The decomposition of jump risks in individual stock returns *

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Abstract

This paper proposes a new GARCH-jump mixed model for individual stock returns that takes into account four types of risks associated to: the systematic and idiosyncratic time-varying arrival of jumps and the systematic and idiosyncratic time-varying diffusive volatility. By considering a general pricing kernel with all underlying risk factors, we decompose the expected stock return into four risk premiums related to the four types of risks. Empirically, we estimate the model jointly for daily stock returns and market returns and investigate the asset pricing consequences. We find that the idiosyncratic jump intensity accounts for about 82.25% of the total jump intensity on average, and idiosyncratic variance accounts for about 66.70% of the total diffusive variance. Over

*The paper was preciously circulated by the title “Systematic and Idiosyncratic Jump Risks in the Expected Stock Returns”. The authors thank Casper de Vries, Ton Vorst, Dick van Dijk, Christian Schlag and Dennis Karstanje for or helpful discussion and comments. Views expressed are those of the authors and do not necessarily reflect official positions of De Nederlandsche Bank.

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the past 50 years, the contribution of idiosyncratic risks decreases. By sorting the stocks into five quintiles by estimated risk premiums, we find that the systemic jump risks, idiosyncratic diffusive risks and idiosyncratic jump risks are significantly priced in the cross section of our sample.

Keywords: jump-diffusion model, GARCH filtering, asset pricing

JEL Classification: C13, C61, G11, G12

1 Introduction

Jumps in stock returns of individual firms are triggered by either systematic events or idiosyncratic shocks. During crisis events such as oil crisis in 1973, the black Monday in 1987, the dot-com crash in 2000 and the subprime crisis from 2007 to 2009, the financial market witnessed large jumps in most traded stocks. In addition, individual stocks may occasionally experience jumps due to firm specific events, such as earning surprises, merger and acquisition, etc. This chapter provides a new modeling framework for the individual stocks that allows for the estimation of time-varying systematic and idiosyncratic jump intensities and volatilities. From an asset pricing point of view, it is of both theoretical and practical importance to understand, how the systematic and idiosyncratic jump intensities can be estimated, and how they are related to the equity risk premium. This new model accommodates the joint dynamic structures of individual stock returns and the market returns, while allowing for jumps. Under such a framework, we estimate the dynamics of idiosyncratic and systematic jump intensities and volatilities for individual stocks and investigate the roles of different risks in the dynamics of equity premium.

We model the return innovation by a Generalized Autoregressive Conditional Heteroskedastic (GARCH)-jump mixture model in the spirit of Maheu et al. (2013). Maheu et al. (2013) only focuses on the market returns. We intend to investigate the dynamics of individual excess stock returns and allow the stock innovations to be affected by the market innovations. To be more specific, the market innovation has two components, which we call "market jump" and "market diffusion". The jump component follows a compound Poisson-normal distribution with autoregressive jump intensities. The diffusive component is governed by an asymmetric two-component GARCH process. The stock innovation has four components: "systematic jump" , "idiosyncratic jump", "systematic diffusion" and "idiosyncratic diffusion". The systematic jump in the stock innovation is triggered by the market jump with a certain probability. The systematic diffusion component loads on the market diffusion component governed by "beta", similar to that in the Capital Asset Pricing Model (CAPM). The idiosyncratic components are directed by similar dynamic structures as in the market components, but are independent from their systematic counterparts. To estimate the model, we provide a filter that can filter daily excess stock returns into large (jump) versus small (diffusion) components, as well as systematic and idiosyncratic counterparts in each of them.

In addition, our model allows for the decomposition of the dynamic equity premium by assuming a general pricing kernel with all underlying risk factors in the economy. The traditional CAPM suggests that the idiosyncratic risk is diversified away and not priced. However, empirical studies find that idiosyncratic risks matter not only in predicting the time series of stock market return[1], but also in pricing in the cross-section of stock returns[2].

[1]For example, Goyal and Santa-Clara (2003) and Guo and Savickas (2006).

[2]For example, Ang et al. (2006), Fu (2009), Ang et al. (2009), Huang et al. (2010) and Stambaugh et al. (2015).

Theoretically, the pricing of idiosyncratic risks can be explained by the fact that investors in reality do not hold perfectly diversified portfolios. Levy (1978) and Merton (1987) show that under-diversified investors demand a return compensation for bearing idiosyncratic risk. In asset pricing model based on prospect theory, where investors are loss averse over the fluctuations of their own stocks, Barberis and Huang (2001) also provide an explanation for the relation between expected returns and idiosyncratic risk. In this paper, we include the idiosyncratic components of the stock innovations in the pricing kernel and test whether they are significantly priced in the dynamics of the equity premium. The specification of the pricing kernel is similar to that in Gourier (2016) and Bégin et al. (2016). The expected stock return can be consequently decomposed into four risk premiums: premiums on systematic and idiosyncratic diffusion risks and systematic and idiosyncratic jump risks.

We conduct a joint estimation strategy to identify different components in daily returns of 15 stocks from 1963 to 2015. For each stock, we estimate the model for the market return and the stock return simultaneously. We observe that the idiosyncratic jump intensity and volatility account for a large amount of the total jump intensity and volatility for the stocks: idiosyncratic jump intensity accounts for 82.25% of the total jump intensity, and idiosyncratic variance accounts for 66.70% of the total variance on average. Over time, the contribution of idiosyncratic risks is declining, which implied that the firms are more and more affected by the systematic risks over the past 50 years. Further, all four types of risks are related to sizable premium in the expected return of individual stocks over time. The equity premium associated to idiosyncratic (jump) risks contribute to 57.18% (16.25%) of the total equity premium on average. Lastly, the cross-sectional difference in the expected stock returns in our sample can be explained by the difference in the model-implied systematic jump, idiosyncratic diffusive and idiosyncratic jump risk premiums.

The closest econometrics approach in the literature is Maheu et al. (2013). They estimate a GARCH-jump mixed model for the market returns with time-varying jump and diffusive risk premiums. We extent their framework to accommodate the estimation for individual stock returns, i.e. the need for estimating systematic and idiosyncratic counterparts in both jump and diffusive components. Our work is also related to Maheu and McCurdy (2004). They estimate the dynamics of jump and diffusive components in stock returns with constant equity premium. The difference between their model and ours is that we introduce the systematic and idiosyncratic counterparts in each components and consider their roles in the time-varying equity premium. On the technical side, we provide a procedure to filter out the four components in stock innovation. This is comparable to the one in Christoffersen et al. (2012), who estimated different specifications of dynamic jump model for the S&P 500 index.

Our paper complements the recent studies that intend to disentangle the four types of risks in equity premiums and in higher order risk premiums. Using stock return and option prices, Gourier (2016) and Bégin et al. (2016) conduct a joint estimation using both stock and option data to decompose the four risk premiums associated with systematic and idiosyncratic diffusive and jump risks. They both find that idiosyncratic risks contribute to more than 40% of the total equity premium on average after 1996 and that idiosyncratic risk mostly comes from the jump risk component. Though we have different data sets and methodology, we obtain some similar results. We use 50 years of daily stock returns to identify the dynamics of jump risks. Using the time series of stock returns, we are able to further investigate the evolution of the contribution of different risks over a long period, in contrast to the fact that their analysis started from 1996 due to the availability of option data. In addition, to better capture the contribution of systematic risks in equity premium in a long time frame, we let the exposures of the stock to the market jump and diffusion risks to be time varying, related to the business cycle variable. An additional difference between our approach and the methodology in the previous two papers is that we jointly estimate the model using the market return and the stock return, while they first estimate the market parameters and then estimate the parameters for individual stocks using the estimates from the market returns. To the best of our knowledge, this is the first study to model dynamic jump and diffusion components, while considering decomposition towards systematic and idiosyncratic risks based only on time series of stock returns.

The remainder of the paper proceeds as follows. Section 2 presents our model setup and discusses the expected stock return under our model. Section 3 discusses the estimation methodology. Section 4 provides the data and the estimation results. Section 5 discusses the implications on asset pricing. Section 6 concludes. The technical derivations are postponed to the Appendix in Section 7.

2 Model

Our model builds on Maheu and McCurdy (2004) and Maheu et al. (2013). The former presents mixed GARCH-jump models for individual stocks, while the latter considers time-varying equity premium in the market returns. We aim at accommodating both systematic and idiosyncratic risks in individual stock returns. In section 2.1, we first present the model on the market return and the dynamics of volatility and jump intensity. Then we discuss the model on individual stock returns in section 2.2. Lastly, we specify a pricing kernel and derive the expression of expected returns of individual stocks in section 2.3.

2.1 The dynamic of market returns

We model the continuously compounded market return by the combination of a normally distributed diffusion component and a jump component.

Assume the following decomposition of the market return:

$$\text{Market:}\;\; R^m_{t+1} = log(\frac{S^m_{t+1}}{S^m_t}) = \alpha^m_{t+1} + y^m_{t+1} + z^m_{t+1}, \tag{1}$$

where S^m_{t+1} denotes the market price at the close of day $t+1$ and α^m_{t+1} is related to model-implied market equity premium expected for period $t+1$, given the information set Φ_t. We will derive the expression of α^m_{t+1} in Section 2.3. The log return is driven by two stochastic processes: a jump component, y^m_{t+1} and a diffusive component, z^m_{t+1}. They are assumed to be independent conditional on the information available at time t. Due to the dynamic interaction between the two terms, they are not unconditionally independent.

The jump innovation is governed by a conditional Poisson jump-arrival process combined by a normal jump-size distribution. Define the discrete-valued number of jumps in the market return over the time period t to $t+1$ as N^m_{t+1}. The conditional distribution of N^m_{t+1} is a Poisson distribution with jump intensity $h_{y,t+1}$,

$$P(N^m_{t+1} = j|\Phi_t) = \frac{\exp(-h_{y,t+1})h^j_{t,t+1}}{j!},\;\; j = 0, 1, 2, ...$$

The conditional arrival rate of jumps, $h_{y,t+1}$ is the expected number of jumps for period $t+1$ given information at time t, that is,

$$h^m_{y,t+1} = E(N^m_{t+1}|\Phi_t).$$

As in Maheu and McCurdy (2004) and Maheu et al. (2013), we parameterize the dynamics of conditional jump intensity $h^m_{y,t+1}$ as responsive to past intensity and "news",

$$h^m_{y,t+1} = w^m_y + b^m_y h^m_{y,t} + a^m_y \zeta^m_t, \tag{2}$$

where b^m_y measures persistence of jump intensity dynamic and a^m_y is the news-impact coefficient, associated with the jump innovation ζ^m_t, defined as follows. The jump intensity innovation ζ^m_t is the forecast update of the number of jumps N^m_t, when the information set at t is available:

$$\begin{aligned}\zeta^m_t &= E[N^m_t|\Phi_t] - E[N^m_t|\Phi_{t-1}] = E[N^m_t|\Phi_t] - h^m_{y,t} \\ &= \sum_{j=0}^{\infty} jP(N^m_t = j|\Phi_t) - h^m_{y,t}. \end{aligned} \tag{3}$$

The first part in Equation (3), the ex-post probability of j jumps at time t given information at t can be calculated as:

$$P(N^m_t = j|\Phi_t) = \frac{f(R^m_t|N^m_t = j, \Phi_{t-1})P(N^m_t = j|\Phi_{t-1})}{f(R_t|\Phi_{t-1})}, \tag{4}$$

where $f(.)$ refers to the conditional density of the market return. Note from this definition that the expectation of ζ_t^m conditional on information set Φ_{t-1} is zero. The jump intensity process is directed by the jump innovation rather than the squared-return innovations. This allows the the impact of time-varying jump intensity on expected variance dynamics to be different from that captured by the GARCH component of variance.

We assume that the jump size follows a normal distribution $N(\theta^m, (\delta^m)^2)$, where θ^m refers to the mean of jump size and $(\delta^m)^2$ is the variance, and the jumps occur independently. That is, the jump components in the return process is given by:

$$y_{t+1}^m = \sum_{j=1}^{N_{t+1}^m} x_{t+1}^j,$$

where x_{t+1}^j, $j = 1, 2, \cdots$ are independently and identically distributed (i.i.d.) random variables drawn from $N(\theta^m, (\delta^m)^2)$. Therefore, the conditional mean and variance of the jump component y_{t+1}^m are $h_{y,t+1}^m \theta^m$ and $h_{y,t+1}^m((\theta^m)^2 + (\delta^m)^2)$, respectively. The conditional mean of market return is thus expressed as $E[R_{t+1}^m|\Phi_t] = \alpha_{t+1}^m + \theta^m h y_{y,t+1}$.

Further, the diffusion term z_{t+1}^m is assumed to follow a normal distribution $N(0, h_{z,t+1}^m)$ with conditional variance $h_{z,t+1}^m$, i.e.

$$z_{t+1}^m = \sqrt{h_{z,t+1}^m}\epsilon_{t+1}^m, \ \ \epsilon_{t+1}^m \sim \mathrm{N}(0,1),$$

where $h_{z,t+1}^m$ is governed by a two-component GARCH model with feedback from jumps. We adopt the specification in Maheu et al. (2013):

$$h^m_{z,t+1} = h^m_{z1,t+1} + h^m_{z2,t+1}, \tag{5}$$

$$h^m_{z1,t+1} = w^m_z + b^m_{z1} h^m_{z1,t} + g^m_1(\Phi_t)(R^m_t - E[R^m_t|\Phi_{t-1}])^2 \tag{6}$$

$$h^m_{z2,t+1} = b^m_{z2} h^m_{z2,t} + g^m_2(\Phi_t)(R^m_t - E[R^m_t|\Phi_{t-1}])^2 \tag{7}$$

The long-run component is captured by $h_{z1,t+1}$ and the transitory moves are modeled by $h_{z2,t+1}$.

The generalized news impact coefficient $g^m_k(\Phi_t)$ $(k = 1, 2)$ for the kth GARCH component is given as,

$$g^m_k(\Phi_t) = \exp(\tau^m_{k1} + I^m_t(\tau^m_{k2} E[N^m_t|\Phi_t] + \tau^m_{k3})), \quad k = 1, 2.$$

$$I^m_t = \begin{cases} 1 \text{ if } R^m_t - E[R^m_t|\Phi_{t-1}] < 0, \\ 0 \text{ otherwise.} \end{cases}$$

Both Maheu et al. (2013) and our empirical analysis below confirm that it is essential to specify a two-component GARCH process to capture the diffusive volatility. It also helps to estimate jumps precisely. Otherwise, the noise and transitory part of the diffusive volatility could be potentially sorted as jumps[3]. This model allows for asymmetric impacts from good and bad news and feedback from jump innovations. The term $R^m_t - E[R^m_t|\Phi_{t-1}]$ is the total return innovation observable at time t. $E[N^m_t|\Phi_t]$ is the filtered number of jumps at time t given Φ_t.

[3]We plot the long-run and short-run components from April 2011 to November 2012 in Figure 2, which shows that the long-run diffusive variance move more slowly than the short-run component.

Similar to Christoffersen et al. (2012), the filtered jump and diffusion terms, $\tilde{y}_t^m$ and $\tilde{z}_t^m$, can be obtained given parameters, which is illustrated in details in Appendix 7.3.

2.2 The dynamic of individual stocks returns

Next, we propose a model on the dynamic of individual stock returns, which involves the dependence between the return processes of individual stocks and the market. First, the individual stock return is modeled by a similar structure as that for the market return:

$$\text{Firm: } R_{t+1}^i = log(\frac{S_{t+1}^i}{S_t^i}) = \alpha_{t+1}^i + y_{t+1}^i + z_{t+1}^i, \tag{8}$$

For the jump component y_{t+1}^i, we assume that when there is a jump in the market, the probability that the market jump will trigger a jump in individual stock i is p^i. Further, there is an idiosyncratic jump process that is independent from the market jumps. In other words, conditional on having N_{t+1}^m jumps in the market, the number of individual jumps N_{t+1}^i equals to the sum of a binomial distributed random number $B^i(N_{t+1}^m, p^i)$ and an independently Poisson distributed random number N_{t+1}^ϵ with intensity $h_{y,t+1}^\epsilon$. Then the number of individual jumps N_{t+1}^i follows a Poisson distribution with intensity $h_{y,t+1}^i$ as:

$$h_{y,t+1}^i = p^i h_{y,t+1}^m + h_{y,t+1}^\epsilon. \tag{9}$$

The jump component in the individual return, y_{t+1}^i, is therefore given as,

$$y_{t+1}^i = \sum_{j=1}^{N_{t+1}^m} x_{i,t+1}^j 1_{\text{market jump j triggers a jump}} + \sum_{j=N_{t+1}^m+1}^{N_{t+1}^m+N_{t+1}^\epsilon} x_{i,t+1}^j$$

Here, the first part consists of the jumps triggered by the market jumps, while the second part consists of the idiosyncratic jumps that are independent from the market jumps. We assume that the jump size of individual stock $x_{i,t+1}^{j}$, $j = 0, 1, 2, \cdots$, are i.i.d. and follow a normal distribution $N(\theta^i, (\delta^i)^2)$. When a jump is triggered by a market jump, the size of the triggered jump and the corresponding market jump are assumed to be correlated with a correlation ϕ^i.

The dynamics of the intensity $h_{y,t+1}^{\epsilon}$ has the following structure:

$$h_{y,t+1}^{\epsilon} = w_y^i + b_y^i h_{y,t}^{\epsilon} + a_y^i \zeta_t^{\epsilon}, \tag{10}$$

where the jump innovation term ζ_t^{ϵ} for period t is defined as

$$\zeta_t^{\epsilon} = E[N_t^{\epsilon}|\Phi_t] - E[N_t^{\epsilon}|\Phi_{t-1}] = E[N_t^{\epsilon}|\Phi_t] - h_{y,t}^{\epsilon}. \tag{11}$$

Notice that the ex-post expected number of idiosyncratic jumps is proportional to that of total jumps in the individual stock returns:

$$E[N_t^{\epsilon}|\Phi_t] = \frac{E[N_t^i|\Phi_t]\, h_{y,t}^{\epsilon}}{h_{y,t}^i}. \tag{12}$$

Here, the ex-post expected number of total jumps in individual stock return $E[N_t^i|\Phi_t]$ can be calculated based on the total jump intensity $h_{y,t}^i$ and the conditional density of the stock return R_t^i:

$$E[N_t^i|\Phi_t] = \sum_{j=0}^{\infty} j \Pr(N_t^i = j|\Phi_t) = \sum_{j=0}^{\infty} j \frac{f(R_t^i|N_t^i = j, \Phi_{t-1}) \Pr(N_t^i = j|\Phi_{t-1})}{f(R_t^i|\Phi_{t-1})} \tag{13}$$

The jump innovation term ζ_t^ϵ is then determined by Equation (11) to (13).

Next, in the spirit of CAPM, we model the total diffusion component of the individual stock as the sum of systematic and idiosyncratic diffusion component:

$$z_{t+1}^i = \beta^i z_{t+1}^m + z_{t+1}^\epsilon, \tag{14}$$

where β^i is the factor loading of stock i on systematic diffusive risk and the idiosyncratic diffusive component z_{t+1}^ϵ follows a normal distribution $N(0, h_{z,t+1}^\epsilon)$. Further, z_{t+1}^m and z_{t+1}^ϵ are independent from each other.

The dynamic of idiosyncratic conditional variance has a parallel structure as that of the market. In addition, we assume that only the idiosyncratic innovation affects idiosyncratic conditional variance as follows:

$$h_{z,t+1}^\epsilon = h_{z1,t+1}^\epsilon + h_{z2,t+1}^\epsilon, \tag{15}$$

$$h_{z1,t+1}^\epsilon = w_z^i + b_{z1}^i h_{z1,t}^\epsilon + g_1(\Phi_t)(R_t^i - E[R_t^i|\Phi_{t-1}])^2 \tag{16}$$

$$h_{z2,t+1}^\epsilon = b_{z2}^i h_{z2,t}^\epsilon + g_2(\Phi_t)(R_t^i - E[R_t^i|\Phi_{t-1}])^2. \tag{17}$$

The generalized new impact coefficient $g_j(\Phi_t)$ ($j = 1, 2$) for the jth GARCH component allows asymmetric impact from good and bad idiosyncratic news:

$$g_j(\Phi_t) = \exp(\tau^i_{j1} + I_t(\tau^i_{j2} E[N^\epsilon_t|\Phi_t] + \tau^i_{j3})), \;\; j = 1, 2.$$

$$I_t = \begin{cases} 1 \;\text{ if } R^i_t - E[R^i_t|\Phi_t] < 0; \\ \\ 0, \;\text{otherwise.} \end{cases}$$

The dynamic of the idiosyncratic diffusive component is driven by the innovation of the stock return:

$$R^i_t - E[R^i_t|\Phi_t] = R^i_t - \alpha^i_t - \theta^i h^i_{y,t}.$$

Under our model setup, the conditional variance of the individual stock return can be derived as:

$$Var(R^j_{t+1}|\Phi_t) = (\beta^i)^2 h^m_{z,t+1} + h^\epsilon_{z,t+1} + (p^i h^m_{y,t+1} + h^\epsilon_{y,t+1})((\theta^i)^2 + (\delta^i)^2). \tag{18}$$

In this paper, we only use the return data to estimate the jump and volatility dynamics of both the market and the stocks. Hence, long term time series from 1963 to 2015 that contain several extreme movements are used to identify the parameters that govern the dynamics of the infrequent jumps. When estimating for such a long time series, we allow the exposures of the individual stocks to the market risks, namely β^i and p^i, to vary with business condition in the spirit of conditional CAPM. Motivated by Avramov and Chordia (2006), we model the conditional β^i and p^i as:

$$\beta_t^i = \beta_1^i + \beta_2^i BC_t,$$

$$p_t^i = p_1^i + p_2^i BC_t,$$

Following Jagannathan and Wang (1996) and Avramov and Chordia (2006), we focus on the default spread as a proxy of the business condition variable BC_t, which has been shown to have the best predictive power for future business conditions in the literature. The default spread is defined as the yield differential between the Baa and Aaa corporate bonds.

2.3 The pricing kernel and the expected returns

To facilitate our analysis of how various risk factors are priced, we introduce a parametric pricing kernel to price all four risk factors, including both volatility risk and the jump risk. With the assumed pricing kernel, we derive the expected returns of the individual stocks and the market. In the absence of arbitrage, a pricing kernel M_t is a positive stochastic process such that $M_t S_t$ is a martingale for any stock price process S_t. In a discrete-time setting, this condition is represented by the following identity:

$$E_t[\frac{M_{t+1}}{M_t}\frac{S_{t+1}}{S_t}] = 1.$$

Naik and Lee (1990) demonstrated that the market is incomplete when jumps are present in stock prices. Such market incompleteness implies the absence of a unique pricing kernel. Consequently, we adopt one candidate pricing kernel that prices the four sources of risks in our model on stock returns: systematic jump shock, systematic diffusion shock, idiosyncratic jump shock and idiosyncratic diffusion shock. We specify a standard log linear pricing kernel as:

$$\log(\frac{M_{t+1}}{M_t}) = -r - \mu_{t+1} - \Lambda^m z^m_{t+1} - \Lambda^m y^m_{t+1} - \sum_{i=1}^{J} \Lambda^i z^\epsilon_{i,t+1} - \sum_{i=1}^{J} \Lambda^i y^\epsilon_{i,t+1}, \tag{19}$$

where r is risk-free rate, Λ^m, Λ^i, $i = 1, ..., J$ are related to the prices of different risk factors. Here, J denotes the total number of stocks in the economy. Recall that the terms z^m_{t+1} and y^m_{t+1} are market diffusive and jump components, and the terms $z^\epsilon_{i,t+1}$ and $y^\epsilon_{i,t+1}$ are idiosyncratic risk components in stock i, independent from the market risks. In such a pricing kernel, we implicitly assume that investors' portfolio are not well-diversified, and idiosyncratic diffusive and jump components are potentially priced. Hence, this general structure allows all possible risks in the market and individual stocks to be priced. In Maheu et al. (2013), they specify a nonlinear pricing kernel to capture the risks due to dynamics of the higher order moments. More parameters would add the richness of the model, but also increase the difficulty of the estimation. To keep the simplicity of the model and to address the main research question in the paper, we use the linear model to specify the pricing kernel.

The coefficient μ_{t+1} is a normalizing constant to ensure that $E_t[\frac{M_{t+1}}{M_t}] = e^{-r}$. Thus, it is derived as:

$$\mu_{t+1} = \log E_t[\exp(-\Lambda^m z^m_{t+1} - \Lambda^m y^m_{t+1} - \sum_{i=1}^{J} \Lambda^i z^\epsilon_{t+1} - \sum_{i=1}^{J} \Lambda^i y^\epsilon_{t+1})].$$

We apply the pricing kernel in (19) to price all stocks in the economy. In the absence of arbitrage, we have the following equations hold:

$$E_t[\frac{M_{t+1}}{M_t} e^{R^m_{t+1}}] = 1 \text{ and } E_t[\frac{M_{t+1}}{M_t} e^{R^i_{t+1}}] = 1, \tag{20}$$

for all i's. This leads to the following proposition on the expected returns of stocks. The proof is left to Appendix 7.4.

Proposition 1 *Under our model on the dynamics of the market and individual returns with the pricing kernel in Equation (19), the discrete-time conditional expected returns of the market and individual stock can be written as:*

$$E_t[\exp(R^m_{t+1})] = \exp(r + \lambda_z h^m_{z,t+1} + \lambda_y h^m_{y,t+1})$$

$$E_t[\exp(R^i_{t+1})] = \exp(r + \beta^i \lambda_z h^m_{z,t+1} + p^i(e^a(1-e^b) + e^{\theta^i + \frac{1}{2}(\delta^i)^2} - 1) h^m_{y,t+1} + \lambda_{zi} h^{\epsilon}_{z,t+1} + \lambda_{yi} h^{\epsilon}_{y,t+1}.)$$

where $a = -\Lambda^m \theta^m + \frac{1}{2}(\delta^m)^2 (\Lambda^m)^2$ *and* $b = \theta_i + \frac{1}{2}(\delta^i)^2 - \phi^i \delta^i \delta^m \Lambda^m$.

In addition, the parameters in the pricing kernel and in the dynamics of the market and individual stock returns should satisfy the following equations:

$$\lambda_z = \Lambda^m, \;\; \lambda_y = \xi^m(1) + \xi^m(-\Lambda^m) - \xi^m(1-\Lambda^m),$$

$$\lambda_{zi} = \Lambda^i, \;\; \lambda_{yi} = \xi^i(1) + \xi^i(-\Lambda^i) - \xi^i(1-\Lambda^i),$$

where $\xi^m(\psi) = exp(\theta^m \psi + \frac{(\delta^m)^2 \psi}{2}) - 1$ *and* $\xi^i(\phi) = exp(\theta^i \phi + \frac{(\delta^i)^2 \phi}{2}) - 1$.

From Proposition 1, we get the expected continuously compounded market return as:

$$\begin{aligned}\alpha^m_{t+1} &= r + (\Lambda^m - \frac{1}{2})h^m_{z,t+1} + (\xi^m(-\Lambda^m) - \xi^m(1-\Lambda^m))h^m_{y,t+1} \\ &= r + (\lambda_z - \frac{1}{2})h^m_{z,t+1} + (\lambda_y - \xi^m(1))h^m_{y,t+1}. \end{aligned} \tag{21}$$

For the individual stock i, the expected continuously compounded return is then:

$$\alpha^i_{t+1} = r + (\beta^i\Lambda^m - \frac{1}{2}(\beta^i)^2)h^m_{z,t+1} + p^i e^a(1-e^b)h^m_{y,t+1} +$$
$$(\Lambda^i - \frac{1}{2})h^\epsilon_{z,t+1} + (\xi^i(-\Lambda^i) - \xi^i(1-\Lambda_{yi}))h^\epsilon_{y,t+1},$$

Note that λ_z, λ_{zi}, λ_y and λ_{yi} are the market prices for loading on four types of risks, which are related to the parameters Λ^m and Λ^i in the pricing kernel. The derived expected stock return from our model can be interpreted in the following ways. First, with complete diversification and in the absence of jumps, only the first term in the expected return of individual stocks remains. The expected return derived under our model reduces to that from the CAPM.

Second, when the correlation between jump sizes in the market and individual stock returns ϕ^i is zero, the second term in the expected return of individual stocks is reduced to $p(e^a - 1)(1 - e^b)h^m_{y,t+1}$. Because the average jump size for the market return is generally estimated as negative in the literature with dynamic jump intensity,[4] the parameter a is therefore positive. The sign of the premium thus depends on the sign of the parameter b. As stated in Jiang and Oomen (2008), individual stock price jumps tend to be idiosyncratic

[4] Christoffersen et al. (2012) estimate the average jump size of SP 500 index as -0.174 from the DVSDJ model. In Maheu and McCurdy (2004), the jump size mean θ is significantly negative for the three indices (DJIA, Nasdaq 100 and TXX)

and predominantly positive, presenting an interesting contrast to mostly negative jumps in market portfolios. Maheu and McCurdy (2004) also find that the jump size mean is centered around zero for most of the individual firms. From the empirical evidence, we conjecture that θ^i is centered around zero or slightly higher than zero. Hence, b is positive, which implies that $(e^a - 1)(1 - e^b)$ is negative. Therefore, the expected return of the individual stock is decreasing with respect to the probability that market jump can trigger an individual jump p.

Third, when the correlation between jump sizes of market and individual stock returns ϕ^i is not zero, stocks whose jump sizes are more correlated with the market earn higher returns. Finally, idiosyncratic diffusive and jump risk premiums are included in the expected return because the pricing kernel allows all underlying risk factors n the economy. We will test their statistical and economic significance in our empirical study.

3 Estimation methodology

In this section, we discuss our methodology to estimate the model described in Section 2. We apply a joint estimation strategy, i.e. to estimate the parameters for the market and the stock return dynamics together for each stock. An alternative method is to use a two-step estimation strategy: first estimate parameters in the market dynamics and then estimate the parameters in the individual stocks by substituting the estimated parameters and dynamics of $\hat{h}_y^m$ and $\hat{h}_z^m$ into the dynamics of h_y^i and h_z^i in the individual stock returns. There are two reasons why we prefer the joint estimation. First, in Equation (9), both $h_{y,t+1}^i$ and $h_{y,t+1}^\epsilon$ are latent processes. Therefore, it would be difficult to identify the parameters in the systematic and idiosyncratic component and make sure they are independent with each other in the second stage. Second, with the joint estimation methodology, the program can achieve a higher likelihood. Due to the two reasons, the joint estimation strategy is essential for identifying the parameters in the idiosyncratic and systematic components.

First, we provide the likelihood function for the market model. Given the parameters $\Theta^m = (\Lambda^m, w_z^m, b_{z1}^m, \tau_{11}^m, \tau_{12}^m, \tau_{13}^m, b_{z2}^m, \tau_{21}^m, \tau_{22}^m, \tau_{23}^m, w_y^m, b_y^m, a_y^m, \theta^m, \delta^m)$, we can get the time series of conditional market variance $h_{z,t+1}^m$ and market jump intensity $h_{y,t+1}^m$ by iterating from the starting time.

The likelihood function is given as follows. First, conditional on having $N_{t+1}^m = j$ jumps during time t to $t+1$, the jump component follows a normal distribution $N(j\theta^m, j(\delta^m)^2)$. The conditional density function for stock return can be written as:

$$f(R_{t+1}^m | N_{t+1}^m = j) = \frac{1}{\sqrt{2\pi(h_{z,t+1}^m + j(\delta^m)^2}} \exp \frac{(R_{t+1}^m - \alpha_{t+1}^m - j\theta^m)^2}{2(h_{z,t+1}^m + j(\delta^m)^2)}. \tag{22}$$

Second, since the number of jumps during time t and $t+1$ follow a Poisson distribution, we get that

$$\Pr(N_{t+1}^m = j) = \frac{(h_{y,t+1}^m)^j}{j!} \exp(-h_{y,t+1}^m). \tag{23}$$

Hence, the unconditional density of the return can be written as:

$$f(R_{t+1}^m) = \sum_{j=0}^{\infty} f(R_{t+1}^m | N_{t+1}^m = j) \Pr(N_{t+1}^m = j). \tag{24}$$

Then, the likelihood function can be constructed as:

$$L^m(R^m_{t+1}, \Theta^m) = \sum_{t=1}^{T} \log f(R^m_{t+1}), \tag{25}$$

where Θ^m includes 15 parameters for the market as stated above. We truncate the potential number of jumps in Equation (24) to a finite number. Maheu and McCurdy (2004) find that the conditional jump probability is zero for $N_{t+1} \geq 10$. The maximum number of jumps in a day is estimated as 5 in Christoffersen et al. (2012). Similarly, we also assume that the maximum number of jumps from t to $t+1$ is 5.

The likelihood function for the individual stock returns has the similar structure as that for the market returns. Substituting the subscript of Equation (22) to Equation (25) from m to i, we have the likelihood function for the individual stocks:

$$L^i(R^i_{t+1}, \Theta^i) = \sum_{t=1}^{T} \log f(R^i_{t+1}), \tag{26}$$

in which the parameter set Θ^i has 19 parameters: $\Theta^i = (\Lambda^i, w^i_z, b^i_{z1}, \tau^i_{11}, \tau^i_{12}, \tau^i_{13}, b^i_{z2}, \tau^i_{21}, \tau^i_{22}, \tau^i_{23}, w^i_y, b^i_y, a^i_y, \theta^i, \delta^i, \beta^i_1, \beta^i_1, p^i_1, p^i_2)$.

We estimate the 34 parameters by maximizing the sum of the likelihood function of the market returns and that of the stocks returns together for each stock. The joint likelihood function $L(R_{t+1}, \Theta)$ is:

$$L(R_{t+1}, \Theta) = L^m(R^m_{t+1}, \Theta^m) + L^i(R^i_{t+1}, \Theta^m, \Theta^i).$$

Based on the estimation result, the systematic jump and diffusion terms and the idiosyncratic jump and diffusion terms can be filtered out using the procedure in Appendix 7.3.

We are aware of the potential drawback that the estimation result for the market return may be different for each stock. As a justification of the methodology, in Section 4 we first estimate for the market alone by maximizing $L^m(R^m_{t+1}, \Theta^m)$ and estimate jointly for the market and the stock by maximizing $L(R_{t+1}, \Theta)$. We find that the estimated parameters of the market dynamic for all stocks are close to the estimated parameters from estimating the market return alone. To the best of our knowledge, joint estimation is the best solution to identify the parameters in each components of the stock return and to guarantee a small correlation between the idiosyncratic and systematic components.

4 Estimation results

This section presents first the data used in our empirical study and then the estimation results. Based on the results, we discuss the risk premiums related to the four potential risk factors.

4.1 Data

We use a dataset consisting of daily returns of the S&P500 index and 15 individual stock prices for the period Jan 3rd, 1963 to December 31st, 2015. The returns of the S&P500 index are regarded as the proxy of the market. These returns are adjusted for all applicable splits and dividend distributions and converted to continuously compounded daily returns. We have two criteria to select the stocks. First, the stocks are included in the S&P100 index by the end of 2015, which represent the largest and most established companies in the index.

Second, the stocks are traded during the sample period from Jan 3rd, 1963 to December 31st, 2015. We select 15 stocks which satisfy the two criteria. The data are obtained from the Center for Research in Security Prices (CRSP) database.

Table 1 provides summary statistics for the daily continuously compounded returns of the 15 firms. The kurtosis ranges from 8.7 to 70.1, which shows strong evidence of non-normality in the stock returns. Such a non-normal feature calls for modeling jump risk. When plotting the time series of returns on S&P500 index in Figure 1, we observe evidence of discontinuous large changes reflecting jumps as well as the pattern of volatility clustering. These features call for modeling the dynamics in diffusive and jump risks. In order to get numerically stable estimates, we scale the daily return by 100, similar to Maheu et al. (2013). The details of the scaling procedure is presented in Appendix 7.1. The results in Table 1 to Table 7 are for the scaled return.

4.2 Estimation Results for the market returns

Table 2 presents the estimated parameters for the returns on S&P500 index. In the column "Single estimation", we show the estimated parameters for estimating the market alone. First, we observe that the magnitude of jumps captured by the jump size estimates has an estimated mean -0.29 and variance 1.156. Both of the estimates are significant different from zero. In Figure 1(a), we observe that the negative jumps occur more frequently than the positive ones. This is in line with the positive market price of jump risk, since investors will be compensated for bearing potential negative price changes. Second, during the sample period, the jump arrival frequency matches the history of crisis. The pattern of $h^m_{y,t+1}$ in Figure 1(b) and the filtered jump components in Figure 1(c) are consistent with the timing of the crises during the sample period. For example, the Asian crisis in 1997, the burst of internet bubble in 2000 and the recent subprime crisis in 2008. The unconditional expected value of the dynamic jump intensity $w_y^m/(1-b_y^m) = 0.15$ is comparable to the estimate in Maheu et al. (2013)[5].

From the estimation on the market return, we observe a positive price for the systematic risk: the estimated market price of systematic risk Λ^m is 3.513, statistically higher than zero. Figure 3 illustrates the time varying equity premium for the market returns. The average equity premium is 0.05, showed by the dotted line.

In the columns called "Joint estimation" in Table 2, we report the estimated market parameters from the joint estimation. Since we jointly estimate for the stock and market returns, the estimation results for the market daily returns are not exactly the same for each stock. From the table, it shows that the results from joint estimation are close to that from the single estimation. For all market parameters, the average value across individual stocks is within the 95% confidence interval obtained from maximizing the market returns alone. The evidence suggest that the joint estimation methodology provides reasonable estimates for the market parameters. We ex-ante choose the joint estimation because of the advantages discussed earlier in this section, but from the comparison we conclude that the joint estimation is not such a necessity for this paper.

[5]The unconditional jump intensity can be estimated differently in different models due to specification of variance dynamics and intensity dynamics. In the DVSDJ model in Christoffersen et al. (2012), the estimate is around 0.025 for S&P500 and in Maheu and McCurdy (2004), it is 0.135 for DJIA.

4.3 Estimation Results for the individual returns

The model on individual stock return is estimated for the 15 stocks in our sample. We first show several general features of the model in Figure 4 to Figure 6 by taking the stock ADM (Archer Daniels Midland Company) as an example. We present the original times series plot for the daily returns of ADM in Figure 4(a). The decomposition of the total conditional jump intensity is shown in Figure 4(b) and (c). The systematic jump intensity contributes to 13.14% of the total conditional jump intensity on average. Hence, it is important to model idiosyncratic jump risk and study whether it is priced. When comparing diffusive and jump risks, we decompose the total conditional variance given in Equation (18) into the conditional variances of the diffusive and jump components. The decomposition of the total conditional variance is shown in Figure 5(b) and (c), respectively. On average, the contribution of conditional variance of the jump component to the total conditional variance is 45.66%.

Using the filtering procedure in Appendix 7.3 and the estimated parameters, we filter out the systematic and idiosyncratic jump and diffusion components for ADM and show the plots in Figure 6. The solid lines are jump components and the dotted lines are diffusion components. The figures show that the model for individual stocks in Section 2.2 can capture large negative and positive outliers which are important for modeling the heavy-tailedness of stock returns. In addition, it shows that the model can differentiate the idiosyncratic and systematic components. For instance, the correlation between systematic and idiosyncratic diffusive components is 0.17 and the correlation between systematic and idiosyncratic jump components is 0.33. The patterns demonstrated in these figures agree with the findings in

Ornthanalai (2014) and Li et al. (2008) that the daily return data favor small-sized jumps that occur frequently over the current practice that typically model jumps as large and rare event.

Next, we discuss the estimates of the model parameters. Since we present the summary statistics for the market parameters in Table 2, we only present the parameters for the idiosyncratic components and the exposure parameters to the market risks in Table 3 and Table 4 for each stock. From the cross-sectional comparison, we obtain the following stylized facts.

First, there is evidence on the persistence of conditional jump intensity for all firms. The persistent parameters, b_y^i, for the jump intensity are significantly positive from 0.874 to 0.999[6].

Second, the importance of revision to the conditional idiosyncratic jump intensity is similar as that to the conditional systematic jump intensity. The parameter a_y^i, which captures the the effect of the most recent intensity residual (the change in the conditional forecast of number of jumps due to last day's information) ranges from 0.036 to 0.362. The estimated a_y^m for the market is 0.055.

Third, the negative idiosyncratic return innovation has significantly positive effect on the long-run and short-run conditional diffusive variance: τ_{13}^i and τ_{23}^i are both positive for 14 out of 15 stocks. The filtered number of jumps $E[N_t|\Phi_t]$, on the other hands, has negative effect on the long-run conditional diffusive variance for all stocks, and positive effect on the short-run conditional diffusive variance for 11 out of 15 stocks. Therefore, we can conclude

[6]The estimated persistent parameters are very close to 1, but they are all significantly different from 1 at the 5% level. The unit-root test rejects the null hypothesis of non-stationary jump intensity series. From the figure that shows the time series of the jump intensity, we observe that a shock does not have a persistent effect on future jump intensities.

that the jump innovations increase the conditional diffusive variance in the short run and decrease that in the long run.

Fourth, the jump characteristics of the individual stocks are different from that of market jumps. The estimated jump size mean θ^i are negative for 2 out of 15 stocks, and positive for the other firms. This result is consistent with Jiang and Yao (2014), who find that individual stock price jumps tend to be idiosyncratic and predominantly positive, presenting an interesting contrast to mostly negative jumps in market portfolios. The result also supports Duffee (1995)'s conjecture that there is a negatively skewed market factor and a positively skewned idiosyncratic firm factor. Further, the volatility of the idiosyncratic jump size δ^i for individual firms are all higher than that for the market. We present estimates of θ^i and δ^i for each stock in Table 3 and Table 4.

The dependence structure between individual stocks and the market return is captured by the loadings on the two types of systematic risks, β_t^i and p_t^i. The summary statistics of the two variables are shown for each individual stocks in Table 5. From Table 3 and Table 4, we find that there are 13 out of 15 stocks with β_t^i positively related to the default spread (β_2^i is positive) and 4 out of 15 stocks with p_t^i positively related to the default spread. In general, for most stocks the time-varying exposure to the market risk is countercyclical: stock returns of those firms react more to the market returns during economy downturns than during upturns. Two exceptions in our sample are Exxon Mobil (XOM) and Colgate-Palmolive Company (CL), whose exposure to the market risk is negatively related to the default spread ($\beta_2^i < 0$ and $p_2^i < 0$). These two stocks are generally considered as defensive stocks. Exxon Mobil, a large oil producer with light leverage is expected to react less to the market movement during a recessionary period. In addition, Colgate-Palmolive is less sensitive to the market movement during bad times than during good times, because it is a geographically diversified nondurable consumer brand,

Finally, Figure 8 present the time-varying contribution of idiosyncratic jump intensity to the total jump intensity (Figure 8(a)) and the time-varying contribution of the idiosyncratic volatility to the total volatility (Figure 8(b)) for the 15 stocks on average. Idiosyncratic jump intensity (variance) contribution to 82.25% (66.70%) of the total jump intensity (variance). The declining patterns in the two figures show that the systematic jump intensity and volatility play increasingly important roles during the past 50 years.

5 Pricing jump risk in the expected stock returns

In this section, we analyze how the four types of risks are priced in the expected stock returns in our sample. First, we calculate the four premiums on the four types of risks and investigate their contribution to the total expected stock return during the sample period. Second, we sort the stocks according to the four risk premiums and construct portfolios representing stocks with low to high premium. By comparing portfolio performance over the entire sample period, we check whether the four types of risks are priced in the cross-section of expected stock returns.

Note that in order to calculate the expected stock return in Proposition 1, we need to first calculate the parameters in the pricing kernel, Λ^m and Λ^i. From Proposition 1, we get that:

$$\lambda_z = \Lambda^m, \ \ \lambda_y = \xi^m(1) + \xi^m(-\Lambda^m) - \xi^m(1 - \Lambda^m),$$

Hence, λ_y and λ_z are calculated based on the estimated Λ^m from the joint model. Similarly, we can calculate the idiosyncratic risk parameters λ_{yi} and λ_{zi} for individual stocks using estimated Λ^i.

5.1 Decomposing the expected stock return

To understand the economic significance of the four risk premiums related to systematic and idiosyncratic jump risks and systematic and idiosyncratic jump risks, we decompose the model-implied expected return into four corresponding risk premiums and evaluate their contributions. We aggregate idiosyncratic diffusive and jump premiums into the idiosyncratic risk premium for each stock. The average contribution of the idiosyncratic risk premium across the 15 stocks is presented in Figure 7(a). We find that on average the idiosyncratic risk premium contributes to 57.18% of the model-implied expected return. This confirms that idiosyncratic risks are economically important pricing factors in the expected stock return over time.

Further, the idiosyncratic risk premiums decrease dramatically during crises, i.e. in the Asian crisis in 1997 and in the recent subprime crisis in 2008. This can be explained by the fact that during the crisis, systematic events, such as the bankruptcy of Lehman Brothers, drive the stock prices more than idiosyncratic events, such as earning surprises. Therefore systematic risk premium accounts more during the crises.

We also aggregate the systematic and idiosyncratic jump risk premiums into a jump premium. We show the average contribution of the jump risk premium across the 15 stocks to the total expected return in Figure 7(b). The jump risk premium accounts for almost 16% of the model-implied expected returns. In addition, it remains around a stable level over our sample period. This shows that prices of jump risks are of great economic importance in studying the expected stock return.

5.2 Portfolio performance

If the risk premiums that we recover represent the reward for bearing risk, stocks with a higher risk premium should have higher expected returns than their peers. To check whether this is the case, we conduct a portfolio performance analysis, by sorting based on the estimated risk premiums. Denote the systematic diffusive risk, systematic jump risk, idiosyncratic diffusive risk, and idiosyncratic jump risk as SD, SJ, ID and IJ. When the information on the conditional premiums is available at t, we use them to sort the stock return one day ahead at $t+1$. In this sense, the sorting is a "pseudo out-of-sample" approach. It is not completely out-of-sample because we use all information over the whole sample to estimate the model. The stocks are sorted into five portfolios with 3 stocks in each of them, according to the risk premium on each type of risks. In each portfolio, we assign equal weights to the stocks and calculate the portfolio return over our sample period.

As shown in Proposition 1, the sorting based on the SD and the SJ premiums will be identical to the sorting based on the level of β and $p(1-e^{b})$, respectively. Thus, if the market exposures are assumed to be constant, then the sorting based on the two risk premiums remain constant throughout the sample period. We relax this restriction by making the exposure parameters vary with economic indicator. Hence, the sorting for market risk premium is time varying. Sorting based on the ID and IJ premiums are also time-varying because the conditional idiosyncratic volatility, $h^{\epsilon}_{z,t}$, and conditional idiosyncratic jump intensity, $h^{\epsilon}_{y,t}$, vary over time. The annualized portfolio returns for the constructed portfolios are calculated in the first five rows in Table 7. The sixth row reports the difference between the average returns of the fifth and the first portfolios. The last two rows report t-statistics and corresponding p-values when testing the null hypothesis that the difference is equal to zeros. The t-statistics are calculated based on the Newey-West standard error.

From Table 7, we find that when we sort the total conditional equity premium of the stocks, the future return increase from 1.7% from the first quintile to 12.2% in the fifth quintile. This supports the setup of model that it is able to capture the variation of the equity premium both over time and across different stocks. When we sort the stocks based on the ID and IJ premiums, the average returns are increasing from the low to high premium portfolios. The difference between the average returns of the fifth and the first portfolios are 8.6% and 5.7%, respectively, and both of them are statistically significant at 5% confidence level. Hence, the idiosyncratic diffusive and jump risks are both priced in the cross-section of expected stock returns in our sample. However, when the stocks are sorted based on the SD, the average returns of the three portfolio return are not lining up in a particular order. Further, the difference between the average returns between the average returns of the third and the first portfolio is negative and not significantly different from zero. As a robustness check, we sort the stocks according to beta obtained from OLS regression, the results are similar as sorting on SD. This suggests that the result is not due to systemic estimation error in SD. When we sort the stocks on SJ, the systematic jump risk premium, however, the difference between the fifth and first portfolio return is statistically significant at 5%.

This observation that systematic diffusive risk is not priced in our sample contradicts the classic CAPM. Due to the complexity of the model in this paper, we only estimate the model for 15 stocks over a 50 years time period. The use of small sample of stocks in this paper may limit the generosity of these results. Alternatively, it is possible that the result is in line with the low-volatility anomaly, which has been found in the United States over an 85-year period and in global markets for at least the past 20 years. The low-volatility anomaly says that portfolios of low-volatility and low-beta stocks have produced higher risk-adjusted returns than portfolios with high-volatility stocks in most markets studied, for instance, by Blitz and Van Vliet (2007), Blitz et al. (2013), Baker et al. (2011), and Frazzini and Pedersen (2014). Our results shed light on the low volatility anomaly by showing that the anomaly comes from the systematic diffusive risk rather than the systematic jump risk.

To summarize, when we decompose the model-implied expected return to the four risk premiums, we find that both the systematic and idiosyncratic risk premiums are of economic significance over time. When sorting the stocks based on the four risk premiums, we find that systematic jump risk, idiosyncratic diffusive risk and idiosyncratic jump risk are priced in the cross-section of the expected stock return.

6 Conclusion

In this paper, we propose a novel econometric framework for modeling the jump risk in individual stock returns. It distinguishes not only jump and diffusion components, but also systematic and idiosyncratic components. All four types of risks along with their associated risk premiums are time-varying. The model also allows for time-varying loadings on systematic diffusive and jump risks. Consequently, we decompose the stock return and study different sources of risk, especially jump risks. The study addresses two questions: (1) How to estimate the four sources of time varying risks in a jump diffusion model for individual stock returns? (2) How different types of risks are priced in equity premium over time and in the cross-section?

We estimate the model with dynamic conditional variance and dynamic jump intensity on 15 stocks returns. We find that (1) the model is able to identify different types of risks only using daily stock returns; (2) Idiosyncratic jump intensity and idiosyncratic diffusive variance account a large amount in the total jump intensity and diffusive variance, i.e. on average 82.25% and 66.7% respectively. The contribution of systematic risks increases over the past 50 years. For the pricing of risks, we find that systematic jump, idiosyncratic diffusive and idiosyncratic jump risk are significantly priced in the cross-section of expected stock returns in our sample.

7 Appendix

7.1 Scaling returns

Empirically, we find that we need to scale the daily return R^m and R^i to get numerically stable estimates. We only present the scaling procedure for the market returns. Individual stock returns can be scaled in a similar pattern. Suppress the time index for convenience of notation, and recall from Section 2.1 that

$$R^m = \alpha^m + y^m + z^m,$$

$$\alpha^m = r + (\lambda_y^m - \xi^m(1))h_y^m + (\lambda_z - 0.5)h_z^m,$$

where $\xi^m(\phi) = exp(\theta^m\phi + \frac{(\delta^m)^2\phi}{2})$, and y^m follows a compound Poisson distribution with parameters (h_y^m, θ^m and δ^m). Scaling R^m by 100 and denoting the scaled return by R_{100}^m:

$$R_{100}^m = \alpha_{100}^m + y_{100}^m + z_{100}^m,$$

in which parameters with subscript 100 are for the scaled returns R_{100}^m:

$$\alpha_{100}^m = 100\alpha^m = 100(r + (\lambda_y^m - \xi^m(1))h_y^m + (\lambda_z - 0.5)h_z^m),$$

$$y_{100}^m = 100y^m \text{ and } z_{100}^m = 100z^m.$$

One can verify that $z_{100}^m \sim N(0, h_{100,z}^m)$, where $h_{100,z}^m = 100^2 h_z^m$, and y_{100}^m follows a compound Poisson distribution with parameters (h_y^m, θ_{100}^m, δ_{100}^m), where $\theta_{100}^m = 100\theta_m$ and $\delta_{100}^m = 100\delta^m$. The original return is thus written as:

$$R^m = \frac{\alpha_{100}^m}{100} + \frac{y_{100}^m}{100} + \frac{z_{100}^m}{100}.$$

Similarly, the log linear pricing kernel is:

$$log(\frac{M_{t+1}}{M_t}) = -\frac{r_{100}}{100} - \frac{\mu_{100}}{100} - \Lambda_z \frac{z^m_{100}}{100} - \Lambda_y \frac{y^m_{100}}{100} - \sum_{i=1}^{J} \Lambda_{zi} \frac{z^{\epsilon}_{100,i}}{100} - \sum_{i=1}^{J} \Lambda_{yi} \frac{y^{\epsilon}_{100,i}}{100}.$$

In the absence of arbitrage, we have the following equality:

$$E_t[\frac{M_{t+1}}{M_t} e^{R^m}] = 1.$$

Henceforth, the expression for equity premium in terms of scaled terms is:

$$\alpha^m_{100} = r_{100} + (\Lambda_z - 0.5)\frac{h^m_{100,z}}{100} + 100(\xi^m_{100}(1 - \Lambda_y) - \xi_{100}(-\Lambda_y))h^m_y,$$

where $\xi^m_{100}(\phi) = exp(\frac{\theta^m_{100}}{100}\phi + \frac{(\delta^m_{100})^2\phi}{20000}) - 1$.

7.2 Filter jump and diffusion terms from the market return

Given the 15 parameters for the market return: $\Theta^m = (\Lambda^m, w^m_z, b^m_{z1}, \tau^m_{11}, \tau^m_{12}, \tau^m_{13}, b^m_{z2}, \tau^m_{21}, \tau^m_{22}, \tau^m_{23}, w^m_y, b^m_y, a^m_y, \theta^m, \delta^m)$, we get the time series of conditional variance $h^m_{y,t}$ and conditional jump intensity $h^m_{z,t}$[7]. Next, we discuss how to filter out the normal component of the return z_t. The filtration of z^m_t involves solving the expectation $\hat{z}^m_t = E[z^m_t|\Phi_t]$. Note that if market return and number of jump are known at time t, we can express z^m_t as:

$$z^m_t(R^m_t, N^m_t = j) = \sqrt{\frac{h^m_{z,t}}{h^m_{z,t} + j(\delta^m)^2}}(R^m_t - \alpha^m_t - j\theta^m).$$

[7]We set starting value of the jump intensity $h^m_{y,1}$ to the unconditional value as $E[h^m_{y,t}] = \frac{w_y}{1-b_y}$ and the starting value of the conditional variance as $var(data) - h^m_{y,1}((\theta^m)^2 + (\delta^m)^2)$, where $var(data)$ is the variance of the market return.

The expectation $E[z_t^m|\Phi_t]$ can then be solved using the following summation:

$$\hat{z}_t^m = E[z_t^m|\Phi_t] = \sum_{j=0}^{\infty} z_t^m(R_t^m, N_t^m = j)\,\Pr(z_t^m, N_t^m = j),$$

where $\Pr(z_t^m, N_t^m)$ is the joint probability of z_t^m and $n_t^m = j$ given that R_t^m is known. Using Bayes' rule, we can write the filtering density $\Pr(z_t^m, N_t^m)$ as:

$$\Pr(z_t^m, N_t^m) = \Pr(z_t^m|R_t^m, N_t^m = j)\,\Pr(N_t^m = j). \tag{27}$$

The second term on the right-hand of Equation (27) is given by Equation (4), and the first term is the probability of of z_t^m given that R_t^m and $N_t^m = j$ are known. Hence, we can write the expected ex post normal component of the return as

$$\begin{aligned}\hat{z}_t^m &= \sum_{j=0}^{\infty} z_t^m(R_t^m, N_t^m = j)\,\Pr(z_t^m|R_t^m, N_t^m = j)\,\Pr(N_t^m = j)\\ &= \sum_{j=0}^{\infty} \frac{h_{z,t}^m}{h_{z,t}^m + j(\delta^m)^2}(R_t^m - \alpha_t^m - j\theta^m)\,\Pr(N_t^m = j).\end{aligned}$$

Once $\hat{z}_t^m$ is known, we can directly infer the filtered jump term $\hat{y}_t^m$ by noting that $\hat{y}_t^m = R_t^m - \alpha_t^m - \hat{z}_t^m$. The time series of filtered $\hat{z}_t^m$ and $\hat{N}_t^m$ from estimated parameters are used in the procedure of maximizing likelihood function for individual stocks.

7.3 Filter jump and diffusion terms from individual stock returns

Given the parameters for the market return and stock return, Θ^m and Θ^i, the time series of $h_{y,t}^i$, $h_{z,t}^i$, $h_{y,t}^\epsilon$, and $h_{z,t}^\epsilon$ can be computed according to the dynamics in Equations (9),

(**??**), (10) and (15). In this section, we discuss how to filter out the unobserved diffusion components and jump components for individual stocks.

Similar as filtering procedure for the market, if the number of jumps in the individual stock return $n_t^i = j$ and stock return R_t^i are known at time t, we can express z_t^i as:

$$z_t^i(R_t^i, n_t^i = j) = \sqrt{\frac{h_{z,t}^i}{h_{z,t}^i + j(\delta^i)^2}}\,(R_t - \alpha_t^i - j\theta^i),$$

where the expression for α_t^i is given in Appendix 7.4. Since $z_t^i(R_t^i, n_t^i = j)$ depends on the discrete number of jumps $n_t^i = j$, the expectation $E_t[z_t^i]$ can be solved by summing up all possible number of jumps:

$$\hat{z}_t^i = \sum_{j=0}^{\infty} \Pr(n_t^i = j | R_t^i)\; z_t^i(R_t^i, n_t^i = j).$$

Once $\hat{z}_t^i$ is known, we can infer $\hat{y}_t^i$ from the relation that $\hat{y}_t^i = R_t^i - \mu_t^i - \hat{z_t^i}$, given the information at time t. After we obtain $\hat{y}_t^i$ and $\hat{z}_t^i$, the filtered idiosyncratic jump component $\hat{y}_t^\epsilon$ and $\hat{z}_t^\epsilon$ can be calculated by,

$$\tilde{z}_t^\epsilon = \tilde{z}_t^i - \beta\tilde{z}_t^m, \quad \tilde{y}_t^\epsilon = \tilde{y}_t^i - p\tilde{h}_{y,t}^m\theta^i.$$

7.4 Expected return for the market and individual stocks

In this section, we provide the proof for Proposition 1. In the absence of arbitrage, the martingale condition in Equation (20) should be satisfied for the market index and individual stock return. First, substituting the pricing kernel in Equation (19) and market dynamic in

Equation (1) into the Equation $E_t[\frac{M_{t+1}}{M_t}e^{R^m_{t+1}}] = 1$, we have:

$$\frac{E_t[\exp(-r - \Lambda^m z^m_{t+1} - \Lambda^m y^m_{t+1} - \sum_{j=1}^J \Lambda^j z^\epsilon_{j,t+1} - \sum_{j=1}^J \Lambda^j y^\epsilon_{j,t+1} + \alpha_m + z^m_{t+1} + y^m_{t+1})]}{E_t[\exp(-\Lambda^m z^m_{t+1} - \Lambda^m y^m_{t+1} - \sum_{j=1}^J \Lambda^j z^j_{t+1} - \sum_{j=1}^J \Lambda^j y^j_{t+1})]} = 1.$$

Since y^m_{t+1} and z^m_{t+1} are independent, $E_t[exp(-\Lambda^m z^m_{t+1} - \Lambda^m y^m_{t+1})]$ can be calculated as:

$$E_t[\exp(-\Lambda^m z^m_{t+1} - \Lambda^m y^m_{t+1})] = exp(\frac{1}{2}(\Lambda^m)^2 h^m_{z,t+1} + h^m_{y,t+1}(\exp(-\Lambda^m\theta^m + \frac{1}{2}(\Lambda^m)^2(\delta^m)^2) - 1)),$$

where we use the moment generating function of normal distribution and compound Poisson distribution. Since there is no correlation between z^m_{t+1}, y^m_{t+1}, $z^\epsilon_{j,t+1}$ and $y^\epsilon_{j,t+1}$, we can get the expression for α_m from Equation (21):

$$\alpha_m = r + (\Lambda^m - \frac{1}{2})h^m_{z,t+1} + (\xi(-\Lambda^m) - \xi(1-\Lambda^m)h^m_{y,t+1},$$

where $\xi(\phi) = \exp(\theta\phi + \frac{\delta^2\phi}{2}) - 1$.

Next, if we substitute the pricing kernel in Equation (19) and the dynamics of individual stock return in (8) into the equation $E_t[\frac{M_{t+1}}{M_t}e^{R^i_{t+1}}] = 1$, we have:

$$\frac{E_t[\exp(-r - \Lambda^m z^m_{t+1} - \Lambda^m y^m_{t+1} - \sum_{j=1}^J \Lambda^j z^\epsilon_{j,t+1} - \sum_{j=1}^J \Lambda^j y^\epsilon_{j,t+1} + \alpha_i + z^i_{t+1} + y^i_{t+1})]}{E_t[\exp(-\Lambda^m z^m_{t+1} - \Lambda^m y^m_{t+1} - \sum_{j=1}^J \Lambda^j z^j_{t+1} - \sum_{j=1}^J \Lambda^j y^j_{t+1})]} = 1. \tag{28}$$

The nominator can be written as:

$$
\begin{aligned}
& e^{-r+\alpha_i} E_t[\exp((-\Lambda^m+\beta) z_{t+1}^m)] E_t[\exp(-\Lambda^m y_{t+1}^m + y_{t+1}^i(p h_{y,t+1}^m))] \times \\
& E_t[-\sum_{j \neq i}^J \Lambda^j z_{j,t+1}^\epsilon - \sum_{j \neq i}^J \Lambda^j y_{i,t+1}^\epsilon - (\Lambda^i - 1) z_{i,t+1}^\epsilon) - (\Lambda^i - 1) y_{i,t+1}^\epsilon)].
\end{aligned}
$$

What we focus on is the second part: $E_t[\exp(-\Lambda^m y_{t+1}^m + y_{t+1}^i(p h_{y,t+1}^m))]$. The jump components for the individual and the market are $y_{t+1}^m = \sum_{j=0}^{N_{t+1}^m} x_j^m$ and $y_{t+1}^i = \sum_{j=0}^{N_{t+1}^i} x_j^i$ respectively. For each market jump, the probability that it triggers a jump in individual stock return is p. Conditionally on N_{t+1}^m jumps in the market, we assume there are T jumps in the individual stocks which are triggered by market and N_ϵ idiosyncratic jumps independent with the market. Conditional on the information of N_m, T and N_ϵ, y_{t+1}^i and y_{t+1}^m follow normal distributions: $N((T+N_\epsilon)\theta_i, (T+N_\epsilon)\delta_i^2)$ and $N(N_m\theta_m, N_m\delta_m^2)$, respectively. Assume that the correlation between x_i which are triggered by the market and x_m is ϕ, and then the conditional covariance between y_{t+1}^i and y_{t+1}^m is $T\phi\delta_i\delta_m$. Using the moment generating function for binomial distribution, the conditional expectation can be written as:

$$
\begin{aligned}
& E_t[\exp(-\Lambda^m y_{t+1}^m + y_{t+1}^i(p h_{y,t+1}^m))|N_m, T] \\
= & \exp((-\Lambda^m \theta_m + \frac{1}{2}\delta_m^2 (\Lambda^m)^2) N_m + (\theta_i + \frac{1}{2}\delta_i^2 - \phi\delta_i\delta_m\Lambda^m)T).
\end{aligned}
$$

While T and N_m is still correlated, we use the fact that T follows binomial distribution $B(p, N_m)$ conditional on N_m. Let $a = -\Lambda^m\theta_m + \frac{1}{2}\delta_m^2(\Lambda^m)^2$ and $b = \theta_i + \frac{1}{2}\delta_i^2 - \phi\delta_i\delta_m\Lambda^m$, and use the law of iterated expectation, the unconditional expectation of $\exp(aN_m + bT)$ can be expressed as:

$$E_t[E_t[\exp(aN_m + bT)|N_m]] = E_t[\exp(aN_m)E_t[\exp(bT)|N_m]]$$
$$= E_t[\exp(aN_m + \log(1 - p + pe^b)N_m)]$$
$$= \exp(h^y_m((1 - p + pe^b)e^a - 1).$$

Substituting everything back to the Equation (28) and taking log of the two sides, we have

$$\alpha_i = r + (\beta\Lambda^m - \frac{1}{2}\beta^2)h^m_{z,t+1} + pe^a(1 - e^b)h^m_{y,t+1} + (\Lambda^i - \frac{1}{2})h^\epsilon_{z,t+1} + (\xi(-\Lambda^i) - \xi(1 - \Lambda^i))h^\epsilon_{y,t+1}.$$

If we let $\lambda_z = \Lambda^m$, $\lambda_{zi} = \Lambda_{zi}$ and $\lambda_{yi} = \xi(1) + \xi(-\Lambda_{yi}) - \xi(1 - \Lambda_{yi})$, the expression for the discrete-time equity premium can be expressed as:

$$E_t[\exp(R^i_{t+1})] = \exp(r + \beta\lambda_z h^m_{z,t+1} + p(e^a(1 - e^b) + e^{\theta_i + \frac{1}{2}\delta_i^2} - 1)h^m_{y,t+1} + \lambda_{zi}h^\epsilon_{z,t+1} + \lambda_{yi}h^\epsilon_{y,t+1}). \tag{29}$$

We can see that the expected stock return is increasing in β and ϕ because $\lambda_z > 0$ and $\lambda_y > 0$ and it is increasing in p when $e^a(1 - e^b) + e^{\theta_i + \frac{1}{2}\delta_i^2} - 1 > 0$.

The price of market diffusive risk λ_z and market jump risk λ_y can be obtained from estimating the model for the market index and the price of equity-specific diffusive risk can be estimated from the model for each stock. The form of expected stock return in Equation (29) is comparable with the continuous time expression in Yan (2011).

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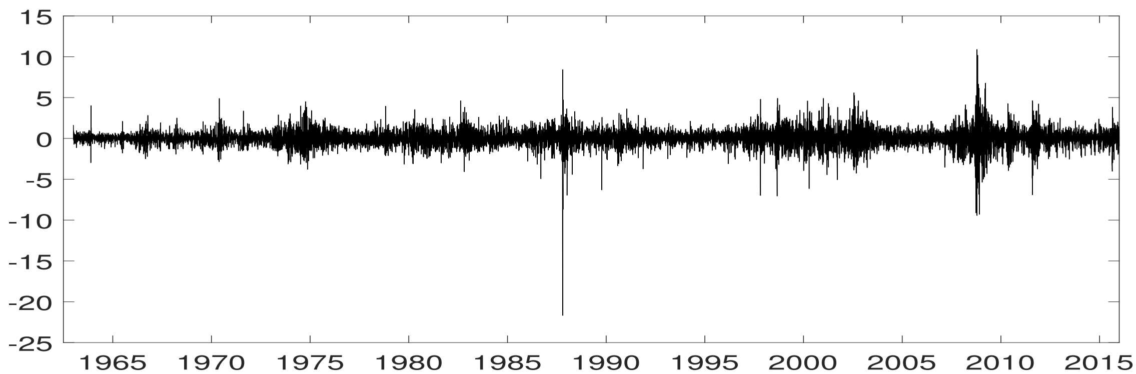
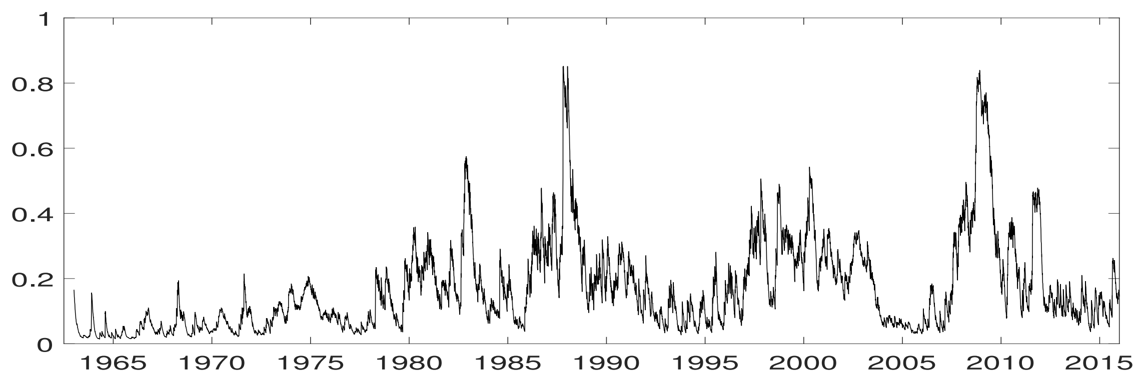
Figure 1: Conditional jump intensity and filtered jump component (S&P500)

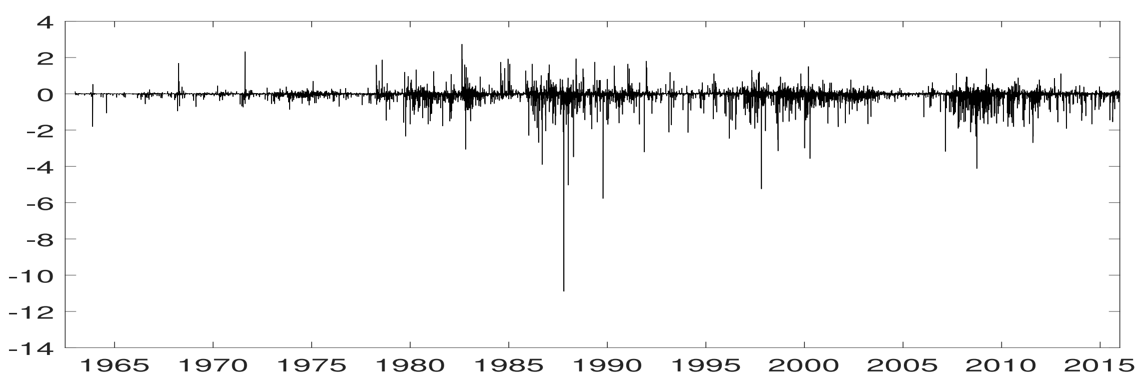
(a) Daily return



(b) Conditional jump intensity

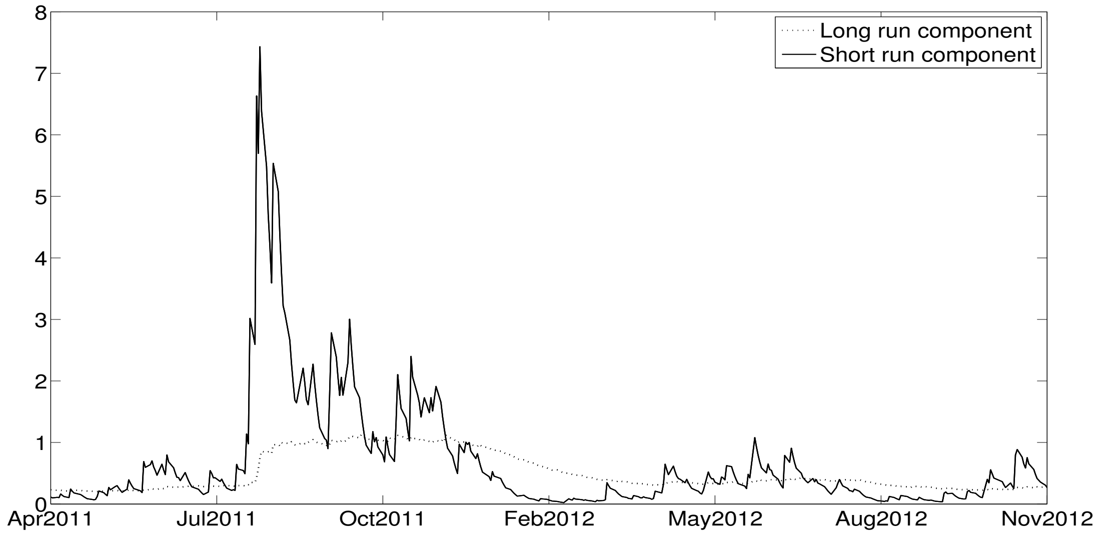


(c) Jump Component



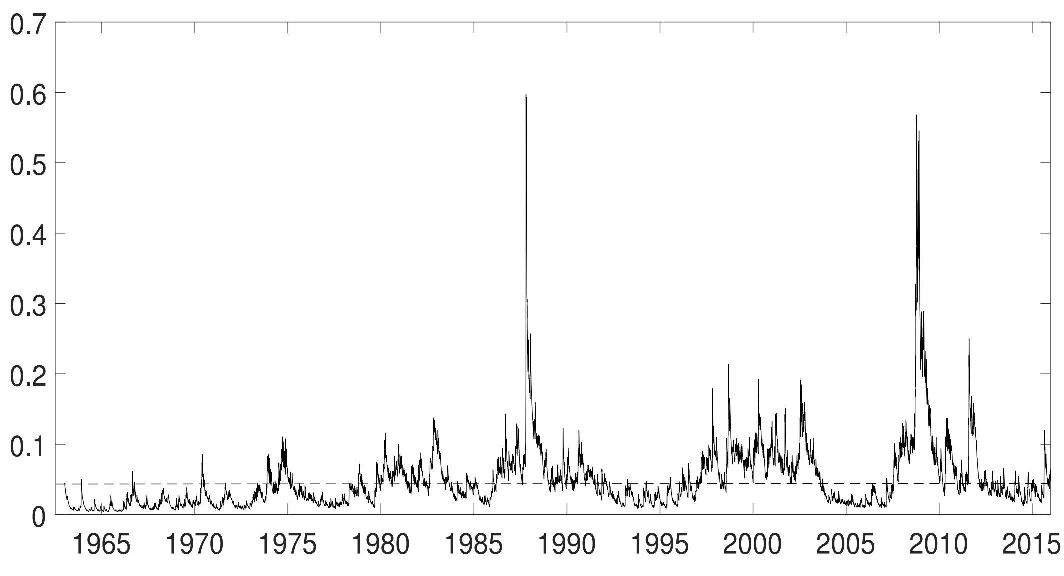
Note: Figure (a) plots the daily return of the S&P 500 index from Jan 1963 to December 2015. By estimating the model in Section 2.1, we filter out the conditional jump intensity $h^{m}_{y,t+1}$ and plot in Figure (b). The filtered jump component is presented in Figure (c). The returns are scaled by 100.

Figure 2: Long run and short-run components of the diffusive variance (S&P 500)



Note: This figure shows the long-run and short-run components of the diffusive variance for market returns from 1911 to 2012.

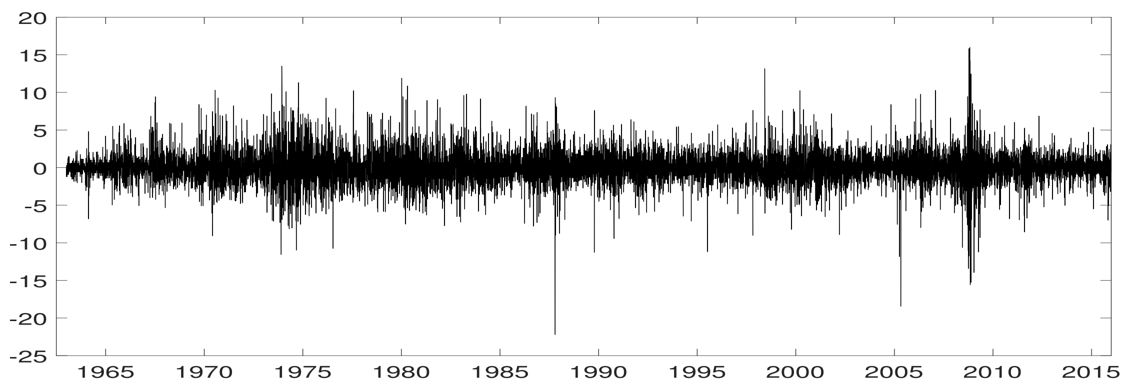
Figure 3: Conditional equity premium (S&P 500)



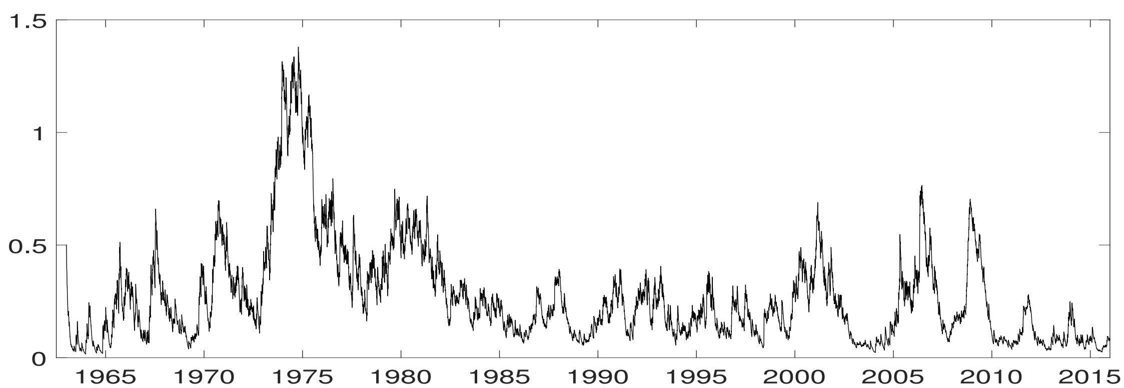
Note: This figure shows the estimated time series of the daily conditional equity premium from 1962 to 2015. The dotted line represents the level of the unconditional mean of the equity premium.

Figure 4: Decomposition of conditional jump intensity (ADM)

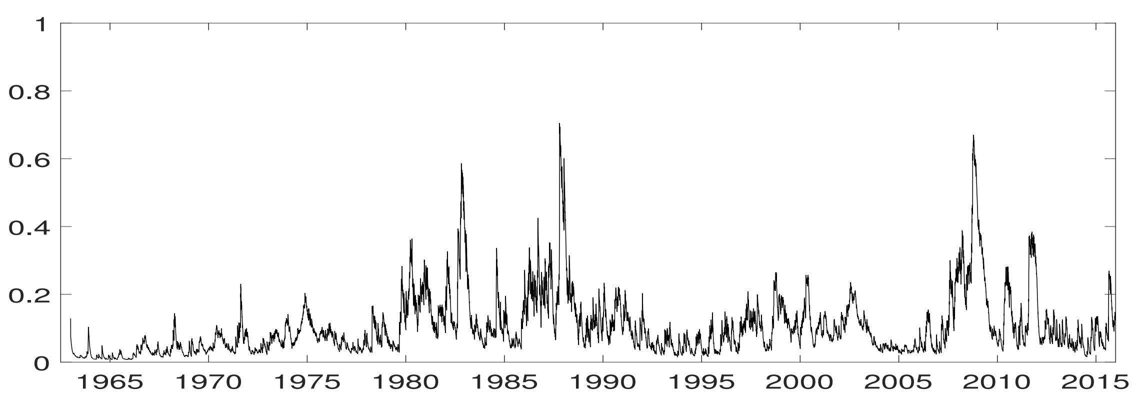
(a) Daily return



(b) Conditional idiosyncratic jump intensity



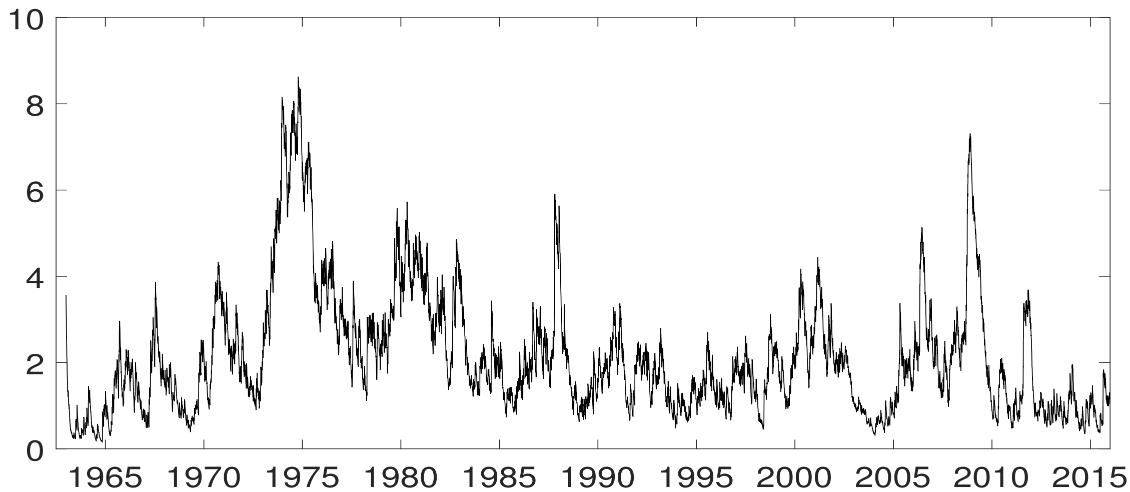
(c) Conditional systematic jump intensity



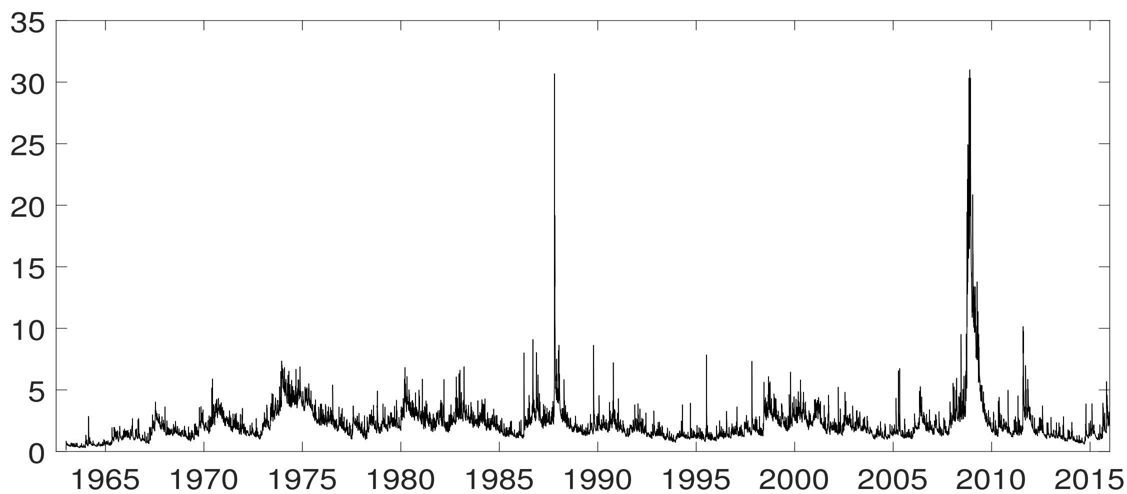
Note: Figure (a) plots the daily return of ABT from Jan 1963 to December 2015. Estimating the joint model for ADM in Section 2.2, we filter out the conditional jump intensity $h^i_{y,t+1}$ and decompose it into two parts: idiosyncratic jump intensity $h^{\epsilon}_{y,t+1}$ and systematic jump intensity $\beta^i_t h^m_{y,t+1}$. They are plotted in Figure (b) and (c).

Figure 5: Decomposition of conditional variance (ADM)

(a) Jump part of the conditional variance



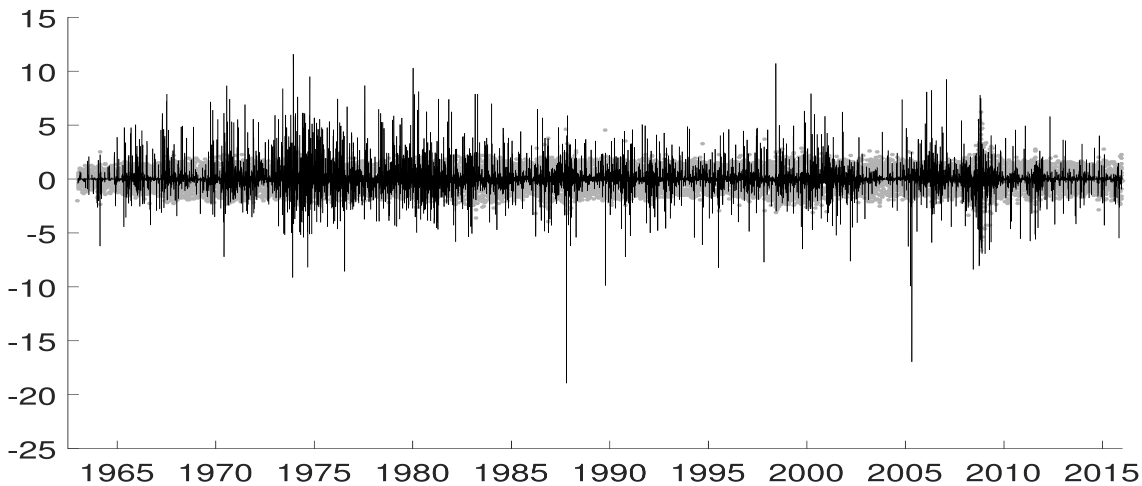
(b) Diffusive part of the conditional variance



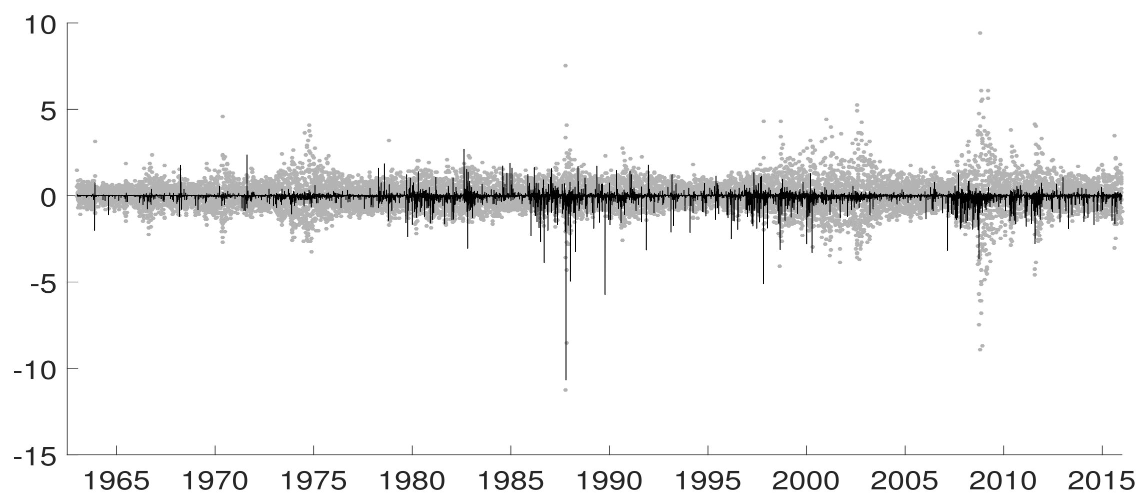
Note: We decompose the total conditional variance of ADM given in Equation 18 into the diffusive and jump variance components. The diffusive and jump components of the total variance are given in Figure (a) and (b).

Figure 6: Filtered diffusion and jump components (ADM)

(a) Filtered idiosyncratic jump and diffusion components for ADM



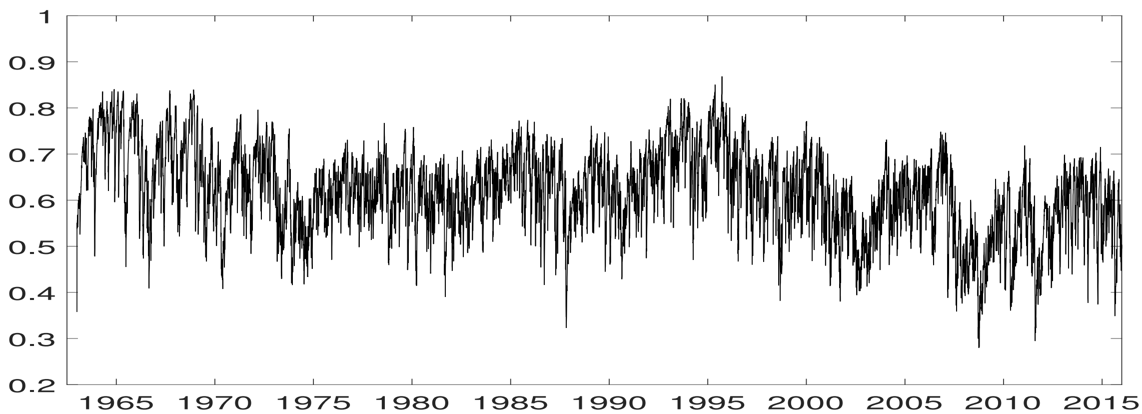
(b) Filtered systematic jump and diffusion components for ADM



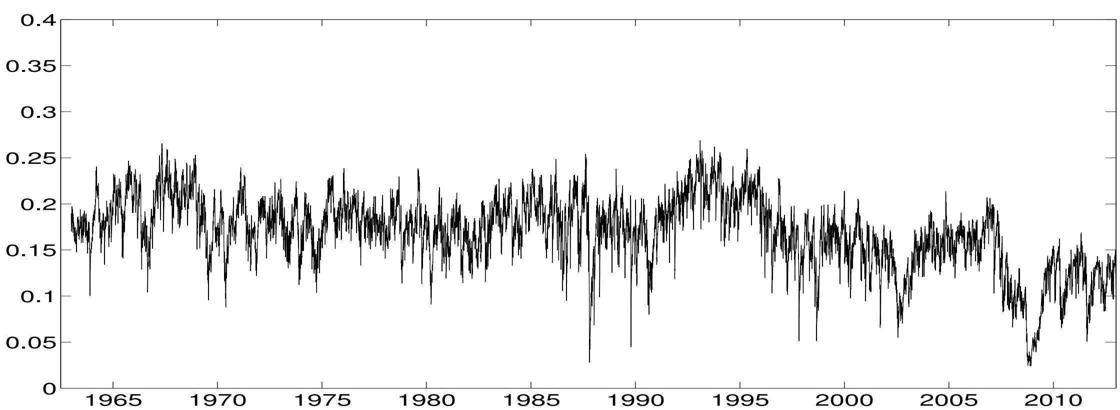
Note: This figure shows the filtered jump and difussion components of ADM stock returns using the procedure in Appendix 7.3. Figure (a) shows filtered idiosyncratic jump (z_t^ϵ) and difussion components (y_t^ϵ) and Figure (b) shows systematic jump and diffusion components. The solid lines plots the jump components and the the gray markers represent the diffusion components.

Figure 7: Decomposition of risk premium over time

(a) Contribution of the total idiosyncratic risk premium



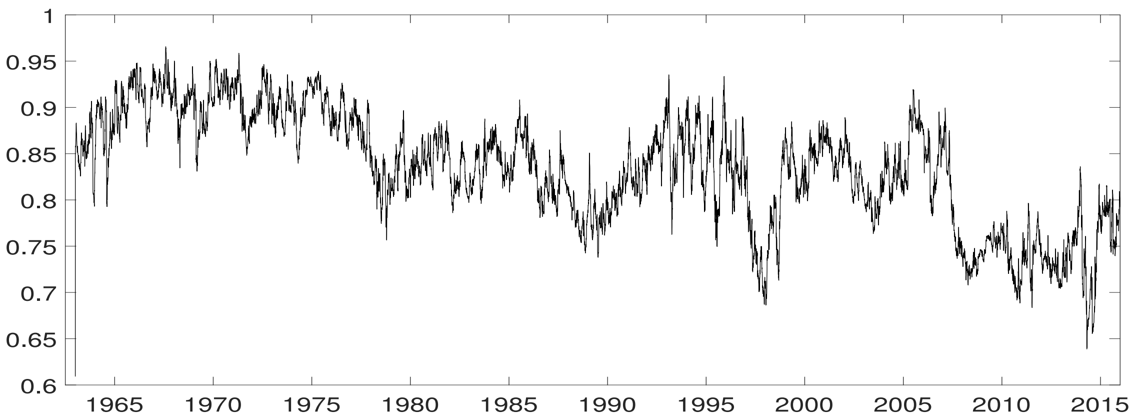
(b) Contribution of the jump risk premium



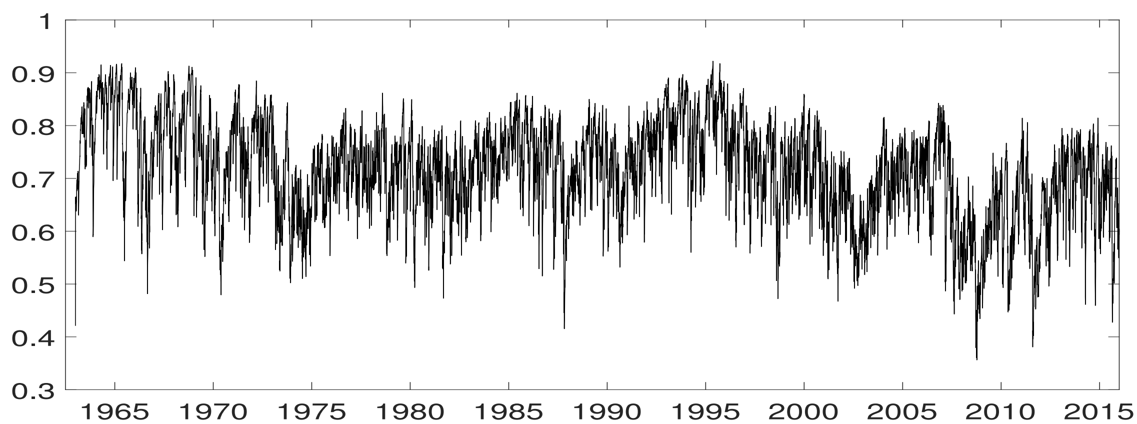
Note: We first calculate the four risk premiums for each stock and take the cross-sectional average on each day. We aggregate the idiosyncratic diffusive and jump risk premiums and show their contribution in the total expected return in Figure (a). We aggregate the systematic and idiosyncratic jump risk premiums and show their contribution in the expected stock return in Figure (b).

Figure 8: Decomposition of jump intensity and volatility over time

(a) Contribution of idiosyncratic jump intensity contribution



(b) Contribution of idiosyncratic volatility



Note: Figure (a) shows the average contribution of idiosyncratic jump intensity in the total jump intensity (hy_t^i/hy_t) for all stocks. Figure (b) shows the average contribution of idiosyncratic diffusive volatility in the total diffusive volatility (hz_t^i/hz_t) for all stocks.

Table 1: Descriptive statistics for the daily stock return

	Mean	Std	Min	Max	Skewness	Kurtosis
ADM	0.029	2.048	-22.210	15.986	0.056	9.214
BAX	0.030	1.849	-30.504	16.465	-0.708	17.587
CL	0.031	1.621	-21.495	18.482	0.028	11.915
DD	0.013	1.600	-20.209	11.196	-0.151	9.119
DOW	0.022	1.832	-21.495	16.916	-0.222	11.403
EMR	0.027	1.588	-17.376	14.319	-0.144	10.092
GE	0.023	1.620	-19.251	17.984	-0.051	11.500
IBM	0.017	1.583	-26.119	12.351	-0.251	15.425
MMM	0.023	1.445	-30.114	10.899	-0.652	20.962
PEP	0.034	1.541	-15.448	14.951	0.021	8.919
PG	0.026	1.385	-37.687	20.029	-2.180	70.666
T	0.010	1.538	-23.920	20.837	-0.098	20.621
UTX	0.029	1.752	-33.213	12.789	-0.463	16.195
XOM	0.029	1.382	-26.726	16.455	-0.371	21.580
XRX	0.007	2.238	-29.801	32.963	-0.500	22.075
S&P500	0.009	1.017	-21.679	10.894	-0.9131	26.818

Table 1 presents the summary statistics for the daily returns of the 15 individual stocks and the S&P500 index. The dataset starts from January 3rd, 1963 and ends at December 31st, 2015. The daily returns are scaled by 100.

Table 2: Parameter estimates for the market returns

	Single estimation		Joint estimation			
	parameters	t stats	mean	std	min	max
Λ^m	2.539	2.231	2.663	0.419	1.610	3.160
w_z^m	0.000	1.652	0.000	0.000	0.000	0.001
b_{z1}^m	0.976	61.341	0.976	0.003	0.971	0.978
τ_{11}^m	-5.013	-23.387	-5.041	0.178	-5.268	-4.716
τ_{12}^m	-1.611	-4.024	-1.354	0.350	-1.850	-0.780
τ_{13}^m	0.954	2.963	0.905	0.415	0.059	1.378
b_{z2}^m	0.859	13.368	0.857	0.007	0.833	0.859
τ_{21}^m	-14.008	-13.153	-13.854	0.233	-14.000	-13.041
τ_{22}^m	-0.295	-3.442	-0.339	0.088	-0.479	-0.214
τ_{23}^m	11.992	11.646	12.036	0.229	11.236	12.184
w_y^m	0.000	2.249	0.001	0.001	0.000	0.002
b_y^m	0.997	321.276	0.996	0.003	0.985	0.999
a_y^m	0.055	2.551	0.053	0.028	0.020	0.120
δ^m	-0.288	-3.421	-0.237	0.051	-0.302	-0.158
θ^m	1.156	11.420	0.983	0.123	0.805	1.164

Note: Table 2 shows the estimation results on the daily returns of S&P500 index, from January 1963 to December 2015. The column called "Single estimation" reports the results for estimating the market return alone with log likelihood 16052.803. The column called "Joint estimation" shows statistics of the 15 sets of estimated parameters for the market dynamics from joint estimation using both stock returns and market returns. In parentheses we report the t statistics. Note that we report estimation results with the return data multiplied by 100.

Table 3: Estimated parameters for the stock returns

	ADM	BAX	CL	DD	DOW	EMR	GE	IBM
Λ_z^i	2.421	2.717	2.457	0.274	1.672	4.347	1.376	1.003
	(1.876)	(3.142)	(2.113)	(0.184)	(1.499)	(2.315)	(0.855)	(0.734)
w_z^i	0.000	0.000	0.003	0.041	0.001	0.002	0.000	0.005
	(4.403)	(0.321)	(1.669)	(1.598)	(0.615)	(2.13)	(0.077)	(2.906)
b_{z1}^i	0.985	0.972	0.967	0.783	0.977	0.971	0.945	0.966
	(4.426)	(831.788)	(175.454)	(5.67)	(454.626)	(171.757)	(95.781)	(242.418)
τ_{11}^i	-4.829	-4.418	-4.797	-2.938	-5.085	-9.501	-4.296	-4.430
	(-13.5)	(-37.569)	(-18.955)	(-14.09)	(-13.435)	(-0.965)	(-11.252)	(-18.567)
τ_{12}^i	-4.661	-3.417	-0.610	-0.727	-1.111	-0.786	-0.604	-2.515
	(-2.583)	(-4.858)	(-2.42)	(-5.753)	(-2.199)	(-4.209)	(-2.686)	(-2.976)
τ_{13}^i	0.882	1.315	0.825	0.637	1.462	5.998	1.319	0.853
	(2.738)	(5.518)	(1.772)	(2.201)	(2.497)	(0.63)	(2.68)	(3.318)
b_{z2}^i	0.688	0.641	0.306	0.051	0.324	0.000	0.303	0.795
	(0)	(7.745)	(3.463)	(0.057)	(2.273)	(0.007)	(1.74)	(14.067)
τ_{21}^i	-14.147	-14.294	-2.715	-15.862	-15.314	-15.135	-14.674	-14.311
	(-0.168)	(-1.351)	(-13.556)	(-0.056)	(-0.185)	(-0.101)	(-0.245)	(-1.057)
τ_{22}^i	-0.618	-0.009	0.339	0.450	1.494	1.618	0.504	-0.414
	(-0.03)	(-0.256)	(1.457)	(0.161)	(3.074)	(3.108)	(0.902)	(-2.071)
τ_{23}^i	11.817	11.648	0.202	10.135	10.683	10.862	11.324	11.678
	(0.01)	(1.134)	(0.716)	(0.036)	(0.135)	(0.073)	(0.199)	(0.903)
w_y^i	0.001	0.002	0.000	0.001	0.005	0.035	0.001	0.000
	(2.118)	(8.497)	(1.006)	(0.156)	(2.491)	(1.933)	(2.088)	(3.133)
b_y^i	0.999	0.989	0.998	0.998	0.971	0.875	0.998	0.996
	(690.339)	(686.008)	(113.016)	(94.485)	(94.282)	(15.86)	(290.105)	(115.194)
a_y^i	0.078	0.066	0.041	0.061	0.208	0.373	0.044	0.025
	(5.82)	(5.51)	(3.574)	(0.356)	(3.793)	(4.215)	(7.469)	(62.857)
θ_i	0.255	0.173	0.187	0.193	0.064	0.134	0.112	0.157
	(2.878)	(1.08)	(4.25)	(1.545)	(1.389)	(2.97)	(1.804)	(1.357)
δ^i	2.368	2.519	1.781	1.472	2.153	1.352	1.316	2.973
	(21.332)	(25.105)	(25.21)	(3.349)	(13.702)	(14.796)	(20.889)	(12.633)
β_1^i	-1.908	-0.472	-0.094	-0.220	-0.149	0.015	-0.153	-0.742
	(0.717)	(-3.536)	(-2.804)	(-0.461)	(-2.784)	(0.841)	(0.089)	(-4.423)
β_2^i	0.521	-0.187	-0.381	0.129	-0.042	-0.095	-0.008	0.071
	(1.212)	(1.739)	(-0.741)	(1.054)	(4.257)	(0.175)	(0.748)	(3.305)
p_1^i	-1.722	0.010	437.937	-2.717	-4.634	8.907	-2.700	1.715
	(-0.102)	(1.081)	(4.128)	(-0.077)	(-0.937)	(1.473)	(-0.19)	(1.223)
p_2^i	2.472	-2.669	-651.153	2.143	2.699	-7.118	-2.956	-5.128
	(-0.051)	(-1.676)	(-4.211)	(0.102)	(0.863)	(-1.472)	(-0.285)	(-2.273)
lgl	42745.685	41547.098	39435.002	39338.968	40701.499	39324.554	39071.487	38957.459

Note: Table 3 shows the estimation results on the daily returns of the 8 out of 15 stocks, from January 1963 to December 2015. T statistics are shown in the parentheses. lgl is the maxmized log likelihood for each stock.

Table 4: Estimated parameters for the stock returns (continued)

	MMM	PEP	PG	T	UTX	XOM	XRX
Λ_z^i	1.259	3.583	2.258	0.272	2.148	2.879	0.992
	(0.802)	(2.293)	(2.076)	(0.306)	(1.718)	(1.562)	(1.335)
w_z^i	0.000	0.049	0.006	0.022	0.004	0.085	0.013
	(0.595)	(2.55)	(2.431)	(3.513)	(1.123)	(3.658)	(3.039)
b_{z1}^i	0.989	0.759	0.932	0.798	0.953	0.726	0.957
	(716.75)	(12.914)	(114.195)	(23.665)	(94.559)	(13.187)	(174.391)
τ_{11}^i	-5.073	-3.010	-3.925	-2.569	-4.230	-2.918	-4.020
	(-45.912)	(-10.199)	(-19.826)	(-16.366)	(-14.625)	(-14.261)	(-20.192)
τ_{12}^i	-18.320	-0.243	-1.053	-0.133	-1.423	-0.098	-1.136
	(-2.281)	(-1.646)	(-4.783)	(-0.833)	(-2.584)	(-1.109)	(-3.516)
τ_{13}^i	0.838	0.718	1.366	-0.045	0.935	0.434	0.683
	(2.67)	(2.603)	(6.233)	(-1.795)	(2.62)	(0.629)	(2.634)
b_{z2}^i	0.000	0.000	0.168	0.000	0.398	0.108	0.276
	(0)	(-0.373)	(1.158)	(0.001)	(3.223)	(0.609)	(2.416)
τ_{21}^i	-14.636	-16.188	-14.480	-14.942	-14.559	-16.544	-14.277
	(-0.173)	(-0.091)	(-0.265)	(-0.198)	(-0.281)	(-0.145)	(-74.179)
τ_{22}^i	0.560	1.613	0.436	0.850	0.604	1.908	-0.716
	(1.025)	(1.674)	(2.281)	(7.364)	(2.152)	(1.675)	(-2.157)
τ_{23}^i	11.350	9.806	11.454	11.056	11.437	9.455	11.708
	(0.135)	(0.056)	(0.212)	(0.151)	(0.222)	(0.083)	(63.075)
w_y^i	0.003	0.000	0.000	0.000	0.000	0.002	0.000
	(2.572)	(0.722)	(1.916)	(0.631)	(1.926)	(1.424)	(16.509)
b_y^i	0.980	1.000	0.998	0.999	0.999	0.993	0.997
	(267.539)	(448.247)	(159.528)	(236.416)	(100.171)	(282.223)	(196.02)
a_y^i	0.356	0.066	0.037	0.049	0.036	0.128	0.019
	(3.914)	(6.855)	(9.241)	(9.965)	(4.665)	(2.733)	(6.958)
θ_i	0.017	0.084	0.107	0.180	0.176	-0.028	-0.213
	(0.867)	(2.559)	(1.993)	(4.948)	(2.805)	(-0.816)	(-0.976)
δ^i	1.785	1.378	1.638	1.457	1.918	1.239	4.191
	(18.417)	(32.815)	(21.345)	(29.234)	(16.81)	(13.729)	(27.807)
β_1^i	-0.347	-0.401	-0.824	-0.672	-0.093	-0.230	-0.063
	(-3.382)	(-4.239)	(-5.952)	(-7.034)	(-0.191)	(-1.802)	(-0.054)
β_2^i	-0.030	-0.100	-0.054	0.133	-0.042	-0.018	-0.123
	(0.704)	(0.252)	(2.648)	(2.982)	(0.845)	(-0.425)	(0.927)
p_1^i	0.682	-3.119	33.411	0.317	-0.320	-0.506	-0.313
	(0.914)	(-1.971)	(0.676)	(0.122)	(-0.196)	(-0.264)	(-1.71)
p_2^i	-0.400	2.216	-45.767	-1.776	-0.132	-0.181	-1.174
	(-0.849)	(1.804)	(-0.718)	(-0.606)	(-0.099)	(-0.162)	(-1.318)
lgl	38251.068	38828.660	36857.448	37229.895	40849.123	37444.384	42899.222

Note: Table 4 shows the estimation results on the daily returns of the 8 out of 15 stocks, from January 1963 to December 2015. T statistics are shown in the parentheses. lgl is the maxmized log likelihood for each stock.

Table 5: Summary statistics of the time varying β_t^i and p_t^i

	β_t^i				p_t^i			
	mean	std	min	max	mean	std	min	max
ADM	0.262	0.080	0.174	0.903	0.652	0.172	0.278	0.999
BAX	0.516	0.041	0.326	0.589	0.088	0.063	0.000	0.306
CL	0.623	0.095	0.242	0.809	0.171	0.374	0.000	1.000
DD	0.918	0.057	0.835	1.257	0.380	0.190	0.114	0.991
DOW	0.825	0.015	0.745	0.850	0.178	0.188	0.022	0.991
EMR	0.921	0.038	0.730	0.985	0.729	0.332	0.000	0.999
GE	0.852	0.003	0.836	0.857	0.005	0.005	0.000	0.026
IBM	0.513	0.017	0.487	0.610	0.082	0.100	0.000	0.531
MMM	0.686	0.009	0.638	0.700	0.567	0.045	0.330	0.636
PEP	0.605	0.026	0.473	0.649	0.318	0.191	0.081	0.990
PG	0.415	0.010	0.363	0.431	0.248	0.397	0.000	1.000
T	0.587	0.037	0.532	0.810	0.205	0.094	0.003	0.442
UTX	0.873	0.016	0.788	0.900	0.388	0.014	0.315	0.411
XOM	0.780	0.006	0.746	0.790	0.334	0.018	0.243	0.363
XRX	0.828	0.044	0.612	0.903	0.234	0.020	0.142	0.342

Note: Table 5 shows summary statistics of the time varying β_t^i and p_t^i for each stock. Note that we report estimation results with the return data multiplied by 100.

Table 6: Summary statistics of the dynamic jump intensities and volatilities

	hy_m	hz_m	hy_i	hz_i	Var(Jump)/Var	Var(Idio) /Var	Var(Idio Jump)/Var
ADM	0.150	0.799	0.271	2.005	0.474	0.816	0.337
BAX	0.168	0.779	0.181	1.802	0.368	0.868	0.344
CL	0.154	0.799	0.291	1.239	0.389	0.804	0.368
DD	0.132	0.814	0.343	0.880	0.380	0.686	0.332
DOW	0.148	0.792	0.164	1.772	0.294	0.776	0.255
EMR	0.138	0.843	0.280	1.048	0.344	0.658	0.263
GE	0.218	0.832	0.369	1.259	0.326	0.761	0.325
IBM	0.138	0.789	0.064	1.567	0.257	0.829	0.231
MMM	0.208	0.839	0.134	0.862	0.365	0.587	0.164
PEP	0.136	0.800	0.543	0.860	0.451	0.756	0.408
PG	0.128	0.806	0.194	1.043	0.326	0.807	0.299
T	0.185	0.835	0.374	1.048	0.384	0.719	0.335
UTX	0.118	0.796	0.243	1.417	0.362	0.733	0.303
XOM	0.206	0.837	0.269	0.803	0.295	0.654	0.229
XRX	0.223	0.834	0.066	2.895	0.267	0.867	0.267

Table 7: Portfolio performance

	Total Premium	SD	SJ	ID	IJ
1	0.033	0.058	0.039	0.042	0.043
2	0.038	0.045	0.047	0.027	0.053
3	0.061	0.082	0.069	0.077	0.068
4	0.051	0.055	0.044	0.042	0.061
5	0.141	0.081	0.087	0.137	0.096
5-1	0.108	0.023	0.049	0.095	0.053
t stat	3.462	0.791	1.750	3.119	1.926
p value	0.000	0.215	0.040	0.001	0.027

Note: We denote the systematic diffusive risk, systematic jump risk, idiosyncratic diffusive risk, and idiosyncratic jump risk as SD, SJ, ID and IJ. The stocks are sorted in ascending order into five portfolios with 3 in each of them, according to the risk premia on each type of risks. We also sort the stocks according to the total risk premia. In each portfolio, we assign equal weights to the stocks and calculate the portfolio return over the sample period. The annualized portfolio returns for the constructed portfolios are presented in the first five rows in Table 7. The last two rows report t-statistics and corresponding p-values when testing the null hypothesis that the difference between the return of the fifth portfolio and the first portfolio is equal to zero. The t-statistics are calculated based on the Newey-West standard error.