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# Moment Risk Premia and Stock Return Predictability \*

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# Moment Risk Premia and Stock Return Predictability

## Abstract

We study the predictive power of option-implied moment risk premia embedded in the conventional variance risk premium. We find that while the second moment risk premium predicts market returns in short horizons with positive coefficients, the third (fourth) moment risk premium predicts market returns in medium horizons with negative (positive) coefficients. Combining the higher moment risk premia with the second moment risk premium improves the stock return predictability over multiple horizons, both in-sample and out-of-sample. The finding is economically significant in an asset allocation exercise, and survives a series of robustness checks.

**JEL Classification:** G12, G13, C22

**Keywords:** Moment risk premia; Variance risk premium; Option-implied moments; Stock return predictability; Predictive regression.

# I. Introduction

The issue of whether the stock market returns are predictable has been one of the most discussed topics in financial economics. Until a few decades ago, the widespread view was that market returns are unpredictable if the market is efficient. It has now been generally accepted that expected returns are time-varying and partially predictable even in an efficient market (See, for example, [Campbell and Shiller \(1988\)](#), [Fama and French \(1989\)](#), [Kothari and Shanken \(1997\)](#), and [Cochrane \(2008\)](#)). Ample empirical evidence have shown that variables including financial ratios and macroeconomic variables can predict variation of stock returns over business cycle and multi-year horizons. More recent studies uncover that predictors extracted from options data forecast market returns at horizons as short as a few months. This paper contributes to the time series predictability of the stock market returns over short horizons by exploiting new predictive information in equity index options.

A typical example of a short-term predictor extracted from the option market is the variance risk premium (see, e.g., [Bollerslev, Tauchen, and Zhou \(2009\)](#)), which has been shown to strongly predict the market return over horizons up to 6-month. In fact, the conventional variance risk premium, defined as the difference between the squared VIX and the realized return variance, is a quasi variance risk premium (henceforth QVRP), since it not only has a second order component, the pure variance risk premium (henceforth

PVRP), but also contains higher moment premium components. In this paper, we seek to investigate the predictability of moment risk premia embedded in QVRP over different forecasting horizons.

Following Bakshi and Madan (2000) and Bakshi, Kapadia, and Madan (2003), we compute the risk neutral moments of returns using portfolios of out-of-the-money European call and put options. Matching the risk-neutral moments with their realized counterparts, we calculate the pure variance risk premium (PVRP), risk premium on the third moment of returns (M3RP), and risk premium on the fourth moment of returns (M4RP), in a model-free fashion.

Using S&P 500 index and its option data from 1990 to 2019, we investigate the predictability of the market return afforded by the option-implied moment risk premia over different horizons using predictive regressions. We find that higher moment risk premia, M3RP and M4RP, are similar to each other but have different statistical features from the second moment risk premium, PVRP. In particular, PVRP and M3RP are only moderately correlated and their means have different signs. By contrast, M3RP and M4RP contain overlapping information and are highly correlated. These evidence suggest that much information in the higher moment risk premia is unspanned by PVRP and aggregating them may lead to substantial information losses. As a consequence, there is room for potential improvement in predicting the market equity return using options by considering these moment risk premia separately.

We evaluate the predictability of each moment risk premium using predictive regressions for 1- to 24-month excess returns on the S&P 500 index both in-sample and out-of-sample. We have three main findings. First, we find that the predictive performance of PVRP dominates that of QVRP at all horizons with higher t-statistics and larger in-sample and out-of-sample  $R^2$ 's. This confirms that PVRP, a cleaner measure of the variance risk premium after removing the higher moment risk premia from QVRP, is a better predictor than the conventional variance risk premium.

Second, we find that while PVRP predicts market returns at short-term, M3RP and M4RP predict market returns at medium-term. At 6- to 24-month horizons, M3RP (M4RP) predicts market returns with highly significant coefficients and higher in-sample and out-of-sample  $R^2$ 's than PVRP. We show that M3RP remains statistically significant after controlling for stock return predictors in [Welch and Goyal \(2008\)](#) and short-term predictors, such as aggregate short interest in [Rapach, Ringgenberg, and Zhou \(2016\)](#), average skewness in [Jondeau, Zhang, and Zhu \(2019\)](#), and left jump probability in [Andersen, Fusari, and Todorov \(2015\)](#).

Finally, combining moment risk premia improves both in-sample and out-of-sample predictability of QVRP over multiple horizons. In particular, the adjusted  $R^2$ 's of the joint regressions with PVRP and M3RP are 9.7%, 6.1%, and 4.5% at 6-, 9-, and 12-month horizons, in contrast to 3.7%, 0.9%, and 0.3% of the univariate regressions with QVRP. The out-of-sample  $R^2$ 's of the forecast combination with PVRP and M3RP are 10.9%,

6.9%, and 4.5% at 6-, 9-, and 12-month horizons, compared with 2.0%, -2.4%, and -7.9% in the univariate regressions with QVRP. Our main findings survive various robustness checks.

We further examine the economic value of the predictability offered by the moment risk premia via an asset allocation experiment. The different predictability contained in moment risk premia can be exploited by forming strategic portfolios. Consistent with our findings on the predictive regressions, the portfolios formed on PVRP result in higher certainty equivalent in shorter terms and those formed on M3RP or M4RP result in higher certainty equivalent in longer terms. In addition, portfolios that combine the predictability from PVRP and M3RP (M4RP) generate higher out-of-sample utility gains in medium horizons than those based on QVRP alone.

Our paper is related to the literature that study option-implied moments and different measures of risk-neutral variance. [Martin \(2017\)](#) proposes an option-implied variance of simple returns and relates it to the lower bound of the expected market return. [Kozhan, Neuberger, and Schneider \(2013\)](#) construct a measure of skewness risk premium, which can be interpreted as the profit to a dynamic trading strategy. In a similar spirit, [Bondarenko \(2014\)](#) defines an alternative variance risk premium, which is robust to sampling frequencies and price discontinuities. Another related paper is [Ait-Sahalia, Karaman, and Mancini \(2018\)](#), who identify a large and time-varying jump component by comparing the variance swaps rates and the VIX index and postulate a parametric model that generates the empirical patterns of these price jumps.

Our paper contributes to the literature on return predictability from the variance risk premium and components of the variance risk premium. Since the seminal work by [Bollerslev et al. \(2009\)](#), who show that the variance risk premium predicts market returns for up to a few months horizon, many papers investigate how different components of variance risk premium contribute to the return prediction. For instance, [Bollerslev, Todorov, and Xu \(2015\)](#) decompose the total variance into its continuous and jump variance components and find that much of the predictability in variance risk premium may be attributed to the jump tail component. [Feunou, Jahan-Parvar, and Okou \(2018\)](#) study the predictability of the downside variance risk premium. [Kilic and Shaliastovich \(2019\)](#) show that the good and bad variance risk premiums can jointly predict stock and bond returns. [Buss, Schönleber, and Vilkov \(2019\)](#) identify a correlation risk premium in the variance risk premium and find considerable predictability in the correlation risk premium. While these papers analyze components within the variance risk premium, we focus on the higher moment risk premia, which, although embedded in the quasi variance risk premium, is beyond the second moment risk premium. We show that higher moment risk premia contain complementary predictive power to the second moment risk premium.

Our paper also contributes to the literature on the predictability of higher moments of returns. Many papers have shown that skewness is related to future stock returns (see, e.g., [Chang, Christoffersen, and Jacobs \(2013\)](#), [Conrad, Dittmar, and Ghysels \(2013\)](#), [Amaya, Christoffersen, Jacobs, and Vasquez \(2015\)](#), and [Stilger, Kostakis, and Poon](#)

(2017)). Most of these studies focus on the individual stock level. An exception is [Jondeau, Zhang, and Zhu \(2019\)](#), who use a weighted average of realized skewness of individual stocks to predict market returns. While [Jondeau et al. \(2019\)](#) use realized skewness to predict returns in the next month, we use the option-implied higher moment risk premia, which has a natural forward-looking component, to predict the market return over one- to 24-month horizons.

The rest of the paper is organized as follows. Section II defines the QVRP and moment risk premia. Section III explains the data used in the empirical analysis. Section IV reports the predictive regression results for the market return on the moment risk premia, along with a series of robustness checks. Section V studies the out-of-sample predictability of moment risk premia in terms of out-of-sample  $R^2$ 's and asset allocation implications. Section VI concludes.

## II. Separating the Moment Risk Premia

The conventional variance risk premium is defined based on the Chicago Board of Options Exchange (CBOE) VIX index, such as in [Bollerslev et al. \(2009\)](#) and [Bekaert and Hoerova \(2014\)](#). The CBOE VIX index is a popular measure of investors' fear, which is constructed from a portfolio of out-of-the-money S&P 500 index call and put options. If

the call and put options have a continuum of strike prices from 0 to  $\infty$ ,  $\text{VIX}_t^2$  is defined as,

$$(1) \quad \text{VIX}_t^2 \equiv \frac{2}{T-t} \int_0^\infty \frac{\Theta_t(K, T)}{K^2} dK,$$

where  $\Theta_t(K, T)$  denotes the time- $t$  value of an out-of-the-money option with strike price  $K > 0$  and maturity  $T$ . Puts are used for low strikes ( $K \leq F_t(T)$ ) and calls are used for high strikes ( $K \geq F_t(T)$ ), since out-of-the-money options are more liquid than in-the-money options. Here,  $F_t(T)$  is the forward price of the underlying asset at time  $t$  with maturity  $T$ .

As noted by Carr and Wu (2009), when used as the option-implied expectation of stock volatility, the VIX given by Equation (1) has an approximation error induced by return discontinuities. As a matter of fact, Kozhan et al. (2013) show that  $\text{VIX}_t^2$  is the risk-neutral expectation of  $g(r(t, T))$ ,

$$(2) \quad \text{VIX}_t^2 = \frac{1}{T-t} \mathbb{E}_t^Q [g(r(t, T))],$$

with  $g(r) \equiv 2(e^r - 1 - r)$ . Here,  $r(t, T)$  denotes the log return on the forward prices from  $t$  to  $T$ :  $r(t, T) = \log F_T(T) - \log F_t(T)$ . Note that we define  $r(t, T)$  using forward prices rather than the spot prices to avoid complications with interest rates and dividends, similar to Bondarenko (2014).<sup>1</sup>

Let  $\{t, t + \Delta, \dots, t + N\Delta\}$  be a partition of  $[t, T]$ , and denote  $r(t + i\Delta, t + (i + 1)\Delta)$

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<sup>1</sup>Returns thus defined are excess returns. In other words, the expectation under which Equation (2) is evaluated is the forward  $Q$ -measure.

as  $r_i$  for simplicity. To obtain a coherent realized counterpart for  $VIX^2$ , we consider the following expression

$$(3) \quad \text{QRV}_T \equiv \frac{1}{T-t} \sum_{i=1}^N g(r_i) = \frac{1}{T-t} \sum_{i=1}^N 2(e^{r_i} - 1 - r_i).$$

We denote the quantity defined in (3) the “Quasi Realized Variance” (QRV), since the function  $g(r)$  differs from  $r^2$  only in higher-order terms. To see this, we apply Taylor expansion to  $g(r)$  and get

$$(4) \quad g(r) = r^2 + \frac{1}{3}r^3 + \frac{1}{12}r^4 + o(r^4).$$

Taking the difference between the squared VIX and QRV gives the “Quasi Variance Risk Premium” (QVRP):

$$(5) \quad \text{QVRP}_t \equiv \text{VIX}_t^2 - \mathbb{E}_t[\text{QRV}_T].$$

We use the word “quasi” to distinguish our definition of the variance risk premium from those in the prevailing literature. Many papers use different formulations of realized variance other than QRV as the realized counterpart of  $VIX^2$ . For instance, Carr and Wu (2009) use the realized squared simple returns  $(\frac{1}{T-t} \sum_{i=1}^N (e^{r_i})^2)$ , and Bollerslev et al. (2009) use the realized squared log returns  $(\frac{1}{T-t} \sum_{i=1}^N r_i^2)$ .

The higher-order terms in Equation (4) are nontrivial when returns can jump. Empirical literature has presented strong evidence of jumps in the S&P 500 index return, for example, Bakshi, Cao, and Chen (1997), Andersen, Benzoni, and Lund (2002), Pan (2002), Eraker, Johannes, and Polson (2003), and Christoffersen, Jacobs, and Ornthalai (2012), among others. If the higher-order terms on the right hand side of Equation (4) are non-negligible, QRV serves as the only consistent realized counterpart of  $VIX^2$ , regardless of the presence of jumps. This internal consistency between the option-implied moments and their realized counterparts in QVRP facilitates the identification of the higher-order risk premiums within QVRP as follows,

$$\begin{aligned}
\text{QVRP}_t &\equiv \text{VIX}_t^2 - \mathbb{E}_t[\text{QRV}_T] \\
&= \underbrace{\frac{1}{T-t} \left( \mathbb{E}_t^Q[r(t, T)^2] - \mathbb{E}_t \left[ \sum_{i=1}^N r_i^2 \right] \right)}_{\text{PVRP}_t} + \underbrace{\frac{1}{3} \frac{1}{T-t} \left( \mathbb{E}_t^Q[r(t, T)^3] - \mathbb{E}_t \left[ \sum_{j=1}^N r_j^3 \right] \right)}_{\text{M3RP}_t} \\
&\quad + \underbrace{\frac{1}{12} \frac{1}{T-t} \left( \mathbb{E}_t^Q[r(t, T)^4] - \mathbb{E}_t \left[ \sum_{j=1}^N r_j^4 \right] \right)}_{\text{M4RP}_t} + \frac{1}{T-t} \sum_{i=5}^{\infty} \frac{2}{i!} \left( \mathbb{E}_t^Q[r(t, T)^i] - \mathbb{E}_t \left[ \sum_{j=1}^N r_j^i \right] \right) \\
&\approx \text{PVRP}_t + \frac{1}{3} \text{M3RP}_t + \frac{1}{12} \text{M4RP}_t.
\end{aligned}$$

PVRP, M3RP, and M4RP represent risk premiums associated with the second, third, and fourth moments of returns, respectively. In the online appendix, we derive the moments of returns in a jump-diffusion model as an example to illustrate the potential sources of

higher moments.

The risk neutral components in the moment risk premia can be constructed by using the quadratic, cubic, and quartic contracts introduced by Bakshi et al. (2003). We denote them as the implied variance (IV), the implied third moment (IM3), and the implied fourth moment (IM4):

$$\begin{aligned}
 (6) \quad IV_t &= \frac{1}{T-t} \mathbb{E}_t^Q [r(t, T)^2] = \frac{2}{T-t} \int_0^\infty \frac{1 + \log(F_t/K)}{K^2} \Theta_t(K, T) dK, \\
 (7) \quad IM3_t &= \frac{1}{T-t} \mathbb{E}_t^Q [r(t, T)^3] = \frac{1}{T-t} \int_0^\infty \frac{6 \log(K/F_t) - 3(\log(K/F_t))^2}{K^2} \Theta_t(K, T) dK, \\
 (8) \quad IM4_t &= \frac{1}{T-t} \mathbb{E}_t^Q [r(t, T)^4] = \frac{1}{T-t} \int_0^\infty \frac{12(\log(F_t/K))^2 - 4(\log(K/F_t))^3}{K^2} \Theta_t(K, T) dK.
 \end{aligned}$$

The realized variance (RV), realized third moment (RM3), and realized fourth moment (RM4), corresponding to IV, IM3, and IM4, are, respectively,

$$(9) \quad RV_t \equiv \frac{1}{T-t} \sum_{j=1}^N r_i^2, \quad RM3_t \equiv \frac{1}{T-t} \sum_{j=1}^N r_i^3, \quad RM4_t \equiv \frac{1}{T-t} \sum_{j=1}^N r_i^4.$$

The pure variance risk premium (PVRP), the third moment risk premium (M3RP), and the fourth moment risk premium (M4RP) are defined as the differences between the

risk-neutral and physical expectation of realized moments of log returns:

$$(10) \quad \begin{aligned} \text{PVRP}_t &= \text{IV}_t - \mathbb{E}_t[\text{RV}_T], \\ \text{M3RP}_t &= \text{IM3}_t - \mathbb{E}_t[\text{RM3}_T], \\ \text{M4RP}_t &= \text{IM4}_t - \mathbb{E}_t[\text{RM4}_T]. \end{aligned}$$

Here, we use the term “risk premium” to indicate that the variables in Equation (10) are differences between Q- and P-expectations.<sup>2</sup> In the next section, we show how to construct QVRP, PVRP, M3RP, and M4RP empirically using option prices and stock returns.

### III. Data Source and Risk Premiums

#### A. Data Source and Variable Construction

We use the S&P 500 index option data from the CBOE, starting from January 1990 and ending in July 2019. Our data sample includes the highest closing bid and the lowest closing ask prices of all call and put options, strike prices, and expiration dates. We obtain monthly one-month risk free rates from the CRSP. These rates are based on the treasury bill that has a minimum of 30 days to maturity, and is the closest to 30 days to maturity.

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<sup>2</sup>Strictly speaking, these moment risk premia are not profits from a trading strategy hence do not qualify as risk premiums in the economics sense, as pointed out by [Kozhan et al. \(2013\)](#).

We obtain monthly dividends rates of S&P 500 index from Compustat, which are the anticipated annual dividend rate.

We apply standard filters to select the option sample. First, we delete all options with zero open interest, zero bid prices, and missing implied volatility. Second, following the literature on model-free implied volatility, such as [Jiang and Tian \(2005\)](#) and [Carr and Wu \(2009\)](#), we only keep out-of-the-money and at-the-money options. A put (call) option is regarded out-of-the-money if the strike price is lower (higher) than the forward price. The one-month forward price at time  $t$  is defined as  $F_t = S_t e^{(r_{f,t} - q_t)\tau}$ . Here,  $S_t$  is the S&P 500 index spot price,  $\tau = 1/12$  denotes the time-to-maturity of one-month,  $r_{f,t}$  is the risk free rate, and  $q_t$  is the dividend rate at time  $t$ . Third, we only keep options with less than 365 days of expiry. After applying the filters, we have 5,503,043 option-day data points. Similar to the construction of VIX index provided by CBOE, we work with the best bid and ask closing quotes. The option price is the average of the highest closing bid and the lowest closing ask prices.

At the end of each month, we construct the annualized  $VIX^2$ , IV, IM3 and IM4

using the discretized version of Equation (1), (6), (7), and (8). That is,

$$\begin{aligned}
 (11) \quad \text{VIX}_t^2 &\approx \frac{1}{T-t} \sum_{i=2}^{m_{t,\tau}} [f(t, T, K_i) + f(t, T, K_{i-1})] \Delta K_i, \\
 \text{IV}_t &\approx \frac{1}{T-t} \sum_{i=2}^{m_{t,\tau}} [f_v(t, T, K_i) + f_v(t, T, K_{i-1})] \Delta K_i, \\
 \text{IM3}_t &\approx \frac{1}{T-t} \sum_{i=2}^{m_{t,\tau}} [f_3(t, T, K_i) + f_3(t, T, K_{i-1})] \Delta K_i, \\
 \text{IM4}_t &\approx \frac{1}{T-t} \sum_{i=2}^{m_{t,\tau}} [f_4(t, T, K_i) + f_4(t, T, K_{i-1})] \Delta K_i,
 \end{aligned}$$

where  $\Delta K_i = K_i - K_{i-1}$ . Here,  $m_{t,\tau}$  is the number of available out-of-the-money options on day  $t$  with maturity  $\tau = T - t$  after we filter the options data. Therefore,  $m_{t,\tau}$  varies by date  $t$  and maturity  $\tau$ .  $f$ ,  $f_v$ ,  $f_3$  and  $f_4$  are defined as,

$$\begin{aligned}
 f(t, T, K_i) &= \frac{\Theta_t(K_i, T)}{K_i^2}, \\
 f_v(t, T, K_i) &= \frac{1 + \log(F_t/K_i)}{K_i^2} \Theta_t(K_i, T), \\
 f_3(t, T, K_i) &= \frac{6 \log(K_i/F_t) - 3(\log(K_i/F_t))^2}{2K_i^2} \Theta_t(K_i, T), \\
 f_4(t, T, K_i) &= \frac{12(\log(F_t/K_i))^2 - 4(\log(K_i/F_t))^3}{2K_i^2} \Theta_t(K_i, T),
 \end{aligned}$$

where  $F_t$  denotes the forward price and  $\Theta_t(K, T)$  denotes the time- $t$  value of an out-of-the-money option with strike price  $K$  and maturity  $T \geq t$ . Following the construction of VIX provided by CBOE, we select two maturities of options—the shortest

maturity with more than 30 days of expiry and the longest maturity with less than 30 days and more than 7 days of expiry. The annualized VIX<sup>2</sup> in Equation (11) is then calculated for these two maturities. Next, we interpolate the 30-day VIX<sup>2</sup> using VIX<sup>2</sup> of the two maturities with linear interpolation. The same procedure applies to the calculation of IV, IM3, and IM4 with 30 days of expiration.

Following the recent literature (e.g. [Bollerslev et al. \(2009\)](#) and [Buss et al. \(2019\)](#) among others), to approximate the expectations under the physical measure, we use daily S&P 500 index prices to calculate quasi realized variance (QRV), realized variance (RV), realized third moment (RM3), and realized fourth moment (RM4) for each calendar month. In accordance with the risk neutral moments that are constructed based on the forward prices, the realized moments are also computed using forward prices. Specifically, we assume that the risk free rate and dividend rate are constant within a month. Given month  $t$ , we denote the forward price on the  $n^{\text{th}}$  day of the month as  $F_t^n$ . Here, the subscript  $t$  denotes month and superscript  $n$  denotes day-of-the-month.  $F_t^n$  is calculated as

$$F_t^n = S_t^n \exp\left((r_{f,t} - q_t)(N_t - n)/(12N_t)\right),$$

where  $S_t^n$  is the spot price on day  $n$  of month  $t$ ,  $r_{f,t}$  and  $q_t$  are the annualized risk-free rate and dividend rate of month  $t$ , and  $N_t$  is number of trading days in month  $t$ . We calculate

daily excess log returns as

$$r_t^{n+1} = \log(F_t^{n+1}) - \log(F_t^n).$$

Realized moments are then computed as

$$\begin{aligned} \text{QRV} &= \sum_{i=1}^N 2(e^{r_i} - 1 - r_i), & \text{RV}_t &= \sum_{i=1}^N (r_t^i)^2, \\ \text{RM3}_t &= \sum_{i=1}^N (r_t^i)^3, & \text{RM4}_t &= \sum_{i=1}^N (r_t^i)^4. \end{aligned}$$

Notice that the implied moments (VIX, IV, IM3, and IM4) are calculated using OTM options at the last trading day of the month, but the realized moments (QRV, RV, RM3, and RM4) are calculated with daily returns *within* the month  $t$ . In other words, we use the realized moments of  $t - 1$  as an estimator for expected realized moments of  $t$ . This formulation has the advantage that the risk premiums are ex ante and model-free. Since both implied and realized moments are available at time  $t$  without relying on any specific model, this facilitates the return forecasting exercise in Section IV.

## B. Summary Statistics of Moment Risk Premia

Table 1 reports the summary statistics of risk neutral moments, realized moments, and moment risk premia. Summary statistics of the risk neutral moments, VIX<sup>2</sup>, IV, IM3, IM4, and those of the realized moments, QRV, RV, RM3, and RM4, are reported in Panel

A. Comparing risk neutral and realized moments, we observe that the sample mean of risk neutral moments are larger in magnitude than their realized counterparts. The risk neutral and realized third moments are both negative. IM3 is larger in magnitude, has larger standard deviation, and is more left skewed than RM3. IM4 and RM4 follow a similar pattern with an opposite sign. The mean of  $VIX^2$  is slightly lower than that of IV because  $VIX^2$  is a linear combination of IV, IM3, and IM4.

[Insert Table 1 here.]

Panel B reports summary statistics of the moment risk premia. Consistent with the existing literature, QVRP is on average positive with a mean of 0.95%. M3RP is on average negative, which explains why PVRP has a slightly larger mean than QVRP. All risk premiums are significantly different from zero at 1% level. Compared with QVRP and PVRP, M3RP and M4RP have relatively lower standard deviation and higher autocorrelation.

Panel C reports the correlation matrix among the risk premiums. The correlation between QVRP and PVRP is as high as 0.99, implying that PVRP is the major component of QVRP. There is also substantial comovement between PVRP and higher moment risk premia, with a correlation coefficient of 0.47 for M3RP and -0.55 for M4RP. M3RP and M4RP almost always move in opposite directions with a correlation coefficient of -0.98.

Figure 1 plots the time series of QVRP and PVRP. The dynamics of QVRP and PVRP are almost indistinguishable. Both QVRP and PVRP fluctuate between positive

and negative values and display moderate variations as well as occasional spikes. Despite the fact that both QVRP and PVRP are on average positive, as shown by the summary statistics, there are a couple of extreme negative values in late 2002, 2008, and 2011. These negative spikes may be attributed to the downward volatility jumps as proposed by [Amengual and Xiu \(2018\)](#) or heightened uncertainty as proposed by [Hu, Pan, Wang, and Zhu \(2019\)](#), associated with resolutions of policy uncertainties. Figure 2 plots the time series of M3RP and M4RP. Compared with QVRP or PVRP, M3RP and M4RP have less fluctuations but sharper spikes. Spikes in M3RP and M4RP coincide with volatile periods in PVRP.

[Insert Figure 1 here.]

[Insert Figure 2 here.]

## IV. Predictive Regression Analysis

In this section, we analyze the predictability of stock market returns using the moment risk premia embedded in QVRP. We run predictive regressions of the market return of different horizons on each moment risk premium separately and on multiple moment risk premia jointly. Section [IV.A](#) reports the baseline predictive results. Section [IV.B](#) reports the prediction results for weighted least squares. In Section [IV.C](#) and [IV.D](#), we control for the established long-term and short-term predictors, respectively.

## A. Predicting the Market Return

As shown by [Bollerslev et al. \(2009\)](#), [Drechsler and Yaron \(2011\)](#), and [Bekaert and Hoerova \(2014\)](#), variance risk premium has significant predictive power for future market returns at quarterly horizon. In this section, we show that while QVRP predicts short-term market returns up to 6 months, higher moment risk premia, M3RP and M4RP, predict medium-term market returns up to 24 months. We also show that at any horizon from 1- to 24-month, separating M3RP and M4RP from PVRP yields better predictive results.

Let  $X_t$  be a vector of predictive variables, containing end-of-month values. We use the following specification for predictive regressions,

$$(12) \quad R_{t,t+h} = \alpha_h + \beta_h' X_t + \varepsilon_{t,t+h},$$

where  $R_{t,t+h}$  is the market excess return from the first day of next month  $t + 1$  to the last day of month  $t + h$ . We use simple excess return on the S&P 500 index as a proxy of market excess return.<sup>3</sup>

As shown in the summary statistics in [Table 1](#), M3RP and M4RP are correlated with PVRP. To investigate the predictive information in higher moment risk premia orthogonal to PVRP, we first regress M3RP and M4RP on PVRP and a constant to obtain

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<sup>3</sup>Here, we use the S&P 500 returns instead of aggregate stock market returns because moment risk premia are only available for the former. An important difference from the traditional aggregate market return is that the S&P 500 is a price index, so that returns do not include dividends.

a time series of M3RP and M4RP residuals, denoted as  $M3RP^\perp$  and  $M4RP^\perp$ . We then use the residuals  $M3RP^\perp$  and  $M4RP^\perp$  as predictors. In the univariate regressions,  $X_t = QVRP_t, PVRP_t, M3RP_t^\perp$ , or  $M4RP_t^\perp$ , respectively. In the joint regressions, we consider  $X_t = (PVRP_t, M3RP_t^\perp)'$  and  $(PVRP_t, M4RP_t^\perp)'$ . We use Newey-West standard errors to correct for the autocorrelation and heteroskedasticity in error terms.

The predictive regression results are reported in Table 2, including univariate regressions using QVRP, PVRP,  $M3RP^\perp$ , and  $M4RP^\perp$ , respectively, and multivariate regressions using PVRP and  $M3RP^\perp$ , and PVRP and  $M4RP^\perp$  jointly. Consistent with the literature, in the univariate regressions of QVRP (first column of each horizon), the coefficients on QVRP are positive and highly significant for horizons of up to 6-month. QVRP achieves a maximum adjusted  $R^2$  of over 9% at the 3-month horizon. The predictive power of QVRP tapers off as the prediction horizon gets longer. As a cleaner measure of variance risk premium, PVRP has better predictive performance than QVRP in all horizons, with larger  $t$ -statistics and  $R^2$ 's.

[Insert Table 2 here.]

The predictive power of the higher moment risk premia (the third and fourth columns of each horizon) has a different pattern. The coefficients on  $M3RP^\perp$  are negative across all horizons. At the short end (1- and 3-month), the predictive regressions on  $M3RP^\perp$  feature small  $t$ -statistics and low  $R^2$ 's. At medium horizons (6- to 24-month), by contrast,  $M3RP^\perp$  is significantly negative.  $R^2$ 's of  $M3RP^\perp$  from 6- to 24-month range from

3.55% to 5.95%. Univariate regressions of  $M4RP^\perp$  exhibit a similar pattern, except that the coefficients on  $M4RP^\perp$  are positive.  $M3RP^\perp$  and  $M4RP^\perp$  share similar levels of predictive coefficients,  $t$ -statistics, and  $R^2$ 's. This is not surprising because  $M3RP$  and  $M4RP$  are highly correlated with a linear correlation coefficient of -0.98.

The multivariate predictive regressions reveal interesting findings on the higher moment risk premia. First, the coefficients on  $PVRP$  and  $M4RP^\perp$  are always positive, and those on  $M3RP^\perp$  are always negative. Since  $PVRP$  and  $M3RP^\perp$  predict future returns with opposite signs, the predictive power of  $QVRP$  is substantially hindered as a result of the negative prediction by  $M3RP$  canceling out the positive prediction by  $PVRP$ . This could explain why  $QVRP$  is not as a strong predictor as  $PVRP$  at short horizons and has less predictive power at medium horizons than  $M3RP$ .

Second, different from the univariate regressions, where the higher moment risk premia are only significant at longer horizons,  $M3RP^\perp$  and  $M4RP^\perp$  coefficients are statistically significant at all horizons in the joint regressions. At short horizons,  $M3RP^\perp$  and  $M4RP^\perp$  coefficients turn highly statistically significant in the multivariate regressions despite their insignificance in the univariate regressions. The  $t$ -statistics of  $M3RP^\perp$  is -2.7 in the joint regression for the 1-month horizon, and -3.5 for the 3-month horizon. Across all horizons, most of the  $t$ -statistics of  $M3RP^\perp$  and  $M4RP^\perp$  coefficients in the joint regressions are larger in magnitude than those in the univariate regressions.

Finally, combining higher moment risk premia and  $PVRP$  leads to improvements in

$R^2$ 's. The  $R^2$ 's of joint regressions are always higher than those of the univariate regressions across all horizons. For example, at the 6-month horizon, the  $R^2$  of the joint regression with PVRP and  $M3RP^\perp$  is as high as 9.7%, while the univariate regression of QVRP only has an  $R^2$ 's of 3.7%. A more impressive example is the 9-month predictive results, in which case the joint regression of PVRP and  $M3RP^\perp$  produces an  $R^2$  of 5.5%, more than five times of QVRP (0.9%).

To compare the predictive power of different moment risk premia over different horizons, we plot the graph of adjusted  $R^2$ 's as a function of forecasting horizons in Figure 3. Panel (a) shows  $R^2$ 's of the univariate regressions of moment risk premia. Panel (b) shows  $R^2$ 's of QVRP and the joint regression of PVRP and  $M3RP^\perp$ . Panel (a) shows that PVRP is a strong predictor at short end. After reaching its peak at 3-month horizon, the  $R^2$  tapers off and remains low after 6-month. We see a less bumpy curve in the higher moment risk premia.  $R^2$ 's of  $M3RP^\perp$  and  $M4RP^\perp$  are at similar magnitude. Both of them reach their highest at 6- to 10-month horizons and remain at moderate levels until 24-month. In terms of  $R^2$ 's, PVRP outperforms the higher moment risk premia at horizons shorter than 6-month and underperforms them ever after.

Panel (b) illustrates the improvement in prediction power across different horizons when we combine the predictability of moment risk premia. We observe that  $R^2$ 's of the joint regression stay above those of QVRP across all horizons. The improvement is more pronounced over longer horizons. The evidence illustrates that the higher moment risk

premia contain complementary predictive power to PVRP. As a result, separating the moment risk premia in QVRP and including them in a joint regression effectively combine the short-term predictability of PVRP and the medium-term predictability of the higher moment risk premia.

[Insert Figure 3 here.]

Note that we use the lagged realized moments as proxies for the physical moments in the next month in this section. The advantage of this specification is that both the risk-neutral moments and the lagged realized moments are available ex ante without specifying any forecasting model. However, by using the lagged realized moments, we implicitly assume that the realized moments are random walks. In the online appendix, we discuss two additional robustness checks, in which we use predicted realized moments and intraday moments to construct moment risk premia. The moment risk premia are then used to predict aggregate stock returns.

It is worth noting that the high-frequency second moment and the high-frequency higher moments have different properties. Under reasonable assumptions, utilizing intraday return data provides a more consistent and efficient estimator for the return variance than using daily returns, but this is generally not the case for realized higher moments. As shown in [Neuberger \(2012\)](#), skewness estimates of long-horizon log returns can be very different from those of the high-frequency log returns due to the leverage effect. For simple returns, skewness estimates of long-horizon returns shall be different from those of

short-horizon returns due to compounding even in the absence of the leverage effect (see Bessembinder (2018)). In the online appendix, we also derive the sources of higher moments of long-horizon log returns in an illustrative example. As shown in the online appendix, our results remain qualitatively similar when using moment risk premia constructed by intraday or predicted moments.

## B. Predicting the Market Returns with Weighted Least Squares

Time-varying market return volatility might create heteroskedasticity in time-series of the error term in the return predictability regressions. Indeed, Johnson (2019) finds that the return predictability afforded by the conventional variance risk premium is not robust and is driven by several extreme observations with high variance. To deal with potential heteroskedasticity, we consider the weighted least squares (WLS) in addition to ordinary least squares (OLS) in this section.

We estimate the regression coefficients in Equation (12) using WLS in two steps. In the first step, we estimate  $\hat{\sigma}_{t,t+h|t}^2$ , the conditional variance of the market return from  $t$  to  $t+h$ . Following Johnson (2019), we estimate  $\hat{\sigma}_{t,t+h|t}$  using realized variance in the past month and in the past year:

$$\hat{\sigma}_{t,t+h|t}^2 = \hat{a} + \hat{b}\sigma_{t-1,t}^2 + \hat{c}\sigma_{t-11,t}^2,$$

where  $\sigma_{t-1,t}^2$  is the sum of squared daily market returns in the past month and  $\sigma_{t-11,t}^2$  is the sum of squared daily market returns in the past year.  $\hat{a}$ ,  $\hat{b}$ , and  $\hat{c}$  are the estimated coefficients in a regression of  $\sigma_{t,t+h}^2$  on a constant,  $\sigma_{t-1,t}^2$ , and  $\sigma_{t-11,t}^2$ .

In the second step, we estimate the following predictive regression for predictor  $X_t$  using the following regression,

$$(13) \quad R_{t,t+h}/\hat{\sigma}_{t,t+h|t} = \alpha_h/\hat{\sigma}_{t,t+h|t} + \beta_h' X_t/\hat{\sigma}_{t,t+h|t} + \varepsilon_{t,t+h}.$$

[Insert Table 3 here.]

Table 3 reports the WLS predictive regression results. We confirm with [Johnson \(2019\)](#) that the  $t$ -statistics of WLS estimators are smaller in absolute value across different horizons. Nevertheless, the predictive coefficients, significance, and  $R^2$ 's are qualitatively similar to those reported in Table 2.

### C. Control for Stock Return Predictors in [Welch and Goyal \(2008\)](#)

To relate our findings to the voluminous literature on market return predictability, we consider a set of predictors documented in the previous literature as control variables. Specifically, we consider 11 variables used in [Welch and Goyal \(2008\)](#): dividend price ratio (DP), dividend yield (DY), log earnings-price ratio (EP), book-to-market ratio (BM), interest rate on a three-month Treasury bill (TBL), difference between Moody's BAA- and

AAA-rated corporate bond yields (DFY), long-term government bond yield (LTY), net equity expansion (NTIS), inflation calculated from the CPI for all urban consumers (INFL), long-term government bond return (LTR), and difference between the long-term corporate bond return and the long-term government bond return (DFR).

Since the higher moment risk premia, M3RP and M4RP, are very similar in terms of predictability, as shown in the baseline results of Table 2, we only report results for the joint regressions of PVRP and M3RP in this section to save space. We report the results of return regressions on PVRP and M3RP for 1-month (Panel A) and 12-month horizons (Panel B) in Table 4 with each of the 11 predictors as control variable in each column.<sup>4</sup> Table 4 shows that the coefficients on PVRP and M3RP are both statistically significant in all regressions. In Panel A, only DP has significant coefficients among the 11 control variables. The adjusted  $R^2$ 's of 1-month prediction range from 5% to 6.5%, similar to the baseline results.

[Insert Table 4 here.]

In Panel B, DP is the only significant predictor at 12-month horizon. The  $R^2$  of the regression with DP as the control variable increases from 5% in the baseline results to 16.5%. Despite insignificant coefficients, BM and NTIS also substantially increase the 12-month adjusted  $R^2$ 's of the baseline results to 13% and 8%, respectively. This is

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<sup>4</sup>The results for 3-, 6-, 9-, 24-month as well as for the joint regression of PVRP and M4RP are qualitatively similar. The results are available upon request.

consistent with [Welch and Goyal \(2008\)](#), who find that these predictors perform better at yearly horizons.

## D. Control for Short-term Predictors

Control predictors considered in Section [IV.C](#) are known to contain predictability over multi-year horizons. Since we focus on the short-horizon predictability of moment risk premia, we control for a set of established short-term predictors in this section. We consider short interest (SI) from [Rapach et al. \(2016\)](#) and the cross-sectional book-to-market factor ( $BM_{KP}$ ) from [Kelly and Pruitt \(2013\)](#), which are shown to contain short-term predictability for market returns. In addition, since M3RP is closely related to jumps and skewness, we consider several jump- or skewness-related predictors: realized signed jumps (RSJ) from [Guo et al. \(2019\)](#), value-weighted average skewness ( $SKEW_{VW}$ ) and equal-weighted average skewness ( $SKEW_{EW}$ ) from [Jondeau et al. \(2019\)](#), and left jump probability (LJP), which is the probability of a 10% weekly down move from [Andersen et al. \(2015\)](#).<sup>5</sup>

Table [5](#) reports the correlation matrix of M3RP and the aforementioned predictors.

While M3RP is related to jumps and skewness, the correlations between M3RP and RSJ,

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<sup>5</sup>The book-to-market predictors are calculated with the data and codes from Seth Pruitt's website. Since these factors are data-driven, we use different BM factors for different predictive horizons, as implemented by [Kelly and Pruitt \(2013\)](#). Specifically, for each predictive horizon, we first extract BM factors using the [Kelly and Pruitt \(2013\)](#) data that date back to 1930. Then we use the BM factors from 1990 onwards in the controlled regressions. The LJP series is downloaded from <https://tailindex.com/index.html>. Numbers of observations vary in each regression depending on the availability of the control variables.

$SKEW_{VW}$ , and  $SKEW_{EW}$  are as small as -0.08. The largest absolute correlation (-0.61) is between M3RP and LJP, as both are related to option-implied jumps.

[Insert Table 5 here.]

Table 6 reports the predictive regression results with these control variables. Similar to Section IV.C, we add one control variable at a time and report the regression results in each column. We report the regression results for 1- (Panel A) and 12-month (Panel B) horizons. The table shows that the predictive coefficients on M3RP remain significantly negative after we include these control variables.

[Insert Table 6 here.]

Among these control variables, SI, RSJ, and  $BM_{KP}$  have significant coefficients in both 1- and 12-month horizons, implying that M3RP and these predictors contain orthogonal information for future market returns. The highest  $R^2$  for both horizons is achieved in the joint regression of M3RP, PVRP and  $BM_{KP}$ . Predictive coefficients on average skewness and LJP are not significant in the joint regression with PVRP and M3RP. Therefore, while the economic intuition of M3RP may partially overlap with the existing jump or skewness-related variables, the coefficient of M3RP remains significant after controlling for these variables.

## V. Out-of-sample Performance

While many variables can significantly predict stock market returns in sample, most of them perform poorly in the out-of-sample (OOS) tests. Several studies, such as Drechsler (2013), Kilic and Shaliastovich (2019), and Buss et al. (2019), have shown that the predictability of the traditional variance premium survives out-of-sample tests. In this section we investigate the out-of-sample predictability of moment risk premia. In Section V.A, we report OOS  $R^2$ 's of the baseline regressions as well as various regressions in Section IV. We conduct asset allocation analysis in Section V.B and show that the predictability of moment risk premia can be exploited by investors to improve portfolio performance.

### A. Out-of-sample $R^2$

For each univariate predictive regression, we calculate the return forecast at time  $t$ , using only the data available up to time  $t$ :

$$\hat{R}_{t,t+h} = \hat{\alpha}_{t,h} + \hat{\beta}_{t,h}X_t, \quad t \geq T_0,$$

where  $\hat{\alpha}_{t,h}$  and  $\hat{\beta}_{t,h}$  are OLS estimates from regression (12). We use observations in the first half sample as the initial sample and construct the first return forecast. Then we construct the remaining return forecasts using an expanding window until the end of the sample.

As pointed out by [Rapach, Strauss, and Zhou \(2010\)](#), combining forecasts from individual predictors can yield less volatile and more reliable forecasts. It is particular useful in our case to reduce noisy signals from higher moment risk premia at short horizons and those from PVRP at longer horizons. We construct our forecast combination of PVRP and M3RP<sup>⊥</sup> (M4RP<sup>⊥</sup>) as

$$\hat{R}_{t,t+h}^{\text{PVRP}+\text{M3RP}^\perp} = w_{h,t}^{\text{PVRP}} \hat{R}_{t,t+h}^{\text{PVRP}} + w_{h,t}^{\text{M3RP}^\perp} \hat{R}_{t,t+h}^{\text{M3RP}^\perp},$$

where  $w_{h,t}^x$  is the ex ante combining weights on predictor  $x$  formed at time  $t$  for forecast horizon  $h$ ,

$$w_{h,t}^x = \frac{(\text{CSE}_{h,t}^x)^{-1}}{(\text{CSE}_{h,t}^{\text{PVRP}})^{-1} + (\text{CSE}_{h,t}^{\text{M3RP}^\perp})^{-1}}, \quad t \geq T_0 + 1.$$

$\text{CSE}_{h,t}^x$  is the cumulative squared forecast error of the univariate predictive regression with predictor  $x$ ,

$$(14) \quad \text{CSE}_{h,t}^x = \sum_{l=T_0}^t (R_{l,l+h} - \hat{R}_{l,l+h}^x)^2, \quad x = \text{PVRP}, \text{M3RP}^\perp.$$

At time  $T_0$  when the first forecast is made, there is no history of prediction errors to differentiate the two models. Hence, we set the initial weights  $w_{T_0,h}^x$  to 1/2. The forecast combination of PVRP and M4RP<sup>⊥</sup> follows a similar procedure.

Following [Welch and Goyal \(2008\)](#), we define OOS  $R^2$  for horizon  $h$  and prediction

$x$  as

$$(\text{OOS } R^2)_h^x = 1 - \frac{\text{CSE}_{h,T}^x}{\text{CSE}_{h,T}^{\text{bm}}},$$

where  $\text{CSE}_{h,T}^{\text{bm}}$  is the cumulative squared forecast error of a benchmark prediction, which uses the average excess return from the beginning of the sample through month  $t$  as the return forecast for the next period. The OOS  $R^2$  measures the forecast accuracy relative to the historical average return. A positive OOS  $R^2$  implies that the predictor outperforms the naive forecast using the historical mean, and a negative OOS  $R^2$  implies under-performance.

The results of out-of-the-sample forecasts are reported in Table 7, including univariate regressions using QVRP, PVRP, M3RP<sup>⊥</sup>, and M4RP<sup>⊥</sup>, respectively, and forecast combinations using PVRP and M3RP<sup>⊥</sup>, and PVRP and M4RP<sup>⊥</sup>. In the baseline regression (Panel A), QVRP and PVRP provide positive OOS  $R^2$ 's at horizons up to 6-month in the univariate regressions, with the latter slightly higher. The higher moment risk premia, M3RP<sup>⊥</sup> and M4RP<sup>⊥</sup>, give positive OOS  $R^2$ 's from 6- to 24-month. The out-of-sample predictive performance of higher moment risk premia reaches a peak at 9-month horizon, with an OOS  $R^2$  of 7.07%. The forecast combinations of PVRP and M3RP<sup>⊥</sup> (M4RP<sup>⊥</sup>) deliver positive OOS  $R^2$ 's at all horizons, with a maximum of over 10% at 3-month horizon. In most cases, the OOS  $R^2$ 's forecast combinations fall between those of the two

univariate regressions, with the exception of 24-month forecasts. Overall, in terms of out-of-sample performance, QVRP and PVRP perform well up to 6-month horizon, while higher moment risk premia perform well in longer horizons. Forecast combination incorporates the advantages of the two and provides positive OOS  $R^2$  across all horizons.

[Insert Table 7 here.]

Panel B reports the OOS  $R^2$ 's of the WLS regressions in Section IV.B. For every  $t$ , we re-estimate  $\hat{\sigma}_{t,t+h}^2$  using information only up to time  $t$ . Then we run WLS regression of returns from month  $h + 1$  to month  $t$  scaled by  $\hat{\sigma}_{t,t+h|t}^2$  on candidate predictors from the beginning of the sample to month  $t - h$  scaled by  $\hat{\sigma}_{t,t+h|t}^2$  to obtain WLS coefficients. The return forecasts are constructed the same way as in an OLS regression, except that the predictive coefficients are WLS estimators. Compared with Panel A, we find that the OOS  $R^2$  for every prediction and every horizon increases when applying WLS. This confirms the finding of Johnson (2019) that predictors perform better out-of-sample using the WLS estimator.

[Insert Figure 4 here.]

Figure 4 compares the out-of-sample  $R^2$ 's of predictions over different horizons of the baseline results. Similar to Figure 3, we present OOS  $R^2$ 's of moment risk premia in Panel (a), and OOS  $R^2$ 's of QVRP and the forecast combination of PVRP and M3RP $^\perp$  in Panel (b). Panel (a) shows that the patterns of OOS  $R^2$ 's of moment risk premia are

similar to those of the in-sample ones. OOS  $R^2$  of PVRP is the highest at 3-month horizon and drops to negative values as the forecasting horizon increases. M3RP $^\perp$  and M4RP $^\perp$ , to the contrary, have negative OOS  $R^2$ 's at the short end and positive ones at medium horizons. Panel (b) shows that the forecast combination of PVRP and M3RP $^\perp$  substantially improves OOS  $R^2$ 's at medium horizons. Comparing with PVRP in Panel (a), the positive OOS  $R^2$ 's of the forecast combination at 6-month and beyond mostly come from the higher moment risk premia.

## B. Asset Allocation

In this section, we evaluate the economic gain of moment risk premia from the asset allocation perspective. Similar to [Campbell and Thompson \(2007\)](#), [Rapach et al. \(2010\)](#), and [Rapach et al. \(2016\)](#), we consider a mean-variance investor who allocates her wealth between a stock and a risk-free asset. At the end of month  $t$ , she invests  $w_t$  of her wealth in the market portfolio and the rest in risk-free assets and hold the portfolio for  $h$  months. The optimal weight of the stock is determined by  $w_t = \frac{\hat{R}_{t+h}}{\gamma \hat{\sigma}_{t+h}^2}$ , where  $\gamma$  is the investor's relative risk aversion.  $\hat{R}_{t+h}$  and  $\hat{\sigma}_{t+h}^2$  are the forecast of excess return and return variance  $h$ -month ahead. VIX $^2$  is used as the forecast of return variance, because it reflects investors' expectation of the return variation in the future. We calculate certainty equivalent return (CER) for this investor:  $\text{CER} = R_p - 0.5\gamma\sigma_p^2$ , where  $R_p$  and  $\sigma_p^2$  are the mean and variance of the portfolio return over the forecasting evaluation period.

We consider the scenario that the portfolio weights are larger than zero and smaller than 1, i.e., no short sales or leverage.<sup>6</sup> We annualize the CER so that it can be interpreted as the annual portfolio management fee that the investor would be willing to pay to have access to the predictive regression forecast.

[Insert Table 8 here.]

The results of the out-of-sample CER are reported in Table 8. We also compute the CER for the buy-and-hold strategy as a benchmark. Similar to the calculation of OOS  $R^2$ , we employ an expanding window to estimate the predictive regression parameters. The first half of the sample is used to estimate the first set of regression parameters. In the table, we report CER of the investment strategy based on QVRP, PVRP, M3RP<sup>⊥</sup>, and M4RP<sup>⊥</sup> in the univariate predictive regression and forecast combination for PVRP & M3RP<sup>⊥</sup> and PVRP & M4RP<sup>⊥</sup>.

We consider three levels of the risk aversion parameter:  $\gamma = 3, 5, 7$ . We observe that in most cases PVRP outperforms QVRP in terms of higher CERs, suggesting that a cleaner PVRP has more economic value for a risk-averse investor than QVRP. QVRP and PVRP perform well up to 6-month horizon, while M3RP<sup>⊥</sup> and M4RP<sup>⊥</sup> perform well in longer horizons. When  $\gamma = 3$ , the forecast combinations yield higher CERs than QVRP from 3-month to 12-month horizons. All predictors and forecast combinations provide higher CERs than that the buy-and-hold strategy. The results are similar for  $\gamma = 5$  and  $\gamma = 7$ .

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<sup>6</sup>The case that allows short sales ( $w_t \in [-1, 1]$ ) gives similar results.

Overall, the results in this section suggest that different moment risk premia show advantages in terms of CERs across different horizons. Combining predictability in different moment risk premia has substantial economic value for a risk-averse investor.

## VI. Conclusion

This paper investigates how to use option-implied information to improve market return predictability over short to medium horizons. The conventional variance risk premium has been shown to be a strong predictor for market returns in the recent literature. Since the conventional variance risk premium contains higher moment premia besides the second moment risk premium, we exploit the predictive power of each moment risk premium separately and jointly.

Using the S&P 500 index and options data, we run predictive regressions of 1- to 24-month excess returns of the market equity index on moment risk premia. We find that, (1) PVRP, the pure variance risk premium, is a better predictor than QVRP, which is contaminated by higher moment risk premia; (2) PVRP contains short-term predictive power for market returns with statistically significant positive coefficients, whereas M3RP (M4RP) contain medium-term predictive power for market returns with statistically significant negative (positive) coefficients. (3) When M3RP and M4RP are separated from QVRP, PVRP and M3RP (M4RP) jointly deliver higher in-sample and out-of-sample  $R^2$ 's than QVRP, across all horizons from 1- to 24-month. The predictability afforded by M3RP

(M4RP) survives a series of robustness checks.

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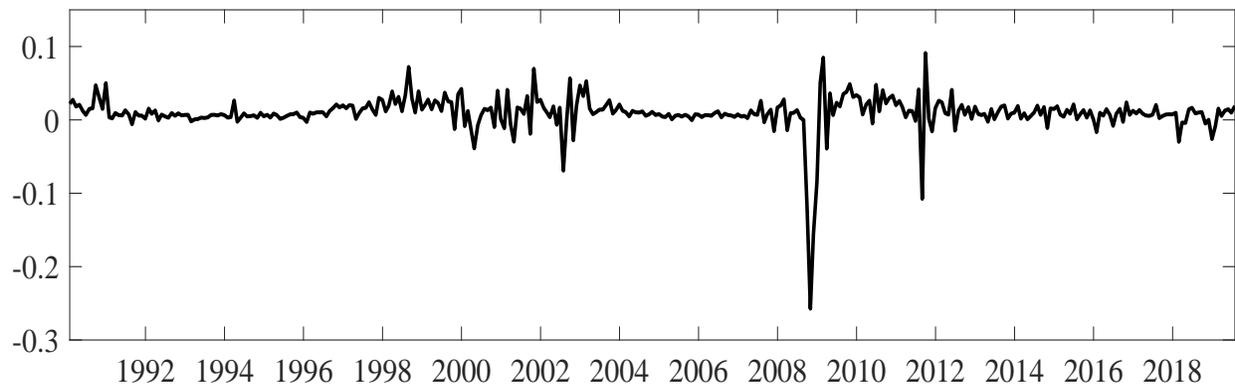
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Figure 1: Time Series of VRP and PVRP

This figure shows time series of quasi variance risk premium (QVRP) and pure variance risk premium (PVRP) from January 1990 to July 2019.

(a) QVRP



(b) PVRP

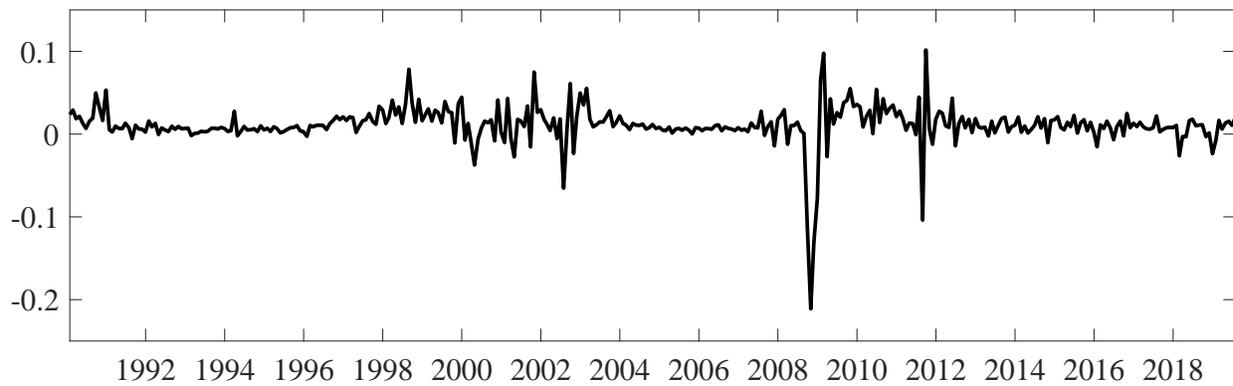
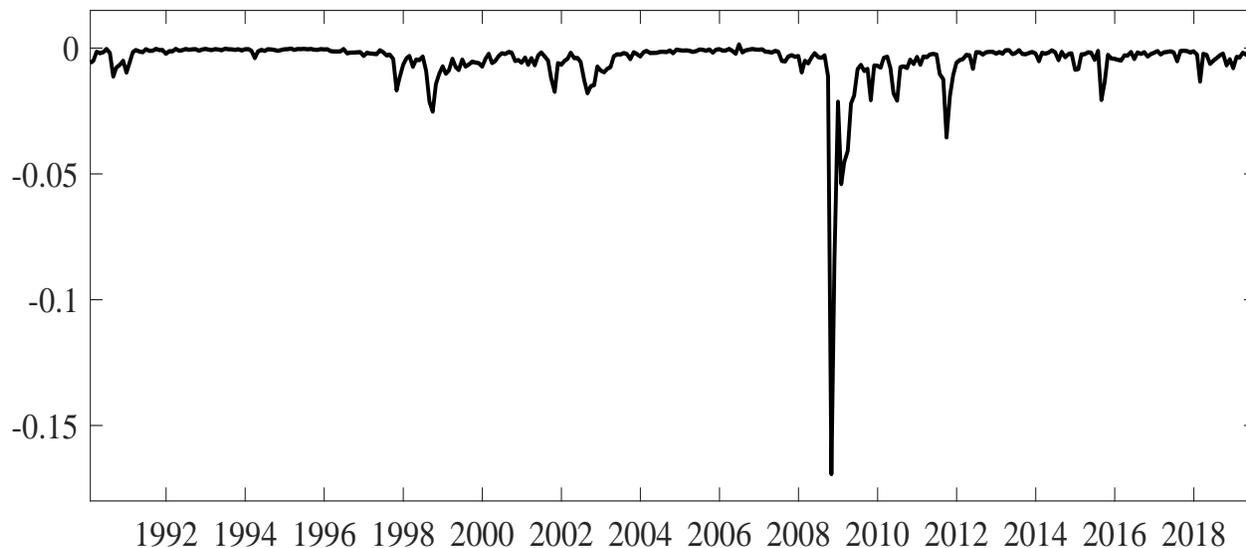


Figure 2: Time Series of M3RP and M4RP

This figure shows time series of the third moment risk premium (M3RP) and the fourth moment risk premium (M4RP) from January 1990 to July 2019.

(a) M3RP



(b) M4RP

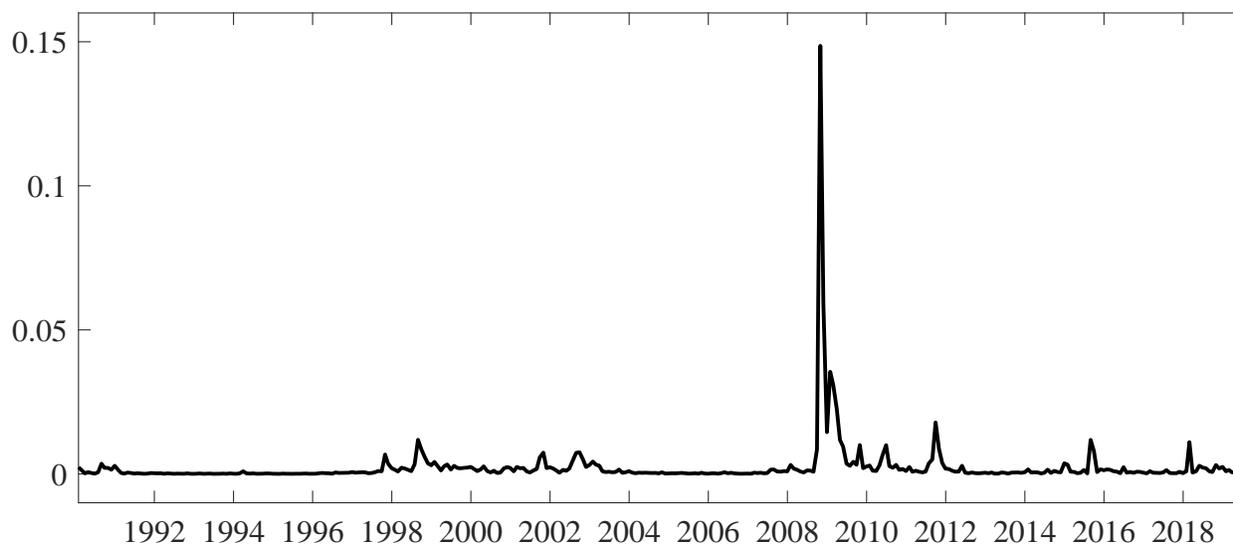
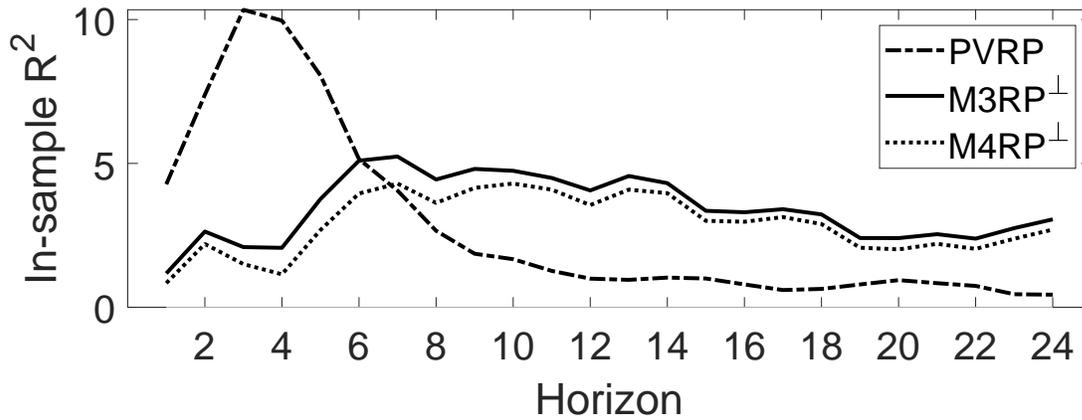


Figure 3: In-sample  $R^2$  of Predictive Regressions

The top panel plots the in-sample  $R^2$  (in percentage) of the predictive regressions for the S&P 500 return afforded by moment risk premia in the univariate regressions, as a function of forecasting horizon (in months). We consider the pure variance risk premium (PVRP), the residual of the third moment risk premium after regressing on PVRP ( $M3RP^\perp$ ), and the residual of the fourth moment risk premium after regressing on PVRP ( $M4RP^\perp$ ). The bottom panel plots the in-sample  $R^2$  (in percentage) of the predictive regressions for the market equity return afforded by the quasi variance risk premium (QVRP) in the univariate regression and by PVRP and  $M3RP^\perp$  in the joint regression, as a function of forecasting horizon (in months).

(a) In-sample  $R^2$  of Moment Risk Premia



(b) In-sample  $R^2$  of QVRP and PVRP &  $M3RP^\perp$  Jointly

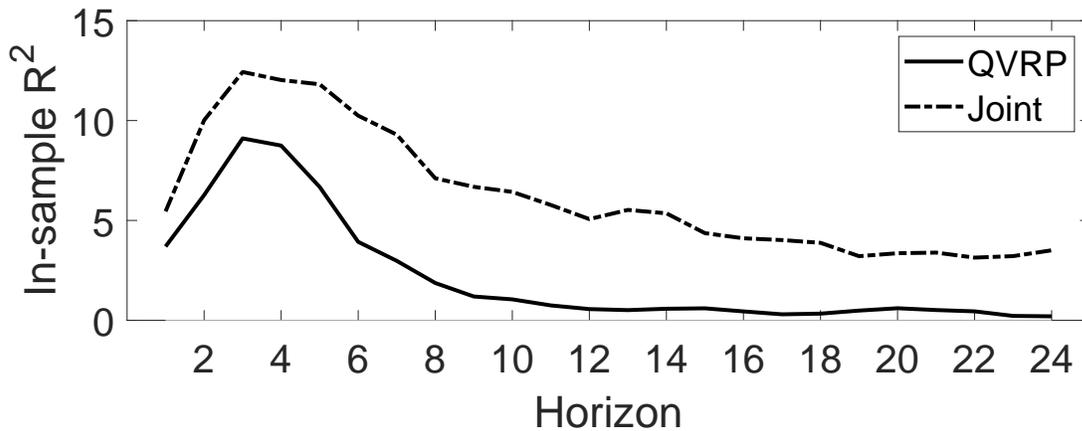
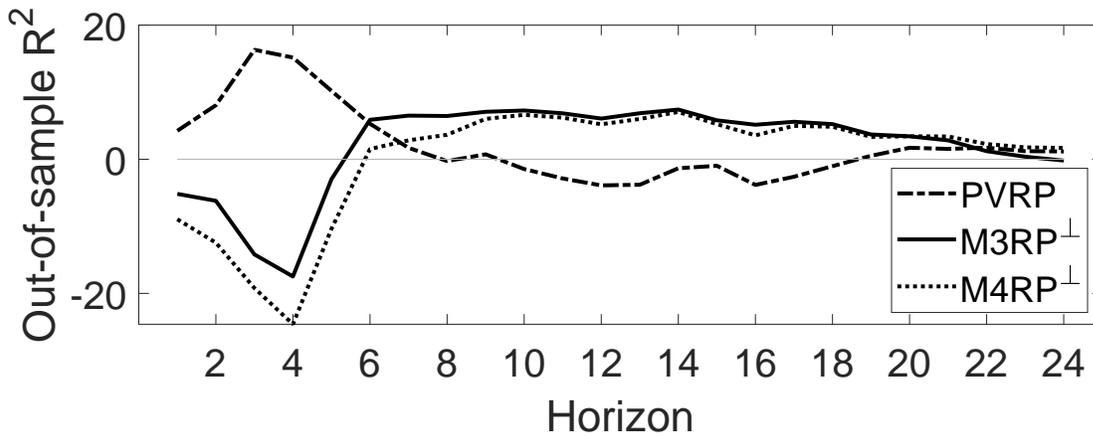


Figure 4: Out-of-sample  $R^2$  of Predictive Regressions

The top panel plots the out-of-sample (OOS)  $R^2$  (in percentage) of the predictive regressions for the S&P 500 return afforded by moment risk premia in the univariate regressions, as a function of forecasting horizon (in months). We consider the pure variance risk premium (PVRP), the residual of the third moment risk premium after regressing on PVRP ( $M3RP^\perp$ ), and the residual of the fourth moment risk premium after regressing on PVRP ( $M4RP^\perp$ ). The bottom panel plots the OOS  $R^2$  (in percentage) of the quasi variance risk premium (QVRP) in the univariate regression and the forecast combination of PVRP and  $M3RP^\perp$ .

(a) Out-of-sample  $R^2$  of Moment Risk Premia



(b) Out-of-sample  $R^2$  of QVRP and PVRP &  $M3RP^\perp$  Combination

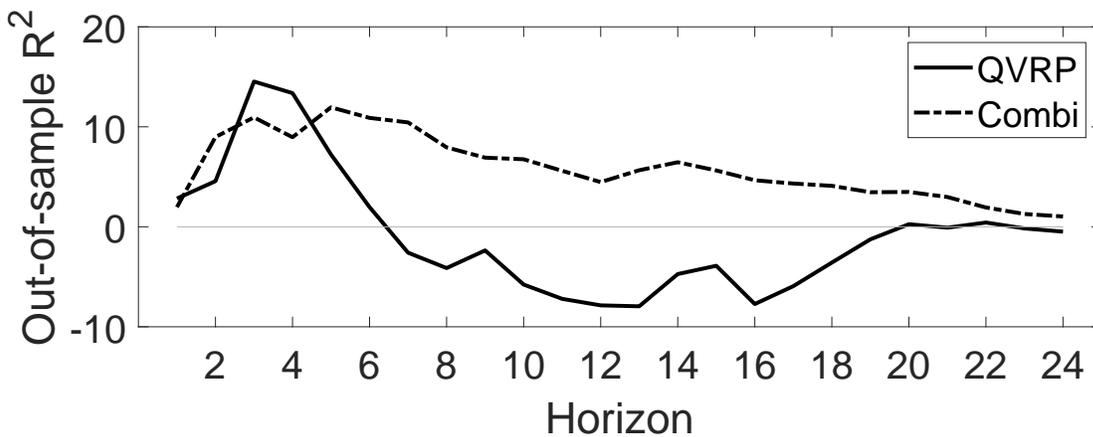


Table 1: Summary Statistics of Moment Risk Premia

Panel A reports mean, standard deviation (Std. Dev.), median, 5 percent quantile (5th Pctl), and 95 percent quantile (95th Pctl) of risk neutral and realized moments. Risk neutral moments include  $VIX^2$ , IV (implied variance), IM3 (implied third moment), and IM4 (implied fourth moment). Realized moments include QRV (quasi realized variance), RV (realized variance), RM3 (realized third moment), and RM4 (realized fourth moment). Panel B reports mean, standard deviation (Std. Dev.), median, 5 percent quantile (5th Pctl), and 95 percent quantile (95th Pctl), and autocorrelation coefficient (AR(1)) of the moment risk premia: QVRP (quasi variance risk premium), PVRP (pure variance risk premium), M3RP (third moment risk premium), and M4RP (fourth moment risk premium). Moment risk premia are differences between the risk neutral and realized moments. Panel C reports correlation matrix among moment risk premia. All variables are denoted in percentage per annum. The sample period extends from January 1990 to July 2019.

Panel A: Risk neutral and realized moments				
	$VIX^2$	IV	IM3	IM4
Mean	3.86	4.02	-0.51	0.23
Std. Dev.	3.77	4.06	1.14	0.93
Median	2.75	2.79	-0.24	0.06
5th Pctl	1.05	1.08	-1.80	0.01
95th Pctl	9.64	10.08	-0.04	0.79
	QRV	RV	RM3	RM4
Mean	2.91	2.91	$-7.15 \times 10^{-3}$	$4.10 \times 10^{-3}$
Std. Dev.	5.07	5.07	0.09	0.03
Median	1.54	1.54	$9.02 \times 10^{-4}$	$3.23 \times 10^{-4}$
5th Pctl	0.41	0.41	-0.07	$2.13 \times 10^{-5}$
95th Pctl	8.93	8.97	0.04	$9.28 \times 10^{-3}$
Panel B: Moment risk premia				
	QVRP	PVRP	M3RP	M4RP
Mean	0.95	1.10	-0.51	0.23
(t-stats)	7.05	8.61	-8.33	4.78
Median	0.90	1.00	-0.23	0.06
5th Pctl	-1.45	-1.23	-1.72	0.01
95th Pctl	3.96	4.25	-0.04	0.79
AR(1)	0.38	0.34	0.55	0.46
Panel C: Correlation matrix				
	QVRP	PVRP	M3RP	M4RP
QVRP	1.00	0.99	0.47	-0.55
PVRP	0.99	1.00	0.36	-0.45
M3RP	0.47	0.36	1.00	-0.98
M4RP	-0.55	-0.45	-0.98	1.00

Table 2: Market Return Predictive Regressions at Different Horizons

The table reports estimated regression coefficients and  $R^2$  of the predictability regressions for one to 24-month excess returns on the S&P 500 index. Heteroskedasticity- and autocorrelation-robust t-statistics are reported in the parenthesis. For each horizon, we report predictive regression results of the univariate regressions on the quasi variance risk premium (QVRP), the pure variance risk premium (PVRP), the residual of the third moment risk premium after regressing on PVRP ( $M3RP^\perp$ ), the residual of the fourth moment risk premium after regressing on PVRP ( $M4RP^\perp$ ), bivariate regression on PVRP and  $M3RP^\perp$  jointly, and bivariate regression for PVRP and  $M4RP^\perp$  jointly. Returns are observed monthly with the sample period ranging from January 1990 to July 2019.

	1-month						3-month					
QVRP	0.31 (3.76)						0.85 (6.26)					
PVRP	0.35 (3.86)		0.35 (3.41)		0.35 (3.50)		0.95 (5.30)		0.95 (3.31)		0.95 (3.46)	
$M3RP^\perp$	-0.41 (-1.38)		-0.41 (-2.69)				-0.96 (-1.37)		-0.96 (-3.49)			
$M4RP^\perp$			0.46 (1.10)		0.46 (2.35)				1.08 (1.22)		1.08 (3.03)	
$R^2$	3.69	4.28	1.18	0.84	5.45	5.12	9.11	10.34	2.09	1.50	12.43	11.84
Adj. $R^2$	3.42	4.01	0.90	0.56	4.92	4.58	8.85	10.08	1.81	1.22	11.93	11.34
	6-month						9-month					
QVRP	0.82 (5.50)						0.57 (2.24)					
PVRP	0.99 (5.66)		0.99 (2.79)		0.99 (2.94)		0.75 (2.91)		0.75 (2.43)		0.75 (2.45)	
$M3RP^\perp$	-2.21 (-2.71)		-2.21 (-3.96)				-2.69 (-3.39)		-2.70 (-3.55)			
$M4RP^\perp$			2.57 (2.40)		2.57 (4.04)				3.31 (3.28)		3.32 (4.06)	
$R^2$	3.93	5.15	5.09	3.95	10.24	9.10	1.19	1.86	4.81	4.15	6.68	6.03
Adj. $R^2$	3.65	4.88	4.82	3.68	9.73	8.58	0.91	1.58	4.53	3.87	6.14	5.49
	12-month						24-month					
QVRP	0.47 (1.42)						0.46 (1.11)					
PVRP	0.65 (1.98)		0.66 (2.22)		0.66 (2.21)		0.72 (1.59)		0.73 (1.45)		0.74 (1.47)	
$M3RP^\perp$	-2.96 (-3.39)		-2.96 (-3.37)				-4.27 (-2.48)		-4.28 (-2.42)			
$M4RP^\perp$			3.66 (3.40)		3.66 (3.90)				5.31 (2.76)		5.32 (2.85)	
$R^2$	0.56	0.99	4.06	3.55	5.07	4.57	0.20	0.44	3.06	2.70	3.50	3.15
Adj. $R^2$	0.27	0.71	3.78	3.27	4.51	4.01	-0.10	0.14	2.76	2.40	2.92	2.56

Table 3: Market Return Predictive Regressions at Different Horizons (WLS)

The table reports estimated regression coefficients and  $R^2$  of the predictability regressions using weighted least squares (WLS) for one to 24-month excess return on the S&P 500 index. Heteroskedasticity- and autocorrelation-robust t-statistics are reported in the parenthesis. For each horizon, we report predictive regression results of the univariate regressions on the quasi variance risk premium (QVRP), the pure variance risk premium (PVRP), the residual of the third moment risk premium after regressing on PVRP (M3RP $^\perp$ ), the residual of the fourth moment risk premium after regressing on PVRP (M4RP $^\perp$ ), bivariate regression on PVRP and M3RP $^\perp$  jointly, and bivariate regression for PVRP and M4RP $^\perp$  jointly. Returns are observed monthly with the sample period ranging from January 1990 to July 2019.

	1-month						3-month					
QVRP	0.33 (3.19)						0.83 (4.31)					
PVRP	0.35 (3.13)		0.30 (2.83)		0.30 (2.83)		0.89 (3.90)		0.77 (2.52)		0.78 (2.65)	
M3RP $^\perp$	-0.57 (-1.92)		-0.30 (-1.73)				-1.43 (-2.51)		-0.85 (-2.36)			
M4RP $^\perp$			0.78 (1.71)		0.40 (1.71)				1.90 (2.36)		1.05 (2.63)	
$R^2$	4.01	4.66	1.16	0.51	5.75	5.45	9.50	10.72	1.43	0.48	12.43	11.92
Adj. $R^2$	3.73	4.38	0.87	0.22	5.47	4.90	9.24	10.46	1.14	0.19	12.17	11.40
	6-month						9-month					
QVRP	0.90 (4.72)						0.72 (2.65)					
PVRP	1.01 (4.30)		0.78 (2.37)		0.79 (2.31)		0.85 (2.95)		0.60 (2.18)		0.59 (1.89)	
M3RP $^\perp$	-2.36 (-2.96)		-1.85 (-2.59)				-2.53 (-2.55)		-2.16 (-2.14)			
M4RP $^\perp$			3.16 (3.07)		2.43 (3.11)				3.56 (3.18)		3.07 (2.96)	
$R^2$	3.96	5.21	4.85	3.63	9.85	8.93	1.13	1.86	4.74	4.13	6.42	5.97
Adj. $R^2$	3.68	4.93	4.57	3.35	9.59	8.39	0.83	1.56	4.45	3.85	6.14	5.41
	12-month						24-month					
QVRP	0.66 (1.92)						0.69 (1.36)					
PVRP	0.80 (2.29)		0.53 (1.82)		0.52 (1.60)		0.91 (1.61)		0.56 (1.11)		0.56 (1.00)	
M3RP $^\perp$	-2.73 (-2.38)		-2.43 (-2.05)				-4.16 (-1.80)		-3.90 (-1.73)			
M4RP $^\perp$			3.84 (3.03)		3.43 (2.85)				5.64 (2.28)		5.28 (2.28)	
$R^2$	0.41	0.87	3.93	3.47	4.75	4.39	0.16	0.42	3.02	2.67	3.42	3.11
Adj. $R^2$	0.11	0.58	3.64	3.18	4.46	3.81	-0.15	0.11	2.71	2.36	3.12	2.50

Table 4: Predictive Regressions Controlling for Predictors in Welch and Goyal (2008)

This table reports regression results of the pure variance risk premium (PVRP), the third moment risk premium (M3RP), and control variables at 1-month and 12-month horizons in Panel A and B. In each column of the table, we add one control variable to the regression of PVRP and M3RP. The “Control” variable is specified in the column head of the table. The control variables are defined in Section IV.C. Variables are obtained from Amit Goyal’s website. The Newey-West t-statistics are reported in the parenthesis. We report  $R^2$  and adjusted  $R^2$  in the last two rows. The sample period is from January 1990 to December 2018.

	DP	DY	EP	BM	TBL	DFY	LTY	NTIS	INFL	LTR	DFR
Panel A: 1-month horizon											
Constant	0.07 (2.07)	-0.01 (-1.64)	0.03 (1.00)	-0.02 (-1.99)	0.00 (-0.65)	0.00 (0.27)	0.00 (0.26)	0.00 (-1.40)	0.00 (-0.86)	-0.01 (-1.48)	0.00 (-1.42)
M3RP	-0.38 (-2.51)	-0.41 (-2.72)	-0.55 (-3.64)	-0.41 (-2.79)	-0.40 (-2.61)	-0.56 (-3.61)	-0.40 (-2.65)	-0.44 (-2.81)	-0.36 (-2.36)	-0.41 (-2.72)	-0.41 (-2.71)
PVRP	0.44 (3.89)	0.43 (3.75)	0.46 (4.16)	0.43 (3.77)	0.42 (3.60)	0.43 (3.51)	0.42 (3.62)	0.42 (3.60)	0.44 (3.50)	0.45 (3.17)	0.44 (3.52)
Control	0.02 (2.23)	0.00 (1.14)	0.01 (1.15)	0.04 (1.66)	-0.06 (-0.67)	-0.69 (-1.18)	-0.11 (-1.06)	0.06 (0.53)	-0.72 (-1.56)	0.17 (1.49)	-0.08 (-0.68)
$R^2$	7.27	5.89	6.75	6.47	5.76	5.94	5.89	5.75	5.96	7.05	5.75
Adj. $R^2$	6.46	5.07	5.93	5.66	4.94	5.12	5.07	4.93	5.14	6.24	4.93
Panel B: 12-month horizon											
Constant	0.83 (2.27)	0.02 (0.35)	0.31 (1.12)	-0.13 (-1.12)	0.06 (1.90)	0.03 (0.42)	0.06 (0.93)	0.02 (0.55)	0.05 (1.52)	0.03 (0.91)	0.03 (1.01)
M3RP	-2.54 (-3.09)	-2.94 (-3.36)	-3.95 (-5.15)	-2.88 (-3.42)	-2.68 (-3.17)	-2.86 (-2.86)	-2.88 (-3.31)	-3.65 (-3.11)	-2.50 (-2.80)	-2.96 (-3.25)	-2.98 (-3.43)
PVRP	1.33 (3.54)	1.20 (3.07)	1.41 (3.93)	1.25 (3.01)	1.16 (2.86)	1.17 (2.99)	1.17 (2.89)	1.01 (2.61)	1.30 (2.52)	1.21 (2.76)	1.13 (2.96)
Control	0.20 (2.18)	0.00 (0.44)	0.09 (1.03)	0.57 (1.62)	-0.81 (-0.77)	0.51 (0.09)	-0.45 (-0.39)	1.74 (1.10)	-6.21 (-1.46)	0.28 (1.12)	0.34 (0.57)
$R^2$	17.27	5.43	8.87	14.07	6.38	5.11	5.37	9.74	6.56	5.35	5.21
Adj. $R^2$	16.53	4.58	8.05	13.30	5.54	4.26	4.51	8.93	5.71	4.50	4.36

Table 5: Correlation Matrix of Short-term Predictors

This table reports the correlation matrix among the third moment risk premium (M3RP) and other short-term predictors. We consider short interest (SI) from [Rapach et al. \(2016\)](#), realized signed jump (RSJ) in [Guo et al. \(2019\)](#), value-weighted average skewness ( $\text{SKEW}_{\text{VW}}$ ) and equal-weighted average skewness ( $\text{SKEW}_{\text{EW}}$ ) from [Jondeau et al. \(2019\)](#), left jump probability (LJP) from [Andersen et al. \(2015\)](#).  $\text{BM}_{\text{KP-1m}}$  and  $\text{BM}_{\text{KP-12m}}$  are the cross-section book-to-market factors, extracted using the three-pass-regression filter following [Kelly and Pruitt \(2013\)](#).

	M3RP	SI	RSJ	$\text{SKEW}_{\text{VW}}$	$\text{SKEW}_{\text{EW}}$	LJP	$\text{BM}_{\text{KP}}^{1m}$	$\text{BM}_{\text{KP}}^{12m}$
M3RP	1.00	-0.08	-0.07	-0.07	-0.10	-0.61	0.11	0.00
SI	-0.08	1.00	-0.07	-0.01	-0.04	0.04	-0.11	0.09
RSJ	-0.07	-0.07	1.00	-0.03	-0.05	0.09	0.02	0.01
$\text{SKEW}_{\text{VW}}$	-0.07	-0.01	-0.03	1.00	0.80	0.02	-0.10	-0.09
$\text{SKEW}_{\text{EW}}$	-0.10	-0.04	-0.05	0.80	1.00	0.09	-0.27	-0.31
LJP	-0.61	0.04	0.09	0.02	0.09	1.00	-0.01	-0.04
$\text{BM}_{\text{KP}}^{1m}$	0.11	-0.11	0.02	-0.10	-0.27	-0.01	1.00	0.69
$\text{BM}_{\text{KP}}^{12m}$	0.00	0.09	0.01	-0.09	-0.31	-0.04	0.69	1.00

Table 6: Predictive Regressions with Short-term Control Variables

This table reports regression results of the third moment risk premium (M3RP), pure variance risk premium (PVRP), and control variables at 1-month and 12-month horizons in Panel A and B. In each column of the table, we add one control variable to the regression of M3RP and PVRP. The “Control” variable is specified in the first row of the table. The Newey-West t-statistics are reported in the parenthesis. We report  $R^2$  and adjusted  $R^2$  in the last two rows in each panel.

	SI	RSJ	SKEW <sub>VW</sub>	SKEW <sub>EW</sub>	LJP	BM <sub>KP</sub>
Panel A: 1-month horizon						
Constant	0.00	0.00	0.00	0.00	-0.01	0.02
	(-1.56)	(-1.12)	(-0.83)	(-0.52)	(-1.27)	(2.07)
M3RP	-0.43	-0.35	-0.44	-0.44	-0.45	-0.31
	(-2.67)	(-2.29)	(-2.84)	(-2.94)	(-2.70)	(-2.03)
PVRP	0.42	0.38	0.43	0.44	0.44	0.37
	(3.64)	(3.54)	(3.14)	(3.44)	(3.46)	(2.88)
Control	-0.02	0.39	-0.03	-0.03	0.00	0.02
	(-2.21)	(3.14)	(-0.40)	(-0.29)	(0.17)	(2.56)
$R^2$	7.83	9.81	6.55	6.50	7.34	5.16
Adj. $R^2$	6.89	8.93	5.67	5.62	6.34	4.01
Panel B: 12-month horizon						
Constant	0.03	0.03	0.03	0.06	0.03	0.65
	(0.90)	(0.80)	(0.85)	(1.17)	(0.81)	(5.22)
M3RP	-3.36	-2.90	-3.11	-3.15	-3.82	-3.04
	(-3.83)	(-3.15)	(-3.43)	(-3.48)	(-2.52)	(-3.00)
PVRP	1.02	1.10	1.15	1.09	1.32	1.41
	(2.66)	(2.81)	(2.65)	(2.30)	(2.90)	(3.09)
Control	-0.28	0.79	-0.17	-0.58	-0.02	0.81
	(-1.96)	(2.77)	(-0.61)	(-0.81)	(-0.35)	(4.95)
$R^2$	20.40	6.50	5.87	6.55	9.82	26.46
Adj. $R^2$	19.57	5.55	4.95	5.64	8.80	25.53

Table 7: Out-of-sample  $R^2$  of the Moment Risk Premia

The table reports out-of-sample  $R^2$  of the predictability regressions using baseline regression (Panel A) and weighted least squares (WLS) regression (Panel B). For each horizon, we report OOS  $R^2$ 's of the univariate regressions for the quasi variance risk premium (QVRP), pure variance risk premium (PVRP), the residual of the third moment risk premium after regressing on PVRP ( $M3RP^\perp$ ), the residual of the fourth moment risk premium after regressing on PVRP ( $M4RP^\perp$ ), forecast combination for PVRP and  $M3RP^\perp$  (PVRP &  $M3RP^\perp$ ), and forecast combination for PVRP and  $M4RP^\perp$  (PVRP &  $M4RP^\perp$ ). The sample period is from January 1990 to July 2019.

	QVRP	PVRP	$M3RP^\perp$	$M4RP^\perp$	PVRP & $M3RP^\perp$	PVRP & $M4RP^\perp$
Panel A: Baseline Regression						
1-month	2.83	4.24	-5.16	-8.95	1.97	0.84
3-month	14.53	16.32	-14.19	-19.21	10.95	9.76
6-month	1.97	5.29	5.88	1.53	10.89	9.65
9-month	-2.36	0.74	7.07	6.02	6.92	7.12
12-month	-7.86	-3.90	6.06	5.22	4.47	4.69
24-month	-0.48	1.16	-0.20	1.69	1.03	2.05
Panel B: WLS Regression						
1-month	5.81	6.91	-1.47	-4.31	4.43	3.47
3-month	16.32	17.69	-7.03	-12.10	12.71	11.40
6-month	5.56	8.10	8.71	5.56	11.77	11.01
9-month	-1.16	2.39	10.04	8.52	9.54	9.63
12-month	-5.06	-1.09	9.24	7.90	7.41	7.45
24-month	1.32	2.59	0.29	2.30	1.89	2.91

Table 8: Out-of-sample CER Gains

The table reports the annualized certainty equivalent return (CER) in percent for a mean-variance investor who allocates between S&P 500 index and risk-free assets using a predictive regression based on the predictive variable in the first column. We report results for relative risk aversion coefficient  $\gamma$  of 3, 5 and 7. The portfolio weights are constrained to be from 0 and 1. “Buy-and-hold” corresponds to the investor passively holding the market portfolio. We consider horizons from 1-month to 24-month. We use non-overlapping returns such that the forecast horizon and rebalancing frequency coincide.

		1-month	3-month	6-month	9-month	12-month	24-month
$\gamma=3$	QVRP	4.06	5.87	6.50	5.96	5.17	4.33
	PVRP	4.25	5.90	6.37	5.97	5.24	4.34
	M3RP $^\perp$	0.94	5.86	5.89	6.00	5.72	4.27
	M4RP $^\perp$	0.68	5.89	6.16	6.04	5.84	5.31
	PVRP & M3RP $^\perp$	3.20	6.60	7.05	6.05	5.42	4.01
	PVRP & M4RP $^\perp$	3.22	6.54	7.03	6.13	5.61	4.47
	Buy-and-hold	1.72	2.97	3.48	3.04	3.59	2.45
$\gamma=5$	QVRP	2.43	5.17	4.81	4.13	3.46	2.77
	PVRP	2.52	5.14	4.76	4.17	3.48	2.79
	M3RP $^\perp$	0.57	4.00	4.49	4.33	4.33	2.81
	M4RP $^\perp$	0.32	4.01	4.43	4.38	4.28	3.47
	PVRP & M3RP $^\perp$	1.84	5.34	5.20	4.28	3.70	2.62
	PVRP & M4RP $^\perp$	1.80	5.32	5.10	4.30	3.77	2.91
	Buy-and-hold	-0.33	1.35	1.58	1.07	1.51	-0.38
$\gamma=7$	QVRP	1.71	4.36	3.70	3.00	2.48	1.98
	PVRP	1.86	4.31	3.74	3.04	2.49	1.99
	M3RP $^\perp$	0.40	2.93	3.51	3.13	3.18	2.01
	M4RP $^\perp$	0.23	2.72	3.40	3.16	3.19	2.48
	PVRP & M3RP $^\perp$	1.31	4.28	4.02	3.10	2.66	1.87
	PVRP & M4RP $^\perp$	1.28	4.27	3.92	3.12	2.70	2.08
	Buy-and-hold	-2.37	-0.28	-0.31	-0.90	-0.57	-3.20