Agent-Based Modelling in the Insurance Industry: an Exploration of Emergent Systemic Risk

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To my Mum,

My whole life I have always known that whatever else happened, you would always be on my team, and so you have been throughout these difficult last few years. My biggest supporter and my role model, you inspired me with your analytical mind and love of learning - and your ubiquitous spreadsheets!

It means so much to me that you died knowing that I had finished writing this. I know it gave you joy. You told me you were proud of me.

You won't be here to see the next adventure. But for every new peak I climb, I will know that I reached the mountain because I first stood on your shoulders.

You loved and are loved. Always.

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Declaration

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Abstract

The insurance market contains systemic sources of risk and bias which emerge from the interactions of the agents operating within that market. This type of risk is often not considered in traditional statistical or competitive pricing models. In this thesis, agent-based simulation models (ABMs) are used to investigate and analyse sources of emergent systemic risk.

Customers' opinions of insurer service quality influence their loyalty and are often spread via word-of-mouth networks. An ABM is used to examine patterns that might arise from this phenomenon and parameterised with empirical data. The existence of the network acts as a persistent memory, causing a systemic bias whereby an insurer's early reputation achieved by random chance tends to persist and leads to unequal market shares. This occurs even when information transmission rates are low. This suggests that newer insurers might benefit more from a higher service quality as they build their reputation. Insurers with a higher service quality earn more profit, even when customer preference for better service quality is small. The impact of this systemic effect is exacerbated under a new regulation which bans the practice of charging renewing customers more than new customers.

The winner's curse is a systemic under-estimation of risk caused by imperfect information. Insurers that have under-estimated risk are more likely to be willing to offer lower prices and therefore win more business. The systemic estimation bias caused by the winner's curse also impacts stochastic capital models commonly used by insurers to assess risks and manage capital. This leads to capital requirements which are more often underestimated than overestimated. ABM simulations show that there is increased parameter uncertainty in capital estimation when there are either more competitors or fewer customers. Features such as higher customer heterogeneity, higher renewal rates, and increased customer tendency to seek quotes from a greater number of insurers, all functionally create a similar situation and worsen the impact of the winner's curse. An insurer should consider the impact of the systemic estimation bias caused by the winner's curse when setting risk and capital management strategies.

An ABM is used to investigate heterogeneous insurer strategies for a market where premium is determined by the balance of supply and demand. Insurers follow either a boundedly rational strategy, or a chartist strategy where market premium is extrapolated from recent trends. As the presence of chartists is increased, the model demonstrates that the market becomes more volatile. Chartist insurers often take better advantage of this disruption and make a higher profit than the rationalists. However, chartist performance is also notably much more volatile. As a result, rationalists remain the dominant choice in an adaptive market where agents may dynamically select strategies. This model suggests that which strategy is 'best' depends on the current situation in the market. For insurers primarily driven by profit, a chartist strategy may be optimal. Insurers who value stability may prefer a rationalist strategy.

Finally, an ABM is constructed as an extension to the model produced by Taylor [*North American Actuarial Journal*, 12(3): 242–26 (2008)] with the aim of establishing a market framework with minimal parameters for use with future work. The model allows for entrants and exits, customer loyalty and price sensitivity, as well as regulatory interventions such as solvency requirements. It also allows for insurers choosing to move either towards or away from the market average, and a strategy where insurers are more willing to take risks when they have a higher capital adequacy. The insurance market simulated by this ABM retains similar dynamics to actual insurance markets including reasonable market premium rates with emergent cyclicality along with stable individual insurer assets. However, the cycles display a slower periodicity than a real-world market.

Chapter 1

Introduction

1.1 Background

There are several definitions of systemic risk. For the purposes of this thesis, systemic risk refers to risks which increase the likelihood of widespread failures and thus threaten the overall stability of the market. Systemic risk arises from the structure and purpose of the system and the connections between the entities operating within that system.

Systemic risk is often considered to be an important issue within the insurance industry. This type of risk is often viewed within the context of specific scenarios which impact multiple companies within the market (Clissitt, 2021).

Financial markets—including the insurance market—often also include systemic sources of risk and bias which emerge from the interactions of the entities (or 'agents') which operate within that market. Danielsson and Shin, 2003 demonstrate the existence of risks arising from the emergent coordinated strategies of the agents within the financial markets. They suggest that the use of standardised models by the agents may lead to systemic risk as agents act in a concerted pro-cyclical fashion.

Bostrom et al., 2015 provide an overview of systemic risk in the insurance market and describe the autopilot problem. This is where the level of trust in model results leads to human agents exercising less judgement when applying the models. Sandberg (2015) expands on this concept by discussing a variety of potential sources of systemic risk related to the modelling process. The author groups this type of systemic risk into three categories:

- Modelling: It is common for insurance companies to purchase catastrophe and economic scenario models from a small number of specialist vendors. The autopilot problem occurs when too much trust is placed in these off-the-shelf models. It also means that a large proportion of insurers are likely to have similar limitations to their models relating to their estimation of this kind of risk.
- Organisational: Regulatory requirements and other market-wide conventions may impact the way that insurers construct their risk models. For example, the Solvency II directive requires insurers to be able to justify their modelling choices (EU, 2009). This encourages insurers to adopt a standard approved approach. Again, this is likely to lead to insurer models which contain similar weaknesses. Additionally, insurer

strategies in response to regulatory requirements may become synchronised.

- Behavioural: There are a number of known biases in human behaviour which may cause human agents to display similar boundedly rational model-based decision-making. For example, a Chief Underwriting Officer who receives risk reports daily instead of quarterly is likely to place too much emphasis on daily fluctuations, causing a systemic market overreaction to rate changes.

It is usual for insurers to estimate risk and base strategy on an internal analysis of their own historical data in isolation from market-wide effects (Parodi, 2014; Kravych, 2013; Taylor, 2012). As a result, these kinds of emergent biases are not well-understood or included in these traditional models.

This thesis aims to investigate and model some potential sources of this kind of systemic risk in the insurance industry and their impact on risk estimation and business strategy.

1.2 Game Theory Approach to Competition

One commonly used approach to analysing competition is game theory, which models interactions as a logical game between rational agents to calculate a long term Nash equilibrium solution. For example, a life insurer must set the rates of acceptance or rejection of policyholders, who may be healthy or unhealthy and can be asked to take a medical exam. The insurer can adjust the underwriting process to vary probabilities of acceptance or rejection to maximise payoff, which occurs when the expected payoffs are equal for healthy and unhealthy insureds (Lemaire, 1980).

Under a simple Bertrand model, there are two insurers making identical products for identical marginal costs and setting prices simultaneously. Customers buy from the lowest price, or randomly select among multiple lowest if the insurers offer the same price. Under this model the best response for both companies is to charge just less than the other's price, with a minimum of their marginal cost. Equilibrium occurs when both firms are setting a premium equal to the cost (Bertrand, 1883).

The probability of renewal can also be assumed to have some 'stickiness' to it, where customers have preferences and are reluctant to switch products. In that case it is possible to find an equilibrium that is higher than the break-even premium. The equilibrium point is found to increase with break-even premium, solvency coefficient, claims volatility, and expense rate; it decreases with a decrease in lapse rate and capital (Dutang et al., 2012).

Since customers have preferences, if an insurer can introduce an innovation that is more attractive to a lower risk group of customers, it will change the equilibrium point for both insurers in the market. It is also possible for an insurer to make a strategic move and manipulate the game to gain an advantage through the use of commitments, threats, or promises; the effectiveness will depend on the insurer's credibility. If the consequence function used by an insurer looks beyond a single year to assess its maximum lifetime value given customer loyalty, this can result in a situation which does not generate an equilibrium point (Warren et al., 2012).

The application of game theory analysis requires the definition of a utility function to be maximised. In the earlier examples, this is taken to be the expected level of profit. It could also include an allowance for the amount of capital tied up in reserves. This is found to give an optimal premium mainly based on: the break-even premium; the company's volume of business of the preceding year; and the expectation of the market average premium, so there is some allowance for competition (Pantelous and Passalidou, 2017).

In practice however, rationality is bounded by imperfect information. Insurers do not know either the underlying risk or the demand behaviour of their customers and must estimate this information from historical data. In that case, it is possible that an equilibrium solution does not exist, which suggests that the game theory approach may be insufficiently robust for modelling real markets (Rothschild and Stiglitz, 1978).

The existence of limited information, differing and unknown lifetime goals, and market manipulation strategies limit the use of game theory as a tool for examining extreme scenarios. An equilibrium assumption does not predict the existence of underwriting cycles, which are a well known feature of insurance market premium.

Takahashi and Terano (2003) use an agent-based simulation model (ABM) containing both rational fundamentalist traders and trend-following technical traders. These traders select their willingness to supply or demand based on their heterogeneous strategies. They find that the trend followers destabilise the equilibrium and are able to take advantage of the resulting volatility, outperforming the fundamentalists in the long term. Similarly, traders displaying a bias towards overconfidence have a large effect on the market and can also obtain excess returns. This suggests that following a rational stance may not be best strategy.

Ingram, Tayler, et al. (2012) and Ingram and Bush (2013) use psychological theories to introduce the idea of plural rationality. They show that it is possible to model four different attitudes towards risk, and that each of these attitudes perform better or worse under particular market conditions.

These papers suggest that a strictly rationalist approach may not be the best way to model insurer strategy. Additionally, the analysis of the game theory approach often requires unrealistic assumptions about the nature of the market in order to find a mathematically tractable solution. This limits the use cases of this approach.

1.3 Network Approach to Interactions

Network theory examines how the properties of networks affect the dynamics of interactions between agents. As such, a network-based approach may be a useful tool in analysing emergent systemic risk.

Networks are particularly important when dealing with reinsurance markets. In the late 1980s, many Lloyd's syndicates who sold excess-of-loss reinsurance contracts were also purchasing excess similar contracts on their own losses. This resulted in a network with many connections known as the 'LMX Spiral'. When several large catastrophe claims occurred, these connections meant that the same risks came around again to the same reinsurers, and they did not have as much protection as they believed (Bell, 2014).

Reinsurance defaults can spread across reinsurance networks in the same way as credit defaults. Lin et al. (2015) develop a network model of an insurer and multiple possible reinsurers in order to evaluate the cost of this kind of contagion risk. This model includes the benefit of 'social capital', where reinsurers that are better connected are more likely to be reliable and trustworthy. This model is used to estimate an optimal reinsurance strategy for the insurer.

By representing investment companies as vertices on a graph with a capital value depending on the level of investment in other companies, it is possible to analyse the spread of contagion when a company defaults. This is done through use of a simulation method to discover the asymptotic properties of the network. This approach indicates that the key measure for a network's resilience is not dependent on the number of links itself, but by the interconnectedness of nodes with a high number of contagious links (Amini et al., 2016). Similarly, a network model of inter-bank lending finds that although such relationships can improve stability by sharing risk, the systemic risk caused by larger economic shocks are worsened by the existence of interconnected nodes (Ladley, 2013).

The shape of a network and the way that links form clusters can also affect how the network behaves. For example, in a fitness network, the probability that a new node links to an existing node depends on the number of existing links. This results in a power law distribution of the number of links for each company. The shape of a network can affect herding behaviour and the nature of competition and co-operation between nodes (Bargigli and Tedeschi, 2014). In the example of an insurance market, the links would represent which insurance companies that a customer asks for a quote before selecting a policy. In that case, this changes the nature of the competition, as an insurer is only competing against a select group of peers.

Network analysis is a useful tool for analysing how shocks and information travel through interconnected agents. However, on its own, a network-based approach does not incorporate decision-making or strategies which might change the network dynamics as events unfold. Furthermore, if the network shape is more complex or if the equations governing behaviour are not simple and linear, then this kind of analysis can become mathematically intractable. An additional tool will be needed to capture the key elements of a dynamic insurance market with decision-making agents.

1.4 Agent-Based Models (ABMs)

There is an ongoing debate about the use of traditional equilibrium-based models in finance. The assumptions of efficient markets, rationality, and assumed equilibrium, while being true at the macro level most of the time, are not appropriate during times of crisis (The Economist, 2010).

One alternative to equilibrium assumptions is the use of agent-based modelling (ABM). This is a computational technique that makes use of Monte Carlo simulations to project the decisions of interacting heterogeneous agents, allowing the results of each time step to affect the next. The simulated output often displays complex emergent behaviour patterns, which are highly relevant to systemic effects.

Although concepts similar to ABMs have been in existence for many years - for example, Neumann (1966) describes a self-replicating machine model first proposed by the author in the 1940s - the earliest true agent-based models came about in the 1970s. One of the earliest was a model demonstrating how segregated housing neighbourhoods could arise from even a small preference to be near similar neighbours (Schelling, 1971). In the 1980s, a Prisoner's Dilemma tournament designed and run by Axelrod (1997) popularised the use of ABMs to explore the success of competing strategies in dynamic simulations based on game theory. However, the possibilities of agent-based modelling have risen along with computing power, enabling researchers to simulate more complex models within reasonable timeframes.

ABMs have been used to explore many ideas in social sciences, such as investigating the mechanisms involved in producing co-operative behaviour. These have included mechanisms for establishing trustworthy reputations and the social punishment of defectors; the effect of social inequality on creating hierarchies of neighbourhoods; and the development of segregation when households have only a small preference for neighbours similar to themselves (Bianchi and Squazzoni, 2015). One well-known example is the patterns of culture formed when interaction increases the spread of ideas according to the similarities of agents (Axelrod, 1997).

There is a growing and substantial body of work on the application of agent-based models to finance. ABMs have been used by financial companies themselves to inform strategy—for example, a model used by a company on Wall Street for pre-payment patterns in the housing market (Geanakoplos et al., 2012). In economics, ABMs have been used to explore the dynamics of barter systems, the labour market, international trade, and the "tragedy of the commons" (Hamill and Gilbert, 2015).

ABMs can be combined with other tools and informed by existing theories and studies. For example, game theory approaches can be used to inform the calculations used in ABMs. It is also possible to combine ABMs with evolutionary algorithms, where a selection process is used to whittle down the available agents to find the 'best' population (Sarker and Ray, 2011). Networks can be incorporated into ABMs in order to capture the effects of network shapes on interactions (Bargigli and Tedeschi, 2014). Liu et al. (2020) used an ABM to extend a contagion network in order to examine contagion within the context of a network which is changing dynamically in response to market conditions.

There is much less development in general insurance applications, but there is rising interest among the actuarial community in the possibilities of agent-based computational techniques from the field of artificial intelligence and whether these can be applied to actuarial problems (Panlilio et al., 2018).

It can be argued that the general insurance market is a natural fit for using a computational intelligence paradigm as an analysis tool: it features heterogeneous intelligent agents making decisions over time to maximise some reward function based on past experience, for risks that change over time and information that is gathered gradually, features which limit the usefulness of a classical game theory approach (Parodi, 2012).

ABMs have several advantages as tools for exploring this type of systemic risk. They can capture a number of features, such as heterogeneity and irrationality, which are not well reflected in traditional modelling approaches. They can show how collective phenomena come about from interactions (e.g. movement of a traffic jam), and isolate the critical behaviour that leads to the emergent pattern. ABMs are also flexible and can easily be implemented within a modular framework, allowing alternate assumptions to be explored in order to guide policymakers (Pyka and Fagiolo, 2005).

ABMs are useful when there is a possibility of emergent behaviour, such as when (1) individuals behave non-linearly, (2) they can learn and adapt over time, (3) they interact heterogeneously, (4) there is the possibility of instability from larger perturbations, (5) we want to examine properties of the system itself (Bonabeau, 2002). These are characteristics that feature in the problem being explored in this thesis.

The difficulty with ABMs is that the results are very dependent on decision-making assumptions, and the output can be complex to analyse and difficult to validate. There is a trade-off between more realistic features and tractability. ABMs are often used to model human agents, who are potentially irrational, subjective, and complex. This can make ABMs difficult to quantify, calibrate and justify (Bonabeau, 2002). However, this feature is also why ABMs are useful for modelling such systems.

There are three main approaches: history-friendly, indirect calibration, and Werker-Brenner (Fagiolo et al., 2007). The history-friendly approach uses historical data to constrain empirically-consistent initial parameters, interactions, and decision rules. The indirect calibration approach uses comparison of the output with stylised facts to constrain the parameter space to be consistent with the valid output. The Werker-Brenner approach uses a Bayesian method to assess the likelihood of possible model specifications based on the percentage of theoretical outputs comparable with each empirical output; this is a promising approach but requires a lot of data.

Where appropriate, efforts will be made within this thesis to parameterise and validate models from empirical data. Due to these limitations, ABMs are used in this thesis primarily as a tool for exploring patterns and as critical indicators of behaviour rather than as statistical prediction models.

The following sections contain some examples of ways in which ABMs have been used to investigate the problem of systemic risk in insurance and financial markets.

1.5 Modelling Sources of Systemic Risk

Owadally, Zhou, and Wright (2018) use an ABM to examine the underwriting cycle. In each modelled time step, insurers offer prices based on the marginal cost of the business, plus an allowance for the estimated elasticity of demand. Customers select a policy based on their cheapest cost, accounting for preferences for insurers that are closer to them on a market space, excluding insurers that have reached capacity. Losses and premium are then generated for the year and the market loss ratios updated. Under this ABM, underwriting cycles are generated endogenously due to the emergent patterns in the estimated demand elasticity. This demonstrates that imperfect information can alter the behaviour of the market and prevent the market from settling into a static state.

Heinrich et al. (2022) use an ABM to investigate systemic risk within catastrophe insurance and reinsurance markets. The overwhelming majority of insurers within these markets purchase data about the estimated risk of these events from the same same three providers. This paper simulates a catastrophe insurance and reinsurance market and examines the effects of different scenarios of varying diversity of information where the available catastrophe models each underestimate a particular type of loss. They find that lower model diversity increases the rate of bankruptcies and decreases the overall levels of market capital, implying a possible source of systemic risk within the real-world catastrophe insurance markets

The Winner's Curse GIRO working party report (Chan et al., 2009) use a simple simulation model of insurers competing for a single customer. In this model, insurers do not know the underlying risk distribution and must estimate this from historical data. The insurers then offer a price based on the estimated loss distribution. This model demonstrates an effect called the 'winner's curse', where insurers which have underestimated the risk are more likely to win the business of the customer. This leads to a systemic bias whereby loss ratios are consistently higher than expected.

1.6 Insurance Regulation and Systemic Risk

Dubbelboer et al. (2017) implement an ABM focussed on flood risk management with an insurance component. This model is used to explore scenarios of public and private flood risk cover within a London borough and the subsequent effect on homeowners, and is intended as a tool to allow regulators to test strategies for managing flood risk. This paper focuses mainly on the housing market, including homeowner and developer agents, and a government flood reinsurance. There is no insurance market or competitive aspect to the insurance, which is provided by a single insurer agent which prices its business purely on the level of risk.

Aymanns and Farmer (2015) use an ABM to explore the ramifications of a simple financial market where investors make decisions within a regulator-imposed capital constraint based on the value-at-risk (VaR) measure. This leads to a leverage cycle, where market downturns are exacerbated by feedback loops arising from investor decisions. This is not unlike the capital constraints experienced by insurers, and may suggest a similar mechanism related to underwriting cycles. The authors use this model to suggest alternative regulatory regimes.

Owadally, Zhou, Otunba, et al. (2019) extend their earlier model (Owadally, Zhou, and Wright, 2018) (described above) with a framework aimed at testing and analysing model outputs under current market conditions. This is intended to assist regulators in monitoring and responding to cycles by running simulations of various regulation and brand strategy scenarios.

Yusgiantoro et al. (2019) build an ABM of an insurance market with a similar set-up to that used by Owadally, Zhou, and Wright (2018). They extend this model by including an intermediary finance firm and regulation of maximum and minimum pricing rates. This paper finds regulatory strategies that can decrease both volatility of the premium rates and the uneven distribution of market shares in the insurance market.

1.7 Behavioural Bias in Insurance Markets

A key assumption behind many analyses of the effects of competition is that agents act rationally, or at least that any bias is randomly distributed around the 'true' approach and thus will cancel out on average. However, humans possess a number of innate biases in their thinking, particularly around risk assessment, that tend to act in the same direction.

Behavioural finance is a relatively new discipline that aims to examine and quantify the possible effects of behavioural biases in the world of finance. A recent survey was carried out by a working party of the Institute of Actuaries to discover whether actuaries display evidence of similar biases, which would imply that this is also something that could affect insurance (Byrne and Cook, 2016). Overall, the results found that actuaries do show the same biases as other people. However, actuaries show less bias in some areas, and specifically less overconfidence bias and bias related to probability and statistics. In general, age, education, and experience seem to be factors in reducing the impact of bias.

Prospect theory is an alternative to expected utility theory in explaining behaviour. Studies indicate that humans perceive the impact of losses based on the change in wealth given a starting reference point instead of considering only the end result, and that reference point may not be the same as their current level of wealth. Because people tend to use the deductible as a reference point, then a lower deductible is often overvalued by customers in comparison to the gain. This is combined with a distortion in evaluation of loss probabilities and a tendency to evaluate risks in isolation, leading to myopic risk aversion (Kunreuther et al., 2013). This means that a standard loss aversion utility-based model of agent is often not a good match for empirical data of customer behaviour.

The goal-based model of choice is another alternative decision-making theory where people focus on achieving weighted goals rather than maximising some sort of value as in utility or prospect theory. Examples of goals might be: collect money (seeing insurance as an investment); satisfy regulatory requirements; reduce anxiety about risk; reduce regret of disappointment, or sadness at the loss of something; appear prudent or fit in with social norms. Other biases may include: status quo bias, where people resist change; availability bias, where people assess probability by how easy it is to remember examples; or short-term availability of funds leading to a very short-term view of cash flows (Kunreuther et al., 2013).

After large catastrophe events, insurers often raise premiums, seemingly more concerned by recent losses than future probabilities. In one case, regulators stepped in temporarily to slow down rate increases. By the time that insurers were able to revert to high premiums, they refrained from doing so, indicating bias rather than a change in expected probability. In other cases, insurers stopped offering coverage rather than acquiesce to rate restrictions, even though they should expect to make a profit from such prices. This suggests increased anxiety about losses immediately following a large loss, and echoes the customers availability bias, or possibly anxiety-related goals. If it has been some time since a catastrophe has occurred, reinsurance rates tend to decrease (Kunreuther et al., 2013; Vasiljevic et al., 2013; Kleindorfer and Kunreuther, 2000; Pitthan and De Witte, 2021). Nevertheless, insurer behaviour is not yet thoroughly understood. It is possible that more standard risk aversion and utility definitions combined with an aversion to ambiguity can explain some apparent anomalies in the insurance market (Zweifel, 2014).

There are a few examples of ABMs developed to investigate behavioural finance. Haer et al. (2017) use an ABM to analyse flood risk under different assumptions of household behaviour. They find that using a model consistent with prospect theory in place of the more traditional utility-based model has a considerable impact on the resulting levels of risk. The authors conclude that it is important to use an accurate model of human behaviour when modelling underlying risk.

Bertella et al. (2014) use an ABM of a finance market with a single asset where agents set their willingness to supply or purchase based on either a fundamentalist or a chartist trading strategy. Among other uses, this model was used to explore the impact of overconfidence bias by allowing traders to underestimate volatility if they have been performing well in recent years. This paper finds that the inclusion of the bias is able to reproduce key features of real-world markets, such as the existence of market bubbles and crashes. This bias is also likely to exist in insurance markets, particularly around estimation of rare events such as catastrophes as described above.

1.8 Structure of Thesis

In this thesis, ABMs are used to investigate and analyse sources of emergent systemic risk due to interactions between agents in the insurance industry.

Chapter 2 is a reproduction of a paper that was published in the *Journal of Artificial Societies and Social Simulations* (England et al., 2022). This chapter examines the systemic emergent risk caused by interactions between customers. An agent-based model is constructed with two types of agents: customers and insurers. Insurers are price-takers who choose how much to spend on their service quality, and customers evaluate insurers based on premium, brand preference, and their perceived service quality. Customers are also connected in a small-world network and may share their opinions with their network.

This model is used to examine some of the systemic effects of the word-of-mouth network on patterns of customer behaviour, and investigate possible implications of a proposed change in UK insurance regulation. By modelling the network explicitly, the impact of realistic network dynamics and in particular the repeated feedback of word-of-mouth information back into the network can be explored.

The model used in chapter 2 is focussed primarily on customer behaviour, and the insurers are modelled as price-takers. However, in practice insurers will base their premium strategy on their estimation of customer risk, and may offer differing prices to potential customers. Chapter 3 focuses on the insurer behaviour and systemic bias in risk estimation due to an effect called the 'winner's curse'. In this model, insurers interact with each other indirectly by competing for individual customers. Each customer purchases from the insurer offering the lowest price.

The insurer premium is based on their independent estimates of the underlying risk. Consequently, insurers that have under-estimated risk are also more likely to be willing to offer lower prices and win more business. The systemic estimation bias caused by the winner's curse will also impact insurers' capital models, causing estimated capital requirements to become more often underestimated than overestimated. The model results show the resulting capital estimation errors. Additionally, the model is extended to include some market features which impact customer purchase decisions. These are: policyholder heterogeneity; renewal rates; and network shapes, where customers consider premium from a subset of the insurers only.

The model used in chapter 3 assumes that insurers set their prices based solely on the estimated risk-based premium. In practice, although the risk premium is highly influential on insurer premium, insurance premium is also informed by an analysis of the actions of competitors. In chapter 4, an ABM is built which focusses on competitive market supply strategies within a market where the premium moves according to the balance between supply and demand. In this model, insurers have imperfect information about both customer demand and underlying risk distributions. The ABM contains two types of insurers. One type follows a rational strategy within the bounds of imperfect information. The other type also seeks to maximise their utility gain, but base their market expectations on a chartist strategy. Under this strategy, market premium is extrapolated from trends based on past insurance prices. The relative performances of the different strategies are then compared.

Chapter 4 addresses some of the shortcomings of previous chapters by examining competitive insurer strategies under imperfect information. However, this model does not include the influence of customer-to-customer interactions as in chapter 2. The models used so far have been individually designed and do not easily combine into an overall model of a market. There are also some market-level features missing, such as market entrants and exits. Additionally, the insurer strategy does not account for capital management. In practice, insurers become more risk-averse when their capital adequacy is lower.

Chapter 5 begins with a model which captures the dynamics of an insurance market with a minimal number of parameters. This is initially based on the Taylor model (Taylor, 2008), which includes a number of features such as premium cycles and market entrants and exits. However, while the Taylor model replicates realistic market features in the aggregate, it also tends to produce unstable individual insurer premium rates and asset values. To address these limitations, the model is extended by using a new premium mechanism. In the new model, insurers set their premium using the solvency ratio to determine the direction of premium movement, with the size of the change depending on competitive pressures. This extension also allows for more complex insurer strategies whereby an insurer may deliberately choose to move away from the market average.

Finally, the investigations that have been carried out are drawn together and suggestions for further work are described in chapter 6.

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Chapter 2

An Agent-Based Model of Motor Insurance Customer Behaviour in the UK with Word of Mouth

Note: This chapter is a reproduction of a paper that was published in the *Journal of Artificial Societies and Social Simulations* (England et al., 2022).

2.1 Introduction

In this chapter, we begin our exploration into emergent systemic risk by considering first the customer side.

Attracting and retaining loyal customers is a key driver of insurance profit. An important factor is the customers' opinion of an insurer's service quality. If a customer has a bad experience with an insurer, they will be less likely to buy from them again.

However, customer opinions are not formed in isolation. Customers who are seeking a new insurer are likely to seek recommendations. Opinions are shared on review websites. And customers who have a bad experience with an insurer are particularly likely to tell others. In this way, word-of-mouth networks allow information to spread between customers.

In this chapter we build an agent-based model with two types of agents: customers and insurers. Insurers are price-takers who choose how much to spend on their service quality, and customers evaluate insurers based on premium, brand preference, and their perceived service quality. Customers are also connected in a small-world network and may share their opinions with their network.

We find that the existence of the network acts as a persistent memory, causing a systemic bias whereby an insurer's early reputation achieved by random chance tends to persist and leads to unequal market shares. This occurs even when the transmission of information is very low. This suggests that newer insurers might benefit more from a higher service quality as they build their reputation. Insurers with a higher service quality earn more profit, even when the customer preference for better service quality is small.

The UK regulator is intending to ban the practice of charging new customers less than renewing customers. When the model is run with this scenario, the retention rates increase substantially and there is less movement away from insurers with a good initial reputation. This increases the skewness in market concentrations, but there is a greater incentive for good service quality.

2.2 Background

Insurance is a service whereby providers offer compensation payouts if the customer suffers a loss due to a specified type of event. In many countries, some types of insurance are mandatory: for example, all motorists in the UK must have motor insurance against legal responsibility for damage to another motorist's person or property. Insurance is a substantial part of the financial services industry and an important part of social fabric.

It is common for general (i.e. non-life) insurance contracts to cover a fixed term, usually one year. At the end of the term, customers receive a renewal offer from their current insurance providers. Since searching for new quotes costs both time and effort, many customers choose to accept their renewal price without searching. UK motor insurance customers consider the cost of searching for a new insurer to be worth 15% of the average cost of their policy (FCA, 2020). It costs insurers less to retain an existing customer than to attract and process a new one, and insurance is a highly competitive industry. Attracting and then retaining loyal customers is therefore often a better strategy than competing on price alone. It is common practice for insurers to offer a discount for new customers, often pricing below the odds, then rely on loyalty and gradually increase the price. In essence, loyal customers are used to cross-subsidize new customers (FCA, 2020).

The probability of renewal is commonly modelled as a logit relationship, and existing literature has employed various techniques to identify which customers are likely to be loyal. Smith et al. (2000) compares the performance of logistic regression, decision trees, and a neural net in classifying and predicting the loyalty of motor insurance customers. Günther et al. (2014) builds on the logit regression model, using generalised additive models to allow for non-linear relationships. Zhang et al. (2017) combines a neural net with a generalised linear model to take advantages of the strengths of both approaches.

Customer service is considered an important factor in customer retention in many industries, and this link has been confirmed in a number of empirical case studies in insurance (Ansari and Riasi, 2016; Ghodrati and Taghizad, 2014; Tsoukatos and Rand, 2006). These papers indicate that though price remains the main factor in customer choice, customer service is a significant part of customer decision making. Customers are also highly influenced by word-of-mouth recommendations from friends (Ghodrati and Taghizad, 2014; Tsoukatos and Rand, 2006). Berger (1988) investigates this using a model where insurance customers remain with a firm unless they have a bad experience, which happens infrequently as claims are also infrequent. When they decide to switch, they choose a new insurer when a friend makes a recommendation. Interestingly, as the rate of recommendation transmissions is increased, the number of dissatisfied customers decreases but the average quality decreases. This is because a higher number of dissatisfied customers switch providers, including to lower quality firms. Because customers usually do not have a customer experience, the low quality is not often discovered, and so many of these customers felt satisfied. However, this paper does not include an allowance for pricing considerations or other customer preferences, and does not model the network explicitly.

Berger et al. (1989) bases customer decisions on price as well as service. In this model, customers have imperfect information and only gradually discover the available prices through word-of-mouth. Customers favour renewals unless they believe there is a sufficiently big price differential, and disatisfied customers requires a lower price gain to be willing to switch. By fitting this model to real-world data, the authors estimate that customers have a high speed of information transmission, but a high reluctance to switch. This paper does not explicitly model the network, choosing instead to use a formula for the rate of information spread. Since this paper was written, it has become easier for customers to get hold of information, and for social networks to allow the transmission of word-of-mouth information.

Conventional analytical approaches may be insufficient to capture network effects, as some features are emergent properties of non-linear interactions. Agent-based models (ABMs) have been used within sociology to model the spread of market innovations and social opinions (Bianchi and Squazzoni, 2015; Squazzoni, 2012). Kowalska-Styczeń and Sznajd-Weron (2016) use an ABM to examine the effect of different word-of-mouth patterns on the resulting market shares. Goldenberg et al. (2001) use a cellular approach to simulate 'strong' ties within a group and 'weak' ties between cells, finding that though strong ties have higher influence within groups, weak ties are as important as strong as they are responsible for new word-of-mouth information into the groups.

The general insurance market features interacting heterogeneous agents making decisions over time to maximise some reward function based on past experience. This seems to be a promising fit for an agent-based modelling approach (Palin et al., 2008; Mills, 2010; Parodi, 2012). There are currently very few examples of ABM literature in the field of insurance, though the possibilities of ABMs have attracted the interest of several actuarial practitioners.

Crabb and Shapiro (1996) build a simulation game with the aim of educating students by allowing them to set the strategy of a motor insurance company and compete against other agents. Insurance World 2 (Gionta, 2000) is a simulation built by the AI analysis company Complexica for a consortium of insurance and reinsurance companies to examine the consequences of different strategies in a catastrophe reinsurance market. Alkemper and Mango (2005) build an ABM of a property-casualty reinsurance market where capital requirements act as a capacity constraint on supply and the price is then calculated from a demand-supply curve. This simple setup produces price cycles from the competitive interactions. However, it is not possible to obtain a detailed description of these models and their parameterisation.

Dubbelboer et al. (2017) implement an ABM focussed on flood risk management with an insurance component. This model is used to explore scenarios of public and private flood risk cover within a London borough and the subsequent effect on homeowners. This paper focuses mainly on the housing market, including homeowner and developer agents, and a government flood reinsurance. There is no insurance market or competitive aspect to the insurance, which is provided by a single insurer agent which prices its business purely on the level of risk.

Owadally, Zhou, and Wright (2018) use an ABM of an insurance market to investigate possible mechanisms for the cyclical behaviour exhibited by real-world insurance premiums. This model contains two types of agents: insurers and customers. Insurers adjust their initial risk-based premium according to an estimation of the current elasticity of demand. Customers select their preferred insurers based on a combination of the cost and their own preference for particular brand or product features. This ABM was found to produce cycles similar to those seen in the real-world as an endogenous feature of the competitive mechanism. Owadally, Zhou, Otunba, et al. (2019) further extend this model with a framework aimed at assisting regulators in monitoring and responding to cycles by running simulations of various regulation and brand strategy scenarios parameterised with the current market position and introducing extensive time-series analysis of the outputs. These models are mainly concerned with the premium behaviour of insurers, and do not explore the impact of insurer quality or the network effects of social influence on customer decisions. However, these papers are notable in the field of insurance ABMs for introducing a simple yet credible model of both consumer and insurer behaviour within a competitive system, and producing outputs which are validated against historical real-world market level premium and loss data.

Heinrich et al. (2022) use ABMs to investigate systemic risk within catastrophe insurance and reinsurance markets. The overwhelming majority of insurers within these markets purchase data about the estimated risk of these events from the same same three providers. This paper simulates a catastrophe insurance and reinsurance market and examines the effects of different scenarios of varying diversity of information where the available catastrophe models each underestimate a particular type of loss. They find that lower model diversity increases the rate of bankruptcies and decreases the overall levels of market capital, implying a possible source of systemic risk within the real-world catastrophe insurance markets.

The examples mentioned so far are focussed on the insurers or regulation rather than on customer renewal and insurer selection. Boucek and Conway (2003) suggests a model where customers will renew with an insurer if their new premium has decreased or increased by only a small amount, and become more likely to seek further quotes the more their premium has increased. Customers are heterogeneous and possess various factors which insurers might use to assess their risk. e.g. age, gender, level of education. This model does not include other factors such as satisfaction with service, though the author does note its potential importance. The paper also mentioned the need for industry data and does not parameterise the model, though it does demonstrate some example scenarios. Ulbinaite and Le Moullec (2010) propose a similar ABM for life insurance customer behaviour, though again this model is described in theory but neither parameterised nor implemented. In this paper, purchase decision is two stage: firstly, the customer decides whether to purchase insurance at all, based on a linear combination of various factors which influence their perception of the value of the insurance versus its affordability. In the second stage, customers decide which insurer to purchase from based on their opinion of the quality of the insurer. Though this paper includes interaction with social networks as a factor in customer decision, it does not specify how this interaction would be calculated or how such a network would be modelled.

In this chapter, we develop an ABM to simulate a word-of-mouth network within an insurance network where customer choices are influenced by their opinion of customer service quality, and parameterise the model with data from the UK motor insurance market. We will use this model to examine some of the systemic effects of the network on patterns of customer behaviour, and investigate possible implications of a proposed change in UK insurance regulation. By modelling the network explicitly, we can explore the impact of realistic network dynamics and in particular the repeated feedback of word-of-mouth information back into the network. This allows us to explore systemic effects not captured in early models such as those of Berger (1988) or Berger et al. (1989).

We find that the existence of the network acts as a persistent memory, causing a systemic bias whereby an insurer's early reputation achieved by random chance tends to persist and leads to market concentration with a few insurers holding large market shares. We demonstrate that it only takes a very low rate of word-of-mouth transmission for this effect to significantly impact market-wide customer decision-making. In a market where insurers are of varying quality, we discover that higher quality insurers make a higher profit despite offering higher prices. This occurs when customers have only a weak preference for better service quality. Finally, we explore the impact of a new regulation change and discover that this may lead to lower competition and an increasingly skewed market concentration, but potentially also incentivise higher service quality.

Based on these findings, we can conclude that the potential impact of the word-of-mouth network on customer decision-making and the resulting systemic biases is a significant one. These findings should be considered by both insurers considering strategies for attracting and retaining customers, and regulators who are assessing possible impacts of a change in the regulation of insurance pricing practices.

2.3 Model

2.3.1 Overview

The aim of this model is to explore patterns of insurance customer choices arising from the existence of a word-of-mouth mechanism. As such, the design focuses on features observed by the customers. The model therefore does not attempt to replicate the internal workings or processes affecting the strategic decision-making processes of the insurance companies such as reinsurance or credit risk. We also assume that there is no claims fraud and that policies have similar cover such that all insurers will sample from the same claims distribution.

The ABM contains two types of agents: insurers and customers. These act within the environment of a motor insurance market. At each simulation, the model undergoes the following steps:

1. Network generation: At the start of the simulation, the model generates a small world network of social links between the customers, and randomly assigns each customer to an initial insurer (sec. 2.3.4).

Then in each timestep:

2. Insurer spending: Insurers choose how much to spend per customer on their level of customer service up to some maximum level (sec. 2.3.3). The more they spend, the greater the chance that any given customer interaction will be a positive and not a negative experience for the customer (sec. 2.3.3).
3. Insurer premium: As this model does not focus on insurer premium strategy, the market premium is set exogenously and follows a simple cyclical pattern similar to those found in existing research (Fenn and Vencappa, 2005) fitted to empirical data, with a stochastic error term (sec. 2.3.2). Insurers will also add a margin to cover their spending cost and profit markup (sec. 2.3.3). Prices for new customers are discounted relative to prices for renewing customers (sec. 2.3.3).
4. Customer purchases: Customers decide whether to renew based on a logit probability function (Günther et al., 2014) based on the change in cost over the previous year (sec. 2.3.4). This is parameterised to give an average chance of renewal that matches empirical data. If they do not renew, they will purchase from the insurer that offers them the lowest total cost (sec. 2.3.4).
5. Claims: Loss events—e.g. theft or traffic accidents—are modelled probabilistically using a Poisson frequency and Gamma severity (sec. 2.3.4). If a customer experiences a loss, they make an insurance claim. Their interaction with their insurer's customer service which may generate a good or bad experience with probability based on the amount spent on customer service. (sec. 2.3.3)
6. Customer word-of-mouth information sharing: Customer service experiences tend to perpetuate across networks as customers tell their friends of their experiences or experiences they've heard about (sec. 2.3.4). The influence of these opinions is calculated using a method similar to many opinion dynamic models (Deffuant et al., 2002) (sec. 2.3.4).
7. Customer cost calculations: The customers re-assess insurers based on a cost function. Similar to Owadally, Zhou, and Wright (2018), this is not just based on pure price. There is an allowance for preferences, and a cost factor based on a customer satisfaction assessment of each insurer (sec. 2.3.4).

Figure 2.1 is a swimlane diagram representing the flow of processes in the model and

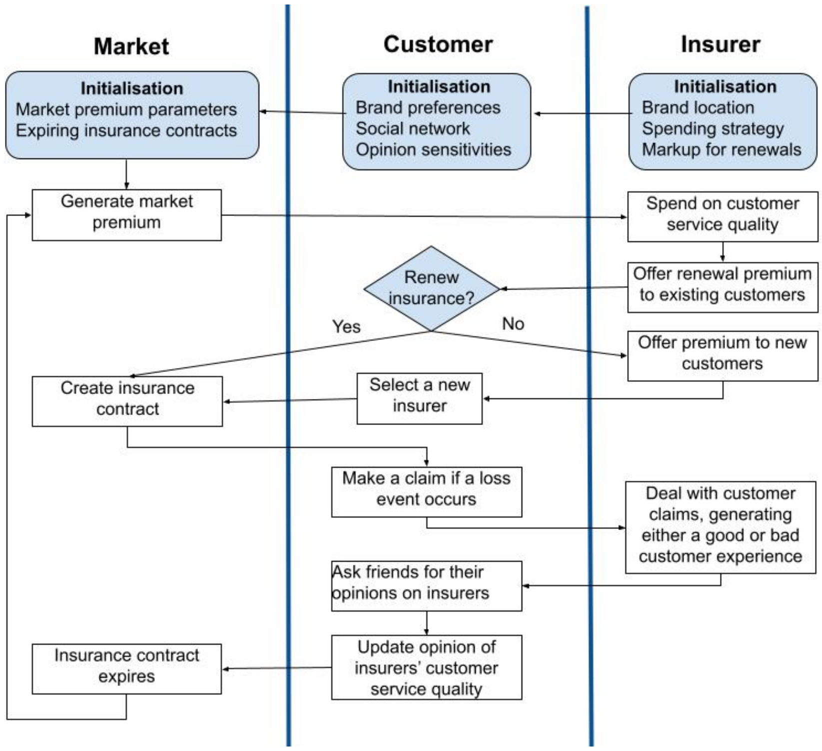

Fig. 2.1 – Swimlane overview of the processes in the customer word-of-mouth ABM

which agent or environment is responsible for each step. The calculations carried out at each step are described in more detail below, followed by an explanation of the data and model parameterisation.

Figure 2.2 shows the pseudocode for the initialisation at the start of the simulation, and Figure 2.3 demonstrates the pseudocode for a timestep. The parameter notation has been kept consistent with the model descriptions below. The pseudocode for the functions referenced are shown alongside the relevant sections. Additionally, the code has been made available on CoMSES (England et al., 2021).

2.3.2 Market

Description

In this model, agents interact within the environment of an insurance market. The market contains two types of agents: insurers and customers. Each timestep, all customers will create an insurance contract with one insurer.

The model is based on the UK motor insurance market. In the UK, it is mandatory for all motorists to purchase motor insurance, so demand is stable. It is therefore reasonable to assume that all customers will purchase an insurance contract each timestep. There are no exits or entrants in the model.

The expiration of the insurance contracts at the start of each timestep triggers the customer agents to seek premium quotes and select an insurer with which to form a new contract. The premium quotes are described further in section 2.3.3, and the customers' insurer

```
Function INITIALISE(P-1, P-2, θ0, θ1, θ2, σM, Ei, E, R, T, a, b, kQ, kD, kW, ϕ, μf, μs, σs, pW, nK, β)

links ← GENERATE_WATTSSTROGATZ_NETWORK(nk, β)
for each Insurer i
  LI,i ← 2*i/nI
  set Insurer i Parameters(Ei, E, R, T, LI,i)
end for
for each Customer j
  LI,i ← 2*j/nC
  set Customer j Parameters(a, b, kQ, kD, kW, φ, μf, μs, σs, pW, LC,j, links[j])
  chosen_insurerj0 ← randomly selected insurer
  for each Insurer
    Dij ← Min(|LI,i - LC,j|, 2 - |LI,i - LC,j|)
    Qij0 ← 0
  end for
end for
set Market Parameters(P-1, P-2, ϑ0, ϑ1, ϑ2, σM)
```

Fig. 2.2 – Pseudocode for initialisation at the start of each simulation. See Figure 2.3 for the pseudocode describing the actions carried out in each timestep t.

```
Function RUNTIMESTEP()

Pt ← ϑ0 + ϑ1Pt-1 + ϑ2Pt-2 + NORMAL(0,σM)
for each Insurer i
  for each Customer j
    Pijt ← (Pt + Ei)(1 + R)^Min(T, no. years customer j has been with insurer i)
  end for
end for
for each Customer j
  chosen_insurerjt ← Customer[j].SELECTINSURER(List of Pijt)
  claimsjt ← Customer[j].GETCLAIMS()
  Customer[j].UPDATEWOM()
end for
for each Customer j
  List of Qijt ← Customer[j].UPDATEOPINIONS()
end for
```

Fig. 2.3 – Pseudocode for running a single timestep. This function is carried out for each timestep after the model has been initialised (see Figure 2.2 for the initialisation pseudocode). Figures 2.4, 2.5, 2.6, and 2.7 show pseudocode for the Customer object functions *SELECTINSURER*, *GETCLAIMS*, *UPDATEWOM*, and *UPDATEOPINIONS* respectively.

selection process in section 2.3.4.

As this model does not focus on premium behaviour, the model does not attempt to replicate the individual insurer strategy that would produce the patterns seen in competitive premium rates. Instead, the market premium is assumed to follow an exogenous stochastic process. The insurers are assumed to be price takers, and they use the market premium as a basis for selecting their own premium to offer to existing and new customers.

Inputs and Initialisation

- n_C: The number of customer agents seeking to purchase insurance.
- n_I: The number of insurer agents operating in the marketplace.
- $\overline{P}_{-1}$: The market premium (price per customer) in the year prior to the first time step.

- $\overline{P}_{-2}$: The market premium in the year two years prior to the first time step.
- θ_0: The constant term in the market premium calculation.
- θ_1: The dependency of the current market premium on the previous year's market premium.
- θ_2: The dependency of the current market premium on the market premium two years previously.
- σ_M: The stochastic variance of the market premium around its expected value.

At initialisation, each customer is randomly assigned to an existing insurer. This is the insurer they will select if they choose to renew their contract in the first time step.

Calculations

Setting the market premium rate

It is common to model insurance market premium as an AR(2) process (Owadally, Zhou, and Wright, 2018; Owadally, Zhou, Otunba, et al., 2019; Boyer and Owadally, 2015; Fenn and Vencappa, 2005; Harrington and Niehaus, 2000; Cummins and Outreville, 1987). Using this formulation, the market insurance price per customer $\overline{P}_t$ for timestep t is calculated exogenously according to equation (2.1) below.

$$\begin{aligned} &\overline{P}_t = \theta_0 + \theta_1 \overline{P}_{t-1} + \theta_2 \overline{P}_{t-2} + \varepsilon_t \\ &\text{where } \varepsilon_t \sim N(0, \sigma_M) \end{aligned} \tag{2.1}$$

Outputs

- $\overline{P}_t$: The (exogeneous) market premium in timestep t, calculated according to equation (2.1).
- $\overline{\Pi}_t$: The market profit in timestep t. This is the sum of the profits made by the insurer agents in time t.
- $\overline{R}_t$: The market renewal rate in timestep t. This is calculated as the proportion of customers in time t who choose to renew their insurance contract with their existing insurer.

2.3.3 Insurers

Description

The insurer agents provide motor insurance cover for loss events experienced by their customers, such as theft or damage caused by a car accident. As their aim is to make a profit, insurers will usually charge customers more than the expected value of their losses. Because they have pooled the risks from many customers, the relative volatility of an insurer's total loss is less than that of an individual customer.

Insurers are modelled as price takers, setting their premium according to the prevailing market premium. They also charge a margin for their expenses, and an additional margin for profit. To entice new customers, insurers remove the extra profit margin for new quotes, effectively using the renewals of existing customers to subsidise new customers.

When a customer makes a claim, they interact with the insurer's customer service department. This interaction can be either a good or bad experience for the customer, and this outcome will directly alter the customer's opinion of the insurer. The more money the insurer has chosen to spend on their customer service quality, the higher the probability that this interaction will be a positive experience for the customer.

Inputs and Initialisations

The insurer agents are indexed by i.

- E: The maximum effective insurer spend per customer on customer service quality.
- E_i: The amount spent by insurer i per customer on customer service quality at the start of each timestep.
- R: The profit markup factor applied to existing customers renewing their insurance contracts.
- T: The maximum time over which the renewing markup factor applied to existing customers is increased.

Similarly to Owadally, Zhou, and Wright (2018), the insurers are also spaced out evenly across a 1-dimensional abstract preference space in a random order. This represents differences in branding or product design features. As in the Owadally, Zhou, and Wright (2018) model, this space is circular so that the value bounds have zero distance between them. The location on this space is valued between $[0,2)$. This has been scaled so that the distance between any two points must lie within $[0,1]$. The location assigned to an insurer i is denoted $L_{I,i}$.

Insurer i spends an amount E_i on customer service quality per customer at the start of each timestep t. The higher this spend, the better the quality of customer service. It is assumed that there is an upper bound past which extra spending will have little effect on the outcome of customer service interactions; this is the maximum spend E.

In the real world, insurers might choose to vary this spend, though we might expect that a particular brand would not generally wish to vary this by a large amount every year. In this ABM, the focus is on customer behaviour rather than insurer strategy, so each insurer i is assigned a single value which they spend every year.

Calculations

Setting the price of insurance for new customers

Insurers are modelled as price takers, so they base their premium on the market premium. They also add an expense margin to cover the cost of their customer service expenditure. Thus, the base premium charged to a new customer by insurer i in time step t is set according to equation (2.2). Consequently insurers trade-off between attracting customers with a lower premium or with a higher service quality.

$$P_{it} = \overline{P}_t + E_i \tag{2.2}$$

Offering renewal price to existing customers

```
Function Customer.GETCLAIMS()
  freq ← POISSON(μ_f)
  sevBeta ← μ_s/σ_s^2
  sevAlpha ← μ_s*sevBeta
  sev ← GAMMA(freq*sevAlpha, sevBeta)
  newService ← 0
  for i in 0 to freq -1
    if(UNIFORM(0,1) <= E_i/E for chosen insurer i)
      newService ← newService + 1
    else
      newService ← newService -1
    end if
  end for
  return severity
```

Fig. 2.4 – Pseudocode for claim generation and service experience

It is common practice for insurers to entice new customers with a lower initial premium before then increasing the premium gradually on renewal to make a profit (FCA, 2020). The FCA report into insurer pricing practices (FCA, 2020) shows that most of the increase in prices for renewing customer take place over the first few years.

To mimic this increasing markup, P_{ijt}, the premium offered by insurer i to an existing customer j at time step t, is calculated according to equation (2.3) where T_{ijt} is the number of consecutive years that customer j has been a customer of insurer i at the start of time t. R is the renewal markup applied to the base premium cost.

$$P_{ijt} = P_{it}(1+R)^{Min\left(T_{ijt},T\right)} \tag{2.3}$$

Note that equation (2.3) simplifies to equation (2.2) for new customers since $T_{ijt} = 0$ for new customers. We can therefore use the signifier P_{ijt} to indicate the premium offered by insurer i for both new and renewing customers.

Dealing with customer claims

When a customer experiences a loss, they interact with their insurer's customer services. The probability p_i of having a good experience will depend on how much the insurer chooses to spend on their customer service, up to the maximum amount. This probability is given by equation (2.4).

$$p_i = \frac{Min\left(E_i, E\right)}{E} \tag{2.4}$$

Figure 2.4 shows the pseudocode for the claims process. Each customer generates a loss frequency and severity from a Poisson-Gamma aggregate distribution. For each claim event, the customer records either a positive or negative outcome from their selected insurer.

Outputs

- Π_{it}: The profit made by insurer i in timestep t. This is equal to the total premium in minus the claims paid out and minus the total expenditure on customer service quality per customer.
- R_{it}: The market renewal rate in time step t. This is calculated as the proportion of

existing customers of insurer i who chose to renew their insurance contract in timestep t.

- $m_{n,it}$: Number of insurance contracts that insurer i sold to new customers in timestep t.
- $m_{s,it}$: Total number of insurance contract sales made by insurer i in timestep t.
- $m_{-,it}$: Number of negative customer service experiences experienced by customers of insurer i in timestep t.
- $m_{+,it}$: Number of positive customer service encounters experienced by customers of insurer i in timestep t.
- z_{it}: Average output of customer service encounters experienced by customers of insurers i in timestep t. Calculated as $z_{it} = \frac{m_{+,it} - m_{-,it}}{m_{+,it} + m_{-,it}}$ (or set as 0 if there were no customer service interactions).

2.3.4 Customers

Description

Customer agents seek to purchase a motor insurance contract from an insurer agent. Each year, they decide whether to renew their existing contract with the same insurer. The greater the increase in the perceived cost of the contract, the lower the probability that they will decide to renew. If they choose not to renew their contract, they seek quotes from all available insurers, and select the provider with the lowest perceived cost.

A customer's assessment of perceived cost is not just based on premium, but also includes an adjustment for branding and product preferences, and an adjustment for their opinion of an insurer's customer service quality. We should note that customer survey data suggests that, though service quality has a significant influence on customer decisions, price remains the largest factor (Ansari and Riasi, 2016; Ghodrati and Taghizad, 2014; Tsoukatos and Rand, 2006). However, insurers in this model are assumed to offer similar prices, differentiated only by their margin for service expenses and any renewal markup (see equations 2.2 and 2.3). We would therefore expect that though changes to the market premium will have a significant impact the customer's decision to renew, the customer's product preferences and opinion of customer service quality will have a higher impact on their selection of a new insurance provider.

During the ensuing year, a customer might experience event, such as a traffic accident or theft. If this occurs, the customer makes a claim from their insurer by interacting with their customer service. This experience could be a positive or a negative interaction for the customer. Additionally, customers may ask each contact on their social network for their opinion on each insurer. This gossip is interpreted as extra information about customer service quality.

The customers update their opinion on each insurer in light of new information, either from their friends or based on their own experiences, though they give more weight to their own experiences. This will be used as their basis for selecting an insurer in the following year. In turn, they will also use it to spread word-of-mouth information to their friends during the course of the following year. Thus, opinions will spread through friendship groups.

Inputs and Initialisations

The customer agents are indexed by j.

- n_K: Average number of links each customer has on their social network.
- β: Probability of re-wiring used to construct the social network.
- a: Sensitivity of renewal probability to change in cost.
- b: Baseline renewal probability parameter.
- k_Q: Sensitivity of customers' cost assessment to insurer customer service quality
- k_D: Sensitivity of customers' cost assessment to insurer branding.
- φ: Rate at which old information about insurers is forgotten as customers place more emphasis on recent information.
- k_W: Influence of social network on customers' opinion of insurer service quality.
- μ_f: A customer's average number of claims per year.
- μ_s: Average size of an individual customer claim.
- σ_s: Standard deviation of the size of an individual customer claim.
- p_W: Probability of a customer obtaining word-of-mouth information from a friend who has information about an insurer.

To represent a social network, the Watts-Strogatz algorithm is used to generate a 'small world' network between the customers (Watts and Strogatz, 1998) at initialisation. This algorithm generates clustered groups with enough links between the clusters to create a small path size, and is commonly used to model real-world social networks. We note that although this algorithm produces the small path size and high level of clustering seen in real-world networks, it does not produce a very varied degree distribution. In real-world social networks, it is common for a small number of agents to be very well connected and have a large amount of influence over the network (Garcia et al., 2017).

The network could instead be modelled using a preferential attachment model such as the Barabási-Albert algorithm (Barabási and Albert, 1999), which generates a scale-free network with a few extremely well-connected hubs. However, this algorithm does not generate the high levels of clustering seen in real-world social networks. Further work could be done to examine the impact of using different types of networks.

The algorithm is implemented using the following steps:

1. Each customer j is linked to their n_K nearest neighbours, $n_K/2$ on each side, wrapping around to the start of the list at the end. This results in a regular ring-shaped network, with a total of n_K links per customer.
2. For every customer j, each $n_K/2$ right-hand links (j,k) are rewired with a probability β. The new link (j,k^*) must not replicate an existing link. Additionally, a customer cannot be linked to itself.

As with the insurer agents, customers are randomly spaced along the preference space.

The location assigned to a customer j is denoted $L_{C,j}$. By the definition of the preference space (see section 2.3.3), the shortest distance D_{ij} between customer j and insurer i is calculated according to equation (2.5) below:

$$D_{ij} = Min\left(\left|L_{I,i} - L_{C,j}\right|, 2 - \left|L_{I,i} - L_{C,j}\right|\right) \tag{2.5}$$

The variable Q_{ij0} represents customer j's estimate of insurer i's quality of service at time $t = 0$. At initialisation, these values are all set to 0, representing a neutral opinion.

Calculations

Assessing the cost of insurance

Similarly to the method used by Owadally, Zhou, and Wright (2018) and Owadally, Zhou, Otunba, et al. (2019), customers assess the cost of an insurance policy using not just the premium P_{ijt}, but additional factors which matter to them. This total cost can be regarded as a disutility function.

Owadally, Zhou, and Wright (2018) and Owadally, Zhou, Otunba, et al. (2019) included the distance in a preference location space, calculated in a similar fashion to the distance D_{ij}. This model also includes an allowance for service quality. Specifically, a customer j evaluates the cost of an insurance policy offered by insurer i at time t as a linear combination of: (a) the quoted premium P_{ijt} (b) the customer's current (subjective) estimate of insurer i's quality of service Q_{ijt} (described further in equation(2.10) below) and (c) the preference cost D_{ij} the customer has for the insurer based on their relative positions in the preference landscape. This is captured in equation (2.6) below:

$$C_{ijt} = P_{ijt} - k_Q Q_{ijt} + k_D D_{ij} \tag{2.6}$$

Deciding whether to renew

Searching and comparing insurer premium quotes carries with it a cost in time and energy. Customers therefore commonly prefer to renew unless they believe they can obtain a significant decrease in cost by searching elsewhere (FCA, 2020).

The decision whether or not renew an existing insurance contract is modelled as a probability. The value of this probability depends on the perceived change in the value of the contract. This probability is modelled using a logit function, which is a common choice (Günther et al., 2014). The probability of renewal r_{jt} for customer j at time t according to equation (2.7) where $\delta C_{ijt} = \frac{C_{ijt} - C_{ijt-1}}{C_{ijt-1}}$ is the rate of increase in customer j's estimated cost of their renewed insurance contract with insurer i at time t (see equation (2.6)).

$$r_{jt} \; = \; 1 \Big/ \left(1 + e^{a\delta C_{ijt} + b}\right) \tag{2.7}$$

Selecting a new insurer

If a customer decides not to renew their existing contract, they will seek premium quotes from all insurers in the market and calculate the total cost C_{ijt} for all insurers. This

```
Function Customer.SELECTINSURER(List of P_ijt)

  C_ijt ← P_ijt - k_Q*Q_ijt + k_D*D_ij for i of last chosen insurer
  delta ← C_ijt - C_ijt-1 for i of last chosen insurer
  r_jt ← 1/(1+exp(a*delta + b))
  if(random number from UNIFORM(0,1) <= r_ijt)
    return chosen_insurer_jt ← chosen_insurer_jt-1
  else
    C_0jt ← P_0jt - k_Q*Q_0jt + k_D*D_0j
    newCost ← C_0jt
    newInsurer ← 0
    for i in 1 to n_C -1
      C_ijt ← P_ijt - k_Q*Q_ijt + k_D*D_ij
      if(C_ijt < newCost OR (C_ijt ==newCost AND UNIFORM(0,1)<=0.5 ) )
        newCost ← C_ijt
        newInsurer ← i
      end if
    end for
    return chosen_insurer_jt ← newInsurer
  end if
```

Fig. 2.5 – Pseudocode for a Customer agent selecting which insurer to purchase from

cost is calculated as in equation (2.6) and includes allowances for brand preference and estimated quality of service. The customer will then aim to minimise their disutility by purchasing from the insurer with the lowest total cost. If the lowest cost corresponds with their existing insurer and renewal price, then they will decide to renew after all. If there are multiple possible insurers offering the lowest cost, they select one of these insurers at random.

Figure 2.5 shows the pseudocode used to carry out the insurer selection. First, the customer agent calculates the total renewal cost and probability of renewal, and then decides whether to renew and return their existing insurer. Otherwise, they calculate the total cost for each insurer in turn, and select the lowest available option.

Making a claim

If a customer suffers a loss event which is covered by the insurance contract, they will make a claim from their insurer. The frequency of these claims are modelled using a Poisson distribution, and the size is modelled using a Gamma distribution. These are common distributions used to model insurance claims (Jørgensen and Paes De Souza, 1994).

When a customer makes a claim, they interact with their insurer's customer service department. Consider a pair (i, j) consisting of customer j and insurer i. Define:

$$s_{kt} = \text{the outcome of the } k^{th} \text{ service experienced by customer } j \text{ interacting directly with insurer } i \text{ in the year } (t-1, t)$$

In the description of the derivation of Q_{ijt} described below, we suppress dependence on i, j to simplify the notation.

The value of each interaction s_{kt} is $+1$ if the customer had a positive experience, and

```
Function Customer.UPDATEWOM()

for each Insurer i
  newCount[i] ← 0
  newInfo[i] ← 0
  for each Customer k in list links[j]
    if(Customer k has any information about insurer i && UNIFORM(0,1) <= pW)
      newCount[i] ← newCount[i] +1
      newInfo[i] ← newInfo[i] + Qikt
    end if
  end for
end for
```

Fig. 2.6 – Pseudocode for a Customer agent obtaining word-of-mouth information

-1 if the interaction was negative. Note that a customer can only collect more experiences with a particular insurer in time t if it has an insurance contract with them.

Word of mouth from the social network

Consider a pair of customers, j and j', who are linked by the social network generated at the start of the simulation. During a year, there is a probability p_W that customer j will share Q_{ijt}, their current opinion of insurer i, with their friend j'. Similarly, j' will share their own opinion $Q_{ij't}$ with the same probability. The word-of-mouth opinions act as another source of information for customer j.

As with the service experiences, we can simplify the notation for a pair (i, j) consisting of an insurer i and customer j. Define:

$$\begin{aligned} w_{kt} &= \text{the value of the } k^{th} \text{ word-of-mouth opinion received by customer} \\ &\quad j \text{ interacting indirectly with insurer } i \text{ in the year } (t-1, t) \end{aligned}$$

Note that if a customer has no opinion about an insurer because they have received no word-of-mouth information and also have had no direct experiences themselves (as will be the case at the start of the first time step), they will not pass on any information.

Figure 2.6 shows the pseudocode run by each customer agent in order to obtain and record word-of-mouth information during a timestep.

Opinion of insurer quality

As before, consider a pair (i, j) consisting of an insurer i and customer j. In any given year $(t-1, t)$, j receives information about i from a finite number of direct and indirect interactions. It is convenient to collect them in two vectors of finite length:

$$s_t = \{s_{1t}, s_{2t}, s_{3t}, \dots\}' \tag{2.8}$$

$$w_t = \{w_{1t}, w_{2t}, w_{3t}, \dots\}' \tag{2.9}$$

where prime indicates a transpose, so that the above are column vectors.

We define two vector functions, $d : \mathbb{R}^k \mapsto \mathbb{R}$ and $a : \mathbb{R}^k \mapsto \mathbb{R}$ for some $k \in \mathbb{N}$. The former describes the length of a vector, and the latter the sum of all the elements of a vector. Consequently, $d(s_t)$ is the number of direct claims interactions that customer j has in relation to insurer i in year $(t-1,t)$ and $a(s_t)$ is the sum total of the outcome of these interactions. If customer j has no direct experiences with insurer i in year $(t-1,t)$, then s_t is empty, and we assume that $d(s_t) = a(s_t) = 0$. Likewise for w_t if the customer receives no word-of-mouth information about this insurer during the year.

It is a common assumption that agents in a dynamic market will weight newer information more highly than old information (Sutton and Barto, 2018). This is consistent with the fading of human memory and a sensible approach when parameters and conditions may change over time. Though the insurer agents in this model maintain a constant customer service quality, insurers in the real market may enact dynamic strategies. It is therefore reasonable to weight each piece of information according to a memory factor $\varphi^{t-\tau}$ where t is the current time and τ is the time at which the information was received and $0 < \varphi < 1$. The closer φ is to 0, the less weight the agents will place on older information.

It is also usual for humans to weight their own experience more highly than the opinions of others. For example, agents in opinion dynamic models take their own opinions as a starting point and move in the direction of other opinions during interactions with other agents with a weight proportional to the agents' affinities for each other (Deffuant et al., 2002). Similarly, we will place a higher weight on an insurer's own experiences s_t than on the indirect information w_t.

By weighting each piece of information as described at the end of each time period, customer j updates their opinion Q_{ijt} of insurer i's quality of service according to the equation (2.10) below:

$$Q_{ijt} = \frac{(1-k_W)\sum_{\tau=1}^{t}\varphi^{t-\tau}a(s_\tau) + k_W\sum_{\tau=1}^{t}\varphi^{t-\tau}a(w_\tau)}{(1-k_W)\sum_{\tau=1}^{t}\varphi^{t-\tau}d(s_\tau) + k_W\sum_{\tau=1}^{t}\varphi^{t-\tau}d(w_\tau)} \tag{2.10}$$

At $t = 0$, $Q_{ij0} = 0$; i.e. all customers begin with a neutral opinion of all insurers until one of them makes a claim and has either a good or bad experience. As each customer will pass on their Q_{ijt} values by word-of-mouth, information is passed around clusters of friends and will be assimilated into their own estimates at the end of the timestep. Note that as Q_{ijt} is ultimately a weighted average of claim interaction experience outcomes, it is bounded $(-1,+1)$.

In this model, there are no entrants or exits, and all insurers begin at the same time. However, if an insurer were to enter the market, it would begin with a neutral reputation. If the existing market insurers are of generally negative service quality, we would expect such an insurer to attract a significant enough proportion of switching customers to establish a reputation and compete in the market. However, if the existing insurers have a generally positive reputation, then a new insurer would not attract many new customers. In order to compete, a new insurer would likely need to offer a premium lower than the market to attract customers.

```
Function UPDATEOPINIONS()

 for each Insurer i
   oldWOMCount ← womCount[i]
   oldServiceCount ← serviceCount[i]
   oldWOMSum ← womSum[i]
   oldServiceSum ← serviceSum[i]
   newWOMCount ← oldWOMCount*φ + newCount[i]
   newWOMSum ← oldWOMSum*φ + newInfo[i]
   if(i == chosen_insurer_jt)
     newServiceCount ← oldServiceCount*φ + freq
     newServiceSum ← oldServiceSum*φ + newService
   else
     newServiceCount ← oldServiceCount*φ
     newServiceSum ← oldServiceSum*φ
   end if
   womCount[i] ← newWOMCount
   womSum[i] ← newWOMSum
   serviceCount[i] ← newServiceCount
   serviceSum[i] ← newServiceSum
   totalSum ← (1-k_W)*newServiceSum + k_W*newWOMSum
   totalCount ← (1-k_W)*newServiceCount + k_W*newWOMCount
   Q_ijt ← totalSum / totalCount
 end for
 return List of Q_ijt
```

Fig. 2.7 – Pseudocode for a Customer agent updating their opinions Q_{ijt} based on information gained during a timestep t

Figure 2.7 shows the pseudocode of a customer agent updating their opinion. Note that instead of re-calculating the complete sum of all information each time, Q_{ijt} can be calculated as an update to the existing value Q_{ijt-1} by keeping track of the totals used in both the numerator and denominator. The pseudocode demonstrates this update and subsequent calculation for each insurer i.

Outputs

- S_{ijt}: Customer j's opinion of insurer i at the start of time t based only on their own experiences. This is defined by:

$$S_{ijt} = \frac{\sum_{\tau=1}^{t} \varphi^{t-\tau} a(s_\tau)}{\sum_{\tau=1}^{t} \varphi^{t-\tau} d(s_\tau)} \tag{2.11}$$

- W_{ijt}: Customer j's opinion of insurer i at the start of time t based only on the word-of-mouth information received from their social network. This is defined by:

$$W_{ijt} = \frac{\sum_{\tau=1}^{t} \varphi^{t-\tau} a(w_\tau)}{\sum_{\tau=1}^{t} \varphi^{t-\tau} d(w_\tau)} \tag{2.12}$$

- Q_{ijt}: Customer j's overall opinion of insurer i at the start of time t based on a mixture of their own experiences and word-of-mouth information.
- Q_{jt}: Customer satisfaction. This is customer j's opinion of their current insurer at time t.
- δ_{jt}: An indicator variable which is equal to 1 if customer j decided to renew at time t, and 0 otherwise.

As the model will contain a large number of customer agents, these values are outputted and analysed as a mean and points along a distribution.

2.4 Data, Parameterisation and Validation

2.4.1 Data

This model is based where possible on data from the UK motor insurance market. However, in some places this data was not available, and US motor data has been used as a proxy. These data sources are listed here.

- The FCA report into insurer pricing practices (FCA, 2020) is used as a data source regarding renewal behaviour. This report includes information about how insurance prices for existing customers seeking a renewal compare with the prices quoted for new customers, and the likelihood that customers choose to renew their existing contracts. It also mentions a proposed regulatory change, which is used here as an alternative scenario.
- The market level data of premium and losses is taken from a summary of EIOPA Solvency I submissions (EIOPA, 2016). This data is for the years 2006-2015.
- To adjust these values to a comparable level, CPI data is used to inflate the historical values. Note that as the data is in Euros, the European CPI data is used (King, 2021).
- Summary statistics from a dataset of facebook social circles collected by Stanford (Leskovec and Krevl, 2014) are used to parameterise the word-of-mouth-network.
- A set of statistical data of US insurance companies' historical advertising spending has been taken from Statista (Guttmann, 2020), a market and consumer data company.
- Statista was also used to obtain data on the market share of the top ten insurers in the UK motor insurance market (Statista, 2020).
- Data from the Insurance Information Institute about how often customers tend to make insurance claims per year (III, 2019).
- This was supplemented by the Allstate claims information available on Kaggle (2016), which provided information about the shape of the severity distribution of motor claims (note: this is US data but was used as a proxy for the variation of claims).
- Information about the number of motorists in the UK was taken from the Society of Motor Manufacturers & Traders (SMMT, 2020).

2.4.2 Parameterisation

Market Parameters

Simulating 35 million customers as individual agents would require a prohibitive amount of computing power. However, the network will have a similar clustering co-efficient if it is sufficiently large to satisfy $n_c >> n_K$ (Barrat and Weigt, 2000), and remain connected if $n_K >> \ln n_c >> 1$ (Watts and Strogatz, 1998). Based on this requirement, the number of customers n_c is set at $10,000$.

Owadally, Zhou, Otunba, et al. (2019) use 20 insurer agents, noting that the top twenty insurers hold a significant majority of the market share in the UK motor insurance market and is a reasonable number for producing computationally tractable yet realistic simulation dynamics. Similarly, the number of insurers n_I is set to 20.

As n_C does not represent the actual number of individual customers, we choose to rescale the inflation adjusted market data from Europa so that the average loss per representative customer agent is 100. An AR(2) curve is fitted to the rescaled market premium to find the values of θ_0, θ_1, and θ_2. The parameter σ_M is set as an unbiased estimate of the standard deviation of the residuals. Finally, the P_{-1} and P_{-2} are set as the last two available rescaled market premium values.

Insurer Parameters

The maximum effective spending level is based on the largest level of spend on advertising per premium in a set of statistical data of US insurance companies (Guttmann, 2020). This gives a maximum service spend of approximately 5% of the expected premium level per premium.

As herding behaviour is common in markets, the base model assumes homogeneous service quality where all insurers spend the same amount per customer on customer service. Later the model is run with heterogeneous markets where insurers are assigned differing levels of service spend. The base model uses a value of $E_i = 0.8$. This indicates that most insurance interactions are generally positive, but insurers are not perfect and about 1 in 5 experiences are negative.

The FCA report into insurer pricing practices (FCA, 2020) indicates that most of the increase in prices for renewing customer take place over the first 5 years, so we will set maximum term of markup increase $T = 5$. Additionally, the FCA report indicates that that the total average increase over that time is 30% of the customer premium, giving us an annual increase of R approximately 5%.

Customer Parameters

Approximately one in two people in the UK own a registered car (SMMT, 2020). Additionally, studies demonstrate that the median number of social links per person is approximately 231 (McCarty et al., 2001). Based on these data, the average number of links per customer n_K is set at 100.

For a Watts-Strogatz network, the global clustering coefficient is approximately equal to $\frac{3(n_K-2)}{2(2n_K-1)}(1-\beta)^3$ for large n_c. Stanford examined various online social networks and found a global clustering coefficient of approximately 0.2647 (Leskovec and Krevl, 2014). Based on this value and the chosen value of the parameter n_K the probability of rewiring β is set at 0.3.

For renewal behaviour, the parameters a and b are calibrated using two reference points. The average retention rate for the UK motor insurance market is approximately 50% (FCA, 2020), so the standard renewal increase of 5% is assumed to correspond with a renewal probability of 50%. For the second reference point, note that this report also tells us that customers consider the cost of searching for a new insurer to be worth £42, or 15%

of the average cost of an insurance policy. Based on this piece of data, the probability of renewal if the cost increases by 15% is set at half the usual rate, or 25%.

The average claims frequency per customer μ_f will dictate how many times a customer will interact with an insurer's service quality and thus is an important parameter. This is set based on data from the Insurance Information Institute (III, 2019). The average severity μ_s is then calculated such that the total average loss per customer is 100 to match the market rescaling. Finally, to find a coefficient of variation, a distribution is fitted to the Allstate claims information available on Kaggle (2016); this determines the parameter σ_S. These pieces of information are then used to parameterise a Gamma severity distribution.

Behavioural Parameters

The remaining parameters are behavioural and thus are difficult to parameterise based on data:

- k_Q, the sensitivity of a customer to an insurer's customer service quality
- k_D, the sensitivity of a customer to an insurer's brand
- k_W, the relative influence of word-of-mouth versus direct experience
- φ, the rate at which old information is forgotten
- p_W, the probability of a customer passing on word-of-mouth about a particular insurer to a friend

These factors are given reasonable estimates for a base model based on judgement. As these are judgement based, the model is also run for different values of the sensitivity parameters to test the effect of different assumptions on the results.

Customer survey data suggests that, though service quality has a significant influence on customer decisions, price remains the largest factor (Ansari and Riasi, 2016; Ghodrati and Taghizad, 2014; Tsoukatos and Rand, 2006). The sensitivity coefficients are therefore set according to a reasonable increase in insurance prices. This is calculated as a year's renewal markup on a premium increase of two standard deviations. This places limits on these values as:

$$0 < k_D < k_Q < 32.65$$

This is explored further in the validation section below.

k_W is set to 20%. This means that a piece of information s_{kt} obtained from direct experience is given a weighting of 80% relative to the weighting 20% of an indirect piece of word-of-mouth information w_{kt}. Thus, it would take contrary information from at least four friends to counterbalance an opinion based on one piece of direct experience.

φ is a memory factor. The higher the value of φ, the greater the weight a customer places on older pieces of information. If $\varphi = 0$ then all estimates are based on the latest information only, and if $\varphi = 1$ the customer places equal weight on all pieced of information regardless of when they occurred. For the 'base' model, a value of $\varphi = 60\%$ is used. This means a given piece of information which is now five years old is given a weighting just less than 10% of that given to recent information.

p_W is set equal to 5%. As the average number of links n_K is set to 100, this indicates than on average, customers will seek an opinion from 5 friends a year on a particular insurer.

2.4.3 Validation

In the real world, firm size often follows an uneven distribution, with a few insurers taking a significant proportion of the available market share (Gabaix, 2009). This pattern can also be seen in the real world market share of the top 10 UK motor insurers (Statista, 2020). This data can therefore be used to validate the model by comparison with the modelled distribution of insurer share.

Preliminary regression tests indicate that the key parameter values which determine the shape of this distribution are the relative ratios of the customer cost sensitivity parameters k_D and k_Q. Variations of the base model were run while this ratio was varied, and a ratio of $\frac{k_Q}{k_D} = 2.2$ was found to minimise the squared distance of the average simulated market shares and the empirical market data to within two significant figures.

To validate this output, we perform a Kolmogorov-Smirnoff test[1] with the initial hypothesis

$$\begin{aligned} H_0 : &\text{ the real-world top 10 market shares are drawn from the same} \\ &\text{ distribution as the top 10 market shares in the model} \end{aligned}$$

To apply this test, we compare the sample of ten real-world values with the distribution of market shares for the top 10 insurers across 300 simulations. The results are ignored for the first 20 timesteps as the model is still settling into equilibrium, and otherwise included for a further 80 timesteps, giving a total of 120,000 datapoints.

This gives us a test statistic of 0.231, at a p-value of 0.584. This is not enough to reject the initial hypothesis at even a strict 80% level. We thus accept that this model produces market share outputs which follow a similar distribution to those produced in the real-world.

From this validation exercise, k_Q is equal to $2.2k_D$. From our earlier reasoning, this puts an upper bound on k_D of 15. For the base model, $k_D = 10$ and $k_Q = 22$.

Looking at the customer's estimation of service quality (equation 2.10), we might naively conclude that this is an unbiased estimate of the expected outcome of an interaction with an insurer in the event of a claim. As all word-of-mouth information is equal to a friend's own opinion Q_{ijt-1}, the value of Q_{ijt} is calculated as a weighted average of experiences (unless the customer has no information at all as yet).

However, the existence of the word-of-mouth network can lead to systemic bias. To see this, consider a simple example where there are two customers and two insurers. The value of k_w is 20%, $\varphi = 60\%$, and both customers always pass information between them.

[1] While we would prefer to run a test with a larger set of data, we do not have access to the data for other insurers (Statista, 2020). However, we note that the top ten insurers cover a total market share of 85.8%, which is a significant majority of the total market.

To start with, the two customers have a neutral opinion; customer 1 selects insurer 1, and customer 2 selects insurer 2. Both customers have an 80% chance of good customer service, and 20% chance of bad customer service.

In the first time step, customer 1 makes a claim, and by chance experiences bad service. Customer 2 also makes a claim, but has a good experience. As a result, customer 1 decides not to renew. Their opinion of insurer 1 is now at -1 based on their only information, so they select insurer 2 over insurer 1. The two friends then exchange word-of-mouth information. They now both hold an opinion of -1 on insurer 1, and $+1$ on insurer 2. After they have both been with insurer 2 for a further three years, customer 1 has a bad experience with insurer 2.

However, customer 1 and 2 have been exchanging word-of-mouth information every year, reinforcing their established opinion by repeating it between themselves. This new information means that customer 1's opinion of insurer 2 is now:

$$Q_{214} = \frac{0.8(-1)+0.2(+1+0.61+0.6^2 1)}{0.8+0.2(1+0.6+0.6^2)} = -0.34$$

From this we can see that this is only a third as negative as their opinion of insurer 1. In this case, the gain in premium from becoming a new customer is not enough to offset the perceived decrease in quality, and customer 1 decides not to switch.

If both customers remain with insurer 2, then over time, they will amass enough information to get an accurate opinion. Although the initial opinion will be passed around as with insurer 1, the long term equilibrium state tends toward the true value as we would expect. This will solidify the perceived gap between insurer 2 and insurer 1, and it would be very rare and fleeting for either customer to ever again purchase from insurer 1.

The persistence of negative opinions through the word-of-mouth network and the rarity of obtaining corrective information causes a similar effect on a large scale in the model, causing a systemic bias in the estimated service quality. We thus see that the expected value of Q_{ijt} is in fact a little lower than the true value, unless the true value is either 0 or 1, in which case there is no variance in outcomes and all opinions are correct. The size of the bias will depend on the shape of the network and how efficiently information is spread through it.

Without this effect, we would expect that in a market where insurers are spending the same amount on customer service, they would take an equal share of the customers. However, this systemic effect can cause some opinions based on a small number of experiences to persist in the market, causing some insurers to be unfairly favoured over others. As a result, the market share follows a distribution, with some insurers taking a greater or lesser market share.

2.4.4 Scenarios

Base Model

The model is implemented in C#. The inbuilt Random class was used to generate random numbers and is based on a modified version of Knuth's subtractive random number generator algorithm (Knuth, 1997). The starting seed is specified by an input file, and is used consistently across different sensitivity tests and scenarios, making these results directly comparable.

The base model outputs demonstrate that the market takes approximately six years for the initialisation phase to end and insurer reputations to become established throughout the market. The market opinion continues to show significant changes for a further 20 years. While customer opinion and the skewness of market shares continue to show a slow pattern of change, this indicates that the output patterns are valid examples of an established market hereafter. In practice, a real-world market does not have time to settle into a truly long-term pattern, since circumstances are always changing and dynamic competitive strategies mean that insurers may change their customer service quality. As such, 100 time steps are deemed sufficient to capture the model results. All models are therefore run for 100 time steps, and 300 simulations.

The parameters used in the base model are taken from the parameterisation and validation process described above and listed in Table 2.1.

Sensitivity Tests

Additionally, several sensitivity tests are run to explore the effect of changing some of the key behavioural parameters. The range of these tests are described in Table 2.2.

Heterogeneity of Insurer Spending

This model does not allow for insurer strategies around service quality spend. However, possible variations can be examined by running models with heterogeneous service quality. This is done by randomly assigning each insurer a constant value of E_i in increments of $0.05E$. In order to get a large enough range of scenarios generated in this way, this model is run for $1,000$ simulations.

Additionally, the heterogeneous model is run for varying values of k_Q as in the sensitivity tests.

Regulation Change Scenario

The FCA report (FCA, 2020) also proposed an enforced change to insurer pricing whereby a customer who renews their policy must not be charged more than if they were a new customer. This scenario is also run in the model, and the results compared with those of the base model.

Without the expected discount for new customers, the insurers will increase their premium overall. To account for this, we model the customers' renewal states as a Markov chain with a 50% chance of returning to 0 years from each state (see Figure 2.8). Multiplying these out by the appropriate renewal premium markup give us an expected overall

Parameter	Description	Value
n_I	No. of insurers	20
n_C	No. of customers	$10,000$
k	No. of links per customer	100
β	Rewiring probability in social network	30%
$\overline{P}_{-1}$	Market premium in year -1	91.85
$\overline{P}_{-2}$	Market premium in year -2	89.28
θ_0	Coefficient in AR(2) process for market premium	76.03
θ_1	Coefficient in AR(2) process for market premium	0.6675
θ_2	Coefficient in AR(2) process for market premium	-0.3580
σ_M	Stochastic variability for market premium	14.19
E	Max insurer spending on customer service per customer	5.83
E_i	Level of annual insurer spend on customer service per customer	4.664
R	Renewal markup	5%
T	Max renewal term	5
a	Renewal probability scaling	10.986
b	Renewal probability shift	-0.5493
k_Q	Customer service sensitivity	22
k_D	Customer preference sensitivity	10
φ	Memory factor	60%
k_W	Word-of-mouth influence factor	20%
μ_f	Average loss frequency per customer	13%
μ_s	Average loss severity	755.8
σ_s	Standard deviation of loss severity	730.1
p_W	Probability of word-of-mouth transmission	5%

Table 2.1 – Table of parameter values for base model

premium of: $\overline{P}_t \sum_{i=0}^{i=5} \frac{1}{2^{\min(i+1,5)}} 1.05^{\min(i,5)}$. This gives an increase factor of 5.05%.

2.5 Results

2.5.1 Base Model

As described earlier, the market opinion of an insurer based on experiences in the first few years persists as it gets passed around friendship groups and reinforced each time a customer

Parameter	Values
E_i/E	0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.9
k_Q	14, 16, 18, 20, 24, 26, 28, 30
k_D	6, 8, 12, 14, 16, 18, 20, 22
k_W	0.05, 0.1, 0.15, 0.25, 0.3, 0.35, 0.4, 0.45
φ	0.1, 0.2, 0.3, 0.4, 0.5, 0.7, 0.8, 0.9
p_W	0.02, 0.03, 0.04, 0.06, 0.07, 0.08, 0.09, 0.1

Table 2.2 – Table of varied parameter values for sensitivity model

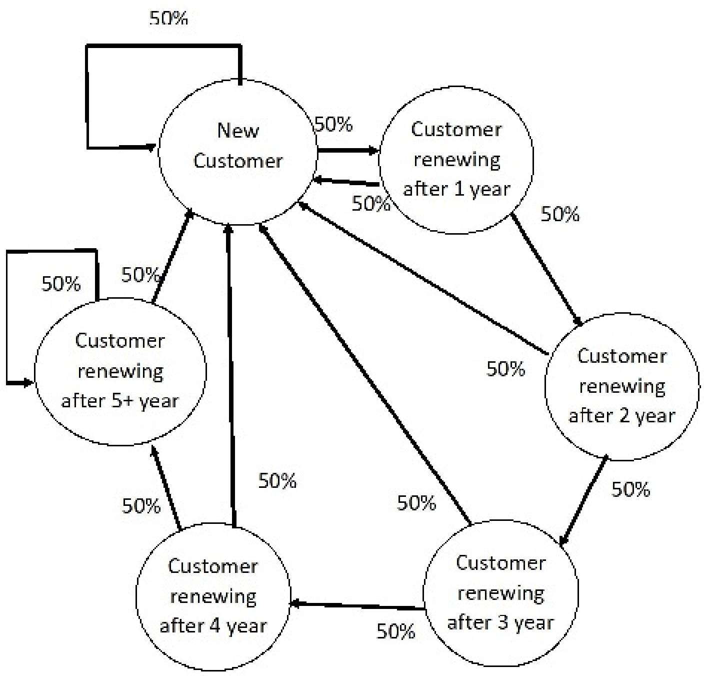

Fig. 2.8 – Diagram of the Markov chain states and transition probabilities between customer renewal states

hears the same information from a friend. This phenomenon, alongside the comparative rarity of new experiences, makes it difficult for an insurer who had an unfortunate early record to attract enough new customers to override the low opinion.

Regression analyses are carried out on the simulated model outputs to elicit an understanding of the variables that drive its behaviour. Table 2.3 shows the results of a regression on the average outputs across each simulation versus the average quality of experiences in the first time step for each insurer. This table shows that as expected from the systemic bias described above, the initial performance is a significant factor with a high goodness of fit on an insurer's subsequent reputation, market share, and renewal rate.

Output y_{it}	Intercept α_0	Slope α_1	p-value	R^2
Insurer reputation Q_{ijt}	0.146	73.3%	0	0.802
Insurer market share $m_{s,it}/n_c$	-18.4%	39.1%	0.000	0.611
Insurer renewal rate R_{it}	-25.5%	108.4%	0.000	0.385

Table 2.3 – Table of regression results for individual insurer outputs regressed on the quality of the customers' experiences with the insurer in the first timestep according to the equation: $y_{it} = \alpha_0 + \alpha_1 z_{i1} + \varepsilon_{it}$

Intercept α_0	Slope α_1	p-Value	R^2
144.7%	-0.876%	0.000	0.239

Table 2.4 – Table of regression results for market retention rate dependency on market premium using the equation: $\overline{R}_t = \alpha_0 + \alpha_1 \overline{P}_t + \varepsilon_{it}$

As price remains the largest factor in customer purchasing decisions, although the average retention rate is close to 50%, it varies throughout the simulation along with the change in market premium. When the premium is increasing, retention rates go down as customers see the increase and seek out new quotes. When it is decreasing, retention rates increase as even with a markup, the renewal premium is not a significant increase. This relationship can be seen in the results of a regression carried out on each individual timestep for each simulation of the base model. These results are shown in Table 2.4.

In this model, insurers have a fixed service quality through a simulation. The possibilities of a dynamic strategy are beyond the scope of this model, though could be explored in future work. However, the above results imply that insurers who maintain good service quality while premiums are rising may suffer a decline in renewal rates if they also pass that expense along to their customers, yet are unlikely to benefit if their reputation has already been well established.

2.5.2 Sensitivity Tests

Service Spend

As we would expect, the main impact of increasing the insurer's chosen customer service spend relative to the maximum spend is to increase the customer's opinion of the insurers. This is because the higher the spend, the higher the insurers' service quality. Table 2.5 shows the p-values and goodness of fit of this regression relationship for both the customer satisfaction — which is the customers' opinion of their own insurer — and also for the average customer's opinion of all insurers. From this table, we see evidence of the bias that causes the unequal market concentrations: the customer satisfaction regression line lies just above the true value, and the average opinion of all insurers lay below it. This can be seen in Figure 2.9. Additionally, the retention rates show a greater variation as E_i is increased. This is because the main driver of retention rates is the change in premium, which increases

Output y_k	Intercept α_0	Slope α_1	p-value	R^2
Average of Q_{it} (Customer satisfaction)	-0.943	2.004	0.000	0.997
Average of Q_{ijt} (Customer opinions of all insurers)	-0.1.016	2.001	0.000	0.975
Standard deviation of $\overline{R}_t$ (market renewal rate)	27.3%	5.11%	0.000	0.383

Table 2.5 – Table of results for the outputs across all timesteps within a single simulation regressed on the insurer service spend E_i/E according to the equation: $y_k = \alpha_0 + \alpha_1 E_i/E + \varepsilon_{it}$

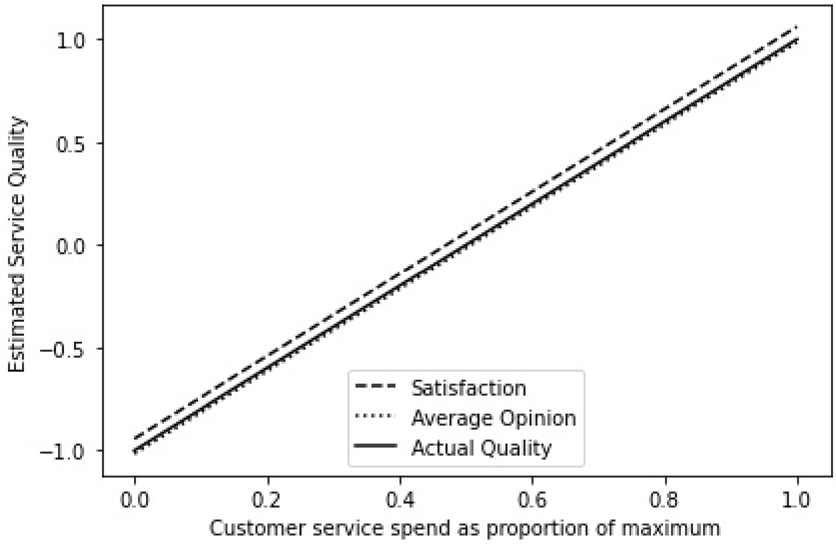

Fig. 2.9 – Comparison of fitted regression lines of customer estimate of service quality versus actual service quality. The customers' opinion of their own insurer is higher than their true quality, and the average opinion is slightly lower, reflecting the small bias caused by the lack of new information about insurers which customers perceive to be lower quality.

along with service spend.

Preference and Service Sensitivities

As discussed in the validation section, the main impact of these parameters is on the concentration of the market shares between the insurers. When k_D is high and k_Q is low, customers are more heavily influenced by their location on the brand preference space. Since the agents were evenly spread out among this space, this creates a market concentration close to even (i.e. 5% for each of the 20 insurers). The higher the value of the service sensitivity k_Q relative to the preference sensitivity k_D, the more the customers' choice of insurer is influenced by their opinion of an insurer's customer service quality, leading to a more unequal market concentration (Figure 2.10). Table 2.6 demonstrates the results of a regression on

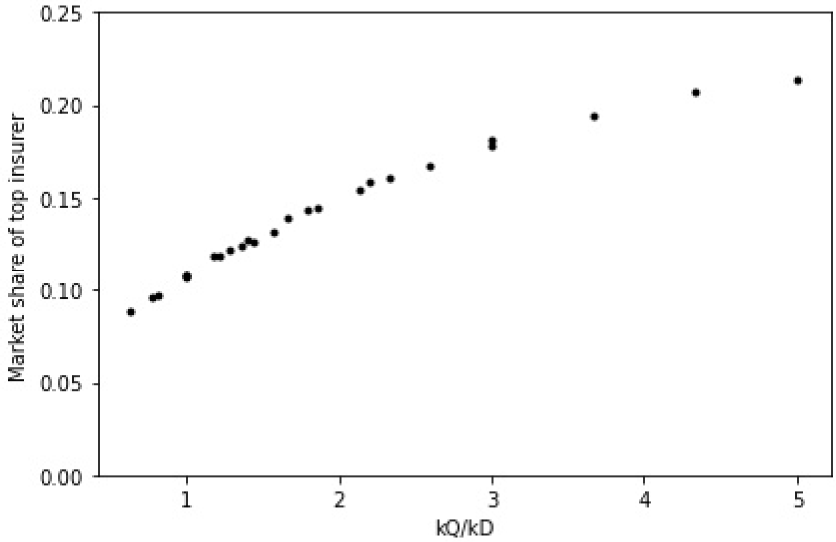

Fig. 2.10 – Market concentration versus ratio of customer sensitivity to service quality (k_Q) to brand preference sensitivity (k_D). Market concentration is measured by the market share of the insurer with the largest market share. The ratio k_Q/k_D is the primary driver of market share inequality.

Intercept α_0	Slope α_1	p-Value	R^2
8.21%	3.02%	0.000	0.675

Table 2.6 – Table of results for the average market share across all timesteps within a single simulation regressed on the ratio of the customer sensitivities to service quality and brand preference according to the equation: $\frac{1}{t}\sum_t m_{s,it}/n_c = \alpha_0 + \alpha_1 k_Q/k_D + \varepsilon_{it}$

the value of the market share of the top insurer respective to the ratio of these sensitivities, and shows that this is a significant relationship which gives a strong fit.

Influence factor and Transmission rate

If either the influence factor k_W or the word-of-mouth transmission rate p_W is set to zero, then there is no transmission of word-of-mouth information. In that case, the market concentration becomes more evenly spread, giving a top market share of 5.5%. Additionally, the average customer opinion of an individual insurer is close to zero as most customers do not have any information about a particular insurer. However, it does not require a very high value before information reaches saturation and the consensus market opinion becomes close to the true value. This is seen in Figure 2.11.

Table 2.7 displays the results of regression on the word-of-mouth influence factor k_W and on the word-of-mouth transmission rate p_W. We see that although k_W and p_W are significant factors for both the top insurer's market share and the customer satisfaction, the goodness of fit R^2 is low. Additionally, the slope is quite shallow in comparison with the size of the parameter change. The regression model suggests that the top insurer's market

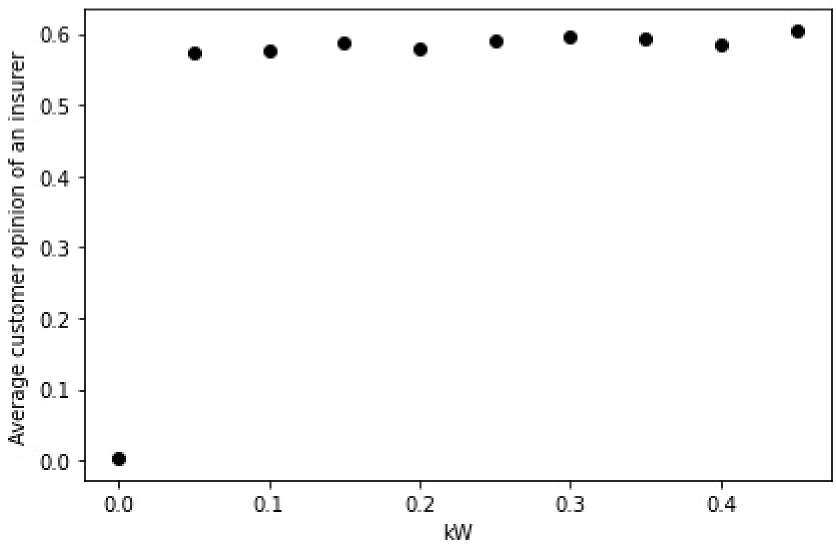

Fig. 2.11 – The average of customers' opinion of an individual insurer is close to zero when word-of-mouth is turned off due to lack of information, but quickly becomes close to the true value of 0.6 as the word-of-mouth influence factor k_W increases

Output y_t	Input x_k	Intercept α_0	Slope α_1	p-Value	R^2
Top insurer market share	k_W	12.6%	12.7%	0.000	0.279
Customer satisfaction	k_W	64.2%	6.95%	0.000	0.168
Top insurer market share	p_W	13.0%	47.0%	0.000	0.182
Customer satisfaction	p_W	63.2%	42.0%	0.000	0.228

Table 2.7 – Table of results for regressing average of outputs across all timesteps within a single simulation on the inputs k_W and p_W according to equation: $\frac{1}{t}\sum_{t \text{for sim} k} y_t = \alpha_0 + \alpha_1 x_k + \varepsilon_{it}$

share varies between 13% and 18% for the k_W sensitivity tests, and the average customer satisfaction varies between 65% and 67%. For the p_W sensitivity tests, the top insurer's market share varies between 14% and 18%, and the average customer satisfaction varies between 64% and 67%.

Memory factor

Without the word-of-mouth network, we would expect that a low memory factor φ would lead to a more equal market share as old experiences do not have a high influence on opinion. However, although a regression shows that φ is a significant factor in the market share of the top insurer, the regression coefficient for the slope is low (3.45%), and the intercept is not close to an equal split of 5% (see Table 2.8). This reflects the effect of the word-of-mouth network, which circulates the experiences and maintains a systemic memory.

Intercept α_0	Slope α_1	p-Value	R^2
13.8%	3.45%	0.000	0.118

Table 2.8 – Table of results for the average market share of the insurer with the highest share across all timesteps within a single simulation regressed on the value of the input memory factor φ according to the equation: $\frac{1}{t}\sum_t \max\{m_{s,it}\}/n_c = \alpha_0 + \alpha_1\varphi + \varepsilon_{it}$

2.5.3 Heterogeneity of Customer Service

The base model was simulated 1,000 times. In each simulation, each insurer i is randomly assigned a customer service spend E_i which is constant over time t and which is drawn equally likely from the set $\{0, 0.05E, 0.1E, 0.15E, \cdots, E\}$.

The most significant factor in a customer's assessment of cost is the price. As a result, changes in the market premium have a large impact on the renewal rates as already seen in the base model (table 2.4). However, when a customer is selecting a new insurer, the differences in the premium offered by each insurer are smaller than the overall change in premium. As a result, we would expect the impact of product preferences and service quality reputation to become more significant when selecting a new insurer. In this model, insurers pass on the cost of their customer service spend to their customers. We would therefore expect that the preference between insurers of different qualities to depend on the balance between the difference in expense and the customer's sensitivity to the differences in perceived service quality, as determined by the size of the sensitivity parameter k_Q.

Looking at the profit averaged across all scenarios, broken down by E_i, we find that the companies assigned a value of E_i less than 75% of the maximum spend E are usually unable to compete with the insurers who spent more money. Despite the increased cost that is passed on to the customer, under the base model parametrisation, the optimal setting is consistently set at the maximum value E. Higher quality insurers attract both a higher renewal rate and a higher number of new sales.

Figure 2.12 shows that, as customer service spend increases, the average insurer profit increases, except at very low levels of customer sensitivity to service quality k_Q. In general, Figure 2.12 demonstrates that the highest average profit is achieved by insurers when they spend the most on customer service.

2.5.4 Regulation Change

We also consider the possible implications of the proposed regulation change by comparing the results of this model with those of the base model, including the results of the sensitivity tests. This was done by performing a series of regressions using indicator variables and testing these variables for significance; i.e.: $y_k = \alpha_0 + \alpha_1 x_k + \alpha_2 I_k + \alpha_3 I_k x_k + \varepsilon_k$ where I_k is equal to 0 for an output from the base model and 1 for an output from a model with the regulation change. If the coefficient α_2 is significant, then it indicates a change in the average value of the output between the two models. If α_3 is significant, then it indicates a change in the relationship between the sensitivity parameter and the tested output.

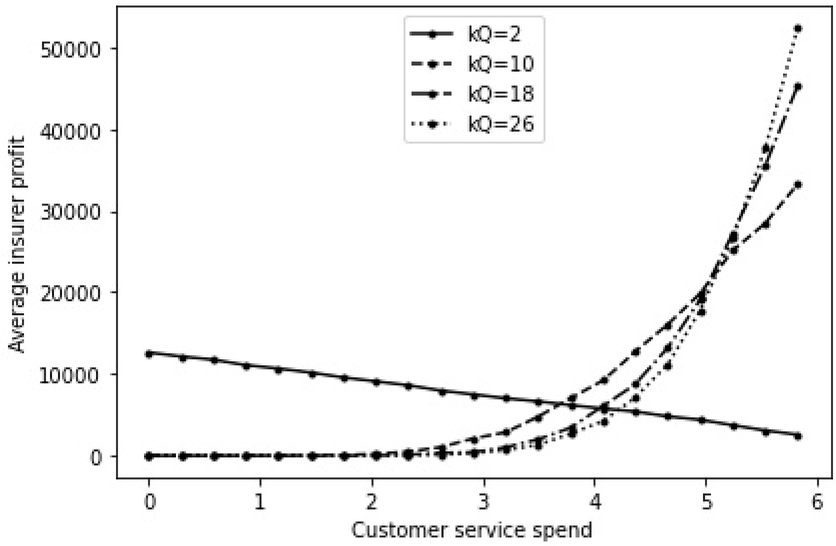

Fig. 2.12 – Average insurer profit versus customer service spend for different levels of customer sensitivity to service quality (k_Q). In general, the more insurers spend on customer service, the greater the average profit that they make. At very small values of kQ such as kQ=2, the customers' preference for better quality is no longer high enough to overcome the price change, and the lower quality insurers attract more business.

Table 2.9 shows the significant results of this regression run excluding the sensitivity tests; i.e. the pure change between the base model and the regulatory scenario. Note that for these regressions, there is no variable x_k, and so $\alpha_1 = \alpha_3 = 0$. As we would expect, these results show a large increase in the average renewal rates for the regulation scenario and a decrease in their variability. There is also an increase in the top market share, suggesting a more unequal market concentration and thus a reduction in market competition. This is because in the base model, there was a small but steady number of customers willing to switch to a lower reputation insurer nearer their brand preference. After the regulation change, the rate of switching is much lower. As a result, the original experiences have more time to circulate among clusters, while there is simultaneously less new information available to the network to correct the initial bad impression.

This result gives us an additional clue as to the likely outcome if a new insurer were to enter an established market. As we would expect the rate of customers willing to switch to the new insurer to be lower than in the initialisation phase, we would expect to see the same effect as increased renewal rates. Therefore, we would expect that if the initial experiences of a new insurer are negative, there would be less opportunity to correct the first impression, and their reputation would be more strongly established by the network.

Table 2.10 shows the regression results for the sensitivity tests run on both the base model and the regulation scenario. Without the noise in renewal rates caused by changing market premium, a relationship can now be detected between renewal rates and the word-of-mouth influence parameters k_W, M, and p_W. These parameters also have a higher impact

Output y_k	Intercept α_0	Impact on intercept α_2	p-Value	R^2
Average of top insurer's market share	15.8%	3.8%	0.000	28.1%
Standard deviation of top insurer's market share	1.25%	0.974%	0.000	16.8%
Average of market renewal rate	48%	46.9%	0.000	99.9%
Standard deviation of market renewal rate	31.4%	−27.6%	0.000	99.3%

Table 2.9 – Table of regression results for regulatory change scenario versus base model using indicator variables for the equation: $y_k = \alpha_0 + \alpha_2 I_k + \varepsilon_k$

Output y_k	Input x_k	Intercept α_0	Slope α_1	p-Value for α_1	Impact on intercept α_2	p-Value for α_2	Impact on slope $\alpha 3$	p-Value for α_3	R^2
Top share	k_W	12.6%	12.7%	0.000	1.95%	0.000	9.69%	0.000	47.6%
Renewal rate	k_W	48.1%	0.0287%	0.806	45.7%	0.000	5.16%	0.000	99.9%
Top share	φ	13.8%	3.45%	0.000	2.77%	0.000	2.22%	0.000	35.5%
Renewal rate	φ	48%	0.0596%	0.314	43.7%	0.000	5.34%	0.000	99.9%
Renewal rate	p_W	48%	0.517%	0.386	45.3%	0.000	25.9%	0.000	99.9%

Table 2.10 – Table of regression results on average output values for sensitivity tests run on regulatory change scenario versus base model according to the equation: $y_k = \alpha_0 + \alpha_1 x_k + \alpha_2 I_k + \alpha_3 I_k X_k + \varepsilon_k$

on market concentration. The higher these values, the more information the customers are using to evaluate insurers, and the higher the renewal rates and unequal market concentration.

We also run a heterogeneous version of the regulatory change model. As with the base model, the optimal position is the maximum spend on customer service quality. By plotting the profit as a proportion of the market average, it can be seen that the relative advantage for the regulation scenario is larger than for the base model (see Figure 2.13).

This is because without the additional premium increase from the renewal markup, customers are far less likely to be motivated to switch due to a change in price. Customer choices become instead much more influenced by their estimates of customer service quality. This implies that the regulatory change could also be an incentive to increase customer service quality.

2.6 Conclusions

An ABM was constructed to explore the patterns that might arise in an insurance market due to customers passing on their opinion of their insurer to their social network. The model is not intended to be a complete model of all of the features of an insurance market, nor should it be taken as a predictive model. Instead, it has been designed to focus on the particular features of customer renewal decisions when a word-of-mouth network is present, and to be

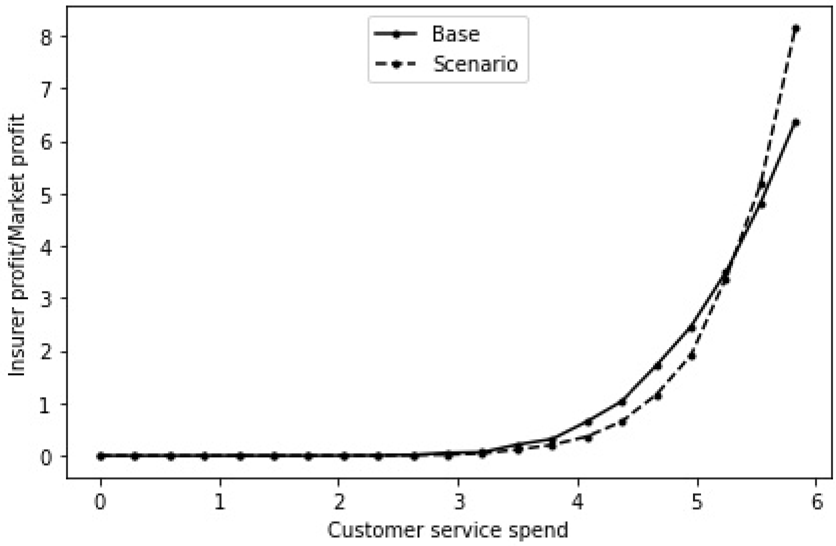

Fig. 2.13 – Average individual insurer profit as a proportion of average market profit versus customer service spend for the base model and the regulation change scenario. In general, the more insurers spend on customer service, the higher their profit relative to their rivals. Under the regulation change, this difference becomes even more pronounced, suggesting that this move could increase incentives for good customer service.

informative of patterns and emergent dynamics.

Empirical data was used to parameterise the model, though this was not always possible for behavioural parameters. In these cases, reasonable values were proposed, and sensitivity tests carried out for these parameters. A validation check was carried out by comparing the model output of market concentrations against real world data.

The ABM notably replicates a key feature of the real-world market: it produces an uneven concentration of market share by insurer. This is because early variations in customer service experiences persist in the market opinion as information is repeatedly repeated and passed around within social groups. Because interaction is rare, it takes some time for an individual to correct a biased perception once they select a new insurer. As a result, some insurers gain a better or worse reputation than others even when they have the same service quality. This phenomenon also causes the average customer's opinion of their own insurer to be higher than their true service quality. This suggests that new insurers particularly benefit from having a high service quality as they establish their reputation.

As we would expect, the biggest driver in customers' decision whether to renew is the change in market premium. This is exacerbated when the insurers are charging a larger margin for service quality. As a result, insurers who maintain good service quality while premiums are rising may suffer a decline in renewal rates if they also pass that expense along to their customers, yet are unlikely to benefit if their reputation has already been well established.

If the insurers are allowed to have different service qualities, their relative success depends on the customers' sensitivity to service quality. In the base model, insurers with higher service quality do better as they both attract and retain more customers. If the customers are less sensitive to customer service relative to the cost, then the extra premium mitigates this effect, until eventually a higher customer quality is a detriment. However, only a small sensitivity value is required for higher customer service quality to become beneficial.

The UK regulatory authority (FCA, 2020) has recently proposed a change in regulations that would prevent UK motor insurers from charging renewing customers a different amount than they would if they were new customers. This change would be expected to cause a large increase in renewal rates as customers' prices change less each year and they can expect less benefit from searching for quotes. Additionally, the relative advantage to insurers attempting to entice new customers with better quality customer service has increased. This implies that the proposed regulation change could also increase the incentive for better customer service quality to compete against rival insurers. However, when the retention rate is very high, customer choices are less influenced by the market cycle. There is much less movement away from insurers with a good reputation and fewer customers deciding to purchase from the lower valued insurers. As a result, the market concentration becomes more skewed. This indicates that the proposed regulatory change might decrease market competition.

Based on these findings, we can conclude that the potential impact of the word-of-mouth network on customer decision-making and the resulting systemic biases is a significant one. These findings should be considered by both insurers considering strategies for attracting and retaining customers, and by regulators who are assessing possible impacts of a change in the regulation of insurance pricing practices.

In future work, we intend to expand the model to allow the insurers to employ a competitive premium-setting strategy and vary their service quality or premium dynamically. The model also contains some implicit behavioural assumptions: for example, good and bad experiences are given the same weight, whereas many studies indicate that people are more sensitive to negative than positive experiences. Additionally, the word-of-mouth information in this model does not include a measure of uncertainty around the customers' opinions. This could potentially change the network dynamics which lead to such a high persistence of opinions within social groups. Some experiments could also be carried out with different types of network and network sizes, to investigate if the current number of customer agents is sufficient to replicate the rate of information saturation and investigate how the word-of-mouth effects vary at different market scales.

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Chapter 3

The Effect of the Winner's Curse on an Insurer's Estimated Capital Requirement

3.1 Introduction

In the previous chapter, we examined systemic bias to insurer reputations caused by networked interactions between customers. As chapter 2 was focussed primarily on customer behaviour, the insurers were modelled as price-takers. However, in practice insurers will base their premium strategy on their estimation of customer risk, and may offer differing prices to potential customers. In this chapter, we will examine an effect called the 'winner's curse' and its impact on an insurer's capital management. The winner's curse is a systemic under-estimation of risk caused by imperfect information in a competitive market. Insurers interact indirectly when they compete for business. Since their premium is based on their independent estimates of the underlying risk, insurers that have under-estimated risk are also more likely to be willing to offer lower prices and win more business.

Insurance companies are increasingly making use of stochastic capital models to assess risks and manage capital. These are Monte Carlo-based stochastic models of the capital flows into and out of the insurance company, capturing the major risks that it faces, most notably the uncertainty surrounding policy claims. It is standard practice among most general insurance companies to make use of an internal capital model, both for regulatory purposes, and also to inform business strategy. The systemic estimation bias caused by the winner's curse will also impact these models, causing estimated capital requirements to become more often underestimated than overestimated.

In this chapter results are generated by an ABM designed to explore the effect of the winner's curse on capital estimates. This model includes two types of agents: insurers and customers. Insurers set premium for each customer based on a simple risk-based premium rule. Customers purchase from the cheapest price. The insurers then estimate their capital requirement. The model results show the resulting capital estimation errors. Additionally, the model is extended to include some market features which impact customer purchase decisions. These are: policyholder heterogeneity; renewal rates; and network shapes, where customers consider premium from a subset of the insurers only.

This chapter finds that more insurers and fewer customers increase the percentage gap between the actual and estimated capital. This effect is mitigated when the model contains renewal rates. When the market include two types of customers, capital estimates are at their worst when roughly half of insurers are aware of the heterogeneity and half remain unaware. When customers seek offers from a subset of the insurers, the gap between actual and estimated capital widens when an insurer has a greater number of rivals and also when insurers tend to share the same customers.

3.2 Background

The winner's curse is a phenomenon that arises in auctions where an item is worth the same to all bidders, and the bidders have imperfect information about the value of the item for which they are bidding (Capen et al., 1971). The bidders will make a bid based on their estimate, and the item is sold to the highest bidder. If the estimates are distributed with an expected value equal to the true value, then the bidder that wins the auction is likely to have overestimated its value (Thaler, 1988).

In the example of insurance, this arises when companies compete for customers by offering a premium. In this case, a company that significantly overestimates the riskiness of a customer is likely to offer a substantially higher premium than its competitors and is therefore unlikely to have its product purchased. However, if the company significantly underestimates the riskiness of a customer, it may offer a much lower premium, and it is therefore more likely that the customer will buy its product. The greater the number of competitors, the greater the chances of the winner having significantly under-estimated the risk, and the worse this effect becomes (Bulow and Klemperer, 2002).

In an insurance market, the underlying risk of an individual policyholder is estimated from past claims data and used to set premium strategy (Parodi, 2014). Due to the heterogeneous nature of the policyholders, lack of data for particular rare events, and the constantly changing nature of the underlying risks, these estimates can contain significant levels of parameter uncertainty. The insurance market is therefore vulnerable to the impact of the winner's curse.

It is common for insurance companies to use an internal capital model to assess risks (Sheaf et al., 2017). These are stochastic models which use Monte Carlo methods to simulate the capital flows into and out of the insurance company. Internal capital models aim to capture the major risks faced by the insurance company, most notably the uncertainty surrounding policy claims (Kravych, 2013).

Under the Solvency II directive, regulatory capital is set according to a 99.5% value-at-risk (99.5% VaR) measure of the insurer's change in equity over a one year time horizon. Additionally, companies must "demonstrate that the internal model is widely used in and plays an important role... in particular their risk management... and their decision-making process" (EU, 2009). Because of this, insurers are increasingly making use of their capital models to inform business strategy.

The parameters of the loss distributions used in the capital model are estimated based on the same loss data as the premium. This introduces an error into the distribution and therefore into the risk measure used to calculate the capital. As the loss data is also used when the insurer sets its premium rates, the parameter misestimation will be subject to the bias caused by the winner's curse. This is particularly relevant for the regulatory capital measure as VaR measures are particular impacted by parameter uncertainty (Cont et al., 2010)

Mata (2000) examines the impact of parameter uncertainty on excess-of-loss reinsurance layers using extreme value distributions. Borowicz and Norman (2006) build on this approach using a Bayesian approach with extreme frequency/severity models. They find that the impact of parameter uncertainty on this type of insurance business is significant and should thus be accounted for in insurers' risk models.

Bignozzi and Tsanakas (2016a) define a residual estimation risk measure used to quantify the impact of parameter uncertainty on a risk measure such as VaR. For example: if an insurer is seeking the 99.5th percentile of a loss distribution, this measure represents the additional amount which should be added to their estimated value in order to be 99.5% certain that the estimate is at least as large as the true 99.5th percentile of the loss distribution.

Bignozzi and Tsanakas (2016b) use this measure to calculate the impact on capital estimated from a random claims history. This is done by taking example distributions and using them to generate sample outputs. The parameters are then estimated based on the outputs, and the capital estimated from applying the risk measure to the estimated distribution. These values are compared to the real capital values to indicate the difference caused by this error for different sample sizes. While these papers aim to quantify parameter misestimation for a capital measurement, they do not account for estimation bias caused by the winner's curse effect.

Yan and Pryor (2018) model an insurance market where customers can be either 'good' risks or 'bad' risks, and insurers estimate which when deciding whether to accept or reject a potential policyholder. Under this model, when a new insurer enters the market, the residual customers still seeking insurance are more likely to be 'bad' risks than 'good'. The new insurer is thus more likely to encounter a higher claims rate than existing insurers. Testing against empirical data shows that this model is consistent with product liability insurance but not with homeowners' insurance. This could reflect the greater homogeneity of risks in a personal lines market.

Mumpower (1991) calculates a formula for setting insurance premium so as to account for the winner's curse effect. In this case, including a margin for ambiguity—as is common practice amongst pricing actuaries—is shown to be rational behaviour for a risk-neutral insurer since this approach offers some protection against the winner's curse.

The Winner's Curse GIRO working party report (Chan et al., 2009) describes the winner's curse and its implications for insurance. This report uses a simple simulation model involving a single customer. The model assumes a simple rule for premium setting and for customer preference based entirely on price, and is used to quantify the effect of the winner's curse on loss ratios within a single time step. Although this is an informative demonstration, the model fails to link the effect to the insurers' estimated capital. Chan et al. (2009) also mention brand preference and loyalty but do not go into this in detail, though they do consider the effect of brokers and aggregators on the loss ratios.

In this chapter, a simple market simulation model is used with a similar setup to Chan et al. (2009). This model is used to examine the resulting parameter uncertainty in capital estimates in a manner similar to Bignozzi and Tsanakas (2016b). There are some implicit assumptions in the calculation of capital: that all customer losses are independent; that there is no runoff from past business; that all business is sold at the start of the year and is fully earned with all loss amounts known by the end of the year; and finally, that each insurer has enough capital that it will not go bankrupt. The focus is also solely on underwriting risk, and there is no allowance for investment and other risks.

The model includes the market features listed below. These features are not well-explored in the existing literature on the winner's curse, but they impact customer purchase decisions and are therefore also pertinent to an investigation of the winner's curse effect:

- Heterogeneity: In real-world markets, customers are heterogeneous, and the insurers' ability to tell them apart is crucial to their ability to offer competitive premium rates. In this model we allow for the existence of two types of customers which we call 'low risk' and 'high risk' in a manner similar to Yan and Pryor (2018) and allow a specified fraction of the insurers to identify which customer belongs to which category. We anticipate that this will reduce the number of insurers that are willing to compete over a particular customer and therefore increase the effect of the winner's curse.
- Renewal: It is common for insurance customers to renew an existing policy instead of searching for a new insurer (FCA, 2020). In this model we allow for a fraction of the customers choosing to renew. We expect this to effectively reduce the number of customers seeking a quote from insurers and therefore to decrease the effect of the winner's curse.
- Networks: As noted in the Winner's Curse GIRO working party report (Chan et al., 2009), many customers have preferences regarding brands and products. Additionally, customers may use a broker or a particular price comparison website. Searching for further quotations carries a cost in time and effort. As a result, most customers will not consider quotes from every insurer. In this model we allow customers to build a network of links to insurers which they will consider. Three types of networks are considered to reflect multiple ways in which customers may make their decision.

3.3 Model Specification

3.3.1 Overview

A simple agent-based model is constructed using C# to demonstrate the effect of the winner's curse on an insurer's capital estimation. It is expected that when there are more com-

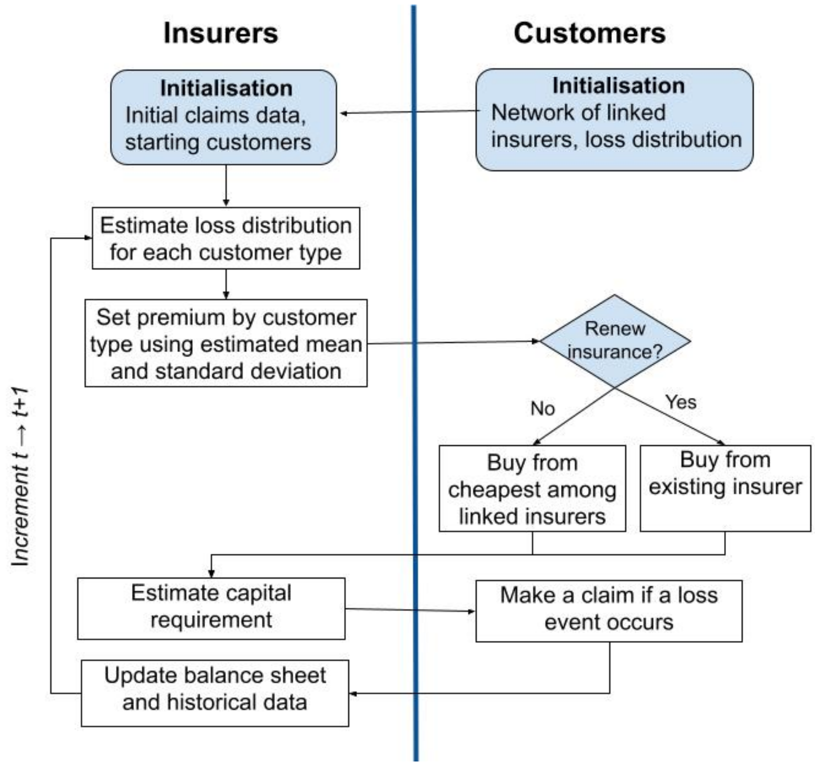

Fig. 3.1 – Swimlane overview of the processes in the winner's curse ABM

petitors, the residual estimation risk will be higher, because the winner's curse means that the business is won by the insurer that has the lowest risk estimate.

The model contains n_I insurers, each labelled as either 'aware' or 'unaware' depending on their ability to correctly differentiate between different types of customers. There are n_J types of customers. For each customer type j, there are n_{j_K} distinct customers of that type.

For each simulation, the agent model begins by simulating starting data for each insurer, and generating a network of linked insurers for each policyholder.

It then undergoes the following steps in each time period:

1. Insurers use past loss data to estimate parameters for loss distributions.
2. Insurers offer each linked customer a contract using the standard deviation premium principle.
3. If a customer renews, it will choose its existing insurer. Otherwise, the customer selects the cheapest option from among its linked insurers.
4. Insurers with customers calculate their capital using 99.5% VaR method.
5. Losses are generated for each customer and recorded by the covering insurer.
6. Output results are calculated for each insurer.

Figure 3.1 is a swimlane diagram representing the flow of processes in the model and which agent is responsible for each step. The code has been made available on CoMSES (England, 2022).

Results are first obtained from a simple base model with homogeneous customers who select purchase from a single insurer with the lowest risk estimate. Further models are then explored containing customer heterogeneity, renewal probability, and incomplete networks. The calculations carried out at each step are described in more detail below.

3.3.2 Definitions

i = Insurer index
j = Index of customer type
k = Customer index of customers of specified type
t = Timestep
$n_{D_{i,t}}$ = Number of loss data points recorded by insurer i at the start of timestep t
$n_{D_{i,j,t}}$ = Number of loss data points recorded by insurer i of customer type j at the start of timestep t
n_I = Number of insurers
n_{I_A} = Number of aware insurers
n_{I_U} = Number of unaware insurers
n_J = Number of customer types
n_{j_K} = Number of customers of type j
q = Network parameter for Poisson network
F = Network parameter for Fitness network
C = Network parameter for Cost-Benefit network
$D_{i,t}$ = Degree network measurement of insurer i at timestep t
$N_{i,t}$ = Neighbourhood network measurement of insurer i at timestep t
$E_{E_{i,t}}[\cdot]$ = Mean average of a function using parameters estimated by insurer i at timestep t
$E_{A_{i,t}}[\cdot]$ = Mean average of a function using actual parameters for insurer i at timestep t
$Var_{E_{i,t}}[\cdot]$ = Variance of a function using parameters estimated by insurer i at timestep t
$Var_{A_{i,t}}[\cdot]$ = Variance of a function using actual parameters for insurer i at timestep t
$\eta_{E_{i,t}}[\cdot]$ = 99.5% VaR of a function using parameters estimated by insurer i at timestep t
$\eta_{A_{i,t}}[\cdot]$ = 99.5% VaR of a function using actual parameters for insurer i at timestep t
$P_{i,j,t}$ = Premium offered by insurer i to customers of customer type j during timestep t

$$\left.\begin{matrix}\mu_{i,j,t}\\ \sigma^2_{i,j,t}\end{matrix}\right\} = \begin{matrix}\text{Lognormal parameters for distribution of losses in a single timestep}\\ \text{for a single customer of type } j \text{ as estimated by insurer } i \text{ at timestep } t\end{matrix}$$

$$\left.\begin{matrix}\mu_{j}\\ \sigma^2_{j}\end{matrix}\right\} = \begin{matrix}\text{Lognormal parameters for actual distribution of losses}\\ \text{in a single timestep for a single customer of type } j\end{matrix}$$

$L_{j,k,t}$ = Total losses generated by customer k of customer type j during timestep t
$L_{i,t}$ = Total losses generated for insurer i during timestep t
R = Renewal probability
$CGR_{i,t}$ = Capital gap ratio for insurer i at the end of timestep t
$NR_{i,t}$ = Normalised residual for insurer i at the end of timestep t

3.3.3 Starting Data

Each insurer generates a set of starting data that it can use to find starting estimates for loss parameters. It is a common assumption to model losses using a lognormal distribution (Bahnemann, 2015). Accordingly, losses are modelled as a single lognormally distributed severity distribution for each individual customer.

Each insurer i begins with a random sample of size $n_{D_{i,0}}$. This gives the starting loss data as:

$$\left(Z_\ell, \ell = 1, \ldots, n_{D_{i,0}}\right) \tag{3.1}$$

where $Z_\ell \sim$ i.i.d. $Lognormal\left(\mu_j, \sigma_j^2\right)$. For each datapoint, the associated customer type j is drawn at random from the available customer types.

3.3.4 Networks

Since searching for premium quotes carries a cost, customers are unlikely to search for quotes from every available insurer. The possible impact of this is examined through a network, where customers seek quotations only from insurers to which they are linked.

The network is generated once for each network parameter, and the simulations run on the resulting shape. Additionally, since customers must always purchase a policy, the result is rejected and a new set of links is generated if a customer has no links.

Most of the models use a simple complete network. This is where each customer has a link to every available insurer. For the network model, three common types of network structures are used to investigate network effects: Poisson, Fitness, and Cost Benefit. These three models have different characteristic properties and structures (Bargigli and Tedeschi, 2014).

Note that if a node generates zero links, the result is discarded and re-drawn, as the model assumes that all consumer agents will purchase insurance. This means that the resulting link distribution will display a bias - for example, the Poisson network instead reflects a zero-truncated Poisson distribution.

- Poisson: This type of network reflects a scenario whereby customers seek a small set of quotes from a random subset of insurers. For each customer, a link to each insurer is generated according to the network connection probability q.
- Fitness: This type of network reflects a scenario similar to the use of price aggregation sites or brokers, where particular insurers are likely to have a larger base of potential customers depending on their choice of distribution channel. In this network formation, when a new customer is added to the network, links are more likely to be formed with insurers that already have many links. The probability of forming a link to an insurer is equal to F multiplied by the proportion of existing links to the insurer out of all links in the network. Here, F is a 'fitness multiplier' parameter. The higher this parameter, the more links are expected to form. This type of network produces more variation in the degrees of the nodes, resulting in a skewed power law distribution.
- Cost Benefit: This type of network reflects the assumption that customers will be

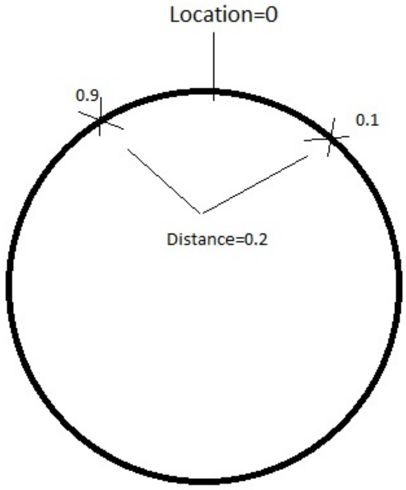

Fig. 3.2 – Distance between two locations in preference space

drawn to certain insurers more than others based on personal preferences or targeted marketing. This is represented using a circular 1D ‘preference’ space similar to that used by Owadally et al. (2018). Insurers and customers are given a random location between 0 and 1. As the location space is circular, the distance between locations 0 and 0.1 and the distance between locations 0 and $0.9 = (1-0.1)$ are both equal to 0.1, and the distance between 0.9 and 0.1 is $0.1+0.1 = 0.2$ (see Figure 3.2). The distance between the location $\gamma_{j,k}$ of the kth customer of the jth customer type and the location γ_i of the ith insurer is then calculated as $\gamma_{i,j,k} = \min\left(|\gamma_i - \gamma_{j,k}|, 1-|\gamma_i - \gamma_{j,k}|\right)$. If the distance between the insurer and the customer is less than the network parameter C, which reflects the cost of searching an additional insurer, then the link is added. This type of network produces a bipartite ring, where insurers that are close to each other tend to form links with the same customers, and similarly for customers who are close neighbours. The lower the value of C, the fewer the number of links, and the fewer links customers and insurers will have in common with their neighbours. The higher the value of C, the closer the network is to a complete network.

The shape of a network impacts the interactions between agents. Two key network properties are the distribution of degree [where degree is the number of links to a node] and the level of clustering, which indicates the formation of small interconnected communities (Amini et al., 2016). Correspondingly, two quantities are calculated for each insurer i:

1. Degree $D_{i,t}$: This is equal to the number of links from insurer i at time t.
2. Neighbourhood Measurement $N_{i,t}$: This measures whether policyholders and insurers group together into neighbourhoods where policyholders tend to have links to the same insurers. It is calculated as:

$$N_{i,t} = \frac{1}{n_I - 1} \sum_{i^* \neq i} N_{i,i^*,t} \tag{3.2}$$

where

$$
\begin{aligned}
N_{i,i^*,t} &= \frac{1}{D_{i,t}} \sum_{l=1}^{D_{i,t}} \Delta_{i,i^*,l,t} \\
\Delta_{i,i^*,l,t} &= \begin{cases} 1 & \text{if insurers } i \text{ and } i^* \text{ both have links to customer } l \text{ at time } t \\ 0 & \text{otherwise} \end{cases}
\end{aligned}
\tag{3.3}
$$

The model outputs are then compared for differing values of these properties.

3.3.5 Parameter Estimation

An insurer i estimates the loss parameters in timestep t by applying a maximum likelihood approach to its past data, $\left(Z_\ell,\, \ell = 1, \ldots, n_{D_{i,0}}\right)$, by each customer type.

$$
\mu_{i,j,t} = \frac{1}{n_{D_{i,j,t}}} \sum_{\text{all } l \text{ belong to customer type } j} \log Z_l \tag{3.4}
$$

$$
\sigma^2_{i,j,t} = \frac{1}{n_{D_{i,j,t}}} \sum_{\text{all } l \text{ belong to customer type } j} (\log Z_l - \mu_{i,j,t})^2 \tag{3.5}
$$

Note that, for heterogeneous customers, there are two types of insurers: 'aware' and 'unaware'. The aware insurers estimate the parameters for each customer type separately, and so will correctly identify data as being of type j and estimate a separate $\mu_{i,j,t}$ and $\sigma^2_{i,j,t}$ for each j. Unaware insurers are unable to differentiate between the different customer types, and therefore incorrectly identify all datapoints as belonging to the same group, giving the same values of $\mu_{i,j,t}$ and $\sigma^2_{i,j,t}$ for each customer type j.

In a real market, it is common for insurers to make use of a no-claims discount to identify low-risk customers (Lemaire, 1988). This is not possible here as claims amounts are approximated as a total lognormal distribution, which does not allow for the possibility of zero claims. However, the use of no-claims discounts would offer unaware insurers the possibility of a crude identifier of low-risk customers, and would therefore potentially enable them to offer more competitive premiums to the low-risk customers. Note however that, if such an insurer could attract lower-risk customers, it would not adapt its premium as quickly for the highe- risk customers as it would have a mixture of high- and low-risk loss experience.

3.3.6 Premium

The focus of this model is on estimation rather than premium strategy. To keep the model simple, the insurers do not incorporate a competitor analysis into their premium. Instead, premium is set based on a risk premium using a simple standard deviation method (Dickson, 2016).

Using this principle, the premium offered by insurer i to customers of type j during time step t is:

$$
P_{i,j,t} = E_{E_{i,t}}[L_{j,k,t}] + a\sqrt{Var_{E_{i,t}}[L_{j,k,t}]} \tag{3.6}
$$

Based on the results of the parameter estimation process, the insurer calculates this as:

$$P_{i,j,t} = \exp(\mu_{i,j,t}\frac{\sigma^2}{2}) + a[\exp(\sigma^2) - 1]\exp(2\mu_{i,j,t}\sigma^2) \tag{3.7}$$

3.3.7 Customer Allocation

Searching for quotes takes time and effort on the part of the policyholder, which creates a disutility. If this disutility is expected to be higher than the utility gained from finding a lower premium, then a rational customer will choose to renew their contract instead (Dutang et al., 2012).

To reflect this, policy renewal is modelled as a simple Bernoulli trial. The kth customer, belonging to customer type j, renews with their existing insurer at time t if $r_{j,k,t} < R$ where R is the policy renewal probability and

$$r_{j,k,t} \sim Uniform(0,1) \tag{3.8}$$

If $r_{j,k,t} \geq R$, the customer seeks premium quotes from all insurers in its network, and chooses to purchase from the insurer offering the lowest premium.

Some types of insurance, such as third-party motor insurance, are mandatory. However, studies show that, even in voluntary markets, demand for insurance tends to remain steady and is not materially affected by price (Daykin et al., 1993). The total number of customers is therefore set as a constant across all timesteps.

3.3.8 Capital Estimation

For the insurer's estimated capital requirement, the Value-at-risk measure of the 99.5% loss is used. This mimics regulation such as Solvency II (EU, 2009).

In practice, most insurers make use of Monte Carlo simulation to produce their capital estimate. However, using this method would greatly increase the computation time. In order to run the model within a reasonable time frame, it is assumed that the sum of the customers' losses is also lognormal. The parameters are estimated by matching the first two moments (Fenton, 1960). This gives the following calculation for insurer i's estimated capital requirement in timestep t:

$$\eta_{E_{i,t}}[L_{i,t}] = \eta_{E_{i,t}}\left[\sum_j \sum_k \delta_{i,j,k,t} L_{j,k,t}\right] \tag{3.9}$$

where

$$\delta_{i,j,k,t} = \begin{cases} 1, & \text{if customer } k \text{ in group } j \text{ has a contract with insurer } i \text{ during time step } t. \\ 0, & \text{otherwise.} \end{cases} \tag{3.10}$$

Thus, this approximation can be calculated by equating the mean and variance of the sum of the estimated independent distributions, denoted as $M_{E_{i,t}}$ and $V_{E_{i,t}}$ respectively. This gives the following equation:

$$\begin{aligned} M_{E_{i,t}} &= \sum_j \sum_k \delta_{i,j,k,t} \exp\left(\mu_{i,j,t} + \frac{\sigma_{i,j,t}^2}{2}\right) \\ V_{E_{i,t}} &= \sum_j \sum_k \delta_{i,j,k,t} \exp(2\mu_{i,j,t} + \sigma_{i,j,t}^2)[\exp(\sigma_{i,j,t}^2) - 1] \end{aligned} \tag{3.11}$$

Thus:

$$\eta_{E_{i,t}}[L_{i,t}] \approx \eta_{E_{i,t}}[X_{i,t}] \tag{3.12}$$

where:

$$X_{i,t} \sim Lognormal\left(\log\left[\frac{M_{E_{i,t}}}{\sqrt{1+\frac{V_{E_{i,t}}}{M_{E_{i,t}}^2}}}\right], \log\left[1+\frac{V_{E_{i,t}}}{M_{E_{i,t}}^2}\right]\right) \tag{3.13}$$

Similarly, the actual capital requirement for insurer i during time step t is calculated as:

$$\eta_{A_{i,t}}[L_{i,t}] \approx \eta_{A_{i,t}}[Y_{i,t}] \tag{3.14}$$

where:

$$Y_{i,t} \sim Lognormal\left(\log\left[\frac{M_{A_{i,t}}}{\sqrt{1+\frac{V_{A_{i,t}}}{M_{A_{i,t}}^2}}}\right], \log\left[1+\frac{V_{A_{i,t}}}{M_{A_{i,t}}^2}\right]\right) \tag{3.15}$$

and:

$$\begin{aligned} M_{A_{i,t}} &= \sum_j \sum_k \delta_{i,j,k,t} \exp(\mu_j + \frac{\sigma_j^2}{2}) \\ V_{A_{i,t}} &= \sum_j \sum_k \delta_{i,j,k,t} \exp(2\mu_j + \sigma_j^2)[\exp(\sigma_j^2) - 1] \end{aligned} \tag{3.16}$$

3.3.9 Losses

Losses are generated for each customer k of customer type j using a lognormal distribution as follows:

$$L_{j,k,t} \sim Lognormal(\mu_j, \sigma_j^2) \tag{3.17}$$

The losses for each insurer i are then obtained by adding up the losses of all of its customers:

$$L_{i,t} = \sum_j \sum_k \delta_{i,j,k,t} L_{j,k,t} \tag{3.18}$$

Each insurer then adds their new losses incurred to their history of losses. The insurer then has a larger data sample to re-estimate the loss distribution parameters in the next time step.

3.3.10 Calculate Output

The model outputs two main results: the capital gap ratio and the normalised residual.

The capital gap ratio (CGR) is calculated for each insurer i as:

$$CGR_{i,t} = (\eta_{E_{i,t}} - \eta_{A_{i,t}})/\eta_{E_{i,t}} \tag{3.19}$$

where $\eta_{E_{i,t}}$= insurer i's estimated capital during time step t, and $\eta_{A_{i,t}}$ is the actual 99.5th percentile of insurer i's total customer loss distribution. This is a measure of the ratio by which the estimated capital should be increased in order to get to the actual capital.

When examining the average across all simulations, results are excluded when the estimated and actual capital were both zero. The average is this a measure of the gap in estimated capital for an insurer which is successfully selling business.

The normalised residual (NR) is calculated as:

$$NR_{i,t} = (L_{i,t} - \eta_{E_{i,t}})/(\eta_{A_{i,t}} - E[L_{i,t}]) \tag{3.20}$$

where $L_{i,t}$ is the value of the total loss generated for insurer i in the time step t. This output can be used to replicate the normalised residual risk (NRR) used by Bignozzi and Tsanakas (2016a) by taking the 99.5th percentile of the NR. Again, when calculating the NRR, results are excluded where the insurer did not win any customers, so that this is a measure of residual estimation risk for an insurer who is selling business.

3.3.11 Parameter Values

The model is run for four different sets of parameter values.

The first model is the Base Model. This model uses only one type of customer, a complete network, and 0% chance of renewal. To explore how the level of competition and parameter estimation risk affects the gap in the estimated capital the following parameters are varied:

- Number of insurers
- Number of policyholders
- Number of initial data points
- Loss coefficient of variation

The second model is the Heterogeneous Model. In this model there are two types of customers: ‘Low risk’ and ‘High risk’ customers, with a 50% split between the two. The percentage of aware versus unaware insurers is then varied. As before, the network is complete and there is 0% chance of renewal.

The third model is the Renewal Model. Here, the number of insurers is held stead, but the number of policyholders and chance of renewal are both varied to explore the impact of introducing the chance of customers choosing to renew their existing policies instead of searching for new quotes. The customer types are homogeneous as in the Base Model, and the network is complete.

Finally, a Network Model is run. For this model, the choice of network shapes and the network parameter values are varied. In each model run, a single network shape is generated for each network parameter and simulations are run for each of these shapes. All other inputs are held steady so as to concentrate on the network related effects.

No. Customer types	$n_J = 1$
Loss Mean	100
Loss CV	0.1 and 0.5
No. of Insurers	n_I = 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10
No. of Policyholders	n_{1_K} = 1, 2, 4, 8, 16, 32, 64, 128, 256, 512, and 1,000
Size of starting data samples	$n_{D_{i,0}}$ = 20, 50, 100
Network	Complete
Prob. of Renewal	R = 0%

Table 3.1 – Input parameter variations for Base Model

No. Customer types	$n_J = 2$
Loss Mean	(80, 120)
Loss CV	(24, 60)
No. of Insurers	n_I = 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10
No. of Aware Insurers	n_{I_A}=All values between 1 and no. Insurers
No. of Policyholders of each type	$n_{1_K} = n_{2_K}$ = 2, 4, 8, 16, 32, 64, 128, 256, and 500
Size of starting data samples	$n_{D_{i,0}}$=20, 50, 100
Network	Complete
Prob. of Renewal	R = 0%

Table 3.2 – Input parameter variations for Heterogeneous Model

No. Customer types	$n_J = 1$
Loss Mean	100
Loss CV	0.5
No. of Insurers	$n_I = 10$
No. of Policyholders	n_{1_K} = 2, 4, 8, 16, 32, 64, 128, 256, and 512
Size of starting data samples	$n_{D_{i,0}} = 20$
Network	Complete
Prob. of Renewal	R = 0%, 10%, 20%, 30%, 40%, 50%, 60%, 70%, 80%, and 90%

Table 3.3 – Input parameter variations for Renewal Model

No. Customer types	$n_J = 1$
Loss Mean	100
Loss CV	0.5
No. of Insurers	$n_I = 10$
No. of Policyholders	$n_{1_K} = 10$
Size of starting data samples	$n_{D_{i,0}} = 20$
Network	Poisson, Fitness, and Cost Benefit
Network Parameters	q = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9 F = 1, 2, 3, 4, 5, 6, 7, 8, 9 C = 0.95, 0.9, 0.85, 0.8, 0.75, 0.7, 0.65, 0.6, 0.55
Prob. of Renewal	R = 0%

Table 3.4 – Input parameter variations for Network Model

3.4 Results

3.4.1 Base Model

As described in the introduction, the normalised residual estimation risk is examined. Bignozzi and Tsanakas (2016b) described the residual estimation risk as $RR_{i,t} = \eta(L_{i,t} - \eta_{E_{i,t}}(L_{i,t}))$; i.e. the 99.5th percentile of the generated losses minus the estimated capital requirement. This is a measure of the extra capital that should be added to the estimated capital in order to be 99.5% certain of holding at least as much capital as the true 99.5th percentile of the total loss distribution. The residual risk can be normalised to give us the normalised residual estimation risk $NRR_t = \eta(NR_t)$ where $NR_t = (L_{i,t} - \eta_{E_{i,t}}(L_{i,t}))/(\eta_{A_{i,t}} - E[L_{i,t}])$ and the NRR is calculated by taking the percentile of the NR across 100,000 simulations.

In Table 3.5, values are tabulated for the normalised residual risk. The first two columns describe the number of insurers and the number of customers in the market respectively, and results are given for different coefficient of variation in the loss distributions and for a differing number of starting data samples. The top row shows the NRR values found by Bignozzi and Tsanakas (2016b); these are comparable with the model results for one insurer and one customer.

		CV=0.1			**CV=0.5**		
n_I	$n_{J=1}$	$n_{D_{i,0}=20}$	$n_{D_{i,0}=50}$	$n_{D_{i,0}=100}$	$n_{D_{i,0}=20}$	$n_{D_{i,0}=50}$	$n_{D_{i,0}=100}$
B&T		0.156	0.066	0.034	0.212	0.098	0.052
1	1	0.155	0.066	0.032	0.193	0.089	0.056
1	1000	6.073	3.558	2.338	5.510	3.374	2.265
10	1	0.315	0.188	0.110	0.440	0.313	0.194
10	1000	7.924	4.814	3.223	6.987	4.460	3.092

Table 3.5 – Normalised Residual Risk of Capital Estimation from 100,000 simulations of the Base Model and Comparison with Bignozzi and Tsanakas (2016b) (denoted by B&T)

From Table 3.5 it can be seen that the model results are comparable with those found by Bignozzi and Tsanakas (2016a) for a single customer and insurer. Similarly to their findings, the normalised residual estimation risk is greater when the loss distribution is more variable, and when there are fewer initial data samples to inform the parameter estimation.

The normalised residual estimation risk is then found to significantly increase for a greater number of policyholders, as parameter misestimation is amplified. As expected, the effect of the winner's curse causes an increase in the residual estimation risk for a greater number of competitors.

Therefore, an insurer faced with competitors would likely need an even greater increase in their capital in order to cover their losses with 99.5% certainty than the amounts calculated by Bignozzi and Tsanakas (2016b).

Figure 3.3 shows the average value of the capital gap ratio. The x axis shows the effect of varying the number of competing insurers, and the different series show the number of customers. The average actual capital is higher than the insurers' estimates. The capital gap ratio increases for a larger number of competitors and a smaller number of policyholders.

This is a similar pattern found by the GIRO Winner's Curse working party (Chan et al., 2009), which investigated the effect of more competitors on the market loss ratio, but did not examine the connection with capital estimation.

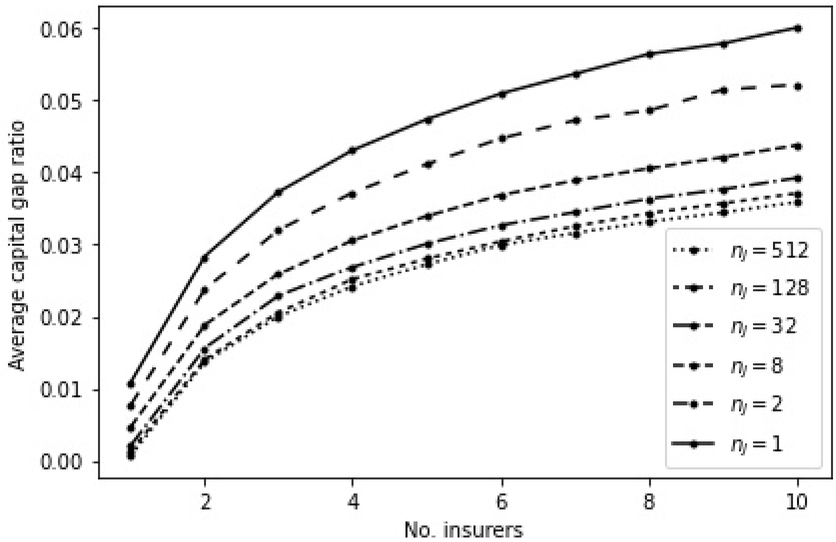

Fig. 3.3 – Average CGR of Base Model: $n_{D_{i,0}} = 10$. Error in capital estimation increases for more insurers or fewer customers.

3.4.2 Heterogeneity

The GIRO Winner's Curse working party (Chan et al., 2009) investigated the effect of insurers with superior models, finding that such insurers showed a lower loss ratio but tended to attract less business. However, the paper did not examine heterogeneous customers. In the heterogeneity model, the 'aware' insurers have superior models and are able to attract a higher share of the lower risk business. Once again, values are produced for the normalised residual estimation risk. This time the total number of insurers is kept at 10, with 5 aware and 5 unaware insurers.

		5 Aware Insurers		
Insurer Type	n_{j_K}	$n_{D_{i,0}}$=20	$n_{D_{i,0}}$=50	$n_{D_{i,0}}$=100
Aware Insurers	1	0.574	0.324	0.193
	500	7.637	4.615	3.066
Unaware Insurers	1	0.662	0.547	0.501
	500	6.777	5.303	4.618

Table 3.6 – Normalised Residual Risk of Capital Estimation from Heterogeneous Model split by Aware and Unaware Insurers

As before, Table 3.6 shows that a higher number of customers increases the normalised

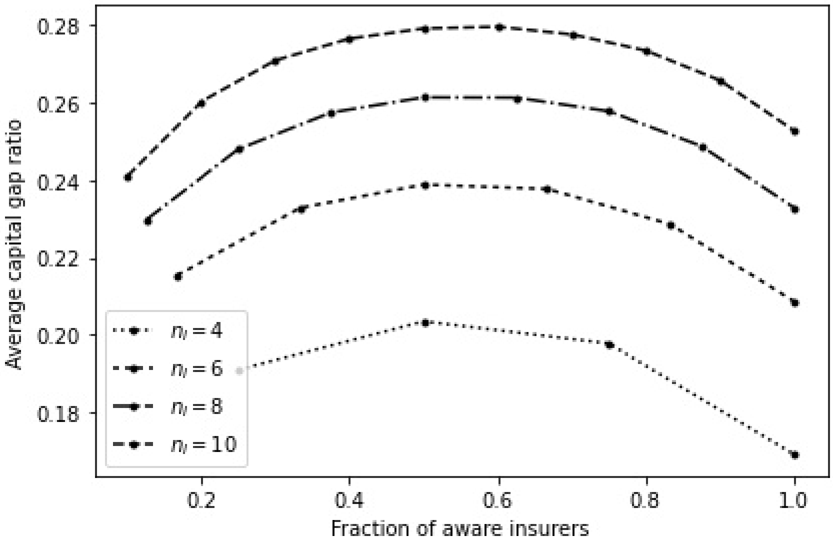

Fig. 3.4 – Average CGR of Heterogeneous Model: $n_{D_{i,0}} = 20$, $n_{j_K} = 256$. The error in capital estimation is worse for aware insurers when there are more aware insurers in the market, and worse for unaware insurers when there are more unaware insurers in the market. Overall, the total error is largest when there are equal numbers of each type of insurer.

residual estimation risk caused by parameter misestimation, as does a lower value of n. Additionally, as would be expected, the unaware insurers have a much higher normalised residual risk than the aware insurers in almost all cases.

However, it should be noted that although the aware insurers have more information about the true nature of their customers, they have less data with which to estimate parameters when split by customer type. This results in a wider variation around their loss estimates. For $n_{D_{0,t}} = 20$ and $n_{j_K} = 500$, this effect is large enough that the aware insurers display a higher normalised residual estimation risk than the unaware insurers.

Figure 3.4 shows shows the average capital gap ratio by the fraction of insurers that are aware, for each total number of insurers, where n_{j_K} = 256. This graph presents as a horseshoe shape; as the fraction of aware insurers increases, the results seem to worsen before improving again. The overall market results are at their worst when approximately half of the insurers are aware.

The aware insurers are much more likely than the unaware to win the business of the lower risk customers because they are able to offer a lower premium. Meanwhile, the unaware insurers are more likely to win the business of the higher risk customers; their results are much worse than for the aware insurers.

Here, the winner's curse operates on the two groups. The greater the number of aware insurers in the same market, the worse the capital gap ratio is likely to be for the insurer who attracts the low risk customers. Similarly, the greater the number of unaware insurers, the worse the capital gap ratio is likely to be for the insurer who attracts the high risk customers.

Therefore, when the ratio of aware to unaware insurers is low, the impact of the win-

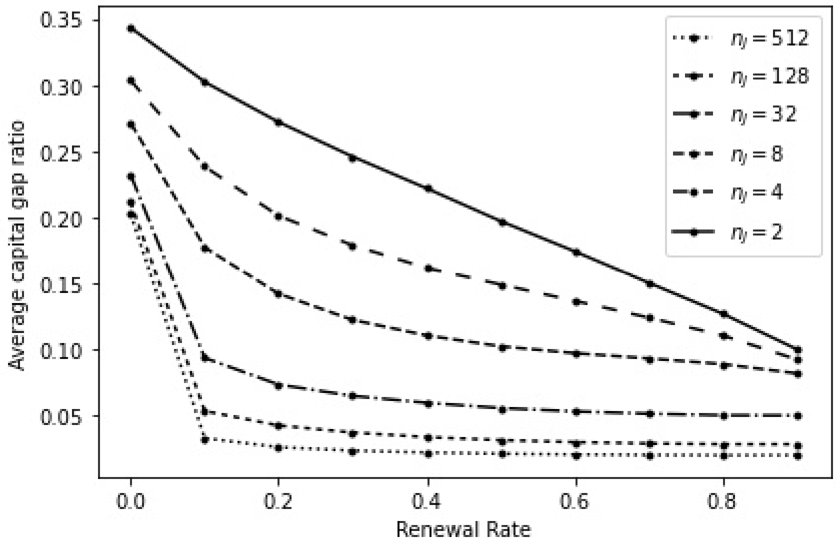

Fig. 3.5 – Average CGR of Renewal Model for different renewal rates: $n_I = 10$, $n_{D_{i,0}} = 20$. Renewal rates decrease the error in the capital estimation.

ner's curse is lower for the aware insurers and higher for the unaware group. When the ratio is high, the impact of the winner's curse is higher for the aware insurers, and lower for the unaware insurers. Overall, the combined impact of the winner's curse for both sets of customers is at its highest when the market is composed of 50% aware and 50% unaware insurers. This is what causes the horseshoe shape seen in figure 3.4.

3.4.3 Renewal

The GIRO Winner's curse model (Chan et al., 2009) did not examine the possible mitigating effects of customer's renewing rather than seeking a new insurer. This possibility is investigated here. Figure 3.5 shows the average capital gap ratio for each total number of customers, for n_I = 10. Increasing the renewal rate slows down the rate of customer jumps to the insurer with the lowest current estimate and thus mitigates the effect of the winner's curse, reducing the average gap between the estimated and actual capital requirement.

Examining the results over 100 timesteps shows that over time, the difference between a model with a 50% chance of renewal and one with zero chance of renewal closes as the customers gradually migrate towards insurers with lower premium estimates. With both models, the effect of the winner's curse reduces over time as insurers learn from experience and the winning insurers update their estimates and increase their premium (Figure 3.6).

However, although the renewals market begins well, an increasing number of insurers jump to the insurer with the lowest premium offer in the first few years. This means that the capital gap ratio increases initially as the winner's curse has an increasing influence on the results.

Additionally, the winning insurers take longer to learn from experience since they do not have as high a share of the market. This means that the improvement over time in the

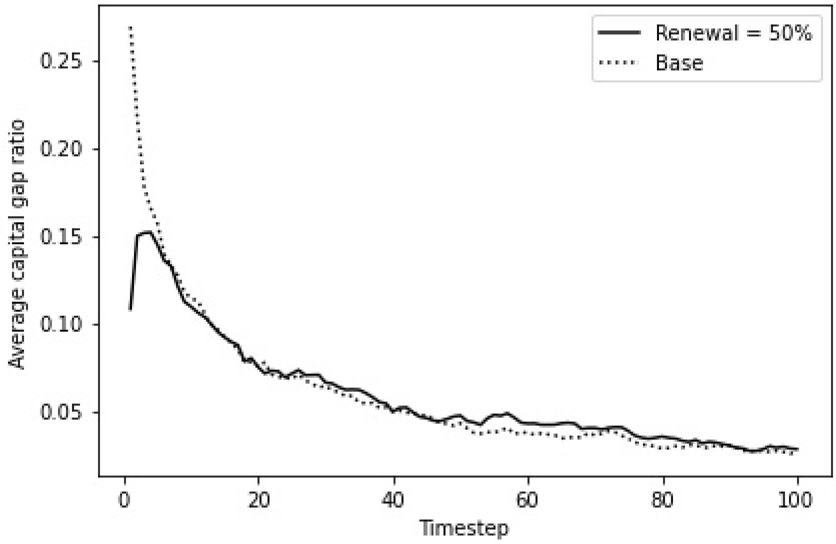

Fig. 3.6 – Average CGR of Renewal Model over time: $n_I = 10$, $n_{D_{i,0}} = 20$, $R = 50\%$ and $R = 0\%$. Over time, the results of the renewal model approaches that of the base model.

renewals market does not quite match the speed of the improvement over time in the control market.

3.4.4 Networks

The Network models are used to allow us to explore the possibility of applying aspects of network theory to the model. The GIRO Winner's Curse working party (Chan et al., 2009) model does not consider networks; however, it does examine the possibility of brand strength allowing an insurer to charge a higher premium while maintaining market share, which can mitigate the effect of the winner's curse. This can be seen as analogous to an insurer in the network which has many links but comparatively few rivals, allowing them to be considered by more customers and thus indicating increased popularity.

Figures 3.7, 3.8, and 3.9 show the average capital gap ratio by the value of the network parameter for the three different network types (q, F, or C respectively). The average capital gap is low for the lowest value of the network parameter as insurers have few rivals for each customer; this is comparable to the Chan et al. (2009) results where all insurers have a stronger brand but with a smaller section of the market. As the parameter is increased, the network approaches the complete network and there are more competitors for each link, causing the winner's curse to increase the capital gap.

Figure 3.10 shows that the cost benefit network demonstrates a weakly positive correlation between degree and the capital gap ratio for a cost benefit model. For Poisson and Fitness network types, there is a stronger correlation between a higher neighbourhood measure and a higher average capital gap ratio, as demonstrated in Figures 3.12 and 3.11.

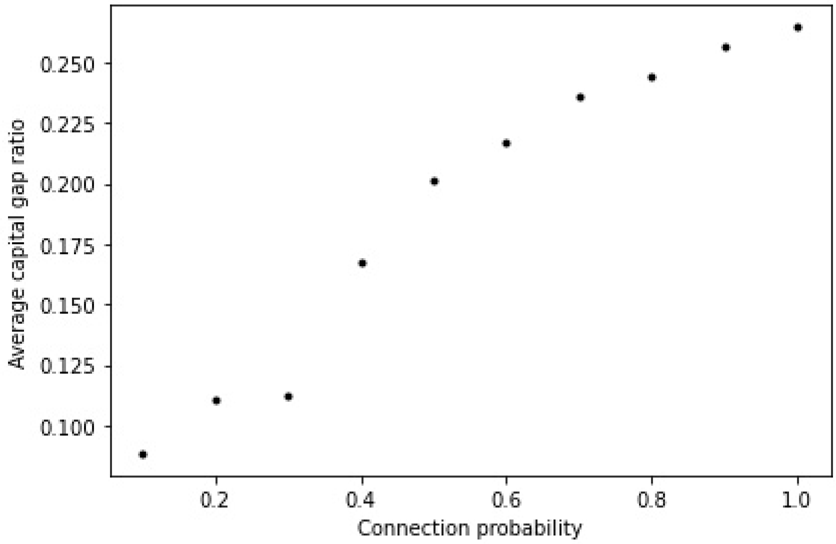

Fig. 3.7 – Average CGR of Network Model: Poisson network, $n_I = 10$, $n_{D_{i,0}} = 20$, $n_{J=1} = 10$. The error in the capital estimation increases when the connection probability is higher and each transaction has a greater number of rivals.

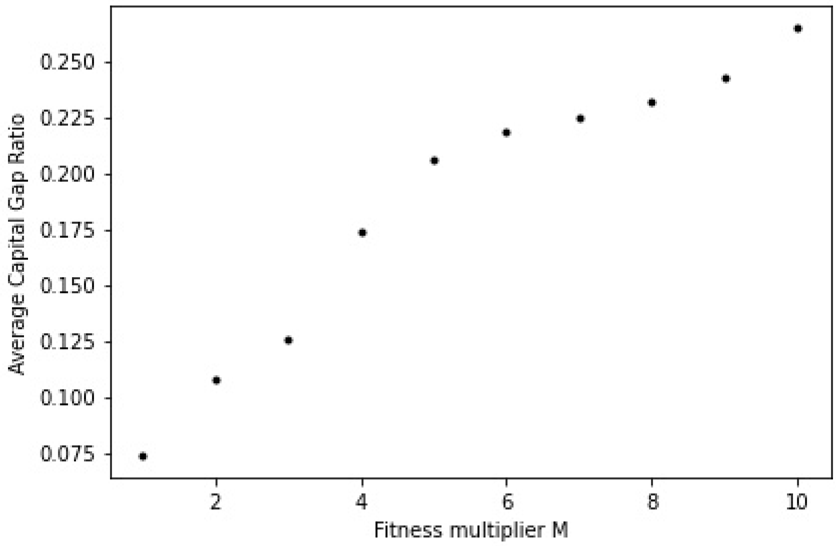

Fig. 3.8 – Average CGR of Network Model: Fitness network, $n_I = 10$, $n_{D_{i,0}} = 20$, $n_{J=1} = 10$. The error in the capital estimation increases when the fitness parameter is higher and each transaction has a greater number of rivals.

3.5 Conclusion

A simple simulation model was built to explore the winner's curse. In the context of insurance, this occurs when the insurers that have underestimated the risk offer lower premium and therefore win more business. This model allowed for imperfect information, where insurers estimate loss distributions based on randomly generated past data. Insurers then calculate their capital requirement based on their estimated distributions, and a capital gap

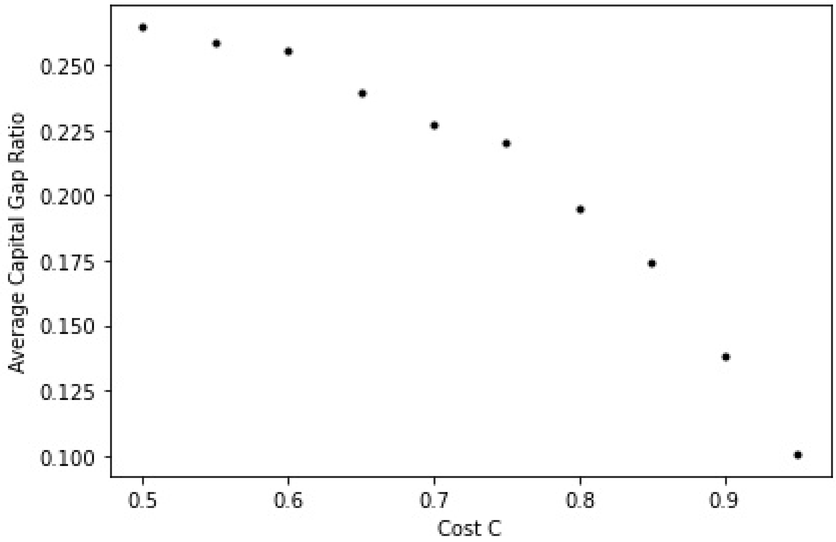

Fig. 3.9 – Average CGR of Network Model: Cost-Benefit network, $n_I = 10$, $n_{D_{i,0}} = 20$, $n_{J=1.} = 10$. The error in the capital estimation decreases when the cost parameter is higher and each transaction has a smaller number of rivals.

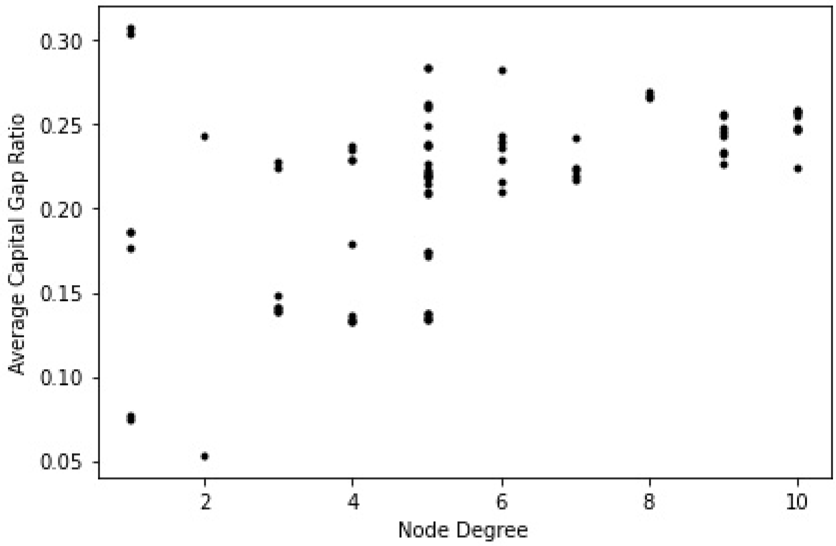

Fig. 3.10 – Average CGR of Network Model split by Insurer Degree: Cost Benefit network, $n_I = 10$, $n_{D_{i,0}} = 20$, $n_{J=1} = 10$

ratio is calculated based on the gap between the estimated capital and the capital based on the actual parameters. In this chapter, the focus is on the impact of the VaR measure of capital, as this is the measure commonly used with current regulation (Solvency II). However, the results could be expanded to consider the impact on other risk measures, such as TVaR or CVaR.

Outputs indicate that the winner's curse increases the estimation risk due to parameter uncertainty when there are more competitors, leading to a higher gap between the estimated

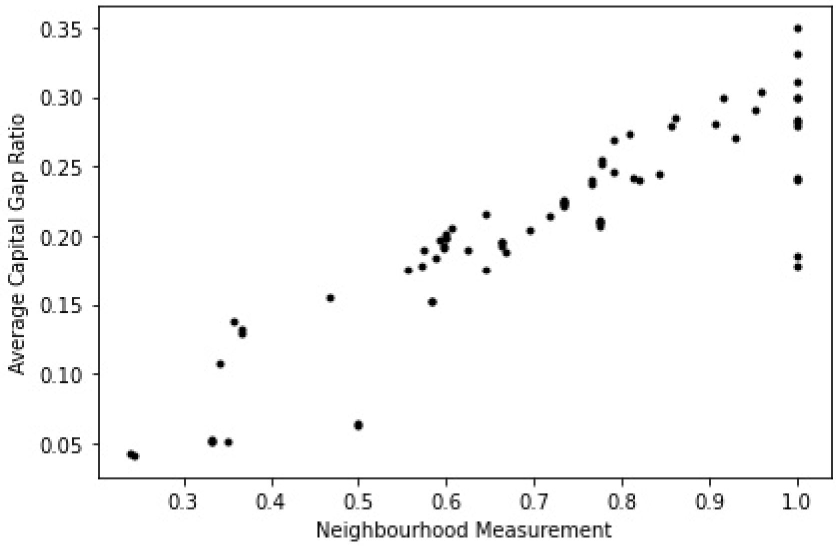

Fig. 3.11 – Average CGR of Network Model, split by Insurer Neighbourhood Measure: Fitness network, $n_I = 10$, $n_{D_{i,0}} = 20$, $n_{J=1} = 10$

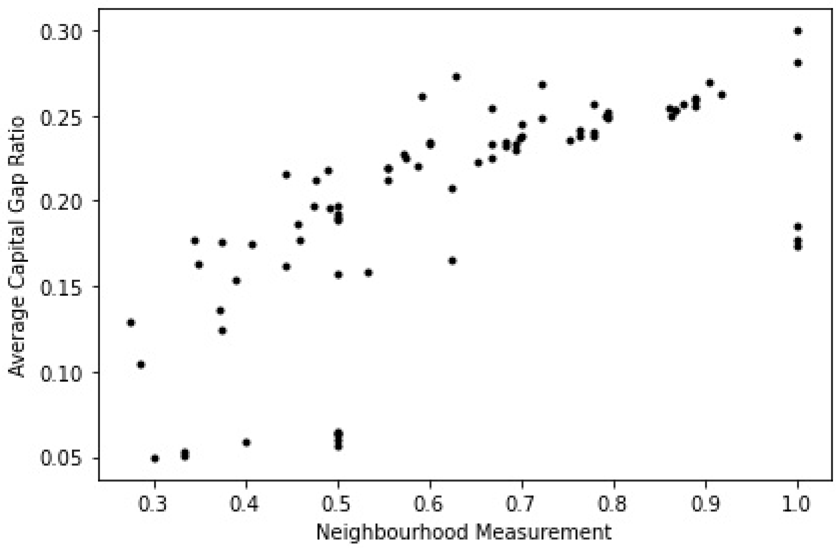

Fig. 3.12 – Average CGR of Network Model, split by Insurer Neighbourhood Measure: Poisson network, $n_I = 10$, $n_{D_{i,0}} = 20$, $n_{J=1} = 10$

and actual capital, and an increase in the parameter estimation risk considered by Bignozzi and Tsanakas (2016b). The capital gap also increases when there are fewer customers. This is similar to the conclusions reached by the GIRO Winner's curse working party (Chan et al., 2009); however, they examined the effect on the market loss ratio, and did not consider the implications for capital estimation.

In a heterogeneous market, the capital gap was worse for unaware than aware insurers, similar to the effect of possessing a superior model investigated by the GIRO Winner's curse working party (Chan et al., 2009). However, although the aware insurers have more information about the true nature of their customers, they have functionally less data with which to estimate parameters, thus resulting in a wider variation around their loss estimates.

The average capital gap ratios across the market as a whole are at their worst when when there are the greatest number of competitors for both an aware and an unaware insurer, and thus gives the highest combined winner's curse effect; this was not an effect previously considered by the GIRO Winner's curse working party.

The second extension to the simple model was to add in renewal rates. The higher the renewal rate, the smaller the capital gap, as there are more insurers with renewals-driven customers instead of winner's curse driven customers. Long term, although the renewals market begins well, an increasing number of insurers jump to the insurer with the lowest premium offer in the first few years. Additionally, the insurer with the lowest premium attracts fewer customers per time step and so doesn't learn as fast. This means that after a while, the market with renewals does slightly worse than the non-renewal market.

In the final model extension, a network was used to represent which insurers a given customer will ask for premium quotes. When the network is sparse, there are fewer competitors per customer and insurers are less affected by the winner's curse. When the network is dense, each insurer has more competitors, and the capital gap ratios is close to the higher outputs of the complete network. In the simulated markets, the capital gap ratio is higher for insurers with a higher degree for cost benefit network, and for insurers with a higher neighbourhood measurement in a poisson or fitness network.

Overall, the implication is that an insurer should increase their capital estimate in order to expect to cover their true capital requirement. The greater the effect of competition, the more this adjustment is required.

This chapter demonstrates the interaction of imperfect information with a number of key market features. However, the premium is based entirely on actuarial risk assessment. Under this model, insurers do not account for competitive forces. This means that an insurer who underestimates risk does not increase offered premium based on its competitors. Similarly, an insurer who fails to win business due to overestimation does not decrease its premium to be competitive. In practice, it is often believed that premium levels are driven by a competition-driven underwriting cycle. In this case, the winner's curse becomes less obvious, and mostly manifests in differences in the market volumes each insurer is willing to seek at different premium levels. Future work should be done to combine the model in this chapter with a competitive premium mechanism in order to examine the link with underwriting cycles.

The winner's curse effect is often assumed by modellers to have negligible impact as insurers have access to a lot of data. However, there are common circumstances that increase the significance of imperfect information, such as: high levels of heterogeneity, underlying risk distributions that change over time, and very rare event such as catastrophes. An exploration of threshold levels which might cause the winner's curse to become more significant could be a useful direction for future work.

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Chapter 4

An Agent-based Model of Insurance Market Price Dynamics with Heterogeneous Market Supply Strategies

4.1 Introduction

In the previous chapter, we explored systemic bias to risk estimation due to insurers competing under imperfect information. In chapter 3, this resulted in an effect known as the 'winner's curse', whereby insurers who win market share are more likely to have underpriced.

However, in practice insurance premium will be based on a competitive analysis of current market conditions and not the pure risk estimates. In this chapter we build an ABM focussed on competitive market supply strategies within a market where the premium moves according to the balance between supply and demand. In this model, insurers have imperfect information about both customer demand and underlying risk distributions.

It is common for traditional models to assume that agents are rational decision-makers. Under this premise, insurers set their supply with the aim of maximising their expected utility gain whilst assuming the market as a whole will do the same. In this chapter, the ABM contains two types of insurers. One type follows a rational strategy within the bounds of imperfect information. The other type also seeks to maximise their utility gain, but base their market expectations on a chartist strategy. Under this strategy, market premium is extrapolated from trends based on past insurance prices.

As the presence of chartists in the market is increased, increased volatility is observed in the market. When this occurs, the chartist insurers often outperform the rationalist insurers. However, their income is significantly more volatile. Because of this volatility, rationalists are found to remain as the dominant presence in an adaptive market where agents may dynamically select their strategies.

Overall, which strategy is 'best' depends on the current situation in the market, including both the current position in the market cycle and the spread of insurers following other strategies in the market. For insurers who are primarily driven by profit, a chartist strategy such as following a medium-term trend might be a better option. However, insurers who value stability more might prefer to follow a rationalist strategy even though the average profit is lower.

This chapter addresses some of the shortcomings of previous chapters by examining competitive insurer strategies under imperfect information. However, this model does not include the influence of customer-to-customer interactions as in chapter 2. There are a number of market features missing from this model such as: market entrants; market exits; catastrophe losses; more realistic loss distributions. Additionally, the insurer strategy does not account for capital management. In practice, insurers become more risk averse when their capital adequacy is lower. In chapter 5, we seek to find and extend a model which captures the dynamics of an insurance market with a minimal number of parameters.

4.2 Background

It is common to assume that agents in financial markets are rational utility maximisers, and that the prices in financial markets contain all available information.

However, some papers have questioned this assumption. Takahashi and Terano (2003) use an agent-based simulation model (ABM) containing both rational fundamentalist traders and trend-following technical traders. These traders select their willingness to supply or demand based on their heterogeneous strategies. They found that the trend followers destabilise the equilibrium and are able to take advantage of the resulting volatility, outperforming the fundamentalists in the long term. Similarly, traders displaying a bias towards overconfidence have a large effect on the market and can also obtain excess returns. This paper suggests that following a rational stance may not be best strategy.

Bertella et al. (2014) builds on this work by comparing the simulated market features produced by a similar ABM with empirical real-world market data. They find that a heterogeneous market containing technical traders is a better match than the pure fundamentalist market, and is able to explain the excess volatility often observed in the financial market. Additionally, the overconfidence bias is able to reproduce other features of the real-world data, such as the existence of market bubbles and crashes.

Brock and Hommes (1997) introduce the idea of 'adaptively rational agents'. Rather than setting their strategy based on a fundamentalist approach alone, agents use a rational choice model to decide on which strategy to follow based on how well agents performed using those strategies in the previous time step. They found that the market becomes more stable as more agents become fundamentalists, but the incentive to switch strategies become greater, producing chaotic bifurcation dynamics.

Brock and Hommes (1998) build on the work done by Brock and Hommes (1997) by examining some other common market strategies and analysing the bifurcation dynamics in more detail. Dieci et al. (2006) explore the concept of a 'market mood' by including non-adaptive agents with a fixed trading strategy. Chiarella et al. (2013) later expand the adaptively rational agent market to include multiple risky assets, finding that such an approach captures features not explained by a standard CAPM approach such as cycles and persistent volatilities. Jackson and Ladley (2016) allow for more varied technical trading strategies. Contrary to earlier models, this paper finds that technical traders can often act as a stabilising force, reducing volatility in the market.

In a manner similar to the more standard modelling approach to financial markets, insurance literature usually also assumes that insurers are rational agents. There is a substantial body of work which takes a game theoretical approach to determine the rational competitive strategy, e.g. Boonen et al. (2018), Dionne (2013), Wu and Pantelous (2017).

However, there is also evidence to suggest that insurance agents may not behave rationally. Investigations into behavioural bias suggests that agents are subject to overconfidence, hindsight bias, a preference for the status quo, emotional considerations such as trust or regret, and a myopic short-term perspective (Raghuram, 2019; Suter et al., 2017; Richter et al., 2019). Catastrophe markets in particular demonstrate a puzzle whereby customers tend to underpurchase cover for extreme events. However, premium rates go up following a catastrophe event and then gradually decrease, despite no change in the estimated proabilities (Kunreuther et al., 2013). This may be due to several biases, in particular the tendency of customers to underweight the risk of extreme events but to overweight when there is recent available experience of an event (Kunreuther et al., 2013; Vasiljevic et al., 2013; Kleindorfer and Kunreuther, 2000; Pitthan and De Witte, 2021).

Ingram, Tayler, et al. (2012) and Ingram and Bush (2013) use psychological theories to introduce the idea of plural rationality. These papers show that it is possible to model four different attitudes towards risk, and that each of these attitudes perform better or worse under particular market conditions.

To investigate the implications of non-rational strategies within an insurance market, an ABM is used to model an approach similar to Takahashi and Terano (2003) and Bertella et al. (2014). The primary difference between the equity market modelled in these papers and an insurance market is that whereas the finance market contains one kind of agent that decides whether to be a buyer or seller, an insurance market contains two distinctly different agents: insurers who provide the supply, and customers who are the source of the market demand.

There are currently very few examples of ABM literature in the field of insurance, though the possibilities of ABMs would seem to be a promising fit (Mills, 2010; Palin et al., 2008; Parodi, 2012)).

Owadally, Zhou, and Wright (2018) use an ABM of an insurance market to investigate possible competitive-driven mechanisms for the cyclical behaviour exhibited by real-world insurance premiums. Owadally, Zhou, Otunba, et al. (2019) further extend this model with a framework aimed at assisting regulators in monitoring and responding to cycles. These papers also include imperfect information as agents must estimate demand functions and loss distributions from their past data. However, they assume that all insurers are rational within the bounds of imperfect information. These papers do not include agents using heterogeneous strategies.

England et al. (2022) use an ABM to investigate the behaviour of insurance customers in an insurance market where insurers vary in customer service quality and customers are connected in a word-of-mouth network which spreads opinions about insurers amongst the customers. This paper finds that the network acts as a form of collective memory such that early experiences dominate customer opinion and generate a persistent consensus reputation. However, this paper concentrates primarily on consumer dynamics and does not explore the selling strategies of the insurer agents.

In this chapter, an ABM is used to create a stylised insurance market containing both consumer agents and two types of insurer agents. The first type of insurer follows a rational strategy bounded by imperfect information, and the second follows a more technical strategy. The model is run first with only the boundedly rational agents, and then with increasing numbers of technical insurers. The impact on the market premium and performance of the two types of agents are examined. The model is then run using adaptive agents in a manner similar to Brock and Hommes (1997) and Brock and Hommes (1998) to examine the resulting strategy dynamics and analyse the make-up of the evolving market.

4.3 The artificial insurance market: outline

4.3.1 Basic assumptions

The artificial insurance market consists of a number of insurers selling non-life insurance contracts to consumers. The contracts are purchased at the start of the year and expire at the end of the year. Insurance losses are stochastically generated on each policy, and insurers indemnify their policyholders at the end of the year.

Consumers are risk-averse and informed of their own risk of loss. They decide on the amount of insurance to buy by maximizing expected utility of year-end wealth. Consumers are also variously referred to as policyholders or customers or insureds.

Insurers are risk-averse firms and set their target exposure to the insurance market in order to maximize the expected utility of their year-end wealth. The movement of the market price of insurance depends on the balance between the consumers' demand to purchase insurance and the insurers' willingness to compete to supply contracts to policyholders.

Whereas consumers are homogeneous in the model, insurers are heterogeneous and are differentiated by their expectations formation and their competitive strategies.

The economy consists of insurance products only, with the only risk present being a pure insurance risk. No other asset is available. For simplicity, the risk-free rate is zero. The only stochastic input driving the artificial market is an exogenous insurance claims process. Insurers interact by competing in the market.

4.3.2 Outline and notation

It is helpful to outline the market process and set out the key notation here, with more details appearing in the later sections of this chapter. It is assumed that time $t \in \mathbb{Z}_0^+$ is discrete, representing yearly intervals. In the artificial insurance market, there are a number

N of insurers, indexed by $i \in \mathscr{I}$ where $\mathscr{I} = \{1, 2, \ldots N\}$. There are also a number M of insurance consumers. During each time step t, a subset M_t of these customers are willing to purchase insurance contracts at the given market premium P_t.

During each year $(t, t+1)$, the market undergoes the following steps:

1. Each insurer i decides on their target number of policies sold N^*_{it}. N^*_{it} is set by the insurer to maximize its expected utility, and depends on the expectation $\mathbb{E}^i_{t-1}\widetilde{P}_t$ formed by the insurer about the future average market price.
2. The market premium P_t is determined using a market impact function based on the balance between the total supply $N_t = \sum_i N^*_{it}$ and the total demand M_{t-1}.
3. The new level of customer demand M_t is determined using a logistic choice function based on the difference in the customer's expected utility of either purchasing or not purchasing insurance.
4. If $N_t \leq M_t$, then each insurer i acquires $N_{it} = N^*_{it}$ customers who pay premium P_t. If $N_t > M_t$, then $N_{it} = \frac{N^*_{it}}{N_t} M_t$.
5. Each customer suffers a random loss of L during the year, and claims this amount from their insurer. An insurer's profit is equal to the total premium revenue of $P_t N_{it}$ reduced by its random insurance losses.
6. Based on the experience during the year, each insurer i updates its estimates of: a)the expected loss per customer; and b) $\mathbb{E}^i_t\widetilde{P}_{t+1}$, the expected value of the market premium in the next year

The market process iterates again as above. Figure 4.1 is a swimlane diagram representing the flow of processes in the model and which type of agent is responsible for each step. The calculations carried out at each step are described in more detail below, followed by an explanation of the data and model parametrisation.

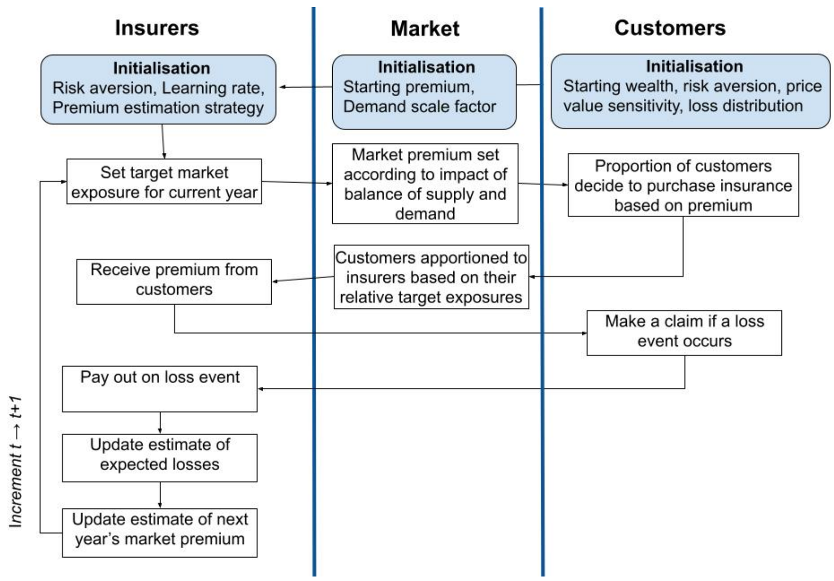


Fig. 4.1 – Swimlane overview of the processes in the heterogeneous market supply strategies ABM

The code is written in C# and has been made available on CoMSES (England, 2022).

4.4 Insurance demand in the artificial market

Insurance customers are homogeneous with identical constant relative risk aversion (CRRA) preferences with risk aversion coefficient γ. There is considerable theoretical support and empirical evidence in favour of the CRRA utility for individuals: see for example Eeckhoudt et al. (2005, p. 21) and Abellán-Perpiñán et al. (2006). At the start of every year, each customer has known wealth W_0, and suffers a random insurable loss L during the year, leading to random end-of-period wealth W at the end of the year. The utility of end-of-period wealth is

$$U(W) = (W^{1-\gamma} - 1)/(1-\gamma), \tag{4.1}$$

where $\gamma > 1$ and $W > 0$. As $\gamma \to 1$, logarithmic utility is recovered.

As this model is focussed on the premium dynamics, the wealth dynamics of individual consumers are not modelled here: there is no labour income, investment return and non-insurance consumption. At the start of the following year, customers' wealth returns to W_0. This is in keeping with the fact that, in the artificial market, customers are homogeneous and representative of a wider population of policyholders.

Customers may choose to purchase an insurance contract. This means that their insurer will indemnify them in the event of a loss in exchange for a premium paid at the start of the year. No other cash flow is modelled. Wealth is not invested and there is zero interest rate in the market. Thus,

$$W = \begin{cases} W_0 - P & \text{if insurance purchased} \\ W_0 - L & \text{otherwise} \end{cases} \tag{4.2}$$

The loss L is assumed to be an identically distributed Bernoulli random variable,independent over the customer population and over time:

$$\mathbb{P}(L = \ell) = 1 - \mathbb{P}(L = 0) = \pi, \tag{4.3}$$

with $\ell > 0$ and $0 \leq \pi \leq 1$.

At the start of year $(t, t+1)$, suppose that the market premium is P_t. The associated *value* function V_t gives the expected utility for the consumer's end of period wealth after purchasing insurance as:

$$V_t = \frac{1}{1-\gamma}\Big((W_0 - P_t)^{1-\gamma} - 1\Big). \tag{4.4}$$

and the expected utility for the consumer's end of period wealth without purchasing insurance as:

$$V_0 \; = \; \frac{1}{1-\gamma}\Big(\pi(W_0-\ell)^{1-\gamma}+(1-\pi)(W_0)^{1-\gamma}-1\Big). \tag{4.5}$$

It might be expected that customers would always purchase insurance if the value of purchasing is greater than that of not purchasing. In practice, the demand tends to increase at lower premium and decrease at higher premium. In such cases of consumer decisions, it is common to use a multinomial logit model. This is also known as the logistic choice or softmax function, and is often used in machine learning or for multi-armed bandit decision models (Gao and Pavel, 2018).

The softmax function is scaled such that the total value of M_t is equal to the maximum level M when the premium is equal to the expected value of L (i.e. $P_L = \pi\ell$), and the total value of M_t is equal to 0 when the value V_t of purchasing the insurance is equal to the value V_0 of not purchasing insurance.

Under this model, the number of customers M_t who choose to purchase insurance in time t is equal to

$$M_t \; = \; M\frac{e^{\alpha V_L}+e^{\alpha V_0}}{e^{\alpha V_L}-e^{\alpha V_0}}\frac{e^{\alpha V_t}-e^{\alpha V_0}}{e^{\alpha V_t}+e^{\alpha V_0}} \tag{4.6}$$

where: $V_L = \frac{1}{1-\gamma}\left((W_0-\pi\ell)^{1-\gamma}-1\right)$, V_0 is defined as in equation 4.5, and α is a customer price sensitivity parameter which determines how quickly the demand drops as prices are raised.

Insurance demand changes only in response to a new market premium as consumers do not anticipate changes in the market. Thus, the demand influencing the start of the period is based on the last known price of the insurance from the previous time period.

4.5 Insurance supply in the artificial market

Each insurer agent sets a target market exposure N_{it}^* by maximising the expected utility of their end-of-period wealth - i.e:

$$\max E\left[U\left(W_{i,t}\right)\right] \tag{4.7}$$

It is straightforward to model the risk preferences of consumer agents. By contrast, insurers may be mutuals or stock corporations and the risk preferences of their decision-making bodies, acting on behalf of their principals, are difficult to capture: see for example Gollier (2013, p. 118) and Kunreuther et al. (2013, p. 146). In order to maintain consistency and comparability with the model of Bertella et al. (2014), insurers are modelled with identical constant absolute risk aversion. This gives the utility function of wealth as

$$U\left(W_{i,t}\right) \; = \; -e^{-\lambda W_{i,t}} \tag{4.8}$$

This gives us an expected end of period utility of:

$$E[U\left(W_{i,t}\right)] \; = \; -e^{-\lambda[E(W_{i,t})-\frac{1}{2}\lambda\sigma^2_{W,i,t}]} \tag{4.9}$$

where $\sigma^2_{W,i,t}$ is the variance in the end of period wealth of insurer i at time t.

The wealth of insurer i at time t is equal to

$$W_{i,t} = W_{i,t-1} + N_{i,t}P_t + \sum_{N_{i,t}} L_{j,t} \tag{4.10}$$

Thus:

$$E(W_{i,t}) = W_{i,t} + N_{i,t}(P_t - \pi l) \tag{4.11}$$

Also, the variance becomes:

$$\sigma^2_{W,i,t+1} = N^2_{i,t}\sigma^2_P + N_{i,t}\sigma^2_L \tag{4.12}$$

where σ^2_L is the variance of L and σ^2_P is the variance of P_t. Note that $\sigma^2_L = \pi(1-\pi)\ell^2$. The value of σ^2_P is estimated by each insurer based on experience.

From equations 4.11 and 4.12, equation 4.9 can be maximised by maximising the value of:

$$N_{i,t}(P_t - \pi l) - \frac{\lambda}{2}\left(N^2_{i,t}\sigma^2_P + N_{i,t}\sigma^2_L\right) \tag{4.13}$$

Equation 4.13 is maximised when:

$$N^*_{i,t} = \frac{P_t - \pi l - \frac{1}{2}\lambda\pi(1-\pi)\ell^2}{\lambda\sigma^2_P} \tag{4.14}$$

Each insurer therefore selects their target market exposure based on their estimates of the expected market premium P_t, the value of π, and the variance σ^2_P.

Using the same estimation method employed by Bertella et al. (2014), the perceived variance of the premium estimated by each insurer is updated at the end of each time period as:

$$\hat{\sigma}^2_{P,t+1} = (1-\theta)\hat{\sigma}^2_{P,t} + \theta\left[P_t - E_i(P_t)\right]^2 \tag{4.15}$$

where θ is a learning rate parameter.

Similarly, the estimate of π is updated at the end of each time period as:

$$\hat{\pi}_{t+1} = (1-\theta)\hat{\pi}_t + \theta\frac{\sum_{N_{i,t}}\delta_{j,t}}{N_{i,t}} \tag{4.16}$$

where $\delta_{j,t} = 1$ if their j_{th} customer makes a claim and 0 otherwise.

4.6 Market premium

The market premium evolves each period according to a market impact function, similar to the price adjustment calculation developed by Farmer and Joshi (2002) and used in the market model created by Bertella et al. (2014). This function is based on the balance of insurance demand and supply, and gives a resulting market premium at the end of each time step as follows:

$$P_{t+1} = P_t e^{(M_t - \sum_i N_{i,t})/\beta} \tag{4.17}$$

for a demand impact scale parameter β.

4.7 Insurer market strategies

4.7.1 Boundedly Rational insurers

Suppose all insurers are rational within the bounds of imperfect informaion and form unbiased estimates of π and σ_P. Then a boundedly rational insurer i would expect the expected average supply set by all market insurers to equal their own optimum supply $N^*_{i,t}$.

Insurance demand is neither perfectly price-elastic, nor perfectly price-inelastic. Insurers can thus establish a demand schedule (Petersen and Lewis, 1999). As the insurers do not know the exact form of the demand function, they assume the relationship between customer numbers and market premium can be modelled as approximately linear for a sufficiently stable market with small changes to market supply. This is consistent with the often-used linear demand function derived from a quasilinear quadratic utility model when individual insurer premium is equal to the market premium (Choné and Linnemer, 2020), and is similar to the linear approximation for an exponential price elasticity of demand as used by Zhou (2013).

Then the premium can be modelled as approximately linearly related to $\overline{N_t}$ the average market supply per insurer according to the equation:

$$P_t \;=\; a - b\overline{N_t} \tag{4.18}$$

Then equation 4.14 becomes:

$$N^*_{i,t} \;=\; \frac{a - bN^*_{i,t} - \pi l - \frac{1}{2}\lambda\pi(1-\pi)\ell^2}{\lambda\sigma_P^2} \tag{4.19}$$

Rearranging this equation gives the bounded rationalist target market exposure as:

$$N^*_{i,t} \;=\; \frac{a - \pi l - \frac{1}{2}\lambda\pi(1-\pi)\ell^2}{\lambda\sigma_P^2 + b} \tag{4.20}$$

The values of a and b can be estimated from the most recent values of $N^*_{i,t}$ and P_t, and smoothed with the learning parameter θ as with the other estimates:

$$\hat{a}_{t+1} \;=\; (1-\theta)\,\hat{a}_t + \theta\frac{P_t N_{t-1} - P_{t-1}N_t}{N_{t-1} - N_t} \tag{4.21}$$

$$\hat{b}_{t+1} \;=\; (1-\theta)\,\hat{b}_t + \theta\frac{P_t - P_{t-1}}{N_{t-1} - N_t} \tag{4.22}$$

If $N^*_{t-1} = N^*_{t-2}$, then the insurer does not update their estimates.

4.7.2 Chartist insurers

For the initial results, all insurers are assumed to be rational. For the rest of the simulations, insurers follow one of two strategies to form expectations about the average market price of insurance.

1. A proportion of the N insurers in the market employs a boundedly rational strategy as described above
2. The remaining proportion of insurers follows a chartist strategy and extrapolate from insurance price trends in the market.

This approach is similar to that used by Bertella et al. (2014), and the existence of entities following a chartist strategy is well-established in financial markets. There is much less research into the existence of insurers following similar strategies. However, it is worth noting that insurance underwriters are commonly trained by professional bodies such as the Chartered Insurance Institute to look for and follow underwriting cycles when setting competitive premium (Burnell, 2022). Additionally, the change in customer demand to changing levels of price is often unknown, and estimation can be both complex and contain a high degree of uncertainty. As such, it is not unreasonable to assume that similar strategies could emerge in the insurance markets.

The modelled chartists estimate a market trend parameter φ from recent history in a manner similar to that used by Bertella et al. (2014):

$$\varphi = \frac{1}{n}\sum_{j=1}^{n}(P_{t-j}/P_{t-j-1} - 1) \tag{4.23}$$

There are two types of chartists: trend followers and contrarians. The trend followers believe that trends will continue, and estimate the market premium as:

$$E_i(P_t) = P_{t-1}(1+\varphi) \tag{4.24}$$

However, the contrarians believe that the market tends towards a central value, and will move in opposition to the trend. Contrarians estimate the market premium as:

$$E_i(P_t) = P_{t-1}(1-\varphi) \tag{4.25}$$

Both of these groups of chartists can be further subdivided according to the time horizon used to estimate the trend. $n = 1$ for short-term chartists, $n = 5$ for medium-term chartists, and $n = 10$ for long-term chartists. The time horizons are taken from previous financial research papers (Bertella et al., 2014; Takahashi and Terano, 2003))

To explore the effect of technical insurers on a heterogeneous market, the model is run with a varying proportion of technical to fundamental insurers.

4.8 Data and Parametrisation

The following data sources are used to parametrise the model:

- The value of M is set at $10,000$ to get a large number of customers wile remaining computationally feasible. The number of insurers is set at 20 because this represents a significant majority of the market share in a personal lines insurance market Owadally, Zhou, and Wright (2018) and is a reasonable number for producing computationally tractable yet realistic simulation dynamics.
- The market level data of premium and losses is taken from a summary of EIOPA Solvency I submissions for the UK property market (EIOPA, 2016). This data is for the years 2006-2015.
- To adjust these values to a comparable level, CPI data is used to inflate the historical values. Note that as the data is in Euros, the European CPI data is used (King, 2021).
- The FCA data "General Insurance value measures data - year ending 31 August 2016" (FCA, 2016) is used to estimate average claims frequency for the UK property market by taking a weighted average of the centre of the given frequency ranges per insurer.
- The ratio of average household savings to the cost of property insurance is based on information from the price comparison index from 'MoneySupermarket.com' (MoneySupermarket.com, 2021)
- The resulting market parameters are rescaled per representative customer such that the average loss per customer is 100, and the loss severity is set accordingly.

This leaves the behavioural parameters, which cannot be directly estimated from the data. These values are set as follows:

- If the insurer learning rate is too high, the estimated variances vary substantially in each time period. If it is too low, the insurer will never change the estimate as the market changes. The insurer learning rate was selected such that the estimated variances were smooth but responsive to change. For this rate, it takes approximately six years (close to the medium term time horizon) before the influence of an estimate is halved.
- Customer risk aversion and price sensitivity both impact the customer demand curve. At extremes, this becomes close to a 'step' function. The values are chosen from a range which gives a curve with a noticeable step change for each whole-number premium.
- The insurer risk aversion is then selected such that the minimum average adjusted historical premium rate is the expected premium value at which the total supply is equal to the total number of customers in the market.
- The premium demand scale factor is then set such that a standard step change in the market premium for a market where all insurers are boundedly rational will be similar to the historical premium changes.

The resulting parameter values are given in Table 4.1.

For the initial model simulations, all insurers are bounded rationalists. The model

Variable	Symbol	Std Dev
No. Customers	M	10000
No. Insurers	N	20
Loss Frequency	π	0.056
Loss Severity	ℓ	1786
Initial Premium	P_0	120
Premium at t=-1	P_{-1}	115
Initial Consumer Wealth	W_0	5711
Insurer Risk Aversion	λ	0.00007
Insurer Learning Weight	θ	0.1
Customer Risk Aversion	γ	3.9
Customer Price Sensitivity	α	2×10^{13}
Premium Demand Scale Factor	β	$100,000$

Table 4.1 – Input parameters used for model

is then run with a mix of only two types of agents: bounded rationalists and one type of chartist. The number of chartists is varied from 1 to 19.

4.9 Simulation Results

4.9.1 Bounded Rationalist Market

Takahashi and Terano (2003) and Bertella et al. (2014) began with a market in which all agents were fundamentalists, acting on the assumption that the market acts rationally and thus reflects the fundamental price. For the initial market simulations, all agents are bounded rationalists, which are equivalent to the financial fundamentalist agents.

The Takahashi and Terano (2003) and Bertella et al. (2014) fundamentalist simulations produced results where the resulting price followed the fundamental price with some estimation error. In this model, the demand estimation process in the bounded rationalist market produces a regular premium cycle arising endogenously from the insurers' demand estimates (Figure 4.2. These results are similar to the results found by Owadally, Zhou, Otunba, et al. (2019), and mimics the cycles found in real-world insurance markets.

It is common to approximate insurance market premium as an AR(2) process (Owadally, Zhou, and Wright, 2018; Owadally, Zhou, Otunba, et al., 2019; Boyer and Owadally, 2015; Fenn and Vencappa, 2005; Harrington and Niehaus, 2000; Cummins and Outreville, 1987). An AR(2) equation can be fitted to the market premium generated by the bounded rationalist model with an R^2 value of 0.99. The residuals have a mean of 0 and a small standard deviation (Table 4.2). From this it can be concluded that the AR(2) model

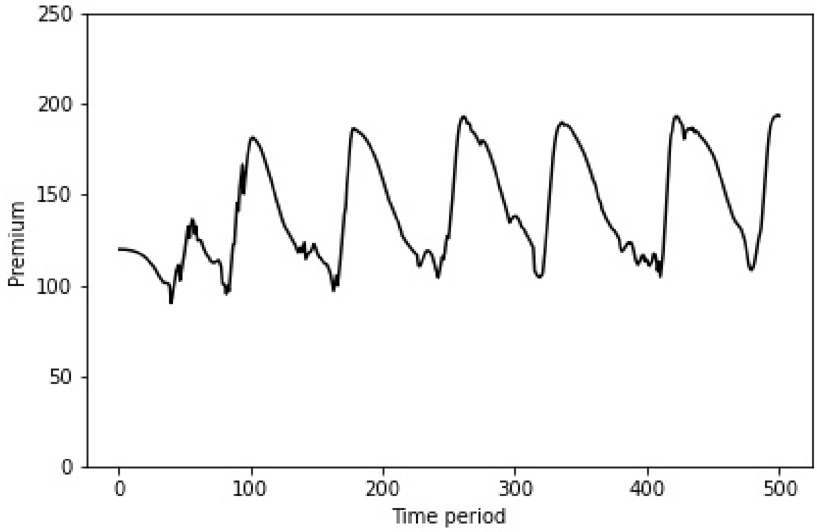


Fig. 4.2 – Market premium by time for single sim of a boundedly rationalist market. Cycles are seen arising from the market dynamics, a feature which is also present in real-world markets.

Output	Mean	Std Dev	Skew	Kurtosis
Premium	141.23	29.4	0.52	−0.97
AR(2) Errors	0.00	2.46	−0.29	12.77

Table 4.2 – Summary statistics of market premium and errors in fitted AR(2) model for market where all agents are boundedly rational. The AR(2) model is a good ft for this case.

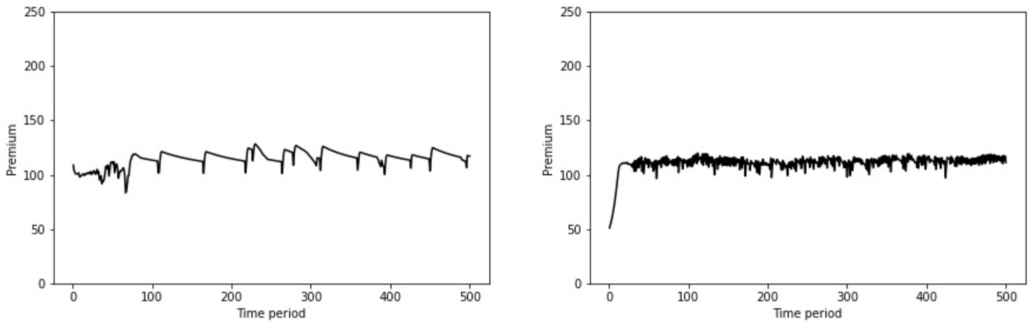
is a good fit for the model output. This suggests that simple chartists may be able to take advantage of this pattern, either by anticipating the upwards or downwards trends (trend followers) or anticipating the change points (contrarians).

4.9.2 Market with Chartists

The model is then run for each of the six types of chartist with a mix of only bounded rationalists and the chartist type. In each instance, the number of chartists is varied from 1 to 19, and the total number of insurers is 20.

Bertella et al. (2014) finds that as the presence of chartists increases, the market becomes more volatile and demonstrates increasingly negative skew and increasing kurtosis. However, the market becomes less volatile when the number of chartists decrease below 25%. In the insurance market simulation, a similar pattern is seen (see Table 4.3 for the medium-term trend follower results and Table 4.4 for the long-term contrarian results).

In the rationalist-only market, the premium showed a clear cyclical pattern (see figure 4.2). Introducing a small number of chartists disrupts this established market pattern, causing an overall decrease in the standard deviation. As the number of chartists is increased, the market premium becomes noisier, causing the overall premium variance to increase again.



(a) Market with one medium-term trend follower. (b) Market with ten medium-term trend followers.

Fig. 4.3 – Market premium from a single simulation where the market contains one and ten medium-term trend followers. The existence of the chartists disrupt the cyclical pattern seen in the rationalist-only market and an increasingly noisy market emerges.

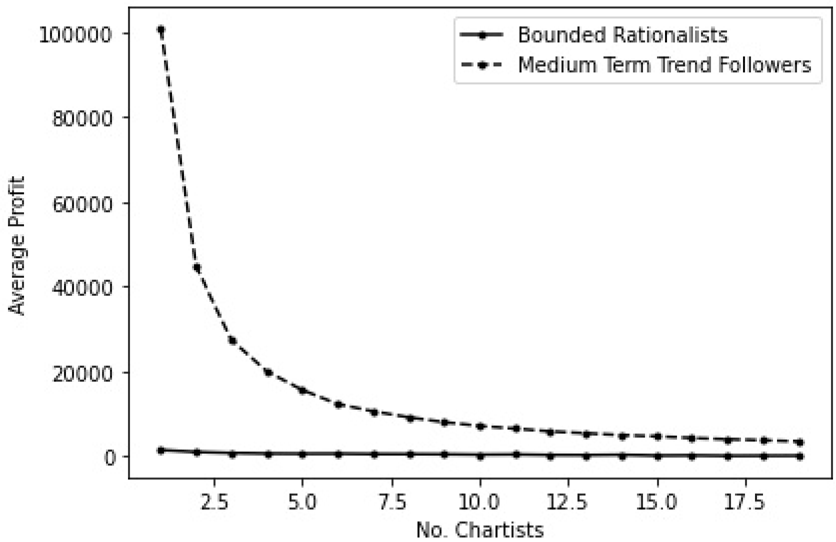


Fig. 4.4 – Average profit of the two types of insurers in the market for different number of medium term trend followers. Chartists outperform rationalists in the disrupted market.

When the market is dominated by chartists, the premium becomes more volatile and less predictable (see figure 4.3).

As predicted, the chartists are able to take advantage of the patterns in the market premium to earn more profit (Figure 4.4). However, their profits are much more volatile as they are caught off-guard when the pattern changes (Figure 4.5). As the number of chartists increase, the disruption to the market premium becomes larger and the bounded rationalists' standard deviation increases. Both types of insurers earn less profit on average, and the difference in profit between the two types of insurers narrows.

Overall, similar patterns emerge for all types of chartists. Longer term chartists tend to cause less volatility in the market as the agents' estimates are smoothed out more over time. Longer terms also tend to result in a greater negative skew in the market premium, and as a

No. Chartists	Std Dev	Skew	Kurtosis
1	7.2	0.1	2.6
2	5.7	−0.7	2.1
3	5.3	−1.2	2.6
4	5.4	−1.5	3.8
5	5.6	−1.8	5.9
6	5.9	−2.2	9.4
7	6.2	−2.8	13.3
8	6.5	−3.2	17.4
9	6.8	−3.7	21.3
10	7.4	−4	24
11	7.6	−4.3	26.1
12	8	−4.4	27.5
13	8.5	−4.7	29
14	8.7	−4.8	30.8
15	9.2	−5	31.3
16	9.5	−5	31.4
17	10	−5.1	31.3
18	10.2	−5.1	31.8
19	10.6	−5.1	31.1

Table 4.3 – Market premium statistics for varying number of medium-term trend follower chartists. As the number of chartists is increased, the premium cycle is disrupted, resulting in a more volatile market. The premium also becomes increasingly negatively skewed, and the kurtosis increases.

result, less profit for either type of agent (see table 4.4).

4.10 Adaptively Rational Agents

4.10.1 Adaptive strategy

Finally, a model is run containing adaptively rational agents. These are based on the design used by Brock and Hommes (1997) and built upon by later authors (Brock and Hommes, 1998; Dieci et al., 2006; Chiarella et al., 2013).

These agents select from among each available strategy k using a probability $q_{i,k,t}$ calculated from a softmax choice function based on the average profit obtained by insurers

No. Chartists	Std dev	Skew	Kurtosis
1	7.8	0.6	4.2
2	6.1	−0.1	3.3
3	5.5	−1	3.5
4	5.3	−1.1	3.4
5	5.3	−1.5	4.6
6	5.4	−1.8	7.3
7	5.4	−2.3	10.3
8	5.7	−2.6	13.4
9	5.8	−3.1	17.2
10	5.8	−3.3	20.2
11	6.2	−3.9	25.9
12	6.4	−4.2	28.4
13	6.4	−4.3	30.1
14	6.8	−4.6	31.6
15	7.2	−5	36
16	7.2	−5	36.2
17	7.6	−5.3	39.3
18	8	−5.5	39.4

Table 4.4 – Market premium statistics for varying number of long-term contrarian chartists. This shows similar patterns to the medium-term trend follower example. Longer term chartists tend to cause less volatility and a greater negative skew in the market premium than for the short and medium-term chartists.

using each strategy in the previous time period.

This is calculated using equation (4.26) as follows:

$$q_{i,k,t} = \exp \omega \hat{\Pi}_{k,t-1} / z_{i,t} z_{i,t} = \sum_k \exp \omega \hat{\Pi}_{k,t-1} \tag{4.26}$$

where ω is an adaptive choice parameter and $\hat{\Pi}_{k,t-1}$ is the average profit obtained by insurers using strategy k in time period $t-1$. Note that if no insurers are using strategy k in time period $t-1$ then $\hat{\Pi}_{k,t-1} = 0$, and there remains a probability of selecting this option.

For each strategy k, the insurer maintains a running estimate of the required variances. These estimates are used to calculate the required market share $N^{*}it$ for each available

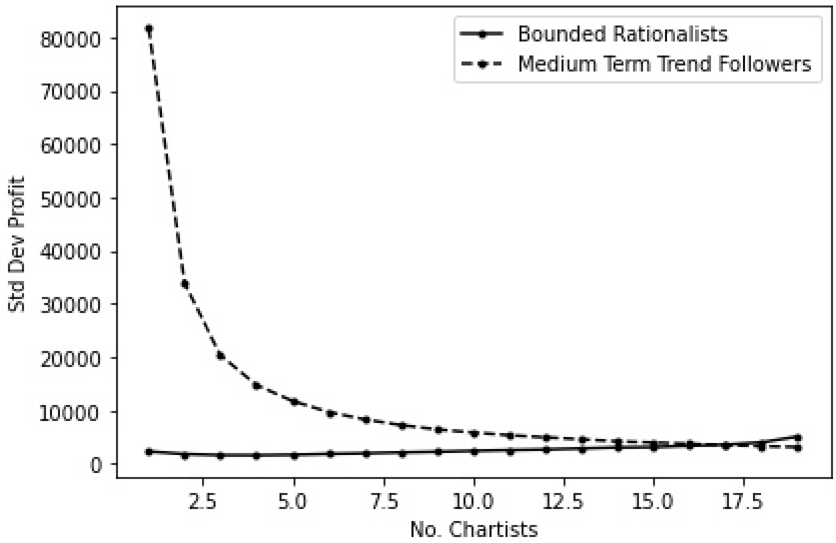


Fig. 4.5 – Standard deviation of the profit of the two types of insurers in the market for different number of medium term trend followers. While the chartists earn higher profits on average, their profits are much more volatile as they are caught off-guard when the pattern changes.

strategy k, which in turn are used to estimate the variance in the following time period for strategy k.

4.10.2 Results

When the choice parameter ω is too small, the decision becomes essentially random. It would be expected that the insurers would be evenly split between all strategies. If it is too high, then the insurers become reluctant to test different strategies at all unless they are making a loss, and the market becomes stuck on a single strategy. In between these extremes, there emerges a market where different strategies dominate for short periods and then wane as the actions of the agents cause the market to push against prevailing expectations and other types start to do better.

Figure 4.6 show an example of counts for each insurer type where the choice parameter $\omega = 0.001$. The counts are averaged across $1,000$ simulations for each time period t. At this value, the medium trend chartists tend to become the dominant market strategy.

Increasing the parameter ω leads to a domination of rationalist despite the profit advantages of chartist strategies (Figure 4.7). This is because the chartist profits have a much higher volatility. As a result, the agents all eventually have a bad year and switch to the more stable bounded rationalist strategy, where they stay because they don't have a bad enough year to overcome the reluctance to switch types.

4.11 Conclusion

It is common to assume that all agents in financial markets are rational utility maximisers. However, a number of financial papers have used market simulation models to investigate

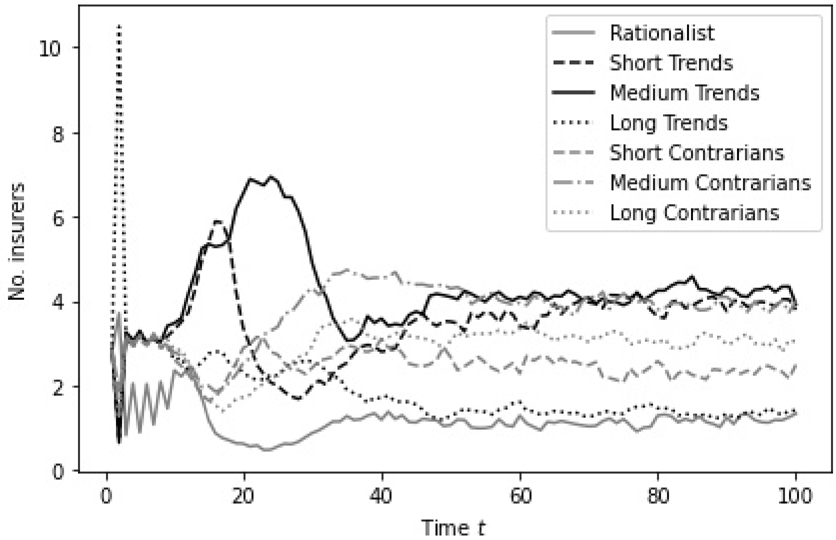


Fig. 4.6 – Number of each insurer type by time period averaged across 500 simulations for choice parameter = 0.001. After an initial period of instability, the medium term chartists tend to dominate the numbers.

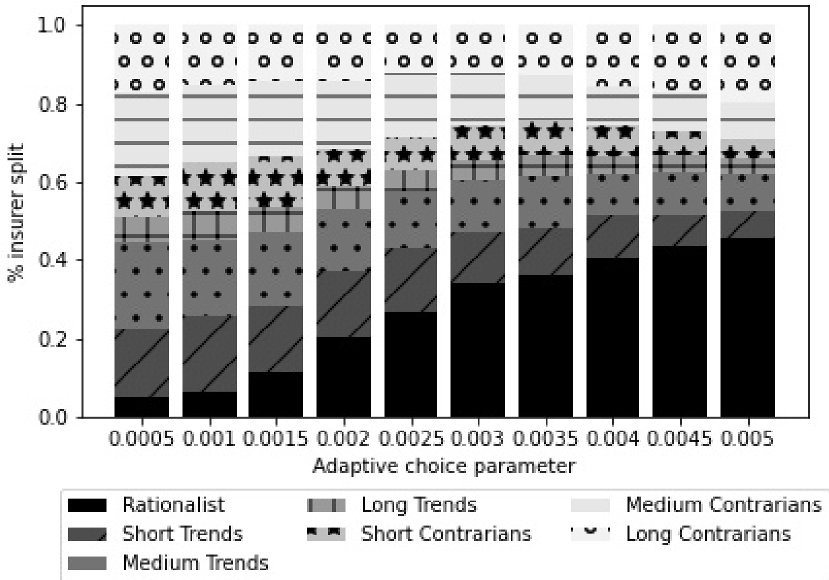


Fig. 4.7 – Number of each insurer type at t=100 for varying values of choice parameter ω. After an initial period where chartists and particularly medium term trend followers tend to dominate, the agents become increasingly wary of the higher volatilities, and the bounded rationalist strategy tends to dominate the numbers.

the presence of trend-following technical traders as well as rational fundamentalist agents within financial markets and discovered that the more technical trading strategy can outperforms the rationalist traders in particular market conditions (Takahashi and Terano, 2003; Bertella et al., 2014; Brock and Hommes, 1997; Brock and Hommes, 1998).

To investigate the implications of non-rational strategies within an insurance market, an ABM is used to model an approach similar to Takahashi and Terano (2003) and Bertella et al. (2014). In this model, movements in market premium is determined by the balance of supply and demand. The customer demand is based on a logit choice model applied to the expected utility gain of purchase at the current market premium. The available supply is set by each insurer individually by maximising their expected utility gain based on their expectation for the new market premium value.

Similarly to Bertella et al. (2014), this model includes two types of insurers, both bounded by imperfect knowledge of the underlying customer demand and of the loss distributions. The first type are bounded rationalists. The second are chartists, who follow either a trend-following or a contrarian assumption about the market premium.

When the model is run for a market of only bounded rationalists, the market produces a regular premium cycle arising endogenously from the insurers' demand estimates. This premium cycle can be well-fitted to an AR(2) process. These results are similar to the results found by Owadally, Zhou, Otunba, et al. (2019), and mimics the cyles found in real-world insurance markets.

As with the financial papers that used similar methods, the market simulation indicates that the existence of insurers following a more chartist based strategy tends to disrupt the market, leading to increased volatility, negative skew, and increased kurtosis in the market premium. These chartist insurers are often better able to take advantage of this disruption and make a higher profit, and thus often out-perform the bounded rationalist insurers. However, their performance is also notably much more volatile than the rationalist insurers.

The model was then run with adaptively rational insurers. This type of of insurer chooses between the available strategies in each time period with probability determined by a logit choice model applied to the relative profits of each strategy in the previous time period in a manner similar to (Brock and Hommes, 1997).

Although much of the existing insurance literature assumes that a rationalist approach is the optimal solution to setting premium, these results indicate that this is not necessarily the case, particularly when operating under imperfect information which necessitates the estimation of the behaviour of both customers and competitors. Instead, it can be seen from the market containing adaptively rational agents that which strategy is 'best' depends on the current situation in the market, including both the current position in the market cycle and the spread of insurers following other strategies in the market. For insurers who are primarily driven by profit, a chartist strategy such as following a medium-term trend might be a better option. However, insurers who value stability more might prefer to follow a rationalist strategy even though the average profit is lower.

This model makes the primary assumption that premium is set as a balance of supply and demand and insurers will all charge the same resulting premium. In practice, although insurance premium is heavily influenced by the rest of the market, there exists variation in individual insurer premium. Further work could be done to create a similar model for pricing strategies instead of focussing on supply and demand dynamics.

There are a number of market features missing from this model such as: market entrants; market exits; catastrophe losses; and more realistic loss distributions. Many of these would be expected to impact levels of market competition. However, they also increase the complexity of the rationalist calculation, making it difficult to impossible to find an analytical solution.

Additionally, the insurer strategy does not account for capital management. In practice, insurers become more risk averse when their capital adequacy is lower. This feature is believed to be a key driver of the underwriting cycle in insurance markets. Further work could be done to incorporate this feature.

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Chapter 5

An Extension of the Taylor Model of Insurance Market Dynamics

5.1 Introduction

Chapter 2 focussed on customer choices and the impact of word-of-mouth networks on insurers' reputations. In this model, insurers were passive price-takers and the model did not allow for supply and demand dynamics. Chapter 3 focussed on risk mis-estimation and the resulting bias in risk-based pricing strategies, but did not allow for competitive strategies. And chapter 4 examined competitive insurer strategies under imperfect information.

Models of the dynamics that drive an insurance market is a key component in investigating these kind of emergent systemic patterns. However, the models used so far have been individually designed and do not easily combine into an overall model of a market. There are also some market level features missing, such as market entrants and exits and the link between capital adequacy and insurer risk aversion.

This chapter seeks to find and extend a model which captures the dynamics of an insurance market with a minimal number of parameters. One such model is suggested by Taylor (2008). In Taylor's model, insurers set premium according to their solvency ratio, adjusted according to the recent prices set by competitors. The Taylor model produces results that replicate realistic market features; however, there are large unstable swings in an individual insurer's premium rates and net assets.

An extension is created to Taylor's model by adjusting insurers' premium using the solvency ratio to determine the direction of premium movement, with the size of the change depending on competitive pressures. A base model is run along with a series of sensitivity tests similar to Taylor's original paper and the results of the two models are compared.

The new model produces market results with similar features to Taylor's model but more stable individual asset values, potentially enabling its use with a range of market environments for other simulation-based models. However, the more stable individual premium rates cause the market to be slower to respond to changes and produces cycles that are less pronounced with a longer periodicity.

In this chapter, the new model is parametrised in order to allow for easy comparison with the results in Taylor's model. An investigation into parametrisation was inconclusive as the algorithm was unable to find a reasonable set of parameters which produce values that are a close fit with empirical data. Further work could be done to find appropriate environmental and dynamical parameter values, and to compare the results of a standard parametrisation exercise with different potential mechanisms.

5.2 Background

5.2.1 Insurance Market dynamics

In this chapter, a simulation model of market dynamics is extended in order to account for some limitations in an existing model.

A significant component in the field of insurance is that of the dynamics that drive an insurance market. This is a key part of understanding causes of underwriting cycles, determining strategies, investigating the effects of interactions, and estimating the possible impacts of regulation policies. It is therefore useful to find a model of insurance market dynamics that can produce realistic market features, yet is flexible enough to be applicable across different market assumptions and environments.

Most features of market dynamics arise from insurer premium strategies. If an insurer sets its premium too high, it will attract fewer customers. It its premium is too low, then it will not be able to cover its liabilities. Empirical tests indicate that as an insurer's capital decreases, its premium rates are increased, and vice versa (Choi et al., 2002). Additionally, there is a large body of evidence and related theory demonstrating the existence of cycles in insurance market premium rates (Cummins and Outreville, 1987; Fung et al., 1998; Lazar and Denuit, 2012). These two features are inconsistent with a premium based on a pure actuarial assessment of risk, such as setting premiums based on the expected loss plus a margin for the variability (Parodi, 2014). It would therefore be expected for an insurer to consider the effect of competition when setting premium rates.

One common approach to solving this problem is to make use of equilibrium based pricing methods, where supply and demand exist in equilibrium and insurers are assumed to choose a premium strategy which maximises their expected utility given a function that links premium rates with demand. Berger (1988) use this method to construct a simple model where an insurer's concern for its probability of ruin can lead to the increase of premium when its surplus capital is low and vice versa. This results in underwriting cycles as more profit leads to more competition followed by declining profits. More recently, Emms (2012) develops a differential model which finds that the competition from new market entrants can produce an optimal strategy where insurers alternate between high and low premium rates. Henriet et al. (2016) investigate a model with recapitalisation frictions, which produces cycles related to insurer capital. Boonen et al. (2018) allow for competition by seeking a Nash equilibrium for a game where the exposure flows between insurers are related to solvency ratios. This also produces cycles, and demonstrates that an insurer with a lower market share should offer a lower premium in order to attract customers.

An equilibrium based approach is a useful and well established method, and as referenced above can produce both underwriting cycles and the link between premium and capital. However, each solution is devised for use under a set of specific assumptions and market environments. In practice, the insurance market possess many elements that can complicate the use of this approach, such as cross-subsidising and adverse selection (Chiappori and Salanié, 2008). Warren et al. (2012) note that as customers are often willing to pay a higher price to renew an existing policy rather than seek out a new one, insurers should consider the lifetime value of a customer. Since this requires an assessment of future actions by competitors as well as an accurate estimate of customer behaviour, this can be difficult to calculate and some assumptions may prevent the existence of an equilibrium solution. Rothschild and Stiglitz (1978) find that the existence of imperfect information - something which is common in insurance markets, where risks are estimated based on past data - can also prevent equilibrium or cause equilibria to have undesirable properties. This limits the wider applicability of an equilibrium based method when these factors are explored.

Some papers deal with the effect of imperfect information by using an approximation to elasticity of demand to calculate an insurer's premium (Kelsey, 1998; Guven and McPhail, 2013; Yao, 2015). Owadally et al. (2018) use a similar approach in an agent-based model which included customer preferences. They discover that a simulation based on these dynamics could lead to underwriting cycles arising endogenously from the interactions of competing insurers. However, although this approach is flexible and produces cycles, it does not include the link with surplus capital that is a feature of empirical markets.

Taylor (2008) proposes a simple dynamic market model that produces cycles, links premium rates with insurer capital levels, and is simple to transport for use in other simulation models. However, this model contains some limitations that can cause instability under some market environments. In this chapter, an extension to th Taylor model is considered in order to address some limitations. The Taylor model and the new extended model are described in further detail below.

5.2.2 Taylor's Model

Taylor (2008) presents a simulation model of an insurance market with the aim of reproducing the main competitive market features using a minimal number of parameters. The paper then shows simulation results for a set of base parameter values and then sensitivity to variations in the parameters.

Taylor's model begins with a market of 20 insurers and an equal share of the total market exposure, which is assumed to remain constant throughout the simulation. The model then simulates a series of time periods, each dependent on the outcome of the previous time step. A single time step t goes through the following steps:

1. Each insurer i sets their target premiums per exposure according to the equation

$$T_{i,t} = P_0 \exp\left[-k_1(S_{it} - S_0)\right] \tag{5.1}$$

where P_0 = present value of the expected losses per exposure, S_{it} = current solvency ratio of insurer i, S_0 = target solvency ratio, and k_1 = premium to solvency sensitivity parameter. As the insurer obtains more capital, it will decrease its target premiums to attract more business; if its capital decreases, it will increase its target premiums to reduce the chance of insolvency.

2. The insurers then calculate competitive premium using the equation

$$P_{i,t} = \max(k_{11}P_0,\ \min(k_{13}T_{it}[T_{it}/\overline{P}_{i,t-1}]^{-k_2} + (1-k_{13})P_{i,t-1},\ k_{12}P_0)) \tag{5.2}$$

where $\overline{P}_{i,t-1}$ = average last known competitive premium of the insurer's nearest neighbours, and k_2, k_{11}, k_{12}, and k_{13} are dynamical parameters. The competitive premium adjusts the target premium according to its relativity to the prices set by its competitors, smoothed over time and subject to an upper and lower bound. This is the premium that will be brought to the market.

3. Losses are generated for each insurer according to a Poisson/Gamma compound distribution, where the Poisson mean is proportional to an insurer's exposure.
4. Insurance profit is calculated for each insurer based on the premium income, losses paid, and asset returns. This model does not include an allowance for investment variation. Dividends are then paid out according to the insurer's current assets relative to a target level.
5. The final net assets are set equal to the starting net assets plus insurance profit minus dividends.
6. Insurers whose solvency has fallen below the minimum level exit the market. If the market profit margin is higher than a threshold, it will attract new entrants into the market.
7. Finally, the market exposure is rebalanced between the insurers. These flows are calculated from each insurer to another with a lower premium using a set of transfer functions, which are scaled according to the difference in competitive premium and proportional to the receiving insurer's existing market share. Insurers whose market share falls below a threshold now exit the market

5.2.3 Examination of the Taylor Model

The Taylor model produces a set of base results demonstrating stable premium rates and solvency (Figure 5.1), a diversity of premium rates, and a slowly increasing Herfindahl-Hirschmann Index[1]. Adjusting the parameters which determine the insurer's sensitivity to competition and solvency when setting premium shows the possible emergence of underwriting cycles (Figure 5.2).

These results look reasonable at a market wide level, which is the stated aim of the

[1] The Herfindahl-Hirschmann Index is a measure of market concentration, and is calculated as $H = \sum_{i=1}^{N} s_i^2$ where s_i is the market share of firm i, and N is the number of firms. For a given value of N, the index becomes smaller if the market share is evenly split between the participants, and larger if a small number of firms dominate.

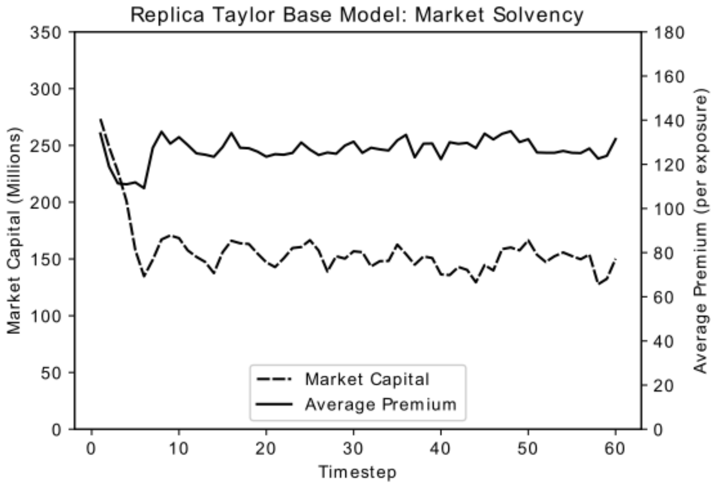


Fig. 5.1 – Market capital and premium by time step from Taylor replication base model.

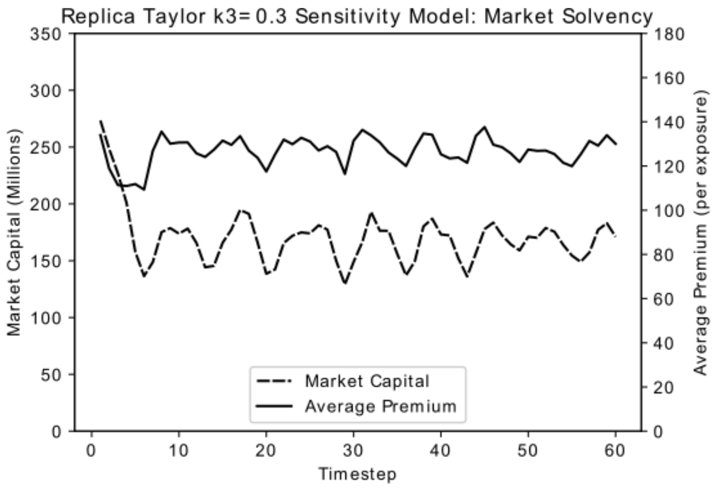


Fig. 5.2 – Market capital and premium from Taylor replication where $k_3 = 0.3$.

paper. However, by replicating the Taylor model and outputting the results for an individual insurer, it can be found that the individual premium rates demonstrate much larger swings that would be expected when compared with the market average. This is not necessarily a concern in itself in light of the papers which suggest that this might be an optimal strategy; however, in this case, the premium swings are driven by large changes in the level of capital (Figure 5.3). This is more of a source of concern, particularly since it can be noted that the replicated net assets would often fall short of a standard regulatory capital requirement to hold sufficient capital to cover 99.5% of the insurer's total loss distribution.

Additionally, it should be noted that there are two causes for a low insurer return: one is that the premium is too low, and the other is that the premium is too high. Although the Taylor setup takes account of capital size, this is done separately to the calculation related to competition. This means that the competition related mechanism does not allow for premium to move deliberately away from the market to achieve higher returns.

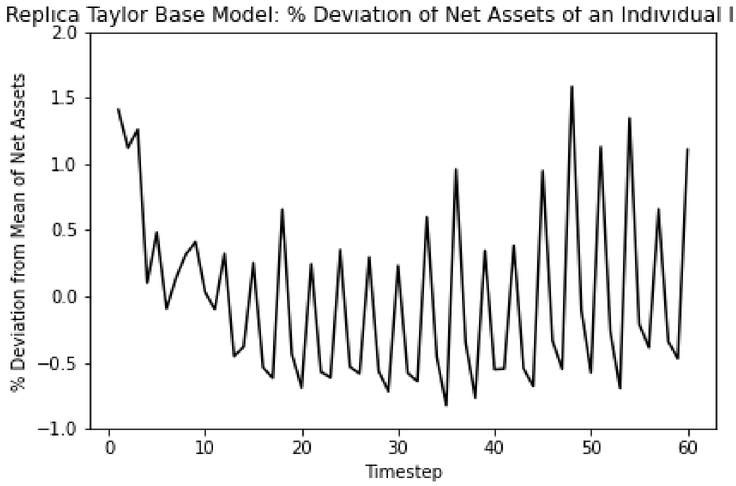


Fig. 5.3 – Single insurer net assets from Taylor replication base model.

In this chapter, a new premium setting process is proposed for Taylor's model which attempts to account for these issues. A base model is run using a similar setup to Taylor's model in order to compare the results. Finally, a sensitivity analysis is performed on the new parameters in a similar way to the original Taylor paper, and the results of these tests are compared between the two models.

5.3 Model Specification

5.3.1 Outline

A simple model is constructed using C# in order to to capture key features of market dynamics, while also producing reasonable results for the individual insurers. A replication of Taylor's model is also constructed in order to compare results. To enable the comparison of the two premium mechanisms, the rest of the model set-up will be maintained as in the Taylor model. As in the Taylor example, homogeneous customers represented as a total market exposure which is allocated amongst the insurers.

Some types of insurance, such as third-party motor insurance, are mandatory. However, even in voluntary markets, demand for insurance tends to remain steady and is not materially affected by price (Daykin et al., 1993). It is therefore appropriate to continue to use the Taylor assumption of a constant total market exposure across all time periods.

In each time period, the model undergoes the following steps:

1. Insurers set competitive premiums per exposure unit
2. Losses are generated based on each insurer's share of the market exposure
3. Accounting results are calculated for each insurer
4. Insurers enter or exit the market based on the accounting results
5. Exposure is reallocated among the insurers based on their competitive premiums

Note that by placing the reallocation of exposure at the end of the time step, this set-up includes an intrinsic time lag, as the effects of a change in premium on exposure and thus

t = time step

i = insurer index where insurers are ordered in ascending size of market share at the start of the time step

$P_{i,t}$ = Competitive premium set by insurer i during time step t

P_0 = Pure premium = Net present value of expected loss per exposure

$E_{i,t}$ = Exposure allocated to insurer i at the start of time step t

E = Total market exposure = $\sum_{all\, i} E_{i,t}$ for all t

$m_{i,t}$ = Competition markup calculated by insurer i during time step t

$K_{i,t}$ = Net assets of insurer i at the start of time step t

$S_{i,t} = \frac{K_{i,t}}{P_{i,t-1}E_{i,t}}$ = Solvency ratio of insurer i at the start of time step t

S_0 = A 'steady state' target solvency ratio

r_{F} = risk free rate of return

r_{M} = stock market expected rate of return

π_{it} = Insurance profit realized by insurer i during time step t

$D_{i,t}$ = Dividend paid out by insurer i during time step t

$L_{i,t}$ = Total losses incurred by insurer i during time step t

λ_{C} = Expected catastrophe claim frequency across entire market in a single time step

λ_{N} = Expected non-catastrophe claim frequency per exposure unit in a single time step

μ_{N} = Expected non-catastrophe claim severity for single claim

σ_{N} = Standard deviation of claim severity for single non-catastrophe claim

μ_{C} = Expected catastrophe claim severity for single claim across entire market

M_{C} = Minimum value of claim severity for single catastrophe claim across entire market

Table 5.1 – Definitions of environmental parameters used in the new model

the rate of capital growth will not be seen until the following time step.

The code has been made available on CoMSES (England, 2022).

5.3.2 Definitions

A set of parameters describing the market environment is defined as in Table 5.1.

Additionally, a series of dynamical parameters is defined in Table 5.2. Parameters k_3 to k_{13} are defined in the same as those used in Taylor's model; k_1 and k_2 have been chosen to have analogous functions to Taylor's first two parameters.

Insurer Behaviour Parameters

k_1 = Baseline markup adjustment

k_2 = Competition intensity

k_{10} = Dividend excess payout ratio

k_{13} = Lack of competitive inertia

Customer Behaviour Parameters

k_7 = Market price sensitivity

k_8 = Market presence limit

Market Participation Requirements

k_4 = New entrant threshold attraction profit margin

k_5 = New entrant attraction per unit market profitability

k_6 = New entrant capitalization

k_9 = Minimum viable market share

Regulatory Controls

k_3 = Floor solvency ratio

k_{11} = Competitive premiums lower bound

k_{12} = Competitive premiums upper bound

k_{13} = Lack of competitive inertia

Table 5.2 – Definitions of dynamical parameters parameters used in the new model

5.3.3 Competitive Premium

Taylor's model sets new premium using an exponential factor based on the solvency ratio (equation 5.1), adjusted according to their competitors' premium rates (equation 5.2). Rather than using an exponential factor, the extended model uses the target solvency ratio to determine the direction of movement in premium, and adjust the size of the change according to the insurer's premium relative to its competitors.

In the new model competitive premium per unit exposure for insurer i in time step t is set by applying a markup function to the actuarial premium. As with Taylor's model, this is subject to a smoothing factor k_{13} and lower and upper bounds represented by k_{11} and k_{12} respectively.

$$P_{i,t} = \begin{cases} \max(k_{11}P_0, \min(P_0 e^{m_{i,t}}, k_{12}P_0)) & \text{for insurer's first period of existence} \\ \max(k_{11}P_0, \min(k_{13}P_0 e^{m_{i,t}} & \\ \qquad + (1-k_{13})P_{i,t-1}, k_{12}P_0)) & \text{otherwise} \end{cases} \tag{5.3}$$

The markup function in each time step is adjusted accorded to the equation:

$$m_{i,t} = m_{i,t-1} + g(E_{i,t})h(P_{i,t-1}, g(E_{i,t})) \tag{5.4}$$

The function $g(E_{i,t})$ is an indicator function indicating the direction of the movement in the premium markup. It is defined by:

$$g(S_{i,t}) = \begin{cases} -1, & \text{if } S_{i,t} > S_0. \\ +1, & \text{otherwise} \end{cases} \tag{5.5}$$

From the above definition, it can be seen that if the insurer's solvency ratio is higher than the target solvency ratio, then the value of the markup and therefore the competitive premium is decreased. This is expected to lead to an increase in the insurer's market share. If its solvency ratio is less than or equal to the target solvency ratio, then the insurer will instead increase its premium, which will lead to a greater expected income per unit exposure but a lower market share. This is consistent with empirical data (Choi et al., 2002).

The function $h(P_{i,t-1}, g(S_{i,t}))$ is calculated using the equation:

$$h(P_{i,t-1}, g(S_{i,t})) = k_1 \exp(g(S_{i,t})k_2(\overline{P}_{i,t-1} - P_{i,t-1})/P_0) \tag{5.6}$$

where $\overline{P}_{i,t-1}$ = average of $P_{i-2,t-1}$, $P_{i-1,t-1}$, $P_{i,t-1}$, $P_{i+1,t-1}$, and $P_{i+2,t-1}$; if any of these values do not exist they are deleted from the average. From the definition of index i, this is an average of the insurer's nearest neighbours according to market share. This definition matches that used in the Taylor model during the competitive adjustment stage.

If $g = +1$ then the increase in markup will be scaled up if the insurer's previous premium was less than its competitors, and scaled down if it was already greater than its immediate competitors. If $g = -1$, then the reverse is true. Thus, the change in markup is of a larger value if the insurer is moving towards its competitors, and smaller if it is moving away. Additionally, when the insurer is moving towards its competitors, the movement will be smaller the closer the insurer already was; when an insurer is moving away from its competitors, it will move less the further away it already was from the competitor average.

If the insurer's previous premium is equal to the average of those its closest peers, then the markup will be changed by an amount equal to k_1. From this, it can be seen that k_1 is a measure of the baseline markup adjustment, and k_2 is a competition intensity parameter. Of the parameters used in Taylor shows that Taylor's k_1 was a premium-to-solvency sensitivity, and Taylor's k_2 was also a measure of competition intensity, though scaled differently.

k_{13} is a measure of a lack of competitive inertia. It is used here in the same way as in Taylor's premium calculations. Similarly, the lower and upper bounds represented by k_{11} and k_{12} are used in the same way as in Taylor's calculation.

5.3.4 Losses

For comparison with Taylor's model, the same loss distributions are used. Catastrophe claims are drawn across the entire market from a Poisson frequency and the total market severity drawn from a Pareto distribution. Non-catastrophe claims are drawn for each insurer from a Poisson frequency and a Gamma severity.

This gives:

$$L_{i,t} = \sum_{j=1}^{n_{\mathrm{N}_{i,t}}} x_{\mathrm{N}_{i,t,j}} + \frac{E_{i,t}}{E} \sum_{k=1}^{n_{\mathrm{C}_{i,t}}} x_{\mathrm{C}_{t,k}} \tag{5.7}$$

where:

$n_{\mathrm{N}_{i,t}} \sim Poisson(E_{i,t}\lambda_{\mathrm{N}})$

$n_{\mathrm{C}_{i,t}} \sim Poisson(\lambda_{\mathrm{C}})$

$x_{\mathrm{N}_{i,t,j}} \sim Gamma(\frac{\mu_{\mathrm{N}}^2}{\sigma_{\mathrm{N}}^2}, \frac{\mu_{\mathrm{N}}}{\sigma_{\mathrm{N}}^2})$

$x_{\mathrm{C}_{t,k}} \sim Pareto(\frac{\mu_{\mathrm{C}}}{\mu_{\mathrm{C}} - M_{\mathrm{C}}}, M_{\mathrm{C}})$

However, as the sum of gamma distributions is also gamma, $L_{i,t}$ can instead be calculated using:

$$L_{i,t} = x_{\mathrm{N}_{n_{\mathrm{N}_{i,t}}}} + \frac{E_{i,t}}{E} \sum_{k=1}^{n_{\mathrm{C}_{i,t}}} x_{\mathrm{C}_{t,k}} \tag{5.8}$$

where:

$x_{\mathrm{N}_{n_{\mathrm{N}_{i,t}}}} \sim Gamma(n_{\mathrm{N}_{i,t}} \frac{\mu_{\mathrm{N}}^2}{\sigma_{\mathrm{N}}^2}, \frac{\mu_{\mathrm{N}}}{\sigma_{\mathrm{N}}^2})$

From this, P_0 is equal to:

$$\begin{aligned} P_0 &= E\left[\frac{L_{i,t}}{E_{i,t}}\right] / (1 + r_{\mathrm{F}}) \\ P_0 &= (\lambda_{\mathrm{N}}\mu_{\mathrm{N}} + \lambda_{\mathrm{C}}\mu_{\mathrm{C}}/E)/(1 + r_{\mathrm{F}}) \end{aligned} \tag{5.9}$$

As in the Taylor model, catastrophe losses are very rare events, and the base model uses a simulation where no catastrophe losses occurred. For the sensitivity tests, an example including a catastrophe loss is considered.

5.3.5 Accounting Results

The insurance profit includes the asset return, the total premium in, and the losses paid out. It is assumed that the premium is invested in risk free assets during the year as a reserve to cover the losses, and the initial net assets are invested in a representative market-based portfolio. This gives the following equation:

$$\pi_{i,t} = K_{i,t} r_{\mathrm{M}} + E_{i,t} P_{i,t} (1 + r_{\mathrm{F}}) - L_{i,t} \tag{5.10}$$

As in the Taylor model, investment risk is not included in the market dynamics, which removes an element of volatility.

The dividend is also calculated as in Taylor's model:

$$D_{i,t} = \max(0, \min[K^*_{i,t} - k_3 P_{i,t} E_{i,t}, k_{10}(K^*_{i,t} - S_0 P_{i,t} E_{i,t})]) \tag{5.11}$$

where $K^*_{i,t}$ = New net assets pre-dividend = $K_{i,t} + \pi_{i,t}$.

Thus, the dividend is paid by applying the dividend ratio k_{10} to the value of $K^*_{i,t}$ that is in excess of the target solvency level, subject to a floor solvency ratio k_3.

Finally, the closing net assets at the end of the time period are set equal to:

$$K_{i,t+1} = K_{i,t} + \pi_{i,t} - D_{i,t} \tag{5.12}$$

5.3.6 Entry and exit of insurers

The entry and exits of insurers is calculated as in the Taylor model.

An insurer i exits if its final capital is below the floor solvency ratio used in the dividend equation; i.e. if:

$$K_{i,t+1} < k_3 E_{i,t} P_{i,t} \tag{5.13}$$

For the entrants, define:

$$\begin{aligned} \pi_t = \sum_i \pi_{i,t} = \text{ Total market insurance profit} \\ P_t = \sum_i E_{i,t} P_{i,t} / E = \text{Market average premium per exposure unit} \end{aligned} \tag{5.14}$$

Then there are new entrants to the market if:

$$\pi_t / E_t P_t > k_4 \text{ and } \pi_{t-1} / E_{t-1} P_{t-1} > k_4 \tag{5.15}$$

In that case, m_{t+1} new insurers are introduced into the market, where

$$m_{t+1} = \lfloor k_5 (\pi_t / E_t P_t - k_4 + \pi_{t-1} / E_{t-1} P_{t-1} - k_4) \rfloor, \tag{5.16}$$

and $\lfloor z \rfloor$ denotes the greatest integer less than or equal to z.

Thus, the number of new insurers is proportional to the sum of the current and previous excess profit margins (including asset return) over the threshold. The constant k_4 indicates the new entrant threshold attraction profit margin, and k_5 represents the new entrant attraction per unit market profitability over the threshold.

Each new insurer is given a starting competition markup of

$$m_{i,t} = \log\left(\frac{P_t}{P_0}\right) \tag{5.17}$$

and a starting capital of

$$K_{i,t+1} = k_6 \sum_j K_{j,t} \tag{5.18}$$

This gives the starting premium per exposure unit equal to the average market premium per exposure, and the starting capital as a k_6 proportion of the total market capital. The starting exposure is 0.

5.3.7 Reallocation of market share

Again, for comparison with Taylor, the same reallocation mechanism is used. Taylor assumes that the total market exposure remains constant; however, the customers do not all purchase from the insurer with the lowest premium. Instead, the market share flows between insurers according to their relative premium and a rate of customer sensitivity to price in a manner based on a logit choice model. This reflects both that customers have preferences, and also will often renew an existing policy rather than immediately moving to find a new insurer. As a result, insurers can charge a premium higher than the average and still attract a market share.

The unscaled transfer function of market exposure from insurer i to insurer j at the end of time step t is defined as:

$$\alpha_{i,j,t} = \max\left(0, P_{i,t} - P_{j,t}\right) \max\left(k_8, \frac{E_{i,t}}{E}\right) \tag{5.19}$$

The unscaled transfer rate is then calculated as:

$$\tau_{i,j,t} = [1 - \exp(-k_7 \alpha_{i,t})]\alpha_{i,j,t}/\alpha_{i,t} \tag{5.20}$$

where:

$$\alpha_{i,t} = \sum_j \alpha_{i,j,t} \tag{5.21}$$

The unscaled transfer rates are applied to the existing exposure amounts to get the new unscaled exposure amounts:

$$E^*_{i,t+1} = E_{i,t}[1 - \sum_j \tau_{i,j,t}] + \sum_k E_{k,t}\tau_{k,i,t} \tag{5.22}$$

There is now a second round of possible exits. If an insurer's market share $E^*_{i,t+1}/E$ is less than the minimum viable market share k_9 then it exits the market.

Finally, exposures are rescaled so as to ensure they will add up to the total market exposure:

$$E_{i,t+1} = E^*_{i,t+1}\frac{E}{\sum_j E^*_{j,t+1}} \tag{5.23}$$

where j includes all insurers remaining in the market at the end of the time step t.

From the above, exposure is transferred from an insurer i to an insurer j if and only if insurer j is offering a lower premium. The transfer function is proportional to the market price sensitivity parameter k_7 and to the difference in premium and to insurer j's existing market share, reflecting the influence of marketing and word of mouth on an insurer's

Base Model Parameter Values			
No. Starting Insurers	20	k_1	0.1
$E_{i,0}$	75,000	k_2	1.8
E	1,500,000	k_3	0.1
m_0	$log(\frac{160}{P_0})$	k_4	0.2
$K_{i,0}$	13,500,000	k_5	30
S_0	$145 * (1 + r_F)/P_0$	k_6	0.000333
r_F	4%	k_7	0.1
r_M	12%	k_8	0.01
λ_C	2%	k_9	0.0006
λ_N	12%	k_{10}	0.7
μ_N	1,000	k_{11}	0
σ_N	1,000%	k_{12}	1,000
μ_C	120,000,000	k_{13}	0.75
M_C	100,000,000		

Table 5.3 – Table of base model parameter values

attractiveness, with k_8 as a limiting parameter to this effect.

5.3.8 Parameter Values

The parameter values have been chosen to allow for an easy comparison with the results in the Taylor model (Taylor, 2008). Table 5.3 shows the parameter values for the base model. Table 5.4 shows the parameter values used to run the sensitivity analysis.

5.4 Results

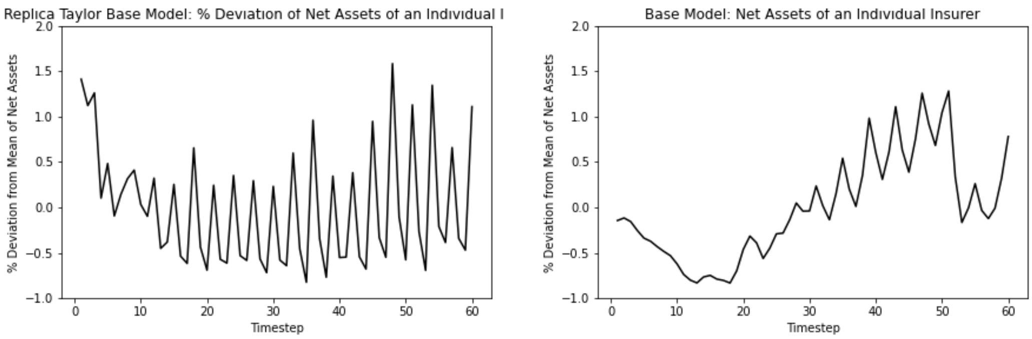
For ease of reference, a summary of the results can be found in table 5.5.

5.4.1 Base Model Results

Figure 5.4 is based on a typical insurer's net assets according to a replication of Taylor's model and the new premium model, and shows the percentage deviation from the mean value over time. Taylor's model sets new premium based on an exponential factor based on the solvency ratio (equation 5.1) which produces large, rapid oscillations in the individual insurer net asset values. The new model does not use this exponential factor, instead choosing to use the target solvency to indicate only the direction of the change in premium (equation 5.5). As a result, the individual insurer's net assets shown in Figure 5.4 is much smoother. The new model's individual insurer assets have a coefficient of variation value

Sensitivity Models Parameter Values	
Testing Parameter	Parameter Values
k_1	0.04, 0.06, 0.08
k_2	1, 6, 10
k_3	0.2, 0.3
k_4	0.15, 0.175, 0.24
k_5	45
k_6	0.001, 0.02
k_7	0.04, 0.17
k_8	0.001
k_9	0.0025
k_{10}	0.8, 0.9
k_{11}	0.65, 0.8, 0.95, 0.97, 0.99, 1
k_{12}	1.05, 1.2
k_{13}	0.4, 0.9

Table 5.4 – Table of sensitivity models parameter values

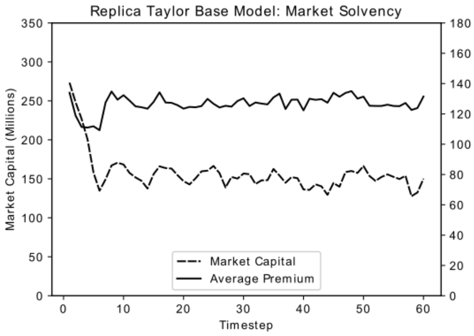


(a) Replica of Taylor's model. (b) New model.

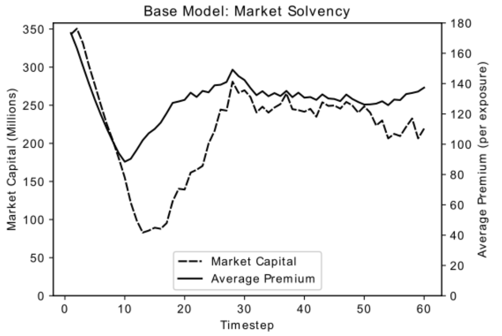
Fig. 5.4 – Percentage deviation from the mean of a single insurer net assets from Taylor replication and new model base models.

of gives a coefficient of variation value of 57% compared with the Taylor model's value of 66%. Additionally, the new model displays only 22 peaks and troughs, whereas the Taylor model shows 39 within the same timeframe.

Figure 5.5 shows the average market premium per exposure and the total market capital over time from the new model and the replica of Taylor's model. As in the Taylor model, there are stable premium rates and market capital once the market has recovered from the

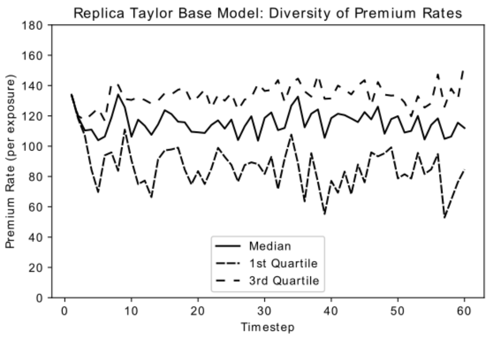


(a) Replica of Taylor's model.

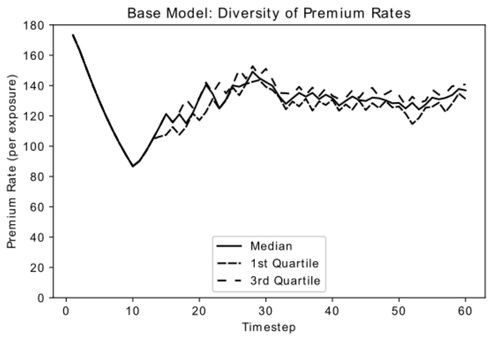


(b) New model.

Fig. 5.5 – Market capital and premium from Taylor replication and new model base models.



(a) Replica of Taylor's model.



(b) New model.

Fig. 5.6 – Premium diversity from Taylor replication and new model base models.

initial 'kick'. The new model produces smaller swings in premium than the Taylor model, and is thus slower to readjust.

In both models, the number of insurers in the market remains generally stable. However, the new base model demonstrates more pronounced cyclical behaviour, which leads to small spikes as new insurers enter the market and then exit again when the premium decreases.

Figure 5.6 shows the median and upper and low quartiles of the premium rates per exposure on offer by time step for both the new model and Taylor's model. Taylor's model produced significant diversity in premium rates caused by constant large oscillations in the individual insurers' premium rates. Since the new model is more stable and does not contain the same large movements, it demonstrates a much lower diversity in premium rates than Taylor's model.

5.4.2 Insurer Behaviour Parameters

k_1: Baseline Adjustment

This parameter represents the baseline amount by which the markup would move if the insurer's last premium was equal to the market average (equation 5.6). Since this defines

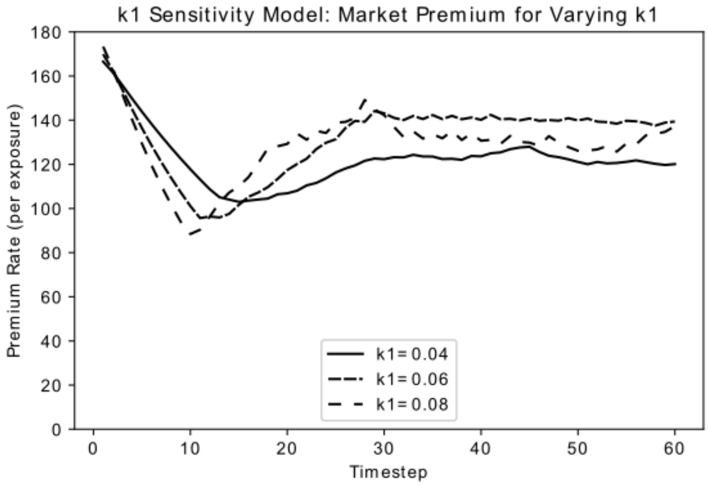


Fig. 5.7 – Market capital and premium for k_1 sensitivity model with varying k_1.

how fast premium rates can change in response to its solvency ratio, it is a key parameter in the new model.

In Taylor's model, k_1 determines the premium sensitivity to the solvency ratio. This is analogous to the k_1 in the new model, but produces larger swings as it is multiplied by the excess solvency ratio (equation 5.1).

As might be expected, increasing k_1 increases the diversity of premium rates as insurers take further steps away from the market average. However, the diversity remains lower than that produced by the Taylor model.

Figure 5.7 shows the average market premium rates over time for k_1 = 0.04, 0.06, and 0.08. Since a large k_1 results in more rapid changes in premium as well as larger swings, the market premium dips further but corrects for the initial anomaly at earlier time periods for larger values of k_1; the premium also becomes less smooth.

A small value of k_1 means smaller changes to premium and therefore less movement in the number of participants. As it increases, there is an onset of new insurers and eventually cycles as the competition drives the numbers back down.

k_2: Competition Intensity

In Taylor's mode, the k_2 parameter represented the willingness of insurers to compete (equation 5.2), so increasing Taylor's k_2 results in cyclical behavior. In the new model, k_2 plays a similar role (equation 5.6). Figure 5.8 shows the autocorrelation in the market premium produced by out model for varying values of k_2. As a general trend, increasing k_2 increases the intensity of cyclicality and also decrease periodicity of the cycles; however, the exact relationship is unclear and is masked at lower values by individual variation in the simulations.

In general, the new extended model produces longer periodicity than Taylor, which is consistently 6-7 years. The periodicity of the extended model varies between approximately 20-30 years. This is larger than empirical results suggest (Cummins and Outreville,

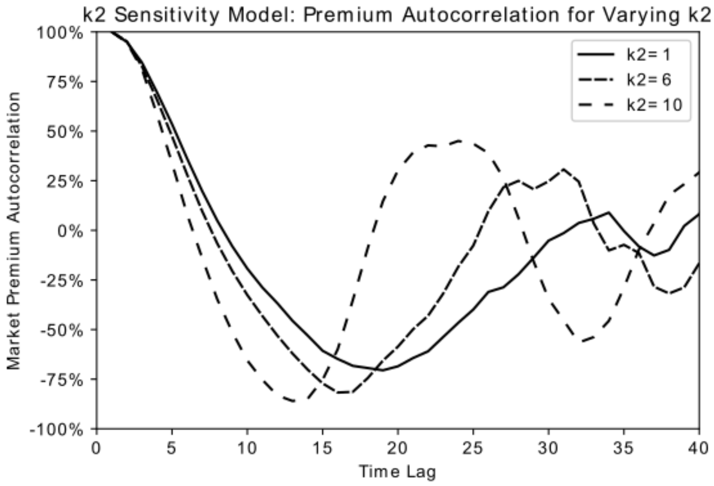


Fig. 5.8 – Market premium autocorrelation for k_2 sensitivity model with varying k_2.

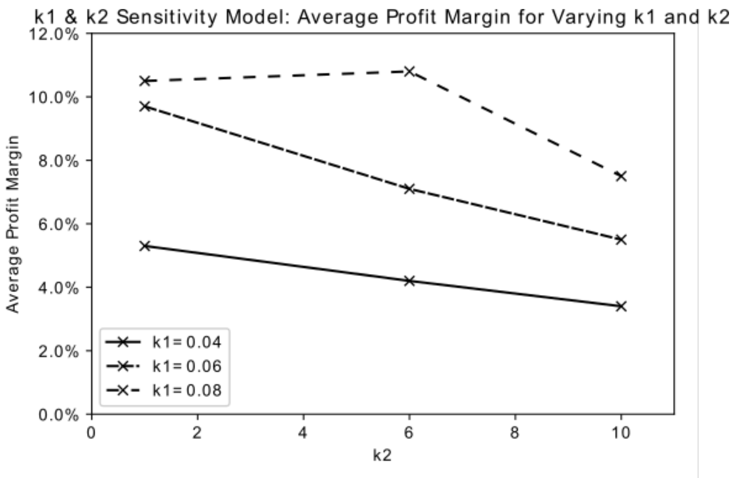


Fig. 5.9 – Average market profit margin for k_1 and k_2 sensitivity models with varying k_1 and k_2.

1987), and implies that further work should be done to investigate appropriate values for the environmental and dynamical parameters.

Figure 5.9 shows the average profit margin for varying values of both k_1 and k_2. As k_2 increases, so does competition, and this causes the profit margin to decrease. As k_1 increases, so does the insurer's concern with their solvency level, and this causes the profit margin to increase.

k_{10}: Dividend Excess Payout Ratio

In Taylor's setup, the dividend payout calculation acts as a stabilizing force to bring capital back towards the steady state. As a result, the number of market participants and concentrations are strongly affected by k_{10}. Taylor found that increasing k_{10} caused an increase in numbers and a small reduction in concentration index. This is followed by a crash as new entrants cannot be supported by the market. In the new extended model, the use of k_{10} has

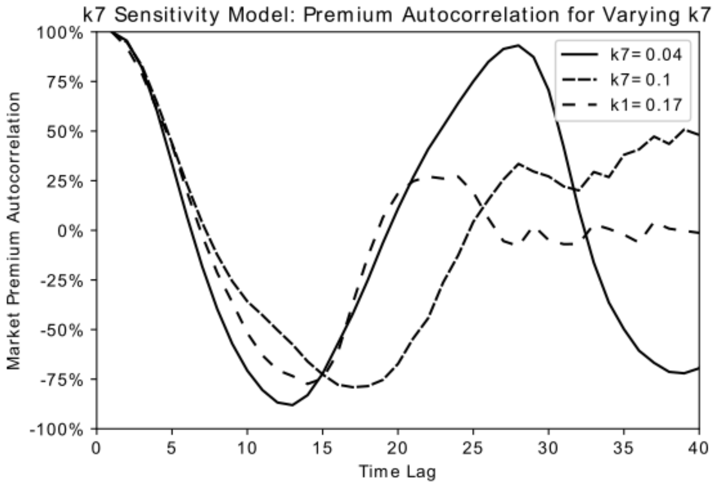


Fig. 5.10 – Market premium autocorrelation for k_7 sensitivity model with varying k_7.

not changed. The results therefore indicate a similar pattern.

k_{13}: Lack of Competitive Inertia

Taylor found that increasing k_{13} reduced premium stability and produced cycles as it reduced premium smoothing. Decreasing k_{13} caused a reduction in the number of market participants as insurers moved premium rates too far, resulting in a non-viable solvency level.

In the new model, the use of the markup includes an inherent smoothing factor, and the use of k_{13} becomes confused with this effect. Although decreasing k_{13} does cause a smoother premium, the results of this are unpredictable and the pattern is unclear. However, increasing k_{13} does cause a marked increase in the premium rate diversity, as might be expected.

5.4.3 Customer Behaviour Parameters

k_7: Customer Price Sensitivity

This parameter represents the customer tendency to respond to high premiums by moving between insurers. In Taylor's model, increasing $k_7$7 caused an increase in the diversity of premium rates. The new model does not show a significant impact on premium rate diversity, which depends more on the direction of movements in exposure than on the size.

However, there are cyclical patterns emerging as k_7 is decreased (Figure 5.10).

k_8 Market Presence Limit

The parameter k_8 determines the extent to which an existing market share can entice new customers. Decreasing k_8 leads to a decrease in insurer numbers as it becomes harder for insurers to attract more exposure share. This is the same effect that Taylor found.

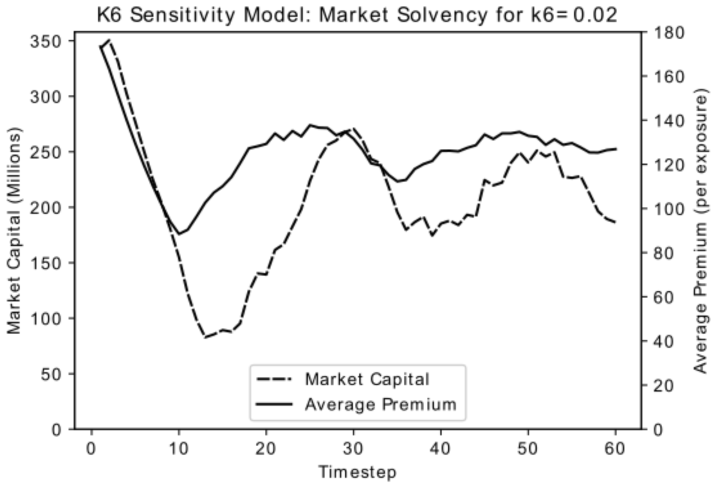


Fig. 5.11 – Market solvency and premium for k_6 sensitivity model where $k_6 = 0.02$.

5.4.4 Market Participation Requirements

k_4: New Entrant Threshold Attraction Profit

Decreasing the threshold parameter k_4 represents a lower barrier to entry for new participants, and therefore leads to an initial increase in insurer numbers. However, the new entrants occur when the market has a lower average viability, and so the effect is temporary. This is same pattern found by Taylor.

k_5: New Entrant Attraction per Market Profitability

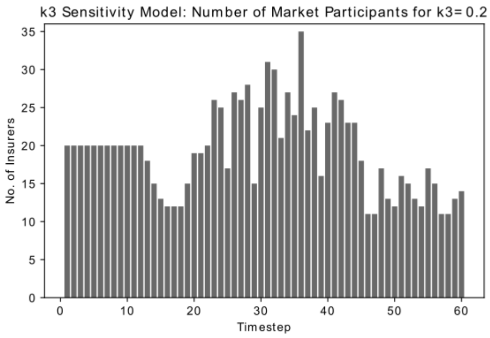
Increasing the parameter k_5 increases the number of new entrants that join the market in a time period. However, this high competition is unsustainable, leading to cycles and large number of exits. As a result, the number increases are not permanent. Taylor's model demonstrated the same pattern.

k_6: New Entrant Capitalization

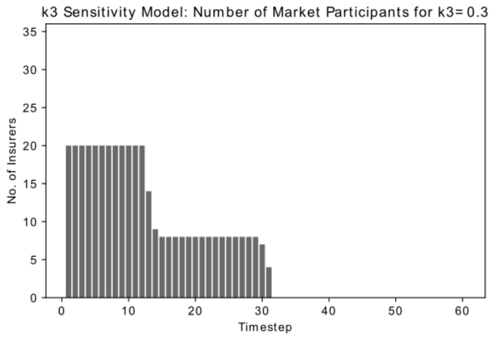
Taylor found that increasing the new entrant capitalization led to the introduction of rapid market cycles as new entrants are more competitive. As before, the new model is slower to respond, and does not seem to be as sensitive to changes in k_6 as the Taylor model was. When k_6 is increased to the level used in Taylor's test, there is an increase in participant numbers as new entrants are able to survive for longer. The value of k_6 must be significantly increased in order to induce visibly significant competitive cycles (Figure 5.11).

k_9 Minimum Viable Market Share

Taylor found that increasing k_9 caused more market exits as more insurers fail to meet the required market share. The new model finds the same pattern, leading to a decrease in insurer numbers.



(a) $k_3 = 0.2$.



(b) $k_3 = 0.3$.

Fig. 5.12 – Insurer Numbers for k_3 sensitivity model with $k_3 = 0.2$ and $k_3 = 0.3$.

5.4.5 Regulator Controls

k_3: Floor Solvency Ratio

Taylor found that increasing the required minimum solvency ratio k_3 drives out a number of insurers, setting off a period of intense competitive cycles. As seen before, the new model is slower to react and cycle lengths are longer. Thus, there is not the same level of sensitivity and resulting oscillations in market premium as in the Taylor results.

However, there are oscillations in insurer numbers reflected in the market participation. Additionally, as seen earlier, the market takes longer than the Taylor model to adjust to the initial instability, which causes a lower drop in market capital. Because of this, participation is more sensitive to changing k_3. For the sensitivity test $k_3 =$, the minimum solvency ratio is now sufficiently high that it has caused all insurers to exit the market (Figure 5.12).

Examining the autocorrelation shows that the peaks are more strongly correlated for higher k_3, implying a stronger cyclical relationship.

k_{11}: Premium Lower Bound

As would be expected, an increase in the minimum premium k_{11} attracts new entrants and reduces the large premium drop near the start, stabilizing the market. However, new entrants' survival is short and numbers decline again. For a high enough k_{11}, no entrants remain in the market long enough to make a difference to the numbers. This is the same pattern as in Taylor's model.

k_{12}: Premium Upper Bound

As with the Taylor model, changing k_{12} in the new model does not have as significant an impact on the numbers as changing k_{11}. Instead, as would be expected, the main effect of introducing an upper bound is to reduce the diversity of premium numbers. Despite the lower diversity of the new model, this effect can be seen by running the same parameter tests as Taylor.

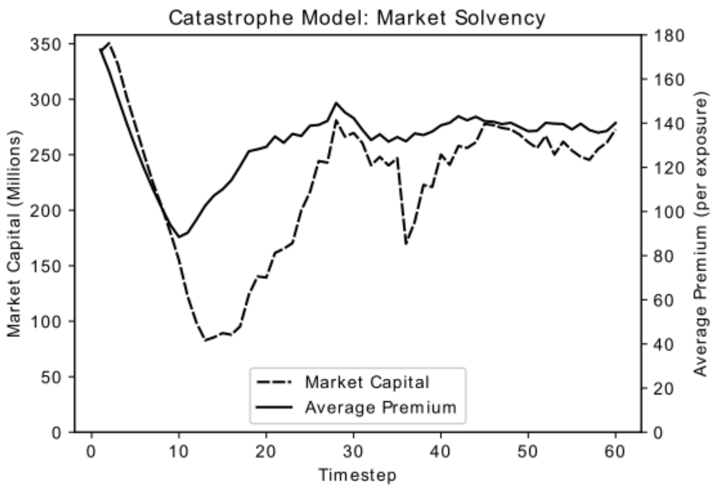


Fig. 5.13 – Market Premium and Capital for simulation with a catastrophe loss.

5.4.6 Catastrophe Loss

For the final test, a version of the base model is run where a cat loss is simulated in time period $t = 36$ with a total market cost of $107m. There is an initial dip in the number of insurers to approximately 15, though the new model is not as sensitive as the Taylor model. This leads to an increasing market premium, though the new model does not produce the immediate sharp increase of Taylor. Instead the rise is spread out over several time periods (Figure 5.13).

As a result, there is the same instigation of a series of cycles as increased rates lead to new entrants which lead to more competition, the new model produces cycles which are less pronounced and have a longer period.

5.5 Conclusion

A new model for competition premium is constructed to address some of the limitations of Taylor's model by reducing the large oscillations leading to instability in individual insurer net assets, and by amending the competitive adjustment to account for the direction of the premium movement. This is implemented within a model that remains otherwise unchanged in order to allow comparison with Taylor's results.

For many of the parameters, the new model displays similar patterns to the Taylor model. Additionally, the individual insurer assets are more stable and the premium mechanism allows for the insurer to choose to move away from the market average. However, the more stable individual premium rates cause the model to be more sluggish in response to changes, and cycles are less pronounced and have a longer periodicity. To address the slow response time in premium changes, it might be possible to use some combination of aspects of the new model and the solvency adjustment used in Taylor's model.

This model produces competitive cycles, which are an important market feature. A key parameter is the sensitivity of competitive premium setting to rival premium. Additionally,

Parameter	Taylor model	New model
Base model	Stable market premium and capital after initial phase, rapidly oscillating individual insurer capital	Stable market premium and capital, more pronounced cycles, smaller premium swings, smoother individual capital
k_1: Baseline adjustment	Increasing k_1 produces larger and more rapid changes in premium and increases diversity of premium	Similar, though diversity remains lower and swings are not as pronounced
k_2: Competition intensity	Increasing k_2 increases intensity and rapidity of cycles	Similar, but periodicity is much longer, and exact relationship less clear for low values
k_{10}: Dividend excess payout ratio	Increasing k_{10} causes increase in numbers followed by a market crash	Similar pattern
k_{13}: Lack of competitive inertia	Increasing k_{13} reduces premium stability. Decreasing causes reduction in market participants.	Parameter has much smaller impact on results as markup factor already inherently smoothed
k_7: Customer price sensitivity	Increasing k_7 causes increase in diversity of premium rates	No significant impact on rate diversity, but decreasing k_7 increases cyclical behaviour
k_8: Market presence limit	Decreasing k_8 leads to decrease in insurer numbers	Similar pattern
k_4: New entrant threshold attraction profit	Decreasing k_4 initially increases market participants but effect is temporary	Similar pattern
k_5: New entrant attraction per market profitability	Increasing k_5 increases no. of new entrants causing increased competition and exits	Similar pattern
k_6: New entrant capitalization	Increasing k_6 introduces rapid market cycles	Slower to respond and not as sensitive to changes in k_6
k_9: Minimum viable market share	Increasing k_9 causes more exits in the market	Similar pattern
k_3: Floor solvency ratio	Increasing k_3 causes more exits and subsequent intense competitive cycles	Slower to respond and resulting cycles are longer
k_{11}: Premium lower bound	Increasing k_{11} attract more new entrants but the effect is temporary	Similar pattern
k_{12}: Premium upper bound	Increasing k_{12} reduces diversity of premium rates	Similar pattern
Catastrophe loss	Drop in market numbers triggers increase in premium and pronounced competition driven cycles, which leads to market entrants	Impact is dampened, premium rise spread out over several time periods, and resulting cycles slower and less pronounced

Table 5.5 – Summary of key results and comparison between the new model and the Taylor model

the average profit decreases with increased levels of competition and decreases as insurers become more sensitive to their solvency level. Cycles are also affected by the customer's sensitivity to price as insurer premium responds to changes in their exposure share. The introduction of a shock such as a catastrophe event can also instigate cycles.

Insurer numbers can be effected temporarily by a number of parameters. However, lowering market participation requirements do not generally make a sustainable difference as the new participants are of lower quality and are driven out by triggering competitive cycles. Key parameters for long term market participation are the limit to the influence of existing market share on customer attraction and the minimal viable market share; decreasing the first or increasing the second leads to more difficulty in maintaining adequate market share.

As it is the direction of the premium change and not the amount of the change that is driven by solvency, premium changes in the new model are slower than in the Taylor model. As a result, price reactions to shocks such as a catastrophe loss are spread out and the resulting cycles are less pronounced. However, as it also takes longer for the model to stabilise after the initial capital shock, the model is also more prone to insurer exits due to low solvency before the premium is able to increase again. An increase to the baseline adjustment parameter or less premium inertia can both lead to faster premium changes.

Like Taylor's model, this dynamic insurance market model uses a minimal number of market parameters to produce results with realistic market features including a negative correlation between capital and premium rates and cycles arising out of competition and loss shocks. It addresses some of the limitations of the Taylor model and produces more stable individual insurer assets, which potentially enables its use within a range of market environments for other market simulation based models.

In this chapter, the new model was parameterised in order to allow for easy comparison with the results in Taylor's model. An investigation into parameterisation was carried out in which environmental parameters were based on market data. In order to set behavioural parameters, a particle swarm algorithm was carried out based on some market outputs such as loss ratio means, loss ratio volatilities, and premium autocorrelation values. However, this investigation was inconclusive as the algorithm was unable to find a reasonable set of parameters using this market model. Further work could be done to find appropriate environmental and dynamical parameter values, and to compare the results of a standard parameterisation exercise with different potential mechanisms.

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Chapter 6

Conclusions and Future Work

6.1 Conclusions

The insurance market—like the financial markets—include systemic sources of risk and bias which emerge from the interactions of the entities (or 'agents') which operate within that market (Danielsson and Shin, 2003). However, this type of risk is often not considered in traditional statistical models used for pricing or setting capital (Parodi, 2014; Kravych, 2013). Typically, competition is modelled using equilibrium models under the assumption that all agents are rational (Boonen et al., 2018; Dionne, 2013; Wu and Pantelous, 2017). This kind of model may be inappropriate for the insurance market where there is imperfect information, the underlying risk is constantly changing, and agents may be irrational.

In this thesis, this kind of systemic risk is investigated through the use of agent-based models, (ABMs). This kind of model is a computational simulation of decision-making 'agents', producing outputs sampled from a large number of possible timelines. The simulated output often displays complex emergent behaviour patterns, which are of use when exploring systemic effects in financial markets (Bonabeau, 2002).

ABMs are a good tool for situations where agents are heterogeneous, mechanisms may be non-linear, agents display behavioural biases or are bounded by imperfect information, and decision-making is impacted by the outcome of interactions with other agents. These are all features of insurance markets and indicate that ABMs might be a good fit (Parodi, 2012). The existing insurance literature making use of ABMs is currently limited, but includes: the paper by Dubbelboer et al. (2017) examining government flood risk management and reinsurance strategies within a housing market; the work by Owadally et al. (2019) to produce an ABM informed framework for regulators to monitor and respond to market cycles; and the investigation by Heinrich et al. (2022) into systemic risk caused by insurers tendency to purchase catastrophe models from the same few sources.

However, ABMs are difficult to validate, particularly for behavioural mechanisms. Due to these limitations, ABMs are used in this thesis primarily as a tool for exploring patterns and critical indicators of behaviour rather than as statistical prediction models.

- In chapter 2, an ABM was constructed to examine the patterns that might arise in an insurance market due to customers passing on their opinion of their insurer to their social network. Empirical data was used to parameterise the model where possible. The work in this chapter was also published in the *Journal of Artificial Societies and Social Simulations* (England et al., 2022).

The existence of the network was found to act as a persistent memory, causing a systemic bias whereby an insurer's early reputation achieved by random chance tends to persist and leads to unequal market shares. This occurs even when the transmission rate of information is very low. This suggests that newer insurers might benefit more from a higher service quality as they build their reputation. Insurers with a higher service quality earn more profit, even when the customer preference for better service quality is small. The impact of this systemic effect is exacerbated under a new regulation which bans the practice of charging renewing customers more than new customers.

These findings should be considered by both insurers considering strategies for attracting and retaining customers, and by regulators who are assessing possible impacts of a change in the regulation of insurance pricing practices.

- In chapter 3, a simple ABM was built to explore the impact of the winner's curse caused by bidding for customers under imperfect information on an insurer's estimated capital requirement. Outputs indicate that the winner's curse increases the estimation risk due to parameter uncertainty when there are more competitors or fewer customers, leading to a higher gap between the estimated and actual capital, and an increase in the parameter estimation risk considered by existing literature on parameter mis-estimation risk.

  Extensions which functionally decrease the number of customers attracted by a particular bid or increases the number of competitors for particular customers both worsen the impact of the winner's curse for that interaction. For example: increasing customer heterogeneity; increased renewal rates; and increased customer tendency to seek quotes from a greater number of insurers.

  Modellers may assume that the winner's curse effect would have a negligible impact as insurers have access to data from a lot of customers. However, there are common circumstances that increase the significance of imperfect information, such as: high levels of customer heterogeneity, underlying risk distributions that change over time, and very rare event such as catastrophes. Overall, this could have a significant impact on an insurer's willingness to take on risk which they have under-estimated. An insurer should increase their capital estimate in order to expect to cover their true capital requirement.
- In chapter 4, an ABM was used to investigate heterogeneous market strategies for a market where premium is determined by the balance of supply and demand, and in particular to compare the outcomes of insurers following chartist assumptions about the market with insurers assuming a more boundedly rational approach.

  Simulation outputs indicate that chartist insurers tend to disrupt the market, causing increased volatility. The chartist insurers are often better able to take advantage of this disruption and make a higher profit than the rationalists, though their performance is also notably much more volatile.

It is common to assume that a rationalist approach is the optimal solution. However, this model suggests that which strategy is 'best' depends on the current situation in the market, particularly when there is imperfect information about the actions of customers and competitors. For insurers who are primarily driven by profit, a chartist strategy such as following a medium-term trend might be a better option. However, insurers who value stability more might prefer to follow a rationalist strategy even though the average profit is lower.

- In chapter 5, a dynamic market model was built based on the Taylor model (Taylor, 2008) with the aim of establishing a market framework with minimal parameters for use with future work. This chapter introduced a new premium mechanism in order to address some limitations of the original Taylor model, and sensitivity tests carried out in order to compare the results of the new model with the behaviour of the Taylor model.

  The model displays useful market dynamics. In particular, it produces stable premium rates, which display an emergent cyclicality. It includes allowances for market entrants and exits, and the customer demand flows are based on the choice logit models with premium-sensitive renewals used in earlier chapters. Additionally, there are some parameters which reflect possible regulatory intervention, such as premium limits and solvency requirements.

  The new extended premium mechanism corrects for some limitations of the original Taylor model. Specifically, the new model produces stable individual premium rates and insurer assets. It also allows for insurers choosing to move either towards or away from the market average, and a strategy where insurers are more willing to take risks when they have a higher capital adequacy. The new model maintains the fundamental shape of the market dynamics. However, the cycles do display a slower periodicity.

6.2 Future Work

The customer choice model with word-of-mouth network used in chapter 2 models insurers as price-takers. In future work, this could be expanded and combined with the work in chapters 4 and 5 to allow insurers to employ a competitive premium-setting strategy. This would create a more realistic market setup on the insurer side.

Insurers in this model are also set at a fixed service quality throughout each simulation. The insurers could be allowed to dynamically set their service quality in each time period as part of their competitive strategy. This would allow for the exploration of the effectiveness of different strategies in light of the systemic bias and resulting persistence of insurer reputations.

The model also contains some implicit behavioural assumptions: for example, good and bad experiences are given the same weight, whereas studies indicate that people are more sensitive to negative than positive experiences (Tversky and Kahneman, 1981). Additionally, the word-of-mouth information in this model does not include a measure of uncertainty around the customers' opinions. This could potentially change the network dynamics which lead to such a high persistence of opinions within social groups.

Some experiments could also be carried out with different types of network and network sizes, to investigate if the current number of customer agents is sufficient to replicate the rate of information saturation and investigate how the word-of-mouth effects vary at different market scales.

Similarly, although the imperfect information model in chapter 3 does allow insurers to set a risk-based premium, the insurers do not account for competitive forces. In practice, an insurer who is winning a lot of business is likely to increase their prices, and an insurer who is failing to win business is likely to decrease its prices. In this case, the winner's curse becomes less obvious, and mostly manifests in differences in the market volumes each insurer is willing to seek at different premium levels. Future work should be done to combine the model in this chapter with a competitive premium mechanism such as those used in chapters 4 and 5 in order to better examine and quantify the resulting estimation bias and how this varies along with underwriting cycles.

As mentioned above, some common circumstances that increase the significance of imperfect information, include: high levels of customer heterogeneity, underlying risk distributions that change over time, and very rare event such as catastrophes. An exploration of threshold levels which might cause the winner's curse to become more significant could be a useful direction for future work with this model. This could be combined with market data to find a more accurate estimation of the likely impact on real-world markets.

The model used in chapter 4 makes the primary assumption that premium is set as a balance of supply and demand and insurers will all charge the same resulting premium. In practice, although insurance premium is heavily influenced by the rest of the market, there exists variation in individual insurer premium. Additionally, insurers become more risk averse when their capital adequacy is lower, which is not reflected in this model. Further work could be done to incorporate these features, for example by making use of the extended premium mechanism proposed in chapter 5.

In chapter 5, the new model was parametrised in order to allow for easy comparison with the results in Taylor's model. An investigation into parametrisation was carried out in which environmental parameters were based on market data. In order to set behavioural parameters, a particle swarm algorithm was carried out based on some market outputs such as loss ratio means, loss ratio volatilities, and premium autocorrelation values. However, this investigation was inconclusive as the algorithm was unable to find a reasonable set of parameters using this market model. Further work could be done to find appropriate environmental and dynamical parameter values, and to compare the results of a standard parametrisation exercise with different potential mechanisms. This work could be useful for establishing a validation framework for establishing a market framework which captures key features and can be used to bring together and investigate systemic effects.

Once a generalised market design and validation procedure has been established, this can be used to investigate some other systemic effects. For example: Aymanns and Farmer (2015) demonstrated how a VaR-based capital requirement in the finance market can exacerbate instead of mitigate market downturns when a shock causes investors to simultaneously pull out of certain companies. This is not unlike the capital constraints experienced in many insurance markets. This suggests that an equivalent ABM of the insurance-reinsurance market with a VaR based capital requirement might find a similar systemic effect. This could be used to examine regulation strategies that might limit systemic risk or act to smooth out market cycles.

Insurers may choose more tailored strategies based on their individual risk appetites. Rather than specifying the format the premium strategy should take, To reflect this and to allow for the construction of more tailored strategies, an ABM could be built with insurers that are able to tune the parameters of a neural network or other machine learning algorithm based on their goals of risk versus return.

Insurance practitioners do not include just new insurance business risk in their capital models. To investigate the systemic biases in these models, a model could be constructed based on a more standard capital model design. This would include reinsurance, reserve risk, catastrophe risk, and investment risk as well as the new business risk investigated in this thesis (Kravych, 2013). The differences could then be examined between an insurer's standard estimated capital model results and the results generated for the whole-market capital model. Note that such a model is likely to be impacted by both the imperfect information effects investigated in chapter 3 and also by the systemic risks associated with all insurers using the same sources for their catastrophe and economic scenario models as investigated by Heinrich et al. (2022).

This thesis has also not considered the impact of behavioural biases, which could potentially change the supply and demand dynamics. Further work could consider the impact of such biases. For example, Bertella et al. (2014) modelled the impact of overconfidence bias on investors in a financial ABM similar to the model used in chapter 4.

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