"Integrating Credit and Market Risk: An Empirical Study for the Swap Market"

by

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Abbreviations

ATM	At the money
BIS	Bank of International Settlements
CE	Credit Exposure
CDF	Cumulative Distribution Function
CSI	Credit Spread Index
CSIO	Credit Spread Index Option
CSO	Credit Spread Option
DF	Discount Function
ECB	European Central Bank
ECE	Expected Credit Exposure
EL	Expected Loss
EWMA	Exponential Weighted Moving Average
GARCH	Generalised ARCH
HS	Historical Simulation
IID	Independent and Identically Distributed
IRS	Interest Rate Swap
JLT	Jarrow, Lando and Turnbull Credit Risk Model
JT	Jarrow and Turnbull Credit Risk Model
LHS	Left Hand Side
LS	The Longstaff & Schwartz Model (1992)
LS (1992)	The Longstaff & Schwartz Model (1992)
MC	Monte Carlo
MTM	Mark to market

OLS	Ordinary Least Squares
PrDef	Probability of Default
PDE	Partial Differential Equation
PDF	Probability Density Function
RHS	Right Hand Side
RR	Recovery Rate
SD	Standard Deviation
S&P	Standard & Poor's
UL	Unexpected Loss
VaR	Value at Risk

Important Symbols

α	LS (1992) parameter
β	LS (1992) parameter
$B(t,T)$	Default Free Bond Price
C	Call Option
δ	LS (1992) parameter
δ	Recovery Rate
$D(t,T)$	Defaultable Bond Price
ε	LS (1992) parameter
$\varepsilon 1$	iid
$\varepsilon 2$	iid
γ	LS (1992) parameter
η	LS (1992) parameter
ht	Conditional Variance
Λ	Generator Matrix
$N()$	Normal Distribution
P	Put Option
$Q(t,T)$	Probability of Default
$Q(t,T)$	Rating Transition Matrix
ρ	Correlation
r	Short Rate
σ	Volatility
$s(t,T)$	Short Credit Spread
V	Volatility of short rate

W	Wiener Process or Brownian Motion

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Declaration

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Abstract

This dissertation proposes an integrated measure of credit and market risk for interest rate swap portfolios. Our research is based upon the use of EUR interest rate and credit spread data for the period 2001 – 2004. There are three self-contained but seemingly related projects in this dissertation. The objectives of this research are: 1) to price interest rate sensitive and credit spread options under the Longstaff & Schwartz 1992 framework; 2) to devise an integrated measurement approach of credit and market risk; 3) to extent the proposed integrated approach in measuring economic loss and compare it with the current standard approach.

The mean reverting and GARCH characteristics of EUR credit spread indices were investigated between 2001 and 2004. We find evidence of significant GARCH effects in the EUR credit spread indices and mean reversion which is dependent on the frequency of the time series. These properties of the EUR credit spread indices suggest a stochastic term structure volatility model would be suitable to model their evolution. The model of choice was used to estimate discount curves based upon the observed interest rate term structure. The range of yield curve shapes fitted accurately was extensive suggesting the model would be suitable in fitting the more complex credit spread curves. The estimation of yield curves over a period of time suggested that the volatility of the model parameters is reduced substantially with the use of weekly data. The model was able to match the market implied volatility in pricing interest rate options with greater accuracy in the pricing of short-term options. The LS model was quite successful in the fitting of various credit spread curves Although the pricing of credit spread options using the LS model is internally inconsistent evidence suggested that it prices short term spread options with good accuracy. The direct link between credit spreads and default probabilities was fully exploited by estimating implied default probabilities. Evidence suggested that the implied default probabilities did not violate the no-arbitrage conditions of credit risk pricing. A time series examination showed only in one occasion that a lower rating had a lower probability of default than its immediate higher rating. Also the historical transition matrix of S&P proved to be quite far from the expectations of the credit markets.

A dynamic approach to manage the risks associated with 10 different rated hypothetical interest rate swap portfolios was proposed based upon a hedging methodology. The proposed dynamic hedging of the swaps default risk is done by taking offsetting exposure related positions in respective credit spread index options. The efficiency of the hedging methodology shows strong linkages between the swap exposures and the credit spread index options. The integrated measure is proved to be higher at all times than the market VaR of swaps. Evidence suggest that the credit risk part of the integrated measure is not correlated with its respective market risk. The approach illustrates in a single overall market VaR measure both the market and the implicit credit risk run by a portfolio of swaps over a specified time horizon and confidence level.

The proposal of the integrated measure was put further to the test by performing a comparison between the existing methodology of integrated credit risk measurement and the proposed analytic "integrated" methodology. The comparison was performed on an actual swap portfolio taken from a medium sized European Bank. The comparison yielded similar results with the integrated approach measuring higher economic loss. The link of the expected credit exposure to the credit spread index option was evident. The use of historical simulation (HS) over a multi-step Monte Carlo (MC) simulation to measure expected credit exposures over a 3 month period is proved to be less accurate but not substantially different across all cases suggesting that over small time intervals the HS method is a fast and efficient way of measuring expected credit exposures.

CHAPTER 1 Introduction and Review of Literature

1.1 Introduction

This dissertation is a quantitative study whose primary objective it to propose an integrated measure of credit and market risk of derivatives portfolios with a special focus in interest rate swaps. Our research is based primarily upon the use of interest rate, credit spread index and credit spread data in the Euro area for the period 2001 to 2004. The first chapter of the dissertation introduces the background of the study, specifies the problems of the study and describes its significance

1.2 Background of the study

Financial risk quantification, analysis and control have evolved dramatically over the last decades. Significant advances in academic theory have had a major impact on practical risk measurement and financial institutions around the world are investing heavily in systems and personnel for measuring risks. The main driving forces behind this phenomenon arises from the demand for up-to-date information for regulators, shareholders and of course senior executives. The question "how much risk we are running" is one that most executives fail to answer without producing a set of figures rather than a single one. There should be a point where this question should be answered by a single figure.

After most major financial crises the regulators and the financial markets participants worked together to agree on procedures and measures that will safeguard the financial system. This led to the identification of most common risks associated with the financial markets. Following that, common and well-thought measures were devised to capture each one particular risk. However, none of these measures account for the interaction of different risks. Ever since the introduction of derivatives in the financial marketplace, the complexity of risks being introduced among market participants has increased exponentially. As a result each type of risk affects the magnitude and the direction of other risks. For example, the lack of liquidity in a traded stock which could appear out of the investor's preference could have a serious effect on the market price of the stock. The lack of transparency on a company's accounts could lead to the deterioration of both its credit rating and stock/bond price.

The integration of risks in the financial marketplace has intensified since the beginning of the 1990s. The introduction of credit derivatives has facilitated substantially the active risk management of "traditional" loan books and bond portfolios. On the other side, it introduced the "pass the parcel" or in this case "pass the risk" mentality to someone else rather than the originator of credit risk creating process i.e. the lender. Even before credit derivatives made their appearance risks were integrated. For example the interest rate swaps market is the biggest OTC derivatives market by turnover. Swaps bear both market and credit risk for both parties entering the deal. The market risk of a swap is quite straightforward. The existence of credit risk is due to the possibility of one party defaulting, then it will not be able to fulfil its obligations to the other i.e. payments. When dealers which often represent financial institutions enter into swap agreements, they usually hedge the market risk of the swap either by entering into an offsetting bond position or by using short term interest rate futures and bond futures. By doing that the dealer is trying to immunise its positions to adverse movements in the underlying rates. This "diversification" of delta risk is the norm among financial institutions, which enter into thousands of swap agreements with counterparties of different ratings every day. This technique reduces market risk and subsequently dealers can take even larger positions creating more exposure for their institution. This results in an increase of credit risk that an institution is running. This is managed by setting credit &

settlement limits usually per counterparty. The process of setting limits is being overseen by the credit risk management function of each financial institution. The limits are being set using a pool of criteria:

- Credit rating
- Financial ratios
- Company outlook
- Internal rating
- Credit spread of companys' debt
- Credit analysts' view
- Nature of business

These are some of the criteria which lead to the internal rating, which leads to the allocation of limits for counterparties. In that way no single dealer can enter into unlimited deals with any counterparty, they can only do so subject to the allocated limits. This is one way of accounting for credit risk and protecting the institution from excess credit risk. Other techniques to protect for credit risk are collateralisation, options for early termination and other features in OTC agreements. These techniques were introduced in the late 1990s when hedge fund activity and investment grade companies activity increased exponentially in the OTC market, especially in the interest rate swap market. Up to date there is no single official risk measurement which takes into account both market and credit risk in a swap deal.

1.3 The Problem

This dissertation proposes an integrated methodology of market and credit risk measurement. In this section we will state the objectives of each of the four projects separately.

1.3.1 Objectives of the First Research Paper

The main objective in this study is to test the performance of the chosen term structure model, which is the Longstaff and Schwartz (1992a) two-factor interest rate model. The efficiency with which the model fits observed term structures is examined and a time series analysis of estimated model parameters is performed to determine the data frequency we need to use for historical simulation purposes.

The efficiency of pricing interest rate options is part of the objective since it will provide evidence to price options on LS (1992a) estimated credit spreads. Hence, the accurate fitting of credit spreads is another objective in this study. As a final objective we will imply default probabilities under two similar credit risk models of the reduced form approach the Jarrow and Turnbull (1995) and Jarrow, Lando and Turnbull (1997) models. The stability and consistency of the default probabilities will also be examined using historical credit spread curves.

1.3.2 Objectives of the Second Research Paper

Using the LS valuation framework as our pricing engine, an integrated measure of credit and market risk will be proposed. The integrated measure is based on a dynamic hedging technique which eliminates the default risk of the swap portfolio subject to its potential future exposure. The effectiveness of hedging is evaluated and the overall integrated measure is compared to the market VaR of the swap portfolio. The final objective is to determine if the relationship between the two parts of the integrated measure are correlated and if there is an alternative to the methodology.

1.3.3 Objectives of the Third Research Paper

The objective of this study is to further extend the use of the proposed integrated measure by using it to estimate the total economic loss which arises form an actual swap portfolio. The total economic loss calculation of the integrated approach should be compared to the standard approach currently employed by some financial institutions. The final objective is to determine if the Historical Simulation method of calculating credit exposure is comparable to the standard multi-step Monte Carlo simulation which is most favoured by practitioners.

1.4 Significance of the study

The aim of this study was to create an integrated measure of market and credit risk for derivatives using a single pricing framework. In the process of achieving that the following were studied separately for the first time:

- Direct fitting of EUR credit spreads to a two-factor interest rate model (LS).
- Pricing of credit spread options using the LS model.
- The estimation of implied probabilities of default using credit spreads.
- The estimation of implied rating transition matrix based on credit spreads.
- The integrated approach was formulated.
- The validity of the newly proposed integrated method was tested using a hypothetical portfolio.
- The integrated approach was further extended to estimate economic capital.
- Subsequently there was a comparison to current practice based on a realtime swap portfolio.

The research significance of each of the self contained projects individually is summarised next.

1.4.1. Significance of the First Research Paper

Interest rate models have been used to model the dynamics of credit spreads as in Duffie and Singleton (1994), who model the evolution of credit spreads using an Ornstein-Uhlenbeck process. There are a number of reasons and all have to do with the credit spread characteristics which have been studied extensively. Credit spread series show strong mean reversion as in Pringent, Renault, and Scaillet (2001)

The use of the LS model to fit credit spread data is the first model of this type to investigate the evolution of credit spreads and price credit spread options. The first factor of the two-factor model will be the short credit spread rate, or in other words the rate at which corporations arrange their short term financing. The second factor would be its volatility in the same sense as with interest rate volatility. This way a stochastic volatility model will be used as a "spread based" model to price European Credit Spread Options (CSOs).

The estimation of the credit spread curves using the LS (1992a) model will help further in implying the probabilities of default using the JT (1995) model specification and the transition rating matrix using the JLT (1997) model. These implied probabilities will be implied using credit spreads rather than bond prices like in JLT (1997) and Arvanitis, Gregory and Laurent (1999).

1.4.2. Significance of the Second Research Paper

The proposal of the integrated measure in this project is the first of its kind. Its simplicity stands out relative to other current methods of integrating market and credit risk Barnhill, Papanagiotou and Schmacher (2002) and Medova and Smith (2003). The proposed measure is based upon the dynamic hedging of the default risk of a hypothetical interest rate swap portfolio. The methodology provides a framework of active management of both credit and market risks in interest rate swaps.

Since the resulting measure can be estimated by calculating the VaR of the integrated portfolio the measure can be used for capital at risk purposes. Furthermore, the measure can be extended to measure the economic loss of a swap portfolio.

The integrated measure consists of two parts: the market risk part and the credit risk part which is nominally related to the market risk of the portfolio. In practical applications the relationship of these two separate risks can be exploited by applying limits to the level of co-movement i.e. when the two risks become highly correlated then a trigger of reducing exposure should be given. In this way risk managers can better manage their exposures.

1.4.3. Significance of the Third Research Paper

The work presented in the second project (Chapter 3) is extended to the estimation of total economic loss run by an "actual" OTC swap portfolio. There has been very little research based on actual OTC transactional data apart from Cossin and Pirotte (1998) and Mozumdar (2001), who have used swap transaction data from swap dealers in order to estimate the default risk in the swap spread.

The current approach of integrated credit risk measurement applied by most financial institutions Mark (1999) is compared to the extension of the integrated approach proposed in Chapter 3. The integrated approach has a number of advantages over the current standard approach. Ease of implementation, time horizons of less than a year[1], backtesting is quite easy and all is done under one term structure model. Essentially

[1] 1-year is usually the time horizon in estimating economic loss since there is a time mismatch between market and credit risk. Although this is true, a shorter time would be desirable in order to allocate capital more efficient.

the proposed integrated approach replaces the modeling of the probabilities of default with the modeling of the credit spread indices.

1.5 Organisation of the Dissertation

In the next few sub-sections of chapter 1 we will review the related literature of market, credit risk and integrated market and credit risk. Also an essay on *Derivative credit risk* will illustrate the importance of our study. The next chapters are the three different but seemingly related projects. In chapter 2 we introduce the LS model and using it we price interest rate options and credit spread options. In chapter 3 we will propose the integrated approach and in chapter 4 we further extend and test our proposed integrated approach.

1.6 Main Risks

Assessing the risks associated with being a participant in the financial markets has become the focus of intense study by a number of institutions. Certain risks such as counterparty defaults have always figured at the top of most banks' concerns. Others such as market risk (the potential loss associated with market behaviour) have only gotten into the limelight over the past few years. The sudden interest emanates from the significant changes that the financial markets have undergone over the last two decades.

There have been significant developments in conceptualising a common framework for measuring market risk. The industry has produced a wide variety of indices to measure return, but little has been done to standardise the measure of risk. Over the last 15 years many market participants have developed concepts for measuring market risks. Over the last 5 years, two approaches have evolved as a means to measure market risk. The first approach involves forecasting the portfolios' return distribution using probability and statistical models. The second approach is referred to as scenario analysis. The methodology simply re-values a portfolio under different values of market rates and prices.

On the other hand, there has also been a lot of effort along the same lines for credit risk. However, as we will see later there are more obstacles to overcome there. These are the two main types of risks, which arise from participating in the financial markets. Other types of risks rise due to liquidity constraints and operations and are not the focus of this thesis.

1.7 Market Risk measurement

Measuring the market risk of a financial firm's book is an essential part of managing the overall risk run by a firm on a daily basis. This section concentrates on how institutions capture overall risk arising from unexpected changes in prices or rates as a result of new information. Credit risk is only viewed as a component of market risk but in most of the following discussion is assumed to be zero.

The day-to day management of market risk serves the welfare of shareholders in the firm, including employees, pension holders, and others. In due process there are specific objectives which need to be achieved:

- Aggregate all outstanding positions and quantify all market risks related with them.
- Measure exposure in different aggregates (desk, trader, trade).
- Charge each position a cost of capital appropriate to its market value and risk.
- Provide information on the firm's financial integrity and risk management technology and contractual counterparties, regulators, auditors and rating agencies, the financial press an others whose knowledge might improve the

firm's terms of trade, or regulatory treatment and compliance Duffie and Pan (1997).

- Measure relative performance using capital at risk, to evaluate different risk taking units.
- Protect the firm from financial distress costs.

Achieving all the above using a unified risk measure is the dream of every risk professional and senior manager. A few years ago a standard benchmark measure called Value at Risk (VaR) was adopted. This is a firmwide measure of risk which was developed in order to capture the inherent correlations among risks. However, this is only one of the many reasons for the adoption of this unified framework. VaR is a benchmark, which is also used by financial institutions to determine if they hold sufficient capital. The current regime, which is determined by the Bank for International Settlements (BIS), typically sets a 10-day VaR with confidence interval of 99%. Many firms use overnight VaR, instead of the 10-day BIS.

1.7.1 Market VaR methodology

"Value at Risk is a singe statistical summary figure which describes the potential profit or loss of a portfolio of assets over a specified time horizon (t) at a given confidence interval (1-p; BIS p = 99%). " This is the only definition which should be understood fully by any of its users before basing any decisions upon it. VaR is not a panacea. It should be used accordingly at each level of risk management otherwise it could lead to misconceptions about the risks one is running.

Approaches to VaR can be basically classified into two groups. The first group is based on local valuation; the best example is the delta-normal method. The second group uses full valuation. Full valuation is implemented in the historical simulation method and the structured Monte Carlo method. Each of these are discussed in the following sections.

1.7.1.1 Computing VaR

The first step towards the measurement of VaR is the choice of two quantitative factors: the length of the holding horizon, and the confidence level. Both are somewhat arbitrary. As an example, the internal model approach of the Basle Committee defines a 99% confidence interval over 10 days. Then it's the choice of methodology based on the portfolio content and cost. This will become apparent following the description of the different methodologies.

1.7.1.2 Variance-Covariance Method

This method which is known as variance-covariance (VCV), is a parametric approach of calculating VaR which was first developed by RiskMetrics (JPMorgan). It is a linear method which assumes that the portfolio's profit and loss profile is linear and that the portfolio's returns are normally distributed. Assumptions, which have firstly been introduced in Sharpe's Capital Asset Pricing Model, are used. Some approaches have been suggested to avoid using the normal distribution in the Delta-Normal model, in order to capture the "fat tails" characteristics displayed by many financial time series. For example Wilson (1997) models successfully both financial time series and portfolio returns with a student-t distribution, implying that large market rate movements will occur with greater frequency than would be predicted by the normal distribution. Similar results have been achieved by assuming that market rate innovations are generated by a mixture of normal distributions, also generating kurtosis or fat tails in the implied market rate innovation time series.

Another line of research focuses on improving the estimate of the covariance matrix needed to calculate VaR using single or multi factor GARCH techniques. The RM approach uses an exponential weighted moving average (EWMA) approach to calculate the VCV matrix.

1.7.1.3 Historical simulation

This method makes very few assumptions about the market price processes generating the portfolio's returns. The main assumption is that the future movements of market rates are being drawn from the same empirical distribution as the historical rates. Other assumptions such as the composition of the portfolio remains static over the period of the simulation are not as important as the one mentioned above. Since the actual distribution is used there is no associated parametric estimation and the steps required to estimate VaR are very simple. However, it's often difficult to implement this methodology in a cost effective manner, since it requires a huge amount of historical data.

The benefits from using a historical distribution is that problems encountered with modelling the evolution of market prices are eliminated. Market prices tend to have "fatter tails" and be slightly more skewed than predicted by the normal distribution. Also, when using this methodology one avoids the assumptions associated with the price functions, since the actual price functions are being used. This leads in the avoidance of local approximations, hence model risk is being eliminated in that way.

Finally, backtesting of this methodology can be facilitated by a simple comparison to the variance-covariance approach. This comparison lies in the distributional assumptions of the two methods, where the full simulation method uses the actual distribution of market rates in comparison to the VCV method, which assumes normality. The difference between the two distributions should be between 2 – 5 % at all times, since this is the difference at the fat tails of the two distributions.

1.7.1.5 Monte Carlo Simulation

Like to the historical simulation method, the MC simulation method estimates VaR by using the same 3-step procedure described earlier, except that this time the evolution of the market rates is drawn from pre-specified distributions. The joint evolution of each market risk factor must be modelled in detail in order to implement this method. This approach is quite powerful but comes under the same criticism as the VCV approach e.g. fat tails, symmetrical distributions, parameter estimation. Furthermore, model risk is inherent in this approach since market rates are being generated rather than taken as observed. This method is quite challenging to implement both in terms of complexity and systems requirements plus the necessity for skilled resources to carry out the implementation.

However, this method is usually applied in practice only when there is an MC simulation model already in use to risk manage complicated structures. This is when its benefits become apparent to the end-users.

1.7.2 Comparison of the standard market VaR approaches

The following table will summarise the differences of each method by considering only the main methodologies and not any of their derivatives:

Variance-Covariance (Delta Normal)	**Historical Simulation**	**Monte Carlo Simulation**

• Parametric approach based on normality assumptions	• Based on the actual distribution of market rates	• Choice of distribution for the market rates
• Linear Valuation • Only Linear Assets	• Uses Full Valuation • Handles non Linear Assets	• Uses Full Valuation • Handles non Linear Assets
• Extreme events not captured	• Some extreme events could be captured	• Some extreme events could be captured
• Ease of computation, easy to communicate • Non linearities can not be accounted for and neither extreme events.	• Easy to communicate, accounts for non linearity • Computationally intensive and costly in terms of systems and maintenance	• Good for complex structures and non linearities • Difficult to implement, inherent model risk

1.7.3 More recent approaches

There have been other approaches, which estimate market VaR and they are somewhat different than the aforementioned. The most popular is the one based on the use of asymptotic *Extreme Value Theory* to calculate VaR. This approach is based on modelling directly the extreme value of the price of a given product or portfolio and estimating the relevant parameters directly rather than using a building block approach to build the portfolio distribution from the joint distribution of the underlying positions.

Longin (1994) proposes a method for calculating the optimal margining requirement for futures markets based on the asymptotic extreme value distribution. The potential of the optimal margin requirement is directly related to the concept of VaR i.e. is the amount of capital needed to support a given risky position over some time horizon. Longin used extreme value theory that gives the exact form for the asymptotic distribution of the minimum of a random variable, and as such is independent of the distribution of daily price changes. Different processes of daily price changes imply different parameters, but the same functional form of the asymptotic extreme value distribution.

The functional form of the extreme distribution is relatively robust, covering serially correlated price changes, innovations that exhibit fat tails, ARCH processes and mixtures of jump-diffusion processes. The potential of this theory is further discussed in Bassi, Embrechts and Kafetzaki (1996).

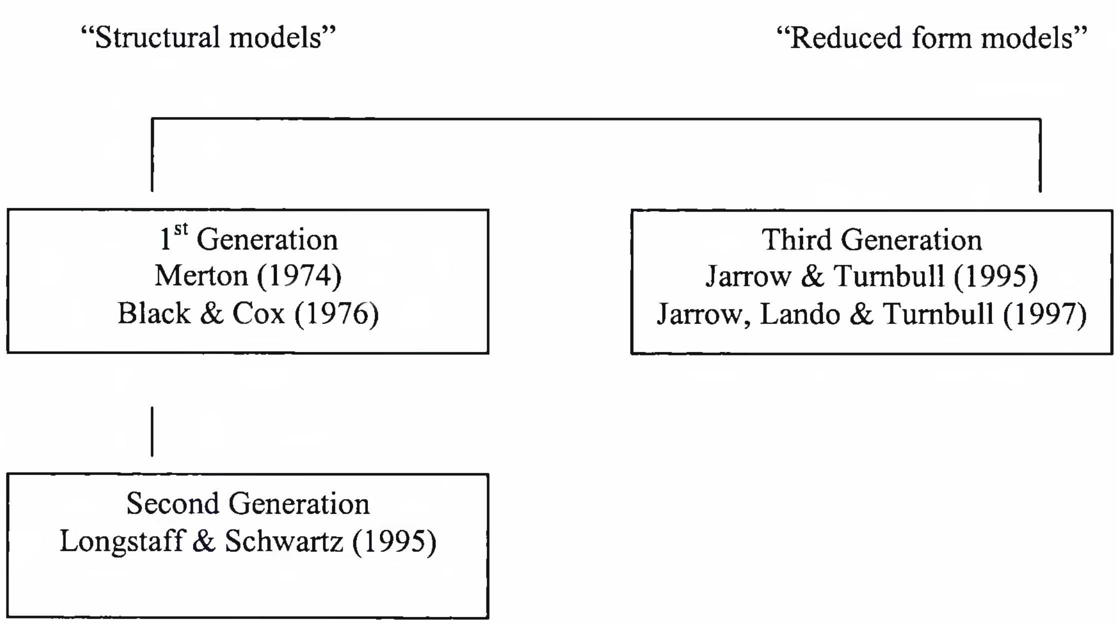
1.8 Credit Risk Models

The initial interest in credit risk models stemmed from the desire to develop more rigorous quantitative estimates of the amount of economic capital needed to support the bank's risk-taking activities. The evolution of credit risk modelling was facilitated by a continuous changing environment, which made its measurement more important than ever before. Some of the factors which contributed to that are: 1.the increased number of bankruptcies, 2. more competition in the lending business, 3. declining value of firm collateral, 4. a dramatic growth of OTC derivatives.

However, of the above models none has managed to capture the inherent credit risk run by an institution when they trade off-balance sheet instruments (swaps, interest rate options, credit derivatives) which have seen a dramatic explosion in volumes over the last 20 years. Furthermore, an important aspect of economic theory which has not yet been captured, not even within the context of the more recent and sophisticated credit risk models, is the fact that market and credit risk are intrinsically related to each other and more importantly they are not separable (i.e. if the market value of the firm's asset changes, generating market risk, which in turn affects the probability of default, consequently generating credit risk). The relation between market and credit risk is of great importance, as it affects the risk adjusted return on capital and therefore should be treated accordingly.

1.8.1 Recent approaches to Credit Risk Modelling

There is a wide range of practices among financial institutions which varies in terms of both methodology and implementation Basle Committee on Banking Supervision [9]. In spite, of the observed methodological differences there are two general frameworks for valuing risky claims, each with different approaches to data limitations and the economics of default. Both frameworks emanate from the pioneering work of Merton (1974) but they differ substantially in form.



The first one, is known as the *structural* because it requires firm-specific inputs to model the default process. Typically, the cause of default is a decline in the value of the firms' assets such that it no longer can pay its fixed claims. The other type of framework models default as being an underlying process by a suitable statistical distribution. This is called the *reduced form approach* which estimates the risk-neutral probability of default over a given interval from prevailing credit spreads, without reference to the cause of default.

1.8.2 Structural Approach

Under the structural approach default is related to the underlying assets of the firm. This was best explained by Merton (1974), who considers an individual firm as the unit of analysis. This firm has a simple capital structure, issuing only one type of debt, a zero coupon bond with a face value B and maturity T. Default is assumed to occur only if the value of assets falls below the book value of debt. The equity holders are long a call option on the firm's assets. If the value of the assets at maturity of the debt issue is greater than the face value of the debt (resembles the exercise price F, of the call option), then the owners of the firm will pay the debt holders and keep the remaining value. However, if assets are insufficient to pay the debt, the owners of the equity will exercise their "default option", and put the remaining assets to the debt holders.

Default is not associated with any cost and priority is given in the repayment of debt. Hence, the debt holders take over the firm and the value of equity is zero, assuming limited liability. Based on that, Merton showed that the value of risky debt, is given by:

$$V_1(t, T)= F\,B (t, T) - p\,[V(t)]$$

where,

V_1 : the value of the risky debt

B(t, T) : the time value of a zero coupon bond that pays one dollar for sure at time T

V(t): the time t value of the firm's assets, and

p[V(t)]: the value of a European put option on the firm's assets that matures at time T with a strike price of F.

1.8.3 Reduced Form Approach

Reduced form models abstract from firm-specific explanations of default. Instead, they describe the process governing the time a default occurs. This type of models avoid the problems associated with the structural models like unobservable asset values and complex capital structures. However, this simplicity comes at a price, since the whole approach relies on credit spread data to estimate the risk-neutral probability of default. The basic reduced-form model relies on an economic argument which can be derived using standard arbitrage-free derivatives pricing arguments:

$$B_{risky}(T) = DF_{risk\text{-}free}(T) * [(1 - q) * \$1 + q * \$ER]$$

Where q is the risk-neutral probability of default prior to maturity and ER is the expected recovery rate in case of default prior to maturity.

Reduced-form models demonstrate their elegance in the above pricing equation. Assuming that we can construct yield curves using observable data of both risky (***B_{risky}***) and risk-free bonds (***$DF_{risk\text{-}free}(T)$***), and the recovery rate is known, then the risk-neutral (market implied) default probability can be obtained as a function of the two discount factors. This makes sense since the credit spread contains the markets expectation of default. Given enough data, it is possible to relax the assumption of a fixed recovery rate and estimate both q and ER as a function of spreads on different classes of security.

Jarrow and Turnbull (1995) give the earliest example of this approach. They allocate firms to credit risk classes and default is modelled as a point process. As described earlier, the term structure of credit spreads is used for each credit class to infer the expected loss over (t, t + Δ_t), which is the product of the conditional probability of default and the recovery rate under the equivalent martingale (risk-neutral) measure, and then price the credit risk derivatives. In order to model the volatility of credit spreads, a more detailed specification is required for the intensity or/and the recovery function. The specification of the recovery process is a very important component in the reduced form approach. If default occurs i.e. on a zero coupon bond, then the bond holder is assumed to receive a known fraction of the bond's face value at the maturity date. This face value is determined with the default free term structure of interest rates.

Further work on the same issue by Das and Tufano (1996) examined a deterministic intensity function together with the assumption that the recovery rate is correlated with the default free spot rate. They assume that the default rate depends on the state of the economy and is subject to idiosyncratic variation. The Das Tufano approach can also be generalised be allowing the probability of default to depend upon the default free rate of interest, and they develop an efficient algorithm, in order to infer the martingale probabilities of default.

The main development in the JT (1995) model was performed by Lando (1994) and (1997) who assumes that the intensity function depends upon different state variables, and this is referred to as a Cox process. [2] Lando derived three main results which all

[2] When the Cox process is dependent upon state variables acts as a Poisson process.

take the form of an expectation operator under the equivalent martingale measure of the expected discounted payoff where the discount rate is (sum of risk free interest rate and the intensity function) adjusted for the default probability. The three different cases represent a security which:

1. pays some random amount X at time T provided that default has not occurred, zero otherwise.
2. pays a cash flow X(s) per unit time at time s provided that default has not occurred, zero otherwise.
3. pays X(A) if default occurs at time A, zero otherwise.

The recovery rate process is an important component in the reduced form approach, since the default free term structure is used to determine the present value of a bond in the event of default. An alternative simple representation for the value of a risky bond by assuming that in default the value of the bond is equal to some fraction of the bond's value just prior to default is derived by Duffie and Singleton (1994).

Empirical observations by Duffee (1999), and Das and Tufano (1996), showed that by modelling the intensity function as a Cox process, the credit spread depends on both the default free term structure and an equity index. In addition, the work of JT (1995) Duffee and Singleton (1994), and Lando or JLT (1997) implies that for many credit derivatives, only the expected loss needs to be modelled, i.e. the product of the intensity and the loss function.

Further empirical work has revealed relationships between credit spreads and default free interest rates Duffee (1998). However, the quality and origin of the data used has to be evaluated carefully at each case since there is evidence that some of these results could be biased forcing relationships that don't exist Longstaff & Schwartz

(1995). More specifically LS used annual data from 1977 to 1992, to fit the following regression:

$$\Delta\, Spread_t = b_o + b_1\, \Delta\, Yield_t + b_2\, \Delta\, I_t + e_t$$

where, Δ Yield$_t$ is the change in the 0.25-year treasury, I_t is the return on the appropriate equity index and e_t is a zero mean unit variance random term. For Aaa, Aa, A and Baa industrial both the estimated coefficients are negative. This is not surprising since an increase in the treasury bill rate, increases the expected rate of return on a firm's assets, and hence lowers the probability of default. This in turn increases the price of the risky debt and thus lowers its yield. Irrespective of the bond's maturity , the coefficients b_1 and b_2 increase in absolute magnitude as the credit quality decreases. However, as argued by Duffee (1998), Longstaff and Schwartz's results must be treated cautiously as their data includes bonds with embedded options which can bias the regression results.

Altman (1992) uses first order differences, and as explanatory variables the percentage change in real GNP, the percentage change in the money supply, the percentage change in S&P index and the percentage change in the new business formation. The results were that there exists a negative correlation between changes in these variables and changes in the aggregate number of business failures. The reported R-squares are substantially low.

In conclusion, we can state with certainty that credit spreads are affected by economic underlying influences. However, one has to be cautious before arriving to conclusions when carrying empirical work, because it can lead to misleading results.

1.8.4 Recent Developments in Credit Risk Framework

As it was mentioned earlier banks are allowed to use their own internal models for assessing capital requirements for market risks. Since Basle 1998 this has been extended to allow banks to use internal models, which can assess regulatory capital for both market and credit risk. Recently advances have been made in modelling credit risk in lending portfolios. These new models are designed to quantify credit risk on a portfolio basis, and therefore have applications in control of risk concentration, evaluation return on capital at the customer level, and are proved to help more active management of credit portfolios. These models include Credit Metrics (CM) (1997), developed by JP Morgan. A portfolio credit risk management model, which uses probability transition matrices, in order to measure the marginal impact on individual bonds on the risk and return of the portfolio. CM is based on credit migration analysis, i.e. the probability of moving from one credit quality to another, including default within a period of one year – chosen arbitrarily. Interest rates are assumed to follow a deterministic process. The credit VaR of a portfolio is then derived in a similar fashion as for market risk, i.e. it is the percentile of the distribution corresponding to the desired confidence level.

In Credit Risk + (1997), as developed by CSFB, there are only two end of period states for the obligor: default and non-default. In the event of default the lender suffers a loss of fixed size, i.e. its exposure to the obligor. CR+ applies an actuarial approach for the derivation of the loss distribution of the loan/loss portfolio. In CR+, no assumptions are made about the causes of default,

The KMV idea of applying the option pricing model to the valuation of risky bonds and loans can be dated back to Merton's model (1974). He noted that when a bank makes a loan its compensation is isomorphic to writing a put option on the assets of the borrowing firm. In particular, as there are five variables which enter the Black-Scholes-Merton (BSM) model of a put option for stock valuation, the value of default of a loan or bond will also depend on five similar variables[3].

The Credit portfolio view (CPV) as presented by McKinsey (1998), is a multi-factor model, which is used to simulate the joint conditional distribution of default and migration probabilities for different rating categories, belonging to different industries and countries, based on a set of macro-economic variables such as the GDP growth, the level of foreign exchange and interest rates, the aggregate savings rate and others.

However, all these named models are inappropriate to measure credit risk for derivative products. The main reason for that is that they assume deterministic interest rates and exposures. The next generation of models should allow at least for stochastic interest rates and relationships between credit events and economic conditions.

1.9 A Brief Comparison of Credit Risk Models

Having mentioned the basic and most influential credit risk models, which have been developed in the last years, it would be important to consider difference and similarities among those models as well as the advantages and disadvantages of each of those models. The following table summarises the four basic models together with their main framework of comparison:

Framework for comparisons	CM	CR+	CPV	KMV
Basis for Credit risk	Market Value of	Expected Default Rates	Macro-economic	Market value of assets

[3] The five variables included in BSM model of a put option are the original interest rate on the swap(the strike price), the current interest rate (the current underlying price), the volatility of interest rates, the sort term interest rates and the time to maturity of the swap.

	assets		variables	
Interpretation of Risk	MTM (mark-to-market)	Default Model	MTM or Default Model	MTM or Default Model
Volatility of micro & macro variables	Constant	Variable	Variable	Variable
Correlation of dependent and independent variables	Multivariate Normal Asset Return	Expected Default Rate or Independence assumption	Factor Loadings	Multivariate Normal Asset Returns
Recovery Rates	Random	Constant within a band	Random	Constant or random
Numerical Approach	Simulation or Analytic	Analytic	Simulation	Analytic

As shown in the table above these models have some similarities and differences. The first and most popular model used by most financial institutions was the CM model. Subsequently the KMV corporation came up with a model which estimates expected default frequencies (EDFs) based on observable data and superseded the CM model which was a bit general in the sense that it would provide an accurate picture over rating classes rather than individual companies. In particular the next sections discuss briefly just that:

1.9.1 Basis of Credit Risk

Both CM and KMV models use the market value of assets and the volatility of assets in order to derive the their credit risk, i.e. they are based on a Merton-type model.

CPV's risk on the other hand is driven by a set of macro-economic variables, while in CR+ the risk is driven by the mean level of default and its volatility. However, it could be argued that all the above models could be linked to each other, since the volatility of the market value of assets as proposed by CM and KMV is linked to stock returns. In turn, stock returns are affected by macroeconomic factors – systematic – as well as non-systematic factors. CPV is also driven by a set of macro economic factors and unsystematic- shocks- in the economy; while CR+ is driven by the mean default rate in the economy, which could also be linked to the state of economy. Therefore, each of the above models can be viewed as being linked to some macro-economic variables and effectively to the state of the economy – directly or indirectly.

1.9.2 Definition of Risk

Some of the models discussed above calculate the VaR based on changes in market values, which are the mark-to-market, and allow for downgrades /upgrades as well as defaults; whereas some others concentrate only on two states of the economy the default and non-default.

1.9.3 Volatility of the variables

In CM, the probability of default is assumed to be fixed based on historical data. In CR+ the probability of default is assumed to follow a Poisson distribution around a mean default rate, which in turn is assumed to follow a gamma distribution, which results in fat tailed distributions. CPV models the probability of default as a logistic function of a set of macroeconomic factors and shocks, which follow a normal distribution and therefore as the macro economy advances, so will the probability of default and the transition matrix. KMV model is based on the EDF which changes as new information is absorbed in stock prices.

1.9.4 Correlation of credit events

The correlations, in all the four models could be seen as correlation between an individual or portfolio of loans or bonds, and the state of the economy, as aforementioned.

1.9.5 Recovery Rates

Generally speaking, the distribution of losses and the VaR calculation have proved to be rather volatile and dependent not only on the probabilities of default but also in the losses, once default has occurred. Therefore, modelling a volatile recovery rate can increase the VaR calculation or the unexpected loss rate. The four models view recovery rates as follows:

CM allows recovery rates to be random. In the normal distribution version of the model, the estimated standard deviation of recovery is included into the VaR calculation. In the actual distribution version, recovery rates are assumed to follow a beta distribution. In CPV recovery rates are estimated through a Monte Carlo simulation, while under CR+ recovery rates are assumed to be constant, but within a specified band.

1.9.6 Numerical Approach

CM uses both the analytic and the simulation approach for calculating VaR. This happens because since the number of loans in a portfolio increases, the analytic approach becomes very complex and thus Monte Carlo simulation approach is more advantageous and produces an approximate aggregate distribution of the portfolio loan values and thus VaR.

CPV also uses a Monte Carlo simulation to generate macro shocks and the distribution of losses on a portfolio. On the other hand, CR+ based on a Poisson distribution for individual loans and a gamma distribution for the mean default rate, generates an analytic solution for the probability density function of losses. KMV also uses an analytic approach in order to generate the probability density function of losses.

The most common assumption made on all the aforementioned credit risk models is that of deterministic interest rates and exposures. This is acceptable for vanilla loans and bonds, but when one has to deal with more complex securities than these then these models become useless in credit risk measurement. For example, when dealing with derivative instruments such as swaps, interest rate options, etc., which are among the simplest derivatives in the financial markets, one has to examine the term structure of interest rates in order to obtain an accurate estimate of the real exposure.

A general framework which might be acceptable to account for credit risk for derivatives instruments will have to take into account the term structure of interest rates. Hence, new approaches should combine stochastic interest rates with default and migration probabilities which will depend on factors such as the level of interest rates and other market indices, possibly from the stock markets.

1.10. Derivative Credit Risk

Previous sections have given an overview of market and credit risk measurement as two separate risks. Although these two major types of financial risk are quite different the relationship between them is quite intricate. There have been many examples in the past which have shown that the risk of default is directly linked to financial markets hence creating market risk and vice versa (Enron, Worldcom in the US and Parmalat in Europe just a few of the recent defaults). The two risks are often related through observed market variables. There is increasing evidence that corporate yield spreads can be explained by a number of market variables, such as equity indices, interest rate spreads, interest rate volatility and in some cases macroeconomic factors Dufresne, Goldstein and Martin (2001). The recent examples of Enron and Worldcom have also shown that their specific credit spreads were affected by their stock price since the stock price slid for a number of months before the spread started to widen. One possibility for that could be that stock market participants had a better view or "knowledge" of each companys' finances or a simpler reason could be that the easiest way to bet on a companys' default is by selling short its stock. Although this is quite recent, it shows the degree of integration of financial markets across all asset classes and all types of risk. This integration creates residual risks which are not being observed directly.

The interest rate market, -a huge market by annual turnover- is a prime example of risk integration. Although equity markets react to credit events and -credit events could be triggered by a deterioration of companys' stock or bond price simply because credit is a "reputable" business- default risk in the interest rate market is a direct issue and needs to be addressed by all market participants on a more frequent basis. All OTC contracts are between two parties of certain credit quality and both counterparties bear credit risk to each other. This credit risk is often higher as maturity of the contract increases. Hence, the existence of term structure of credit risk. The most actively traded instrument in the OTC interest rate market is the interest rate swap (IRS). Notably interest rate swaps, which form a major part of the interest rate market, bear substantial default risk. *In an efficient market, one would expect market* *swap rates to incorporate the risk of default. Hence, one would expect the swap rates to be sensitive to the credit ratings of the counterparties. For instance, a swap dealer who pays a floating rate and receives fixed payments in exchange would require a BBB-rated counterparty to pay a higher fixed rate compared to a AAA-rated counterparty. Conversely, the dealer would be willing to make lower fixed payments in exchange for floating rate payments to a BBB-rated counterparty than a AAA-rated counterparty (Eom Subrahmanyam 1997).*

There is little research to suggest that the swap rates quoted in the market vary depending on the counterparty credit rating. Duffie & Huang (1998) in their attempt to propose a pricing model for valuing claims subject to default by both contracting parties, which captures the possibility of default. Swaps were used for illustration purposes. According to their proposed framework, counterparty credit rating plays a role in swap rates but a lesser one in terms of magnitude. For example, when they switched to a lower rating counterparty on an ongoing swap agreement, the swap rate increased by roughly one basis point. This credit impact on swap rates was found to be approximately linear within a range of normally encountered credit quality. They conclude that the impact of credit risk on swap rates may be less than indicated by their results given industry practices designed to reduce default risk, such as collateralisation, and options for early termination among others. Previous to Duffie & Huang, others attempted to price swap default risk. For other approaches see Abken (1993), Cooper & Mello (1991) and others.

Empirical research on swap credit risk hasn't been in huge supply apart form Cossin & Pirotte (1998) and Mozumdar (2001). Using real-time swap data from two interest rate swap dealers with different credit ratings (AAA and A) their aim was to examine the relationship between the dealers' credit rating and the swap bid-offer quotations.

Their main findings show that spreads between AAA swap offers and Treasury yields are significantly positive and they increase significantly with maturities. Also, bid-offer spreads of swap dealers are sensitive to their credit reputations. The A dealers' swap rates appear to be bracketed by the AAA dealers' swap rates.

Cossin & Pirotte (1998) analyse swap credit risk on actual transaction data provided by a medium sized European Bank, a small operator in the swap markets. The primary goal of their study was to examine the presence of credit risk in the pricing of swap deals. They examined 55 IRS (Interest Rate Swap) deals and 201 CS (Currency Swap) deals, which took place throughout 1990-1994.

The counterparties involved in the deals were of different credit rating, ranging from Aaa to unrated. Their methodology was to compare the transaction data available to end of day bid or offer quotes out of Datastream, which represent quotes to the interbank market. Their findings suggest that there is pricing of credit risk in their sample of IRS especially during 1994. They also find that there is a strong difference of spreads with unrated companies. Unrated firms have much less favourable prices on the swap market but no worse terms in terms of maturities and amounts. Almost the same results were found for the CS. The only major difference between the two is that maturity makes a huge difference in CS.

Mozumdar (2001) shows that the impact of default risk should not be taken lightly. Most research so far has shown that swaps bear default risk but is quite negligible. For example, both Duffiee and Singleton (1997) find only week empirical evidence that default risk is important in swap markets. Similarly, as was mentioned earlier, Duffiee & Huang (1998) report that a 100 bps difference in debt rates corresponds to 1bp difference in swap rates, and they estimate the expected annual loss rate to be 0.00025% of the notional principal. Mozumdar (2001), stipulates that the issue of

default risk in swaps needs to be addressed by examining the structure of the swap market. He shows that the default risk of swap drastically increases if is used for speculation in contrast to its use for hedging purposes. He also shows that when firms have such private information, price-based mechanisms are unable to control default risk. Attempting to compensate the swap dealer for the default risk of speculative swaps by raising the cost of swapping to the counterparty would only increase the speculative intent of the pool of counterparties, thereby further increasing default risk.
The academic literature described has specified that default risk is already in the price of a swap but it seems to be negligible when theoretical models are applied. Empirical evidence show that only if the credit rating of a company is below investment grade or unrated will affect default risk the swap rates that the company will be quoted by dealers. This shows that the industry uses other information and techniques to mitigate the default risk of the swaps. However, it doesn't prove that once a swap agreement is entered that default risk is being "priced in". As Mozumdar (2001) finds, dealers need to rely upon additional exposure information or credit enhancement devices in order to preserve equilibrium. The main reason for that is that firms can use swaps for either speculative or hedging purposes with the former bearing higher risk of default. Therefore, when a dealer looks at the overall risk of a swap book s/he has to assess default risk per counterparty.

Assuming that not all counterparts use credit enhancement devices, simply marking to market and netting agreements are not sufficient to mitigate credit risk. Hence, additional techniques needs to be applied in order to assess and measure default risk. For example, if the dealer wishes to hedge the swap book in terms of market risk then he can probably do so using the liquidity available in both the money markets for the short end and the government bond markets for the long end. If the dealer wishes to hedge its credit risk there are a number of ways he/she can do it. Collateralisation or other margin requirement at the initiation of the swap is a very popular way of doing so, although credit risk is not fully hedged and needn't be. Triggers for early termination in the International Swaps and Derivatives Association (ISDA) (1998) agreement, penalties for rating downgrades are less popular due to the intense and non standardised legal aspects.

However, recent changes in the regulatory regime, require that dealers measure expected and maximum exposures over the life of all their outstanding derivatives contracts. They also have to assign initial exposures. A recent survey by the Group of 30 asked dealers to indicate the initial exposure that would be assigned to a variety of derivative transactions with varying maturities. The results reveal that most dealers differentiate sharply by the type of transaction and the maturity of the transaction but not according to the credit rating of the counterparty they have the exposure with. The fact that credit rating is not an accountable factor by dealers is explained by the actual procedure that banks carry on with that part of their business which is limit setting. This is the usual credit process which allows dealers to trade with almost any counterparty of any rating by allocating bigger limit to the highest rated and lower limit to the lowest rated or unrated counterparty. Of course this allows for continuous and alarmingly increased trading between major dealers.

This is an actual fact and it was flagged up by the Federal Reserve chairman Alan Greenspan who expressed concern over the risks posed to financial markets by the concentration of the $142 trillion derivatives in the hands of a few investment banks (Reuters). The same year Warren Buffet warned that large amount of credit risk has become concentrated within just a few major dealers (Financial Times). The limit setting methodology is tied in with allocating resources based on the relevant exposure, purely the combination of the two resembles the insurance approach. The study by the Group of 30 informs us about this resource allocation per derivative instrument. For instance, the average dealer assigns an initial exposure of about 22% of notional amount fro a 3-year currency swap and about 5% of notional amount fro a 3-year interest rate swap. This differentiation according to the type of transaction would be expected by the difference in the exposure profiles that the two instruments have over time. The study also informs us of how end-users treat exposures like that. Twenty-six percent of end-users measure exposure according to the notional amount alone. An additional 39% of end-users measure the exposure as a percent of notional amount, but differentiate by type and maturity of transaction. Fourteen percent of end-users measure exposure on a mark-to market basis.

Evidence such as this lead into taking a closer look on how to measure and manage credit risk on a portfolio of derivatives. Quite recently the "art" of calculating potential and worst case exposure has been in place by most major dealers. Usually a Monte Carlo engine will generate potential scenarios which can be projected in the future. As a result the potential and worst case exposure can be calculated. This technique has been further enhanced by examining the probability by which the potential or expected exposure can be achieved. Hence, deriving the expected loss or expected worst case loss which is simply the product of the potential exposure multiplied by the probability of default. There are various ways that the probability of default can be calculated and often major banks have big credit departments dealing with that issue in depth by examining credit spreads, bond ratings and by empirical evidence on actual defaults over a long period.

That was a brief description on how credit risk is handled by most major banks of the world. However, none of these approaches integrate or take into account the point of market and risk integration. The purpose of this study is to identify a technique which can evaluate both the market and credit risk elements in interest rate sensitive instruments from a theoretical perspective. The idea is that credit risk could be hedged if possible in the same way as market risk by using credit derivatives. However, this is quite an expensive procedure and many firms prefer not to hedge unless they feel is absolutely necessary. Hedging could be triggered by a number of factors including qualitative factors and observed market variables. One observed market variable which is almost always used in order to make a hedging decision is the spread between the yield of the corporate bond over its respective government bond, simply known as the credit spread. By monitoring the level of the credit spread and its volatility, dealers can make informed decisions on whether they need to hedge a specific exposure or not. Consider a dealer who wants to hedge a credit exposure, or in other words transfers the credit risk of his/her portfolio then s/he needs a specific credit derivative agreement. The specification of this will match the size and maturity of the exposure without adding any further counterparty risk. Instruments which can be used to replicate that range from credit default swaps, to total return swaps and credit spread options (CSO)[4]. Since the risk of default increases as the credit spread of the counterparty widens it is logical to use either a CDS or a CSO. The perfect credit hedge for a swap would be if its exposure profile matched the credit spread. In this study we examine the use of CSOs as hedging instruments for swap credit risk. The natural way to do that is by purchasing credit spread call options, in order to protect for any spread widening.

[4] Credit spread options (CSOs) are derivative instruments introduced in the beginning of the 1990s. CSOs are options on a particular reference credit spread in the loan or bond market over a standard rate. One party pays a premium at initiation in return for a payment in the event that the reference credits' spread crosses a certain strike.

However, as it was mentioned earlier this is a costly technique in an illiquid market. It can be used though to calculate the risk of default or the market price of risk that a swap bears throughout its life. By calculating the cost of dynamic hedging of the risk of default, one can probably estimate the market price of risk of default or otherwise how much credit risk a counterpart takes by entering into a swap agreement. Furthermore, the portfolio of the swap and the CSO or otherwise the credit hedged portfolio can form the basis of calculating an integrated measure of market and credit spread risk.

By running market VaR on the swap portfolio and market VaR on the spread options and then integrating the two by looking either their long-tem relationship one can create a single measure for both risks. Another way could be by examining the relationship of the credit hedged portfolio versus the credit spread and the running market VaR. In the following sections we will explain the reasons for choosing each technique used starting form the choice of the LS model and why it was suitable for fitting credit spread curves. Then we'll describe the LS methodology and how it was calibrated using market rates in order to price consistently interest rate options and credit spread options. The next steps will show a brief statistical analysis on the credit spread data following that how the historical simulation VaR was calculated for the IRS portfolios and the credit spread options. For the final part of this analysis the relationship between the two VaR figures will be examined in order to draw the necessary conclusions.

1.11 Integrated Market & Credit Risk

In general one cannot isolate one risk form the other in pricing and risk management. This is quite obvious in corporate bonds which are subject to both credit and market risk. In other financial markets such as interest rate derivatives one may face both risks and others such as liquidity risk in a more complex way. The academic literature usually treats market and credit risk as separate. There are only a few papers which attempt to integrate the two risks.

Most of this type of research is simulation based. Large portfolios which contain a number of different financial instruments, ranging from corporate bonds to equity and equity derivatives to credit derivatives. Then all market rates directly affecting the portfolio are being simulated. Others attempt to integrate market and credit risk using a dynamic asset allocation perspective Barnhill, Papanagiotou and Schmacher (2002). However, this approach has a slightly different target than the one in this paper. They investigate how financial institutions who face both market and credit risk in addition to equity risk would allocate their financial resources in a dynamic set-up. Our purpose is to deduce a measure which integrates both risks. From a capital allocation perspective this would be useful since investors would use capital at risk measurements such as this one to deduce their earnings at risk over different time horizons. This could also create a dynamic asset allocation framework.

Barnhill, Papanagiotou and Schmacher (2002) present a numerical solution based on a simulation model that explicitly links changes in the relevant variables that characterize the financial environment and the distribution of possible future bank capital ratios. Their model was applied to the study of the risk profile of various hypothetical banks operating in the South African financial environment. Their study demonstrated that their model is able to capture the impact of correlated market and credit risk on the potential losses that a bank can suffer due to interest rate, foreign exchange rate, equity price and real estate price changes as well as client defaults and downgradings. They also find that extreme volatility events (fat tails) were not captured and they also haven't looked at the effect of large derivative exposures.

A framework of integrated risk was proposed by Medova and Smith (2003). They use a Monte Carlo methodology to introduce a single platform for both market and credit risk models. Their methodology is based in calculating distributions of future values of a portfolio at a series of time horizons. They provide examples on a variety of instruments including FX swaps. Their methodology is based on having two states in the world, default and non-default. They simulate both states and their measure is one which discounts default to today at all times. This methodology is simply the combination of Merton's corporate model together with any other market risk pricing model. An interesting feature of their framework is that the integrated risk measure is less than the market risk measure.

The integration of risks in one measure is something that many researchers envisaged achieving. So far most of the effort has been on the banking book level. The issue of derivative credit risk hasn't been addressed with a joint market and credit risk measure. The only way to achieve this for all securities and rates is by simulation only according to current research. Our view is that integration of risks can be achieved without carrying out Monte Carlo simulations. We assume that the information content of the credit spread is sufficient to provide an accurate view of a corporate's status and vice versa. Based on that we will attempt to tie credit exposures to credit spreads in such a manner that one is a function of another. This will facilitate in creating an integrated measure, which is based on theoretical relationships rather than a generated or even historical simulation. Essentially we will create a credit neutral portfolio of OTC derivatives where the derivative exposure is linked to a credit derivative. By running a market VaR on that portfolio we produce a measure which accounts for both the market and credit risk of the OTC derivatives.

CHAPTER 2: Pricing Interest Rate Sensitive Securities and Credit Spread Options using the Longstaff & Schwartz (1992) Model

Abstract

This study uses the valuation framework of Longstaff and Schwartz (1992a) to price European Interest Rate and Credit Spread Options. Credit spreads are assumed to be stochastic variables following the two-factor process as proposed by LS (1992a) with one factor being the level of the credit spread and the second factor being the volatility. The parameters of this process are easily estimated using observable data. The model allows the fitting of various curve shapes including the complex credit curves. Calibration of credit spread options prices is done using a replicating option strategy. The estimated credit spread curves are used to imply default probabilities under the Jarrow and Turnbull (1995) and Jarrow, Lando and Turnbull (1997) credit risk models.

2.1 Introduction

2.1.1 Background of the study

The term structure of interest rates is an important source of information for market participants as well as central banks since it provides information on, among other things, the markets' expectations concerning future monetary policy. Specifically, the estimated implied instantaneous forward interest rate curve can, given adequate assumptions, be interpreted as the expected short-term interest rate, which is directly or indirectly controlled by the central bank. However, implied forward rates do not provide any information on the uncertainty associated with the expected future short-term interest rate.

Interest rate models serve the purpose of providing accurate estimates of spot yield and forward curves in order to perform pricing and risk management functions. Interest rate modelling is in a well advanced stage at the moment offering state of the art models. A good review of most well known interest rate models can be found in Rebonato (1996). The Longstaff and Schwartz (1992a) two-factor interest rate model was one of the first equilibrium interest rate models to include a second factor, the volatility of the interest rate. This model has several advantages that make it attractive as a candidate for estimating the term structure of interest rates. First, the two factors in the LS model – the short-term interest rate itself and its volatility – are intuitively reasonable as determinants of the interest rate process, and they have also been found to be important in empirical work. Second, the LS model is a general equilibrium model which makes it appealing from a theoretical perspective. Third, from a practical point of view, the model belongs to the affine class of models, and therefore provides closed-form solutions for zero-coupon bond prices which facilitates estimation of the term structure using cross sectional data.

Interest rate models have also been used to model the dynamics of credit spreads Duffie and Sinlgeton (1994), which model the evolution of credit spreads using an Ornstein-Uhlenbeck process. There are a number of reasons and for all relate these reasons to the credit spread characteristics which have been studied extensively. Credit spread series show strong mean reversion Pringent, Renault, and Scaillet (2001). In this chapter we will show that most of our credit spread index data show mean-reversion. Furthermore, we successfully fitted the credit spread indices in GARCH (1,1) models since it has been noted by academics and practitioners through empirical work that the volatility of credit spreads changes over time. Duffie (1999) found that the volatilities of yields change continuously i.e. display GARCH-like effects. "A stochastic volatility model can capture the skewness naturally embedded in credit data" Jacobs K., Li X. (2003). Longstaff & Schwartz (1995) following an empirical investigation into credit spreads proposed a mean reverting model for the logarithm of the credit spread. The specification of the model is such that it provides closed form valuation expressions for risky bonds as well as risky floating rate debt. It is a two-factor model where one factor is the default free rate and the second factor is the default risk rate. In essence two yield curves can be estimated simultaneously based on observed default free and default bond prices. The model allows for correlation between the two. However, the term structure of credit spreads has multiple and complex shapes. LS 1995 allows for monotone increasing or hump shaped curves to be estimated. This feature of the model constrains the fitting of more complex shaped curves, whereas the LS 1992 model allows for more complex curve shapes to be fitted. The proposition of using LS 1992 to estimate credit spread curves comes at a time that the credit derivative market has grown exponentially over the last decade. Volatility is one of the fundamental factors in pricing credit derivatives as was noted by Schonbucher (1998). LS 1992 a stochastic volatility model, offers closed form valuation expressions for pricing interest rate derivatives.

2.1.2 Problem of the study

Pricing of a European spread option should resemble the pricing of an interest rate option. The payoff function must be dependent on a risk-neutral expectation derived from the distribution of the spread. Deriving the distribution of the spread is the major step in deriving a closed formula solution for the pricing of credit spread options. The two major credit risk modelling approaches[5] fail to provide a closed-form solution for

[5] Structural and Reduced Form Approach.

the spread distribution. For that reason most research into pricing of credit spread options has concentrated in obtaining numerical solutions for the spread distribution. Using a "spread based" model where the credit spread is modelled as a stochastic variable provides a solution to the spread distribution problem. There is a variety of models that can be used to capture the evolution of credit spreads. Modelling the credit spread as a stochastic variable is internally inconsistent. However, it is the quickest and less complicated way of to price credit spread options.

2.1.3 Significance of the study

The LS model will be used to fit cross sectional credit spread data in order to estimate the term structure of credit spreads and their respective volatilities. The first factor of the two-factor model will be the short credit spread rate, or in other words the rate at which corporations arrange their short term financing. The second factor would be its volatility in the same sense as with interest rate volatility. This way a stochastic volatility model will be used as a "spread based" model to price European Credit Spread Options. The fact that the LS (1992) model is used to price credit spread options provides a unified pricing framework for both interest rate sensitive and credit sensitive securities.

The estimation of the credit spread curves using the LS (1992) model will facilitate further in implying the probabilities of default using the JT (1995) model specification and the transition rating matrix using the JLT (1997) model. The major difference to Arvanitis, Gregory and Laurent (1999) is that the implied probabilities will be estimated using credit spreads rather than bond prices.

2.1.4 Overview of the study

This chapter is organised as follows: (a) a brief statistical and econometric analysis of the credit spread index data (b) a description of the economic justifications of the model plus the main assumptions and steps to its derivation, (c) implementation of the model using a cross section of interest rate data, (d) pricing of interest rate derivatives, (e) a comparison in using daily and weekly data to estimate yield curves, something that will facilitate our analysis in chapter 4, (f) estimation of credit spread curves and pricing of credit spread options, (g) estimation of implied default probabilities and of implied rating transition matrix.

2.2 Methodology

2.2.1 EUR Credit Spread Indices

Credit spread indices have been analysed in empirical work over the past few years to determine their dynamics. Empirical research in corporate credit spreads has been increasingly revealing on what determines their evolution and behaviour. One of the first papers, which initialised a wave of empirical papers was by Pedrosa and Roll (1998). They analysed a variety of corporate bond credit spread indices pooled by rating and sectors. They examined US corporate credit spread indices supplied by Bloomberg for a number of years. Their main findings were:

- Most of the series examined showed non-stationarity in levels.
- When the 60 different credit spreads were examined together, they showed a high degree of cointegration.
- Credit spread changes exhibit non-Gaussian distributional properties.
- Credit spreads can be modelled using Gaussian mixtures.

- In most credit spreads, the persistence of volatility is quite simple; for a few series, however, there is evidence of longer-term volatility persistence.

That was the first paper which showed that there is systematic risk in corporate bond credit spreads and it argues that credit spread risks are not diversifiable. Others tried to identify the underlying factors which explain credit spread movements.

Christiansen (2000) stipulates that during macroeconomic announcement days credit spreads and interest rates are uncorrelated whereas, in general, credit spreads and interest rates are negatively correlated. An interesting result, which can be seen in our results as well is that the process for the conditional variance of the credit spread is highly persistent. Huang J., Kong W. (2003), examined monthly and weekly US credit spread data in relation to major financial market variables such as the Russell 2000 index historical return volatility and the Conference Board composite leading and coincident economic indicators. They showed that these variables have significant power in explaining credit spread changes especially for high yield indexes. Furthermore, they showed that in total eight variables can explain approximately 68% and 61% of credit spread changes for the B- and the BB- rated indexes, respectively. Finally their analysis showed that credit spread changes for high-yield bonds are more closely related to equity market factors.

In others similar studies, Brown (2001) examined the explanatory power of the 10-year Treasury yield, consumer confidence, the VIX[6] index and a Treasury bond liquidity measure on credit spread changes. His findings show that these variables can explain approximately up to 33% of spread changes. Kao (2000), shows that implied volatility of OTC interest rate options, the yield curve slope, the interest rate level and

[6] VIX is an implied volatility index based on the S&P500 cash index options.

the Russell 2000 index return have significant explanatory power for changes in the credit spread index level.

This section will serve as a primer to what follows in this chapter. We will illustrate the major statistical properties of Credit Spread Indices. The area of interest is the Euro area. The FMC function in Bloomberg was used to collect credit spread indices of various ratings (Appendix 1). The length of the data is not as long as we would have liked but is sufficient to provide a good insight into their dynamics. By examining the dynamics of EUR credit spread indices we will be able to decide if a theoretical interest rate model such as that of LS (1992a) is appropriate to model their evolution.

2.2.1.1 GARCH in EUR Credit Spread Indices

The volatility of credit spreads is reported to be time varying Pedrosa and Roll []. Volatility seems to be increasing when past innovations are positive but the same is not happening when past innovations are negative. Using the collected data we tested the 2Y, 5Y and 10Y credit spread indices per rating for GARCH(1,1) effects. The PCGive software was used to examine for the existence of GARCH effects using a (1,1) specification. This uses an optimisation algorithm to determine the maximum likelihood ratio. The optimisation was carried out using the following constraints:

$$\alpha_0 > 0,\ \alpha 1 >= 0,\ \beta >= 0 \quad (3.1)$$

and

$$\alpha_1 + \beta_1 < 1 \quad (3.2)$$

$$r_t - r_{t-1} = \mu + \varepsilon_t, \qquad \varepsilon_t / \Omega_{t-1} \sim N(0, h_t) \quad (3.3)$$

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta h_{t-1}, \quad (3.4)$$

where, Ω_{t-1} denotes the information set at time t-1.

2.2.1.2 Testing for mean reversion in EUR Cedit Spread Indices

Mean reversion is a well-known characteristic found in interest rates. For that reason one of the tests we need to do is the test of mean reversion in our series. In empirical research as Credit Spreads, Pedrosa and Roll (1998), and other credit related spreads such as MBS spreads Koutmos (2002) seem to be non-stationary in levels, clearly indicating the existence of mean reversion. Testing for mean reversion involves a unit root test. This test which has been defined by Dickey and Fuller[7] (DF test) involves the testing of the following hypothesis for equation (3.5):

$$Y_t = \rho Y_{t-1} + u_t \quad (3.5)$$

Null Hypothesis :if ρ is equal to1
based on the τ (tau) Test statistic.

If the computed absolute value of the τ statistic exceeds the DF absolute critical values, then we do not reject the hypothesis that the given time series is stationary. If, on the other hand, it is less than the critical value, the time series is non-stationary[8]. Usually, for theoretical and practical the DF test is applied to regressions of the following form:

$$\Delta y_t = \beta_1 + \beta_2 t + \delta Y_{t-1} + \alpha_i \sum_{i=1}^{m} \Delta Y_{t-i} + \varepsilon_t \quad (3.6)$$

Null hypothesis if $\delta = 0$ based on the τ (tau) statistic

When the DF test is applied to models like (3.6) is called augmented DF (ADF) test. The ADF statistic has the same asymptotic distribution as the DF statistic, so the same critical values can be used[9].

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[7] D. A. Dickey and W. A. Fuller, "Distribution of the Estimators for Autoregressive Time Series with a Unit Root, " *Journal of the American Statistical Association, vol. 74, 1979, pp. 427-431.* See also W.A. Fuller, *Introduction to Statistical Time Series,* John Wiley & Sons, New York, 1976.

[8] Since we run the regression in the form of (3.5) the estimated τ statistic usually has a negative sign. Therefore, a large negative τ value is generally an indication of stationarity.

[9] Damodar N. Gujarati, "Basic Econometrics" McGraw-Hill International Editions, 1995.

2.2.2 The LS (1992a) model

The LS model considers a stylised version of the economy as a whole in which interest rates are obtained endogenously rather than received from empirical observation. In their model agent (investors) are faced, at each point in time, with the choice between investing or consuming the single good produced in the economy. If C(t) represents consumption at time t, the goal of the representative investor is to maximise, subject to budget constraints, his additive preferences of the form

$$E_t[\exp(^{(-\rho s)\ln C(s)ds})] \quad (3.7)$$

Consumption at time s is 'discounted' to the present time t by a utility discounting rate ρ, which present-values the 'pleasure' of future consumption C(s). Et[] is the conditional expectation operator, i.e. investors maximise their expectation, subject to information available up to time t, of the discounted future consumption. In other words, by deferring consumption the investor can reinvest the single good in the economy so as to realize a greater consumption at a later time. The utility discount factor accounts for the reduction in satisfaction due to delayed consumption. Furthermore, a logarithmic utility function is assumed. Consumption or reinvestment decisions have to be made subject to budget constraints that, given the assumptions above, have the from

$$dW = W\frac{dQ}{Q} - Cdt \quad (3.8)$$

i.e. the infinitesimal change in wealth W over time dt is due to consumption (-Cdt) and returns from the production process (dQ / Q), scaled by the wealth invested in it (hence the constant-return-to-scale technology assumption). The returns on the physical investment (the only good produced by the economy) are in turn described by a stochastic differential equation of the form

$$\frac{dQ}{Q} = (\mu X + \theta Y)dt + \sigma X dz_1 \quad (3.9)$$

were dz_1 is the increment of a Brownian motion, μ, θ and σ are constants, and X and Y are two state variables (economic factors) chosen in such a way that X is the component of the expected returns unrelated to production uncertainty (i.e. to dz_1), and Y is the factor correlated with dQ. Both X and Y are Wiener processes described by stochastic differential equations

$$dX = (a - \beta X)dt + \gamma X dz_2 \quad (3.10)$$

$$dY = (\delta - \varepsilon Y)dt + \phi Y dz_3 \quad (3.11)$$

Given the assumptions made, there is no correlation between the processes dz_1 and dz_2, on the one hand, and between dz_2 and dz_3 on the other, i.e. $E[dz_2\, dz_1]$ and $E[dz_2\, dz_3]$ are both equal to zero. It is not easy to find an intuitive interpretation for the two factors described above; rather than economic intuition or plausibility, the main justification for the description of the economy embedded in equations (3.7) to (3.11) is analytical tractability, as will become apparent later on.

If one accepts that the optimal consumption, given the assumption above, is ρW (see Cox Ingersol and Ross (1985a) for a proof), direct substitution of (3.9) and of the optimal consumption in the budget constraint equation (3.8) gives for wealth the stochastic differential equation

$$dW = (\mu X + \theta Y - \rho)W dt + \sigma W Y dz_1 \quad (3.12)$$

2.2.2.1 The PDE obtained by contingent claims

Having obtained the stochastic differential equation obeyed by the process for the wealth of the representative investor, two results from CIR (1985b) can be drawn upon to obtain that the partial differential equation obeyed by any contingent claim H is:

$$\frac{\partial^2 H}{\partial x^2}\frac{x}{2}+\frac{\partial^2 H}{\partial y^2}\frac{y}{2}+(\gamma-\delta x)\frac{\partial H}{\partial x}+(\eta-(\xi+\lambda)\gamma)\frac{\partial H}{\partial y}-rH=\frac{\partial H}{\partial \tau} \qquad (3.13)$$

where $x = X / c^2$, $y = Y / f^2$, $g = a / c^2$, $e = \xi$, $\delta = b$, $\eta = d / f^2$, r is the instantaneous riskless rate, and the market price of risk has been endogenously derived to be proportional to y, rather than exogenously assumed to have a certain functional form.

The set of equations and assumptions described above provide a general equilibrium model for the economy as a whole. Contingent claims are priced in this framework as endogenous components of the economy, and their prices are therefore equilibrium prices. The same cannot be said, in general, for no-arbitrage models, which dispense with any description of the economy. While this added feature of the CIR (1985b) and LS (1992a) models is certainly intellectually interesting, it should be kept in mind that their claim of providing a general equilibrium model is only valid within the context of the very stylised economy they assume (only one good produced, no role for money in the economy, no trade with foreign economies, etc.). As is the case with all equilibrium models, the investors' utility function or, more specifically, the market price of risk, will appear in the PDE describing the price of any contingent claim. Since, of course, the parameters (including the market price of risk) of the model cannot in practice be determined a priori, the user will be faced with the need to estimate them either by a suitable best-fit procedure to some observable set of data (typically a yield curve, but the term structure of volatilities and the correlation between rates might also have to be taken into account), or by a means of a mixed historical-implied' approach.

2.2.2.2 The equilibrium term structure

As mentioned in 3.2.1.1 any security traded in the economy described by the LS model must satisfy the PDE (3.13). In particular, this equation will have to be satisfied by a zero-coupon bond, i.e. a security with terminal condition F(r, V, 0) = 1. When this boundary condition is imposed, and a separation of variables approach is followed, the resulting expression for the value of a discount bond, F, t years before expiry turns out to be given by:

$$F(r,V,\tau) = A^{2\gamma}(\tau)B^{2\eta}(\tau)\exp(\kappa\tau + C(\tau)r + D(\tau)V) \qquad (3.14)$$

The expression for the continuously compounded yield of a zero-coupon bond, which can be directly obtained as the negative of log(F(T)) / T; a simple calculation gives:

$$Y(T) = \frac{-\kappa T + 2\gamma\log A(T) + 2\eta\log B(T) + C(T)r + D(T)V}{T} \qquad (3.15)$$

2.2.2.3 Term structure of volatilities

For practical option pricing applications, achieving a good fit to the term structure of volatilities can be as important as fitting the yield curve correctly. The volatility of rates of different maturities can be obtained by deriving the volatility of zero-coupon bond prices for different maturities, and then applying Ito's lemma to convert the price volatility to yield volatility. Hence the instantaneous volatility of bond returns is

$$Var[dF(T)] = \sigma^2_{F(T)} = r\left[\frac{\alpha\beta\psi^2(e^{\varphi T}-1)A^2(T) - \alpha\beta\varphi^2(e^{\psi T}-1)B^2(T)}{\varphi^2\psi^2(\beta-\alpha)}\right] + V\left[\frac{-\alpha\psi^2(e^{\varphi T}-1)A^2(T) - \beta\varphi^2(e^{\psi T}-1)B^2(T)}{\varphi^2\psi^2(\beta-\alpha)}\right] \qquad (3.16)$$

2.2.3 Estimation of the Longstaff-Schwartz Model

The academic literature proposes that a mixed (historical/implied) parametrisation Hordahl (2000), Rebonato (1996), procedure should be used for the calibration of the LS model, which is the approach used in this study. A purely implied approach is the one which regards the two state variables and the six parameters as fitting quantities, whereas a historical/implied approach involves the estimation of the short-rate volatility using time series and applying to the model regarding only the six parameters as fitting quantities.

In practice, this kind of model is frequently estimated using cross-sectional data on bills and bonds/swaps at some specific time. This results in a new set of parameters each time the model is estimated. Using cross-sectional data rather than a time series approach to estimate the parameters in the model could possibly capture changes in the dynamics of the term structure in a much more timely manner. While this approach violates the equilibrium set-up of the model, it is nevertheless used in order to fit the model to observed bond prices as closely as possible.

2.2.3.1 Estimation

The estimation procedure relies on (3.14), which provides a closed form solution for the discount function. Using this expression, the six parameters of the LS model can be estimated with cross-sectional bond price/swap rate data, given initial values of the two state variables r and V. As a first step the initial values of r and V are determined. The short rate r, is represented by the average of the yield of the most liquid short term instrument, i.e. a T-bill over the examined period. The next step is to estimate the initial value of the variance in interest rate changes. This is done using a simple GARCH(1,1) model, assuming a constant conditional mean:

$$r_t - r_{t-1} = \mu + \varepsilon_t, \qquad \varepsilon_t / \Omega_{t-1} \sim N(0, h_t) \quad (3.15)$$
$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta h_{t-1}, \qquad (3.16)$$

where, Ω_{t-1} denotes the information set at time t-1.

Once the r and V were estimated the next step was to estimate the six model parameters using cross-sectional data on T-Bills and Bonds across the Euro area for the specified date, which was the first trading day of May-01. Specifically, the official 3 month T-Bill, a 6 month T-Bill and 11 benchmark bonds of 1, 2, 3, 4, 5, 6,7, 8, 9, 10 and 15 years maturity, were used in the estimations. Hence a total of around 13 observations along the yield curve were used. A standard bootstrapping approach with linear interpolation was used to calculate the discount curve implied by the observed bond prices (for each day from 2 May 2001 -when the ECB main refinancing rate was 4.25%- to 06 June 2001). The reason we used linear rather than cubic spline interpolation is that we have enough points on the curve in order to estimate it. Also, previous experience of using cubic spline interpolation has resulted in unusual spikes in the curve, which didn't reflect accurately the observed bond yields.

Next, it is assumed that the observed market prices of these instruments differ from the prices produces by the LS model by an error term with expected zero value. This implies an assumption that the LS model provides the true financial form for pricing bonds, or at least that it is sufficiently flexible to be able to price all bonds correctly.

The estimates for the parameters in the LS model are obtained by minimizing the distance between the observed market prices and the model's theoretical prices of bills and bonds, using the following:

$$\Theta = \arg\min_{\Theta} \sum_{i=1}^{n} [P_i - P_i(r, V, \Theta)]^2 \quad (3.17)$$

P_i denotes the observed price of bill/bond(i) among the n available securities with different maturities, while P_i (r, V, Θ) is the corresponding LS price given the current values of r,V and the parameter vector. First an unconstrained optimisation was carried out by minimising the sum of the squared deviations between the model and the observed prices for maturities from 0.5 to 15 years at semi-annual intervals.

2.2.4 Option pricing using the Longstaff-Schwartz Density

One of the most appealing features of the LS model is its ability to price discount bond options in analytic form. LS give the solution to the problem of a call expiring at time t and struck at K on a discount bond with T further periods to maturity. Furthermore, in a later paper LS (1992b) pointed out a separation of variables of technique, which is applicable to the fundamental differential equation (3.13), which makes their approach extendable to European options with generic payoffs.

$$V(r,t) = \text{Max}[r - X, 0] = \text{Max}[(ax + by) - X, 0] \quad (3.18)$$

where X is as usual, the strike (LS 1992b).

$$q(x, y, \tau / x_0 y_0) = \frac{4}{\alpha(\tau)c(\tau)}\left(\frac{x}{b(\tau)x_0}\right)^{\gamma-\frac{1}{2}}\left(\frac{y}{d(\tau)y_0}\right)^{\eta-\frac{1}{2}}$$

$$\exp\left[\frac{-2}{a(\tau)}\left(x + b(\tau)x_0\right)\right]\exp\left[\frac{-2}{c(\tau)}\left(y + d(\tau)y_0\right)\right]$$

$$I_{2\gamma-1}\left(\frac{4}{a(\tau)}\sqrt{b(\tau)xx_0}\right)I_{2\eta-1}\left(\frac{4}{c(\tau)}\sqrt{d(\tau)yy_0}\right)$$

where

$$a(\tau) \equiv \frac{2\phi(\exp(\phi\tau) - 1)}{\phi[(\delta + \phi)(\exp(\phi\tau - 1) + 2\phi]}$$

$$b(\tau) \equiv \frac{4\phi^2 \exp(\phi\tau)}{[(\delta + \phi)(\exp(\phi\tau - 1) + 2\phi]^2}$$

$$c(\tau) \equiv \frac{2\psi(\exp(\psi\tau) - 1)}{\psi[(\xi + \psi)(\exp(\psi\tau - 1) + 2\psi]}$$

$$d(\tau) \equiv \frac{4\psi^2 \exp(\psi\tau)}{[(\xi + \psi)(\exp(\psi\tau - 1) + 2\psi]^2}$$

$$\phi \equiv \sqrt{2\alpha + \delta^2}$$

$$\psi \equiv \sqrt{2\beta + \xi^2} \qquad (3.19)$$

This technique allows the factorisation of the solution function H(x,y,t) in the product of a discount function F(x,y,t) and M(x,y,t) where M(.) gives the forward value of the claim. The forward value of the claim is the expectation of the payoff function V(x,y,t), taken over the probability distribution of x and y. For a caplet on the short rate, the payoff function has the form

LS show that the dynamics of (3.10) and (3.11) imply a specific joint density for the two state variables x and y (LS 1992a,b), which is a bivariate non-central chi-square density with closed form, given initial values of the state variables as above, where $I_p(.)$ is the modified Bessel function of order p. By using the transformations:

$$r = ax + by \quad (3.20)$$

$$V = a^2x + b^2y \quad (3.21)$$

we can obtain the bivariate non-central chi-squared density of the two transformed variables $q(r,V,\tau / r_0 , V_0)$. Since the variables r and V are correlated, the transformed density is more elliptical (in the x – y plane) then the density of the two state variables x and y which are uncorrelated by construction.

Hence, the joint density of r and V can be integrated over all values of V to obtain the one-dimensional marginal distribution of the short term interest rate,

$$q(r,\tau / r_0) = \int_{\beta r}^{\alpha r} q(r,V,\tau / r_0 V_0) dV \quad (3.22)$$

where the limits of the integral are due to the condition:

$$\alpha r \leq V \leq \beta r \quad (3.23)$$

which is set in order to avoid complex values of r and V. Hence, by estimating the parameters of the LS model as shown in the previous section, and then using the parameter estimates to calculate the above, we can obtain an estimate of the future short term interest rate distribution implied by the underlying process of r and V.

However, when this was tried it turned out to be problematic. For some combinations of parameters and state variable values, the numerical evaluation of the modified Bessel function or the numerical evaluation of the bivariate density breaks down. This seems to be a known problem also pointed out by Rebonato (1996), pp326. He notes that there are combinations of parameter values that make the probability function tend to infinity in the limit as one of the arguments goes to zero. Furthermore, it appears that this problem occurs frequently when the model is estimated using actual data, which suggests that an alternative, more robust approach to the evaluation of the closed-form density expression is warranted in practice.

Consequently, the density was estimated using a different approach Hordahl (2000). The strategy employed in that paper is to use a Monte Carlo method instead of the closed form solution to obtain the density. This is done by using discretised versions of the processes for the short rate and its variance to simulate possible future realisations of r and V. Specifically, by using an Euler approximation and assuming weekly time steps, discrete versions of the continuous-time dynamics are obtained as follows:

$$r_{t+\Delta t} - r_t = \left(\alpha\gamma + \beta\eta - \frac{\beta\delta - \alpha\xi}{\beta - \alpha} r_t - \frac{\xi - \delta}{\beta - \alpha} V_t \right) \Delta t + \alpha \sqrt{\frac{\beta r_t - V_t}{\alpha(\beta - \alpha)}} \sqrt{\Delta t}\, \varepsilon_{1,t+\Delta t}$$

$$V_{t+\Delta t} - V_t = \left(\alpha^2\gamma + \beta^2\eta - \frac{\alpha\beta(\delta - \xi)}{\beta - \alpha} r_t - \frac{\beta\xi - \alpha\delta}{\beta - \alpha} V_t \right) \Delta t + \alpha^2 \sqrt{\frac{\beta r_t - V_t}{\alpha(\beta - \alpha)}} \sqrt{\Delta t}\, \varepsilon_{1,t+\Delta t} + \beta^2 \sqrt{\frac{V_t - \alpha r_t}{\beta(\beta - \alpha)}} \sqrt{\Delta t}\, \varepsilon_{2,t+\Delta t} \quad (3.24)$$

where $\Delta t = 1/52$, while $\varepsilon_{1,t+\Delta t}$, $\varepsilon_{2,t+\Delta t}$, are drawn from two independent standard normal distributions. Note that since we have assumed that the local expectations hypothesis holds, the above processes are approximations of the risk neutral dynamics of r and V.

The above equations were used to simulate future values of r and V, in a recursive manner starting form the initial values r_0 and V_0, estimated in the previous section. In this study, a time horizon of up to half a year is chosen, which means that 26 future values of r and V are simulated, given the choice of $\Delta t = 1/52$. This process is then repeated 20,000 times with the same parameter values. Hence, for each of the 26 future weeks following the date of estimation, the described procedure produces a simulated sample consisting of 20,000 r's and 20,000 V's. The next step was to obtain an estimate of the distribution of the future short-term interest rate at each time, which was done by using a simple histogram. Since the simulations were performed using the risk neutral dynamics, the resulting density estimate for some forecast horizon is the risk neural density, RND, implied by the observed bond prices, and their volatility and assuming that the LS model holds.

Using the simulated density of the short rate we can price European Call and Put options on bonds using:

$$C(t,T) = E_t^Q[\exp(-\int_t^T r_s ds)\max(0, r(T) - X)], \quad (3.25)$$

under the risk neutral measure Q and

$$P(t,T) = E_t^Q[\exp(-\int_t^T r_s ds)\max(0, X - r(T)] \quad (3.26)$$

where C(t,T) is the price of a call and P(t,T) is the price of a put. Hence, using the Monte Carlo simulation we can use the discrete approximations of (3.25) and (3.26) to calculate option prices:

$$C(t,T) = \frac{1}{N}\sum_{i=1}^{N}\exp\left(\sum_{s=t}^{T-1} r_{i,s}\Delta t\right)\max(0, r_{i,T} - X), \quad (3.27)$$

$$P(t,T) = \frac{1}{N}\sum_{i=1}^{N}\exp\left(\sum_{s=t}^{T-1} r_{i,s}\Delta t\right)\max(0, X - r_{i,T}) \quad (3.28)$$

where Δt is the time step in the Monte Carlo simulation. So using the LS estimated parameters and (3.24) the simulation produced the risk neutral distribution of the short rate. Then using (3.27) and (3.28) we obtained prices for 7 different call options using the appropriate discounting factors from the LS (1992) estimation as of the 16/05/01. The strikes on the 7 call options were the ATM, which, was the implied forward and 3 strikes above and below the ATM at 25bp intervals. The prices were compared with calculated bond option prices using Black's formula.

2.2.5 Credit Spread Curves & the LS (1992) model

In chapter 2 it was shown that some credit spreads and credit spread indices exhibit characteristics found in default free rates. These are the *mean reversion*, *term structure* of their respective *volatilities,* the *time variation of their volatility* and their jump characteristics. Hence, we are going to use the two-factor LS (1992a) to model the evolution of the short credit spread. Credit spreads are observed quantities and it is possible to fit these observed quantities to an equilibrium model such as the LS. The same assumptions[10] will be carried forward for the evolution of the credit spread. This approach was first introduced by Ramaswamy and Sundaresan (1996) who used a direct assumption about the stochastic process followed by the credit spread. They assumed that the credit spread evolves according to the square root model of Cox Ingersoll and Ross (1985) and the Local Expectation Hypothesis, according to which expected returns on all default-free bonds over an infinitesimally short period of time are equal to the risk free rate. In the same context we used the stochastic volatility model of LS 1992 to model the short credit spread. This two-factor model is a near perfect tool to capture the dynamics of credit spreads since it can capture both the mean reversion and the time varying volatility of credit spreads (and credit spread

[10] As described in section 3.1 the main assumptions about the economy will hold.

indices), as observed in chapter 2. Thus, we assume that the level of the short spread and its instantaneous volatility follows

$$
\begin{aligned}
dspread_t &= (\alpha - \beta spread_t) + \gamma\sigma_t dZ_{1,t} \\
d\sigma_t &= (\delta - \varepsilon\sigma_t) + \nu\sigma_t dZ_{2,t}
\end{aligned}
\qquad (3.29)
$$

The next section describes how credit spread curves where estimated by assuming that the spread process is a contingent claim satisfying equation (3.13). Using the same methodology as in section 3.2 we will estimate credit spread curves based on observed bootstrapped spread curves.

2.2.5.1 Estimation of credit spread curves

In this section we estimate credit spread curves using bond prices of various European corporates and EUR denominated government benchmark bonds. A list of the bonds obtained from Bloomberg can be found in Appendix 1. The same bootstrapping methodology[11] as in section 3.2.1 was applied to the benchmark and corporate bonds in order to obtain market zero coupon discount factors. Once the discount factors of all the bonds were obtained the spread discount factors were calculated by:

$$\textit{spread}\text{ zero Disount Factor} = P(0,T) - P^i(0,T) \quad \text{should be} > 0 \quad (3.30)$$

$P(0,T)$ is a risk free zero coupon bond paying 1 at maturity

$P^i(0,T)$ is a risky zero coupon bond of rating i under the assumption of paying 1 at maturity if there is no default or δ (recovery rate) in case of default.

which is the difference between the two discount factors, as observed in the market. The spread zero discount factor should always be positive. This is mainly insured by the fact that the benchmark discount curve is usually higher than the risky discount

[11] The maturities 1,2,3,4,5,7,8,9 and 10 were used.

curves. In cases where this difference is less than zero then a potential mispricing has occurred.

Next, the volatilities of the risk free short rate and of the respective short credit spreads were estimated using GARCH. The 6M Euro-LIBOR was used as a proxy to the risk free short rate[12]. The relevant proxies were used for the short credit spreads. They are the observed 6M term spreads at which corporates arrange their short-term financing. The volatility of the short credit spread was found to be a few orders of magnitude higher than the volatility of the short risk free rate. That was to be expected because of the jump characteristics of credit spreads, rendering abnormal returns in the data.

The optimisation was carried out using initial values for all parameters based on the volatility condition (3.23). Hence, the parameter values of alpha and beta were in effect determined by the level of the short credit spread and its respective stochastic volatility.

2.2.6 Pricing Credit spread options

Credit risk modelling, has been extensively examined and there are two major approaches currently[13]. The first approach is called *structural approach* and the second one is the *reduced form* approach. The structural approach developed by Black and Scholes (1973) and Merton (1974) is based on the value of the company, where default is regarded as an endogenous decision made by the holders of the company's equity. These models are often called firm-value models. The default is being modelled as an option and as a result the same principles used for option pricing can be applied in the valuation of credit risky instruments.

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[12] Two years of daily data were collected from Bloomberg for the 6M Euro-LIBOR, 6M AAA, AA+, AA-, BBB, BB and B rates.

[13] Which have been fully described in chapter 1

The reduced form approach accounts for default as being both exogenous and a surprise in contrast to the firm value models. This group of models, most notably the Jarrow-Trunbull (1995) and its extension by Lando (1997) and the Duffie and Singleton (1994) model directly the likelihood of default or likelihood of a change in rating. Both models are arbitrage free and employ the risk-neutral measure to price securities.

There have been other attempts to price credit risk such as *spread-based* models and *hazard* models. Both models can be considered extensions to the structural approach. *Spread based* models are quite similar to barrier structural models[14], where default is detected by a bond spread crossing a specific barrier. Default is exogenous in both the barrier structural and spread based models and not a surprise. The spread dynamics are modelled using a log normal distribution and default is being priced by specifying a default boundary in terms of a spread. *Hazard* models take an econometric approach to derive default probabilities. Shumway (2001) proposed such a model, which incorporates both theoretical and empirical factors. The functional form of the model is derived by maximising the likelihood function between the survival probability and a collection of estimated empirical parameters[15].

The pricing of credit derivatives has been as a result been thoroughly studied in the literature. The fractional recovery of Duffie and Singleton (1994), (1997), and Schonbucher (1996), (1998) and the other intensity-based models such as the pioneering Jarrow and Turnbull (1995), Jarrow, Lando & Turnbull (1997), Madan and Unal (1994), are regularly used in order to price credit derivatives. Most of these methodologies including Hull and White (2000) are being used for pricing credit

[14] Black and Cox [ ] introduced these models by modeling default as a knockout (down and out barrier) option where default occurs the moment the firm value crossed a certain threshold.

[15] The functional form is $f(t,x;\theta) = h(t,x;\theta)\ S(t,x;\theta)$ where h is the hazard function and S is the survival function and x is a collection of explanatory variables and θ is a collection of parameters to be estimated.

derivatives such as credit default swaps, total return swaps, credit linked notes, CDOs and other second-generation credit derivatives.

Pricing of a European spread option resembles the pricing of an interest rate option. The payoff function is dependent on a risk-neutral expectation derived from the distribution of the spread. Hence, if we are able to derive the distribution of the spread then we can derive its payoff function. In chapter 1, we outlined the Jarrow, Lando & Turnbull (1997) model. It can be used to price a range of credit derivatives. Using that model and its notation lets consider an example. For a zero recovery, zero-coupon bond the survival probability of default is Fabozzi F.J., Choudhry, Anson and Chen. (2004) pp 265 – 270:

$$Q(t,T) = E\left(\exp\left[-\int_t^T \lambda(u)du\right]\right) \quad (3.31)$$

Also from chapter 1 the spread is given by Fabozzi F.J., et. al. pp 265 – 270 (2004):

$$s(t,T) = -\frac{\ln Q(t,T)}{T-t} \quad (3.32)$$

Hence, the intensity parameter of the Poisson process, λ(t), is deterministic if the spread s(t,T) is deterministic Fabozzi F.J., et. al. pp 265 – 270 (2004):

$$s(t,T) = -\frac{\ln Q(t,T)}{T-t} = \frac{1}{T-t}\exp\left(\int_t^T \lambda(u)du\right) \quad (3.33)$$

Now if we assume that λ(t), is stochastic, then the distribution of the spread depends on the distribution of the survival probability, Q(t,T). If we assume that λ(t) follows a mean reverting Gaussian (Ornstein-Uhlenbeck) process then the survival probability will be log normally distributed. Hence, the spread is normally distributed. The normality assumption creates an important problem. If λ(t) is normally distributed then the survival probability can exceed 100% (for a negative λ(t)). This violates the no-arbitrage condition Fabozzi F.J., et. al. pp 265 – 270 (2004).

There have also been attempts to model the intensity process with a square root process such as the one proposed by Cox, Ingersoll and Ross (1985), in order to avoid negative λ(t). Using a CIR process is feasible to get a closed form solution for the survival probability but the distribution of the survival probability is unknown, hence, the distribution of the spread is unknown. Since the spread can be log transformed by the survival probability, there is no distribution than can lead to a closed-form distribution for the spread Fabozzi F.J., et. al. pp 265 – 270 (2004).

Even if the *structural approach* is used where the spread of a zero-coupon bond is:

$$
\begin{aligned}
s(t,T) &= -\ln\frac{D(t,T)}{P(t,T)} \\
&= -\ln\{A(t)[1-N(d_1)] + P(t,T)KN(d_2)\} + \ln P(t,T)
\end{aligned}
$$

where A(t) is the asset price at time t, K is the face value of debt and (3.34)

$$
d_1 = \frac{\ln A(t) - \ln K - \ln P(t,T) + V/2}{\sqrt{V}} \quad \text{and } d_2 = d_1 - \sqrt{V} \qquad \text{Fabozzi F.J., et. al. pp 265 270 [27]}
$$

The spread variable has no closed-form solution for its distribution. Since neither of the two main approaches of credit risk modelling reach to a closed form distribution for the spread most research has concentrated in obtaining numerical solutions for spread option prices[16].

Thus, opting for simpler "spread based" models would be a relatively sensible idea if we are looking for a quick and relatively uncomplicated method. Spread based models rely on modelling the spread as a stochastic process. For example, if we directly assume a log normally distributed spread then we can use Black's (1976) framework to price credit spread options. This in fact generates internal inconsistency because we have assumed that all spreads irrespective of rating and type (asset swap spreads, default swap spreads and bond spreads) are distributed in the same way.

---

[16] See Schonbucher 1999, "A Tree Implementation of a Credit Spread Model for Credit Derivatives" where he prices credit spread options using a mean reverting Gaussian process for λ(t).

In the same context we have assumed that credit spreads evolve based on equations (3.10) and (3.11) and their implied distribution equates to (3.22). Hence, we will use the estimated credit spread curves from section 3.5.1 to price credit spread options. Of course the inconsistency mentioned still exists by choosing the LS (1992) models, but there is a closed form solution for the distribution of the spread and also we have taken into account the variability of the spreads' volatility –second factor of LS (1992)-.

2.2.7 Pricing of Credit Spread Options using LS (1992)

Credit spread options are contracts, which "bet" on the potential movement of corporate bond yields relative to the movement of government bond yields. Credit spreads can be thought of as the compensation investors receive to accept all the incremental risks inherent in holding a particular bond instead of some "riskless" benchmark. Spread options are often used in the market for speculation and hedging of credit risk. One needs to keep in mind that they are still rather exotic instruments traded OTC only. Moreover the contracts are very rich in detail. Hence, ever since credit risk was commoditised they rank low in investors/speculators preferences. Still in theory assuming they are well defined contractually they are the best instrument to replicate spread movements with controlled downside risk depending on the employed hedging strategy. In the following section we will price credit spread options which their payoff is dependent on the movement of the underlying spread.

Using the same methodology as in section 3.4 we attempted to price credit spread options. The lack of credit spread option prices led to the engineering of the payoffs of credit spread options. In the same fashion as Abken (1993) proposed the decomposition of a given swap into two yield options we assumed that the price of a credit spread option is equivalent when at the money with two vanilla options written on two assets. One asset being a government bond and the other asset being a corporate bond. Hence, for a call on a credit spread we assumed that the following holds at the money:

$$Spread(i) = yield(i) - yield_{Benchmark}$$

$$Call\ on\ spread(i) = Call\ on\ yield(i) + Put\ on\ yield_{Benchmark} \quad (3.35)$$

This approximation was carried out for ***calibration purposes only*** and it holds if both the risk free and the defaultable bonds are discounted by the same risk free curve Please note that the inverse holds for the bond prices. Using the above relationship we were able to examine the pricing power of the LS model when credit spread options were priced.

To illustrate, consider a call on the AA+ spread. This call option would be replicated by going long a call on a AA+ corporate yield and long on a put on the respective government benchmark yield. This just replicates the position on the underlying which is long the AA+ credit spread or just long AA+ calls.

The methodology used in this section could be useless unless it can be replicated by using underlying instruments, such as two reference bonds. In our case it is possible to do that because we can use the implied default probabilities as obtained in section 2.2.8 and price a corporate bond. Based on that an options portfolio priced using the LS 1992 can be replicated by a portfolio of default free and default bonds.

2.2.8 Implied Default and Transition Probabilities

Essentially one of the end results of credit risk modelling is to imply the survival probabilities and hence the associated credit spreads in order to price securities subject to credit risk. Credit spreads have been closely linked[17] to the survival probabilities by many researchers such as Jarrow and Turnbull (1995), Jarrow, Lando and Turnbull (1997), Madan and Unal (1994). In these models the intensity process which governs the probability of default is specified by a Poisson process. Following from Arvanitis, Gregory and Laurent (1999) and Schonbucher (1999) one can use this close relationship in order to imply probabilities of default using as inputs credit spread data. However, before we use our estimated credit spreads in order to derive the survival probabilities[18] we need to specify the credit risk model we are using.

2.2.8.1 An iterative procedure to imply default probabilities from credit spreads

The credit risk model used is based on Jarrow and Turnbull (1995) and the default free rate process is based on LS (1992). The model is set up in a filtered probability space $(\Omega, f, (f_t)_{(0 \le t \le \tau)}, Q)$. There is a unique Q equivalent martingale measure[19] making all the default-free and risky zero-coupon bond prices martingales. The economy is frictionless with a finite horizon [0,τ] and trading can be discrete or continuous. Further assumptions are markets are arbitrage free and complete. We also assume that the default free spot rate r(t) and the spread s(t) both evolve under the LS (1992) framework. Let B(t,T) be the time t price of a default-free zero-coupon bond paying 1 at time T. The money market account accumulates returns at the spot rate as:

$$B(t,T) = \exp \int_0^t r(s)ds \qquad \text{in the continuous case}^{20} \qquad (3.36)$$

Let D(t,T) be the time t price of a risky zero-coupon bond promising to pay 1 at time t if there is no default before time T and at default it pays the recovery rate δ < 1. Since

[17] In Chapter 1 there is a full description of one of these equivalent recovery models

[18] Since in these models first the implied probabilities are being implied and then the credit spreads.

[19] Q is an equivalent martingale measure with respect to the money-market account B(t), see Harrison and Kreps (1979) or Harrison and Pliska (1980).

[20] In the discrete time case $B(t) = \exp(\sum_{i=o}^{t-1} r(i))$

we are using the Jarrow and Trunbull (1995) which is an equivalent recovery model, the recovery rate δ is taken to be an exogenous constant. Following the assumption that the stochastic process for default-free spot rates and the bankruptcy process are statistically independent under Q we arrive at the standard equation:

$$\begin{aligned} D(t,T) &= E_t\left(\frac{B(t)}{B(T)}\right)E_t(\delta 1_{\{\tau^* \leq T\}} + 1_{\{\tau^* > T\}}) \\ &= B(t,T)(1 - Q_t(t,T) + Q_t(t,T)\delta) \end{aligned} \quad (3.37)$$

So assuming that default hasn't already occurred the survival probability is:

$$Q_t(t,T) = \frac{1 - D(t,T)/B(t,T)}{(1-\delta)} \quad (3.38)$$

And since D(t,T) and B(t,T) are two zero-coupon bonds we can rewrite the equation by replacing the difference between the two zeros with their instantaneous spread s(t,T):

$$Q_t(t,T) = \frac{s(t,T)}{(1-\delta)B(t,T)} \quad (3.39)$$

in continuous time with a minor re-arrangement this relates the default probabilities with the forward credit spreads:

$$Q_t(t,T)(1-\delta) = 1 - e^{-\int_t^T s(t,\tau)d\tau} \quad (3.40)$$

Based on (3.32) and (3.33) for a small time dt the short-term credit spreads are directly related to local default probabilities q(t,t+dt). Local default probabilities are the probability of default between t and t + dt, conditional on not having defaulted before t. In addition, we can relate the probability of default to the intensities of the default process Arvanitis, Gregory and Laurent (1999):

$$\lambda(t) = \frac{q(t,t+dt)}{dt} \quad (3.41)$$

Clearly using (3.33) for a small period of time we can use the spread discount factors obtained using the LS (1992) model in order to derive the local default probabilities and subsequently the conditional default probabilities. The following iterative procedure will be used in order to imply the survival probabilities out of the term structure of credit spreads.

For $t < \tau < T$

$$q(t,\tau) = \frac{B(t,\tau) - D(t,\tau)}{(1-\delta)B(t,\tau)} = \frac{s(t,\tau)}{(1-\delta)B(t,\tau)}$$

$$q(t,\tau+1) = \frac{\{B(t,\tau+1) - D(t,\tau+1)\} - \{B(t,\tau) - D(t,\tau)\}}{(1-q(t,\tau)(1-\delta)B(t,\tau+1)}$$

$$= \frac{s(t,\tau+1) - s(t,\tau)}{(1-q(t,\tau)(1-\delta)B(t,\tau+1)}$$

•   •   •   •   •

•   •   •   •   •

$$q(t,T) = \frac{\{B(t,T) - D(t,T)\} - \{\sum_{\tau=\tau+1}^{T} B(t,\tau) - D(t,\tau)\}}{\{\prod_{\tau=\tau+1}^{T}(1-q(t,\tau)\}(1-\delta)B(t,T)} \qquad (3.42)$$

$$= \frac{s(t,T) - \{\sum_{\tau=\tau+1}^{T} s(t,\tau)\}}{\{\prod_{\tau=\tau+1}^{T}(1-q(t,\tau)\}(1-\delta)B(t,T)}$$

Following the determination of the local default probabilities we can now determine the cumulative survival probabilities taking into account again the two possibilities at each time point: default and no-default.

$$Q(t,\tau) = q(t,\tau)$$

$$Q(t,\tau+1) = q(t,\tau) + (1-q(t,\tau))q(t,(\tau+1) \qquad (3.43)$$

$$Q(t,\tau+2) = q(t,\tau) + (1-q(t,\tau))q(t,(\tau+1) + (1-q(t,\tau))(1-q(t,\tau+1))q(t,\tau+2)$$

Next, using the K possible ratings model of Jarrow, Lando and Turnbull (1995) we will demonstrate how the current risk premia as estimated using the LS (1992) model can be used to derive an implied rating transition matrix.

2.2.8.2 Implying transition probabilities from credit spreads

The simple model of default/no default of Jarrow and Turnbull (1995) was expanded to include more rating classes in Jarrow Lando and Turnbull (1997). The credit rating dynamics of K possible credit ratings, are represented by a Markov chain. The first state of the Markov chain corresponds to the best credit quality and the (K-1) state the worst before default. The Kth state represents default and is an absorbing state, which pays the recovery rate δ times the par value at maturity.

The dynamics of credit ratings are characterised by a set of transition matrices Q'(t,T)[21] (k x k matrices) for any period between time t and T. Each of the elements $q_{ij}(t,T)$ of these matrices represents the probability of migrating from rating i at time t to rating j at time T. The last column of Q'(t,T) (which is the $q_{ik}(t,T)$) gives the default probabilities.

In JLT (1997) and Arvanitis, Gregory and Laurent (1999), the transition probability matrix is expressed exponentially:

$$Q'(t,T) = \exp[\Lambda(T-t)] \quad (3.44)$$

This is based following the specification of a continuous time Markov chain by JLT (1997). Matrix Λ (a k x k matrix) is called the generator of transition matrices Q'(t,T)

[21] The matrix notation has been dropped.

and is assumed to be diagonalisable[22]. JLT (1997) by assuming that the generator matrix under the equivalent martingale probability is:

$$\overline{\Lambda}(t) = U(t)\Lambda(t) \quad (3.45)$$

The U(t) is the vector of the risk premia which transform the historical generator matrix to the risk neutral. The elements of the generator matrix are directly related to the short-term probabilities. The probability of staying to the same rating i from time t to t + dt is 1 + λ_{ii} dt. The probability of going from rating i to rating j (i≠j) is λ_{ij} dt and the probability from rating i to default is λ_{ik} dt (j≠k). The transition probabilities are constrained by further assumptions is order to ensure the proper evolution of credit spreads[23].

Since the generator matrix is diagonalisable and using (3.44) we get:

$$Q'(t.T) = \Sigma \exp[D(T-t)]\Sigma^{-1} \quad (3.46)$$

where D represent the eigenvalues of the generator matrix and Σ represent its' eigenvectors. Using (3.41) the probabilities of default are being expressed:

$$q_{iK}(t,T) = \sum_{j=1}^{K-1} \sigma_{ij}\sigma_{ij}^{-1}[\exp[d_j(T-t)-1] \quad (3.47)$$

where σ_{ij} are the elements of Σ and σ_{ij}^{-1} are the elements of Σ^{-1} for $1 \leq i \leq K-1$

Hence, using (3.32) and (3.42) we can write:

$$D(t,T,\Lambda) = B(t,T)(1 - q_{iK}(t,T) + q_{iK}(t,T)\delta) \quad (3.48)$$

Using (3.43) and (3.42) we can solve for the credit spread of rating i:

---

22 For example $\Lambda = \Sigma D \Sigma^{-1}$ where D is a diagonal matrix

23 λ_{ij} >= 0 always, The sum of transition probabilities $(1+\lambda_{ij}) + \sum_{\substack{i=1 \\ i\neq j}}^{K} \lambda_{ij} = 1, \quad j = 1,.....,K$, is equal to one. The k-th state is absorbing $\lambda_{kj} = 0$ and finally state I+1 is always more $\sum_{j\geq k} \lambda_{ij} \leq \sum_{j\geq k} \lambda_{i+1,j}$ risky than a state i

$$s^i(t,T,\Lambda) = (\delta - 1)B(t,T)\sum_{j=1}^{K-1} \sigma_{ij}\sigma_{ij}^{-1}[\exp[d_j(T-t)-1]$$

or (3.49)

$$= (\delta - 1)B(t,T)q_{iK}$$

This is how credit spreads are related to the eigenvectors and eigenvalues of the generator matrix. This equation provides the term of credit spreads based on a given generator matrix. The estimation of the current generator matrix is based on the actual (historical). JLT (1997) have showed a procedure that is used to estimate such a matrix using as inputs market prices of default free zero-coupon bonds, risky zero-coupon bonds and the historical generator matrix. The procedure involves estimating the risk premia using the observed market prices and multiplying the risk premia diagonal matrix ($diag(\pi_1, \ldots\ldots \pi_{K-1}, 1)$) to the historical generator matrix. In this way the Q'(0,t) matrix is calculated and subsequently the Q'(0,t+1) matrix can be calculated using:

$$Q'(t,T) = \exp(diag(\pi_1, \ldots. \pi_{k-1}, 1)\Lambda(T-t)) \quad (3.50)$$

This iterative procedure produces the risk neutral transition matrix based on current risk premia. Their next step was to minimise the risk premia by minimising the difference between the theoretical risky zero-coupon bond prices estimated using the risk-neutral transition matrix and the observed risky zero-coupon bond prices.

In the same context, following the estimation of the credit spreads (section 3.5.1) we minimised the difference between the LS estimated spreads and the spreads derived (using 3.41) using the historical transition matrix[24] as published in JLT (1997). Essentially we used the observed risk premia as derived from the LS spread curves in order to derive the risk neutral generator matrix.

[24] This is the historical transition matrix as published by Moody's. Other rating agencies such as S&P also publish matrices like that on a regular basis.

2.3 Data and Description

2.3.1 EUR Credit Spread Indices

Weekly EUR credit spread indices were retrieved covering the period of first week of May 2001 to the first week of May 2004[25]. The S&P ratings for which data was available were the following: AAA, AA+, AA, AA-, A+, A, A-, BBB+, BBB, BBB+. For each rating, the credit spread for the following maturities was collected: 0.25, 0.5, 1, 2, 3, 4, 5, 7, 8, 9 and 10 years. The data series comprise of 205 weekly and 820 daily observations. Univariate statistics (% week) were calculated across all ratings and maturities. The sample we have mainly originates from the financial and industrial sectors. The mean of the daily series and its standard deviation were also estimated in order to determine the term structure of credit spread indices and their volatility based on our sample of data. Table 2.1 shows the univariate statistics of the credit spread indices.

Table 2.1 (% Week), Univariate Statistics CSI changes May-01 – May-04

	Standard Mean	Standard Error	Standard Deviation	Kurtosis	Skewness
2YAAA	0.010	0.010	0.147	7.255	1.652
3YAAA	0.048	0.040	0.576	16.964	1.242
4YAAA	0.030	0.025	0.363	14.013	1.071
5YAAA	0.009	0.009	0.125	5.799	0.083
7YAAA	0.005	0.008	0.108	2.967	-0.202
8YAAA	0.004	0.006	0.085	2.007	0.037
9YAAA	0.002	0.006	0.092	1.013	0.124
10YAAA	0.001	0.006	0.085	0.856	0.367
2YAA+	0.011	0.011	0.158	4.463	1.206
3YAA+	0.014	0.012	0.170	4.475	0.803
4YAA+	0.012	0.011	0.159	16.755	1.970
5YAA+	0.008	0.009	0.123	3.095	0.233
7YAA+	0.007	0.008	0.119	1.707	0.365
8YAA+	0.006	0.008	0.112	1.521	-0.026
9YAA+	0.004	0.007	0.103	1.191	0.168
10YAA+	0.001	0.006	0.089	1.644	0.233

[25] Bloomberg calculates yield curves for different ratings categories by applying cubic splines on a daily basis to large cross-sections of similarly rated corporate bonds.

2YAA	0.006	0.008	0.114	2.949	0.868
3YAA	0.008	0.009	0.132	14.177	1.532
4YAA	0.008	0.010	0.140	43.433	3.940
5YAA	0.004	0.007	0.100	6.349	-0.004
7YAA	0.003	0.006	0.086	3.240	-0.092
8YAA	0.002	0.006	0.079	2.606	-0.276
9YAA	0.001	0.005	0.076	2.161	-0.073
10YAA	-0.001	0.004	0.062	0.441	0.415
2YAA-	0.006	0.008	0.109	2.664	0.839
3YAA-	0.006	0.007	0.104	4.923	0.432
4YAA-	0.005	0.007	0.097	9.413	0.570
5YAA-	0.003	0.006	0.087	5.528	-0.050
7YAA-	0.002	0.005	0.076	3.136	0.175
8YAA-	0.001	0.005	0.074	1.997	0.317
9YAA-	0.000	0.005	0.070	1.590	0.207
10YAA-	0.002	0.006	0.087	7.384	0.366
2YA+	0.008	0.010	0.103	0.676	0.226
3YA+	0.008	0.012	0.117	3.752	0.205
4YA+	0.007	0.011	0.107	7.531	0.649
5YA+	0.002	0.007	0.073	0.746	-0.331
7YA+	0.003	0.007	0.073	2.336	-0.441
8YA+	0.004	0.008	0.076	1.197	-0.445
9YA+	0.003	0.007	0.074	0.104	-0.038
10YA+	-0.001	0.007	0.069	0.873	-0.067
2YA	0.004	0.011	0.098	1.986	-0.081
3YA	0.003	0.011	0.096	1.836	-0.138
4YA	0.000	0.009	0.078	1.395	-0.506
5YA	-0.003	0.008	0.070	0.114	-0.083
7YA	-0.005	0.008	0.074	0.740	-0.312
8YA	-0.006	0.008	0.072	1.054	-0.315
9YA	-0.005	0.008	0.076	0.850	-0.139
10YA	-0.007	0.007	0.067	1.511	-0.260
2YA-	0.002	0.008	0.076	0.808	0.049
3YA-	0.001	0.009	0.093	7.579	0.652
4YA-	-0.002	0.009	0.090	14.265	1.492
5YA-	-0.005	0.006	0.063	-0.208	0.033
7YA-	-0.003	0.006	0.064	1.251	0.068
8YA-	-0.003	0.007	0.069	2.148	0.380
9YA-	-0.003	0.008	0.076	3.275	0.014
10YA-	-0.001	0.011	0.110	13.674	2.214
2YBBB+	0.001	0.006	0.073	1.297	0.411
3YBBB+	0.000	0.005	0.066	3.742	-0.162
4YBBB+	0.000	0.005	0.059	3.129	-0.010
5YBBB+	0.000	0.004	0.052	3.018	-0.317
7YBBB+	-0.002	0.003	0.039	2.036	-0.484
8YBBB+	-0.002	0.003	0.039	4.807	-1.117
9YBBB+	-0.002	0.003	0.043	2.670	-0.280
10YBBB+	-0.002	0.004	0.051	3.661	0.412
2YBBB	0.002	0.007	0.092	3.984	0.879
3YBBB	0.000	0.007	0.084	4.670	0.281
4YBBB	0.000	0.006	0.076	8.068	-0.120
5YBBB	0.000	0.006	0.073	9.731	-0.133

7YBBB	-0.001	0.005	0.058	17.796	-2.227
8YBBB	-0.001	0.005	0.066	16.778	-0.808
9YBBB	-0.001	0.006	0.074	14.444	-0.337
10YBBB	-0.001	0.006	0.072	15.486	-0.281
2YBBB-	0.006	0.011	0.136	14.688	2.048
3YBBB-	0.008	0.009	0.120	15.410	2.218
4YBBB-	0.007	0.008	0.104	11.703	1.943
5YBBB-	0.006	0.007	0.094	9.949	1.268
7YBBB-	0.005	0.007	0.092	12.674	1.631
8YBBB-	0.005	0.007	0.093	9.160	1.260
9YBBB-	0.005	0.007	0.093	10.587	1.497
10YBBB-	0.004	0.007	0.092	9.411	1.318

The means of the levels were also estimated over the same period for the weekly data, in order to give us a graphic representation of the term of credit spreads.

Table 2.2 Means of Credit Spread Index Levels (Weekly)

	0.25	0.5	1	2	3	4	5	7	8	9	10
AAA	5	22	22	28	25	27	33	27	41	26	32
AA+	8	12	25	28	31	33	36	33	32	33	38
AA	13	17	31	33	37	39	43	39	37	38	49
AA-	17	22	37	39	42	44	48	43	41	43	32
A+	29	33	49	53	57	62	65	62	61	64	75
A	34	39	54	57	61	67	69	68	68	71	80
A-	47	52	68	78	95	100	102	102	108	111	123
BBB+	32	38	61	80	90	93	100	113	111	105	104
BBB	47	38	61	80	90	93	100	113	111	105	104
BBB-	208	212	247	289	302	331	334	345	365	358	357

2.3.2 Data description for the LS estimation

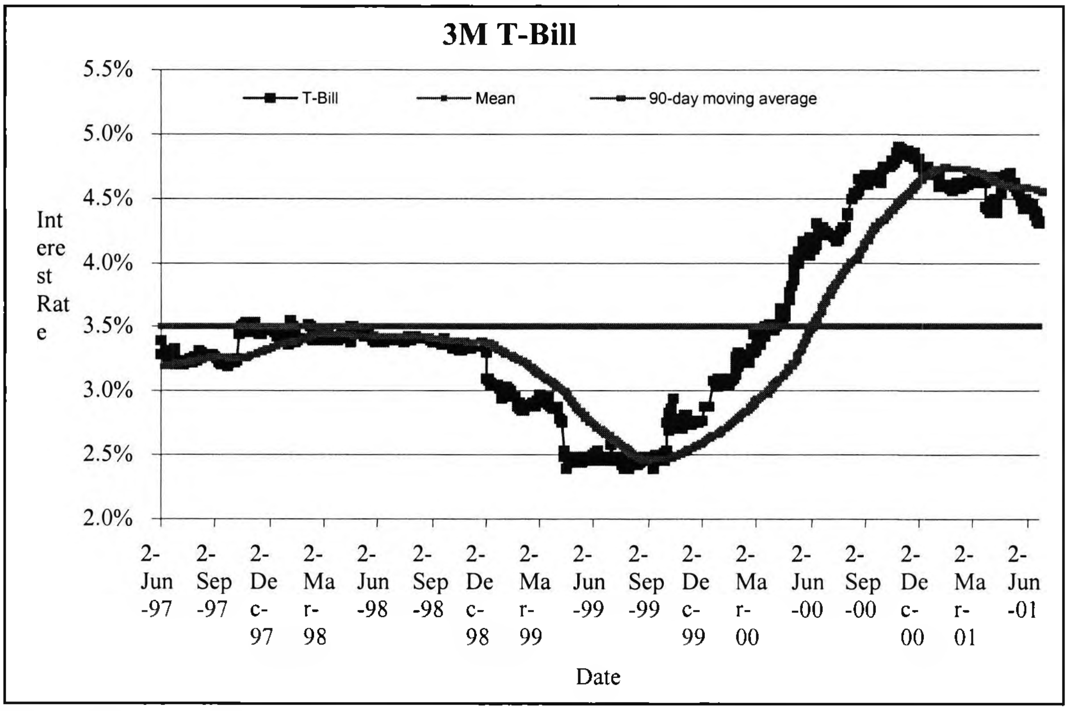
The short rate chosen was the most liquid T-bill in issuance at the time the estimation was carried out. That was the French 3 month t-bill. The GARCH model was fitted to daily changes in the French 3 month T-Bill rate since the repo rate remains constant over long periods of time, making it unsuitable to estimate V. The daily data of the T-Bill was collected from Bloomberg (Jun-97 – Jun-01) and the GARCH model was estimated using Microfit 4.0. The estimated r and V are shown in Table 2.3.

Table 2.3 GARCH (1,1) and mean of French 3M T-Bill

Mean (Jun-97 – Jun-01)	GARCH (1,1)
3.51%	0.36%

In figure 2.1 the French 3M T-Bill rate is plotted over a 2 year period. The red line shows the level of r used for the estimation of the yield curve. The first trading of May-01 was chosen as the start date of our estimation. Cross-sectional data of T-Bills and Bonds from the Euro area were collected. Specifically, the 3 month T-Bill rate, the 6 month T-Bill and 11 benchmark bonds of 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 and 15 years maturity approximately. The details of the bonds including the bootstrapping routine are shown in Appendix (2).

Figure 2.1 3M French T-Bill Rate Jun-97 – Jun-01



2.3.3 Credit Spread Data

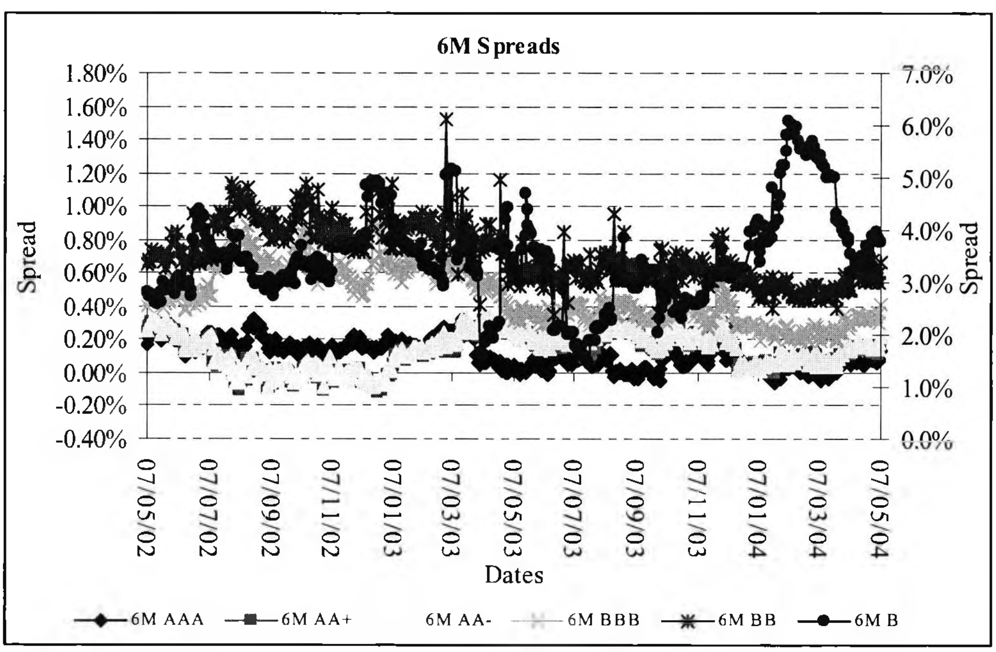
This set of data was used to estimate credit curves using the LS (1992) model. Two years of daily data were collected from Bloomberg between 07/05/02 – 07/05/04 for the 6M Euro-LIBOR rate, and 6M AAA, AA+, AA-, BBB, BB, B credit spreads. All the corporate bonds where from the industrial sector apart from the AAA which was from the financial sector. The date, which we fitted the 6 different credit spread curves was the 7th of May 2004. The bonds were stripped to create the zeroes using the same bootstrapping procedure as in 3.3.1. All the bonds used to fit the credit spread curves are listed in Appendix (1).

Table 2.4 Mean and GARCH (1.1) for 6M credit spreads

Rating	Mean	GARCH (1,1)
AAA	0.11%	27.72%
AA+	0.12%	12.87%
AA-	0.14%	12.21%
BBB	0.46%	8.33%
BB	0.72%	10.13%
B	3.44%	5.53%

The GARCH (1,1) was performed using the PCGIVE software this time, as opposed to MICROFIT 4.0 in section 3.3.1.

Figure 2.2 6M credit spreads (May-02 – May-04)



In Figure 2.2 the 6M credit spreads are plotted over time. It is quite clear that the 6M B spread is the highest of all. The lower rated spreads such as the 6M AAA and 6M AA+, 6M AA- are often quite low, as low as zero. This usually occurs when there is a huge amount of liquidity in the corporate market, which makes the short term financing among highly rated institutions "almost" risk free. This increased liquidity is related to the monetary policy by the European Central Bank, which reduced the level of interest rates during that period. In periods of low interest rates there is an increased risk appetite since the cost of borrowing is quite low.

2.3.4 Frequency comparison

The first approach that the LS was implemented was done over a small number of random consecutive days. It was safely assumed then that the parameters of the LS model could well be different. However, when the European discount curves were examined more closely on a day-by day basis it was observed that some days had quite irregular shape. Unfamiliar humps and twists which are not usually seen with European yield curves. A possible reason for this could be a liquidity effect between particular maturities on the curve. This is something that occurs often in the short end part of the yield curve where the short term financing of central banks and corporations takes place, leaving the curve slightly distorted when it occurs. Another reason for this could be the volatility of the short rate, which is quite high when examined with daily data.

Running the calibration of the LS model on a daily basis it was observed that the volatility of the LS parameters was quite high. Although the unconstrained optimisation produced zero SSD (sum of squared differences), the shape of the LS yield curve was not exactly the same as the market discount curve. For example, there were points on the curve that over- or underestimated the bond yield at long or short end of the yield curve. Based on that we run a constrained optimisation where the volatility condition of the model was preserved, hence the initial parameters of alpha and beta were easy to deduce based on the r and V of the particular day. This helped to produce better fitting of the market discount curve to the LS model.

Table 2.5 Squared Differences

	0.25 year	0.5 year	1 year	2 years	3 years	4 years	5 years	7 years	8 years	9 years	10 years	Sum
Unconstrained	0.000036	0.000059	0.000022	0.000014	0.000047	0.000058	0.000037	0.000021	0.000001	0.000023	0.000081	**0.000401**
Constrained	0.000000	0.000001	0.000003	0.000000	0.000002	0.000002	0.000003	0.000002	0.000000	0.000001	0.000006	0.0000217
Reduction	-99%	-98%	-84%	-99%	-97%	-96%	-91%	-91%	-99%	-95%	-93%	-95%

The table above shows the improvement in the fitting of the curve, which is improved by approximately 100%. Examining the daily charts of the LS parameters over a period of two and a half years showed that their volatility was quite high. This is something that affects option pricing using the LS model. The methodology used to carry out option pricing would produce complex numbers if the eta and ni parameters are higher then 0.5. This occurred a number of times using the daily data. This produces discontinuity in the historical time series. These missing points would have to be interpolated in order to calculate VaR. A simple way to avoid this -unnecessary interpolation, which could distort the end result- is to treat the data over a larger time interval and compare the two sets of parameters. Hence, the data was treated over weekly time intervals and subsequently compared.

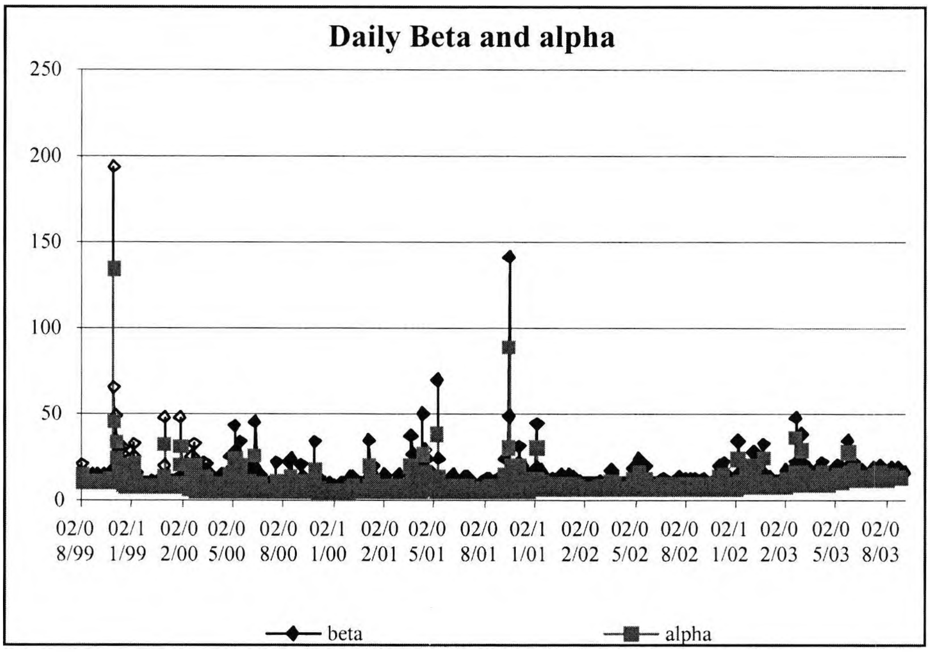
2.3.4.1 The Daily-Weekly comparison

The same methodology was carried out as in section 3.2.1. The only difference is that a longer time period was examined. Daily LIBOR and swap (annualised) rates were examined for the period 04/08/99 – 04/06/03 for the following terms: 3m, 6m, 1Y, 2Y, 3Y, 4Y, 5Y, 6Y, 7Y, 8Y, 9Y, 10Y and 15Y. A constrained optimisation was performed based on the **volatility condition** for the short rate:

$$Ar <= V <= br \quad (3.51)$$

All the 6 parameters found acceptable values following the optimisation. It is worth noting that since the above constraint is used the alphas and the betas have almost the same magnitude as the ratio of the volatility of the short rate relative to the short rate itself. Hence, at periods of high volatility and low rates these two parameters will be quite high and vice versa. As the following chart shows these two parameters are fully correlated over time and they exhibit quite high volatility. However, the high degree of correlation between the short rate and its volatility was expected since it is and

Figure 2.3 Daily alpha and beta parameters



intrinsic assumption of the LS model. This is a condition, which needs to be maintained, since it's a precondition for option pricing.

A first view of figures 2.3 and 2.4 shows that the alpha and beta parameters are relatively uncorrelated to the gamma, delta, eta and ni parameters. On a closer look we can say that they exhibit higher volatility than the previous two, apart from the eta parameter which is not correlated with the movement of the rest of the parameters. These results were used to compare the derived LS parameters over weekly time intervals in order to deduce which datasets are more volatile.

The same technique was applied as before, with different datasets. The results show that the LS parameters are smoother, less volatile and lack the erratic swings observed with the daily LS parameters. The best way to understand how the two datasets differ is to calculate and compare their respective descriptive statistics as shown in Tables (2.6) and (2.7):

Figure 2.4 Daily parameters gamma, delta and eta

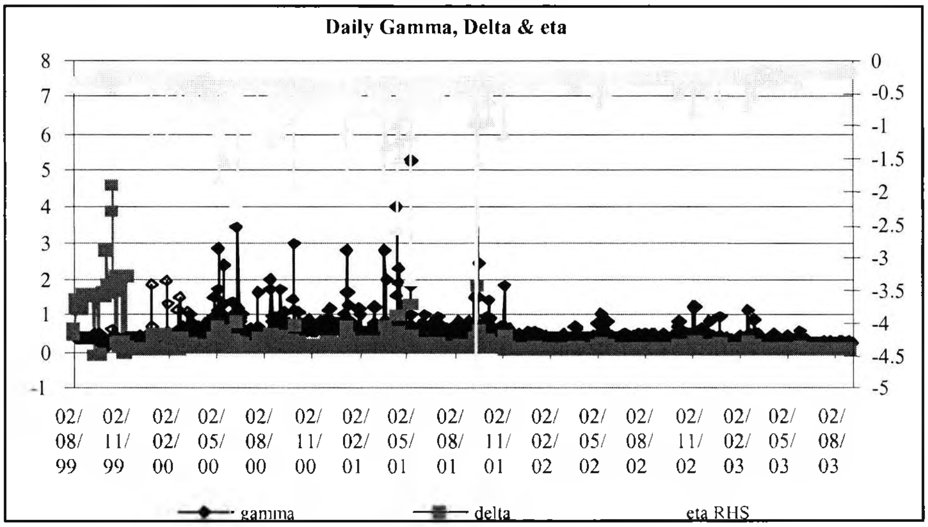


Table 2.6 Descriptive statistics of the 6 LS (1992) parameters (Daily)

	alpha	Beta	Gamma	delta	Eta	ni
Mean	8.71	12.78	0.53	0.23	-0.34	-1.30
Standard Deviation	6.25	9.14	0.42	0.41	0.27	1.41
Median	7.55	10.84	0.42	0.12	-0.29	-1.16
Kurtosis	183.04	186.11	88.16	26.57	89.12	43.14
Skewness	10.62	11.13	7.47	4.57	-7.52	-3.63

The yield curves were estimated over a period of approximately 4 years (02/08/1999 – 19/09/2003) for the two different frequencies, daily and weekly. As it can be seen the volatility of the daily parameters is quite high as expected relative to the weekly parameters.

The standard deviation calculated for all the weekly parameters is approximately twice the volatility of the volatility of the daily parameters. In addition the range of the weekly parameters –min to max- is much narrower than in the daily parameters.

Table 2.7 Descriptive statistics for LS parameters (Weekly)

	alpha	beta	Gamma	delta	eta	ni
Mean	9.55	14.09	0.46	4.54	-0.33	4.86
Standard Deviation	3.77	5.00	0.16	0.20	0.15	1.85
Median	8.74	13.71	0.43	5.10	-0.29	5.57
Kurtosis	5.19	3.23	-0.87	3.96	-0.65	2.99
Skewness	1.46	0.83	0.41	-2.24	-0.56	-2.14

This justified to us the choice of weekly data in order to run weekly VaR rather than daily since it would produce "continuous" – in other words no missing points in the time series- results of time series. Hence, slightly better VaR results, especially for options.

The only disadvantage of reducing the data size is that the results of our subsequent historical regressions will be less powerful. However, we will still have sufficient data points in order to deduce useful conclusions, since the total number of points will be over 100 points. Our point of view is that the more the better, but a good starting point in regression analysis is 100 data points. If there is any kind of explanatory power between one random variable to the other it will -probably- be revealed.

2.3.5 Transition Rating Matrix and Recovery Rate

The transition rating matrix used was taken form the seminal paper of Jarrow Lando and Turnbull (1997). Its a 1-year historical transition matrix which was estimated by Standard and Poor's Credit Review (1993). JLT (1997) modified this matrix I order to remove the "not rated" and created a modified matrix (Table 3.6).

The recovery rate used was the historical recovery rate from Moody's Special Report (1992). That is a weighted recovery rate based on a number of different types of debt. The value of the recovery rate used was **0.3265**.

Table 2.8 1-Year modified historical transition matrix

	AAA	AA	A	BBB	BB	B	CCC	D
AAA	0.8779	0.1078	0.0087	0.0021	0.0034	0.0001	0.0001	0.0001
AA	0.0144	0.8339	0.1252	0.0166	0.0049	0.0049	0.0002	0.0002
A	0.0016	0.0522	0.8014	0.1165	0.0181	0.0081	0.0002	0.0017
BBB	0.0008	0.0055	0.0854	0.7951	0.0839	0.0208	0.0023	0.0059
BB	0.0002	0.0013	0.0048	0.0459	0.8573	0.0665	0.0081	0.0154
B	0.0001	0.0014	0.0023	0.0052	0.0396	0.8656	0.0333	0.0525
CCC	0.0000	0.0000	0.0035	0.0036	0.0062	0.0258	0.8801	0.0793
D	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000

2.4 Results and Analysis

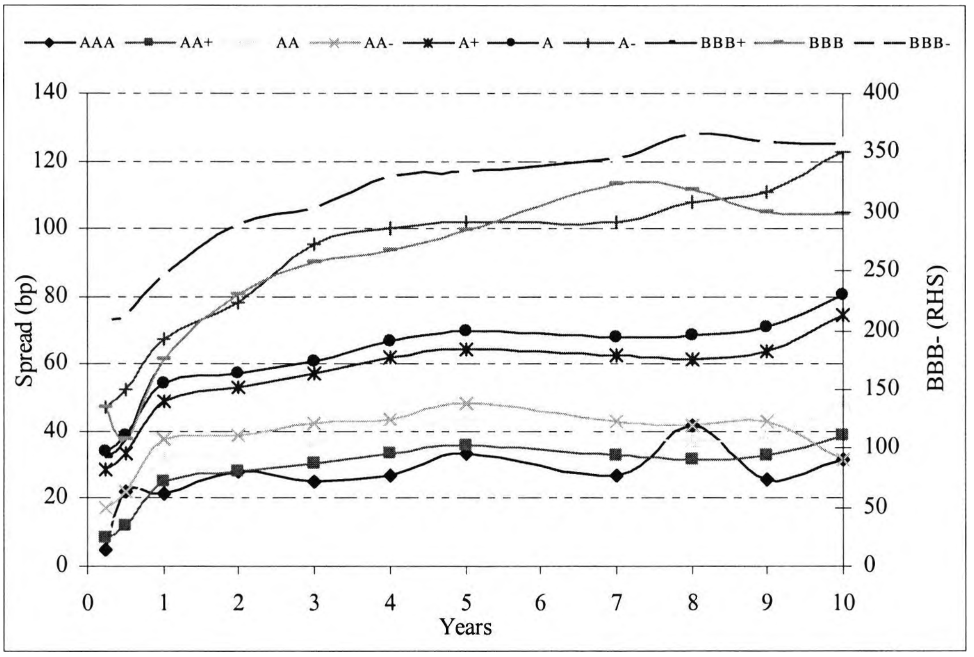
2.4.1 Term Structure of Credit Spread Indices

The term structure of credit spreads evident in the means of the respective spreads is on average upward sloping (Figure 2.5), unlike their volatilities, which seem to be humped. Litterman and Iben (1991) observe the same upward sloping term structure based on US data, unlike Sarig and Warga (1989) who examined spreads for US corporates over the period Feb-95 to Sep-87. They observed that high and low-quality spread curves were respectively upward and downward-sloping and they argued that this agreed with the predictions by Merton (1974).

The lower rated spreads seem to exhibit more convexity and also show a bigger slope than the rather flatter curves of the higher ratings. Most of the short-term financing of the financial institutions occurs over the money markets with little or zero credit risk considerations when rates are quoted hence the lack of liquidity in these credit spread indices, and as a result huge swings in the respective credit spreads resulting in increased volatility. The shapes of the volatility curves are mostly downward sloping with the 1Y maturity having the highest and the 10Y the lowest volatility. There is a lot of variability in the volatilities among the medium sized maturitties. The 3 to 5Y sector is the most active area in most credit markets and it seems to be the case in the European market as well.

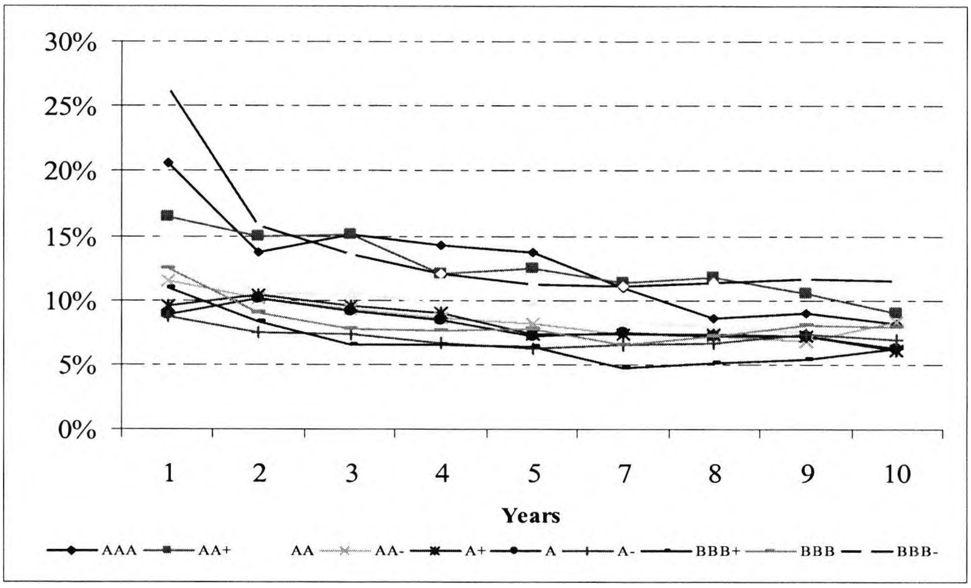
In Figure 2.6 the volatilities of the credit spread indices per maturity sector and rating are shown.

Figure 2.5 Term Structure of Credit Spread Indices Means per maturity sector



The volatility term resembles the humped volatility structure observed in interest rates. It is quite remarkable that the volatilities vary so little across some different ratings categories. The big jump in volatility in the BBB ratings is due to the fact that these indices are from the industrial sector whereas all the rest of the credit spreads indices from the financial and banking sector. Plotting the terms of the means and the volatilities of credit spread indices is more of a qualitative bit of analysis which gives an illustration of the levels of credit spread indices and their historical volatilities and how they differ across ratings and maturities. The observed declining slope of the volatility curves is probably due to the liquidity effects found in the credit spread index data, indicating a difference between the humped volatility curves of interest rates and the importance of liquidity.

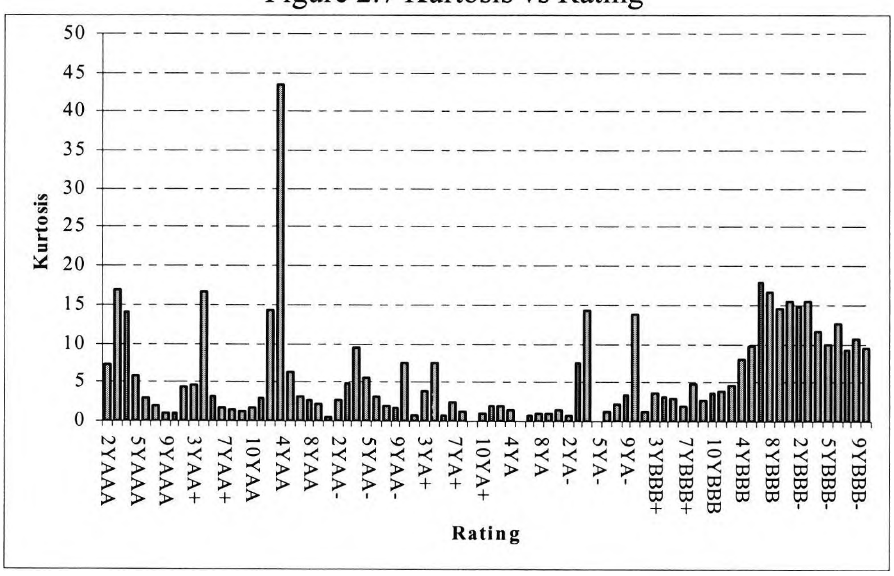
Figure 2.6 Term Structure of Volatilities of Credit Spreads



2.4.2 Distributional Properties of EUR Credit Spread Indices

It is quite clear from Table 2.1 that EUR Credit Spread Indices exhibit non-Gaussian distributional properties. The skewness and kurtosis of the weekly changes differ substantially from the respective moments of the normal distribution. The indices seem to be highly skewed a property also found in credit spreads. The kurtosis per rating exhibits a pattern with the first four ratings of AAA, AA+, AA, AA- (Figure 2.7). The kurtosis decreases with maturity for these ratings with the highest being between the 2 – 5Y sector.

Figure 2.7 Kurtosis vs Rating



The fact that kurtosis differs across rating and maturity substantially has important implications for risk management.

2.4.3 GARCH in credit spread indices

The simple statistical analysis in section 2.3.1 demonstrates that EUR credit spread indices exhibit term structure in their historical volatilities. Historical volatilities of some ratings present a flat term structure whereas in other ratings the term structure is somewhat more complex. Since the historical volatilities are a product of "historical averaging" of observed prices, it avoids identifying important price actions in the past since it assigns an equal weight to all the observed prices. Examining the GARCH effects of the credit spread indices will give us an indication of the stochasticity of the series.

Table 2.9 shows a summary of our optimisation results using the PCGive software. The first observation is that some of the series examined did not exhibit GARCH(1,1) effects since there was no convergence. These series were examined for higher orders of heteroskedasticity (1,2 etc.) but still there was no convergence. However, all series converged when the (1,0) was performed. It was somehow surprising that not all series did not show heteroskedasticity even at the lowest level (1,1).

Table 2.9 GARCH (1,1) parameters

	AAA_5Y	AA+_2Y	AA+_5Y	AA-_5Y	BB+_2Y	BB+_10Y	BB_2Y	BB_10Y	BB-_10Y
X_1	0.76488	-0.00936	0.82752	0.75367	0.73668	0.88497	0.87357	0.95732	0.88736
Constant	0.07790	0.28460	0.05791	0.09323	0.12746	0.08574	0.12047	0.02897	0.10833
α_0	0.00082	0.00026	0.00000	0.00156	0.00084	0.00000	0.00301	0.00187	0.00419
α_1	0.35012	-0.05168	0.03214	0.04186	0.15688	0.00742	0.45855	0.85327	1.02757
β	0.01721	0.79797	0.95353	-0.04186	-0.07536	0.97597	-0.05060	0.14673	-0.02757
Log-likelihood	140.443	143.734	130.595	130.463	150.046	90.300	83.214	67.938	50.984

The credit spread indices which proved to follow a GARCH (1,1) process show an acceptable log-likelihood ratio. The estimated parameters confirm the volatility trend which is higher for higher ratings and higher for longer maturities. The α_0 and α_1 have an upward trend starting from AAA 5Y towards BB- 10Y as expected.

The fact that not all the credit spread indices showed generalised ARCH effects doesn't prove that there is no heteroskedacticity in the series and vice versa. It only proves that during the period that these indices were examined some of them (shown in the table above) exhibit GARCH effects and others didn't. Clearly, if one wants to fully characterise the heteroskedasticity effects in credit spread indices a larger sample of data needs to be examined in order to reach a proper conclusion. However, I think it is safe to assume that a stochastic volatility model would probably be appropriate to model the evolution of credit spreads.

2.4.4 Mean reversion in credit spread indices

Figure 2.8 shows the spread level of the AA 5Y credit spread index. As we can see the spread level reverts to the level of 30bp after 3 years. Many spread levels examined resembled that one, suggesting just from that pure eyeballing that credit spreads exhibit mean reversion.

Of course, the best way to examine that is by performing ADF tests for each of the time series. The ADF test results (Table 2.10) at the 95% level of confidence show that most of the examined credit spread indices exhibit non-stationarity. Only the bolded results in Table 2.10 seem to be an exception to the previous statement because the ADF test statistic is higher than the ADF critical value at the 95% level (in absolute terms). A possible reason for these exceptions is lack of liquidity during the examined period.

Figure 2.8 Weekly Plot of the 5Y AA Credit Spread index

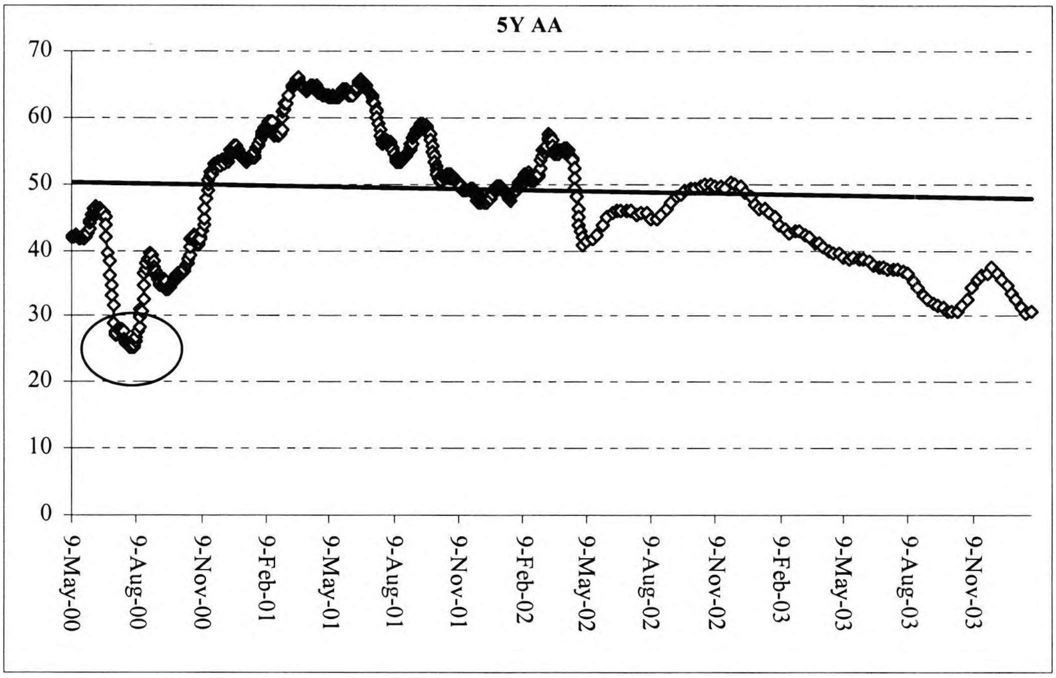


Table 2.10 Augmented Dickey-Fuller Tests (weekly data)

Maturity	Rating	ADF 1	ADF 2
2Y	AAA	**-4.13**	**-5.20**
5Y	AAA	-2.64	-2.93
10Y	AAA	-1.01	-2.49
2Y	AA+	**-3.70**	**-5.34**
5Y	AA+	-2.55	-2.98
10Y	AA+	-1.16	-2.62
2Y	AA	**-3.64**	**-5.35**
5Y	AA	-2.40	-2.88
10Y	AA	-0.66	-2.30
2Y	AA-	**-4.17**	**-4.78**
5Y	AA-	-2.20	-2.73
10Y	AA-	-1.09	-1.67
2Y	A+	-2.31	-2.45
5Y	A+	-1.55	-2.36
10Y	A+	-0.91	-3.12
2Y	A	-2.16	-2.15
5Y	A	-1.15	-1.75
10Y	A	-0.39	-2.84
2Y	A-	-1.35	-2.22
5Y	A-	-0.43	-2.43
10Y	A-	-0.46	-2.13
2Y	BBB+	-2.49	-3.03
5Y	BBB+	-1.72	-3.18
10Y	BBB+	-0.60	-3.00
2Y	BBB	-3.15	**-3.46**
5Y	BBB	-2.20	**-3.57**
10Y	BBB	-1.51	**-3.86**

2Y	BBB-	-1.85	-1.72
5Y	BBB-	-2.27	-2.20
10Y	BBB-	-2.07	-1.98

95% ADF Critical Value	**-2.88**	**-3.43**
ADF 1	Intercept no trend	
ADF 2	Intercept and trend	

This clear evidence of existence of mean reversion in credit spread indices was taken into account in our choice of interest rate model, hence the main purpose of this brief econometric study.

2.4.5 Estimation of LS (1992) May-01 – Jun-01

The first estimation of the LS (1992) model was performed using data from the period between May-01 to Jun-01. In the course of the minimisation, the coefficients were capable of 'naturally' finding acceptable regions. Most of the fitted parameters seemed quite stable over the examined period, especially alpha, beta and eta (Figures 2.9 and 2.10). The gamma and eta parameters show a seasonal pattern, which could well be the case or just forcing the model too far. An interesting observation is that the parameters exhibit high "volatility" in their levels from day to day. Hence, the predictive ability of the model is constrained to give a more accurate view on the market as a whole, rather than being used as a realtime trading tool[26] (day in day out). More evidence to support that is when one tries to price interest rate options by integrating the non-central chi square probability function where the "wrong" combination of parameters could result in complex numbers. For example unless the gamma and eta parameters are not higher than ½ the modified Bessel function would yield a complex number (equation 3.19).

[26] This is one of the main reasons why this model hasn't been used much in practice.

Figure 2.9 Daily plot of the beta, gamma and ni LS parameters

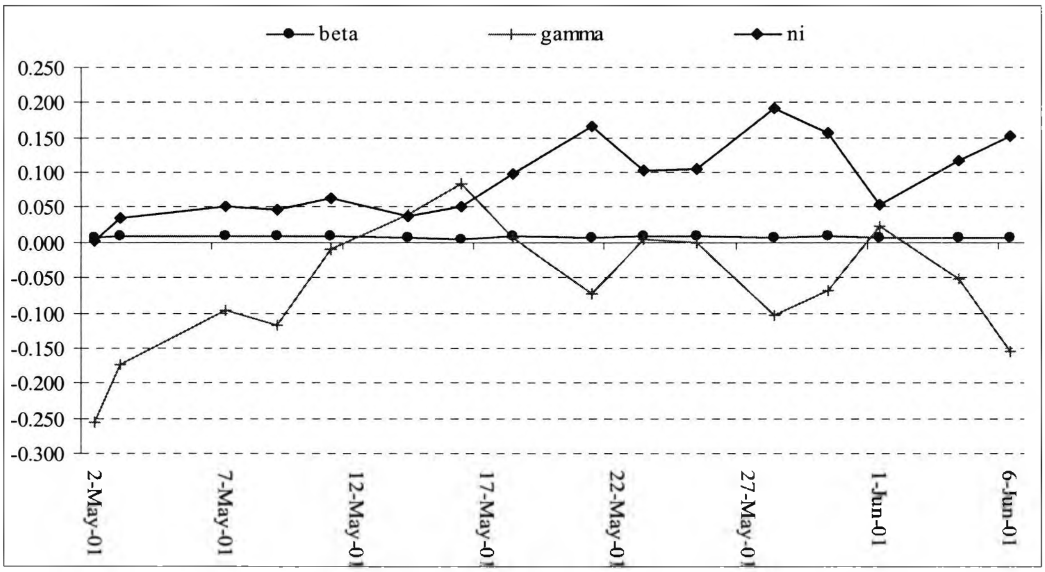


Figure 2.10 Daily plot of the alpha, delta and eta LS parameters

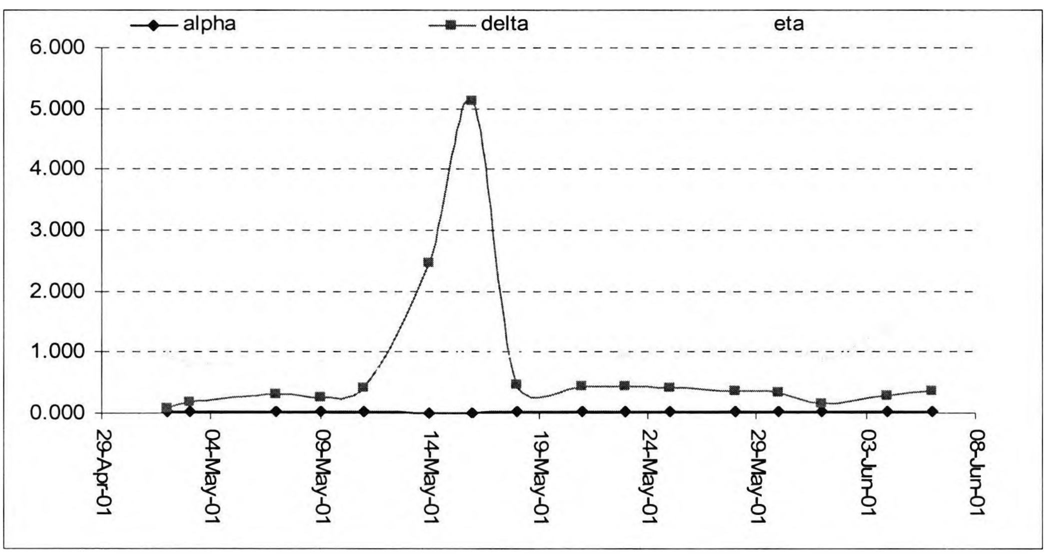
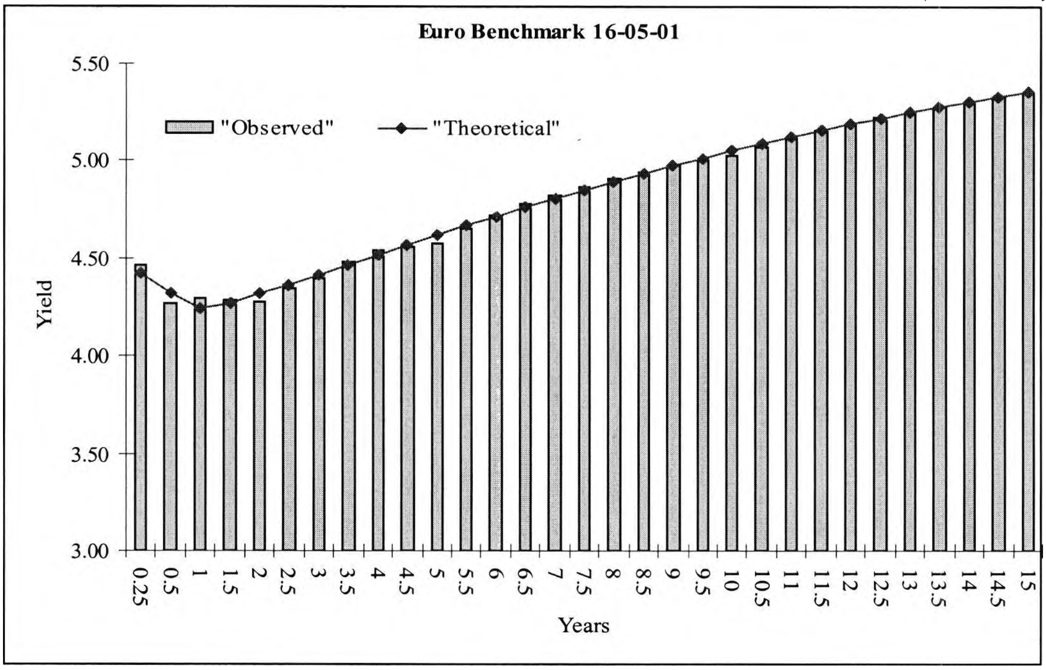


Figure 2.11 shows the observed European benchmark bond curve and the theoretical curve as estimated for the 16th of May 2001. The shape of that yield curve is quite complex since there are two opposed curvatures within the curve, one at the short end and one at the long end. These are signs of a yield curve which could well steepen as soon as there is either monetary easing by the central bank or signs of reduced

economic growth. Our interpretation of the shape of this yield curve (Figure 2.11) is that of an economy at the end of a transitional period.

Figure 2.11 Observed and LS Estimated Euro Benchmark Yield Curve (16/05/01)



The short end of the curve seems to be moving according to the markets expectation of a rate cut by ECB and the long end reflects the current and future expected inflation in the euro area. Also, the long end of the curve is pricing expected growth of the economy in the long run, hence the opposing curvatures.

The term structure of volatility was also obtained using the estimated parameters (Figure 2.12). The shape of the volatility curve has the humped structure where the short maturity volatilities are higher than the long term volatilities. The level of volatility is quite low for the T-bill/bond market. The reason for this is that these volatilities represent the volatility of bond yields, which in reality are quite low, in contrast to the volatility of their prices.

The LS (1992) model makes the assumption of correlation of 1 between the short rate r and its' volatility V and this is evident in Figure 2.13 where the correlation between r and V is plotted against time. The correlation was calculated based on the estimated parameters of the model. This is also an implementation test for the LS (1992) model since the dynamics of r and V have been accurately captured in the LS world.

Figure 2.12 Term Structure of Volatility (16/05/01)

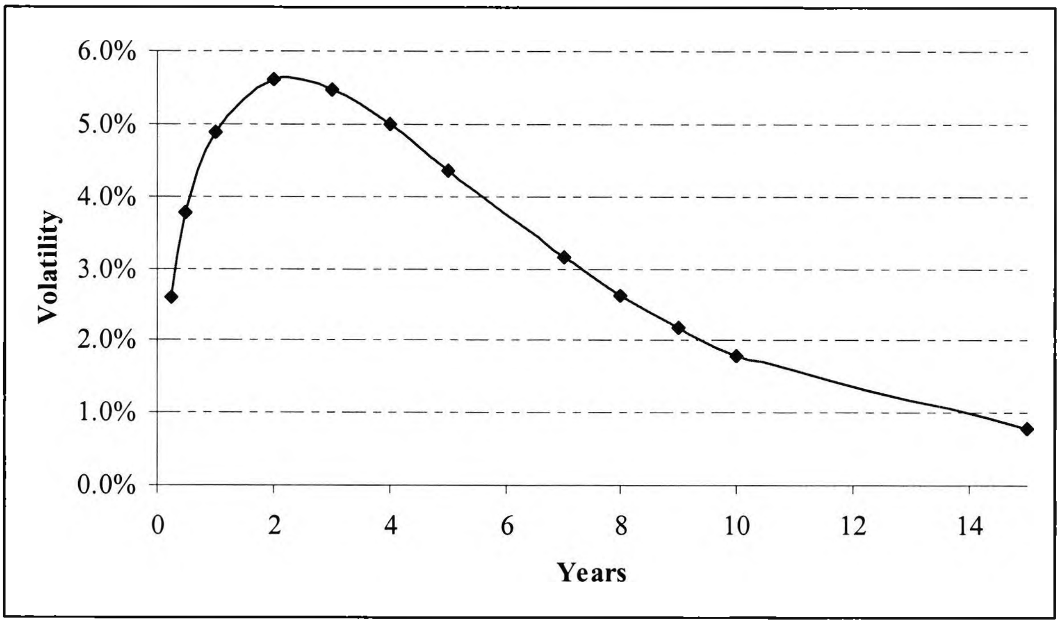
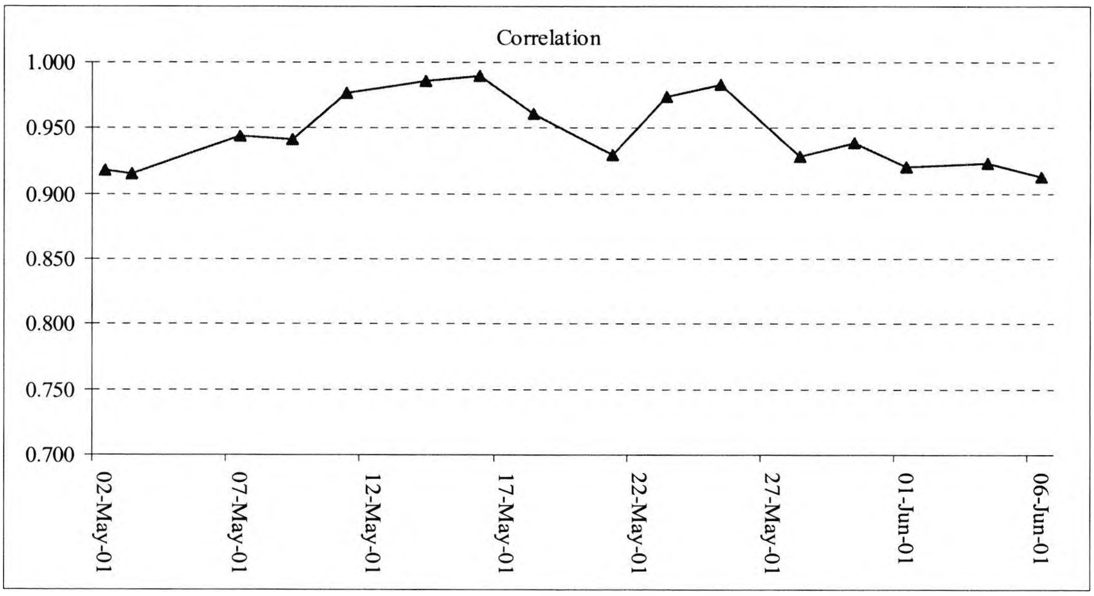


Figure 2.13 Correlation between r and V



2.4.5.1 Option Pricing

The LS (1992) has a closed form solution for the pricing of vanilla zero coupon bond options (the equivalent of caplets/floorlets). Using a Monte Carlo simulation (section 3.2.3) we priced a number of options on a 5Y zero coupon bond. Before we actually describe its pricing power we will examine the implied distribution of the r conditional on its volatility V. The distributional properties of both short and long-term interest rates at 7 days intervals were implied using a Monte Carlo simulation. In that way, one can take views on the movement of future interest rates based on that information. The density obtained using this approach is also comparable to the more common option-implied density, which is also risk-neutral.

The best way to show the results, in relation to the scope of this thesis, is to plot the PDFs[27] for all horizons up to half a year in a three-dimensional as shown below. The number of weeks ahead in time is displayed on the x-axis, the level of the short-term interest rate is on the y-axis, and the probability density is on the z-axis. It can be clearly seen that the distribution converges as the horizon increases.

The x-axis shows the interest rate and the y-axis the probability that the x-month interest rate will be as it is viewed by the market on the day of the calibration. As it can be seen more clearly in Figure 2.14, the 6-month probability distribution is broader and the probability that rates could be almost unchanged is much lower compared to the 3-month probability distribution. The same shape remains as the number of months increasing, demonstrating the stability of the distribution. Using (3.24) and the implied distribution of the short rate we priced European call options of various maturities on a 5Y zero coupon bond. The prices obtained were compared with option prices calculated using the Black's formula. In Table 2.11 the implied forward rates used for the ATM strikes are listed including the respective volatilities and LS option prices. Also the Black's option prices and their difference is shown. In table 2.11, 7 different options prices are shown as estimated using the LS model.

[27] Probability Density Function

Figure 2.14 Estimated RND of future short-term interest rate (16/05/01)

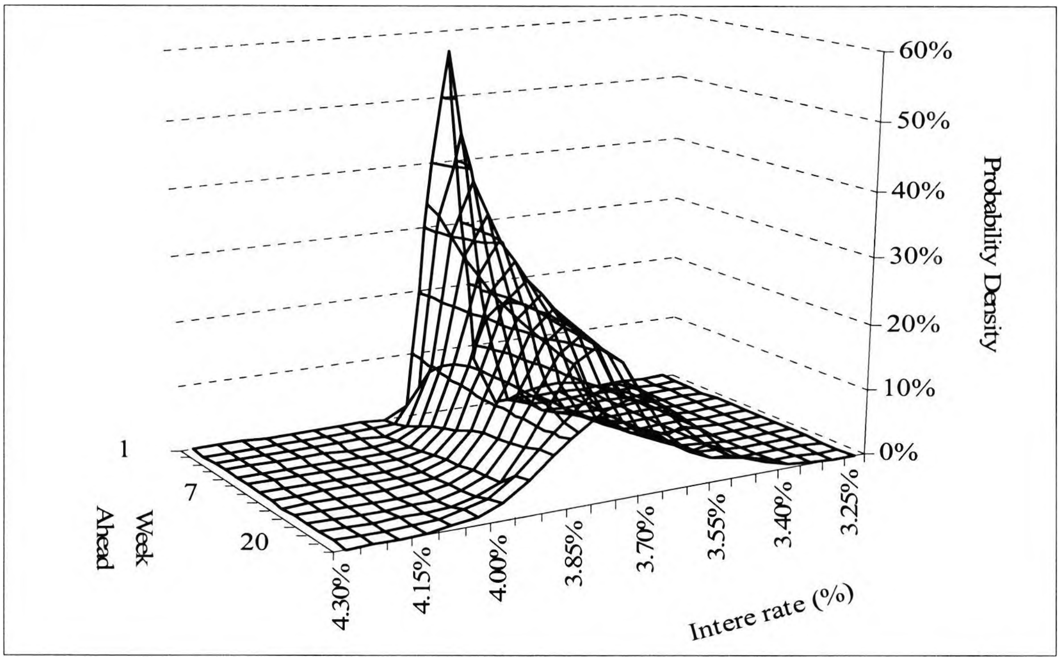


Figure 2.15 3M and 6M RND of future short-term interest rate (16/05/01)

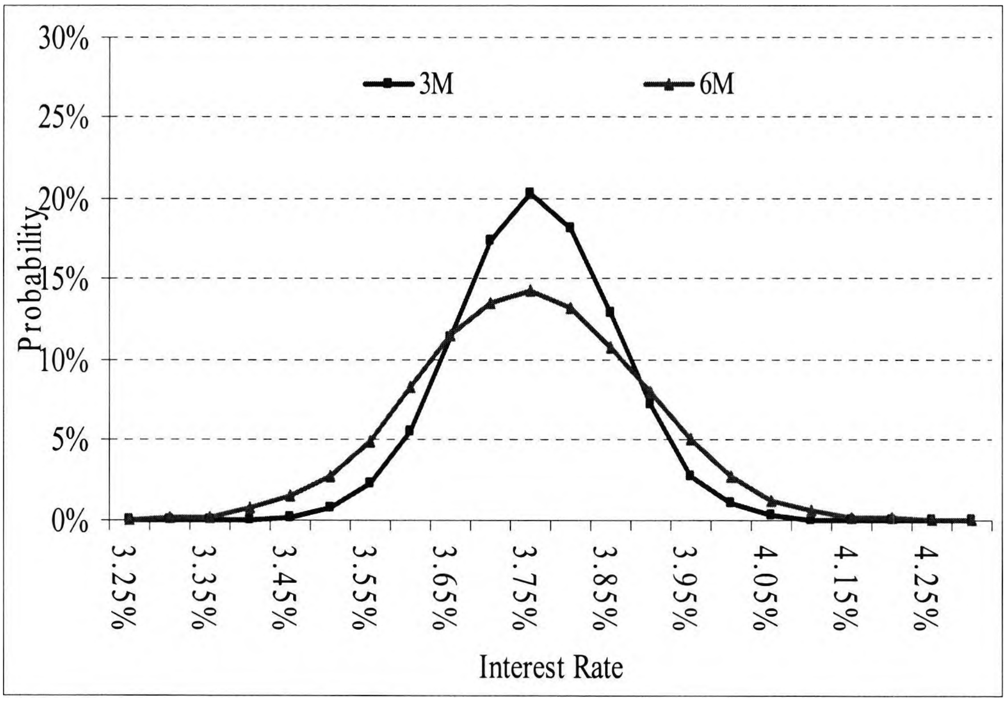


Table 2.11 Comparison of Option Prices (16/05/01)

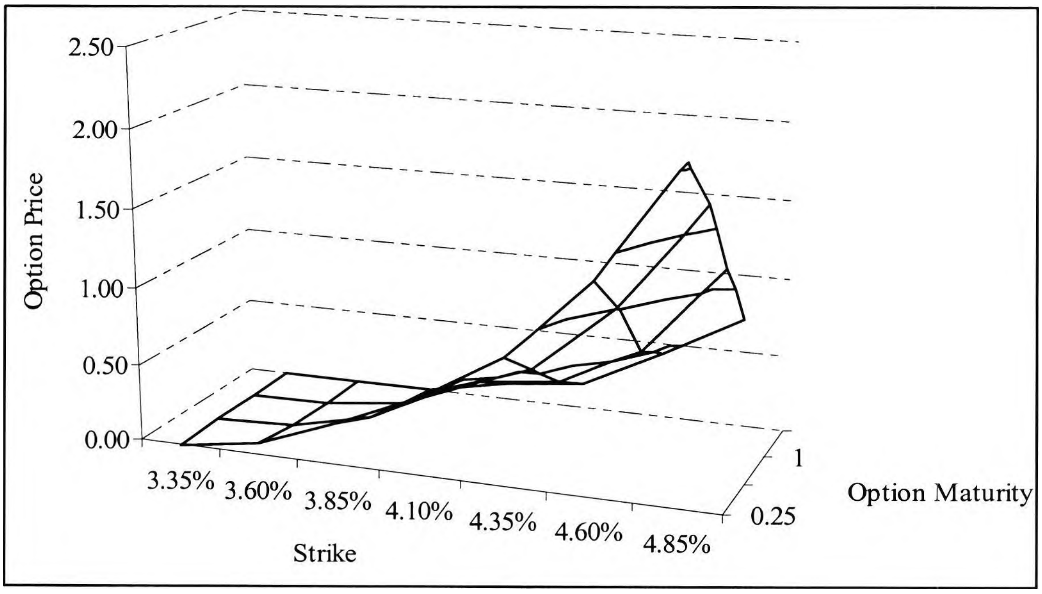
Option Maturity, Bond Maturity	5Y Implied Forward Rates	LS ATM Price	Black's price	Difference
0.25Y,5Y	4.10%	0.4970	0.496	0.001
0.5Y,5Y	4.14%	0.2978	0.296	0.002
1Y,5Y	4.24%	0.1856	0.174	0.011
2Y,5Y	4.44%	0.0530	0.090	-0.037

Table 2.12 Option Prices on the 5Y zero coupon bond (16/05/01)

Option Maturity / Strike	3.35%	3.60%	3.85%	4.10%	4.35%	4.60%	4.85%
0.25	0.0020	0.0800	0.2744	0.4970	0.8080	1.3180	2.0540
0.5	0.0015	0.0242	0.1446	0.3921	0.5771	1.0190	1.6860
1	0.0011	0.0048	0.0720	0.2832	0.3316	0.5910	1.1770
2	0.0001	0.0006	0.0014	0.1424	0.1045	0.1630	0.7100

There are minor differences (Table 2.11), which indicates that calibration to market discount factors could be reliable to price these options. Options of longer maturity seem to have larger differences to the Black's prices. This could be due to the nature of the implied distribution obtained. The implied distribution of the short rate becomes wider and wider as the maturity of the option is increased. As a result the at the money (ATM) options tend to become worthless as maturity of the option increases. This makes it impossible to price the full matrix of all possible combinations of options. This is another reason why practitioners avoid the use of LS to calculate the MTM of their books or even to market make in interest rate options. However, it has great theoretical attributes and it can still be used for most vanilla and non-standard swaps plus short dated options. Using a model such this to price short dated options instead of Blacks' has the advantage of taking into account stochastic volatility and the theoretical term structure of interest rates.

Figure 2.16 Option Prices surface based on table 3.7 (16/05/01)



2.4.6 Estimating Credit Spread Curves and pricing credit spread options using LS (1992).

The date of our analysis is the 7th of May 2004. After a long time of low credit spreads and continuous spread tightening we see the first signs of a reversal in the credit markets. The first 5 months of 2004 have been quite interesting since the increased liquidity and the global reflation theme has taken its toll. Investment in 2004 its not a "sure thing" anymore across all asset classes and especially in credit where careful selection of investments in credit superseded the theme of going long in all types of corporate bonds. Thus, the period we examined the credit spread curves shows the first signs of spread widening.

Table 2.13 LS (1992) Estimation Parameters

	AAA Finance	**AA+ Industrials**	**AA- Industrials**	**BBB Industrials**	**BB Industrials**	**B Industrials**
Short Spread	0.11%	0.12%	0.14%	0.46%	0.72%	3.44%
V	27.72%	12.87%	12.21%	8.33%	19.55%	5.53%
α	0.114	0.085	0.123	0.089	0.042	0.097
β	18.515	11.973	14.995	21.995	22.000	15.759
γ	0.070	0.184	0.100	0.167	0.052	0.112
δ	-0.179	0.109	-0.086	-0.001	-0.307	-0.432
ε	0.001	-0.001	-0.001	-0.002	0.000	-0.001
η	3.459	1.691	2.346	0.768	0.741	-21.009

The estimation of the credit spread curves was performed in the same way as if we were estimating interest rate curves. The initial spread level and their volatilities are shown in Table 3.7, including the estimated parameters.

The alpha and beta parameters are substantially higher than the same parameters estimated in section 3.4.1. The main reason for that is the level of the volatilities which are a few orders of magnitude higher than the volatilities of interest rates. The rest of the parameters reflect the characteristics of the model where the spread reverts to its long term mean and so its volatility.

Table 2.14 Squared Differences for EUR AA+ spread curve (07/05/04)

Maturity /years	Observed spread discount Factors	Estimated spread discount Factors	Squared Difference	Estimated credit spreads in %
0.25	0.00051	0.00124	0.00000	0.510%
0.5	0.00093	0.00163	0.00000	0.364%
1	0.00105	0.00170	0.00000	0.211%
2	0.00306	0.00272	0.00000	0.152%
3	0.00514	0.00429	0.00000	0.164%
4	0.00632	0.00708	0.00000	0.189%
5	0.01009	0.00983	0.00000	0.214%
7	0.01352	0.01282	0.00000	0.254%
8	0.01449	0.01436	0.00000	0.268%
9	0.01362	0.01690	0.00001	0.281%
10	0.02210	0.01976	0.00001	0.291%

Table 2.14 shows the estimated spread discount factors and credit spreads based on the estimated parameters. The optimisation has worked quite well since the squared differences are quite small. In Figure 2.17 we observe the difference between the observed B Industrial credit spread curve and the LS (1992) estimated credit spread curve.

The fit is surprisingly good even for a complex curve like that, enforcing our choice of model. Liquidity effects on the examined corporate bonds could be responsible for that shape or simply a reflection of expectations.

Figure 2.17 EUR B Industrial Credit Spread Curve (07/05/04)

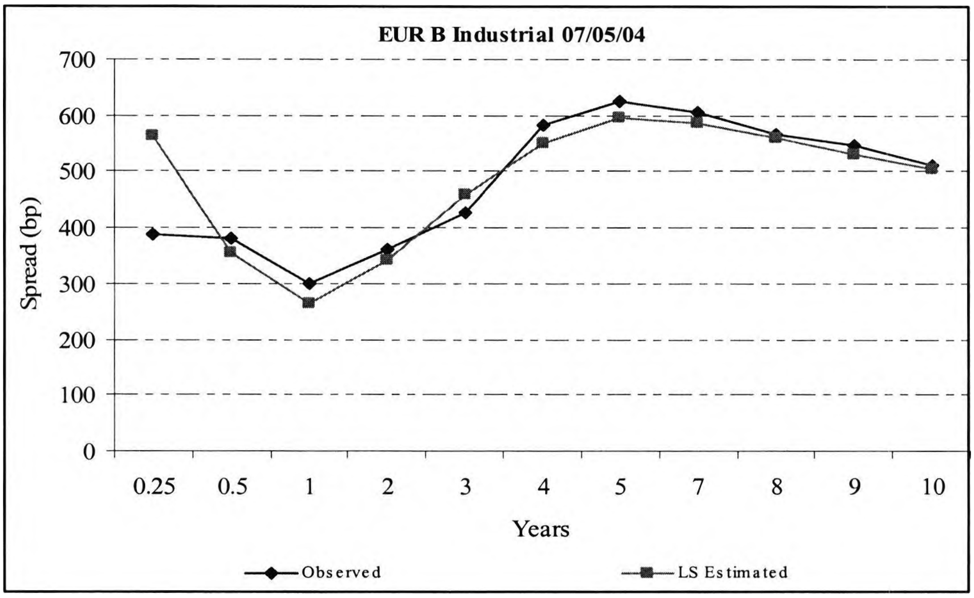


Figure 2.18 Zero Discount Factors (07/05/04)

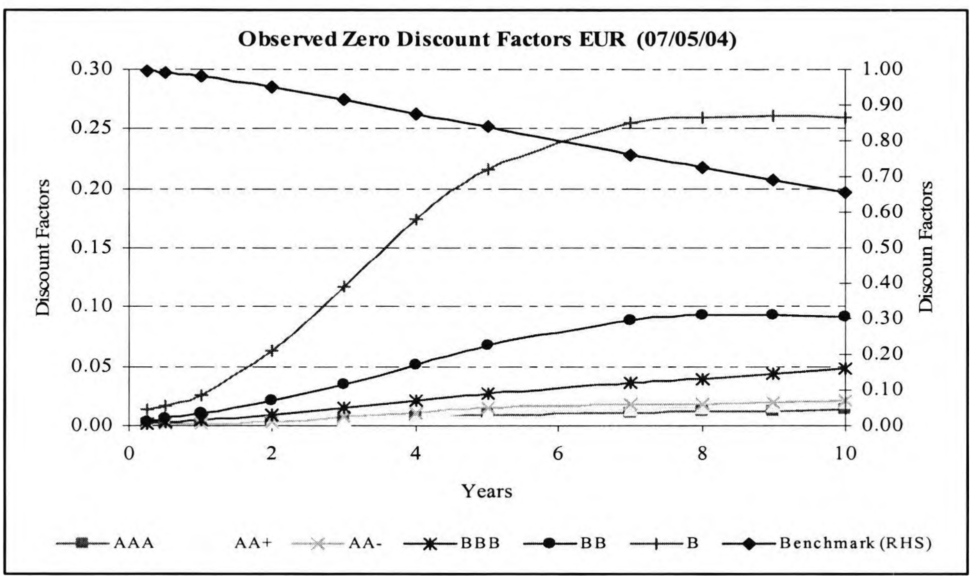


Figure 2.18 shows the estimated discount factors per credit spread curve including the estimated benchmark discount factor curve. The natural observation is that the spread zero discount factors are upward sloping curves instead of downward sloping in comparison to the bond discount factor curves. The absence of monotonically decreasing discount functions in the credit spread curves is striking.

The shape of the credit curves is quite complex. Possible reasons are liquidity, which is always a major factor in the cash market of fixed income securities and potential arbitrage opportunities due to market mis-pricing. All estimated curves however, are mainly upward sloping reflecting the term structures of corporate and government bonds.

The estimated credit spreads are also very close to the observed spreads. The only serious mis-pricing observed based on our results is on the AAA curve. During the 2 – 7 year period the spread is higher than AA-. This is something that does not occur in the observed credit curves since the level of risk of holding a AAA asset compared to a AA- asset is always lower. A possible explanation is that the volatility of the AA+ and AA- curves is quite low relative to the AAA volatility for the period examined and vice versa. Another reason could be that the 2 to 7 year parts of the AA+ and AA- curves has not moved for some time, i.e. lack of liquidity or even lack of activity. Clearly the same estimation should be carried over a period of time in order to establish what might have been the cause, or if it was just an outlier.

The fact that the part of the AAA curve yields higher than the AA+ and AA- shows that although the LS model can fit complicated (Figure 2.19) curve shapes and takes into account the volatility of credit spreads but there is no guarantee that first: the credit spread discount curves are strictly positive, second: the premium of holding a higher rated security is lower than the premium of holding a lower rated security.

The main reason for that is that the LS model is only concerned with fitting an observed credit spread term and its volatilities. However, the high quality of fitting spread curves can be used to our advantage since as we will see later, there is quite a lot of information that can be extracted out of the estimated credit spread curves.

Figure 2.19 Estimated Credit Spread Curves (07/05/04)

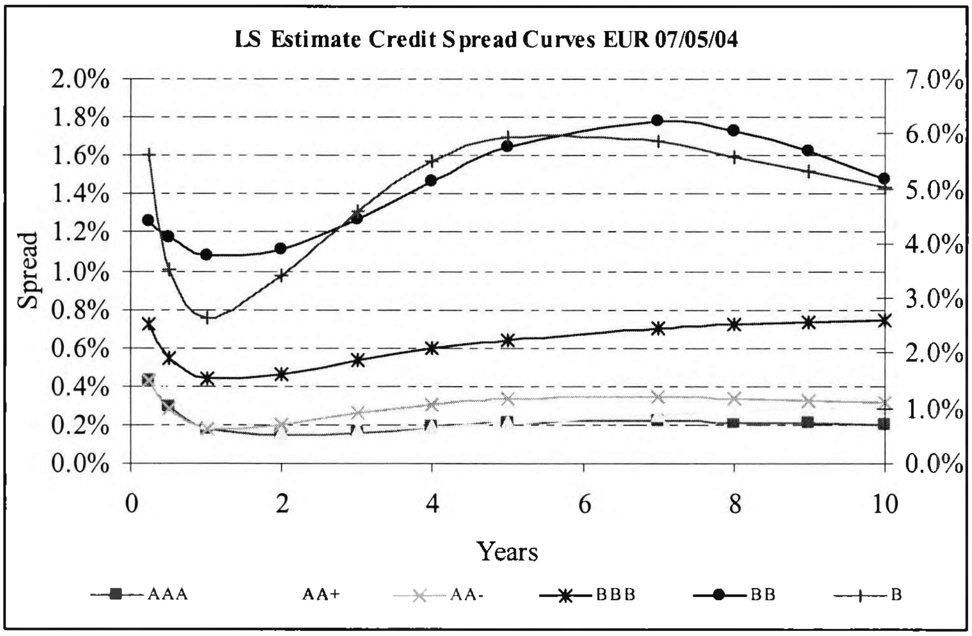
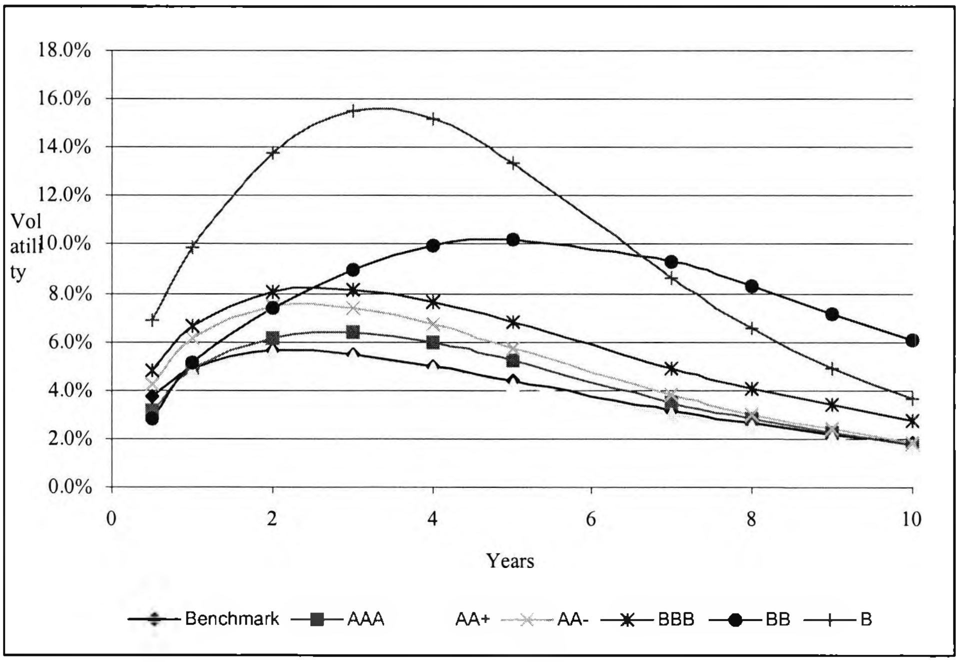


Figure 2.20 Credit Spread Volatility Curves (07/05/04)



Furthermore, the estimated credit spread curves can give us good insight in where the "equilibrium" of the short credit spread might be and also where the level of the forward credit spreads are.

2.4.6.1 Credit Spread Option Pricing

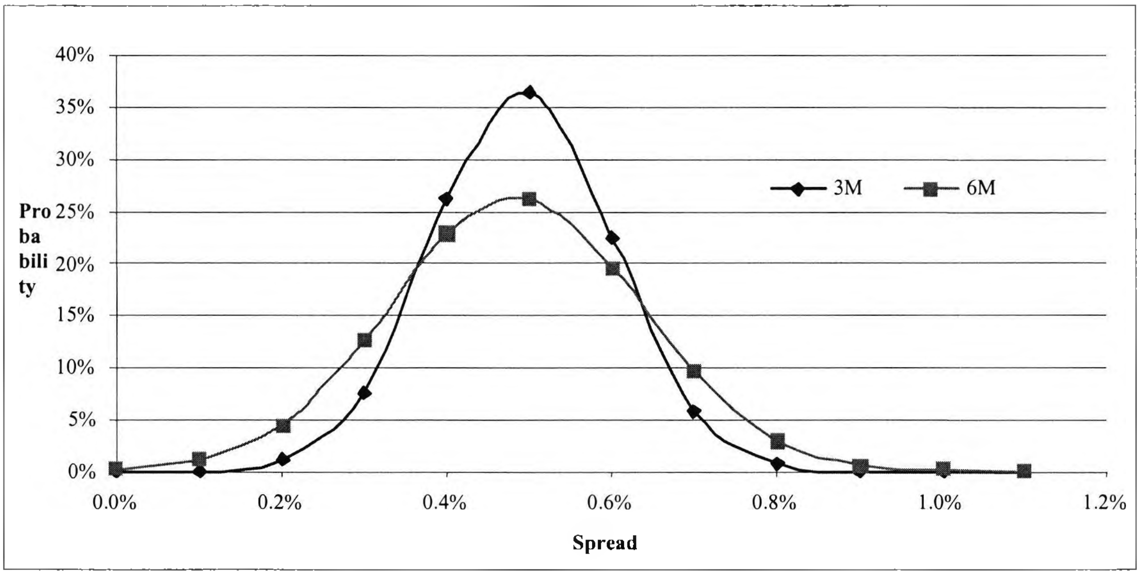
The estimated credit spread curves were used to provide the forward structure of the credit spread in order to price credit spread options by assuming the spread is a stochastic variable which follows the diffusion equations of 3.26. The implied probability density for different time intervals behaves in the same way as with the risk free rates (Figure 2.21). This is a useful property of the model because it reflects the uncertainty on the spread level as time increases. One of the advantages of using the LS (1992) to model the dynamics of credit spreads is that it combines the level of the credit spread with stochastic volatility. This is quite important in option pricing applications because the dynamics of the spread's volatility can be successfully matched. The existence of a closed form solution for European options on bonds under the LS (1992) framework is also advantageous, since most of the credit risk models used for pricing don't provide closed form solutions for spread instruments.

Using the pricing methodology of section 3.2.6 we priced options on different credit spreads. The "calibration" of the credit spread options was performed via replication. The ATM spread options were replicated by buying 1 unit of an ATM corporate bond of rating (i) and buying ½ a unit of a government bond[28], thus replicating the greeks and the payoff of the credit spread options.

The two bonds were chosen to be of similar duration (Table 2.15) and the options were struck at the ATM implied forward as estimated from the observed discount curve of the 07/05/04. The respective deltas were 0.5009 for the calls and -0.4999 for the puts.

[28] For example the combination of the VMG bond and the OBL government bond options will yield a Total Delta = (1x Duration(i) x Delta of option) + (0.5 x Duration x Delta of Option), i.e. Total Delta of strategy = 1 x 2.574 x 0.5009 + 0.5 x 2.276 x (-0.499) = 0.50254, which is matched but the ATM spread option. For different option moneyness the appropriate weights should be used to match the delta.

Figure 2.21 RND of future short credit spread AA+ Industrials (07/05/04)



The underlying maturity of the bonds were approximately 3Y and 2Y. The option maturities examined were 3M. The credit spread options priced were 3M options on 3Y and 2Y underlying respectively. The spread options were struck at the same spread strike as the replicating strategy and the option implied volatility of the strategy was matched.

Table 2.15 Underlying Government and Corporate Bonds

Strike Call	Strike Put	Government Bond	Coupon	Corporate Bond	Coupon	Duration Government	Duration Corporate
2.61	2.52	OBL	4	VMG	4	2.574	2.276
2.77	2.57	BKO	2	Total	3.875	1.728	1.864
2.96	2.57	BKO	2	Bosch	5.25	1.728	1.964
3.27	2.57	BKO	2	Renault	5.125	1.728	1.972

The total cost of the strategy (Table 2.16) was expected to be higher than respective option prices out of the LS (1992) model. The reason is that the fractional difference in the time value between the combination of the two options compared to the single spread option would increase the overall premium by almost the same amount. This expected price difference between the LS spread options and the strategy is shown in table 2.16. The magnitude of the difference is quite small considering we used two very different models to price spread options. The advantage of the LS (1992) over Black's model is mainly that the LS model accounts for the true nature of volatility and its stochasticity.

Table 2.16 Credit Spread Option Prices

Rating	Option Maturity/ Spread Maturity	Price Difference	LS Price (decimals)	Cost of Strategy (decimals)	Government Bond Put Option	Corporate Bond Call Option	Spread Strike (bp)
AA	3M_3Y	0.008	0.553	0.545	0.499	0.295	9
AA-	3M_2Y	-0.006	0.367	0.374	0.315	0.216	20
BBB	3M_2Y	-0.003	0.403	0.406	0.315	0.248	39
BBB	3M_2Y	-0.013	0.541	0.555	0.315	0.397	70

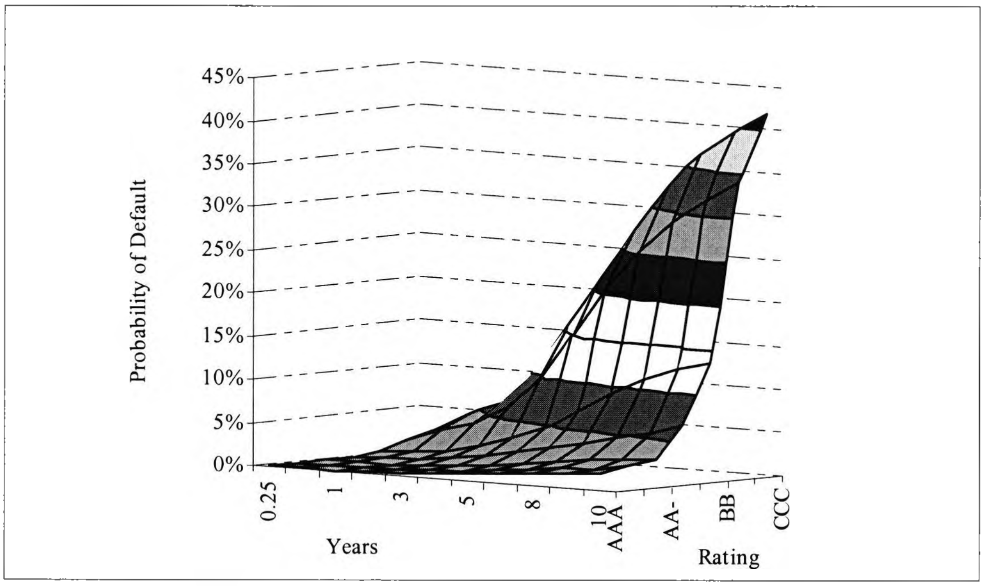
Longstaff and Schwartz (1995) arrive to an interesting result about credit spread options. Based on their proposed model which assumes that credit spreads are conditionally log-normally distributed they conclude that the value of call credit spread options can be less than the their intrinsic value. Based on our results this not true when using the LS (1992) to model the credit spread as a stochastic process. The value of the credit spread options is higher than its intrinsic value. The reason is that the pricing of the credit spread options is no different than the pricing of interest rate option when using the LS (1992) model.

2.4.7 Implied Probability of Default

By estimating zero-coupon spreads and given a historical recovery rate we implied default probabilities in the same way as one would use risky zero-coupon bonds and default free zero-coupon bonds. Using the estimated spread curves (section 3.4.2.1), the relevant Bloomberg data[29] and the weighted recovery rate (0.3265) as used in the JLT paper[30], we were able to imply the cumulative probabilities of default. These probabilities are based on the two state model of Jarrow and Turnbull [46].

The trend of the cumulative default probabilities follows the risk premia as obtained from the estimated spread discount factors. The probability of default is higher for lower ratings and also increases with time. If we take a closer look at Figures 2.22 and 2.23 as an example we will realise that the probabilities are not straightforward exponential curves as one would expect, instead there is a higher level of convexity. A potential reason for that is that the inclusion of the volatility of the spread when the curves were estimated (section 3.4.2).

Figure 2.22 Cumulative Default Probabilities



Hence, the shape of the implied probability of default, which shows that the term of default is directly related to the term structure of the credit spread curves.

The evolution of these default probabilities over time should resemble the evolution of credit spreads.

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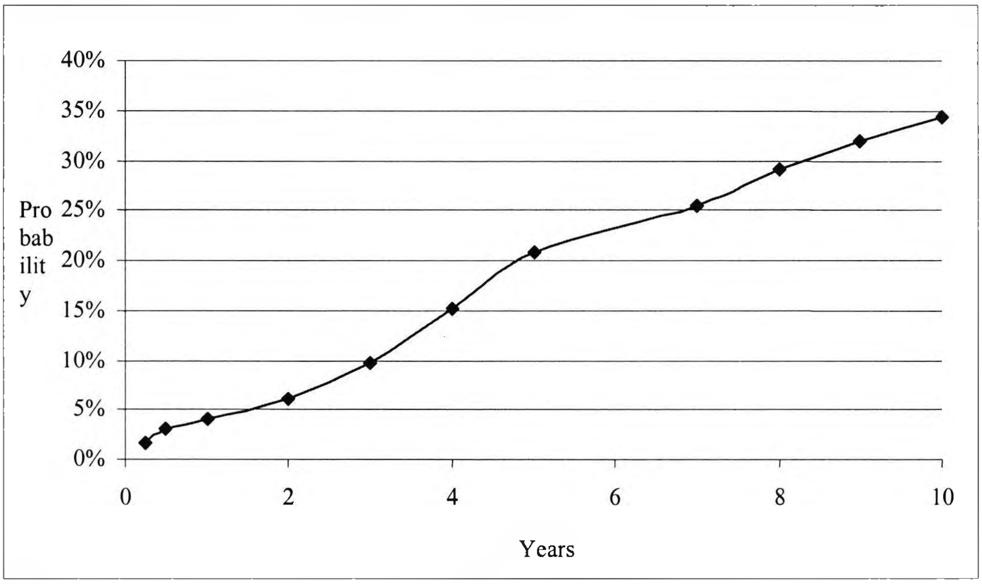
[29] See appendix 2 since the default probability curves were obtained for the 7th of May 2004.

[30] The level of the weighted recovery rate in the Jarrow, Lando & Turnbull [46] paper.

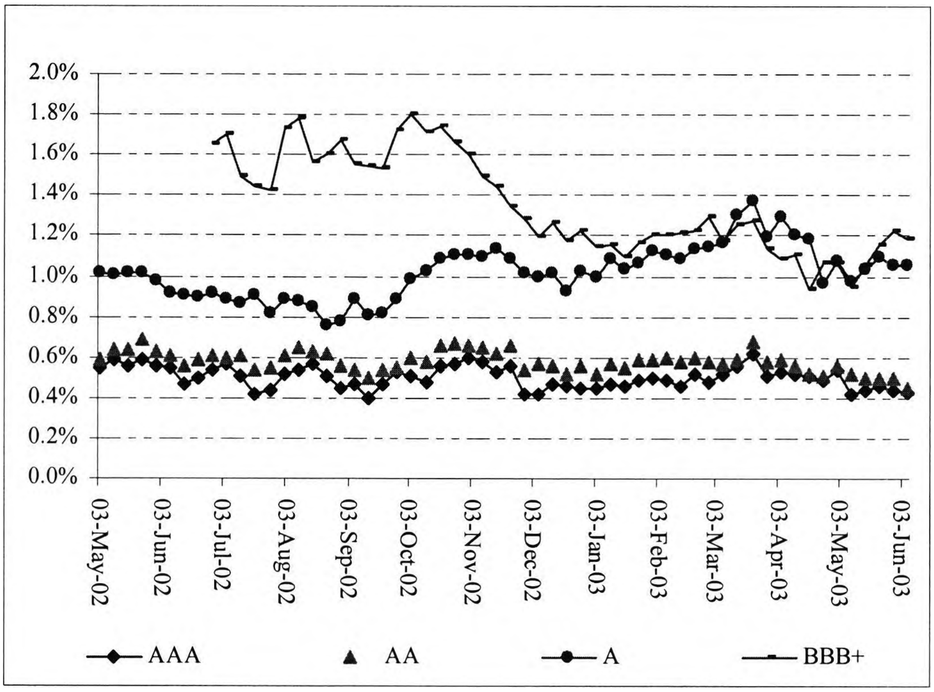
Figure 2.23 Implied Cumulative Default Probability Curve of Rating B



This is easily deduced since our inputs were the credit spreads themselves. In figure 2.24 weekly time series of the implied default probabilities are being plotted.

The series show the existence of relative low default risk over the last 2 years. This is in accordance with the environment of low interest rates (US and Euro area), which allows corporations to borrow money at historical low interest rates, thus making their debt servicing cost very low. An interesting observation is that for a brief period of time the probability of default of rating BBB+ as implied from the estimated credit spreads was lower than the probability of default of rating A which is a lower rating. This is clearly violation of no-arbitrage if one was using a reduced-form model. However, in our case this could be a mis-pricing which it didn't last long. Alternatively it could be a case of the market pricing in a rating upgrade event for the sample of bonds in question. This could make sense since a number of corporations (again fuelled by a low interest rate environment) were placed on a positive outlook by major rating agencies.

Figure 2.24 Time series of 2Y implied default probabilities



2.4.7.1 Implying the Transition Rating Matrix using the LS (1992) estimated credit spread curves

In tables 2.16 the 1-year historical transition matrix as obtained from JLT [] is shown. Following the JLT (1997) paper and Arvanitis, Gregory and Laurent (1999) we implied the transition rating matrix using the spread risk premia as estimated using the LS (1992) model (section 3.4.2).

Table 2.16 (see JLT (1997))

	AAA	AA	A	BBB	BB	B	CCC	D
AAA	-0.1154	0.1020	0.0083	0.0020	0.0032	0.0000	0.0000	**0.0000**
AA	0.0091	-0.1043	0.0787	0.0104	0.0031	0.0031	0.0000	**0.0000**
A	0.0009	0.0308	-0.1172	0.0688	0.0107	0.0048	0.0000	**0.0010**
BBB	0.0006	0.0046	0.0714	-0.1711	0.0701	0.0174	0.0020	**0.0049**
BB	0.0004	0.0023	0.0086	0.0814	-0.2531	0.1181	0.0144	**0.0273**
B	0.0000	0.0020	0.0034	0.0075	0.0568	-0.1929	0.0478	**0.0753**
CCC	0.0000	0.0000	0.0126	0.0131	0.0223	0.0928	-0.4319	**0.2856**
D	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	**0.0000**

There are striking differences between the two generator matrices especially in the last column, which is also the implied cumulative probability of default. The historical matrix seems to weigh much higher probabilities of default in comparison to the implied generator matrix. This is easily explained by the fact that the level of credit spreads during the examined date (07/05/04) is very low. The implications is that default risk as being viewed by the market (risk premia) is quite low but seems that it could be on the increase, since the probabilities of being at the same rating after 1 year are lower than the historical transition probabilities. Table 2.18 shows the input spreads as estimated using the LS (1992) model in comparison to the spreads implied using the risk neutral generator matrix (Table 2.17).

The differences are quite small since an optimisation was carried out, but an interesting point is that the 1Y spread as implied from the rating matrix is 1 where the LS estimated is six and ten times higher.

Table 2.17 Implied Transition Rating Matrix (07/05/04)

	AAA	**AA**	**A**	**BBB**	**BB**	**B**	**CCC**	**D**
AAA	-0.1302	0.1148	0.0092	0.0022	0.0035	0.0001	0.0001	**0.0001**
AA	0.0158	-0.1815	0.1370	0.0180	0.0052	0.0053	0.0001	**0.0002**
A	0.0017	0.0584	-0.2214	0.1298	0.0203	0.0090	0.0002	**0.0018**
BBB	0.0007	0.0057	0.0954	-0.2292	0.0938	0.0233	0.0027	**0.0065**
BB	-0.0001	0.0013	0.0055	0.0495	-0.1540	0.0718	0.0087	**0.0165**
B	-0.0001	0.0018	0.0025	0.0056	0.0425	-0.1445	0.0356	**0.0563**
CCC	-0.0001	0.0003	0.0043	0.0041	0.0066	0.0275	-0.1278	**0.0845**
D	0.0000	-0.0001	-0.0001	0.0000	0.0001	0.0001	0.0000	**0.0000**

However, this is very close to as if we were using the historical generator matrix. The spreads implied using the historical generator matrix are very much different as it can be seen by looking at the default probabilities (last column in table 2.16). Using the historical generator matrix the implied spreads are very much different from the risk neutral spreads.

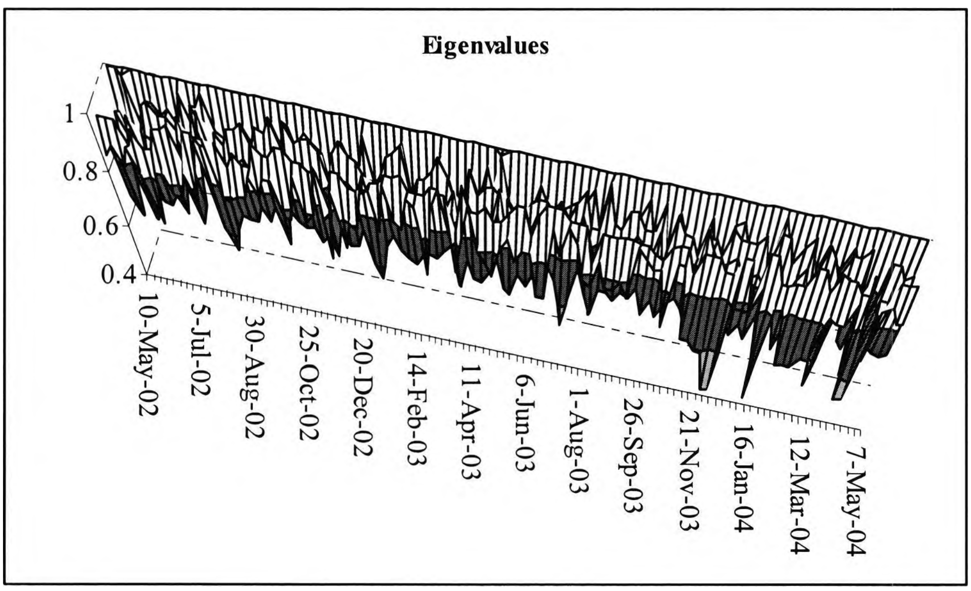
This shows again that the current risk premia are much different than the historical average, hence the big difference between the spreads.

Table 2.18 Credit Spreads used for calibration to imply the Transition Rating Matrix

Rating	Spread as estimated from historical matrix in bp	Spread as estimated from risk-neutral matrix in bp	LS Spread in bp	Difference in bp
AAA	21	1	7	6
AA	29	1	11	10
A	46	12	12	0
BBB	50	44	41	-3
BB	89	112	107	-5
B	472	386	375	-11
CCC	1698	586	578	-8

This difference between the spreads needs to be thought carefully when one is measuring risk and even when one is marking to market. As we've seen historical default data show that the "fair value" of credit spreads is much higher than where they are at the moment. However, one cannot measure risk based on that "fair value" because market rates fluctuate according to market expectations.

Figure 2.25 Weekly time series of eigenvalues



What is mainly important when measuring risk is the time horizon, the volatility of the underlying rates over the specified horizon and the details of the risky position (maturity, size, direction) the risk is calculated for. Thus we estimated transition matrices over a period of time based on weekly credit spreads as obtained form the LS model. The main reason for that is to identify how the risk neutral transition probability matrix shapes up over time. The best way to visualise that is by examining the eigenvalues over a period of time as shown in Figure 2.25.

2.5 Summary

The distributional and statistical properties of the EUR credit spread indices examined have given us a fairly accurate picture of their dynamics. They exhibit mean-reversion and GARCH effects and they are highly skewed. These observations coupled with the empirical literature justifies choosing an interest rate model to model the dynamics of credit spread indices, especially a model, which can capture both the evolution of the short credit spread index and its volatility. Such a model is the affine two-factor term structure model of Longstaff and Schwartz (1992) which was used to fit credit spread data directly.

The two-factor interest rate model by LS (1992) was examined in order to test its ability to fit yield curves and price interest rate options. It was shown that, as a theoretical model it is capable of providing a fairly accurate pricing framework, where complex yield curve shapes can be estimated to reflect observed market rates and the term of the volatility can also be fitted simultaneously, making the model quite appealing for option pricing applications. However, although there is a closed form solution for pricing options the complexity of the final distribution function make it difficult to reflect the changes in all the estimated parameters either daily or weekly. Features like these make the LS model a great theoretical rather than trading tool to the fixed income/interest rate markets.

The ability of the LS model to fit complicated curve shapes and to model volatility stochastically was explored by fitting credit spreads directly. It was also shown that credit spread options can be priced under the LS framework implying that it can be used as a “spread based” model. Furthermore the credit spread curves were used to imply default probabilities showing an alternative way of estimating probabilities based on spread data rather than bond data.

CHAPTER 3 Integrated Risk Measurement

Abstract

This study presents a dynamic approach to manage the risks associated with interest rate swaps. The proposed methodology reflects the implicit price of default risk implied by a hypothetical swap portfolio. The proposed dynamic hedging of the swap default risk by taking offsetting positions in credit spread index options provides an integrated measure of market and default risk for interest rate swap portfolios. This measure will facilitate in the proactive management of the risks associated with interest rate swaps. The methodology can be easily extended to cover other financial instruments, which bear both marker and credit risk.

3.1 Introduction

3.1.1 Background of the study

A number of well publicised losses (Table 4.1) due to derivatives have made end users and financial practitioners alike to rethink the way they account for their risks. Hence, the credit exposure generated from derivative transactions has become the focus of many academics, practitioners and regulators around the world.

Table 3.1 Well publicised derivatives losses

Company	Loss Amount	Related Derivatives
Orange County	US$ 1.7 billion	Leverage and structures notes
Metallgesellschaft	US$ 1.3 billion	Oil Futures
Barrings	+ US$ 1 billion	Equity and interest rate futures
Procter & Gamble	US$ 157 milion	Leveraged Currency Swaps
Air Products & Chemicals	US$ 113 milion	Leveraged interest rate and currency swaps
Italian Postal Bank	~ US$ 105 milion	Leveraged exotic interest rate swaps
Dell Computer	US$ 35 milion	Leveraged interest rate swaps
Arco Employees Savings	US$ 22 milion	Money Market derivatives
Gibson Greetings	US$ 20 milion	Leveraged interest rate swaps
Mead	US$ 12 milion	Leveraged interest rate swaps

In the past financial institutions and regulators were happy to treat the different types of risks separately. However, a number of reasons has altered this view lately. The globalisation and integration of financial markets has led financial institutions to consolidate their risk under global integrated measures. The tremendous two-fold growth in complexity and turnover of financial products has broken down many boundaries between the different types of risk. A financial institution has the ability to eliminate banking risk by swapping credit risk for market risk or even eliminate both through tailored made derivatives. A corporation can eliminate market risk from its balance sheet by entering into derivatives contracts with financial institutions. It is not only large corporations which enter into derivatives contracts with financial institutions but all customers with various credit qualities. Prudent risk management and understanding of the risks associated with such transactions is of great importance to any financial institution who is in the derivatives business.

These changes have transformed financial institutions into the "risk managers" of the financial system as a whole. Hence, their great need to assess, price, hedge, monitor, diversify and report all risks accurately across all their business units on an integrated basis.

3.1.2 The Problem Statement

There has been a lot of effort in creating an integrated measure of risk, especially for market and credit risk. Jobst and Zenios (2002), Medova and Smith (2003), and Mark (1999) propose an integrated framework which accounts for both market and credit risk factors in portfolios of derivatives. Both approaches are quite similar in the sense that both use a Monte Carlo simulation to generate multiple scenarios of credit states and market rates. The difference is that Jobst and Zenios (2002) use an arbitrage-free model for credit risk pricing (Jarrow, Lando and Turnbull (1997)), whereas Medova and Smith (2003) use option pricing theory to price credit risk (Merton (1974)). Mark (1999) suggests that the best way to account for both types of risks simultaneously is by simulation. These methodologies if run successfully could give an accurate picture of a financial institution's (or a portfolio's) market and credit risks.

Although such techniques could prove to be highly valuable, they do have a number of disadvantages. The computational effort in running all the necessary scenarios and understanding the outcome is quite huge in terms of the amount of time, cost and human capital. Because of the distinctive difference of time horizons between market and default risk all these measures are no shorter than one year. And history will tell us that default events have been triggered due to substantial unexpected market events. So, the time horizon of a credit event could well be shorter than one year.

Kealhofer (1999) has suggested that *the efficient diversification of a portfolio of derivatives requires active rather than passive management because of the dynamic nature of the risks*. Thus, active management of risk should be designed to respond to market wide commonly held information on the changing nature of default risks. The management of risk could be done in a number of ways:

- By maintaining margin accounts as in organised exchanges;
- Quick settlement;
- Reduce the time to maturity;
- Actively managing credit exposure by buying or selling;

3.1.3 The Significance of this Study

The purpose of this study is to propose and evaluate a dynamic hedging methodology of the default risk of a hypothetical interest rate swap portfolio. Based on that methodology an integrated measure of market and credit risk will be proposed. This methodology will assist in the active management of risk in interest rate swaps and other financial instruments with similar exposure profiles.

The methodology is based on taking an opposing position in default risk via a credit spread index[31] options. Suitable[32] credit spread index options were priced under the LS framework and were used for the proposed hedging methodology. The value of that portfolio is sensitive to both the movements of market rates and related credit spreads. The correlation between the two types of market rates is only inherent if a historical simulation is used. Hence, the distribution of returns of that portfolio is directly linked to credit spread risk[33] as they are to market risk. By measuring the volatility of these returns using the VaR measure we are able to deduce a VaR measure which accounts for the market risk of the swap portfolio and the credit spread risk which is directly related to default risk. Thus an integrated measure of market and credit risk is proposed which can be extended to other financial instruments other than swaps.

3.1.4 Overview of the study

In section 2 the methodology is outlined, in section 3 the data was described and in section 4 and 5 the results and conclusions are analysed.

3.2 Methodology

This study presents an integrated measure of market and credit risk for a hypothetical interest rate swap portfolio. It is based on the dynamic management of risks over a short time horizon. This work utilises the LS (1992) framework which was examined in chapter 3.

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[31] Credit spread indices were mostly available to us rather than credit spread data over a period of time. Hence the use of credit spread indices in this analysis.

[32] Meaning that options on the available credit spread indices.

[33] The term structure of credit spreads is directly related to the term structure of default rates as it was explained in chapter 3.

3.2.1 Dynamic Value of Interest Rate Swaps

Interest rates swaps are agreements between two counterparts by which the two parties agree to exchange payments based on fixed/floating interest rates periodically for a period of time in the future. By market convention, the fixed-rate payer that has a long swap position in a fixed/floating interest rate swap is called the *buyer* of the swap, while the fixed-rate receiver has a short swap position in the fixed/floating interest rate swap and is called the *seller* of the swap. At the date of contract initiation of a fixed/floating interest rate swap, the swap contract is usually executed *at* a *par value* (or *at-the-money* or *at-market)* swap because there is no initial cash exchange between the two counterparts. Thus, at the date of contract initiation, an interest rate swap contract is neither an asset nor a liability to either counterpart. However, subsequent to its initial date of agreement, the market value of the interest rate swap may become positive or negative. These value changes are stochastic in nature and are primarily driven by stochastic variations of the term structure of interest rates. Hence, the market value of the swap position can become positive to one counterpart and negative to the other counterpart. For instance, a fall in market rates (r) of the fixed/floating interest rate swaps (expressed in terms of the fixed rate of interest on a swap) will make the existing swap contract a liability to the counterpart with *long* swap position (i.e., the fixed-rate *payer in* the swap) and an asset to the counterpart with a *short* swap position (i.e., the floating-rate *payer* in the swap). Conversely, a rise in the market rates of the fixed/floating interest rate swaps will bring a gain to the counterpart with a *long* swap position (the *buyer)* and a loss to the counterpart with a *short* swap position (the *seller).*

3.2.2 Pricing of Interest Rate Swaps

The calculation of the swap market values requires the generation of a discount curve and subsequently a forward curve. The swap market value is the discounted difference between the fixed and floating payments. In the case that the holder of the swap is receiving floating and paying fixed, equation 3.1 would give us the Swap Value (MTM) of the swap[34]. The unknowns are the discounting rates, which differ between fixed and floating in terms of their maturity since the reset times between fixed and floating might differ. The future floating rate is also another unknown and it's the one derived off the forward curve. By future floating rate we mean the floating rate, which starts after the first fixing.

$$SwapValue = \text{floating value - fixed value}$$

$$= notional * \left(\sum_{i=1}^{n} rfl_i e^{-r_i \delta t_i} - \sum_{k=1}^{m} rfixe^{-r_k \delta t_k} \right) \qquad 3.1$$

rfl = the forward rate
$rfix$ = the fixed rate
$e^{-r_i \delta t_i}$ = the disocunt factor for the floating payments
$e^{-r_k \delta t_k}$ = the discount factor for the fixed payments

The observed EUR discount curve was bootstrapped and it was fitted to the LS (1992) model. After we estimated the theoretical discount curve (as in chapter 2) we subsequently calculated the forward curve using the relationship which relates forward rates to discount factors:

$$f(i-j,j) = \frac{e^{-r_i t_i} / e^{-r_j t_j} - 1}{t_j - t_i} \qquad \text{for } j > i \quad 3.2$$

[34] And if the holder of the swap was paying floating and receiving fixed then equation 4.1 would be Swap Value = fixed value – floating value

The forward curve was used to derive the appropriate forward rates for each swap in the portfolio. Hence all the unknowns of equation 3.1 were recovered and the MTM[35] of the swap portfolio was calculated for that date. The procedure of marking to market a swap book is highly automated in many financial institutions if not in all of them.

3.2.3 Historical Simulation Value at Risk

A universally accepted[36] way to measure the overall market risk with a single measure of any financial instrument is the use of Value at Risk. The case for interest rate swaps is no different, so we using the historical simulation Value at Risk methodology we calculated the weekly VaR for the hypothetical swap portfolio and the swap sub-portfolios[37].

Historical simulation requires a large number of historical data. Often lack of data availability make this methodology quite intensive. In our case, we were able to collect the necessary historical data in order to run the simulation for our swap portfolio. Historical yield curves were estimated using the LS (1992) model for the period 23/02/99 – 07/05/04. The swap portfolio was re-valued for each date and the weekly movement was recorded. The historical weekly changes were ordered for every 104 data points (2 years) and the second worst (negative) and best (positive) weekly movement was recorded. The average of the two was the weekly VaR at each data point at the 99% confidence interval (or 2.33 standard deviations).

In more detail the VaR calculation for the interest rate swaps using historical simulation was quite straightforward. Using the time series of the discount curve, we

[35] Mark to market (MTM) is the swap value.

[36] By practitioners, academics and regulators especially after Basle (I) in 1998.

[37] Swap sub-portfolios as defined in the Data section are portfolios of swaps with counterparts of the same rating.

discounted the fixed cash flows at each time point. Naturally the fixed rate was used to generate the fixed cash flows. These cash flows were summed up and then they were netted with the floating side cash flows. The floating side cash flows were being calculated using the forward curve at each week. Prior to that the forward curve was calculated using the discount curve. This is the most laborious way one can use to calculate historical NPVs for interest rate swaps. A faster way would be to apply the appropriate LIBOR rate for each payment date to the notional and then discount and some up all these future cash flows. Of course there is a degree of error to that which increases significantly as the maturity of the swap increases. The main reason being that the LIBOR rate could change substantially between the date the NPV is calculated to maturity. Nevertheless this is a quick way of obtaining VaR for a swap.

Testing if the VaR model in use is working is to backtest its performance. This was possible in this study because we assumed that the portfolio had started in the past and we now have realised profit and loss (P/L). Since VaR is a type of a "P/L predictor" by comparing the weekly VaR and the weekly realised P/L (hypothetical) we can test the model's efficiency. Thus, the weekly P/L was recorded and the weekly VaR was run recursively between May-99 to May-04. The use of historical simulation makes the backtesting procedure slightly easier since we know what is the acceptable result. Historical simulation uses the actual distribution of market rates to estimate VaR as opposed to the risk neutral distribution. The risk neutral distribution is usually the normal distribution. The difference between the actual distribution and the normal distribution is in the observed fat tails of the actual distribution. This difference is could be between 2 – 5 %. Thus, by backtesting the weekly VaR we would expect the weekly P/L to be out of the weekly VaR "limits" only between 2 – 5% at all times, that is art least for 30 weeks.

3.2.4 Swap Default Risk

In swap contracts, there are two most basic forms of risk: price risk and default risk. The price risk as it was explained earlier arises due to the movement of the underlying index so that the default free present value of the future payments changes. The price risk can be hedged by taking offsetting positions using related derivative instruments, like interest rate futures, currency futures, etc. The default risk is defined to be the exposure to the risk of payment failure of the counterpart. Unlike forward contracts, swaps are over-the-counter (OTC) contracts so they are not backed by the guarantee of a clearing house or an exchange. Swap default may be due to early termination of the swaps contract, or defaulting on some other obligation or filing for bankruptcy. Early termination may be due to the non-performance of obligations under the swap contract, for example, defaulting on a swap payment. The swap may include clauses that trigger early termination, say, the credit rating of either party falling below a certain class, or failure to meet margin payment when required on a marking to market basis.

Assessment of termination damages in case of premature termination is based on the replacement cost of the swap. An estimate of the swap value is obtained from quotes from several established swap dealers. The average of these quotes is used as the replacement value. The cost of default is related to the replacement cost of the contract, and this depends on the rule for sharing claims in default. There are two basic rules of settlement. In the full two-way settlement, if the swap has positive value to the defaulting party, the counterpart pays the full replacement value of the swap. However, in the limited two-way settlement, the non-defaulting party is not liable to pay the defaulting party even if the swap is in-the-money to the defaulting party based on the rationale that there has been a breach of contract by the defaulting party. In real life, non-defaulting parties have typically settled out of court by paying part of the replacement cost to the defaulting parties.

The common tools of default risk analysis of swaps use either the structural models or the reduced form models. The structural models of Cooper and Mello (1991); Li (1995) employ the contingent claim approach where the firm values of the swap parties are assumed to follow some stochastic processes. Default occurs when either firm value cannot meet its liabilities. The payment streams are incorporated as source terms in the governing partial differential equation and the settlement rules are modeled as auxiliary conditions. In the reduced from models Duffie and Huang (1998), JT (1995), default arrives suddenly as point process. Valuation of defaultable securities is characterized by an effective discount rate, which is above the default free rate by a premium that is related to the arrival rate of default and recovery rate upon default.

All theoretical analyzes in search of a credit spread in swaps show that the difference in swap rates between two counterparts of different credit ratings is much less than the difference in their debt rates. For example, Duffie and Huang (1998) found that for a 5- year interest rate swap between a given party paying LIBOR and another party paying a fixed rate, the replacement of the given fixed-rate counterpart with a lower quality counterpart whose bond yields are 100 basis points higher, increases the swap rate by roughly 1 basis point. For a 5-year currency swap, with volatility on the exchange rate of 15%, their model shows the impact of credit risk asymmetry on the market swap rate to be roughly 10-fold greater than that for interest rate swaps. This is consistent with the actual market practices that swap dealers quote the same rates to all counterparts, irrespective of their credit ratings. The insensitivity of swap rates to credit ratings may be attributed to the very nature of a swap that it can be either an asset or liability to either counterpart. Also, the multi-period nature effectively mitigates the impact of default risk. In real market environment, several non-price devices to control default risk are commonly used in swap markets to limit the ability of bad firms to shift risk via swaps. Some of these common techniques include credit trigger, collateral, netting and marking-to- market.

The overall swap portfolio was examined closely in order to understand how much default risk it runs. The first step was to breakdown the portfolio per counterpart credit rating. The portfolio was assumed to have exposure to 11 different counterparts with different credit ratings. Based on that we created 11 sub-portfolios, which contain swaps with counterparts of the same rating. By summing all the counterpart exposures per rating we've made the following assumption. T*he financial institution that holds the hypothetical swap portfolio has netting agreements with all 11 counterparts.* This has been a standardised approach lately among financial institutions in order to minimise their exposure within counterparts. In the past financial institutions used to measure exposures per deal instead of counterpart. This would overestimate the default risk and would lead to inefficient capital allocation. Hence, the standardisation of ISDA netting agreements which are being countersigned between two parties which are willing to enter into an OTC agreement.

In most financial institutions counterparts are being analysed and assessed by the credit departments before any trading or dealings initialise. Credit departments are looking at both quantitative and qualitative data in order to assess their creditworthiness and rate them according to their internal ratings model. Subsequently if they are satisfied they initiate the ISDA agreement negotiations. Once this is done, settlement and counterpart trading limits are being set using past trading history of similar rated counterparts. One important factor used in that process is the spread, which the bonds of that counterpart are trading. If there are no tradable long-term borrowings[38] of that form by the counterpart then a credit spread index of the same rating is being used as a reasonable proxy. The volatility of that index is also quite important and especially the choice of maturity. Usually the 10Y sectors are being chosen, but in my opinion the 5Y is quite informative since this is the most liquid sector of the credit curve[39]. However, a term structure approach would be more appropriate unless the majority of the exposure is concentrated around one maturity. The exposures from the sub-portfolios were further analysed and broken down by maturity in order to give us a better idea of the extent of the current exposure.

3.2.5 Hedging Swap Default Risk

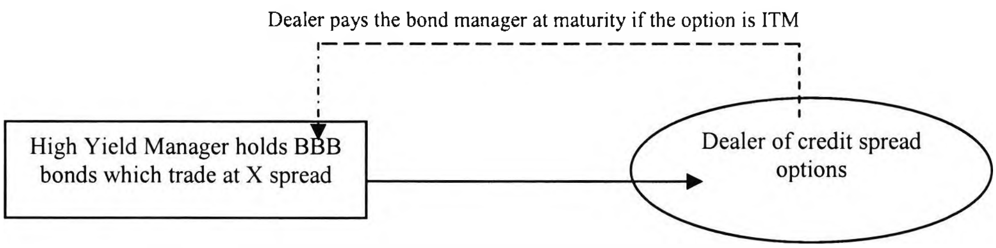
Many money managers and banks have always tried to find ways to eliminate their credit risk so they can increase their capacity to lend. A very popular technique is to use credit risk transfer. This technique involves the undertaking of a credit derivative deal, which protects or immunises from potential credit events, thus reducing or even eliminating credit risk by transferring to someone else. This is highly adopted by banks, which try to free up capital for further investments or even for pure credit risk management purposes. Money managers who invest in high yield also require protection for their portfolios in response to credit events, i.e. in case that credit spreads widen or narrow depending on their positioning. There are various credit derivative products, which will facilitate the needs of both money managers.

For example, assume the high yield money manager holds a bond portfolio of bonds which trade at an X spread to the government bonds. The manager is very bullish on that economy and wants to stay in the bonds long-term. However, the money manager

[38] If there are no traded default securities under the counterpart's name, then suitable proxies are being carefully chosen to match the dynamics of the missing security.

[39] It has been the most liquid part of the credit default swap curve.

is worried of short-term event risk, which could lead to spread widening. The way to hedge that short-term event risk is by buying spread widening protection. This can be done through short-term credit spread options struck either at the money or further out depending on the anticipated move. The way this works is by paying an upfront premium to a dealer for a spread option, which is exercised at maturity. The following diagram illustrates the idea:



Pays y% premium for credit spread option struck at X

This simple way of protecting against credit spread movements has been used and is currently in use by many money managers. This hedging technique is being derived using option pricing theory. Of course, this is not a day-to-day practice because credit events don't occur as often plus the cost of hedging could be quite high. So to hedge or not to hedge credit risk is not a daily operation performed by money managers. However, what is being done daily is the measurement of their market and credit risk exposure. This is done usually by relying on VaR methodologies for both market and credit risk.

The previous example of a bond money manager trying to hedge its credit exposure against credit spread movements can be used to draw useful parallels on how the same technique can be applied to hedge credit spread risk in our hypothetical swap portfolio (section 3.2). In theory, hedging derivative credit risk is not a straightforward process. It requires the establishment of a unified pricing framework where both credit risky and credit risk free securities can be priced. Beumee, Hilberink, Patel and Walsh

(1999), have shown how a derivative credit risk can be hedged using the Jarrow and Lando (1997) model specification. Using the Black-Scholes-Poisson option pricing framework they proposed a hedging methodology which creates a credit risk neutral portfolio, assuming of course that the derivative used to hedge the existing exposure bears no credit risk.

Assuming that a dealer wants to hedge the default risk of a swap portfolio, the first step would be to examine the sensitivity of its aggregate swap exposures per credit spread. Likewise to the money manager example the dealer could examine the effect that credit spread widening/tightening would have to the swap portfolio. The difference between bond and swaps is that credit risk does not come in the pricing equation at all. So, it's not as straightforward to find out the sensitivity to credit spread movements as it is for bonds. However, what we can say is that as the swap exposure increases so is the credit exposure and thus credit risk and vice versa. What it can also be said is that when there is credit spread widening credit risk increases even if the swap exposure hasn't increased over the same period. Based on that rationale and the money manager example we propose that if one would like to hedge its current swap exposure in a dynamic setting it can be done by taking an offsetting position in credit spreads. This can be done using credit derivatives and in this study we used credit spread index options, which "bet" on the future widening or tightening of a credit spread index. In that way even if the swap exposure remains unchanged and credit spreads widen the current exposure remains default risk free. Even in cases where swap exposures decline and eventually become negative -in that case we could over-hedge-, if the credit spread widens the result could still be positive.

At this point a few assumptions have to be made before we continue. *Since we will be using credit spread indices instead of counterpart credit spreads, the first one is that* *the credit spread of a counterpart is almost fully correlated to its respective credit index.* The correlation between credit spreads and their respective index is time varying according to market forces. Our view is that the correlation is at the highest when there is market stress. For example, when a financial crisis is looming or has just happened, the correlation among market rates is quite high. At times like these the probability of a credit event happening is at the highest. For these reasons we assumed that the credit spread index is a suitable proxy to capture individual credit spread movements. As an alternative the relationship between corporate spreads and their respective indices could be examined at regular time intervals in order to establish if the assumption is too strong. However, that should be neither a daily nor a weekly procedure.

Some credit risk measurement methodologies require to take into account the transition probabilities. This is a matrix of probabilities which describes the probability of a counterpart of a certain rating can be either downgraded/upgraded or stay at the same rating over a specified period of time. This matrix can be generated using as inputs default free zero coupon bonds, an exogenous recovery rate and a historical transitional matrix (usually provided by the major credit rating agencies) JLT (1997). The change of credit rating of a counterpart has a significant effect on its debt spread and assets in general. Hence, accounting for credit risk without examining the transition probability of a counterpart would not be sound practice especially when one is examining loan or corporate debt exposures. The credit spread of a counterpart operates in a much more efficient market setting. If there is a credit event looming a counterpart then the credit spread of its debt is more efficient in pricing the event than the efficiency of a rating agency, which will assign lower/higher rating since this is a discrete event and the credit spread movements is continuous.

The second important assumption is that counterparts of the same rating have the same rating transition probabilities. This assumption facilitates in aggregating all the credit rating related specific exposures to a total and examining the overall spread risk of each sub-portfolio[40]. It also assists us in using credit spread index options instead of counterpart related credit spread options.

Using credit spread index options as a hedging instrument for swap default risk, it directly means that we take additional credit risk with the seller of these options. Of course, part of it can be avoided by selling options rather than buying them. *For this reason we need to assume that the credit risk of the option seller is much, much smaller than the credit risk generated by the swap portfolio, and we will treat it as negligent.*

Making hedging decisions in any financial market environment is a well thought process and often well-timed. We wouldn't expect any financial manager to be hedged at all times –unless they are using passive strategies only- because this is in the expense of its profit and loss. The timing, the amount and the time horizon and the financial manager's view are all important factors in any hedging decisions. The timing though has to be the most important of all.

By creating the swap sub-portfolios according to each counterparts rating we could examine the swap default risk in a more detailed way. A graphic illustration would show the level of default risk per rating the swap book is running. Relating these swap exposures to credit spread movements was part of our goal. By looking at the proposed hedging strategy this is the first step towards it. Taking an offsetting position in credit risk by buying credit spread index call options, essentially what you do is buying protection against credit spread widening. Of course not all sources of

[40] We will call sub-portfolios the portfolios generated by aggregating all exposures with counterparts of the same rating. In our hypothetical case we have 11 sub-portfolios.

credit risk are being hedged using that strategy. For example, the risk of rating downgrades isn't captured, but if a counterpart is downgraded during the holding period of the options then current options are being terminated and lower rating credit spread index options are purchased. Alternatively if all the counterparts in our swap sub-portfolios have tradable bonds we don't have to do that because the rating change will be reflected in the credit spread of their bonds. This is actually more efficient since credit spreads usually discount such events before they actually happen. The main reason is that rating agencies are quite inert in announcing rating changes. Since we are using credit spread indices we will employ a short-term dynamic hedging strategy. Thus we will employ 3M options instead of longer maturity options. There is a very low chance of a credit event happening more than once a year, let alone four times a year[41]. The choice of the 3M options was also due to the fact that the LS model prices more efficiently short-term options than long-term[42]. In addition, we wanted to have as small as possible time value. Also it makes more sense to hedge with short-term options in terms of cost. The maturity of the option underlying is the matched to the maturity of the biggest exposure.

The notional of credit spread index options, needed to hedge the swap exposures, should be a function of the total exposure and the payoff of the option. The total exposure of the swap portfolio will be the current swap exposure and the potential exposure over the same period that the option is been written i.e. 3 months. For example, if we look at the AA- exposure, which currently stands at approximately 5.8mio EUR (07/05/04), one would think that hedging that exposure would be an expensive strategy. The first step in order to do that is to examine the maturity of the exposure, i.e. when the exposure ceases to become an uncertainty. For the AA-

[41] Since we are using 3M options, it means that if the strategy was to be employed dynamically we would enter into 4 different option contracts throughout the year.
[42] Note the higher price differences as the maturity of the options increases (section 3.6.1).

exposure this is close to 2 years. Hence, we need to look at the closest spread maturity for that rating. The history of the AA- 2Y spread and its volatility are used in order to decide the credit spread index option strike. The strike of the option is simply decided by looking at the historical mean of the specific spread index (2Y AA-). In our example the long term mean stands at 39bps then the strike of the option would be at 39bps. This would be a reasonable level since credit spread indices are mean reverting (chapter 2) and the spread would fluctuate over time above and below that long term mean moving the option from in the money to out of the money. An important point to be made is that this is the long-term mean level given the sample of the observed data. Using a larger sample would probably give us a different mean, depending on how far back in history one wants to go.

The option notional is an important quantity, which will be linked to the total swap exposure of the portfolio over the 3M period rather than the swap notional as it would be the case if we were dealing with corporate bonds. Going back to our example, we used the LS (1992) pricing framework and priced 3M call options on the 2Y AA- spread index, struck at 39bps. If the price of that option is at Y then the option notional is derived by multiplying the value of the 39bp call option to the total potential exposure of the AA- swap sub-portfolio. The total potential exposure is the current exposure as of the date in question plus the 2.33SD potential exposure over the next 3 months, which is the maturity of the option.

$$SwapMTM_i + SwapVaR(3M)_i = TotalPotentialExposure(3M)$$

(3.3)

$$OptionNotional = \frac{TotalPotentialExposure(3M)}{3MCreditspreadindexoptionprice}$$

We used as the potential exposure the weekly VaR scaled by the square root of time[43] in order to get the 3M VaR[44]. By dividing the total exposure with the option premium we arrive at the required notional for the option. This is the amount of options need to be bought in order to hedge the total potential exposure if the counterpart defaults in 3M time and there is no recovery. The same methodology was carried out for all sub-portfolios.

However, our calculation of the required hedging amount was based on the fact that the counterpart will default rather than just a credit event happening. If the counterpart was defaulting then its credit spread would probably move more than 4SD. In that case even if the exposure doesn't increase, the probability of default is so high that the loss from the exposure in near certainty. Furthermore, if the spread moves higher than its usual volatility over a period of 3M then the option is still in-the-money and would still pay for the potential exposure. If the spread moves lower or stays at the same level then it means that the counterpart has a much smaller chance of defaulting.

3.2.6 Integrated Risk Measure

The hedging of swap default risk based on the methodology of section 3.2.5 can be used in order to calculate a single risk measure of market and credit risk for swaps. What we mean is by creating a portfolio of swaps and credit spread index options, essentially we created a default free portfolio. The part of the portfolio sensitive to market risk, which is the swaps can be measured in a straightforward manner by calculating a VaR type of measure. The part of the portfolio sensitive to default risk, which is the credit spread index options represents the associated default risk of the swap portfolio and can also be measured by a VaR type of measure. Hence, by

[43] Multiplied by the square root of 12.

[44] As calculated in section 4.2.1

assessing the market value distribution of the overall portfolio we can deduce the amount of risk subject to integrated market and credit risk at a given confidence interval. Thus, by running a VaR on the overall portfolio we can deduce a VaR measure which accounts for both market and credit risk.

The credit spread index option VaR was calculated using the same methodology as in section 3.2.3. The data range is 08/02 – 05/04 which overlaps with the swap data used to run the VaR for the swap portfolios. Thus, there is an intrinsic correlation among all the market rates used to run the VaR for the swaps and the options. The VaR results for the option portfolios were also backtested in the same way as the VaR of the swap sub-portfolios (section 3.2.3).

Subsequently, the total VaR of each sub-portfolio was run. The total VaR represents the maximum amount of money that could be made or lost in a week's time with 99% probability assuming market rates moves by a maximum of 2.33SD even if the counterpart defaults and there is no recovery. The difference to the market risk VaR is the amount of risk that accounts for the swap default risk:

$$\text{Total VaR}_i = \text{Market VaR}_i \text{ of Swaps} + \text{Market VaR}_i \text{ of CSOs.}$$

(only for historical simulations, since the correlation is the actual)

i = counterpart rating

The first term of the above expression represents the "predicted" 1-week P/L of the swap portfolio of rating (i) due to market risk. The second term represents the amount of default risk that the specific swap exposure runs.

However, it would be useful to know if there is any co-dependence between the two VaR measures in order to devise a more standardised methodology. That standardisation would involve multiplying the swap VaR with the respective credit spread index options VaR, without possibly running the credit spreadit index options VaR itself. Hence, a simple linear regression was run between the swaps VaR and the

CSIOs VaR per rating. The regression was estimated using OLS. The dependent variable in that case was the swaps VaR. Effectively what is being examined is the "volatility" of the credit spread against the volatility of the swap exposure.

$$\text{Swap VaR}(i) = a * \text{CSO VaR}(i) + e$$

A different but quite similar way to examine the relationship between the two risks would be to look at the two volatilities. The volatility of the respective spreads to the volatility of the respective swap spreads. A swap's NPV is the discounted cash flows between the fixed and floating payment side. These cash flows are being determined by the forward curve and its relative steepness, flatness or its overall shape. By looking at the volatility of the respective swap spread which reflects the relative shape of the forward curve is the approximately the same to looking at the volatility of swap NPVs. Using that is easy to relate to the volatility of credit spreads. This is only a quick method that can be run as an alternative to regressing the two VaRs, in order to establish a fair idea of how the two "risks" relate to each other.

3.3 Data and Description

3.3.1 Hypothetical Swap Portfolio

The swap portfolio was constructed in such a way in order to simulate a bank's typical swap portfolio. The hypothesis is that this portfolio belongs to an asset and liability department where most of the positioning is one sided, either long or short and not neutral. It was hypothetically initiated throughout 1999-2000 with different counterparts -hence, the different fixed rates for similar swap maturities-. The fixed strikes were distributed around the actual swap rates at the time and the positioning (long/short) was in such a way as to generate positive (credit) exposures. In addition, the assumed ratings per counterpart are almost evenly distributed according to the overall exposure.

The portfolio was sorted by maturity and is overall short since we are receiving fixed in most cases. This means that if interest rates go up then the portfolio is bound to lose more money since the PVBPs (present value of a basis point) are negative for a shift in the underlying rate by 1 basis point (bp). Fortunately for the portfolios' value, interest rates moved to historical lows in the EURO area over the holding period that the portfolio was examined (02/99 – 05/04). If the portfolio was only partly hedged as it often happens (market risk) then a great deal of profit would be made. However, the credit risk of the portfolio was increasing as the NPVs were becoming more positive.

Table 3.2 Hypothetical Swap Portfolio in EUR

Maturity	Notional	Fixed Rate	Fixed Frequency	Floating Frequency	Pay Fixed/Floating
15-Nov-03	9,000,000	4.67%	Annual	Semi	Receive Fixed
15-Nov-03	7,000,000	4.12%	Annual	Semi	Receive Fixed
25-Nov-03	6,000,000	4.32%	Annual	Semi	Receive Fixed
25-Mar-04	7,000,000	5.60%	Annual	Semi	Pay Fixed
09-Jun-04	11,000,000	5.45%	Annual	Semi	Receive Fixed
11-Jun-04	12,000,000	4.35%	Annual	Semi	Receive Fixed
10-Jul-04	7,500,000	5.78%	Annual	Semi	Receive Fixed
12-Sep-04	12,000,000	6.00%	Annual	Semi	Receive Fixed
27-Sep-04	10,000,000	4.71%	Annual	Semi	Receive Fixed
14-Oct-04	17,500,000	6.25%	Annual	Semi	Receive Fixed
30-Oct-04	2,000,000	4.65%	Annual	Semi	Receive Fixed
13-Nov-04	1,750,000	4.67%	Annual	Semi	Receive Fixed
03-Apr-05	200,000,000	4.98%	Annual	Semi	Pay Fixed
23-May-05	17,500,000	5.78%	Annual	Semi	Receive Fixed
20-Jul-05	115,000,000	4.11%	Annual	Semi	Receive Fixed
15-Aug-05	2,250,000	4.19%	Annual	Semi	Receive Fixed
22-Aug-05	5.500,000	5.70%	Annual	Semi	Receive Fixed
22-Sep-05	8,500,000	4.78%	Annual	Semi	Receive Fixed
16-Oct-05	14,500,000	4.31%	Annual	Semi	Receive Fixed
23-Feb-06	5,000,000	4.62%	Annual	Semi	Receive Fixed
18-Apr-06	25,000,000	4.43%	Annual	Semi	Pay Fixed

12-Jun-06	17,500,000	5.96%	Annual	Semi	Receive Fixed
12-Jun-06	5,000,000	5.75%	Annual	Quarterly	Receive Fixed
05-Aug-06	7,500,000	5.34%	Annual	Semi	Pay Fixed
06-Sep-06	5,000,000	5.23%	Annual	Semi	Pay Fixed
20-Nov-06	35,000,000	5.64%	Annual	Semi	Receive Fixed
22-Dec-06	2,500,000	4.50%	Annual	Semi	Receive Fixed
02-Feb-07	50,000,000	4.34%	Annual	Semi	Receive Fixed
03-Feb-07	22,500,000	4.89%	Annual	Semi	Receive Fixed
20-Mar-07	17,500,000	5.45%	Annual	Semi	Receive Fixed
20-Mar-07	10,500,000	5.43%	Annual	Semi	Receive Fixed
20-Jun-07	25,000,000	5.12%	Annual	Semi	Pay Fixed
20-Jun-07	8,000,000	5.52%	Annual	Semi	Receive Fixed
20-Aug-07	7,500,000	5.23%	Annual	Semi	Receive Fixed
20-Aug-07	6,000,000	5.22%	Annual	Semi	Receive Fixed
12-May-08	2,500,000	4.99%	Annual	Semi	Receive Fixed

3.3.2 Historical Data

The historical data for the simulations were collected from Bloomberg. Cross sectional data of interest rates was collected between 27/08/99 – 07/05/04. The interest rates used were the 3M and 6M EUR LIBOR, and the 1Y, 2Y, 3Y, 4Y, 5Y, 7Y, 8Y, 9Y and 10Y EUR swap rates on a weekly frequency. These rates were used to bootstrap the zero discount curves. The short rate used for the LS (1992) estimation (chapter 3) was the 3M EUR LIBOR and its GARCH(1,1) volatility at each week. The GARCH volatility was estimated using the PCGIVE software.

Credit spread index data were also collected from Bloomberg (chapter 2). The short credit spread index used varied from 3M to 2Y since the GARCH estimation wouldn't converge for all the 6M credit spread indices. The respective short credit spread indices used were: 6M AAA, 6M AA+, 6M AA, 6M AA-, 3M A+, 6M A, 3M A-, 2Y BBB+, 2Y BBB and 1Y BBB-.

The statistical properties of all these indices can be found in chapter 2. The mean of and the GARCH volatility of the 2Y credit spread indices is shown in Table 3.3.

Table 3.3 Selected statistics of the 2Y spread indices

	GARCH (1,1)	2.33 * GARCH (1,1)	Mean	Mean * (1 + 1SD)	Mean * (1 + 2.33SD)
AAA	32.97%	76.83%	28	37	50
AA+	33.03%	76.96%	28	37	50
AA	34.54%	80.49%	33	44	60
AA-	**32.98%**	**76.83%**	**39**	**52**	**69**
A+	33.24%	77.45%	53	71	94
A	33.27%	77.51%	57	76	101
A-	38.48%	89.66%	78	108	148
BBB+	35.84%	83.51%	81	110	149
BBB	42.31%	98.59%	95	135	189
BBB-	60.56%	141.10%	300	482	723

The mean of the 2Y indices was used as the strike of the credit spread index options in this study. As a description of our credit spread index data we examined the actual probability of the 2Y credit spread index movement per rating. Based on the time series collected we constructed the following Table 3.4.

Table 3.4 Actual Probability Distribution of 2Y indices

Spread Level bp	AAA	AA+	AA	AA-	A+	A	A-	BBB+	BBB	BBB-
10	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
36	**82.44%**	**71.71%**	**57.07%**	23.90%	3.96%	1.18%	0.00%	0.00%	0.00%	0.00%
37	7.80%	1.95%	9.27%	6.83%	1.98%	2.35%	0.00%	0.00%	0.00%	0.00%
42	8.78%	15.61%	15.12%	**34.15%**	18.81%	9.41%	0.00%	0.00%	0.00%	0.00%
52	0.98%	10.73%	18.05%	32.20%	33.66%	37.65%	12.87%	0.00%	0.00%	0.00%
71	0.00%	0.00%	0.49%	2.93%	**40.59%**	**48.24%**	38.61%	30.00%	12.43%	0.00%
76	0.00%	0.00%	0.00%	0.00%	0.99%	1.18%	6.93%	14.12%	8.28%	0.00%
108	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	**41.58%**	**54.12%**	**59.17%**	2.37%
110	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	1.18%	1.78%	1.18%
135	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.59%	17.75%	14.79%
555	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.59%	**81.66%**

Table 3.4 is a representation of the actual probability distribution for the 2Y spread indices. The actual probability is measured using a frequency calculation. It was calculated to illustrate how the spreads moved over the period examined and the low spread environment. The points of the highest frequency are also shown in the first column. These levels were subsequently chosen to be the strikes for the credit spread index options.

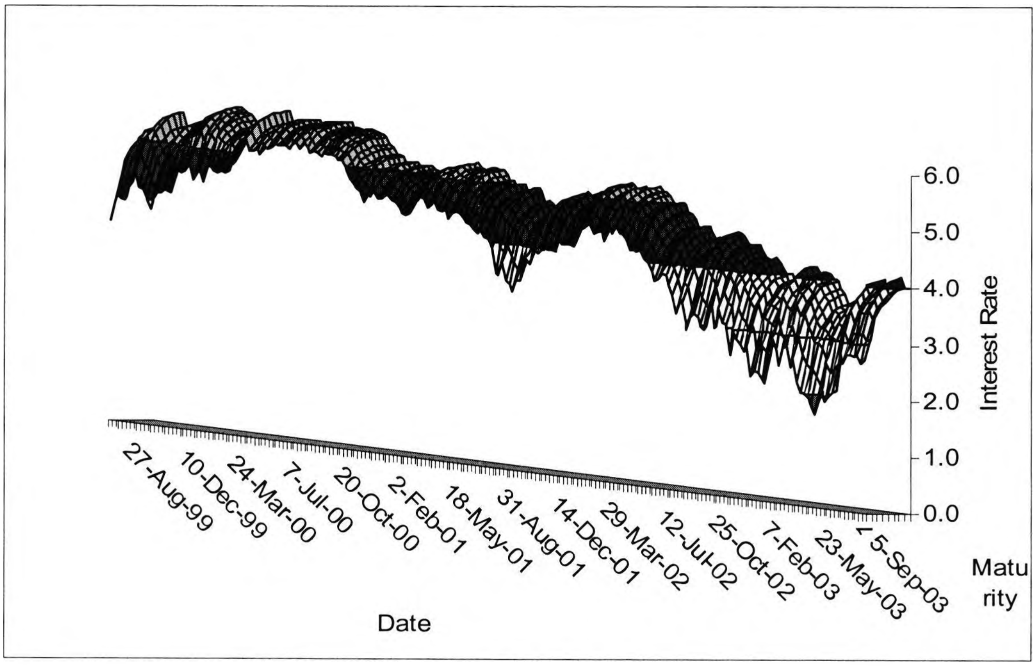
3.4 Results and Analysis

3.4.1 Swap Sub-Portfolios Valuation and Historical Simulation

The weekly LS (1992) estimated yield curves are being shown in Figure 3.1. A first look at the time series of the EUR yield curves shows that interest rates shifted to historic lows. That was due to the slowdown in global economic growth, which led the major central banks of the world including the European Central Bank (ECB) to lower their benchmark interest rate quite aggressively. Only recently there were signs that the major economy of the world (US) and of the Euro area would rebound hence the small rebound in the interest rates. This was fuelled by market expectations of where future interest rates might be.

The hypothetical swap portfolio was simulated using the estimated yield curves assuming that the portfolio remained unchanged. A time series of its market value is plotted in Figure 3.2.

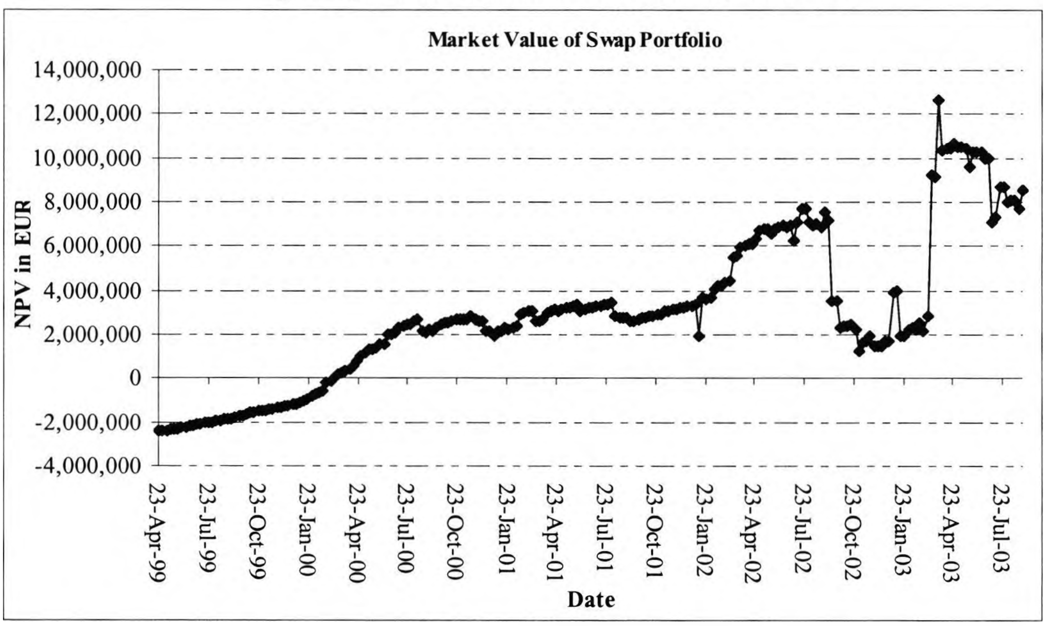
Figure 3.1 Weekly LS (1992) Estimated Yield Curves



The sudden drops on the overall NPV is a result of net cash flows either being paid or received which occur either simultaneously for a few swaps in the portfolio or by extreme moves of the yield curve.

During the period Jul-02 to Oct-02 there were two rate cuts by the European Central Bank (ECB) which resulted in wild fluctuations in the market value of the swap portfolio. The more recent swing resulted mainly by the reduction of future cash flows on the swap portfolio. This is the nature of the market value of swaps which exhibits a specific asymmetry with relation to time to maturity. Examining the market value of the swap portfolio over time we can see that it becomes more positive.

Figure 3.2 Time series of the Portfolio's MTM



As an example we will examine the VaR of a single swap and then we will look at the overall swap portfolio. The swap considered was a near 5-year interest rate swap between a given party paying floating rate such as the London Inter Bank Offered Rate (LIBOR) in EUR and another counterpart paying a fixed rate. The following table shows the exact specification of the contract:

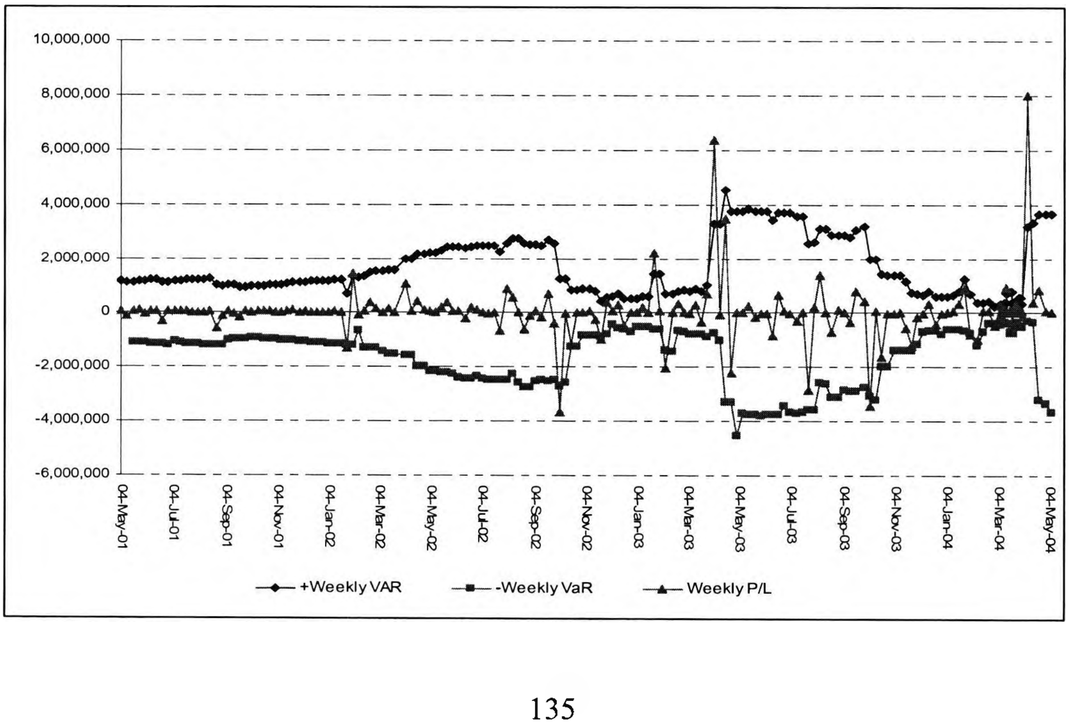
Table 3.5 Interest Rate Swap Example

Maturity	Notional	Fixed Rate	Fixed Frequency	Floating Frequency	Floating Rate
12-Sep-04	12,000,000	6.00%	Annual	Semi-Annual	6M EURIBOR

The NPV of the above swap as priced using the estimated discount curve as of the 19/09/03 was standing at 438,396 EUR, positive to the party receiving fixed and negative to the party receiving floating. This is the otherwise known sum of the discounted net payments between the two parties of the swap agreement. As we can see from the Figure 3.2, the "moneyness" of the swap increases up to 1 year before maturity. The reason for that is that interest rates have been falling since the swap agreement was entered. The sudden decreases on the swaps' NPV reflect the reduction of future net cash flows. We would expect the swaps' value to decrease earlier than a year before maturity but the party which receives the fixed rate is so far in the money that the market value goes up even after a number of cash flows go through. The sudden drop is a result of huge cash flow going through on the 12th of September of 2003.

In Figure 3.3 the backtesting of the VaR model is performed. The major reason for doing this is to check our pricing capability. The weekly P/L is higher/lower than the + weekly VaR/ - weekly VaR only in 6 out of the 172 weeks. This gives is a ratio of 3.5% which is well within the accepted 2 – 5 % boundaries.

Figure 3.3 Backtesting of Overall Swap Portfolio



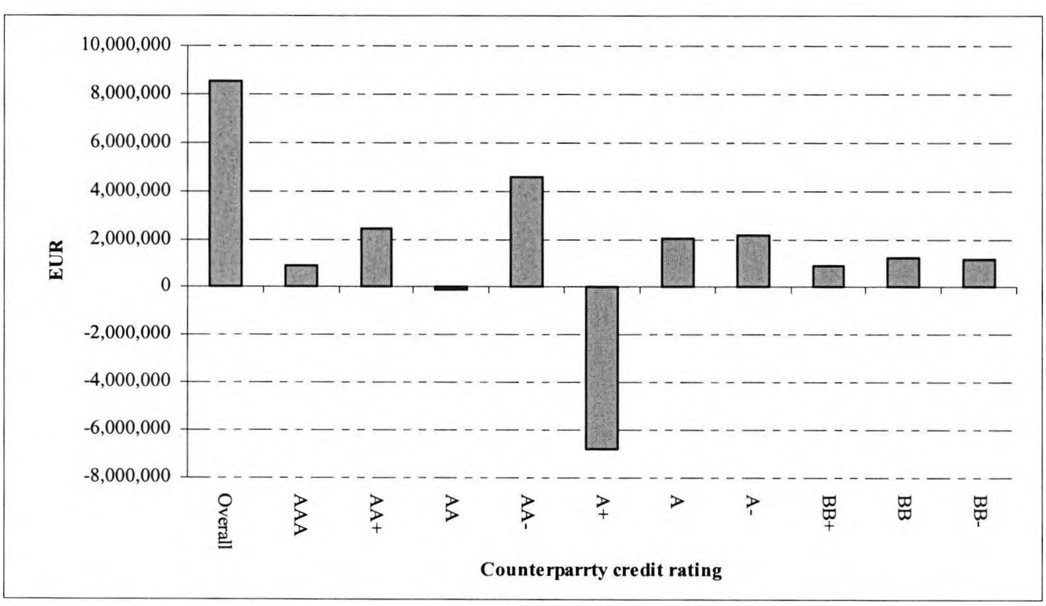
3.4.2 Swap sub-portfolios

The credit exposure increased over time to reach extreme highs (Figure 3.2). Together with this huge increase in the swap NPVs the risk of default also reached historic highs for the portfolio. However, the risk of default differs for each counterpart since each counterpart is of different quality and could well have different credit rating. The portfolio constructed is assumed to contain counterparts with at least 10 different ratings. In order to quantify the default risk our portfolio is running we need to classify and assess the swap exposures as they occur.

The overall exposure is around +80 million EUR, which is a substantial amount subject to credit risk. Since, the amount of credit risk is quite large, if we were to allocate capital for that portfolio it would be in the area of 5 – 10 % of total exposure (Group of 30) throughout the life of the portfolio. This approach though, does not discriminate between different counterparts. We know that the relative riskiness of each exposure is not the same for all the counterparts. The reason is that they are of different credit quality and hence credit rating. Figure 3.4 shows the exposures per rating. The total exposure, which is the sum of the individual exposures is quite high. The individual exposures are all positive apart from the AA and A+ exposures, which are negative. However, since we are dealing with swaps we have to note that these are only the current exposures. Swap market values can move quite erratically in any direction creating positive NPVs rather fast. Due to the asymmetric nature of the swap profile all swap exposures can be treated as potential credit exposures. Hence, negative exposures, which currently bear no credit risk could become potentially positive subject to the movements and volatility of the underlying rates. This is also a major factor affecting counterpart limit-setting decisions[45]. The volatility of certain rates and the properties of some financial instruments together with the size of positions are major determinants of potential exposures. Breaking down the exposures by maturity gives us a better idea of the current exposures and also of the potential exposures. Most exposures lies at the 2Y area starting in 1 week up to and including the 5Y area. The biggest single swap exposure lies at the 2Y - 3Y maturity sector. Figure 3.5 shows the overall exposure by maturity.

If we wanted to hedge the market risk we would do so by taking opposing positions in financial instruments which provide negative/positive cash flow at specific times in the future. Usually short-term interest rate futures and options are used in order to hedge the short-term cash flow of a swap book. The long-term part of the swap book is usually being hedged by taking offsetting positions in government bonds. In practice dealers do not hedge their positions all the time depending on their market views. Even when they have hedged their market risk they are still exposed to default risk.

Figure 3.4 Exposure per Rating

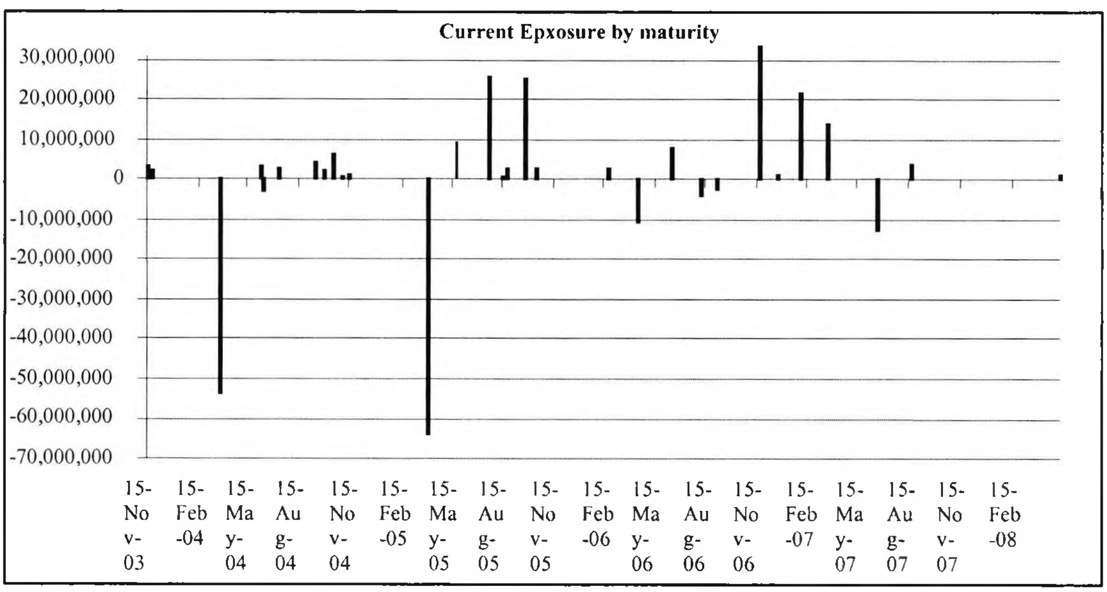


[45] Implying the counterpart limit setting decisions taken by credit risk departments in financial institutions.

Actually the exposure together with the offsetting position could become even bigger if a counterpart defaults because the market risk hedged position becomes more risky and in some occasions could become highly unprofitable. It is like having suddenly a mismatch between your assets and liabilities.

Hence, dealers -mainly their risk departments- are looking at the dynamic evolution of the exposures they are running at all times. This is mainly done by simulating all available market rates and applying them on to the current positions. Different scenarios are produced and their results are analysed in order to derive potential exposures and the probability of them happening. These techniques are based on Monte Carlo simulation. There has not been a full analytic technique so far which could measure the replacement cost of financial instruments which create exposures.

Figure 3.5 Exposure by maturity

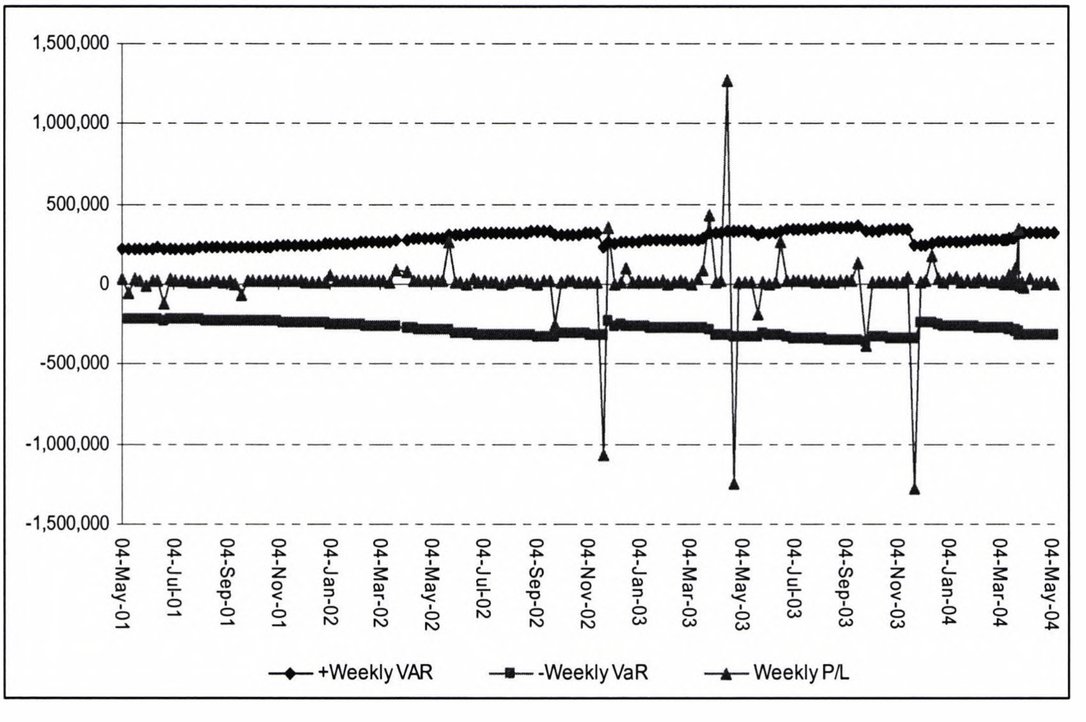


In this study we used as potential exposure the 3M VaR per sub-portfolio. We have 10 sub-portfolios which we run the weekly VaR for each one of them separately using the historical simulation method as described in section 3.2.3. In Table 3.6 we show the backtesting results per sub-portfolio. That is the number of times the weekly P/L was out of the wekly VaR limits. On average the weekly P/L exceeds the weekly VaR by 3.87%, which is within the acceptable limits. Hence, the historical simulation seems to have worked. The highest divergence out of the VaR limits occurs with the AA- sub-portfolio. The weekly P/L is out of the weekly VaR 4.07% of the times. Figure 3.6 shows just that. The big differences between the P/L and the VaR come from the rolling off of cash flows.

Table 3.6 Backtesting Results per sub-portfolio

Swap Portfolio	%
AAA	4.00%
AA+	2.98%
AA	3.91%
AA-	**4.07%**
A+	4.02%
A	4.02%
A-	3.97%
BB+	3.81%
BB	3.89%
BB-	4.02%
Average	3.87%

Figure 3.6 Backtesting of AA- sub-portfolio

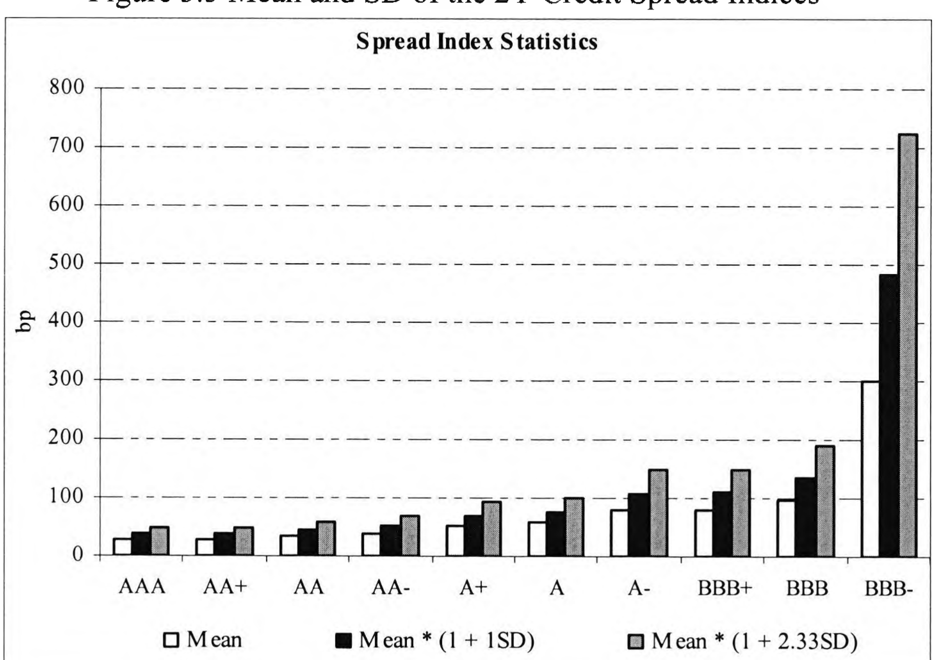


3.4.3 Hedging swap default risk with credit spread index options

Making hedging decisions in any financial market environment is a well thought process and often well-timed. We would not expect any financial manager to be hedged at all times –unless they are using passive strategies only- because this is at the expense of its profit and loss. The timing, the amount and the time horizon and the financial manager's view are all important factors in any hedging decisions. The timing though has to be the most important of all.

In Section 3.4.2 we created the swap sub-portfolios according to each counterparts rating as seen in Figure 3.4. That illustration shows the current replacement cost (MTM) per rating. Relating the current replacement cost and potential exposure to credit spread movements was part of our goal.

Figure 3.5 Mean and SD of the 2Y Credit Spread Indices



Most exposures in the swap sub-portfolios are concentrated in the 2Y maturity sector. In Figure 3.5 we show the mean of the 2Y credit spread indices. The mean represents the strike of each option we used to hedge the default risk per sub-portfolio. Under the LS (1992) framework the respective credit spread index options were priced as of the

07/05/04. Table 3.5 shows the option prices in decimal form and also the total potential exposure based on the 3M VaR per sub-portfolio. Two of the sub-portfolios, AA and A+, have been omitted since they have negative current exposures and their total potential exposures do not exceed the current MTM, hence these are the so called wrong way exposures. The hedging amount for these sub-portfolios is currently zero. Using equation (3.3) we calculated the option notionals as required to hedge default risk assuming there is no recovery rate over a period of 3M (Table 3.7).

Table 3.7 Hedging Strategy

7-May-04	**AAA**	**AA+**	**AA-**	**A**	**A-**	**BB+**	**BB**	**BB-**
Option Strike	28	28	39	57	78	81	95	300
Option Price ATM	0.633	0.559	0.661	0.712	0.511	0.546	0.270	0.177
Swap sub-portfolio MTM(t)	697,303	5,207,339	4,033,640	1,695,122	1,634,680	37,298	1,018,504	1,427,820
Swap sub-portfolio 3M VaR at 99%	193,242	1,443,100	1,117,835	469,766	453,016	10,336	282,256	395,689
Total swap sub-portfolio exposure	890,546	6,650,440	**5,151,475**	2,164,888	2,087,696	47,634	1,300,761	1,823,510
So if counterparty defaults in 3M the amount of options need to be bought today are	1,406,865	11,897,030	7,788,474	3,040,572	4,088,940	87,173	4,822,027	10,287,826
Initial cost of options	890,546	6,650,440	**5,151,475**	2,164,888	2,087,696	47,634	1,300,761	1,823,510

The initial cost of options is quite high. This is due to our assumption of the extreme case that counterparts will default and the recovery is zero. This shows the actual default risk run by the swap portfolios through the options market or through the "protection" market. Of course, exposures can be reduced by termination, through structured deals, by posting risk-free collateral etc. etc. But these precautions are usually taken once the credit spreads have moved towards to what might be a credit event. Hence, this dynamic approach of linking exposures to credit spreads is more forward looking than any other current approach. Finally, this hedging approach shows what is the market price of default risk of the swap exposures in question.

If we were to run the effectiveness of the hedging strategy we would have to run a portfolio of 3M credit spread index options which rollover until the swap exposure is zero. As an example we assessed the cost of the hedging strategy against the AA- portfolio. Assuming that transaction costs (bid-offer spreads and commissions) are absent and funding occurs at LIBOR flat, then we can price an on the run 3M credit spread index option struck at 39bp (for the AA- swap portfolio). The premium for that option over a period of two years is compared with the relative movement of the swap portfolio. Since we will be using 3M options we look at the 3M change of the MTM of the swap portfolio. In Figure 3.5 we show the history of running a rolling of the hedging strategy. The LHS Y-axis shows the relative change of the AA- swap portfolio and the RHS Y-axis shows the price of a rolling 3M credit spread call option following the 2Y AA- credit spread index. I am using the 3M change of the swap exposure in order to map the time horizon of the protection to the swap exposure. The relative market value of the swap portfolio initially declines as the credit spread option price declines (which shows that the credit spread index tightened over the same period), showing an overall reduction in credit risk.

This could be purely coincidental since the credit exposure "just happened" to decrease at the same time as the specific credit spread tightened. At a later stage the swap exposure starts to pick up again whereas the credit spread index seems to be relatively stabilised. Just eyeballing Figure 3.5, we can say that the volatility of the credit spread index option seems to be comparable to the volatility of the swap exposure.

Next we examined the 3M movements of the A- swap portfolio which shows correlation to the credit spread option price especially when the exposure starts to decline towards the end of 2003 (Figure 3.6). Again, this might be coincidental since the swap portfolio is mainly receiving fixed in a period of low global growth where interest rates are falling.

Figure 3.6 3M Changes of AA- sub-portfolio vs 3M, 2Y AA- 40C option

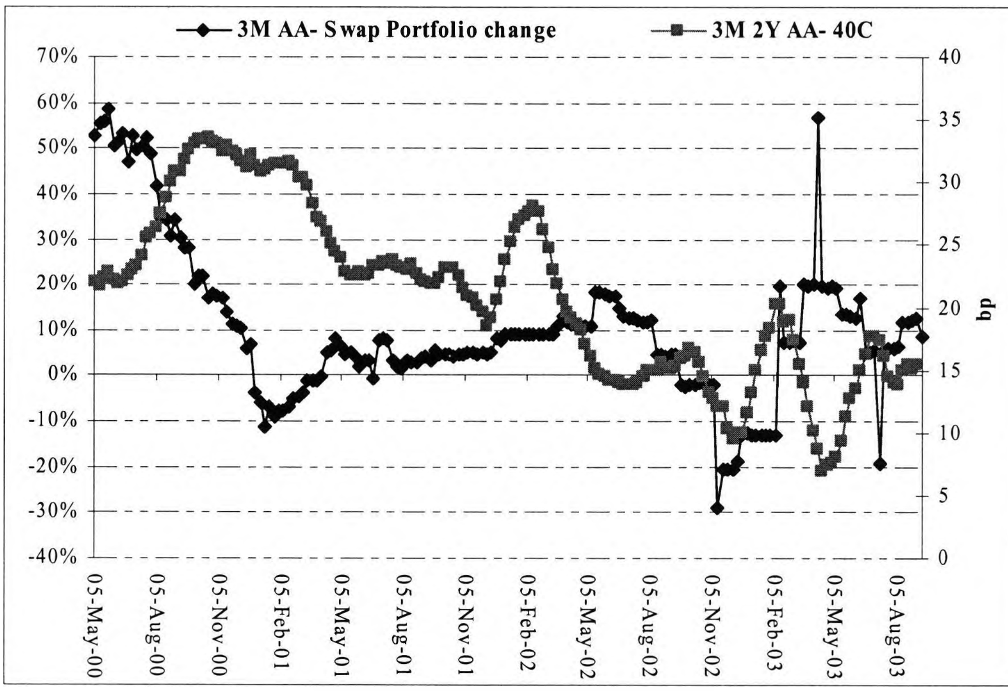
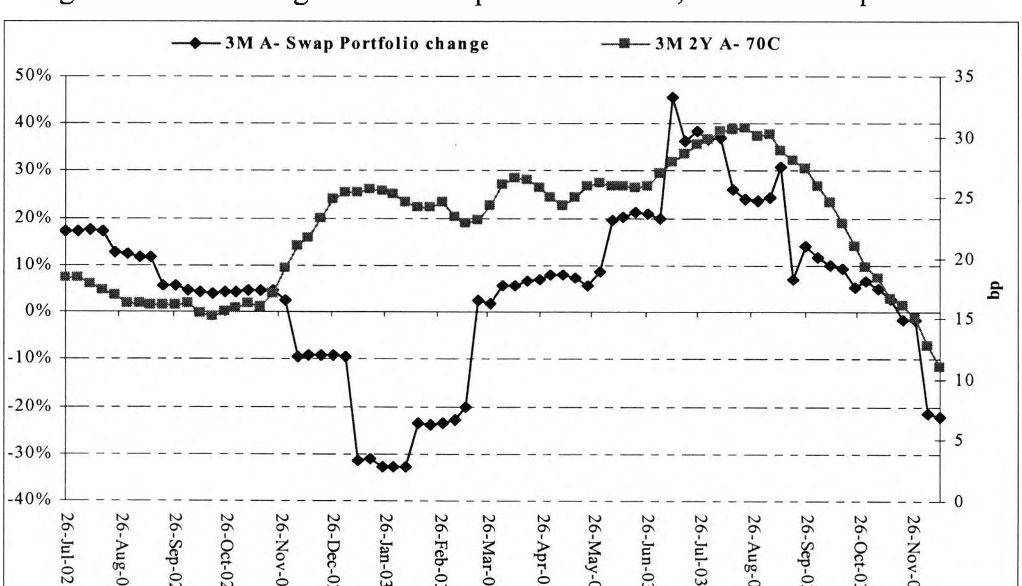


Figure 3.7 3M Changes of A- sub-portfolio vs 3M, 2Y A- 70C option

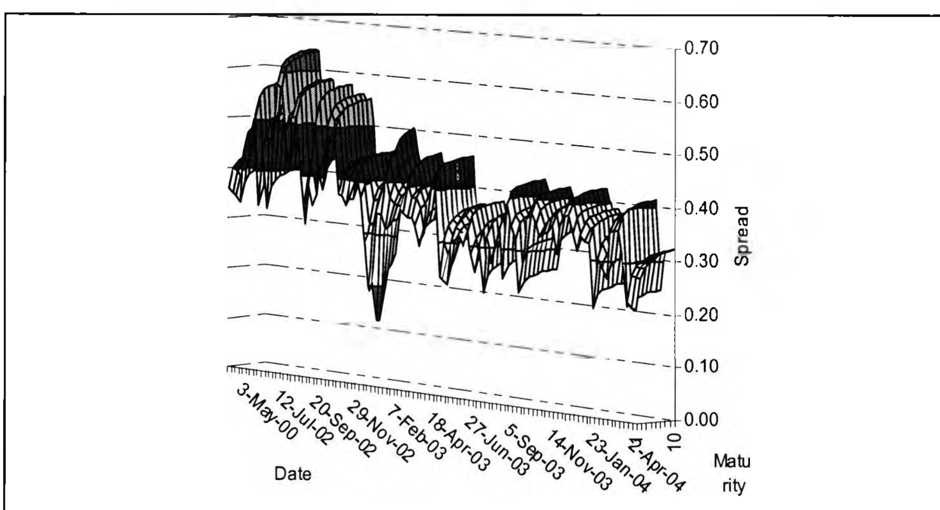


In that environment where you are overall short in swap rates and corporate debt servicing costs are low, swap exposures are increasingly positive and credit spreads are tightening.

3.4.5 Integrated Measure of swap market and credit risk

Credit spread index curves were estimated between 04/05/2000 – 07/05/2004 on a weekly basis as in Chapter 2 (as an example Figure 3.8 shows the AA spread index curves). The credit spread index option VaR was calculated using the same methodology as in section 3.2.3. The data range overlaps with the swap data used to run the VaR for the swap portfolios. Thus, there is an intrinsic correlation among all the market rates used to run the VaR for the swaps and the options. The VaR results for the option portfolios were backtested (for example Figure 3.9 shows the BB- weekly VaR to be backtested) in the same way as for the VaR of the swap portfolios (section 3.2.1). The VaR model seems to work across all cases examined[46].

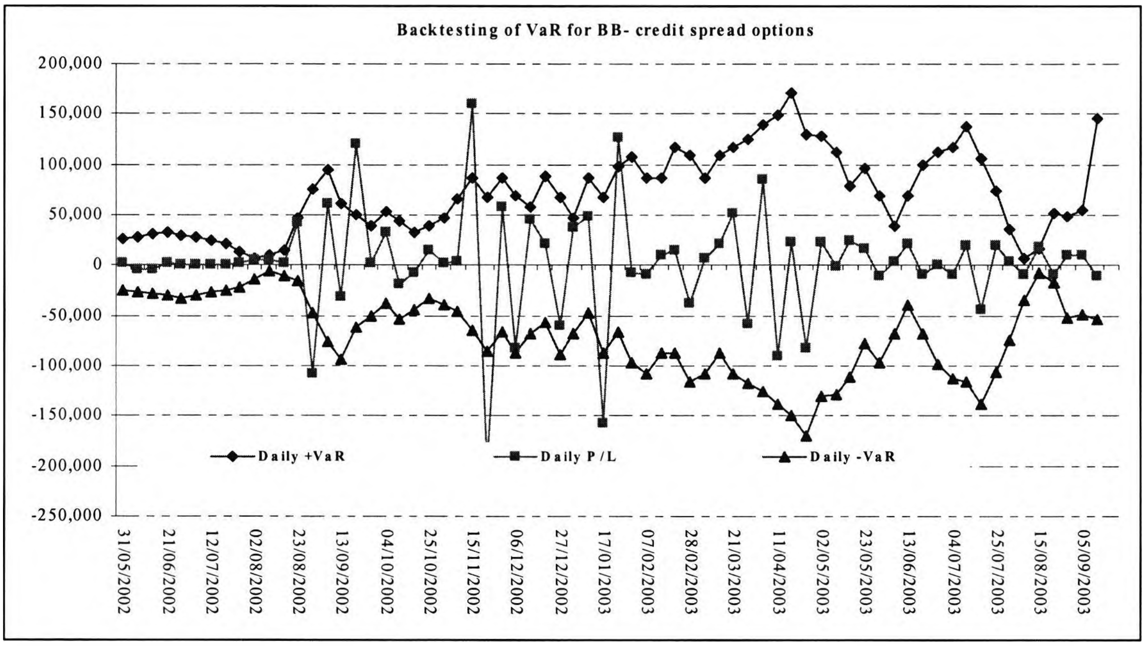
Figure 3.8 LS (1992) estimated AA spread index curves



[46] Figure4.7 shows the BB- case as an example.

Since the VaR model works for the credit spread options we can now run the total VaR for the overall portfolio per rating. The total VaR represents the maximum amount of money that could be made or lost in a week's time with 99% probability assuming market rates moves by a maximum of 2.33SD even if the counterpart defaults. In Figure 3.8 the contribution of the spread option market VaR to the overall VaR is plotted against the market value of the swap and the underlying 2Y

Figure 3.9 Backtesting of weekly VaR of BB- credit spread index options



spread[47] over time. The 2Y spread index has been relatively stable over a long period but during the first 6M of 2004 experienced a sudden increase, and hence the spread VaR increased. This is due to the fact that macroeconomic data out of the US have shown that the Federal Reserve is ready to start tightening its monetary policy. This had as an effect the unwinding of huge leveraged positions, so called global "carry" which affected corporate spreads across the globe. Hence, the theoretical cost of the credit spread index options in our portfolios increased substantially signalling that a period of low credit spreads may soon be over. This is an important phenomenon

[47] The 2Y spread index is also plotted to show how the VaR is moving against it. They are all weekly figures.

since a similar event has happened before[48]. The level of VaR contribution is at its highest when the 2Y credit spread index and the swap exposure is at its highest. The group of 30 study suggests that most dealers reserve 5 –10% of the MTM against such positions irrespective of the counterparts rating. Based on that methodology dealers can now have an overall measure of exposure which is linked to the volatility of a counterparts credit spread. When the overall figure is substantially higher to the market VaR of the swap portfolio it could mean that credit spreads have moved substantially higher, suggesting hedging of default risk. Of course this rule of thumb should always be scrutinised since the overall VaR could be higher simply because the volatility of the credit spread is quite high. The historical average ratio (over the same period of the historical simulation) of the credit spread index option VaR to the market VaR of the swaps (Table 3.3) differs across ratings since each exposure is different and the volatility of the underlying credit spread index differs. These ratios show how much more risk is held by running swap positions with counterparts of these ratings. The highest ratio comes with the AA+ rated swap portfolio indicating that the volatility of the spread and /or the exposure is quite high. We have to note that these ratios are not increasing exponentially like the probabilities of default and they should not, because they are directly linked to the movement of the underlying exposure which is different form rating to rating. A financial manager could assign threshold levels on these ratios in order to have a proactive hedging decision model. For example, if the ratio reaches at 50% then the exposure could be under review, and if the ratio reaches 70% then action has to be taken either to reduce the exposure or to hedge it.

[48] The LTCM crisis in 1998, which caused market rates to move in a very small period of time up to 4SD.

Effectively what is being examined is the "volatility" of the credit spread against the volatility of the swap exposure. It would be very surprising if the two variables were fully correlated since a brief examination between swap spreads i.e. 10Y – 6M was not correlated with any credit spread index (Appendix 2).

The swap VaR and the credit spread option VaRs were regressed so that their relationship is examined. The regression was carried out on the levels of the series of the VaRs. The VaR data counts between 89 and 102 points. The dependent variable is the swap VaR and the independent is the CSO:

$$\text{Swap VaR}(i) = a * \text{CSO VaR}(i) + e$$

None of the r-squared values are above 50% showing a weak relationship between the two VaRs. The highest one is at 42.36%, which comes with the AA rated swap portfolio and the AA credit spread index. There is no particular pattern of relationship emerging based on the regressions. We do not observe an increase or decrease according to higher or lower rating. However, each swap portfolio behaves differently to the respective spreads. This is the point where we observe that the swap exposure is linked to the volatility and level of the credit spread. The average R^2 of all the regressions is 12.40% (quite low) and only in one case the r-squared is close to 50%. The AA+ CSO VaR seems to be the one mostly correlated on the VaR levels. This is could be a highly important observation because this could give another signal to the dealer/risk manager that the portfolio needs to be hedged since its risks are highly correlated. The dependence of the swap VaR to the VaR resulting from the portfolio of credit spreads is quite important since it illustrates that there is an interdependence resulting form both market and credit risk. This relationship needs to be updated at regular time intervals in order to keep track on the level of correlation. The correlation of risks is a dangerous issue since this is the point that event risk could actually move rates in one direction with a catastrophic result. This is of course an added advantage of using this methodology to capture the default risk of a swap position. The regressions of the two risks have shown us the relationship (if any) between the volatility of credit spreads and swap exposures. Of course the relationship is not perfect and in some cases quite imperfect. However, it is a starting point where the swap exposures are being linked and expressed as a function of credit spreads. Table 3.7 shows the correlation between the 12-week rolling volatility of the 2Y to 6M swap spread against the 12-week rolling volatility of the 2Y respective credit spread index. The data period and frequency are as described in section 4.2 The reason that this method was not used was simply because the volatility of the credit spread derived by using the two factor LS model is not the same to the historical volatility. The LS volatility is determined by the model dynamics and is based on the observed market rates. Essentially, the described model where we deduce an integrated VaR measure was based on the fact that we can accurately price interest rate and credit spread options using the current term structure of interest rates, credit spreads and their respective stochastic volatility. This methodology is more fundamentally sound than just looking at the historical volatility between the two risky variables the swaps spreads and the credit spreads.

This regression needs to run at least once every 3-months in order to know the degree of correlation between the two risks, i.e. regression-wise. In that way the decision of "to hedge or not to hedge" is assisted by two extra important figures which are directly related to size, direction (exposure) and the "view" of the financial market of the credit situation of specific counterparts or better still the "view" in the credit market. The potential advantage of using a methodology such as this is that a bank could be better prepared before a big event happens. Because when a big event such as: LTCM crisis, Russian default, Asian crisis, Enron/Worldcom/Parmalat collapse happens, every single financial institution would need to get out of their positions. This immediately creates an overcrowding in the exit resulting in lack of liquidity, which eventually pushes market rates at levels, which no standard VaR model can ever predict. All this occurs because as it was mentioned earlier risks are highly correlated especially at periods of extreme market stress. Being proactive and informed it could be one way of protecting one's financial viability.

Figure 3.10 AA- sub-portfolio Market Value vs Spread VaR contribution

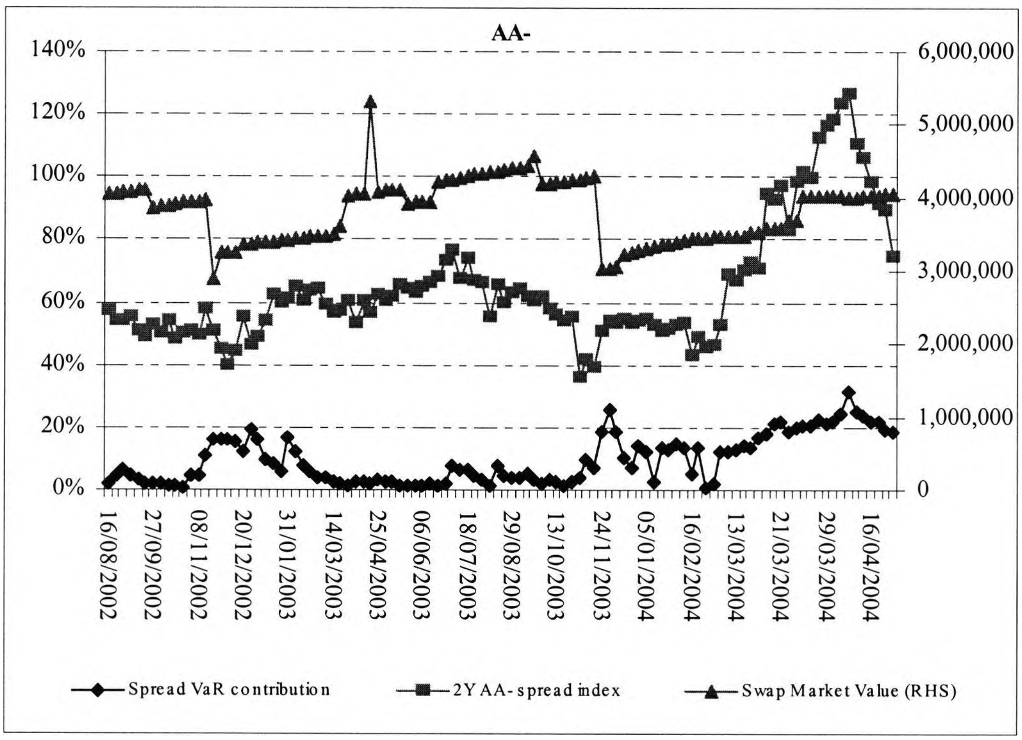


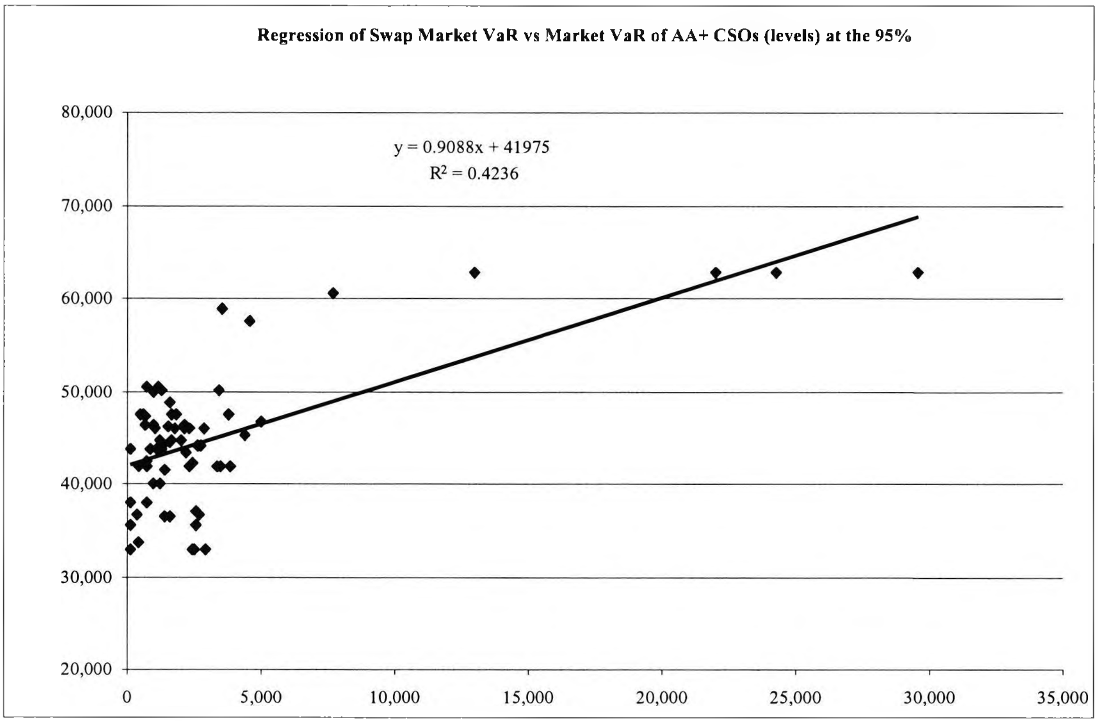
Table 3.8 Average Spread VaR contribution per sub-portfolio

Rating	AAA	AA+	AA-	A	A-	BB+	BB	BB-
Average Spread VaR contribution over 2Y	4.78%	32.74%	9.57%	2.9%	15.7%	17.3%	10.3%	32.5%

Table 3.9 R-Square Results

	R Square	SE	b	a
AAA	13.43%	7,271	112,380	8.43
AA+	6.63%	39,591	630,803	-0.85
AA	**42.36%**	**5,525**	**41,975**	**72.70**
AA-	17.05%	26,813	508,474	-1.00
A+	1.53%	91,573	889,804	-0.20
A	0.53%	18,189	325,868	-0.62
A-	5.55%	16,636	236,885	0.50
BB+	12.08%	11,059	84,296	-0.72
BB	1.20%	8,732	178,927	0.21
BB-	23.68%	4,244	212,368	0.23

Figure 3.11 Regression between AA+ Swap VaR against the VaR of AA+ CSIO



3.5 Summary

This study proposed an integrated measure of credit and market risk for interest rate swap portfolios. Using a dynamic hedging methodology we created a portfolio, which is sensitive to both market and credit risk factors. The market value distribution of that portfolio is the distribution one can use to derive a market VaR measure which accounts for both market and credit risk of interest rate swaps.

The hedging methodology was based on hedging the 3M exposure amount which arises from the potential future movement of interest rates. This potential 3M exposure was measured using a VaR historical simulation. The simulation was performed by fitting observed data to the LS model over time. The instrument of hedging the potential exposure is a credit spread index option, which was priced using the LS (1992) model. Under the same framework the options were simulated historically over the same period to determine the efficiency of hedging to the swap exposures. The relative change of the swap market value to the credit spread option market value was had low correlation showing that the hedging of the default risk was efficient.

The integrated measure is encapsulated in an overall market VaR figure of the market VaR of swaps plus the market VaR of credit spread index options (for HS only). The relationship between the two VaR figures was investigated using a linear regression. The relationship proved to be weak but a useful conclusion was drawn. The level of correlation per sub-portfolio could provide a "signal" to the risk manager of the degree of correlation between the two risks. This "signal can be used to either set trigger points where hedging actually takes place or by reallocating a credit line.

CHAPTER 4 Integrated Credit Risk Measurement

Abstract

This study presents a comparison between the existing methodology of integrated credit risk measurement and the proposed analytic "integrated" methodology (Chapter 3). The current approach combines expected credit exposures with default and recovery rates to generate loss distributions which are being used in capital allocation decisions. The proposed integrated approach combines the expected credit exposures with appropriate credit spread options and an exogenous given recovery rate to produce a loss distribution. Effectively the probability of default is replaced by an equivalent proxy, the market risk of the credit spread option over a pre-specified time horizon, which can be as short as 3 months. The comparison was performed on an actual swap portfolio taken from a medium-sized European Bank. The difference between using a historical simulation (HS) over a multi-step Monte Carlo (MC) simulation to measure expected credit exposures over a 3 month period is also being proven to be small but significant, concluding that the multi-step MC is a better tool than the HS to calculate expected credit exposures.

4.1 Introduction

4.1.1 Background

Financial institutions greatly need to integrate their approach to risk measurement. Under the 1998 Bank of International Settlements (BIS) accord, financial institutions are allowed to use their own internal risk measurement models subject to approval. Based on their models they are allowed to measure market and credit risk according to a VaR style methodology. Financial markets integration, complexity in financial products and huge growth of transactions mean that this approach is no longer adequate. In 1999 the Capital Adequacy Directive conceptual paper suggests that financial institutions need to integrate risk measurement across the trading and banking book to ensure that risk is consistent with one overall minimum regulatory capital risk management framework. The integration of market and credit risk measurement is inevitable. Both types of risks are computed from the same market value distributions taken at selected points in time over the life of a transaction.

Credit risk measurement for derivatives or counterpart credit risk exposure measurement has taken many forms and shapes to reach its current form today. The first stage was the so-called "notional approach". This approach defined the credit exposure of a transaction in terms of the notional amount. This unrealistic approach was quickly superseded by applying a static fixed percentage on the notional value of a transaction. The next step was to use a dynamic approach which measured future exposures and added to the MTM of the transaction, called the add-on methodology. The latest and most sophisticated methodology is the one which utilizes sophisticated systems to simulate potential credit exposures in probability terms.

Most financial institutions are measuring exposures using the third methodology (add-on), generating the add-on using different techniques. A number of institutions also use the more sophisticated fourth methodology.

4.1.2 The Problem

Integration in practice comes from the allocation of credit lines per counterpart or otherwise called counterpart exposure limits. These limits are being allocated mainly on credit grounds. Hence, if the exposure limit is being used up then no more transactions are being done James J. (1999). The monitoring and measurement of exposures is based on the current MTM plus a potential exposure or by simulating future exposures -using a Monte Carlo simulation-. Using these simulated exposures and probabilities of default a loss distribution is calculated. The resulting loss distribution provides the loss amounts at a given confidence interval over a pre-specified time period which is always no less than a year. This poses a restriction in the active management of default risk of a derivatives portfolio since capital allocation decisions have to be made based on the 1 year loss distribution and not less. The speed however of market events and occasionally of credit events is much faster than a year, requiring decisions on capital allocation decisions with a less than 1 year time horizon.

Also, there has to be a dynamic setting where financial managers know exactly how they can hedge their default risk arising from potential future exposures, which is linked to the credit markets.

4.1.3 The Significance of this Study

The probability of default has a dynamic structure which changes with time. The proposed integrated approach (Chapter 3) essentially replaces the probability of default with the amount of risk that the appropriate credit spread option bears. This chapter extends this methodology to estimate economic capital. The proposed approach in question is being tested for its efficiency by comparing it to current approaches of estimating economic capital.

We will show that the integrated approach gives similar results to current standard techniques. Also the ease of implementation in comparison to running a full Multi-step Monte Carlo can set this approach as the number one alternative.

4.1.4 Overview of the study

In this section we will use an actual swap portfolio taken from a medium-sized European Bank to extend our methodology in calculating economic loss. First we introduce the actual swap portfolio and a historical simulation is run in order to examine its market value over time. The portfolio is then split in sub-portfolios which contain swap deals with counterparts of the same rating. Using the hedging methodology proposed in Chapter 3, we theoretically hedge each of the sub-portfolios. Finally, we run historical simulation VaR of all the sub-portfolios including the credit spread index options used for hedging. In the second part we describe how many financial institutions model exposures arising from derivatives using a Monte Carlo simulation. In view of that we describe how financial institutions through the use of sophisticated risk measurement systems implement a multi-step Monte Carlo simulation in order to calculate potential future exposures. Subsequently we run a multi-step Monte Carlo for the swap portfolio, using the LS (1992) model as the diffusion process for the short rate and calculate the exposure profile of the swap portfolio. Using the Monte Carlo generated exposures we then calculate the expected and unexpected losses on the swap portfolio over a pre-specified time horizon.

The third part calculates the economic losses for the same swap portfolio as they would appear if one was using a VaR type methodology for exposure calculation instead of a multi-step Monte Carlo. We also compare the different outcomes of using the historical probabilities obtained from Moody's or the implied probabilities obtained using the LS (1992) credit spread curves. Finally we compare how our proposed hedging methodology compares in terms of calculating total loss of a swap portfolio. The comparison shows that for small time horizons a VaR type approach of generating exposures is not as inaccurate as it could be when one is looking at longer time horizons

4.2 Methodology

This section describes the methodology used in order to arrive at the two-fold comparison:

1. Between the historical and multi-step Monte Carlo simulations in calculating potential exposures.
2. Between the "standard" and "integrated" approaches in calculating economic capital.

The first comparison aims to show that based on the simulation set-up the potential exposures could vary, hence the amount subject to default risk can vary. The question though is by how much. The second comparison which is the main one aims to show the structural differences between the proposed approach and the standard approach. Furthermore, it is a great test of the integrated approach since the standard technique is being used for a number of years. The steps taken to arrive in the two comparisons are:

- Historical simulation using weekly data on the swap sub-portfolios.
- Estimation of weekly and 3M VaR for Swaps and Credit Spread Index Options.
- Multi-step Monte Carlo of the swap sub-portfolios including the 3M time-step.

At that point the economic capital measurement calculations are being introduced. There is the standard method of estimating Economic Capital and we've called that the standard method and is governed by equations (4.10) & (4.11) and the integrated approach which is an extension of the proposed integrated approach of Chapter 3. The integrated approach is based on the joint distribution[49] of the portfolio exposure and the price of the credit spread index option to estimate the expected and unexpected

[49] Both variables are independent

loss, where *Expected Loss* is the mean of that distribution and the *Unexpected Loss* is its standard deviation.

In the subsequent sections we describe the methodologies used in this Chapter.

4.2.1 Historical Simulation

4.2.1.1 Historical Simulation of Swaps

A historical simulation was carried out using the swap yield curves generated in chapter 4. The only difference was that the sample was extended to start from the 1^{st} of February 1999 and the end date this time is the 2^{nd} of April 2004 which is the date we carried out the Monte Carlo simulation in the subsequent section. The historical yield curves were estimated using the LS (1992) model for the period 27/08/99 – 02/04/04 as described in chapter 2. For each week the 6 parameters were estimated using the observed zero discount curve. The observed zero discount curve was bootstrapped as explained in chapter 2. For each weekly yield curve the appropriate forward curves were calculated using equation 3.2 as in Chapter 3. By appropriate we mean the forward curve required to price the swaps of the sub-portfolios. Of course the assumption is that the swap sub-portfolios remain unchanged throughout the historical simulation.

4.2.1.2 VaR of Credit Spread Index Options

A historical simulation was carried out using the credit spread index curves generated in chapter 3. The sample was extended to start from the 1^{st} of February 1999 and the end date this time is the 2^{nd} of April 2004. Each option was priced using the LS credit and volatility curves as in Chapter 2. Of course the assumption is that the swap sub-portfolios remain unchanged throughout the historical simulation.

4.2.1.3 Historical Simulation of Implied Default Probabilities

The risk of default is a dynamic process which takes a different shape every day. It is a process which is affected by many factors and is directly linked to credit spreads (Chapter 2). Using the same methodology as in chapter 2 of implying the transition matrix based on a given credit spread discount curve and the Moody's historical transition matrix, we implied default probabilities over a period of 2 years using the relevant estimated LS (1992) credit spread discount curves (Chapter 2). The historical simulation will demonstrate how the probabilities of default change every week (the simulation was performed with weekly data) according to market expectations captured by the dynamics of the term of credit spreads.

The implied transition matrix estimated in chapter 2 was with the assumption that default might occur only once at every time step i.e. once every year. The transition matrix was implied using the risk premia observed in the term of the credit spreads for every week between Apr-02 and May-04. For each week the eigenvalues and eigenvectors were implied and the respective generator matrices were obtained. A plot of the eigenvalues can be found in chapter 2, Figure 2.26. Using the implied generator matrices and the eigenvalues for every week it was straight-forward[50] to derive the implied default probabilities for any maturity just by reading the last column of the generator matrices.

4.2.2 Multi-Step Monte Carlo

Monte Carlo simulation is a simulation technique which has been in use for a long time in many industries. In the finance and banking industry it is predominantly used

[50] Since the eigenvalues can be multiplied by the square root of time we can deduce the probabilities of default for any maturity.

for pricing complex financial products and for scenario generation. The technique has been near perfected over that last few years and is being used by a large number of financial institution for a number of tasks: pricing of complex financial instrument where there is no closed form solution, Value at risk, scenario generation and for calculating expected credit exposures. Using a pure VaR type approach to calculate expected credit exposures as in previous sections it poses a few problems. The overall VaR figure might not incorporate events which take place in the tails of the distribution. Also the VaR figure is usually measured over a static time horizon (1 week or 1 day) and then multiplied with the square root of time to transform it to a VaR over a longer time horizon. This is not exactly correct because by doing that we are assuming that all market rates follow a Markov chain. Thus, a Monte Carlo simulation is a more appropriate method to use when one is calculating expected credit exposures, since its possible to create an abundance of scenarios over many different time horizons each time.

There are a few general steps that are taken to ensure a successful result. Here, we will describe briefly the industry standard and subsequently we will describe the methodology we used to generate scenarios.

4.2.2.1 General Multi-Step Monte Carlo

Monte Carlo scenario generation for market rates is based on a discrete version of a multivariate process of $\log(S(t)) = \log(S_1(t)), \log(S_2(t)), \ldots \log(S_N(t))$ risk factors Sj, $j = 1, 2, \ldots, N$. The risk factors could be anything from interest rates, FX rates to equity indices. Their evolution in time is governed by the stochastic differential equation

$$d\log(S_j(t)) = dW_j(t) \quad (4.1)$$

Here, W is a multivariate Gaussian diffusion process such that, at time t, $W(t)$ has a covariance matrix $L \cdot t$, where L is associated with a given correlation matrix ρ and volatilities σ_j of the log-return of the risk factor S_j. Typically the σ_j, σ_i and ρ are estimated historically[51] (for a given confidence interval) The correlations among the risk factors are incorporated through ρ.

The discrete-time process is usually determined by the stochastic difference equation

$$\log(S_j(t+\Delta t)) - \log S_j(t)) = K_j\sqrt{\Delta t} \quad (4.2)$$

The vector K on the right side of equation (4.2) is *i.i.d.* with $K \sim N(0,Q)$. The main task is to simulate this random vector. For each j, $j = 1, 2, ..., N$, the random number K_j is calculated. The change between the simulated rate and the original rate at this risk factor node follows as:

$$S_j(t+\Delta t) - S_j(t) = S_j(t)(e^{K_j\sqrt{\Delta t}} - 1) \quad (4.3)$$

The general approach to generate a random number x from an arbitrary continuous distribution with cumulative distribution function $F(x)$ is to generate y uniformly in [0,1] then to solve $y=F(x)$. However, for generating a normal random number, the Box-Muller algorithm (Sheldon R (1994)) is used instead of solving the inverse problem. The Box-Muller method was modified using a transformation method in order to simplify the calculations Appendix(4).

Since the generated normally distributed numbers are not correlated they need to be converted in correlated random numbers (Jacobi method see Appendix 5 for VB code). The statistical volatilities and correlation matrix of the log-returns of the risk factors are σ_j and ρ, respectively. Set L to be the associated covariance matrix:

[51] There are a number of financial software providers which use the RiskMetrics datasets, which are volatilities and correlations estimated using an exponential weighted moving average method (EWMA).

$$L_{ij} = \sigma_i \sigma_j \rho_{ij} \quad (4.4)$$

The eigenvalues and eigenvectors of the variance covariance matrix L were calculated. The decomposition of a symmetric matrix L is given by:

$$L = VDV^T \, (4.5)$$

where V is an NxN orthogonal matrix, i.e. $VV^T = I$. The V^T matrix is the transpose of V and D is an NxN matrix with the N eigenvalues of L along its diagonal and zero elsewhere. The eigenvalues need to be checked to see if any negatives are being produced and in such cases zero should be used. Next we describe how we run multi-step Monte Carlo for our swap sub-portfolios.

4.2.2.2 Multi-Step Monte Carlo using the LS (1992) model

The simulation framework described is used in order to generate different scenarios on multiple market risk factors. In our case we would only consider one risk factor, which is the short interest rate. In that case equation 4.3 would be different since a mean reverting model would be used to simulate the evolution of the short interest rate

$$dr_t = \kappa(\theta - r_t)dt + \sigma\sqrt{r_t dz} \quad (4.6)$$

Equation 4.6 is a one-factor equilibrium model CIR (1985), which is driven by the movement of the short interest rate dr_t. Again the discrete version of that model would be used to simulate the change in the short interest rate in order to estimate the full yield curve. Often other models with more than one factor are being used to simulate for the short rate and in this section we used the LS (1992) two-factor equilibrium model as described in Chapter 2. The discretised version of the model is given by:

$$r_{t+\Delta t} - r_t = \left(\alpha\gamma + \beta\eta - \frac{\beta\delta - \alpha\xi}{\beta - \alpha} r_t - \frac{\xi - \delta}{\beta - \alpha} V_t \right) \Delta t + \alpha \sqrt{\frac{\beta r_t - V_t}{\alpha(\beta - \alpha)}} \sqrt{\Delta t} \varepsilon_{1,t+\Delta t}$$

$$V_{t+\Delta t} - V_t = \left(\alpha^2\gamma + \beta^2\eta - \frac{\alpha\beta(\delta - \xi)}{\beta - \alpha} r_t - \frac{\beta\xi - \alpha\delta}{\beta - \alpha} V_t \right) \Delta t + \alpha^2 \sqrt{\frac{\beta r_t - V_t}{\alpha(\beta - \alpha)}} \sqrt{\Delta t} \varepsilon_{1,t+\Delta t} + \beta^2 \sqrt{\frac{V_t - \alpha r_t}{\beta(\beta - \alpha)}} \sqrt{\Delta t} \varepsilon_{2,t+\Delta t}$$

(4.7)

where the ε_1 and ε_2 are normal i.i.d random variables. The two i.i.ds were randomly generated using the routine in Appendix 4. The LS model in its nature estimates yields curves for any given r and V. In the LS model the r and V are almost fully correlated (Chapter 2). For our purposes we will incorporate the actual correlation between r and V as observed. Hence, using the 3M EUR-LIBOR and estimating its volatility with GARCH(1,1) as described in chapter 2 we obtained a 4 year time series of r and V. It was clear that the correlation between r and V was time-varying. However, it was not difficult to observe that it was reverting to a mean level. That was the level of ρ we used to correlate the randomly generated variables. Using the Cholesky decomposition we transformed the two random variables to be correlated. In this case where we have two variables is easy to decompose the correlation matrix and arrive to the following:

$$\begin{aligned} &\varepsilon_1 = \mu_1 \\ &\varepsilon_2 = \rho\mu_1 + (1 - \rho^2)^{1/2} \mu_2 \\ &\textit{where } \rho \textit{ is the correlation between } \varepsilon_1 \textit{ and } \varepsilon_2 \\ &\rho \textit{ was at } 31.32\% \end{aligned} \quad (4.8)$$

The Monte Carlo simulation carried out in Chapter 2 was a single step, providing an implied distribution for the short rate conditional on its volatility. Our aim here is to produce a distribution of values at discrete time steps in the future, in order to generate yield curve scenarios. These yield curve scenarios were used at each time step as it occurs in the future to re-value each swap.

Let t_0 denote the current time, and let T denote the end date of simulation: $T = t_0 + (n - 1)\cdot\Delta t$, where Δt, is the step length and n is the number of steps. The intermediate simulation dates are $T = t_0 + (i - 1)\cdot\Delta t$, for $i = 1,\ldots,n$. This is what is otherwise known as a multi-step Monte Carlo simulation. At each time step 1000, scenarios were performed. An important step when one is using a normal distribution to generate interest rate yield curves is to check for negative interest rates. This was done after the generated curves were estimated and any negative rates produced were removed.

The simulated time step was chosen to be 3 months or to be exact 90 days on a 360 days per year. The main reason for that choice was to facilitate previous analysis[52] (section 4.1 and chapter 3). Usually financial institutions use different time-steps[53]. At the first step of the simulation the mean of the distribution of the short interest rate and its volatility was recorded and values at the 2.3SD. Then two yield curves were estimated based on these values and the swaps were revalued based on these estimated yield curves, yielding two MTM values. The first one was the average of the MTM and the second one was the MTM at 99% confidence interval. The next step was similar to the first one with only the time difference being brought forward by 3M. Hence, all the swaps were re-valued as if the value date was plus 3M. For example, a swap with outstanding maturity of approximately 2 years at value date is re-valued on the second time-step[54] as a swap with time to maturity of 1.75 years adjusting for any missing cash flows. Once this is done that forward MTM is

[52] We need to have the same time horizon to the historical simulation
[53] Usually 2D, 1W, 2W, 1M, 3M, 6M, 9M, 1Y, 2.5Y, 5Y and 10Y. These time-steps closely match the time buckets used for measuring market risk for fixed income securities. The 2D scenario is used to calculate settlement risk. For our purposes the 3M is being used because we want to make a comparison between the Historical Simulation and the Monte Carlo and also because our integrated measure is based on 3M intervals.
[54] Since the first time-step is the value date.

discounted to the value date using the value date curve. These steps were repeated each time until the date of the longest maturity of the swaps sample[55]. Finally, the aggregation was done on the level of receiving and paying cash flows rather than the level of the overall swap[56].

4.2.3 Hedging the swap exposures

Using the same methodology as in chapter 3 we will calculate the option premium required to hedge these exposures. The main difference between the example taken in chapter 3 is the different maturities. Each sub-portfolio consists of swap deals, which mature in half a year to 9 years instead of the 2 years generic example in chapter 3. This poses a practical "problem" in implementing the hedging strategy which is the question of which maturity of credit spread index we have to use in order to carry out our methodology. There are two answers to that question. First, we can examine each swap separately and hedge that exposure by matching[57] its maturity using the appropriate tenure of credit spread index. The second answer is instead of matching everything deal by deal, we can chose the point where the largest exposure lies and match the maturity of that exposure using a single maturity of credit spread index. The first solution has the advantage that, hedging of the counterpart default risk becomes more accurate but in practice it accumulates a number of credit spread index options. Furthermore, implementation becomes more laborious both in practical and theoretical terms. The second solution has not the advantages of the first one but it is

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[55] The longest maturity was in sub-portfolio 5 with 8.91 years.

[56] All the paying swaps and all the receiving swaps were pooled separately per sub-portfolio and after they were re-priced they were netted.

[57] In chapter 4 we explained that the hedging methodology was designed to hedge the movements of the swap exposures using 3M options written on X years credit spread indices. The maturity of the credit spread index would have to match the maturity of the exposure, i.e. the options were 3M on 2Y underlyings.

also a good approximation in terms of risk management since what we are trying is to test if the integrated measure of risk works in a real-time example.

Hence, the spread index maturities chosen per sub-portfolio were matched with the maturity of the largest exposure per sub-portfolio (table 4.4). Their long-term mean as of the 2nd of April of 2004 was estimated in order to calculate the option notionals. The exposures used were the sum of the exposures up to the highest exposure. For example, in sub-portfolio 5 (AA) the total current exposure used is 3,455,018 EUR which is the sum of all the swap exposures up to 8.91 years of maturity. In this case, by coincidence the sum of the total current exposure of sub-portfolio 5 is the same figure. Hence, the credit spread index maturities used are the 2Y AAA, the 9Y AA, the 2Y A and the 2Y BBB.

Running a historical simulation weekly VaR (99% confidence interval) as in chapter 4 we have calculated the potential movement of the swap sub-portfolios at the 99% confidence interval. By adding the VaR figure to the current MTM, we obtain the total potential exposure or the expected credit exposure (E(CE)) of the 4 sub-portfolios in 3M[58]. These are the exposures we need to hedge of default risk. Again we have to make the following assumption: *All counterparts of the same rating bear the same probability of default*[59]. Hence, using the strike prices (table 5.4), we priced[60] the following 3M options: 28C on 2Y AAA, 56C on 9Y AA, 78C on 2Y A

[58] The weekly VaR is being scaled by the square root of time in order to increase the magnitude of the weekly VaR to the 3M VaR.

[59] This is an unavoidable assumption due to the lack of data. Given the data we wouldn't use credit spread indices but counterpart related credit spread data. The main reason is that this assumption has proven to be too rigorous in many occasions. However, using credit spread indices was the only way to demonstrate our idea.

[60] The pricing was done under the LS (1992) framework.

and an 81C on the 2Y BBB. Thus, neutralizing the default risk of each sub-portfolio we need to buy an equivalent amount of options to the total expected exposure.

This hedging methodology is designed to protect against the worst outcome which is full default and zero recovery. However, in reality default is treated in terms of probabilities and usually expected losses are measured based on these probabilities. Also the loss given default or otherwise known recovery rate is taken into account. If we assume that there is recovery under the equivalent recovery model of JT (1995) then we can apply a recovery rate in the total expected exposure and recalculate the amount of options need to be bought in order to hedge the swap exposures (Table 4.5). The amount required –option notional (2)- is substantially smaller across all sub-portfolios. This is a more realistic assumption since the recovery is not zero, instead it's the weighted recovery rate as calculated by Moody's over different classes of debt on actual historical default.

4.2.4 Economic Capital Measurement

4.2.4.1 Standard Approach

At this stage we need to introduce two measures, which have been used lately by practitioners to calculate total economic loss due to credit risk. The total economic loss consists of two components, the *expected loss* and the *unexpected loss.* The expected loss is governed by the distribution of three major factors, i.e. credit exposure, default rate and recovery rate data. It is generally accepted that if these three distributions are combined then one can integrate across the combined function to estimate the expected loss at a given time point:

$$EL = \int\int\int (CE)(\Pr Def)(1 - RR)F(CE, PrDef, RR)\, dCE\, dPrDef\, dRR \quad (4.9)$$

$$\begin{aligned}&\textit{for a single exposure}\\&EL = E(CE) * (1 - RR) * \Pr[Def(t_i, t_{i+1})]\\&\textit{where } E(CE) \textit{ is the expected credit exposure}\end{aligned} \quad (4.10)$$

where CE is the credit exposure, dCE is the derivative of CE, PrDef is the default rate, dPrDef is the derivative of PrDef, RR is the recovery rate and dRR is the derivative of RR and F(CE,PrDef,RR) is the multivariate probability density function. If we assume statistical independence between CE, PrDef and RR, then if the recovery rate is zero then one can multiply the probability of default to the expected credit exposure to calculate the economic loss. If however the assumption is that the probability of default is 100% and the recovery rate is x, then the economic loss is the expected credit exposure multiplied by (1 - the recovery rate).

The unexpected loss for a binomial event for a single exposure is given by:

$$UL = ECE * (1 - RR) * \sqrt{\Pr[Def(t_i, t_{i+1})](1 - \Pr[Def(t_i, t_{i+1})])} \quad (4.11)$$

which is the volatility of the expected loss at a given confidence interval and at a given time in the future.

The standard approach[61] being used by financial institutions utilises equations (4.10) and (4.11) in order to calculate the expected and unexpected losses of the swap sub-portfolios. Both equations involve the probability of default. The probability of default will provide variants of the standard approach depending on which probability is being used. Hence:

- ***Method 3*** uses the probability of default taken from the last column of the historical transition rating matrix (Moody's).
- ***Method 4*** uses the implied probability of default estimated using the methodology described in Chapter 2.

[61] There is no specific name for this methodology; we will call it the standard approach in order to differentiate from the integrated approach.

- ***Method 5*** uses the 2.3SD implied probability of default.

4.2.4.2 Integrated Approach

The probability of default is a factor which can be described by the credit spread of a counterpart. In chapter 2 we saw that one can use the term structure of credit spreads to imply probabilities of default using the binomial model of JT (1995) and the Markov chain model of JLT (1997). Hence, any change in the term structure of credit spreads is reflected in the probability of default of a counterpart.

Our integrated measure, which assumes dynamic hedging of swap exposures using credit spread options is an attempt to link the expected credit exposure to the probability of default through the dynamics of credit spreads. The relationship between the two is not statistically significant since the VaR regressions in chapter 4 yielded low r-squared. However, in terms of economic loss the exposure and probability of default are linked since the likelihood of a loss increases with an increase in the credit spread and the size of the loss increases if the exposure increases. Thus, the notion of using credit spread options in order to "replace" economic loss of a swap sub-portfolio is quite appealing. Thus expected loss can be given by the mean of the portfolios of credit spread index options.

As with the expected loss, we will equate the unexpected loss to the potential movement of the credit spread index options at a given point in time, i.e. for our purposes the 3M VaR of the credit spread index options based on the exposure calculated option notionals.

Using option notional (1) (Table 4.5) and option notional (2) we run a historical simulation of each of the credit spread index options in order to calculate the average movement and the 2.3SD movement over 3M. The 3M time horizon is chosen because this is the maturity of the credit spread index options.

The integrated approach is based on a dynamic hedging methodology which uses credit spread options to hedge swap exposures. The idea is that as the exposure fluctuates so is the amount of options required to hedge the exposure. Also, as the credit spread widens the moneyness of the call options used to hedge the credit exposure increases protecting from an increasing probability of default[62]. The end result is a creation of a portfolio of swaps and credit spread options which has zero default risk when its rebalanced continuously. The market value distribution of that portfolio is sensitive to both market and credit risk factors. The economic capital calculation of that portfolio then becomes the total of the expected and unexpected loss due to the credit part of the portfolio. This is due to the fact that the credit part of the portfolio is linked to the potential credit exposure of the swap portfolio through its notional and the probability of default is represented by the price of the credit spread option. The integrated approach has two variants which depend on the recovery rate:

- ***Method 1*** assumes recovery rate of 1 and
- ***Method 2*** assumes recovery rate of 0.3265.

with the first being a worst-case scenario of zero recovery and the second being the standard case where there is recovery.

4.3 Data and Description

There is a data overlap between chapter 4 and this chapter apart from the swap portfolio. The swap portfolio was provided by a medium sized European Bank, which is active across all asset classes with a substantial business in the EUR interest rate

[62] We have previously shown that the probability of default is directly related to the credit spread of a counterpart, hence when the credit spread of a counterpart widens then the probability of default deteriorates and vice versa.

swaps market. However, this is not the overall swap portfolio of the bank but is only one of the actively traded books.

4.3.1 Swap portfolio

This study uses a unique actively traded OTC portfolio of swaps taken from a medium sized European Bank. The actual swap portfolio contains 53 vanilla interest rate swaps denominated in EUR. The reporting currency for the medium sized European bank is in EUR hence all the aggregated figures will be in EUR. Overall the portfolio is short which means that if rates move down the portfolio makes money and if rates move up then the portfolio loses money. This is observed by looking at the overall DV01 (short for the sensitivity of the swap position to 1bp movement in the underlying interest rates) in Table 4.1. The portfolio contains deals with various European and US counterparts of different rating. Although the bank in question has an approved internal rating system[63] we used the S&P long-term rating to rate each swap deal according to the rating of the counterpart. In total, there are 15 different counterparts with 7 different ratings. They are all rated at the investment grade spectrum, i.e. AAA, AA, AA-, A+, A, A- and BBB. The medium-sized European bank has netting agreements with all the counterparts in the portfolio. This is important because we can sum up all the exposures per counterpart. We also summed up all the exposures based on the counterpart rating and created the sub-portfolios which we were used to calculate the integrated measure and economic loss. This summation was based on the assumption that counterparts of the same rating follow the same default process. The portfolio consists of deals in EUR currency only and the counterparts that the deals have been made with are mainly European. There are

[63] According to BIS 1998 banks can use only approved internal rating systems for all their customers if and only if they have approved risk management systems. This is done mainly to maintain consistency in the way each bank calculates credit risk in their portfolios.

only 4 US counterparts which issue bonds in their name in both US and Europe. Hence, for our subsequent hedging purposes we only need to use European credit spread indices.

The market risk per sub-portfolio in terms of parallel yield curve shift of one bp differs across the sub-portfolios. Generally speaking if interest rates move upwards in the Euro-area then most of the sub-portfolios will lose money. In contrast, the credit exposure of these portfolios will be reduced since the actual credit exposure is defined as the maximum of zero or a positive Mark-to-market is at time t, where t is equal to the value date[64].

Table 4.1 Actual Swap Portfolio taken from Medium Sized European Bank 02/04/04

DV01 (EUR)	Sub-Portfolio No	S+P rating	Notional	Long/Short	Time to maturity (Years)	Currency
475	1	A+	15,115,950	Long	0.56	EUR
	1	A+	25,564,000	Short	0.66	EUR
	1	A+	10,225,838	Long	1.52	EUR
	1	A+	35,790,432	Long	4.41	EUR
	1	A+	5,000,000	Long	4.54	EUR
	1	A+	15,000,000	Short	4.73	EUR
	1	A+	25,000,000	Short	5.74	EUR
	1	A+	10,000,000	Long	0.06	EUR
	1	A+	10,000,000	Long	0.08	EUR
	1	A+	25,000,000	Short	0.63	EUR
	1	A+	50,000,000	Short	1.01	EUR
	1	A+	45,000,000	Long	3.08	EUR
	1	A+	10,000,000	Short	3.59	EUR
	1	A	10,000,000	Long	3.96	EUR
	1	A	11,000,000	Short	4.23	EUR
	1	A	10,000,000	Short	4.94	EUR
	1	A	10,000,000	Short	4.98	EUR
	1	AA	10,000,000	Long	5.07	EUR
	1	AA	5,000,000	Long	9.68	EUR
13,879	2	AA	30,000,000	Short	0.39	EUR
	2	AA	25,000,000	Short	0.46	EUR
	2	AA	50,000,000	Long	0.89	EUR
	2	AA	14,500,000	Long	1.15	EUR

64 Typically value date is the date at which OTC deals are being settled in each currency. For the EUR currency value date is T+2.

	2	AA	10,000,000	Long	3.21	EUR
	2	AA	15,000,000	Long	4.58	EUR
-506	3	AA	9,810,833	Short	0.14	EUR
	3	AA	14,534,567	Long	0.61	EUR
	3	AA	7,267,283	Long	0.66	EUR
	3	AA	14,897,931	Short	1.16	EUR
14,561	4	AAA	30,677,513	Long	0.80	EUR
	4	BBB+	25,564,594	Short	4.48	EUR
	4	A+	9,447,468	Long	2.64	EUR
	4	A+	15,988,024	Long	0.61	EUR
	4	A+	51,129,188	Long	1.38	EUR
	4	A+	10,000,000	Long	5.92	EUR
	4	A+	10,000,000	Long	9.19	EUR
	4	A+	12,000,000	Short	2.59	EUR
	4	A+	26,000,000	Long	3.83	EUR
	4	A+	10,000,000	Short	8.90	EUR
-33,282	5	A+	20,000,000	Short	3.44	EUR
	5	A+	15,000,000	Short	1.41	EUR
	5	A+	35,000,000	Short	3.44	EUR
	5	A+	10,000,000	Long	3.98	EUR
	5	A+	30,000,000	Long	4.88	EUR
	5	A	20,000,000	Long	1.14	EUR
	5	A	15,000,000	Short	4.25	EUR
	5	A	10,000,000	Short	4.89	EUR
	5	A	10,000,000	Short	4.98	EUR
	5	AA	10,000,000	Short	5.24	EUR
	5	AA	10,000,000	Short	8.71	EUR
	5	AA	10,000,000	Short	2.98	EUR
-2,200	6	AA	10,000,000	Short	2.20	EUR
-1,551	7	AA	10,225,838	Short	1.52	EUR

4.3.2 Historical Yield Curves

The time period of our analysis conveniently overlaps with the time period in Chapter 3. Market rates used in Chapter 4 were also used in this chapter with one minor difference in the final date. In this chapter the time series of 3M EUR LIBOR, 6M EUR LIBOR rates and 1Y, 2Y, 3Y, 4Y, 5Y, 7Y, 8Y, 9Y, 10Y EUR Swap rates end in 02/04/04. The reason is that the 2nd of April of 2004 was the date that the multi-step Monte Carlo was performed.

4.3.3 Credit Spread Indices and credit spread curves

The credit spread index data collected from Bloomberg as noted in Chapter 4 were used. The final date is the 2nd of April 2004 for the reason explained in 5.3.2. However, in this chapter we used 4 credit spread indices since we only need to measure the default risk of positive exposures.

Table 4.2 Long-term mean of CSIs and their GARCH(1,1) volatility

Rating / Maturity	GARCH(1,1)	2.33 Vol	Long-term mean	Mean * (1 + 1SD)	Mean * (1 + 2.33SD)
AAA 2Y	32.97%	76.83%	28	37	50
AA 9Y	29.76%	69.34%	56	73	95
A 2Y	38.48%	89.66%	78	108	148
BBB 2Y	35.84%	83.51%	81	110	149

4.3.4 Historical Transition Matrix and recovery rate

The historical transition matrix was obtained from the JLT (1997) paper, including the recovery rate (Chapter 2).

4.4 Results and Analysis

4.4.1 Historical Simulation

4.4.1.1 Historical Simulation of swap sub-portfolios

For credit risk measurement purposes, if there were no netting agreements in place we would have to disregard the swap deals with a negative net present value (NPV). In that case only the positive NPVs would be considered and the risk profile would be much different (Table 4.3). The DV01s are much different per sub-portfolio compared to the DV01s shown in Table 4.1. All the DV01s are negative indicating that all the exposure is one way, thus if interest rates were to go down the overall credit exposure would increase rather substantially. However, this is not the case since the netting agreements allow negative and positive NPVs to be summed, thus reducing the overall exposure.

Table 4.3 Current MTM of Sub-portfolios

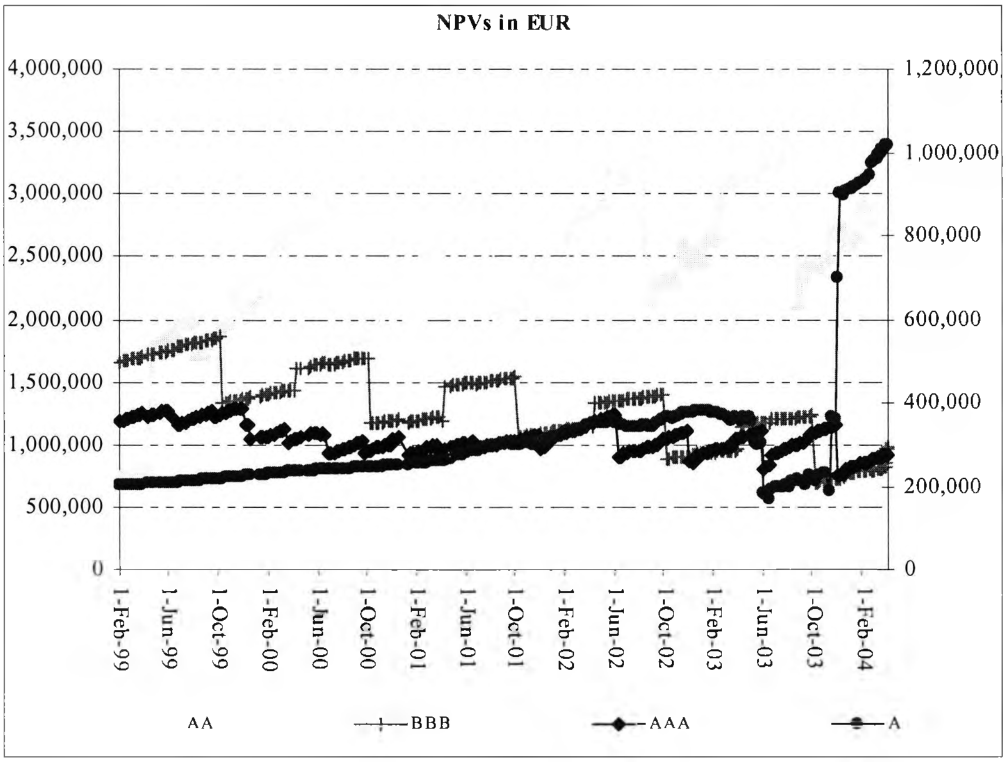
Sub-Portfolio No	S+P rating	Long/Short	Currency	DV01	Actual Credit Exposure
6	AAA	Short	GBP	-2,200	**276,664**
4	AA	Short	EUR	-23,447	**3,452,793**
5	AA-	Short	EUR	-29,674	-2,490,101
1	A+	Short	EUR	-7,497	-1,478,964
2	A	Short	EUR	-2,343	-1,152,408
3	A-	Short	EUR	-1,869	**1,017,100**
7	BBB+	Short	EUR	-1,551	**972,223**

At that date only 4 out of the 7 sub-portfolios had an overall positive NPV. The time series of their NPV is shown in Figure 4.1. The sudden drops observed at regular time intervals are outgoing cash flows, since we haven't included any of the accumulated accrued or realized P/L of the swaps. However, all time series seem to be increasing over time which is what we would expect. The huge increase of NPV in the A sub-portfolio comes from the change in the NPV of a single swap[65]. Throughout the last 4 years these swaps were running a positive market value bearing actual credit exposure. The time series of the A- market value is a clear example of how the swap market values can change substantially over time. This is one of the reasons that stochastic interest rates are used in order to model future expected exposures. We can also verify the amortisation effect of the swap exposures by looking the AA swap market values. There are regular drops in the market values, which they would have

[65] It's a EUR swap which receives fixed at 5.6% and the LIBOR at that time goes below 2% for the first time, making the forward curves to steepen unexpectedly.

resulted in a zero market value at the maturity of the swaps. But because we have a portfolio of swaps with different maturities, the amortisation effect will kick in when all the swaps left in the portfolio are passed their half-life. The dramatic increase in the NPV of the A sub-portfolio is due to monetary changes by the ECB during that period.

Figure 4.1 Time series of the market value of sub-portfolios



The sub-portfolios created following our assumption of netting all exposures which arise form counterparts of the same rating are shown in table 4.4. Only 4 of the sub-portfolios exhibit positive exposure. The other 3 have zero exposure since their current mark-to-market is negative. Negative MTM plus potential exposure is termed as wrong-way exposure. Not accounting in this study for these exposures does not mean that we disregard them completely. It just happens that the total expected credit exposure after 3 months is still negative.

Table 4.4 Current credit exposure of sub-portfolios

Sub-Portfolio No	S+P rating	Notional	Long/ Short	Time to maturity (Years)	Currency	Actual Credit Exposure EUR
6	AAA	10,000,000	Short	**2.20**	EUR	**276,664**
						276,664
5	AA	20,000,000	Short	3.44	EUR	753,806
5	AA	15,000,000	Short	1.41	EUR	1,005,502
5	AA	35,000,000	Short	3.44	EUR	749,534
5	AA	10,000,000	Long	3.98	EUR	-443,384
5	AA	30,000,000	Long	4.88	EUR	208,614
5	AA	20,000,000	Long	1.14	EUR	-523,641
5	AA	15,000,000	Short	4.25	EUR	424,754
5	AA	10,000,000	Short	4.89	EUR	-79,203
5	AA	10,000,000	Short	4.98	EUR	-66,293
5	AA	10,000,000	Short	5.24	EUR	223,671
5	AA	10,000,000	Short	**8.71**	EUR	**1,099,999**
5	AA	10,000,000	Short	2.98	EUR	101,659
						3,455,018
3	A	9,810,833	Short	0.14	EUR	427,862
3	A	14,534,567	Long	0.61	EUR	-285,740
3	A	7,267,283	Long	0.66	EUR	-266,297
3	A	14,897,931	Short	**1.16**	EUR	1,132,680
						1,008,505
7	BBB	10,225,838	Short	**1.52**	EUR	964,798
						964,798

4.4.1.2 Hedging the Expected Credit Exposure of the swap sub-portfolios and estimating economic loss under the integrated measure

The proposed integrated approach as in Chapter 3 was implemented for the swap sub-portfolios. The expected credit exposure of each sub-portfolio (E(CE)) is the sum of the current MTM plus the 3M VaR of the sub-portfolio. This represents the amount at risk in case of default and if there is no recovery (Method 1).The respective amounts in options (Option Notional 1 and 2) need to be bought to hedge the default risk of each sub-portfolio under the assumption of zero recovery[66] and of 0.3265 recovery are shown in Table 4.5.

Table 4.5 Hedging Details per sub-portfolio

2-Apr-04	2Y AAA	9Y AA	2Y A	2Y BBB
Option Strike (bp)	28	56	78	81
Option Price (decimals) 3M	0.583	0.489	0.601	0.682
MTM	276,664	3,452,793	1,017,100	972,223
3M VaR (99%)	260,792	1,848,815	76,641	102,812
E(CE) (expected credit exposure)	537,456	5,301,608	1,093,741	1,075,035
Recovery Rate	0.3265	0.3265	0.3265	0.3265
E(CE) (after recovery rate)	361,977	3,570,633	736,635	724,036
Option Notional (1)[67]	921,880	10,841,735	1,818,589	1,576,298
Option Notional (2)[68]	620,886	7,301,908	1,224,820	1,061,637

The total economic loss was then estimated (Table 4.6) fro methods 1 and 2. The total economic loss represents the amount of at risk under a certain probability of default and a pre-specified recovery rate. It is the sum of the expected loss and the unexpected loss. The expected loss is estimated from the historical simulation of the credit spread index options and is equal to the mean of the resulting profit and loss distribution. The unexpected loss is 2.33 times the standard deviation of the same distribution. This is the major difference to current practices which model the probability of default separately. In this integrated approach the probability of default and the size of each exposure are "integrated" into the price and the notional of the

[66] We will define this as method (1).
[67] Assuming 100% probability of default and recovery rate of 1.
[68] Assuming 100% probability of default and recovery rate of 0.3265.

credit spread index option. The recovery rate is modeled as an exogenous variable as in the standard approach and it can vary significantly. We have used a long-term historic actual recovery rate.

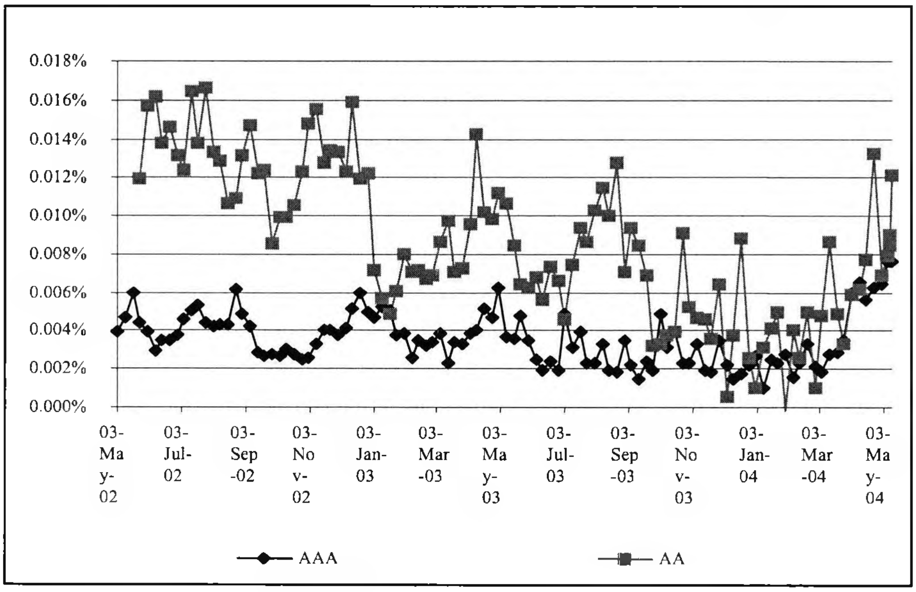
Table 4.6 Economic Loss comparison between methods (1) and (2) under the integrated approach

EUR	**AAA**	**AA**	**A**	**BBB**
EL (1)	133	2,251	633	135
EL (2)	90	1,516	426	91
UL (1)	10,776	28,518	68,781	27,804
UL (2)	7,258	19,207	46,324	18,726
TL (1)	10,909	30,769	69,413	27,939
TL (2)	7,347	20,723	46,750	18,817

4.4.1.3 Historical simulation of implied default probabilities and estimation of economic loss under the standard approach

The weekly time series of the 3M AAA, AA, A and BBB implied default probabilities are shown in Figures 4.2 and 4.3. They are quite volatile and it seems that they started to rise during the first 6 months of 2004 (Figures 4.2 and 4.3). As explained in chapter 3 this could be due to massive liquidations of bond portfolios at this time.

Figure 4.2 Time series of 3M implied default probabilities, AAA and AA



There is an implicit correlation between the time series since the simulation was historical. It can also be observed that the implied correlation between the default probabilities varies with time.

There is a strong directional correlation in terms of direction between all four 3M implied default probabilities, especially in 2004. However, a better way to examine their correlation is to estimate it based on the historical data obtained. The correlation between the implied default probabilities seems to be quite low when examined in levels. When the correlation is calculated using the logarithmic differences the correlations is even lower. This suggests that the 3M implied default probabilities in our sample have low correlation.

The historical simulation of the default probabilities can give a good indication of how the probability of default will fluctuate over a specified period. Since we have a two year sample we can measure the level of the implied probability of default at the 99% confidence interval.

Figure 4.3 Time series of 3M implied default probabilities, A and BBB

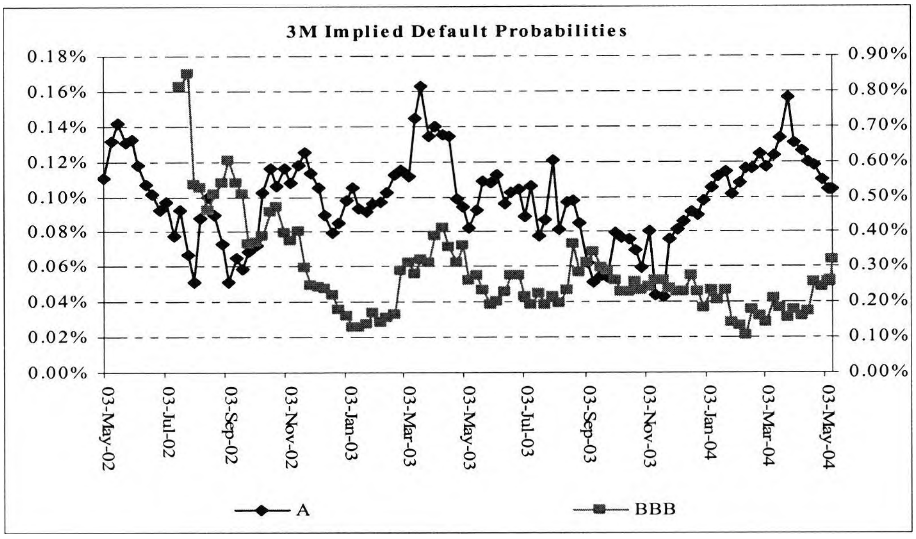


Table 4.7 Historical Correlation matrix of the 3M implied probabilities of default

Differences					Levels				
3M	**AAA**	**AA**	**A**	**BBB**	**3M**	**AAA**	**AA**	**A**	**BBB**
AAA	1	-0.042	0.133	0.217	**AAA**	1	0.308	0.145	0.329
AA	-0.042	1	-0.044	0.057	**AA**	0.308	1	-0.017	0.214
A	0.133	-0.044	1	-0.060	**A**	0.145	-0.017	1	0.052
BBB	0.217	0.057	-0.060	1	**BBB**	0.329	0.214	0.052	1

This is probably explained by the fact that the joint probabilities of default are quite low. The joint probabilities are utilised when looking at the overall swap portfolio, i.e. the sum of the sub-portfolios.

Table 4.8 Implied Default Probabilities 02/04/04

Rating	AAA	AA	A	BBB	BB	B	CCC
Prob of Default	0.0026%	0.0028%	0.0312%	0.1345%	0.3651%	1.3720%	2.0959%

Subsequently, we will use the expected credit exposure and equations 4.10 and 4.11 to measure the total economic loss. The 2.33SD value of the implied default probabilities per rating is listed in Table 4.9 together with the calculations of total economic loss. As a comparison we have also calculated the total economic loss using the implied probability of default as of the 2nd of April of 2004 and the historical probabilities of default as they appear in the Moody's transition matrix. The 3M probabilities of default as implied using the risk premium of the 02/04/04 are shown in Table 4.8.

The highest estimated economic loss comes when the 2.3SD implied default probability is used since this is the highest default rate over the period of the historical simulation. The level of probability of default can differ quite a lot depending on what method we used to estimate it whereas the option prices of the credit spread options do not differ as much even if a different model was used for pricing.

Table 4.9 Economic Loss calculations using probabilities of default

	AAA	AA	A	BBB
3M Moody's Historical Probability of Default (3)	0.0003%	0.0034%	0.0166%	0.0894%
3M Implied Probability of Default as of 02/04/04 (4)	0.0026%	0.0028%	0.0312%	0.1345%
3M 2.3SD Implied Probability of default (5)	0.0076%	0.0472%	0.1022%	0.2340%
E(CE)	**537,456**	**5,301,608**	**1,093,741**	**1,075,035**
RR	32.65%	32.65%	32.65%	32.65%
EL(3)	1	120	122	648
EL(4)	10	95	230	974
EL(5)	27	1,686	753	1,694
UL(3)	644	20,711	9,477	21,643
UL(4)	1,861	18,408	13,017	26,536
UL(5)	3,155	77,565	23,539	34,981
TL(3)	645	20,831	9,599	22,291
TL(4)	1,870	18,503	13,247	27,510
TL(5)	3,182	79,251	24,291	36,676

In the next section we will replicate the same analysis only this time we will use a multi-step Monte Carlo to calculate the expected credit exposures created by the swap sub-portfolios.

4.4.2 Multi-step Monte Carlo simulation of the swap sub-portfolios and economic loss

4.4.2.1 Multi-step Monte Carlo simulation of the swap sub-portfolios

The exposure profiles of the sub-portfolios are reported in Appendix 5 and Figure 5.4. Both the diffusion and amortisation effect are observed in the exposure profiles of all 4 sub-portfolios. The exposure declines as the swaps in the sub-portfolios reach near maturity and eventually goes to 0 when the last swap matures. The highest and longest exposure comes from the AA sub-portfolio, which at the starting date

(02/04/04) the exposure stands at 7,914,055 EUR. This is substantially higher than the current MTM, which stands at 3,455,018. In chart 5.4 we can see the difference between the mean of the exposure and its 2.3SD, which is substantially higher.

Figure 4.4 Potential Credit Exposures after netting of AA- sub-portfolio

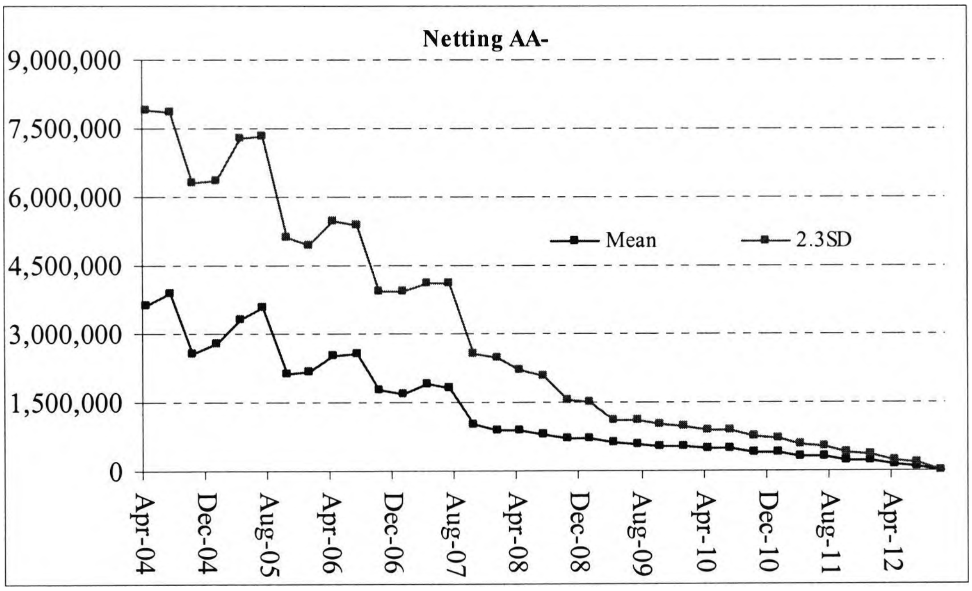
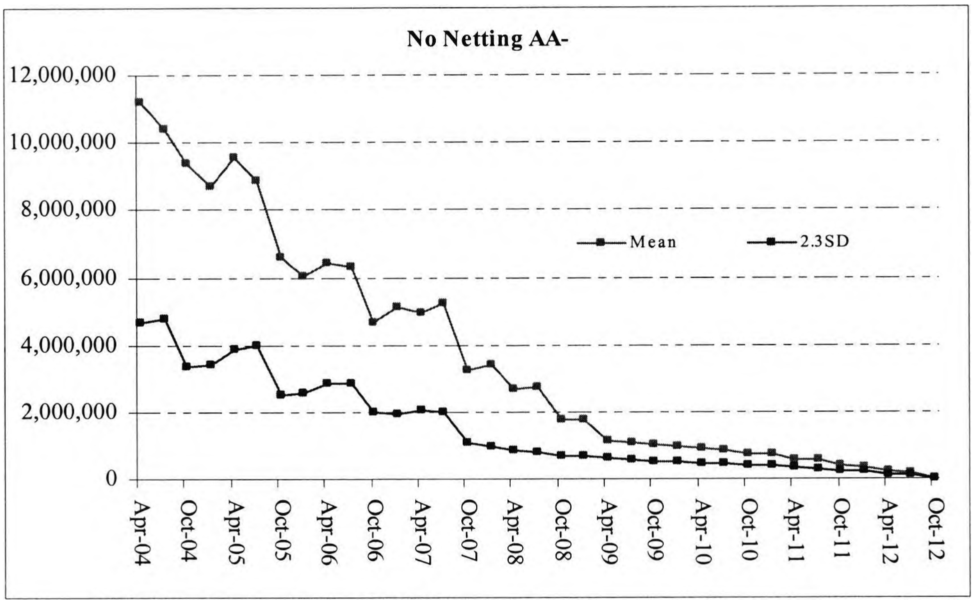


Figure 4.5 Potential Credit Exposure of AA- sub-portfolio without netting



Furthermore, this is a netted sub-portfolio, an assumption, which is very clear to understand since we have taken into account wrong-way exposures[69] as well. If there were no netting agreements in place then the exposure profiles would look different, since only the positive MTM swap would be taken into account (Figure 4.5).

As we can see the maximum exposure stands at 11,228,652 EUR compared to the 7,914,055 EUR estimated under the netting assumption. In the next section we will calculate the total expected loss that stands after 3M based on the exposure profiles obtained from the multi step Monte Carlo. These 3M exposures are listed in Table 4.10. These are the MTM values of the respective swap sub-portfolios which will be used to calculate the total loss per swap sub-portfolio in the next section.

Table 4.10 Expected Credit Exposures per sub-portfolio at 2.33SD

Rating	3M
AAA	433,410
AA	7,838,484
A	626,039
BBB	1,228,376

4.4.2.2 Economic loss based on the Monte Carlo expected exposures

The total loss that can be experienced by a portfolio of derivatives is a function of three variables (section 4.1.2): the expected credit exposure, the probability of default and the rate of recovery. In this section we will replace the expected credit exposure used in section 4.1.2 with the expected credit exposure as obtained form the multi-step Monte Carlo (Table 4.10). Based on these new expected credit exposures we will apply the same calculations as in section 5.1.2 in order to measure the total economic

[69] Wrong-way credit exposure is an exposure with negative current MTM, which could evolve to become a positive exposure due to the nature of the instrument i.e. in swaps.

loss. Thus, by using all five different methodologies[70] we will calculate the different economic losses. So using the credit spread options methodology and equations 4.1 and 4.2 for the calculation of the expected and unexpected loss we can calculate the total loss of each swap sub-portfolio in 3M subject to each method. In Table 4.11 we show the economic loss calculated using the "integrated approach" which uses credit spread index options to measure the economic loss of the swap sub-portfolios. The total losses between the two differ substantially since in method (1) we assume recovery rate of 1 and in method (2) we've assumed recovery rate of 0.3265.

In Table 4.12 the economic losses based on the default probabilities are calculated. The difference in the probabilities of default makes substantial difference to the overall result since the total loss increases substantially.

Table 4.11 Methods (1) and (2) using the MC expected credit exposures

EUR	**AAA**	**AA**	**A**	**BBB**
Option Strike	28	56	78	81
Option Price ATM	0.583	0.489	0.601	0.682
MC ECE	433,410	7,838,484	626,039	1,228,376
MC ECE * (1-RR)	291,902	5,279,219	421,637	827,312
EL (1)	107	3,329	362	154
EL (2)	72	2,242	244	104
UL (1)	8,690	42,164	39,369	31,770
UL (2)	5,853	28,397	26,515	21,397
TL (1)	8,797	45,492	39,731	31,924
TL (2)	5,925	30,639	26,759	21,501

This is purely due to the big difference between the two probabilities. This result shows that the probabilities of default based on the historical transition matrix (chapter 2) are quite far away from the default probabilities as implied by the market potential economic loss of the swap sub-portfolios.

[70] Method (1) was merely the one where the option notional were measured based on the ECE assuming 100% default rate and 1 as the recovery rate. Method (2) was exactly the same only this time the recovery rate was 0.3265. Method (3), (4) and (5) are using default rates instead of credit spread options and in particular the Moody's Historical Probability of Default, the implied probability of default as of 02/04/04 and the 2.3SD Implied probability of default respectively.

Table 4.12 Methods (3), (4) and (5) based on the MC expected credit exposures

EUR	AAA	AA	A	BBB
Moody's Historical Probability of Default (1)	0.0003%	0.0034%	0.0166%	0.0894%
Implied Probability of Default as of 02/04/04 (2)	0.0026%	0.0027%	0.0312%	0.1345%
2.3SD Implied Probability of default (3)	0.0076%	0.0472%	0.1022%	0.2340%
E(CE)	**433,410**	**7,838,484**	**626,039**	**1,228,376**
RR	32.65%	32.65%	32.65%	32.65%
EL(3)	1	120	122	648
EL(4)	10	95	230	974
EL(5)	27	1,686	753	1,694
UL(3)	644	20,711	9,477	21,643
UL(4)	1,861	18,408	13,017	26,536
UL(5)	3,155	77,565	23,539	34,981
TL(3)	645	20,831	9,599	22,291
TL(4)	1,870	18,503	13,247	27,510
TL(5)	3,182	79,251	24,291	36,676

This result also shows that the potential credit loss is proportionate to the credit risk premium that the liabilities of a counterpart bear. Since this extra risk is captured by the credit spread, next we will compare the proposed methodology to account for both market and credit risk with the total loss calculations performed in this section.

4.4.3 Comparison between the different methodologies

In this section we will perform a two-fold comparison on how the economic loss of the swap sub-portfolios can be calculated. The first level of comparison will be the difference in using a VaR style approach to estimate expected potential exposure to a multi-step Monte Carlo approach. The second level will be a comparison between the integrated approach developed in this study versus the standard approach currently employed.

As mentioned in the beginning of this section we will perform a two-fold comparison. The first one is on how the expected credit exposures and subsequently the total economic loss is compared when a VaR style approach (historical simulation) is used against the multi-step Monte Carlo. Table 4.13 shows the total economic loss per swap sub-portfolio using the different expected credit exposures as derived from the historical simulation and the multi-step Monte Carlo.

These two different simulation approaches have given different results as expected. For methods (1) and (2), the economic loss for sub-portfolios AA and BBB is higher when the Monte Carlo simulated exposures are used and vice versa for methods (3), (4) and (5). This is inconclusive since there is no clear pattern if one of the simulations yields higher or lower results. The reason is that the time horizon is small enough for the historical simulation to produce adequate results.

Table 4.13 HS – MC comparison

	AAA	AA	A	BBB
HS (1)	10,909	30,769	69,413	27,939
MC (1)	5,129	22,246	23,895	21,772
% Change	**-19%**	**48%**	**-43%**	**14%**
HS (2)	7,347	20,723	46,750	18,817
MC (2)	3,454	14,982	16,093	14,663
% Change	**-19%**	**48%**	**-43%**	**14%**
HS (3)	520	30,799	5,494	25,470
MC (3)	645	20,831	9,599	22,291
% Change	**24%**	**-32%**	**75%**	**-12%**
HS (4)	1,508	27,357	7,582	31,434
MC (4)	1,870	18,503	13,247	27,510
% Change	**24%**	**-32%**	**75%**	**-12%**
HS (5)	2,566	117,174	13,904	41,907
MC (5)	3,182	79,251	24,291	36,676
% Change	**24%**	**-32%**	**75%**	**-12%**

For longer time horizons, i.e. more than 1 Year the historical simulation would probably yield poorer results against the Monte Carlo simulation simply because the base of the exposure is the weekly VaR figure extracted from historical time series of only two years. In contrast the multi step Monte Carlo generates 1000 scenarios per time step i.e. the equivalent of approximately 20 years.

The next and more important level of comparison is between the two different methods of calculating economic capital. We will only look at the figures calculated using the Monte Carlo simulation since this seems to be the most appropriate simulation methodology to estimate expected credit exposures.

The highest economic loss comes with sub-portfolio AA using method (5). The AA sub-portfolio has the highest expected credit exposure and the 3M 2.3SD implied default probability stands at 0.0472%, which is a substantial increase from the historical default probability estimated by Moody's which is 0.0034%. Looking at the MC(1) and MC(2) results we can say that for a novel methodology the economic loss estimates are quite different from the rest (Table 4.14).

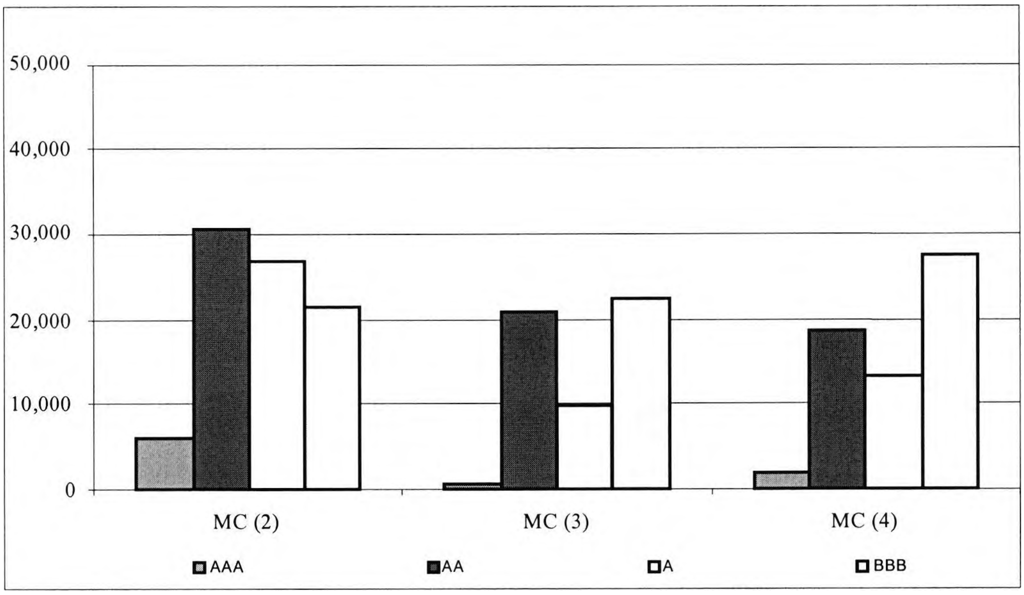
Table 4.14 Comparison of the 5 methods using the MC expected credit exposure

	AAA	AA	A	BBB
MC (1)	8,797	45,492	39,731	31,924
MC (2)	5,925	30,639	26,759	21,501
MC (3)	645	20,831	9,599	22,291
MC (4)	1,870	18,503	13,247	27,510
MC (5)	3,182	79,251	24,291	36,676

If we exclude the two most extreme methods, which are (1) and (5) we can easily compare the economic loss calculations (Figure 4.6). It is quite clear that methods (3) and (4) are very much governed by the upward sloping structure of the default rates, since the economic loss profiles are increasing as the credit rating deteriorates. There is an exposure effect which can be seen by looking at the AA sub-portfolio where the total economic loss is higher than the total economic loss for sub-portfolio A.

This exposure effect, is much more obvious with the integrated method. The highest economic loss comes with the AA sub-portfolio which has the highest expected credit exposure compared to the rest of the sub-portfolios. We also have to note that using the integrated approach the total economic loss per sub-portfolio is higher apart form the BBB sub-portfolio. Overall the differences between methods (3) and (4)[71] seem to be quite high especially when they are used to calculate the ratio of capital per sub-portfolio. This was calculated by dividing the total economic loss to the current MTM of the swap sub-portfolios. Only the BBB sub-portfolio has a comparable ratio and then the AA sub-portfolio. This means that the integrated approach based on these results would allocate a higher capital per swap sub-portfolio, hence reducing the return on capital. Although this is not desired by any financial manager it could be true. A possible reason is that due to its dynamic nature the proposed integrated approach is much faster in capturing changes in the probability of default through the evolution of credit spreads.

Figure 4.6 Comparison of Total Economic Loss



The integrated approach is a "mixed blessing" because the volatility of the total economic loss figure will be in accordance to the volatility in interest rates and credit spreads which is highly advantageous since the volatility of market rates is captured

[71] We used (3) and (4) firstly because they're very similar since in method (2) the option VaR is calculated for the 2nd of April of 2004 and the implied default probabilities in method (4) are implied using the risk premia for the same date.

directly, but its not great for capital allocation decisions since a continuous approach has to be adopted.

Table 4.15 Comparison of (2) and (4) methods

	AAA	AA	A	BBB
MC (2)	5,925	30,639	26,759	21,501
MC (4)	1,870	18,503	13,247	27,510
MC (2) / MTM	2.1%	0.9%	2.6%	2.2%
MC (4) / MTM	0.7%	0.5%	1.3%	2.8%

This new methodology is proven to be comparable to the existing one and one of its potential advantages is that back-testing it is quite straightforward. Since we are dealing with financial instruments which we can price accurately can always check how well the methodology has performed over any time horizon given we have the necessary historical data. In contrast the standard methodology, which uses probabilities of default is not easy to back-test its efficiency since we need to observe actual defaults and calculate the rate at which they happen. This is a process, which takes years to implement.

4.5 Summary

The proposed integrated measure is put to the test by estimating the economic loss for an actual swap portfolio. The methodology devised in Chapter 4 was designed to capture the worst case scenario of full default in 3M time and zero recovery. We further extend the methodology to include the recovery rate in the measure. Under the proposed integrated measure the three different distributions required to measure economic loss are captured by the expected and unexpected movements of the credit spread index options. Since the notional of the credit spread index option is tied with the 3M expected credit exposure and its recovery, then by calculating the expected exposure of the options and their 2.3SD exposure over 3M we can arrive to the total economic loss.

The comparison between the standard approach and the integrated approach proved to be interesting. Using the simple proposed method of integrating credit and market risk we arrive at very similar results with the use of a single term structure model. Furthermore this methodology has the ability to extend or reduce the standard time horizon of at least 1-year of the standard approach. These are of course the main advantages of the integrated methodology. However, there is a disadvantage related to our choice of hedging instrument.

Credit spread index options are not as liquid as one would think. They represent approximately 8% of the credit derivatives market [72] and the bid offer spread is quite high. Hence, the cost of dealing in credit spread options is a parameter which might need to be taken into account. Of course from a theoretical point of view is a parameter which can be ignored since our methodology assumes hedging of swap default risk but not actually doing it unless the necessary signals are there

Finally the comparison between the HS and multi-step MC to estimate expected credit exposures show that for time steps such as 3M the two methods could be comparable. However, for longer time periods it would seem that the accuracy of the HS simulation to estimate expected credit exposures is decreasing. Hence, the multi-step MC is probably the best available method to estimated such future distributions.

[72] Various credit derivatives dealers.

CHAPTER 5 Summary, Discussion and Suggestions for Further Research

5.1 Introduction

This chapter serves as an aid to the reader by restating the research problems in this study. A summary of the results is given and recommendations for future research are made.

5.2 Statement of the problem

In chapter 2 we assessed the statistical properties of EUR credit spread indices using daily and weekly data. The objective was to decide on a suitable model to capture the dynamics of credit spread indices. The appealing features of the two-factor interest rate model by Longstaff and Schwartz (1992) made this choice easier.

In chapter 2 we evaluated the performance of the LS (1992) model, which was used for all our pricing applications in chapters 3, and 4. The objectives were:

- To investigate the curve fitting ability of the LS model based on observed bond prices and the stability of the estimated model parameters over different time intervals.
- To test the efficiency of the LS model in pricing interest rate sensitive securities, with special focus on European Bond Options.
- To test the ability of the model to describe the evolution of the short credit spread by assuming the credit spread is a stochastic variable.
- To price credit spread options by using the LS model as a "spread based" model.

- To study the ability of the LS generated credit spread curves to imply probabilities of default and rating transition matrices.

In Chapter 3 we analysed the risks inherent in a hypothetical swap portfolio in order to devise an integrated measure of market and credit risk. The objective was to use the same valuation framework to price the implied default risk of a swap portfolio and deduce an integrated measure of market and credit risk.

In chapter 4, the aim was to further extend the use of the proposed integrated measure to provide an integrated credit risk measure for an actual swap portfolio. The second objective was to compare this newly proposed measure with current standard practices. The final objective was to investigate the efficiency of multi-step MC used for exposure calculations over a short time period.

5.3 Summary of the Results

Throughout our research we have implemented a number of methodologies and examined a number of empirical issues. This section summarizes the results.

The first study (Chapter 2) *"Pricing Interest Rate Sensitive Securities and Credit Spread Options using the Longstaff & Schwartz (1992) Model"*, starts by analysing statistical and distributional properties of EUR credit spread indices using weekly and daily data between May-01 to May-04.

This analysis suggested that the evolution of credit spread indices shares many properties with the evolution of interest rates. For example, the term structure of their volatilities, mean reversion, and the time variation of volatility. The following were observed based on the brief statistical analysis of the credit spread indices:

1. EUR Credit Spread Indices are mean-reverting hence for statistical purposes the first differences should be used to avoid spurious results.

2. Their distribution is highly skewed and there is a tendency for the skewness to be higher in higher maturities and for kurtosis to be lower in higher maturities apart from the lower ratings where there is no clear pattern.
3. The mean of the one-day percentage changes is highly correlated to the respective standard deviation.
4. The kurtosis and the standard deviation have low correlation, suggesting a Gaussian probability will not be sufficient for risk management purposes.
5. The volatility of most credit spread indices exhibit GARCH (1,1) effects.

Subsequently, the interest rate model (LS 1992) of choice was implemented using cross sectional data of short and long-term interest rates. The yield curve estimation was performed using the observed discount function. The results show that the LS 1992 model can fit quite complex yield curve shapes and not just upward, downward or humped yield curve shapes. Subsequently, the volatility of the estimated parameters was assessed over a period of 4 years, and a comparison between data of weekly and daily frequency was performed. The volatility of the six estimated parameters was reduced substantially suggesting the use of weekly data for historical simulation purposes.

Interest rate options were priced under the specifications of the LS model. The option prices were compared with prices as calculated using the Blacks' formula yielding very low differences for short-term options. Longer-term options exhibited higher differences to the option prices obtained using Blacks' formula.

Using the same estimation methodology we then estimated the spread discount function based on observed spread discount factors. Quite complex credit spread curves were fitted successfully for different ratings given initial values of the short spreads and their GARCH(1,1) estimated volatility. Interestingly, the credit spread curve discount function was monotonically increasing.

Subsequently, 3M ATM options on respective credit spreads were priced and they were compared to an option strategy with the same delta. The option strategy consisted off two bond options: a call on a corporate plus ½ a put on a government bond. The pricing result proved to be quite accurate relative to the option strategy, since small differences were observed.

Based on the estimated credit spread curves and using the JT (1995) model first and then the JLT (1997) models we implied default probabilities per rating and maturity. The estimation of the spread curves was also performed over a period of over two years, providing a time series of implied default probabilities.

In the next study (Chapter 3) *"Integrated Risk Measurement"*, using the LS (1992) as our pricing framework, an integrated measure of market and credit risk was proposed for a hypothetical portfolio of swaps. The measure is a result of hedging the default risk arising from the 3M exposure of each sub-portfolio. The process of eliminating the default risk of each swap sub-portfolio is quite straightforward and easy to implement.

The 3M swap exposure is matched by an offsetting position in respective credit spread index options assuming full default and zero recovery. The efficiency of this hedging technique was examined through a historical simulation and proved to be good. Combining the swap sub-portfolios with their respective CSIOs a portfolio free of default risk was created. The market VaR of which represents the integrated market and credit risk measure since the distribution of that portfolio is sensitive to market and credit risk factors. Furthermore, the credit part of the overall portfolio is directly linked to the exposure of the swap sub-portfolio, which is subject to market risk.

The relationship of the two types of risk was further analysed by regressing the market VaR of the swap sub-portfolio against the CSIO VaR. The relationship proved to be quite weak, hence a standardised approach where a fixed percentage of the swap VaR is applied to represent default risk was not possible. The relationship of the volatility of swap spreads against the volatility of credit spread indices was analysed as an alternative. The relationship proved to be quite weak.

The next study (Chapter 4) *"Integrated Credit Risk Measurement"*, builds upon the proposed integrated measure to account for economic loss calculations. The integrated approach is compared to current standard practices of calculating the economic loss. Current approaches combine the market value distribution of credit exposure with the probability of default and the loss given default (Recovery Rate). These three variables are treated as being statistically independent. The expected credit exposure is derived from the market value distribution of the portfolio or position in question, whereas the probability of default is modelled using either the structural Merton [] or the reduced form approach JLT (1997). The recovery rate is either taken as a constant or implied from historical data using pareto distributions. The integrated approach proposes the replacement of the modelling of the probability of default. Measuring the expected credit exposure and applying the recovery rate on that exposure the loos amount given default is estimated over any time period in the future without being restricted by the 1 year minimum of the standard approach. Subsequently the estimated loss amount is used to determine the weight of the CSIO. Using that weight (notional) we can measure the expected and unexpected exposure created by the CSIO over a pre-specified period of time. The unexpected exposure can be easily measured by the market VaR of the CSIO. Thus, the expected and unexpected loss is the expected and unexpected exposure created by the CSIO.

The comparison between the two methods yielded similar results in the sense that total expected loss measure per sub-portfolio was a function of the credit exposure and rating of the counterparts. The level of the losses was different, with the integrated approach yielding a higher loss for most of the sub-portfolios. Furthermore, the effect of using various default probabilities was examined. When the historical default probabilities were used based on the S&P historical rating transition matrix the loss increased inversely as the rating counterparts increased. When the implied default probabilities were used the same result was observed only this time expected credit exposure had much more of an effect to the estimated total loss.

A further comparison was made on the way expected exposures are calculated. The 3M exposures were measured using a HS and a multi-step MC. The resulting exposures were different across all sub-portfolios, with the multi-step MC being higher in some cases (AAA and A sub-portfolios) and lower in others (AA and BBB).

5.4 Discussions of the Results

The purpose of this research was to create an integrated measure which accounts for both market and credit risk in derivatives portfolios. As a special case interest rate swaps were examined.

The starting point was the evaluation of the characteristics of credit spread indices in the Euro area. The study of the properties of the EUR credit spread indices gave us a fairly accurate picture of their dynamics. The high persistence of volatility in their time series coupled with their mean reverting characteristics suggested a term structure stochastic volatility model which can accurately capture the complicated curve shapes formed by credit spread indices and the underlying credit spreads.

The distributional properties of the credit spread indices were also found to be deviating from the Gaussian distribution. The high degree of skewness and kurtosis in the data as evident in credit spreads as well, Pedrosa & Roll (1998) implies that standard risk management techniques such as VaR under the normality assumption could be far fro accurate. In essence, this result shows that the market risk is not diversifiable using standard portfolio theory.

The next step was to investigate the term structure stochastic volatility model of choice. It was quite clear that the LS model can fit many different yield curves consistent with the volatility structure. The complex yield curves shapes that can be fitted with LS model worked to our advantage since the term structure of credit spreads are quite complex, supporting even further the use of the LS model to describe the evolution of credit spreads. A problem in using such a model for the evolution of credit spreads is that in theory lower ratings could have lower spreads than higher ratings. The only guarantee is the level of the short credit spread. If the credit spread is always higher for lower ratings then the resulting spread curves will be in accordance with the no-arbitrage conditions, which state that for a lower rated security a higher level of return is required.

The LS 1992 model proved to be suitable for pricing interest rate options since it provides a closed form solution for discount bond options. The closed form solution involves the modified Bessel function of order $(\eta - 1/2)$ and $(\gamma - 1/2)$ where η and γ are two of the six parameters which are easily estimated by fitting the observed discount function. However, fitting the observed discount function over a 4 years using both weekly and daily data, yielded lower than ½ values for these two parameters. Hence, the modified Bessel functions produced complex numbers. Thus, a different method, which approximates the dynamics of the short rate was used to price interest rate options. The methodology was based on first implying the risk neutral probability density of the short rate conditional to its volatility using a Monte

Carlo simulation, and then using the implied probabilities to obtain the discounted risk neutral option prices. The price comparison to the Black's formula seemed that pricing was quite good especially for short-term options. For longer-term options the differences were greater.

The estimate implied default probabilities based on the LS credit spread curves share the same volatility with the credit spreads. The time series of implied default probabilities show that no-arbitrage conditions are maintained throughout the examined period. For example, higher ratings exhibited lower implied probabilities of default apart from a very short period of time where the implied probability of default of rating A was higher than the BBB and the no-arbitrage condition was violated. The interesting issue with this violation is that it can be linked to the period of examination. Spread levels were generally quite low, lower than their long term averages. This could result in mispricings since investors and financial managers become complacent about risk, thinking that this low spread environment could go on for a long time, hence arranging their short-term borrowings with different rated counterparts than usual, thereby shifting liquidity from their regular counterparts to others.

The implied transition matrix was clearly different to the historical and the time series of eigenvalues show that there are fluctuations in their values but they are closely correlated to the credit market rather than past default history. The use of the LS estimated spread curves to imply transition matrices is rather interesting because both the level of credit spread and its volatility are reflected in the result, hence providing a more dynamic implied estimate than by simply using corporate bond prices for estimation.

Once the pricing framework was analysed the next step was to devise an integrated measure of credit and market risk for derivatives using as an example interest rate swaps. In search of a dynamic approach of managing exposures generated by holding IRS portfolios the integrated measure was constructed. The hedging of the exposures created by an IRS portfolio was the starting point. The dynamic hedging of different rated swap portfolios with credit spread options provided a new portfolio of swaps and credit spread options. The new portfolio consists of the market value of the swaps and the implied price of default risk of holding the specific swap portfolio relative to the counterparts' credit rating. The swap exposures and the credit rating of a counterpart are related through the relevant hedging amount applied to hedge the default risk of the swap portfolio. The relationship between the two is proportional and dynamic for a given time horizon. The market value distribution of the "new portfolio" is sensitive to both market i.e. interest rates and credit risk factors i.e. credit spreads. Thus the proposed integrated measure was a VaR measure of the new portfolio for any given time horizon.

The advantage of this measure is that it is quite easy to implement in theory. The reason is that most financial institutions run market VaR as a daily operation so all the infrastructure is already in place. The point where the swap exposure is linked to the notional of the required option is where the implied default risk of each swap sub-portfolio is being measured. This method is water tight in many respects because default risk is directly linked to the credit spread and the probability of default is linked to the credit spread and vice versa. So if the spread widens the probability of default increases and vice versa. Now if the exposure remains quite low and the credit spread widens then the implied probability of default increases and the option premium increases, thus accounting for the increases in the probability of default.

The relationship between the market VaR of the swap sub-portfolio to the credit spread option VaR is quite interesting because it can give signals to a risk manager of increasingly high correlation between the two. This would also serve as a signal for hedging decisions.

After the proposal of the integrated measure we extended our methodology in accounting for the total expected loss of an actual swap portfolio. Up to date the only way to account for both market and credit risk in OTC derivatives portfolios is through running Monte Carlo simulations for multiple time steps in the future and assign counterpart exposure limits. Although, this is an overall accurate way of accounting for both types of risk is also computationally intense and requires vast investment in human capital and systems. In comparison the integrated approach provides a close result to this standard approach assumed mainly by large financial institutions. Both approaches provided closely related results. The structure in both approaches was such that the total economic loss was a function of the expected credit exposure and probability of default, and it was observed by the relevant amounts. This means that highest expected loss was observed with the highest expected exposure and the probability of default prevailed when the sub-portfolios had similar expected credit exposures. The advantage of the integrated approach is that is a simple and quite accurate way (as proved by the comparison) of estimating expected economic losses in IRS portfolios over a number of time horizons. The problem with the standard approach is that it solely relies in the method by which the default probabilities are estimated whereas the proposed integrated approach relies on the accurate pricing of credit spread options.

The HS and multi-step MC comparison led us to two conclusions. Multi-step MC works better for larger time periods and for shorter time periods i.e. 3M they yield comparable results.

The understanding of risks is of primary importance to financial practitioners and regulators alike. Ever since BIS 1998 was published and enforced, financial institutions and other market participants have been investing heavily in research and risk management systems. A different force, which led to this huge investment was the increase in the complexity of risks. This growth in complexity was matched by the growth of transactions especially in the derivatives OTC market. The new accord, which was first published in 2000, was an attempt for creating greater transparency in these markets. With transparency came the proposal of more realistic and accurate treatment of all risks associated with transacting in the OTC derivatives markets. It is a well-known fact among practitioners and academics that exposures created by OTC derivatives are subject to a number of risks including the risk of default. The interaction of this risk of default with the market risk of OTC derivatives in particular and how it is measured is the focus of many financial institutions and academics since it is one of the main requirements of the new capital accord. This research attempted to shed some light in how integrated risk measurement is currently performed and also proposed and alternative methodology, which can be implemented by most financial institutions with a good degree of accuracy.

5.5 Recommendations of Further Research

The statistical overview (Chapter 2) should be extended in examining the half-life of mean reversion over longer time periods. The evolution of credit spread indices and their underlying credit spreads can be examined even further by using an extended version of the Longstaff and Schwartz (1993) model. Koutmos G., (2002) modelled the dynamics of MBS spreads using an extended version of LS (1992) in order to facilitate the better management of mortgage portfolios.

Although the LS (1992) model provides a sound theoretical framework for pricing interest rate sensitive securities, its calibration is still an important issue. The goodness of fit of the generated theoretical curves to the observed is quite good when a mixed historical-implied approach was used. However, using the closed form solution to price interest rate options is not always possible by using the estimated parameters. Thus, a calibration method, which yields the suitable parameters for the option pricing formula should be found. This will facilitate in transforming the LS (1992) to a real-time trading tool.

Using an interest rate model to describe the dynamics of credit spreads is something that has been tried before. Duffie and Singleton (1994) used the CIR (1985) square root process to model the evolution of credit spreads. The use of the LS (1992) model for the same purpose provides a better framework since it's a two-factor mean reverting model able to capture the dynamics of credit spreads. It is quite important to investigate the model's ability to price credit spread options by comparing it to current approaches of pricing credit spread options. Schonbucher (1998) used a numerical method based on the two-factor Hull and White model incorporating stochastic volatility. A comparison between the LS (1992) and the proposed pricing methodology by Schonbucher (1998) would be interesting because Schonbucher is modelling the intensity function and not the credit spread directly as we did.

Currently there are two methods of estimating joint default probabilities. One is using historical equity correlations (structural approach) and the other is by using a historical joint rating transition matrix and implying the joint default probabilities based on market prices (reduced form approach). In the same way implied default probabilities were estimated based on the spread curves, implied joint default probabilities can be estimated based on the implied correlation across the term structure of spread curves. This implied estimation will provide a better overall picture of the probability of default as estimated based on the observed spread rather than observed bond prices.

Since the main purpose of this dissertation was to devise an integrated measure for derivatives, the methodology proposed should be extended to other asset classes such as FX and equity. By using the appropriate stochastic models for the evolution of the respective market rates one should be able to apply the same principles of the integrated approach to create an integrated credit and market risk measure for these asset classes as well. This will provide a total firm-wide measure of market and credit risk. Furthermore, the relationship between the two risks as being measure in Chapter 4 should be investigated further using longer time series of credit spread indices in order to fully characterise the relationship from an econometric standpoint.

Appendices

Appendix 1 List of Bonds 07/05/04

Coupon	Coupon Frequency	Maturity	Time to Maturity	Name	Price	Moody's Rating
2	1	20-Oct-04	1.48	GERMAN TREASURY BILL	99.066	Benchmark
2.5	1	18-Mar-05	1.89	BUNDESSCHATZANWEISUNGEN	100.225	Benchmark
2	1	10-Mar-06	2.88	BUNDESSCHATZANWEISUNGEN	98.971	Benchmark
4	1	16-Feb-07	3.84	BUNDESOBLIGATION	102.754	Benchmark
4.25	1	15-Feb-08	4.85	BUNDESOBLIGATION	103.42	Benchmark
3.25	1	17-Apr-09	6.03	BUNDESOBLIGATION	98.596	Benchmark
5.375	1	4-Jan-10	6.76	BUNDESREPUB. DEUTSCHLAND	108.367	Benchmark
5.25	1	4-Jan-11	7.78	BUNDESREPUB. DEUTSCHLAND	107.77	Benchmark
5	1	4-Jan-12	8.79	BUNDESREPUB. DEUTSCHLAND	105.994	Benchmark
4.5	1	4-Jan-13	9.81	BUNDESREPUB. DEUTSCHLAND	102.044	Benchmark
4.25	1	4-Jan-14	10.82	BUNDESREPUB. DEUTSCHLAND	99.555	Benchmark
6.25	1	4-Jan-24	20.96	BUNDESREPUB. DEUTSCHLAND	117.212	Benchmark
4.75	1	4-Jul-34	31.61	BUNDESREPUB. DEUTSCHLAND	96.144	Benchmark
4.5	1	11-Aug-04	0.27	LEASE ASSET BACKED SECS	100.69	AAA
5	1	28-Jan-05	0.74	VAUBAN MOBILISATION GAR	101.94	AAA
7.4	1	13-Apr-05	0.95	CSSE DE REF DE L'HABITAT	104.54	AAA
3.625	1	19-Sep-05	1.39	CDC IXIS	101.47	AAA
2.75	1	6-Mar-06	1.86	CIF EUROMORTGAGE	100.07	AAA
6	1	6-Jun-06	2.11	CSSE DE REF DE L'HABITAT	106.21	AAA
6.75	1	24-Jul-06	2.24	FI MORTGAGE SECURITIES	108.10	AAA
4	1	30-Oct-06	2.52	VAUBAN MOBILISATION GAR	102.18	AAA
4	1	30-Jul-07	3.28	VAUBAN MOBILISATION GAR	101.98	AAA
3.5	1	12-Nov-07	3.57	CIF EUROMORTGAGE	100.74	AAA
5.375	1	28-Jan-08	3.78	VAUBAN MOBILISATION GAR	106.43	AAA
5	1	25-Apr-08	4.03	CSSE DE REF DE L'HABITAT	105.67	AAA
2.75	1	26-Jun-08	4.20	CDC IXIS	97.22	AAA
5	1	15-Jul-08	4.25	COLONNADE SECURITIES BV	105.49	AAA
4.4	1	9-Oct-08	4.49	CDC FINANCE - CDC IXIS	102.98	AAA
4.5	1	28-Oct-08	4.54	VAUBAN MOBILISATION GAR	103.41	AAA
4.75	1	29-Oct-08	4.54	CIF EUROMORTGAGE	104.76	AAA
4.5	1	12-Nov-08	4.58	SAGESS	103.44	AAA
4.375	1	28-Apr-09	5.05	VAUBAN MOBILISATION GAR	102.27	AAA
4.25	1	15-Jul-09	5.26	COLONNADE SECURITIES BV	101.91	AAA
5.8	1	21-Jul-09	5.28	CDC FINANCE - CDC IXIS	109.16	AAA
4	1	25-Oct-09	5.55	CAISSE REFINANCE HYPOTHE	100.67	AAA
5.875	1	15-Apr-10	6.03	COLONNADE SECURITIES BV	109.96	AAA
5.75	1	25-Apr-10	6.05	CSSE DE REF DE L'HABITAT	108.76	AAA
3.625	1	16-Jul-10	6.28	CIF EUROMORTGAGE	98.21	AAA
6.125	1	2-Aug-10	6.33	CDC IXIS CAPITAL MARKETS	110.48	AAA
4.375	1	25-Apr-11	7.07	CREDIT D'EQUIPEMENT PME	102.04	AAA
4.2	1	25-Apr-11	7.07	CSSE DE REF DE L'HABITAT	100.59	AAA
5.25	1	27-Apr-11	7.07	SAGESS	106.46	AAA
5.375	1	6-Jul-11	7.27	CDC IXIS	107.37	AAA
6	1	28-Oct-11	7.58	VAUBAN MOBILISATION GAR	110.79	AAA
9	1	4-Jun-12	8.19	CDC FINANCE - CDC IXIS	131.62	AAA

5.25	1	30-Jul-12	8.35	VAUBAN MOBILISATION GAR	105.69	AAA
4.625	1	11-Oct-12	8.55	CIF EUROMORTGAGE	102.22	AAA
4.25	1	25-Feb-13	8.93	SAGESS	98.46	AAA
3.75	1	29-Jul-13	9.36	VAUBAN MOBILISATION GAR	94.07	AAA
5	1	25-Oct-13	9.61	CSSE DE REF DE L'HABITAT	104.11	AAA
4.625	1	29-Oct-13	9.62	GE CAPITAL EURO FUNDING	100.35	AAA
4.5	1	10-Dec-13	9.73	CIF EUROMORTGAGE	100.30	AAA
4.25	1	25-Oct-14	10.62	CSSE DE REF DE L'HABITAT	97.54	AAA
5.625	1	5-Oct-04	0.42	TOTAL S.A.	101.27	AA+
5.375	1	2-Jun-05	1.09	TOTAL S.A.	102.9797	AA+
5.75	1	29-Sep-05	1.42	TOTAL S.A.	104.1994	AA+
3.875	1	5-May-06	2.02	TOTAL S.A.	102.025646	AA+
3.5	1	28-Jan-08	3.78	TOTAL CAPITAL SA	100.3972	AA+
6.75	1	25-Oct-08	4.53	TOTAL S.A.	112.359751	AA+
4.5	1	23-Mar-09	4.95	ELF AQUITAINE	103.1601115	AA+
5.125	1	21-Jul-09	5.28	TOTAL S.A.	105.8525402	AA+
3.75	1	11-Feb-10	5.85	TOTAL CAPITAL SA	98.84618	AA+
6	1	15-Jun-10	6.19	DEUTSCHE BAHN FINANCE BV	110.095819	AA+
5.375	1	31-Jul-12	8.35	DEUTSCHE BAHN FINANCE BV	106.290683	AA+
4	1	15-Jul-13	9.32	CORES	96.07047325	AA+
5.125	1	28-Nov-13	9.70	DEUTSCHE BAHN FINANCE BV	104.194642	AA+
4.25	1	8-Jul-15	11.33	DEUTSCHE BAHN FINANCE BV	95.409185	AA+
4.75	1	14-Mar-18	14.05	DEUTSCHE BAHN FINANCE BV	99.0625	AA+
5.75	1	25-Jul-05	1.23	BASF AG	103.7633	AA-
5	1	4-Jul-06	2.19	SIEMENS FINANCIERINGSMAT	104.1403	AA-
5.25	1	19-Jul-06	2.23	ROBERT BOSCH GMBH	104.6612	AA-
5.5	1	12-Mar-07	2.89	SIEMENS FINANCIERINGSMAT	107.501	AA-
6	1	12-Nov-09	5.60	AGBAR INTERNATIONAL BV	109.5807945	AA-
6.125	1	9-Jun-10	6.18	ENI SPA	111.2364045	AA-
3.5	1	8-Jul-10	6.26	BASF AG	97.08931	AA-
5.75	1	4-Jul-11	7.26	SIEMENS FINANCIERINGSMAT	106.0491085	AA-
5.25	1	3-Jul-12	8.28	POSTE ITALIANE SPA	106.1753285	AA-
4.625	1	30-Apr-13	9.11	ENI SPA	100.43	AA-
4.375	1	8-Jul-13	9.30	SCHIPHOL NEDERLAND B.V.	98.3458685	AA-
5	1	12-Jul-04	0.18	METRO FINANCE BV	100.459494	BBB
4.375	1	15-Jul-04	0.19	LAFARGE	100.1325	BBB
5.1	1	3-Feb-05	0.76	LAFARGE	101.7728837	BBB
4.625	1	4-Mar-05	0.84	LAFARGE SA	101.662287	BBB
5.875	1	14-Apr-05	0.95	CASINO GUICHARD PERRACH	102.823	BBB
5.875	1	4-Jul-05	1.18	COCA COLA ERFRISCHUNGETR	104.325139	BBB
4.75	1	8-Nov-05	1.53	VEOLIA ENVIRONNEMENT	102.883294	BBB
5.125	1	16-Dec-05	1.63	WOLTERS KLUWER NV	102.755013	BBB
5.8	1	20-Dec-05	1.64	RENAULT S.A.	104.32669	BBB
5.75	1	9-Mar-06	1.86	METRO FINANCE BV	104.4988325	BBB
5.125	1	26-Jun-06	2.17	LAFARGE SA	104.002809	BBB
5.75	1	5-Jul-06	2.19	ACCOR	104.8090935	BBB
4.75	1	6-Jul-06	2.19	CASINO GUICHARD PERRACH	103.253698	BBB
5.25	1	14-Jul-06	2.22	THYSSENKRUPP FINANCE BV	103.273288	BBB
5.125	1	21-Jul-06	2.24	RENAULT S.A.	104.0950895	BBB
5.75	1	4-Dec-06	2.61	REPSOL INTL FINANCE	105.934727	BBB
5	1	20-Dec-06	2.66	ACCOR	103.521971	BBB
6	1	7-May-07	3.04	IMERYS SA	106.9570215	BBB

6.25	1	11-Jun-07	3.14	WOLTERS KLUWER NV	105.646174	BBB
6.375	1	26-Jul-07	3.26	LAFARGE SA	108.0797	BBB
6.375	1	19-Oct-07	3.50	RENAULT S.A.	108.650034	BBB
5.875	1	23-Nov-07	3.60	CASINO GUICHARD PERRACH	106.73	BBB
5.4	1	3-Feb-08	3.80	LAFARGE SA	105.213752	BBB
5.125	1	13-Feb-08	3.83	METRO AG	104.23	BBB
6	1	6-Mar-08	3.89	CASINO GUICHARD PERRACH	107.75	BBB
6.125	1	10-Apr-08	3.98	ARCELOR FINANCE	107.31	BBB
5.875	1	27-Jun-08	4.20	VEOLIA ENVIRONNEMENT	107.6119	BBB
5.875	1	6-Nov-08	4.57	LAFARGE SA	107.2164	BBB
4.5	1	19-Nov-08	4.60	SES GLOBAL SA	102.1875	BBB
4.8	1	22-Dec-08	4.69	LOTTOMATICA SPA	100.2632495	BBB
6.125	1	26-Jun-09	5.21	RENAULT S.A.	108.7227	BBB
5.875	1	10-Jul-09	5.25	SOCIETE DES CIMENTS FRAN	106.76686	BBB
6.35	1	1-Oct-09	5.48	UPM-KYMMENE CORP	109.443937	BBB
5.25	1	28-Apr-10	6.06	CASINO GUICHARD PERRACH	103.8952	BBB
6	1	5-May-10	6.08	REPSOL INTL FINANCE B.V.	108.08	BBB
4.625	1	28-May-10	6.14	RENAULT S.A.	101.1394	BBB
5.125	1	24-Sep-10	6.48	ARCELOR FINANCE	102.1968	BBB
6.125	1	23-Jan-12	7.83	UPM-KYMMENE CORP	108.165776	BBB
5.875	1	1-Feb-12	7.85	VEOLIA ENVIRONNEMENT	107.2922	BBB
6	1	27-Feb-12	7.92	CASINO GUICHARD PERRACH	106.6245	BBB
4.875	1	28-May-13	9.19	VEOLIA ENVIRONNEMENT	99.44154	BBB
5	1	22-Jul-13	9.34	REPSOL INTL FINANCE	99.99	BBB
5.448	1	4-Dec-13	9.72	LAFARGE SA	102.56	BBB
5.375	1	28-May-18	14.26	VEOLIA ENVIRONNEMENT	98.75	BBB
6.125	1	25-Nov-33	29.98	VEOLIA ENVIRONNEMENT	99.76	BBB

Appendix 2 FMC definition (Bloomberg)

FMC curves are created using prices from new issue calendars, trading/portfolio systems, dealers, brokers, and evaluation services which are fed directly into the specified bond sector databases on an overnight basis. All prices are used.

All bonds for each sector are then subject to option adjusted spread (OAS) analysis and the option-free yields are then plotted to from the fair market yield curve without any yields being distorted by embedded calls, puts, or sinks. This allows bonds with very different structures to be compared on an equivalent basis. A best fit curve is then drawn from the option-free yields, resulting in a specific yield curve for each bond category.

Debt issues are divided into hundreds of sectors that are grouped by several variables such as rating or industry type. The sectors are numbered, and an option-free yield curve is constructed daily for each sector. The ratings categories for each sector are expressed as Bloomberg Composite Ratings, which are blends of Moody's Investor Service and Standard & Poor's ratings.

COMP	MOODY'S	**S&P**
AAA	Aaa	AAA
AA1	Aa1	AA+
AA2	Aa2	AA
AA3	Aa3	AA-
A1	A1	A+
A2	A2	A
A3	A3	A-
BBB1	Baa1	BBB+
BBB2	Baa2	BBB
BBB3	Baa3	BBB-
BB1	Ba1	BB+
BB2	Ba3	BB
BB3	Ba3	BB-
B1	B1	B+
B2	B2	B
B3	B3	B-
CCC1	Caa1	CCC+
CCC2	Caa2	CCC
CCC3	Caa3	CCC-

Appendix 3 Bonds and Bootstrapping

Bond List used for the 16/05/01 LS estimation:

Bloomberg Code	Coupon	Coupon Frequency	Currency	Maturity	Time to Maturity/Years
EC236643 Corp	4.5	Annual	EUR	15-Mar-02	0.88

EC357165 Corp	4.25	Annual	EUR	14-Mar-03	1.89
EC103285 Corp	3.25	Annual	EUR	17-Feb-04	2.84
EC199471 Corp	4.25	Annual	EUR	18-Feb-05	3.86
EC284146 Corp	5	Annual	EUR	17-Feb-06	4.87
GG729023 Corp	6	Annual	EUR	4-Jan-07	5.76
GG729377 Corp	5.25	Annual	EUR	4-Jan-08	6.78
EC085455 Corp	3.75	Annual	EUR	4-Jan-09	7.79
EC189167 Corp	5.375	Annual	EUR	4-Jan-10	8.81
EC300211 Corp	5.25	Annual	EUR	4-Jan-11	9.82
ZZ207101 Corp	6	Annual	EUR	20-Jun-16	15.36

Bootstrapping of coupon Bonds:

P_T is the price of the coupon bond. First we compute the P(t,T) and since the coupon frequency is annual and the face value 100:

$$P(tT) = \sum_{i=1}^{T} \frac{c}{(1+y_T)^i} + \frac{100}{(1+y_T)^T}$$

where, y_T is the yield to worst and c is the coupon. Then:

$$P_1 = (c+100)DF(1)$$
$$P(2) = cDF(1) + (c+100)DF(2)$$

.

.

.

$$P(10) = cDF(1) + cDF(2) + ... + (c+100)DF(10)$$

For the 11, 12, 13 and 14 maturities we used linear interpolation. After that we calculated the 15Y zero coupon bond.

Appendix 4 Random Number Generation

The following are two uniform random numbers in [-1,1]:

$$v_1 = 2U - 1 \qquad v_2 = 2V - 1 \quad (5.7)$$

The following was performed in order to generate the required standard normal numbers.

$S = v_1^2 + v_2^2$

if $S \geq 1$ *start again*

if $S < 1$ *then* $v_2\sqrt{((-2\ln S)/S)}$

both $v_2\sqrt{((-2\ln S)/S)}$ *and* $v_1\sqrt{((-2\ln S)/S)}$ *are i.i.d*

Appendix 5 VB code of Jacobi Function

```
Option Explicit
Option Base 1

Function MatrixIdentity(n As Integer)
'  Returns the (nxn) Identity Matrix
   Dim i As Integer
   Dim Imat() As Double
   ReDim Imat(n, n)
   For i = 1 To n
      Imat(i, i) = 1
   Next i
   MatrixIdentity = Imat
End Function

Function MatrixTrace(Xmat)
'  Returns the trace of a matrix (sum of elements on leading diagonal)
   Dim sum
   Dim i As Integer, n As Integer
   n = Xmat.Columns.Count
   sum = 0
   For i = 1 To n
      sum = sum + Xmat(i, i)
   Next i
   MatrixTrace = sum
End Function

Function MatrixUTSumSq(Xmat)
'  Returns the Sum of Squares of the Upper Triangle of a Matrix
   Dim sum
   Dim i As Integer, j As Integer, n As Integer
   n = Sqr(Application.Count(Xmat))
   sum = 0
   For i = 1 To n
      For j = i + 1 To n
      sum = sum + (Xmat(i, j) ^ 2)
      Next j
   Next i
   MatrixUTSumSq = sum
End Function
```

```
Function Jacobirvec(n As Integer, Athis)
'  Returns vector containing mr, mc and jrad
'  These are the row and column vectors and the angle of rotation for the P matrix
   Dim maxval, jrad
   Dim i As Integer, j As Integer, mr As Integer, mc As Integer
   Dim Awork() As Variant
   ReDim Awork(n, n)
   maxval = -1
   mr = -1
   mc = -1
   For i = 1 To n
      For j = i + 1 To n
         Awork(i, j) = Abs(Athis(i, j))
         If Awork(i, j) > maxval Then
            maxval = Awork(i, j)
            mr = i
            mc = j
         End If
      Next j
   Next i
   If Athis(mr, mr) = Athis(mc, mc) Then
      jrad = 0.25 * Application.Pi() * Sgn(Athis(mr, mc))
   Else
      jrad = 0.5 * Atn(2 * Athis(mr, mc) / (Athis(mr, mr) - Athis(mc, mc)))
   End If
   Jacobirvec = Array(mr, mc, jrad)
End Function

Function JacobiPmat(n As Integer, rthis)
'  Returns the rotation Pthis matrix
'  Uses MatrixIdentity fn
'  Uses Jacobirvec fn
   Dim Pthis As Variant
   Pthis = MatrixIdentity(n)
   Pthis(rthis(1), rthis(1)) = Cos(rthis(3))
   Pthis(rthis(2), rthis(1)) = Sin(rthis(3))
   Pthis(rthis(1), rthis(2)) = -Sin(rthis(3))
   Pthis(rthis(2), rthis(2)) = Cos(rthis(3))
   JacobiPmat = Pthis
End Function

Function JacobiAmat(n As Integer, Athis)
'  Returns Anext matrix, updated using the P rotation matrix
'  Uses Jacobirvec fn
'  Uses JacobiPmat fn
   Dim rthis As Variant, Pthis As Variant, Anext As Variant
   rthis = Jacobirvec(n, Athis)
   Pthis = JacobiPmat(n, rthis)
   Anext          =          Application.MMult(Application.Transpose(Pthis),
Application.MMult(Athis, Pthis))
```

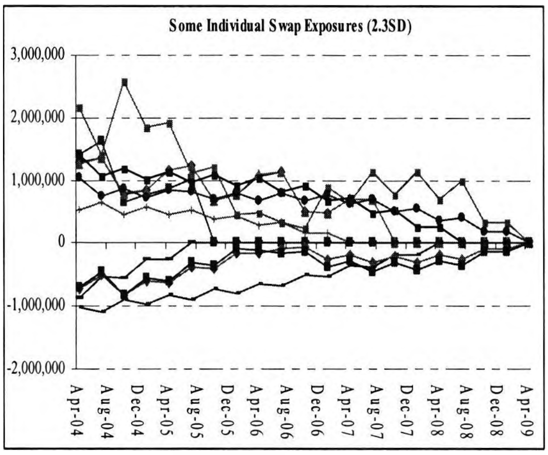
```
   JacobiAmat = Anext
End Function

Function Eigenvaluesevec(Amat, atol)
' Uses the Jacobi method to get the eigenvalues for a symmetric matrix
' Amat is rotated (using the P matrix) until its off-diagonal elements are minimal
' Uses MatrixUTSumSq fn
' Uses JacobiAmat fn
   Application.Volatile (False)
   Dim asumsq
   Dim i As Integer, n As Integer, r As Integer
   Dim evec() As Variant
   Dim Anext As Variant
   n = Sqr(Application.Count(Amat))
   r = 0
   asumsq = MatrixUTSumSq(Amat)
   Do While asumsq > atol
      Anext = JacobiAmat(n, Amat)
      asumsq = MatrixUTSumSq(Anext)
      Amat = Anext
      r = r + 1
   Loop
   ReDim evec(n)
   For i = 1 To n
      evec(i) = Amat(i, i)
   Next i
   Eigenvaluesevec = evec
End Function

Function JacobiVmat(n As Integer, Athis, Vthis)
' Returns Vnext matrix
' Keeps track of the eigenvectors during the rotations
' Uses Jacobirvec fn
' Uses JacobiPmat fn
   Dim rthis As Variant, Pthis As Variant, Vnext As Variant
   rthis = Jacobirvec(n, Athis)
   Pthis = JacobiPmat(n, rthis)
   Vnext = Application.MMult(Vthis, Pthis)
   JacobiVmat = Vnext
End Function

Function EigenvectorsEmat(Amat, atol)
' Uses the Jacobi method to get the eigenvectors for a symmetric matrix
'  Similar to eigenvalue function, but with additional V matrix updated with each
rotation
' Uses MatrixUTSumSq fn
' Uses JacobiAmat fn
' Uses JacobiVmat fn
' Uses MatrixIdentity fn
   Application.Volatile (False)
```

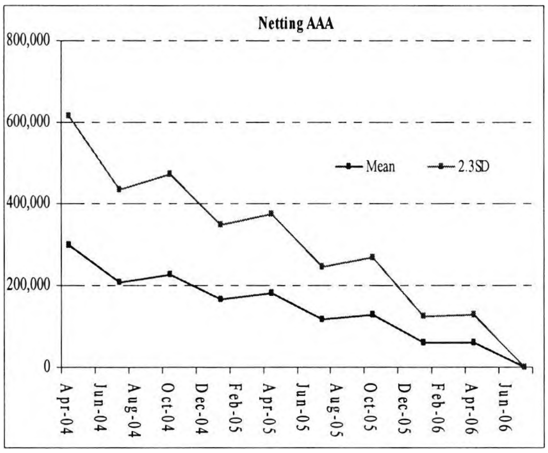
```
  Dim asumsq
  Dim n As Integer, r As Integer
  Dim Anext As Variant, Vmat As Variant, Vnext As Variant
  n = Sqr(Application.Count(Amat))
  r = 0
  Vmat = MatrixIdentity(n)
  asumsq = MatrixUTSumSq(Amat)
  Do While asumsq > atol
    Anext = JacobiAmat(n, Amat)
    Vnext = JacobiVmat(n, Amat, Vmat)
    asumsq = MatrixUTSumSq(Anext)
    Amat = Anext
    Vmat = Vnext
    r = r + 1
  Loop
  EigenvectorsEmat = Vnext
End Function

Function PDcheck(evec)
' Checks definiteness of symmetric matrices using their eigenvalues
' Returns 1 (+ve def), 0.5 (+ve semi-def), -0.5 (-ve semi-def), -1 (-ve def)
  Dim pd, sa, smin, smax
  Dim i As Integer, n As Integer, p As Integer
  Dim svec() As Variant
  n = Application.Count(evec)
  ReDim svec(n)
  For i = 1 To n
    svec(i) = Sgn(evec(i))
  Next i
  sa = Application.sum(svec)
  smin = Application.Min(svec)
  smax = Application.Max(svec)
  If sa = n Then
    pd = 1
  ElseIf sa = -n Then
    pd = -1
  ElseIf sa >= 0 And smin >= 0 Then
    pd = 0.5
  ElseIf sa <= 0 And smax <= 0 Then
    pd = -0.5
 Else
    p = 0
  End If
  PDcheck = pd
End Function
```

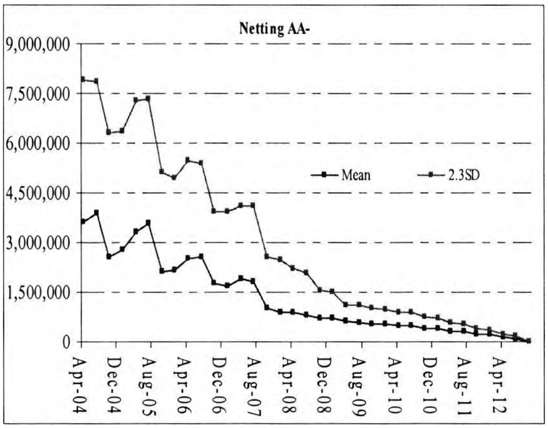
Appendix 6 Multi-Step MC Exposures

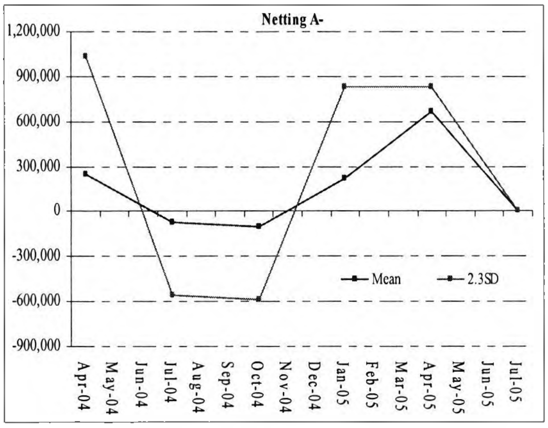
Sub-portfolio	6	5	3	7
Rating	AAA	AA	A	BBB
2-Apr-04	1,324,400	12,537,757	1,544,618	690,159
1-Jul-04	1,125,740	10,657,093	1,004,001	448,603
29-Sep-04	1,069,453	9,591,384	652,601	291,592
28-Dec-04	802,090	9,111,815	619,971	393,649
28-Mar-05	521,358	8,656,224	588,972	338,538
26-Jun-05	182,475	8,223,413	559,524	291,143
24-Sep-05	27,371	6,989,901	0	0
23-Dec-05	1,369	5,941,416	0	0
23-Mar-06	1,163	5,644,345	0	0
21-Jun-06	0	4,797,693	0	0
19-Sep-06	0	3,598,270	0	0
18-Dec-06	0	2,338,875	0	0
18-Mar-07	0	1,988,044	0	0
16-Jun-07	0	1,689,838	0	0
14-Sep-07	0	1,436,362	0	0
13-Dec-07	0	1,220,908	0	0
12-Mar-08	0	1,037,771	0	0
10-Jun-08	0	882,106	0	0
8-Sep-08	0	749,790	0	0
7-Dec-08	0	637,321	0	0
7-Mar-09	0	541,723	0	0
5-Jun-09	0	460,465	0	0
3-Sep-09	0	391,395	0	0
2-Dec-09	0	332,686	0	0
2-Mar-10	0	282,783	0	0
31-May-10	0	240,365	0	0
29-Aug-10	0	204,311	0	0
27-Nov-10	0	173,664	0	0
25-Feb-11	0	26,050	0	0
26-May-11	0	22,142	0	0
24-Aug-11	0	18,821	0	0
22-Nov-11	0	15,998	0	0
20-Feb-12	0	13,598	0	0
20-May-12	0	11,558	0	0
18-Aug-12	0	9,825	0	0
16-Nov-12	0	0	0	0

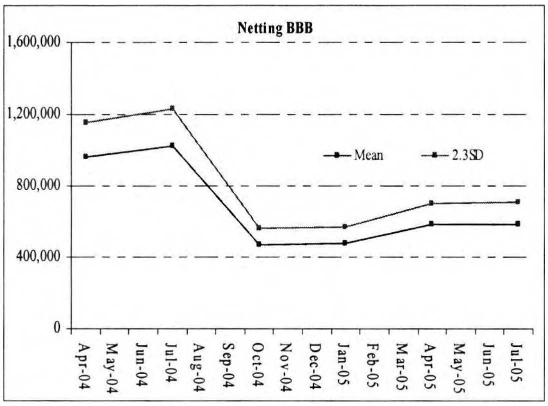
Appendix 7 Figures of Multi-Step MC Exposures

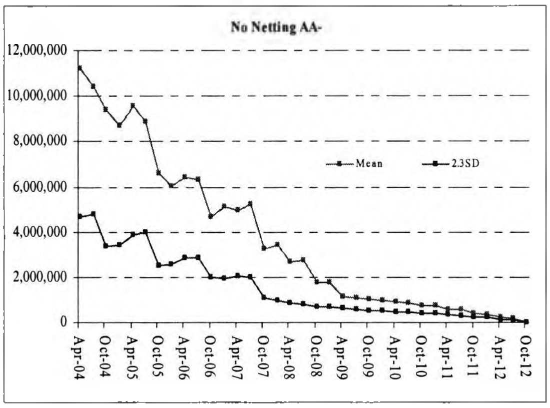


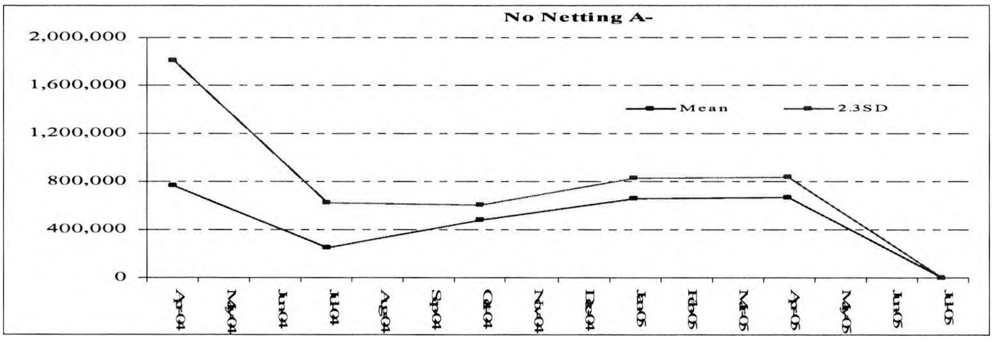












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