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# Build more and regret less: Oversizing H<sub>2</sub> and CCS pipeline systems under uncertainty

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## *Abstract*

*The large-scale deployments of Carbon Capture and Storage (CCS) and hydrogen (H<sub>2</sub>) require the installation of costly carbon dioxide (CO<sub>2</sub>) and H<sub>2</sub> pipeline infrastructures. As the future demands for CO<sub>2</sub> and H<sub>2</sub> pipeline transportation are expected to increase substantially, this paper examines the economics of installing adequately oversized infrastructures. We analytically show that the underlying engineering equations de facto define Cobb-Douglas production functions for both fluids. We then use these production functions to determine the cost-minimizing decisions of a pipeline operator and infer the optimal ratio of oversizing. From a policymaking perspective, this ratio indicates whether the level of oversizing envisioned by the operator is consistent with its expectations of future demand (and thus, by contrast, whether it is attempting to overcapitalize and exploit a regulatory flaw). As the anticipations of the operator may fail to materialize, we opt for a minimax regret perspective and compare the performances of the operator's investment decisions under alternative demand trajectories. Our study shows that the conservative recommendation to build for the proven demand only is systematically regret-maximizing. Consequently, an infrastructure push would likely contribute to overcoming the chicken-and-egg problem and thus support the large-scale deployment of these emerging technologies.*

**Keywords:** *Oversizing, Minimax Regret, Carbon Capture and Storage (CCS), Hydrogen, Pipeline Operator*

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# 1. Introduction

Low-emitting energy technologies such as hydrogen (H<sub>2</sub>) and Carbon Capture and Storage (CCS) are currently experiencing unprecedented momentum in the United States (US) and the European Union (EU). On both sides of the Atlantic, dedicated supporting policies – namely the Inflation Reduction Act (117th Congress 2022) and the Net Zero Industry Act (European Commission 2023) – provide generous funding and accelerated permitting procedures to favor the rise of a creditworthy demand for these new technologies and achieve their widespread diffusion. By nature, the large-scale deployment of H<sub>2</sub> and CCS requires the installation of dedicated pipeline infrastructures connecting production sites to respectively storage sites or end-users. The magnitude of that infrastructure need is substantial. For instance, the European Hydrogen Backbone (EHB) estimates that the European hydrogen pipeline network should cover 28,000 km by 2030 (van Rossum *et al.* 2022), and CCS studies project up to 20,400 km long pipeline network transporting carbon dioxide (CO<sub>2</sub>) in Europe by 2050 (Oei *et al.* 2014).

The decision to invest in a pipeline system is akin to the “chicken and egg” conundrum (Herzog 2011). Convincing potential users to adopt the technology (either carbon capture or H<sub>2</sub> supply or use) can be challenging without a pre-existing transport infrastructure. Conversely, only a critical mass of users generating foreseeable revenues can prompt investment into a capital-intensive pipeline infrastructure. An infrastructure push could contribute to overcoming the chicken-and-egg problem (Brozynski and Leibowicz 2022) and building an adequately oversized infrastructure by anticipating future demand appears as a viable option. Indeed, oversizing aligns with decarbonization objectives, as they require increasing transportation of CO<sub>2</sub> and H<sub>2</sub> volumes. Moreover, given the significant economies of scale that characterize pipeline transportation (Johnson and Ogden 2012, Holz *et al.* 2021), adequate oversizing lowers the cost of fluid transportation over the system’s lifetime (Chenery 1952).

However, the conditions for constructing these sizeable infrastructures ahead of demand remain to be fulfilled. Three objections are recurrently mentioned. First, these infrastructure projects are subject to major sources of uncertainty. Depending on the mitigation scenarios considered, there are considerable variations in the magnitudes and the timings of future CO<sub>2</sub> and H<sub>2</sub> requirements. Regarding H<sub>2</sub>, projected consumption figures vary considerably depending on whether that fuel becomes adopted against alternative technology options in the industry, transport, and heating sectors (Welder *et al.* 2018). Another source of uncertainty regarding the sizing of new H<sub>2</sub> pipelines is the possibility of repurposing natural gas pipelines (Cerniauskas *et al.* 2020). Regarding CCS, the variations in that technology's role in mitigation scenarios do not provide a clear picture of the pipeline infrastructure's requirement (IPCC 2005, 2022, IEA 2009). In this context of deep uncertainty, investors have a low propensity to build infrastructure ahead of demand or even to build at all.

A second issue concerns the lack of experience in building these sizeable infrastructures. Indeed, despite numerous public funding programs in the last two decades, no CO<sub>2</sub> pipeline network has yet been built (Lupion and Herzog 2013, Åhman *et al.* 2018) except for projects connected to enhanced oil recovery operations. Moreover, a high failure rate has cursed previous CCS projects (Wang *et al.* 2021). Regarding H<sub>2</sub>, only a handful of small pipelines are currently in operation, and these infrastructures run relatively short distances to connect producers and consumers in neighboring industrial clusters (IEA 2021).

Third, the profitability of infrastructure projects critically depends on the credibility of current supporting policies since these projects are irreversible and have long lifespans. Such policies must be immune to electoral vicissitudes to prevent future policymakers from reneging on the current commitment to supporting these emerging technologies (i.e., these policies must be time consistent in the sense of (Kydlund and Prescott 1977) such that the optimal policy in subsequent periods is the continuation of the initial optimal plan). Nevertheless, assessing ex-ante whether supporting policies are dynamically consistent can be challenging.

Against this background doomed by radical uncertainty, there is a need for a competent, independent regulator to provide a clear and favorable framework for deploying these infrastructures. However, in contrast with these regulatory needs, the institutional framework governing the provision of CCS and H<sub>2</sub> pipeline infrastructures is fuzzy or inexistent. Regarding H<sub>2</sub>, there is significant uncertainty about the authority in charge of regulating it in the US; and the EU is still in the early stages of establishing a new legislative framework for hydrogen networks.<sup>1</sup> Concerning CCS, fuzziness prevails in both the EU and the US.<sup>2</sup> As an exception, the UK intends to appoint Ofgem as the regulatory entity of its CCS pipeline infrastructures and has exposed its regulatory framework (BEIS 2022). While existing sectoral regulators like Ofgem have accumulated considerable experience in the regulation of gas and electricity networks, providing these agencies with an extended competence on CCS and possibly H<sub>2</sub> raises two concerns. First, the mission assigned to these energy regulators has historically focused on the short-run performance of the regulated firm (e.g., see Newbery (2006) on incentive regulation in the UK). While that goal was suited to the case of former nationalized industries with existing infrastructures, one can wonder whether the prevalent regulatory mindset can be adjusted to handle the case of infrastructures that do not yet exist (ACER and CEER 2021). A second concern is the overall perception of infrastructure oversizing in regulatory debates. The recommendation to build ahead of demand in (Chenery 1952) requires some degree of oversizing, which the regulator can interpret as the operator's attempt to inflate the regulated asset base under rate-of-return regulation. Indeed, standard regulatory textbooks (e.g., (Viscusi *et al.* 2018)) point out the regulated firm's perverse incentive to overcapitalize

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<sup>1</sup> In the US, although a recent study advocates regulation by the Federal Energy Regulatory Commission (FERC) under the Interstate Commerce Act (Bolgiano 2022), a report by an energy law association confers jurisdiction of H<sub>2</sub> pipelines to the FERC under the Natural Gas Act (Diamond *et al.* 2022). In the EU, the European Commission has submitted a proposal to extend the scope of the existing Gas Directive to cover hydrogen networks (European Commission 2021). This is the first stage – out of nine – before adoption of the proposal.

<sup>2</sup> The EU's CCS Directive does not impose any specific form of regulation on its member countries, apart from vague non-discriminatory and third-party access aspects (European Parliament and Council of the European Union 2009). In the US, the regulation of CCS pipeline transport varies greatly from one state to another, with also unclarity at the federal level (Nordhaus and Pitlick 2009, Mack and Endemann 2010, Jacobs and Craig 2017, NETL and Great Plains Institute 2017).

under such regulation, an effect first highlighted by Averch and Johnson (1962). Therefore, the provision of either H<sub>2</sub> or CO<sub>2</sub> pipelines requires a regulator that can distinguish whether oversizing is motivated by Chenery's (1952) virtuous ambition to lower costs or by the latter adverse one. Without clear guidelines, one might suspect the regulator to err on the side of restricting oversizing.

The purpose of this paper is thus to propose an economic model describing the operator's investment dilemma to assist policymakers and regulators about the impacts of infrastructure sizing decisions. The fundamental question addressed here is whether – and if yes, to which extent – a pipeline infrastructure should be oversized in expectation of future demand. First, we clarify the investment decisions of a risk-averse pipeline operator under uncertainty. That analysis aims at determining the amount of capital needed by that operator. The relations between, on the one hand, the regulated firm and, on the other hand, both the regulator in charge of overseeing the sector and the policymakers adopting climate targets are cursed by information asymmetry. Our findings help to handle that asymmetry as our simple analytical results allow regulators and policymakers to identify whether the level of oversizing proposed by a firm is appropriate given its expectations of future demand levels. This analysis also provides useful guidance to policymakers, regulators, and scholars wishing to understand whether the proposed infrastructure developments are aligned with climate targets.

From a methodological perspective, analyzing investment decisions under uncertainty is a recurrent topic in the energy sector (e.g., del Granado et al. 2019). Studies dealing with uncertainty generally favor two approaches: stochastic programming and robust optimization (Chen *et al.* 2014). In the former, uncertainty is resolved by associating a probability distribution to uncertain parameters, thus generating a spectrum of scenarios. The resulting unique solution is optimal for the “expected” outcome and would be suited for most scenarios. However, at the planning stage of infrastructure investments, it can be difficult to assign probabilities to outcomes whose causes are not fully understood (Chen *et al.* 2014, Anderson and Zachary 2022), mainly due to their deep uncertainty (Marchau *et al.* 2019). The robust optimization approach circumvents that issue by dealing with uncertainty through parametric sets (Chen

*et al.* 2014). In this paper, we adopt the minimax regret criterion, a robust optimization technique that determines the best decision in the worst-case scenario (Kouvelis and Yu 1997, Aissi *et al.* 2009, Chen *et al.* 2014, Anderson and Zachary 2022). To the best of our knowledge, the minimax regret criterion has never been applied to CO<sub>2</sub> or H<sub>2</sub> pipeline systems' planning.

Our analysis conveys a series of new findings. First, we analytically prove that Cobb-Douglas production functions can model the technology of H<sub>2</sub> and CO<sub>2</sub> pipeline systems with two inputs – capital and energy – and that both systems exhibit economies of scale. We show that the anticipation by the pipeline operator of a higher load in the future pushes the pipeline operator to oversize. We then introduce the possibility that its anticipations are not fulfilled and adopt a minimax regret perspective. To this end, we start with the simplest case by considering two scenarios and two decisions. We compute the capital investments under the two scenarios as a what-if analysis and use these to compute the minimax regret decision. Under this setting, building ahead of demand is always a regret-minimizing strategy. By extending this framework to a three-case scenario, we find that the conservative recommendation to build only for proven demand is regret-maximizing and thus never advantageous. Lastly, we provide investors and the regulator with a thumb rule to analytically evaluate the level of oversizing that minimizes total infrastructure costs based on the pipeline operator's expectations of future demand.

This paper contributes to the growing literature on CO<sub>2</sub> and H<sub>2</sub> infrastructures. Surprisingly, the uncertainty associated with the magnitude of future demand levels for CO<sub>2</sub> or H<sub>2</sub> transportation and its impacts on infrastructure planning has so far little been examined. Regarding H<sub>2</sub>, the literature on network deployment either assumes exogenously decided demand paths (André *et al.* 2013, 2014) or explicitly overlooks oversizing aspects for computational purposes (Johnson and Ogden 2012). Regarding CO<sub>2</sub>, the static models in the CCS literature (Middleton and Bielicki 2009, Kuby *et al.* 2011, Massol *et al.* 2015) assume a steady quantity of CO<sub>2</sub> to be transported that does not change over time and thus ignore the issue of oversizing. Dynamic models, which assume an increase in transportation



demand due to decarbonization scenarios, assume that the pipeline operator minimizes the total cost of the pipeline infrastructure over a multi-year horizon. Through perfect anticipation of the future demand for transportation, the pipeline operator minimizes the costs of its network by oversizing some pipelines, thus exploiting the economies of scale (Mendelevitch *et al.* 2010, Middleton *et al.* 2012, Morbee *et al.* 2012, Oei *et al.* 2014). As an exception, Wang *et al.* (2014) compare oversizing versus duplication from an engineering point of view, but they also suppose perfect foresight for the pipeline operator concerning future demand. Hence, whatever the fluid under scrutiny ( $H_2$  or  $CO_2$ ), assuming perfect foresight is not credible: There is a need to examine the decisions of a pipeline investor that is not omniscient.

The paper is organized as follows: Section 2 establishes the Cobb-Douglas production function of  $CO_2$  and  $H_2$  pipeline systems' technology based on engineering equations. Section 3 presents the pipeline operator's cost-minimizing optimization problem, establishes the overcapitalization ratio for an anticipating pipeline operator, and presents the minimax regret approach. We present numerical case studies for  $CO_2$  and  $H_2$  in sections 4 and 5. Section 6 discusses the results, and section 7 concludes and provides policy recommendations.

## **2. Background: the technology of $CO_2$ and $H_2$ pipeline infrastructures**

Our approach is stemmed from the engineering-based literature analyzing the microeconomics of a simple point-to-point natural gas pipeline (Chenery 1949, 1952, Yépez 2008, Massol 2011, Perrotton and Massol 2018). The technology of a natural gas system is characterized by two engineering variables: the size of the compressor equipment and the diameter of the pipeline. While the former is directly proportional to the amount of energy the infrastructure will consume, the latter is directly proportional to the square root of the immobilized capital (Perrotton and Massol 2018). In the case of a new pipeline project (i.e., in the long run), there are smooth substitutions between these two variables as they can be combined in various proportions to transport a given flow of natural gas. The engineering relations at hand are such that, in the long run, the ratio of average cost to marginal cost is constant and indicates

the presence of pronounced increasing returns to scale. In the case of an existing pipeline system, the diameter is fixed, and any adjustment in the pipeline output is accommodated by varying the compressor horsepower. The corresponding compressor cost function is monotonically increasing and convex, which can yield decreasing returns to scale in the short run (Perrotton and Massol 2018). Furthermore, this engineering literature has proved the existence of a one-to-one mapping between that technology and a production function of the Cobb-Douglas type that has two inputs: the capital immobilized in the pipeline and the amount of energy needed to power the compressor (Perrotton and Massol 2018).

That said, the engineering literature on H<sub>2</sub> and CO<sub>2</sub> pipelines emphasize a series of marked differences with the case of natural gas, and one can wonder whether these differences affect the validity of the insights gained from the case of natural gas. That observation prompted us to conduct an engineering-based characterization of the technologies governing CO<sub>2</sub> and H<sub>2</sub> pipeline systems. This section presents the outcomes of these analyses that are further detailed in the corresponding appendices.

### *2.1 CO<sub>2</sub> pipeline system*

Unlike natural gas that is transported as a compressed gas, the pipeline transportation of CO<sub>2</sub> requires piping it in a dense-phase state (Wang *et al.* 2016, Bui *et al.* 2018). As a result, CO<sub>2</sub> pipeline systems use pumps, not compressors, to move the fluid (Knoope *et al.* 2013, Wang *et al.* 2014).

Following the logic of Chenery (1949) and Yépez (2008), we consider the simplest CO<sub>2</sub> pipeline system that consists of a point-to-point pipeline and a pumping station. The infrastructure at hand is governed by three engineering parameters: the pumping power, the mass flow or output, and the inside diameter of the pipe. These engineering parameters verify three engineering equations – a pumping equation that gives the amount of pumping power needed to move the fluid, a flow equation describing the friction losses, and a mechanical stability equation. These equations are further described in Appendix A and can be combined to obtain a single production function describing the technology of a CO<sub>2</sub> pipeline system. That technology has two inputs, namely the capital  $K$  immobilized in the pipeline

and the energy needed for powering the pumping equipment  $E$ . That technology is of the Cobb-Douglas type, and the output  $Q$  is:

$$Q^\beta = K^\alpha E^{1-\alpha} \quad (1)$$

where the coefficients  $\beta = \frac{9}{11}$ , and  $\alpha = \frac{8}{11}$  are directly obtained from the engineering analysis presented in Appendix A.

## 2.2 $H_2$ pipeline system

Hydrogen is transported as a compressed gas (Witkowski et al. 2017). The movement of a compressed gas in a pipeline is governed by a flow equation, which relates the pressure drop caused by friction losses to the output, and the geometry of the pipeline (diameter and length of the pipeline).<sup>3</sup> In the case of natural gas (Chenery 1949, Yépez 2008, Massol 2011), the Weymouth flow equation is widely adopted. However, according to Klatzer et al. (2022), that equation may underestimate the flow by a factor close to 21% in the case of hydrogen. Therefore, we implement the Panhandle B equation in this study, which is commonly used in large-diameter, high-pressure transmission lines (Shashi Menon 2005).

We consider a simple  $H_2$  pipeline system consisting of a point-to-point pipeline and a compressor station. From the engineering equations governing that system, we show in Appendix B that this infrastructure can be modeled using the following Cobb-Douglas production function:

$$Q^\beta = K^\alpha E^{1-\alpha} \quad (2)$$

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<sup>3</sup> The flow equation is obtained by combining two elements: (i) a general flow equation derived from thermodynamic reasoning, and (ii) an empirically determined formula yielding the so-called friction coefficient (Mohitpour *et al.* 2003). Several formulas have been proposed and the selection of an appropriate one depends on the chemical composition of the gas, the geometry and fluid mechanics considerations (e.g., the Reynolds number, gas flow regime).

where the values  $\beta = 0.85$  and  $\alpha = 0.71$  are directly obtained from the engineering analysis presented in Appendix B.

### *2.3 Economic implications*

The calibrated production functions above have important implications for planning H<sub>2</sub> and CO<sub>2</sub> infrastructures. In both cases, we observe that the coefficient  $\beta$  is positive and less than one, which confirms that both pipeline systems exhibit economies of scale in the long run.

Furthermore, installing a pipeline in both cases is irreversible as the chosen diameter cannot be modified subsequently. Because of that irreversibility and the presence of economies of scale in the long run, an investor seeking to minimize the present value of the infrastructure's total cost can rationally decide the installation of an oversized pipeline diameter whenever it expects a future expansion of the infrastructure output (Chenery 1952, Manne 1961, Massol 2011, Perrotton and Massol 2019). That oversizing suggests that it can be rational to “build the infrastructure ahead of demand”. The H<sub>2</sub> and CO<sub>2</sub> pipeline technologies are thus compatible with the building ahead of demand behavior evoked in the introduction.

## **3. Model**

### *3.1 Assumptions and notations*

We consider a discrete time representation based on an annual resolution and let  $T$  denote the lifetime of the pipeline infrastructure. For simplicity, the infrastructure construction occurs in year 0, and the pipeline is operated for  $T$  years. The entire planning horizon is thus  $\{0, 1, \dots, T\}$  and is further decomposed into two successive subperiods.

The first subperiod labeled  $A$  covers  $(T_A + 1)$  years including the construction of the infrastructure and the first  $T_A$  years of operations with  $0 < T_A \leq T$ . During that subperiod, the annual flow of fluid

transported by the infrastructure operator  $Q_t$  in year  $t$  is constant and posited equal to the positive level  $Q_A$ . That volume represents the “anchor load” from early infrastructure users and is assumed certain.<sup>4</sup>

The second subperiod labeled  $B$  covers the remaining years. During these years, the annual flow of fluid to be transported is uncertain. At the end of the year  $T_A$ , uncertainty is resolved, and different outcomes can materialize. For simplicity, we assume that  $Q_t$  the annual flow to be transported during that second subperiod remains steady once uncertainty is resolved. For  $t \in \{T_A + 1, \dots, T_A\}$ ,  $Q_t$  is equal to  $Q_B = (1 + \delta)Q_A$  where  $\delta \geq 0$  is the expansion coefficient.

We then consider two polar scenarios: While the demand during the first subperiod  $Q_A$  is certain and invariant in both scenarios, the demand during the second subperiod  $Q_B$  is uncertain and depends on the value of the expansion coefficient  $\delta$ . In the “low demand” scenario there is no expansion of demand (i.e.,  $\delta = 0$ ), which corresponds to an unchanged level of demand during the second subperiod (i.e.,  $Q_B = Q_A$ ). In the “high demand” scenario, we consider a positive coefficient  $\delta > 0$  and the associated inflated demand  $Q_B = (1 + \delta)Q_A$  during the second subperiod. In the sequel, we solve the cost minimization problem twice to determine the least-cost infrastructure design obtained under each scenario.

### *3.2 Least-cost infrastructure planning*

Once immobilized, the capital stock  $K$  is fixed and can no longer be modified. Any subsequent change in output level thus requires adjusting the only variable input: the energy powering the compressing or pumping equipment. Following the discussion in Section 2, we assume a Cobb-Douglas

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<sup>4</sup> Investors usually consider the “anchor load” – i.e., the proven demand emanating from creditworthy early infrastructure adopters (see World Bank 2007, Gerner 2010, ICF International 2012, Perrotton and Massol 2019) – and then forge some expectations regarding the future evolution of that load.

production function of the form  $K^\alpha E^{1-\alpha} = Q^\beta$ , where the technical coefficients  $\alpha$  and  $\beta$  are both positive, less than one, and such that  $\beta > 1 - \alpha$ .

The pipeline operator's problem is determining the capital stock and the path of future energy consumption that minimize the present value of the total cost to build and operate the infrastructure under the posited demand scenario. We thus model the operator's decisions as a deterministic two-subperiod nonlinear programming problem. The first-subperiod decision variables are the capital stock  $K$  and the annual energy use during the first subperiod  $E_A$ . The annual energy use during the second subperiod  $E_B$  is the unique second-subperiod variable. The operator's cost-minimizing problem is:

$$\begin{aligned} \min_{K, E_A, E_B} C &= r K + s a_A E_A + s a_B E_B & (3) \\ \text{s. t. } K^\alpha E_i^{1-\alpha} &= Q_i^\beta, \quad \forall i \in \{A; B\} \\ K \geq 0, E_i &\geq 0, \quad \forall i \in \{A; B\} \end{aligned}$$

where the objective function to be minimized  $C$  is the total costs of the infrastructure,  $r$  is the market price of capital,  $s$  is the market price of energy,  $a_A$  and  $a_B$  are compact notations for the cumulative discounting factors corresponding to subperiods  $A$  and  $B$  (i.e.,  $a_A = \sum_{t=0}^{T_A} (1+d)^{-t}$  and  $a_B = \sum_{t=T_A+1}^T (1+d)^{-t}$ , where  $d$  is the discount rate).

Using the production function, one can define the variable input requirement function  $e_{Q_t}(K)$  that gives the amount of energy needed to transport the given flow  $Q_t$  on a pipeline infrastructure as a function of  $K$  the amount of capital immobilized. That function is smooth, monotonically decreasing, and convex. Using the input requirement function, the operator's problem becomes a convex, single-variable minimization problem:

$$\begin{aligned} \min_K C &= r K + s a_A e_{Q_A}(K) + s a_B e_{Q_B}(K) & (4) \\ \text{s. t. } K &\geq 0 \end{aligned}$$

where  $e_{Q_i}(K) = K^{\frac{-\alpha}{1-\alpha}} \cdot Q_i^{\frac{\beta}{1-\alpha}}$  for  $i \in \{A; B\}$ .

The first-order condition for optimality is:

$$r = -s a_A \frac{de_{Q_A}}{dK}(K) - s a_B \frac{de_{Q_B}}{dK}(K) \quad (5)$$

That condition indicates that, at the margin, the optimal level of capital stock must be such that its marginal cost  $r$  equals the present value of future marginal energy savings.

We now assume that the pipeline operator considers one of the two preceding scenarios and presumes either the low-demand scenario  $\delta = 0$ , or the high-demand scenario  $\delta > 0$ . Solving (5) under each demand scenario and letting  $K^*$  (respectively  $K^{**}$ ) denote the solution obtained in case of an unchanged (respectively inflated) future output level, one obtains:

$$K^* := \left[ \frac{s}{r} \cdot g_0 \cdot \frac{\alpha}{1-\alpha} \right]^{1-\alpha} Q_A^\beta \quad (6)$$

$$K^{**} := \left[ \frac{s}{r} \cdot g_1 \cdot \frac{\alpha}{1-\alpha} \right]^{1-\alpha} Q_A^\beta \quad (7)$$

where  $g_0 = (a_A + a_B)$  and  $g_1 = \left( a_A + a_B (1 + \delta)^{\frac{\beta}{1-\alpha}} \right)$ .

These results provide valuable insights into the magnitude of the oversizing decided by an optimistic operator that anticipates a future demand expansion.

**Proposition 1:** *The oversizing ratio is:*

$$\frac{K^{**}}{K^*} = \left( \frac{g_1}{g_0} \right)^{1-\alpha} = \left( 1 + \frac{a_B}{a_A + a_B} \left[ (1 + \delta)^{\frac{\beta}{1-\alpha}} - 1 \right] \right)^{1-\alpha} \quad (8)$$

and its value is greater than one whenever  $\delta > 0$ . Furthermore, this ratio increases with the demand expansion  $\delta$  and decreases with the discount rate  $d$ . (See Proof in Appendix C).

Consistent with Chenery (1952), that ratio is greater than one when a future demand expansion is expected, indicating that building ahead of proven demand translates into some degree of oversizing. Remark that this oversizing ratio is entirely determined by the technology parameters (i.e.,  $\alpha$  and  $\beta$ ), the value of the discount rate  $d$ , and the magnitude of the future increase in output  $\delta$ . This ratio is thus invariant with the cost of capital  $r$  and that of energy  $s$ . Furthermore, the underlying microeconomics is consistent with intuition. In case of sizeable future demand expansion, so must the amount of capital immobilized. Similarly, a *ceteris paribus* larger discount rate  $d$  lowers the infrastructure's total energy cost in present value terms, negatively affecting the operator's inclination to oversize. For brevity, the proofs are presented in Appendix C.<sup>5</sup>

The discussions in Appendices A and B highlight that the amount of capital stock immobilized is directly proportional to the squared value of the pipeline diameter  $D$ . As the latter is a readily observable element that is regularly used as an index of the scale of pipeline infrastructure in industry talks, one can convert that oversizing ratio in terms of diameter ratio:

$$\frac{D^{**}}{D^*} = \sqrt{\frac{K^{**}}{K^*}} = \left(1 + \frac{a_B}{a_A + a_B} \left[ (1 + \delta)^{\frac{\beta}{1-\alpha}} - 1 \right]\right)^{\frac{1-\alpha}{2}} \quad (9)$$

where  $D^*$  (respectively  $D^{**}$ ) denotes the diameter chosen by an operator expecting an unchanged future output (respectively, a future output expansion).

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<sup>5</sup> From a policymaking perspective, the overcapitalization ratio at hand may look cumbersome. So, we detail in Appendix C approximate rules of thumb that should provide useful guidance to policy makers.



### 3.3 Long- and short-run cost functions

#### 3.3.1 Long-run cost function

Having determined  $K^*$  and  $K^{**}$ , one can readily evaluate the minimum cost incurred under each scenario. Suppose the operator expects no future demand expansion (i.e.,  $\delta = 0$ ) and immobilizes the capital  $K^*$ . If that demand expectation materializes, its total infrastructure cost is the solution of (4) with  $\delta = 0$ , that is:

$$C^* := \frac{s^{1-\alpha} r^\alpha g_0^{1-\alpha}}{(1-\alpha)^{1-\alpha} \alpha^\alpha} Q_A^\beta \quad (10)$$

In the sequel, we leverage that result to define  $LRTC$ , the long-run total cost function that gives the total cost to build and operate the least-cost pipeline designed to transport the steady output  $Q$ . That is:

$$LRTC(Q) := \frac{s^{1-\alpha} r^\alpha g_0^{1-\alpha}}{(1-\alpha)^{1-\alpha} \alpha^\alpha} Q^\beta \quad (11)$$

As  $1 > \beta > 0$ , that long-run cost function is smooth, monotonically increasing and concave.

Similarly, we now examine the case of a pipeline operator expecting a future demand expansion  $\delta > 0$ . That operator immobilizes  $K^{**}$  and, should that expanded demand materializes, its total infrastructure cost will be:

$$C^{**} := \frac{s^{1-\alpha} r^\alpha g_1^{1-\alpha}}{(1-\alpha)^{1-\alpha} \alpha^\alpha} Q_A^\beta \quad (12)$$

To simplify the analysis, we now introduce  $Q_C$  the equivalent steady output level. That output is the constant output over the entire planning horizon such that  $LRTC(Q_C)$  the long-run total cost to serve that output equals  $C^{**}$ . Hence,  $Q_C$  solves  $LRTC(Q_C) = C^{**}$  that is:

$$Q_C := \left(\frac{g_1}{g_0}\right)^{\frac{1-\alpha}{\beta}} Q_A \quad (13)$$

If  $\delta > 0$ , we have  $g_1 > g_0 > 0$ : the flow  $Q_C$  is greater than the anchor load level  $Q_A$ .

The steady output  $Q_C$  is equivalent from a cost perspective to the complex demand schedule that consists of the succession of the “anchor load”  $Q_A$  during the first subperiod and the inflated flow  $Q_B = (1 + \delta)Q_A$  with  $\delta > 0$  during the second one.

As a side remark, note that the amount of capital stock required to serve  $Q_C$  obtained when solving the corresponding instance of problem (4) equals  $K^{**}$ .

### 3.3.2 Short-run cost function

In the short run, the amount of capital stock immobilized is fixed. Hence, we now define the short-run total cost function  $SRTC_K$  that gives the total cost to transport the steady output level  $Q$  on an infrastructure for which the capital stock  $K$  has been immobilized. That function is:

$$SRTC_K(Q) := rK + s(a_A + a_B)e_K(Q) \quad (14)$$

where  $e_K(Q) = K^{-\frac{\alpha}{1-\alpha}} \cdot Q^{\frac{\beta}{1-\alpha}}$  is the variable input requirements function that gives the amount of energy needed to transport the steady flow  $Q$  when the capital is fixed and equal to  $K$ .<sup>6</sup>

As  $\beta > 1 - \alpha$ , the short-run total cost function is monotonically increasing and convex.

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<sup>6</sup> The definition of  $e_K(Q)$  is similar to  $e_{Q_i}(K)$ , as both express the energy input based on the Cobb-Douglas production function. We highlight in the definition of  $e_K(Q)$  that the only variable is the output  $Q$ , while the capital  $K$  is fixed, and vice-versa for  $e_{Q_i}(K)$ .

By construction, the short-run cost function equals the long-run one for the output level such that the immobilized capital stock solves (4) and thus,  $SRTC_{K^*}(Q_A) = LRTC(Q_A) = C^*$  and  $SRTC_{K^{**}}(Q_C) = LRTC(Q_C) = C^{**}$ .

### 3.4 A minimax regret perspective

So far, we have examined how the pipeline operator determines the amount of capital stock to be immobilized, given its anticipations about future demand. We now explore the consequences of these investment decisions by considering the demand scenarios that can materialize. Following Anderson and Zachary (2022), we adopt the minimax regret criterion to describe the uncertainty related to the demand scenarios and the conservative decisions of the pipeline investor.<sup>7</sup>

Similar to Chen et al. (2014), we define for a given decision  $d \in D$ , with  $D$  the set of feasible decisions and a scenario  $s \in U$ , with  $U$  the feasible set of uncertain scenarios, its regret  $R(d, s)$  as the deviation from the decision that minimizes the costs for this scenario that is:  $R(d, s) = C_s(d) - \min_{z \in D} C_s(z)$  with  $C_s(d)$  the cost associated with decision  $d$  in scenario  $s$ . Hence, the regret measures the extra cost incurred when opting for that decision despite the best one for that demand scenario. We define the minimax regret criterion as:

$$\min_{d \in D} \max_{s \in U} (R(d, s))$$

For a given decision  $d$ , the “worst-case” scenario is thus defined as the scenario with the highest regret in the minimax regret criterion.

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<sup>7</sup> We exclude the minimax cost criterion of our study. Thus, the high demand scenario will lead to higher costs, regardless of the decision taken by the operator. As already pointed by Chen et al. (2014), it is more interesting from a decision-making perspective to compare costs relatively for each scenario, i.e., to calculate the regret than comparing costs in absolute terms.

In the case of our study, the decisions  $d$  are the alternatives for initial capital installation – oversizing or not. Thus, the set of feasible decisions  $D$  corresponds to  $\{K^*; K^{**}\}$ . The feasible set of uncertain scenarios  $U$  is composed of our two polar cases  $\{\delta = 0; \delta > 0\}$ . In the following, we will refer to the regret of a capital investment decision  $K$  under the expansion of demand scenario  $\delta$  as  $R(K; \delta)$ .

As a consequence, if the operator anticipates the low-demand scenario  $\delta = 0$  (respectively, the high-demand scenario  $\delta > 0$ ) and immobilizes the capital stock  $K^*$  (respectively  $K^{**}$ ), it will incur the total infrastructure cost  $C^*$  (respectively  $C^{**}$ ) if that demand scenario materializes. However, the operator can be mistaken in its expectations. If the alternative demand scenario materializes, the operator is compelled to serve the demand using a poorly adapted infrastructure and thus incur a total infrastructure cost determined by the short-run total cost function for the materialized demand schedule.

Table 1 presents the cost incurred in the four cases obtained by combining the operator’s decisions with the two demand scenarios that can materialize. The columns show the two building strategies that the pipeline operator can adopt: if it expects a future expansion in output, it installs the capital  $K^{**}$ , while if it opts for the pessimistic view that only considers a steady output equal to  $Q_A$ , it immobilizes  $K^*$ . The rows present the different demand scenarios that materialize.

**Table 1: The total infrastructure costs incurred in the four cases**

	The pipeline operator’s decision	
	Pessimistic pipeline operator $K^*$	Optimistic pipeline operator $K^{**}$
<b>Low demand scenario materializes</b>		
$\delta = 0$	$LRTC(Q_A) = C^*$	$SRTC_{K^{**}}(Q_A)$
<b>High demand scenario materializes</b>		
$\delta > 0$	$SRTC_{K^*}(Q_C)$	$LRTC(Q_C) = C^{**}$

We compute the regret associated with each decision under each demand scenario. We thus identify, for a given demand scenario, the best-case decision (i.e., installing  $K^*$  or installing  $K^{**}$ ), i.e., the one

yielding the lowest cost.<sup>8</sup> To simplify the analytical expressions, we normalize the obtained regrets by dividing each of them by  $LRTC(Q_A)$ , the long-run cost to serve  $Q_A$ . The normalized regrets are summarized in Table 2, to which we add the last row that presents the maximal regret for each strategy.

**Table 2: Normalized regrets**

	The pipeline operator's decision	
	$K^*$	$K^{**}$
<b>Low demand scenario materializes</b>		
$\delta = 0$	0	$\frac{SRTC_{K^{**}}(Q_A) - LRTC(Q_A)}{LRTC(Q_A)}$
<b>High demand scenario materializes</b>		
$\delta > 0$	$\frac{SRTC_{K^*}(Q_C) - LRTC(Q_C)}{LRTC(Q_A)}$	0
<b>Max normalized regret</b>	$\frac{SRTC_{K^*}(Q_C) - LRTC(Q_C)}{LRTC(Q_A)}$	$\frac{SRTC_{K^{**}}(Q_A) - LRTC(Q_A)}{LRTC(Q_A)}$

The Minimax Regret decision rule recommends implementing the decision that minimizes the worst-case regret. As all regrets are normalized with the same value, one has to compare the values of the Max normalized regret presented in Table 2 and pick the smallest one to obtain the minimax regret decision.

**Proposition 2:** *From a Minimax Regret perspective, “building ahead of demand” (oversizing) is always a better alternative than building for the proven demand only since:*

$$\frac{SRTC_{K^*}(Q_C) - LRTC(Q_C)}{LRTC(Q_A)} > \frac{SRTC_{K^{**}}(Q_A) - LRTC(Q_A)}{LRTC(Q_A)} \quad (15)$$

<sup>8</sup> Per definition of the short-run cost curves  $SRTC_{K^*}$  and  $SRTC_{K^{**}}$ , of the long-run cost curve  $C$ , and the fact that  $Q_C > Q_A$ , we obtain directly that  $SRTC_{K^*}(Q_C) > LRTC(Q_C)$  and that  $SRTC_{K^{**}}(Q_A) > LRTC(Q_A)$ .

for all value of increase in demand  $\delta > 0$ , discounting factor  $d > 0$ , and for technological parameters  $\beta < 1$  and  $1 - \alpha < \beta$ . The maximal regret is obtained in the myopic strategy, and one should adopt the optimistic strategy. The proof is in Appendix D.

### 3.5 Extension to a three-case scenario

The two scenarios analysis above indicates that the myopic strategy that consists of building for the proven demand only is dominated. As one can wonder whether that result is robust to the introduction of new scenarios, we now consider an enriched set of uncertain scenarios  $U = \{\delta_0; \delta_1; \delta_2\}$  with  $\delta_0 = 0$  and  $0 < \delta_1 < \delta_2$  and the associated set of investment decisions  $D = \{K^*; K^{**}; K^{***}\}$  corresponding to the cost-minimizing capital investment of each scenario. Similar to section 3.2, we introduce the least-cost capital required to serve the expanded load  $\delta_2$ , that is:  $K^{***} := \left[\frac{s}{r} \cdot g_2 \cdot \frac{\alpha}{1-\alpha}\right]^{1-\alpha} Q_A^\beta$  with  $g_2 = \left(a_A + a_B(1 + \delta_2)^{\frac{\beta}{1-\alpha}}\right)$  and the equivalent steady output level for the expansion coefficient  $\delta_2$ ,  $Q_D := \left(\frac{g_2}{g_0}\right)^{\frac{1-\alpha}{\beta}} Q_A$ . We obtain the following regret in Table 3:

**Table 3: Regret Table for a three-case scenario**

	The pipeline operator's decision		
	$K^*$	$K^{**}$	$K^{***}$
<b>Low demand</b>			
$\delta_0 = 0$	0	$\frac{SRTC_{K^{**}}(Q_A) - LRTC(Q_A)}{LRTC(Q_A)}$	$\frac{SRTC_{K^{***}}(Q_A) - LRTC(Q_A)}{LRTC(Q_A)}$
<b>Middle demand</b>			
$\delta_1$	$\frac{SRTC_{K^*}(Q_C) - LRTC(Q_C)}{LRTC(Q_A)}$	0	$\frac{SRTC_{K^{***}}(Q_C) - LRTC(Q_A)}{LRTC(Q_A)}$
<b>High demand</b>			
$\delta_2$	$\frac{SRTC_{K^*}(Q_D) - LRTC(Q_D)}{LRTC(Q_A)}$	$\frac{SRTC_{K^{**}}(Q_D) - LRTC(Q_D)}{LRTC(Q_A)}$	0
<b>Max Regret</b>	$R(K^*; \delta_2)$	$\max(R(K^*; \delta_0); R(K^*; \delta_2))$	$R(K^{***}; \delta_0)$

Appendix E provides an analytical formulation of Table 3 and the analytical proofs for all regret calculations.

In the case of three scenarios, we note that for the decision to install capital  $K^{**}$ , the maximum value of regret depends on the relative values chosen for the parameters. An intuitive analytical explanation of this result is given in Appendix E. Contrary to the case with only two scenarios, one decision does not minimize the maximum regret for any value of the parameters taken into account: depending on the relative values of  $\delta_0$ ,  $\delta_1$  and  $\delta_2$ , the decision  $K^{**}$  or  $K^{***}$  minimizes the maximum regret, while the decision to not oversize  $K^*$  consistently maximizes the maximum regret. In other words, a minimax regret perspective never supports the decision to install the smallest capital  $K^*$  and thus recommends building ahead of demand.

## 4. Application to CO<sub>2</sub> pipeline infrastructures

In this section, we apply the model of Section 3 to the East Coast Cluster (ECC) in the United Kingdom (UK) to quantify the oversizing that the cost-minimizing and anticipating CCS pipeline operator would develop. We then numerically apply the minimax regret approach to assess the cost difference between each strategy.

### 4.1 Case study: The East Coast Cluster in the UK

The potential for storage in the North Sea for the UK represents 230 Gt of CO<sub>2</sub>, nearly 20 times its current emissions (Karvounis and Blunt 2021). In particular, the ECC plans to exploit the Endurance aquifer in the first instance, which is estimated to have a potential of 450 Mt of CO<sub>2</sub> (BP 2022). The ECC is a collaboration between two carbon-emitting industrial sites, the Net Zero Teesside and the Zero Carbon Humber, and a transportation and storage alliance, the Northern Endurance Partnership. Net Zero Teesside and Zero Carbon Humber have a capture potential of 10 MtCO<sub>2</sub>/y and 17 MtCO<sub>2</sub>/y, representing 50% of the UK's industrial emissions (ECC 2022). This project is particularly relevant for

the government’s Department for Business, Energy & Industrial Strategy (BEIS) since it would allow it to reach its CCUS objectives: capturing 20-30 MtCO<sub>2</sub>/year by 2030 and developing a net zero industrial cluster by 2040 (BEIS 2021a). As such, BEIS has recognized the capture potential of this project by deeming, under its cluster sequencing process program, that the ECC is suitable for deployment as early as the mid-2020s (BEIS 2021b). It is one of the two projects retained and selected in Phase 1 of the program.

#### 4.2 Assumptions

According to these ambitions, we make the following assumptions in our study:

**Table 4: Numerical parameters for CO<sub>2</sub> application**

Parameter	Abbreviation	Value	Assumption
Start of project	$t_0$	2025	ECC is ready for deployment in the mid-2020s, according to BEIS
Quantity of CO <sub>2</sub> captured during the first period	$Q_A$	10 MtCO <sub>2</sub> /y	We suppose that ECC will capture half of the government’s ambitions by 2030
Date of increase in capture rate	$T_A$	15 years	ECC becomes the first net-zero industrial cluster in 2040
Quantity of CO <sub>2</sub> captured during the second period	$Q_B$	27 MtCO <sub>2</sub> /y	-
End of project	$T$	25 years	Lifetime of a CO <sub>2</sub> pipeline infrastructure: 25 years (Allinson <i>et al.</i> 2006, Essandoh-Yeddu and Gülen 2009, Stewart <i>et al.</i> 2014, Wang <i>et al.</i> 2014, Vivid Economics 2020)

Assuming constant monetary units to circumvent the effect of inflation, we choose  $d = 0.05$ , a market price of capital  $r = 2$ , and a market price of energy  $s = 1$ .



### 4.3 Results

Applying equation (8) with the numerical parameters of Table 4 and the technological parameters of a CO<sub>2</sub> pipeline, we obtain the following result:

$$\frac{K^{**}}{K^*} = 1.83$$

which represents an increase in diameter of 35%. We obtain the following regret Table 5 for this CO<sub>2</sub> case study:

**Table 5: Numerical application of the Regret Table for the CO<sub>2</sub> case study**

	<b>Myopic</b>	<b>Optimistic</b>
<b>Low scenario</b>	0	0.38
<b>High scenario</b>	1.38	0
<b>Max Regret</b>	1.38	0.38

The optimistic strategy's maximal regret is more than three times smaller than that of the myopic strategy.

## 5. Application to Hydrogen pipeline infrastructures

This section applies the model from Section 3 to the European Hydrogen Backbone (EHB) initiative. We quantify the level of oversizing of a pipeline operator that anticipates a future increase in demand and derive its regret table.

### 5.1 Case study: The European Hydrogen Backbone

The European Hydrogen Backbone (EHB) initiative is a group of thirty-one energy infrastructures which have proposed a plan for Europe's hydrogen pipeline infrastructure. In particular, the plan proposes a "Southwest corridor", exploiting the low-cost production of renewable hydrogen in Spain

and transporting it to France and Germany (van Rossum *et al.* 2022). Currently, two natural gas pipelines link France to Spain, and the initiative projects to repurpose them into hydrogen pipelines and build a third one that would connect Occitanie to Catalonia by 2030. In a second phase, by 2040, the project proposes a more extensive trunkline system that imports hydrogen from Morocco to Spain (through two pipelines near Tarifa and Almeria). This hydrogen would then transit through France before reaching Germany.

### 5.2 Assumptions

For our study, we retain the parameters in Table 6.

**Table 6: Numerical application for a hydrogen pipeline**

Parameter	Abbreviation	Value	Assumption
Start of project	$t_0$	2025	Own assumption
Quantity of hydrogen transported during the first period	$Q_A$	2 MtH <sub>2</sub> /y	Own assumption
Date of increase in hydrogen transportation	$T_A$	15 years	Own assumption: Expansion in 2040 (van Rossum <i>et al.</i> 2022)
Quantity of hydrogen transported during the second period	$Q_B$	6 MtH <sub>2</sub> /y	Own assumption: three times more hydrogen transportation due to imports from Morocco
End of project	$T$	50 years	Gillette and Kolpa (2008)

The numerical data of this H<sub>2</sub> case study relies mainly on our assumptions: this is a consequence of the lack of data on these projects, which are still in their early stages.<sup>9</sup> In the same way as for the CO<sub>2</sub> pipeline project, we assume a similar discounting rate  $d$ , cost of capital  $r$ , and cost of energy  $s$ .

<sup>9</sup> The assumption of 2 MtH<sub>2</sub>/y is related to the fact that the plan assumes five different corridors to meet the 10 MtH<sub>2</sub>/yr of the European Commission. We therefore assume that the Southwest Corridor accounts for one-fifth of these quantities, and that existing pipelines are negligible compared to the one being built. The assumption of a threefold increase in transportation demand is based solely on the fact that we assume that Morocco's imports are of the same order of magnitude as the

### 5.3 Results

Applying equation (8) with the numerical parameters of Table 6 and the technological parameters of an H<sub>2</sub> pipeline determined in Section 2, we obtain the following oversizing ratio:

$$\frac{K^{**}}{K^*} = 2.21$$

which represents an increase in diameter of 49%. This capital ratio indicates that a pipeline operator with expectations similar to those used here would need to more than double its level of capital compared to a situation where it would only build to meet proven demand. We calculate the regret table for our case study and present the results in the following Table 7.

**Table 7: Numerical application of the Regret Table for the H<sub>2</sub> case study**

	Myopic	Optimistic
<b>Low scenario</b>	0	0.61
<b>High scenario</b>	3.0	0
<b>Max Regret</b>	3.0	0.61

The optimistic strategy divides the maximum regret by more than three compared to a myopic strategy.

## 6. Discussion

### 6.1 Thumb rule of the oversizing ratio

The oversizing ratio calculated in section 3.2 is a major decision tool for investors (public or private) regarding the level of capital the pipeline operator needs to finance its infrastructure. From the regulator's

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transportation demand already present. Since there are two routes from Morocco to Spain, this amounts to a threefold increase in transport.

point of view, this oversizing ratio is a relevant piece of information to consider when choosing the economic regulation model to be implemented. Indeed, with this oversizing ratio, regulators now have a clear vision on how to evaluate oversizing, and eventually distinguish it from an overcapitalizing behavior, which could for instance occur in a cost-plus regulation. However, the oversizing ratio is difficult to assess, as it relies on many parameters; we thus provide some general rules to facilitate its understanding in this subsection. We simplify this ratio for two cases.

On the one hand, let us assume that  $\delta \ll 1$ . This represents the case of a pipeline operator which desires to oversize but believes that future demand will hardly be much greater than the anchor load. A first-order Taylor expansion of the oversizing ratio in equation (8) yields the following rule of thumb for  $\delta \ll 1$ :

$$\frac{K^{**}}{K^*} \sim 1 + \frac{a_B}{a_A + a_B} \cdot \beta \cdot \delta \quad (16)$$

When the pipeline operator anticipates a slight increase in demand, we retain that the cost-minimizing oversizing ratio increases linearly with the demand expansion  $\delta$ . For a policy maker, this provides a “thumb rule” that allows it to quickly estimate whether the pipeline operator's project is consistent with its expectation of future demand.

Conversely, we can assume that the operator anticipates a large increase in future demand (reaching a doubling or even a tripling of demand), and therefore wishes to oversize greatly. In this case, we provide an upper bound on the oversizing ratio, beyond which the oversizing is not justified. The oversizing ratios of a CO<sub>2</sub> and a H<sub>2</sub> pipeline are naturally bounded as  $\frac{a_A + a_B}{a_B} < 1$ ,  $(1 - \alpha) < \frac{1}{3}$  and  $\frac{\beta}{1 - \alpha} < 3$  in both cases:

$$\frac{K^{**}}{K^*} \leq [1 + (1 + \delta)^3]^{\frac{1}{3}} \quad (17)$$

These general rules allow public authorities and private investors to determine the capital required by oversizing quickly. For instance, this thumb rule could provide an easy way for a government to estimate the subsidies it should provide the pipeline operator with in order to meet its decarbonization targets. For regulators, this thumb rule is a simple tool to evaluate the impact on the economic efficiency of the oversizing behavior in specific cases. This simple calculation could lead it to favor one form of regulation over another.

### *6.2 Sunk costs*

We find that it is regret-maximizing to not oversize. Nevertheless, this does not eliminate the risk of a pipeline operator misjudging future demand, ending up with a pipeline larger than necessary: in this case, the oversizing turns into an undesired overcapitalization. The related additional costs are sunk and can be a main barrier to CCS adoption by pipeline project planners. Surmounting this barrier requires the intervention of a public authority that would agree to share the risks ex-ante and cover part of the potential sunk costs ex-post. That intervention can take several forms (e.g., providing guarantees, preferred funding, allowing contracts with take-or-pay provisions). Though our study does recommend a specific form, it provides a proxy for the magnitude of the sunk costs that arise from a wrong anticipation of future demand (see Table 2) and can thus help policymakers design the appropriate support mechanisms. For governments facing hard budget constraints, these considerations will usefully inform the cost-benefit analyses of the envisioned public interventions.

### *6.3 Overcoming the chicken-and-egg problem*

CCS and H<sub>2</sub> industries face a classical “chicken and egg” problem, which has already been emphasized in both literatures more than a decade ago (Melaina 2003, Herzog 2011). Policymakers attempt to overcome this market failure by subsidizing both parties (Brozynski and Leibowicz 2022).

However, in the case of H<sub>2</sub> and CCS industries, the recent EU and US Acts primarily focus on demand pull. Indeed, the Inflation Reduction Act intends to provide more tax credits to industrial sites that capture their CO<sub>2</sub> emissions (enhancement of the 45Q tax credit) or that produce clean hydrogen (117th Congress 2022). Likewise, the Net Zero Industry Act has set new storage objectives in the EU (of reaching an annual 50Mt injection capacity by 2030) and has established the European Hydrogen Bank to promote investment in low-carbon hydrogen production (European Commission 2023). Nevertheless, a study by Uden et al. (2022) recommends an infrastructure push to overcome the chicken-and-egg problem in the CCS industry – and its reverse-engineering methodology and results can be extended to the H<sub>2</sub> industry. By clarifying the investment dilemma facing pipeline project planners, the present analysis can usefully assist policymakers in designing such policies.

## **7. Conclusion and Policy Implications**

Massive H<sub>2</sub> and CCS pipeline infrastructures need to be deployed in the upcoming years to reach the ambitious decarbonization objectives set by policymakers. Economies of scale in CO<sub>2</sub> and H<sub>2</sub> pipelining suggest that pipeline project planners must oversize their pipelines now to absorb future demand for transportation. However, these infrastructures are capital-intensive and high-risk, and it is unlikely that the private sector will engage in these irreversible investment decisions absent a clear vision of current and future regulation. Indeed, investors need to be informed ex-ante on how sectoral regulation will perceive that oversizing and set its allowed revenues. This need contrasts with current regulatory frameworks governing the US and the EU's CO<sub>2</sub> and H<sub>2</sub> pipeline industries, which are fuzzy or non-existent. Furthermore, regulatory mandates are unclear. Following the British example, countries can be inclined to extend the competence of the existing energy regulator so that it also oversees these nascent infrastructures. However, despite the expertise of these authorities in improving the performance of existing infrastructures, they lack experience in incentivizing the emergence of new infrastructure industries. Compared with gas and electricity, there is a need to account for the low maturities of H<sub>2</sub> and CO<sub>2</sub> pipeline sectors. By shedding light on optimal infrastructure oversizing decisions under uncertainty,

this paper informs current H<sub>2</sub> and CO<sub>2</sub> pipelines' regulatory policies and practices that have so far been little studied.

To this end, we first show that the essential features of CO<sub>2</sub> and H<sub>2</sub> pipeline systems are well captured by an engineering-based Cobb-Douglas production function. For both H<sub>2</sub> and CO<sub>2</sub>, the resulting Cobb-Douglas production functions exhibit economies of scale, which confirms our initial assumption that building ahead of demand is cost-minimizing. We then determine the oversizing ratio the operator should undertake based on its expectations of future demand. While oversizing is the cost-minimizing decision when future demand is known with certainty, this is not necessarily the case under uncertainty: if demand does not increase as expected, part of the capacity can be unused, thus leading to substantial sunk costs. For this reason, we introduce uncertainty in the future demand for transportation. To our knowledge, no study has addressed this aspect in the literature on CCS and H<sub>2</sub>.

From a methodological perspective, we propose a minimax regret approach to determine the best investment decision, as pipeline project planners of both industries are most likely risk averse. The key finding of this paper is that deciding not to oversize (and thus building for the anchor load only), is the regret-maximizing decision, no matter the values of the expansion coefficient or the date of the increase in demand. As an application, we detail two numerical case studies that showcase this finding for contemporary H<sub>2</sub> and CO<sub>2</sub> pipeline projects and show that the regret triples when building for the anchor load only.

This key finding has important policy implications. First, to overcome the chicken and egg problem that affects the H<sub>2</sub> and CCS industries, our study indicates that an appropriate degree of oversizing is desirable. Many policy instruments exist to overcome the chicken-and-egg problem, but we note that policymakers have recently only adopted demand-pull incentives by financially supporting CO<sub>2</sub> capture and H<sub>2</sub> production. To complement this demand-pull method, policymakers could promote oversizing of the pipelines to the extent detailed in this study, and thus generate an infrastructure push. The second policy recommendation is to support oversizing by distinguishing an overcapitalizing behavior (i.e., a

pipeline operator seeking an informational rent) from a benevolent oversizing one (i.e., a pipeline operator that oversizes to minimize the total costs of the infrastructure). To this end, the oversizing ratio is a strong analytical tool for policymakers, as it indicates the level of oversizing that the cost-minimizing pipeline project planner should invest in. Because this oversizing ratio can be hard to compute, we relax it into a “thumb rule”, allowing the regulator to quickly estimate the validity of the pipeline project planner’s decisions. Lastly, our study calls for appropriate contract developments, due to the potential sunk costs that can arise. Indeed, the solution that minimizes regret consists in oversizing, and thus risking of having unused capacity. Negotiating a contract that deals with this issue could encourage the pipeline project planner to adopt the desirable level of investment.

At least four possible future research directions can be envisioned. The first research avenue concerns the access pricing system and the tariffication of the infrastructure in a dynamic framework. Indeed, the pipeline operator might charge excessive access rates during the first phases of operation to recover the costs of the initial oversizing of its infrastructure, disincentivizing industrial sites to invest in CO<sub>2</sub> capture or H<sub>2</sub> production. Some studies have already explored this issue in the case of CCS by adopting a cooperative game approach (Massol *et al.* 2015, 2018, Jagu Schippers and Massol 2020, Jagu Schippers *et al.* 2022). However, these earlier studies are based on a static framework, which does not capture the dynamics associated with initial oversizing. Moreover, further research could address the emerging market design issues associated with CCS and H<sub>2</sub> infrastructures. For example, congestion management is a recurrent topic in network infrastructures and market design debates (Singh *et al.* 1998, Pillay *et al.* 2015) but has not yet been addressed for CCS or H<sub>2</sub> pipeline infrastructures. Yet, congestion can occur if the pipeline is built with insufficient capacity and its management, and the treatment of the associated revenues need to be clarified. The second possible research avenue is designing contracts that deal with both risk management and pricing, bridging the gap between law and economics. The study by Banet and O’Brien (2021) could be a starting point, as it inspects how legislation and contracts could manage and mitigate risks in the CCS and H<sub>2</sub> value chains. It also discusses the role of public authorities. Complementing their study with an economic model could provide valuable insights for



regulation and risk-sharing in these pipeline industries. In this regard, combining their study with the model of Cai et al. (2014) could be relevant, as the latter focuses on pricing contracts between a CO<sub>2</sub> capture site and a transportation operator. A third promising avenue for future research concerns the credibility of the supporting mechanisms envisioned for these infrastructures. As pointed out in the introduction, the seminal contribution of (Kydlund and Prescott 1977) has shown that if public authorities (i.e., policymakers or regulators) cannot commit themselves credibly to a course, their decision may be futile and will not be able to provide the required infrastructure. Further research could thus explore whether some institutional arrangements or policies are capable to overcome this time consistency problem. Lastly, the fourth possible extension concerns the representation of decision-making under uncertainty. The multicriteria approach detailed in Trachanas et al. (2022) could represent an appealing point of departure for further studies in that direction.

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## Appendix A – Cobb Douglas production function of a CO<sub>2</sub> pipeline system

We consider a point-to-point pipeline system consisting of a trunkline of length  $L$ , of inside diameter  $D$ , without bends or height variation, and a pump of power  $W_p$ . This system transports a quantity of CO<sub>2</sub>  $Q$  as a dense-phase liquid (Wang *et al.* 2016). Similar to the case of natural gas developed by Chenery (1949) and Yépez (2008), we describe the CO<sub>2</sub> pipeline system through three engineering parameters: the pumping power  $W_p$ , the mass flow or output  $Q$  and the inside diameter of the pipe  $D$ . These engineering variables verify three equations presented in Table A.1. The pumping power equation gives the power required by the pumps to compensate for the pressure drop  $\Delta P$  along the pipeline. The flow equation links the pipeline diameter  $D$  to the pressure drop  $\Delta P$  and the output  $Q$  along the pipeline. Lastly, the mechanical stability equation links the diameter  $D$  to the thickness of the pipe  $\tau$ .

**Table A.1: Engineering equations for a CO<sub>2</sub> pipeline**

Engineering equations	Parameters	Source
<u>Pumping power</u>		
$W_p = \frac{Q \cdot \Delta P}{\rho \cdot \eta_p}$	$\eta_p$ : efficiency of the pump $\rho$ : density of CO <sub>2</sub>	Mohitpour, Golshan, and Murray (2003) Ikeh and Race J.M (2011)
<u>Flow equation</u>		
$D = \left( \frac{4^{\frac{10}{3}} n^2 Q^2 L \rho g}{\pi^2 \rho^2 \Delta P} \right)^{3/16}$	$g$ : gravity constant $n$ : Manning factor $\pi$ : the geometric constant	Vandeginste and Piessens (2008)
<u>Mechanical equation</u>		
$\tau = a \cdot D$	$\tau$ : thickness $a$ : constant	Ruan et al. (2009)

By isolating the pressure drop  $\Delta P$  in the pumping power equation, then injecting this into the flow equation, we can isolate the output  $Q$  and obtain the following engineering equation:



$$Q = A^{1/3} W_p^{1/3} D^{16/9} \quad (\text{A.1})$$

where  $A$  is a technical parameter, i.e.,  $A = \pi^2 \rho^2 \eta_p / (4^{10/3} g L n^2)$ .

Assuming that the capital of the pumping station is negligible compared to the capital of the pipeline, we obtain for an open diameter cylinder pipeline the capital stock equation:

$$K = p_s w_s L \pi D^2 (a + a^2) \quad (\text{A.2})$$

Where  $p_s$  denotes the unit price of steel,  $w_s$  denotes the weight of steel per unit of volume. We suppose that the power required by the pipeline  $E$  can be assimilated to the compression power  $W_p$  such that  $E = W_p$ . Combining the engineering equation (A.1) with the capital stock equation (A.2), and by normalizing the output, we find the following Cobb-Douglas production function for a CO<sub>2</sub> pipeline system:

$$Q^\beta = K^\alpha E^{1-\alpha}$$

with  $\alpha = \frac{8}{11}$  and  $\beta = \frac{9}{11}$ .

## **Appendix B – Cobb Douglas production function of a H<sub>2</sub> pipeline system**

We consider a similar pipeline system for H<sub>2</sub>, whose engineering equations are in Table B.1. We assume that the density  $d$  remains constant during the flow. More precisely, we suppose this density variation is negligible compared with the other forces at work. In particular, for pressures around 10 MPa (Cerniauskas *et al.* 2020), the density of hydrogen remains around a few kilograms per cubic meter.

**Table B.1: Engineering equations for a hydrogen pipeline**

Engineering equations	Parameters	Source
<u>Compressor equation</u>		
$W = 196 \cdot Z_{avg} \cdot \left(\frac{QT_s}{E}\right) \cdot \left(\frac{k}{k-1}\right) \cdot \left(\frac{P_l}{T_l}\right) \cdot \left[\left(\frac{P_0 + \Delta P}{P_0}\right)^{\frac{k-1}{k}} - 1\right]$	<p><math>W</math>: compression power (W)</p> <p><math>Z_{avg}</math>: gas pseudo-compressibility factor (-)</p> <p><math>T_s</math>: suction temperature (K)</p> <p><math>k</math>: ratio of specific heats (-)</p> <p><math>E</math>: overall efficiency (-)</p> <p><math>P_l</math>: standard pressure (Pa)</p> <p><math>T_l</math>: standard temperature (K)</p> <p><math>P_0</math>: suction pressure (Pa)</p> <p><math>\Delta P</math>: net pressure rise (Pa)</p>	<p>Formula adapted from the Engineering Data Book (Gas Processors Suppliers Association 2004)</p>
<u>Flow equation (Panhandle B)</u>		
$Q = 108.08 \cdot \frac{1}{\mu^{0.02}} \cdot \frac{T_n}{P_n} \left[ \frac{(P_0 + \Delta P)^2 - P_0^2}{L d^{0.9608} T_{avg} Z_{avg}} \right]^{0.51} D^{2.53}$	<p><math>\mu</math>: dynamic viscosity of hydrogen (Pa · s)</p> <p><math>P_n</math>: normal pressure (Pa)</p> <p><math>T_n</math>: normal temperature (K)</p> <p><math>d</math>: hydrogen relative density (-)</p> <p><math>T_{avg}</math>: average gas temperature (K)</p>	<p>Blacharski et al. (2016)</p>
<u>Mechanical equation</u>		
$\tau = a \cdot D$	<p><math>\tau</math>: thickness</p> <p><math>a</math>: constant</p>	<p>Ruan et al. (2009)</p>

We suppose that the pressure rise  $\Delta P$  is only a fraction of the transportation pressure  $P_0$  (least than 30%) which leads to the following first-order approximation:

$$W = c_1 \cdot \left(\frac{k-1}{k}\right) \cdot \left(\frac{\Delta P}{P_0}\right) Q \quad (\text{B.1})$$

With  $c_1 = 196 Z_{avg} \frac{T_s}{E} \cdot \frac{k}{k-1} \cdot \frac{P_l}{T_l}$  I.U. The average compressibility factor  $Z_{avg}$  is equal to the mean value of the suction compressibility factor  $Z_s$  and of the discharge compressibility factor  $Z_d$ , so that  $Z_{avg} = \frac{Z_s + Z_d}{2}$ . The ratio of specific heats  $k$  is defined as  $k = \frac{C_p}{C_v}$  with  $C_p$  the heat capacity at constant pressure and  $C_v$  the heat capacity at constant volume. The overall efficiency is equal to  $E = 0.82$  for a high-speed reciprocating compressor (Gas Processors Suppliers Association 2004).  $c_1$  is assumed to be constant during the whole compression stage.

Combining the first-order approximation with the flow equation we obtain:

$$Q = c_2^{0,66} \left( \frac{2P_0^2}{Lc_1} \cdot \frac{k}{k-1} \right)^{0,34} W^{0,34} D^{1,68} \quad (\text{B.2})$$

$$\text{With } c_2 = 108,08 \frac{1}{\mu^{0,02}} \cdot \frac{T_n}{P_n} \cdot \left( \frac{1}{d^{0,9608} T_{avg} Z_{avg}} \right)^{0,51} \text{ I.U}$$

Similar to the CO<sub>2</sub> pipeline system, we suppose that the capital required by the hydrogen pipeline system can be reduced to the capital stock of the pipeline and that the power required by the system is reduced to the power of the compressor (i.e.  $K = p_s w_s L \pi D^2 [a + a^2]$  and  $E = W_p$  respectively). By normalizing the output, we obtain the following Cobb-Douglas production function for a H<sub>2</sub> pipeline system:

$$Q^\beta = K^\alpha E^{1-\alpha}$$

Where  $\beta = 0.85$  and  $\alpha = 0.71$ .

## Appendix C – Oversizing ratio

Table C.1: Partial derivative of capital stock ratio for different parameters

Partial derivative of capital ratio	Analytical formula	
$\frac{\partial}{\partial \delta} \left( \frac{K^{**}}{K^*} \right)$	$\left( \frac{a_B}{a_A + a_B} \right) \cdot \beta \cdot (1 + \delta)^{\frac{\beta + \alpha - 1}{1 - \alpha}} \cdot \left( \frac{a_A + a_B}{a_A + a_B (1 + \delta)^{\frac{\beta}{1 - \alpha}}} \right)^\alpha$	$> 0$
$\frac{\partial}{\partial d} \left( \frac{K^{**}}{K^*} \right)$	$\frac{\partial}{\partial d} \left( \frac{a_B}{a_A} \right) \cdot (1 - \alpha) \cdot \left( \frac{a_A + a_B}{a_A + a_B (1 + \delta)^{\frac{\beta}{1 - \alpha}}} \right)^\alpha \cdot a_A^2 [(1 + \delta)^\beta - 1]$	$< 0$

For the partial derivative of the capital ratio with respect to the discounting rate  $\frac{\partial}{\partial d} \left( \frac{K^{**}}{K^*} \right)$ , we prove that  $\frac{\partial}{\partial d} \left( \frac{a_B}{a_A} \right)$  is negative for all  $d \geq 0$ . Thus, we have that:

$$\frac{\partial}{\partial d} \left( \frac{a_B}{a_A} \right) = \frac{\frac{\partial a_B}{\partial d} a_A - a_B \frac{\partial a_A}{\partial d}}{a_A^2}$$

where:  $\frac{\partial a_A}{\partial d} = \sum_{t=0}^{T_1} \frac{-t}{(1+d)^{t+1}}$  and  $\frac{\partial a_B}{\partial d} = \sum_{t=T_1+1}^{T_2} \frac{-t}{(1+d)^{t+1}}$ .

The sign of  $\frac{\partial}{\partial d} \left( \frac{K^{**}}{K^*} \right)$  therefore only depends on the sign of  $\frac{\partial a_B}{\partial d} a_A - a_B \frac{\partial a_A}{\partial d}$ , since the other terms are positive. We obtain:

$$\frac{\partial a_B}{\partial d} a_A = \left( \sum_{t=T_A+1}^T \frac{-t}{(1+d)^{t+1}} \right) \cdot \left( \sum_{t=0}^{T_A} \frac{1}{(1+d)^t} \right) = \sum_{i=T_A+1}^T \sum_{j=0}^{T_A} \frac{j}{(1+d)^{i+j+1}}$$

$$\frac{\partial a_B}{\partial d} a_A \leq \sum_{i=T_A+1}^T \sum_{j=0}^{T_A} \frac{i}{(1+d)^{i+j+1}}$$

We can reformulate the upper bound of the previous equation as:

$$\sum_{i=T_A+1}^T \sum_{j=0}^{T_A} \frac{i}{(1+d)^{i+j+1}} = \sum_{i=T_A+1}^T \frac{i}{(1+d)^{i+1}} \sum_{j=0}^{T_A} \frac{1}{(1+d)^j} = a_B \frac{\partial a_A}{\partial d}$$

We thus obtain that  $\frac{\partial a_B}{\partial d} a_A \leq a_B \frac{\partial a_A}{\partial d}$ , which implies that  $\frac{\partial}{\partial d} \left( \frac{a_B}{a_A} \right) \leq 0$ . We conclude that  $\frac{\partial}{\partial d} \left( \frac{K^{**}}{K^*} \right) \leq 0$  for all  $d \geq 0$ .

## Appendix D – Maximal Regret analytical proof

In this section, we prove the following theorem:

$$\frac{SRTC_{K^*}(Q_C) - LRTC(Q_C)}{LRTC(Q_A)} > \frac{SRTC_{K^{**}}(Q_A) - LRTC(Q_A)}{LRTC(Q_A)}$$

We find that:

$$\frac{SRTC_{K^*}(Q_C) - C(Q_C)}{LRTC(Q_A)} = \alpha + \left( \frac{g_1}{g_0} \right) (1 - \alpha) - \left( \frac{g_1}{g_0} \right)^{1-\alpha}$$

and:

$$\frac{SRTC_{K^{**}}(Q_A) - C(Q_A)}{LRTC(Q_A)} = \alpha \left( \frac{g_1}{g_0} \right)^{1-\alpha} + (1 - \alpha) \left( \frac{g_1}{g_0} \right)^{-\alpha} - 1$$

We introduce  $\forall x \geq 1, h(x) = \alpha + x(1 - \alpha) - x^{1-\alpha} - \alpha x^{1-\alpha} - (1 - \alpha)x^{-\alpha} + 1$ . Per construction:

$$h\left(\frac{g_1}{g_0}\right) = \frac{SRTC_{K^*}(Q_C) - LRTC(Q_C)}{LRTC(Q_A)} - \frac{SRTC_{K^{**}}(Q_A) - LRTC(Q_A)}{LRTC(Q_A)}$$

In the following, we prove that  $\forall x \geq 1, h(x) \geq 0$ , which concludes the proof. The function  $h$  is twice-differentiable and its second-order derivatives verifies:  $\forall x \geq 1, h''(x) = \frac{\alpha(1-\alpha)(1+\alpha)}{x^\alpha} \left( \frac{x-1}{x^2} \right) \geq 0$ .

The positivity of the second derivative  $h''$  implies that the first derivative  $h'$  increases for  $x \geq 1$ . Since

$h'(1) = 0$ , we conclude that, for  $x$  greater than one,  $h$  increases with  $x$ . Having  $h(1) = 0$ , we conclude that  $\forall x \geq 1, h(x) \geq 0$ .

## Appendix E – Maximal regret for three-case scenario

We write the analytical results for each case and add the max regret for each decision in the following

Table 8:

**Table 8: Analytical results of the Regret Table in a three-case scenario**

		The pipeline operator's decision		
		$K^*$	$K^{**}$	$K^{***}$
<b>Low demand</b>				
$\delta_0 = 0$		0	$\alpha \left(\frac{g_1}{g_0}\right)^{1-\alpha} + (1-\alpha) \left(\frac{g_1}{g_0}\right)^{-\alpha} - 1$	$\alpha \left(\frac{g_2}{g_0}\right)^{1-\alpha} + (1-\alpha) \left(\frac{g_2}{g_0}\right)^{-\alpha} - 1$
<b>Middle demand</b>				
$\delta_1$	$\alpha + (1-\alpha) \cdot \left(\frac{g_1}{g_0}\right) - \left(\frac{g_1}{g_0}\right)^{1-\alpha}$		0	$\alpha \left(\frac{g_2}{g_0}\right)^{1-\alpha} + (1-\alpha) \cdot \frac{g_1}{g_2^\alpha \cdot g_0^{1-\alpha}} - \left(\frac{g_1}{g_0}\right)^{1-\alpha}$
<b>High demand</b>				
$\delta_2$	$\alpha + (1-\alpha) \cdot \left(\frac{g_2}{g_0}\right) - \left(\frac{g_2}{g_0}\right)^{1-\alpha}$		$\alpha \left(\frac{g_1}{g_0}\right)^{1-\alpha} + (1-\alpha) \cdot \frac{g_2}{g_1^\alpha \cdot g_0^{1-\alpha}} - \left(\frac{g_2}{g_0}\right)^{1-\alpha}$	0
<b>Max Regret</b>				
	$\alpha + (1-\alpha) \cdot \left(\frac{g_2}{g_0}\right) - \left(\frac{g_2}{g_0}\right)^{1-\alpha}$		$\max(R(K^*; \delta_0); R(K^*; \delta_2))$	$\alpha \left(\frac{g_2}{g_0}\right)^{1-\alpha} + (1-\alpha) \left(\frac{g_2}{g_0}\right)^{-\alpha} - 1$

In the following we provide the analytical proofs for the results in the max regret row and show that choosing to build for the anchor load – by installing  $K^*$  – maximizes the maximal regret and is thus never the optimal solution from a minimax regret perspective.

Maximal Regret for the decision of installing  $K^*$ :

We need to prove that  $R(K^*; \delta_2) > R(K^*; \delta_1)$ . To this end, we introduce:  $\forall x > 1, f(x) = \alpha + (1 - \alpha) \cdot x - x^{1-\alpha}$ . Per construction,  $f\left(\frac{g_1}{g_0}\right) = R(K^*; \delta_1)$  and  $f\left(\frac{g_2}{g_0}\right) = R(K^*; \delta_2)$ . As  $g_2 > g_1$ , proving that  $f$  increases for  $x$  greater than 1 will conclude the proof. We find that  $f$  is differentiable for  $x > 1$  and we have:  $\forall x > 1, f'(x) = (1 - \alpha) \left(1 - \frac{1}{x^\alpha}\right) \geq 0$ . Thus,  $f$  increases for  $x$  greater than 1.

Maximal Regret for the decision of installing  $K^{***}$ :

We prove that  $R(K^{***}; \delta_0) > R(K^{***}; \delta_1)$ . We find that:

$$R(K^{***}; \delta_0) - R(K^{***}; \delta_1) = \frac{g_1 - g_0}{g_2^{2\alpha} g_0^{1-\alpha}} \left[ -\frac{(1 - \alpha)}{g_2^\alpha} + \frac{g_1^{1-\alpha} - g_0^{1-\alpha}}{g_1 - g_0} \right]$$

The sign of  $R(K^{***}; \delta_0) - R(K^{***}; \delta_1)$  only depends on the terms in brackets. We show that  $\frac{g_1^{1-\alpha} - g_0^{1-\alpha}}{g_1 - g_0} >$

$\frac{(1-\alpha)}{g_1^\alpha}$ . Since  $g_1 < g_2$ , we find that  $\frac{g_1^{1-\alpha} - g_0^{1-\alpha}}{g_1 - g_0} > \frac{(1-\alpha)}{g_2^\alpha}$  which concludes the proof.

Maximal Regret for the decision of installing  $K^{**}$ :

For this decision, the maximal regret will depend on the respective values of  $g_0, g_1$  and  $g_2$ . We provide here an intuition of this result. We want to determine the sign of  $R(K^{**}; \delta_0) - R(K^{**}; \delta_2)$ . We find that:

$$R(K^{**}; \delta_0) - R(K^{**}; \delta_2) = \frac{g_2 - g_0}{g_0^{1-\alpha}} \left( \frac{-(1 - \alpha)}{g_1^\alpha} + \frac{g_2^{1-\alpha} - g_0^{1-\alpha}}{g_2 - g_0} \right)$$

We introduce the function  $f$  that verifies:  $\forall x \in [g_0; g_2], f(x) = x^{1-\alpha}$ .  $f$  is continuous on  $[g_0; g_2]$  and differentiable on  $]g_0; g_2[$ . We have  $\forall x \in ]g_0; g_2[ f'(x) = \frac{1-\alpha}{x^\alpha}$

By the mean value theorem, there exists an intermediate point  $g_i$  so that:  $f'(g_i) = \frac{g_2^{1-\alpha} - g_0^{1-\alpha}}{g_2 - g_0} = \frac{1-\alpha}{g_i^\alpha}$ .

Therefore

$$R(K^{**}; \delta_0) - R(K^{**}; \delta_2) = \frac{(g_2 - g_0) \cdot (1 - \alpha)}{g_0^{1-\alpha}} \left( \frac{-1}{g_1^\alpha} + \frac{1}{g_i^\alpha} \right)$$

The sign of  $R(K^{**}; \delta_0) - R(K^{**}; \delta_2)$  only depends on the term in parenthesis and thus solely depends on the respective values of  $g_1$  and  $g_i$ . Depending on the assumptions made by the pipeline operator, we can have  $g_1 > g_i$  or  $g_1 < g_i$ .

*Proof that installing  $K^*$  is the maximizing maximal regret decision:*

Depending on the values of  $g_0, g_1$  and  $g_2$ , the decision that minimizes the maximal regret may be  $K^{**}$  or  $K^{***}$ . However, the decision to not oversize  $K^*$  is never the decision that minimizes the maximum regret. We prove this aspect here by showing that the maximal regret for the decision to install the capital  $K^*$ ,  $\max_{\delta} R(K^*; \delta)$ , is greater than the maximum regret for the decisions to install  $K^{**}$  or  $K^{***}$ .

First, we prove that the maximal regret for installing  $K^*$  is greater than the maximal regret for installing  $K^{***}$ , that is:  $\max_{\delta} R(K^*; \delta) > \max_{\delta} R(K^{***}; \delta)$ . Based on the above, we have that  $\max_{\delta} R(K^*; \delta) = R(K^*; \delta_2)$  and that  $\max_{\delta} R(K^{***}; \delta) = R(K^{***}; \delta_0)$ . The proof is similar to the analytical proof of the two-case scenario. We introduce  $\forall x \geq 1, h(x) = \alpha + x(1 - \alpha) - x^{1-\alpha} - \alpha x^{1-\alpha} - (1 - \alpha)x^{-\alpha} + 1$  and prove that  $\forall x \geq 1, h(x) \geq 0$ . As  $g_2 > g_0$  and  $h\left(\frac{g_2}{g_0}\right) = R(K^*; \delta_2) - R(K^{***}; \delta_0)$ , this concludes the proof.



Second, we prove that the maximal regret for installing  $K^*$  is greater than the maximal regret for installing  $K^{**}$ ,  $\max_{\delta} R(K^*; \delta) > \max_{\delta} R(K^{**}; \delta)$ . As the maximal regret for installing  $K^{**}$  depends on the numerical values of the parameters, it can either be equal to  $R(K^{**}; \delta_0)$  or  $R(K^{**}; \delta_2)$ . We suppose first that  $\max_{\delta} R(K^{**}; \delta) = R(K^{**}; \delta_0)$  and prove that  $\max_{\delta} R(K^*; \delta) > R(K^{**}; \delta_0)$ . Thus, we have that

$$\begin{aligned} \max_{\delta} R(K^*; \delta) - \max_{\delta} R(K^{**}; \delta) &= R(K^*; \delta_2) - R(K^{**}; \delta_0) = \left( \alpha + (1 - \alpha) \cdot \left( \frac{g_2}{g_0} \right) - \left( \frac{g_2}{g_0} \right)^{1-\alpha} \right) - \\ &\left( \alpha \left( \frac{g_1}{g_0} \right)^{1-\alpha} + (1 - \alpha) \left( \frac{g_1}{g_0} \right)^{-\alpha} - 1 \right) \end{aligned}$$

We introduce:  $\forall x > 1, f(x) = \alpha + (1 - \alpha) \cdot x - x^{1-\alpha}$ . It is obvious that  $f$  is increasing for  $x$  greater than 1. Since  $g_2 > g_1$ ,  $f\left(\frac{g_2}{g_0}\right) > f\left(\frac{g_1}{g_0}\right)$ . Using function  $h$  of Appendix D, we find that:

$$f\left(\frac{g_2}{g_0}\right) - \left( \alpha \left( \frac{g_1}{g_0} \right)^{1-\alpha} + (1 - \alpha) \left( \frac{g_1}{g_0} \right)^{-\alpha} - 1 \right) > f\left(\frac{g_1}{g_0}\right) - \left( \alpha \left( \frac{g_1}{g_0} \right)^{1-\alpha} + (1 - \alpha) \left( \frac{g_1}{g_0} \right)^{-\alpha} - 1 \right)$$

which is equivalent to:

$$R(K^*; \delta_2) - R(K^{**}; \delta_0) > h\left(\frac{g_1}{g_0}\right) > 0$$

Which concludes the first part of the proof. Now we suppose that  $\max_{\delta} R(K^{**}; \delta) = R(K^{**}; \delta_2)$  and prove that  $\max_{\delta} R(K^*; \delta) > R(K^{**}; \delta_0)$ . We have:

$$\begin{aligned} \max_{\delta} R(K^*; \delta) - \max_{\delta} R(K^{**}; \delta) &= R(K^*; \delta_2) - R(K^{**}; \delta_2) = \left( \alpha + (1 - \alpha) \cdot \left( \frac{g_2}{g_0} \right) - \left( \frac{g_2}{g_0} \right)^{1-\alpha} \right) - \\ &\left( \alpha \left( \frac{g_1}{g_0} \right)^{1-\alpha} + (1 - \alpha) \cdot \frac{g_2}{g_1^{\alpha} \cdot g_0^{1-\alpha}} - \left( \frac{g_2}{g_0} \right)^{1-\alpha} \right) \end{aligned}$$

We find that:

$$R(K^*; \delta_2) - R(K^{**}; \delta_2) = -\alpha \left( \left( \frac{g_1}{g_0} \right)^{1-\alpha} - 1 \right) + (1 - \alpha) \left( \frac{g_2}{g_0} \right) \left( 1 - \left( \frac{g_0}{g_1} \right)^{\alpha} \right)$$

We introduce  $\forall x > 1, m(x) = -\alpha(x^{1-\alpha} - 1) + (1 - \alpha)x(1 - x^\alpha)$ . Since  $g_2 > g_1$ , we have per construction of  $m$ :  $R(K^*; \delta_2) - R(K^{**}; \delta_2) > m\left(\frac{g_1}{g_0}\right)$ . It is immediate to prove that for  $x$  greater than one,  $m$  increases with  $x$  and that  $m(1) = 0$ . This proves that the function  $m$  is positive for  $x$  greater than one. As a result,  $R(K^*; \delta_2) - R(K^{**}; \delta_2) > m\left(\frac{g_1}{g_0}\right) > 0$ , which concludes the proof.