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Citation: Rickayzen, B. D. (2007). Some actuarial aspects of health insurance. (Unpublished Doctoral thesis, City, University of London)

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Some Actuarial Aspects of Health Insurance

by

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Submitted for the degree of
Doctor of Philosophy
at City University

Faculty of Actuarial Science and
Insurance, Cass Business School,
November 2007

Some Actuarial Aspects of Health Insurance

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ACKNOWLEDGEMENTS

Much of the material in this thesis has already appeared in papers published in refereed journals. In particular, Chapters 2, 3 and 4 were originally published in Rickayzen (2001), Haberman, Butt and Rickayzen (2004) and Rickayzen and Walsh (2002), respectively. The author is grateful to the editors of the journals involved (Journal of Actuarial Practice, ASTIN Bulletin and British Actuarial Journal) for giving their permission for the papers to appear in the thesis.

The author would like to acknowledge all the helpful comments and suggestions made by his colleague, Professor Steven Haberman, both on early drafts of the original papers and, acting in his capacity as advisor, on drafts of the completed thesis.

The author would also like to acknowledge the work of Zoltan Butt, a Research Assistant, in carrying out the computational work required for Chapter 5. The work in Chapters 3 and 4 was funded by research grants provided by the Institute and Faculty of Actuaries.

The author is also grateful for the support and encouragement that he has received from his father, Gerald, and two colleagues, Professor Richard Verrall and Professor Celia Glass.

Finally, the author owes a great debt of gratitude to his long-suffering wife, Rachel, and their three boys (Adam, Jacob and Jed) for their love and patience.

DECLARATION

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ABSTRACT

The thesis demonstrates some ways in which multiple state models can be used to investigate particular actuarial aspects of two common health insurance products: income protection (IP) insurance and long term care (LTC) insurance. The thesis contains 4 papers: two concerning IP insurance and two concerning LTC insurance.

The first paper investigates the sensitivity of IP insurance premiums to changes in the parameter values used in a multiple state model. Lapses are incorporated within the model, and the net premium for a particular policy is tested for sensitivity to the various parameters used, including their interaction with the lapse rate. One of the conclusions is that the net premium is insensitive to changes in the lapse rate.

The main objective of the second paper is to measure the process error for a portfolio of independent IP insurance policies in a multiple state modelling context. A second objective is to observe the extent to which the process error changes as we increase the volatility and complexity of the models which generate investment returns and inflation.

The results demonstrate that pooling, as implied by the law of large numbers, reduces the process error. However, once we introduce volatility to the investment and inflation experience, we find that the economic process risk is considerably more significant than the demographic process risk.

In the third paper, a multiple state model is developed to project, over the next 35 years, the number of people in the United Kingdom with different levels of disability. These projections assist in identifying the potential demand for LTC by the older population in the future. The projections suggest that the resource implications of having an ageing population in the UK will be ameliorated by a reduction, over the projection period, in the proportion of older people who are severely disabled.

The fourth paper uses the multiple state model described in the third paper to investigate a potential new type of LTC product: a disability-linked annuity. The annuity is purchased while the individual is in reasonable health, and the level of payment is increased as the individual becomes more disabled. The product is analysed under various health scenarios and using different definitions of disability. The premium is compared with that required for the corresponding standard annuity (which does not increase upon the onset of disability). It is found that, given the additional disability benefits provided by the product, the increase in premium required is relatively modest. It is therefore potentially an attractive product, albeit only affordable by the more affluent section of the population.

The overall conclusion of the thesis is that multiple state models have a significant part to play in health insurance in terms of both calculating premiums and reserves, and in measuring risk. Hence, the use of multiple state models should be viewed as being an important technique by health insurance actuaries when considering the broad array of risk management tools at their disposal.

1. INTRODUCTION

1.1 Overview

This thesis comprises four papers in the field of health insurance:

1. A sensitivity analysis of the premiums for a Permanent Health Insurance model

2. Measuring process risk in income protection insurance

3. A multi-state model of disability for the United Kingdom: implications for future need for long-term care for the elderly

4. An analysis of disability-linked annuities

The objective of the thesis is to demonstrate some ways in which multiple state models can be used to investigate particular actuarial aspects of two common health insurance products: income protection (IP) insurance and long term care (LTC) insurance. The first two papers concern IP insurance and the other two papers concern LTC insurance.

The first paper was published in the Journal of Actuarial Practice in 2001 (Rickayzen, 2001). The second paper was joint-authored with Steven Haberman and Zoltan Butt, and was published in ASTIN Bulletin in 2004 (Haberman et al, 2004). The third paper was joint authored with Duncan Walsh, and was published in the British Actuarial Journal in 2002 (Rickayzen and Walsh, 2002). The fourth paper has been produced as part of the series of actuarial research papers published by the Actuarial Research Centre at Cass Business School (Rickayzen, 2007). In view of the fact that the second and third papers are co-authored, I describe my individual contribution to each of those papers in section 1.2 below.

To put the two health insurance products in context, we should begin by describing each product in turn.

An IP insurance product provides a benefit in the form of replacement income in the event that the insured becomes unable to work for a period of time due to illness or injury. The benefit is commonly paid as a monthly annuity until such time as the insured recovers sufficiently to return to work, or the policy term expires. Individuals often choose a policy term expiry date that coincides with the normal retirement age of any pension arrangement they may have. The benefit only commences after the insured has been incapacitated for longer than the deferred period, which is typically 3 to 12 months in the UK. For more information on IP insurance - which was formerly known as Permanent Health Insurance ("PHI") in the UK - see Booth et al (2005).

An LTC insurance product provides a benefit when the insured life is deemed to be too disabled to be able to satisfactorily care for himself/herself. In the UK, a claim can typically be made when an individual is unable to carry out three (or sometimes two) of the six standard Activities of Daily Living (ADLs). These are washing, dressing, feeding, toileting, mobility and transferring. In addition, a claim can be made if an individual suffers serious cognitive impairment, regardless of his/her ability to carry out ADLs. The benefit is usually an annuity payable until the individual recovers or dies. As with IP insurance, the benefit would not commence until after the deferred period has expired. Again, for more information on LTC insurance, the reader is referred to Booth et al (2005).

The papers included in this thesis form part of a continuing research programme which has led to other papers in recent years, particularly with regard to LTC. These will be described in more detail in Chapter 6 (the conclusions) to indicate how the research has been, and is being, continued.

Each of the papers has its own introduction and literature review. Therefore, the purpose of this introduction is to provide a short summary of the more important aspects of the four papers, and to describe the connecting features.

In the first paper, we investigate the sensitivity of IP insurance premiums to changes in the parameter values used in a multiple state model. There are three main features of this work to note: splitting both the healthy and sick states into sub-states; considering lapses in the analysis; and performing a sensitivity analysis of the parameter values described in the 1991 Continuous Mortality Investigation Report No. 12 (CMI Committee, 1991). We now consider each of these features in turn.

To avoid using duration dependent probabilities, both the healthy and sick states are split into sub-states to act as a proxy for duration spent in a particular state. This enables a Markov, rather than semi-Markov, model to be adopted. The approach follows Jones (1994) whereby constant forces of transition are assumed between the states making the calculations tractable.

Having adopted a relatively simple IP model by splitting both the healthy state and the sick state into two sub-states each, it is straightforward to introduce a lapse state and consider the effects of lapses on the net premiums. This is an area where academic research has not been carried out in the past. One of the conclusions of the paper is that the net premium is relatively insensitive to changes in the intensity of the lapse rate.

With the six-state multiple state model developed, it is instructive to examine how sensitive the net premiums are to changes in the parameter values described in CMI Committee (1991) as the lapse rate varies. We find that if the force of transition from the "super healthy" state to the "ultimate" healthy state $\mu_{12}(x)$ is increased, the extent to which the net premium increases depends on the level of the lapse rate within the model. By contrast, when any of the other transition forces are increased, the resultant change in the net premium is largely independent of the lapse rate. This suggests that actuaries concerned with the effect of lapse rates on premiums should focus their attention on obtaining accurate estimates of the force of transition $\mu_{12}(x)$, as opposed to the other forces of transition.

The second paper concerns the use of a different multiple state model in connection with IP insurance. A conventional three state model is considered (ie healthy, sick and dead) in a continuous time framework.

It is well established that models are subject to three broad types of error or risk:

- Model error
- Parameter error
- Process error

In this research, we focus on the process error. The pooling of insured units in a portfolio is a key feature of the insurance process and leads to a reduction in the relative level of variability in the portfolio (ie the risk per unit). This is a statement about process error – the other two types of error listed above are not controllable in a comparable way. We also note that external factors such as investment returns and inflation have a systematic effect which affects all policies at the same time. Therefore, the pooling of risks through writing more of the same business may not reduce the relative variability from these external sources.

Putting these ideas together, we have two main objectives in this paper. Our first is to identify ways of measuring the process error for a portfolio of independent IP policies in a multiple state modelling context. Our second objective is to observe the extent to which the process error changes as we increase the volatility and complexity of the models which generate investment returns and inflation.

We start with a stochastic demographic model (ie random morbidity and mortality transitions) and deterministic economic assumptions. We then increase the volatility of the latter by making the investment experience stochastic but keeping the inflation rates deterministic. Finally, we increase the volatility of the economic factors further by making the both the investment experience and the inflation rates stochastic.

An important and unusual feature of the work is that we make the demographic model stochastic by using the technique of "thinning" which is described in Ross (1990). The methodology was demonstrated in the actuarial literature by Jones (1997) in the context of continuing care retirement communities; however, it does not appear to have been used for IP modelling before.

The results demonstrate that pooling, as implied by the law of large numbers, reduces the process error. Therefore, with a stochastic demographic model and deterministic economic model, the process risk decreases as the size of a portfolio of IP insurance policies increases. However, once we introduce volatility to the investment and inflation experience, we find that the economic process risk is considerably more significant than the demographic process risk. Nevertheless, the length of policy term has a significant influence on the relative weights of the two components of the process risk. Hence, the shorter the policy term, the more the process risk depends on the morbidity rates (and on the size of the portfolio), and therefore the more the insurer is vulnerable to random variations in the morbidity experience.

For the third paper, we turn our attention to long-term care for the elderly. In particular, we build a multiple state model which can be used to project the number of disabled people in the UK over the next 35 years. By focusing on the population who are over age 65 and projected to have severe disabilities, we are able to estimate the future LTC demand over the next three decades.

The model is a 12-state model: healthy, dead and 10 states of disability. It requires three types of data: prevalence rate data, transition rate data and trends data.

We use recent trends in healthy life expectancy data (ie expected future life time spent healthy at a given starting age) to shape the assumptions made regarding changes in morbidity rates of the UK population over time in the future. In particular, recent trends suggest that, at older ages, the time spent free of any disability has been increasing in line with life expectancy; whereas, at least for males, the time spent free of severe disability has been increasing at a faster rate than life expectancy, (ie an improvement) with a less pronounced effect for females.

The trend assumptions which we consider within our transition rate model are as follows:

- The extra risk of mortality imposed on an individual who is severely disabled, as well as general improvements in mortality rates over time;
- The probability that a healthy individual becomes disabled during a year, together with the initial level of severity of the disability;
- The probability that a disabled individual's health deteriorates during a year;
- The probability that a disabled individual's health improves during a year.

In view of the paucity of data, and the error attaching to the parameters of the model estimated from the data, we present projections based on 16 different trend assumptions regarding the morbidity and mortality rates. We therefore present results on a range of deterministic assumptions to illustrate possible future health scenarios. From this, we are able to consider the likely demand for LTC under the most optimistic and most pessimistic assumptions which we feel are plausible, as well as a central set of assumptions.

Overall, our results suggest that although there will be a significant increase in the number of elderly people in the UK over the next 35 years, the implications for the number of elderly people requiring LTC could be ameliorated by a reduction in the proportion of older people who are severely disabled.

In the fourth paper, we investigate a special type of annuity where the annuity is issued to a policyholder who is in reasonable health at the outset; however, if the policyholder subsequently becomes disabled then the annuity payments are increased to a higher level – a “disability-linked annuity”.

We analyse different types of disability-linked annuities (eg single life and last survivor, level and index-linked increases in payment, generous and less generous definitions of “disabled”) in order to examine such annuities’ main characteristics and assess their suitability as potential new products in the annuity and LTC market. In particular, we compare the premium required for the disability-linked annuity with that required for the corresponding traditional annuity to assess the “cost” of converting a traditional annuity into a disability-linked version. The premiums are determined using the multiple state model developed for the third paper. Since the work described in the fourth paper (which is contained within Chapter 5) is designed to be a stand-alone paper, there is inevitably a certain amount of repetition of the description of the LTC model developed in detail in Chapter 4. We use the most optimistic and most pessimistic morbidity assumptions, as well as the central assumptions, in order to carry out a sensitivity analysis.

We conclude that the disability-linked annuity seems to be worthy of attention by insurers as a product providing a certain level of LTC insurance cover, whilst not having the same negative connotations as traditional stand alone LTC insurance appears to have amongst consumers.

The fact that the longevity risks and morbidity risks contained within the product work in opposite directions should make the overall risk more controllable. This facet has other positive effects: the underwriting requirements need not be so stringent, the disability definitions are not so critical and the single premiums appear to compare favourably with their traditional life annuity counterparts when the additional LTC benefits being offered are taken into account.

The finding that the probability that an annuity enhancement will eventually be paid is relatively high, particularly in the case of females, should make the product marketable. In addition, the product appears to have benefited from recent changes in tax regime in the UK and should therefore be even more appealing to consumers.

1.2 Individual contribution to the co-authored papers presented in the thesis

As noted in section 1.1, the thesis comprises four papers, two of which are sole authored and two of which are co-authored. For clarity, I describe below my contribution to each of the co-authored papers.

1.2.1 Personal contribution to the work in Haberman, Butt and Rickayzen (2004)

In 1998, Dr Robert Chadburn (a colleague within the Department of Actuarial Science and Statistics, City University) and I decided to make a joint grant application for the UK Actuarial Profession's Grant Committee to consider.

We believed that the process risk within Income Protection (IP) insurance should be quantified and investigated further, since it could represent a risk to the solvency of insurers which was being overlooked.

Robert and I decided to collaborate on this project since it would combine Robert's practical knowledge of life and IP insurance with my experience with regard to the mathematics used in multiple state models in IP business. The latter had been demonstrated in Rickayzen (2001).

We were successful in our grant application and we recruited a Research Assistant, Zoltan Butt, to work on the project for 50% of his time for a term of 12 months.

Zoltan had a BSc in Mathematics but no knowledge of actuarial science or insurance products. Therefore, as joint supervisors, Robert and I needed to explain both the practical and theoretical concepts behind the work to Zoltan so that he could carry out the computational part of the work. In particular, I took responsibility for ensuring that Zoltan understood the multiple state model described in Section 3.3.2.1, including the required parameterisation from CMI Committee (1991). In addition, Robert and I explained to Zoltan the theory behind the "thinning" process as discussed by Ross (1990) and described in Section 3.3.2.2. Robert and I also agreed upon the policy design (Section 3.3.1), the economic models used for the deterministic and SAM1 models (Section 3.3.3), the cash flow model (Section 3.3.4) and the method to calculate the break even premium (Section 3.3.6). In order to supervise the work most effectively, Robert and I had regular progress meetings with Zoltan.

Robert and I presented the preliminary results from the work at the Healthcare Conference organised by the UK Actuarial Profession at the University of Warwick in May 1999. Soon after, Robert decided to take up employment elsewhere, and Steven Haberman replaced him on the project.

Steven and I refined the economic model to incorporate the alternative stochastic model, SAM2, and we agreed upon the risk measures described in Section 3.3.5. We then discussed the way in which the results and conclusions should be presented in Sections 3.4 and 3.5. Steven produced the first draft of the paper for Zoltan and me to comment upon. We then submitted it to ASTIN Bulletin, and it was published in 2004.

I have presented the work at several conferences. This is discussed in more detail in Chapter 6. The work has also been separately presented by Steven Haberman.

1.2.2 Personal contribution to the work in Rickayzen and Walsh (2002)

In 1997, I became acutely aware that, with an ageing population, projecting future demand for long term care (LTC) in the UK had become an important issue, both socially and politically. Very little research had been done in the UK up to that point. The first major contribution by actuaries to the debate about the future financing of LTC had been made by a Working Party set up by the UK Actuarial Profession, led by Steve Nuttall, which reported its findings in Nuttall et al (1994). However, I felt that whilst this piece of research was a valuable starting point, the modelling approach was too simplistic.

The main drawbacks of the approach used by Nuttall et al (1994) were the following:

- Males and females were not considered separately
- The approach did not enable movements between different states of disability to be considered
- The approach assumed that reaching even low levels of disability had an impact on the associated mortality risk. This seems counter-intuitive since many medical conditions which cause individuals to become slightly disabled are unlikely to be life threatening.

I believed that a more sophisticated multiple state approach would be more useful where all the different levels of disability were contained within the same model. In this way, it would be possible to compute the underlying transition probabilities and consider different possible trends in, say, morbidity rates over time. Having explicit transition probabilities would also be helpful as far as health policy is concerned. For example, if it were decided that resources should be targeted at trying to ensure that moderately disabled people did not become severely disabled then this could easily be explicitly allowed for within the model. This would be effected by reducing the transition probabilities of deteriorating from moderate to severe disability whilst leaving the other probabilities in tact.

At about the same time as I had identified a need for more research into LTC, the UK Actuarial Profession had independently come to the same conclusion. The Profession decided to set up another Working Party, which I was invited to join at inception. This was, initially, led by one of the co-authors of the Nuttall et al (1994) paper, Monica Cornall.

I decided that, to make efficient inroads into the vast amount of research that was needed, I would need to employ a Research Assistant within the Department of Actuarial Science and Statistics (now the Faculty of Actuarial Science and Insurance, within Cass Business School). I therefore wrote a grant proposal for the UK Actuarial Profession's Grant Committee to consider. In this document, I described the kind of work which I intended the Research Assistant to carry out, under my direction, to assist the Working Party.

My grant application was successful and I was able to recruit a Research Assistant, Duncan Walsh, to work on the project for 50% of his time for a term of 12 months.

The way that the Working Party, Duncan Walsh and I operated in practice was as follows. The Working party met approximately every 2 months to consider a progress report which Duncan would have prepared in advance, each time under my guidance and direction. The input from the Working Party would be from the practitioner's perspective since all the members other than Duncan and myself had practical experience of writing LTC insurance.

Examples of where the Working Party influenced the paper are in:

- Section 4.3.2.2 when the extra mortality risk associated with being in OPCS disability categories 6-10 is discussed.

- Section 4.3.4 when recoveries from disability are discussed. For example, it was agreed that assuming a 10% probability of recovering by one OPCS category in a year was appropriate, based on the practitioners' experience.

- Table 4.14 when the various trends in transition rates are discussed. Again, the members of the Working Party were able to confirm that the trend assumptions were all reasonable.

Throughout the project, Duncan and I would have regular meetings together to discuss progress achieved to date, and map out how the research should be progressed. Since Duncan was employed to work half his time on the project, inevitably he would be the one physically carrying out the work, but under my guidance.

During the year of the project, our Working Party was invited to discuss our findings with the Secretariat of the Royal Commission into Long Term Care for the Elderly to inform them when producing their report for the UK Government (Royal Commission on Long Term Care, 1999). We were also asked to present the work at the Healthcare Conference organised by the UK Actuarial Profession at the University of Warwick in May 1998. I presented the work there with three other members of the Working Party, including Duncan.

In view of the importance of the work we had done, and in particular, the fact that the UK Actuarial Profession wanted to submit the research to the Royal Commission as written evidence, the Actuarial Profession took the unusual step of setting up a team of practitioners from the profession to review the model. The team invited a small sub-group of the Working Party, including Duncan and me, to a series of meetings where they could examine the model further. After rigorous scrutiny, the team pronounced that the model we had developed was robust (see The Actuary, 1998).

At the end of the project, just before he left City University, Duncan completed a first draft of the paper for submission to the British Actuarial Journal (BAJ), and I scrutinised and commented upon it. In view of the interest which the work had generated within the profession, we believed that the topic was worthy of a long and detailed paper being published in BAJ. However, the Editor felt that she could not publish such a long paper and I set about shortening the paper and dealing with the anonymous referees' other comments in Duncan's absence. I successfully managed to do this, and the paper eventually appeared in BAJ in 2002.

I have presented the work at several conferences, and the disabled lives' projection model has, subsequently, been used in several other pieces of research. More details regarding both these developments are provided in Chapter 6.

2. A SENSITIVITY ANALYSIS OF THE PREMIUMS FOR A PERMANENT HEALTH INSURANCE (PHI) MODEL *

2.1 Introduction

2.1.1 Overview of the U.K. PHI Business

Permanent health insurance (PHI) has been written in the U.K. for over 100 years. The business was a natural extension of the fraternal (Friendly Society) weekly sickness benefit paid to its members. The rise of the welfare state in the early part of the twentieth century saw the state assume some of the responsibilities of the fraternal societies. Consequently, the amount of business written by private insurers was limited.

The PHI business has increased since World War II, with individual and group business being written by a number of insurers. The market consists of a few specialist direct insurers and reinsurers to support their operations.

The U.K. government still provides a small long-term disability benefit. Recovery rates of state claimants are low; the benefit is a substitute for unemployment benefits. Anyone earning more than national average earnings needs to insure, but there is considerable underinsurance. Increasingly the PHI business is being referred to as *income protection* insurance.

PHI benefits are built around the U.K. pension system and are often expressed in amounts per week or per month. These benefits often cease at state pension age, which is currently age 65 for males and age 60 for females. Some limited benefit period business also is written.

The contracts are similar to those issued in North America, but the terminology differs. For example, the *elimination period* is referred to as the *deferred period* in the U.K. There are similar exclusions, but benefits are paid in full for behavioural

*Chapter 2 is reproduced from Rickayzen (2001) health problems. In addition, benefits are paid whether the cause of disability is due to an accident or to sickness. The major change in the last 20 years has been the switch in individual business from non-cancellable individual business to guaranteed renewable.

The primary difference between group and individual PHI business is the impact of tax on premiums and benefits paid. Under group business, the employer generally pays the premiums, which are tax deductible, and the benefits paid to the employees are taxed as salary. Under the individual business, there is no tax relief on the premiums paid, but the benefits are paid free of income tax. Waiver of premium is included as a benefit provision. The most common deferred periods are one week, four weeks, 13 weeks, 26 weeks, and 52 weeks.

Benefit limitations apply related to pre-disability income. Benefits from all sources are taken into account, including other group and individual insurances and pensions received. Various disability definitions are offered, including inability to follow any occupation.

2.1.2 Objectives

The objective of this paper is to introduce a practical mathematical model of a U.K. style PHI system. Specifically, the PHI system is modelled using a multi-state process in which, as a healthy individual ages, he or she may become sick then recover, become sick again, etc., until death.[1] Thus the individual's health fluctuates between two states (sickness and health) until death. If healthy, sick, and dead are viewed as separate states, the probability that a policyholder moves from the sick state to the dead state or to the healthy state depends on the time spent in the sick state. In other words, the transition probabilities depend on duration in a particular state as well as the age of the policyholder.

[1] For a detailed discussion on the use of multi-state models in disability insurance, see, for example, Haberman and Pitacco (1999).

It is possible to incorporate the duration-dependence aspect in the model, which leads to a much more complicated model. This is the approach used in the 1991 *Continuous Mortality Investigation Report No. 12* (CMIR 12) – see CMI Committee (1991). To obtain numerical values for the transition forces within the PHI model, CMIR 12 splits the sick states into 781 sub-states, each relating to a different duration of sickness. CMIR 12 then calculates probabilities at every 1/156th of a year of age for duration of sickness up to 5 years in all (making 780 sub-states) and all sickness periods beyond 5 years are aggregated. CMIR 12 (Part D) shows how it is possible to obtain numerical values for probabilities, annuities, etc. Clearly, CMIR 12 provides a thorough and complex model.

The approach taken in this paper is to develop a simpler model, one with only three (healthy, sick, and dead) states, then split the sick state into a small number of sub-states. We adopt the approach based on Jones (1994). Though the CMIR 12 technique of splitting the sick states into sub-states pre-dates Jones, Jones' approach is simpler because it uses a constant force of transition assumption for transition from state to state. This maintains the Markov property of the model. Increasing the number of states makes the state space more complicated, but maintaining the Markov process keeps the calculations tractable.

One advantage of using the simpler model described in this paper is that it can easily be used by actuaries who do not have access to complex models such as CMIR 12 or the detailed data required to use such models. It also can be used as an initial practical model for actuaries who are interested in rough estimates for net premiums for PHI models.

The paper is organized as follows: Section 2.2 introduces the model of the various transition probabilities. Expressions are derived for the transition probabilities required to obtain actuarial present values. Section 2.3 explains the connection between the parameters used in the model and those that are derived using data contained in CMIR 12. The data contained in CMIR 12 are used to test the sensitivity of the net premium to some of the parameters involved in the transition probabilities. Section 2.4 describes the results, while Section 2.5 provides a summary and conclusions.

2.2 The Model

2.2.1 The States and Transition Probabilities

The PHI model has six states labelled one to six.

- State 1 (Super Healthy): This is the state in which new policyholders enter the model when their policy commences. Because they have provided satisfactory medical evidence, new policyholders are deemed to be select lives and therefore healthier than other insured lives of the same age. We describe these lives as *super healthy*.

- State 2 (Ultimate Healthy): It is likely that, in time, the selection effect will disappear and that the super healthy lives will move to the ultimate form of the healthy state from which they may become sick enough to make a claim under the PHI policy.

- State 3 (Short-Term Sick): It is possible to recover from the short-term sick state 3 and, therefore, to return to state 2.

- State 4 (Long-Term Sick): It is not possible to recover from the long-term sick state. Death is the only mode of exit from this state.

- State 5 (Lapse): We assume that only super healthy policyholders will lapse their policy because policyholders in any other state would find it worthwhile to continue their PHI policy.

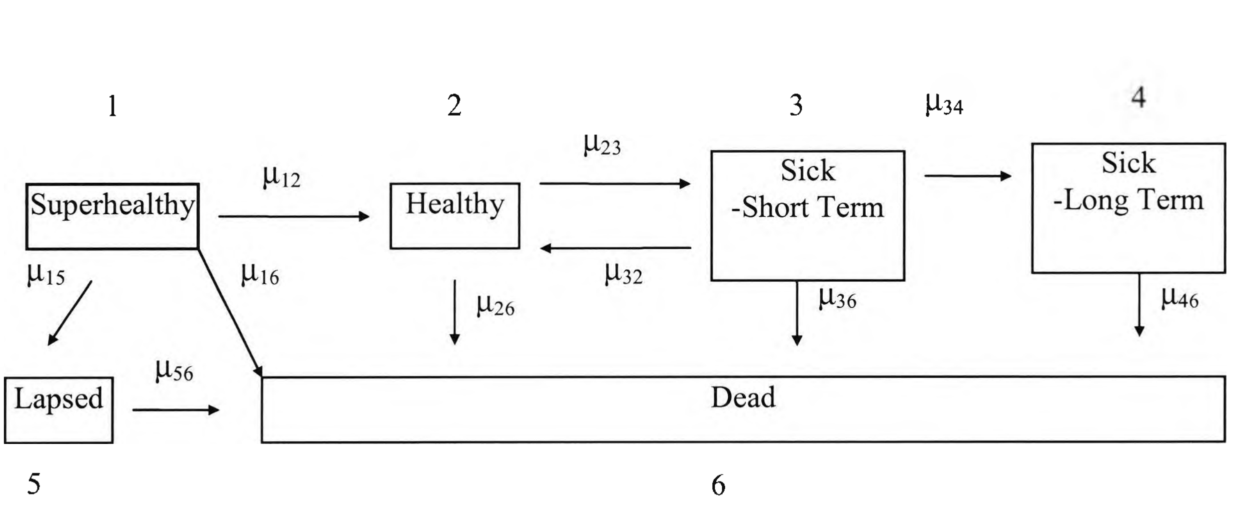
- State 6: Death.

A diagrammatic representation of the multi-state model adopted in this paper is displayed in Figure 2.1.

It is possible to introduce more sickness states as a proxy to a greater number of durations of sickness. This has not been done, however, because it is difficult to choose parameter values for the transition forces between the different sick states. In addition, having more states would increase the computational problems, albeit not insurmountably.

The forces of transition between states in PHI are continuous functions that depend on many factors including such factors as age, sex, income, and the time spent in a state. Though the exact mathematical form of these functions is unknown, we are sure that they are not constant.

Figure 2.1 Outline of PHI Model



Due to the mathematical difficulties inherent in using continuously varying forces, however, we will adopt the general methodology described in Jones (1994), i.e., we assume that the forces of transition are piecewise constant over each age interval instead.

Suppose there are n states labeled 1, 2,... ,n. Let $\mu_{ij}(x + t)$ denote the force of transition from state i to state j at age $x + t$, for $i, j = 1, 2, 3,\ldots, n$, $x = 0, 1, 2,\ldots$, and $0 \leq t \leq 1$. If state j is not linked directly to state i then $\mu_{ij}(x + t) = 0$. It is convenient also to define, for each i,

$$\mu_{ii}(x+t) = -\sum_{\substack{j=1 \\ j \neq i}}^{n} \mu_{ij}(x+t) \tag{2.1}$$

where $i = 1, 2, 3,\ldots, n$, $x = 0, 1, 2,\ldots$, and $0 \leq t \leq 1$.

The piecewise constant force of transition implies that

$$\mu_{ij}(x+t) = \mu_{ij}(x) \text{ for } x = 0, 1, 2, \ldots \quad \text{and} \quad 0 \leq t \leq 1 \tag{2.2}$$

One implication of the piecewise constant transition intensities assumption is that the length of time already spent in the current state has no effect on the future length of time that the policyholder will remain in the state, i.e., a memoryless property exists. [See Haberman (1992) for more on the memoryless property of multi-state processes with constant transition intensities.]

Next, let $p_{ij}(t, x)$ be the probability that a life currently exact age x in state i will be in state j in t years time. The common approach[2] to deriving an expression for $p_{ij}(t,x)$ is to use the Chapman-Kolmogorov backward system of difference-differential equations as contained in Cox and Miller (1965, Chapter 4). The backward system of equations is derived by considering the interval $(0, t+h]$ as comprising subintervals $(0, h]$ and $(h, t+h]$ and letting $h \rightarrow 0$:

$$\frac{d}{dt} p_{ij}(t,x) = \sum_{k=1}^{n} \mu_{ik}(x) p_{kj}(t,x) \qquad (2.3)$$

for $i, j = 1, \ldots, n$, $x = 0, 1, \ldots$, and $0 \leq t \leq 1$.

These equations lead to a set of difference-differential equations. For illustration purposes, some of the differential equations are presented below:

[2] See, for example, Ramsay (1989), Jones (1994), and Haberman (1995).

$$
\left.
\begin{aligned}
\frac{d}{dt}p_{11}(t) &= -(\mu_{12}+\mu_{15}+\mu_{16})\,p_{11}(t)\\
\frac{d}{dt}p_{12}(t) &= -(\mu_{12}+\mu_{15}+\mu_{16})\,p_{12}(t)+\mu_{12}p_{22}(t)\\
\frac{d}{dt}p_{22}(t) &= -(\mu_{23}+\mu_{26})\,p_{22}(t)+\mu_{23}p_{32}(t)\\
\frac{d}{dt}p_{23}(t) &= -(\mu_{23}+\mu_{26})\,p_{23}(t)+\mu_{23}p_{33}(t)\\
\frac{d}{dt}p_{33}(t) &= \mu_{32}p_{23}(t)-(\mu_{32}+\mu_{34}+\mu_{36})\,p_{33}(t)\\
\frac{d}{dt}p_{32}(t) &= \mu_{32}p_{22}(t)-(\mu_{32}+\mu_{34}+\mu_{36})\,p_{32}(t)\\
\frac{d}{dt}p_{44}(t) &= -\mu_{46}p_{44}(t)\\
\frac{d}{dt}p_{46}(t) &= -\mu_{46}p_{46}(t)+\mu_{46}p_{66}(t)\\
\frac{d}{dt}p_{55}(t) &= -\mu_{56}p_{55}(t)\\
\frac{d}{dt}p_{56}(t) &= -\mu_{56}p_{56}(t)+\mu_{56}p_{66}(t)\\
\frac{d}{dt}p_{66}(t) &= 0
\end{aligned}
\right\}
\qquad (2.4)
$$

The easiest way to solve the system of differential equations given in equation (2.3) is to follow the method outlined by Cox and Miller (1965), which involves matrix manipulation. First define the following $n \times n$ matrices

$M(x) = \{\mu_{ij}(x)\}_{i,j=1}^{n}$ = The forces of transition matrix;

$P(t,x) = \{p_{ij}(t,x)\}_{i,j=1}^{n}$ = The transition probability matrix; and

$$P'(t,x) = \left\{\frac{d}{dt}p_{ij}(t,x)\right\}_{i,j=1}^{n}.$$

The Chapman-Kolmogorov backward system of equations may be written as

$$P'(t,x) = M(x)P(t,x) \tag{2.5}$$

for x = 0, 1,... , and $0 \le t \le 1$, with boundary condition $P(0,x)$ = I (where I is the identity matrix).

It is easily seen that equation (2.5) has the solution

$$P(t,x) = e^{tM(x)} = I + \sum_{k=1}^{\infty} \frac{t^k}{k!}(M(x))^k \tag{2.6}$$

If it is known that $M(x)$ has distinct eigenvalues $d_1(x), d_2(x), ..., d_n(x)$, then

$$M(x) = A(x).D(x).A(x)^{-1} \tag{2.7}$$

where $D(x)$ is the diagonal matrix:

$$D(x) = \text{diag}\left(d_1(x), d_2(x), ..., d_n(x)\right)$$

and the i^{th} column of $A(x)$ is the right-eigenvector associated with $d_i(x)$ (Cox and Miller 1965, Chapter 4.5). Equations (2.6) and (2.7) lead to the following expression for $P(t,x)$:

$$P(t,x) = A(x).\text{diag}\,(e^{td_1(x)}, ..., e^{td_n(x)}).A(x)^{-1} \tag{2.8}$$

In this paper, equation (2.8) is used to compute $P(t,x)$.

Once $P(t,x)$ is known for x = 0, 1,... , and $0 \le t \le 1$, we must develop an expression to compute $p_{ij}(t, x)$ for x = 0, 1,... , and $t > 1$. Suppose $t = k + s$ where k = 1, 2,...

and $0 \le s \le 1$. It follows that

$$P(k+s,x)=\left(\prod_{r=1}^{k} P(1,x+r-1)\right)P(s,x+k). \tag{2.9}$$

Next, as premiums and benefits are paid m times per year, we need expressions for transition probabilities at m^{thly} intervals. Consider the form of $p_{ij}(1/m, x + h/m)$ where $h = 0, 1,\ldots, m-1$. Under the piecewise constant assumption of equation (2.2), $p_{ij}(1/m, x + h/m)$ is independent of h for $h = 0, 1,\ldots, m-1$. Let us define $\gamma_{ij}^{(m)}(x)$

$$\gamma_{ij}^{(m)}(x)=p_{ij}\left(\frac{1}{m},x+\frac{h}{m}\right) \tag{2.10}$$

In other words, $\gamma_{ij}^{(m)}(x)$ is the probability that a person currently age $x + h/m$ and in state i will be in state j at age $x+(h+1)/m$ where $h = 0, 1,\ldots, m-1$. We now define the $n \times n$ matrix:

$$\Gamma_{x}^{(m)}=\left\{\gamma_{ij}^{(m)}(x)\right\}_{i,j=1}^{n} \tag{2.11}$$

It follows that, for $t = k + h/m$, $k = 0, 1,\ldots,$ and $h = 0, 1,\ldots, m-1$,

$$P\left(k+\frac{h}{m},x\right)=\left(\prod_{r=1}^{k}\left(\Gamma_{x+r-1}^{(m)}\right)^{m}\right)\left(\Gamma_{x+k}^{(m)}\right)^{h} \tag{2.12}$$

and $p_{ij}(t, x)$ can be determined. There is no real advantage to using equation (2.12) over equation (2.9) except when m is large. If m is large, say $m = 52$ (i.e., weekly payments), we can approximate $\gamma_{ij}^{(m)}(x)$ as follows:

$$\gamma_{ij}^{(m)}(x) = \begin{cases} \frac{1}{m}\mu_{ij}(x) & \text{if } i \neq j \\ 1+\frac{1}{m}\mu_{ii}(x) & \text{if } i = j \end{cases} \tag{2.13}$$

2.2.2 Determination of the Net Premium

Premiums are assumed to be payable weekly in advance. A premium is only payable if the policyholder is either in state 1 (super healthy) or state 2 (ultimate healthy) at the start of the week in the policy year under consideration if premiums are waived during periods of sickness.

The annual net premium P is determined by equating the actuarial (expected) present value of future net premiums and the actuarial (expected) present value of future benefits at policy inception. To determine the net premium we need an expression for the present value of an m^{thly} annuity due payable for z years whenever x is in state j, which is:

$$_{ij}\ddot{a}^{(m)}_{x:\overline{z}|} = \frac{1}{m}\sum_{r=0}^{zm-1} v^{r/m} p_{ij}\left(\frac{r}{m}, x\right) \tag{2.14}$$

and an expression for the present value of an m^{thly} annuity immediate payable for z years whenever x is in state j, which is:

$$_{ij}a^{(m)}_{x:\overline{z}|} = \frac{1}{m}\sum_{r=1}^{zm} v^{r/m} p_{ij}\left(\frac{r}{m}, x\right) \tag{2.15}$$

It follows that the actuarial present value (APV) of the future premium is:

APV of Future Premiums $= P\left({}_{11}\ddot{a}^{(m)}_{x:\overline{z}|} + {}_{12}\ddot{a}^{(m)}_{x:\overline{z}|}\right)$.

The PHI benefit is assumed to be paid weekly during periods of sickness at the rate of *£B* per year. The PHI benefit is only payable if the policyholder is in either state

3 (short-term sick) or state 4 (long-term sick) at the end of the week in the policy year under consideration. Hence, the actuarial present value of the PHI benefits is:

$$\text{APV of Future Benefits} = B\left({}_{13}a_{x:\overline{z}|}^{(m)} + {}_{14}a_{x:\overline{z}|}^{(m)} \right).$$

Therefore, we can find P from

$$P = \frac{B\left({}_{13}a_{x:\overline{z}|}^{(m)} + {}_{14}a_{x:\overline{z}|}^{(m)} \right)}{\left({}_{11}\ddot{a}_{x:\overline{z}|}^{(m)} + {}_{12}\ddot{a}_{x:\overline{z}|}^{(m)} \right)} \qquad (2.16)$$

2.3 PHI Data and Parameter Values

The parameter values used in this model have been influenced by the data contained in CMIR 12. As the data used in CMIR 12 are somewhat outdated, it is not necessary to input into our model precisely the output values emanating from CMIR 12.[3] Therefore CMIR 12 is simply used as a guide to choosing parameter values for this paper.

For convenience the ages are grouped into 5-year age bands with the forces of transition assumed to be constant over each 5-year age band. The age bands are 30–34, 35–39, ..., 60–64. Next we describe the way in which each parameter value has been chosen.

$\mu_{23}(x)$ **(Unstable Healthy → Short-Term Sick):** This parameter is based on the sickness inception rate, σ_x, described in Part C of CMIR 12. We use the values of σ_x for a deferred period of 13 weeks because the data sets for the shorter deferred periods (i.e., one week and four weeks) may be less typical of the general insured population. The values for the deferred period of 13 weeks are found in Table C16 of CMIR 12 (p. 74).

[3] CMIR 12 is based on data collected between 1975 and 1978. Subsequent work by Clark and Dullaway (1995), Haberman and Walsh (1998), and Renshaw and Haberman (2000) have suggested that PHI experience has changed since 1978.

The force of sickness, σ_x , in CMIR 12 should be applied to the whole of the healthy population (i.e. states 1 and 2 combined) whereas $\mu_{23}(x)$ is a force that operates only on lives in state 2 (i.e. the healthy state). It could be argued, therefore, that the values of σ_x taken from CMIR 12 should be adjusted. Because CMIR 12 is being used merely as a guide, no adjustments have been made, i.e., $\mu_{23}(x) = \sigma_x$.

$\mu_{16}(x)$ **(Super Healthy → Dead):** Under CMIR 12 the mortality rate for healthy lives is assumed to be that of male permanent assurances 1979–82, duration 0. The rates are shown in Table E17 (p. 132) under the column headed *m(x)*. In our model, we have divided healthy lives into *super healthy* and *ultimate healthy* states. Because lives in the latter state will experience higher mortality rates than those in the former, we have decided to assume: $\mu_{16}(x) = 0.80m(x)$, i.e., 80 percent of the mortality rates for male permanent assurances of 1979-82, duration 0.

$\mu_{26}(x)$ **(Ultimate Healthy → Dead):** We assume $\mu_{26}(x) = 1.20m(x)$, i.e., 120 percent of the mortality rates for male permanent assurances of 1979-82, duration 0.[4]

$\mu_{32}(x)$ **(Short-Term Sick → Ultimate Healthy):** Recovery rates are described in Section 3, Part B of CMIR 12. On page 34 of CMIR 12 various values of $\rho_{y+z,z}$, the transition intensity from sick to healthy at current age $y + z$ and current duration of sickness z, are displayed. These recovery rates vary markedly by duration of sickness (measured in weeks). In view of the relatively simple approach adopted in our model, we will use a constant parameter value, i.e., $\mu_{32}(x) = 2.5$ at all ages.

[4] The overall effect of the mortality assumptions for $\mu_{16}(x)$ and $\mu_{26}(x)$ can be considered to be broadly consistent with CMIR 12. As suggested by Cordeiro (1995), net premium values are likely to be less sensitive to the parameter values chosen for the forces of mortality.

$\mu_{36}(x)$ **(Short-Term Sick → Dead):** These mortality intensities are described in Section 6, Part B of CMIR 12. On page 39 of CMIR 12 the values of $v_{y+z,z}$ at various ages are displayed where $v_{y+z,z}$ is the transition intensity from sick to dead at current age $y + z$ and current duration of sickness z measured in weeks. For our calculations, we will use the values at 15 weeks duration of sickness, which is when the transition intensities reach their peak, i.e., $\mu_{36}(x) = v_{x,15}$. Interpolated values have been used where necessary.

$\mu_{34}(x)$ **(Short-Term Sick → Long-Term Sick):** CMIR 12 does not provide explicit parameter values for $\mu_{34}(x)$. Having considered the order of magnitude of all the other forces in the model, we assume $\mu_{34}(x) = 0.1$ at all ages.

$\mu_{46}(x)$ **(Long-Term Sick → Dead):** We can again consider the mortality intensities $v_{y+z,z}$ that were described under $\mu_{36}(x)$ above. It seems appropriate to use these intensities at a suitably long sickness duration. We will use the values at duration five years (260 weeks) that are shown on page 39 of CMIR 12,

i.e., $\mu_{46}(x) = v_{x,260}$.

$\mu_{56}(x)$ **(Lapse → Dead):** Because only super healthy policyholders lapse their policies, we will assume that $\mu_{56}(x) = \mu_{16}(x)$

$\mu_{12}(x)$ **(Super Healthy → Ultimate Healthy):** CMIR 12 is not able to provide explicit parameter values for $\mu_{12}(x)$. It seems reasonable, however, to ensure that our estimates of $\mu_{12}(x)$ should be such that the aggregate mortality rates implied within our model approximately reflect the U.K. Male Permanent Assurances 1979–

82 (duration 0) mortality table. The values for $\mu_{12}(x)$ that meet this constraint are, for simplicity, chosen by inspection.

$\mu_{15}(x)$ **(Super Healthy → Lapse):** Finally, having set the other parameters, $\mu_{15}(x)$ is varied in order to investigate its effect on the net premium rate.

Table 2.1 displays the parameter values. Table 2.2 shows the number of lives in each state at various sample ages given 100 super healthy lives entering state 1 at age 30, using the data in Table 2.1 and assuming $\mu_{15}(x) = 0.05$ for all x.[5] For example, Table 2.2 shows that, by age 65, 12.0 percent of the lives would have died, 50.6 percent would have lapsed, and none of the lives would still be in the super healthy state.

The next step is to calibrate the model, i.e., to check if the model can produce the expected proportions of lives that are healthy, sick, or dead at various ages similar to those shown in CMIR 12 (Table E14, page 126). Table 2.3 displays these comparisons. The proportions are similar, particularly up to age 55. In Section 2.4.1 we will make another reasonableness check by comparing the net premium implied by our model with that implied by CMIR 12.

[5] The assumption $\mu_{15}(x) = 0.05$ is consistent with the assumption of Sanders and Silby (1986) who use a lapse rate of 5 percent per annum for policy duration greater than two years.

Table 2.1

Summary of Parameters

Age x	$\mu_{16}(x)$	$\mu_{26}(x)$	$\mu_{46}(x)$	$\mu_{36}(x)$	$\mu_{23}(x)$	$\mu_{12}(x)$
30–34	0.0003	0.0005	0.0172	0.1108	0.1982	0.0270
35–39	0.0004	0.0006	0.0190	0.1180	0.1766	0.0150
40–44	0.0006	0.0010	0.0215	0.1251	0.1560	0.0480
45–49	0.0011	0.0017	0.0239	0.1379	0.1408	0.1100
50–54	0.0019	0.0028	0.0271	0.1507	0.1337	1.1000
55–59	0.0031	0.0046	0.0303	0.1694	0.1375	1.5000
60-65	0.0049	0.0073	0.0343	0.1880	0.1576	2.0000

Notes: We have assumed (i) constant forces of transition over successive 5-year age bands (i.e., age 30–34, 35–39, ... , 60–64); and (ii) $\mu_{56}(x) = \mu_{16}(x)$, $\mu_{32}(x) = 2.5$, and $\mu_{34}(x) = 0.1$ for all x.

Table 2.2

Percentage of Lives in Each State at Sample Ages

	State					
Age	1	2	3	4	5	6
30	100	0	0	0	0	0
31	92.6	2.5	0.1	0	4.8	0
32	85.7	4.7	0.3	0	9.3	0
⋮	⋮	⋮	⋮	⋮	⋮	⋮
50	13.4	30.3	1.5	1.5	50.0	3.3
⋮	⋮	⋮	⋮	⋮	⋮	⋮
65	0	32.4	1.9	3.1	50.6	12.0

Notes: using the data from Table 1 and $\mu_{15}(x) = 0.05$

Table 2.3
Comparing Percentage of Healthy, Sick and Dead Lives Under CMIR 12 (Table E14) with Our Model

	CMIR 12 (Table E14)			**Our Model**		
Age	Healthy	Sick	Dead	Healthy	Sick	Dead
35	98.4	1.1	0.5	98.8	0.9	0.3
40	97.3	1.4	1.3	97.6	1.4	1.0
45	95.8	1.9	2.3	96.0	2.1	1.9
50	93.2	2.8	4.0	93.7	3.0	3.3
55	88.9	4.4	6.7	90.4	4.1	5.5
60	81.6	7.4	11.0	87.1	4.5	8.4

Notes: Our model uses the data from Table 2.1 and $\mu_{15}(x) = 0.05$

2.4 The Main Results

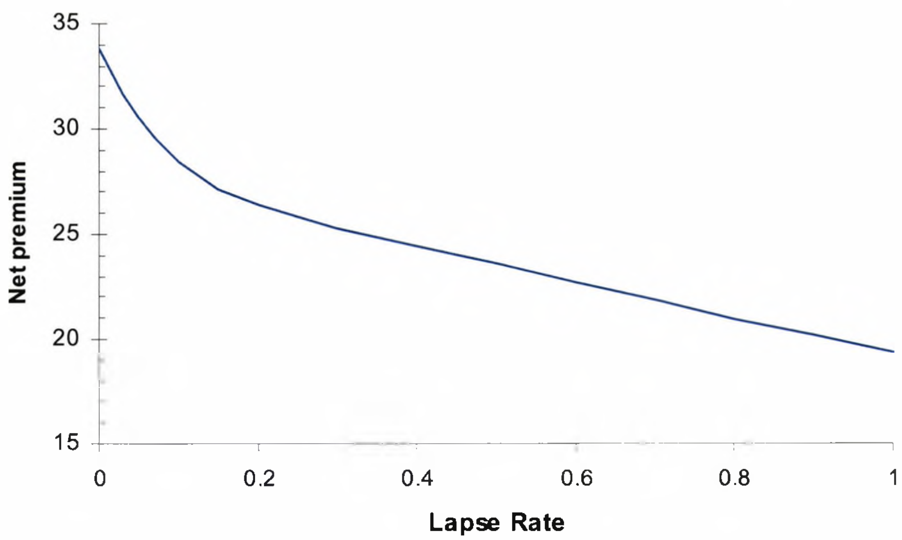
The PHI policy under consideration here is a 35-year term policy issued to a life age 30. The sickness benefit is paid weekly during periods of sickness at the rate of £1,000 per annum. Premiums are paid weekly and are waived during periods of sickness. Benefits are paid on a weekly basis. There is no deferred period, and the benefits and premiums cease at the age of 65. The valuation rate of interest is set to 6 percent per year. The forces of transition used are given in Table 2.1.

2.4.1 Sensitivity of Net Premiums to Various Parameters

Sensitivity of P to $\mu_{15}(x)$: Figure 2.2 shows how the net premium varies as the lapse rate $\mu_{15}(x)$ takes values between 0 and 1. The net premium is relatively insensitive to the lapse rate. For example, the net premium decreases from £33.79 per annum to £26.36 per annum as the lapse rate increases from 0 to 0.2. This relative insensitivity is due to the fact that only super healthy lives lapse their policies, and their reserves are relatively small. Lapse rates of more than 0.4 would be unrealistic.

For example, it can be shown that if $\mu_{15}(x) = 0.4$, over 83 percent of the insured population age 30 at the outset would have lapsed their policy during the first five years of the policy.

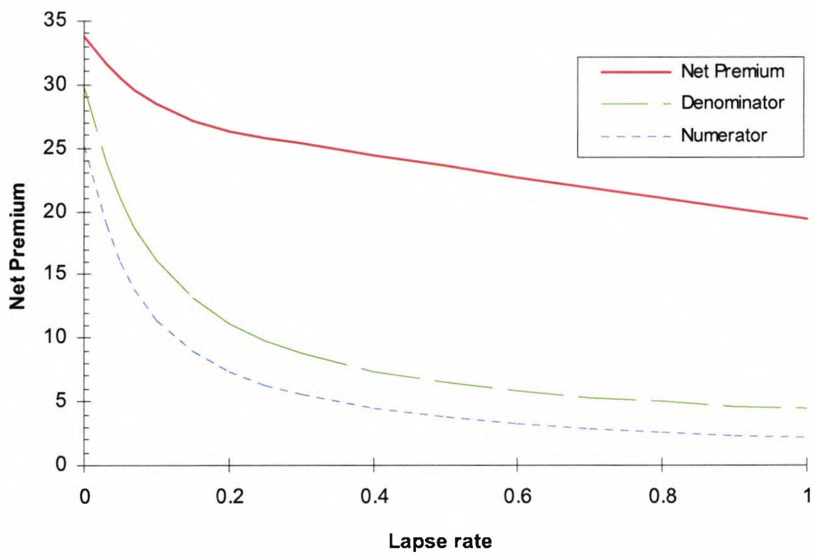
Figure 2.2
Sensitivity of Net Premium to lapse rate, μ_{15}



It is counter-intuitive that the net premium decreases rather than increases as the lapse rate increases. Standard actuarial logic suggests that the net premium should increase, because when the lapse rate is small, there are large numbers of lives in the system who are in the super healthy state and therefore continue to pay premiums without receiving any PHI benefit payments. This tends to suppress the net premium averaged over all the policyholders in the system. As the lapse rate increases, more of the super healthy lives leave the system by lapsing, which will tend to increase the average premium payable in respect of the remaining, relatively unhealthy, insured population.

So why does the net premium decrease as the lapse rate increases? Figure 3 shows how the numerator and the denominator of the right-hand side of equation (2.16) vary as the lapse rate increases. We show scaled versions of the numerator and denominator in order to fit them on the same graph. Both numerator and the denominator decrease, as would be expected, because the effect of lapses is to remove lives from state 1 before they have an opportunity to enter states 2, 3, or 4. The rate of decrease is the result of the complicated interaction between the different forces within the model. It can be seen that the numerator decreases at a faster rate than the denominator, and, therefore, the overall effect is that the net premium decreases.

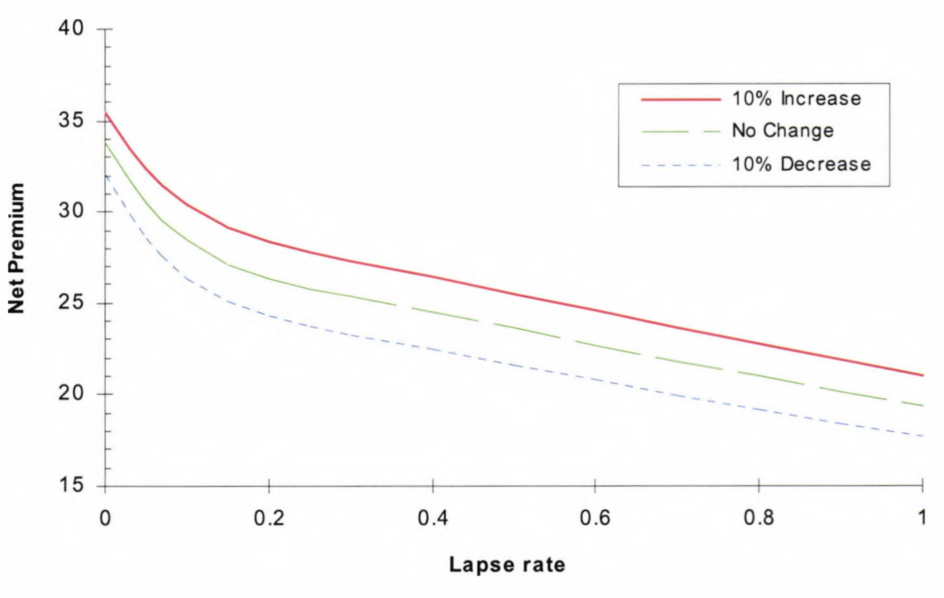
Figure 2.3
Variations in the numerator and denominator
of the Net Premium as the lapse rate increases



Finally, before discussing other sensitivity issues, it is worth comparing the net premiums calculated using the model described in this paper with those derived from the data in CMIR 12. The data contained in Table F1 on page 228 of CMIR 12 suggest that the net premium for a policy similar to that described earlier in this section, but with premium and benefit payments made continuously and with a deferred period of one week, should be £24.24 per annum. The net premium figures shown in Figure 2.2 are of the same magnitude and hence provide some comfort that our model (including the parameter values chosen) is consistent with the model described in CMIR 12.

Sensitivity of P to $\mu_{12}(x)$: Figure 2.4 shows how the net premium changes when the parameter values for $\mu_{12}(x)$ given in Table 2.1 are increased or decreased by 10 percent. If $\mu_{12}(x)$ is increased by 10 percent, the net premium increases by between 4.9 percent (when the lapse rate, $\mu_{15}(x) = 0$) and 8.4 percent (when $\mu_{15}(x) = 1.0$). If $\mu_{12}(x)$ is reduced by 10 percent, the net premium decreases by between 5.1 percent (when $\mu_{15}(x) = 0$) and 8.7 percent (when $\mu_{15}(x) = 1.0$).

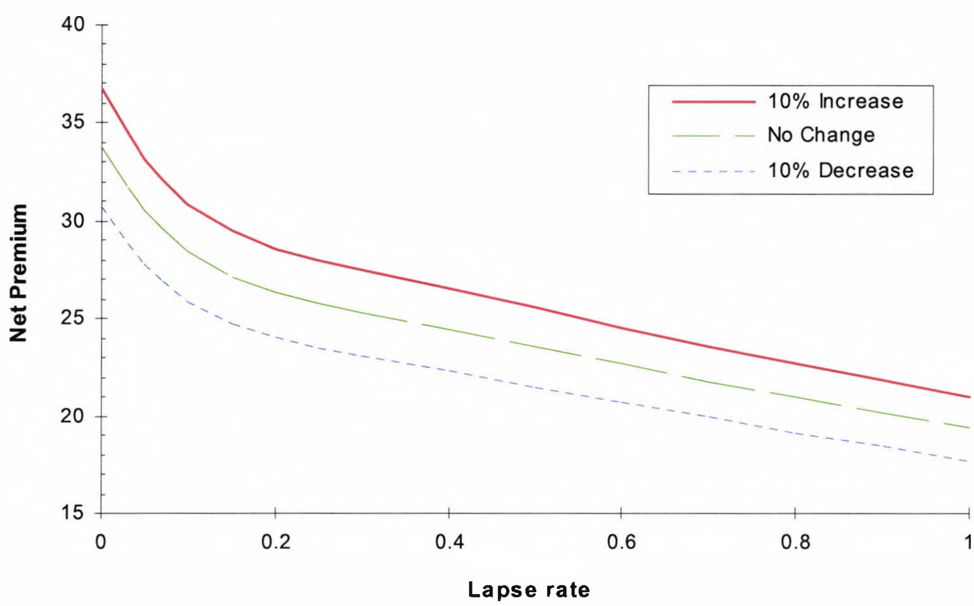
Figure 2.4
Net Premium sensitivity to a 10% change in $\mu_{12}(x)$



As expected, the net premium is expected to move in the same direction as $\mu_{12}(x)$. An increase in $\mu_{12}(x)$ causes more lives to move from the super healthy to the ultimate healthy state where they are exposed to the risk of sickness inception, which, in turn, will lead to an increase in the premium required.

Sensitivity of P to $\mu_{23}(x)$: Figure 5 shows how net premiums change when the parameter values for $\mu_{23}(x)$, the sickness inception rate, are altered by 10 percent. The net premium increases by approximately 8.6 percent when the $\mu_{23}(x)$ values are increased by 10 percent and decreases by approximately 8.9 percent when the $\mu_{23}(x)$ values are decreased by 10 percent. These results (in terms of relative sensitivities) are largely unaffected by the level of lapse rate assumed. As expected, an increase in the sickness inception rate causes an increase in the net premium required.

Figure 2.5
Net Premium sensitivity to a 10% change in $\mu_{23}(x)$



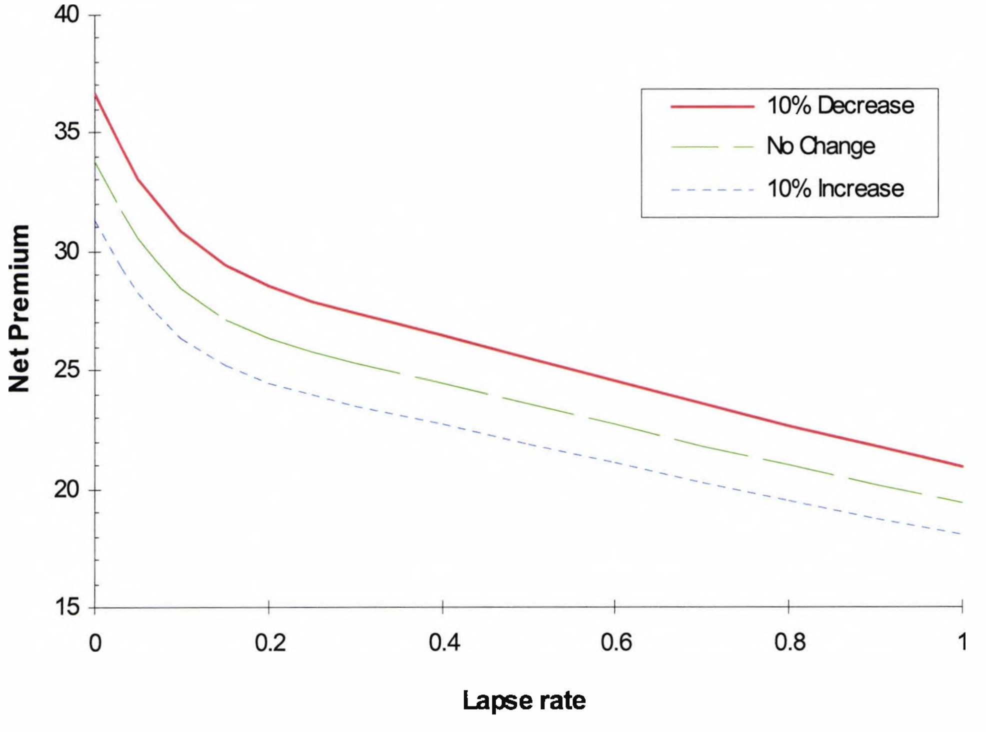
Cordeiro (1995) extends the work described in CMIR 12 by considering the effect on net premiums in changes in the sickness inception rates for various deferred periods and entry ages. Cordeiro finds that, for the CMIR 12 model and data, if the sickness inception rate is doubled, the net premium is approximately doubled. The results of this paper are therefore consistent with those of Cordeiro (1995).

Sensitivity of P to $\mu_{32}(x)$: Figure 2.6 shows how net premiums change when the parameter value for $\mu_{32}(x)$, the recovery rate, is increased or decreased by 10 percent (i.e.,changed from 2.5 at all ages to 2.75 or 2.25, respectively).

The net premium increases by approximately 8.3 percent when the recovery rate is reduced by 10 percent and decreases by approximately 7.2 percent when it is increased by 10 percent. Again, the level of lapse rate has little effect on these relative sensitivities. It is to be expected that an increase in the recovery rate should lead to a reduction in the amount of PHI premium required.

Cordeiro (1995) has investigated the effect that changes in the recovery rates have on net premiums based on the CMIR 12 model and data. Cordeiro discovers that a 10 percent increase in the recovery intensity leads to a 27.6 percent reduction in the net premium for entry age 30 and deferred period one week. Therefore, the net premium is less sensitive to a change in the recovery intensity under the model described in this paper than under the model used by Cordeiro (1995).

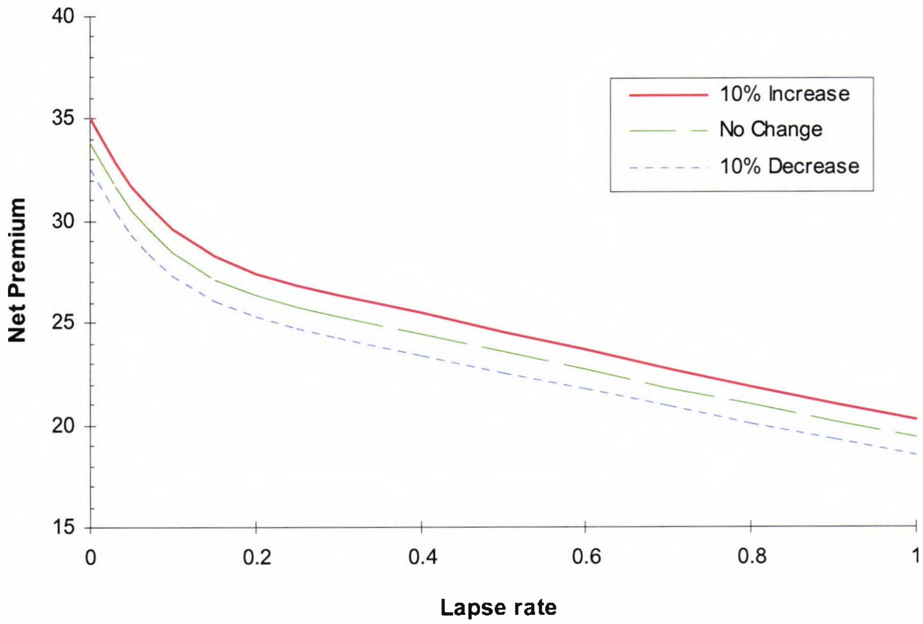
Figure 2.6
Net Premium sensitivity to a 10% change in $\mu_{32}(x)$



Sensitivity of P to $\mu_{34}(x)$: Figure 2.7 shows the changes in net premiums when the parameter values for $\mu_{34}(x)$ are increased or decreased by 10 percent.

It can be seen that the net premium is relatively insensitive to changes in $\mu_{34}(x)$ because a 10 percent increase/decrease in the latter causes only a 4.0 percent increase/decrease in the net premium. As expected, an increase in the long-term sickness inception rate leads to an increase in the net premium required.

Figure 2.7
Net Premium sensitivity to a 10% change in $\mu_{34}(x)$



2.4.2 The Relationship Between $\mu_{12}(x)$ and $\mu_{32}(x)$

In Section 2.3, we explained how the parameter values for μ_{12} are chosen so that the aggregate mortality rates within the model broadly reflect the male permanent assurances 1979–82, duration 0. We now analyse how sensitive the values of $\mu_{12}(x)$ are to a change in the other parameters, in particular to a 50 percent increase in the recovery rate, $\mu_{32}(x)$ In other words, we retain all the parameter values summarized in Table 2.1 except for $\mu_{32}(x)$, which we increase from 2.5 at all ages to 3.75, and $\mu_{12}(x)$, which we need to recalibrate in order to ensure that the aggregate mortality rates still reflect the mortality table mentioned above. The results are summarized in Table 2.4.

Table 2.4

Comparison of $\mu_{12}(x)$ values

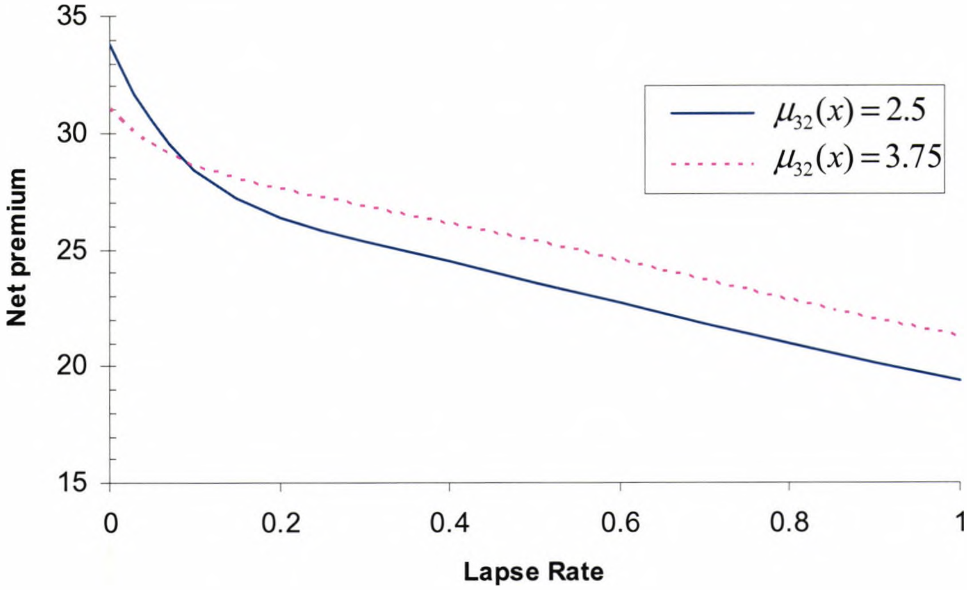
when $\mu_{32}(x)$ increases from 2.5 to 3.75

Age	$\mu_{32}(x)=2.5$	$\mu_{32}(x)=3.75$
30-34	0.027	0.045
35-39	0.015	0.025
40-44	0.048	0.074
45-49	0.110	0.180
50-54	1.100	1.500
55-59	1.500	1.900
60-64	2.000	2.400

A 50 percent increase in $\mu_{32}(x)$ requires an increase in $\mu_{12}(x)$ of approximately the same order of magnitude up to age 50 in order to leave the aggregate mortality rates within the model unaltered.

Figure 2.8 shows the effect on the net premium of changing the values of $\mu_{32}(x)$ and $\mu_{12}(x)$ from the first column of Table 2.4 to the second column. The changes in the two sets of parameter values leave the net premium at approximately the same level as before (i.e., approximately £28 per annum), which is consistent with the net premium calculated using the model described in CMIR 12.

Figure 2.8
Impact on Net Premium of increasing $\mu_{32}(x)$



2.4.3 Relationship Between $\mu_{12}(x)$ and $\mu_{34}(x)$

In order to consider how sensitive the values of $\mu_{12}(x)$ are to a 50 percent increase in $\mu_{34}(x)$ (i.e., $\mu_{34}(x)$ = 0.15 at all ages), we must increase $\mu_{34}(x)$ to 0.15 and recalibrate $\mu_{12}(x)$ to ensure that the aggregate mortality rates reflect the male permanent assurances 1979–82, duration 0.

Table 2.5 shows the impact of $\mu_{34}(x)$ on $\mu_{12}(x)$ values. It can be seen that a 50 percent increase in $\mu_{34}(x)$ has little impact on the values of $\mu_{12}(x)$ required to leave the aggregate mortality rates within the model unaltered.

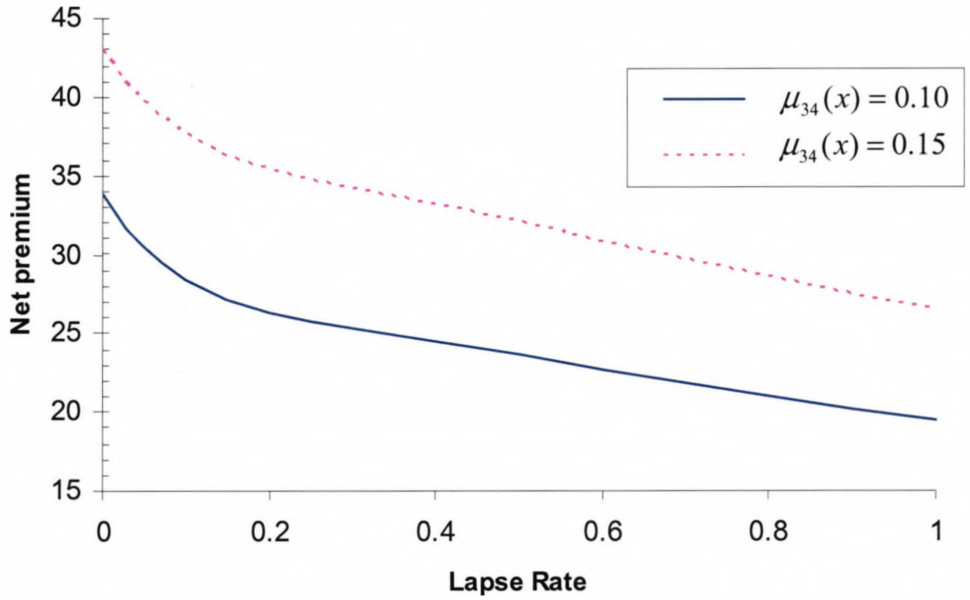
Table 2.5
Comparison of $\mu_{12}(x)$ values
when $\mu_{34}(x)$ increases from 0.10 to 0.15

Age	$\mu_{34}(x) = 0.10$	$\mu_{34}(x) = 0.15$
30–34	0.027	0.031
35–39	0.015	0.017
40–44	0.048	0.056
45–49	0.110	0.129
50–54	1.100	1.200
55–59	1.500	1.600
60–64	2.000	2.100

Figure 2.9 shows the effect on the net premium of changing the values of $\mu_{34}(x)$ and $\mu_{12}(x)$ from the first column of Table 2.5 to the second column. The shape of the two curves is the same. The net premium calculated using the second column of parameter values, however, is approximately 30 percent higher than when the first column of values is used.

This result involving changes to $\mu_{34}(x)$ and $\mu_{12}(x)$ contrasts with the result in Section 2.4.2 where increasing $\mu_{32}(x)$ and recalibrating $\mu_{12}(x)$ has a neutral effect on the net premium. This feature further illustrates how complicated the interaction between the transition intensities is within the model.

Figure 2.9
Impact on Net Premium of increasing $\mu_{34}(x)$



2.5 Conclusion

An objective of this paper is to develop a simple, practical U.K. style PHI model that can be used by actuaries who do not have access to complex models such as CMIR 12 or the detailed data required to use such models, or who are interested in rough estimates for net premiums for PHI models.

One of the main difficulties that needs to be overcome in maintaining the simplicity of the model, however, is that the forces of transition between different states may depend not only on the age of the policyholder, but also on the time spent in the current state. For example, the longer a policyholder remains in the sick state, the less likely he or she is to recover. That is, there is duration-dependence. This factor usually leads to a semi-Markov model being used. However, convenient expressions for the transition probabilities are then hard to obtain.

In this research, the problem of duration-dependence is handled, in part, by increasing the number of states to differentiate between short-term and long-term stays in a particular status. This enables the model to be Markov rather than semi-Markov and therefore leads to tractable solutions. The model also includes lapses.

Using a particular policy, we test the sensitivity of the net premium to changes in the most significant model parameter values ($\mu_{12}(x)$, $\mu_{15}(x)$, $\mu_{23}(x)$, $\mu_{32}(x)$ and $\mu_{34}(x)$). Not surprisingly, the net premium is relatively insensitive to changes in the lapse rate $(\mu_{15}(x))$ because only the most healthy lives are assumed to lapse their policies and they have small reserves. We also find that when any of the forces of transition, $\mu_{23}(x)$, $\mu_{32}(x)$, or $\mu_{34}(x)$ are increased, the resultant change in the level of net premium depends little on the level of the lapse rate. As a result, actuaries may initially ignore lapse rates when considering rough estimates for net premiums for PHI models.

By contrast, however, when the force of transition from the super healthy to the ultimate healthy state ($\mu_{12}(x)$) is increased, the extent to which the net premium increases depends on the level of the lapse rate. This shows that actuaries should probably spend more of their energies trying to obtain accurate estimates of $\mu_{12}(x)$.

3. MEASURING PROCESS RISK IN INCOME PROTECTION INSURANCE*

3.1 Introduction

Describing and quantifying the risks involved in health related products are becoming increasingly more important as insurance companies endeavour to respond to the demand for such products, created by external economic trends (eg the increasing numbers of self-employed) and demographic changes (eg the ageing of the population), noting that this type of business has caused substantial losses in the past. The objective of this paper is to consider the calculation of premiums for a portfolio of simple disability insurance policies in a stochastic environment represented both by random transitions in the underlying multiple state model and random external economic factors (i.e. factors not under the control of the insurer). The disability insurance product which we examine is now called income protection insurance, IPI (or simply IP) but was formerly known as permanent health insurance (PHI) in the UK.

We will consider the risk in the portfolio that emanates from the sickness–claims process (i.e. morbidity) and the risk attributable to the economic uncertainty represented by stochastic investment returns and inflation rates. We will also consider how this level of risk can be managed through, for example, a risk–loaded premium or through a solvency margin (i.e. risk–based capital) both of which can be determined from probabilistic calculations.

* Co-authored with S.Haberman and Z.Butt - Chapter 3 is reproduced from Haberman et al (2004)

We note that all models abstract to some extent from the real world and, as noted by Daykin et al (1994) and other authors, models are subject to three broad types of error or risk:

model error	– because models are not known with certainty and usually are only approximations to the real world;
parameter error	– past observation data are limited in quantity and so parameters are not known with certainty;
process (or stochastic) error	– our target quantities will be subject to random fluctuations about the mean, even when the model and parameters are correct.

Our focus will be on the third component: process error. The pooling of insured units in a portfolio is a critical part of the insurance process and, as demonstrated by Cummins (1991), leads to a reduction in the relative level of variability in the portfolio i.e. the risk per unit. This is a statement about process error – the other two types of error are not controllable in a comparable way. We note that external factors (for example, investment returns and inflation) have a systematic effect which affects all policies at the same time, so that the pooling of risks through writing more of the same business may not reduce the relative level of variability from this source: this is analysed in section 3.4 of the paper.

Before focusing on process error, we will make some comments at this stage about the two other types of error. Modelling a portfolio of disability insurance policies provides us with an example where model error could be important – there are, for example, at least three established methodologies for pricing and reserving of disability insurance products in use in the UK i.e. *Manchester Unity*, *inception rates plus disability annuity* approach and the *multiple state model* approach, although it is well argued that the multiple state model provides a framework within which the different models can be formulated successfully

(CMI Committee (1991), Haberman and Pitacco (1999)). Practitioners in the UK have tended to abandon the Manchester Unity approach because of the model error associated with it and now use one of the other two approaches. For IP portfolios, we might expect that the parameter risk would be more significant than in life insurance because:

a) the underlying data are less reliable. For example:
   - sickness inceptions with durations less than the deferred period are unobservable, making the data more scanty;
   - claim times tend to be a round number of weeks;

b) the definition of sickness and of a claim are less clear cut, and are influenced by policy conditions. Thus, a company will need care in using unadjusted industry wide data, as provided by the CMI Bureau in the UK;

c) sickness claims are often linked with socio–economic conditions and there is scope for moral hazard (Haberman (1987)).

Our main objective will be to identify ways of measuring the process error for a portfolio of independent income protection policies in a multiple state modelling context. A second objective will be to observe the extent to which the process error changes as we increase the volatility and complexity of the models generating investment returns and inflation. Thus, we start with a model that is based on random multiple state transitions and deterministic (i.e. expected) economic assumptions and then increase the volatility of this model by allowing for stochastic economic factors. In this way, we will be able to compare, in a specific case (i.e. for a particular policy), the magnitudes of the relative variability due to demographic process risks and due to demographic and economic process risks combined. Further, we will be able to identify and isolate the effect of different components of the process risks and ultimately to measure their impact on the proposed risk measures.

In life insurance, it is usual to consider the (liability) process risk as being relatively insignificant because claim amounts are fixed and portfolios tend to be large so that the law of large numbers and pooling lead to a reduction in process risk. However, Marceau and Gaillardetz (1999) have demonstrated that this effect operates differentially for different types of life insurance policy, with the mortality process risk being more significant for temporary insurances than for endowment insurances. In income protection insurance, there is variability in terms of claims incidence and size (i.e. measured by duration of time spent sick or disabled beyond the deferred period) and portfolios tend to be smaller than in life insurance, so that we might expect that liability process risk may not be negligible. This will be investigated in subsequent sections.

The paper is organised as follows. Section 3.2 describes the overall methodology. Section 3.3 describes in detail the models used and assumptions made, as well as introducing the four risk measures that will be used and the method of calculation of the premium. Section 3.4 describes and analyses a selection of the results obtained. Section 3.5 provides some concluding comments.

3.2 Methodology

We construct a simple asset–liability model of an insurer with only IP business using a multiple state model approach. Firstly, we simulate independent sample paths for individual policyholders occupying the healthy and sick states and moving between states, which are used to draw up a cumulative yearly experience over the policy term for the cohort of insured lives. Secondly, we simulate annual stochastic investment returns and stochastic inflation rates applying the widely accepted Wilkie model. We then construct a simple cash flow model for the portfolio and use the simulation results to construct useful financial quantities at the portfolio (or indeed company) level (see section 3.3).

In particular, we will consider four overall measures of risk, viz the probability of ruin and relative average shortfall for the portfolio and the levels of risk–based capital or risk–loaded premium to achieve a pre–specified probability of ruin. We will analyse and compare the results for three different models with increasing levels of volatility, that is:

a) stochastic sickness experience and deterministic economic factors, where the latter are obtained as the expected values of their stochastic counterparts: the **Deterministic Assets Model (DAM)**. This class of model is intended to describe the set of simulations where the final outcomes are based on a stochastic morbidity experience and fixed (i.e. deterministic) economic factors (i.e. return on investment and cash-flow inflation rates). Using this model, we will be able to measure the process risk due to the demographic factors and quantify the effect of the demographic volatility in isolation on the important risk measures that we have defined.

b) stochastic sickness experience combined with stochastic investment experience and deterministic inflation rates: the **Stochastic Assets Model 1 (SAM1)**. This second class of model is defined to capture the process risks resulting from the volatile nature of the morbidity experience and the uncertainty in the investment returns, while keeping the premium, benefits and expense inflation at a fixed level. In this type of model, the resulting asset returns are stochastic and less dependent on the size of portfolio. Intuitively, by comparing the outcomes of this group of simulations with those for DAM, we will be able to measure the process risk due to investment returns and evaluate the effect on the risk measures being considered.

c) a fully stochastic model, with all of the above demographic and economic factors randomised: the **Stochastic Assets Model 2 (SAM2)**. In this third class of model, we further emphasize the influence of economic process risk in the proposed simulations by including also the effect of stochastic inflation on premiums, benefits and expenses.

Therefore, we could consider the resulting model to be '*fully stochastic*' having each of its important components randomised and time dependent, so that the model is brought closer to replicating a real IP office experience. However, this is not the actual purpose of this final model, since the results would heavily depend on the chosen parameter values. Instead, our principal objectives are to measure the process risk arising from introducing the extra randomness attributable to these economic factors and to quantify their effect on the above mentioned risk measures.

Figure 3.1 illustrates the above strategy by indicating the type of extra volatility added to each model.

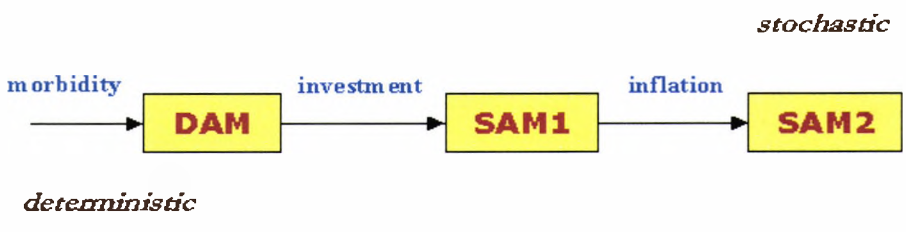


Figure 3.1 Incorporated volatility structure of the defined asset–liability models

3.3 Models And Assumptions

3.3.1 Policy Design

We consider a simple policy design with terminal age 65 and a range of entry ages: $\{30, 40, 50, 60\}$ with zero lapse rates. Premiums are payable while the policyholder is either healthy or sick but for a period less than the deferred period (i.e. a no claim exclusion period). Benefits are payable as an annuity while the policyholder is sick for a duration longer than the deferred period. Various expenses are incorporated. The policy design is such that the premiums and benefits are guaranteed from the outset to increase in line with inflation, but are not altered to take account of the actual demographic or investment experience.

We note that the premium and benefit levels may vary significantly in the case of the stochastic inflation model, although in practice, this effect would be mitigated by market pressures. All regulatory requirements and taxation are ignored. Overall, we assume a homogeneous IP portfolio made up of a cohort of independent policies and there is no possibility of their insurance cover being increased by starting additional contracts during the fixed term.

The detailed assumptions are presented in Table 3.1 and are intended to be representative of IP contracts in the UK at the time of writing.

Table 3.1 Summary of chosen policy design

IP Policy Design			
Deferred period:		(d)	13 weeks
Expenses:	*– Initial*	(e)	£200 at inception of policy (including commission)
	– Renewal	(w_0)	£25 pa if the policyholder is healthy
		(y_0)	£95 pa if the policyholder is sick
	– Claim	(z_0)	£200 at inception of the claim (i.e. claim registration and claim underwriting costs)
Return on investment:		(r_t)	10.7% pa expected rate of investment return for the deterministic approach or Wilkie model for the stochastic approaches (see section 3.3.3.1)
Escalation rates:	*– premium*	(e_1)	3% pa expected inflation of premiums and benefits using both deterministic (i.e. fixed) or stochastic (i.e. time series) approaches (see section 3.3.3.2)
	– expense	(e_2)	4% pa expected inflation of expenses using both deterministic (i.e. fixed) or stochastic (i.e. time series) approaches (see section 3.3.3.2)
Premium:		(p_0)	£220 pa for entry age 30 and similar levels for other entry ages across different models (see section 3.3.6)
Benefits:		(c_0)	£8,000 pa while policyholder is sick, but subject to the requirements of the deferred period
Ages at entry:		(x_1)	30, 40, 50 or 60
Termination age:		(x_2)	65

3.3.2 <u>The Demographic Model</u>

3.3.2.1 Multiple State Model

To describe the transitions, we have used the continuous time and three-state (i.e. discrete) healthy-sick-dead model (H, S, D), with the states labelled $S=\{1, 2, 3\}$ respectively. It should be noted that, given the universality of the multiple state technique, we can readily apply our methodology to other similar three-state models, for example active-disabled-dead, or indeed extend it with additional states like: 'short-term sick' and 'long-term sick' or 'lapsed' (Rickayzen, 2001), or apply it to other types of insurance cover, for example Long Term Care (see Haberman and Pitacco, 1999). The computing power needed to simulate a large number of stochastic transitions using multiple state models is no longer a problem, so we have available an excellent mathematical framework for modelling a complex insurance portfolio and determining the relevant measures of risk.

We consider an insured aged x at entry and let $S(x+t)$ denote the random state occupied by the insured at age $x+t, \quad t \geq 0$. We define the conditional probabilities

$$ {}_t p_x^{ij} = \Pr[S(x+t) = j | S(x) = i] \tag{3.1}$$

for $i, j \in \{1, 2, 3\}$ assuming the Markov property. For the specific applications here, we will only consider new policyholders who are healthy at their entry age x_1 (i.e. $S(x_1) = 1$).

Then we define the transition intensities

$$ \mu_x^{ij} = \lim_{t \to 0} \left[\frac{{}_t p_x^{ij}}{t} \right] \tag{3.2}$$

For the functional form of the transition intensities we have adopted the graduated values obtained by CMI Committee (1991) from fitting to the males 1975–78 experience (for individual policies), as these have wide currency in the UK (although a less complex set of graduations has been proposed by Renshaw and Haberman (1995)).

Our intention is to consider a portfolio of policies with a deferred period of 13 weeks. Where appropriate we have chosen the graduated transition intensities that are consistent with this selection. For the transition intensities from state 2 (sick), there is strong evidence of dependence on sickness duration, z, (i.e. length of stay in state 2) leading to a semi–Markov framework. However, we have ignored this level of complexity and have used the relevant graduated values corresponding to $z = 17$ weeks. These have been chosen because one of the reported features of the 1975–78 data (and other comparable data sets – see Renshaw and Haberman (1995)) is the reduced recovery rates during the first 4 weeks of claim. The choice of $z = 17$ weeks for the 13 weeks deferred period policies, being considered here, avoids this anomaly.

Thus, the functional forms are as follows:

$$\mu_x^{12} = \exp(a_0 + a_1 x + a_2 x^2 + a_3 x^3) \tag{3.3}$$

where $a_0 = -2.722$, $a_1 = 0.1290$, $a_2 = -4.240 \times 10^{-3}$ and $a_3 = 3.888 \times 10^{-5}$

$$\mu_x^{13} = b_0 + b_1 y + \exp(b_2 + b_3 y) \tag{3.4}$$

where

$$y = \frac{x-70}{50},\ b_0 = -4.652 \times 10^{-3},\ b_1 = -4.525 \times 10^{-3},\ b_2 = -3.986,\ b_3 = 3.185$$

$$\mu_x^{21} = c_0 + c_1(x - c_2) \tag{3.5}$$

where $c_0 = 3.086$, $c_1 = -0.0927$, $c_2 = 50.326$

$$\mu_x^{23} = (d_0 + d_1(x - d_2) + d_3(x - d_2)^2)\, d_4 + d_5 \exp(d_6 x) \tag{3.6}$$

where $d_0 = 0.238$, $d_1 = -4.819\times10^{-3}$, $d_2 = 0.326$, $d_3 = 9.587\times10^{-5}$, $d_4 = 0.537$, $d_5 = 7.221\times10^{-3}$, $d_6 = 2.435\times10^{-2}$.

We have carried out some tests for policies with deferred period other than 13 weeks and the results obtained are of a similar character to those reported in section 3.4. It is beyond the scope of this paper to provide any detailed results on the sensitivity to the choice of deferred period, however the results are available from the authors on request.

3.3.2.2 Simulation and Thinning

Our goal is to simulate a random morbidity experience (i.e. length of times spent in different states) for each life in the initial IP portfolio over the term of the policy. A convenient way to achieve this is by "*thinning*" as described in Ross (1990) and utilised (in the actuarial literature) by Jones (1997) for modelling transitions in continuing care retirement communities. This method produces individual experiences (i.e. 'sample paths' – see Figure 3.2) that are independent of each other and identically distributed. The thinning procedure used to simulate an individual sample path is set out in detail in the flow chart in Figure 3 and a brief description of the process is given below.

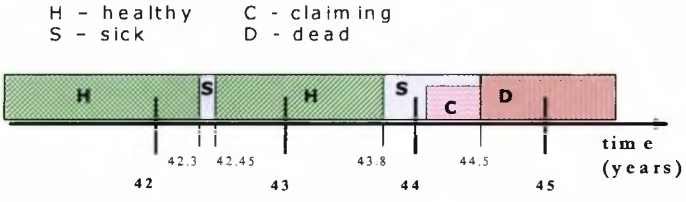


Figure 3.2 Example of a three-state model 'sample path'

We assume that all policyholders are healthy (i.e. in state 1) at entry at age x_1 with the prospect of either surviving the term to the final age (x_2) or dying during the term. For IPI, the policies are designed so that a claim may be made only when the policyholder has been in the sick state for longer than the deferred period.

For a policyholder in state 1 at entry age x_1 at time 0 and a policy with term m years we consider

$$\alpha = \max_{x_1 \le x \le x_1 + m} \left(\mu_x^{12} + \mu_x^{13} \right) \tag{3.7}$$

where the term in parentheses represents the transition intensity out of state 1 at age x, and α is chosen to be the maximum value of this overall intensity throughout the age range $[x_1, x_1 + m]$.

We can generate 'jumps' (i.e. exits out of state 1) using a Poisson process with rate α since the intervals between successive event times would be exponentially distributed with parameter α^{-1}. However, there would be too many such event times (i.e. exits) because of the definition of α in (3.7) and so we need to moderate or *"thin"* out the possible jumps. The probability of the occurrence of a simulated jump at time t_1 is proportional to the ratio:

$$p_1 = \frac{\mu_{x_1+t_1}^{12} + \mu_{x_1+t_1}^{13}}{\alpha}.$$

Thus we accept the first exit time for which a simulated uniform random number $u_2 \sim \mathrm{U}(0, 1)$ is smaller than or equal to p_1. If t_1^* is the time of the first accepted jump, then we need to establish whether the insured jumps to state 2 or state 3. For this purpose, we simulate a third uniform random number $u_3 \sim \mathrm{U}(0,1)$ and provided this is smaller than or equal to the ratio:

$$p_2 = \frac{\mu_{x_1+t_1^*}^{12}}{\mu_{x_1+t_1^*}^{12} + \mu_{x_1+t_1^*}^{13}}$$

we confirm the jump as being to the sick state (i.e. state 2). Otherwise, we accept the jump as being to the dead state (i.e. state 3).

Once the simulation has led to our deciding on the time and state of the first transition, we move forward to the next transition. We then determine α (over the age range $x_1 + t_1^*$ to $x_1 + m$) and proceed iteratively until we have a jump that takes the insured life beyond time m or until death, whichever is sooner.

We note that there is a choice regarding a simulated jump time, t, which takes the path beyond time m: we can accept this either with or without testing it using a random number u_2 against p_1, as in the standard thinning process. We have chosen the latter on the grounds of simplicity and have checked whether the former approach would markedly affect the results. Our conclusion is that there is little difference between the two approaches as far as the cumulative claim times (i.e. joint experience of all policyholders in the portfolio) in the vicinity of the terminal age x_2 are concerned.

THINNING FLOW CHART

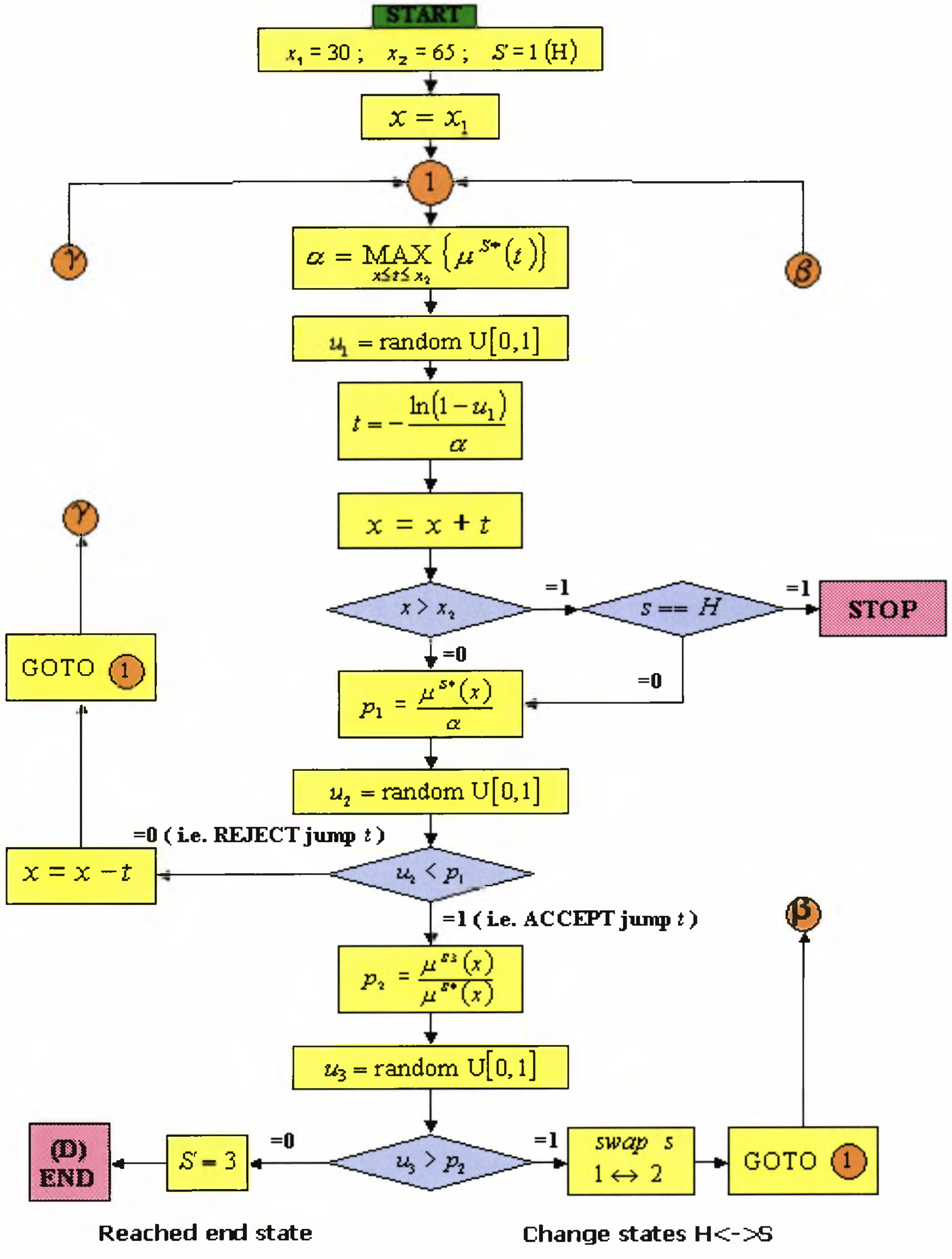


Figure 3.3 Logical Scheme of Simulation with Thinning

3.3.3 The Economic Model

3.3.3.1 Investment Returns Model

In order to generate random investment returns for the SAM1 and SAM2 cases, we will use the widely accepted Wilkie model for equity returns and returns on index–linked government bonds. We use the structure of the model as in Wilkie (1995) and the initial parameter settings recommended therein. We consider an asset portfolio comprising 50% equities and 50% index–linked government bonds, with annual rebalancing (and assuming that investment income received for each asset class is re-invested in the same asset class) so that the asset allocation strategy is static.

For the DAM case, in order to maintain a consistent level of investment return (with the SAM1 and SAM2 cases), we need to estimate the annual interest rate (assumed to be constant over the whole term) from a large number of trials based on the underlying stochastic asset model. This is obtained from running 1,000 simulations of the Wilkie model over an appropriate time horizon and calculating the equivalent mean annual compound rate of return. This method results in a constant rate of return of

$$r = 10.7\ \%\ \text{pa}$$

for the first entry age (30) and minor adjustments have been made to allow for the effect of different entry ages used for later simulations.

3.3.3.2 Inflation Rate Model

Further volatility from the external economic environment is incorporated by allowing for the effects of random annual rates of inflation on the expenses (per unit of exposure) experienced in the portfolio and on the levels of premiums and benefits. For consistency with the stochastic investment return model, we apply the same type of first order autoregressive series model for the force of inflation as suggested in Wilkie (1995). However, we set different expected values (rather arbitrarily) for the time series. Thus, we assume a mean annual increase of 3% in the premium and benefits levels and a mean annual increase of 4% in the renewal and claim expenses, with corresponding variances determined by the simulations of the Wilkie model. We note that this configuration implies an indexed type of policy design (see section 3.3.1).

In summary, inflation has been incorporated within the three models as follows:

- For the DAM case, deterministic inflation rates are applied to the premium and benefit levels (at 3% pa) and to the claim and renewal expense levels (at 4% pa). These are consistent with the predicted means of the formulated time series models.
- For the SAM1 case, deterministic inflation rates are applied to the premiums, benefits and expense levels (as above) but the investment returns are assumed to be stochastic (as described in section 3.3.3.1).
- For the SAM2 case, stochastic inflation rates are applied to the premiums, benefits and expense levels (with the investment returns also assumed to be stochastic).

3.3.4 Cash Flow Model

For any policy year $(t, t+1)$, the cash flow is assumed to occur mid–way through the year (for a mathematical treatment, that is equivalent to uniform incidence over a year) and is defined to be the difference between the simulated income and outgo i.e. for simulation j, the cash flow per cohort is:

$$CF_t^j = P_t^j - C_t^j - \underbrace{\left(W_t^j + Y_t^j + Z_t^j\right)}_{\text{expenses}} \qquad (3.8)$$

where for the policy year $(t, t+1)$ and simulation j the components are:

P_t^j = total premiums paid by all policyholders,

C_t^j = total claim payments payable to policyholders who are sick and for whom the duration of sickness exceeds the deferred period of the policy,

W_t^j = total premium-related expenses,

Y_t^j = total regular claim expenses,

Z_t^j = total initial claim expenses (payable at the commencement of a claim).

In equation (3.8), the terms P_t^j and W_t^j are proportional to the simulated time spent in the healthy state or in the sick state if less than the deferred period. Similarly, the terms C_t^j and Y_t^j are proportional to the simulated time spent claiming (i.e. continuous sick time in excess of the deferred period) and finally, the term Z_t^j is proportional to the simulated number of new claims initiated in year $(t, t+1)$.

In the above model, we assume an instantaneous occurrence of all cash–flows so that there are no administrative (or other) delays. The policy funds then accumulate as follows:

$$A_{t+1}^j = A_t^j \cdot (1 + r_t^j) + CF_t^j \cdot (1 + r_t^j)^{\frac{1}{2}} \tag{3.9}$$

where A_t^j represents the accumulated assets (in respect of all current policyholders) at time t for simulation j and r_t^j is the simulated rate of return on investment for year $(t, t+1)$. In our calculations we assume that $A_o^j = 0$ for all j (i.e. there are zero initial assets per policy) apart from when we consider risk-based capital requirements in section 3.4.

For the case where rates of return on investment are assumed to be non–stochastic, r_t^j is the estimated constant value referred to in section 3.3.3.1 (eg $r_t^j = 10.7\%$ for all t and j for entry age 30).

3.3.5 Risk Measures

We have considered four types of risk measures relevant to our model IP portfolio that we use in order to quantify the risks faced by an insurer with IP liabilities. These measures are defined as follows:

1. *Probability of Ruin*

   This is evaluated as the proportion of outcomes with negative residual assets at the termination of the contract (i.e. $A_m < 0$, where m corresponds to the termination of the contract) from a predetermined number of sample simulations (with each simulation having an identical number of policies issued at entry age). That is:

   $$\Pr(ruin) = \frac{k}{N}$$

   where k represents the number of insolvent cases and N is the total number of simulations (usually 500). Note that the actual size of the ruin is irrelevant for this risk measure.

2. *Mean Shortfall*

   The mean shortfall is the mean of A_m (the residual assets at the termination of the contract), conditional on $A_m < 0$. This is estimated by the average shortfall in a predetermined number of simulations (N) relative to the (fixed) number of policies at entry:

   $$ruin_{rel} = \frac{1}{n}\left[\frac{1}{k}\sum_{j=1}^{k} A_m^j\right] \quad \text{where } A_m^j < 0 \text{ for } j=1\ldots,k$$

   n is the number of policies issued at x_1 (constant across the trials) and k is the number of insolvent cases. To facilitate comparison between cohorts of different sizes, we express this index relative to n.

Although it is theoretically possible to have a simulation j such that $A_t^j < 0$ for some $t < m$ but $A_m^j \geq 0$, this situation does not in fact arise for any of our simulations.

As demonstrated by Artzner et al (1997) and Artzner (1999), risk measures based on the mean shortfall approach are superior to the more commonly used Value-at-Risk (VAR) measures. In particular, they satisfy the coherence requirement by being sub-additive (whereas VAR measures need not satisfy this property) and are less sensitive to changes in the tail of the distribution being investigated.

3. *Risk–Based Capital*

   The amount of capital (i.e. assets) required at outset $A_0(\varepsilon)$ in order to secure at most a pre-specified probability of ruin ε (assuming a 'break-even' premium level, see section 3.3.6). Note that this is expressed in multiples of the break-even premium.

4. *Risk–Loaded Premium*

   The percentage of additional premium $\lambda(\varepsilon)$ (to be added to the 'break-even' premium, see section 3.3.6) required to secure at most a pre-specified probability of ruin ε.

3.3.6 Calculation of the "*Break-Even*" Premium

In order to calculate the "*break-even*" initial premium per policy for a particular class of models (i.e. DAM, SAM1 or SAM2), we consider a portfolio of 1,000 identical policies and run $N = 500$ simulations of the residual policy funds at the termination of the contracts (i.e. at age 65) for a given initial premium. We then use iteration to derive a value for the initial premium such that

$$\Pr\ (ruin) = 0.5 \qquad (3.10)$$

where $\Pr(ruin)$ is the probability of ruin defined in section 3.3.5.

The values that are obtained (rounded to the nearest pound) for the different entry ages under consideration are summarised in the following table:

Table 3.2 Summary of break-even initial premiums

Age at Entry	Break-even initial premium (£)	
	SAM1 (DAM)	SAM2
30	220	235
40	272	280
50	336	338
60	462	462

The models DAM and SAM1 lead, not unexpectedly, to almost identical break-even premiums for the 1,000 policies at issue, and continue to be close even for significant changes in the size of the initial cohort (see Figure 3.11, later). We recall that the SAM1 approach uses simulated annual investment returns while the DAM approach uses an equivalent mean annual compound rate of return. However, there is an increase (about 7% for entry age 30, see Figure 3.12, later) in the break-even premium level when we move from the SAM1 to the SAM2 model. This arises because the SAM2 approach includes an extra level of variability compared to the SAM1 approach through the impact of stochastic inflation on premiums, benefits and expenses. This effect becomes less significant with increases in the entry age (i.e. as the insurance term reduces) due to the reduction in the cumulative effect of the stochastic components.

This approach is different from the conventional approach based on the equivalence principle where we equate expected present values of policy income and outgo. In future work, we will investigate the effect of different choices for the calculation method for the premium.

The implications of using (3.10) are that:

a) the premium does not include any margin for profit or adverse experience,

b) the company expects to "break even" at the termination of the contracts, assuming that all profits are retained internally and

c) any projection that leads to ruin (of the portfolio) can be attributed to "process error" only.

3.4 Simulation Results

Because of the limited space available, we will present only a selection of the results. A fuller set of results is provided in an accompanying working paper: Haberman et al (2001).

Figures 3.4 to 3.6 show the simulated distribution of the assets for a portfolio of n policies issued at age 30 for the respective cases n=1,000 and 10,000, based on 500 simulations. These figures summarise the distribution at each age through the use of box–plots with the 25^{th}, 50^{th} and 75^{th} percentiles clearly identified. Figure 3.4 is based on the DAM asset model i.e. random morbidity and deterministic economic variables. It is clear that as n increases, the distributions become more compact, showing that the demographic process risk is reduced by the pooling of policies as discussed in section 3.1.

Figures 3.5 and 3.6 repeat the presentation but allow for stochastic investment returns and inflation, based on the SAM1 and SAM2 approaches respectively. Generally the distributions in Figures 3.5 and 3.6 are much wider than in Figure 3.4 – as we would anticipate because of the extra source of variability present. Also, the more randomness that is built into the models the larger is the variance across the ages. However, there is an important feature that can be observed: while in the SAM1 type the pooling effect is still present, although reduced compared to DAM (i.e. the variability reduces with the size of the portfolio), in the case of SAM2 the effect is much less apparent. This would suggest that the additional process risk introduced in the SAM2 model by assuming that the inflation rates applied to the premiums, benefits and expenses are stochastic is far more important than the process risk inherent in the morbidity part of the model.

Surprisingly, in the SAM2 case the sample standard deviation of the residual funds increases in size when we move from 1,000 to 10,000 policies assumed at entry age. However, it should be stressed that the increase seems to be due mainly to a small number of extreme cases (i.e. outliers) and the resulting sample distribution of the residual assets for the case of 10,000 policies at entry is, in fact, more peaked at the mean than the one corresponding to the portfolio with 1,000 policies at entry. Indeed, this effect is reduced for the SAM2 model, when the number of simulations is increased to 1,000 and beyond.

Figures 3.7 and 3.8 illustrate the simulated distributions of the residual assets for portfolios of lives with initial ages 30 and 60, respectively, for the SAM1 case with the rates of investment returns generated from the Wilkie model as described earlier. Similarly Figures 3.9 and 3.10 represent the simulated distributions for the DAM and SAM2 cases, respectively, with entry age 30. The following features are noteworthy.

a) increasing the portfolio size from $n = 1{,}000$ to $n = 10{,}000$, when the asset model is deterministic, leads to a reduction in the standard deviation of the residual assets by a factor of approximately $\sqrt{10}$ (as we would expect from the pooling of risks), so that the distribution is sharper (see Figure 3.9);

b) the feature described in a) above whereby the standard deviation of the residual assets reduces as the portfolio size increases is not as pronounced in the stochastic model SAM1 (see Figures 3.7 and 3.8). This is because the pooling of risks does not reduce the impact of investment return variability which affects all policies in the portfolio simultaneously. However, with the reduction of the policy term (as we move from Figure

3.7 to 3.8) the effect of pooling of risks increases in weight and the reduction in the standard deviation from a portfolio size of $n = 1{,}000$ to $n = 10{,}000$ is much more significant. Indeed, the reduction factor approaches $\sqrt{10}$ as the entry age increases from 30 to 60;

c) furthermore, in the case of the 'fully–stochastic' model, SAM2, shown in Figure 3.10, the level of the uncertainty of the outcome is less dependent on the size of the portfolio, and it is possible that increasing the number of policies could increase the process risk;

d) increasing the policy term (i.e. reducing the age at entry) leads to an increase in the skewness of the distribution. This is because the accumulation effect becomes stronger for longer terms and corresponds to the approximate lognormal character of the simulated accumulations obtained from the Wilkie model. (Figures 3.7 and 3.8);

e) in the SAM2 case, the shape of the distribution is normal rather than lognormal (see Figure 3.10).

The implication of changing the entry ages (30, 40, 50 and 60) on the residual assets (and on the probability of ruin) across the defined models is summarised in more detail in Table 3.3 for the $n = 1{,}000$ and $n = 10{,}000$ portfolios. We note that the ratios of the standard deviations of the residual assets for the SAM1 and SAM2 cases relative to the DAM case are inversely related to the initial policy term (i.e. 65 minus entry age). Thus, with a sufficiently short initial policy term (i.e. high entry age) this ratio reduces to 1. For example, for an entry age of 30 the standard deviation of the residual assets for SAM2 is 9.4 times as large as for DAM and for SAM1 is 2.3 times as large as for DAM (for the case of $n = 1{,}000$). But for an entry age of 60, the relative ratios are both close to 1, demonstrating very similar process risks for the three models in this case.

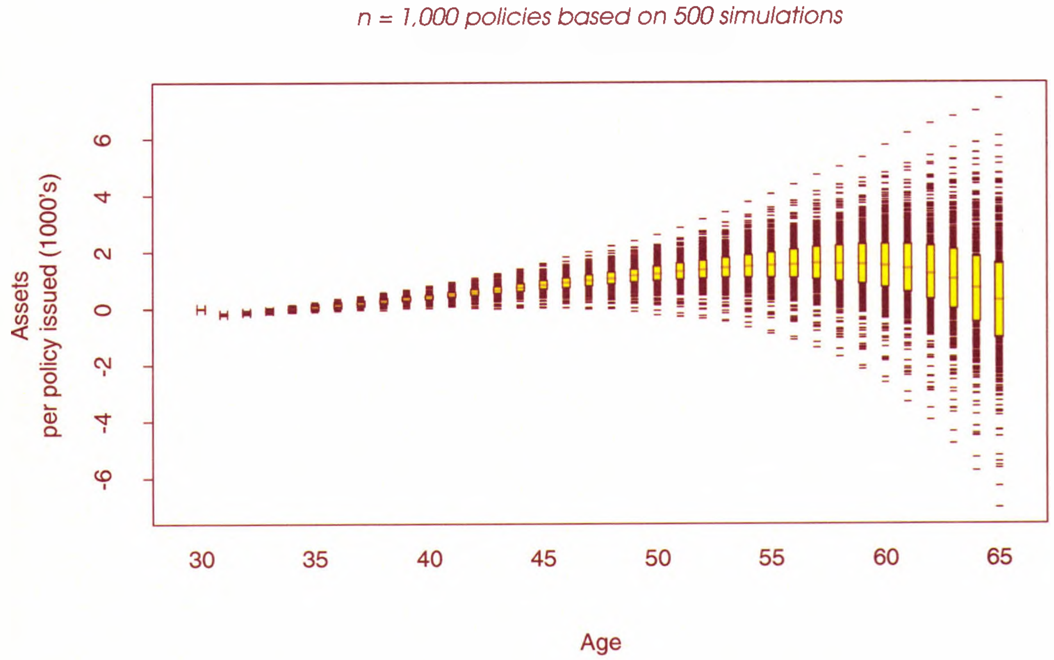


Figure 3.4 Deterministic Assets Distribution over age for 1,000 and 10,000 policies at outset

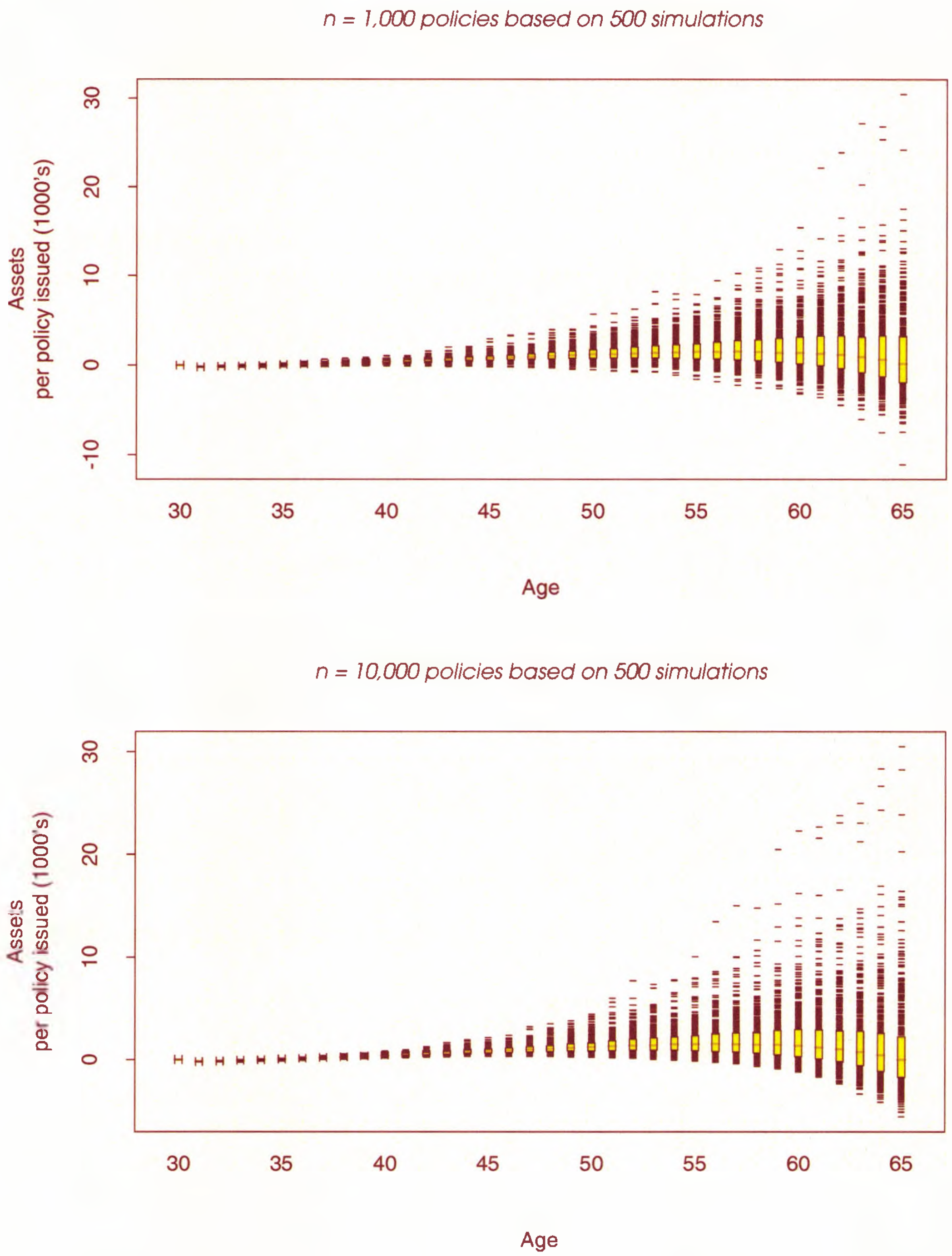


Figure 3.5 Stochastic Assets Distribution (SAM1) over age for 1,000 and 10,000 policies at outset

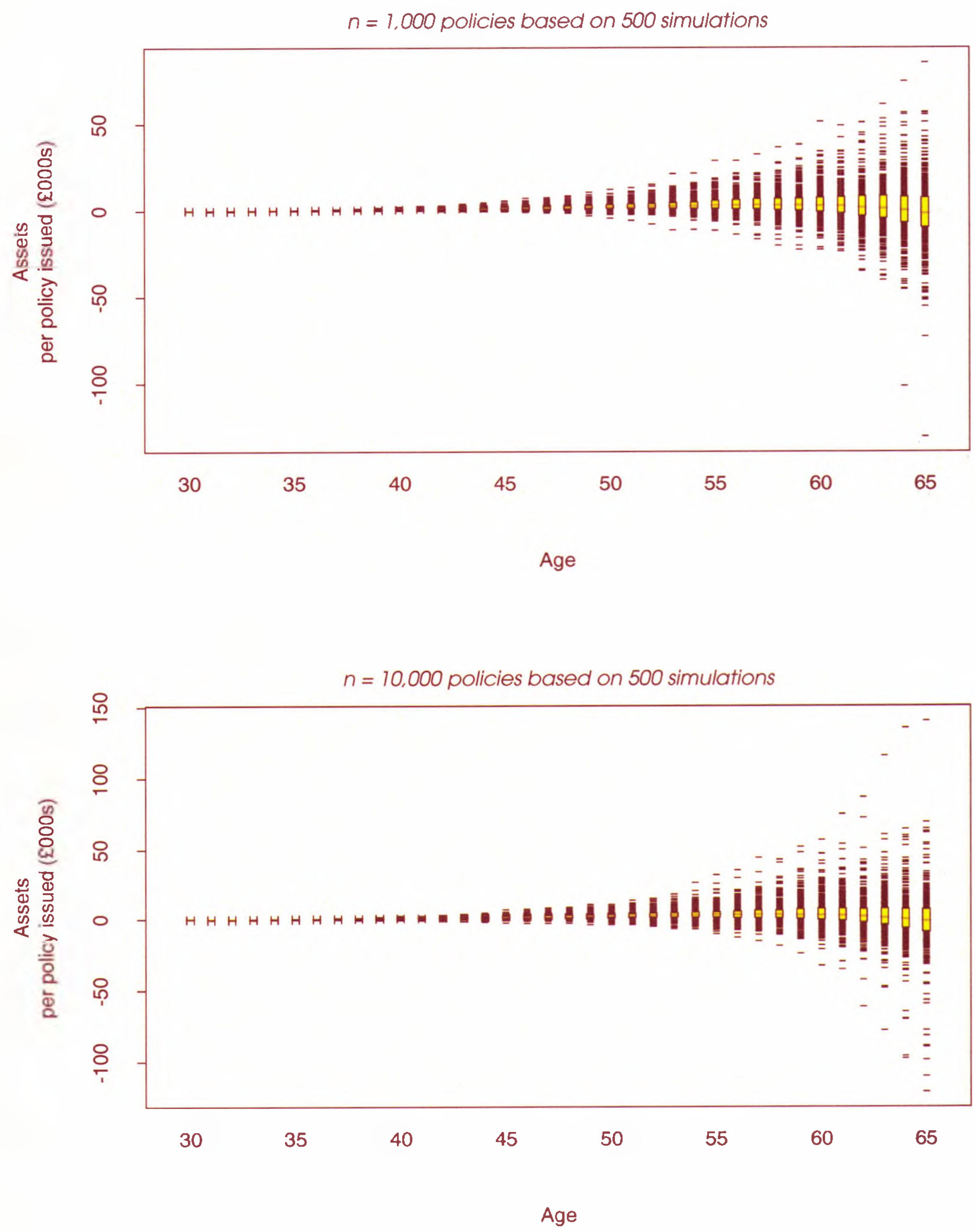


Figure 3.6 Stochastic Assets Distribution (SAM2) over age for 1,000 and 10,000 policies at outset

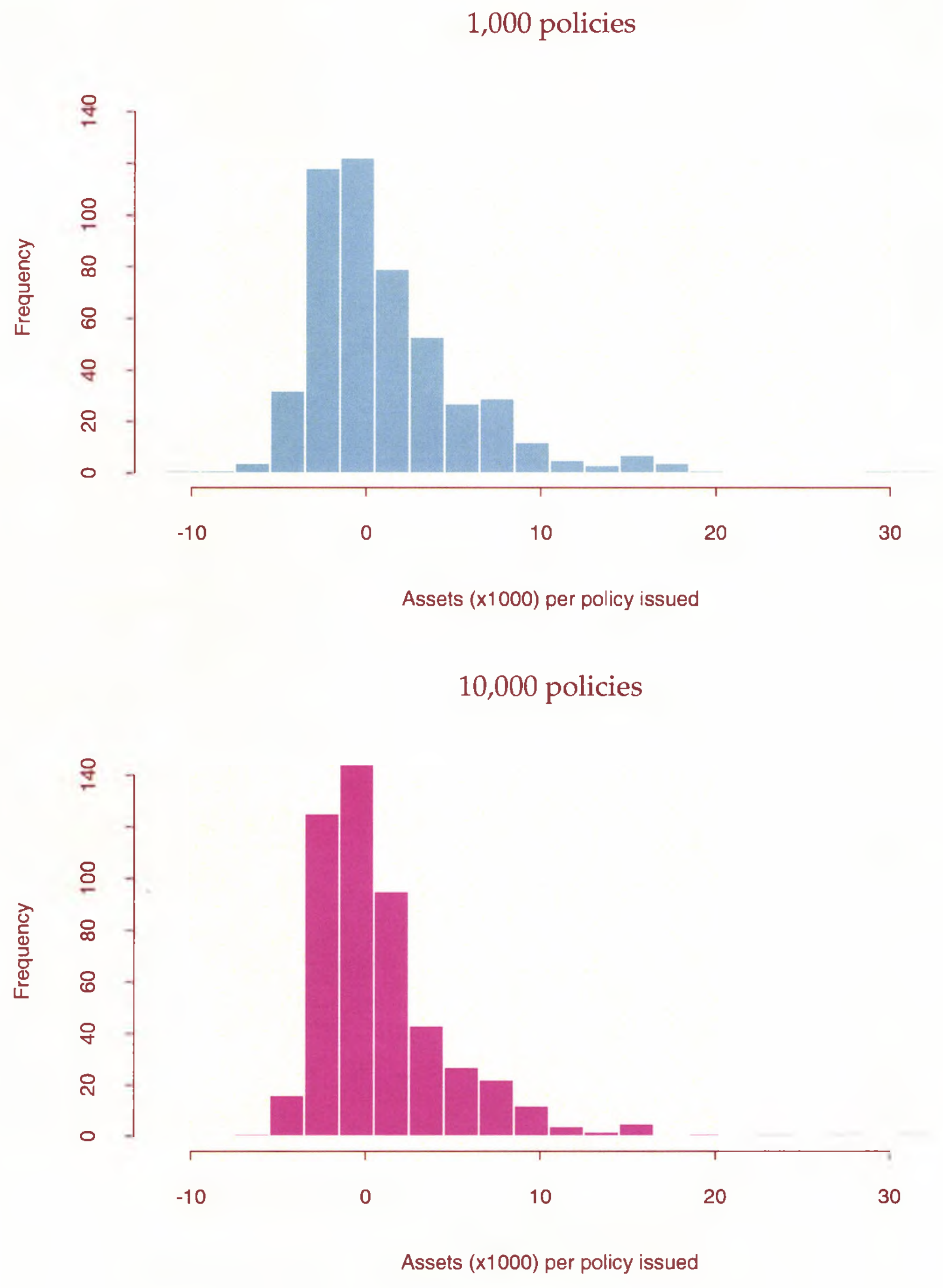


Figure 3.7 SAM1 Residual Assets Distribution in relation to entry age 30

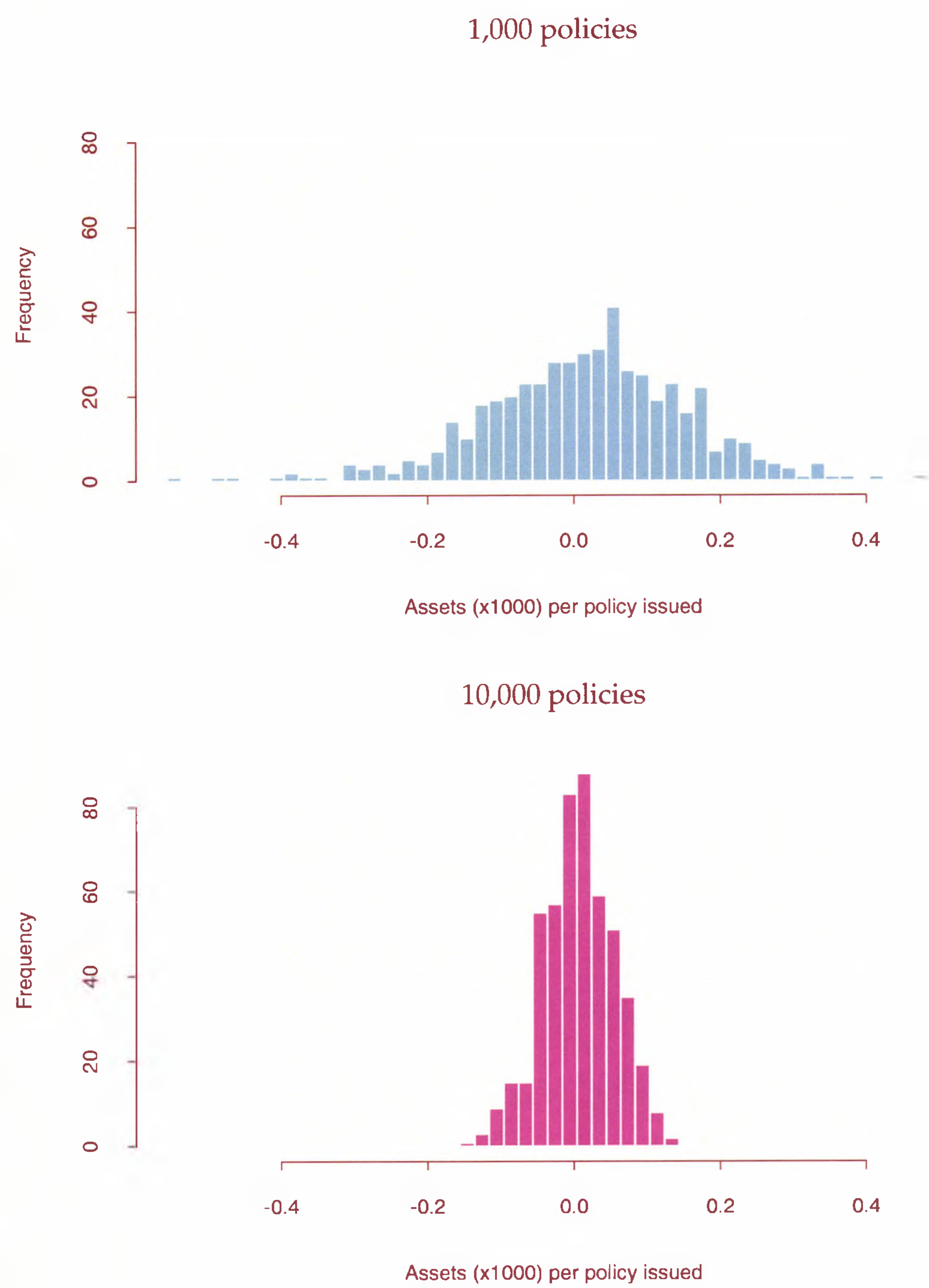


Figure 3.8 SAM1 Residual Assets Distribution in relation to entry age 60

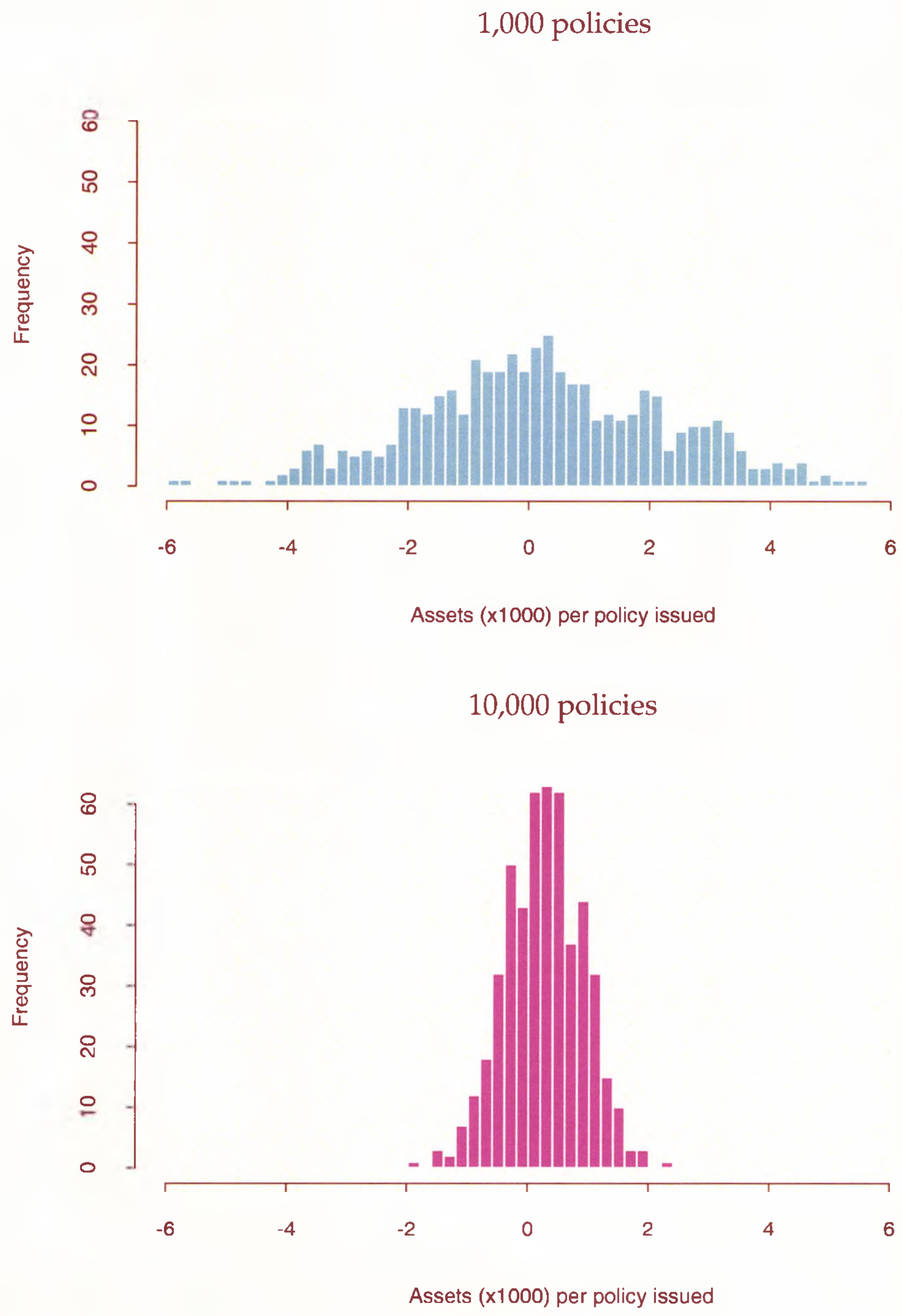


Figure 3.9 DAM Residual Assets Distribution in relation to entry age 30

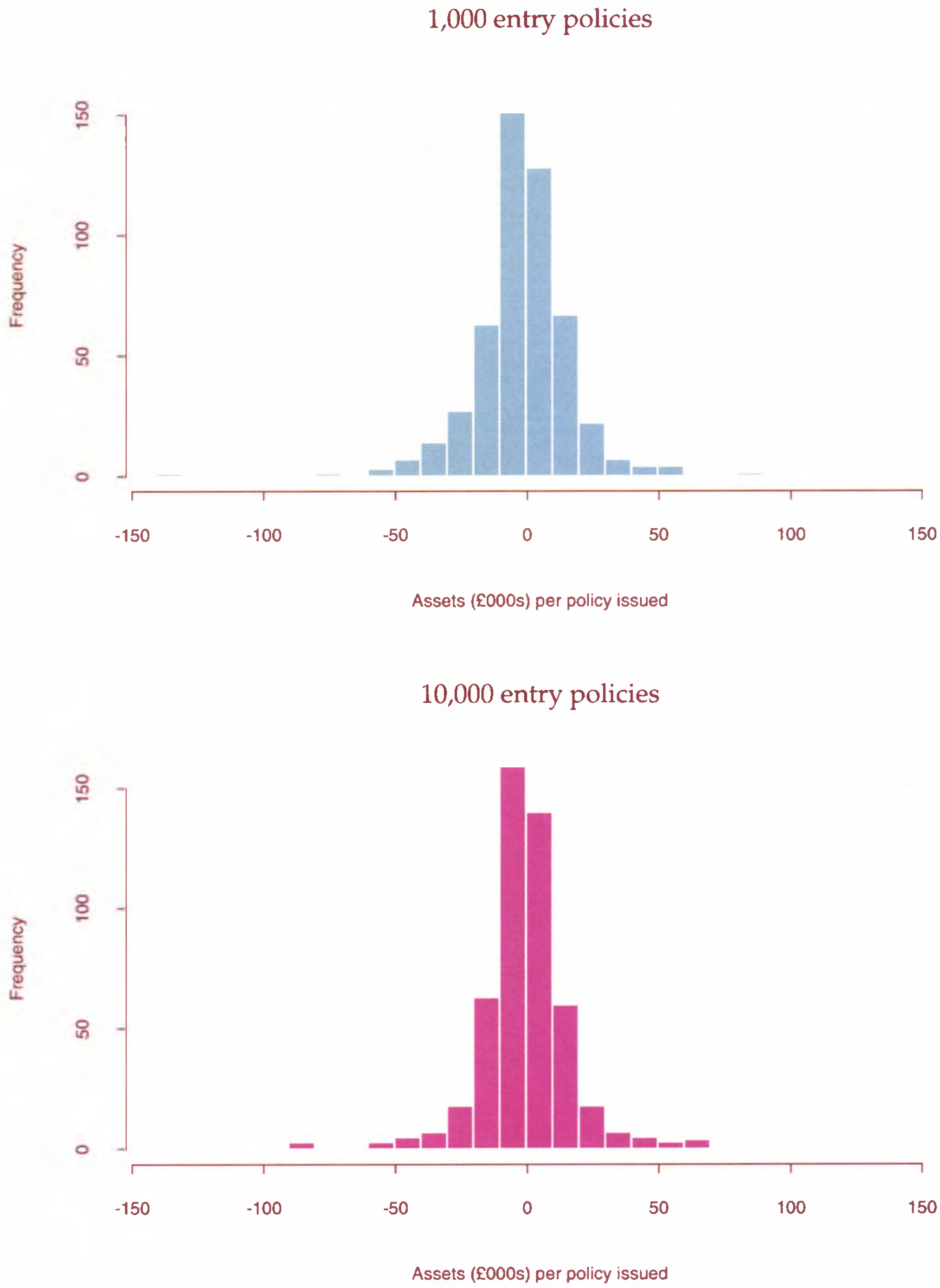


Figure 3.10 SAM2 Residual Assets Distribution in relation to entry age 30

Table 3.3 Residual Assets Distribution and Probability of Ruin for various cohorts

Entry Age	Mean of Assets (£ 000s) per policy issued			Standard Deviation of Assets (£ 000s) per policy issued			Probability of Ruin (%)		
x_1	**1,000 policies issued at entry age**								
	SAM2	**SAM1**	**DAM**	**SAM2**	**SAM1**	**DAM**	**SAM2**	**SAM1**	**DAM**
30	-1.711	1.171	0.204	19.369	4.825	2.065	53.7	51.0	47.0
40	0.012	0.303	0.103	3.979	1.490	0.845	50.3	49.8	45.8
50	-0.005	0.000	-1.985	0.732	0.471	0.411	51.7	50.0	49.2
60	0.003	0.016	0.016	0.147	0.141	0.137	49.5	42.4	42.2
	10,000 policies issued at entry age								
30	-2.355	1.052	0.273	20.119	4.296	0.644	53.0	50.0	33.0
40	0.016	0.187	0.078	3.502	1.044	0.296	53.1	51.2	38.0
50	-0.021	0.073	0.032	0.567	0.231	0.121	52.5	43.6	37.8
60	0.002	0.007	0.008	0.063	0.049	0.045	49.4	44.6	43.6

We now consider the effects of changing the initial assets and the initial premium per policy on the first two defined risk measures (i.e. probability of ruin and mean shortfall). For the case of a portfolio of policies with age at entry 30 and n=10,000, Figure 3.11 shows the effect on the probability of ruin of increasing the initial assets from zero in the upper panel and the effect of charging an initial premium which is different from the break-even premium of £220 in the lower panel. In each case, the curves of the probability of ruin for the stochastic model, SAM1, and for the model with deterministic asset returns, DAM, are presented. We note the following features:

a) from the upper panel, initial assets of zero correspond approximately to a probability of ruin of 0.5 for the stochastic case (and to a slightly lower level for the deterministic case because of there being fewer unfavourable results within the 500 simulations for the DAM model – this effect disappears when we increase the number of simulations);

b) increasing the initial assets per policy leads to a lower probability of ruin. This is more marked for the deterministic case where there is less overall variability;

c) from the lower panel, an initial premium of £220 corresponds approximately to a probability of ruin of 0.5;

d) increasing the premium beyond £220 leads to a lower probability of ruin, which, as in b), is more marked for the deterministic case;

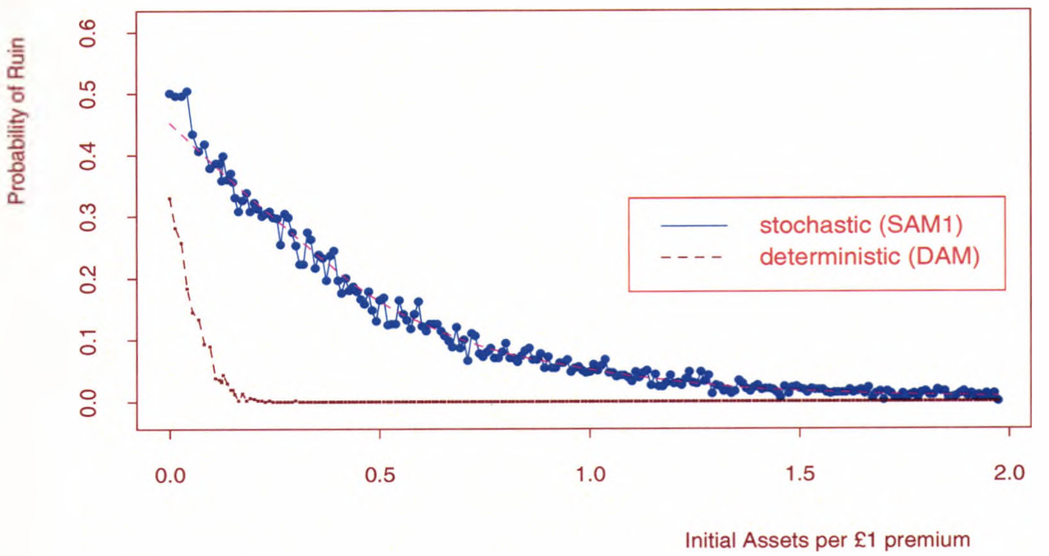
e) decreasing the premium below £220 leads to a higher probability of ruin and we note that, unlike d), the extra variability from the stochastic case (SAM1) may help when the premium is inadequate and may lead to a lower probability of ruin.

We note that the variation of the probability of ruin when considered as a function of either the initial assets or the initial premium is amplified in the case of a smaller portfolio (results not shown). For the deterministic model (DAM), this is particularly dramatic, for example with the tenfold decrease in portfolio size leading to a significant increase in "process risk" and a considerable degree of fluctuation in the curves corresponding to Figure 3.11, for the case of $n = 1{,}000$ policies (results not shown).

Figures 3.12 and 3.13 allow us to compare the probability of ruin and mean shortfall profiles in the case of the different modelling frameworks for a portfolio with $n = 10{,}000$. From Figure 3.12, which compares the SAM1 and SAM2 models, we note that an increase in the size of the break-even premium (calculated as described in section 3.3.6) is required for the SAM2 model, due to the extra variability present in the latter. We observe that, overall, the slopes of the curves decrease with an increase in the volatility in the models, so that, the more volatility that is allowed for in the model, the greater is the increase in the initial premium (or assets) needed to achieve a given reduction in the probability of ruin. This is demonstrated in Table 3.4, where we can see that in order to reduce the probability of ruin from 25% to, say, 5%, the SAM2 model requires a significantly higher capital input than the SAM1, or indeed the DAM, models. Similarly, Table 3.5 shows a substantial increase in the additional premium required in the first year to produce the same probability of ruin (5%) in the SAM1 and SAM2 models.

Figure 3.13 compares the mean shortfall by annual premium for the DAM and SAM2 models. We note that, for both models, increasing the annual premium leads to a reduction in the size of the mean shortfall, as expected. The mean shortfall is larger in size for the SAM2 case than the DAM case – as before, a reflection of the inclusion of process risk from the economic variables. We note the smooth progression in the curve for the DAM case. For the SAM2 case, the mean shortfalls are more erratic as the annual premium is increased. This arises because the number of simulations with $A_m < 0$ reduces as the annual premium is increased and so this feature is a reflection of sampling error – and we have fitted a smoothing curve in an attempt to reduce the impact of this variation.[1] It is interesting to note that there is a steady decrease in the mean shortfall as the initial premium is increased up to the break-even level for both models. The smoothing curve indicates that this trend continues as the premium is increased further, although, as noted above, there is increased variability in the underlying estimates.

[1] We have used a robust local smooth curve generated using the 'lowess' S-Plus smoothing routine, which, in effect, estimates each point of the curve by robust local linear fits from a given (local) proportion of the total observations (see Venables and Ripley 1999).



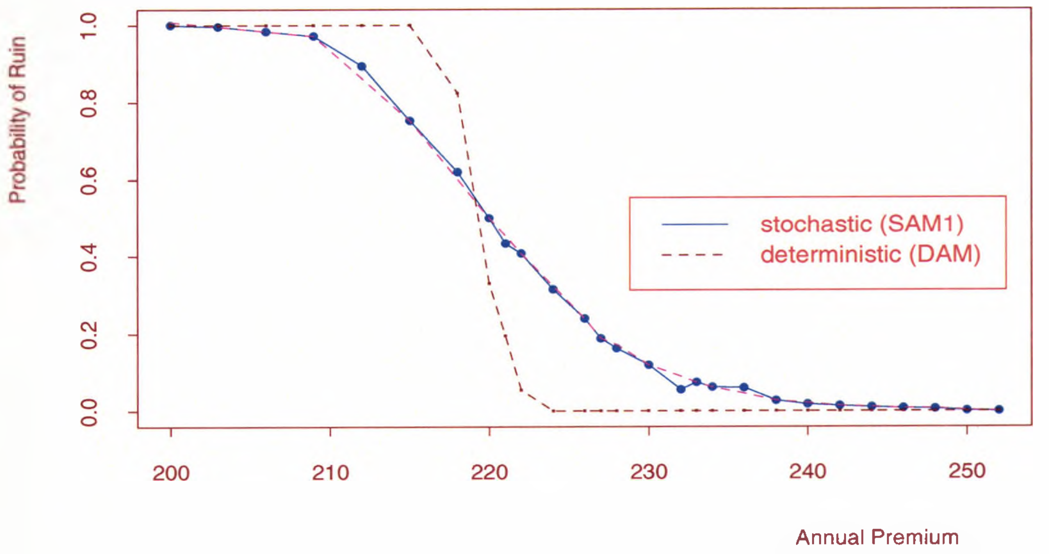


Figure 3.11 Probability of ruin by initial assets and premium level for SAM1 and DAM in relation to 10,000 policies assumed at outset

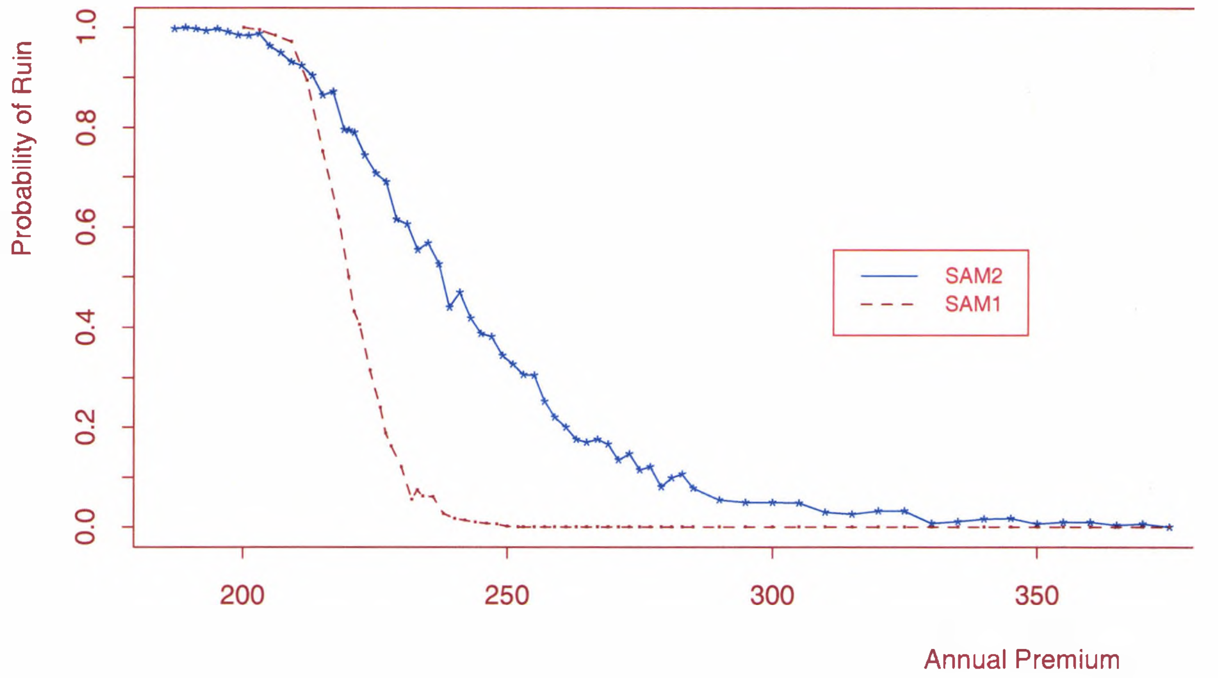


Figure 3.12 Probability of ruin against premium level for SAM1 and SAM2 models in relation to 10,000 policies assumed at outset

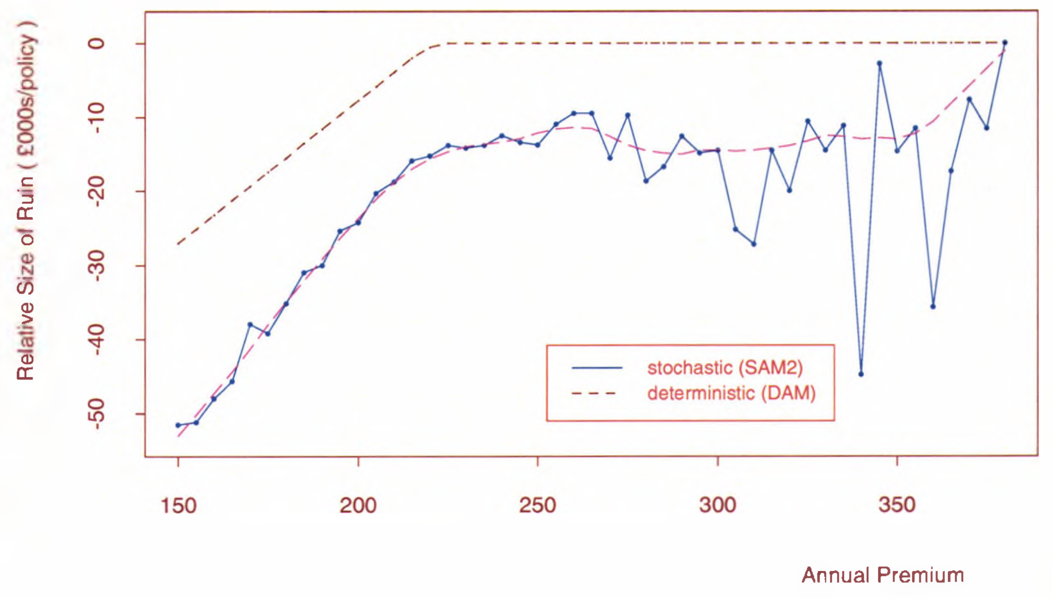


Figure 3.13 Average relative size of insolvency against premium level for SAM2 and DAM models in relation to 10,000 policies assumed at outset

We recall that $A_0(\varepsilon)$ is the level of initial assets required to produce at most a given level of probability of ruin (ε). We now consider Table 3.4 in more detail which presents some examples of $A_0(\varepsilon)$, calculated relative to the annual premium in the first year of the policy, for the cases of $n = 1{,}000$ and 10,000 for the deterministic and stochastic asset models and a range of values of ε. We note that, in the case of the stochastic models, the values in the tables have been determined based on the smoothed probability curves mentioned earlier. The results show that:

a) reducing ε leads to an increase in $A_0(\varepsilon)$, as expected;

b) for the deterministic asset model, DAM, increasing n has a significant effect on $A_0(\varepsilon)$, indicating the extent to which the demographic process risk reduces as the portfolio size increases;

c) for the stochastic asset models, the effect of increasing n has a much weaker effect on $A_0(\varepsilon)$. This indicates that the demographic process risk identified in b) is relatively insignificant when compared with the process risk arising from the economic variables.

Table 3.4 Risk–Based Capital at entry age 30 – initial assets (£000s) required to produce a given ruin probability (per £1 annual premium in year 1)*

	Deterministic Asset Model		Stochastic Asset Models			
	DAM	DAM	SAM1	SAM1	SAM2	SAM2
Pr (Ruin)	$n = 1{,}000$	$n = 10{,}000$	$n = 1{,}000$	n=10,000	$n = 1{,}000$	n=10,000
25%	0.148	0.025	0.366	0.329	1.609	1.398
10%	0.305	0.081	0.787	0.697	3.545	3.471
5%	0.402	0.101	1.173	1.020	5.515	5.737
1%	0.534	0.162	2.022	1.856	13.551	12.556

* where 0 initial assets produce a 50% probability of ruin for the break-even initial premiums defined in Table 3.2.

Table 3.5 presents the results for $A_0(\varepsilon) = 0$ for $\varepsilon = 10\%, 5\% \text{ and } 1\%$ but allowing a proportionate risk loading $1 + \lambda(\varepsilon)$ to the premiums (i.e. the percentage increase in the initial premium required to obtain a maximum probability of ruin of ε% when there are no initial assets per policy). Similar effects are demonstrated.

Table 3.5 Risk-loaded Premium at entry age 30 – additional premium required in year 1 to produce an ε% ruin probability assuming 0 initial assets

	Deterministic Asset Model		Stochastic Asset Models			
	DAM	DAM	SAM1	SAM1	SAM2	SAM2
ε	$n = 1{,}000$	$n = 10{,}000$	$n = 1{,}000$	$n = 10{,}000$	$n = 1{,}000$	n=10,000
10%	2.9 %	0.8 %	6.0 %	5.1 %	20.1 %	19.5 %
5%	3.5 %	1.0 %	7.9 %	7.0 %	29.0 %	27.9 %
1%	5.4 %	1.6 %	11.8 %	11.5 %	51.8 %	50.1 %

3.5 Conclusions

The results presented here (and more extensive investigations not reported) demonstrate that pooling, as implied by the law of large numbers, reduces the demographic process error. We have shown that the economic process error is considerably more significant than the demographic process error for portfolios of income protection insurance policies. We have also shown that the length of term has a decisive influence on the weight of these components of the process risk. Therefore, the shorter the policy term the more the process risk depends on the experienced morbidity and on the size of the portfolio, so that the insurer is relatively more vulnerable to the morbidity experience.

We have demonstrated the methodology for simulating asset/liability profiles in a multiple state context and shown how estimates for risk–loaded premiums and for risk–based capital can be obtained. We have also shown, from the simulation results, the extent to which the risk measures investigated depend on the input parameters (in particular, the level of the initial premium or of the initial capital) for models with different levels of stochasticity.

We have conducted some trials based on changing various aspects of the model, for example the parameterization of the transition intensities, μ_x^{12} and μ_x^{21}, as in (3.3) and (3.5) and the choice of the deferred period in the policy design. These results indicate that the principal conclusions remain valid.

We have investigated a simple policy design. More complex features that could be added would include:

a) lapse rates, possibly dependent on the type of business;
b) effects of policy duration and temporary initial selection;
c) more complex benefit schemes, for example split levels of benefits as in Long Term Care contracts and inclusion of termination and death benefits;
d) reviewable premiums, where the level of premium depends on the recent claims experience at the portfolio level.

As an extension to this study, we are conducting a systematic sensitivity analysis of the results to changes in the transition intensities.

4. A MULTI-STATE MODEL OF DISABILITY FOR THE UNITED KINGDOM: IMPLICATIONS FOR FUTURE NEED FOR LONG-TERM CARE FOR THE ELDERLY*

4.1 Introduction

This paper describes a model which has been developed to project the number of people in the United Kingdom who will be disabled over the next thirty five years. The projections give an indication of the long-term care needs of the UK population in the future.

The number of elderly people in the UK is growing both in absolute terms and in relation to the number of people of working age. According to the projections produced by the Government Actuary's Department (GAD) in 1998 there will be around 4.3 million people aged 80 or more in forty years' time compared with about 2.3 million now. The number of people aged between 20 and 64 is expected to fall slightly from 34.8 million to 33.8 million (Government Actuary, 1998).

At present the prevalence of disability amongst elderly people is much higher than for the rest of the population. As this situation is likely to continue it is possible that there will be very many more people with some degree of disability in the future. Such people will need some form of long-term care.

* Co-authored with D.E.P Walsh – Chapter 4 is reproduced from Rickayzen and Walsh (2002)

The provision of long-term care is expensive (in terms of both time and money). Hence it is very important to be able to estimate the numbers of people who are likely to need long-term care, although it is also necessary to recognise that any such estimate will be subject to much uncertainty. The numbers, while they are fundamental, are only one of several aspects that will affect the provision of long-term care. The other aspects include the connection between the severity and nature of a disability and the cost of caring for someone with the disability; the split between formal and informal provision of care services; and methods of paying for the cost of care provision. This paper focuses on the issue of the number of people who will require care and does not address the other key aspects mentioned above.

The output from the model, for a given set of assumptions, is an estimate of the number of people who are healthy and the number who are disabled. (In this paper we use the terms 'healthy', 'able' and 'not disabled' interchangeably.) The number of disabled people is further split into several categories of disability from relatively mild to very severe. The model covers people aged 20 and over and goes up to the year 2036. The numbers are produced for all combinations of age, year and sex.

In this paper, we describe in some detail the data which we have used as a starting point and the various trends which we have included in the projections. We present the results of projections produced using a range of assumptions. The output is very sensitive to the assumptions incorporated.

This paper follows an earlier one in the UK actuarial literature by Nuttall et al (1994). That paper covered more ground than this one, in particular the financial implications of future demand for long-term care were considered. There are, naturally, some important similarities between the model which we use for the projection of the number of disabled people and the model which Nuttall et al used. There are, though, some key differences as well and we have added considerably to the complexity of the earlier model. Just as important as the new developments in modelling is the existence of new data. The most important new data to be published since Nuttall et al (1994) relates to trends in healthy life expectancy. We discuss this data in detail in Section 4.2.3 and Section 4.3.

The model that we describe in this paper relates to the number of people with disabilities. We do not go into any detail on the associated provision of care services and the cost of these services. Much work has been done in the area of the cost of care. See the review by Darton (1994) for a discussion of the levels of dependency of people in residential care and nursing homes. See Bone (1995) for figures showing how the utilisation of care services depends on levels of disability. The costs of providing care services have been compiled by Netten & Dennett (1997) and have been incorporated within the model described by Wittenberg et al (1998).

There are important financial implications of the split between formal and informal care. This topic is discussed by Nuttall et al (1994) who estimate that the bulk of care provision is informal, i.e. it is carried out by family, friends and neighbours of the disabled people rather than by care professionals. Green (1985) has analysed who it is that provides informal care and Glendinning (1992) discusses some of the implications for the carers. The model used by Wittenberg et al (1998) treats informal care as a function of both the level of disability of an elderly person and whether or not the elderly person lives with other people. The provision of informal care in the future will depend on such statistics as the proportion of the elderly population that is married and how far the children of elderly disabled people live from their parents' home.

Section 4.2 describes the data that feed into our model. We highlight the data which are particularly important to the projected number of disabled people and the main areas of uncertainty relating to the data.

We need data for three parts of the model:

- Prevalence rate data are needed as a starting point for the model. The data must show what proportion of people at each age have disabilities now.

- Transition rate data are needed so that we can follow the current population forward. Transitions include, for example, a healthy person becoming disabled and a moderately disabled person becoming severely disabled. There is not much published data that can help us to set the transition rates used in the model.
- Trends data are needed to indicate how transition rates change over time. For example, are people becoming more or less likely to become disabled at a particular age? There is some information which can be used indirectly to answer this sort of question.

Section 4.3 contains a description of the model and Section 4.4 discusses the way in which trends in healthy life expectancy can be used to determine the trends in transition rates which should be incorporated in the model.

In Section 4.5 we discuss the results from the projection model for three sets of assumptions: the most pessimistic, most optimistic and central set of assumptions. In Section 4.6, we discuss the uncertainties that surround the model and attempt to indicate the extent to which these uncertainties might influence the projections. We draw together our conclusions in Section 4.7.

4.2 Data Sources

4.2.1 Prevalence rate data

The starting point for a model that projects the number of people requiring long-term care in the future is a set of data that shows how many people require long-term care now. There is no completely satisfactory set of data for the UK but there have been a number of disability surveys which are useful.

The data from the surveys are generally presented in terms of the proportion of males and females in a range of age bands who are unable to perform one or more specified activities. The surveys differ from each other in many aspects: the number of people surveyed; the date of survey; the activities which are used to categorise disability; the survey method, such as the use of interviews or questionnaires; and whether the target population includes people in households or institutions or both.

It is essential to recognize that the estimates of the number of people with disabilities in the future that are produced by any projection model will be directly related to the current level of disability rates. This means that the usefulness and accuracy of any projected numbers will inevitably be limited by any problems relating to the initial data. We describe in some detail the data which we use and discuss the limitations associated with them.

Although there have been several disability surveys, we have used one to provide the initial data for the number of people with disabilities. This survey is the OPCS survey of disability in Great Britain (Martin et al, 1988). The reasons for relying on this particular survey are:

- The coverage included both private households and communal establishments;
- The survey was based on interviews rather than responses to a questionnaire;
- The sample was large;
- A wide range of disabilities was covered;
- The survey covers all adults whereas some surveys cover only people over 65 and therefore miss a significant number of disabled people;
- The survey report presents the data in a useful form involving several disability categories and age groups.

This survey was conducted as follows. For private households, a sample of 100,000 addresses was chosen for screening. A short questionnaire was either posted to these addresses or taken along by an interviewer. Questionnaires which indicated that there was a disabled person at the address led to a full interview. 14,308 adults were interviewed. The screening and interviewing took place in 1985.

For the survey of disabled people in communal establishments, 1,408 institutions were contacted. This resulted in a sample of 570 institutions in which interviewing took place. 3,775 adults were interviewed. The screening and interviewing took place in 1986.

The report on the survey allocates disabled people into one of ten categories with category 1 being the least severe and category 10 being the most severe. The categorisation process was developed to handle the data collected from the survey interviews. There have been a few subsequent surveys which have used the same disability scale. However, most surveys do not use the same procedures and definitions and the results from these cannot be compared directly with this large disability survey.

Tables 4.1a and 4.1b show the estimated number of disabled adults in Great Britain. These tables are taken from Appendix 5 of Dullaway & Elliott (1998). The numbers are based on the OPCS survey but the original report did not show males and females separately.

Table 4.1a OPCS Estimates of the number of disabled females (thousands)

		OPCS Disability Category									
Age	Able	1	2	3	4	5	6	7	8	9	10
20–29	4,102	21	13	14	21	18	18	11	10	8	6
30–39	3,660	36	15	23	27	24	18	15	12	7	4
40–49	2,958	50	28	27	34	30	25	20	15	9	3
50–59	2,604	87	54	57	55	55	36	28	22	19	5
60–69	2,266	138	111	94	86	90	55	49	34	37	11
70–79	1,427	161	151	132	116	122	112	86	66	57	34
80+	364	86	72	80	79	106	96	111	84	100	79

Source: Dullaway & Elliott (1998)

Table 4.1b OPCS Estimates of the number of disabled males (thousands)

		OPCS Disability Category									
Age	Able	1	2	3	4	5	6	7	8	9	10
20–29	4,235	24	15	14	16	13	13	9	8	5	7
30–39	3,717	42	16	22	20	18	13	11	10	4	5
40–49	3,015	57	30	25	25	21	18	15	11	6	4
50–59	2,577	100	58	53	41	40	25	21	18	12	6
60–69	1,956	173	116	81	69	58	32	32	30	27	11
70–79	1,020	152	117	86	71	60	46	38	38	29	13
80+	137	55	39	37	38	41	29	34	33	38	18

Source: Dullaway & Elliott (1998)

It is helpful to present the same information as prevalence rates per 1,000 of population at each age (i.e. the proportion of males or females of a particular age who have each level of disability, scaled so that the proportions at each age add up to 1,000). Tables 4.2a and 4.2b present the information in this form.

Table 4.2a OPCS Disability prevalence rates for females (per 1,000)

		OPCS Disability Category									
Age	Able	1	2	3	4	5	6	7	8	9	10
20–29	967.0	5.0	3.1	3.3	5.0	4.2	4.2	2.6	2.4	1.9	1.4
30–39	952.9	9.4	3.9	6.0	7.0	6.2	4.7	3.9	3.1	1.8	1.0
40–49	924.7	15.6	8.8	8.4	10.6	9.4	7.8	6.3	4.7	2.8	0.9
50–59	861.7	28.8	17.9	18.9	18.2	18.2	11.9	9.3	7.3	6.3	1.7
60–69	762.7	46.4	37.4	31.6	28.9	30.3	18.5	16.5	11.4	12.5	3.7
70–79	579.1	65.3	61.3	53.6	47.1	49.5	45.5	34.9	26.8	23.1	13.8
80+	289.6	68.4	57.3	63.6	62.8	84.3	76.4	88.3	66.8	79.6	62.8

Table 4.2b OPCS Disability prevalence rates for males (per 1,000)

		OPCS Disability Category									
Age	Able	1	2	3	4	5	6	7	8	9	10
20–29	971.6	5.5	3.4	3.2	3.7	3.0	3.0	2.1	1.8	1.1	1.6
30–39	958.5	10.8	4.1	5.7	5.2	4.6	3.4	2.8	2.6	1.0	1.3
40–49	934.3	17.7	9.3	7.7	7.7	6.5	5.6	4.6	3.4	1.9	1.2
50–59	873.3	33.9	19.7	18.0	13.9	13.6	8.5	7.1	6.1	4.1	2.0
60–69	756.7	66.9	44.9	31.3	26.7	22.4	12.4	12.4	11.6	10.4	4.3
70–79	610.8	91.0	70.1	51.5	42.5	35.9	27.5	22.8	22.8	17.4	7.8
80+	274.5	110.2	78.2	74.1	76.2	82.2	58.1	68.1	66.1	76.2	36.1

Since any projections of the number of people needing long-term care in the future are heavily dependent on the initial data, it is worth considering the key aspects of the OPCS survey data which might cause problems.

All people aged over 80 are put into a single age category. This might be quite a serious problem. Table 4.1 shows how rapidly numbers and prevalence rates change with age and it is very likely that rates which apply to people in their early eighties do not apply to people over 90. The number of people who survive to ages well in excess of 80 is expected to grow rapidly over the next few decades, hence it is very important to have some knowledge of the prevalence of disability amongst the most elderly people. The costs of caring for disabled people at these ages may be very high.

The extent of this problem depends on what the prevalence rates are used for. If the only use of the rates were as a starting point for projections, there would be no problem. In projections to, say, 2020 the people who will be aged 90 or more would have been in their 50s and 60s when the OPCS survey was carried out and it is irrelevant that there is some uncertainty about disability amongst the elderly in the mid 1980s. However, in our projections, transition rates are used and we need, for example, some estimate of the probability that a non-disabled 85 year old female will become disabled in the next year. We will choose this probability, along with a great many others, to be compatible with the prevalence rate data. This means that the prevalence rates of disability in the OPCS survey do feed through into the projected prevalence rates in the future.

The information collected in the survey is sufficient to allow prevalence rates to be calculated for narrower age bands. As far as we know, this information has not been published. There is, however, one graph in Martin et al (1988, Figure 3.3) which does show some information broken down into five year age bands.

Other limitations of the OPCS disability survey prevalence rates include:

- The OPCS disability definitions are not directly linked to cost.
- The process of assigning a disability category is complex and hence errors or peculiarities may have crept in.
- Despite the large sample size, if the data are split into the two sexes, seven age groups and eleven disability categories (including 'able') some errors in the parameter estimates will inevitably be introduced.
- The survey was carried out in 1985 and 1986 and is therefore out of date. We deal with this point in our models by starting all projections in 1986 rather than starting from the present.

Although the disability definitions are not directly linked to care costs, there is some information which shows how much additional expenditure is incurred by disabled people in private households and where the same definitions of disability are used as in the OPCS survey (Matthews & Truscott, 1990). Also, the report on the survey (Martin et al, 1988) does show the proportion of people in each disability category and at each age who were in institutions at the time of the survey. For the people in the more severe categories, it is reasonable to assume that, in most cases, it was the institutions which were providing their care. This, therefore, gives a useful indicator of how care utilisation relates (or, more accurately, related at the survey date) to disability.

Another important source of data is the General Household Survey (GHS) which is carried out annually (see, for example, Thomas et al, 1998). The survey has a large sample size (22,001 in 1996, for example). It includes two questions about disability:

1. *Do you have any long-standing illness, disability or infirmity? By long-standing I mean anything that has troubled you over a period of time or that is likely to affect you over a period of time.*
2. *Does this illness or disability limit your activities in any way?*

The answers to the second of these questions should provide useful information about the level of disability in the population. It is also potentially useful that the survey is carried out every year. We return to this point in Section 4.2.3, when discussing trends.

The survey is confined to households so that there are bound to be differences compared with the OPCS survey which included people in communal establishments. However, there are also very clear differences between the number of people disabled according to the second GHS question given above and the number of people in private households who have any disability according to the OPCS survey. Martin et al (1988) show these differences in their figure 3.4 and table 3.5. Both of these compare the prevalence rates per thousand of population at various ages for GHS survey of 1985 and the OPCS disability survey. The GHS shows substantially more disabled people below the age of 75 and substantially fewer over the age of 75.

The differences below age 75 are explained by Martin et al (1988) as being due to the GHS question allowing any disability to count while the interview based OPCS survey questions related to specific tasks or functions. The suggested reason for the difference amongst people over 75 is that these people may not see themselves as disabled. Any limitations which they have may be thought of as due to old age rather than to disability.

As well as the questions contained each year in the General Household Survey, there are supplementary questions which are repeated every few years. One of the areas in which there are a large number of supplementary questions is the health of people over 65. As a result there is much more information available on the abilities of the elderly in the surveys of 1980, 1985, 1991, 1994 and 1996. The availability of this information is very important in showing changes over time. These GHS surveys provide the key data, discussed in Section 4.2.3, concerning trends.

People are asked about a variety of tasks such as climbing stairs, dressing, shopping and using a vacuum cleaner. The data are summarised in the form of the proportion of men and women in each of five age groups who have difficulty with each task. The proportions are given separately for each task.

An established way of categorising disabilities is to measure the ability of people to perform certain tasks known as activities of daily living (ADLs). There are a few different definitions in use, but six usual ADLs are bathing, dressing, going to the toilet, transferring (to and from a bed or chair), continence and feeding.

Some research work has been done which enables some comparison between the OPCS disability categories and the ADL based categories which are measured by the GHS. Bone (1995, chapter 3) has defined a disability scale based on ADLs and has reported on the disability prevalence rates shown by the people responding to the GHS surveys.

The GHS surveys have regularly covered only four of the six usual ADLs: feeding, transferring to or from bed, going to the toilet and bathing. The surveys also cover some instrumental activities of daily living (IADLs), specifically shopping, cooking, house cleaning, laundry and travel. The disability scale reflects failure in IADLs and ADLs as shown in Table 4.3.

Table 4.3 The dependency scale used by Bone (1995) for analysing GHS data

Dependency	Level	Definition
Independent	1	Manages all ADLs and IADLs without help
Least dependent	2	Cannot manage one or more IADL alone but can manage ADLs
	3	Cannot manage one ADL alone and cannot manage one or more IADL
	4	Cannot manage two ADLs alone and cannot manage one or more IADL
	5	Cannot manage three ADLs alone and cannot manage one or more IADL
Most dependent	6	Cannot manage four ADLs alone and cannot manage one or more IADL

Source: Bone (1995)

With these definitions, the levels of dependency shown in Table 4.4 were found in the 1985 GHS survey (i.e. the one closest to the date of the OPCS disability survey). These figures refer only to people in households and they are combined values for males and females.

Table 4.4 Disability prevalence rates (%) according to the 1985 GHS survey

Age	Category				
	2–6	3–6	4–6	5–6	6
65–69	13	3	1	1	0
70–74	18	4	1	0	0
75–79	30	9	3	1	0
80–84	49	15	2	0	0
85+	77	31	9	5	1

Source: 1985 GHS Survey

The prevalence of disability amongst people in private households according to the OPCS survey, using the OPCS disability categories, is given in Table 4.5.

Table 4.5 Disability prevalence rates (%) according to the OPCS disability survey (in private households only)

Age	Category									
	1–10	2–10	3–10	4–10	5–10	6–10	7–10	8–10	9–10	10
60–69	23.6	18.0	13.9	10.8	8.0	5.3	3.8	2.4	1.3	0.3
70–79	39.5	31.8	25.3	20.0	15.5	11.1	7.4	4.6	2.3	0.6
80+	67.4	58.4	51.6	44.5	37.7	29.1	22.2	14.2	8.5	2.5

Source: Martin et al (1988)

A comparison of Table 4.4 with Table 4.5 suggests, very roughly, that failing an IADL (Bone's category 2) corresponds with OPCS category 3 in terms of the cumulative prevalence rates. Also, the failure of an ADL (Bone's category 3) appears to correspond, roughly, to OPCS category 7. This is, however, quite misleading. As the examples given earlier indicate, a category 7 disability on the OPCS scale is very severe and would be equivalent to the failure of more than one ADL. The reason why there are so many people in the high OPCS disability categories compared with the high GHS disability categories might be that the OPCS definition of disability covers some elements not measured by ADLs.

Another difference between the two sets of data is that the OPCS survey covered Great Britain whereas the figures analysed by Bone (1995) are for England only.

The fact that the OPCS disability scale is difficult to mesh with an ADL based scale has meant that we have relied solely on the OPCS survey results for providing prevalence rate data. However, we have had to rely on GHS surveys to provide information on trends. This is clearly not an ideal situation.

There have been other large surveys which cover disability. We have not used the data from these surveys. We comment on these surveys and, where possible, compare their findings with those of the OPCS disability survey in Walsh and Rickayzen (2000a).

There is a lack of useful data on transition rates in the UK and because there are some transition rate data in the US it would be advantageous to be able to use it. However, an analysis of US data shows that they are very different from English data. This might be due to: different policies as to who receives care in the community and who goes into an institution to receive care, different surveying methods, different definitions of ADL failure or populations with different levels of disability. Whichever is the case, it provides a warning regarding the use of overseas data.

4.2.2 Transition rate data

In order to project forward the number of people with disabilities we use a transition rate model. This model requires assumptions for the likelihood of various transitions occurring. The sort of transitions we are interested in include:

- A healthy person becoming moderately disabled
- A healthy person becoming severely disabled
- A healthy person dying
- A moderately disabled person becoming severely disabled
- A moderately disabled person recovering from their disability and becoming healthy
- A moderately disabled person dying

In practice, we do not limit ourselves to two categories of disability, moderate and severe, but use all ten of the OPCS categories.

We require estimates of the probabilities of these transitions occurring. The probabilities are likely to depend on age and sex. Many probabilities will also depend on which particular disability category or categories is involved. The probabilities may well change over time and we will need trend data to model this (see Section 4.2.3).

There are a great number of transitions in which we are potentially interested but unfortunately there is very little UK information that we can use to estimate the transition rates. There has been no large scale longitudinal survey which tracks a population at frequent intervals over a number of years and records information on disabilities. We have the choice of using small scale UK longitudinal data sets, larger US longitudinal data sets or not using any longitudinal data at all.

Transition rates between various levels of disability have been analysed in the US. A report by the Society of Actuaries Long-Term Care Valuation Insurance Methods Task Force (1995) considers data from the National Long-Term Care Surveys of 1982 and 1984. The surveys cover Medicare enrolees in the community and institutions. In table 5 of that report, the number of people who have transferred between each of several disability states is given. Table 4.6 is based on that data. Transition rates have been found and divided by two to give approximate annual transition probabilities. The numbers in italics are the probabilities of not changing category. These are calculated as 100% minus the sum of the probabilities of moving out of the category.

Table 4.6. US transition rates (% per year)

	Age	Initial status	Status after 2 years				
			0 ADL	1 ADL	2 ADL	3+ ADL	DEAD
Males	65–74	0 ADL	*94.23*	0.45	0.27	0.48	4.57
		1 ADL	17.60	*59.60*	5.20	3.60	14.00
		2 ADL	9.80	9.80	*56.86*	5.88	17.65
		3+ ADL	5.97	1.49	4.48	*66.42*	21.64
	75–84	0 ADL	*89.50*	1.21	0.47	0.92	7.89
		1 ADL	10.98	*54.55*	4.92	5.30	24.24
		2 ADL	5.88	3.92	*56.86*	9.80	23.53
		3+ ADL	3.17	1.59	3.17	*66.67*	25.40
	85+	0 ADL	*81.38*	2.28	1.46	1.79	13.09
		1 ADL	7.14	*59.52*	2.38	10.32	20.63
		2 ADL	0.00	6.25	*54.17*	10.42	29.17
		3+ ADL	2.63	0.00	13.16	*57.89*	26.32
Females	65–74	0 ADL	*96.62*	0.67	0.19	0.30	2.21
		1 ADL	19.06	*62.81*	4.69	4.69	8.75
		2 ADL	10.94	8.59	*57.03*	13.28	10.16
		3+ ADL	4.49	3.21	3.85	*69.87*	18.59
	75–84	0 ADL	*90.78*	2.21	0.64	0.91	5.46
		1 ADL	16.20	*61.97*	3.05	7.98	10.80
		2 ADL	8.67	5.33	*58.67*	12.00	15.33
		3+ ADL	4.79	2.66	5.85	*70.74*	15.96
	85+	0 ADL	*81.44*	4.77	1.58	2.37	9.84
		1 ADL	10.49	*62.94*	3.50	7.69	15.38
		2 ADL	5.00	6.67	*57.50*	9.17	21.67
		3+ ADL	4.12	2.94	2.94	*64.71*	25.29

Source: Society of Actuaries Long-Term Care Valuation Insurance Methods Task Force (1995)

The dependence of death on disability shows the following features:

- The mortality rate increases with the level of disability.
- At the higher ages (i.e. above age 74) there does not appear to be much difference in the mortality rate between people failing 2 ADLs and people failing 3 or more ADLs.
- The ratio of mortality rates for those failing 3 or more ADLs to those failing no ADLs falls with age.

- The differences between the mortality rates of those failing 3 or more ADLs and those failing no ADLs are 17.1%, 17.5% and 13.2% for males (starting with the lowest age group) and 16.4%, 10.5% and 15.5% for females. Very roughly, this is consistent with a constant addition of 0.15 to the mortality rate each year, independent of age and sex.
- For females the difference in mortality rate between those failing 1 ADL and those failing none appears to be independent of age: it is 6.5%, 5.3% and 5.5% for the three age groups.

The following features relating to deterioration in ability are shown in Table 4.6:

- Deterioration is less frequent than death.
- People who fail no ADLs are less likely to fall into the 2 ADL category than those who already fail 1 ADL. This applies to all ages and both sexes.
- People who fail no ADLs are less likely to fall into the 3+ ADL category than those who already fail 1 ADL, and they in turn are less likely to fall into the 3+ ADL category than those who already fail 2 ADLs. This applies to all ages and both sexes.
- Deterioration from no ADL failure increases with age.
- For males, deterioration from 1 ADL failure to 2 ADL failure decreases with age while the deterioration from 1 ADL failure to 3+ ADL failure increases with age.

The following features relating to improvements in ability are shown in Table 4.6:

- Improvements from 1 ADL are more frequent than deaths for males aged 65 to 74 and females aged 65 to 84.
- Improvements rates to 0 ADLs are higher for people who had failed 1 ADL than people who had failed 2 ADLs and, generally, are higher for those failing 2 ADLs than for those failing 3+ ADLs.
- Most improvement probabilities decrease with age but there are some exceptions to this.

- Some improvements are very great, i.e. those from failing 3+ ADLs to failing none.

The existence of a significant number of improvements is consistent with UK population data.

As well as using the information on transitions which we have described in this section, it is possible to use prevalence rate data to determine transition rates. Under a given set of assumptions it is possible to derive transition rates from prevalence rate data. This is, in fact, the approach we have adopted. The approach is described in detail in Section 4.3. Some of the 'shape' of the transition rate model is determined by the data in this section. An example of this is the requirement that the probability of a moderately disabled person becoming severely disabled should be higher than the probability of a non-disabled person becoming severely disabled in the next year. This 'rule' is inferred from the US data, but in our model we include a parameter which describes just how great the difference is. The value for this parameter is determined by looking at UK prevalence rate data.

4.2.3 Trends data

We can use prevalence rate data as a starting point for our projections of the number of people requiring long-term care and we can use the transition rate model to move this population forward. However, there are likely to be changes in the transition rates over time. We have looked for evidence of what changes have happened in the recent past to determine what trends should be included in our model.

The trends assumptions we adopt are important because the number of people who are projected to require long-term care according to our model is very sensitive to them.

The main type of trend information concerns healthy life expectancy (HLE). Just as life expectancy gives a measure of the time someone may expect to live, healthy life expectancy gives a measure of the time someone expects to live and to be healthy.

Like life expectancy, it can be determined by a snapshot of the population rather than actually involving any forecasting. We will be considering this type of HLE. We will also consider disabled life expectancy (DLE) which is a measure of the time someone expects to live whilst in a state of disability.

HLE depends on age and sex. It also depends on the definition of 'healthy'. If the definition is very narrow, so that many conditions count as unhealthy, HLE will be relatively short. On the other hand, if a wide definition is used, many people will be classed as healthy and HLE will be relatively long, and will tend towards the total life expectancy if very few people are counted as unhealthy.

The data which we discuss relate only to people aged 65 and over. We concentrate on these ages because they are the most important in terms of the number of people needing care.

It is important to note that, because of the way HLE is calculated, the time spent unhealthy depends both on how many people ever become unhealthy and on how long they live once they are unhealthy. This matters most for definitions in which anyone counted as unhealthy is in a severe state. It is quite plausible that improvements in medicine and care act both to prevent people ever reaching this severe state and also to prolong the life of anyone who does reach the state. These two effects work in opposite directions in terms of DLE — the former decreases it and the latter increases it.

Before we discuss the data on trends in HLE we will describe how we can use information on HLE trends. HLE is not an input for our projection model but it may be derived from the populations produced by the model. For a given set of input assumptions, including trends in, say, the probability that someone becomes severely disabled, we can examine how HLE changes over time. By adjusting the input trends, we can find a set which is compatible with the externally available HLE trend data.

The HLE data we use are taken from the booklet *Health Expectancy and Its Uses* (Bone et al, 1995) and the discussion paper *Healthy Life Expectancy in England and*

Wales: Recent Evidence (Bebbington & Darton, 1996). The main set of data considered in both of these publications is derived from the General Household Survey. Both publications only consider data for England and Wales. (Some of the data are for 1976 and these are from the Elderly at Home Survey which only covered England.) The two publications are not independent, being based on the same raw data; however, the more recent publication also considers data from a more up-to-date survey.

We will principally be looking at two definitions of 'healthy', but will also make some comments on other definitions. We use the phrases 'free from any disability' and 'disability free' to refer to people who do not have any limiting long-standing illness. We use the phrases 'free from severe disability' and 'severe-disability free' to refer to people who are unable to perform ADLs.

Although the HLE estimates are derived from GHS data, Bone et al (1995) and Bebbington & Darton (1996) have adjusted the data to allow for the fact that part of the population is not resident in households.

The analysis contained within both papers suggests that for both males and females the disability free life expectancy has been increasing and the ratio of disability free life expectancy to total life expectancy has been roughly constant.

As indicated, there is some uncertainty surrounding the interpretation of the trends in healthy life expectancy based on a catch-all definition of disability. The situation is, however, far more confusing as regards severely disabled life expectancy. Bone et al (1995) examine HLE from the Elderly at Home Survey of 1976 and the GHS surveys of 1980, 1985 and 1991. Three definitions of severe disability are considered.

The trends apparent for the three definitions differ in the following ways:

- The time spent severely disabled appears to have been falling if failure of an ADL is used to identify severe disability.

- The time spent severely disabled appears to have been rising if inability to manage stairs and steps is used to identify severe disability.
- The time spent severely disabled shows an erratic pattern if inability to get outdoors is used to identify severe disability.

In summary, the trends in HLE shown by data from the General Household Survey are:

- Life expectancy free from any disability has been slowly increasing.
- The proportion of life spent free from any disability has been roughly constant.
- Severe-disability free life expectancy has been increasing according to an ADL based definition of severe disability.
- The proportion of life spent free from severe disability has been increasing.
- The severely-disabled life expectancy may have been falling, but this is far from clear.

It is important to recognise that all this trend information relates to disabilities recorded in the General Household Surveys. We mentioned in Section 4.2.1 that there are difficulties in aligning the GHS disability categories with those used in the OPCS disability surveys. In discussing Table 4.5, we suggested that there are several types of disability captured by the OPCS definitions which are not measured by the GHS questions. It is quite possible that some of these disabilities, such as those related to behaviour and intellectual functioning, do not follow the same trends as the physical disabilities measured in the GHS. If this were the case, the HLE trend data would not be so useful.

4.3 A Transition Rate Model

4.3.1 Outline

In Section 4.2, we explained that we used the OPCS survey of disability in Great Britain (Martin et al, 1998) to provide the initial data for the number of people with disabilities. We did this by combining the prevalence rate data from the OPCS survey with the number of males and females at each age to give us the number of people at each of the ten disability levels, and the number healthy, in 1986. We need a transition rate model to project this population forward. Each year some people will show improvements in their abilities, some will show no change, some will deteriorate and some will die.

There are many possible transitions, all of which may depend on age and year. We have separate models for males and females. One thing we do not allow for in our model is duration: the probability that a transition takes place is taken to apply to all people in a particular sex/age/year/disability category; we do not take into account how or when someone arrived in that category.

All of the probabilities we use are annual. So, for example, a process that involves deteriorating from healthy to a category 3 disability and then deteriorating further to category 4 during the same year will be regarded as a single healthy-to-category 4 transition.

Thus, the model is a discrete time multiple state model. For a full description of such models and discussion of applications to disability insurance, see Haberman and Pitacco (1999).

4.3.2 Mortality

4.3.2.1 Overall mortality

We use the Government Actuary's Department (GAD) central population projection for the period 1996 to 2036 (Government Actuary, 1998). This gives the projected total number of deaths each year at each age. Our model matches these numbers exactly. Note that the GAD projections include migration as a transition and we also include migration so that the numbers match.

In theory, it might be reasonable not to reproduce the GAD projected population. Future death rates will be closely related to the prevalence of disability in the future. Since we are producing a new model for the prevalence of disability it would be possible to use it to derive the number of deaths in each future year under certain assumptions about the link between mortality and disability. However, we decided that it would be undesirable to produce a population projection which differed from the GAD central projection. Thus, we use the GAD central projection as a constraint on the output of our model.

Since the prevalence rate data apply to 1985 and 1986 there is a ten year gap to fill before the start of the period covered by the current GAD projection model. (We actually assume that the prevalence rate data all apply to 1986.) During these years we use mortality rates which are interpolated between those of English Life Tables No.14 (ELT14) (OPCS, 1987) which are taken to apply in 1980 and the GAD 1996 rates. These rates are used to determine the total population each year (working back from 1996) and also the total number of deaths each year during the ten year period.

4.3.2.2 The dependence of mortality on disability

The mortality rate is higher for people in the severe disability categories and we split the total mortality into two components in order to model this. One of the components applies equally to all healthy and disabled people of a given age and sex in particular year. The other component is higher for people with severe disabilities.

This second component was set by reference to the US data described in Section 4.2.2. Note that the US data only relate to people over 65. The US data were useful in suggesting an overall 'shape' for the dependence of mortality on disability and how this relates to age. We have not attempted to include any of the detail from the US data in our model. The features that we incorporate in our model regarding the disability-related component of mortality are:

- There is only weak age dependence (above age 65) in the disability-related addition to healthy mortality.
- The extra mortality is low at younger ages. This is needed because applying the 65+ rates to the disabled population aged around 35 produces too many deaths. In fact the number of disabled people dying would be more than the total number of deaths according to the GAD model.
- There is no extra mortality compared with healthy people for those with disabilities in category 5 and lower. The description of these disabilities suggests they are not life threatening conditions.
- The extra mortality increases linearly starting with category 6. The US data do not fully support this, but we feel that we do not have enough information to justify a more complex category dependence.
- The maximum extra annual mortality is 0.20.
- The model is the same for males and females.

Once the extra mortality has been chosen, the other mortality component is determined by the requirement that the total number of deaths should match the GAD projection numbers.

The formula we use to express the extra mortality for someone aged x in disability category n (where $n = 0$ means healthy) is:

$$ExtraMort(x, n) = \frac{0.20}{1 + 1.1^{50-x}} \cdot \frac{\text{Max}(n-5, 0)}{5}$$

The form of 'reciprocal of one plus an exponential' is the same as we use for modelling deterioration (see Section 4.3.3). The choice of the pivotal age 50 and the steepness factor 1.1 effect the extra mortality at younger ages. The following table shows illustrative values for this function at a range of ages and disability levels.

Table 4.7 Annual addition to mortality due to disability

Age	Category 6	Category 8	Category 10
20	0.00	0.01	0.01
30	0.01	0.02	0.03
40	0.01	0.03	0.06
50	0.02	0.06	0.10
60	0.03	0.09	0.14
70	0.03	0.10	0.17
80	0.04	0.11	0.19
90	0.04	0.12	0.20
100	0.04	0.12	0.20
110	0.04	0.12	0.20

The extra mortality might change with time. We discuss trends in Section 4.4.

4.3.3 Deterioration

4.3.3.1 Outline

Healthy people can become disabled and the condition of disabled people can become worse. Both of these come under the heading of deterioration. In our model, deterioration is allowed from any state to any more severely disabled state. This results in a huge number of transitions. Owing to the complexity, the model for deterioration is split into three parts, which are dealt with in the next three sections. One part relates to the probability of a healthy person becoming disabled, another relates to the distribution of the severity of new disabilities amongst previously healthy lives and the final part relates to deterioration amongst people who are already disabled.

There are parameters for each part of the deterioration model. The parameter values are chosen so that the transition rate model is able to reproduce the prevalence rate data closely.

In making the comparison between the observed prevalence rates and those produced by the model, we start with twenty-year olds with disabilities matching the OPCS rates. This population is projected forward to produce the model prevalence rates at higher ages. The transition model includes mortality and improvements in health as well as deterioration but these other components are fixed separately — they are not chosen for their ability to reproduce the prevalence rate data.

Note that this approach assumes that there is a stationary population, i.e. transition rates have been constant in the past. This is clearly not correct. We refer to this problem in Section 4.6.3. For convenience, we use a single mortality table during this comparison process (rather than using time dependent rates). The mortality table we use is ELT14.

Tables 4.8a and 4.8b present the ability of the model to reproduce the crude prevalence data for females and males, respectively. They show the difference between the disability prevalence rates according to the OPCS survey (i.e. those shown in Table 4.2) and those produced by the transition rate model.

Table 4.8a. Difference in prevalence rate for females, *data – model* (per thousand)

		OPCS Disability Category									
Age	Able	1	2	3	4	5	6	7	8	9	10
20–29	–2	0	0	0	1	0	1	0	0	0	0
30–39	4	0	–3	–1	1	0	0	0	0	0	0
40–49	5	0	–3	–3	1	–1	0	0	1	0	0
50–59	–5	2	–2	0	2	2	0	0	2	2	–1
60–69	0	0	3	–1	0	0	–3	–2	1	3	–2
70–79	4	–4	7	0	–2	–6	3	–4	3	0	–1
80+	–1	0	0	1	0	0	0	0	0	0	0

Table 4.8b Difference in prevalence rate for males, *data – model* (per thousand)

		OPCS Disability Category									
Age	Able	1	2	3	4	5	6	7	8	9	10
20–29	–3	0	1	1	1	0	1	0	0	0	0
30–39	2	0	–2	0	0	0	0	0	0	–1	0
40–49	3	–1	–1	–1	0	0	1	0	0	–1	0
50–59	–5	0	0	2	0	1	1	0	1	0	0
60–69	–10	3	8	1	0	–1	–2	–1	1	1	1
70–79	15	–8	9	0	–5	–8	1	–4	2	–2	0
80+	0	0	0	0	0	0	0	0	0	0	0

Note that the structure of the data that we are trying to model (i.e. those shown in Table 4.2) is very complex. The prevalence rates do not vary smoothly across categories and the dependence of the prevalence rates on age is quite different for the low disability and high disability categories.

Tables 4.8a and 4.8b are encouraging as the differences between the data and the model are not large. There are some systematic errors but there appear to be no major problems at the highest categories, which are the more important categories as far as care costs are concerned.

The reason why the values in the table are small is that the model of deterioration is complicated. There may be a case for simplifying the model and accepting a poorer fit to the data.

4.3.3.2 The probability of becoming disabled

We use formulae to express the probability of becoming disabled. The probability of becoming disabled is primarily constrained by the observed proportion of people who have no disability. There are only seven age bands for the published disability survey data but we find that a complex model is needed to provide a good fit to the data. The formula we use has four parameters for females and there is an additional one for males. For females, the formula we use is:

$$NewDisab(x) = A + \frac{D - A}{1 + B^{C-x}}$$

where the four parameters are A, B, C and D and *NewDisab(x)* is the probability that a female aged x becomes disabled in a year. We note that this formula is logistic in form and was first proposed by Perks for the graduation of mortality rates (See Benjamin & Pollard, 1993).

For males, the formula we use is:

$$NewDisab(x) = \left(A + \frac{D-A}{1+B^{C-x}}\right) \times \left(1 - \frac{1}{3} \cdot \exp\left[-\left(\frac{x-E}{4}\right)^2\right]\right),$$

where the additional parameter is E.

The parameter A is the limit of the probability of becoming disabled at young ages. D is the limit of the probability of becoming disabled that would apply at extremely high ages. The pair of parameters B and C determine how rapidly the probabilities change between the two extreme values. The extra parameter, E, gives the age at which there is a 'kink' in the *NewDisab(x)* function.

Figure 4.1 shows the logarithm (base 10) of the annual probability of becoming disabled, for males and females. The parameter values used in the figure are the same as were used to produce Table 4.8.

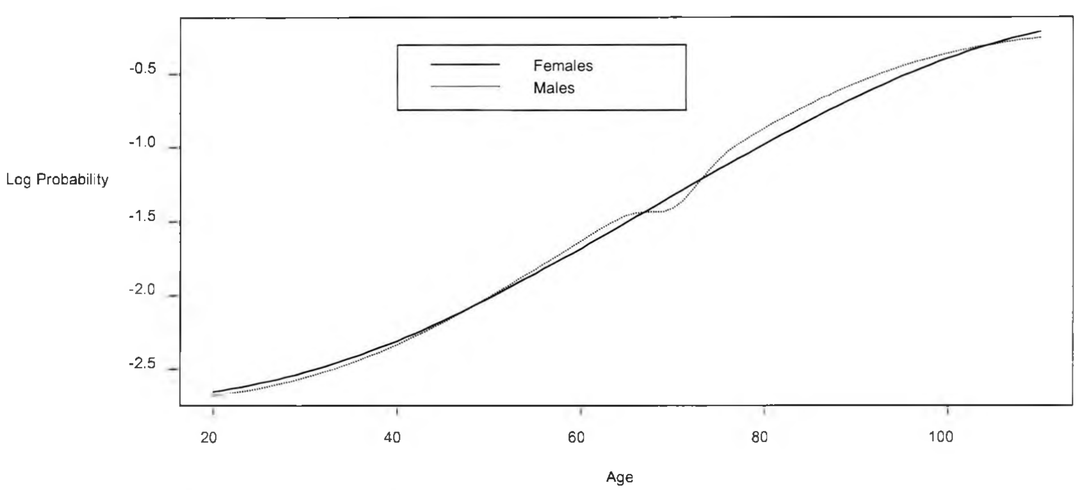


Figure 4.1: (log) Annual Probability of becoming Disabled

The shapes of the curves in the figure are quite complex. The continual oscillations, which show females having the higher probability of becoming disabled at young ages, followed by males in their fifties, followed by further changes, may be traced directly to the data. To get sufficient flexibility in the shapes, an extra parameter was added for fitting the males' data (which is not needed for the females' data).

The parameter values that we use are given in Table 4.9. These parameters have been fitted so that the prevalence rates predicted by the model match the 14 prevalence rates observed from the OPCS survey published results (i.e. seven age bands for each sex). It should be remembered that the 9 parameters are therefore being obtained by reference to prevalence rates which are based on a sample size of over 100,000 lives.

Table 4.9 Parameter values for *NewDisab* (x)

Parameter	Males	Females
A	0.0017	0.0017
B	1.1063	1.0934
C	93.5111	103.6000
D	0.6591	0.9567
E	70.3002	(Not used)

The behaviour of the $NewDisab(x)$ formula above the age of 85 or so is not well constrained. Since the highest age group in the data we use to constrain the model includes all people over 80 and these have an average age of around 85, the probability of becoming disabled could be very different at the highest ages without noticeably changing the prevalence rates in the crude data.

We define the probability of becoming disabled in such a way that it only applies to people who survive the year. This was done for numerical convenience: because mortality and disability rates become high at old ages some technique is needed to avoid total transition probabilities exceeding 1. The device of defining transitions in sequence, i.e. with mortality first, followed by deterioration (which is followed by improvement), achieves this.

4.3.3.3 The severity of new disabilities

Someone becoming disabled from healthy may enter any of the ten disability categories. The relative probability of joining each category may change with age, with the likelihood that the disability is severe increasing for older people. The transition rate model has three parameters covering this age dependence.

An examination of the prevalence rates at ages over 80 shows that the progression is erratic. The rate is higher in category 1 than in category 2, category 2 has a lower rate than category 3 and so on. In fact the rate in every category is either higher than in both the neighbouring categories or lower than in both. A simple model cannot replicate such a pattern. We decided to adopt a model which could reproduce the observed pattern closely. This involves having a separate parameter to represent the 'width' of each category. This approach is not unreasonable given the complex definitions used for each category. Because of the complexity of the definitions, some categories may include more people than others — this is the aspect of the disability prevalence rates that the width parameters are intended to mimic.

The formula for the probability that a person who becomes disabled at age x will have a disability in severity category n is given by:

$$Severity(x, n) = W(n) \cdot f(x)^{n-1} \,/\, Scale(x)$$

$$f(x) = A + \frac{1-A}{1+B^{C-x}}$$

$$Scale(x) = \sum_{n=1}^{10} W(n) \cdot f(x)^{n-1}$$

The category widths are given by $W(n)$. The *Scale* term ensures that the probabilities add up to 1 and its inclusion means that we can arbitarily set W(1)=1. The three parameters relating to the age dependence are A, B and C (they are distinct from the parameters used in the formulae for $NewDisab(x)$).

Figure 4.2 illustrates the age dependence of the relative severity of new disabilities. The figure shows the probability that someone newly disabled will be in category 6 or worse at the end of the year. The shapes of the curves are similar for other categories.

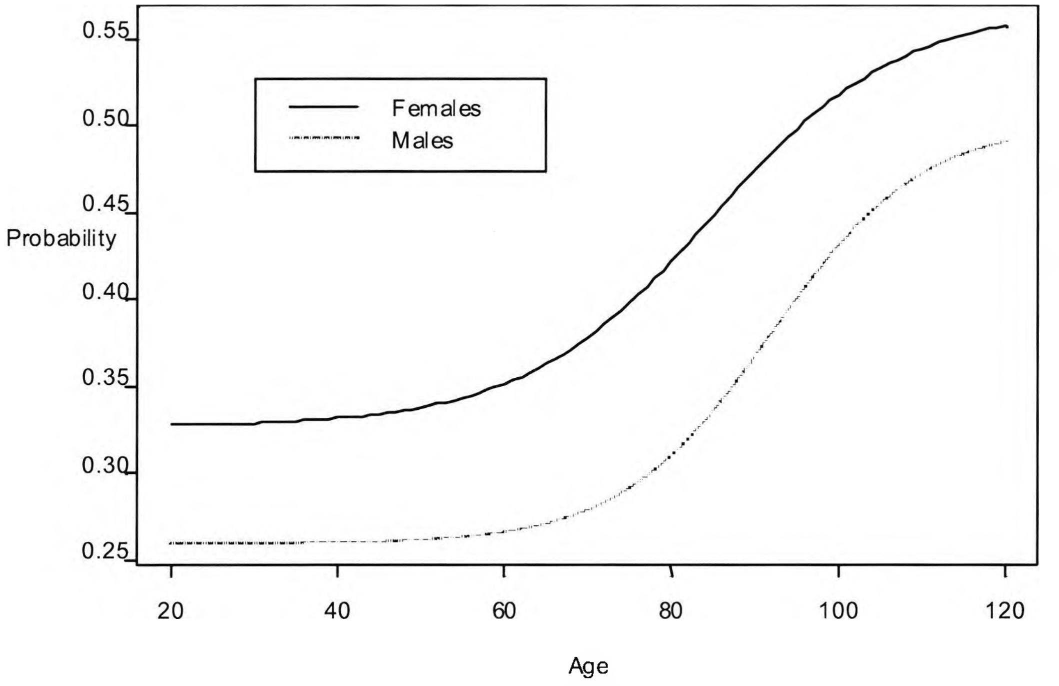


Figure 4.2: Probability of a new disability being category 6 or worse

Figure 4.2 shows that there is a difference between the probabilities for females and males. This was derived from fitting the prevalence rate data. However, it would be plausible to find slightly poorer fits in which there was little difference between the probabilities for males and females (by, for example, changing the likelihood of disabled people deteriorating).

The parameters we use are given in Table 4.10. Note that $W(1) = 1$ is fixed for both males and females.

Table 4.10 Parameter values for *Severity*

Parameter	Males	Females
A	0.8246	0.8180
B	1.1146	1.0911
C	91.7127	85.5099
$W(2)$	0.5250	0.6823
$W(3)$	0.4632	0.8166
$W(4)$	0.4622	0.6656
$W(5)$	0.6066	1.1749
$W(6)$	0.4205	1.0426
$W(7)$	0.6299	1.4203
$W(8)$	0.6370	0.9399
$W(9)$	0.9004	1.2222
$W(10)$	0.4874	1.0674

4.3.3.4 Deterioration from disabled states

People in any disability category can get worse and their new disability level could be any of the more severe categories. These transitions are included by relating them to the probability of deteriorating from healthy (i.e. becoming disabled). We use the following rule: the probability of someone in disability category m deteriorating to category n is F^m times the probability that a healthy person deteriorates to category n. This may be expressed by the following pair of equations:

$$Deteriorate(x, m, n) = Deteriorate(x, 0, n) \times F^m$$
$$Deteriorate(x, 0, n) = NewDisab(x) \times Severity(x, n).$$

The parameter F is required to be greater than 1 in order to reflect the fact that disabled people are more likely to become severely disabled than healthy people. For males we use $F = 1.1561$ and for females we use $F = 1.1830$.

4.3.3.5 The fitting procedure

We have said, in Section 4.3.3.1, that the parameters are set so that the transition rate model can generate a set of prevalence rates that closely matches the OPCS disability survey prevalence rates. In this section we specify how we define 'close matching' and how we obtain a satisfactory fit.

We are trying to model prevalence rates for seven age bands and ten disability categories. This gives 70 'cells'. We simply try to minimise the sum of the absolute values of the differences between the prevalence rates in the data and the prevalence rates produced by our model. In other words, we take the numbers in a table like 4.8a or 4.8b, remove the minus signs and add them up. (The prevalence rate in the 'able' category is automatically 1,000 minus the sum of the other ten prevalence rates at each age, and we do not include it in our error statistic.) Other statistics could have been chosen. For example, extra weighting could have been given to the high ages or high disability categories, or some weights relating to the uncertainties in the cells could have been used.

Our deterioration model has 17 parameters for fitting the females' data and 18 for fitting the males' data. It is difficult to obtain an optimal fit to the data when there are so many parameters to be considered. This is especially true when there are so many local minima encountered in the fitting process. However, we believe that, overall, we have obtained a good fit to the data.

4.3.4 Improvements

As noted in Section 4.2.2, there is evidence that a significant number of disabled people improve to some extent. The US data show some dramatic improvements. It is not clear whether these represent recoveries from long-term disabilities or from temporary disabilities caused by, say, breaking a bone.

We decided not to include a full range of improvements in our transition rate model and, instead, we have adopted a simple assumption: all people, at all ages and in all disability categories have a 10% chance of improving by one category over the course of a year. This 10% probability only applies to those who survive the year and do not deteriorate during the year. The figure of 10% is broadly consistent with the UK data (Goddard, 1998). The approach is not consistent with what is shown by the US data unless those data include some short-term disabilities.

4.4 Assumed Trends

4.4.1 Changing transition rates over time

In Section 4.2.3 we discussed data relating to trends in healthy life expectancy (HLE). We want our projection model to be able to reproduce trends similar to those indicated by the data.

In the projection model, trends are included by changing transition rates over time. The procedure we adopt to identify which transition rate changes correspond to the observed HLE trends is:

- Calculate the healthy life expectancies in 1986 using the definitions of 'healthy' which are related to the disability categories of the OPCS survey (Martin et al, 1988).
- Project the population forward for ten years using a range of assumptions for changes to the transition rates.
- Calculate the healthy life expectancies in 1996 for the various projections.

– Compare the changes in HLE produced by the model with those shown by the data and decide which trends to continue with for projections up to the year 2036.

The reason why we stop in 1996 is because that is the year when the GAD population projections start. The projection model becomes more complicated when it is built around the GAD population model because it must include migration. The year 1996 is therefore a natural break point for the projections.

Table 4.11 shows the healthy life expectancies in 1986. To calculate these, prevalence rates and a life table are needed. We have used the prevalence rates produced by the transition rate model described in Section 4.3. (The model provides prevalence rates at individual ages unlike the published data that give the rates in ten-year age bands.) The life table is an interpolation between ELT14 and the life table corresponding to the 1996 mortality rates in the GAD population projection model. The ELT14 table is taken to apply to 1980.

Table 4.11 Life expectancies in 1986

		HLE(0)	HLE(0)/*e*	HLE(7)	HLE(7)/*e*	DLE(7)
Males	65	7.70	56.95%	12.58	93.09%	0.93
	70	5.17	49.19%	9.57	90.98%	0.95
	75	3.07	38.40%	6.98	87.24%	1.02
	80	1.56	25.99%	4.88	81.19%	1.13
	85	0.68	15.09%	3.26	72.20%	1.26
Females	65	9.14	52.84%	15.61	90.21%	1.69
	70	6.24	45.60%	11.99	87.65%	1.69
	75	3.89	37.16%	8.78	83.85%	1.69
	80	2.15	27.85%	6.03	78.03%	1.70
	85	1.03	18.47%	3.87	69.13%	1.73

The columns in Table 4.11 have the following meanings:

- HLE(0) is the life spent free from any disability, measured in years. HLE trend data suggest that this quantity should increase over time.
- HLE(0)/e is the ratio of the time spent free from any disability to the life expectancy e. This should stay roughly constant over time.
- HLE(7) is the time (in years) spent free from severe disability. Here, 'severe disability' means the OPCS categories worse than category 7. HLE trend data suggest that this quantity should increase over time.
- HLE(7)/e is the ratio of the time spent free of severe disability to the future life expectancy. The ratio appears to have been increasing for males. It may have been either increasing or constant for females.
- DLE(7) is the severely disabled life expectancy, i.e. total life expectancy minus HLE(7). The evidence for trends relating to severely disabled life expectancy is unclear, as discussed in Section 4.2.3. Some data indicate that it has been falling and others indicate it has been rising.

The comments made above about trends indicated by the data for severe disabilities relate to the ADL based definition of severe disability. As noted in Section 4.2.3, different definitions of severe disability show different trends. In terms of inability to manage steps and stairs, the time spent disabled has been roughly constant for males but has lengthened for females. In terms of mobility outdoors, there appears to have been deterioration for both males and females.

The following definitions of life expectancies have been used. Let l_x be the number of lives aged x in a life table and let $l_x^{(n)}$ be the number of lives who are healthy or who have a disability of category n or less. This means that $l_x^{(0)} < l_x^{(1)} < \ldots < l_x^{(9)} < l_x^{(10)} = l_x$. Then, we define:

Complete expectation of life: $$\overset{\circ}{e}_x = \frac{1}{l_x} \cdot \left\{ \left(\sum_{y \geq x} l_y \right) - \frac{l_x}{2} \right\},$$

Complete expectation of life spent in disability categories *0-n* inclusive:

$$\text{HLE}(x,n) = \frac{1}{l_x} \cdot \left\{ \left(\sum_{y \geq x} l_y^{(n)} \right) - \frac{l_x^{(n)}}{2} \right\}$$

Complete expectation of life spent in disability categories more severe than *n*:

$$\text{DLE}(x,n) = \mathring{e}_x - \text{HLE}(x,n)$$

It should be noted that in Table 4.11, and the tables which follow, we have used the following abbreviated expressions: 'HLE(0)', 'HLE(7)' and 'e' for HLE(0,x), HLE(7,x) and $\mathring{e}_x$, respectively.

Changes in the total life expectancy directly affect the healthy life expectancies. The changes in total life expectancy between 1986 and 1996 depend only on the mortality rates in those two years and not on the transition models or trends. These life expectancies are given in Table 4.12.

Table 4.12 Life expectancy: comparing 1996 with 1986 (years)

		1986	1996
Males	65	13.51	14.57
	70	10.51	11.37
	75	8.01	8.67
	80	6.01	6.49
	85	4.52	4.85
Females	65	17.30	17.93
	70	13.68	14.27
	75	10.47	11.03
	80	7.73	8.23
	85	5.59	6.04

The transition rate model has six components — total mortality, extra mortality due to disability, the probability of becoming disabled, the severity of new disabilities, the extra likelihood of disabled people deteriorating as compared with healthy people, and improvements in health. Changes in any of these can affect healthy life expectancies.

For overall mortality we have adopted the central projection of the GAD model produced in 1998. We have not explored the effect of varying this. The GAD projection assumes reductions in the rates of mortality and therefore an increase in life expectancy. If there are no changes to disability prevalence rates, this leads to increases in disabled life expectancy. It also leads to a decrease in the ratio of healthy life expectancy to total life expectancy because the disability prevalence rates are highest at the high ages, so the extra years being gained are *ceteris paribus* years of below average health.

If the extra mortality due to disability declines, perhaps as a result of medical breakthroughs or an improvement in care provision for disabled people, then people will live longer once they become disabled. If there are no other changes, in particular no reduction in the number of people becoming disabled and no increase in the probability of people recovering from their disabilities, then people will spend a greater proportion of their lives with a disability. This would cause the disabled life expectancy and especially the severely disabled life expectancy to rise.

If the opposite happened, i.e. improvements in mortality rates applied more to the total population than to the disabled population, the effect on disabled life expectancy would be to tend to reduce it. It seems unlikely that there could be a substantial widening of the difference between the mortality rates of disabled people and the mortality rates of healthy people. This is because there is not very much room for improvement in the mortality rates of healthy people, so that a significant widening would actually require the mortality rate for disabled people to get worse over time.

We have analysed the effect of changes in the level of extra mortality in two models. In one (model E) the gap between the mortality of healthy people and the mortality of severely disabled people widens and in the other (model F) it narrows. The way the trends are implemented is to replace the quantity 0.20 in the equation for *ExtraMort* (x, n) (see Section 4.3.2.2) in year t by the expression $0.20 + \Delta \cdot (t - 1986) / 10$. In model E, $\Delta = 0.02$ and in model F, $\Delta = -0.02$.

Neither of these trends could continue indefinitely. Where Δ is positive it will eventually lead to a worsening of the mortality of disabled people. Where Δ is negative it will eventually lead to the mortality of disabled people being less than that of healthy people.

If fewer people become disabled then this will tend to increase the healthy life expectancy and decrease the disabled life expectancy. We introduced the probability *NewDisab* in Section 4.3.3.2 to represent the probability of becoming disabled. We can change the parameters in this function to effect changes in the probability of becoming disabled.

We use expressions such as '1 in 10' to describe the changes made to *NewDisab*. A rate of 1 in 10 means that the probabilities that apply to someone aged x in year t will also apply to someone aged $x + 1$ in year $t + 10$, to someone aged $x + 2$ in year $t + 20$ and so on (so, for example, the probability that a 71 year-old becomes disabled in 2010 is the same as the probability that a 70 year-old becomes disabled in 2000). Table 4.13 indicates what a rate of 1 in 10 means in terms of percentage reductions in the probability of becoming disabled. The table shows, for example, $R - 1$ (expressed as a percentage) where R is the ratio of the probability of a sixty year old becoming disabled in year t to the probability of a sixty year old becoming disabled in year $t + 1$. The probabilities in year t are determined by the parameters in Table 4.9.

Table 4.13 Annual reduction in the probability of becoming disabled implied by a '1 in 10' change in *NewDisab(x)*

Age	Males	Females
20	0.19%	0.22%
30	0.39%	0.40%
40	0.64%	0.59%
50	0.83%	0.73%
60	0.90%	0.81%
70	0.74%	0.82%
80	0.81%	0.79%
90	0.59%	0.69%
100	0.34%	0.52%

$NewDisab(x)$ is assumed to affect all of the probabilities of deterioration, including the deterioration from one disabled state to another more severely disabled state (see Section 4.3.3.4). Hence a reduction in $NewDisab(x)$ will reduce the number of people who become severely disabled in two ways: fewer people become disabled and fewer of these deteriorate to severe categories.

Trends in the probability of becoming disabled are included in most of the models we consider. The trends are expressed as rates such as '1 in 10' in Table 4.14.

The severity of new disabilities is one of the components of our transition rate model. If the average severity of new disabilities reduces, this should have a greater impact on severely disabled life expectancy than on disabled life expectancies based on a lower disability threshold. As there is some indication from the healthy life expectancy trend data that there has been an increase in the proportion of life spent free from severe disability but no increase in the proportion of life spent free of all disability, this component could help the model to match the observed trends.

We include trends in the *Severity* formula in the same way as in the $NewDisab(x)$ formula. That is we introduce changes at a rate of 1 in 10, say, so that the probabilities that apply to someone aged x in year t apply also to someone aged $x + 1$ in year $t + 10$

and so on (so that the distribution of the severity of new disabilities for 71 year olds in 2010, for example, is the same as the distribution of the severity of new disabilities for 70 year olds in 2000). Trends in *Severity* are included in models G to J.

In Section 4.3.3.4 we introduced a parameter, F, which relates the probability of deterioration for a person who is disabled to the probability that a healthy person becomes disabled. If this parameter decreases then fewer people should become severely disabled. Its effect should therefore be similar to making new disabilities less severe. Trends in this parameter are included in models K to M.

Changes to the parameter F of the deterioration-from-disabled model are incorporated in a different way to changes in *NewDisab(x)* or *Severity*. Since F ought to be at least 1, we have used the following form for changes to F:

$$F(t) = 1 + \left[F(1986) - 1\right] \times \alpha^{t-1986}.$$

The value of F in 1986 is 1.156 for males and 1.183 for females (see Section 4.3.3.4). When $\alpha = 0.99$ the value of $F(1996)$ is 1.141 for males and 1.166 for females, and the value of F(2036) is 1.094 for males and 1.111 for females. The trends in F are given in Table 4.14 in terms of α. (The absence of a trend means $\alpha = 1$.)

A reduction in the relative likelihood of deterioration for a disabled person might be the result of the targeting of health care resources towards people who are already disabled.

The other ingredient in the transition rate model is the probability that disabled people improve slightly. We consider one model, model P, in which the probability of a disabled person improving increases steadily from 10% per year in 1986 to 12% per year in 1996.

We have considered seventeen combinations of trends in the transition rate model. The trends which we have assumed are listed in Table 4.14. The trends assumed in model Q are in a different form to those used for the other models and these are discussed separately in Section 4.4.3. A dash indicates that no trends are included for the component.

Table 4.14. Transition rate trends in the models

Model	*ExtraMort*	*NewDisab*	*Severity*	*Deteriorate*	*Improve*
A	—	—	—	—	—
B	—	1 in 20	—	—	—
C	—	1 in 10	—	—	—
D	—	1 in 5	—	—	—
E	+2%	1 in 10	—	—	—
F	–2%	1 in 10	—	—	—
G	—	—	1 in 10	—	—
H	—	1 in 10	1 in 10	—	—
I	—	1 in 10	1 in 5	—	—
J	—	1 in 10	1 in 2	—	—
K	—	—	—	0.99	—
L	—	1 in 20	—	0.99	—
M	—	1 in 10	—	0.99	—
N	—	1 in 5	—	0.99	—
O	—	1 in 10	—	0.97	—
P	—	1 in 10	—	—	+2%
Q	Down	Down	—	—	—

4.4.2 Healthy life expectancies for three sample models

Table 4.15 shows the healthy life expectancies in 1996 according to models A, C and N. The form of each table is the same as Table 4.11 which relates to healthy life expectancies in 1986. By comparing Table 4.11 with Table 4.15, we can establish how the trend assumptions contained within models A, C and N alter the computed healthy life expectancy figures between 1986 and 1996.

The reason why these three models have been chosen for illustrative purposes is that models A, C and N represent the most pessimistic, the central and the most optimistic assumptions, respectively, out of the sixteen models A to P.

Table 4.15A. Life expectancies in 1996, Model A

		HLE(0)	HLE(0)/*e*	HLE(7)	HLE(7)/*e*	DLE(7)
Males	65	8.10	55.58%	13.48	92.50%	1.09
	70	5.44	47.87%	10.27	90.31%	1.10
	75	3.24	37.30%	7.50	86.50%	1.17
	80	1.64	25.26%	5.22	80.43%	1.27
	85	0.71	14.71%	3.47	71.52%	1.38
Females	65	9.32	51.95%	16.07	89.63%	1.86
	70	6.38	44.71%	12.41	86.97%	1.86
	75	4.00	36.31%	9.16	83.04%	1.87
	80	2.23	27.12%	6.35	77.11%	1.88
	85	1.08	17.94%	4.11	68.16%	1.92

Table 4.15C. Life expectancies in 1996, Model C

		HLE(0)	HLE(0)/*e*	HLE(7)	HLE(7)/*e*	DLE(7)
Males	65	8.24	56.58%	13.54	92.90%	1.04
	70	5.58	49.10%	10.33	90.85%	1.04
	75	3.37	38.88%	7.57	87.28%	1.10
	80	1.72	26.55%	5.28	81.38%	1.21
	85	0.76	15.62%	3.53	72.70%	1.32
Females	65	9.48	52.87%	16.16	90.12%	1.77
	70	6.52	45.71%	12.49	87.58%	1.77
	75	4.12	37.37%	9.24	83.83%	1.78
	80	2.32	28.16%	6.43	78.14%	1.80
	85	1.14	18.87%	4.20	69.51%	1.84

Table 4.15N. Life expectancies in 1996, Model N

		HLE(0)	HLE(0)/*e*	HLE(7)	HLE(7)/*e*	DLE(7)
Males	65	8.38	57.53%	13.61	93.40%	0.96
	70	5.71	50.23%	10.41	91.53%	0.96
	75	3.51	40.43%	7.66	88.30%	1.01
	80	1.81	27.85%	5.37	82.75%	1.12
	85	0.80	16.53%	3.62	74.58%	1.23
Females	65	9.63	53.73%	16.29	90.83%	1.64
	70	6.66	46.66%	12.62	88.49%	1.64
	75	4.23	38.37%	9.38	85.04%	1.65
	80	2.40	29.15%	6.57	79.78%	1.66
	85	1.19	19.74%	4.33	71.77%	1.70

It is worth commenting on the result for the most pessimistic model, model A (which includes no trends in the transition rates other than in the overall mortality). This model results in a reduction in the ratio of life expectancy free of any disability to total life expectancy. This contradicts the findings of Bebbington & Darton (1996). Also, the severely disabled life expectancy increases which appears to contradict the healthy life expectancy data, at least where disability is defined in terms of ADLs (Bone et al (1995)). A full analysis of the results for models A to P can be found in Rickayzen & Walsh (2000).

4.4.3 Consideration of central model used by Nuttall et al (1994)

The final model that we have considered, model Q, includes trends in a different form to those used in any of the other models. In this model we have set the trends to match, as far as is possible, the assumptions used in the central model of Nuttall et al (1994). In their central model, two trend assumptions were chosen:

- The probability of becoming disabled reduces by 0.5% per year
- Improvements in mortality from the disabled states were 50% more rapid than the improvements in mortality incorporated in the GAD population projection model.

It is straightforward to incorporate the first of these two trends. The change of 0.5% per year in the probability of becoming disabled is somewhat lower than that implied by the '1 in 10' rate of change — see Table 4.13.

However, there is one complication with introducing this change. In our model, all deterioration probabilities are linked to the probability of becoming disabled. Nuttall et al (1994) did not include transitions between disabled states, so the authors did not have to decide whether there should be any changes in these transitions to accompany the change in the probability of becoming disabled. There are two options for treating these other types of deterioration:

Option 1: to reduce the other transition probabilities at the same rate as the reductions in the probability of becoming disabled.

Option 2: to make no changes to the other transition probabilities.

The implication of using Option 2 is that, in the future, someone in disability category 1 would be less likely to become severely disabled than would someone who is healthy. This is clearly counter-intuitive. Consequently, we have chosen Option 1.

There are several complications introduced by trying to implement Nuttall et al's trend in the improvement in mortality for disabled people.

One complication is that population mortality does not improve at all ages in every year according to the GAD central population projection. There are several ages and years when the mortality rates are projected to become worse. Some of this is due to AIDS (this is at low ages and is probably not important). But this worsening of experience also occurs sporadically at the oldest ages. It is not clear what would be meant by an 'improvement' of 1.5 times that of an adverse trend. For reasons of convenience, we have just multiplied all changes in q_x by 1.5 whether they were positive or negative.

In implementing projection model Q, there are some situations where it is impossible to include all the constraints being imposed on the projected mortality experience: the GAD model constrains the total number of deaths; the assumption for the improvements in the mortality of disabled people fixes the proportion of disabled people who die; common sense requires that the mortality rate for disabled people cannot be lower than that of healthy people; and no mortality rate can be negative. Problems can arise in two ways. On some occasions, this number of disabled deaths is higher than the total number of deaths. On other occasions, the number of deaths required of healthy people to make up the difference between the two figures is very high. It can mean that the mortality rate of healthy people has to be higher than that of disabled people (and sometimes over 100%).

In order to avoid these problems, we have imposed the following constraints. We require always that the GAD population is matched exactly. We also require that the mortality rate from healthy is never negative and never greater than the mortality rate from disabled. These restrictions means that it is necessary sometimes to stray away from the Nuttall et al (1994) trend.

In our model, the mortality of people with low levels of disability (up to category 5) is no different from the mortality of healthy people. Hence we cannot have the mortality of these people improving at a rate in excess of the improvement in mortality from healthy.

Table 4.16 shows the life expectancies in 1996 according to the model just described, model Q.

Table 4.16. Life expectancies in 1996, Model Q

		HLE(0)	HLE(0)/*e*	HLE(7)	HLE(7)/*e*	DLE(7)
Males	65	8.13	55.76%	13.44	92.27%	1.13
	70	5.47	48.09%	10.24	90.05%	1.13
	75	3.26	37.57%	7.48	86.18%	1.20
	80	1.66	25.52%	5.20	80.03%	1.30
	85	0.72	14.88%	3.45	71.04%	1.41
Females	65	9.36	52.23%	16.05	89.52%	1.88
	70	6.42	45.00%	12.39	86.84%	1.88
	75	4.03	36.56%	9.13	82.83%	1.89
	80	2.24	27.26%	6.32	76.76%	1.91
	85	1.08	17.92%	4.08	67.56%	1.96

This model produces one of the lowest ratios of HLE(0)/*e* of any of the models considered and the highest values of any of the models for DLE(7). The trends are more 'pessimistic' than no trend at all (i.e. model A) in the sense that projections using these trends will lead to a large proportion of lives being severely disabled. The ratio of life expectancy free of any disability to the total life expectancy would have fallen since 1986 according to these trends. This appears to contradict the data.

4.5. Projections Based on the Transition Rate Model

4.5.1 The projection method

In Section 4.4 we described sixteen different sets of trend assumptions which we decided to incorporate in our model (in addition to the special set for model Q). Before presenting the results arising from some of these sets of assumptions, we provide some details of the projection method used.

For the initial population (in 1986) we need to consider the number of men or women in each disability category at each individual age. Such data are not available for individual ages. To provide the individual age populations we use the prevalence rates derived from the transition rate model discussed in Section 4.3. The population is not fully consistent with the OPCS prevalence data but, as Table 4.8 shows, the differences are small.

Twenty-year-olds are treated differently in the projection model from people of other ages. The disability prevalence rates for twenty-year-olds in each year must be included as assumptions. The assumption that we adopt is that these prevalence rates stay constant — we use the OPCS disability prevalence rates for people aged 16 to 19 as the rate appropriate to twenty-year-olds in all years. This assumption is of no great consequence as there are few disabled twenty-year-olds.

The Government Actuary's Department (GAD) population projection includes migration and we include it in our model too in order to reproduce the same total population as the GAD projection. Migration is included in the GAD projection in the following way:

- Half of the migrations are assumed to occur at the start of the year and half at the end.
- Those immigrating at the start of the year are 'exposed' to the same mortality rates as the rest of the population during the year.

We take the same approach. The immigrants at the start of the year are also 'exposed' to the possibility of deterioration or improvement in health.

We assume that the migrants at age x share the same level of disability as the rest of the population at that age. In the GAD central projection the number of migrants per year does not change beyond 1998. The number does vary with age. In total, there is assumed to be a net immigration per year of roughly 19,500 men aged 20 to 59, 1,250 men aged 60 and over and 22,500 women aged 20 to 59. There is assumed to be a net emigration of roughly 1,500 women aged 60 and over each year.

The following equations describe how the population is moved forward. The equations apply separately to males and females.

Let *Lives*(x, t, n) be the number of lives aged x in year t with a category n disability, where category 0 is taken to mean 'healthy' and let *Migrants*(x, t, n) be the corresponding number of immigrants. *Lives*(x, t, n) is determined by the following equation:

$$\begin{aligned} Lives(x,t,n) = & \left[Lives(x-1,t-1,n) + Migrants(x-1,t-1,n)/2\right] \times \\ & \left[1 - Mortality(x-1,t-1,n)\right] \times \\ & \left[1 - DeteriorateFrom(x-1,t-1,n)\right] \times \\ & \left[1 - ImproveFrom(x-1,t-1,n)\right] + \\ & DeteriorateTo(x,t,n) + \\ & ImproveTo(x,t,n) + \\ & Migrants(x,t-1,n)/2 \end{aligned}$$

The quantity $Mortality(x,t,n)$ represents the probability that a person aged x in year t who is in disability category n dies during the next year.

This quantity can be written as:

$$Mortality(x,t,n) = Mortality(x,t,0) + ExtraMort(x,t,n).$$

The extra mortality due to disability is given by a formula (Section 4.3.2.2) and the mortality rate that is independent of disability is set so that the number of deaths in year t at age x agrees with the GAD projection (see Section 4.3.2.1).

The quantity *DeteriorateFrom* represents a probability. It is related to the expressions in Section 4.3.3 in the following way:

$$DeteriorateFrom(x,t,0) = NewDisab(x,t)$$

and

$$DeteriorateFrom(x,t,m) = \sum_{n=m+1}^{10} Deteriorate(x,t,m,n).$$

where *NewDisab* (x, t) and *Deteriorate* (x, t, m, n) are defined in the following way:

NewDisab (x, 1986) is the same as *NewDisab* (x), as defined in Section 4.3.3.2.

NewDisab (x,t) differs from *NewDisab* (x, 1986) in models that include time dependence in the probability of becoming disabled. Similarly, *Deteriorate* (x, 1986, m, n) is the same as *Deteriorate* (x, m, n), which is defined in Section 4.3.3.4. *Deteriorate* (x, t, m, n) differs from this in models that include time dependence in the probability of becoming disabled or in the extra likelihood of disabled people deteriorating.

The quantity *ImproveFrom* represents the probability that a person who survives a year, and does not deteriorate during the year, improves by one disability category during the year. As explained in Section 4.3.4, in the current projection model this probability is set at 0.1 for all ages and disability classes (but not category 0) and both sexes.

The quantity $DeteriorateTo(x,t,n)$ represents the number of persons aged x in year t who made a transition to disability category n from a lower disability category during the last year. The number is given by:

$$DeteriorateTo(x,t,n) = \sum_{m=0}^{n-1} \{ExposedToDet(x-1,t-1,m) \times Deteriorate(x-1,t-1,m,n)\}$$

where

$$ExposedToDet(x,t,n) = \left[Lives(x,t,n) + Migrants(x,t,n)/2\right] \times \left[1 - Mortality(x,t,n)\right].$$

The quantity *ImproveTo* represents the number of persons aged x in year t who made a transition from disability category $n+1$ to n during the last year. The number is given by:

$$ImproveTo(x, t, n) = ExposedToImp(x-1, t-1, n+1) \times 0.1,$$

where

$$\begin{aligned} ExposedToImp(x, t, n) = {} & \left[Lives(x, t, n) + Migrants(x, t, n) / 2\right] \times \\ & \left[1 - Mortality(x, t, n)\right] \times \\ & \left[1 - DeteriorateFrom(x, t, n)\right]. \end{aligned}$$

(The 0.1 is the probability of improvement from one year to the next).

4.5.2 Results

In the Appendix to this chapter, we present the results of the projections of the disabled population for three of the models: model A, model C and model N. As mentioned in Section 4.4.2, these models represent the most pessimistic, the central and the most optimistic trend assumptions of the sixteen models under consideration. The results for eight of the models are shown in Walsh and Rickayzen (2000b).

Since the number of people in each disability category is closely dependent upon the total number of people, we include the totals in Table 4.17. In this table and subsequent ones, the age category 'All' refers to ages 20 and upwards.

For the five years shown in the table, the adult population under 60 peaks in 2016 and the population aged 60-69 peaks in 2026, reflecting the baby boom generation. For higher ages the size of the population is highest in 2036.

The projected results for the different models vary a great deal from one model to another. However, we believe that the assumptions in the models are generally plausible. Also, as discussed in Section 4.2.3 and Section 4.4, it is hard to rule out models by using data on trends because these data point in two different directions — more time spent severely disabled according to some data and less time according to others. This means that it is not possible to be confident that the results of one model are more realistic than those from another unless some other constraints can be provided on the trend assumptions. We are not aware of any other constraints.

Table 4.17 Projected population (thousands) according to the GAD Model

Age Group	Year	Males	Females
20 – 59	1996	16,097	15,801
	2006	16,578	16,188
	2016	16,680	16,204
	2026	15,867	15,430
	2036	15,266	14,906
60 – 69	1996	2,597	2,822
	2006	2,878	3,039
	2016	3,484	3,634
	2026	4,123	4,163
	2036	3,862	3,855
70 – 79	1996	1,800	2,435
	2006	1,882	2,310
	2016	2,204	2,588
	2026	2,708	3,116
	2036	3,278	3,624
80 – 89	1996	659	1,370
	2006	772	1,386
	2016	890	1,395
	2026	1,126	1,683
	2036	1,400	2,037
90+	1996	67	273
	2006	104	340
	2016	139	374
	2026	184	430
	2036	258	577
All	1996	21,220	22,701
	2006	22,214	23,262
	2016	23,398	24,196
	2026	24,008	24,822
	2036	24,064	25,000

We can comment on the results shown in the Appendix to this chapter, as follows:

Model A has no trends and is therefore the most pessimistic model (in the sense that it is likely to project relatively high numbers of severely disabled lives).

The main features of the projection are as follows:

- For adults aged less than 60 the number who are healthy is projected to fall and the number in each of the disability categories is roughly constant.
- For the higher ages, the number of people in all categories of disability is expected to increase, as is the number who are healthy.
- The number of adult males who are severely disabled (categories 8, 9 and 10) is projected to increase by 321,000 from 384,000 in 1996 to 705,000 in 2036. This increase is made up from a decrease of 1,000 males aged less than 60 and increases of 32,000, 76,000, 131,000 and 82,000 at the higher age groups (60 to 69, 70 to 79, 80 to 89 and 90 plus).
- For adult females the projected increase in the number who are severely disabled is 380,000 — from 689,000 to 1,069,000. This comprises a decrease of 3,000 aged under 60 and increases of 28,000, 75,000, 131,000 and 149,000 at the higher age groups. (These numbers differ from those in the Appendix due to rounding.)
- The overall increase in the number severely disabled is larger for females than males in this projection. The difference is entirely due to the 90 plus age category.

Model N has the strongest trends and is therefore the most optimistic model. The main features of the projection are as follows:

- In the 20 to 59 age group the number of males and females in each disability category, as well as the number who are healthy, is projected to fall between 1996 and 2036.
- In the 60 to 69 age group, the number of healthy people is projected to rise while the number of disabled people is expected to fall (this applies to all disability categories). The changes in numbers in each category over time are not monotonic.
- In the 70 to 79 age group, the number of healthy males and the number of males in disability categories 1 to 7 are projected to rise while the number of males in disability categories 8 to 10 is projected to stay roughly constant. For females in this age group, there is projected to be an increase in the number who are healthy and in the number in disability categories 1 to 4 and a decrease in the number in the higher categories.
- In the 80 to 89 age group, the number of healthy males and the number of males in disability categories 1 to 7 are projected to rise while the number of males in disability categories 8 to 10 is projected to fall. For females in this age group, there is projected to be an increase in the number who are healthy and in the number in disability categories 1 to 5 and a fall in the number in the higher categories.
- For males aged 90 and over, there is projected to be an increase in the number healthy and the number in each disability category. For females there is projected to be an increase in the number who are healthy and the number in disability categories 1 to 8 and a decrease in the number in category 10.
- Combining all of these age groups, there is projected to be an increase in the number of males who are healthy or who are in disability categories 1 to 6 and a decrease in the number of males who are more severely disabled. For females there is projected to be an increase in the number who are healthy or who are in disability categories 1 to 4 and a decrease in the number who are more severely disabled.

The projected numbers produced by the various models can be converted to prevalence rates. Also, healthy life expectancies can be calculated for the projections. The results for both these quantities can be found in Walsh and Rickayzen (2000b) for eight of the models.

4.6 Uncertainties

4.6.1 Outline

In Section 4.6 we discuss some of the uncertainties surrounding the projections. We have discussed the uncertainties due to ambiguous trend data in other sections (Sections 4.2.3, 4.4 and 4.5). Apart from the difficulty of identifying the most likely trends, the most important uncertainty is the subject of Section 4.6.2. This uncertainty relates to the fact that published data for the population over the age of 80 have not been sub-divided into age bands. In Section 4.6.3, we discuss the assumption made that the population is stationary. Section 4.6.4 compares our results with those obtained by Nutall et al (1994).

A fuller discussion of the uncertainties within the model can be found in Walsh and Rickayzen (2000b).

4.6.2 The population over the age of 80

Table 4.17 shows that the number of people over the age of 80 is expected to rise greatly over the next 35 years. The level of disabilities amongst the elderly population will be absolutely critical to the need for long-term care provision and hence any weakness in the model relating to this age group is important.

We have determined the parameters in our transition rate model by using the OPCS disability survey prevalence rates. The oldest age group for which disability prevalence rates are known covers everyone aged 80 and above. The original data for the OPCS survey do include information on the exact ages of the people who were questioned in the survey, so it is possible to gain more detailed information on the disabilities of the elderly population. We believe that this kind of analysis has been carried out but, as far as we know, has not been published.

Since the prevalence rates in the most severe disability categories rise extremely rapidly between people in their seventies and people older than that (see Table 4.2), the prevalence rates may well be very much higher for people in their nineties than for people in their eighties. Our transition rate model does produce rapidly increasing prevalence rates of severe disabilities and, therefore, the results produced by the model are plausible. The results are not, however, well constrained. If the rise in the prevalence rates of severe disability were to tail off at ages around, say, 90 this would have virtually no effect on the reported prevalence rates in the OPCS survey (because there were so few people over 90 at the time of the survey).

There is one graph in the OPCS disability survey report that provides extra information on disabilities at ages over 80 (Martin et al, 1988, Fig 3.3). The following numbers have been obtained from that graph by inspection. Note that the figures are for males and females combined, and that there will be some measurement error in the numbers.

Table 4.18. OPCS Survey disability prevalence rates (per 1,000) in five-year age bands

Age	OPCS Disability Categories					
	Healthy	1 & 2	3 & 4	5 & 6	7 & 8	9 & 10
70 – 74	653	133	93	53	40	27
75 – 79	520	153	107	107	67	47
80 – 84	347	173	147	133	113	87
85+	153	133	120	187	193	213

These numbers show the very rapid increase in severe disabilities. The following pair of tables show the rates derived from our transition rate model for the same age bands (Table 4.19a) and the difference between the rates in the data and the model (Table 4.19b).

Since the figures in Table 4.18 have been obtained fairly crudely, small differences between the data and the model are inevitable.

The difference in the prevalence of the 'healthy' category in the two highest age groups is unexpected. There is no difference between the data and model prevalence rates in the 80+ category in Table 4.8a or Table 4.8b. The only way in which the model can overshoot on both subgroups (80 to 84 and 85+) is if the age structure of the 1986 population that we use is quite different from that used in the OPCS report. Table 4.8 also shows that the prevalence rates of category 1 and 2 disabilities is the same for the model as it is for the data for ages 80 and over. It is strange that the model undershoots the prevalence rates in both of the age subgroups (80 to 84 and 85+).

The increase in the prevalence rates between the 80 to 84 age group and the 85 and over age group in the two severest disability categories is not quite as steep in our model as is indicated by the data.

Note that the changes in the model prevalence rates between ages 80 to 84 and ages 85 and over, whether they are increases or decreases, are smaller in all six categories shown in the tables than the changes shown by the data. This suggests that the prevalence rates produced by the transition rate model are not as sensitive to age as they should be. If this is true, our projections might be underestimating the number of severely disabled people in future years.

Table 4.19a Model prevalence rates (per 1,000) in five-year age bands

Age	OPCS Disability Categories					
	Healthy	1 & 2	3 & 4	5 & 6	7 & 8	9 & 10
70 – 74	638	127	90	74	45	25
75 – 79	513	151	115	104	72	45
80 – 84	365	158	136	142	115	83
85+	188	123	127	171	193	199

Table 4.19b Difference in prevalence rates (per 1,000): *Data – Model*

Age	OPCS Disability Categories					
	Healthy	1 & 2	3 & 4	5 & 6	7 & 8	9 & 10
70 – 74	15	6	3	-20	-5	2
75 – 79	7	2	-9	3	-5	2
80 – 84	-18	15	11	-9	-2	4
85+	-34	11	-7	15	1	14

4.6.3 The stationary population assumption

The transition rates that we use in the population projections are derived from the prevalence rate data. In doing this, an assumption had to be made regarding the underlying population structure. The assumption which we have made is that the population is stationary.

This assumption is clearly not valid. The 1986 prevalence rates would have depended on mortality, deterioration and improvement rates in earlier years. The mortality rates had certainly been changing in those years and the other rates may have been changing as well. The projections show that future prevalence rates are strongly dependent on future changes in transition rates and the same would have been true in the past.

The stationary population assumption was made for two reasons: it is easier to derive transition rates under this assumption than under any other assumption; and we do not have any evidence of what sort of changes had been taking place regarding deterioration and improvement in disability. We would, therefore, not have been any more confident about any transition rates derived from assumptions about past changes than those derived from an assumption of no change.

Projection model A provides some defence for the stationary population assumption. In model A, mortality changes over time but there are no other trends. We have found that disability prevalence rates at ages over 60 change very little for this model. This suggests that the derivation of transition rates may not be very sensitive to any changes in mortality rates prior to 1986.

By using a stationary population assumption, the derived transition rates are effectively averages of transition rates which had applied in preceding years; i.e. they are out of date. If, as is likely, the probabilities of deterioration had been decreasing, the rates of deterioration derived from the prevalence rate data would be too high. The effect of this would be that the transition rates we use in the projections are out of date and therefore pessimistic. The assumption may, therefore, mean that the models overestimate the number of people who will be disabled.

4.6.4 A comparison with earlier projections

In one of our projections, model Q, we included trends which attempt to replicate those used in the central projection of Nuttall et al (1994). In Table 4.20 we compare the output from our projection and Nuttall et al's projection. The numbers for the Nuttall et al projection are taken from table 3 of Nuttall et al (1994). Four categories of care need are considered by Nuttall et al. These categories correspond directly with the OPCS disability categories. 'Low' means disability categories 1 and 2. 'Moderate' means OPCS disability categories 3, 4 and 5. 'Regular' means disability categories 6, 7 and 8. 'Continuous' means disability categories 9 and 10. The Nuttall et al (1994) projections stopped in 2031.

Table 4.20 Projected number of disabled adults in 2031 (thousand)

Care Need	Nuttall et al (1994)	Our Model Q
Low	2,556	2,646
Moderate	2,745	2,931
Regular	2,058	2,312
Continuous	1,184	1,064
Total	8,543	8,952

The projected numbers differ between the two projections. In the most important category, continuous care, our model has 10% fewer people than the earlier projection model produced. There are many possible reasons for the differences:

- The underlying population model is different. We used the 1996 GAD projection while Nuttall et al (1994) used the 1991 version. According to the 1996 projection there will be 49.0 million adults (over 20) in 2031, 18.7 million of whom will be over 60, and according to the 1991 projection there will be 47.8 million adults in 2031, 18.1 million of whom will be over 60.
- We have separate models for males and females, whereas Nuttall et al (1994) used a single model.
- We used a full population projection model (including migration) which exactly reproduces the GAD projection. We believe that Nuttall et al (1994) used a simpler projection.
- There are more transitions in our model. Nuttall et al (1994) did not include movements between disabled states.
- There are more categories in our model. Nuttall et al (1994) used four categories of disability, whereas we used 10.
- Our projections are started in 1986 rather than in 1991.
- There will be a difference in the 'graduation' of the initial prevalence rate data. Although we are using the same data as Nuttall et al (1994) for these rates, the data are given in ten year age bands. Hence, the prevalence rates for individual ages may disagree.
- We were unable to implement the Nuttall et al (1994) trend assumptions exactly (see Section 4.4.3).
- In our model, the mortality rates for people in disability categories 1 to 5 are the same as the rates for healthy people of the same age. In the Nuttall et al (1994) projections, the mortality rates for these disabled people were higher than for healthy people.

As well as making a central projection, Nuttall et al (1994) ran six other models with different trend assumptions. These assumptions led to a range of results, which we now compare with the range produced by a subset of the models that were described in Section 4.4.1.

There were two types of trends in the Nuttall et al (1994) projections. One was in terms of the annual decrease in mortality of people with disabilities compared with the overall decrease in mortality according to the GAD population model. The other trend was in the annual decrease in the probability of becoming disabled. The assumed trends for the seven models considered by Nuttall et al (1994) are listed in Table 4.21. The mortality improvements are given as multiples of the overall mortality improvements. The decrease in the onset of disability is an annual quantity. (Basis A is their central projection, which is the one we have approximated by our model Q.)

Table 4.21 Trends used by Nuttall et al (1994)

Basis	Decrease in disabled mortality	Decrease in onset of Disability
A	1.5 times	0.5%
B	None	0.5%
C	1 time	0.5%
D	2 times	0.5%
E	1.5 times	0.0%
F	1.5 times	1.0%
G	1.5 times	2.0%

The output from the projections based on these assumptions is given in Table 4.22a. These numbers are given in Appendix D of Nuttall et al (1994). The values are the number of disabled per 1,000 of population in 2031 (both the numerator and denominator include only adults). The categories are cumulative (1 to 10, 3 to 10, 6 to 10 and 9 to 10). In Table 4.22b the corresponding numbers are given for nine of the projection models considered in Section 4.4.1 of this paper.

Table 4.22a Number of disabled (per thousand) in 2031 in Nuttall et al (1994)

Category	Basis A	B	C	D	E	F	G
1 to 10	185	155	176	193	203	169	140
3 to 10	130	104	121	137	144	117	95
6 to 10	70	51	64	76	80	62	48
9 to 10	26	16	22	29	30	22	16

Table 4.22b Number disabled (per thousand) in 2031 in our projections

Category	Model A	B	C	D	K	L	M	N	Q
1 to 10	196	179	163	135	197	179	163	136	183
3 to 10	135	121	109	88	134	120	108	88	129
6 to 10	67	58	51	38	64	56	48	37	69
9 to 10	21	17	15	10	18	15	13	9	22

The basis that gives the highest disability prevalence rates amongst those reported in Nuttall et al (1994) is basis E. That model projects a higher number of disabled people than do any of our models. The reason that there are so many disabled people in the Nuttall et al (1994) basis E projection is that there is no reduction in the onset of disability but people live longer when they become disabled.

The basis that gives the lowest disability prevalence rates amongst those reported in Nuttall et al (1994) is basis G, which includes a larger reduction of the probability of becoming disabled than the other bases. However, some of our models lead to even lower prevalence rates. Our models D and N produce lower rates for all of the cumulative disability categories. Models C, L and M also produce a lower rate in disability categories 9 and 10.

Our most pessimistic model in terms of disabilities in categories 9 or 10 is model Q. Several of the bases considered by Nuttall et al (1994) lead to higher projected proportions of severely disabled people. None of our models projects as many people needing continuous care as Nuttall et al (1994) find with their central projection.

The differences between our most pessimistic projected prevalence rates and our most optimistic are similar to the differences between the most pessimistic and optimistic of the Nuttall et al (1994) projections. For disability categories 1 to 10 the gap between the highest and lowest prevalence rates is 61 (per 1,000) among our models and 63 among the Nuttall et al (1994) projections. For categories 3 to 10 the two differences are 47 and 49. For categories 6 to 10 they are 32 and 32. For categories 9 to 10 they are 13 and 14.

In summary, there is much overlap of the ranges of disability prevalence rates in the two sets of models. Our models tend to produce lower prevalence rates. This is especially so in the severe disability categories.

4.7 Conclusions

We draw two main conclusions from the results projected in this paper. The first of these is a cause for optimism. However, the second conclusion means that the latter should be treated with caution:

- Although there will be a large increase in the number of elderly people in the UK the implications for the number of people needing long-term care will be ameliorated to some extent by a reduction in the proportion of older people who are severely disabled.
- The data that have shown changes in the prevalence of severe disabilities among the elderly do not present a clear picture of what has been happening in the recent past. As a result of this lack of clarity, there is a large amount of uncertainty surrounding the results of our projections and it is quite plausible that the first conclusion is wrong.

Fundamentally, the number of people with severe disabilities in 35 years' time will depend on what happens to the probabilities of deterioration and improvement in health and on what happens to the mortality rate of people with severe disabilities. These influences are all included in our projection model. We have tried to make sense of the data on healthy life expectancies as measured at intervals over the past two decades in order to input appropriate trends to the model. The data, however, do not provide a unique message. It is possible to take from them the view that people are spending less time, on average, with severe disabilities. On the other hand, the opposite view can also be taken.

Although we are not experts at interpreting healthy life expectancy data, we have consulted people who are and have read what has been published in this area regarding British data. The conclusions of these researchers, who are more familiar with life expectancy data than we are, seems to be that the situation is improving. At worst, people are spending the same proportion of their lives severely disabled — so gains in life expectancy are split between time spent healthy and time spent in poor health. At best, the trend over the last twenty years has been for the increase in life expectancy to lead to an equal increase in healthy life expectancy and no change in disabled life expectancy.

If we choose assumptions for trends that reflect this optimistic view, the result is that disability prevalence rates fall and consequently the disabled population does not rise in line with the total number of elderly people, and may even fall.

The range for the projected number of severely disabled adults in 2036 (according to one particular definition of severity) is between 0.8 million and 1.8 million for the models we have run. Moreover, some more extreme models may also be compatible with existing trends data. Such a wide funnel of doubt is inevitable when projecting forward for 35 years on the basis of inconclusive data.

There are many other aspects of the projection model which could be refined or even overhauled. However, we do not feel that the model itself is an important source of uncertainty. Indeed, apart from the doubts over trends, the most important shortcoming of the projections is probably the lack of data on the prevalence rates of disability for people over the age of 85. If such data, which do exist, are published it may be possible to improve the reliability of the output from the projection model.

Another theme which underpins the work described in this paper is the lack of reliable data. For example, we described in Section 3 how we derived the transition rates for our multiple state model from prevalence rate data applicable to 1985 and 1986. Future research in the area of long-term care would be greatly assisted if regular national surveys were undertaken which enabled longitudinal data to be collected (i.e. an appropriate cross section of the UK population could be tracked at each survey date so that transition rates could be computed directly from the data). Ideally, the surveys should be undertaken at least biennially since we are most interested in calculating probabilities of transition from one year to the next.

Finally, we have projected, under various assumptions, the disabled population over the next 35 years. The next step would be to assess the care needs of this population, being careful to distinguish between formal and informal provision.

Appendix to Chapter 4

Table A4.1(M). Number of males with disabilities (thousands), Model A

Age Group	Year	OPCS Disability Category										
		Able	1	2	3	4	5	6	7	8	9	10
20–59	1996	15,123	255	143	121	111	102	70	63	50	41	18
	2006	15,502	283	160	135	123	112	77	69	54	44	19
	2016	15,568	294	166	140	127	115	79	71	55	46	19
	2026	14,809	280	158	133	121	110	75	68	53	44	18
	2036	14,271	262	148	125	114	103	71	64	50	41	17
60–69	1996	1,987	167	97	79	70	62	38	36	28	24	9
	2006	2,209	183	107	87	76	68	42	39	30	26	10
	2016	2,657	226	132	108	95	84	52	49	38	33	12
	2026	3,165	263	153	125	109	97	60	56	43	38	14
	2036	2,936	253	147	121	106	94	58	54	42	37	14
70–79	1996	1,077	177	109	93	84	78	48	47	38	35	14
	2006	1,114	186	116	98	90	83	51	50	41	38	15
	2016	1,310	217	135	114	104	96	60	59	47	44	18
	2026	1,583	271	168	143	131	121	76	75	60	57	23
	2036	1,946	323	200	171	155	143	89	87	71	66	26
80–89	1996	194	76	53	50	50	53	37	42	40	44	20
	2006	228	89	62	58	59	63	43	50	46	51	23
	2016	257	102	72	67	68	72	51	58	55	60	27
	2026	327	129	91	85	86	91	64	73	69	76	35
	2036	392	158	112	106	108	115	81	94	89	99	45
90+	1996	5	4	4	4	5	6	5	7	8	11	6
	2006	8	6	6	6	7	9	8	11	13	18	10
	2016	10	9	8	8	10	13	11	15	18	25	14
	2026	13	11	10	11	13	16	14	20	24	33	19
	2036	19	15	14	15	18	23	20	28	34	47	27
All	1996	18,387	679	406	348	321	301	199	195	163	155	66
	2006	19,061	748	450	385	355	334	221	219	184	178	77
	2016	19,803	848	511	437	404	381	252	251	213	208	90
	2026	19,897	952	579	497	460	436	288	291	249	249	109
	2036	19,564	1,012	621	536	500	479	319	327	285	291	130

Table A4.1(F). Number of females with disabilities (thousands), Model A

Age Group	Year	Able	OPCS Disability Category 1	2	3	4	5	6	7	8	9	10
20–59	1996	14,693	212	154	151	138	142	104	86	56	45	22
	2006	14,980	232	170	166	150	154	112	93	59	48	24
	2016	14,975	238	173	169	152	157	114	94	60	49	24
	2026	14,264	225	164	160	145	149	108	89	57	46	23
	2036	13,799	213	155	152	137	142	103	85	54	44	22
60–69	1996	2,149	132	99	94	82	86	60	51	29	26	15
	2006	2,323	140	105	100	87	91	64	54	31	27	16
	2016	2,761	170	128	122	106	111	78	67	38	34	20
	2026	3,182	192	144	137	119	125	88	75	43	37	22
	2036	2,922	182	136	130	113	119	84	71	41	36	21
70–79	1996	1,406	168	131	130	119	135	102	95	58	56	35
	2006	1,324	160	125	124	113	130	98	92	56	54	34
	2016	1,493	179	139	138	126	144	108	102	62	60	38
	2026	1,772	217	169	168	154	177	134	126	77	75	47
	2036	2,089	250	195	194	176	202	152	143	87	84	53
80–89	1996	441	100	83	90	89	117	102	113	80	89	65
	2006	448	101	85	91	90	118	103	114	81	90	65
	2016	446	102	85	91	91	119	105	116	83	92	67
	2026	541	122	102	110	109	143	126	139	99	110	81
	2036	636	147	123	132	132	175	154	172	124	139	102
90+	1996	31	12	11	13	14	22	24	34	31	43	39
	2006	35	14	12	15	16	26	29	42	39	57	54
	2016	39	15	14	16	18	29	32	46	43	63	60
	2026	44	17	15	18	20	33	36	52	50	73	70
	2036	58	23	21	24	27	43	49	70	67	99	95
All	1996	18,719	624	477	477	441	503	392	379	254	259	176
	2006	19,111	648	496	495	457	520	407	394	267	276	192
	2016	19,713	704	538	536	493	560	437	424	287	297	208
	2026	19,803	774	594	593	547	627	492	481	327	342	243
	2036	19,504	815	630	632	586	680	542	541	374	402	293

Table A4.2(M). Number of males with disabilities (thousands), Model C

Age Group	Year	OPCS Disability Category										
		Able	1	2	3	4	5	6	7	8	9	10
20–59	1996	15,141	249	140	119	110	100	69	62	49	40	17
	2006	15,562	266	151	128	117	106	73	65	50	41	17
	2016	15,675	264	151	128	116	105	72	64	49	40	17
	2026	14,949	240	137	117	107	96	66	58	45	37	16
	2036	14,432	217	124	106	97	88	60	53	41	33	14
60–69	1996	2,002	163	95	78	68	60	37	35	27	23	9
	2006	2,261	170	100	81	71	62	38	35	27	23	9
	2016	2,768	199	116	95	83	72	44	40	31	27	10
	2026	3,350	216	127	103	89	77	47	43	32	28	10
	2036	3,164	196	115	93	81	70	42	38	29	25	9
70–79	1996	1,097	174	108	91	83	75	47	45	36	33	13
	2006	1,170	177	110	93	84	76	47	45	35	32	13
	2016	1,417	199	123	104	93	83	51	48	38	34	13
	2026	1,777	238	147	123	110	98	60	56	44	40	15
	2036	2,228	273	167	139	124	110	66	62	48	43	17
80–89	1996	204	76	53	50	50	52	36	41	38	41	18
	2006	263	90	62	57	57	58	40	44	40	43	19
	2016	327	103	70	64	63	64	43	47	43	46	20
	2026	455	128	86	78	76	76	50	54	48	51	22
	2036	599	157	105	94	91	90	59	64	56	59	26
90+	1996	6	5	4	4	5	6	5	7	8	11	6
	2006	10	7	6	7	8	10	8	11	12	16	9
	2016	16	11	9	9	11	13	11	14	16	20	11
	2026	24	15	12	13	14	17	14	18	20	25	13
	2036	39	22	18	18	20	24	18	24	26	33	17
All	1996	18,449	667	400	343	316	295	194	189	156	148	63
	2006	19,265	711	429	367	337	313	205	200	165	156	66
	2016	20,203	775	469	400	366	338	220	214	176	167	71
	2026	20,554	838	509	434	396	365	236	229	189	181	77
	2036	20,462	865	528	451	412	381	246	241	200	193	83

Table A4.2(F). Number of females with disabilities (thousands), Model C

Age Group	Year		OPCS Disability Category									
		Able	1	2	3	4	5	6	7	8	9	10
20–59	1996	14,709	208	152	149	136	140	102	84	55	44	22
	2006	15,037	221	163	159	144	147	107	87	56	45	22
	2016	15,076	218	160	156	142	144	104	85	54	43	21
	2026	14,398	199	147	143	130	132	95	77	50	40	19
	2036	13,957	183	135	132	120	122	88	71	46	36	18
60–69	1996	2,165	129	97	92	80	84	59	49	28	25	14
	2006	2,374	132	100	94	82	84	58	49	28	24	14
	2016	2,869	153	116	109	95	97	67	55	31	27	15
	2026	3,353	164	124	116	101	102	70	57	32	28	16
	2036	3,130	148	112	105	90	91	62	51	29	24	14
70–79	1996	1,429	167	130	129	117	133	99	91	56	53	33
	2006	1,390	155	120	119	108	121	89	82	50	47	29
	2016	1,623	166	129	127	114	126	92	83	50	47	29
	2026	1,999	195	151	148	133	146	106	95	57	53	33
	2036	2,424	215	166	161	144	156	112	99	59	54	33
80–89	1996	456	101	84	90	90	116	101	110	77	85	61
	2006	496	103	86	91	90	114	97	104	72	78	55
	2016	532	104	86	91	89	112	94	98	68	72	51
	2026	691	125	103	107	104	129	106	110	74	79	55
	2036	875	150	123	128	123	151	123	127	85	90	62
90+	1996	33	12	11	13	15	23	24	33	30	41	37
	2006	43	16	14	16	18	28	30	41	37	51	46
	2016	54	18	17	19	21	31	33	44	38	52	46
	2026	71	23	20	23	25	36	37	49	42	56	49
	2036	108	32	28	32	34	49	49	63	53	69	60
All	1996	18,792	617	474	473	438	495	384	368	246	247	166
	2006	19,341	627	482	480	442	495	382	362	242	244	166
	2016	20,153	660	508	502	461	511	389	366	242	242	163
	2026	20,512	706	544	538	493	545	414	388	255	255	171
	2036	20,494	727	563	558	512	569	434	410	271	274	186

Table A4.3(M). Number of males with disabilities (thousands), Model N

Age Group	Year		OPCS Disability Category									
		Able	1	2	3	4	5	6	7	8	9	10
20–59	1996	15,158	244	138	118	108	99	68	61	47	39	17
	2006	15,614	251	144	123	112	101	69	61	47	38	16
	2016	15,760	240	138	118	108	97	66	58	44	35	15
	2026	15,050	212	123	105	96	86	59	51	39	31	14
	2036	14,539	187	109	94	86	77	53	46	35	28	12
60–69	1996	2,017	160	94	77	67	59	36	33	25	22	8
	2006	2,310	158	93	76	66	57	35	31	24	20	7
	2016	2,867	174	103	84	72	62	37	33	24	21	8
	2026	3,500	178	105	86	73	62	37	32	24	20	7
	2036	3,334	152	90	73	62	52	31	26	19	16	6
70–79	1996	1,114	171	106	90	82	74	45	43	34	30	12
	2006	1,214	170	106	90	80	71	43	40	31	27	10
	2016	1,500	184	114	95	85	74	44	40	31	27	10
	2026	1,917	211	129	108	95	82	49	44	34	29	11
	2036	2,445	228	138	114	99	85	50	45	34	30	11
80–89	1996	213	77	54	50	50	52	35	39	35	38	16
	2006	299	90	62	57	55	55	36	38	33	34	14
	2016	399	100	68	61	58	56	36	37	31	31	13
	2026	580	120	80	71	66	61	38	37	30	29	12
	2036	786	142	94	82	75	68	41	40	32	30	12
90+	1996	6	5	4	5	5	7	5	7	8	10	5
	2006	12	8	7	8	9	10	8	10	11	13	7
	2016	22	13	10	11	12	14	10	13	13	15	7
	2026	38	18	15	15	16	18	13	15	14	16	7
	2036	68	27	21	20	21	23	16	18	17	18	8
All	1996	18,507	657	396	339	312	290	190	183	150	139	58
	2006	19,449	678	412	353	322	295	191	181	146	132	55
	2016	20,548	711	434	370	335	302	194	180	143	128	53
	2026	21,086	738	452	384	346	309	195	180	141	126	51
	2036	21,172	735	451	383	344	305	191	175	137	122	49

Table A4.3(F). Number of females with disabilities (thousands), Model N

Age Group	Year		OPCS Disability Category									
		Able	1	2	3	4	5	6	7	8	9	10
20–59	1996	14,725	205	150	147	135	138	101	82	54	43	21
	2006	15,087	212	157	153	139	141	102	82	53	42	20
	2016	15,158	202	150	147	134	134	96	77	49	39	19
	2026	14,499	180	134	131	120	120	85	68	44	34	16
	2036	14,066	161	120	118	109	108	77	61	40	31	14
60–69	1996	2,180	127	96	91	79	82	57	47	27	23	13
	2006	2,420	125	95	90	78	79	53	43	24	20	11
	2016	2,959	139	106	99	86	85	57	46	25	21	12
	2026	3,486	142	108	101	87	84	56	44	24	20	11
	2036	3,282	122	93	87	74	71	46	36	20	16	9
70–79	1996	1,451	165	129	128	117	130	96	87	53	49	30
	2006	1,451	150	117	116	105	114	82	72	42	39	23
	2016	1,736	155	122	118	106	112	78	67	39	35	21
	2026	2,187	174	136	132	117	121	83	70	40	36	21
	2036	2,687	182	142	136	119	120	81	67	38	33	19
80–89	1996	471	102	86	92	92	117	100	106	73	78	54
	2006	541	105	89	94	93	113	92	93	61	63	42
	2016	611	105	89	93	90	106	83	81	52	51	34
	2026	831	124	103	106	102	116	88	82	51	49	32
	2036	1,094	144	119	122	115	128	94	86	53	50	32
90+	1996	35	13	12	14	16	24	25	33	29	38	32
	2006	51	18	16	19	22	32	32	40	34	42	34
	2016	71	22	21	24	26	37	35	41	32	37	29
	2026	101	28	26	29	32	42	38	42	31	35	26
	2036	165	40	36	41	43	55	47	50	35	38	27
All	1996	18,862	612	472	472	438	492	379	356	236	231	151
	2006	19,549	609	474	472	437	479	361	331	215	205	131
	2016	20,535	624	487	481	442	474	349	311	197	183	113
	2026	21,103	647	507	499	456	483	350	306	191	174	106
	2036	21,294	649	511	503	459	482	346	300	185	169	102

5. AN ANALYSIS OF DISABILITY-LINKED ANNUITIES

5.1 Introduction

As in many other developed countries around the world, the UK population has been ageing over recent decades, and is expected to continue to do so in the future. For example, the proportion of the population over age 65 is expected to increase between 2000 and 2030 from 16.0% to 23.1% (Karlsson et al, 2005). With an ageing population, an increasing number of people need to consider how they will meet their standard of living requirements in retirement both in good health, and in poor health when they might require long term care (LTC).

In this paper we investigate a special type of annuity where the annuity is issued to a policyholder who is in reasonable health at the outset; however, if the policyholder subsequently becomes disabled then the annuity payments are increased to a higher level depending on the level of disability.

Throughout this paper we will refer to such an annuity as a “disability-linked annuity”. Clearly, the definition of “disabled” is important for this product and will be discussed in detail in the body of the paper. Since the highest level of annuity payment will be reached when the individual requires LTC, the annuity is sometimes referred to as being an integrated life annuity and LTC insurance product. For a fuller account of LTC, and of LTC insurance in general, the reader is referred to Booth et al (2005).

This class of annuity business has been considered by Warshawsky et al (2002), Murtaugh et al (2001), Spillman et al (2003), Ferri and Olivieri (2000), Olivieri and Pitacco (2001), Pitacco (2004) and Warshawsky (2007). The salient issues arising from these papers are described briefly below.

Warshawsky et al (2002) identified the following positive features of a disability-linked annuity:

- The pooling of the longevity risk (associated with the life annuity) with the morbidity risk (associated with the LTC insurance component) should result in a lower overall risk for the insurer to manage since the two risks are working in opposite directions. In other words, the longer an individual stays healthy and receives the standard life annuity, the lower the present value of the LTC annuity enhancement; whereas, the earlier the individual becomes severely disabled (thereby triggering the LTC annuity enhancement), the shorter the overall term of the annuity since the individual's life expectancy is likely to be compromised by the illness. This is explored in more detail in Section 5.6.2.

- The annuity enhancement would help to meet the additional care costs associated with severe disability and thereby support any bequest motive for the individual's children.

- The annuity is more flexible than a standard annuity since it increases to help meet LTC costs when required. Indeed, such an annuity could enable the purchaser to continue to live, and receive care, in their own home rather than having to move into an institution such as a residential home. The fact that large unexpected medical expenses can be met, to a certain extent, by the annuity enhancement helps to meet a major concern of retirees that they will have insufficient savings to defray such costs in the future (Panis, 2003)

- From a marketing perspective, the annuity should be attractive to consumers since its two components can be presented in a positive fashion: a life annuity payable whilst the individual is healthy and an enhancement to this annuity payable should the individual suffer very poor health. Stand alone LTC policies have tended not to sell well in the past because, by definition, they force prospective purchasers to dwell on the unsavoury prospect of requiring LTC at some point in the future.

The first advantage listed above suggests that a combined life annuity and LTC product would require less stringent underwriting procedures than those required if the two components had been offered as stand alone products. This is because the opposing longevity and morbidity risks are being pooled together.

Murtaugh et al (2001) explored this point using data taken from the USA. They compared the potential demand for the combined product, after minimum underwriting had removed the applicants unsuitable for this product (which were those people who would immediately be able to claim the disability benefit), with the potential demand for the two stand alone products, after current underwriting procedures had removed the applicants unsuitable for those products. They found that the relaxation in underwriting procedures would enable 98% of 65 year olds to be considered for the integrated product as opposed to 77% for the stand alone products. In addition, they found that the increase in the size of the pool would lead to a reduction in premium of approximately 3.5% for the integrated product compared to the total premium payable in respect of the two stand alone products.

The authors pointed out that, with such a small proportion of people being excluded from cover for underwriting reasons, the idea of developing a mandatory State scheme which offers such an integrated product to every citizen regardless of current health status becomes quite feasible (Spillman et al, 2003).

Ferri and Olivier (2000) considered the risks to which the providers of such an annuity would be exposed. In particular, they analysed the risks presented by demographic changes (ie future trends in mortality and morbidity rates). Olivieri and Pitacco (2001) extended the analysis to assess the minimum solvency reserve required with this type of product, and discussed the ways in which stop-loss reinsurance could be used to reduce the minimum solvency margin. Pitacco (2004) demonstrated the way in which, as the number of policyholders increases, the process risk (ie the risk from random fluctuations within the model) reduces but the systematic risk (ie the longevity risk emanating from fundamental changes in future mortality and morbidity rates) does not.

Warshawsky (2007) makes a strong case for introducing a disability-linked annuity product in the USA. In particular, he argues that some of the burden of the LTC liability could successfully be transferred from the government welfare programs, Medicare and Medicaid, to individuals and occupational pension schemes if there were sufficient incentives available for the latter groups to use such products.

A type of disability-linked annuity was launched in the early 1990's by an insurance company in the UK. However, this product had to be withdrawn from the market after an objection from HM Revenue & Customs (the UK tax authority formerly known as the Inland Revenue). They argued that such a product could not be treated as a "pension" (with the all important accompanying tax concessions) since it was a change in health status (ie the individual becoming disabled) which caused the annuity to increase.

Other insurance companies have launched similar products in the UK over the last few years which were not offered within the company's pension business. However, without the tax concessions for which a pension product would be eligible, these types of disability-linked annuities have tended to be unattractive to consumers. The take-up rate, therefore, for such annuities has been very low. The tax position, however, appears to have improved as a result of the simplification of the tax regime post A-Day (6^{th} April 2006). This is considered in more detail in Section 5.3.8.

It is interesting to note that this type of annuity was alluded to in Chapter 12 of the final report prepared by the Wanless Social Care Review team in March 2006 (Wanless Social Care Review, 2006). The report was a result of a wide ranging review of social care arrangements in England prepared for the health and social care think tank, the Kings Fund.

In this paper, we will examine disability-linked annuities by considering, by way of illustration, a particular class of disability-linked annuity where the annuity is increased to a certain level once the annuitant has reached a particular state of disability, and a higher level once the annuitant has reached a more severe state of disability. In other words, there are two levels of annuity enhancement due to onset of disability.

The main objective of the research is to consider the characteristics of disability-linked annuities under a range of assumptions (eg from pessimistic to optimistic morbidity assumptions, and from narrow to wide definitions of disability). In particular, under the different assumptions, we wish to compare the single premium required for the disability-linked annuity with the single premium in respect of the corresponding traditional whole life annuity which is not enhanced upon disability. In addition, we wish to consider the expected times spent receiving each level of annuity enhancement and the probabilities of reaching each of the associated states of disability.

This paper has two main intentions from a practical perspective: firstly, to help inform the debate on annuities which is currently occurring in the UK partly prompted by the Government's consultation paper "Modernising Annuities" (Inland Revenue, 2002); and secondly, to consider whether disability-linked annuities offer a viable alternative to traditional means of obtaining LTC insurance cover.

Our main conclusion is that disability-linked annuities have a number of favourable qualities from both the insurer's and the policyholder's point of view.

The longevity risks and morbidity risks contained within the combined life annuity and LTC product act in opposite directions which should minimise the overall risk to the insurer. In consequence, the insurer can be more flexible over the underwriting requirements and disability definitions used. In addition, the premiums appear to compare very favourably with those offered for traditional life annuities when the additional disability benefits provided with the former are taken into account.

The relatively high probability that at least one level of annuity enhancement will be reached, particularly in the case of females, should make the products marketable. Recent changes in tax regime could have increased the product's appeal to consumers further.

Although by no means everyone will be able to afford the premiums for such products, our research suggests that a significant number could, and would be in sufficiently good enough health to satisfy the eligibility requirements.

This paper is organised as follows. Section 5.2 describes the single life disability-linked annuities being considered as examples to be explored within the remainder of the paper. Section 5.3 describes the model and assumptions. Section 5.4 examines the expected times spent in different states of disability whilst Section 5.5 considers the associated probabilities of reaching each of these states. Section 5.6 presents the results using the central set of morbidity assumptions, and Section 5.7 considers the results if more optimistic or pessimistic assumptions are adopted instead. Section 5.8 revisits the results if a wider definition of disability is employed. Section 5.9 investigates the underwriting considerations and Section 5.10 considers another possible product: a last survivor disability-linked annuity. Section 5.11 explores the potential demand for the products described in the paper and Section 5.12 concludes.

5.2 The annuity products under consideration

5.2.1 The level 1/1.5/2.5 annuity

We carry out our analysis of disability-linked annuities by determining the characteristics of particular examples. We begin by investigating the following level disability-linked annuity:-

The annuity commences at a rate of £10,000 per annum. It increases to £15,000 per annum once the annuitant has become moderately disabled and £25,000 per annum once the annuitant has become severely disabled. It is assumed that the annuity is purchased by single premium by an annuitant who is healthy at outset. The terms "healthy", "moderately disabled" and "severely disabled" will be discussed in Section 5.3.5 below. For illustrative purposes, we consider alternative starting ages for males and females of 60, 65, 70, 75 and 80.

For convenience, we assume that the annuity is payable annually in arrears. Therefore, it is the individual's state of health at each anniversary of the commencement of the policy which determines the level of annuity payable. In practice, such an annuity is more likely to be paid more frequently. However, for the purposes of comparing the single premium required for this annuity with that required for a standard annuity (also assumed to be paid annually in arrears), it is acceptable to adopt this simplified approach.

The transition probabilities described in Section 5.3.1 below are based on annual transitions so, clearly, it is more convenient to assume that the annuities are paid annually. Separate calculations, not shown in this paper, confirm that assuming that payments are made annually rather than more frequently does not distort the findings. It also needs to be borne in mind that, in theory, at each annuity payment date the health status of the individual needs to be verified in case the level of payment should be changed. Clearly, this would be impractical if payments were made very frequently, say, monthly.

The disability-linked annuity being considered in this paper is assumed not to be guaranteed for any period (ie the annuity payments cease immediately upon death of the insured life). In practice, it is likely that the annuity would be guaranteed for at least 5 years since, otherwise, a purchaser who dies immediately after the annuity commences would leave no benefits in respect of the product to his/her estate having just paid a substantial premium to the insurer. The reason why we have not allowed for a guaranteed period is that we wish to compare a traditional life annuity with a disability-linked annuity without the results being distorted by the guarantee period.

The annuity amounts quoted throughout this paper are assumed to be gross. However, tax considerations are discussed in Section 5.3.8.

The £10,000 / £15,000 / £25,000 per annum structure leads us to describe this as a "level 1/ 1.5/ 2.5" annuity. To put these annuity amounts in context, the average annual costs of private residential care and private nursing care in the UK in 2003 were £17,100 and £23,700, respectively (Laing and Buisson, 2006).

5.2.2 The index-linked 1/1.5/2.5 annuity

The second type of annuity we consider has the same structure as the annuity described in Section 5.2.1 except that the annuity payments increase in line with price inflation each year. The enhanced levels of £15,000 per annum and £25,000 per annum are also increased in line with price inflation between date of commencement of policy and date when payment at the enhanced rate is due. In other words, this version of the product can be described as follows:

The annuity commences at £10,000 per annum and increases each year in line with the level of price inflation which prevailed over the previous 12 months. Once the annuitant becomes moderately disabled, the annuity is increased by 50% (ie from considering the ratio 15,000: 10,000); and once the annuitant becomes severely disabled, the annuity is further increased by 67% (ie from considering the ratio 25,000: 15,000).

Since this annuity is an index-linked version of the annuity described in section 5.2.1, we describe this as an "index-linked 1/ 1.5/ 2.5" annuity.

5.2.3 The 1 /1.8 /3 annuities

For illustrative purposes, we also consider the annuities which correspond to those described in Sections 5.2.1 and 5.2.2 that have the 1/ 1.8/ 3 structure. In other words, the level version of this annuity will commence at £10,000 per annum and increase to £18,000 per annum and £30,000 per annum on the annuitant becoming moderately and severely disabled, respectively.

5.3 The model and assumptions

5.3.1 Disability/recovery rates

Since the annuities described in Section 5.2 are enhanced once an individual becomes moderately or severely disabled, we need to make assumptions regarding disability and recovery rates when calculating the single premiums required.

We use the disability and recovery rates obtained from the long term care UK population projection model described in Rickayzen and Walsh (2002). The model is a 12 state multiple-state model and is depicted, together with the possible annual transitions, in Figure 5.1.

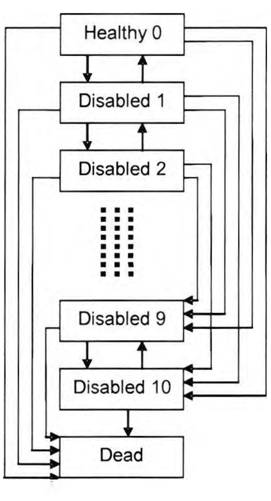


Figure 5.1 The multiple state long term care model

The 10 disability states correspond to those emanating from the OPCS survey of Great Britain which took place in 1985 and 1986. These are described in more detail in Martin et al (1988). State 1 and State 10 are the least, and most, severely disabled states, respectively. State 0 is the healthy state and State 12 is the dead state.

The arrows pointing downwards in Figure 5.1 indicate that it is assumed that an individual can deteriorate to any other worse state of disability in a year. The arrows pointing upwards show that it is assumed that an individual can only improve by at most one disability category in a year. The allowance made for recovery is considered in more detail in Section 5.9.

Rickayzen and Walsh (2002) describe how the model was developed to project the number of disabled lives over the next 40 years from the base year of 1996. Importantly, assumptions regarding trends in future disability rates were made in the model which were based on recent trends in healthy life expectancy data. Given the paucity of the data, 16 alternative sets of trend assumptions were considered. As part of the sensitivity analysis carried out in this paper, we will consider the effect of using the most optimistic and most pessimistic sets of those assumptions, as well as the central set of assumptions, in our calculations. These are considered in Section 5.7.

It should be borne in mind that the disability/recovery rates we are using are based on population data. However, as pointed out by Dullaway and Elliott (1998), we would expect individuals purchasing health insurance products to experience lighter morbidity rates than the general population due to underwriting and the fact that such individuals are likely to come from a relatively high socio-economic grouping. Dullaway and Elliott suggest ways in which general population data can be adjusted to become suitable for insured populations in Chapter 5 of their paper. However, as they acknowledge, this is not only a complicated task but is also fairly arbitrary given the limited amount of data involved.

For the type of disability-linked annuity under consideration in this paper, it is not necessarily the case that the higher the morbidity rates assumed, the higher the single premium required. This is because, whilst higher disability rates will indeed lead to greater numbers of lives receiving enhanced annuities, such disabled lives will also be subject to higher mortality rates (see Section 5.3.2 below). This would mean that the total period for which the lives receive the annuity from the outset of the policy will be relatively short. The net effect on the single premium of assuming high disability rates coupled with higher mortality rates amongst more severely disabled people could be neutral. This important point is considered in more detail in Section 5.6.2.

It should be noted that one of the problems encountered by Rickayzen and Walsh in developing their model is that the published report on the OPCS survey (Martin et al, 1988) shows disability prevalence rates for all lives aged over 80 aggregated together. As acknowledged by Rickayzen and Walsh, not being able to split the data into narrower age bands above age 80 may mean that the morbidity rates at the older ages (eg above age 90) may be higher in practice than those assumed in the model. However, the trade-off between mortality rates and morbidity rates mentioned in the previous paragraph, and analysed in Section 5.6.2, suggests that this should not materially affect the premiums shown in this paper.

5.3.2 Mortality

We have used the analysis set out in Section 5.3.2 of Rickayzen and Walsh (2002) to shape the mortality assumptions adopted for this piece of research.

In particular, we assume that the mortality rate at any age has two components: one which applies equally to healthy and disabled people of a given age and sex in a particular year, and one which increases linearly from OPCS disability category 6 to disability category 10, but does not apply at all to people in disability categories lower than 6. The rationale (which is described more fully in Rickayzen & Walsh (2002)) is that people in the lower disability categories are not experiencing life threatening conditions.

For our purposes, the formula used to express the second component for someone aged x in disability category n is:

$$\text{ExtraMort}(x,n) = \frac{0.10}{1+1.1^{(50-x)}} \cdot \frac{\text{Max}(n-5,0)}{5} \qquad (5.1)$$

The level mortality component has been chosen at each age in such a way as to ensure that the overall mortality at each age (ie when the two mortality components are combined) replicates the IL92 mortality table. For this purpose, we assume that the proportion of lives in each disability category at each age is in accordance with the prevalence rates calculated in Rickayzen & Walsh (2002). Details of the IL92 mortality table can be found in Continuous Mortality Investigation (CMI) Report No. 17 (CMI Committee, 1999).

The ExtraMort (x,n) component given in equation (5.1) is half of that used in Rickayzen and Walsh (2002). However, the latter involved general population mortality whereas we are concerned with insured population mortality in this paper. It is therefore appropriate to re-scale the extra mortality component.

The author has investigated using fractions of the ExtraMort (x,n) component adopted in Rickayzen & Walsh (2002) other than 0.5 (in particular, 0.25 and 0.6) and found that the results are relatively insensitive to this fraction. For example, using a fraction of 0.25 rather than 0.5 would have the effect of increasing the premiums involved by approximately 2.5%. Fractions greater than 0.6 were not considered since these would lead, at some ages, to the level mortality rate component at low disability levels needing to be negative in order that the overall mortality rate for that age should accord with the IL92 Table.

5.3.3 Expenses

We assume that the expenses associated with this product will be as follows:

- 2.5% of the basic part of each annuity payment (ie the part which is payable even while the individual is in good health).

- 15% of the annuity enhancement due to the onset of a disability (ie the part of each annuity payment which is in excess of the basic annuity amount).

We assume that the expenses attached to the uplift component will be relatively high since individuals receiving an enhanced level of payment will need to have their health monitored by the provider to ensure that they remain eligible for the enhancement.

5.3.4 Valuation rate of interest

We use a valuation rate of interest of 7% per annum for the level annuity and 3% per annum for the index-linked annuity. We are, therefore, implicitly assuming a price inflation rate of 3.9% per annum.

5.3.5 Definition of "disabled"

Since the disability-linked annuity product being investigated pays out different annuity amounts depending on whether the policyholder is "healthy/slightly disabled", "moderately disabled" or "severely disabled", it is important to define these terms carefully.

Before considering these definitions further at this stage, it is helpful to consider when claims are paid in relation to conventional LTC insurance products. In the UK, it is common practice for a claim to be payable if a policyholder is unable to perform a certain number of the six benchmark Activities of Daily Living ("ADLs") or has suffered significant cognitive impairment. The six standard ADLs are: washing oneself, dressing oneself, mobility, toileting, feeding and transferring (from, say, a chair to a bed). It is usual for a full claim to be paid if the individual fails 3 ADLs and, in addition, some policies include provision for a partial payment to be made if the individual fails 2 ADLs. A full claim is always paid if the individual fails the cognitive test regardless of whether or not they have any physical disabilities.

Relating this information to the disability-linked annuity under consideration, it seems reasonable to define "severely disabled" to be when the individual fails 3 or more ADLs and "moderately disabled" to be when he/she fails 2 but not 3 ADLs. However, the data we are using are based on OPCS disability categories rather than ADLs. We must therefore, in turn, relate the OPCS disability categories to ADL failures. This can only be done approximately.

Dullaway and Elliott (1998) suggest that, for both sexes, the number of lives failing 2 ADLs or the cognitive test would, approximately, equate to the number of lives in OPCS categories 7-10. They also suggest that the number of lives failing at least 3 ADLs or the cognitive test would, approximately, include all the lives in categories 9 and 10 and half the lives in category 8.

For our central set of assumptions, we will therefore define the terms "healthy/slightly disabled", "moderately disabled" and "severely disabled" for claims purposes as follows:

"Healthy/slightly disabled" = Lives in OPCS categories 0-6 inclusive.

"Moderately disabled" = Lives in OPCS category 7 and half the lives in OPCS category 8.

"Severely disabled" = Lives in OPCS categories 9 and 10 and half the lives in OPCS category 8.

Since the above definitions of disability are relatively narrow, we also wish to examine the effect of making the disability definitions more generous. After all, there is no reason why the disability-linked annuity should be enhanced only when the individual has failed 2 ADLs. The more generous the definition of disability used, the more attractive such an annuity is likely to be, provided the premium is still affordable to the policyholder. We will investigate this point further in Section 5.8.

To put the above disability categories in context, a working party of the UK Actuarial Profession assumed that the relationship between OPCS categories and the care needs of individuals would be as summarised in Table 5.1 (Nuttall et al, 1994).

Table 5.1 Care needs according to OPCS category

OPCS categories	Care needs	Hours per week of care required
1-2	Low	5
3-5	Medium	15
6-8	Regular	30
9-10	Continuous	45

Source: Nuttall et al (1994)

5.3.6 The central set of assumptions

For the purposes of our calculations we have adopted the central set of trend assumptions that were used in Rickayzen and Walsh (2002) (ie Model C described in Table 14 of that paper).

The trend assumptions incorporated within this set relate to improvements in both overall mortality and disability rates over time. Regarding the latter, we assume that the probability that a healthy life aged x in year t becomes disabled during year t is the same as the probability that a healthy life aged x+1 in year t+10 becomes disabled during year t+10 (eg the probability that a healthy 61 year old becomes disabled in 2018 is the same as the probability that a healthy 60 year old becomes disabled in 2008). Since the underlying assumption is that the probability that a person becomes disabled increases with age, this trend assumption represents an improvement (ie reduction) in disability rates. The one year shift in age every 10 years of time leads to the trend assumption being referred to as "1 in 10" in the aforementioned paper.

In Section 5.7 we consider the results if more optimistic or more pessimistic trend assumptions are adopted.

5.3.7 Base year for calculations

Although 1996 was the base year used for the projections described in Rickayzen and Walsh (2002), we have carried out our calculations for this paper assuming that annuities were purchased during 2005. This has been done by allowing for changes in the mortality and morbidity rates between 1996 and 2005 in accordance with the particular trend assumptions adopted.

5.3.8 Tax considerations

Since the disability-linked annuity product described in this paper is not currently available, it is not possible to be precise about the taxation aspects. However, correspondence with the HM Revenue & Customs suggests that such an annuity could either be offered as a pension product or be treated as a combination of a Purchase Life Annuity (PLA) and a Permanent Health Insurance (PHI) benefit – the latter being more commonly known as Income Protection.

If the annuity were treated as a pension product (which would not have been possible prior to 6th April 2006, but now seems allowable as a result of the simplification changes in tax regime which have occurred with effect from that date) the product could be purchased out of the individual's pension account. The contributions made by the individual towards this fund would have been made out of pre-tax income. The premium would accumulate within the insurer's fund without being subject to capital gains or income tax (other than tax on any UK dividends received by the fund) and the annuity payments would be subject to income tax in the hands of the individual. It should be noted that it is unlikely that the insurer would be allowed to reduce any annuity enhancement even if the individual were subsequently to recover to a healthier grouping. This could be a substantial disadvantage to offering the annuity as a pension product since recoveries are expected to occur (see Section 5.9).

If, on the other hand, the annuity were treated as a combination of separate PLA and PHI products then the tax position is very different. The premium would be purchased by the individual out of post-tax savings. Regarding benefit payments, both the capital element of the PLA component of each annuity payment and the whole of the PHI benefit would be payable tax-free whilst the interest element (ie non-capital element) of the PLA component would be subject to income tax.

As mentioned in Section 5.2.1, for the purposes of our calculations, we assume that all amounts considered in this paper are gross of tax.

5.4 <u>Expected time spent disabled</u>

Based on the assumptions set out in Section 5.3, we can calculate the expected time that an individual will spend healthy/slightly disabled, moderately disabled or severely disabled in the future.

This has been done as follows:

Let:

l_x = the number of lives aged x in a life table

ω= the limiting age of the life table

l_x^h = the number of lives aged x in the life table who fall under the "healthy/slightly disabled" definition

l_x^m = the number of lives aged x in the life table who fall under the "moderately disabled" definition

l_x^s = the number of lives aged x in the life table who fall under the "severely disabled" definition where $l_x = l_x^h + l_x^m + l_x^s$

Then, the complete expectation of life for a person aged x is, as an approximation to the underlying integral, given in (5.2):

$$\overset{\circ}{e}_x = \frac{1}{l_x}\left\{\left(\sum_{y=x}^{\omega} l_y\right) - \frac{l_x}{2}\right\} \tag{5.2}$$

For an individual who is healthy (ie in OPCS category 0) at age x, the expected number of years spent "healthy/slightly disabled", "moderately disabled" and "severely disabled" in the future (depicted by $\overset{\circ}{e}{}_x^h$, $\overset{\circ}{e}{}_x^m$ and $\overset{\circ}{e}{}_x^s$, respectively) are approximately:

$$\overset{\circ}{e}{}_x^h = \frac{1}{l_x}\left\{\left(\sum_{y=x}^{\omega} l_y^h\right) - \frac{l_x}{2}\right\} \tag{5.3}$$

$$\overset{\circ}{e}{}_x^m = \frac{1}{l_x}\left\{\sum_{y=x+1}^{\omega} l_y^m\right\} \tag{5.4}$$

$$\overset{\circ}{e}{}_x^s = \frac{1}{l_x}\left\{\sum_{y=x+1}^{\omega} l_y^s\right\} \tag{5.5}$$

where x is the starting age of the life table so that $l_x = l_x^h$.

Table 5.2 shows, for each sex, the expected number of years spent in each disability category for the five different initial ages under consideration. The calculations have been done using the central set of assumptions described in Section 5.3.6. In each case, the life is assumed to be healthy (ie in State 0) at outset.

Table 5.2 Disabled life expectancies (in years) – central assumptions

	Initial Age	$\mathring{e}_x^h$	$\mathring{e}_x^m$	$\mathring{e}_x^s$	$\mathring{e}_x$
Males	60	21.7	1.1	1.5	**24.3**
	65	17.3	1.0	1.4	**19.7**
	70	13.4	0.9	1.3	**15.6**
	75	9.8	0.8	1.3	**11.9**
	80	7.1	0.7	1.2	**9.0**
Females	60	23.6	1.6	2.2	**27.4**
	65	19.0	1.5	2.2	**22.7**
	70	14.8	1.3	2.1	**18.2**
	75	11.2	1.1	1.9	**14.2**
	80	8.1	0.9	1.7	**10.7**

The following features should be noted:

- Since in each case the life is healthy at outset, the life will, on average, spend most of his/her future remaining life falling under the "healthy/slightly disabled" definition with relatively short periods of time spent moderately or severely disabled. For example, a female who is healthy at age 65 is expected to spend 19.0 years "healthy/slightly disabled", 1.5 years "moderately disabled" and 2.2 years "severely disabled".

- Not only are females expected to live longer than males but they are expected to spend more time both moderately and severely disabled than males of the same initial age. This finding accords with observations made by others - for example, Murtaugh et al (2001).

- For both sexes, the proportion of future lifetime expected to be spent severely disabled doubles (approximately) as the initial age moves from age 60 to age

80 (ie from 6.2% to 13.3% in the case of males, and from 8.0% to 15.9% in the case of females).

- For both sexes, the period of time expected to be spent severely disabled is, to a large extent, independent of initial age. The amount of time expected to be spent severely disabled is approximately 1.3 years in the case of males and 2.1 years in the case of females, regardless of initial age. This feature whereby proximity to death is roughly constant and independent of age is consistent with the findings of Himsworth and Goldacre (1999) and Seshamani (2004).

5.5 Claim probabilities

Potential consumers will need to be convinced that there is a strong likelihood that they will eventually be paid an annuity enhancement if they are to consider purchasing a disability-linked annuity. Therefore, in considering the marketing of such a product, it is helpful to consider the probability that an individual taking out such a product will eventually become so disabled that they satisfy the eligibility requirements to receive either of the enhanced levels of annuity.

Table 5.3 shows these probabilities for individuals who take out such a product at different starting ages, based on the central set of assumptions. In each case (and in all the following tables, other than Tables 5.24 - 5.26), the individual is assumed to be healthy (ie in State 0) at commencement of the annuity.

Table 5.3 Claim probabilities – central assumptions

	Age at outset	Prob (remains healthy/slightly disabled)	Prob (becomes moderately but not severely disabled during remaining life)	Prob (becomes severely disabled during remaining life)
Males	60	0.627	0.123	0.250
	65	0.631	0.120	0.249
	70	0.633	0.117	0.250
	75	0.624	0.117	0.259
	80	0.621	0.114	0.265
Females	60	0.489	0.159	0.352
	65	0.493	0.155	0.352
	70	0.501	0.149	0.350
	75	0.512	0.142	0.346
	80	0.528	0.134	0.338

It can be seen that the probability of eventually claiming an enhanced annuity is substantially higher for females than for males. This is particularly true when considering the higher level of enhancement and will be reflected in the respective premiums charged in relation to the sexes. It is also interesting to note that the probabilities vary very little with age at purchase.

The relatively high probability that an individual will eventually receive the second level of annuity enhancement, particularly in the case of females, should make such a disability-linked annuity attractive to consumers. A second reason why such an annuity might appeal to females in particular is that wives usually outlive their husbands; there is therefore less opportunity for females to receive informal care provision from their spouse when they require long term care.

5.6 Results using central assumptions

5.6.1 Single premiums required for each type of annuity

Table 5.4 and Table 5.5 set out the single premiums required for different commencement ages using the central set of assumptions for the level and index-linked annuities, respectively. The 1/1.5/2.5 and 1/1.8/3 annuities were described in Section 5.2 and the 1/1/1 annuity is the standard whole life annuity to which these two annuities are being compared. In all cases, the initial level of annuity paid to the policyholder is £10,000 per annum.

Table 5.4 Single premium (in £) required for different types of level annuity – central assumptions

	Age at commencement	1/1/1	1/1.5/2.5	1/1.8/3.0
Males	60	120,697	128,118	131,004
	65	109,418	118,082	121,429
	70	96,431	106,310	110,095
	75	82,272	94,236	98,787
	80	68,507	81,600	86,539
Females	60	127,943 **(6.0%)**	138,167 **(7.8%)**	142,162 **(8.5%)**
	65	118,031	130,420	135,228
	70	105,993	120,441	126,006
	75	92,255	108,296	114,420
	80	77,655 **(13.4%)**	94,421 **(15.7%)**	100,762 **(16.4%)**

Table 5.5 Single premium (in £) required for different types of index-linked annuity – central assumptions

	Age at commencement	1/1/1	1/1.5/2.5	1/1.8/3.0
Males	60	176,259	192,331	198,539
	65	152,065	168,952	175,436
	70	127,495	144,925	151,571
	75	103,575	122,299	129,392
	80	82,453	101,071	108,070
Females	60	192,238 **(9.1%)**	215,462 **(12.0%)**	224,449 **(13.1%)**
	65	168,833	193,773	203,372
	70	144,139	170,095	180,020
	75	119,299	145,251	155,099
	80	95,687 **(16.1%)**	120,384 **(19.1%)**	129,677 **(20.0%)**

From the size of the premiums, it can be seen that disability-linked annuities are unlikely to be affordable for the less affluent members of the population. The latter would be more concerned with maximising their initial level of income than purchasing an annuity enhancement should they suffer ill health in the future. The potential market for such a product, taking means into account, is considered in more detail in Section 5.11.

As expected, for each of the annuity products, the single premium is greater for females than males. The percentages shown for females age 60 and 80 represent the increase in single premium required for females as compared with males at those ages. It can be seen that the difference is more pronounced in the case of the 1/1.8/3 annuities than the 1/1/1 annuities. For example, with the index-linked version of the annuities, the premium at age 60 is 9.1% higher for females than males for the 1/1/1 annuity (ie comparing £192,238 with £176,259); whereas the difference is 13.1% for the 1/1.8/3 annuity (ie £224,449 compared with £198,539). This reflects the greater expected time spent disabled by females compared with males, as demonstrated by Table 5.2. In addition, the percentages increase with initial age. This reflects the fact that life expectancy for females reduces with initial age more gradually than for males.

For example, using the figures shown in Table 5.2, the life expectancy at age 60 is 12.8% greater for females than males (ie 27.4 years compared to 24.3 years) but 18.9% greater at age 80 (ie 10.7 years compared to 9.0 years)

It should be borne in mind that the single premiums shown in these tables will not necessarily be those that would actually be charged by an insurance company. This is because the disability rates used in the calculations have been extracted from general UK population data whereas an insurance company would tend to use lower rates since they would have been derived from insured population data. However, since the purpose of this paper is to calculate the percentage increase in the single premium that would be required (rather than the absolute values of the premium amounts) when comparing each disability-linked annuity with the standard whole life annuity, the latter point should not detract from the analysis. Nevertheless, we mentioned in Section 5.3.1 that using higher disability rates than might be appropriate for an insured population would not necessarily lead to higher single premiums being required for the type of disability-linked annuity under consideration in this paper. This point is now considered in more detail in Section 5.6.2.

5.6.2 <u>The link between disability and mortality rates</u>

To illustrate the effect on the single premium of having relatively high morbidity rates and, in consequence, high overall mortality rates, we consider initially a level 0/1.5/2.5 annuity. This is an annuity which is consistent with the 1/1.5/2.5 annuity described in Section 5.2.1 (ie nothing is paid until the policyholder becomes moderately or severely disabled in which case the annuity is £15,000 or £25,000 per annum, respectively).

In Section 5.3.6, we referred to the "1 in 10" new disability trend assumption. Let us now consider the single premium required for a female purchasing a 0/1.5/2.5 level annuity at age 60 under the following alternative new disability trend assumptions:-

(1) “1 in 20”

(2) “1 in 10”

(3) “1 in 5”

Assumptions (1), (2) and (3) (which are equivalent to Models B, C and D in Rickayzen and Walsh (2002)) represent progressively lighter disability rate assumptions. For example, the probability that a 60 year old becomes disabled in 2008 is equal to the probability that a 61 year old becomes disabled in 2028 under assumption (1) and in 2013 under assumption (3).

Table 5.6 shows the corresponding single premiums required under assumptions (1), (2) and (3).

Table 5.6. Single premium for female aged 60 purchasing level 0/ 1.5/ 2.5 annuity under alternative disability trend assumptions

Assumption		
(1)	**(2)**	**(3)**
£21,339	£18,915	£14,671

We notice that the single premium required decreases as the disability rate assumption becomes lighter. This is to be expected as nothing is paid whilst the policyholder remains healthy / slightly disabled (which is the majority of her remaining life according to Table 5.2). Hence, the more likely the policyholder is to remain relatively healthy (which occurs under assumption (3)), the lower the premium she should be charged. Table 5.6 indicates that the premium decreases by £6,668 from assumption (1) to (3).

Let us now consider the single premiums required under assumptions (1), (2) and (3) for the level 1/ 1.5/ 2.5 annuity (ie the 0/ 1.5/ 2.5 annuity but with

£10,000 p.a. paid, in addition, whilst the policyholder is healthy/ slightly disabled). These are set out in Table 5.7.

Table 5.7 Single premiums for female aged 60 purchasing level 1/ 1.5/ 2.5 annuity under alternative disability trend assumptions

Assumption		
(1)	**(2)**	**(3)**
£138,981	£138,167	£136,747

We notice that the premium required decreases by a smaller amount (ie £2,234) than in the 0/ 1.5/ 2.5 case as we move from assumption (1) to (3). This demonstrates that when the disability-linked annuity product includes annuity payments which are made whilst the policyholder is healthy/ slightly disabled, the upward pressure on the single premium of assuming relatively high disability rates (with associated high levels of annuity payments) is ameliorated to some extent by the downward pressure caused by the fact that such lives will spend less time receiving the basic level of annuity.

This point is also illustrated by considering the expected time spent in the three states: "healthy/slightly disabled", "moderately disabled" and "severely disabled" under assumptions (1) and (3). This information is set out in Table 5.8 below:

Table 5.8: Disabled life expectancies (in years) for female age 60

	$\mathring{e}^{h}_{60}$	$\mathring{e}^{m}_{60}$	$\mathring{e}^{s}_{60}$	$\mathring{e}_{60}$
Assumption (1)	22.8	1.7	2.6	**27.1**
Assumption (3)	25.0	1.3	1.7	**28.0**

It can be seen that, since assumption (1) involves the heavier disability rates, the expected time spent receiving £15,000 p.a. and £25,000 p.a. is greater than under assumption (3) (ie by 1.3 years in total). However, since the model under assumption (3) involves the lighter overall mortality rates (as the disability assumptions are lighter), the expected time spent receiving the basic £10,000 p.a. annuity is much greater than for the model under assumption (1) (ie by 2.2 years).

We can conclude, therefore, that assuming heavier disability rates in our calculations than are likely to be applicable in practice for an insured population will lead to single premiums which are not very different from those actually required for the insured population.

5.6.3 Comparison of disability-linked annuities with standard life annuities

Table 5.9 shows the percentage increase in single premium required if a disability-linked level annuity as opposed to a traditional level life annuity is purchased. Table 5.10 shows the corresponding information for the index-linked annuities.

Table 5.9 Percentage increase in single premium required, when comparing a disability-linked level annuity with a standard level annuity- central assumptions

	Age at commencement	1/1.5/2.5	1/1.8/3
Males	60	5.5%	7.7%
	65	7.1%	9.9%
	70	9.2%	12.8%
	75	13.1%	18.1%
	80	17.2%	23.6%
Females	60	7.2%	10.0%
	65	9.5%	13.1%
	70	12.3%	17.0%
	75	15.7%	21.6%
	80	19.4%	26.8%

Table 5.10 Percentage increase in single premium required when comparing a disability-linked index-linked annuity with a standard index-linked annuity – central assumptions

	Age at commencement	1/1.5/2.5	1/1.8/3
Males	60	8.2%	11.4%
	65	10.0%	13.8%
	70	12.3%	17.0%
	75	16.3%	22.5%
	80	20.3%	27.9%
Females	60	10.9%	15.1%
	65	13.3%	18.4%
	70	16.2%	22.4%
	75	19.6%	27.0%
	80	23.2%	31.9%

The most significant point emerging from these tables is that the increase in the single premium in all cases is relatively small given the substantial extra benefits being provided by the disability-linked annuity if the policyholder were to become disabled. This should make such an annuity product relatively attractive to consumers. For example, being able to upgrade the standard level annuity to the 1/1.5/2.5 level annuity for males in return for an additional 5.5% of the standard single premium could be of great interest to males age 60 wishing to purchase an annuity.

The reason why the increases are so low is that, as Table 5.2 demonstrates, individuals who are healthy at outset are expected to spend relatively short periods in a state of moderate or severe disability. Furthermore, such periods will tend to be towards the end of a person's life so that the annuity enhancement will be heavily discounted and therefore have a relatively low present value to be added to the standard single premium.

Other observations that can be made about Tables 5.9 and 5.10 are as follows:

- The percentage uplift in premium increases monotonically as the age at commencement increases from age 60 to age 80. This is because, as noted in connection with Table 5.2, the period of time expected to be spent disabled is, to a large extent, independent of age at commencement. This means that individuals are expected to spend an increasing proportion of future lifetime disabled as the age at commencement increases. The disability-linked annuity therefore becomes more valuable relative to the standard annuity the older the policyholder is at outset.

- The percentage uplift in premium is greater for females than males. This is because females are expected to spend more time disabled than males as illustrated by Table 5.2.

- The percentage uplift in premium is greater for the index-linked annuity than the level annuity. The reason for this is that index-linking causes the effective rate of interest used to discount the benefits to be reduced and so the enhanced payments (which are paid toward the end of life) become relatively more valuable.

The author has compared the results shown in Tables 5.9 and 5.10 with those of Murtaugh et al (2001). The latter considered a 1/ 3/ 4 annuity for an individual age 65, used a rate of interest of 6% pa, assumed mortality and morbidity rates based on US data, included a 10 year guarantee and paid the higher disability enhancement on the failure of 4+ ADLs (rather than 3+). They analysed both the level version of the annuity and an increasing version where the life annuity component increased by 3% pa and the disability part increased by 5% pa (ie significantly higher increases on the disability component than assumed in this paper). The authors found that the increase in unisex premium required for the level annuity was 11.5% and for the increasing annuity was 23% when compared with standard life annuities. Separate calculations (not shown) which allow for the differences between the two products and assumptions confirm that the results in this paper are consistent with those of Murtaugh (2001).

5.7 Adoption of pessimistic and optimistic assumptions

It is accepted that life expectancy has been increasing in most countries over the last few decades (see, for example, Dorling et al, 2006). However, there is considerable debate over how the extra years have been spent with regard to state of health. Two main hypotheses, one at each end of the spectrum, have emerged: the compression of morbidity and the expansion of morbidity.

The compression of morbidity hypothesis, espoused by Fries (1980), is that periods of disability are deferred by the extra years of life expectancy. In other words, on average, people spend the extra years in a healthy state and, therefore, a decreasing proportion of their total lifetime in a state of disability. This is an optimistic assumption.

By contrast, the expansion of morbidity hypothesis, suggested by Gruenberg (1977), is that increases in life expectancy are due to a decline in accident rates. Consequently, as fewer people die due to accidents, an increasing number die from chronic disease. Hence, on average, people spend an increasing proportion of their lifetime in a state of disability. This is a pessimistic supposition.

An alternative hypothesis, which sits between these two extremes, is known as the dynamic equilibrium hypothesis. Manton (1987) suggested that changes in both mortality and morbidity rates could lead to a conclusion that the extra years of life expectancy are a mixture of years spent in good health and years spent disabled. MacDonald et al (2006) postulated that recent data collected for England and Scotland supported this theory in both those countries. By contrast, Khoman and Weale (2006) found that, applying an incidence-based rather than prevalence-based approach to calculating healthy life expectancy figures, recent data for the whole of the UK supported the expansion of morbidity hypothesis. This divergence of opinion amply demonstrates the lack of agreement over which theory currently applies.

Karlsson et al (2006) discussed how the sets of assumptions described in Rickayzen and Walsh (2002) relate to the three hypotheses described above. They concluded that Model C of that work reflects the dynamic equilibrium hypothesis, whilst Model A and Model N reflect the expansion and compression theories, respectively.

The assumptions underlying Model C have already been described in Section 5.3.6 above as they are the central set of assumptions used in this paper. To illustrate the effect of using pessimistic and optimistic trend assumptions in this work, we use the morbidity assumptions underlying Model A and Model N, respectively.

Full details of the assumptions can be found in Rickayzen and Walsh (2002). However, the differences can be summarised as follows: under Model A, we assume no improvement in the morbidity rates over time; under Model N, we assume a "1 in 5" trend assumption as regards improvement in morbidity rates (see Section 5.3.6) and a slight reduction in the probability that a disabled person becomes even more disabled during the following year.

5.7.1 <u>Results with pessimistic assumptions</u>

Table 5.11 shows the time which an individual will expect to spend in each health status over their future lifetime assuming that the pessimistic morbidity assumptions apply, and that they were healthy (ie in State 0) at the initial age.

As expected, with stronger morbidity assumptions, the individual can expect to spend longer in both the moderately and severely disabled states as compared to Table 5.2. In addition, in every case, the individual's overall life expectancy is reduced since the expected time spent in the relatively healthy state is substantially reduced.

We can note how the proportion of expected time spent disabled (ie in categories "m" or "s") increases as the morbidity rates become higher. For example, for females age 75, the proportion increases from 21.1% to 25.2%.

Table 5.11 Disabled life expectancies (in years) – pessimistic assumptions

	Initial Age	$\overset{\circ}{e}{}_x^h$	$\overset{\circ}{e}{}_x^m$	$\overset{\circ}{e}{}_x^s$	$\overset{\circ}{e}_x$
Males	60	20.6	1.3	2.0	**23.9**
	65	16.4	1.1	1.8	**19.3**
	70	12.6	1.0	1.7	**15.3**
	75	9.2	0.9	1.6	**11.7**
	80	6.7	0.8	1.4	**8.9**
Females	60	22.0	1.8	2.9	**26.7**
	65	17.8	1.6	2.7	**22.1**
	70	13.8	1.5	2.5	**17.8**
	75	10.4	1.2	2.3	**13.9**
	80	7.5	1.0	2.0	**10.5**

Table 5.12 sets out the probabilities associated with each different claim being made under the pessimistic assumptions.

When compared with Table 5.3, we note that the probability that the annuity will eventually become enhanced due to the onset of disability increases from approximately 0.50 to 0.58 for females (and approximately 0.37 to 0.44 for males) regardless of initial age. For both sexes, the increase is mainly attributable to the increase in probability that the higher level of enhancement will eventually be paid.

Table 5.12 Claim probabilities – pessimistic assumptions

	Age at outset	Prob (remains healthy/slightly disabled)	Prob (becomes moderately but not severely disabled during remaining life)	Prob (becomes severely disabled during remaining life)
Males	60	0.536	0.141	0.323
	65	0.551	0.135	0.314
	70	0.562	0.130	0.308
	75	0.563	0.128	0.309
	80	0.573	0.122	0.305
Females	60	0.389	0.169	0.442
	65	0.405	0.163	0.432
	70	0.424	0.156	0.420
	75	0.446	0.147	0.407
	80	0.470	0.138	0.392

Tables 5.13 and 5.14 show the single premiums required for the level and index-linked annuities, respectively, under the pessimistic assumptions.

The single premiums are lower for the 1/1/1 annuity than in Tables 5.4 and 5.5 where the central assumptions are applied. This is because this annuity is the standard life annuity and, with higher disability rates assumed in Tables 5.13 and 5.14, the total life expectancies are reduced (as shown by comparing Table 5.2 with Table 5.11). Such annuities are therefore less valuable.

The premiums are higher for the 1/1.5/2.5 and 1/1.8/3.0 types of annuity with the pessimistic assumptions than with the central assumptions. This reflects the fact that the value of the annuity enhancement on disability more than compensates for the reduction in value due to reduced overall life expectancy. However, it is interesting to note that the net effect is that the premiums increase by at most 3.2% (female, age 80, 1/1.8/3.0 index-linked annuity) even though we have moved from the dynamic equilibrium theory to the expansion of morbidity theory.

As with Tables 5.3 and 5.4, the percentages shown for females age 60 and 80 represent the increase in single premium required for females as compared with males at those ages. It can be seen that the percentages are all similar to those in Tables 5.3 and 5.4, thus demonstrating that moving to the more pessimistic scenario affects premiums for males and females equally.

Table 5.13 Single premium required for different types of level annuity – pessimistic assumptions

	Age at commencement	**1/1/1**	**1/1.5/2.5**	**1/1.8/3.0**
Males	60	119,912	129,566	133,291
	65	108,635	119,689	123,927
	70	95,538	108,235	113,068
	75	81,467	96,193	101,765
	80	67,966	83,235	88,964
Females	60	126,799 **(5.7%)**	139,856 **(7.9%)**	144,911 **(8.7%)**
	65	116,859	132,324	138,273
	70	104,871	122,509	129,245
	75	91,263	110,432	117,693
	80	76,857 **(13.1%)**	96,493 **(15.9%)**	103,867 **(16.8%)**

Table 5.14 Single premium required for different types of index-linked annuity – pessimistic assumptions

	Age at commencement	**1/1/1**	**1/1.5/2.5**	**1/1.8/3.0**
Males	60	174,184	195,211	203,257
	65	150,243	171,798	180,005
	70	125,740	147,828	156,192
	75	102,192	124,952	133,523
	80	81,600	103,186	111,255
Females	60	189,149 **(8.6%)**	218,702 **(12.0%)**	230,014 **(13.2%)**
	65	166,083	197,037	208,830
	70	141,827	173,300	185,221
	75	117,487	148,286	159,873
	80	94,377 **(15.7%)**	123,120 **(19.3%)**	133,853 **(20.3%)**

5.7.2 Results with optimistic assumptions

Table 5.15 shows the time which an individual will expect to spend in each health status over their future lifetime assuming that the optimistic assumptions apply, and that they were healthy (ie in State 0) at the initial age.

It can be seen that, particularly at the younger initial ages, the time expected to be spent healthy increases as compared to Table 5.2. As expected, the figures for the average time spent disabled are lower than those for Table 5.2, particularly for the more severely disabled category. It is also worth noting that, for this category, the average time spent disabled is virtually constant regardless of initial age.

Table 5.15. Disabled life expectancies (in years) – optimistic assumptions

	Initial Age	$\mathring{e}_x^h$	$\mathring{e}_x^m$	$\mathring{e}_x^s$	$\mathring{e}_x$
Males	60	22.9	0.8	1.0	**24.7**
	65	18.3	0.8	1.0	**20.1**
	70	14.2	0.7	1.0	**15.9**
	75	10.5	0.7	1.0	**12.2**
	80	7.5	0.6	1.0	**9.1**
Females	60	25.4	1.3	1.5	**28.2**
	65	20.5	1.3	1.5	**23.3**
	70	16.0	1.2	1.5	**18.7**
	75	12.1	1.0	1.4	**14.5**
	80	8.8	0.9	1.3	**11.0**

Table 5.16 sets out the claim probabilities under the optimistic assumptions. Compared to Table 5.3 (when central morbidity rates are assumed), the probability of remaining healthy/ slightly disabled is much higher. Most of this increase is due to the reduction in probability that the individual becomes severely disabled. Indeed, moving from the pessimistic to the optimistic

assumptions (ie comparing Tables 5.12 and 5.16) has very little impact on the probability that the individual will become moderately, but not severely, disabled. Even under the optimistic assumptions, there is still a fairly high probability that an individual will receive an annuity enhancement (ie approximately 0.28 for males and 0.39 for females).

Table 5.16. Claim probabilities – optimistic assumptions

	Age at outset	Prob (remains healthy/slightly disabled)	Prob (becomes moderately but not severely disabled during remaining life)	Prob (becomes severely disabled during remaining life)
Males	60	0.737	0.101	0.162
	65	0.729	0.102	0.169
	70	0.722	0.102	0.176
	75	0.705	0.105	0.190
	80	0.686	0.107	0.207
Females	60	0.629	0.142	0.229
	65	0.618	0.143	0.239
	70	0.611	0.141	0.248
	75	0.608	0.138	0.254
	80	0.610	0.132	0.258

Tables 5.17 and 5.18 show the single premiums required for the level and index-linked annuities, respectively, under the optimistic assumptions.

The premiums for the 1/1/1 annuities are all higher than under the central assumptions, reflecting higher overall life expectancy under the optimistic assumptions.

By contrast, the premiums for the disability-linked annuities are all lower than under the central assumptions. This reflects the fact that the individuals are less likely to trigger the annuity enhancement which more than offsets the fact that the annuity is payable for longer. It is interesting to note that the net effect is that the premiums decrease by at most 4.2% (female, age 80, 1/1.8/3.0

index-linked annuity) even though we have moved from the dynamic equilibrium theory to the compression of morbidity theory. Indeed, if we move from the expansion of morbidity theory to the compression of morbidity theory (ie comparing Tables 5.12 and 5.13 with Tables 5.17 and 5.18), we find that the premium only decreases by at most 7.2%.

As with the previous tables of single premiums, the percentages shown for females age 60 and 80 represent the increase in single premium required as compared with males at those ages. Since the percentages are similar to those in the previously tables, it can be concluded that moving to the optimistic scenario affects the premium rates for males and females equally.

Table 5.17 Single premium required for different types of level annuity – optimistic assumptions

	Age at commencement	**1/1/1**	**1/1.5/2.5**	**1/1.8/3.0**
Males	60	121,376	126,774	128,901
	65	110,113	116,526	119,038
	70	97,185	104,457	107,281
	75	83,152	91,965	95,358
	80	69,128	79,607	83,607
Females	60	129,031 **(6.3%)**	136,395 **(7.6%)**	139,322 **(8.1%)**
	65	119,196	128,324	131,928
	70	107,159	118,064	122,337
	75	93,331	105,741	110,561
	80	78,560 **(13.6%)**	91,852 **(15.4%)**	96,965 **(16.0%)**

Table 5.18 Single premium required for different types of index-linked annuity – optimistic assumptions

	Age at commencement	1/1/1	1/1.5/2.5	1/1.8/3.0
Males	60	178,104	189,387	193,819
	65	153,742	165,919	170,673
	70	129,076	141,788	146,711
	75	105,127	119,034	124,376
	80	83,456	98,368	104,048
Females	60	195,292 **(9.7%)**	211,619 **(11.7%)**	218,077 **(12.5%)**
	65	171,674	189,768	196,882
	70	146,629	166,049	173,628
	75	121,332	141,329	149,067
	80	97,216 **(16.5%)**	116,777 **(18.7%)**	124,275 **(19.4%)**

5.8 Adopting wider definitions of disability

As suggested in Section 5.3.5, the wider the definition of "disabled" used by an insurer, the more attractive to a potential consumer the product is likely to be (provided the premium is still affordable). We now consider the impact on the results if we make the definitions of "disabled" more generous.

In this section, we will redefine the three definitions of healthy/disability to be as follows:

"Healthy/slightly disabled" = Lives in OPCS categories 0-4 inclusive.

"Moderately disabled" = Lives in OPCS categories 5-7 .

"Severely disabled" = Lives in OPCS categories 8-10.

These are groupings which have been used by others (see, for example, Mayhew (2001)). We now investigate the results when using these wider definitions of disability as compared to those shown in Sections 5.4, 5.5 and

5.6. We use the central morbidity assumptions throughout this section to be consistent with the earlier sets of results.

Table 5.19 shows the life expectancy and disabled life expectancy figures which should be compared to Table 5.2.

As expected, although the total life expectancy figures are the same in the two tables, the expected time spent in the disabled categories is considerably longer with the wider definitions and the expected time spent healthy/slightly disabled is correspondingly shorter. Of the two disabled categories, the largest impact is on the moderately disabled category. For example, females aged 60 would be expected to spend an extra 1.6 years moderately disabled (ie from 1.6 to 3.2 years) but only an extra 0.5 years severely disabled (ie from 2.2 to 2.7 years). As a result, they are expected to spend 2.1 years fewer in a healthy/ slightly disabled state. Overall, they are expected to spend 21.5% of future lifetime in a moderate or severe state of disability, as compared with 13.9% using the narrower definitions of disability. Similar results can be found for the other initial ages, and for males.

Table 5.19 Disabled life expectancies (in years) – central assumptions and using wider disabled definition

	Initial Age	$\mathring{e}_x^h$	$\mathring{e}_x^m$	$\mathring{e}_x^s$	$\mathring{e}_x$
Males	60	20.4	2.1	1.8	**24.3**
	65	16.1	1.8	1.8	**19.7**
	70	12.3	1.6	1.7	**15.6**
	75	8.9	1.4	1.6	**11.9**
	80	6.3	1.2	1.5	**9.0**
Females	60	21.5	3.2	2.7	**27.4**
	65	17.2	2.9	2.6	**22.7**
	70	13.3	2.5	2.4	**18.2**
	75	9.9	2.1	2.2	**14.2**
	80	7.1	1.6	2.0	**10.7**

Table 5.20 sets out the claim probabilities under the wider definitions of disability. This should be compared with Table 5.3.

It can be seen that, at all initial ages and for both sexes, the probability that the life remains healthy/slightly disabled reduces by approximately 0.1; the corresponding increase in probability of becoming moderately or severely disabled is fairly evenly split between the two disability categories. The overall effect is that, with the wider disability definitions, the probability of receiving an enhanced annuity increases from approximately 0.50 for females (0.37 for males) to 0.61 for females (0.46 for males). The fact that, with the wider definitions of disability, it becomes more likely than not that females will receive at least one level of annuity enhancement (and almost as likely as not for males) could make the product of great interest to consumers.

Table 5.20 Claim probabilities – central assumptions and using wider disabled definition

	Age at outset	Prob (remains healthy/slightly disabled)	Prob (becomes moderately but not severely disabled during remaining life)	Prob (becomes severely disabled during remaining life)
Males	60	0.531	0.172	0.297
	65	0.537	0.167	0.296
	70	0.543	0.162	0.295
	75	0.533	0.162	0.305
	80	0.532	0.158	0.310
Females	60	0.375	0.219	0.406
	65	0.382	0.214	0.405
	70	0.393	0.206	0.401
	75	0.409	0.197	0.394
	80	0.430	0.186	0.384

Table 5.21 shows the single premiums required with the wider disability definitions for the level version of the product. This should be compared to Table 5.4.

It can be seen that the premiums for the 1/1/1 products are identical since the annuity payments are not dependent on disability. The numbers in parentheses in the 1/1.5/2.5 and 1/1.8/3.0 columns at sample ages 60 and 80 represent the percentage increases in the premiums compared to Table 5.4.

The premium increases are relatively modest even when the disability definitions have been widened quite substantially and the most generous (1/1.8/3.0) version of the product is being considered (e.g. the maximum increase is 7.1% for females age 80). The reason for this is the same as the reason why the premium increases for the two disability-linked annuities compared to the 1/1/1 versions are so modest – the annuity enhancements due to disability are expected to commence only towards the end of life (ie are discounted many years into the future) and are not expected to be paid for very long. This suggests that if a disability-linked annuity product were to be launched, the most sensible strategy would be to incorporate a relatively wide definition of disability. In this way, the increase in premium required for widening the definition is relatively low but the appeal of the product should increase considerably since it is very likely that, at some point, at least one level of enhancement would be paid to the policyholder.

The percentage increase in premium as a result of widening the disability definition increases with age. This is because, at older starting ages, individuals spend a greater proportion of their future life in a state of moderate or severe disability. Therefore, the disability definition becomes more significant.

Table 5.21 Single premium required for different types of level annuity – central assumptions and using wider disabled definition

	Age at commencement	1/1/1	1/1.5/2.5		1/1.8/3.0	
Males	60	120,697	131,337	**(2.5%)**	135,763	**(3.6%)**
	65	109,418	121,570		126,565	
	70	96,431	109,919		115,380	
	75	82,272	98,243		104,625	
	80	68,507	85,568	**(4.8%)**	92,289	**(6.6%)**
Females	60	127,943	142,609	**(3.2%)**	148,810	**(4.7%)**
	65	118,031	135,374		142,613	
	70	105,993	125,688		133,790	
	75	92,255	113,518		122,128	
	80	77,655	99,270	**(5.1%)**	107,879	**(7.1%)**

Table 5.22 sets out the corresponding premium information for the index-linked versions of the annuities.

It can be seen that similar comments apply to those made regarding Table 5.21. In addition, it should be noted that the premium increases in Table 5.22 are all greater than their counterparts shown in Table 5.21. The reason for this is that the effect of allowing for index-linked increases to benefits in payment is to use a lower effective rate of interest to discount the payments. Therefore, the annuity enhancements, which are expected to occur towards the end of life, become more valuable when index-linked increases are applied. A similar point was made in Section 5.6.3 to explain the reason why the percentage increase in the premium for disability-linked annuities compared to the traditional life annuity was higher for index-linked than level annuities.

Table 5.22. Single premium required for different types of index-linked annuity – central assumptions and using wider disabled definition

	Age at commencement	1/1/1	1/1.5/2.5		1/1.8/3.0	
Males	60	176,259	198,801	**(3.4%)**	208,064	**(4.8%)**
	65	152,065	175,300		184,746	
	70	127,495	150,933		160,336	
	75	103,575	128,270		138,064	
	80	82,453	106,479	**(5.4%)**	115,883	**(7.2%)**
Females	60	192,238	224,581	**(4.2%)**	238,028	**(6.0%)**
	65	168,833	202,891		216,901	
	70	144,139	178,809		192,893	
	75	119,299	153,143		166,701	
	80	95,687	127,120	**(5.6%)**	139,529	**(7.6%)**

5.9 Effect of changing initial health status

Until now, it has been assumed that the underwriting being employed by the insurer has been such that every individual accepted for cover is healthy (ie in State 0) at the outset. It is interesting to revisit this assumption and analyse the effect on the premium which should be charged if the individual were, in fact, in State 4 at the outset. For this part, we use mortality assumptions which are consistent with those described in Section 5.3.2, but applicable to a life in State 4. We also assume throughout this analysis that the central morbidity assumptions apply and that the original (narrow) disability definitions are being used.

Table 5.23 shows the disabled life expectancy figures assuming that individuals are in State 4 at commencement. When compared to Table 5.2 (when lives are assumed to be in State 0), it is perhaps surprising that the figures are not smaller. The reason they are not is that there is an assumption within the morbidity basis that 10% of lives who do not deteriorate in health during a year will recover by one OPCS category during that year. This relatively strong assumption is based on data in both the UK and the USA suggesting that a number of people do recover from disabilities during a year. For the full discussion and justification of this assumption, the reader is referred to Rickayzen & Walsh (2002).

The impact of this assumption is that, with a group of lives starting in State 4, a reasonable number of them will be expected to recover by at least one category during the first few years of the policy.

When compared with Table 5.2, it can be seen that expected time spent in the two disabled categories increases since the life is starting in a state of disability closer to the moderately disabled grouping (OPCS categories 7- 8.5) than if they were initially in State 0. The overall life expectancy reduces as a result, and the impact of this new assumption is greater on females than males. For example, females aged 60 are expected to spend 0.9 years more either moderately or severely disabled, and with a reduced overall life expectancy of 0.8 years; whereas, males aged 60 are expected to spend only 0.5 years more either moderately disabled or severely disabled, and with a 0.5 year reduction in overall life expectancy.

Table 5.23 Disabled life expectancies (in years) – Initial State 4

	Initial Age	$\overset{\circ}{e}{}_x^h$	$\overset{\circ}{e}{}_x^m$	$\overset{\circ}{e}{}_x^s$	$\overset{\circ}{e}_x$
Males	60	20.8	1.3	1.8	**23.9**
	65	16.4	1.2	1.7	**19.3**
	70	12.5	1.1	1.7	**15.3**
	75	9.0	1.0	1.6	**11.6**
	80	6.3	0.9	1.5	**8.7**
Females	60	21.9	2.0	2.7	**26.6**
	65	17.4	1.9	2.6	**21.9**
	70	13.3	1.7	2.5	**17.5**
	75	9.7	1.5	2.4	**13.6**
	80	7.0	1.2	2.1	**10.3**

Tables 5.24 and 5.25 show the single premiums that should be charged to a life in State 4 at each initial age for the level and index-linked versions of the annuity, respectively. These should be compared to Tables 5.4 and 5.5 which show the premiums that would actually be charged according to the assumptions in Section 5.3. The figures in parentheses for the two disability-linked annuities show the percentage increase in premium theoretically required, given the individual's actual state of health.

Table 5.24 Single premium required for different types of level annuity – Initial State 4, central assumptions

	Age at commencement	**1/1/1**	**1/1.5/2.5**		**1/1.8/3.0**	
Males	60	119,570	129,538	**(1.1%)**	133,429	**(1.9%)**
	65	108,208	119,835		124,343	
	70	95,240	108,401		113,459	
	75	80,973	96,954		103,048	
	80	67,270	84,790	**(3.9%)**	91,411	**(5.6%)**
Females	60	126,063	139,998	**(1.3%)**	145,470	**(2.3%)**
	65	115,870	132,821		139,429	
	70	103,684	123,504		131,167	
	75	89,983	112,023		120,465	
	80	75,613	98,684	**(4.5%)**	107,431	**(6.6%)**

Table 5.25 Single premium required for different types of index-linked annuity – Initial State 4, central assumptions

	Age at commencement	1/1/1	1/1.5/2.5		1/1.8/3.0	
Males	60	173,809	194,106	**(0.9%)**	201,981	**(1.7%)**
	65	149,676	171,167		179,455	
	70	125,333	147,587		156,101	
	75	101,451	125,589		134,759	
	80	80,606	104,794	**(3.7%)**	113,906	**(5.4%)**
Females	60	187,954	217,450	**(0.9%)**	228,927	**(2.0%)**
	65	164,405	196,488		208,898	
	70	139,865	173,633		186,599	
	75	115,480	149,576		162,559	
	80	92,546	125,292	**(4.1%)**	137,647	**(6.1%)**

The premiums for the 1/1/1 version of the annuity are all lower for Tables 5.24 and 5.25 than Tables 5.4 and 5.5. This is as expected since this annuity is the standard life annuity and is expected to be paid for a shorter period of time if the individual's initial health status is poor.

As usual in this analysis, the 1/1.5/2.5 and 1/1.8/3.0 versions of the annuity are of more interest. It can be seen that the increases in premium required are of a similar order of magnitude for the level and index-linked annuities. The percentage increases with age since, at older ages, individuals are expected to spend a greater proportion of their future lifetime in a state of disability. The percentage increase is greater for females than males (again, since the proportion of time spent disabled tends to be greater for females than males – see Section 5.4).

The percentage increase for females purchasing the level 1/1.8/3.0 annuity ranges from 2.3% at age 60 to 6.6% at age 80. These percentages are relatively modest, given the difference between State 0 and State 4 health status, for two reasons: firstly, as explained at the start of this section, the allowance for recovery being made; and secondly, the effect of starting in a state of disability (and therefore increasing the likelihood that enhanced annuity payments will be made) will be mitigated, to some extent, by the fact that the overall life expectancy will be reduced.

The conclusion which can be made following this analysis is that the insurer can afford to be relatively flexible as regards the underwriting requirements. This view is consistent with that of Murtaugh et al (2001) mentioned in the introduction to this paper (Section 5.1).

5.10 Consideration of last-survivor disability-linked annuities

Karlsson et al (2006) recognised that an individual's requirements for formal long term care depend, inter alia, on whether or not the individual has access to informal care support from a partner. For example, if an individual whose health is poor has a spouse who is willing and healthy enough to provide informal care, then the individual will not need to purchase as much formal care to satisfy his/her care needs.

This observation leads to the concept that individuals might wish to consider purchasing another type of annuity: a disability-linked last-survivor annuity. This is a last survivor annuity where the income level depends on the health statuses of the two individuals involved. It is a natural extension of the disability-linked annuity considered earlier in this paper. Pauly (1990) acknowledges that couples are likely to find LTC insurance products attractive which enable one life to receive costly nursing care without severely depleting the income of the other. This type of annuity should meet that objective.

To analyse the product further, we consider, for illustrative purposes, an annuity having the features described in Table 5.26.

Table 5.26 Last survivor disability-linked annuity payment schedule

Health statuses of the two lives	Total amount payable (£ p.a.)
Both lives in OPCS 0-7	10,000
One life in OPCS 0-7, one in OPCS 8-10	25,000
Both lives in OPCS 8-10	40,000
One life in OPCS 0-7, one dead	10,000
One life in OPCS 8-10, one dead	25,000

With this product design, it can be seen that a standard last survivor annuity of £10,000 p.a. is payable whilst both lives are in reasonable health. The annuity increases by £15,000 p.a. to £25,000 p.a. whilst one life is in serious ill health (ie OPCS categories 8-10), and by a further £15,000 p.a. to £40,000 p.a. whilst both lives are in serious ill health.

Table 5.27 sets out the periods of time which lives are expected to spend in different states of disability for different initial ages, assuming that the two lives are the same age and are both healthy (ie in State 0) at the outset. It is assumed that mortality between the two lives is independent. The central set of morbidity assumptions has been adopted.

Table 5.27 Expected time (in years) spent by two lives in different states of disability – central assumptions

Age of both lives at commencement	Health Statuses of the two lives					
	Both 0-7	One 0-7 One 8-10	Both 8-10	One 0-7 One dead	One 8-10 One dead	**Total**
60	18.437	1.991	0.154	8.365	2.240	**31.187**
65	14.267	1.799	0.156	7.742	2.237	**26.201**
70	10.626	1.577	0.156	6.893	2.198	**21.450**
75	7.539	1.389	0.158	5.848	2.120	**17.054**
80	5.217	1.154	0.151	4.707	1.964	**13.193**

It can be seen, for example, that if both lives are healthy and age 60 at the outset, the expected length of time before the death of the second life is 31.187 years. During this period, 18.437 years are expected to be spent with both lives in OPCS categories 0-7, 8.365 years with one life in OPCS categories 0-7 and the other no longer alive, and 4.385 years with at least one life in poor health.

The expected times shown in the 3rd, 4th and 6th columns are fairly constant. In other words, the times spent with at least one life in OPCS categories 8-10 are reasonably impervious to the age at commencement. For example, the expected time spent in OPCS categories 8-10 by a widow/widower is approximately 2.2 years regardless of the age at commencement.

Table 5.28 sets out the single premiums required for the level disability-linked last survivor annuity described in Table 5.26 compared with the level standard last survivor annuity. Table 5.29 sets out the same information for the index-linked version of the annuity.

Table 5.28 Single premium (in £) required for level last survivor annuity – central assumptions

Age of both lives at commencement	Standard	Disability-linked	% increase in premium
60	136,107	153,194	12.6
65	128,087	148,567	16.0
70	117,731	141,526	20.2
75	105,055	132,610	26.2
80	90,828	120,361	32.5

Table 5.29 Single premium (in £) required for index-linked last survivor annuity – central assumptions

Age of both lives at commencement	Standard	Disability-linked	% increase in premium
60	211,064	249,393	18.2
65	188,907	229,858	21.7
70	164,724	207,389	25.9
75	139,334	183,475	31.7
80	114,417	157,399	37.6

The first thing to note is that the percentage increase in standard premium required to support the disability-linked annuity is substantially higher than in the case of the single life disability-linked annuity considered earlier. Leaving aside the fact that the benefits offered by the two annuities are not directly comparable, it is to be expected that the percentage increase for last survivor disability-linked annuities will tend to be relatively high. This is because, with two lives involved, there is an increased likelihood that an annuity enhancement will eventually be made since it only requires one of the lives to become disabled for this to occur. Notwithstanding this level of percentage increase in premium required, last survivor disability-linked annuities could still be attractive to consumers since they do allow for the health status of each partner in considering future long term care provision.

It can be seen that the percentage increase in the standard premium required for the disability-linked annuity increases substantially with age at commencement. This reflects the fact that the proportion of time spent receiving the basic level of annuity (ie when neither life is in OPCS categories 8-10) decreases with age. For example, Table 5.27 shows that the proportion of expected time spent with neither life severely disabled decreases with age at commencement from 86% at age 60 to 75% at age 80.

5.11. Affordability of products

It must be recognised that the premiums required for the products described in this paper represent significant lump sums. It is appropriate, therefore, to consider the extent to which such premiums would be affordable to the target group: the older members of the UK population.

Using the information contained within the English Longitudinal Study of Ageing (ELSA) regarding wealth and disability, it is possible to estimate the wealth distribution of the UK population who are healthy and over the age of 65. For information about the first two waves of the ELSA dataset, see Marmot et al (2006).

Table 5.30 shows, for each asset band (with the value of the house excluded in the first set of columns and included in the second set), the estimated numbers of people over age 65 who report that they have no ADL impairments. The information is shown separately for single people and couples. The percentages shown are the estimated proportions of the UK population over age 65 within each asset bracket which these numbers represent. In other words, we estimate that 80% of the UK population over age 65 with assets (including housing wealth) of over £1 million have no ADL impairments.

Table 5.30 Estimated UK population over age 65 who have no ADL impairments in different wealth brackets (in 000s)

Assets (£000s)	Excluding housing				Including housing			
	Single	Married	Total	% of population	Single	Married	Total	% of population
0-50	2,071	2,209	4,280	70	1,084	673	1,757	66
51-200	442	861	1,303	81	1,026	1,411	2,437	74
201-400	87	233	320	78	392	883	1,275	79
401-600	21	93	114	83	80	274	354	77
601-800	5	37	42	82	30	106	136	77
801-1000	2	13	15	83	10	55	65	80
>1000	7	26	33	94	13	70	83	80
Total	**2,635**	**3,472**	**6,107**	**73**	**2,635**	**3,472**	**6,107**	**73**

It should be noted that the percentage generally increases with the level of assets held. This pattern is consistent with the well known link between socio-economic status and health - see, for example, Fuchs (2004). It is interesting to observe that such a high proportion of older people are in good health, particularly at the wealthier end of the spectrum.

It can be seen that, for example, we estimate that 524,000 people over age 65 have assets in excess of £200,000 if housing wealth is excluded, and this number increases to 1,913,000 if housing wealth is included. When assessing potential demand for the disability-linked annuity, the latter is particularly relevant since there is a tendency for pensioners to down size their property once their children have left home (ie release some equity in property by moving to a smaller and cheaper house). These figures, together with the fact that such a high proportion of the elderly appear to be in good enough health to be eligible to purchase a disability-linked annuity, suggests that the potential demand for such a product could be reasonable.

Similar comments can be made about the potential demand for the type of last survivor product discussed in Section 5.10 by considering the information contained for couples within Table 5.30. It should be noted that these figures represent the estimated number of individuals who are married and have no

ADL impairments - the figures do not represent the number of people who are married where both people have no ADL impairment (as would be required to purchase the last survivor product described earlier). Nevertheless, in view of the high percentage of the population who are in good health and the relatively high proportion of people who are married (as demonstrated in Table 5.30), we can again conclude that there would be a number of people with the means and necessary health status to purchase such a last survivor product.

5.12 Conclusions

In this paper, we have analysed different types of disability-linked annuities, both single life and last survivor, in order to examine their main characteristics and assess their suitability as potential new products in the annuity and LTC market. In view of the ageing of the UK population over recent decades, and the expectation that this will continue in the future, it is important that innovative insurance products should be developed which might enable individuals to meet their future LTC requirements.

Some types of disability-linked annuity products have been issued in the UK in the past; however, they could not be issued in a tax efficient way and were therefore unattractive to consumers.

The simplification in tax regime that has occurred in the UK post-6th April 2006 suggests that the type of product being described in this paper could be issued more tax efficiently. Correspondence with HR Customs and Revenue suggests that it could either be issued as a pensions product (in which case it could be purchased out of a consumer's pensions account, which has been built up out of contributions from pre-tax income) or it could be issued as a combination of a Purchase Life Annuity and a PHI benefit (in which case a substantial proportion of the benefits could be paid free of income tax). Clearly, the precise tax position would need to be ascertained before such a product could be issued.

We would endorse the views of Warshawsky et al (2002) that such products possess the following attractive qualities:

- The fact that the longevity risk (associated with the life annuity component) and the morbidity risk (associated with the LTC component) operate in opposite directions helps to minimise the overall risk of adverse selection. This is turn enables the underwriting requirements to be relaxed to some extent, and helps to reduce the size of the premiums.

- The annuity enhancement upon onset of disability will help to defray the associated care costs. This should offer reassurance to the policyholder that the care costs can be met if and when they are required, and should support any bequest motive in respect of the policyholder's beneficiaries. Furthermore, the enhancement might enable the policyholder to remain living in their home to receive care, rather than having to move into a residential care home.

- As a way of obtaining some LTC insurance cover, a disability-linked annuity is likely to be viewed in a more positive light than traditional stand alone LTC insurance since the individual knows that they will receive a benefit whilst they are healthy, with an increase in the benefit occurring should their health deteriorate markedly. In other words, a change in emphasis occurs with this type of product.

Initially, we used a central set of assumptions - in particular, incorporating the central set of morbidity trend assumptions used by Rickayzen and Walsh (2002) - to calculate various quantities. These included the expected times spent in different states of disability, the probabilities of each level of annuity enhancement being eventually triggered and the single premiums required.

We found that, at all initial ages, individuals are expected to spend the majority of their lives in the healthy/slightly disabled state, with a short period of more serious disability towards the end of life. As a result, the increase in premium required for a disability-linked annuity as compared to a standard life annuity without enhancement on disability is relatively modest. The period of time spent severely disabled tends to be impervious to the initial age at around 1.4 years for males and 2 years for females.

We found that the premiums are higher for females than males since, not only are females expected to live longer than males, but they tend to spend more time in moderate and severe states of disability.

The probabilities that an annuity enhancement would eventually be paid were relatively high (eg approximately 0.37 for males and 0.50 for females) which should make such a product attractive to consumers, particularly females.

We then considered the results if pessimistic assumptions consistent with the expansion of morbidity were used. We found that the probabilities of the enhancement eventually being paid increased significantly to approximately 0.44 for males and 0.58 for females. However, the effect on the single premium was relatively small. This is because, although there is upward pressure on the premium from the increased likelihood that an individual will become disabled, the overall term of the annuity is shortened due to the consequent reduced life expectancy.

Similar arguments applied when optimistic assumptions consistent with the compression of morbidity were used. The probabilities reduced significantly to approximately 0.28 and 0.39 for males and females, respectively; however, the premiums did not decrease substantially since the expected term expands due to increased life expectancy.

We considered the case if wider, more generous, definitions of disability were adopted than those originally assumed which had been incorporated as a proxy to current LTC claim trigger points.

The product should become more attractive to consumers since it becomes more likely that at least one level of annuity enhancement will be paid, and the individual would be expected to spend longer receiving each level of enhancement under these wider definitions. For example, for females aged 60, the probability that an annuity enhancement will be paid increases from 0.51 to 0.62, and the times spent moderately and severely disabled are expected to increase by 1.6 years and 0.5 years, respectively.

We found, however, that the knock-on effect on the single premium was modest. This is, again, due to the fact that the periods of time spent disabled are relatively short and concentrated towards the end of life (and therefore heavily discounted).

These results suggest that if a disability-linked annuity product were to be launched, the most sensible strategy would be to incorporate a relatively wide definition of disability. In this way, the resulting increase in premium required for widening the definition is relatively small but the appeal of the product increases considerably since it is very likely that, at some point, at least one level of enhancement would be paid to the policyholder.

In order to analyse the importance of the underwriting function, we considered the effect on the premium theoretically required if the individual was, at commencement, already slightly disabled. We found that the implied percentage increase in premium was surprisingly low for two reasons. Firstly, an allowance for recovery is made in the assumptions; and secondly, the effect of starting in a state of disability (and therefore increasing the likelihood that enhanced annuity payments will eventually be made) will be mitigated, to some extent, by the fact that the overall life expectancy is reduced.

Since a key factor for individuals in deciding their formal LTC needs is the extent to which they can rely on informal care from a spouse, we analysed a last survivor type of disability-linked annuity.

We found that the expected time spent with at least one life being severely disabled was almost constant, regardless of the initial ages of the lives. We compared the premiums for this type of product with those for the corresponding standard last survivor annuity and found that the percentage difference increases substantially with age at commencement. This reflects the fact that the proportion of time spent receiving the basic level of annuity (ie when neither life is severely disabled) decreases with age.

Finally, we examined the potential market for disability-linked annuity products. Our analysis suggests that a high proportion of the older UK population are in sufficiently good health to be eligible to take out such a policy. This is particularly true amongst the wealthier citizens (ie the group most able to afford the premiums).

Clearly, the premiums will be large since the product provides a much higher level of benefits than a more traditional life annuity, and by no means everyone will be able to afford such premiums. However, we found that the premiums appear to be affordable for a reasonably large number of people, particularly if individuals are willing to trade down their property to release sufficient equity.

In conclusion, the disability-linked annuity seems to be worthy of attention by insurers as a product providing a certain level of LTC insurance cover, whilst not having the same negative connotations as traditional stand alone LTC insurance appears to have amongst consumers.

The fact that the longevity risks and morbidity risks contained within the product work in opposite directions should make the overall risk more controllable. This facet has other positive effects: the underwriting requirements need not be so stringent, the disability definitions are not so critical and the single premiums appear to compare favourably with their traditional life annuity counterparts when the additional LTC benefits being offered are taken into account.

The fact that the probability that an annuity enhancement will eventually be paid is relatively high, particularly in the case of females, should make the product marketable. In addition, the product appears to have benefited from recent changes in tax regime and should therefore be even more appealing to consumers.

Perhaps the most interesting features of the disability-linked annuity are as follows:

- Individuals using such annuities to ring-fence their care needs can be more certain that the rest of their assets can be used to satisfy any bequest motives. Hence, the disability-linked annuity is, potentially, a powerful tool with regard to inheritance tax planning.

- As shown in Section 5.9, the initial health status of an individual purchasing a disability-linked annuity has little impact on the premium which should be paid. The product could, therefore, be incorporated within either an occupational pension scheme or the State pension scheme with minimal risk of adverse selection.

6. CONCLUSIONS

In this thesis, we have seen examples of the way in which multiple state models can be used in analysing health insurance products – in particular, IP and LTC insurance.

Chapters 2 and 3 are concerned with IP insurance and Chapters 4 and 5 relate to LTC insurance. Since each of these chapters is a self-contained paper, including its own set of conclusions, we will not repeat those conclusions in this chapter. Instead, we will discuss the importance of the work, both in terms of the different audiences it has reached and the ways in which the research has been utilised in further work carried out by the author, and by others.

6.1 Uses to which research contained in Chapters 2 to 5 have been put

6.1.1 IP research in Chapter 2

The work contained within Chapter 2 has been published in the Journal of Actuarial Practice in 2001 (Rickayzen, 2001). The results were referred to in the widely used textbook on actuarial models for disability insurance, Haberman and Pittacco (1999).

The work was presented in 1997 at both the Actuarial Research Centre, City University, and at the Actuarial Teachers' Conference held at University of Heriot Watt.

6.1.2 IP research in Chapter 3

The work described in Chapter 3 was published in ASTIN Bulletin in 2004 (Haberman et al, 2004). The thesis author presented the research at the Healthcare Conferences organised by the UK Actuarial Profession in both 1999 and 2000. The work has also been presented by one of the co-authors, Steven Haberman, at the 31st ASTIN Colloquium in Sardinia in 2000.

6.1.3 LTC research in Chapter 4

The author has tended to focus his research interests in recent years on the topic of long term care since there has been a great deal of interest in the work contained within both Chapter 4 and Chapter 5.

The work described in Chapter 4 was published in the British Actuarial Journal in 2002 (Rickayzen and Walsh, 2002). Due to the importance of the LTC projection model, the author has been invited to present the work at several conferences and seminars. These include:

- The Healthcare Conference organised by the UK Actuarial Profession at the University of Warwick in May 1998 – the author presented the work, together with three members of the UK Actuarial Profession's Working Party.

- Réseau Espérance de Vie en Santé (REVES) 11th Conference on Healthy Life Expectancy in London in April 1999.

- One day seminar on LTC organised by the London School of Economics, London in April 1999 - the author presented the work with another member of the UK Actuarial Profession's Working Party.

- The Healthcare Conference organised by the UK Actuarial Profession at the University of Warwick in May 1999 – the author presented the work with another member of the UK Actuarial Profession's Working Party.

- Actuarial Teachers' Conference at the University of Cambridge in 2000

- International Institute for Applied Systems Analysis (IASSA) Health Conference in London in 2001.

- International Union for the Scientific Study of Population (IUSSP) World Population Conference in Salvador, Brazil in 2001.

- Long Term Care Workshop organized by London School of Hygiene and Tropical Medicine, London in 2001.

- A specially convened seminar at the actuarial consultancy firm, Watson Wyatt Worldwide, at their Reigate Office in 2002.

In addition to the presentations listed above, the work has attracted the attention of various interested parties, and spawned further research. These activities are described below.

In December 1997, the then Secretary for Health announced that he was setting up an independent Royal Commission on Long Term Care. The intention was to investigate the options, both short term and long term, for finding a sustainable system for the UK to fund LTC for the elderly. During 1998, the Secretariat of the Royal Commission invited the author, together with a few members of the UK Actuarial Profession's Working Party, to discuss the projection model and findings as part of their fact-finding exercise. The UK Actuarial Profession subsequently submitted the research to the Royal Commission as written evidence and the Royal Commission published its final report in March 1999 (Royal Commission on Long Term Care, 1999).

In 2005, the author was commissioned by the Institute for Public Policy Research (IPPR) to update the projection work so that it could be incorporated within a report which the IPPR had been asked to prepare for the Disability Rights Commission (Rickayzen and Karlsson, 2005). The results are included in the final report which was published in March 2007 (IPPR, 2007). The updated healthy life expectancy data suggested that the central set of assumptions had moved from Basis C to Basis O. The difference in the two sets of assumptions is that, under Basis O, we assume that the probability that a person who is currently disabled deteriorates in health during the following year decreases over time; under Basis C, we assume that this probability is constant over time.

The author has also used the model commercially to advise a large insurance company over the securitisation of equity release mortgages for a major equity release initiative they were about to embark upon. Indeed, the work described in Chapter 4 was used to satisfy the regulators that the insurer's arrangements were satisfactory. On another occasion, the author has advised an actuarial consultancy over particular LTC problems their clients were facing.

In September 2007, the author was invited to attend a meeting with representatives from the Strategy Unit of the Cabinet Office and the Department of Health to discuss the LTC projection model. The attendees wanted to understand better the model that had been used, and use the work to inform their thinking regarding a new Government initiative in respect of the funding of LTC in the UK.

As mentioned above, the research described in Chapter 4 has given rise to further research carried out by the author, and by others. This is described below.

As well as using the model to analyse disability-linked annuities in Chapter 5, the author has used the model in three other co-authored papers: Karlsson et al (2006a), Karlsson et al (2006b) and Karlsson et al (2007).

In Karlsson et al (2006a), the projection model was used to estimate the demand patterns of LTC by the older UK population over the next four decades. By considering the likely levels of informal care provided over the period (eg unpaid care carried out by family and friends), it is possible to deduce the expected level of formal care required. From this, the expected burden on public finances caused by LTC can be determined. The results suggested that the demand for long term care in the UK will increase substantially in 2010, and will reach a peak in around 2040. According to the research, the most significant increase in the demand for LTC will be amongst people requiring informal care (from around 2.2 million today to 3.0 million in 2050). When the more pessimistic assumptions were adopted, the results showed that it is quite possible that there would be a shortage of informal carers available over the period. The results described in this paper are, therefore, potentially quite significant from a public health policy perspective.

In Karlsson et al (2006b), the authors consider a theoretical new type of LTC insurance product where an individual's personal circumstances are taken into account within the policy (ie the person's levels of income and assets, and whether the individual has a spouse capable of providing informal care should the individual need it). The benefits payable from the policy then bridge the gap between the level of care which the individual's circumstances allow him or her to obtain, and the required level of care. The morbidity rates implicit within the LTC projection model were used in the calculation of the theoretical premiums that should be charged. As expected, such a top-up product, when compared with a standard LTC insurance policy, would result in more affordable premiums. The theoretical premiums measure the extent to which individuals' own circumstances mean that the individual is, effectively, over-insured when taking out a standard LTC policy.

In Karlsson et al (2007), the LTC systems used by the UK, Sweden, Japan and Germany were compared. The LTC projection model was used to calculate the effects of importing each of the other three countries' systems for financing LTC into the UK. The resulting position was then considered from a taxation point of view. In addition, the distributional consequences (ie comparing the relative impact of each system on the two sexes and on the different generations) were assessed. The results suggested that the German system could be discounted as an improvement on the UK system since it uses a regressive method of financing. The debate over viable alternatives to the UK system was therefore restricted to considering a general tax-based system as used in Sweden or a compulsory insurance system as used in Japan. The research suggested that if any of the other three systems were introduced in the

UK, the rate of taxation required to fund the LTC arrangements would need to increase.

Other researchers have used the LTC projection model in their work: Rossi (2004) and Leung (2004). These pieces of research are discussed briefly below.

In work supervised by the author, Rossi (2004) used the morbidity rates implicit within the model to assist him with his research. In particular, Rossi was measuring the relative importance of process risk and parameter risk for an insurer writing a portfolio of disability-linked annuities similar to the type of product described in Chapter 5. The approach adopted followed the same lines as the work described in Chapter 3 for IP business.

Leung (2004) adapted the LTC projection model to apply it to the population of Australia. He was then able to project the future LTC costs in Australia over the period from the present to 2051.

6.1.4 LTC research in Chapter 5

There has been a great deal of interest in the work described in Chapter 5. The preliminary results were presented to the actuarial consultancy firm, Watson Wyatt Worldwide, in 2002. The final results, as quoted in Rickayzen (2007), were presented at the Healthcare Conference organised by the UK Actuarial Profession in Manchester in May 2007. Both presentations generated a significant amount of positive comment, and there was some interest from reinsurers present at the latter event in investigating the disability-linked annuity product further.

In addition, as mentioned in 6.1.3, the author was invited to attend a meeting at the Strategy Unit of the Cabinet Office and the Department of Health to discuss the ways in which LTC could be provided in the future in the UK. The people present were interested in the product as they could recognise that it could be an effective tool for moderately to highly affluent individuals in making their own satisfactory LTC provision.

The representatives were also interested in pursuing the author's suggestion of incorporating the disability-linked annuity within the State pension. This could be done in a cost neutral way, for example, by reducing the initial level of State pension to compensate for the enhancement on future disability. As demonstrated in section 9 of Chapter 5, the initial health status of an individual is relatively unimportant since the longevity and morbidity risks operate in opposite directions.

Alternatively, the State could subsidise the enhancement element of a disability-linked annuity offered by an insurance company by allowing tax relief on the single premium payable. If this happened, then it would be quite feasible for the arrangement to be extended so that occupational pension schemes could offer disability-linked annuities to retirees as an alternative to traditional life annuities.

This approach could very easily be incorporated within a defined contribution arrangement which, in recent years, has become the most common type of occupational pension scheme to be offered in the UK. Individuals would then be offered the option of using their pension account to purchase a disability-linked annuity, with the associated tax relief, when they retire.

For the disability-linked annuity to be offered by insurers, the following must occur:

- the tax aspects of the premium and benefits payable from the product need to be clarified by the UK tax office, HM Revenue & Customs (HRMC)

- the claims management issues need to be satisfactorily resolved by the insurer (eg the way in which individuals claiming the enhancement should be assessed and monitored).

Provided these issues can be dealt with successfully, the disability-linked annuity could be a very attractive and innovative product for both individuals and the State in meeting future LTC requirements.

It has been seen that the LTC model described in Chapter 4 has proved to be a powerful tool in tackling various LTC-related research projects (eg the work described in Chapter 5). However, it should be noted that the data used to obtain the morbidity rates date back to the mid-1980s. They might therefore be considered out of date. Nevertheless, due to the size of the OPCS dataset and the fact that the disability information was obtained through objective tests rather than self-reporting, the dataset remains valuable. Indeed, the author understands that re-insurance companies still use the OPCS data in their pricing and reserving work. However, in recent years, an English Longitudinal Study of Ageing (ELSA) has been set up at the Institute for Fiscal Studies. This is a longitudinal study which tracks a cohort of lives born before 1952 and analyses their life experiences (including health status) every 2 years. To date, two waves of data have been produced (ie. for 2002/3 and 2004/5). It is hoped that, in time, this dataset will be a valuable aid in refining further the parameters incorporated within the Rickayzen and Walsh (2002) LTC projection model.

6.2 Overall summary

The four papers contained in this thesis illustrate some of the ways in which multiple state models can be used by actuaries in the field of health insurance. The level of detail obtained from calculating transition intensities, and associated probabilities, enable sensitivity analyses to be performed in a straightforward manner. Such models are particularly powerful for products which depend in some way on the time spent in each state.

The overall conclusion of the thesis is that multiple state models have a significant part to play in health insurance in terms of both calculating premiums and reserves, and in measuring risk. Hence, the use of multiple state models should be viewed as being an important technique by health insurance actuaries when considering the broad array of risk management tools at their disposal.

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