial solvency ratio of about 1.80 does not need a 50% level of quota share reinsurance treaty in order to remain solvent, irrespective of the choice of free assets allocation proportion (ρ_m) to meet its expansion plans. The above also holds for mean shortfall risk measure.

Figure 4.18 also shows that, for a given contour line (e.g. 5% constant insolvency risk curve), the free assets allocation proportion increases with the percentage of business ceded. This reflects the fact that the financial strength being provided by the quota share reinsurance treaty increases with the percentage of business ceded. This is because the reinsurance commission covers in full the ceding company's expenses (and hence the new business strain is also covered in respect) of the reinsured portion of the business. Thus, it enables the ceding company to allocate a higher proportion of its free assets to write new business in order to increase its market share, if all other things remain the same.

Where the level of new business financing (e.g. reinsurance commission) received is sufficient to cover in full the expenses (i.e. the full new business strain) related to the reinsured business, it may be inappropriate for a company to control the quantity of new business written in a year e.g. not to exceed a 85% of the quantity of new business demanded, i.e. $\beta_m^j(t) \leq 0.85$ as defined in equation (7), when the reinsurance treaty is still in-force. This is because the control of new business growth does not allow the extra free assets created by the financial reinsurance to be used for writing a profitable business, and this may be seen as an inefficient use of the available capital even though it produces a very low level of probability of insolvency (see quota share reinsurance with and without new business control in figure 4.19).

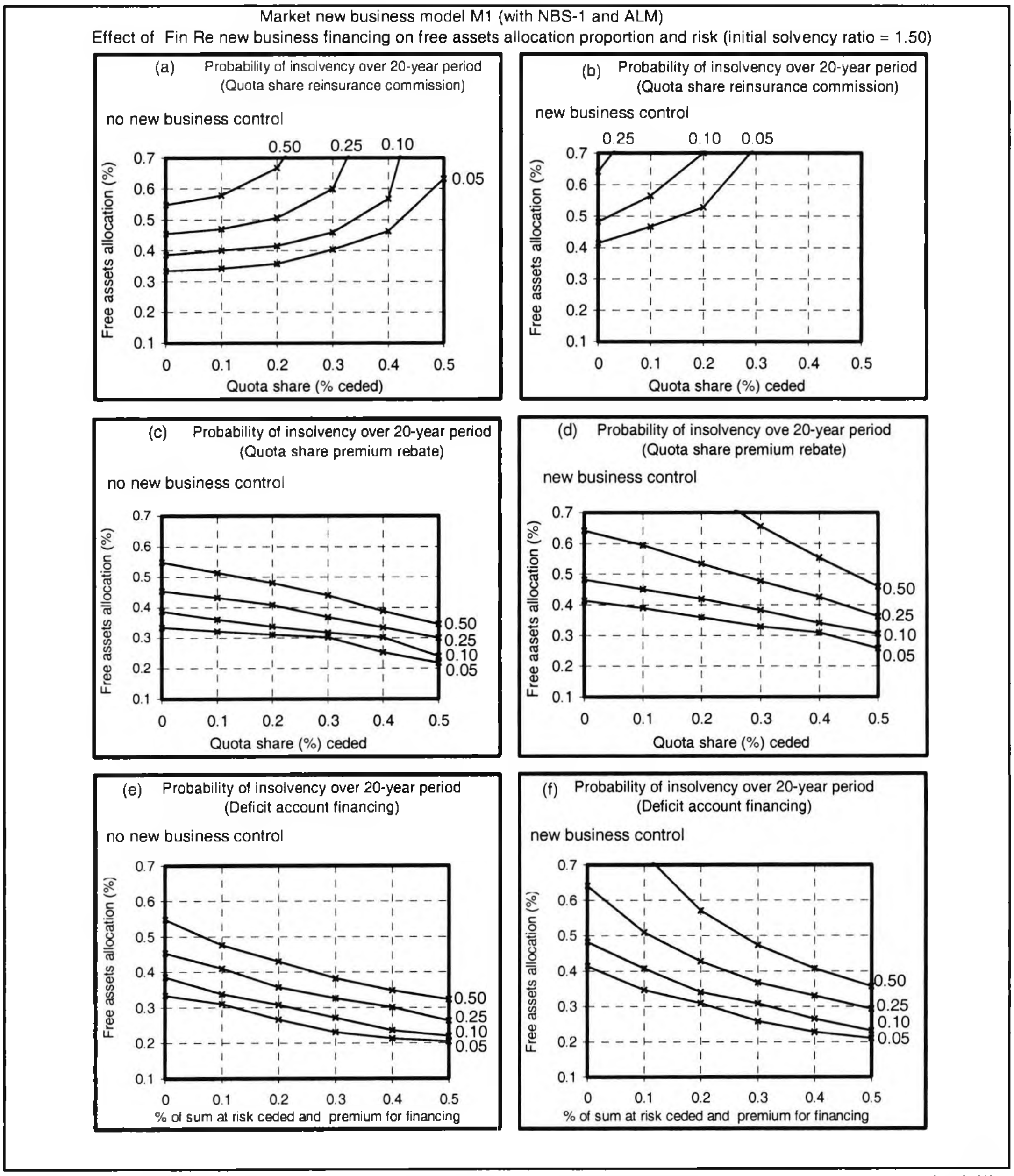


Figure 4.19: Effect of Fin Re methods on new business plans and probability of insolvency under with and without new business control.

Comparing figure 4.19(a) with figure 4.19(b) shows that the contour lines for quota share reinsurance commission with control of new business growth are higher than those without the control of new business, which indicates that the probability of insolvency is lower when the new business written is controlled than when it is not controlled. However, a company may prefer to write more profitable business with its available free assets if it will not have an adverse effect on statutory solvency requirements.

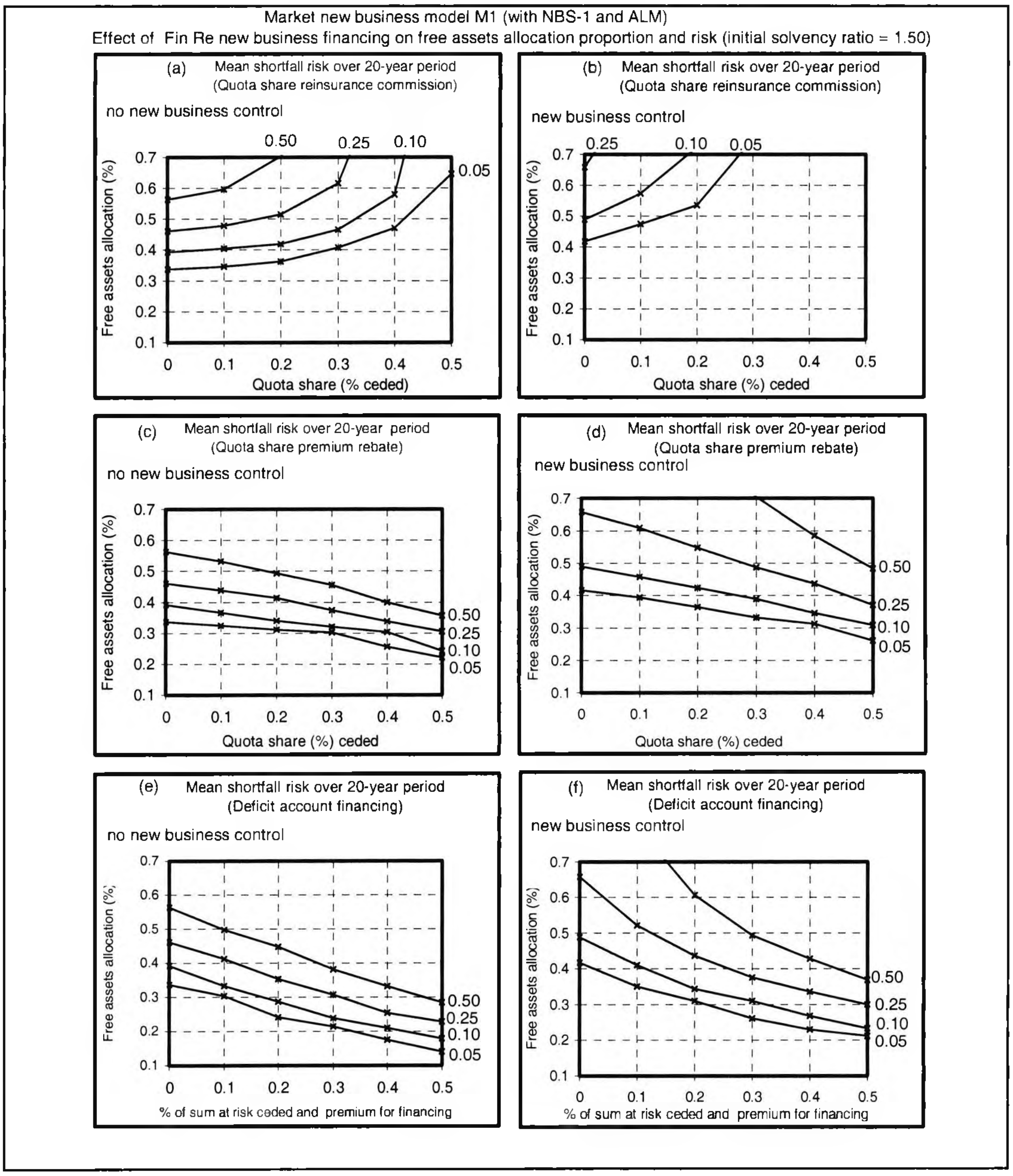


Figure 4.20: Effect of Fin Re methods on new business plans and mean shortfall risk under with and without new business control.

Secondly, the level of new business financing provided by a quota share premium rebate may affect a ceding company's new business plans differently when compared with a reinsurance commission. Figures 4.19(c) and 4.19(d) show that, as the percentage of business ceded increases the proportion of free assets allocated to write new business decreases if the new business financing is being provided by a premium rebate. This is because the premium rebates received from the re-insurer (e.g. 90% of the reinsurance premium) over the first four years' of the policy are not sufficient to cover in full the initial and renewal expenses (including commission) related to the reinsured business, irrespective of whether the company controls its future new business growth or not.

As more business is being reinsured more free capital is used up to offset the difference between the premium rebate received from the re-insurer and the expenses incurred by the ceding company which relate to the reinsured business. Thus, a company may decide to control the new business growth under this scenario in order to reduce the use of further extra free assets which may also reduce the probability of insolvency.

The above is reflected in figures 4.19(c) and 4.19(d) which show that the contour lines for quota share reinsurance premium rebate with control of new business growth are higher than those without control of new business growth, which is an indication of an improvement in the solvency position. The control on new business growth may be seen as writing an appropriate volume of new business leading to an efficient management of available free assets. These results also hold for the mean shortfall risk measure shown in figures 4.20(c) and 4.20(d).

Thirdly, the effect of new business financing with a deficit financing method (cash advance) on a company's new business plan is similar to that of quota share premium rebate, in that they do not cover adequately the expenses in respect of the reinsured portion of the policy. Figures 4.9(e) and 4.19(f) also show that that, as the percentage of business ceded increases the proportion of free assets allocated to write new business decreases if the new business financing is provided by deficit account (loan) financing.

On the other hand, the deficit account (loan) financing method differs from the quota share premium rebate method because the cash advance is a loan which will be repaid from the premiums with interest, and hence they provide different levels of financing as shown in figure 4.17. As the deficit account financing method provides less financing than the quota share financing methods, it requires the use of more free assets (and hence a lower proportion of free assets to be allocated) in writing new business over the projection period than the quota share reinsurance. Comparing figure 4.19(c) with figure 4.19(e) shows that the contour lines for deficit account financing are lower than those under quota share reinsurance. This reveals that it is also appropriate to control the new business growth under a deficit account (loan) financing method. The above also holds for mean shortfall risk measure shown in figure 4.20.

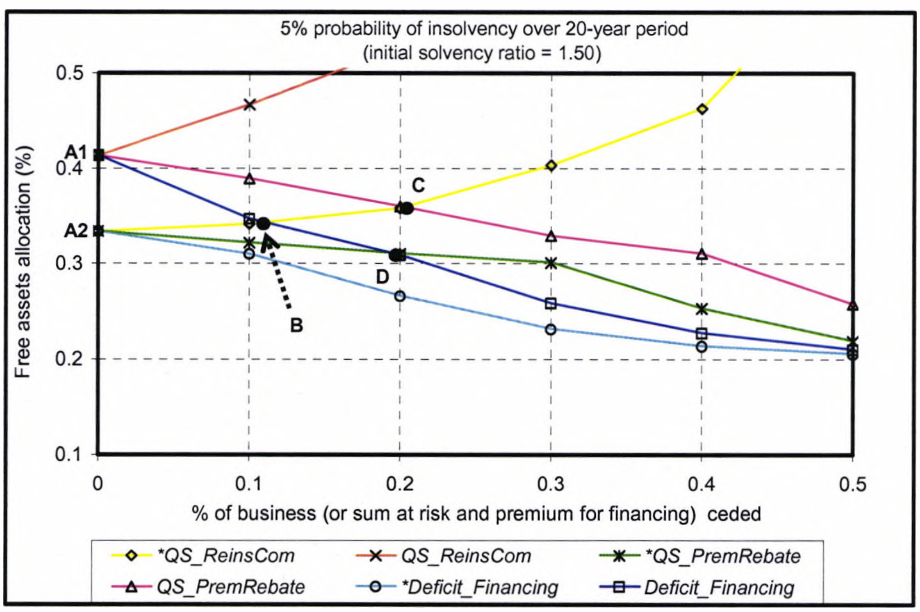
4.6.6 How do the financing methods compare?

In this section, we compare the different new business financing methods under with and/or without new business control.

Firstly, we compare the three financing methods on the basis of either with or without new business control. In section 4.6.5, figure 4.19 clearly shows that the new business financing by quota share reinsurance commission performs better (in terms of the proportion of free assets allocated to write new business) than the other methods, for a given percentage of business ceded over a 20-year period. This is a reflection of the high level of financing (reduction in new business strain) being provided. In the same vein, the quota share premium rebate performs better than the deficit account financing.

On the other hand, figure 4.19 also shows that the quota share reinsurance commission financing method produces the lowest probability of insolvency than the other methods, for a given percentage of business ceded over a 20-year period. Further, the deficit account financing produces the highest probability of insolvency than the other methods. The ranking of riskiness in financing methods also holds for mean shortfall risk measure shown in figure 4.20.

Secondly, the above ranking of riskiness in financing methods may change if the impact of with and without control of new business growth on a company's free assets is also taken into account. Figure 4.21 shows the 5% probability of insolvency contour lines for both with and without control of new business growth as in figure 4.19. The first set consists of three contour lines for the case of control of new business growth which start at point A1 whilst the second set of three contour lines for the case of no control of new business growth start at point A2. Thus, points A1 and A2 show the proportions (about 41% and 33%) of free assets allocated to write new business for with and without controlling the new business growth respectively, in the absence of a financial reinsurance arrangement (i.e. 0% of business ceded).



Note :

QS_ReinsCom = Quota share reinsurance (with reinsurance commission)
QS_PremRebate = Quota share reinsurance (with premium rebate)
Deficit_Financing = Risk premium reinsurance (with deficit account financing)
The star (*) indicates a reinsurance arrangement without new business control (see point A2) and without star (*) indicates an arrangement with new business control (see point A1)

Figure 4.21: Comparison of financial reinsurance methods with 5% probability of insolvency contour lines.

In each set of contour lines (e.g. start at point A1 for with control of new business growth) the contour line for a quota share reinsurance premium rebate method is lower and higher than the contour line for a quota share reinsurance commission and a deficit account financing method respectively. In the presence of a financial reinsurance arrangement (i.e. % of business ceded is greater than zero), the contour lines for control of new business growth (starting at point A1) may intersect with the contour lines for without control of new business growth (starting at point A2) at different points in the graph. Each point of intersection indicates a combination of free asset allocation proportion and percentage of business ceded. For example, figure 4.21 shows that the contour line for quota share reinsurance commission method without control of new business growth (starting at point A2) intersects with the contour line for deficit account financing method with control of new business growth (starting at point A1) at point B. Similarly, figure 4.21 also shows that, the same contour line for quota share reinsurance commission method without control of new business growth (starting at point A2) intersects with the contour line for quota share reinsurance premium rebate with control of new business growth (starting at point A1) at point C.

Thus, at point B, the company can choose between deficit account financing with control of new business growth and quota share reinsurance commission without control of new business control growth, if it allocates about 34% of free assets to write new business and cedes about 10% of new business written over the projection period. Thus, the company is indifferent (in terms of the 5% level of insolvency risk) in choosing between the two financing methods. This feature can be explained as follows.

As the company cedes 10% of it business (or sum at risk), the financial strength provided by the reinsurance commission is used up in writing more new business in the absence of a control of new business growth. On the other hand, the free assets required to offset the shortfall in financing the new business strain by deficit account financing reduce as less new business is written under the control of new business growth. By allocating 34% of free assets to write new business, more free assets are released to improve the company's solvency position under the deficit account financing with control of new business growth than under the reinsurance commission without the control of new business growth. Thus, both scenarios will produce the same 5% level of probability of insolvency, see point B. This argument is also applicable to the next two examples below.

Similarly, at point C, the company is indifferent (in terms of the 5% level of insolvency risk) in choosing between quota share premium rebate with control of new business growth and quota share reinsurance commission without control of new business control. At point C, the company needs to allocate a 36% of free assets to write new business whilst ceding a 20% of new business written over the projection period.

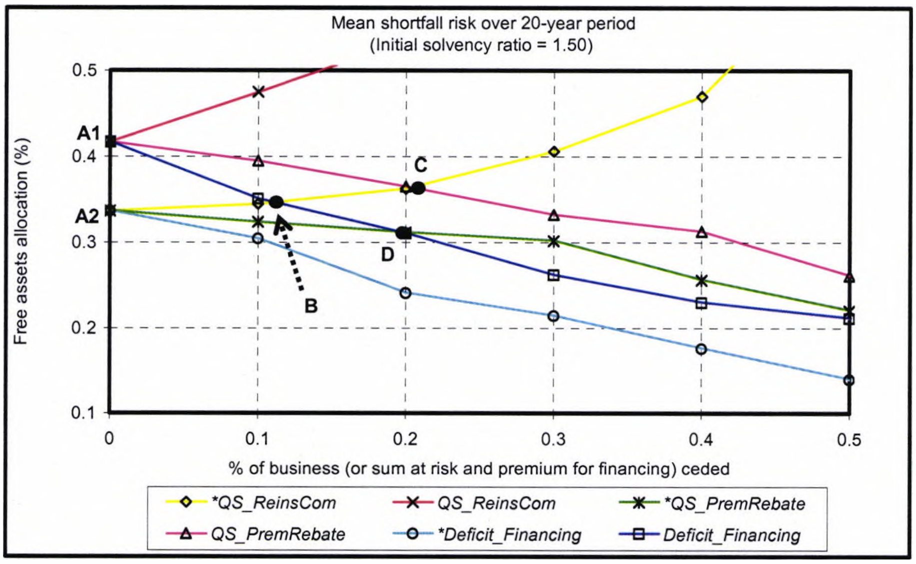


Figure 4.22: Comparison of financial reinsurance methods with 5% mean shortfall contour lines.

Figure 4.21 also shows that, at point D, the company is indifferent (in terms of the 5% level of insolvency risk) in choosing between deficit account financing with new business control and quota share premium rebate without new business control. At point D, the company needs to allocate about 31% of free assets to write new business whilst ceding about 20% of new business written over the projection period.

Thus, figure 4.21 reveals that a company has a choice of new business financing methods, which depends on the company's level of control of new business growth. This holds also for mean shortfall risk shown in figure 4.22. As the percentage of business ceded increases (e.g. above 20%), the financing methods tend to produce different results. This is particularly true for mean shortfall risk, as reflected in the contour lines shown in figure 4.22.

4.6.7 The effect of Fin Re (with mix of contract types) on company's risk

As different contract types produce different levels of new business strain, so also the different financial reinsurance treaties provide different levels of new business financing under the same policy type. Figure 4.23 (using portfolio MP6) shows the mean of the distributions of new business strain per policy for a temporary insurance policy (policy type 1 with four year commission period) and a non-profit endowment insurance policy (policy type 4 with one year commission period) under with (50% of business ceded) and without financial reinsurance.

Figure 4.23 shows that, for the cases of with and without financial reinsurance, the new business strain per policy under a non-profit endowment insurance is slightly higher than that of a temporary insurance over the projection period, except for deficit account financing. The difference in new business strain between non-profit endowment insurance and temporary insurance is slightly higher under the quota share premium rebate than the other financing methods.

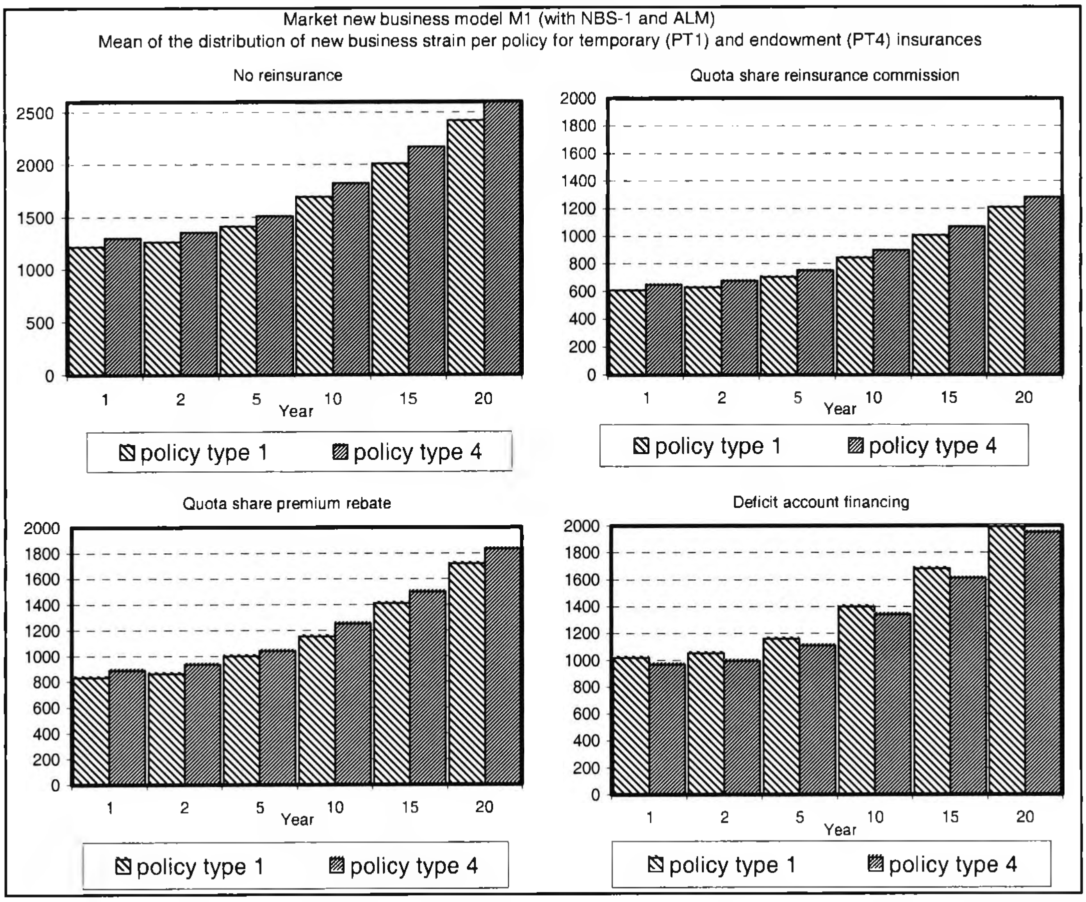


Figure 4.23: Distribution of new business strain for policy types 1 and 4 under financial reinsurance methods.

The effect of Fin Re methods on a company's new business plan and risk can change if there is mix of business by contract type in a portfolio. Thus, we consider market portfolios MP3 and MP10 in table 4.7 in section 4.4.3.1 in order to investigate the effect of Fin Re on a portfolio with mix of business by contract type. Market portfolio MP3 (also considered in section 4.6.5) consists of 10,000 temporary insurance policies issued to lives aged 35 and 55 (5,000 in each group) in year 0.

Market portfolio MP10 consists of both temporary (policy type 1) and non-profit endowment (policy type 4) policies issued to lives aged 35 and 55 at entry in year 0, and out of 10,000 policies issued in year 0, 2000 are non profit endowment policies (1000 in each age group) while 8000 are temporary policies (4000 in each age group). We assume that the two portfolios under consideration have initial solvency ratios of 1.5, and allocate 40% of free assets to write new business over the projection period.

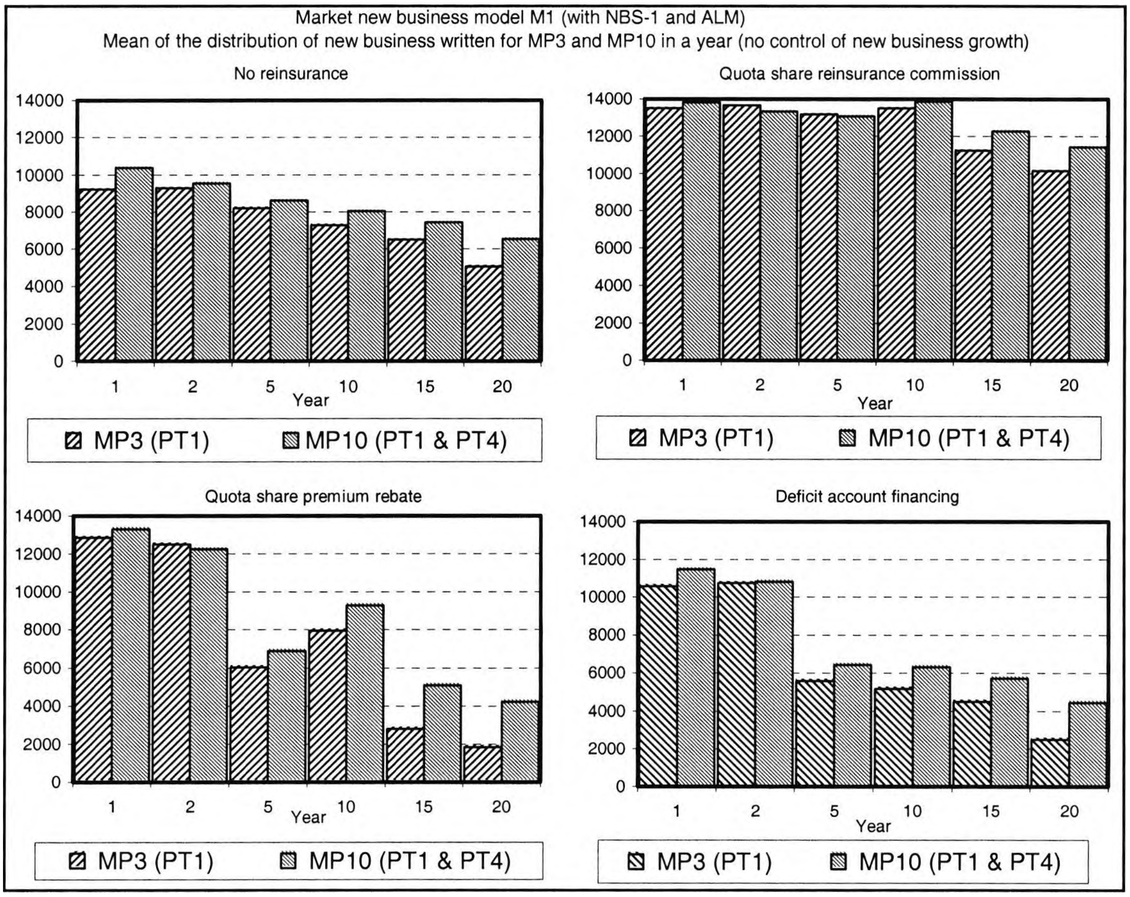


Figure 4.24: Distribution of new business written for market portfolios MP3 and MP10 under financial reinsurance methods.

Figure 4.24 shows the mean of the distribution of new business written for market portfolios MP3 and MP10 (under no control of new business growth). The figure shows that market portfolio MP10 writes more new business volume than market portfolio MP3 in the absence of reinsurance and therefore the drain on free assets is likely to be more in market portfolio MP10 than in market portfolio MP3.

The initial assets at projection date (defined as a multiple (e.g. initial solvency ratio) of the reserves of in-force policies as at that date) for market portfolio MP10 (with a mix of temporary insurance and non-profit endowment insurance policies) are higher than those of market portfolio MP3 (with temporary insurance policies), since the reserves for endowment insurance policies are higher than those of temporary insurance policies.

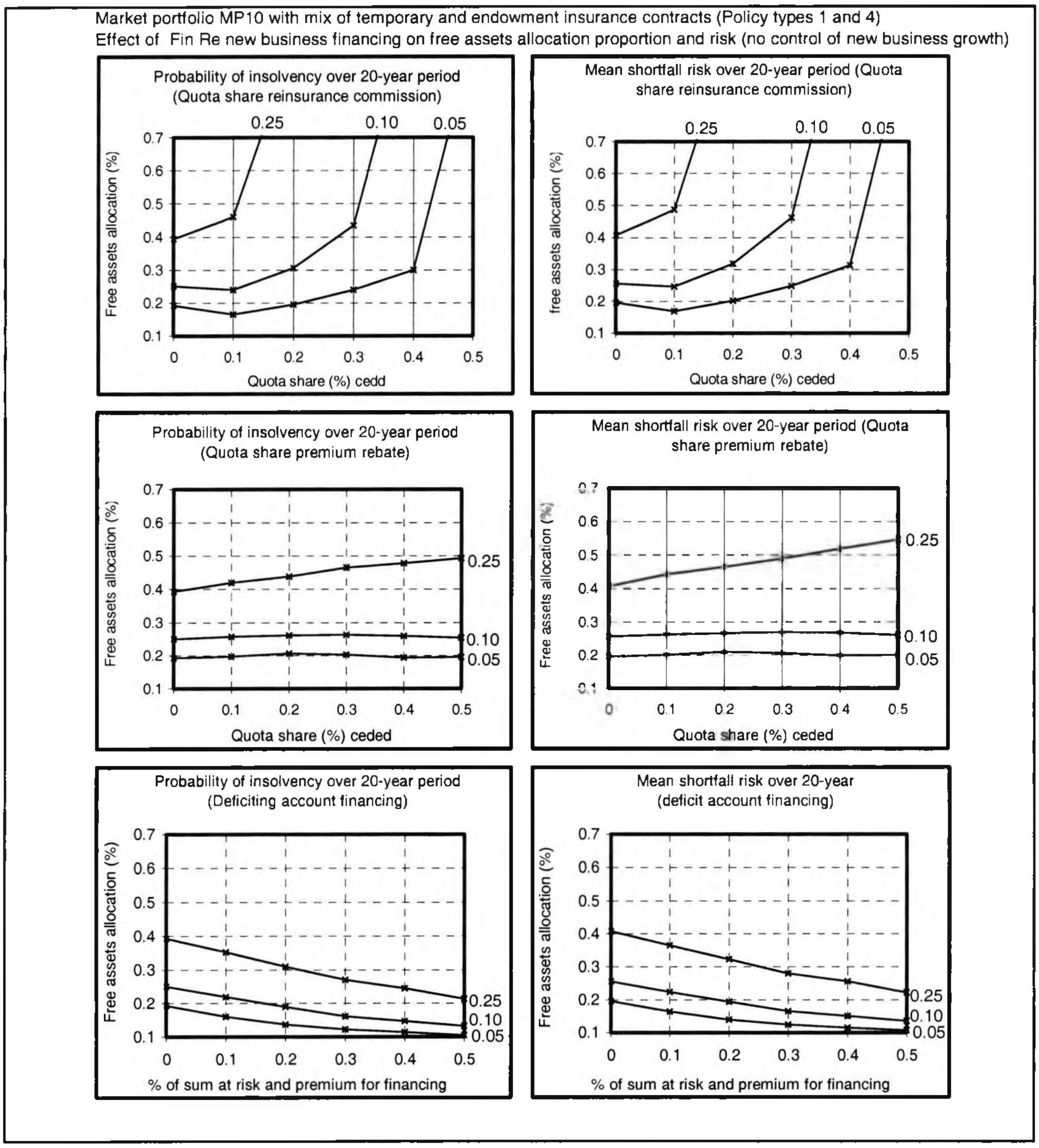


Figure 4.25: Effect of Fin Re on new business plans and insolvency in market portfolio MP10 without new business control.

Figure 4.25 shows the effect of financial reinsurance methods on new business plans and insolvency risk for market portfolio MP10 without new business control. The features in figure 4.25 can be fully explained by comparing it with figure 4.19 under no control of new business growth.

Comparing figure 4.19 with figure 4.25 for market portfolio MP3 and market portfolio MP10 respectively shows that, in the absence of reinsurance (i.e. 0% of business ceded) and without control of new business growth, the market portfolio MP3 (with only policy type1) has a lower probability of insolvency than market portfolio MP10 (with mix of policy types 1 and 4). This is because more capital is required in writing a non-profit endowment insurance policy (and hence more drain on free assets) than writing a temporary insurance policy, and market portfolio MP10 writes more new business with its high initial free assets.

Figure 4.24 shows that for a quota share reinsurance commission method, both market portfolios write high volume of new business over the projection because of the high level of financing being provided, and there is little or no difference in the quantity of new business written between the market portfolios, particularly in the early years of the projection. As more capital is required in writing endowment insurance policies, a larger drain on free assets is likely in market portfolio MP10 than in market portfolio MP3. This leads to a higher probability of insolvency in market portfolio MP10 than in market portfolio MP3 as the proportion of business ceded increases, comparing figure 4.19 with figure 4.25. This is reflected in the contour lines which are higher in figure 4.19 under no control of new business growth than in figure 4.25. Thus, market portfolio MP10 needs to cede more than 40% of its business in order to remain solvent irrespective of the proportion of free assets allocated to write new business (see 5% probability of insolvency contour line in figure 4.25). Thus, the level of financing provided becomes adequate when the proportion ceded exceeds 40%.

Figure 4.25 also shows that, for the quota share premium rebate method, the proportion of free assets allocated to write new business increases or remains the same as the proportion of business ceded increases (see 25% contour lines). However, in figure 4.19 the proportion of free assets allocated to write new business decreases as the proportion of business ceded increases under the quota share premium rebate. The reason for the above is explained below.

Figure 4.24 shows that, for both market portfolios, there is little or no difference in the new business volume written in the early years of the projection, and the new business volume written over the medium and long term is relatively low under the quota share premium rebate method. Thus, the high initial free assets and investment returns for market portfolio MP10 as explained above may not be used up in writing new business in the early years of the projection. This may create investment freedom leading to an improvement in the solvency position. Thus, the above shows that quota premium rebate is more appropriate for portfolio with a mix of non-profit endowment and temporary insurance policies.

Figure 4.24 also shows that, under the deficit account financing method, market portfolio MP10 writes more new business than market portfolio MP3 particularly in the short and medium term. Since the level of financing provided is also inadequate under the deficit account financing, writing more new business leads to a higher probability of insolvency in market portfolio MP10 than in market portfolio MP3, as shown in comparing figure 4.19 with figure 4.25.

This above discussion holds also for mean shortfall risk, by comparing figure 4.20 with figure 4.25.

4.6.8 Conclusion

The Original terms reinsurance on a quota share basis can support a company's new business expansion plans effectively if the expenses relating to the reinsured business are adequately covered by the level of new business financing (rebate) being provided. A company is likely to increase the proportion of free assets allocated to finance new business as the percentage of business ceded increases if the rebate (e.g. reinsurance commission) does cover in full the expenses of the reinsured portion of the business, and still remain solvent. Thus, the ceding company can substantially increase (may double) the volume of business written for the same new business strain or free capital without having insolvency problems.

On the other hand, a company may need to reduce the proportion of free assets allocated to write new business as the percentage of business ceded increases, if the rebate (e.g. premium rebate) does not cover adequately the expenses of the reinsured business (as the control of new business growth becomes appropriate). As an alternative, company may reduce the proportion of quota share reinsured. Thus, the choice of Fin Re methods depends on a company's desired level of control of new business volume, for a given combination of proportion of free assets allocated to write new business and the proportion of business ceded.

The above confirms the findings of Brett and Cowley (1993) that "if the new business is expanding faster than expected the quota share can be increased and if the new business is progressing slower than expected the quota share [reinsured] can be reduced".

The level of new business financing also depends on the mix of new business by contract type, and the quota share premium rebate method may be suitable for a portfolio with mix of endowment and temporary insurance policies.

On the other hand, the deficit account financing method may only be appropriate where there is a control of new business growth. Thus, a life office which is expanding rapidly is unlikely to use a deficit account method.

Chapter 5

Conclusion

5.1 Brief Overview

The insolvency risk arising from the uncertainty in fulfilling a company's future new business plans is central to our investigation in this research. In determining the level of risks inherent in new business plans, the study employs the simulated results from asset/liability models for the market and/or company to determine the effects of new business risk on a life insurance company in a competitive and stochastic environment.

The research adopts new business models and new business strategies, with sensible parameter values obtained using sensitivity analysis in order to investigate the risk in new business plans. The investigation provides important results in relation to the objectives established in section 1.3 of chapter 1. Before the findings are discussed we wish to briefly capture the salient issues outlined in the various chapters.

In chapter 1, we note that the quantity of new business written is distinct from the quantity of new business demanded because in practice, in contrast with the literature, the amount of new business actually written is constrained by the available capital to write new business. We further note that a company's pricing policy assumes a particular level of new business volume to be written for it to be profitable. Where the quantity of new business volume to be written under a given new business strategy with its pricing policy is not realized due to insufficient free assets, this could affect the company's future profitability and could result in the risk of insolvency.

The new business strategies formulated and models adopted that provide the framework for the simulated results are discussed in chapter 2. The new business strategies are formulated in the context of different pricing policies and they include:

- New Business Strategy 1 - A company reduces (increases) its premium relative to the profit-tested premium to attract more (less) quantity of new business demanded in a particular projection year if its solvency ratio in the previous year is above (below) a new business decision factor (NDF).
- New Business Strategy 2 - A company increases (reduces) its premium relative to the profit-tested premium to attract less (more) quantity of new business demanded in a particular projection year if its solvency ratio in the previous year is above (below) a new business decision factor (NDF).
- New Business Strategy 3 - A company reduces its premium relative to the profit-tested premium to attract more quantity of new business demanded in any particular projection year if its previous year's solvency ratio is either above or below a new business decision factor (NDF).
- New Business Strategy 4 - A company increases its premium relative to the profit-tested premium to attract less quantity of new business demanded in any particular projection year if its solvency ratio is either below or above a new business decision factor (NDF).
- The level of new business decision factor (NDF) is arbitrarily determined based on the company's degree of risk aversion relative to the desire for market share. New business strategies 1 and 2 have $NDF = 1.0$ whilst for new business strategies 3 and 4, $NDF > 1.0$.

The following investment strategies are also applied in the evaluation process of new business risks as discussed in chapter 2.

- Asset-Liability Matching (ALM) Investment Strategy - All liabilities are backed by fixed securities. Only free assets are invested in equities.

- Static Investment Strategy - Fixed proportion (50%) of the assets is invested in equities and the other 50% is invested fixed interest securities (gilts).
- Dynamic Asset Allocation Strategy (DS1 & DS2) - Funds are allocated to invest on fixed securities and equities based on the degree of solvency position of the company in the previous year.

In modelling the market new business, we assume that the new business volume demanded is determined by the disposable income of prospective policy holders (which may be linked to real wage inflation) and the price of the product. Similarly, we develop new business models for the company to interact with the market model based on the company's price elasticity of demand function as its office premium relates to that of the market office premium taking into account both the regulators control of new business volume or management decision. As an integral part of the market, the company would only write a portion of the quantity of new business demanded.

The overall effect of these new business models developed in our study produces only the quantity of new business demanded. But given our focus in this research that the actual new business written is not only dependent on the quantity of new business demanded but also on the free assets available and allocated to finance the new business, the study develops a model for the actual quantity of business written in section 2.2.1.4.

Chapter 3 considers the measures of new business risk and uses the simulated results to:

- Investigate a company's free assets allocation proportion to finance new business over a chosen time horizon with the aim of producing a fixed level of insolvency risk for a given initial solvency ratio under a desired new business strategy and investment strategy.

- Evaluate the sensitivity of the risk measures to the new business models and model parameters.
- Show the influence of the factors affecting new business plans on the risk measures.
- Demonstrate the effect on risk measures of various types of company model (which differ by new business strategy and/or investment strategy) interacting with the market model in a competitive environment.

In the investigation, a company' decision concerning the allocation of free assets to write new business under a given initial free assets and new business strategy with its pricing policy is taken as a proxy for the company's new business plans.

The sensitivity analysis of the new business models using the concepts of hypothesis testing has been carried out with the application of the new business strategy 1 and the asset-liability matching investment strategy to determine which market new business model would serve as a benchmark for representing the term assurance market and which parameters are most influential in affecting the new business plan. It has also been used to show how the effectiveness of a new business plan to meet a given level of insolvency risk is dependent on the choice of values for the most influential parameters.

We further consider the factors affecting the new business plans in chapter 3 and note the important role being played by the free assets available and allocated to finance new business. The study is based on the assumption that there would be no further demand for capital from the shareholders to write new business in future projection years aside from the initial capital provided at the projection date other than internally generated funds from reserves released and surpluses from existing business.

In chapter 4, the study restricts its evaluation to the effects of market new business Model M1 and/or company new business Model C1, new business strategy 1 and assets-liability matching investment strategy (being the most intuitive strategies) and risk measures. The emerging new business risks from the study are noted to be controlled by the application of the following tools: underwriting, product design, mix of new business, constrained new business growth and reinsurance.

5.2 Discussion of Findings

5.2.1 We have set out to examine the sensitivity of a company's new business risks to factors affecting new business. In examining this, we have sought to determine, using new business strategy 1 and the asset-liability matching strategy, which new business model could serve as a benchmark and which of the parameters affecting new business plans could be most influential as well as the effectiveness of the new business plan to meet a given level of insolvency risk.

Using a sensitivity analysis, our investigation reveals that the models differ mainly by the quantity of new business demanded. Therefore, the true sensitivity of the models may only be known with certainty if a company has sufficient free capital to write any quantity of new business demanded. Thus, the sensitivity of the models may increase with increased free assets allocated to write new business. This shows that model sensitivity is affected by the free assets allocated. Nonetheless, Model M1, considered in terms of the price effect only which is a prominent factor in this market, could be taken to represent the term assurance market since it has the least variability in the quantity demanded. In addition, Model M1 (exponential price elasticity of demand function) is more sensitive to insolvency risk than models with constant price demand functions. The study further shows that there is not a significant difference in the probabilities of insolvency between the company new business models. However, the company new business models which allow regulators' control have more impact on a company with low free assets and yet allocate a high proportion of free assets to write new business. And given that the models with exponential price elasticity of demand function is more sensitive to the risk of insolvency than the models with constant price elasticity of demand function as earlier noted, we choose company model C1 to interact with the market new business model M1 on the basis of price effect only.

Also, the study notes that the free assets allocation proportion to finance new business is the most important factor that affects a life company's level of solvency in the future. Since this could help with achieving a company's strategic aim of sustainable growth in size, it needs to be remarked that a substantial increase in this parameter could lead to writing an inappropriate volume of business. This could drain available free assets and constrain investment freedom and the meeting of statutory solvency requirements. Therefore, for new business plans effectively to meet the demands of statutory solvency requirements, there would need to be a choice of combination of measures of premium response to a company's solvency in the pricing policy and free assets allocation under a given level of initial capital.

5.2.2 Where no further demand for capital from shareholders to write new business in future projection year is allowed, the study reveals that initial capital adequacy is critical to cover emerging new business strains and to avoid the peaks and troughs in profitability. This is because the insolvency risk for any given projection period decreases as the initial financial strength of the company increases although this depends, according to our investigation, on the level of business written in the projection period. This reveals that, despite the fact that underwriting cycles are generally known to be absent in life insurance markets, a company underwriting a life insurance product with a particular policy design (for example policy type 1) is likely to expect periods of high and low profitability (i.e. the emergence of peaks and troughs in profitability in projection years), if a particular new business plan is being maintained over the projection period.

The investigation reveals that the set of constant insolvency risk curves shown in fig. 3.2 will allow a company to choose a sustainable growth rate from its own internal resources in order to produce a fixed level of insolvency risk. We observe that a company requires a lower constant proportion of free assets to finance new business over a longer projection period than over the short run in order to maintain a given level of insolvency risk except for very weak companies. Thus, companies would require more free assets to invest in equities over the longer term by writing a smaller volume of new business at least in the short term.

The study also shows a preference for business strategy 1 for a company with a desire for market share even though this could result in solvency problems in the long-run and this may create the need for a strategy on allocation of free assets to finance new business. Similarly, the asset-liability matching investment strategy (ALM) seems to be the preferred investment strategy as it offers the lowest risk of insolvency for a given combination of constant proportion of free assets to finance new business and the initial size of the company. Therefore writing profitable non-participating business and investing assets in matched positions (using the ALM strategy) would enable a company to write more new business over the longer term than other investment strategies. The ALM strategy will thus enable the company to meet its long term strategic aim since the company will invest its free assets on less risky securities like gilts. The decision to allocate a constant proportion of free assets to finance new business to produce a particular level of insolvency risk over a given time horizon would require a balance between the company's new business strain risk and investment risk.

In meeting its strategic aim, the company needs to adopt a valuation basis that would enable its new business plans meet the desired solvency level. We note that the insolvency risk for any projection period is lowest under the cautious basis and highest under the optimistic basis when considering the non-

Zillmerised net premium valuation. From our investigation, we discover that the cost of changing the valuation basis to the optimistic valuation basis from either premium basis or cautious basis for an on-going company in order to write more new business may lead to a drain on free assets. The valuation basis may also have a second order effect on the actual volume of new business written.

5.2.3 We note in our investigation that a company needs strong investment and new business policies to be supported by adequate initial capital in order to have an improved competitive position. Specifically, the new business strategy with appropriate pricing policy and investment strategy should provide for reasonable investment freedom.

The study has adopted the probability of relative insolvency to examine the degree of risk of the variant life offices in a competitive and stochastic environment. We note that the combination of asset-liability matching investment strategy (ALM) and any new business strategy other than new business strategy 2 offers less risk than any other combination, particularly under the situation where there is insufficient initial free capital. We also note that, where there is sufficient initial free capital with options of investment in equities, the combination of static investment strategy and new business strategy 2 will do better in the long-run.

We also note that an increase in initial capital which is not sufficient to cover the new business strains in the early projection years will affect the future solvency of the company. This may generate peaks and troughs in profitability. Where an increase in initial free capital covers the new business strains in the early projection years, sufficient free assets would be available for investment in equities. Our investigation concludes, therefore, that a company with a substantial initial solvency ratio at the projection date provides a powerful stabilizing factor which enables it not only to withstand an adverse future condition but also to provide free assets needed to finance future investment and new business.

5.2.4 We note that for a given level of free assets allocation proportion, new business model M2 (price and income effects) generates a greater quantity of new business and hence a larger drain on free assets than new business model M1 (price effect only). This is in line with the result that the demand for life insurance is positively related to the income of prospective policyholders (as noted by Diacon, 1980, and Browne and Kim, 1993). The increase in sales may result in a drain on the free assets which could lead to an insurer's insolvency as against the (ex ante) expectation that a large volume of new business demanded is positively related to an insurer's performance. This hypothesis is only true for a risk tolerant company with sufficient free assets and allocating a high constant proportion of free assets to write new business.

5.2.5 Our investigations show that life offices are not completely insulated from the risk of insolvency even if there are adequate free assets available to write the quantity of new business demanded. The desire for increased market share and profitability through increased quantity of business demanded and volume of business written could result in a drain of available capital and consequently expose the company to the risk of insolvency. We have therefore identified some mitigating factors, drawn from the available literature that companies need to apply effectively.

5.2.5.1 Underwriting

Our investigation shows that a pricing method with risk classification as part of underwriting process will enable a company to have an efficient use of its free capital in writing more new business without having solvency problems. The effects of not having risk classification tend to be more adverse on small companies than large ones. The relevance of classification as revealed in our study is such that a failure to use risk classification constitutes a systematic error which extra pooling of risks will not remove. On the other hand, a company with the strategic aim to grow at a sustainable rate may necessarily need to apply more stringent underwriting procedures to high-risk proposers in a portfolio with a mix of high and low risks as this will not only improve the company's mortality experience of insured lives but also will provide sufficient free assets for investment and writing appropriate volume of new business.

5.2.5.2 Policy design

We further note that the shorter the initial commission payment period in a product design, the more likely for a company to meet its solvency requirements. This would enable a company to recoup faster the initial capital invested. Hence a company should aim to reduce its new business volume proportionately in line with an increase in initial capital strain arising from a contract design in order to remain solvent.

5.2.5.3 Mix of New Business

Using the mix of new business as a tool of managing risk, the study reveals that the mix of business by contract can significantly reduce a company's capital required to finance new business if a company writes more of a contract with a lower level of new business strain. Accordingly, where a company targets more low-risk and medium risk consumers in the life insurance sales process, this will reduce a company's capital needs for financing new business and improve its statutory and relative solvency position. Therefore, we note that a company needs to balance the new business strain risk and the mortality risk associated with the contract types within its portfolio in order to remain solvent.

5.2.5.4 Constrained New Business Growth

The study further reveals that a company needs to control its new business growth, particularly where there are insufficient free assets, to remain solvent. Where there is no reasonable or direct control, a company's existing block of business with substantial initial solvency ratio at projection period provides a powerful stabilizing factor that enables it to have an indirect control of its insolvency risk arising from writing an inappropriate volume of new business.

5.2.5.5 Financial Reinsurance

We further note that the quota share basis of original term reinsurance provides greater security for a company than the deficit account financing with risk premium reinsurance, since it enables the company to write more volume of business with any given level of capital. This corroborates the available literature. For example, McGaughey et al (2001) note that the Original terms reinsurance on a quota share basis supports higher volumes of new business with the same amount of free assets. Thus, we note that the decision concerning the allocation of free assets to finance new business needs to be taken not only alongside the choice of an appropriate investment strategy, but also the choice of an appropriate volume of business to be ceded. However, our investigation further notes, that it is only possible for the ceding company to write a large volume of business if the level of financing received from the re-insurer is able to cover fully the initial and renewal expenses of the reinsured part of the business. Where the level of financing is not adequate to cover the expenses related to the reinsured business the company needs to reduce the proportion of free assets allocated to write new business as the proportion of business ceded increases for it to remain solvent. As an alternative, the company may reduce the proportion of business reinsured. Other hand, the study reveals that the deficit account financing method may only be appropriate where the level of new business growth is controlled by management.

5.3 Main Contributions of the Thesis

- A framework for modelling quantity of new business demanded and written separately in a dynamic environment.

- Simultaneous analysis of new business decisions (with contour lines) by setting the initial solvency ratio and free asset allocation proportion as free variables under a given premium setting structure.
- The free assets allocation decision to meet a desired level of solvency depends on the collective decision on investment policy, new business policy and financial reinsurance.

5.4 Future Research

This investigation is based on the assumption that no future capital from shareholders will be allowed to write new business demanded in future projection years except funds that are internally generated from releases of reserves from expired policies and surpluses from existing business. Our work also relates only on non-profit non-linked life insurance products with the assumption that the expense loadings match the actual acquisition and management costs of a company operating in the market. It would therefore be interesting to look at the effects of future capital injections in new business plans as well as products designs that incorporate profit. In this regard, therefore, we recommend the following for future research.

- The impact of future injection of new capital from shareholders on a company's new business plans and insolvency risk.
- The effects of bonus distribution policy on a company's new business plans and insolvency risk on with profit portfolios.
- The effects of any additional costs above the marginal cost that is directly related to the sale of a contract on the company's capital requirement. In other words, any extra costs associated with using multiple distribution channels.
- The effect on new business risk of using a dynamic strategy for allocation of free assets to finance new business.

- The effect of charging a relatively higher premium for high risks so that there is a bigger safety loading in order to reduce the insolvency risk resulting from high risks in addition to underwriting expenses.
- The model parameter values are to be be simulated (e.g. using Bayesian approach) to allow for parameter risk.
- The indirect effect of underwriting standards on sales could be modelled using other methods than logistic function (e.g. by simulation).

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Appendix 1

1.1 Market Model

1.1.1 Definition of Market Liability Model Variables

The notation for the variables and constants in the market liability model is given below.

Variables

$AdjF_m^j(t)$ = Market office premium adjustment function for a new business strategy in year t for simulation j.

$ANM^j(t)$ = Quantity of actual of new business written (in terms of number of policies) in the Market in projection year t for simulation j.

$ANM(0) = NM(0)$ = Quantity of new business (quantity demanded and written) a year before projection date (e.g. $ANM(0) = NM(0) = 10{,}000$ when t = 0).

$\beta_m^j(t)$ = The ratio of the allocated free assets to the total expected new business strain in the market in projection year t for simulation j.

$CDF_m^j(t)$ = Constant price elasticity of demand function for the market in projection year t for simulation j.

$DF_m^j(t)$ = Demand function for the market in projection year t for simulation j.

$EDF_m^j(t)$ = Exponential price elasticity of demand function for the market in projection year t for simulation j.

$ENBS_m^j(t)$ = Total expected business strain for the market (i.e. capital required to write new business volume demanded) at start of projection year t for simulation j.

$FAN_m^j(t)$ = Free assets allocated to write new business in year t for simulation j.

$F'A_m^j(t)$ = Market's statutory free assets (including reserves released from expired policies) at the end of projection year t for simulation j.

$InfacF_p^j(t)$ = Accumulation of price inflation up to the end of projection year t for simulation j.

$\inf_p^j(t)$ = Price inflation rate for the projection year (t-1, t) for simulation j.

$NM^j(t)$ = Quantity of new business demanded (in terms of number of new policies) in the market in projection year t for simulation j.

$OP_m^j(t)$ = Market office premium for new business issued in projection year t for simulation j.

$PRF_m^j(t)$ = A company's premium response to solvency function in the market in projection year t for simulation j.

$RWP^j(t)$ = Proportion of volume of exposure (measured in terms of quantity of new business demanded in year *t - 1*) to be demanded in year *t* in the market due to the effect of real wage inflation only, for simulation j.

$RWIR^j(t)$ = Real wage inflation rate for the projection year t for simulation j.

$SR_m^j(t)$ = Market's solvency ratio at the end of projection year t for simulation j.

$SR_m(0)$ = Market's initial solvency ratio at projection date (e.g. $SR_m(0) = 1.50$).

${}^jW_x^u(t)$ = Withdrawal rate for a life aged x at entry with duration u years in the year projection t for simulation j.

The following parameters are assumed to be constants over time:

a = The intercept term in equation (2) that is not linked to real wage inflation.

b = Rate of change in new business volume demanded with respect to real wage

$C_x(u)$ = Lapse rate at policy duration for a policy issued to a life aged x at entry.

d = Rate of change in policy lapses with respect to disposable income.

EK_m = Exponential price elasticity of demand function parameter.

CK_m = Constant price elasticity of demand function parameter.

$l_m(\pm ve)$ = A factor that measures the response of a company's premium to changes in its worsening (or improving) solvency ratio in the market.

NDF = New business decision factor.

$\overline{OP_m}$ = Profit-tested office premium for the market in section 2.2.5 in chapter 2 (the same as premium for existing business at projection date i.e. $OP_m(0) = \overline{OP_m}$).

ρ_m = Constant proportion of free assets allocated to write new business over a projection period.

1.1.2 Market New Business Models

Model M1: Exponential Demand Function (No Real wage Inflation Effect).
Model M2: Exponential Demand Function and Real wage Inflation Effect.
Model M3: Constant Price-elasticity of Demand Function (No Real wage Inflation Effect).
Model M4: Constant Price-elasticity of Demand Function and Real wage Inflation Effect.

1.1.3 New Business Strategies

- New Business Strategy 1 - A company reduces (increases) its premium relative to the profit-tested premium to attract more (less) quantity of new business demanded in a particular projection year if its solvency ratio in the previous year is above (below) a new business decision factor (NDF).
- New Business Strategy 2 - A company increases (reduces) its premium relative to the profit-tested premium to attract less (more) quantity of new business demanded in a particular projection year if its solvency ratio in the previous year is above (below) a new business decision factor (NDF).
- New Business Strategy 3 - A company reduces its premium relative to the profit-tested premium to attract more quantity of new business demanded in any particular projection year if its previous year's solvency ratio is either above or below a new business decision factor (NDF).
- New Business Strategy 4 - A company increases its premium relative to the profit-tested premium to attract less quantity of new business

demanded in any particular projection year if its solvency ratio is either below or above a new business decision factor (NDF).

- The level of new business decision factor (NDF) is arbitrarily determined based on the company's degree of risk aversion relative to the desire for market share. New business strategies 1 and 2 have $NDF = 1.0$ whilst for new business strategies 3 and 4, $NDF > 1.0$.

1.1.4 Investment Strategies

- Asset-Liability Matching (ALM) Investment Strategy - All liabilities are backed by fixed securities. Only free assets are invested in equities.
- Static Investment Strategy - Fixed proportion (50%) of the assets is invested in equities and the other 50% is invested fixed interest securities (gilts).
- Dynamic Asset Allocation Strategy (DS1 & DS2) - Funds are allocated to invest on fixed securities and equities based on the degree of solvency position of the company in the previous year.

1.1.5 New business model allowing for underwriting effect

Model M1, Exponential Demand Function (No Real wage Inflation Effect) is modified by using logistic function (see section 4.5.3.1).

1.2 Company Model

1.2.1 Definition of Company Liability Model Variables

The notation of variables and constants over time in the company liability model is given below.

Variables

$\alpha_R^j(t)$ = A factor specified by the regulator(s) to reduce or increase the company's proportion of the quantity demanded in the market in projection year t for simulation j, if the solvency ratio of the company at the end of year t-1 is less than one.

$CDF_c^j(t)$ = Constant price elasticity of demand function for the company in projection year t for simulation j.

$DF_c^j(t)$ = Demand function for the company in projection year t for simulation j.

$EDF_c^j(t)$ = Exponential price elasticity of demand function for the company in projection year t for simulation j.

$ft_c^j(t)$ = The company's proportion of the quantity demanded in the market in projection year t for simulation j, allowing for the effect of both the changes in a company's premium relative to the market premium and either the company's management decision or the regulator's control due to the company's level of relative solvency.

$ft_c(0)$ = The company's proportion of the quantity demanded and written in the market in year 0 (i.e. a year before projection date).

$F_R^J(t)$ = Weighting factor specified by the regulator(s) to damp down the growth or fall in the company's proportion of the quantity demanded in the market in projection year t for simulation j, if the solvency ratio of the company at the end of year t-1 is greater than or equal to one.

$NC^j(t)$ = Company's quantity of new business demanded by prospective policyholders (in terms of number of new policies) in projection year t for simulation j.

$OP_c^j(t)$ = Company's office premium for new business issued in projection year t for simulation j.

$PMS^j(t)$ = Company's proportion of the quantity demanded in the market in projection year t for simulation j due to the effect of either the company's management decision or the regulator's control.

$SR_c^j(t)$ = Company's solvency ratio at the end of projection year t for simulation j.

The following parameters are assumed to be constants over time:

$NBConF_c$ = The company assumed maximum proportion of the quantity demanded in the market in projection year t.

$\overline{OP_c}$ = Profit-tested office premium for the company (the same as for the market) for existing business at projection date (i.e. $\overline{OP_c} = \overline{Op_m}$).

$\operatorname{Re} gConF$ = A factor specified by regulators to wind up a company if the company's future solvency ratio is less than this factor.

1.2.2 Company New Business Models

Model C1: Exponential Demand Function and Management Decision

Model C2:_Exponential Demand Function and Regulator's Control

Model C3:_Constant Price-elasticity Demand Function and Management Decision
Model C4:_Constant Price-elasticity Demand Function and Regulator's Control

1.3 Stochastic Return and Inflation Rates

We use Wilkie AR(1) (auto-regressive process of order 1) model (1995) to generate random inflation rates and rates of investment return.

1.3A Force and Rates of Inflation

Haberman et al (2004) adopt the following approach to generate three types of inflation rates:

The force of inflation $I_k(t)$ over the year *(t −1, t)* described by:

$$I_k(t) = QMU_k + QA.(I_k(t-1) - QMU_k) + QE_k(t)$$

where $QMU_k = \begin{cases} 0.03 & -\textit{for price escalation rates;} \\ 0.047 & -\textit{for expense escalation rates;} \\ 0.047 & -\textit{for investment return rates;} \end{cases}$ and $\begin{cases} QA = 0.58 \\ QSD = 0.0425 \\ I_0 = QMU_k \end{cases}$

Where $k = (p, e, i)$ corresponding to the three cases of inflation. The corresponding rate of inflation (for three separate cases) is given by

$$\inf_k(t) = \exp(I_k(t) - 1$$

Then the value of inflation index at time t, $Q_k(t)$ is given by:

$Q_k(t) = Q_k(t-1).\exp(I_k(t))$

$\inf_p(t)$ = *Price escalation Rate for year (t-1, t).*

$\inf_e(t)$ = *Expense escalation Rate for year (t-1, t)*

$\inf_i(t)$ = *Investment Return inflation rate for year (t-1, t)*

$QE_k(t) = QSD.QZ_k(t)$

$QZ_k(t) \sim iid \quad N(0,1)$

The following sections in this appendix will be affected only by $I_i(t)$. For simplicity, we drop the index *i.*

1.3B Wage Inflation

Wilkie's model for the wages index $\boldsymbol{W(t)}$ is of the form:

$W(t) = W(t-1).\exp\{J(t)\}$

$J(t) = \ln W(t) - \ln W(t-1)$ is the force of wage inflation over the year *(t-1, t)* and

$J(t) = WW1.I(t) + WW2.I(t-1) + WN(t)$

$WN(t) = WMU + WA.(WN(t-1) - WMU) + WSD.WZ(t).$

$WZ(t) \sim iid \quad N(0,1)$

Wilkie suggested the parameters:

WW1 = 0.69; *WMU* = 0.016; *WA* = 0; *WSD* = 0.0244

1.3C Share Dividend Yields

Wilkie's AR(1) model for share dividend yield $\boldsymbol{Y_e(t)}$ is of the form:

$\ln Y_e(t) = YW.I(t) + YN(t)$

$YN(t) = \ln(YMU) + YA.(YN(t-1) - \ln(YMU)) + YSD.YZ(t).$

and $YZ(t) \square \; iid \; N(0,1)$

Wilkie suggested the parameters:

YW = 1.8; *YA* = 0.55; *YMU* = 0.0375; *YSD* = 0.155

1. 3D Dividends

The Wilkie's MA(1) model for the force of share dividend growth is:

$$K(t) = DW.DM(t) + DX.I(t) + DMU + DY.YE(t-1) + DB.DE(t-1) + DSD.DZ(t)$$

The value of dividend index $D(t)$ at time t is:

$$D(t) = D(t-1).\exp[K(t)]$$

and *DZ(t)* $\sim iid \quad N(0,1)$

Wilkie suggested the parameters:

DX = 0.42; *DD* = 0.13; *DMU* = 0.016; *DY* = -0.175; *DB* = 0.57; *DSD* = 0.07; *DW* = *(1-DX)*

Then value of equity price index at time *t,* $P(t)$ is given by:

$$P(t) = \frac{D(t)}{Y(t)}$$

and the value of the 'rolled-up' equity index at time t, $PR(t)$, is given by:

$$PR(t) \quad PR(t-1).\frac{P(t)+D(t)}{P(t-1)}$$

1. 3E Long Term Interest Rate (Consols Yield)

Wilkie's AR(1) model for consols yield *C(t)* is of the form:

$$C(t) = CW.CM(t) + CMU.\exp[CN(t)$$

$$CM(t) = CD.I(t) \quad (1-CD).CM(t-1)$$

$$CN(t) = CA1.CN(t \quad 1) + CY.YE(t) + CE(t)$$

$$CE(t) = CSD.CZ(t)$$

CZ(t) $\sim iid \quad N(0,1)$

Wilkie suggests the parameters:

CW = 1; *CD* = 0.045; *CMU* = 0.0305; *CA* = 0.9; *CY* = 0.34; *CSD* = 0.185.

Then the value of 'rolled-up' undated fixed fixed-interest gilts index at time t, $CR(t)$, is given by:

$$CR(t) = CR(t-1).\ \frac{1}{C(t)} \quad 1\ .C(t-1)$$

1. 3F Starting Conditions suggested by Wilkie (1995)

$$
\begin{array}{lll}
I(0) & = QMU & = 0.047 \\
J(0) & = QMU \quad WMU & = 0.06189 \\
Y(0) & = \exp(YW.QMU).YMU & = 0.040811 \\
DM(0) & = QMU & = 0.047 \\
C(0) & = QMU \quad CMU & = 0.0775 \\
CM(0) & = QMU & = 0.047
\end{array}
$$

1. 3G Normally Distributed Random Numbers

Normally distributed random numbers can be generated by the so-called log-and trig formula (Rubinstein, 1981) as described in *Daykin et al (1994).*

$$x_1 = \sqrt{-2\ln(r_1)}\ .\cos(2\pi\ r_2)$$

$$x_2 = \sqrt{-2\ln(r_1)}\ .\sin(2\pi\ r_2)$$

Where r_1, r_2 is a pair of uniformly distributed random numbers from the interval (0, 1) and the generated pair x_1, x_2 consists of normally distributed mutually independent random numbers having mean 0 and standard deviation 1.

Appendix 2

2.A Estimation of Initial Parameters for Market New Business Models

Table 2A.1: Inflation Indices and New Business Data (for Non-Linked Regular Premium)

Year	Retail Prices Inflation		Wage inflation		Real Wage Inflation Rate	Non Mortgage Related Term Assurance Policies		
(t)	Index	% change over 12 months I(t)	Index	% change over 12 months J(t)	RWIR(t) = {exp[J(t) - I(t)]} - 1	No. of Policies	Premiun income £000's	Average Annual Premium (£)
(1)	(2)		(3)		(4)	(5)	(6)	(7)
1990	126.1		79.4					
1991	133.5	5.87	85.6	7.81	0.01959	416,000	127,000	305.29
1992	138.5	3.75	90.7	5.96	0.02237	445,000	137,000	307.87
1993	140.7	1.59	93.6	3.20	0.01622	450,000	139,000	308.89
1994	144.1	2.42	97.0	3.63	0.01223	495,000	159,000	321.21
1995	149.1	3.47	100.0	3.09	-0.00376	503,000	148,000	294.23
1996	152.7	2.41	103.5	3.50	0.01091	530,000	182,000	343.40
1997	157.5	3.14	108.0	4.35	0.01212	537,000	179,000	333.33
1998	162.9	3.43	113.5	5.09	0.01678	594,000	186,000	313.13
1999	165.4	1.53	119.0	4.85	0.03367	652,000	213,000	326.69
2000	170.3	2.96	117.9	-0.92	-0.03812	750,000	244,000	325.33
2001	173.4	1.79	123.1	4.41	0.02654	809,000	275,000	339.93
2002	176.2	1.63	127.5	3.57	0.01961	977,000	372,000	380.76

Sources: *(2) National Financial Statistics Yearbook: Index (Jan, 1987 = 100)*
(3) International Financial Statistics Yearbook: Index (Jan 1995 = 100)
(5) ABI and SIAS Paper, " A Life Term", May 22nd 2001.

Table 2A.2 : Estimation of Initial parameters for market new business model M2

Assumption:
85% of business in year (t-1) to be written in year t if the market premium in yeay t is 20% higher than the market premium in year (t-1)
Exponential Demand Function: EK_m = 0.81
Intercept a = *0.97*
Slope b = *4.81*

Years (t)	No. of Policies NM(t)	$\frac{NM(t)}{NM(t-1)}$	Annual Premium $OP_m(t)$	$\left(\frac{OP_m(t)}{OP_m(t-1)}-1\right)$	$EXP(-EK_m * (5))$	Real Wage Inflation Rate RWIR(t-1)	Actual values (3) / (6)	Predicted Values
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1991	416000		305.29					
1992	445000	1.070	307.87	0.00844	0.99316	0.01959	1.077	1.041
1993	450000	1.011	308.89	0.00333	0.99730	0.02237	1.014	1.029
1994	495000	1.100	321.21	0.03990	0.96810	0.01622	1.136	1.057
1995	503000	1.016	294.23	-0.08399	1.07063	0.01223	0.949	1.075
1996	530000	1.054	343.40	0.16708	0.87304	-0.00376	1.207	1.149
1997	537000	1.013	333.33	-0.02930	1.02410	0.01091	0.989	1.081
1998	594000	1.106	313.13	-0.06061	1.05048	0.01212	1.053	1.076
1999	652000	1.098	326.69	0.04329	0.96543	0.01678	1.137	1.054
2000	750000	1.150	325.33	-0.00414	1.00337	0.03367	1.146	0.977
2001	809000	1.079	339.93	0.04485	0.96421	-0.03812	1.119	1.307
2002	977000	1.208	380.76	0.12012	0.90700	0.02654	1.331	1.009

Average growth rate 1.082

Model M2

$$\frac{NM(t)}{NM(t-1)} = Exp\left(\frac{-Ek_m(OP_m(t)-OP_m(t-1))}{OP_m(t-1)}\right)\left(a+b \quad RWIR(t-1)\right)$$

$$Actual\ Value = \frac{NM(t)}{NM(t-1)} \div Exp\left(\frac{-Ek_m(OP_m(t)-OP_m(t-1))}{OP_m(t-1)}\right)$$

$$\text{Predicted Value} = a + b * RWIR(t-1)$$

Model M4

$$\frac{NM(t)}{NM(t-1)} = \left(\frac{OP_m(t)}{OP(t-1)}\right)^{-CK_m} \times \left(a + b * RWIR(t-1)\right.$$

$$\text{Actual Value} \quad \frac{NM(t)}{NM(t-1)} \quad \left(\frac{(OP_m(t)}{OP_m(t-1)}\right)^{-CK_m}$$

$$\text{Predicted Value} = a + b * RWIR(t-1)$$

Table 2A.3 : Estimation of Initial parameters for market new business model M4

Assumption:

85% of business in year (t-1) to be written in year t if the market premium in yeay t is 20% higher than the market premium in year (t-1)

Exponential Demand Function: $CK_m =$ ***0.89***

Intercept a = 0.96
Slope b = 5.04

Years (t)	No. of Policies NM(t)	$\frac{NM(t)}{NM(t-1)}$	Annual Premium $OP_m(t)$	$\left(\frac{OP_m(t)}{OP_m(t-1)}\right)$	$\{(5) \wedge -CK_m\}$	Real Wage Inflation Rate RWIR(t-1)	Actual values (3) / (6)	Predicted Values
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1991	416000		305.29					
1992	445000	1.070	307.87	1.00844	0.99254	0.01959	1.078	1.040
1993	450000	1.011	308.89	1.00333	0.99705	0.02237	1.014	1.028
1994	495000	1.100	321.21	1.03990	0.96573	0.01622	1.139	1.056
1995	503000	1.016	294.23	0.91601	1.08133	0.01223	0.940	1.074
1996	530000	1.054	343.40	1.16708	0.87134	-0.00376	1.209	1.147
1997	537000	1.013	333.33	0.97070	1.02687	0.01091	0.987	1.080
1998	594000	1.106	313.13	0.93939	1.05731	0.01212	1.046	1.075
1999	652000	1.098	326.69	1.04329	0.96293	0.01678	1.140	1.053
2000	750000	1.150	325.33	0.99586	1.00371	0.03367	1.146	0.976
2001	809000	1.079	339.93	1.04485	0.96164	-0.03812	1.122	1.305
2002	977000	1.208	380.76	1.12012	0.90383	0.02654	1.336	1.009

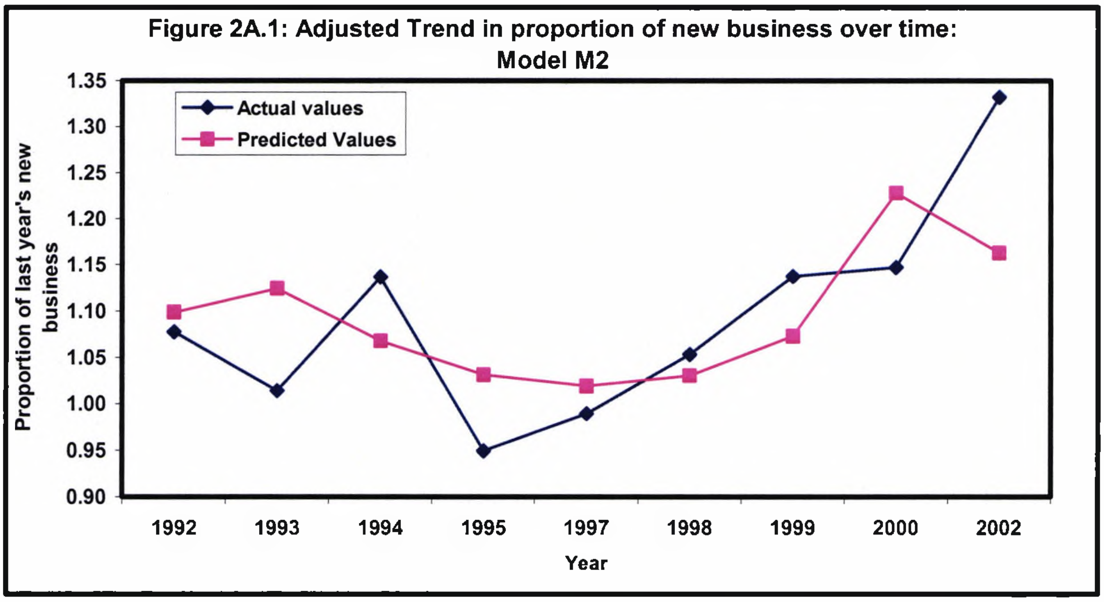
Table 2A.4: Initial Parameter Estimation for Market New Business Business Models

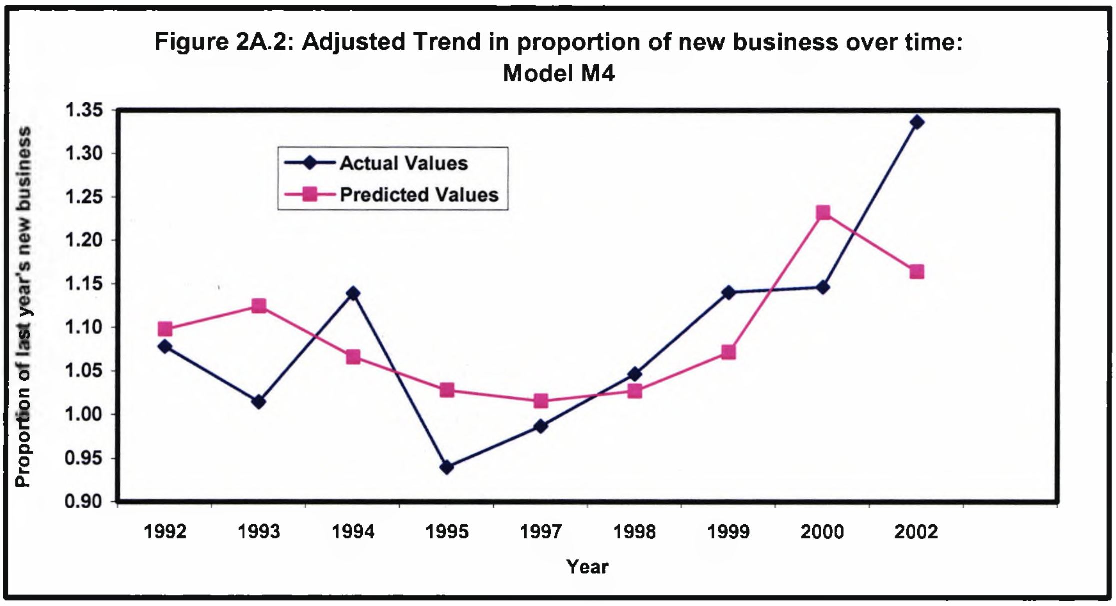
Model M2

% increase in market premium	% Reduction in New Business demanded	DF Parameter EK_M	Correlation Coefficient Multiple R	Residual Sum of Squares	Intercept a	Slope b	P-value for b	Width of 95% C.I. For b
20	80	1.12	0.574	0.024	0.95	5.45	0.14	15.55
15	80	1.49	0.577	0.030	0.93	6.23	0.13	17.61
10	80	2.23	0.562	0.050	0.89	7.72	0.15	22.70
20	85	0.81	0.560	0.020	0.97	4.81	0.15	14.23
15	85	1.08	0.573	0.023	0.95	5.39	0.14	15.39
10	85	1.63	0.576	0.033	0.92	6.51	0.14	18.47

Model M4

% increase in market premium	% Reduction in New Business demanded	DF Parameter CK_M	Correlation Coefficient Multiple R	Residual Sum of Squares	Intercept a	Slope b	P-value for b	Width of 95% C.I. For b
20	80	1.22	0.583	0.025	0.94	5.76	0.13	16.03
15	80	1.60	0.584	0.032	0.92	6.55	0.13	18.21
10	80	2.34	0.568	0.054	0.88	8.08	0.14	23.41
20	85	0.89	0.571	0.020	0.96	5.04	0.14	14.47
15	85	1.16	0.582	0.024	0.95	5.63	0.13	15.71
10	85	1.71	0.582	0.035	0.92	6.78	0.13	18.91





Figures 2A.1 and 2A.2 show the trend in proportion of actual and predicted (using market new business models M2 and M4 respectively) new business over time.

2B. Guidelines for Profit-Testing Technique

Forfar and Gupta (1986) describe in detail the process of cash-flow techniques in product pricing. In simple terms, Hylands & Gray (1990) describe the process as "one of calculating, on a given set of assumptions, the cash-flows to and from the office for a specimen policy. The calculations are normally performed on a month-by-month basis. Cash-flows to the office each month arise from premiums paid and investment income received, net of tax where appropriate, and cash-flows from the office arise from death claims, maturity payments, surrender value payments and expenses, allowing for tax relief where appropriate". The increase in the reserves over the month held by the office is taken into account.

The premium basis in table 2.1 in chapter 2 of the thesis enables a profit-testing technique to be used to determine the monthly office premium to ensure that a specified value of a profit criterion is met. The profit criterion chosen is the net present value (NPV) which is the discounted value of the profit signature at the risk discount rate (e.g. r_D = 10% p.a.). The net present value being the chosen profit criterion is expected to be zero (or close to zero) at the start of the policy. The risk discount rate is the minimum rate of return on capital required by the providers of the risk capital required in writing the policy. The profit signature of a contract is the sequence of profits over time from inception to termination of the contract. The net present value as profit criterion is expected to be zero at the start of the policy. This means that the internal rate of return on the cash flows arising from the policy is equal to the risk discount rate.

The guidelines for calculation of monthly office premium for all existing business as at projection date using the profit-testing technique adopted in Hylands & Gray (1990) stated below:

(1) Reserve in-force at the end of month t = $V(t)$

(2) Total premium income in month t = $PI(t)$

(3) Total gross commission in month t = $Com(t)$

(4) Total gross expenses in month t = $Exp(t)$

(5) Office taxable income in month t = $Taxablinc(t)$

(6) Tax rate = Tr

(7) Tax payable by office in month t = $Tax(t)$

(8) Total death benefit in month t = $DB(t)$

(9) Increase in fund in month t = $IncFund(t)$

(10) Increase in reserve in month t = $Inc\,\mathrm{Re}\,serve(t)$

(11) Profit at the end of month t = $profit(t)$

(12) Solvency margin in-force at the end of month t = $solvm(t)$

(13) Increase in solvency margin in month t = $IncSolvm(t) \quad Solvm(t) - Solvm(t-1)$

(14) Interest on Solvency margin in month t = $SolvmInt(t)$

(15) Tax on interest on solvency margin in month t = $SolvmTax(t)$

(16) Profit allowing for solvency margin at the end of month t = $prosolvm(t)$

(17) Discount rate = r_D

(18) Value of future profit with solvency margin at start of month t = $Valprosolvm(t)$

$$IncFund(t) = \{(PI(t) \quad Exp(t) \quad Com(t) + Taxablinc(t) \quad Tax(t) \quad DB(t)) \qquad (1)$$
$$Taxablinc(t) = V(t \quad 1) + PI(t) \quad Exp(t) \quad Com(t) \qquad (2)$$
$$Tax(t) = Tr \times \{Taxablinc(t) \quad Exp(t) \quad Com(t) \qquad (3)$$
$$Inc\,\mathrm{Re}\,serve(t) = V(t) \quad V(t \quad 1) \qquad (4)$$
$$profit(t) = IncFund(t) \quad Inc\,\mathrm{Re}\,serve(t) \qquad (5)$$
$$proSolvm(t) = \mathrm{Pr}\,ofit(t) + SolvmInt(t) \quad SolvmTax(t) \quad IncSolvm(t) \qquad (6)$$
$$ValproSolvm(t) = \{Valprofitsolvm(t+1) + profitSolvm(t)\} \times Discont\ factor \qquad (7)$$

Note:

1. **Office taxable income in month t =** $Taxablinc(t)$ is calculated as the twelfth root of the gross rate of interest earned on reserves and premium.
2. **Decrement Rates and Numbers**

(a) Annual mortality rates
(b)

$q_{[x]}$ = TM92 (with 5-year select period) mortality rate for life age x at entry

(b) Monthly mortality rates for a life age x with duration u

$$q^{M}_{[x]+u} = 1 - [(1 - q_{[x]})\wedge(1/12)]$$

(c) Lapse rates for a life age at entry with policy duration u

$C_x(u)$ = Annual lapse rate (see table 2.1 in the thesis)

$C_x^M(u)$ = Monthly Lapse Rates

$$C_x^M(u) = 1 - [(1 - C_x(u))\wedge(1/12)]$$

(d) Number of Deaths in the Month

No. Of Deaths = No. In-force at start $\times q_{[x]+u}^M \times [1 \quad 1/2 \times C_x^M(u)]$

(e) Number of Lapses in the Month

No. Of Lapses = No. In-force at start $\times C_x^M(u) \times [1 - 1/2 \times q_{[x]+u}^M]$

(f) Number in-force at End of Month

No. In-force at End = No. In-force at Start $\times (1 - q_{[x]+u}^M) \times [1 - C_x^M(u)]$

3. Reserves Calculation

Monthly policy reserves are calculated using gross premium valuation method with the premium basis in table 2.1 of the thesis.

3. Calculation of initial commission and payment period (LAUTRO Rules)

(PIA-Adopted rules: LAUTRO, obtained from IFA)

LAUTRO Initial commission per period = initial percentage times premium. The initial % for term insurance is 35% and 25% for endowment contracts. See table showing initial commission period for temporary insurance policy in section 4A of appendix 4.

2C. UK Life Offices Solvency and Free Asset Ratios (Long-Term Trends)

Below are tables showing UK Life insurance Companies solvency ratios and free asset ratios for the period 1994 to 2002.

Table 2C.1(a) : Free asset Ratios of a set of Uk Life offices (Long Term Trends)

No.	Group of Life Offices	Year 2002 Free Asset Ratio in %	Year 2001 Free Asset Ratio in %	Year 2000 Free Asset Ratio in %	Year 1999 Free Asset Ratio in %	Year 1998 Free Asset Ratio in %	Year 1997 Free Asset Ratio in %	Year 1996 Free Asset Ratio in %	Year 1995 Free Asset Ratio in %	Year 1994 Free Asset Ratio in %
	Proprietary Companies									
1	Barclays Pensions MGT Ltd	0.1	0.1	1.001			0.1	0.2	0.2	
2	Prudential Ass co.Ltd	7.8	10.9	14.4	22.2	17.2	18.5	21.4	20.5	15.9
3	American Life Ins Co	3.7	4.2	4.1	3.5	3.3	3.7	4.8	6.5	8.6
4	Legal & General Ass Soceity	10.9	10.6	14	18.1	14.6	17	17.3	17.3	15.6
5	Norwich Union Life & Pens Ltd	4.2	4	4	9.5	16.7	18.6			
6	Clerical Medical Investmet Grp	2.1	3.2			13.4	17.2	15.8		
7	Allied Dunbar Ass	1.5	1.5	1.8	2	2.5	3	3.7	3.6	4.1
8	Philips & Drew Life Ltd			0.1	0.1	0.1				
9	CGU Life Assurance Ltd					24.6	24.4	21	24.2	23.9
10	Scottish Mutual Ass Soc	5.1	6.6	5.1	15.4	9.1	12.6	10.4	11.4	7
11	AXA Equity & Law Life Ass Soc	12.1	13.3	65.7	63	22.8	17.9	16.9	18	17.2
12	Financial assurance	2	2.6	2.3	2.2	6	4.5	7.2	11.5	11.3
13	Pearl Assurance	6.6	8.5	14.1	21.2	19.4	18.9	21.6	26	19.6
14	Royal & Sun Alliance life & Pens	7.1	10.3	11.2	16.1	15	18.4	16.9	17.2	12
15	Abbey Life Ass	2	2.6	2.3	2.2	1	3	4.3	4.3	4.4
16	Scottish Equitable	0.5	0.5	0.4	0.3	11.4	15.1	16.3	17	14.4
17	Skandia Life Ass Co	1.5	0.9	1	1.1	1.4	1.4	1.5	1.5	1.9
18	Eagle Star Life Ass	5.9	7.9	10.7	13.2	12.2	16.4	16.8	16.2	9.8
19	Scottish Amicable life		2.3	1.8	2.4	2.9	0.9			
20	M & G Pens & Annuity Co					1.5	2	2.2	1.1	1.2
21	J Rothschild Ass		1	1.2	1.1	2.2	2.6	2.2	4.1	1.1
22	J P Morgan Life Ass	0.5	0.5	0.3	0.4	1.1	3.9			
23	Britannic Ass	6.3	10	10.2	19.8	21.7	29.2	27	33	31.2
24	Mercury Life ass				1	0.9	0.5	0.6	0.5	1.1
25	Guardian Ass		9.2	11.2	12.9	12.3	13.5	12.5	10.9	9.5
26	Winterthur Life Uk Ltd	1.9	1.9	1.9	2	4.8	8.3	8.2	5	5.6
27	Colonial Life Ltd					14.6	17.9	16.7		
28	Lincoln Ass	2.5	1.9	1.7	0.7	2.4	3	3.5	3.6	3.1
29	Morgan Grenfell Life & Pens					2.7				
30	Hermes Liberty Int. Pens Ltd					0.1	0.2			

Sources: DTI Returns Only the main company from each group is shown.
Free Asset Ratio = Free assets / total assets (the ratio includes required minimum margin)
Free Asset = total assets + future profits - total liabilities.
Total assets: Form 9, rows21 and 22. **Future profits**: Form 9, row 31. **Total Liabilities**: Form 9, rows 23 and 24

Table 2C.1(b) : Free asset Ratios of a set of Uk Life offices (Long Term Trends)

No.	Group of Life Offices	Year 2002 Free Asset Ratio in %	Year 2001 Free Asset Ratio in %	Year 2000 Free Asset Ratio in %	Year 1999 Free Asset Ratio in %	Year 1998 Free Asset Ratio in %	Year 1997 Free Asset Ratio in %	Year 1996 Free Asset Ratio in %	Year 1995 Free Asset Ratio in %	Year 1994 Free Asset Ratio in %
No.	**Mutual Offices**									
1	Equitable Life ass	5.7	6.2	4.8	11.7	8.9	8.9	9.1	10.3	8.6
2	Standard Life Ass	6.4	8	17.5	19.7	19.3	21.3	20.5	20.2	17.4
3	Scottish Widows' Fund & Life As[1]	11.1	10.3	16		17.6	19	19.5	19.2	18.3
4	Friends Provident life Offices[2]	7.6	11	12.8	21.8	12.7	18.3	17	12.3	7.4
5	National Provident Institution[3]	2.3	4.3	5.3	7.9	7	10.5	9.6	11	9.2
6	Co-Operative Ins Soc	8.5	13.4	12.3	18.3	17	18.7	21.9	22.1	17.5
7	Scottish Life Ass					9.5	10.6	11.4	9.7	9.8
8	Scottish Provident Institution[4]	4	9.4	10.6	14.9	10.6	15.4	13.1	10.4	9.2
9	Royal London Mutual Ins Soc	7.4	8.8	19.1	37.3	23.1	29.3	32.1	33.3	30.1
10	National Mutual Life Ass[5]	3.8	4	5.7	11.8	9.9	13.4	13.1	14.4	15.5
11	Canada Life Ass					6.3	15.1	12.6	9.5	18.2
12	Sun Life Ass Co. of Canada[6]	7.7	8.3	9.4	2	20.7	21.9	26.4	28.6	31
13	National Farmers Union Mutual	12.8	17.4	18.1	19.9	15.6	13.7	20.8	19.7	18.3
14	Wesleyan Ass soc	7.4	16.4	19.4	22.8	19.3	23.4	30	30	29.6
15	Royal Nat Pens Fund For Nurse					6.2	10.7	8.5	13.5	12.7
16	Marine & General Mutual Life ass	7.7	6.1	10	13.1	9.5	11.5	9.8	10.1	9.5
17	Reliance Mutual Ins Soc		13.8	10.1	4.4	9	6	9.9	11.2	9.9
18	Forester Life Ltd	10	14.5	21.5	20.3	19.4	22.7	24.2	26.2	
19	Ecclesiastical Ins Office				18.3	16.8	18.2	12.5	13.2	8.2
20	London aberdeen & Noth Mutual			11.1	12.8	6.7	9	9.8	11.3	6.4
No.	**Banks/ Building Societiy Companies**									
1	National West Life ass	3	2.6	2.3	1.6	1.9	3.4	4.2	5.1	6.5
2	Britannia Life Ltd					2.6	3.1	4.4	4.7	5.9
3	Lloyds TSB Life Ass	6.5	3.3	4.2	2.7	3.4	5.8	6.6	5.9	5.8
4	Barclays Life Ass	3.2	13.5	2.7	2	2	2.2	2.8	2.4	2.4
5	Midland Life Ass (HSBC life (UK) Ltd)	5.7	4	3.7	3.1	3.1	4.5	4.9	5.3	4.9
6	Royal Scottish Ass	3.7	2.5	1.9	2.1	1.4	1.1	1.7	2.7	3.6
7	Abbey Nat Life	10.4	7.2	6.7	6.2	9.5	6.6	4.9	5.6	9.7
8	Halifax Life		3.1	6.1	6.4	6.4	7.5	14.9	55.3	
9	Nationwiode Life	8.8	6.9	5	3.9	4.1	5.7	0.4	61	
10	Woolwich Life Ass	7.9	22.6	8.9	6.7	8	8	8.6	9.3	10.2

Note:

1. Scottish Widows' Fund & Life Assurance Society business was transferred to Scottish Widows Plc after 1999
2. Friends Provident Life Office business was transferred to Friends Provident Life & Pensions Ltd in July 2001
3. National Provident Institution business was transferred to NPI Ltd after 1999
4. Scotish Provident Institution business was transferred to Scotish Provident Ltd in July 2001
5. National mutual Life Assurance Society business was transferred to GE Pensions Ltd in March 2002
6. Sun Life Ass. Co. of Canada (UK branch) business was transferred to Sun Life Ass. Co. of Canada (UK) Ltd in 2000

Table 2C.2 (a): Solvency Ratios of a set of Uk Life offices (Long Term Trends)

No.	Group of Life Offices	Year 2002 Solvency Ratio	Year 2001 Solvency Ratio	Year 2000 Solvency Ratio	Year 1999 Solvency Ratio	Year 1998 Solvency Ratio	Year 1997 Solvency Ratio	Year 1996 Solvency Ratio	Year 1995 Solvency Ratio	Year 1994 Solvency Ratio
	Proprietary Companies									
1	Barclays Pensions MGT Ltd	1.001	1.001			1.000	1.001	1.002	1.002	
2	Prudential Ass co.Ltd	1.085	1.122	1.168	1.285	1.208	1.227	1.272	1.258	1.189
3	American Life Ins Co	1.038	1.044	1.043	1.036	1.034	1.038	1.050	1.070	1.094
4	Legal & General Ass Soceity	1.122	1.119	1.163	1.221	1.171	1.205	1.209	1.209	1.185
5	Norwich Union Life & Pens Ltd	1.044	1.042	1.042	1.105	1.200	1.229			
6	Clerical Medical Investmet Grp	1.021	1.033			1.155	1.208	1.188		
7	Allied Dunbar Ass	1.015	1.015	1.018	1.020	1.026	1.031	1.038	1.037	1.043
8	Philips & Drew Life Ltd			1.001	1.001	1.001				
9	CGU Life Assurance Ltd					1.326	1.323	1.266	1.319	1.314
10	Scottish Mutual Ass Soc	1.054	1.071	1.054	1.182	1.100	1.144	1.116	1.129	1.075
11	AXA Equity & Law Life Ass Soc	1.138	1.153	2.915	2.703	1.295	1.218	1.203	1.220	1.208
12	Financial assurance	1.020	1.027	1.024	1.022	1.064	1.047	1.078	1.130	1.127
13	Pearl Assurance	1.071	1.093	1.164	1.269	1.241	1.233	1.276	1.351	1.244
14	Royal & Sun Alliance life & Pens	1.076	1.115	1.126	1.192	1.176	1.225	1.203	1.208	1.136
15	Abbey Life Ass	1.020	1.027	1.024	1.022	1.010	1.031	1.045	1.045	1.046
16	Scottish Equitable	1.005	1.005	1.004	1.003	1.129	1.178	1.195	1.205	1.168
17	Skandia Life Ass Co	1.015	1.009	1.010	1.011	1.014	1.014	1.015	1.015	1.019
18	Eagle Star Life Ass	1.063	1.086	1.120	1.152	1.139	1.196	1.202	1.193	1.109
19	Scottish Amicable life		1.024	1.018	1.025	1.030	1.009			
20	M & G Pens & Annuity Co					1.015	1.020	1.022	1.011	1.012
21	J Rothschild Ass		1.010	1.012	1.011	1.022	1.027	1.022	1.043	1.011
22	J P Morgan Life Ass	1.005	1.005	1.003	1.004	1.011	1.041			
23	Britannic Ass	1.067	1.111	1.114	1.247	1.277	1.412	1.370	1.493	1.453
24	Mercury Life ass				1.010	1.009	1.005	1.006	1.005	1.011
25	Guardian Ass		1.101	1.126	1.148	1.140	1.156	1.143	1.122	1.105
26	Winterthur Life Uk Ltd	1.019	1.019	1.019	1.020	1.050	1.091	1.089	1.053	1.059
27	Colonial Life Ltd					1.171	1.218	1.200		
28	Lincoln Ass	1.026	1.019	1.017	1.007	1.025	1.031	1.036	1.037	1.032
29	Morgan Grenfell Life & Pens					1.028				
30	Hermes Liberty Int. Pens Ltd					1.001	1.002			

Sources: DTI Returns Only the main company from each group is shown.
Free Asset Ratio = Free assets / total assets (the ratio includes required minimum margin)
Free Asset = total assets + future profits - total liabilities.
Total assets: Form 9, rows21 and 22. **Future profits**: Form 9, row 31. **Total Liabilities**: Form 9, rows 23 and 24
Solvency Ratio = Total assets / total liabilities = 1/(1- Free Asset Ratio/100)

Table 2C.2(b) : Solvency Ratios of a set of Uk Life offices (Long Term Trends)

	Group of Life Offices	Year 2002	Year 2001	Year 2000	Year 1999	Year 1998	Year 1997	Year 1996	Year 1995	Year 1994
		Solvency Ratio	Solvency Ratio	Solvency Ratio	Solvency Ratio	Solvency Ratio	Solvency Ratio	Solvency Ratio	Solvency Ratio	Solvency Ratio
No.	**Mutual Offices**									
1	Equitable Life ass	1.060	1.066	1.050	1.133	1.098	1.098	1.100	1.115	1.094
2	Standard Life Ass	1.068	1.087	1.212	1.245	1.239	1.271	1.258	1.253	1.211
3	Scottish Widows' Fund & Life As[1]	1.125	1.115	1.190	1.000	1.214	1.235	1.242	1.238	1.224
4	Friends Provident life Offices[2]	1.082	1.124	1.147	1.279	1.145	1.224	1.205	1.140	1.080
5	National Provident Institution[3]	1.024	1.045	1.056	1.086	1.075	1.117	1.106	1.124	1.101
6	Co-Operative Ins Soc	1.093	1.155	1.140	1.224	1.205	1.230	1.280	1.284	1.212
7	Scottish Life Ass					1.105	1.119	1.129	1.107	1.109
8	Scottish Provident Institution[4]	1.042	1.104	1.119	1.175	1.119	1.182	1.151	1.116	1.101
9	Royal London Mutual Ins Soc	1.080	1.096	1.236	1.595	1.300	1.414	1.473	1.499	1.431
10	National Mutual Life Ass[5]	1.040	1.042	1.060	1.134	1.110	1.155	1.151	1.168	1.183
11	Canada Life Ass					1.067	1.178	1.144	1.105	1.222
12	Sun Life Ass Co. of Canada[6]					1.261	1.280	1.359	1.401	1.449
13	National Farmers Union Mutual	1.147	1.211	1.221	1.248	1.185	1.159	1.263	1.245	1.224
14	Wesleyan Ass soc	1.080	1.196	1.241	1.295	1.239	1.305	1.429	1.429	1.420
15	Royal Nat Pens Fund For Nurse					1.066	1.120	1.093	1.156	1.145
16	Marine & General Mutual Life ass	1.083	1.065	1.111	1.151	1.105	1.130	1.109	1.112	1.105
17	Reliance Mutual Ins Soc		1.160	1.112	1.046	1.099	1.064	1.110	1.126	1.110
18	Forester Life Ltd	1.111	1.170	1.274	1.255	1.241	1.294	1.319	1.355	
19	Ecclesiastical Ins Office				1.224	1.202	1.222	1.143	1.152	1.089
20	London aberdeen & Noth Mutual			1.125	1.147	1.072	1.099	1.109	1.127	1.068
No.	**Banks/ Building Societiy Companies**									
1	National West Life ass	1.031	1.027	1.024	1.016	1.019	1.035	1.044	1.054	1.070
2	Britannia Life Ltd					1.027	1.032	1.046	1.049	1.063
3	Lloyds TSB Life Ass	1.070	1.034	1.044	1.028	1.035	1.062	1.071	1.063	1.062
4	Barclays Life Ass	1.033	1.156	1.028	1.020	1.020	1.022	1.029	1.025	1.025
5	Midland Life Ass (HSBC life (UK) ltd)		1.042	1.038	1.032	1.032	1.047	1.052	1.056	1.052
6	Royal Scottish Ass	1.038	1.026	1.019	1.021	1.014	1.011	1.017	1.028	1.037
7	Abbey Nat Life	1.116	1.078	1.072	1.066	1.105	1.071	1.052	1.059	1.107
8	Halifax Life		1.032	1.065	1.068	1.068	1.081	1.175	2.237	
9	Nationwiode Life	1.096	1.074	1.053	1.041	1.043	1.060	1.004	2.564	
10	Woolwich Life Ass	1.086	1.292	1.098	1.072	1.087	1.087	1.094	1.103	1.114

Note:

1 Scottish Widows' Fund & Life Assurance Society business was transferred to Scotish Widows Plc after 1999

2 Friends Provident Life Office business was transferred to Friends Provident Life & Pensions Ltd in July 2001

3 National Provident Institution business was transferred to NPI Ltd after 1999

4 Scotish Provident Institution business was transferred to Scotish Provident Ltd in July 2001

5 National mutual Life Assurance Society business was transferred to GE Pensions Ltd in March 2002

6 Sun Life Ass. Co. of Canada (UK branch) business was transferred to Sun Life Ass. Co. of Canada (UK) Ltd in 2000

Appendix 3

3A The Market Model

3A.1 Existing business in projection year t

A n–year temporary insurance (level annual premium) policies of identical size (sum assured) are assumed to be issued at the start of each year over the past 10 years before the projection date to policyholders aged x (e.g. age 55) at entry in the market. 10,000 of such policies are assumed to be issued a year before projection date. The past new business is assumed to grow at a constant rate (equal to past inflation rate, e.g. 3% per annum).

$\eta_m^j(s,t)$ = Number of policies issued at the start of year s and still inforce at the end of year t for simulation j.

$\eta_m^j(t)$ = Total number of inforce policies at the end of year t for simulation j.

$$\eta_m^j(t) = \sum_{s=-9}^{t} \eta_m^j(s,t) \mathrm{I}_{u(t-s)} \qquad (3A.1)$$

$$\mathrm{I}_{u(t-s)} = \begin{cases} 1 & if \ (t-s) < n \\ 0 & if \ (t-s) \geq n \end{cases} \qquad (3A.2)$$

$\mathrm{I}_{u(t-s)}$ = Indicator function for policy duration u to ensure that no cash flows are generated beyond the term n of each policy in a particular group s.

$s \leq t$, $1 \leq t \leq T$ and $-9 \leq s \leq T$

s = Policy issue year.

t = Projection year

T = Projection period in years (e.g. 20 years).

n = Policy term (e.g. 10 years)

$u = (t - s)$ = Policy duration

3A.2 Office premium and sum assured per policy issued in projection year t

At projection date t = 0, the profit tested premium denoted by $\overline{OP_m}$ is payable to an existing policy and the corresponding sum assured is $SA_m(0)$.

$OP_m^j(s) = OP_m(0) = \overline{OP_m}$ for $s = 0, -1, -2, \ldots\ldots\ldots\ldots, -9$)

$SA_m^j(s) = SA_m(0)$ for $s = 0, -1, -2, \ldots\ldots\ldots\ldots, -9$)

The office premium per new policy issued in projection year t is a function of price inflation and the new business strategy that determines the measure of premium response to solvency, i.e. $l_m(+ve)$ if new business strategy 1 is considered. The equation (3A.3) below is from equations (9) to (12) in section 2.2.3 of chapter 2.

$$OP_m^j(t) = \overline{OP_m} \times \prod_{k=0}^{t-1}\left[1+\inf_p^j(t)\right] \times \left[1 - \frac{l_m(+ve)}{SR_m^j(t-1)}\left(SR_m^j(t-1) - NDF\right)\right] \qquad (3A.3)$$

Given that $\inf_p^j(0) = \inf_p(0)$ and $SR_m^j(0) = SR_m(0)$

For example, $\inf_p(0) = 3\%$, $SR_m(0) = 1.50$, $\overline{OP_m} = £874.20$, $SA_m(0) = £100{,}000$

$$SA_m^j(t) = \prod_{k=0}^{t-1}\left[1+\inf_p^j(t)\right] \times SA_m(0) \qquad (3A.4)$$

3A.3 New business demanded and written

(a) New business issued at the start of year s before projection date, $-9 \le s \le 0$

$NM(0) = ANM(0) = 10{,}000$, and $\inf_p(0) = 3\%$

$$ANM(s) = \frac{ANM(0)}{\left(1+\inf_p(0)\right)^s} \qquad (3A.5)$$

(b) New business demanded and written in projection year t, $s \le t$, $1 \le t \le T$

Market quantity of new business demanded under an exponential price elasticity of demanded function.

$$\frac{NM^j(t)}{NM^j(t-1)} = \exp\left[-EK\left(\frac{OP^j(t)}{OP^j(t-1)} - 1\right)\right] \qquad (3A.6)$$

Let $g^j(t) = -EK\left(\frac{OP^j(t)}{OP^j(t-1)} - 1\right)$ (3A.7)

Given that $NM^j(0) = NM(0) = 10{,}000$ and $NM(0) = ANM(0)$

$$NM^j(t) \quad NM(0) \quad \prod_{k=1}^{t} \exp\left[g^j(k)\right]$$

$$NM^j(t) = NM(0) \quad \exp \sum_{k=1}^{t} g^j(k) \qquad (3A.8)$$

$$ANM^j(t) = \frac{\left(\left(F^j A^j_m(t-1) \quad \rho_m\right.\right.}{ENBS^j_m(t)} \quad NM^j(t), \text{ for } 0 \le ANM^j(t) \le NM^j(t) \qquad (3A.9)$$

3A.4 Withdrawals in projection year t

$$\eta w^j_m(t) = \sum_{s=-9}^{t} \eta w^j_m(s,t) \mathrm{I}_{u(t-s)} \qquad (3A.10)$$

$\eta w^j_m(s,t)$ = Number of policies issued at the start of year s and lapse in year t for simulation j.

$\eta w^j_m(t)$ = Total number of withdrawals in year t for simulation j.

3A.5 Death benefits in projection year t

$$\eta d^j_m(t) = \sum_{s=-9}^{t} \eta d^j_m(s,t) \mathrm{I}_{u(t-s)} \qquad (3A.11)$$

$$DB^j_m(t) = \sum_{s=-9}^{t} \left(\eta d^j_m(s,t) \mathrm{I}_{u(t-s)} \times SA^j_m(s)\right) \qquad (3A.12)$$

$\eta d^j(s,t)$ = Number deaths in projection year t for policies issued at the start of year s for simulation j.

$\eta d^j_m(t)$ = Total number of deaths in year t for simulation j.

$DB^j_m(t)$ = Total death benefit in year t for simulation j

$C_m^j(t) = DB_m^j(t)$, for temporary insurance policies.

3A. 6 Total expenses in projection year t

$$E_m^j(t) = \sum_{s=-9}^{t-1}\left[\eta_m^j(t-1,s)\left(\mathrm{I}_{c(t-1-s)} \times OP_m^j(i) + \mathrm{I}_{e(t-s)}\right)\right] + ANM^j(t)\left[OP_m^j(t)\left(ICR_m + IER_m\right) + IEP_m^j(t)\right]$$

(3A.13)

$$\mathrm{I}_{c(t-s)} = \begin{cases} ICR_m & if \quad 0 \le (t-s) \le 3 \\ RCR_m & if \quad 3 < (t-s) < n \end{cases} \tag{3A.14}$$

$$\mathrm{I}_{e(t-s)} = \begin{cases} IEP_m^j(t) & if \quad (t-s) = 0 \\ REP_m^j(t) & if \quad 1 \le (t-s) < n \end{cases} \tag{3A.15}$$

$$IEP_m^j(t) = \prod_{k=0}^{t-1}\left[1 + \inf_p^j(t)\right] \times IEP_m(0) \tag{3A.16}$$

$$REP_m^j(t) = \prod_{k=0}^{t-1}\left[1 + \inf_p^j(t)\right] \times REP_m(0) \tag{3A.17}$$

ICR_m = Market initial commission rate (e.g. 125% of premium).

RCR_m = Market renewal commission rate (e.g. 2.5% of premium).

IER_m = Market initial expense rate (e.g. 5% of premium).

$IEP_m(0)$ = Market initial expense per policy at projection date (e.g. £115).

$IEP_m^j(t)$ = Market initial expense per policy in projection year t for simulation j

$REP_m(0)$ = Market renewal expense per policy at projection date (e.g. £15).

$REP_m^j(t)$ = Market renewal expense per policy in projection year t for simulation j.

$E_m^j(t)$ = Total expenses incurred at the start of the year t for simulation j.

3A.7 Total premium income at the start of projection year t

$$P_m^j(t) = \sum_{s=-9}^{t-1}\left(\eta_m^j(t-1,s)\mathrm{I}_{u(t-s)} \times OP_m^j(i)\right) + ANM^j(t) \times OP_m^j(t) \tag{3A.18}$$

$P_m^j(t)$ = Total premiums received at the start of year t under all policies for simulation j.

3A.8 Statutory reserves per policy at the end of projection year t

$$V_x^J(t,s) = SA_m^J(s) \times A^{\,1}_{x+u+1:\overline{n-u-1}|} - NP_m^J(s) \times \ddot{a}_{x+u+1:\overline{n-u-1}|} \tag{3A.19}$$

$$NP_m^j(s) = \frac{SA_m^j(s) \times A^{\,1}_{x:\overline{n}|} + Z}{\ddot{a}_{x:\overline{n}|}} \tag{3A.20}$$

$$SM_x^j(s,t) = 0.04 \times V_x^j(t,u) + 0.003 \times [SA_m^j(s) - V_x^j(s,t)] \tag{3A.21}$$

$$^S V_m^j(s,t) = \left(V_x^j(s,t) + SM_x^j(s,t)\right) \tag{3A.22}$$

Z= maximum allowable Zillmer

$NP_m^j(t)$ = Net Premium for policies issued in the market in projection year t = s for simulation j.

$V_x^j(s,t)$ = Statutory reserve per policy at the end of the projection year t with duration $(u+1)$ for life aged x at entry in year s for simulation j.

$SM_x^u(s,t)$ = Statutory solvency margin per policy at the end of projection year t for life aged x at entry in year s for simulation j.

$^S V_m^j(s,t)$ = Statutory reserve plus solvency margin per policy at the end of projection year t for policies issued in year s for simulation j.

3A.9 New business strain per policy

New business strain per policy is the capital required to write a policy. New business strain is the premium paid at the start of a contract, less the initial expenses including commission payments, is not sufficient to cover the mathematical reserve that the company needs to set at that point,

$$NBS^j(t) = \left(OP_m^j(t) - \left(OP_m^j(t) \times \left(ICR_m + IER_m\right) + IEP_m^j(t)\right) - {}^S V_x^j(t,0^+)\right) \tag{3A.23}$$

$$V_x^j(t,0^+) = SA_m^j(s) \times A^{\,1}_{x:\overline{n}|} - NP_m^j(s) \times (\ddot{a}_{x:\overline{n}|} - 1) \tag{3A.24}$$

$$SM_x^j(t,0^+) = 0.04 \times V_x^j(t,0^+) + 0.003 \times [SA_m^j(s^+) - V_x^j(t,0^+)] \tag{3A.25}$$

$$^{S}V_x^j(t,0^+) = \left(V_x^j(t,0^+) + SM_x^j(t,0^+)\right) \tag{3A.26}$$

$SM_x^j(t,0^+) =$ Statutory solvency margin per policy for life aged x at entry in projection year t immediately after the payment of the first premium for simulation j.

$V_x^j(t,0^+)$ = Statutory reserve per policy for life aged x at entry in projection year t immediately after the payment of the first premium for simulation j.

$^{S}V_x^j(t,0^+)$ = Statutory reserve plus solvency margin per policy for life aged x at entry in projection year t immediately after the payment of the first premium for simulation j.

$NBS_m^j(t)$ = New business strain per policy in the market (i.e. capital required to write a new policy) at start of projection year t for simulation j.

3A.10 Total statutory value of liabilities and new business strain

$$\eta_m^j(t) = \eta_m^j(t-1) + ANM^j(t) - \eta d_m^j(t) - \eta w_m^j(t) \tag{3A.27}$$

$$L_m^j(t) = \sum_{s=-9}^{t} \left(\eta_m^j(s,t) \mathrm{I}_{u(t-s)} \times {}^{S}V_x^j(s,t)\right) \tag{3A.28}$$

$$ENBS_m^j(t) = NM^j(t) \times NBS_m^j(t) \tag{3A.29}$$

$ENBS_m^j(t)$ = Total expected new business strain for the market (i.e. capital required to write new business demanded) at start of projection year t for simulation j.

$L_m^j(t)$ = Total market statutory reserve plus solvency margin at the end of projection year t for simulation j.

3A.11 Total assets at the end of year t for simulation j

$$A_m^j(t) = [A_m^j(t-1) + P_m^j(t) - E_m^j(t)](1 + r^j(t)) - C_m^j(t) \tag{3A.30}$$

$A_m^j(t) =$ Total statutory value of assets at the end of time t years for simulation j.

$r^j(t)$ = Total return on assets over the year (t-1, t) for simulation j.

$C_m^j(t)$ = Total benefits paid under death claims (or maturity claims for endowment non profit policies if considered) at the end of year t for simulation j.

3A.12 Solvency ratio in projection year t

$$SR_m^j(t) \quad \frac{A_m^j(t)}{L_m^j(t)} \tag{3A.31}$$

$$FA_m^j(t) \quad A_m^j(t) - L_m^j(t)$$

$SR_m^j(t)$ = Solvency ratio at time t for simulation j.

$FA_m^j(t)$ = Statutory free assets (surplus) at time t for simulation j.

$L_m^j(t)$ = Total statutory value of the liabilities at the end of year t for simulation j.

3B. Description of how contour lines are obtained

For the j-th simulation, $SR_m^j(t)$, $t = 0, 1, 2, ...,T$, gives a stochastic development of solvency ratios over the projection period. Repeating this N times gives N simulated realisations of the solvency ratios. In this investigation a simulation is deemed to have produced an insolvent outcome if the projected solvency ratio (or surplus) falls below one, (or zero) i.e. i.e. $SR_m^j(t) < 1$ or $FA_m^j(t) < 0$ at least once during the T-year projection period. The risk of insolvency (e.g. probability of insolvency) can be measured by considering the insolvency cases arising from N simulated realisations. The probability of insolvency is defined as the proportion of outcomes with statutory insolvency from a predetermined number of simulations (see equation (20) in chapter 3).

For a given initial solvency ratio $SR_m(0)$, a constant proportion of free assets allocation strategy to finance new business (e.g. ρ_m) is required to produce a fixed level of insolvency risk (e.g. $\varepsilon = 5\%$) over a time horizon of T years and in all simulations. For example, $P\left\{\min\left(SR_m^j(t) < 1: \quad t, j\right)\right\} = \varepsilon$

As expected, the constant proportion of free assets to finance new business increases with the initial solvency ratio for a given level of insolvency risk. In this investigation, simulations are carried out for 11 initial solvency ratios, i.e. 1.0, 1.1, 1.2,..............,1.9, 2.0; and 11 free assets allocation proportions, 0.5, 0.1, 0.2,.........0.9, 1.0. For a given initial solvency ratio, 11 free assets allocation proportions are considered. We then measure the insolvency risk associated with a particular combination of initial solvency ratio and free assets allocation proportion. Each curve in figure 3.2 shows all the combinations of initial solvency ratio and free assets allocation proportion that lead to a given insolvency risk over a time horizon. It is not possible to consider all combinations of initial solvency ratio and free assets allocation proportion to finance new business in the computer simulations, due to computer run time. Hence, for a given insolvency risk interpolations are carried out, where necessary, to calculate the combinations along the curves.

Each point on the graph can be identified by three coordinates (x, y, z), where x is the initial solvency ratio, y is the free assets allocation proportion, and z is the insolvency risk. For example, in figure 3.2 of section 3.3.2 in chapter 3, the point A has coordinates (1.90, 100%, 0.75) and point B has coordinates (1.40, 31%, 0.05). This means that, if a company with an initial solvency ratio of 1.90 allocates 100% of its free assets to finance new business over a 10-year period, then we expect a probability of insolvency of 0.75. On the other hand, if a company with an initial solvency ratio of 1.40 allocates 31% of its free assets to finance new business over a 10-year projection period, then we expect a probability of insolvency of 0.05.

3C Graphs

Below shows the figures related to chapter 3.

Figure 3C.1: Market new business models (NBS-1 and ALM investment strategy)
Effect of models on allocation of free assets to finance new business and insolvency risk

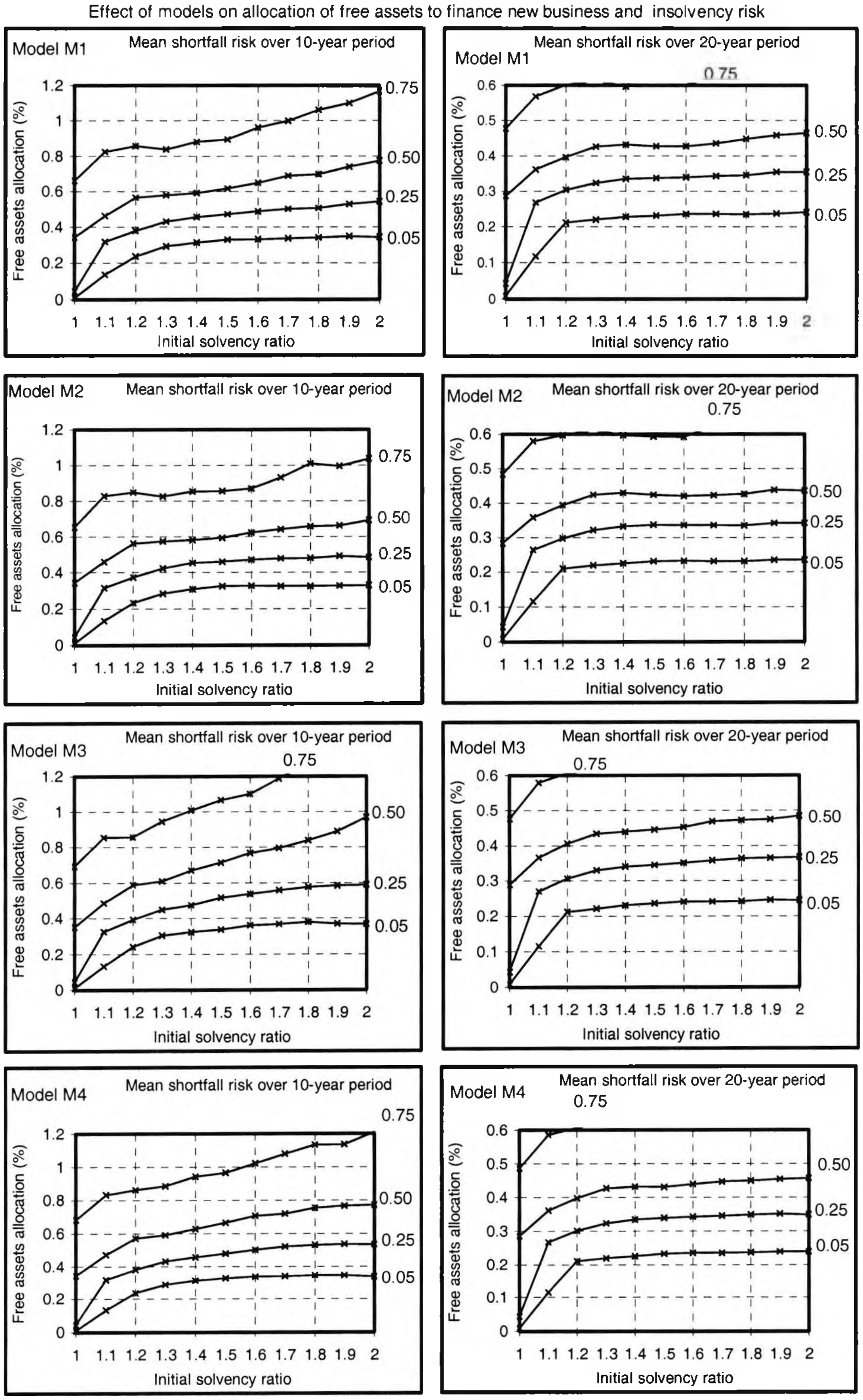


Figure 3C.2: Company new business models (NBS-1 and ALM investment strategy)
Effect of models on allocation of free assets to finance new business and insolvency risk

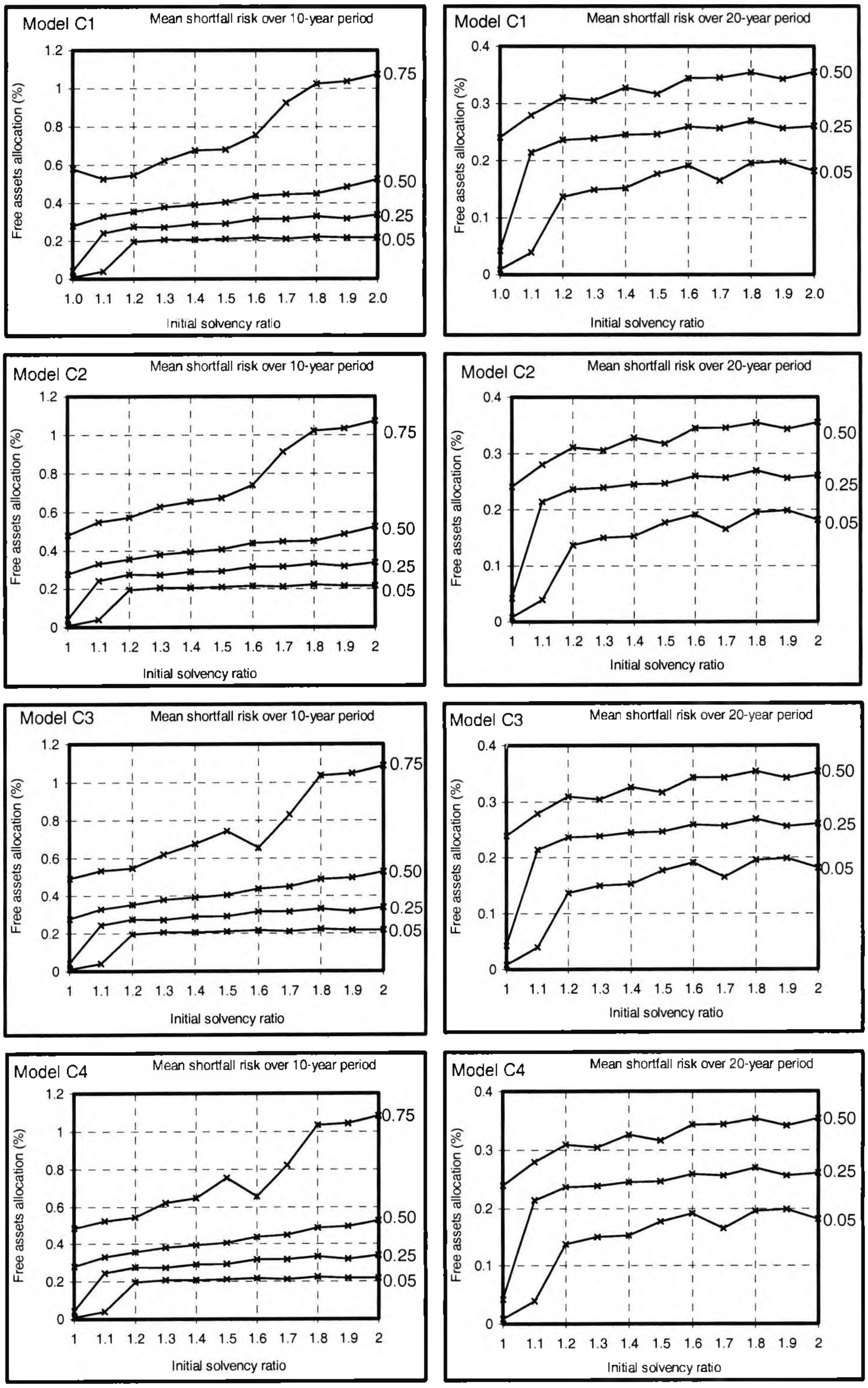


Figure 3C.3: Market new business model M1 (Exponential Demand Function)
Allocation of free assets to finance new business and ALM investment strategy

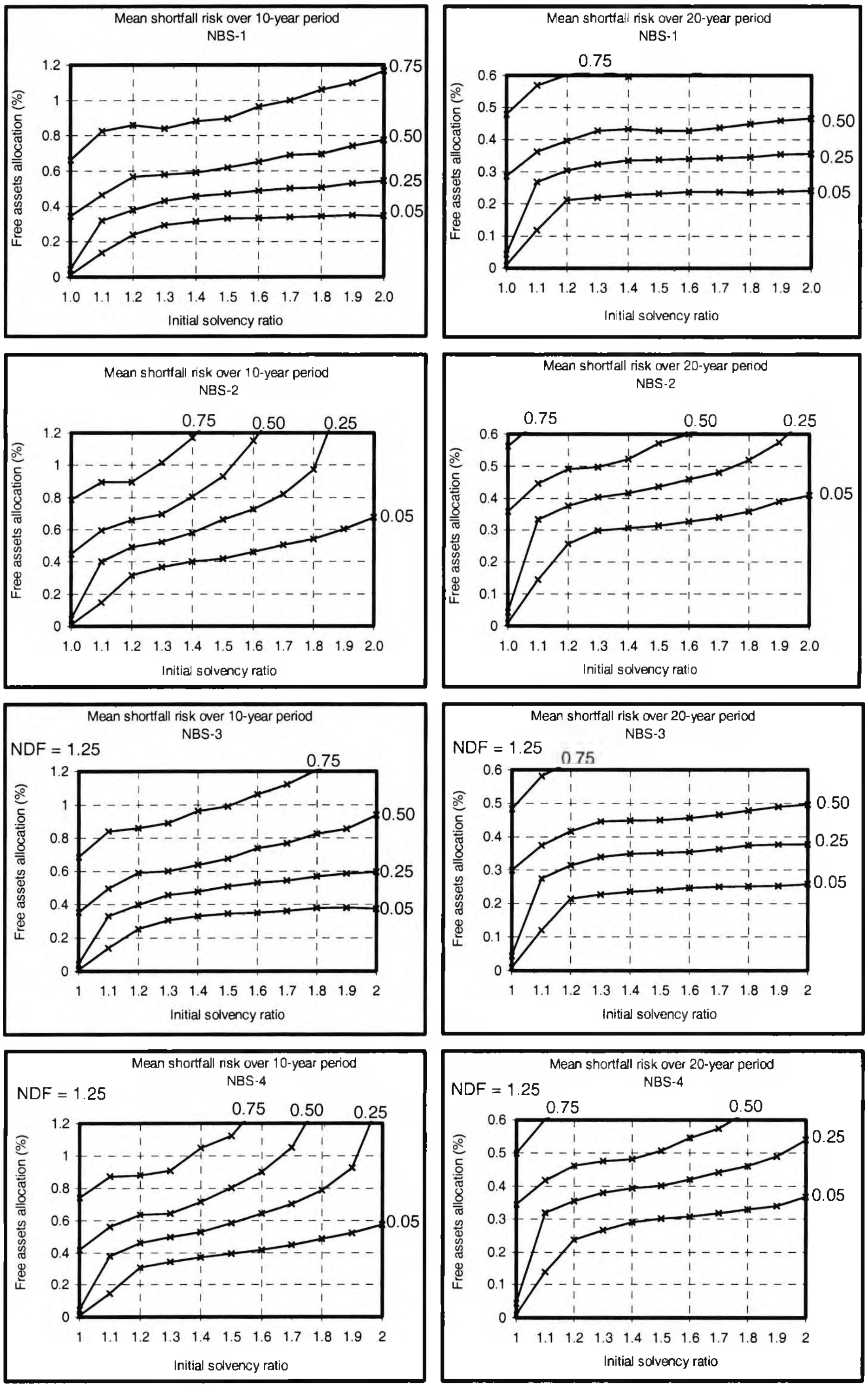


Figure 3C.4: Market new business model M1 (Exponential Demand Function)
Effect of investment strategies on allocation of free assets to finance new business (NBS-1)

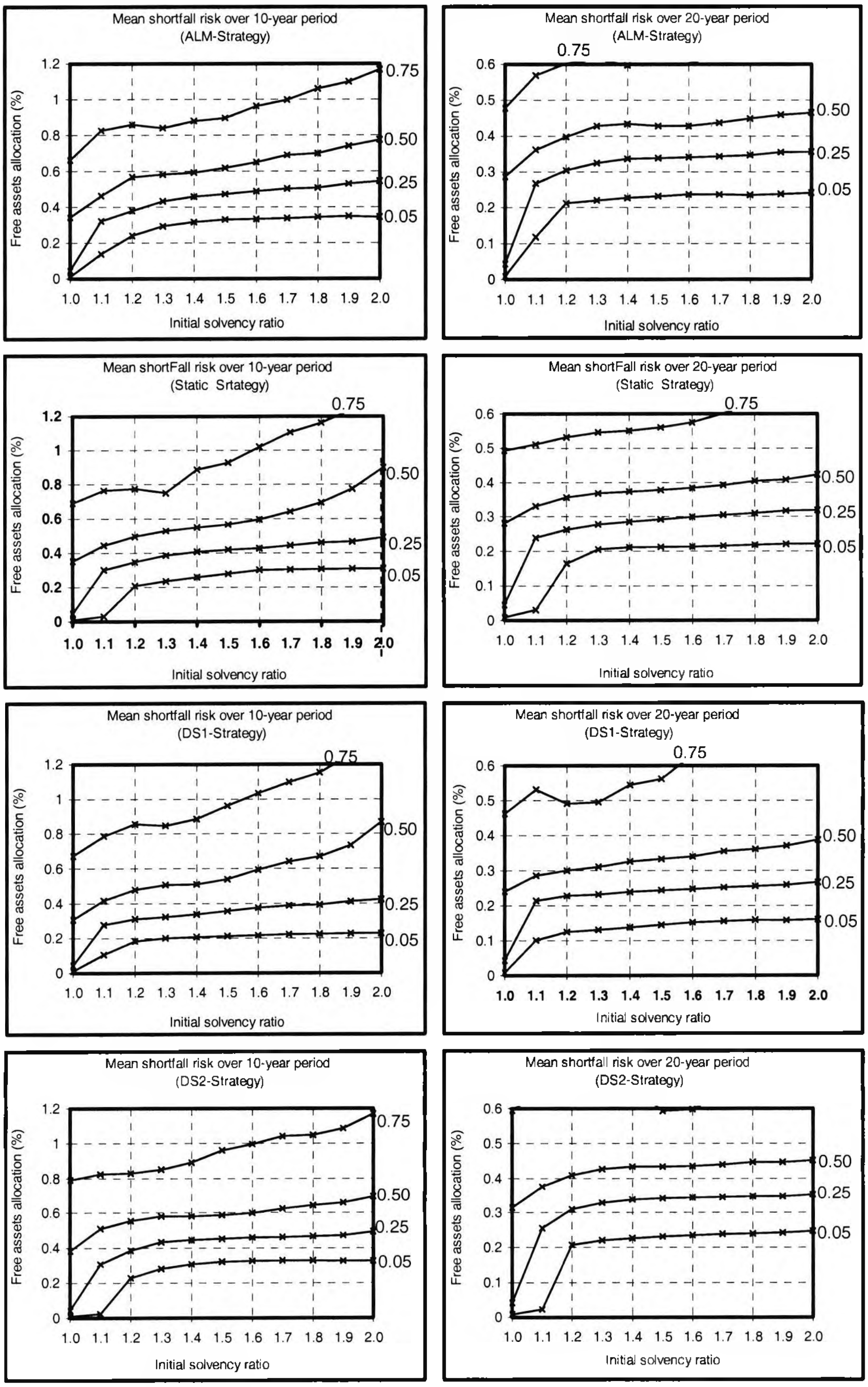


Figure 3C.5: Market new business model M1 (Exponential Demand Function)
Effect of changes in valuation basis on new business plan and insolvency risk under NBS-1

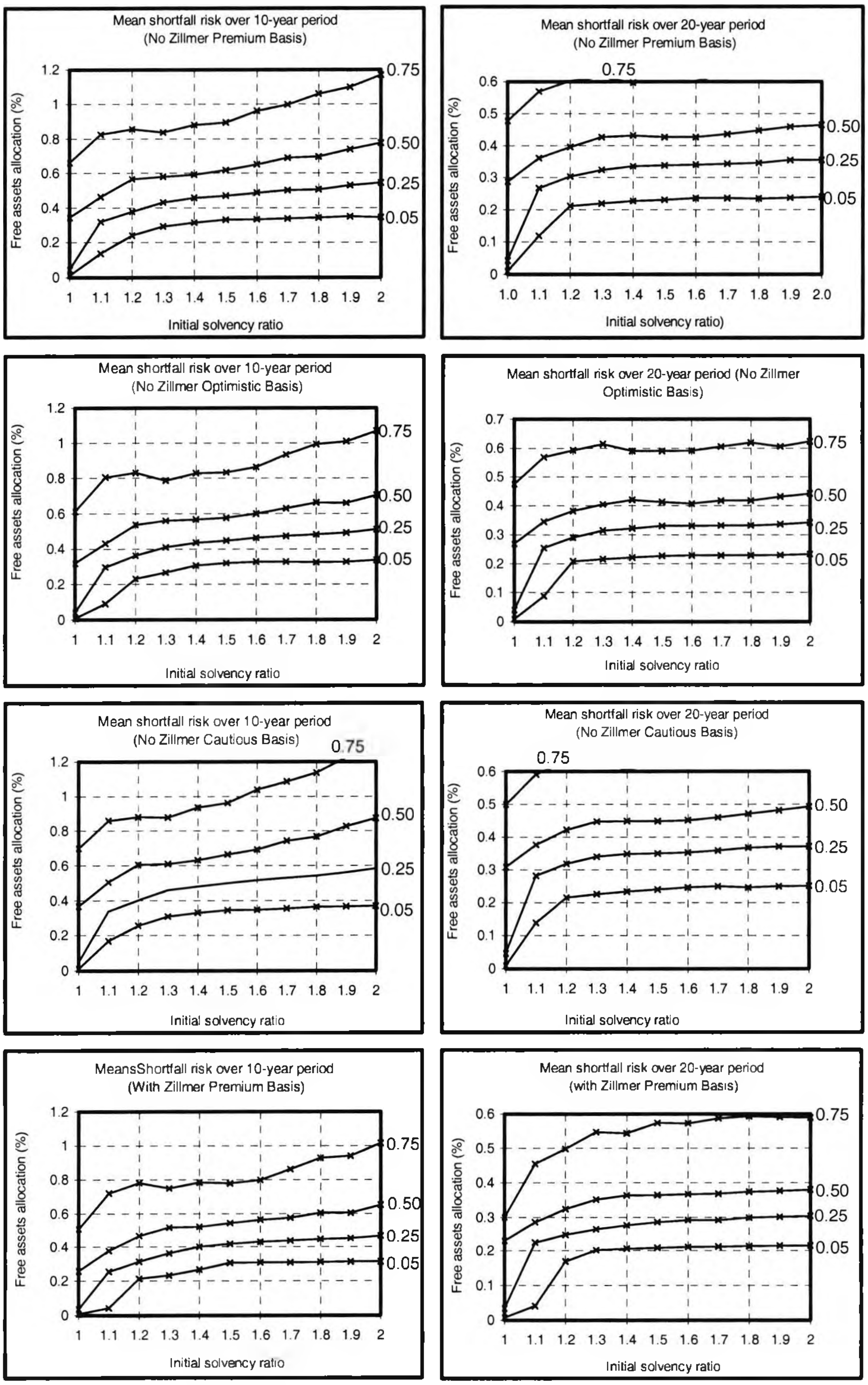
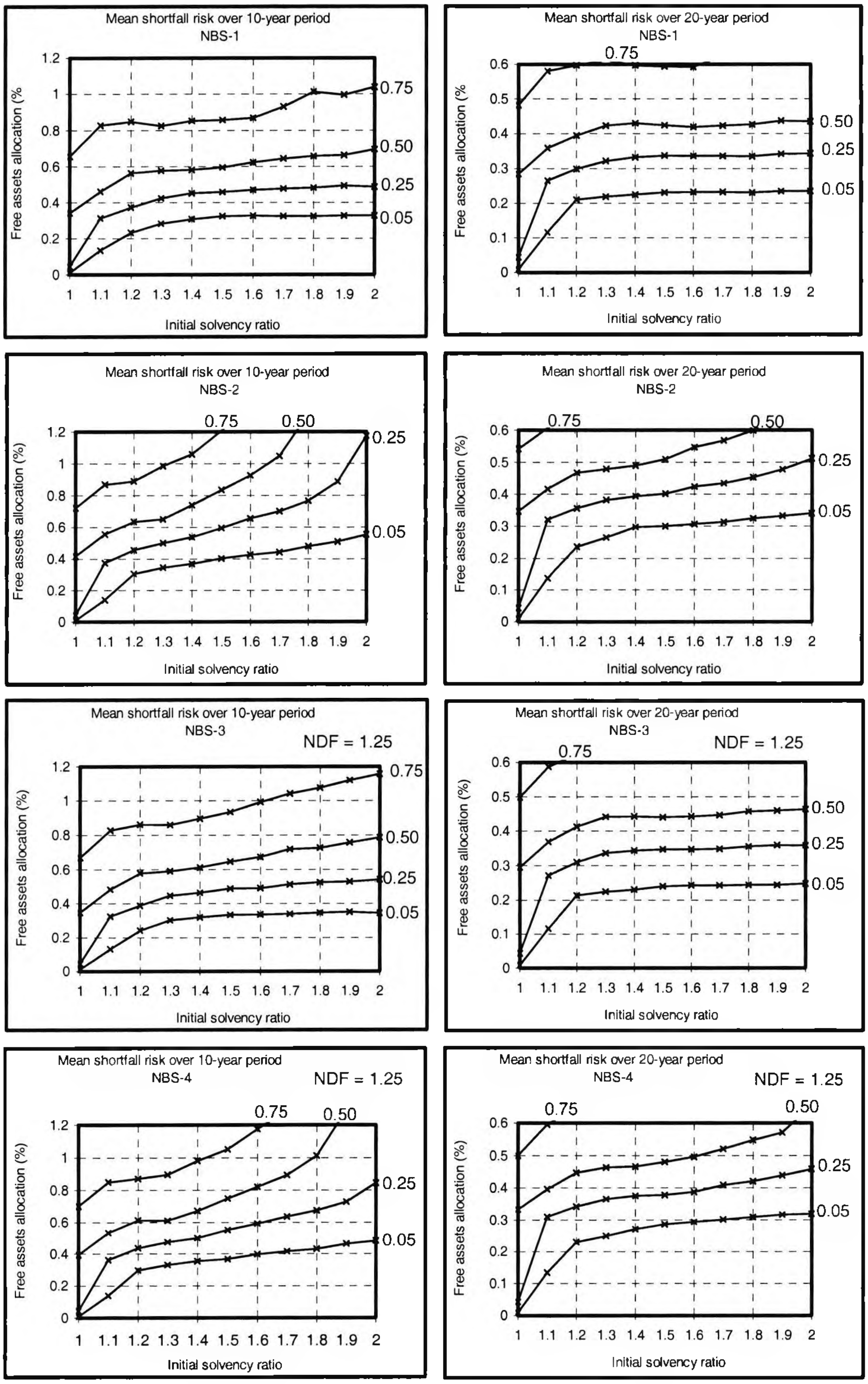


Figure 3C.6: Market new business model M2 (Exponential Demand Function and Real Wage Infaltion)
Allocation of free assets to finance new Business and new business strategies (ALM investment strategy)



Appendix 4

4A Product Design

In this investigation, we consider two types of contract, namely temporary insurance and non profit endowment insurance policies, and for each contract type we design two policy types which differ mainly by initial commission terms and/or expenses.

In practice, the initial commission for regular premium business may be payable over an initial period or as a single payment at commencement of the policy. The later mode of initial commission payment may allow the discounting (e.g. at a compound interest rate of 1% per month) of the initial commission instalments over the initial period to a single payment (indemnity commission). If the policy terminates (other than death) before the end of the initial commission period, the insurance broker must repay the unearned part of the indemnity commission with due allowance for the discount rate, Hylands et al (1990).

Table 4A.1

Initial commission payment periods (LAUTRO Rules)

Policy Type	Premium payment term	Initial Period	Initial commission % of premium	Indemnity commission
Temporary Ir	10	43 months	35%	103%
Endowment I	10	16 months	25%	31%

Source: Hylands et al (1990)

We illustrate below how the details in table 4A.1 for a temporary insurance policy are found:

4.3X = I, (PIA-Adopted rules: LAUTRO, obtained from IFA)

where **X** = number of years or part of years of the effective premium term, or 11 years 3 months, whichever is the less

I = number of months constituting the initial period, where the result is not a whole number, it shall be rounded to the nearest whole number, a half being rounded up.

LAUTRO initial commission per period = initial commission percentage times premium. In practice, companies may pay more than LAUTRO initial commission.

Indemnity commission = Initial commission % times annuity factor @ 1% per month (e.g. $0.35 \times \left(\frac{1}{12}\ddot{a}_{\overline{43|}} @ 1\% \right. = 1.025)$. The indemnity commission is expressed as percentage of one year's premium.

For simplicity, we consider contract designs with initial commission payable on non-indemnity terms over an initial period (in complete years only), and even if the initial commission instalments are discounted to provide a single payment at commencement date, no allowance is made for unearned commission to be clawed back.

4A.1 Temporary Insurance Policy Types 1 and 2

Table 2.1 in chapter 2 shows the policy details for a temporary insurance contract type 1 (PT1) with initial commissions payable over 4 years. We consider a new temporary insurance contract, policy type 2 (PT2), which has same contract details (including the annual premium, £874.20) as policy type 1 except for the initial commission rate and initial commission payment period. For policy type 2, the initial commission is payable at commencement date, and the initial commission rate (426% of LAUTRO commission) is the rate required to produce the same annual premium as in policy type 1.

The Initial commission rate as a % of LAUTRO commission for policy type 2 can be investigated by carrying out a sensitivity analysis of the discounted profit (in a profit testing) to changes in the initial commission rate. For a given premium, the required initial commission rate will produce a discounted profit approximately equal to zero. Thus, policy type 2 with an annual premium of £874.20 will produce zero discounted profit if a 425.67% of LAUTRO commission is used, if all other things remain the same. This is approximately equal to an indemnity commission of 426 % $\left(=1.25\times\left(\frac{1}{12}\ddot{a}_{\overline{48}|}\ @0.7\%\right)\right.$) of LAUTRO commission. In calculating an indemnity commission, MCGaughey et al (2001) use 0.75% per month.

4A.2 Non Profit Endowment Insurance Policy Types 3 and 4

We consider two non profit endowment insurance policies (policy type 3 and policy type 4). Table 4A.2 below shows the a non typical contract details for a non profit endowment insurance policy type 3 (PT3) which are similar to policy type 1 except for the maturity and surrender benefits payable under the endowment insurance policy. The non profit endowment insurance policy type 4 (PT4) differs from PT3 only in the following:

- Initial commission: 125% of LAUTRO commission is payable at the start of the first year
- Renewal commission: 2.5% of premium payable each year starting from the second year.
- Initial expenses: £150 per policy at the start of first year
- Renewal expenses: £40 each year with inflation starting from second year.

Thus, the office premiums for ages 35, 45 and 55 are £859.25, £866.00 and £890.75 respectively for policy type 4, using the above details.

As the reserves for endowment insurance policy are expected to be higher than those for temporary insurance policy, the corresponding initial new business strain for the former policy is also likely to be higher than that for the latter policy. In order to reduce the effect of the new business strain to an endowment insurance portfolio, we assume that $l_m(\pm ve)=0.20$ and $EK_m=0.2$ for the non profit endowment insurance market. This will reduce the quantity of new business demanded in the non profit endowment insurance market relative to the temporary insurance market.

Table 4A.2 Policy design for endowment (non profit) contract (Policy Type 3)

Ages at Entry :		x	35, 45 and 55
Term of Contract		n	10 years
Annual Premium		OP(0)	£1,281.40 (age 35), £1,294.05 (age 45) and £1,340.45 (age 55)
		OP(t)	for policies issued in projection year t
Sum Assured:		SA(0)	£10,000 per policy for policies issued before projection date
		SA(t)	Sum assured per policy for policies issued in projection year t
Expenses	Initial	Ie(0)	£115 per policy at start of first year
		Ier(0)	5% of annual premium at start of first year
	Renewal	Re(0)	£15 p.a (for Profit Testing -payable yearly from year 2 with inflation)
Commission	Initial	Icr(0)	125% of LAUTRO commission payable for 4 years on non-indemnity terms (LAUTRO rules)
	Renewal	Rcr(0)	2.5% of premium each year after first 4 years
Inflation rates:	Expenses	$inf_e(0)$	4.7% p.a. for period before projection date
		$inf_e(t)$	expected inflation of expenses (per policy amount) in projection year t generated using Wilkie model
	Price	$inf_p(0)$	3% p.a (assumed initial rate at projection date)
		$inf_p(t)$	expected inflation of premium and sum assured in projection year t generated using Wilkie model
Tax rate:		Tax	31% (on a net/net basis without a deferral period-company will be taxed on investment income and obtain tax relief on expenses before projection date) . We ignore taxation in future projection periods.
Gross Investment Income:		r_t	7.75% p.a. (adjusted in line with Wilkie model)
Risk Discount Rate:		r_D	10% p.a. (adjusted in line with Wilkie model)
Mortality:		AM92 Table	90% of AM92, 2-year select period (male)
Benefits:		SA(0)	£10,000 (Sum assured payable at the end of year of death for policies issued before projection date
		SA(t)	Sum assured payable at the end of projection year of death for policies issued in projection year t
Lapse Rates :		c(y)	10% in first year of policy, 8% in 2nd year, 6% in 3rd Year, 4% in 4th year and 2% thereafter.
Withdrawal Benefits		SV(t)	85% of reserves held at start of year t
Valuation Basis:		Gross premium prospective policy values for profit testing: Valuation rate of interest = 4.5% p.a. Mortality = 110% of Male mortality. Expense Reserve = 20% of all renewal expenses held. Solvency capital = 4% of reserves plus 0.3% of capital at risk	

4B Financial Reinsurance

4B .1 Deficit account financing

The theoretical background of deficit account financing (risk premium reinsurance with a loan) given in section 4.6.3 of chapter 4 is based on (Brett et al, 1993). We give below a recursive formula for deficit account balance in a projection year t, and the figures stated against the variables defined are chosen arbitrarily.

In considering risk premium reinsurance with financing arrangement, we assume that the re-insurer makes provision for mortality in excess of that for direct office. The ceding company will pay to the reinsurer for both the higher anticipated mortality on reinsured policies and the reinsurer's operating expenses. In addition, the ceding company incurs the expense of handing the reinsurance cessions which is financed by a loan (reinsurance commission) from the re-insurer to be repaid through the reinsurance financing premiums.

The re-insurer's risk premium rate (denoted by $RQ_x^j(s,t)$), which consists of an appropriate expected mortality rate, increased by expense and profit loadings is applied to the reinsured sum at risk. Thus, the reinsured sum assured and the risk premium for year t can be calculated, based on Booth et al (2005), as follows:

$$SAR_x^j(s,t) = \alpha_{RP} \times \left(SA_m^j(s) - V_x^j(t-s)\right) \quad (4B.1)$$

$$RP_x^j(s,t) = RQ_x^j(s,t) \times SAR_x^j(s,t) \quad (4B.2)$$

$$RQ_x^j(s,t) = SAR_x^j(s,t) \times v \times q_{x+u}^r + \left(E_r + \pi_r\right) \times RQ_x^j(s,t) \quad (4B.3)$$

$$RQ_x^j(t,s) = \frac{1}{1-E_r-\pi_r}\left[SAR_x^j(t,s) \times v \times q_{x+u}^r\right] \quad (4B.4)$$

$$q_x^r = \kappa_r \times q_x^c \quad \text{and} \quad q_x^c = \kappa_c \times q_x \quad (4B.5)$$

$$v = \left(\frac{1}{1+r_{RP}}\right) \quad (4B.6)$$

$s \leq t$, $u \geq 0$, $1 \leq t \leq T$ and $-9 \leq s \leq T$

Where,

s = Policy issue year.

t = Projection year

$u = (t - s)$ = Policy duration

x = age at entry

T = Projection period in years (e.g. 20 years)

q_x = TM92 ultimate mortality rate for a life aged x

α_{RP} = Proportion reinsured under risk premium method (e.g. 50%)

$SAR_x^j(s,t)$ = Reinsured sum assured per reinsured policy in year t for a life aged x at entry in year i for simulation j

$SA_m^j(s)$ = Sum assured for a policy issued in year s for simulation j (e.g. £103,000).

$V_x^j(s,t)$ = Statutory reserve per policy at the start of projection year t with duration u for life aged x at entry in year s for simulation j.

$RQ_x^j(s,t)$ = Risk premium rate per unit sum at risk per policy in year t for a life aged x at entry in year s for simulation j.

$RP_x^j(s,t)$ = Risk premium per policy in year t for a life aged x at entry in year s for simulation j.

q_{x+u}^r = Re-insurer's expected mortality rate in year t with duration u for life aged x at entry in year s.

q_{x+u}^c = Ceding company's expected mortality rate in year t with duration u for life aged x at entry in year s (e.g.70% of TM92 5 year select).

E_R = Re-insurer's expense loadings per unit of risk premium (e.g. 25%).

π_r = Re-insurer's profit loadings per unit of risk premium (e.g. 10%).

κ_R = Percentage of ceding company's mortality rate for the re-insurer (e.g. 105%)

κ_c = Percentage of standard mortality rate (TM92) for the ceding company (e.g. 70%).

r_{RP} = Re-insurer's assumed total return on assets (e.g. 7.75).

Deficit Account

We consider a tranche of new policies (denoted by $ANM^{j}(s)$) issued in year s with identical characteristics such as premium, age at entry, sum assured and term of policy. We assume that the maximum cash advance per policy is equal to one and half times the direct writer's initial commission (denoted by $CD^{j}(s)$) to cover the initial commission in the first year. However, a ceding company will receive a proportion of the maximum cash advance (denoted by $LR^{j}(s)$) as a loan which depends on the proportion of business ceded ($0 \le \alpha_{RP} \le 1$). In this investigation, we assume that a proportion ceded does not exceed 50% ($0 \le \alpha_{RP} \le 0.5$).

In addition to the risk premium $RQ_{x}^{j}(t,s)$, the re-insurer also receives α_{RP} of the annual office premium in each projection year t (denoted by $\alpha_{RP} \times OP_{m}^{j}(t)$) from the ceding company to offset the deficit account balance (denoted by $DA^{j}(t)$) on the loan $LR^{j}(s)$ made in the year of policy issue s, where $s \le t$. The reinsurance is recaptured by the ceding company once balance of the deficit account ($DA^{j}(t)$) is zero. The loan will be repaid before the tranche of policies expires (using the figures chosen) as the loan amount is small.

Let $DA^{j}(t)$ = Deficit account balance at the start of year t for simulation j.

i_{DA} = interest rate on loan (assumed to be fixed) over projection period (e.g. 35%).

$C_{x}(u)$ = lapse rate (assumed to be fixed) for a life aged x at entry with duration u

$P_{x+u}^{c}(s,t)$ = Probability that an insurance policy issued in year s to a life aged x at entry with duration u will remain in the ceding company's portfolio in year t.

$OP_m^j(t)$ = Ceding company's annual office premium for a policy issued in year t. (e.g. £810.38 in year 1).

$ANM^j(s)$ = Number of nee policies issued in year s (e.g. 4226 policies in year 1)

$\eta_x^j(s,t)$ = Number of policies issued to lives aged x at the start of year s and still in-force at the end of year t for simulation j.

$IC_m^j(s)$ = Ceding company's initial commission in year s

$$P_{x+u}^c(s,t) = \left(1 - q_{x+u}^c\right) \times \left(1 - C_x(u)\right) \tag{4B.7}$$

$$\eta_x^j(s,t) = \begin{cases} ANM^j(s) \times P_{x+u}^c & \text{if } s = t \\ \eta_x^j(s,t-1) \times P_{x+u}^c & \text{if } t > s \end{cases} \tag{4B.8}$$

$$IC_m^j(s) = 1.25 \times 0.35 \times OP_m^j(s), \quad CD^j(s) = 1.5 \times IC_m^j(s) \tag{4B.9}$$

$$LR^j(s) = \alpha_{RP} \times CD^j(s) \tag{4B.10}$$

$$DA^j(t) = \begin{cases} ANM^j(s) \times \left[\quad LR^j(s) \quad \left(\quad {}_{RP} \times OP_m^j(s) \right.\right. & \text{if } s = t \\ DA^j(t \quad 1) \times \left(1 \quad i_{DA} \quad \left(\quad {}_{RP} \times OP_m^j(s) \times \eta_x^j(s,t \quad 1) \right.\right. & \text{if } t \quad s \end{cases} \tag{4B.11}$$

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