Three Studies in Credit Risk Modelling

by

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Table of Contents

List of Tables v
List of Figures vi
Acknowledgements vii
Declaration viii
Abstract ix

Chapter 1. General Introduction

Abstract of Chapter 1 1
1.1. Risk Management 2
1.2. The Importance of Credit Risk Modelling 3
1.3. Research Objectives and Contribution of the Thesis 5
1.4. Organisation of the Thesis 10

Chapter 2. A Survey of Credit Risk Modelling

Abstract of Chapter 2 11
2.1. Introduction 12
2.2. Credit Pricing Models 13
2.2.1. Structural Approach 14
2.2.1.1. Extensions to the Merton Model 17
2.2.1.2. Testing Structural Models 19
2.2.2. Reduced Form Approach 20
2.2.2.1. The Application of the Poisson Process 21
2.2.2.2. Modelling Default 22
2.2.2.3. Testing Reduced Form Models 24
2.3. Credit Risk Management Models 25
2.3.1. Traditional Approaches to Measure Credit Risk 26
2.3.2. Credit Portfolio Models 27
2.3.2.1. CreditMetrics 28
2.3.2.1.1. Advantages and Disadvantages of the Model 30
2.3.2.2. KMV Model 30
2.3.2.3. CreditRisk+ 32
2.3.2.3.1. Derivation of the Distribution of Losses 33

2.3.2.3.2. Advantages and Disadvantages of the Model.................... 34
2.3.3. Other Streams in Credit Risk Management.. 35
2.4. Conclusions and Further Research.. 36

Chapter 3. Can Structural Models explain Prices of Sovereign Bonds?

Abstract of Chapter 3.. **40**
3.1. Introduction 41
3.2. Literature Review 44
3.3. Data Description 45
3.4. Pricing Sovereign Bonds 46
3.4.1. A Pricing Formula for Brady Bonds 46
3.4.2. An Extended Structural Model:
The Cathcart and El-Jahel Approach (CEJ).. 48
3.4.3. Description of a Naïve Model with Constant Probability of Default (NM)..... 50
3.5. Implementation of the Models and the Kalman Filter 51
3.5.1. Estimation Results 55
3.5.2. Goodness-of-Fit and Model Comparisons In-Sample............................. 58
3.6. The Economic Interpretation of the Distance-to-Default 59
3.6.1. Performance of the Economic Proxy of the Distance-to-Default................ 63
3.7. Conclusions and Further Research... 64
Appendix A: The Estimation of the Risk-Free Term Structure 77

Chapter 4. What drives Credit Risk in Emerging Markets? The Role of Country Fundamentals and Market Co-movements

Abstract of Chapter 4.. **81**
4.1. Introduction 82
4.2. Literature Review 85
4.3. The Model 86
4.3.1. An Extended Structural Model of a Risky Zero Coupon Bond 87
4.3.2. The Pricing of a Par Brady Bond 90
4.4. Data 90
4.5. Implementation of the Model 92
4.5.1. Estimation Results and the Distance-to-Default 94
4.5.2. Descriptive Statistics of the Distance-to-Default 97
4.6. The Theoretical Determinants of the Distance-to-Default 98
4.6.1. Global Factors 98
4.6.2. Country-Specific Factors 100
4.7. Why do Credit Risk evolve together across Countries? The Empirical Results 102
4.8. Conclusions and Implications.. 106

Appendix A: Correlation Analysis of the Stock Market Returns.. 119

Chapter 5. A Systematic Comparison of two Approaches to Measuring Credit Risk: CreditMetrics versus CreditRisk+

Abstract of Chapter 5 .... **122**
5.1. Introduction 123
5.2. Description of the Models 125
5.2.1. CreditMetrics 125
5.2.2. CreditRisk+ 126
5.3. Literature Review on the Comparison of Models 127
5.4. A Common Framework for CreditMetrics and CreditRisk+ 130
5.4.1. Default Rates and Migration Rates 131
5.4.2. Conditional Distributions of Portfolio Default Rate and Portfolio Migration Rates 134
5.4.3. Aggregation 135
5.4.4. Consistent Parameterisation of the Models 136
5.5. Implementation of the Models 137
5.5.1. Generation of the Default Rate Distribution and Migration Rates Distributions 137
5.5.2. Generation of the Distribution of Losses 139
5.6. Analysis of the Differences in CVaR between the Models 142
5.6.1. Effect of Individual Factors that explain the Differences in CVaR............ 142
5.6.2. Global Effect of the Factors that explain the Differences in CVaR........... 145
5.7. Conclusions and Implications 147
Appendix A: Derivation of the Migration Rates for CreditMetrics 157
Appendix B: Monte Carlo Simulations 161
Appendix C: Differences of CVaR between CR+3 and CreditMetrics 165

References **166**

List of Tables

3.1. Summary Statistics of the Mexican Brady Par ... 68

3.2. Summary Statistics of US Government Bond Yields ... 68

3.3. Estimation Results for the Parameters of the CEJ Model ... 69

3.4. Analysis of the Standardised Residuals of the Models ... 69

3.5. Comparison of the Models ... 70

3.6. Economic Interpretation of the Distance-to-Default ... 71

3.7. Predictive Power of the Proxy of the Distance-to-Default ... 72

A.3.1. Estimation Results for the CIR Model ... 80

4.1. Characteristics of Brady Par Bonds ... 109

4.2. Summary Statistics of Monthly Brady Prices and their Returns ... 109

4.3. Estimation Results and Diagnostic Tests ... 110

4.4. Summary Statistics of the Distance-to-Default ... 111

4.5. Determinants of the Distance-to-Default ... 112

A.4.1. Correlations between Variations in the Distance-to-Default and Stock Market Returns ... 121

A.4.2. Principal Component Analysis of the Market Returns ... 121

5.1. Unconditional Transition Probabilities ... 151

5.2. Statistics of the Distributions of Default for CreditMetrics and CreditRisk+ ... 151

5.3. Statistics of the Loss Distributions for CreditMetrics and CreditRisk+ ... 151

5.4. The Effect of Migration Risk on CreditMetrics ... 152

5.5. The Effect of the Distributions of Default of CreditRisk+ and CreditMetrics ... 152

5.6. The Effect of the Exposure in CreditRisk+ ... 152

5.7. Differences of CVaR between CR+2 and CreditMetrics Low Quality Portfolio ... 153

5.8. Differences of CVaR between CR+2 and CreditMetrics High Quality Portfolio ... 154

List of Figures

2.1. Distribution of the Credit Quality of the Firm 29

3.1. Prices of the Mexican Brady Par 6.25 of 2019 73
3.2. Implicit Probabilities of Default ρ_t 73
3.3. Distance-to-Default implied by the Models 74
3.4. 95% Confidence Interval for the Distance-to-Default 74
3.5. Actual against Fitted Prices 75
3.6. One-step-ahead Residuals calculated for the CEJ Model 75
3.7. Forecasted Prices using a Proxy of the Distance-to-Default and the CEJ Model.. 76
3.8. Forecasted Pricing Errors of the CEJ Model 76

4.1. Market Prices of Brady Bonds 113
4.2. Observed Prices vs. Theoretical Prices 114
4.3. One-step-ahead Residuals 115
4.4. The Distance-to-Default implied by the CEJ Model 116
4.5. Credit Rating Index vs. the Distance-to-Default 117
4.6. Stability Tests 118

5.1. Consistent Parameters for CreditMetrics and CreditRisk+ 155
5.2. Default Rate Distributions for CreditMetrics 156
A.5.1. Distribution of the Credit Quality of the Firm 157
A.5.2. Effect of the Economic Factor on Migration Rates 160

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Declaration

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Abstract

This thesis presents three studies in Credit Risk Modelling. The first two studies are exploratory in nature. They investigate whether structural models can be used to price sovereign debt and obtain a creditworthiness variable for countries. In the first study, we test the ability of an extended structural model proposed by Cathcart and El-Jahel (2003) to capture the dynamics of the Mexican Brady Par. Using market prices and a Kalman Filter methodology, we estimate the model and obtain the distance-to-default, which is an implicit variable that drives the country's creditworthiness. The model is slightly superior to one which assumes that the distance-to-default follows a random walk. We find that approximately 80% of the distance-to-default can be explained by just a few economic factors. When this variable is approximated from these factors and substituted back into the models, the Cathcart and El-Jahel model still performs better than the naïve model both in sample and out-of sample, albeit only by a small margin.

The second study extends the above analysis and investigates the dynamics and co-movements of the distance-to-default across different countries, specifically: Argentina, Brazil, Mexico and Venezuela. We find that a few country fundamental variables, and external variables, including a variable that measures market sentiment, are able to explain up to approximately 80% of the distance-to-default of each country. Although country-specific factors are statistically significant in explaining the distance-to-default, external factors (such as the US stock market index, interest rates and market interdependence across countries) are much more important in explaining the dynamics of this variable.

The last study makes a comparison between two Credit Portfolio Risk Models: CreditMetrics versus CreditRisk+. The paper builds on work done by Koyluoglu and Hickman (1998), but we make a significant extension by assessing the impact of migration risk on credit-risk. We make a very careful comparison of Credit-Value-at-Risk for the two models using Monte Carlo techniques on standardised portfolios of bonds. The conclusion is that for regulators, which model is used matters very little. This is because regulators are concerned with extreme values, and loss distributions of both models capture information about defaults at very high confidence levels. However, for internal purposes, where rating migrations matter more than default, CreditMetrics can generate higher estimates of risk.

Chapter 1.

General Introduction

Abstract of Chapter 1

This chapter sits the context of the research by discussing briefly the importance of research on Credit Risk Modelling. We then present the objectives and the main contributions of the whole thesis.

1.1. Risk Management

The importance of Risk Management has become increasingly evident within the context of evolution and globalisation of financial markets. With more open and deregulated markets, new products and services have been developed. Competition among intermediaries has increased and more investors have access to markets. All these changes in the industry have generated new types of risks and stimulated the development of new ways to manage risks.

In recent history, economic and financial crises have cost billions of dollars to both governments and the private sector. Examples of such crises are the Great Depression in the early 30's, the crash of the stock markets in the 80's, the disintegration of the European Monetary System in the early 90's, the Asian crisis of 1997 and the Russian crisis of 1998. During these crises, investors experienced increasing volatility in economic and financial variables. They suffered dramatic losses due to adverse changes in exchange rates, stock and bond prices. One lesson of all these crises has been that market participants should be equipped with the proper tools for measuring and managing risks.

For a practitioner the concept of Risk Management should be understood as a process or set of activities whose main objective is to measure risks in order to monitor and control them (Bessis, 1998). The risk management process should be seen as an integral process that considers several activities:

1) The <u>identification of risks</u>. For example, a portfolio composed of corporate bonds can be exposed to changes in the risk-free interest rate or changes in the borrower's credit quality.
2) The <u>pricing of securities</u>, that takes account of the fact that their value is affected by changes in the identified risk factors.
3) The <u>measurement of risk</u> or quantification of losses due to changes in risk factors.

4) The administration of risks, which consists of monitoring and controlling risks that could potentially cause losses. For example, to reduce the interest rate risk in a portfolio, risk managers may decide either to hedge the portfolio by using short positions in bonds or derivatives, or simply to reduce their exposure to specific securities.

In this thesis we are particularly interested in pricing and measuring risks in securities subject to credit risk. Efficient risk management can give competitive advantages to the firm or bank, by improving its decision-making process and optimising capital allocation. This is because risk management can provide a global overview of potential losses and allow corrective measures to be taken in order to diminish them.

1.2. The Importance of Credit Risk Modelling

In the analysis of financial risks, we can conventionally identify three major groups: market risk, credit risk and liquidity risk. Most literature has been focused on the first two types of risks, with liquidity risk becoming a topic of more active research only recently. Traditionally, research on these types of risks has been carried out separately: although they seem to be very closely related, their joint modelling still represents a major academic challenge.

Credit risk has typically been defined as the risk that a borrower, issuer or counterparty might default on its promised obligations[1]. Consequently, many credit risk models consider default risk as the only credit risk factor that can cause losses. However, the definition of credit risk has long been a matter under discussion. A wider definition considers that credit risk is the risk that the security holder will not realise the expected value of the security owing to a deterioration in the credit quality of the borrower, issuer or counterparty (Caouette, et al., 1998). In this case, "deterioration"

[1] In the Risk Management literature, counterparties in a contract, loan borrowers or issuers of securities are named "obligors". Here, we will refer to them as borrowers, which is the most common term.

does not necessarily imply default, but any increase in the borrower's default probability. This type of credit risk is known as <u>migration risk</u> and the probabilities associated with changes from one credit quality to another are called transition probabilities or migration rates.

Credit risk is one of the most important quantifiable risks to which financial institutions are exposed. In practice, any transaction in the market involves credit risk. Any security not issued by the government has default risk[2]. Also, any transaction carried out in OTC markets, such as transactions with swaps or forwards, involves credit risk. Only in markets such as futures or options can credit risk be eliminated, through the intervention of agents called "Clearinghouses". These act as the investors' counterparties and have zero default risk.

Credit derivatives have been introduced in the market to reduce or transfer credit risks. This market is growing rapidly and the growth is expected to continue. These instruments have become very popular because they provide a unique and effective tool for hedging and controlling credit risk. Consequently, credit risk modelling and the pricing of not just credit derivatives but also other simpler securities such as corporate bonds and swaps, has become an area of intense research. Its increasing importance for both academics and practitioners may be attributed not only to the success of credit derivatives, but also to the volatility and complexity of the financial markets.

Financial Authorities have also boosted research on credit risk modelling. In 1988, The Basle Committee[3] agreed to establish similar supervisory regulations governing the capital adequacy of international banks. This put in place the first guidelines to determine regulatory capital in terms of banking credit exposures. Subsequently, "The 1996 Amendment" extended the rules to include market risks. The Basle regulations were widely criticised by the banking industry. One of the main criticisms was that

[2] In practice, many governments are considered non-default free.
[3] The Basle Committee is an entity formed by senior officials from Central Banks of 10 developed countries.

they did not account for diversification effects in the calculation of risk exposures. This situation motivated banks to look for effective ways of estimating risk exposures and calculating regulatory capitals.

In January 1998, "The 1996 Amendment" was formalised. Regulators realised that many banks had developed sophisticated risk management systems and were implementing more complex risk measurement models to estimate market risk. Regulators declared Value-at-Risk (VaR) to be the official methodology for assessing market risk exposures in banks. They also allowed banks to implement their own internal market VaR methodologies, subject to proper supervision.

In addition, authorities have put pressure on banks to measure credit risks and to keep risk exposures at an acceptable level. Consequently, banks have developed several tools for measuring and controlling credit risk. For example, in 1997, JP Morgan released CreditMetrics, the first publicly available model for measuring and managing portfolio default risk. Finally, the New Basle Accord released recently will allow banks to implement their own credit methodologies subject to supervisory review. This will encourage further research in the area.

1.3. Research Objectives and Contribution of the Thesis

This research has two main objectives: firstly, to investigate the application of structural models to price sovereign debt and analyse the determinants of the distance-to-default for countries; and secondly, to understand the differences between the performance of two credit portfolio models in measuring Value-at-Risk in portfolios subject to credit risk.

In order to fulfil these objectives, the core of the research consists of three studies. Each of them can be seen as an independent paper, although they are closely related.

The objectives of the first study are threefold. Firstly, we examine if prices using an extended structural model, suitably adapted, are consistent with market prices of the Mexican Brady Bond over a seven-year period. We use the model proposed by Cathcart and El-Jahel (2003), which incorporates both a hazard rate and conventional structural features. Secondly, we compare the in-sample performance of this model with a Naïve Structural Model that assumes that the probability of default in any future period is constant, conditional on no default having occurred yet. Thirdly, we explore the economic determinants of the distance-to-default of the country implied by the models and test their ability to fit market prices in-sample and forecast prices out-of-sample.

We believe that this chapter contributes to the current literature in several ways. Firstly, most of the literature on testing structural or reduced form models is focused on high-grade bonds with investment ratings. Also, very little work has been done on the use of structural models to price sovereign bonds. Here we make an experimental analysis to test the performance of a structural model to price non-investment grade sovereign debt. Secondly, as far as we are aware, the Cathcart and El-Jahel (2003) model has not been tested empirically before, either for companies or countries. One of the major problems in using structural models to price sovereign debt is the definition of both the solvency variable and the barrier beyond which default will occur. In this study, we estimate them as a "latent variable". This is our third contribution, using seven year of market bond prices and a Kalman Filter methodology we are able to extract a latent variable which drives the country's creditworthiness and can be interpreted as the distance-to-default. Finally, we are able to identify a set of economic and financial fundamentals that determine the dynamics of the distance-to-default. Therefore we show that structural models can help us to understand the determinants of credit risk for countries.

In this study we find that the Cathcart and El-Jahel model seems to fit the data slightly better than the Naïve Model, though the hazard rate feature in the model makes no contribution to explaining the bond prices. The distance-to-default is closely related to a set of economic variables, including the Mexican stock-market level, exchange rate and the level and slope of the risk-free term structure. We also find that an increase in interest rates causes spreads to fall, consistent with the literature on corporate bonds. Driving the model forward with these economic variables, the CEJ model performs slightly better out-of-sample than the Naïve Model. This suggests that structural models can explain prices (and spreads) for sovereign bonds, although it remains to be seen whether the same economic variables as in Mexico determine the distance-to-default in other countries.

The previous analysis is extended in the second study of the thesis. Here we explore the determinants of the distance-to-default for a set of emerging countries, explicitly: Argentina, Brazil, Mexico and Venezuela. In particular, our objective is to investigate the extent to which variations in the distance-to-default can be attributed to changes in common factors across countries, leading to contagion effects. Other studies have analysed contagion using credit spreads or bond returns. Here we explore credit co-movements using the distance-to-default of countries. We are able to explain up to approximately 80% of the variance of the distance-to-default of each country, using a small set of country fundamental variables and external factors, including a variable that measures market sentiment. We find that whereas country specific factors are statistically significant in explaining the distance-to-default, global factors (such as the US stock market, interest rates and market interdependence across countries) are much more important in explaining the dynamics of this variable.

The analysis of the joint behaviour of sovereign credit risk is vital for pricing and portfolio valuation. On the one hand we find that systematic factors play a very important role in determining the distance-to-default of these countries. Existing pricing models, specifically structural models, predict that firm-specific or country-specific factors should play more important role than systematic factors. Therefore our results seem to highlight the importance of further research on pricing models that acknowledges this empirical fact. On the other hand, the fact that credit risks in bond markets have an important non-diversifiable component has important implications for the risk management of portfolios and the regulation of financial institutions.

In the above two studies, we have obtained and investigated the content of a measure of creditworthiness implicit in the market prices of sovereign bonds, the distance-to-default. This approach has been discussed by Claessens and Penacchi (1996) and Cumby and Evans (1995), Anderson and Renault (1999). They treated creditworthiness as an unobservable variable that follows a specific process. In the context of structural models this variable can be considered as a function of the firm's asset value. In the case of a country, this can be any set of economic and financial variables driving its credit behaviour. There are two main advantages of trying to identify the content of the distance-to-default as a measure of creditworthiness over trying to explain credit ratings or credit spreads. Firstly, such a measure can be seen as a credit rating index in continuous time. Secondly, there is a potential application of such a measure within the context of structural models. Following Hull and White (2001) and Avellaneda (2001), default correlation can be modelled by correlating the distances-to-default for different companies or countries. Furthermore, knowing the components of the distance-to-default may help to estimate the degree of such correlations. This is relevant for portfolio credit risk and pricing of credit derivatives.

The objective of the third and final study is to compare two credit portfolio models. We investigate the differences between two popular credit risk management models: JP Morgan's "CreditMetrics" versus "CreditRisk+" of Credit Suisse Financial Products. At first sight, the models appear very different, as they rely on different definitions of credit risk. CreditRisk+ assumes that only borrowers' default can cause losses, whereas CreditMetrics includes any deterioration in credit ratings of borrowers. Other studies have compared both models by focusing on the risk of default, whereas our comparison incorporates both the risk of default and the risk which arises from changes in credit ratings.

In this study we build on work done by Koyluoglu and Hickman (1998) and make a systematic comparison of Credit-Value-at-Risk (CVaR) for the two models. We set up a common mathematical framework to compare the loss distributions and use Monte Carlo techniques to implement both models in two simulated bond portfolios. One with high-credit quality and the other with low-credit quality. We then examine the sensitivity of CVaR to changes in parameters. The analysis is restricted to a one-year time horizon.

This chapter contributes to the existing literature in two respects. Firstly, we explore the similarities and differences between two classical models in the literature, which are based on different definitions of credit risk. In other words, we compare a Default Model vs a Credit Rating Model. Secondly, we investigate the implications on risk management and capital adequacy requirements when these models are used to calculate CVaR. In particular, we assess the impact of migration risk on CVaR and identify portfolios for which migration risk is relevant, in order to determine the differences of CVaR between the models.

We find that for both types of portfolio (low- and high-quality) most of the differences in CVaR between the models are due to the underlying assumptions of the distribution of default. However, for high-quality portfolios and low confidence intervals of CVaR, the omission of migration risk is also significant in explaining the differences between the models. These results have important implications for the calculation of capital requirements, since choosing between CreditMetrics and CreditRisk+ seems to be irrelevant. At the extreme tails of the loss distribution, information about default is captured by either of the two models. However, for internal purposes, such as the estimation of reserves, where rating migrations matter more than default, CreditMetrics may be a better approach.

We believe that the results of all three papers will give a better understanding of the performance of some credit pricing models and portfolio models. Furthermore, we believe that the results could have important implications for pricing, hedging and risk management.

1.4. Organisation of the Thesis

The rest of the thesis is organised as follows: The second chapter presents a general overview of the theoretical and empirical literature on Credit Risk Modelling. The more specific literature related to each of the three studies considered in the thesis will be discussed separately in the following chapters. The third chapter investigates whether structural models can price sovereign debt, and the determinants of the distance-to-default for countries. The fourth chapter studies credit co-movements across a sample of countries. The fifth chapter presents a comparison between two credit portfolio models: CreditMetrics versus CreditRisk+. Conclusions and further research regarding each study are also discussed separately in each of the above chapters.

Chapter 2.

A Survey on Credit Risk Modelling

Abstract of Chapter 2

This chapter presents an overview of the literature on Credit Risk Modelling. Literature which is specific to each of the three studies in chapter 3 to 5 will be covered in greater detail there. Research in this area has been developed in two broad streams: Pricing and Risk Management.

The first stream is purely focused on the pricing of individual securities subject to default risk. Within this stream models are classified into two major types of approaches: the Structural Approach and the Reduced Form Approach. Here, we summarise both theoretical and empirical research in this area.

The second stream of research is focused on risk management and its main objective is to determine the potential losses of a portfolio subject to changes in credit risk factors. These models are known as Portfolio Models and are based on a Value-at-Risk framework.

We conclude this survey with a brief discussion of directions for future research.

2.1. Introduction

In the last decade, credit risk modelling has become one of the major research fields in financial economics. Research in this area has been developed across two separate streams. The first stream is focused on credit pricing models or the pricing of individual securities subject to default risk. The second stream is focused on credit risk management models. Research in this stream tries to answer questions such as: What is the default probability of the borrower? What is the probability that the borrower will default in a specific period? How much is a portfolio "likely" to loose when credit risk factors move adversely? The importance of these models lies in their application to the calculation of hedging, assessing potential losses in a portfolio and quantifying capital adequacy requirements.

In the first stream, Credit Pricing Models have been developed along two major lines: the Structural Approach and the Reduced Form Approach. Under Structural Models there is a latent variable which captures the firm's fundamentals and consequently drives the default behaviour of the firm. This approach has its origins in Merton(1974)'s corporate debt pricing model. He introduced a contingent-claim approach to valuing corporate debt using option pricing theory. His analysis has been extended in several ways, by relaxing assumptions or adding more features to his simple framework. The Reduced Form Approach assumes that default is driven by an exogenous variable and occurs as a surprise. Though initially models from this approach lack any economic link with the firm's fundamentals, they offer the advantage of being mathematically more tractable and less restrictive than Structural Models. From the theoretical point of view, several models have been developed to price corporate debt. However, from the empirical side, little work has been done on fitting, testing and comparing such models. Among the reasons for this are the complexity of the models and the lack of sufficient data to estimate the parameters.

Within the second stream of research, Credit Risk Management Models, we will briefly discuss an "old generation" of models which are mainly focused on the qualitative assessment of credit risk. Saunders (1999) calls them Traditional Models. These models use qualitative and statistical techniques to determine the credit quality of borrowers or their probability of default. A popular model of this generation is the Z-Score of Altman et al. (1977). A new generation of models comprises Portfolio Models, which can be seen as an application of pricing models, which are focused on the quantification of risk in credit portfolios. They are based on the concept of Value-at-Risk (VaR). Their objective is to estimate the potential losses in a portfolio due to credit events, such as default or rating changes. A key feature of these models is the way default correlation is modelled, so the portfolio's distribution of losses can be generated.

This chapter is organised as follows. Section 2.2 discusses the literature related to credit pricing models. The Structural Approach and Reduced Form Approach are discussed. Section 2.3 presents the risk management framework to estimate credit risk. Traditional Models and the Portfolio Models are reviewed briefly. Section 2.4 presents the conclusions and further research.

2.2. Credit Pricing Models

For pricing risky debt, three variables are central in the construction of models: 1) the probability that the issuer will default, 2) the risk-free interest rate, and 3) the recovery rate. In general, each of these variables can be modelled as a deterministic variable or as a diffusion process.

The main focus of the models is the determination of the probability of default and the conditions under which it occurs. In this sense, two different approaches have been developed: The Structural Approach and the Reduced Form Approach. Regarding the risk-free interest rate, this can be assumed deterministic or stochastic. The recovery rate is usually considered deterministic and is modelled as a fraction of the debt's face value or of the market value of a default-free security. This variable can be the most difficult to model due to lack of theoretical and empirical research. For example, under financial distress, the firm can avoid bankruptcy by implementing a financial restructuring. In addition, bonds issued by the same or similar borrowers can have a different recovery rate. Empirical literature on this issue is still sparse: see for example Altman and Kishore (1998), and Carty and Lieberman (1996).

2.2.1. Structural Approach

This approach has its origin in the option pricing theory of Black and Scholes (1973). Merton (1974) uses that theory to value a risky bond. The key point of those papers is that equity and debt can be seen as derivatives on the firm's asset value. The firm's asset value is the variable driving all the dynamics of securities prices issued by the firm, including default.

In Merton's simple framework, default can be triggered only at maturity and when the firm's assets fall below the value of the debt. Let V(t) be the value of the firm's assets at time t. The debt of the firm consists of a single-zero coupon bond that pays an amount B at maturity. The dynamics of the firm value V depends on its growth rate, volatility and future payouts. Specifically the value of the firm follows the process:

$$dV = \mu V dt + \sigma V dz \qquad (2.1)$$

where:

μ denotes the instantaneous expected rate of return of the firm's assets per unit time and σ^2 is the instantaneous variance of the return on the firm's assets and dz is a Wiener process.

It is assumed that, at maturity, the value of the debt is the minimum between the face value of the debt B and the market value of the firm. If the assets' value of the firm is larger than its liabilities, then bondholders get back the face value of the debt. Otherwise the firm is unable to meet its liabilities and the bondholders take over the firm. Hence, the payoff to debt-holders can be written as:

$$\begin{aligned} D(T) &= \min[B, V(t)] \\ &= B - \max[0, B - V(t)] \qquad (2.2) \end{aligned}$$

According to the second equation, the value of the firm's debt equals the face value of the debt less a European put option on the value of the firm's assets, with exercise price equal to the face value of the debt.

Assuming no arbitrage opportunities, Merton gets the following partial differential equation for any security F whose market value depends on the value of the firm and time:

$$0 = \frac{1}{2}\sigma^2 V^2 \frac{\partial^2 F}{\partial V^2} + rV\frac{\partial F}{\partial V} + \frac{\partial F}{\partial t} - rF \qquad (2.3)$$

With the appropriate boundary conditions[1], Merton shows that the value of the firm's debt at time t is:

$$E_t\left[e^{-r(T-t)} \min(V_t, B)\right] = Be^{-r(T-t)} - P(t, V_t, B) \qquad (2.4)$$

where $P(t, V_t, B)$ is the value at time t of a put option on the firm's assets and strike price B, and r is the risk free rate. Equation (2.4) can be solved by applying the Black-Scholes formula in the calculation of the put price. Thus the value of the debt at time t is expressed as follows:

$$d(t, V_t, B) = Be^{-r(T-t)}\Phi(-d_2) - V_t\Phi(-d_1) \qquad (2.5)$$

where

$d_1 = \dfrac{\log(V_t / B) + (r + \sigma^2/2)(T-t)}{\sigma\sqrt{T-t}}$, $d_2 = d_1 - \sigma\sqrt{T-t}$ and Φ is the cumulative standard normal distribution.

Using the identity that the debt value of the firm equals the market value of the assets less the market value of equity, Merton also derives the equity value of the firm as a call option on the value of the firm with strike price equal to the face value B. This is one of the main features of Merton's model, i.e., debt and equity are modelled within a consistent framework.

One direct application of a pricing model is the derivation of the term structure for credit spreads. Assuming that the risk-free rate is static, from equation (2.5) the spread on corporate debt is:

$$R(T-t) - r = -\frac{1}{T-t}\log\left[\Phi(-d_2) - \frac{1}{d}\Phi(-d_1)\right] \qquad (2.6)$$

where R(T-t) is the yield to maturity on the risky debt provided that the firm does not default; and $d = Be^{-rT}/V$ is interpreted as the leverage of the firm. This function is a decreasing function of maturity for borrowers who are largely leveraged. For low-quality borrowers, credit spreads decrease with maturity. This is because the value of bonds issued by the firm already have low values, capped by zero; therefore, in the future, only improvements in the credit quality of the firm can be expected, which would reduce the size of the credit spreads. For borrowers with low leverage, the function is humped or upward-sloping.

Though this approach seems very intuitive, it imposes important restrictions: 1) Firms can only issue one type of debt. 2) Firms can default only at the maturity of their debt. In reality, firms usually have more than one class of debt in their capital structure and they can default on any of the intermediate payments. 3) The absolute-priority rule holds; however in practice this is usually violated[2]. 4) Securities trading takes place continuously; however, most firms´ debts are thinly traded or not traded at all. 5) The

[1] The boundary conditions can be written as follows: a) $F(0,\tau) = 0$, i.e., the debt value can only take non-negative values; b) $F(V,\tau) \leq V$ and c) the initial condition for the debt at $\tau = 0$ is $F(V,0) = \min(V,B)$.

[2] That the absolute-priority rule holds means that in case of default, assets are allocated among claimants according to the seniority of the bonds. However, empirical evidence shows that this situation is rarely held in distressed firms. Financial distress is costly, so lenders and borrowers usually try to obtain some agreement to reduce the cost of bankruptcies. Up to now, very few theories have tried to assess the impact of this on risky bond prices or on credit spreads. See Franks and Torous (1994).

risk-free term structure is static and flat. 6) Finally, though Merton claims that the final pricing formula is a function of observable variables, in reality the estimation of the firm's value (and in particular its volatility) are difficult to estimate.

2.2.1.1. Extensions to the Merton Model

Merton's approach is interesting because its structure based on the fundamentals of the firm is very suitable for pricing securities that maintain a close relation with the value of the firm, for example callable bonds or convertible bonds. Also, it allows the analysis of important questions such as optimal capital structure.

Other models have tried to improve Merton's model by making more realistic assumptions. For example, Geske (1977) makes a more realistic treatment of coupons by using compound options. Equity holders must decide whether to pay the coupon or default. Black and Cox (1976) introduce an absorbing barrier to allow for default occurring before maturity. Thus the problem is solved by using a first-passage-time approach, and determining the first time the firm's asset value crosses a default barrier.

Other models have relaxed the assumption that the risk-free interest rate is deterministic. Shimko, Tejima, and Deventer (1993) present a two-factor model where the interest rate follows a Vasicek's process (see Vasicek, 1977). However the correlation between both processes (the asset value process and the interest rate) is assumed zero. This reduces to a great extent the complexity of the problem, though it may be unrealistic. Longstaff and Schwartz (1995) also propose a two-factor model that results in a semi-closed form solution[3]. They assume a non-zero correlation between the asset value and the interest rate, which also follows a Vasicek's process. Here default can occur at any time when the firm value crosses a constant boundary

[3] Rogers (2000) points out that the derivation of this solution contains a flaw. He argues that Longstaff and Schwartz use the theory of first-passage distributions of diffusions processes to solve a process

K. Kim, Ramaswamy and Sundaresan (1993) assume a Cox, Ingersoll and Ross (1985) process for the interest rate and cash flows for the driving variable of default. Under this context, default occurs when cash flows are unable to cover coupons and dividends. Other more complicated models that assume that default can occur prior to maturity are Ericsson and Reneby (1995), Brys and de Varenne (1997), Collin-Dufresne and Goldstein (2001a). Unfortunately these models are even more difficult to implement since they increase the number of parameters to be estimated.

Leland (1994), Leland and Toft (1996), Anderson and Sundaresan (1996) and Mella-Barral and Perraudin (1997) develop another type of model characterised by endogenous capital structure and default barrier. These models determine the optimal asset value at which a firm should declare itself bankrupt. Leland and Toft (1996) conclude that debt structure is a trade-off between tax advantages, bankruptcy costs and agency costs. Though they relax significantly Merton's original framework, they add some other assumptions which are questionable. For example, they implicitly assume that the firm can issue new debt all the time at the same cost, which is not very realistic. In addition, interest rates are kept deterministic.

Structural models have also been criticised for their inability to explain the observed term structure of credit spreads. In particular, they produce very low spreads that go to zero as maturity approaches. Intuitively, this is because the firm's value process follows a diffusion process, which is a "smooth" continuous path that cannot jump. If the value process of the firm is far away from the boundary condition and maturity is close to zero, then it is very unlikely that the firm will suddenly jump to reach the boundary. This phenomenon is observed in all the models based on a first hitting-time diffusion process, in which default can never occur by surprise. In reality, investors think that a sudden default still can occur even when the debt is close to maturing. This happens for example in unexpected devaluations, or sudden catastrophes. Therefore, empirically credit spreads do not decrease to zero as maturity decreases to

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which is not a diffusion. Therefore, the model is still unsolved, though it is considered one of the pillars zero. Zhou (1997) and Schöunbucher (1996) present one solution to this problem by allowing a jump process that creates discontinuities in the firm value process. Another solution is that the models ignore liquidity risk, which should also be estimated (see Ericsson and Renault, 2001).

2.2.1.2. Testing Structural Models

There is a sparse literature on comparing structural models and on their performance in predicting prices or spreads. Jones, Mason and Rosenfeld (1984) apply Merton's model to a sample of firms with simple capital structures and their bond prices during the 1977-1981 period. The paper concludes that Merton's model produces credit spreads significantly lower than actual credit spreads. Jones et al. also find little evidence that Merton's model performs better than a naïve model (which does not consider default) for non-investment bonds. Sarig and Warga (1989) confirm that the empirical shape of the term structures is consistent with that implied by Merton (1974). However, their analysis is simple and does not use rigorous statistical techniques, so their conclusions are not sufficiently strong.

Wei and Wong (1997) compare Merton (1974) with Longstaff and Schwartz (1995) and conclude that the first outperforms the latter. Unfortunately their results are restricted to a small sample (covering only the Eurodollar market in 1992). Anderson and Sundaresan (2000) also implement Merton's model and three strategic default models (Leland (1994), Anderson and Sundaresan (1996) and Mella-Barral and Perraudin (1997)), using aggregated data on corporate bonds. They conclude that models fit reasonably well and default probabilities are consistent with the historical record of Moody's.

More recently Eom, Helwege and Huang (2002) test five structural models: Merton (1974), Geske (1977), Leland and Toft (1996), Longstaff and Schwartz (1995), and

of the Structural Approach.

Collin-Dufresne and Goldstein (2001b), and conclude that contrary to previous research, structural models do not systematically underpredict credit spreads. This depends on the model and the level of riskiness of the bond. Nevertheless, models face problems in trying to predict accurately credit spreads on corporate bonds.

An additional drawback of structural models is that they cannot cope with the pricing of certain Credit Derivatives, as they do not model changes in credit ratings. Adding more complicated features makes models very difficult to solve. In contrast, the Reduced Form framework allows for more complex structures at low mathematical cost.

2.2.2. Reduced Form Approach

In this approach, default is driven by an exogenous variable and not by a firm's fundamentals, so default is more unpredictable than in structural models. These models eliminate the need for any economic variable explaining default. This is their main weakness, but also their main strength since this assumption allows the modelling of more complex features without increasing the mathematical costs. For example, liquidity and the recovery rate are modelled by adding up further diffusion processes, without having to look for a structural interpretation. However, since the fitting of these models relies on the quality of data on credit spreads, parameters are likely to be unstable but still useful for analysing securities for short periods of time. Nevertheless, they are more likely to fit a particular data set better than structural models and price more complex instruments.

The event of default or (default time) is usually modelled as a Poisson process with parameter λ, known as the intensity or hazard rate. This represents the probability that the firm will default over a small time interval. It can be constant, time dependent or driven by exogenous or endogenous variables, such as interest rates. These types of models, only concerned with the modelling of the default time, are referred to as the Intensity-Based Models. Models within this framework include those of Jarrow and

Turnbull (1995), Duffee (1999), Lando (1998), Madan and Unal (1998). The default process can also be modelled as a Markov chain, which incorporates the credit ratings to default timing. This class of models are called Credit-Migration Models. Some such models are Das and Tufano (1996), Jarrow, Lando and Turnbull (1997), Bielecki and Rutkowski (2000).

In contrast with structural models, reduced form models assume that the initial risk-free term structure and the credit spreads term structure are known. Risk-free prices are taken as inputs of the models, whereas credit spreads are used to calibrate the models.

2.2.2.1 The Application of the Poisson Process

Schönbucher (2000) illustrates how to incorporate a Poisson process to price default. Consider a simple zero-coupon bond B, with zero recovery rate and default triggered by the first jump of a Poisson process N with intensity λ. Extending Itô's lemma to include a jump process, we get the following representation of a defaultable bond B.

$$dB(t,r,N) = \frac{\partial B}{\partial t}dt + \frac{1}{2}\frac{\partial^2 B}{\partial r^2}dt + \frac{\partial B}{\partial r}dr - BdN \qquad (2.7)$$

A jump dN=1 in equation (2.7) would mean that $dB = -B$ (the other terms are of lower order), and therefore the bond price would jump to zero. Under the absence of arbitrage and using $E[dN] = \lambda dt$, the pricing equation becomes:

$$0 = \frac{\partial B}{\partial t} + \frac{1}{2}\frac{\partial^2 B}{\partial r^2} + \frac{\partial B}{\partial r}\mu_r - B(\lambda + r) \qquad (2.8)$$

This equation resembles the typical pricing equation for a risk-free security. The only difference is the addition of λ in the final discounting term. Therefore, the price of a zero-coupon bond that matures at time T is:

$$B(t,r) = P(t,r)e^{-\lambda(T-t)} \qquad (2.9)$$

where P(t,r) is the price of a risk-free bond. This is one of the main attractions of this approach, that the price of a risky bond can be calculated using the market risk-free term structure.

Finally, from the above equation, the credit spread is easily derived:

$$R - r = \frac{1}{T - t}(\ln P - \ln B) = \lambda \qquad (2.10)$$

For this simple specification, the credit spread is just equal to the constant intensity parameter λ. Most extensions to reduced-form models focus on different characterisations of this hazard rate. While a constant intensity parameter implies that default is a Poisson arrival process, and makes the model easier to estimate, this is an unrealistic assumption. Firms are likely to change their default intensities depending on the time horizon being considered. Models also make different assumptions about the recovery rate and the risk-free rate.

2.2.2.2 Modelling Default

Jarrow and Turnbull (1995) produce one of the most representative models of the Reduced Form Approach, where the hazard rate is constant. Jarrow, Lando and Turnbull (1997), make a notable extension by incorporating credit ratings. Default time follows a Markov process and it occurs as the first time the Markov chain hits the absorbing state. For instance, if $B(t,T)$ denotes the value of a risky zero-coupon bond of a firm that currently has credit rating "i" (for example, AAA) at time t, then the price of the bond can be expressed as follows.

$$B(t,T) = P(t,T) - P(t,T)\left[(1-\delta)\lambda_i(t,T)\right] \qquad (2.11)$$

where $\lambda_i(t,T)$ is the probability that a bond with current credit rating "i" defaults and δ is the recovery rate. The above formula indicates that the lower the probability of default $\lambda_i(t,T)$, the higher the price of the bond and in consequence the lower the credit spread will be.

Though the model seems realistic, its disadvantage is in its estimation. The Markov chain increases dramatically the number of parameters to estimate. Jarrow et al. suggest the use of historical transition matrices issued by rating agencies. However, the reliability of transition matrices is still an issue. Nickell, Perrauding and Varotto (2000) find that transition matrices should be differentiated by sector and business cycle. In addition, Altman and Kao (1992) find that actual rating transitions are likely to be non-stationary; however, Jarrow et al. assume that they are Markovian.

Duffie and Singleton (1999) assume that the intensity parameter λ depends on the level of interest rates across time. Given that upon default the borrower recovers a fraction δ of the value of the bond, they find that the discounted rate of a risky bond (R) can be decomposed into two components: default and recovery.

$$R = r + \lambda(1-\delta) \qquad (2.12)$$

where the interest rate process r, and the default process λ are determined by an square root stochastic process. Thus the prices of a risky bond can be seen as the same as for a risk-free bond, except that the discounted rate r is adjusted by the expected loss rate:

$$B(t,T) = E_t\left(\exp\left\{ -\int_t^T \left[r_u + \lambda^*{}_u(1-\delta) \right] du \right\} \right) \qquad (2.13)$$

where λ_t^* is the intensity rate under the equivalent martingale measure.

Duffie and Lando (1999) try to integrate the intensity-based approach into the structural approach. They assume that the assets of the firm cannot be observed directly by outsiders, but they may partially solve this information gap by looking at the accounting information releases. This leads to jumps in asset values since they have to adjust their expectations.

In summary, this approach has three major advantages: a) The mathematics of the models is more tractable, so various functional forms can be implemented. Most variables can be assumed stochastic and the model still gets closed form solutions. b) Models use only observable variables as inputs, so risky prices can be calculated by using risk-free prices. c) They can price a wider variety of assets such as credit derivatives.

The most important criticisms to this approach are: 1) The default event does not keep any relation with the fundamentals of the firm. Therefore, it is not possible to interpret the default event in terms of the structure or variables of the firm. 2) The assumption that risk-free assets and debt assets are widely traded is not realistic. 3) Parameters estimated in these models are very unstable.

2.2.2.3 Testing Reduced Form Models

There is little evidence about the performance of reduced form models to explain prices and the evolution of credit spreads. The performance of these models depends to a large extent on the accuracy of the data about credit risk, recovery rates and liquidity. Duffie and Singleton (1997) test their model using swap yields. The discount rate is driven by two square-root diffusions, one representing credit risk and the other liquidity risk. Their model fits the swap rates reasonably well, apart from the short-end of the term structure.

Dufee (1999) finds that models based on Duffie and Singleton (1997) show instability in the parameters. The parameters of the model change dramatically when it is calibrated with firms which have different credit ratings. This suggests that a credit pricing model, such as that of Jarrow, Lando and Turnbull (1997), may be a better choice.

Monkkonen (1998) compares six variations of reduced form models, using Jarrow and Turnbull (1995) as the benchmark model. The alternative models either allow the default probability to depend on the default-free spot rate or the recovery rate to be stochastic or both. Results are very similar across short-maturity bonds but discrepancies are dramatic for long-maturity bonds. He also find that for investment-grade bonds, the results are not sensitive to the specification of the relationship between default, recovery rate and the risk-free rate.

2.3. Credit Risk Management Models

The aim of models for credit risk management is to quantify credit risk in individual securities or portfolios. Several measures to quantify credit risk have been developed across time. For a long time credit ratings were the only measure to characterise credit risk. Later, numerical scores and more sophisticated statistical models were developed. Saunders (1999) distinguishes two generations of models: "Traditional Models" and a "New Generation of Models". The first type of model is characterised by three features:1) the use of qualitative analysis and very few statistical methods rather than elaborated quantitative methods; 2) models are focused on measuring the credit risk of individual securities rather than portfolios; and 3) final outputs are simple risk measures, such as credit ratings or credit scores. Within this category we find Expert Systems, Rating Systems, and Credit-Scoring Systems.

The "New Generation of Models" started in the mid 90´s. These models extend the concept of Value-at-Risk (VaR), widely used to quantify market risks, to the quantification of credit risk or Credit-Value-at-Risk. These models are focused on the estimation of the loss distribution and the modelling of its parameters. Modelling the loss distribution is particularly difficult, since changes in the value of the portfolio due to credit events such as defaults are not normally-distributed. Also, the estimation of some parameters, such as default correlations, represents important complications to be overcome.

2.3.1 Traditional Approaches to Measure Credit Risk

For a long time, banking experts determined the credit risk in individual securities using a subjective analysis. This method of assessing credit risk is called Expert Systems. Expert analysts or officers used to base their decisions on the analysis of factors such as: a) the reputation of the borrower and its repayment history; b) the financial situation of the firm, or its capital structure; c) the capacity to repay or the volatility of their earnings; d) the value of the collateral; and e) the economic conditions, such as interest rates levels or business cycles. The disadvantages of this method are the lack of analysis about the common factors affecting borrowers in a portfolio and the subjectivity of weighing and combining all the factors to produce the final decision.

At the end of the 70´s, statistical tools such as discriminant analysis and logit and probit models that estimate the probability of borrowers' default were commonly used. Altman (1968) develops a Credit Scoring System, called "Z-Score". The model uses a firm's accounting ratios and discriminant analysis to identify which ratios affect the performance of the credit. Some of the disadvantages of this model are that the relationship between performance and accounting ratios is not necessarily linear; and the eligible variables are in general only accounting ratios.

Rating Systems were developed later to provide a more formal procedure to take decisions. Loans are classified into different credit categories, implicitly associated with a default probability. The classification combines a subjective valuation of the quality of the borrower with a quantitative score.

Croates and Fant (1993) implement Neural Networks and look for more complex relationships among variables. Unfortunately the interpretation of these relationships may be very difficult and models may suffer by being overfitted.

2.3.2. Credit Portfolio Models

Traditional approaches are often criticised for their individual assessment of credit risk and the use of accounting data rather than market variables. Modern research on risk management has been focused on the design of models that aggregate individual exposures in a portfolio. The ideas in pricing models developed above have been used by several risk management models and extended to include the concept of Credit-Value-at-Risk (CVaR).

CVaR is defined as the maximum loss in a portfolio, due to credit events, within a known confidence interval over a specific period (Wilson, 1997a). It is also interpreted as the additional economic capital that is needed to cover the unexpected credit losses of a portfolio subject to credit risk.

In order to compute CVaR, a "time horizon" and the "level of confidence" need to be specified apart from the parameters involved in the model itself. In general, risk managers set up confidence levels of 95% or 99% and time horizons of one-year. The long time horizon is because changes in credit risk factors do not occur as often as changes in market risk factors and because it is thought that one year is enough time to take precautionary measures to reduce the credit risk of the portfolio.

Models usually work in a two-step framework to calculate CVaR. In the first step, models estimate the credit exposure of individual securities in the portfolio. In the second step, they estimate the distribution of losses generated by the whole portfolio at the end of a time horizon. To do this, models estimate the correlation of default between borrowers in order to put together the individual losses of each security in the portfolio. Once the loss distribution is generated, CVaR is calculated as the difference between the unexpected losses and the expected losses of the portfolio, where unexpected losses represent a high-quantile in the distribution (usually 95% or 99%).

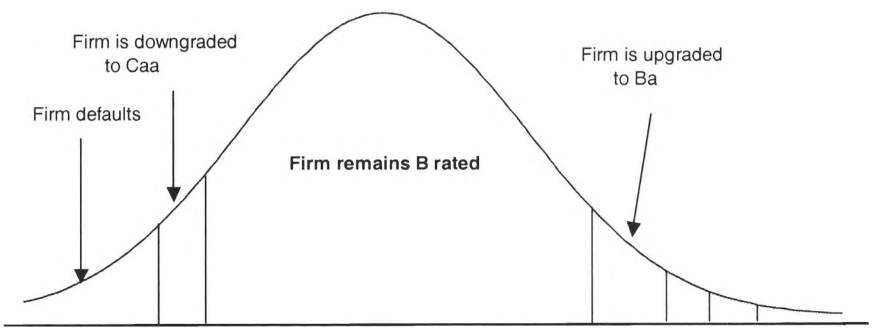
Within this generation of portfolio models, the financial industry has sponsored three important models: CreditMetrics by JP Morgan (1997), Creditor Monitor Model by KMV Corporation and CreditRisk+ by Credit Suisse Financial Services (1997).

2.3.2.1. CreditMetrics

The fundamental idea of CreditMetrics (JP Morgan, 1997) is an application of the Structural Approach proposed by Merton (1974). This view is extended in CreditMetrics by assuming that a firm's asset value determines not only default but also its credit quality. The basic assumptions of the model are:

1. Transition probabilities are stationary and follow a Markovian chain over time.
2. The risk-free term structure and the term structure for each rating class are static and known at the beginning of the time horizon, so we can use forward rates to discount cash flows.
3. All bond issuers are credit-homogenous within the same rating class. Therefore they have the same statistical properties and share the same transition probabilities and credit spreads.
4. The recovery rate is known and deterministic[4].

The value of the assets is transformed into returns, which are assumed normally distributed. Asset's returns are compared against thresholds, mapped from transition probabilities, in the normal distribution to obtain the firm's credit quality at the end of the period. *Figure 2.1* shows the distribution of a firm's final rating, which has been rated in class B at the beginning of the period. For example, the firm has a probability of 9.82% (=2.76+7.06) of suffering deterioration in its credit quality at the end of the time horizon.



Class	Def	Caa	B	Ba	Baa	A
Transition Prob	7.06	2.76	82.7	6.59	0.65	0.17
Thresholds	-1.5	-1.3	1.4	2.4	2.8	3.3

Figure 2.1. Distribution of the Credit Quality of the Firm

The price of the bond is calculated using forward rates to discount the remaining cash flows to time T (the end of the time horizon). Observe that because interest rates and credit spreads are assumed deterministic, no market risk factors are affecting the value of the bond.

Individual securities in the portfolio are aggregated by assuming that firms are correlated through their asset returns. For example, the probability that two borrowers are assigned a specific rating at the end of the time horizon can be calculated as:

$$\Pr ob\left\{Z^i_{j-1} < R < Z^i_j, Z^k_{m-1} < R' < Z^k_m\right\} = \int_{Z(i,j-1)}^{Z(i,j)} \int_{Z(k,m-1)}^{Z(k,m)} \Phi(r, r'/\Sigma)\, dr\, dr' \qquad (2.14)$$

where R and R' are the firms' returns; Z^i_j is the j-th threshold in the normal distribution for a firm which has been rated class i at the beginning of the period; r and r′ are the standardised asset returns, Φ is the density function for a bivariate normal distribution with covariance matrix Σ equal to:

$$\Sigma = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}$$

where ρ represents the correlation coefficient between the assets of the firm.

[4] CreditMetrics has extended the approach to include stochastic recovery rates.

value of the total liabilities and the face value of short-term liabilities. The latter need to be fully financed, otherwise the firm will not be able to operate, whereas the financing for the long-term liabilities may temporarily remain negative.

KMV uses Merton's model as a setting to derive a creditworthiness variable called the Distance-to-Default defined as follows:

$$\text{Distance-to-Default} = \frac{E(V_T) - d^*}{\sigma_V} \qquad (2.15)$$

The asset market value V, and its volatility σ_V, are inferred from market-based information: the market capitalisation of the firm and the volatility of its shares. Let S' be the observed value of the firm's market capitalisation. According to Merton, the price of the equity can be seen as a call option on the value of the firm. Therefore:

$$S' = C(V, \sigma_V; r, T, D) \qquad (2.16)$$

In addition, KMV uses the following theoretical relationship between the volatility of the equity, which is observable, and the volatility of the assets, which is unobservable:

$$S'\sigma_S = V\sigma_V C \qquad (2.17)$$

Putting equations 2.16 and 2.17 together, we have only observable quantities on the left-hand side of the equations and two unknown variables to be found : V and σ_V.

The Distance-to-Default is assumed to summarise all the relevant information about default. This theoretical variable is compared with empirical distances-to-default, which are associated to default probabilities. Empirical probabilities are estimated using a large database, which contains approximately 20,000 firms. Thus the model is able to estimate the default probability of individual firms. The estimated probability is called the "Expected Default Frequency.

The most important advantage of this model is that it incorporates the latest information about the firm to estimate default, since it relies on equity market values. Some of the disadvantages of this approach are:

[5] See Kealhofer (1995), McQuown (1995) and Vasicek (1987).

In practice, CreditMetrics is implemented by using Monte Carlo Simulation, which simplifies enormously the computational work.

2.3.2.1.1. Advantages and Disadvantages of the Model

CM has two main advantages: 1) The model is formulated under the fundamentals of the value process of the firm, which makes the model very intuitive. 2) The model calculates a portfolio's losses due to both default risk and migration risk.

One of the main weaknesses of CreditMetrics is its assumption that credit spreads and interest rates are deterministic. This makes the valuation of instruments such as swaps or futures difficult, since the credit risk exposure of these securities is linked to market variables. Other limitations of the model are the following:

1. CreditMetrics assumes that transition matrices follow a stable Markov process. However, Altman and Kao (1992) find evidence of autocorrelation in the process of migration. If a bond is downgraded, then the probability that the bond will be downgraded again in the next period is higher.
2. It relies on transition matrices calculated by Rating Agencies, which do not differentiate across borrower types or across time.
3. The credit quality of the firm is completely determined by the rating of the Senior Unsecured debt. However, Altman (1989) finds that the age of the debt also affects the probability of default.

2.3.2.2. KMV Model

This is probably the most popular commercial implementation of Merton's approach, developed by S. Kealhofer, M. McQuown and O. Vasicek (KMV)[5]. They define the threshold or barrier (d*) under which default occurs as somewhere between the face

1. Normality assumptions are essential, as well as an ad-hoc liability structure for the firm.
2. The model can only be applied to companies listed in the stock market. For non-listed companies, accounting data must be considered to run the model.
3. Wilson (1998) argues that the model overreacts to market movements, reflecting "market sentiment" instead of market fundamentals.
4. Collateral, covenants or convertibility are not taken into account.
5. The model measures only default risk and leaves out the effect of interest rates
6. Jarrow and Turnbull (1998) argue that the use of historical data to estimate the expected default frequency implicitly assumes a stationarity behaviour in the estimated parameters.

2.3.2.3 CreditRisk+

CreditRisk+ can be seen as an application of the Reduced Form Approach, in which default is driven by an exogenous variable. The approach consists of modelling two sources of uncertainty: the number of defaults in the portfolio, and the severity of the losses. These two pieces of information are then combined to produce the distribution of losses of the portfolio, due to default events.

The simplest implementation of the model assumes that the number of defaults in the portfolio follows a Poisson process with a deterministic intensity parameter. Under this assumption, the model generates a distribution of losses with lower volatility than that observed empirically. By making the intensity parameter stochastic, the model is able to produce distributions with more fat-right tails and skewness. In particular, this intensity parameter is assumed to be Gamma-distributed.

In the event of default, the counterparty incurs a loss equal to the amount owned by the borrower (or the mark-to-market value of the debt), less the recovery rate. Contrasting other models, CreditRisk+ does not model the value of the exposure, and this is considered an input in the model.

To handle information in the portfolio, CreditRisk+ proposes reducing the amount of data by classifying exposures in bands according to their size. The size of each exposure is adjusted, so each band is characterised by a common exposure V_i. Also, to make the model mathematically tractable, the estimation of the distribution of losses is done using Probability Generating Functions rather than Distribution Functions.

2.3.2.3.1 Derivation of the Distribution of Losses

Let Gi(z) be the probability generating function of band i; then the probability of losing V_i units on a portfolio formed by only 1 borrower, must be equal to the probability that the borrower defaults:

$$G_i(Z) = F_i(Z^{V_i}) = \sum_{i=0}^{1} p(\text{no.of}\quad \text{defaults}) z^{V_i} \qquad (2.18)$$

Each band is viewed as a portfolio of individual exposures, whose borrowers are independent. Therefore the probability generating function for any band is by definition:

$$G_j(z) = \sum_{n=0}^{\infty} p(\text{ndefaults}) Z^{nV_j} = \sum_{n=0}^{\infty} \frac{e^{-\mu_j} \mu_j^n}{n!} Z^{nV_j} \qquad (2.19)$$

where μ_j is the expected number of defaults in band j. If the exposures in the portfolio are independent, then the probability generating function of the portfolio can be written as the product of the probability generating functions of each band:

$$G(z) = \prod_{i=1}^{m} G_j(z) \qquad (2.20)$$

From the probability generating function, the loss distribution can be derived as:

$$P(\text{losses}) = \frac{1}{n!} \frac{d^n G(z)}{dZ^n} \Big|_{z=0} \qquad \text{for } n=1,2,\ldots \qquad (2.21)$$

Equation 2.21 has a closed form expression. Once the distribution of losses has been estimated, expected losses and CVaR can be calculated.

CreditRisk+ has been extended in two ways: first the model can be extended to a multi- period framework; second, it can be applied to portfolios where borrowers are correlated through a "background factor", which represents an economic variable or an economic sector. Under a sector analysis, borrowers are divided into bands. The borrowers' default probability in each band is affected by one specific background factor, which follows a Gamma distribution. Thus within each band, borrowers are independent, so the above derivation for the model can be followed.

2.3.2.3.2 Advantages and Disadvantages of the Model.

The main advantages of this model is the simplicity in terms of assumptions and requirements of data. Also, the closed functional forms of the final distributions make this model easy to implement.

There are two main criticisms to the model:

1. The model does not consider migration risk.
2. Interest rates are assumed deterministic; therefore the application of this model to non-linear products such as options and swaps is limited.

2.3.3 Other Streams in Credit Risk Management

The production of Credit Risk Models poses two important questions: What is the accuracy of these models? How different are they? Unfortunately, assessing the accuracy of models is a difficult task. To carry out back- testing analysis requires long time series of data, which are particularly scarce in credit risk. Lopez and Saidenberg (2000) suggest that panel data analysis might allow the validation of the models in the absence of long time series. In addition, we would like to know how sensitive models are to changes in the parameters. This is an important question in credit risk, as quality of data is one of the main concerns in the area. Nickell, et al. (2000) argue that transition matrices should differentiate borrowers by domicile and industrial sectors and taking into account business cycles.

With respect to the comparison of models, Crouhy (2000) applies CreditMetrics, CreditRisk+ and the KMV model to a large diversified benchmark. He finds differences of up to 50% in the CVaR. Using a more structural framework and control of parameters, Gordy (2000) and Koyluoglu and Hickman (1998) find that CreditMetrics and CreditRisk+ yield similar results when they are parameterised in a consistent way. However, both papers only look at the default part of credit risk. Therefore, the relationship between a credit default model, such as CreditRisk+, with a credit rating model, such as CreditMetrics, is still an open question.

Nickell, Perraudin and Varotto (1998) compare a model resembling CreditMetrics with the KMV model. They conclude that both models are similar when they are applied to well-diversified portfolios, but there are important differences when they are applied to less well-diversified and low credit-quality portfolios.

Other papers propose improvements to current models, such as CreditMetrics. For instance, Kiesel, et al. (2000) allow for stochastic spread risk. They find that the omission of this risk seriously understimates CVaR figures.

Though questions such as accuracy and performance of models have not been widely explored, research is looking ahead and addressing more complicated issues. For example, what is the relationship between market risk factors and credit risk factors? How can market risk and credit risk be integrated to produce one unified measure?

Some recent papers have addressed the relationship between market and credit risk factors. Kiesel, Perraudin and Taylor (1999) analyse the relationship between interest rates and rating transitions. They find that negative interest rate changes are associated with fewer upgrades. When calculating CVaR using CreditMetrics and transition matrices based on data from years in which interest rates fell, CVaR figures were higher than the CVaR estimated with transition matrices from other years.

Jarrow and Turnbull (2000) argue that the lack of separability between market risk and credit risk affects the determination of economic capital. To solve this problem, they propose using pricing models in the estimation of credit risk in portfolios.

2.4 Conclusions and Further Research

In this chapter we have presented some results from the literature on Credit Pricing and Risk Management.

With respect to Credit Pricing, we have presented some models that characterise the two main streams in this area: the Structural Approach and the Reduced Form Approach.

Structural Models are attractive because they are consistent with intuition. They are suitable for pricing securities associated with the value of the firm, such as callable and convertible bonds. They are also suitable to answer questions such as capital structure. However, the intuitive framework of the Structural Approach is offset by its inability to fit credit spreads accurately and price more complicated securities, such as credit derivatives. Also, these models seem very difficult to implement. They require many inputs which are difficult to observe or estimate, for example, the asset value of the firm and its volatility: these two pieces of information are particularly difficult to estimate when firms are not tradable or their assets are intangible.

Reduced Form Models try to overcome the fitting problem of Structural Models by defining the probability of default in terms of jump-processes. Thus, default can suddenly occur even when maturity is close. These models can reproduce more realistic credit spreads in the short run, but they lack financial structure and intuition with regard to the default process. Reduced Form Models are also attractive because their mathematics is more tractable; therefore various functional forms can be implemented. Most variables can be assumed stochastic and the model still gets closed-form solutions. Compared with Structural Models, this approach uses only observable variables as inputs and it can price a wider variety of assets including credit derivatives.

Summarising the scope of both streams, we should say that most pricing models seem to consider default as the only credit risk factor and ignore other sources of credit risk, which can also affect the value of the assets, such as migration risk or even liquidity risk. Empirical experience indicates that liquidity drives the size of credit spreads on corporate bonds in an important way (see Ericsson and Renault, 2001). The above indicates that further research needs to be done with respect to modelling. The specification of structural models could still be improved by considering other variables that also seem to be relevant in driving bond prices.

In addition, the nature and importance of some variables and parameters need to be investigated empirically, in order to make proper assumptions within the models. For example, the effect of financial distress in companies, and the correlations between variables such as default, interest rates and recovery rates, need to be explored further. A relatively wide empirical literature on the relationship between credit risk and interest rate risk has been produced recently[6]; however, results are still inconclusive.

A new generation of models is seeking to overcome the disadvantages of both approaches by combining them within a single model. The distinguishing feature of these new models is that they combine the attractive characteristics of both frameworks. For example a hazard rate may be given a meaningful structure in term of economic factors or characteristics of the firm (Madan and Unal, 2000). Cathart and El-Jahel (2003) assume that default can occur either when a latent variable crosses a barrier as in structural models, or when a sudden jump occurs, as in reduced form

[6] See for example Duffe(1998), Morris, et. al (1999) and Collin-Dufresne, et al. (2001b)

models. We believe that the development of this new framework could improve current models.

The basis of credit risk pricing models is still largely theoretical. Empirical results, comparison and testing of models remain scarce. The lack of data is the main impediment to carrying out these analyses. There are few databases on corporate debt available, and most of them are focused only on investment-grade bonds. Further empirical analysis involving non-investment bonds and other types of bonds is needed. For example the modelling and application of models to price sovereign bonds rather than corporate bonds is an area still to be explored[7]. Further empirical results will be crucial to finding the right direction in the construction of new models.

With respect to Risk Management Models we have presented some of the models dedicated to measuring credit risk in individual securities and portfolios. In the last few years, the concept of Value-at-Risk has been extended to Credit Risk. The financial industry has sponsored important models such as CreditMetrics, KMV and CreditRisk+.

One of the main criticisms of all these models is that they fail to incorporate stochastic movements in the interest rate. This feature limits the scope of such models in portfolios such as credit derivatives or other portfolios sensitive to interest rates. Current research is focused on relaxing this important assumption.

In the production of models, two questions are fundamental: What is the performance of the models? Which model is superior? Testing and comparison of models are major concerns of banking authorities, as they are particularly interested in applications to determining capital adequacy requirements. Though these questions seem to be simple, the answers are not straightforward. On the one hand, the lack of data has

[7] Only a few studies, such as Duffie, Pedersen and Singleton (2003), Keswani (2000) and Pages (2001) have been produced in this connection.

restricted the assessment of models, so little research has been done on this area[8]. On the other hand, though some papers have analysed the differences between models, they fail to set up a proper framework of comparison, where parameters of the models are consistent.

Though the above issues have not been widely addressed, more complex questions have already been formulated. For example: How can market risk and credit risk be measured in a unified way? How can one calculate the total risk of a portfolio affected by market and credit risk factors? How do market risk factors affect the calculation of credit risk in portfolios? The integration of credit risk and market risk is a topic of great interest. Clearly there is interdependence between the two types of risks. Intuitively, if market prices change suddenly affecting the market price of a firm's assets, then the firm's probability of default will also be affected, generating credit risk. Likewise, changes in the firm's probability of default are likely to affect the market value of the firm, generating market risk (Jarrow and Turnbull, 2000). One of the problems in producing integrated models is that the relation between market risk factors and credit risk factors is still not well understood. Therefore, further research is needed on the relationship between these variables.

Finally, it is important to keep in mind that lack of data is the most important restriction in the implementation of any credit risk model. Therefore in the formulation of new and more sophisticated models, a trade-off between simplicity and accuracy is always worth pursuing.

[8] See Gordy (2000) and Hickman and Koyluoglu (1998) for the comparison of some models.

Chapter 3.

Can Structural Models explain Prices of Sovereign Bonds?

Abstract of Chapter 3

We test the ability of an extended structural model, originally proposed by Cathcart and El-Jahel (2003), to capture the dynamics of prices for Mexican Brady bonds. In this framework, default is triggered either when a latent variable measuring financial distress falls below a specific threshold (as in structural models), or when a hazard rate causes an unexpected jump (as in reduced-form models).

Using market prices and a Kalman Filter methodology, we estimate the model and extract the implicit "distance-to-default" over a seven-year period. The model is slightly superior to one which assumes that distance-to-default follows a random walk. However, the hazard-rate feature of the model makes no contribution to explaining the dynamics of market prices.

We find that three economic factors explain approximately 80% of the variation in the distance-to-default, namely: the level of the Mexican stock market, the exchange rate and the risk-free term structure. When the distance-to-default is approximated from these variables and substituted back into the models, the Cathcart and El-Jahel model still performs better than the naïve model, not only in-sample but out-of-sample as well. The structural model is therefore supported over simpler alternatives, but only by a small margin.

3.1. Introduction

Within the continuous-time theory of credit pricing, two different approaches have been developed to price risky bonds: the Structural Approach and the Reduced Form Approach. They differ in the way in which default is triggered. Under Structural Models default is determined by the impossibility of making any payments due to solvency problems. Hence default is triggered when a solvency variable crosses a specific threshold. Such a variable has an economic or financial meaning for the firm (see for example, Merton (1974), Leland (1994), Longstaff and Schwartz (1995), Collin-Dufresne and Goldstein (2001a)), but is much less well-defined for a sovereign borrower. These models have been widely criticised for their inability to produce the appropriate size of credit spread near to maturity. One reason is that, close to maturity and given that no default has occurred, the latent variable characterised by a diffusion process can only move "smoothly", therefore the probability of default is practically zero. In order to ameliorate this problem, Zhou (1997) models the asset value of the firm as a jump diffusion process. Unfortunately this leads to a non-closed-form solution and the model becomes mathematically less tractable. Alternatively, Reduced Form Models treat default as an unpredictable event, which occurs with a hazard rate h(t) (see Jarrow, Lando and Turnbull (1997), Lando (1995) and Duffie and Singleton (1997)). These types of models have successfully replicated the size of credit spreads in the short term and they are mathematically more tractable. However, the advantage of Structural Models over Reduced Form Models is that they allow a better understanding of the dynamics of debt pricing, since default is explained by firm-specific variables, and this may lead to better forecasting performance out-of-sample.

This chapter investigates whether structural models, suitably adapted, can provide insight into the pricing of sovereign debt. There are three specific aims of the study. Firstly, we examine the extent to which prices generated by an extended structural model are consistent with market prices of the Mexican Par bond over a seven-year period. The model which we use is that of Cathcart and El-Jahel (2003) (CEJ), which incorporates both a hazard rate and conventional structural features. This model has not previously been implemented in any empirical study. Secondly, we compare the in-sample performance of the model with an alternative, the Naïve Structural Model (NM). Such a model assumes that the probability of default in any future period is constant, conditional on no default having occurred yet. Thirdly, we explore the importance of economic fundamentals in determining the distance-to-default of the country implied by the models and test their ability to fit market prices in-sample and forecast prices out-of-sample.

Several papers have studied the ability of Structural Models to fit spreads on corporate bonds, but only a sparse literature examines whether this approach can be extended to sovereign debt. In principle, the perception of the repayment capacity of countries has to do not only with their ability to pay but also with their willingness to pay. Under the lack of a bankruptcy code for sovereign debt, a government's willingness to pay will be driven by reputation costs, political and economic sanctions, etc. Lenders and borrowers may also negotiate, and final payments might depend on the bargaining capacity of lenders[1]. In this chapter we are interested in Structural Models and in the solvency variables that determine countries' ability to pay. Therefore, we will assume that only these variables determine countries' probability of default.

One of the major problems in using structural models to price sovereign debt is the definition of both the solvency variable and the barrier beyond which default will occur, as there is no equivalent for countries of the insolvency which applies to companies. It is necessary to estimate the barrier as a "latent variable". Our methodology consists of using the CEJ model to recover the latent variable from seven years of market prices, using a Kalman Filter. We then investigate whether the latent variable (interpreted as the distance-to-default) can be approximated over time by a set of economic and

[1] Gibson and Sundaresan (2001) discuss the bargaining game in case of default and reorganisation for countries. They point out that the absence of a bankruptcy code for countries is sufficient to make the optimal default strategies differ significantly from those of companies.

financial fundamentals. We also test the performance of this proxy for the distance-to-default (based on fundamentals) to forecast prices both in-sample and out-of-sample. We use Brady bonds in this study rather than conventional bonds, since they have certain advantages. Firstly, they are the most actively traded bonds from emerging markets, so there is no need to worry about estimating a liquidity premium. Secondly, they facilitate the modelling of the recovery rate, since some of their cashflows are guaranteed by Treasury bonds.

Our main results are as follows. We find that the structural model of CEJ seems to fit the data slightly better than the naïve random-walk model; however the difference is not statistically significant. Having identified a series for the latent "solvency" variable over the seven-year sample, we find that it is quite closely related to a set of economic variables, including the stock-market level, exchange rate and the level and slope of the risk-free term structure. We also find that an increase in interest rates causes spreads to fall, consistent with the literature on corporate bonds. Driving the model forward with these economic variables, the CEJ model performs slightly better out-of-sample than the naïve model. This suggests that structural models can explain prices (and spreads) for sovereign bonds, although it remains to be seen whether the same economic variables as in Mexico determine the solvency in other countries. Finally, the hazard rate in the model makes no contribution to explaining the bond prices, which is rather surprising but may be because (in the CEJ model) it is dependent on the interest rate which is already an input to the structural component of the model.

This chapter is organised as follows. In Section 3.2 we briefly discuss the underlying literature on testing structural models for sovereign debt. Section 3.3 presents a description of the data. Section 3.4 presents a generic framework for the models. The implementation of the models, estimation results, diagnostic checking and comparison in-sample are presented in Section 3.5. In Section 3.6 we investigate whether the distance-to-default implied by the models can be approximated by a set of economic variables. We also test the performance of the models in-sample and out-of-sample when the latent variable has been substituted by an approximation in terms of fundamentals. Section 3.7 gives the conclusions and further research.

3.2. Literature Review

In principle, credit risk models have been designed for pricing corporate debt and there is only a small literature on the performance for sovereign debt. Varga (1998) applies a structural model (based on Longstaff and Schwartz (1995) and Das (1995)) to price a Brazilian Brady Bond. He assumes that the driving variable of default is the level of international reserves. Contrary to what is usually expected from structural models, his estimated credit spreads are much higher than those observed empirically. He argues that other variables such as emergency loans provided by the IMF may also be important for predicting default.

Claessens and Pennacchi (1996) propose a continuous-time pricing model similar to that of Longstaff and Schwartz (1995). They use this structural model and the Kalman Filter to generate a latent variable that determines Mexico's repayment capacity. Though the model allows for anticipated changes of credit spreads, it is very restrictive for describing the dynamics of credit quality.

Keswani (2000) implements both Longstaff and Schwartz (1995) and Duffee (1999) models (a structural model and a reduced form model respectively), to price Brady Bonds from Mexico, Argentina and Venezuela during the period 1993-1996. Using the Kalman Filter and the Longstaff and Schwartz (1995) model, he obtains an implicit distance-to-default variable. However, he does not investigate whether there is any set of economic fundamentals that can determine this implicit variable. His conclusion is that the structural model performs better than the reduced form model only in-sample, for Mexico and Argentina and before the Mexican crisis. Using the reduced form model, he also finds weak evidence of a common factor across emerging markets that drives default and is therefore responsible for contagion effects.

Duffie, Pedersen and Singleton (2003) extend their reduced form model (Duffie and Singleton (1999)) to price Russian sovereign debt. This extension considers some exogenous credit events such as restructuring, renegotiation and illiquidity. Their model seems to fit reasonably well during the sample period 1994-1998. Pages (2001) extends the reduced form model of Duffie and Singleton (1999) to include liquidity for the pricing of the Brazil discount Brady bond. He finds that, allowing for liquidity, the model generates negative probabilities of default and so leads one to question the whole approach.

3.3. Data Description

We use end-of-month prices of the Mexican Par as reported by Datastream from December 1993 through February 2002. The Mexican Brady Par is a dollar-denominated coupon bond issued in April 1990 with an original maturity of 30 years. The bond pays a semi-annual coupon of 6.25% and its maturity is 31st December 2019.

Figure 3.1 displays the prices of the Mexican Par Bond in the period December 1993-February 2002. The dramatic effect of the Mexican crisis can be observed between the last quarter of 1994 and the first quarter of 1995. Another important fall occurs in August 1998, reflecting the Russian default which eroded confidence in all emerging markets. Nevertheless, the price of the Mexican Par rises on average between 1994 and 2002, reflecting a decrease in interest rates and also a possible improvement in the market perception of the probability of default during this period.

Table 3.1. gives descriptive statistics for the Mexican Par prices and returns. The monthly prices and returns of the bond are quite volatile. Prices fluctuate in a range between 47.250 and 96.250 USD over the period, whereas returns vary between –17.4% and 11.5%. Prices show a high persistence, according to the first order autocorrelation coefficient, suggesting that they may be non-stationary.

In order to estimate the parameters of the risk free term structure and its discount factors, we collected yields of stripped US government bonds from 8 maturities from Bloomberg. The properties of the yields between December 1993 and February 2002 are summarised in *Table 3.2*. According to the first order serially correlated coefficient, there is persistence of autocorrelation in each series: all coefficients are above 0.9 (monthly). Also the volatility decreases across maturities and there is excess kurtosis, particularly for short maturities.

3.4. Pricing Sovereign Bonds

In this section, we first introduce a simple formula to price Brady Bonds, which is based on a combination of risky and non-risky zero coupon bonds. Then we discuss the model proposed by Cathcart and El-Jahel (2003). This approach prices a risky zero coupon bond assuming that default is driven by both a signalling process and a hazard rate. In order to test this model, we will compare its performance with a naive model that assumes that default is driven by a random walk process. The properties of this naïve model are also discussed in this section.

3.4.1. A Pricing Formula for Brady Bonds

A Brady Par bond is a bond denominated in dollars, issued by a sovereign borrower that usually pays semi-annual coupons. The principal value is completely collateralised by 30-year US Treasury zero-coupon bonds. In addition, Brady bonds have up to 18 months of rolling guarantee, composed of securities with a credit rating of at least AA. Such securities are deposited in an account at the Federal Reserve Bank of New York and the interest earned on the funds accrues to the debtor country every six months, provided that the country pays the corresponding coupon in full.

The value of a Brady bond B_t at each point in time *t* can be expressed as the sum of three components according to the following equation:

$$B_t = F \cdot P_t(r_t, T) + C \cdot F \sum_{i=1}^{q} P_t(r_t, \tau_i) + C \cdot F \sum_{i=q+1}^{N} (1 - \gamma_t(\tau_i - n)) P_t(r_t, \tau_i) \qquad (3.1)$$

where F is the nominal value of the bond, C is the coupon rate, $P_t(r_t, \tau_i)$ is the price at time *t* of a default free zero coupon bond that matures at time τ_i, q is the number of guaranteed coupons, and $1-\gamma_t(\tau)$ is the survival probability (i.e., the probability at *t* that no default has occurred prior to τ (τ >t)).

The first term in the equation accounts for the receipt of the face value F with maturity T, which is fully collateralised, and so it is discounted at the risk-free rate. The second term corresponds to the present value of q guaranteed coupons, each with maturity τ_i. The Mexican Par has 18 months of guaranteed interest payments so that $q = 3$ and they can also be discounted at the risk free rate. The third component accounts for the value of the risky coupons. These may be valued according to their expected payout, which takes account of default. Following Keswani (2000), if n is the length of the rolling interest guarantee then each coupon with maturity τ_i is paid if and only if default has not occurred before τ_i-n.

In the formula we assume that the recovery rate is zero for any other cashflows not included in the rollover guarantee. This means that we may overstate the true probability of default, so results for bond prices should be considered as a lower bound (as noted by Keswani, 2000).

In the next two sections we discuss a sophisticated and then a naive model, which only differ in the way the probability of default $\gamma_t(\tau)$ is constructed.

3.4.2. An Extended Structural Model: The Cathcart and El-Jahel Approach (CEJ)

The assumptions of the model are as follows:

Assumption 1: Markets are frictionless and trading is carried out in continuous time. There are no taxes, transaction costs or informational asymmetries.

Assumption 2: The risk-adjusted dynamics of the short-term interest rate follow the Cox Ingersoll and Ross-CIR (1985) square root specification:

$$dr_t = \kappa_r(\mu_r - r_t)dt + \sigma_r\sqrt{r}_t dZ_r \qquad (3.2)$$

where μ_r is the long-term mean of the interest rate, κ_r is the speed of adjustment of r_t toward the steady state mean, σ_r is the (constant) volatility and Z_r is a standard Wiener process.

Assumption 3: There is a "signalling variable", X_t, which summarises the set of factors which reflect the creditworthiness of the country. Under the risk neutral measure this variable follows a Geometric Brownian Motion:

$$dX_t = \alpha_X X_t dt + \sigma_X X_t dZ_X \qquad (3.3)$$

where α_X and σ_X are constants and Z_X is a standard Wiener process. Default occurs when X_t hits a barrier X_ℓ for the first time, in line with structural models.

Assumption 4: Default can also occur by surprise as a jump event (as in reduced-form models). The hazard rate is an affine function of the short-term interest rate: $h_t = a_r + b_r r_t$, where a_r and b_r are positive constants.

Assumption 5: If, during the life of the security, either the signalling variable hits the barrier X_ℓ, or a default jump occurs, then the bondholder receives a proportion δ of the bond face-value, where δ is the recovery rate.

Finally, it is important to point out that in order to facilitate solving the model, CEJ makes the assumption that the correlation between the signalling process and the interest rate is zero. In other words, the instantaneous correlation between Z_x and Z_r is zero.

Given the above assumptions, Cathcart and El-Jahel show that the price of a risky discount bond can be expressed as:

$$H(x_t, r_t, \tau) = P_t(r_t, \tau) - P_t(r_t, \tau)(1 - f(x_t, \tau)g(r_t, \tau))(1 - \delta) \qquad (3.4a)$$

where

$$f(x_t, \tau) = \Phi\left(\frac{y + \left(\alpha_x - \frac{1}{2}\sigma_x^2\right)\tau}{\sigma_x\sqrt{\tau}}\right) - \exp\left(\frac{-2\left(\alpha_x - \frac{1}{2}\sigma_x^2\right)y}{\sigma_x^{\,2}}\right)\Phi\left(\frac{-y + \left(\alpha_x - \frac{1}{2}\sigma_x^2\right)\tau}{\sigma_x\sqrt{\tau}}\right)$$

(3.4b)

$$y = \ln(x_t / x_\ell) \qquad (3.4c)$$

$$g(r_t, \tau) = \exp\left(C(\tau) + D(\tau)r_t\right) \qquad (3.4d)$$

and $C(\tau)$ and $D(\tau)$ are solutions to the following system of ordinary differential equations:

$$\frac{1}{2}\sigma_r^2 D(\tau)^2 + \left(\sigma_r^2\tilde{B}(\tau) - \kappa_r\right)D(\tau) - D_\tau(\tau) - b_r = 0 \qquad (3.4e)$$

$$\kappa_r\mu_r D(\tau) - C_\tau(\tau) - a_r = 0$$

subject to the initial conditions C(0)=0 and D(0)=0[2].

The function $1-f(x_t,\tau)g(r_t,\tau)$ can be interpreted as the probability of default due either to the signalling process X_t hitting the default barrier X_ℓ, or to a sudden movement in the interest rate r_t. Hence, within the CEJ context, the survival probability can be expressed as follows:

$$1-\gamma_t(\tau)=f(x_t,\tau)g(r_t,\tau) \qquad (3.5)$$

and the price of a Brady Bond can be calculated by plugging equation 3.5 into equation 3.1.

3.4.3. Description of a Naïve Model with Constant Probability of Default (NM)

An alternative "naïve" structural model, used by practitioners, is as follows. Let us assume that there is a stochastic variable that follows a random walk and drives default or the creditworthiness of the country. If the variable falls below zero for the first time then the country defaults and no other payments will be made apart from the established guarantee.

As default occurs when the variable X_t becomes negative for the first time, this variable can also be interpreted as the distance-to-default. This model is consistent with the fact that there are no expectations of changes in the credit quality of the bond's issuer (as the drift of a random walk is zero). Consequently, the probability at *t* that default will occur at horizon Δt conditional on no default occurring prior to *t* is a

[2] $\tilde{B}(\tau)=\dfrac{2(\exp(\phi_1\tau)-1)}{\phi_4}$ as defined in the CIR model for the risk free term structure. See Appendix A.

constant, $\rho_t = \Phi\left(\frac{-X_o}{\sigma\sqrt{\Delta t}}\right)$. Thus the survival probability (or the probability that no default has occurred before time s) can be expressed as:

$$1-\gamma_t(s) = (1-\rho_t)^s \qquad (3.6)$$

The above formula implies that, regardless of time horizon, the expected probability of default in each period Δt is not expected to increase or decrease. In other words the perception of the market about the solvency capacity of the country is a constant at any point in time when looking forward[3].

In this framework the price of a Brady Bond can be estimated by substituting the survival probability from equation 3.6 into equation 3.1 above. It is then elementary to calculate the probabilities of default ρ_t across time, using the market prices of the bonds. *Figure 3.2.* displays the estimated annualised probability of default at time t during the period December 1993-February 2002, calculated this way. The effects of the Mexican and Russian crises are highly evident: the annual probabilities of default increased from about 5% before the crises, up to approximately 16% and 12% respectively. Other events such as the Asian crisis of 1997 were not as devastating. We also notice that the current levels of the country's perceived credit quality are quite similar to those observed before the crises.

3.5. Implementation of the Models and the Kalman Filter

The use of a Kalman Filter is a natural technique to estimate the model parameters. This technique is appropriate in the case of an evolving system of which only a part is observable. Such is the case for pricing models, in which market bond prices can be

[3] Some academic papers that make this assumption are: Bierman and Hass (1975), Bhanot (1998), and Cumby and Pastine (2001), Anderson and Renault (1999). Merrick (1999) investigates the reliability of this assumption.

observed directly but the latent variable or signalling process that drives default cannot.

The framework of the Kalman Filter is as follows. The data consists of observations of bond prices $\boldsymbol{B}_t$ at times $t_1,\ t_2, \dots t_n$. The relation between the observed variable $\boldsymbol{B}_t$ and the unobserved variable X_t is explained by *the Measurement Equation*:

$$\boldsymbol{B}_t = \mathrm{B}(t, r, X_t, X_\ell; \Psi, \Gamma) + \varepsilon_k \qquad (3.7)$$

where:

X_t is an unobserved variable that satisfies the dynamics of equation 3.3 in the case of the CEJ model, and follows a Random Walk in the case of the NM model. The discretisation of this process is known as the *Transition Equation*;

X_ℓ is zero in the NM model and a positive constant in the CEJ model;

Γ is the set of parameters that determine the movements of the risk free term structure in the CEJ model[4];

Ψ is the set of risky parameters[5].

We assume that prices $\boldsymbol{B}_t$ are measured with error ε_t, which is known as the *Measurement Error*, and is assumed Gaussian-distributed with mean zero and variance σ_ε. Thus the function $\mathrm{B}(\cdot)$ in equation 3.7 is interpreted as the theoretical price of a risky bond. The measurement error will also be an indicator of the adequacy of the model. If the true underlying process is not as in equation 3.3 for the CEJ model, then equation 3.7 will be misspecified and prices estimated theoretically will deviate systematically from the observed prices.

In order to produce an estimate of the unobservable variable and the parameters in the vector Ψ, the econometric estimation of the Kalman Filter is carried out in two

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[4] $\Gamma = \{\kappa_r, \mu_r, \sigma_r\}$ is the set of parameters of the CIR process in the CEJ model.

[5] $\Psi = \{\alpha_x, \sigma_x, a_r, b_r\}$ in the CEJ model, whereas this set is simply σ_x for the NM model.

steps. In the first step, the latent variable X_t is filtered and a set of estimates $\hat{X}_t$ is obtained. The filtering algorithm is constructed as follows: at time *t-1* estimates of the latent variable $\hat{X}_{t-1}$ are known[6]. Thus the Kalman Filter forms an optimal predictor $\hat{X}_{t|t-1}$ by using the distribution of the unobserved variable, conditional on the previous estimated values $\hat{X}_{t-1}$[7]. The filter allows the predicted estimates of the unobserved variable $\hat{X}_t$ to be updated once a new observation $\boldsymbol{B}_t$ is available. In the second step, the errors from the prediction are used to construct the Maximum Likelihood function and estimate the model parameters in the vector Ψ.

When there is a linear relationship between the observed and unobserved variables the estimation of the Kalman Filter guarantees efficient estimates. However, equation 3.7 is not linear in X_t, and we are obliged to apply an approximation method called the *Extended Kalman Filter* that consists of linearising the function $B(\cdot)$ using a first order Taylor expansion. The estimation is then done via Quasi-Maximum Likelihood (QML).

In order to avoid the estimation of the barrier X_ℓ in the CEJ model, instead of filtering the signalling process X_t, we will filter the variable $Y_t = \ln(X_t / X_\ell)$. This new variable can be interpreted as the distance-to-default, as it measures how far the latent variable is from the barrier of default. The definition of Y_t in terms of natural logs has two advantages: Firstly, the error term of the new transition equation will be Gaussian, so the Kalman Filter can be applied. Secondly, the linearisation of function $B(\cdot)$ above in terms of Y_t is simplified enormously, reducing the computing work. Applying Itô's Lemma and given equation 3.3, the transition equation of the new variable Y_t is given by:

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[6] Initial values for $\hat{X}_0$ and its variance need to be supplied.

[7] The variable X_t is assumed to follow a Markovian process, of which the dynamics are governed by the transition density $p(X_t|X_{t-1};\Gamma,\Psi)$

$$Y_{t|t-1} = Y_{t-1} + \left(\alpha_X - \frac{\sigma_X^2}{2} \right) \frac{1}{12} + \sigma_X \sqrt{1/12}\eta_t \qquad (3.8)$$

where $Y_t = \ln(X_t / X_\ell)$ and $E(\varepsilon_t \eta_s') = 0$.

This formulation describes the distance-to-default of the CEJ model as a Brownian Motion with a drift[8], in which the forecast of the next period is the current observation plus the average increase over the sample period. Observe that setting $\alpha_X = \frac{\sigma_X^2}{2}$ would yield a pure random walk, resembling the transition equation of the NM model.

To implement equation 3.7 under both the CEJ and the NM models, we require estimates of the discount factors $P_t(r_t, \tau)$. In the CEJ model the risk-free term structure is modelled using a CIR specification, whereas the naive model does not make any assumptions about the dynamics of the risk-free rate. It is well known that no risk-free model is able to adjust the observed term structure adequately. Hence it is very likely that fitting errors calculated under the CEJ model will also reflect the inability of the CIR model to explain properly the risk-free term structure. Since we are interested in the performance of the models' default features rather than in their risk-free features, we estimate the discount factors by fitting a cubic spline to the observed yield curve. However, in the case of the CEJ model, the estimation of the CIR model is still important, since its parameters and the implicit driving factor of the risk-free term structure are required to calculate the probability of default $\gamma_t(\tau)$.

The implementation of the CEJ model and the inference of its parameters is carried out in two stages (following Duffee (1999) and Keswani (2000)). In the first stage we estimate the parameters of the risk-free process (one-factor CIR model) using also a

Kalman Filter[9]. The advantage of using a Kalman Filter at this stage is twofold. On one hand, according to Geyer and Pitchler (1998) and Duan and Simmonato (1995), we can exploit all the information available, across time and maturities, about the observed term structure. On the other hand, we can extract the implicit factor that drives the dynamics of the risk term structure. Such a factor in the CIR model is interpreted as the instantaneous interest rate and is important for the default model since this determines the dynamics of the hazard rate. In the second stage, we use the resulting parameter estimates for the risk free process and the estimated instantaneous interest rate to estimate the parameters of default. We again use a Kalman Filter to obtain estimates of the distance-to-default and estimate the parameters of the hazard rate.

3.5.1. Estimation Results

We implement the model in the period December 1993 to December 2000[10], leaving a final 14 months over for ex-ante testing. Estimates of the parameters for the CEJ model are given in *Table 3.3.*[11]. In order to simplify the estimation, data are standardised and σ_x is set equal to one. To test the significance of each parameter we use a Log-Likelihood Ratio statistic [12] (LR). This ratio is based on the comparison of the restricted and unrestricted natural log of the Maximum Likelihood Function.

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[8] To estimate the parameters α_x and σ_x, it is convenient to think of equation 8 as the representation of the dynamics of the distance-to-default in the objective measure, rather than in the risk neutral measure, since parameters will be estimated using time series.

[9] The estimation of the parameters of the risk-free term structure is not our main objective, so its estimation is discussed in Appendix A.

[10] We assume that any structural changes in the creditworthiness of the country are absorbed by the distance-to-default; therefore we will estimate the model considering one single period. Nevertheless, when we split up this sample in order to consider a possible structural change around the Russian crisis we found very similar results.

[11] The numerical optimisation routine used to maximise the Maximum Likelihood Function is Powell's Method.

[12] The likelihood ratio statistic (LR) is defined as $LR = -2(\ln L_{UR} - \ln L_R)$, where $\ln L_{UR}$ and $\ln L_R$ are the log likelihood function of the unrestricted and restricted models respectively. The LR statistic has an asymptotic distribution χ^2 with *m* degrees of freedom; where *m* is the number of restrictions. When testing the significance of α_x, m is set equal to 1. To test the significance of the parameters a_r and b_r, we should notice that under Ho, we are restricting the parameters to lie on the boundary of

The first feature to observe from *Table 3.3.* is that the parameters a_r and b_r of the hazard rate are both practically zero. According to the LR test (statistic is shown in brackets) these coefficients are clearly not significant. Hence the hazard rate expressed in terms of the instantaneous risk free rate is irrelevant in determining any change in the credit quality of the country. In addition, α_X is positive and significant (equal to 0.3783). However the drift of the distance-to-default $Y_t = \ln(X_t / x_\ell)$ according to equation 3.8 is given by the difference $\left(\alpha_x - \frac{\sigma_X{}^2}{2}\right)$, which in this case is slightly negative as $\sigma_X{}^2 \cong 1$. This result implies that the market anticipated a slight worsening in the credit quality of the country during the estimated period. Nevertheless when we test for the significance of such a drift, we find that the hypothesis that this difference is equal to zero cannot be rejected. Therefore the distance-to-default implied by the CEJ model is very close to a random walk.

Figure 3.3. displays the distance-to-default extracted from the Kalman Filter (they correspond to the updated estimates $\hat{Y}_t$) for the CEJ and the NM models. The variables resemble each other quite closely. However, their levels are not directly comparable since they arise from adjusting different types of models. A 95% confidence interval for the Distance-to-Default of the CEJ is plotted in *Figure 3.4.* There are two important falls in the series that correspond to the Mexican and Russian crises. Observe that according to both models the Russian default had almost the same dramatic and negative impact on the creditworthiness of the country as the Mexican devaluation. This is quite a surprising result. We attribute this to a liquidity effect surrounding the Russian crisis which dried up the markets. Realistically there are other factors apart from default driving market prices, so it is very likely that the

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their parameter space (i.e., the parameters should be equal to zero); therefore, in this case, LR has the following asymptotic distribution under Ho:

$$\text{LR} \sim \frac{1}{2}\chi^2(0) + \frac{1}{2}\chi^2(1)$$

where the $\chi^2(0)$ distribution is a degenerate distribution with all its mass at the origin. See Harvey(1989, p.236) for details.

estimation of the distance-to-default has captured other effects such as a liquidity premium[13].

Figure 3.5. shows the goodness-of-fit of the CEJ model in terms of bond prices. The predicted prices correspond to the one-step-ahead fitted values $B_{t|t-1}$. The plot also shows an upper sequence which indicates what the bond prices would have been if they had been risk-free.

We check the adequacy of the models by performing diagnostic tests and goodness-of-fit according to Harvey (1989, p. 256). Appropriate tests are based on the standardised residuals $\tilde{v}_t$ of the one-step-ahead prediction errors v_t:

$$\tilde{v}_t = v_t / \sqrt{f_t}, \quad \text{where } v_t = \boldsymbol{B}_t - B_{t|t-1} \text{ and } f_t = var(v_t)$$

Figure 3.6. shows the one-step-ahead residuals v_t in the estimated period for the CEJ model. Apart from a few outliers, the size of the errors is less than 5% across the estimated period. These residuals are conditional on information known at time *t-1*. So an analysis of these should show whether the distribution of prices at time t, conditional on the information about the latent variable at time t-1 ($B_t(y_{t|t-1})$), has the same properties as the realised distribution of market prices $\boldsymbol{B}_t$. A well-specified model requires that the standardised residuals $\tilde{v}_t$ be normal and identically distributed.

Looking at the descriptive statistics for the residuals in *Panel A* of *Table 3.4.*, it is clear that the assumption of normality is rejected by both models, indicating the inability of both models to adjust correctly. This is because the series exhibit negative skewness (-2.0480 for the CEJ model and –2.7811 for the NM model) and high kurtosis (9.6140 and 14.0038 respectively). The lack of normality is more pronounced for the

[13] Some authors have explained episodes of market turbulence in terms of changes in market sentiment rather than liquidity events. In an extension of this chapter we investigate the effect on the distance-to-default of the discount of closed-end country funds (investing in Latin-America) as a measure of market sentiment. This variable is found to be significant, indicating that market sentiment

standardised residuals of the NM model than for the CEJ model, according to the Jarque-Bera test in *Panel B.* Though normality is rejected for v_t, there is no evidence of autocorrelation and heteroscedasticity, which may indicate that the models are well specified as there is no evidence of a missing variable in the residuals.

3.5.2. Goodness-of-Fit and Model Comparisons In-Sample

In order to determine which model performs best, in *Table 3.5.* we summarise some measures of goodness-of-fit based on one-step prediction errors. In *Panel A*, the sums of squared errors (SSE) for both models are quite small, though this statistic for the CEJ model is marginally smaller (0.0865 vs 0.0920). This indicates that the CEJ model is marginally more accurate, which is consistent with the coefficient of determination R^2.

Following Harvey(1989), goodness-of-fit estimates should be calculated keeping in mind that observed Brady prices have shown evidence of a unit root[14]. Thus the coefficient of determination R_D^2 defined in *Table 3.5.* is a better measure than R^2, when time series are not stationary (the larger and more positive R_D^2, the better). This statistic is positive but close to zero for the CEJ model (0.0651), while this figure is even closer to zero (0.0063) for the NM model. Hence there are some gains to be had from implementing the CEJ model rather than a much simpler model, though they seem to be small.

The number of parameters involved in the models also matters. A fairer measure of comparison is therefore the Akaike Information Criterion (AIC). However, with respect to this criterion, both models are similar.

also drives credit risk. The discount may also contain a liquidity premium. This is because when markets become illiquid the size of the discount may increase.

[14] *Table 3.1* showed that Brady prices have a high first order correlation coefficient. In addition, when testing for a unit root using the Dickey Fuller test, we could not reject the hypothesis of a unit root.

We also quantify the goodness-of-fit over the estimation period by computing some forecast evaluation measures. Panel B shows some statistics - the smaller the figures the better. Theil's inequality coefficient lies in the range [0,1], with 0 indicating a perfect fit and 1 indicating a predictive performance as bad as it could possibly be. According to this measure, the two models fit very well, though the CEJ model fits slightly better than the NM model.

In summary the CEJ model seems to fit better than the NM model in terms of goodness-of-fit statistics and forecast evaluation measures. Since the gains of the CEJ seem marginal, we compare the models' forecast accuracy further. Following Diebold and Mariano(1995), we test the hypothesis of equal forecast accuracy for the two models. Using the Sign Test, we find that the NM model is not a statistically significantly worse predictor of bond prices than the CEJ model[15]. Hence the marginally better accuracy of the CEJ model may not justify its greater complexity and the computational costs of its implementation.

3.6. The Economic Interpretation of the Distance-to-Default

In this section we examine the relationship between the distance-to-default generated by each model and a set of country fundamentals. Several studies have suggested a number of variables that can explain other measures of creditworthiness, such as credit ratings or credit spreads. For example, Cantor and Packer (1996) find that per-capita income, inflation, growth rate and the ratio of foreign currency to exports are all factors relevant to explaining the credit rating of a country. Beck (2001) argues that

[15] The loss function associated to the test is the absolute error. Under the null hypothesis of equal forecast accuracy for two forecasts, the median of the difference of the absolute errors should be zero ($\text{median}(\text{abs}(e_t^{NM}) - \text{abs}(e_t^{CEJ})) = 0$). The null could not be rejected at 95% confidence level. See Diebold and Mariano(1995) for details.

credit spreads are affected by different variables at different time horizons[16]. For the determinants of credit spreads in the medium term, he proposes similar variables to those used by Cantor and Packer(1996). He classifies the variables into three categories. One category is formed by country fundamentals, which includes the real GDP growth, the domestic inflation rate and the current account deficit. The other two categories are international interest rates and market variables such as the volatility of capital markets[17].

To choose the variables that best explain the distance-to-default, we use the general-to-specific approach (see Hendry and Doornik, 2001) to find a parsimonious characterisation of the dependent variable. The general model considers several explanatory variables predicted by the theory as determinants of creditworthiness[18]. After testing several variables we found that the distance-to-default of both models can be explained by the same fundamentals: the stock market index, the currency exchange rate and the level and slope of the yield curve. The last two variables are calculated as the first and second principal components respectively, of the risk-free term structure. *Table 3.6.* gives OLS estimates[19] of the relationship between the distance-to-default and these fundamental variables, for both models. Apart from the lag of the stock returns in the NM model, all the variables are significant at least at 95% confidence level for both models. The fact that the distance-to-default of both models can be explained by the same factors is not surprising. However, what is surprising is that these few variables alone are able to explain around 80% of the variance of the distance-to-default for both models. This suggests, among other results, that the stock market may be an important determinant of the bond market

[16] Apart from economic fundamentals, credit spreads are expected to be affected by bond maturity, coupon size, degree of subordination and other bond features such as call structure. Litterman and Iben (1991).

[17] See also Edwards (1984), Ming (1998) and Eichengreen and Mody (1998).

[18] Some of the variables that were considered in the general model are: ratio of external debt to GDP, ratio of exports to industrial production, reserves to industrial production, inflation, depreciation, returns in the stock market (in pesos), several maturities of the risk free term structure. We make use of PcGets (see Hendry and Krolzig, 2001) and start with a general, dynamic and unrestricted linear model for the variations of the distance-to-default.

When considering the credit rating of the country as an explanatory variable (calculated as the average of the ratings issued by S&P and Moody's and converted into a numerical scale), we find that this variable does not convey extra information to that already explained by the chosen model.

behaviour or at least that the two variables (distance-to-default and stock market) are dependent on the same unspecified factors.

On analysing the importance of each variable to explain the variation of the distance-to-default, we find that the return of the Mexican stock market index is the most relevant variable[20]. This variable accounts for approximately 52% of the variance of the dependent variable in the case of the CEJ model. For the NM model this figure is 45%. As would be expected the coefficient is positive, meaning that positive market returns improve the perception of the credit quality of the country. This is consistent with the predictions of structural models, in which the stock market seems to lead the bond market. This result is also consistent with some recent literature on sovereign credit spreads. For instance, Barnhill et al. (2000) find important co-movements between high-yield bonds and equity indices. A possible explanation for this is that financial market conditions seem to capture information about countries' credit quality. Barnhill argues that an increase in the stock market index increases capital gains, leading to an increase in tax revenues and consequently to an increase in the government's ability to service its debt.

The next most relevant variable is the variation of the exchange rate. Its coefficient has the expected negative sign for both models. The results show that a depreciation of 1% of the peso will produce a negative change in the distance-to-default by 0.0221 and 0.0017 units in the CEJ and NM models respectively.

With regard to the role of the risk-free interest rate, we find that variations in the level and slope of the yield curve are significant at 95% and 99% significant levels respectively[21]. These results are also in line with previous results on corporate bonds.

[19] Standard Errors are corrected for heteroscedasticity using White's method.

[20] In order to avoid multicolinearity, the stock market returns have been calculated using the index in pesos and not in dollars. This is because the variable in dollars was highly correlated with the exchange rate.

[21] Recall that the estimation of the interest rate process (CIR) via a Kalman Filter generates an implicit factor that drives the risk-free term structure. Such a factor is theoretically associated to the instantaneous or short-term interest rate. The correlation between this estimated factor and the level of the yield curve (measured as its first principal component) is very high (98%). We find that this factor is

The empirical relationship between credit spreads and interest rates has been widely discussed in the literature. Longstaff and Schwartz (1995) show that the interest rate is an important factor in determining default risk in corporate bonds. They find that an increase in the interest rate leads to a fall in credit spreads. Duffee (1998) looks at the effect of the level and slope of the risk free term structure. He finds that changes in the short-end of the Treasury curve are negatively related to changes in credit spreads. This relationship is more significant for low-rate bonds. A similar result holds for the relationship of spreads with the slope of the yield curve[22]. In our case the measure of credit risk is the distance-to-default, which might be expected to be negatively correlated with credit spreads. In fact our results indicate that there is a positive relationship (the size of the coefficients are 0.0609 and 0.0054 for the CEJ and NM model respectively) between the level of the yield curve and the distance-to-default implied by both models. Following the explanation for corporate bonds, it may be that an increase in the risk-free rate produces an increase in the risk neutral growth rate of the country's wealth and therefore increases the distance-to-default. This result is actually predicted by the sovereign-bond model of Gibson and Sundaresan (2001). Though this result is consistent with empirical findings, it also proves that the assumption of the CEJ Model that the signalling variable and interest rate are uncorrelated, is unrealistic. Furthermore, since the hazard rate was found to be insignificant in explaining the dynamics of the model, it seems that in this period the effect of the interest rate on bond prices could be more appropriately modelled through a diffusion process rather than a jump.

Examining whether these variables are also good predictors of observed credit spreads, we find that they are able to explain around 76% of the variation. This figure is similar for the distance-to-default. The direction of the variables is consistent with the literature and also with the findings about the distance-to-default. This high figure is quite surprising, since in previous empirical literature, the explanatory power of the

significant for the distance-to-default of the NM model and provides the same information as the level of the yield curve in the regression model. However, this factor is weakly significant for the distance-to-default of the CEJ model, and therefore the level of the yield curve seems to be a better predictor.

models has been quite low. Collin-Dufresne et al. (2001b) explain not more than 25% of corporate credit spreads, using those variables predicted by structural models. Gruber et al. (2001) find also that expected default risk explains about the same percentage; and other variables such as tax effects and a risk premium are even more relevant. Using several macroeconomic variables, Westphalen (2002) also explains around 25% of the changes of credit spreads on bonds issued by emerging markets.

3.6.1. Performance of the Economic Proxy of the Distance-to-Default

Here we examine whether we can replace the distance-to-default with its estimated value from the economic variables and still produce accurate price predictions both in-sample and out-of-sample (January 2001 to February 2002).

First we approximate the distance-to-default for each model using the fundamentals found in the previous section and produce a forecast of this variable for the period out-of-sample. The new proxy of the distance-to-default for the whole period (December 93-February 2002) is now used as a latent variable in each model. The forecasted prices and the prediction errors for the whole sample under the CEJ model are plotted in *Figures 3.7.* and *3.8.* The fitting errors, calculated as the difference between the observed price and the forecast, show a systematic pattern. Most of the time the model seems to underestimate the observed prices (forecasted pricing errors are positive in *Figure 3.8.*), apart from the period between February 1994 and February 1996 in which forecasted prices are higher than the observed ones (forecasted pricing errors are negative). This is evidence of a missing variable that should capture information about improvements in the credit quality of the country after the Mexican crisis.

[22] See Morris, Neal and Rolph (1999), for an analysis of the relationship in the long-run between credit spreads and interest rates.

Table 3.7. displays the statistics about the performance of the models, using the proxy of the latent variable. In terms of the sum of squared residuals (SSE), the square root of the mean squared residuals (RMSE) and the mean absolute error (MAE), the figures for the CEJ model are smaller than those for the NM model. This indicates that the CEJ model still performs better than the naive model, both in-sample and out-of-sample, when the distance-to-default has been substituted by a proxy based on economic variables. However, when testing equality of forecast accuracy, using the Sign Test we cannot reject the hypothesis that both models generate similar forecasts.

Finally, credit models can also be tested on their ability to replicate empirical credit spreads. We find that when using the proxy of the distance-to-default, the theoretical credit spreads and bond prices can explain about 79% of the observed series (total sample), according to the R^2 statistic. This shows that in addition to explaining a high proportion of the distance-to-default, we have also been able to explain a high proportion of the empirical credit spreads and bond prices.

3.7. Conclusions and Further Research

This research fits the Cathcart and El-Jahel (2003) (CEJ) model to prices of the Mexican Brady Par bond, using an extended Kalman Filter to estimate the default barrier. The model is appealing for two reasons: firstly because it incorporates two different ways of triggering default – it occurs either when a stochastic variable hits a barrier (in line with structural models) or when a jump event occurs (in line with reduced-form models); and secondly because it provides a semi-closed-form solution for zero coupon bonds, allowing relatively easy calculation.

The empirical results are encouraging in some ways, but discouraging in others. Beginning with the "discouraging" results, we find that the parameters of the hazard rate (jump) component are not significant and so the reduced-form features of the model make no contribution to its performance. There are several reasons why this might occur:

- The rationale for including a jump process in the CEJ model is to explain missing information about the dynamics of default that structural models have not been able to capture through diffusion processes. In this particular application the estimated structural variable has been able to explain most of the impact on Mexican bonds of major credit events over the last decade. Consequently there may be no need for a jump process.

- According to the model, movements of the hazard rate are determined by the short risk-free interest rate. However, we have found that a variable which is highly correlated with the estimate of the short rate, the level of the yield curve, explains part of the variance of the distance-to-default. In consequence of this double role of the risk-free rate, it is not surprising that the hazard rate is unable to explain additional variance, apart from that already captured by the distance-to-default.

- The properties of a hazard-rate process might be more relevant for short-term rather than long-term debt, since, according to the literature, reduced-form models have proved to be more effective than structural models when predicting credit spreads in the short term.

A further discouraging or disappointing result is that, in-sample, the sophisticated structural CEJ model is only slightly superior in terms of goodness-of-fit to an extremely simple structural model which assumes that the distance-to-default follows a random walk. The estimated drift of the distance-to-default for the CEJ model is quite small; therefore it is difficult to distinguish between this model and a random walk. Actually further tests indicate that the predictive power of both models is statistically not different from each other.

Turning to the more positive results, there are two in particular which are worth emphasising:

- We find that country fundamentals do play an important role in explaining most of the dynamics of the distance-to-default. 80% of changes in the latent variable of the CEJ model can be attributed to changes in three fundamental variables: the stock market level, the exchange rate and the shape of the yield curve. Surprisingly, the level of the stock market is found to be the most important determinant of the dynamics of the distance-to-default (accounting for approximately 52% of the total variance). The remaining 20% of unexplained variance could be attributable to external variables which we have not measured, such as contagion effects or international liquidity, or even to other non-quantifiable variables, such as country reputation, willingness-to-pay or bond-market sentiment. The exploration of these missing variables is a subject for further research.

- Changes in the distance-to-default are positively correlated with changes in the risk-free rate. Consequently (and in line with the literature on corporate rather than sovereign bonds) the risk-free rate is negatively related to credit spreads. This is a fundamental characteristic of structural models.

- We find that the same variables that explain the distance-to-default are also able to explain a high proportion of the observed credit spreads (79%). Therefore, it seems that credit spreads and the distance-to-default capture similar information. Further research needs to be done to analyse the theoretical and empirical differences between these two variables. Nevertheless, we believe that there are some advantages in modelling the distance-to-default rather than credit spreads. Firstly, the distance-to-default can be used to price other bonds issued by the same borrower. Following the same methodology we can infer the determinants of sovereign risk using actively traded instruments, then we can combine such determinants to produce the distance-to-default and price new issues or less liquid instruments. Secondly, by analysing the relationship between the distance-to-default of different borrowers, it may be possible to model credit correlations and price portfolios within a structural framework.

The general conclusion is that the factors affecting the prices of Mexican Brady bonds can be identified and combined into a single variable which reflects credit-worthiness. A structural model which uses this latent variable as the default barrier performs slightly better than a random walk, both in-sample and out-of-sample, but the gain is achieved at a considerable cost in extra complexity. Therefore, to answer the question of the chapter's title, structural models can help in explaining the prices of sovereign bonds, but there is still much to discover in this area.

Further research in several directions needs to be done. Regarding the specification of the model, here we have made important assumptions such as no-recovery or renegotiation of the risky coupons upon default. It would be worth analysing the robustness of these results under different scenarios of the recovery rate. Also, the CEJ model assumes no correlation between the risk-free process and the signalling variable driving default. Alternative specifications relaxing this assumption or modelling the hazard rate in terms of other fundamentals rather than the risk-free rate need to be tested. Regarding the determinants of the distance-to-default, other variables such as liquidity, contagion effects and the role of factors that determine the willingness to pay may be worth exploring. Finally, a more extensive analysis using a large sample of countries will give us more insights into the utility of structural models and the relationship of credit risk across countries.

Table 3.1. Summary Statistics of the Mexican Brady Par

	Mean	Std. Dev.	Minimum	Maximum	Skewness	Kurtosis	Autocorrelation
Prices	75.689	11.498	47.250	96.250	-0.266	2.347	0.943
Returns	0.001	0.046	-0.174	0.115	-1.081	5.162	0.005

Data is obtained from Datastream. Returns are calculated using the difference of natural logs of prices.
The sample contains 99 observations in the period December 1993-February 2002.
The Autocorrelation coefficient corresponds to the first order serially correlated coefficient.

Table 3.2. Summary Statistics of US Government Bond Yields

Maturity	Mean	Std. Dev.	Skewness	Kurtosis	Autocorrelation
3-month	0.0486	0.0101	-1.7705	5.8629	0.903
6-month	0.0500	0.0104	-1.6236	5.3386	0.914
1-year	0.0519	0.0107	-1.4292	4.9648	0.919
2-year	0.0542	0.0099	-0.9244	3.7958	0.923
3-year	0.0558	0.0091	-0.5566	3.1775	0.922
5-year	0.0576	0.0084	-0.1967	2.7162	0.927
10-year	0.0610	0.0075	0.1206	2.5191	0.936
30-year	0.0627	0.0072	0.4026	2.4304	0.949

Statistics are calculated from monthly US strips published by Bloomberg in the period December 1993-February 2002. Rates have been converted into continuously compounded rates.

Table 3.3. Estimation Results for the Parameters of the CEJ Model

	Hazard Rate		Latent Variable
	$a_r (x10^3)$	$b_r (x10^3)$	α_x
Parameter Value	0.0020 (0.0015)	0.0319 (0.0015)	0.3783** (26.71)
Log Likelihood	170.784		

The first row of the table represents the estimated values of the parameters. The figures in brackets correspond to the likelihood ratio statistics (LR) that test the significance of the parameter.
** parameters are significant at 99% confidence level.
a_r and b_r are the parameters of the hazard rate defined as $\lambda = a_r + b_r \cdot r_t$.
The critical value for the parameters a_r and b_r at 95% (see text for explanation) is equal to
$\frac{1}{2}\chi^2_{90\%}(0) + \frac{1}{2}\chi^2_{90\%}(1) = 1.35$.
α_x is the drift of the latent variable X_t and has been estimated using the specification of the following transition equation in the Kalman Filter (see text for explanation) :

$$Y_{t|t-1} = Y_{t-1} + \left(\alpha_x - \frac{\sigma_x^2}{2}\right)\frac{1}{12} + \sigma_x\sqrt{1/12}\eta_t, \quad \text{where } Y_t = \ln(X_t / X_\ell),$$

The critical value for α_x at 95% : $\chi^2_{95\%}(1) = 3.84$.
Variables are standardised by setting $\sigma_x \equiv 1$.

Table 3.4. Analysis of the Standardised Residuals of the Models

Panel A. Summary Statistics

	Median	Skewness	Kurtosis
CEJ Model	0.2150	-2.0480	9.6140
NM Model	0.2686	-2.7811	14.0038

Panel B. Diagnostic Tests

	Normality Test Jarque-Bera	Autocorrelation Q-stat (k=1)	Heteroscedasticity
CEJ Model	214.3479 (0.000)	0.1961 (0.274)	11.6935 (0.9971)
NM Model	538.4161 (0.000)	0.8359 (0.361)	5.2009 (0.9999)

p-values appear in brackets.

The standard residuals are defined as: $\tilde{v}_t = v_t / \sqrt{f_t}$, where $v_t = B_t - B_{t|t-1}$
For the description of the heteroscedasticity test see Harvey (pag 259).

Table 3.5. Comparison of the Models

Panel A. Goodness-of-fit Statistics

	SSE	R^2	R_D^2	**AIC**
CEJ Model	0.0865	0.9363	0.0651	0.0011
NM Model	0.0920	0.9323	0.0063	0.0011

The Sum of the Squared Errors is defined as $SSE = \sum_t v_t^2$

The Coefficient of Determination $R_D^2 = 1 - SSE / \sum_{t=2}^{N} (\Delta\ \boldsymbol{B}_t\text{- Mean}(\boldsymbol{B}_t))^2$

and AIC is the Akaike Information Criterion and is equal to AIC= $\tilde{\sigma}^2$ exp(2m/N), where m is the number of parameters to estimate, N is the number of observations in the sample and $\tilde{\sigma}^2$ is the variance of the model.

Panel B. Forecast Evaluation Measures

	RMSE	**MAE**	**MAPE**	**Theil's U**
CEJ Model	0.0317	0.0226	0.0327	0.0216
NM Model	0.0327	0.0229	0.0331	0.0223

Where

RMSE = Square Root of the Mean Squared Error
MAE= Mean Absolute Error
MAPE= Mean Absolute Percentage Error
Theil's U = Theil Inequality Coefficient

Table 3.6. Economic Interpretation of the Distance-to-Default

Dependent Variable: Δ (Distance-to- Default)$_t$

Variable	CEJ Model	NM Model
C	0.0059 (0.0167)	0.0009 (0.0011)
Δ ln(Stock Market Index)$_t$	1.5821** (0.1652)	0.1075** (0.0121)
Δ ln(Stock Market Index)$_{t-1}$	0.3093* (0.1431)	
Δ (Exchange Rate)$_t$	-0.0221** (0.0014)	-0.0017** (0.0002)
Δ (level of yield curve)$_{t-1}$	0.0609* (0.0281)	0.0054* (0.0021)
Δ (slope of yield curve)$_{t-1}$	0.1993** (0.0722)	0.0169** (0.0052)
Dummy$_{Aug-1998}$	-0.5688** (0.0745)	-0.0321** (0.0106)
R-squared	0.8116	0.7993
Durbin-Watson	1.7504	2.1697
F- statistic	54.5665	61.3190

Standard errors in brackets. * and ** mean that parameters are significant at 95% and 99% respectively.
Estimates are OLS where standard errors are adjusted for heteroscedasticity.
The number of adjusted observations is 83 (Feb 1994-Dec 2000).
The variable Dummy takes value one in August 1998 and zero otherwise.

Table 3.7. Predictive Power of the Proxy of the Distance-to-Default

	In Sample (Feb 94- Dec 00)			Out of Sample (Jan 01- Feb 02)		
	SSE	**RMSE**	**MAE**	**SSE**	**RMSE**	**MAE**
CEJ Model	0.0805	0.0310	0.0257	0.0652	0.0659	0.0531
NM Model	0.0902	0.0327	0.0267	0.0736	0.0700	0.0555

SSE= Sum Squared Errors.
RMSE= Square Root of the Mean Squared Errors.
MAE= Mean Absolute Error

Each model is run using a proxy of the distance-to-default. Such a proxy corresponds to the fitted values of the respective regression in *Table 3.6.*

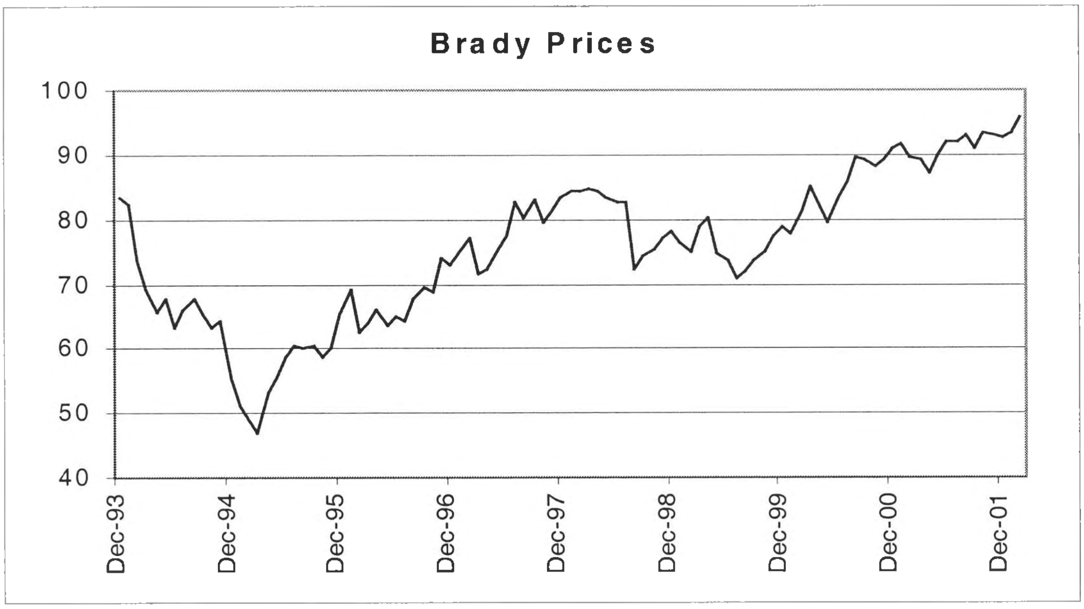

Figure 3.1. Prices of the Mexican Brady Par 6.25 of 2019

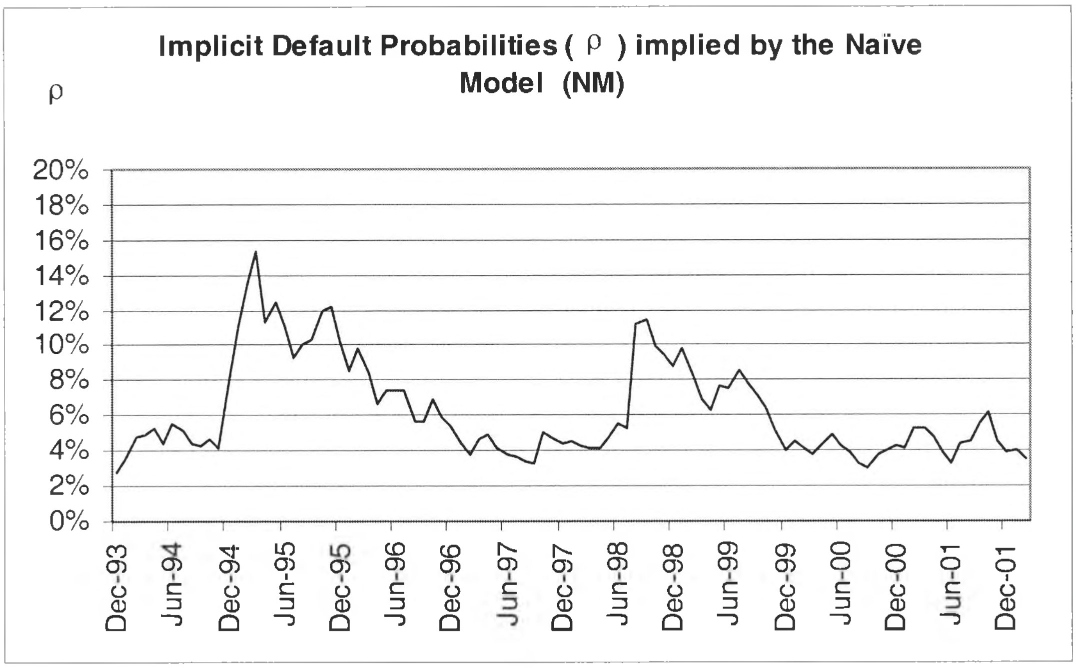

Figure 3.2. Implicit Probabilities of Default ρ_t

They are obtained from a model that assumes that at each point in time the probability of default at the end of period Δt is constant. The plot displays annual probabilities.

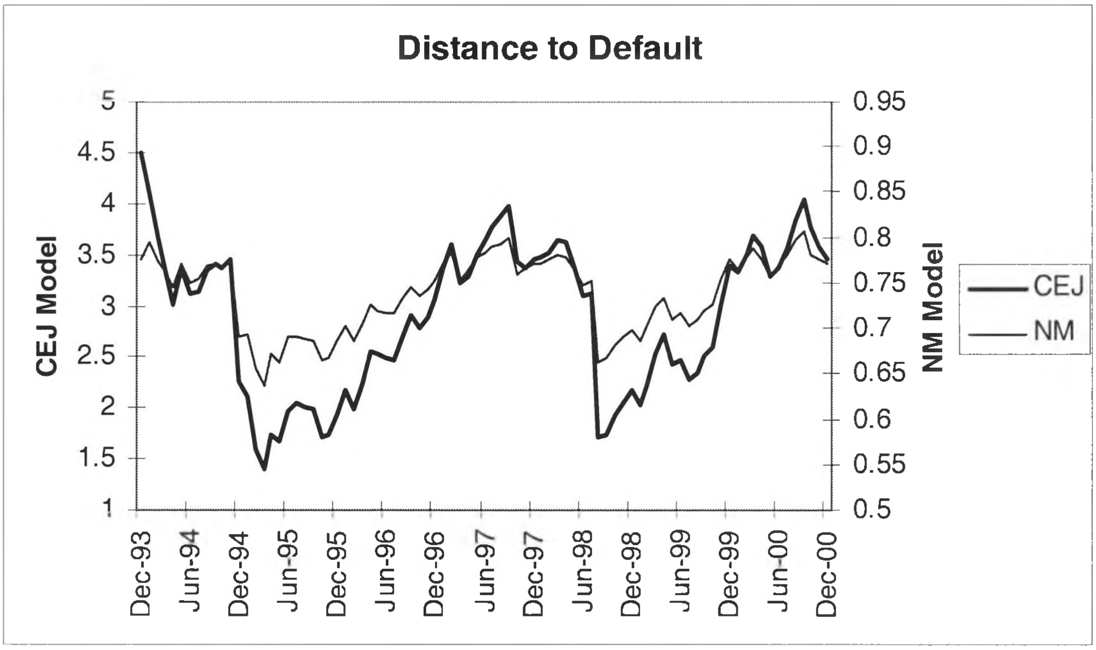

Figure 3.3. Distance-to-Default implied by the Models
The variables correspond to the updated estimates of the Kalman Filter

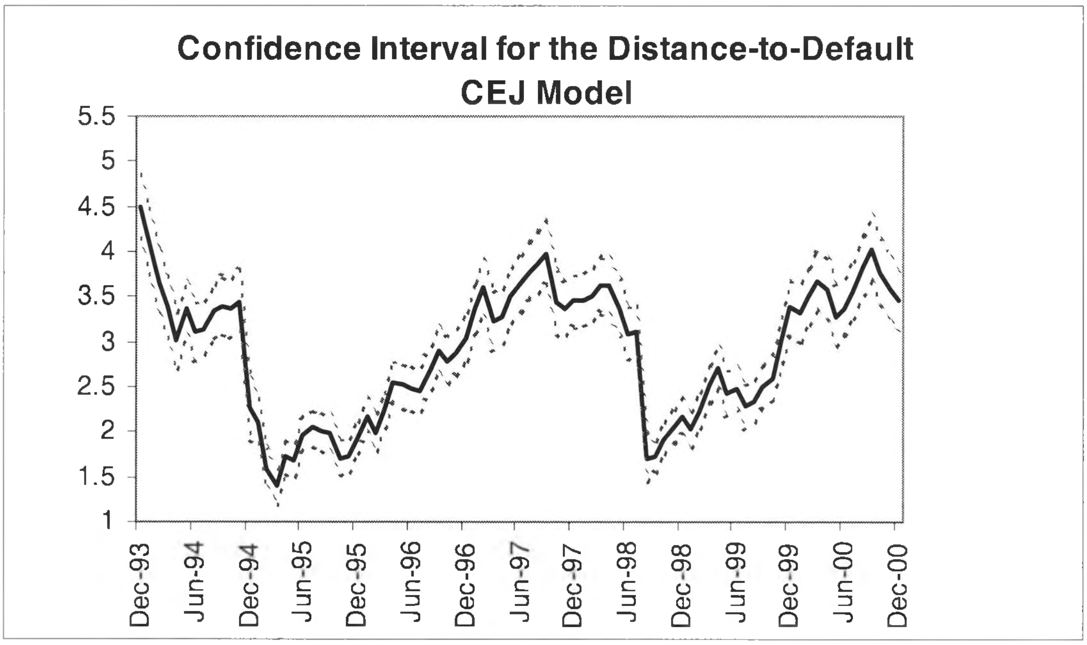

Figure 3.4. 95% Confidence Interval for the Distance-to-Default
Intervals are calculated as $\hat{Y}_t \pm 1.96\sqrt{P_t}$ *, where* P_t *is the variance of the updated estimates.*

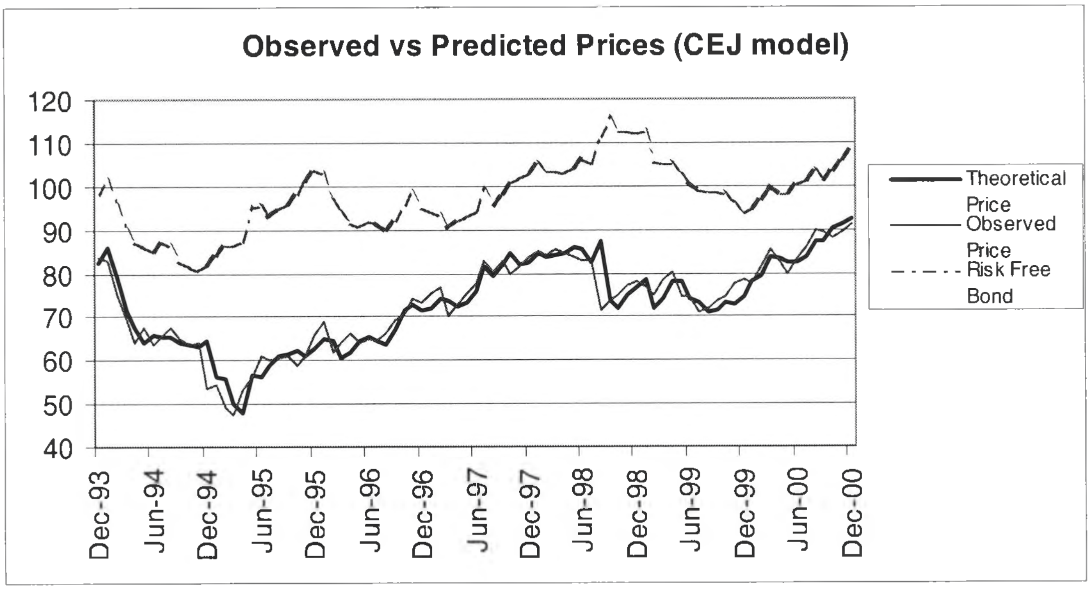

Figure 3.5. Actual against Fitted Prices
The plot corresponds to the actual price B_t (Observed Price), its one-step-ahead fitted values $B_{t|t-1}$ (Theoretical Price) and a Theoretical Risk-Free Bond with the same maturity and coupon as the risky bond.

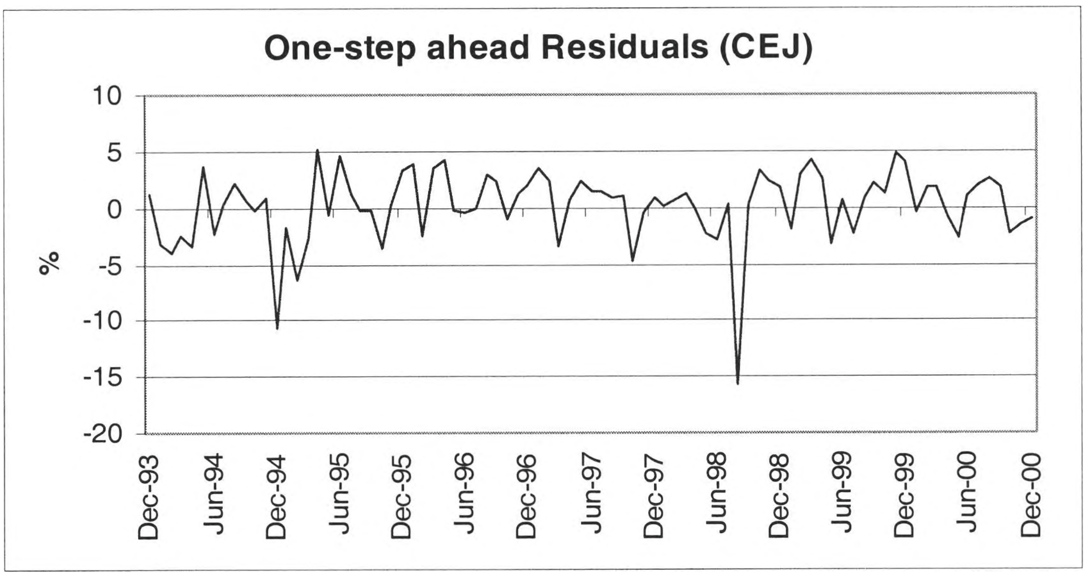

Figure 3.6. One-step-ahead Residuals calculated for the CEJ model.
The residuals are the difference between the observed price and the estimated price calculated with the one-step-ahead prediction: $v_t = B_t - B_{t|t-1}$

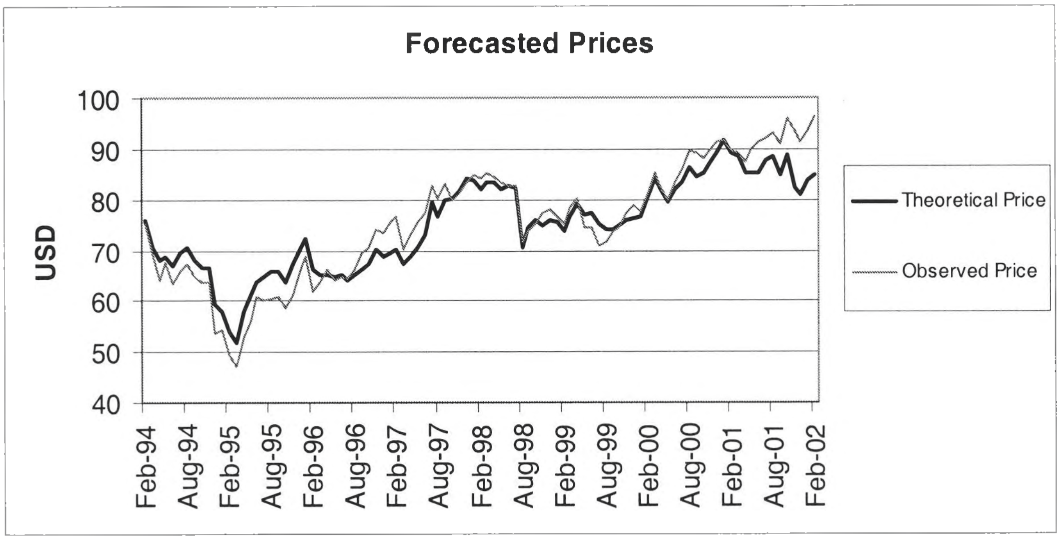

Figure 3.7. Forecasted Prices using a Proxy of the Distance-to-Default and the CEJ Model
The proxy of the distance-to-default is expressed in terms of economic fundamentals.

The proxy of the distance-to-default is defined as:

$$\Delta\ (\text{Distance-to-Default})_t = \underset{(0.0167)}{0.0059} + \underset{(0.1652)}{1.5821}\ \Delta\ \ln(\text{Stock Market Index})_t +$$

$$\underset{(0.1431)}{0.3093}\ \Delta\ \ln(\text{Stock Market Index})_{t-1} - \underset{(0.0014)}{0.0221}\ \Delta\ (\text{Exchange Rate}) + \underset{(0.0281)}{0.0609}\ \Delta\ (\text{level of yield curve})_{t-1}$$

$$+\underset{(0.0722)}{0.1993}\ \Delta\ (\text{slope of yield curve})_{t-1} - \underset{(0.0745)}{0.5688}\ \text{Dummy}_{Aug\text{-}1998}$$

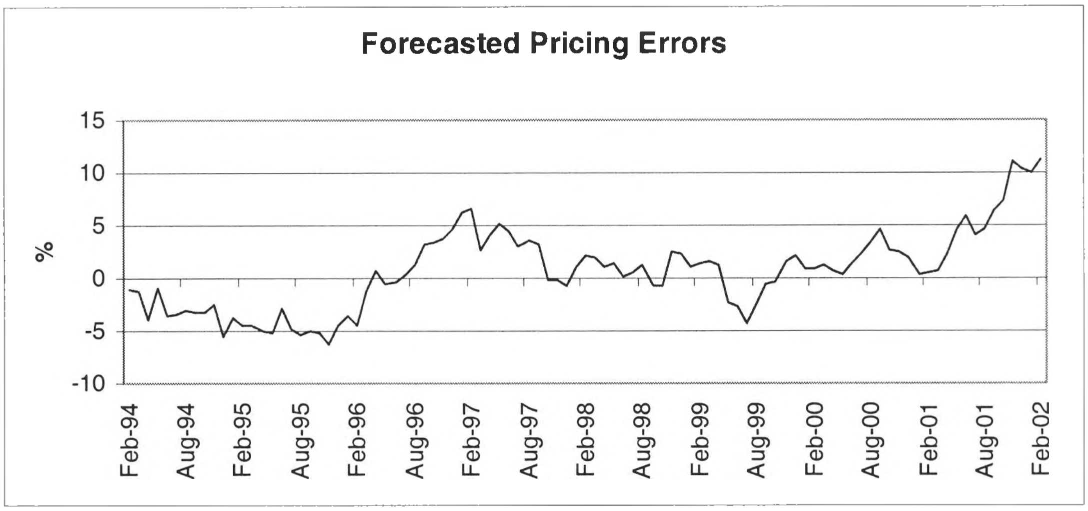

Figure 3.8. Forecasted Pricing Errors of the CEJ model
These are calculated using a proxy of the distance-to-default.

Appendix A: The Estimation of the Risk-Free Term Structure

In order to estimate the dynamics of the instantaneous nominal interest rate under the objective measure, we adopt the following formulation of the CIR model that considers the market price of risk:

$$dr_t = \left(\kappa_r\mu_r - \left(\kappa_r + \lambda_r\sigma_r\right)r_r\right)dt + \sigma_r\sqrt{r_r}\,dZ_r, \qquad r(0) = r_o$$

where Z_r is a Wiener process, μ_r is the long term mean, κ_r is the mean reversion parameter, λ_r is the market price of risk and σ_r is the constant volatility parameter. In addition the condition $2\kappa_r\mu_r > \sigma_r{}^2$ must be satisfied in order to guarantee positive r_t.

According to Cox, Ingersoll and Ross (1985), the nominal price at time *t* of a pure discount bond with face value of one dollar and time to maturity τ is:

$$P_t(\tau) = \tilde{A}(\tau)\exp\left(-\tilde{B}(\tau)r_t\right)$$

where

$$\tilde{A}(\tau) = \left(\frac{2\phi_1\exp(\phi_2\tau/2)}{\phi_4}\right)^{\phi_3}$$

$$\tilde{B}(\tau) = \frac{2\left(\exp(\phi_1\tau) - 1\right)}{\phi_4}$$

$$\phi_1 = \sqrt{(\kappa_r + \lambda_r)^2 + 2\sigma_r{}^2}, \quad \phi_2 = \kappa_r + \lambda_r + \phi_1, \quad \phi_3 = 2\kappa_r\mu_r / \sigma_r \text{ and}$$

$$\phi_4 = 2\phi_1 + \phi_2\left(\exp(\phi_1 T) - 1\right)$$

The yield to maturity at time *t* of a discount bond that matures at time τ is an affine function of the instantaneous interest rate r_t:

$$R_t(\tau) = -\frac{\ln P_t(\tau)}{\tau} = -\frac{\log\tilde{A}(\tau)}{\tau} + \frac{\tilde{B}(\tau)}{\tau}r_t$$

We estimate the parameters of the model by implementing the approach used by Geyer and Pitchler (1998) and Duan and Simonato (1995). They argue that by using a Kalman Filter we can incorporate all available information about the yield curve contained in time series and cross-sections. In their framework the system involves an observed variable which is the observed term structure, and an unobserved factor or variable that drives the dynamics of the term structure. The implementation of the Kalman Filter relies on the transition density of the unobservable variable $p(r_t | r_{t-1}; \Gamma)$, which for the CIR model is a non-central χ^2. The estimation of the model can be carried out by substituting for this transition density with a normal distribution with mean and variance equal to those of the non-central χ^2, and consequently our parameter estimates will be quasi-maximum likelihood.

The dynamics of the measurement equation for the observed yields $R_t(r_t, \Gamma, \tau_K)$ and the transition equation for the one-factor CIR model are defined as follows.

Measurement Equation:

$$\begin{bmatrix} R_t(r_t,\Gamma,\tau_1) \\ R_t(r_t,\Gamma,\tau_2) \\ \vdots \\ R_t(r_t,\Gamma,\tau_M) \end{bmatrix} = \begin{bmatrix} -\ln \tilde{A}(\Gamma,\tau_1)/\tau_1 \\ -\ln \tilde{A}(\Gamma,\tau_2)/\tau_2 \\ \vdots \\ -\ln \tilde{A}(\Gamma,\tau_M)/\tau_M \end{bmatrix} + \begin{bmatrix} \tilde{B}(\Gamma,\tau_1)/\tau_1 \\ \tilde{B}(\Gamma,\tau_2)/\tau_2 \\ \vdots \\ \tilde{B}(\Gamma,\tau_M)/\tau_M \end{bmatrix} r_t + \begin{bmatrix} \varepsilon_{t,1} \\ \varepsilon_{t,2} \\ \vdots \\ \varepsilon_{t,M} \end{bmatrix}$$

where Γ is the set of parameters, τ_K is the time to maturity, $\varepsilon_t \sim NID(0, Q)$ and Q is a diagonal MxM matrix that contains the variance of the errors for each maturity.

Transition Equation:

$$r_{t|t-1} = \frac{\mu_r}{12}(1-\exp(-\kappa_r/12)) + \exp(-\kappa_r/12) r_{t-1} + \eta_t$$

where $\sigma_\eta = \sigma_r^2 \dfrac{1-\exp(-\kappa_r/12)}{\kappa_r}\left(\dfrac{\mu_r}{2}[1-\exp(-\kappa_r/12] + \exp(-\kappa/12) r_{t-1}\right)$.

The implementation of the filter generates all the necessary information to calculate the Quasi-Maximum Likelihood function (QML) (see Harvey 1989, p. 126):

$$\ln L = -\frac{1}{2}N\ln(2\pi) - \frac{1}{2}\sum_{t=1}^{N}\ln F_t - \frac{1}{2}\sum_{i=1}^{t} v_t' F_t^{-1} v_t$$

where N is the number of observations, v_t is a *Nx1* vector of errors

$$v_t = R_t - \hat{R}_t(y_{t|t-1}),\quad F_t = \begin{bmatrix} \tilde{B}(\Gamma,\tau_1)/\tau_1 \\ \tilde{B}(\Gamma,\tau_2)/\tau_2 \\ \vdots \\ \tilde{B}(\Gamma,\tau_M)/\tau_M \end{bmatrix} P_{t|t-1} \begin{bmatrix} \tilde{B}(\Gamma,\tau_1)/\tau_1 \\ \tilde{B}(\Gamma,\tau_2)/\tau_2 \\ \vdots \\ \tilde{B}(\Gamma,\tau_M)/\tau_M \end{bmatrix}' + H;$$ and $P_{t|t-1}$ is the

conditional variance of r_t and $var(\eta_t) = H$.

A.1. Empirical Results

We use US interest rates for 8 maturities observed monthly during the period Dec 1993 to Dec 2000. The estimates of the parameters using Maximum Likelihood are shown in *Table A.3.1.* below. *Panel A* shows the estimates of the parameters μ_r, κ_r, σ_r and λ_r, with their respective p-values in brackets. All the parameters except λ_r are highly significant (at 95% and 99%).

On analysing the one-step-ahead residuals, we find that they are mainly negatively-biased across short maturities, and highly autocorrelated (see *Panel B*, below). The autocorrelation is higher for the short-end and long-end of the curve than for intermediate maturities. *Panel C* also shows the square root of the mean square error in basis points (RMSE). Observe that the RMSE statistic is also higher for the short-term and long-term maturities than for other maturities. In summary, it seems that the one-factor CIR model cannot account for all the dynamics of the term structure. This is actually consistent with the literature. Other empirical results also show that multifactor CIR implementations are also unlikely to fit the observed yield curve properly.

The implementation of the Kalman Filter provides an estimation of the factor that drives the whole term structure. In a one-factor model, such a variable is interpreted as the instantaneous interest rate and we denote it by $\hat{r}_t$. *Panel C* displays the correlations for the series in differences between the driving factor $\hat{r}_t$ and the yields from different maturities. Observe that the latent variable is more correlated with the intermediate maturities of the curve. This may support the fact that the model seems to adjust better for intermediate maturities.

Table A.3.1. Estimation Results for the CIR Model

Panel A: Parameter Estimates

	μ_r	κ_r	σ_r	λ_r
Parameter Value	0.0928** (0.0000)	0.0816** (0.0000)	0.1285** (0.0000)	-0.0280 (0.2444)
Log Likelihood	3397.727			

** Parameters are significant at 99% confidence level. p-values are shown in brackets.
Parameters are calculated using monthly observations of 3 and 6 months and 1, 2, 3, 5, 10 and 30-year US yields during the period December 1993 to December 2002.

Panel B: Statistics of the One-step-ahead Residuals.

Maturity	3M	6M	1Y	2Y	3Y	5Y	10Y	30Y
Skewness	-0.379	-0.286	-0.757	-0.479	0.019	0.370	0.449	0.550
Autocor.	0.837	0.792	0.669	0.366	0.321	0.569	0.772	0.923
RMSE (basispoints)	0.668	0.540	0.400	0.289	0.282	0.354	0.419	0.549

$\text{RMSE} = \sqrt{\text{mean(SSE)}}$, $\text{SSE} = \sum (y_t - \hat{y}_{t-1})^2$.

Panel C: Correlation of the Latent Variable with the Term Structure

Maturity	3M	6M	1Y	2Y	3Y	5Y	10Y	30Y
Correlation	0.389	0.686	0.884	0.963	0.999	0.978	0.918	0.830

Correlations are measured using the first differences of the yields for each maturity.

Chapter 4.

What drives Credit Risk in Emerging Markets? The Role of Country Fundamentals and Market Co-movements.

Abstract of Chapter 4

This chapter investigates how the creditworthiness of Argentina, Brazil, Mexico and Venezuela as reflected by their bond prices, is influenced by both global factors and country-specific fundamentals.

We use an extended structural model suggested by Cathcart and El-Jahel (2003) and a Kalman Filter to obtain the distance-to-default implicit in prices of each country's Brady bonds. We find that a small set of country fundamental variables and external factors, including a variable that measures market sentiment, are able to explain up to approximately 80% of the variance of the distance-to-default of each country. Whereas country specific factors are statistically significant in explaining the distance-to-default, external factors (such as the US stock market index, interest rates and market interdependence across countries) are much more important in explaining the dynamics of this variable. Using principal component analysis we find that there is a common factor for all of the countries which explains approximately 60% of the remaining variance (of the residuals). This common factor is therefore systematic and purely related to the bond market.

4.1. Introduction

Credit spreads are usually seen as a measure of the creditworthiness of corporate bonds or sovereign bonds issuers. Looking at their determinants, Collin-Dufresne et al. (2001b) show that changes in credit spreads on corporate bonds cannot be explained by changes in the expected default risk of the firm and that other identifiable variables, such as interest rates, explain little of the variation. Most of the risk is systematic and cannot be diversified away.

Within the literature on sovereign debt, a number of relatively recent papers have explored the determinants of credit spreads. Macroeconomic variables, such as GDP growth, inflation and US Treasury yields, are found to be important. However, Kamin and Kleist (1999), Eichengreen and Mody (1998) and Cantor and Packer (1996) have pointed out that it is not only country-specific fundamentals and external factors which drive fluctuations in sovereign spreads of emerging markets, but that "market sentiment" also seems to be important.

This chapter builds on the previous one and investigates a measure which is implicit in the market prices of sovereign debt, namely the distance-to-default. In particular we are interested in investigating the extent to which variations in the distance-to-default can be attributed to changes in common factors across countries (leading to contagion effects).

Trying to identify the content of the distance-to-default as a measure of creditworthiness of the country offers two main advantages over trying to explain credit spreads directly. Firstly, such a measure can be seen as a credit rating index in continuous time[1]. By extracting this measure from a credit risk model, we are isolating

[1] This approach is discussed by Claessens and Penacchi (1996) and Cumby and Evans (1995), Anderson and Renault (1999). They treat creditworthiness as an unobservable variable that follows a specific stochastic process. KMV Corporation (which is now part of Moody's) has also developed a creditworthiness variable ("the expected distance-to-default") based on a firm's equity market prices.

default risk from other factors that usually affect credit spreads, such as time to maturity, coupons, amortization schedules. Secondly, there is a potential application of such a measure within structural models (see for example Hull and White (2001) and Avellaneda and Zhou (2001)). Following the same idea from Reduced Form Models, where the correlation between issuers is imposed by correlating their hazard rates, in the case of structural models the relationship between issuers may be modelled by correlating their distances-to-default.

Understanding the variables that determine the dynamics of countries' creditworthiness is important for financial institutions. Furthermore, the analysis of the joint behaviour of sovereign credit risk and the mechanisms of contagion and default is vital for pricing, portfolio valuation, risk management and the regulation of financial institutions.

We use monthly Brady Bond prices of Argentina, Brazil, Mexico and Venezuela, during the period April 1994-October 2001. The advantage of using Brady Bonds is that they are highly liquid instruments. In addition, Brady Bonds are partially collateralised instruments; therefore such collateral can be considered as a proxy for the recovery rate. We use an extended structural model suggested by Cathcart and El-Jahel (2003) and a Kalman Filter to obtain the distance-to-default. As far as we know this model has not been tested empirically before. An additional attractive feature of this study is that most of the literature on testing pricing models has been developed around investment grade corporate bonds. In this chapter we use a structural model to price sovereign Brady bonds. Therefore this study provides further insights about both the use of these types of models to price sovereign bonds and the credit risk behaviour of non-investment rated instruments.

We parameterise the model following the methodology described in the previous chapter. Having extracted the implicit distance-to-default implied by bond prices for each nation, we then investigate its economic determinants. Global factors, such as the US stock market and common shocks transmitted via the stock market, are the most important variables for all of the countries. Other local variables, such as international reserves and inflation, are also significant though their contribution is less important. We also test the significance of a sentiment variable towards emerging markets. According to the literature (see for example De Long et al (1990)), investors who trade on sentiment may have a systematic impact. We construct an index from the discount of closed-end country funds that invest in Latin-American markets, which is believed to capture the dynamics of sentiment of foreign investors. We find that this variable is significant and important for Argentina, Brazil and Mexico, but not for Venezuela.

Another result is that there is a negative relationship between credit spreads and the risk-free interest rate. This confirms that the findings on corporate bonds also apply to sovereign bonds. Finally, we run a principal component analysis on the residuals of the above regressions and find that the first principal component captures approximately 60% of the residual variance. This indicates that, although important co-movements in the bond market are the result of contagion across stock markets, there is still some co-movement which has to do with the bond market only.

This chapter is organised as follows. Section 4.2 reviews the existing literature on the empirical determinants of credit risk. Section 4.3 introduces an extended structural model proposed by Cathcart and El-Jahel (2003) to price risky zero-coupon bonds, and a pricing model for Brady Par bonds. In Section 4.4 we present the data. Methodology and results of the estimated model are shown in Section 4.5. In Section 4.6 we analyse the variables that affect the distance-to-default. In Section 4.7 we present the results of estimating regression equations that explain the variations of the distance-to-default by using OLS. We also investigate the information contained in the residuals of these regressions. Section 4.8 gives the conclusions.

4.2. Literature Review

There is a wide literature which investigates the determinants of credit spreads on corporate bonds and sovereign bonds. In an empirical paper using a regression model, Collin-Dufresne, Goldstein and Martin (2001b) find that the variables predicted to be relevant by structural models, such as leverage ratio, interest rate, volatility and economic environment, explain at most 25% of the changes in credit spreads on corporate bonds. Most importantly, using principal components on the residuals of those regressions they find that there is an unobserved common factor that explains most of the residual variance. However they are unable to find any economic meaning for such a common factor. Elton, Gruber, Agrawal and Mann (2001) also find that default risk explains only around the 25% of corporate bond spreads. Other factors such as tax effects and a risk premium play an important role. They conclude that most of the risk in corporate bonds is systematic and cannot be diversified away.

Turning to sovereign bonds, Cantor and Packer (1996) conclude that per capita income, GDP growth, inflation and external debt are significant determinants of credit spreads for developed and developing countries. Kamin and Kleist (1997), Eichengreen and Mody (1998) and Cantor and Packer (1996) argue that it is not only country specific fundamentals which drive fluctuations in emerging market sovereign spreads, but also changes in market sentiment. For example Eichengreen and Mody (1998) find that changes in spreads are mainly due to shifts in market sentiment rather than in fundamentals.

In an extensive analysis, using spreads from 26 sovereign bonds, Westphalen (2002) finds that variables that are supposed to explain credit risk (according to structural models), explain no more than 20% of total variance. Some of the variables that he investigates are: the spot interest rate, the slope of the term structure and the distance-to-default, proxied as the ratio of debt service to exports. Using principal components, he concludes that there is a systematic factor explaining a significant part of the residual variance (67.2%). However he does not explore further the content of such a factor.

4.3. The Model

On the Credit Pricing literature there are two approaches to valuing risky debt. The Structural Approach has its origins in Merton (1974) (and has been extended in other directions by Black and Cox (1976), Longstaff and Schwartz (1995), Leland (1994) and Collin-Dufresne and Goldstein (2001a) among others). Under this framework default is defined as the first time a solvency variable (the firm's asset value) hits a particular barrier. This approach is conceptually appealing because it provides some insights into the default process of the firm in terms of firm-specific variables. However, an important drawback is that these models seem unable to produce the right size of credit spread close to maturity. One reason may be that the market value of the firm has been modelled as a diffusion process, and therefore the probability that an unexpected default occurs close to maturity is zero and the spread goes to zero. Another explanation is low liquidity, particularly near to maturity (see Ericsson and Renault, 2001).

The second approach, the Reduced Form Approach to valuing risky debt was introduced by Jarrow and Turnbull (1995). This assumes that default occurs by surprise, as the first jump of a Cox process (see also Duffee (1999), Duffie and Singleton (1999)). This approach is less intuitive than structural models, since default is driven by an exogenous variable; however these models are mathematically more tractable and allow calibrating credit spreads more easily.

We are going to use a Structural Model, which is extended by a hazard rate variable characteristic of Reduced Form Models, as explained in the next section.

4.3.1. An Extended Structural Model of a Risky Zero Coupon Bond.

We implement the model proposed by Cathcart and El-Jahel (2003), which is an extension of the original structural model proposed by Longstaff and Schwartz (1995). Apart from structural features, Cathcart and El-Jahel (CEJ) introduce a reduced-form feature: a stochastic hazard rate of default, which is a linear function of the spot interest rate, allowing for unexpected defaults. Thus default can occur smoothly (expectedly) when a signalling variable falls below a specific threshold or suddenly (unexpectedly) when a jump in the risk free interest rate occurs. The assumptions of the model are the following:

Assumption 1: Markets are frictionless and trading is carried out in continuous time. There are no taxes, transaction costs or informational asymmetries.

Assumption 2: The risk-adjusted dynamics of the short-term interest rate follow a CIR (1985) process:

$$dr_t = \kappa_r(\mu_r - r_t)dt + \sigma_r\sqrt{r_t}\,dZ_r \qquad (4.1)$$

where μ_r is the long-term mean of the interest rate, κ_r is the speed of adjustment of r_t towards the mean, σ_r is the volatility and Z_r is a standard Wiener process.

Assumption 3: Following the structural approach, there is a "signalling variable", X_t, which summarises the set of factors that determine the creditworthiness of the country. Under the risk neutral measure this variable follows a Geometric Brownian Motion:

$$dX_t = \alpha_x X_t dt + \sigma_x X_t dZ_x \qquad (4.2)$$

where α_x and σ_x are constants and Z_x is a standard Wiener process. Thus default occurs at the first time the signalling variable X_t hits a constant barrier X_ℓ.

Assumption 4: In line with reduced-form models, default can also occur unexpectedly as a jump event. The hazard rate is a linear function of the short-term interest rate: $h_t = a_r + b_r r_t$, where a_r and b_r are positive constants.

Assumption 5: If, during the life of the security, either the signalling variable hits the barrier X_ℓ, or a default jump occurs, then the bondholder receives a proportion δ of the bond face-value, where δ is the recovery rate.

In addition, the correlation between the signalling process and the interest rate is assumed zero[2]. In other words, the instantaneous correlation between Z_x and Z_r is zero.

Under the above assumptions, CEJ proves that the price of a risky discount bond may be expressed as:

$$H(x_t, r_t, \tau) = P_t(r_t, \tau) - P_t(r_t, \tau)(1 - g(r_t, \tau) f(x_t, \tau))(1 - \delta) \qquad (4.3a)$$

where

$$f(x_t, \tau) = \Phi\left(\frac{y + \left(\alpha_x - \frac{1}{2}\sigma_x^2\right)\tau}{\sigma_x\sqrt{\tau}}\right) - \exp\left(\frac{-2\left(\alpha_x - \frac{1}{2}\sigma_x^2\right)y}{\sigma_x^{\ 2}}\right)\Phi\left(\frac{-y + \left(\alpha_x - \frac{1}{2}\sigma_x^2\right)\tau}{\sigma_x\sqrt{\tau}}\right)$$

(4.3b)

$$y = \ln(x_t / x_\ell) \qquad (4.3c)$$

$$g(r_t, \tau) = \exp(C(\tau) + D(\tau) r_t) \qquad (4.3d)$$

and $C(\tau)$ and $D(\tau)$ are solutions to the following system of ordinary differential equations:

$$\frac{1}{2}\sigma_r^2 D(\tau)^2 + \left(\sigma_r^2 \tilde{B}(\tau) - \kappa_r\right) D(\tau) - D_\tau(\tau) - b_r = 0 \qquad (4.3e)$$

$$\kappa_r \mu_r D(\tau) - C_\tau(\tau) - a_r = 0$$

[2] This assumption facilitates the numerical solution of the model.

subject to the initial conditions C(0)=0 and D(0)=0[3].

The function $1-f(x_t,\tau)g(r_t,\tau)$ can be interpreted as the probability of default due either to the signalling process X_t hitting the default barrier X_ℓ, or to an unexpected jump in the interest rate r_t. Hence, the survival probability can be expressed as follows:

$$1-\gamma_t(\tau)=f(x_t,\tau)g(r_t,\tau) \qquad (4.4)$$

The key feature of the model is that a simple transformation of the signalling variable X_t can be defined as the distance-to-default and can be interpreted as a measure of creditworthiness of the country. Let $Y(t)=\ln(X_t/X_\ell)$ denote the risk-neutral distance-to-default process. Using Itô's lemma and equation 4.2, the risk-neutral distance-to-default satisfies the following diffusion equation:

$$dY_t=\alpha_Y dt+\sigma_Y dZ_Y \qquad (4.5)$$

$$\text{where } \alpha_Y=\left(\alpha_x-\frac{\sigma_X^2}{2}\right) \text{ and } \sigma_Y=\sigma_X \qquad (4.6)$$

We can think of this variable as a function of the asset value in the case of a firm (see Avellaneda (2001) and Hull and White (2001)), or in the case of countries as any combination of economic fundamentals that determines the probability of default. Another perspective is that this measure can be seen as a credit rating in continuous time (see for example the KMV methodology[4]).

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[3] $\tilde{B}(\tau)=\dfrac{2(\exp(\phi_1\tau)-1)}{\phi_4}$ as defined in the CIR model for the risk-free term structure. See Appendix A of Chapter 3.

[4] The KMV methodology was produced by the KMV Corporation in San Francisco. The company joined Moody's in 2002. The methodology has not been fully documented but some documents available are Kealhofer (1995) and Vasicek (1987).

4.3.2. The Pricing of a Par Brady Bond

In this section we discuss how the pricing of a risky zero coupon bond can be used to price a Brady Bond. Following Chapter 3, the price of a Brady Bond B_t can be seen as the sum of three components according to the following equation:

$$B_t = F \cdot P_t(r_t, T) + C \cdot F \sum_{i=1}^{q} P_t(r_t, \tau_i) + C \cdot F \sum_{i=q+1}^{N} (1 - \gamma_t(\tau_i - n)) P_t(r_t, \tau_i) \qquad (4.7)$$

Where F is the face value of the bond, C is the coupon rate, $P_t(r_t, \tau_i)$ is the price of a default free zero coupon bond at time *t* that matures at time τ_i, q is the number of guaranteed coupons, and $1 - \gamma_t(\tau)$ is the survival probability (i.e., the probability at *t* that no default has occurred prior to τ (τ>t)).

The first term of the above equation corresponds to the present value of the face value F with maturity T. The principal is fully guaranteed; therefore it is discounted at the risk-free rate. The second component accounts for the present value of q guaranteed coupons, each with maturity τ_i. The third term corresponds to the value of the risky coupons, so it takes account of the probability of default $\gamma_t(\tau)$. If n is the length of the rolling interest guarantee then each coupon with maturity τ_i is paid if and only if default has not occurred before τ_i-n. Observe that according to the formula we assume that the recovery rate is zero for any other cashflows not included in the rollover guarantee. Finally, in order to incorporate the CEJ model within the Brady pricing formula in equation 4.7 we only need to substitute the survival probability $1 - \gamma_t(\tau)$ by $f(x_t, \tau) g(r_t, \tau)$.

4.4. Data

We use end-of month market bond prices of Par Brady bonds from Argentina, Brazil, Mexico, and Venezuela reported by Bloomberg and Datastream, during the period

April 1994-October 2001. Brady bonds are dollar-denominated coupon bonds which are partially collateralised by highly-rated instruments and were issued by several emerging countries at the beginning of the 1990's under the Brady Plan[5]. There are two features of these bonds that facilitate the empirical implementation of a pricing model: firstly, Brady bonds are highly liquid instruments; secondly, Brady Bonds are partially collateralised. The total amount of the principal, and up to 18 months of rolling coupon payments, are guaranteed by AA or higher-rated securities. Thus such a guarantee may be considered as a proxy of the recovery rate of the bond, which is needed for pricing.

The characteristics of the bonds are displayed in *Table 4.1.* All the bonds were issued with an initial maturity of 20 years with semi-annual payments. In the case of Argentina and Brazil, their bonds were issued with initial coupon of 4%, but this rises in steps over time to reach 6% in the seventh year. For Mexico and Venezuela the rate of the coupon is 6.25% and 6.75% respectively, for the whole life of the instruments. The principal of all the bonds is guaranteed by a Treasury zero-coupon bond, and rolling coupons up to 18 months are also guaranteed.

Figure 4.1. shows the Brady Prices of the four countries. The effect of several crises can easily be observed in the figure. The end of 1994 and the first quarter of 1995 show a price fall which was due to the Mexican peso devaluation. Another dramatic fall occurs around August 1998, when Russia fell into default. Although for most of the time prices for the four countries remain very close to each other, the gap between each other widens at the end of the period.

Table 4.2. gives the descriptive statistics of the monthly bond prices and returns, during the period April 1994-October 2001. *Panel A* indicates that the first order autocorrelation parameter is quite high for all of the bonds, showing that prices may be non-stationary. Running an Augmented Dickey-Fuller test we cannot reject the

[5] The purpose of this plan was to reduce the sovereign debt in emerging countries.

hypothesis of a unit root in any of the cases. Therefore it makes more sense to analyse returns rather than prices. *Panel B* shows similar means and standard deviations for the returns of all the countries. Returns are slightly negative skewed and leptokurtic (kurtosis exceeds 3) in all the cases. The matrix of cross-correlations of the returns in *Panel C* shows how closely prices move together: the correlation coefficients vary between 0.66 and 0.76. Looking at *Figure 4.1.*, this common behaviour is more noticeable before the Russian crisis than after it. Such differences are more dramatic at the end of the sample, where the Argentinean default is approaching and the Mexican bond shows particular strength.

In order to estimate the parameters of the risk free process implicit in the model, we also collected monthly Treasury and Bonds rates with maturities: 3 months, 6 months, 1, 2, 3, 5, 10 and 30 years. Prices correspond to the strip rates published by Bloomberg.

4.5. Implementation of the Model

Following Chapter 3 we implement a Kalman Filter to estimate a latent variable, identified as the distance-to-default, and simultaneously estimate the parameters of the model for each country.

We assume that market bond prices $\boldsymbol{B}_t$ are observed with error ε_t. Thus the relationship between the signalling variable and observed prices is:

$$\boldsymbol{B}_t = \mathrm{B}(t, r_t, Y_t; \Psi, \Gamma) + \varepsilon_t \qquad (4.8)$$

where:

Y_t is the distance-to-default that determines the creditworthiness of the country and satisfies the dynamics of equation 4.5;

Γ is the set of parameters that determines the movements of the risk-free-term structure in the CEJ model[6] and is determined by a CIR process;

Ψ is the set of parameters that determines the risky parameters, i.e., those of the signalling process and of the hazard rate[7].

The error ε_t in equation 4.8 is assumed Gaussian distributed, with mean zero and variance σ_ε. This term is also an indicator of the adequacy of the model. If the true underlying process is not as in equation 4.5 then equation 4.8 will be misspecified and estimated prices will deviate systematically from the observed prices.

Observe that equation 4.8 is non-linear in Y_t and we should therefore apply an Extended Kalman Filter that consists of linearising function $B(\cdot)$ using the first order term of the Taylor expansion. Since it is not possible to apply a simple Kalman Filter our estimates will be Quasi-Maximum Likelihood.

Following Chapter 3 we implement the model using a two-stage procedure. In the first stage we estimate the parameters of the risk free generating process, which according to CEJ follows a one-factor CIR process. Those parameters are needed to calculate the probability of default ($\gamma_t(\tau)$). These parameters are taken from Chapter 3[8] (see Apprendix A of that chapter for the description of their estimation). In the second stage, we use the parameters of the CIR process and estimate the risky parameters, i.e., the parameters of the signalling process and those of the hazard rate, also using an Extended Kalman Filter. In addition, to calculate Brady prices according to equation 4.7 we also need the risk-free discount factors $P_t(r_t, \tau_i)$. We calculate them by fitting a cubic spline to the observed monthly yield-curve.

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[6] $\Gamma = \{\kappa_r, \mu_r, \sigma_r\}$ which is the set of parameters of a CIR process.

[7] $\Psi = \{\alpha_x, \sigma_x, a_r, b_r\}$

[8] Recall that the estimation of the CIR process is done by using an Extended Kalman Filter and Quasi-Maximum Likelihood. The Kalman Filter allows obtaining an implicit factor that drives the dynamics of

4.5.1. Estimation Results and the Distance-to-Default

Estimates of the parameters of the model are given in *Panel A* of *Table 4.3.* Variables are standardised by making ${\sigma_X}^2 \equiv 1$. *Figure 4.2.* shows the observed prices against the one-step-ahead estimated values $B_{t|t-1}$, giving an idea of the performance of the model to replicate market prices in-sample. The fit seems to be quite good. The graphs also show prices of bonds if they had been risk-free.

There are several features to observe in *Panel A* of *Table 4.3.* First, in all of the cases the parameters of the hazard rate are practically zero and not significant. Therefore, in all of the cases, the reduced-form hazard-rate feature does not provide any additional information to the structural framework. Since the hazard rate is not significant, the credit risk dynamics of the bond prices must depend totally on the signalling variable and its barrier. The further the signalling variable is from the default barrier, the greater the distance-to-default and the higher the price. The drift parameter α_X of the signalling variable (X_t) is positive and different from zero in all the cases and varies between 0.18709 and 0.35632. However, as mentioned before, the dynamics of the distance-to-default are represented by equation 4.5. Therefore the predictable component or drift is given by the difference $\left(\alpha_X - \sigma_X^2/2\right)$. Since the variance parameter σ_X^2 has been standardised to one, the long-term drift of the distance to default for the four countries is then negative. This means that lenders were increasingly pessimistic about the long-term future of these economies over time and expected a slight worsening on their creditworthiness. The most negative drift corresponds to the Argentinean distance-to-default, whereas the least negative drift is for the Mexican distance-to-default. This indicates that the market expected Mexico to have better creditworthiness and Argentina worse than the rest of the countries in the long-run.

the term structure, such a factor is interpreted as the short-term interest rate and determines the dynamics of the hazard rate under the CEJ model.

Panel B in *Table 4.3.* shows the diagnostic tests for each country. Test are carried out using the standardised one-step-ahead residuals defined as follows, according to Harvey (1989):

$$\tilde{v}_t = v_t / \sqrt{f_t}\,, \quad \textit{where } v_t = \mathbf{B}_t - B_{t|t-1} \textit{ and } f_t = \mathrm{var}(v_t)$$

Though there is a lack of normality for all of the countries' standardised residuals, according to the Jarque-Bera test, there is no evidence of autocorrelation or heteroscedasticity. Therefore, the models are not misspecified.

Figure 4.3. plots the one-step-ahead residuals for each country. It shows no evident systematic behaviour. Nevertheless the model fails to fit some important events properly, such as the Asian crisis in October 1997 and the Russian crisis in August 1998. Observe that in those events, estimated prices deviate from the observed ones by up to $20 (on a face value of $100).

It is important to point out that since the distance-to-default has been defined as the natural logarithm of the ratio between the signalling variable X_t and its barrier, then it is a standardised variable that can be compared across countries. Looking at *Figure 4.4.*, the implicit distance-to-default of the Argentinean bond seems to exceed consistently those of the other countries until the year 2000, when according to the perception of the market the economy starts showing signs of deterioration. This high credit rating of the Argentinean economy may be attributable to the peg of its currency with the dollar, given the image of a very strong economy. In contrast, the Venezuelan index systematically underperforms the other countries apart from the period that falls between the third quarter of 1996 and the Russian crisis of 1998. Between the Mexican and the Russian crises the four distance-to-default indexes show a sustained and joint recovery. This may have been attributable to the implementation of a series of structural reforms that took place during that period. Such reforms may have changed the perception of risk towards these countries. However, other theories indicate that the decline in credit spreads experienced during this period was the result of market liberalisation that stimulated an increase of capital flows and a willingness of industrial countries to diversify their portfolios. This feature is a puzzle and we will explore it further when we analyse the factors driving the distance-to-default of these countries.

In order to have an idea of the performance of the distance-to-default as a measure of creditworthiness, we will compare it with a Credit Rating Index calculated using the actual credit ratings issued by Standard and Poor's and Moody's. The actual categorical credit ratings produced by those two rating agencies[9] are converted into numerical indexes that go from 1 to 22, where 1 represents the worst credit rating and 22 the best one. The Credit Rating Index is calculated as a simple average of those two numerical indexes. Therefore, the larger the index the higher the credit rating of the country.

Figure 4.5. shows both the Credit Rating Index and the distance-to-default for each country. We can observe a fairly similar trend between the credit rating indexes and the distance-to-default for all of the countries. After the Mexican crisis and before the Russian crisis credit ratings predicted a modest and joint recovery for the four economies. The recovery in this period is more marked when measured by the distance-to-default. After the Russian crisis all the countries suffered a downgrading, according to both measures[10]. The country most affected by a downgrading from rating agencies is Venezuela, which is consistent with its dramatic fall in the distance-to-default.

Observe that whereas the distance-to-default for all of the countries registers a slight fall in October 1997 due to the Asian crisis, credit ratings remain stable. This indicates that the distance-to-default is a more sensitive measure than credit ratings. The predicted deterioration of Argentina is anticipated very much in advanced by the rating agencies. The index starts falling since the early 1999, whereas such deterioration

[9] We use the credit ratings assigned to the long-term debt issued by each country.

actually started to be perceived by the market, according to the distance-to-default in 2000. Other inconsistencies between the two variables can be appreciated from the plots. However this is not the focus of this chapter. A robust comparative analysis between these two measures, and their consistency as forward-looking measures of the ability of the country to pay are left for further research.

4.5.2 Descriptive Statistics of the Distance-to-Default

Table 4.4. gives the descriptive statistics of the distance-to-default for all the four countries. The series in levels have been modelled as non-stationary processes, which is confirmed by their Autocorrelation coefficient in *Panel A*, which is very close to one. Looking at the differences of the distance-to-default in *Panel B*, the statistics look very similar across all four countries. Series exhibit a mean close to zero, small standard deviation but negative skewness and high kurtosis. *Panel C* displays the correlation coefficients between the differences of the distance-to-default across countries. The coefficients indicate a high interdependence of credit risk across markets. Coefficients vary between 0.704 and 0.845.

A more accurate description of the co-movements of the distance-to-default is provided by Principal Component Analysis (PCA). The PCA is a decomposition of the risk of the four variables into uncorrelated factors. *Panel D* in *Table 4.4.* shows that the eigenvalue of the first component is 3.289, a figure very close to four[11], meaning that one factor is able to capture most of the total variability of the four variables, i.e, 82.2%. The coefficients of the first normalised eigenvector in *Panel E* indicate that the first component is a systematic factor, which affects the four indexes of creditworthiness similarly (in terms of impact and direction). These results anticipate that most of the dynamics of the distance-to-default should be attributed to systematic

[10] The fact that countries suffered a downgrading once that Russia defaulted, shows that credit rating failed to anticipate the Russian crisis. Some studies find evidence that credit ratings are backward-looking measures instead of forward-looking. See Kaminsky and Schmukler (2001).

[11] The variables are standardised, so the sum of the eigenvalues should add four.

factors or interdependence among countries rather than to country-specific movements. We will explore this further in the following section.

4.6. The Theoretical Determinants of the Distance-to-Default

In this section we discuss the variables that are relevant to explaining the distance-to-default. They will be introduced in two subsections, in which we will justify their importance. The first set of variables corresponds to global macroeconomic variables such as changes in the US Treasury curve, and the US stock market. The second set of variables consists of country-specific factors such as the inflation rate and changes in the level of international reserves. Within this subsection we will also explain the role of stock returns and stock market volatility. These two variables capture country-specific information but also international shocks[12].

4.6.1. Global Factors

The importance of global factors, in particular the role of US interest rates and US stock returns, in the development of Latin American countries, has been widely discussed in the literature on capital flows (see for example Chuhan (1998) et al, and Calvo et al (1993)). Here we will consider the following variables:

1) Interest Rates. Regarding the effect of interest rates on default, the literature is controversial. On the one hand higher interest rates increase debt-service burdens, decreasing the ability to repay and therefore increasing the possibility of default. Cline and Barnes (1997), in a study of 11 emerging markets, find a positive though insignificant effect of treasury rates on credit spreads during the mid-1990s. Kamin

[12] All data are on a monthly basis from April 1994 to October 2001. The data were taken from several sources including Datastream, Bloomberg, the Central Bank of each country, the US Federal Reserve System and the US Treasury Department.

and von Kleist (1999) find no statistically significant relationship between those variables, though the correlation is negative in some cases. Eichengreen and Mody (1998) examine the primary bond market during the period 1991-1995. They find that when the US treasury rates increase, only countries with good credit quality come to the market to borrow. Since there are few issuers, prices rise and consequently credit spreads fall.

Turning to the literature on companies, Longstaff and Schwartz (1995) and Duffee (1999) find that credit spreads are strongly negatively related to interest rates and that this variable is highly explicative of credit-spread movements on corporate bonds. Longstaff and Schwartz (1995) also conclude that this relation is more important for lower rating classes. The reason for a negative relationship between credit spreads and interest rates is that an increase in the level of the risk free rate implies a higher drift on the value of the firm's assets, so the incidence of default is reduced and consequently the size of credit spreads. Duffee (1998) finds that for all maturities and credit ratings, changes at the short end of the Treasury curve (measured by the 3-month Treasury Bill) are negatively and modestly related to changes in credit spreads. He also finds that the relationship with the slope of the term structure (the difference between the 3-month treasury and the 30 years treasury) is mostly negative and small.

We summarise the information in the yield curve by extracting principal components. We will use the first two components, which have traditionally been interpreted as the level and slope of the yield curve[13].

2) <u>S&P500 returns</u>. Several papers have argued that increased globalisation has increased the dependence of emerging markets on industrial countries. In particular,

[13] Recall that Cathcart and El-Jahel model the yield curve as a one-factor CIR process. Its estimation using a Kalman Filter generates a latent variable that drives the term structure and is identified as the short-term spot rate. We find that variations of this latent variable is highly correlated with the first principal component of the term structure (the correlation coefficient is equal to 0.975).

world economic conditions are likely to affect the creditworthiness of countries. We consider the US stock index as a proxy of the global economic performance.

3) Changes in Market Sentiment. Several authors have pointed out that, in addition to country-specific fundamentals, changes in market sentiment have been important in driving fluctuations in credit spreads on emerging markets debt (see, for example, Cantor and Packer (1996), Eichengreen and Mody (1998) and Kamin and von Kleist (1999)). Eichengreen and Mody (1998) argue that some participants in the bond market do not discriminate in an informed way among borrowers. Since information is costly, investors price bonds based on incomplete information about fundamentals, leading to herding behaviour that makes them behave in a synchronised way during specific periods. The discount of closed-end funds[14] has often been cited in the literature as a measure of sentiment of small investors. We construct an index of sentiment towards these countries, using data from 3 UK closed-end country funds that invest in Latin American shares[15]. An increase in the discount may be understood as a signal of deterioration of the perceived credit quality of the region.

4) Changes in Oil Prices. Oil products constitute an important part of the exports of Venezuela and Mexico. Hence oil prices significantly affect the budget deficit of those countries: the higher the price, the higher the revenues and consequently the higher the distance-to-default. The price considered here is that of Brent Crude.

4.6.2. Country-Specific Factors

We test several domestic variables as possible explanatory factors of the distance-to-default. The set of relevant variables can be summarised as follows:

[14] Discount is defined as the negative value of the premium, which is calculated as (Share Price-NAV)/ NAV, where NAV is the Net Asset Value.
[15] The closed-end country founds considered are: Abeerden Latin America, Deutsche Latin America and F&C Latin America. Historical share prices and NAV were obtained from Datastream.

1) Changes in International Reserves. This is considered as a measure of liquidity that captures all the external dynamics and the ability of the country to pay foreign debt. Thus, the higher the level of reserves the smaller the probability of default and therefore the higher the distance-to-default.

2) Inflation rate. It is often used as an indicator of a country's good management of its monetary policy. High inflation rates usually indicate imprudent policies such as excess borrowing, and therefore a higher possibility of default. Hence a negative effect of this variable on the distance-to-default is expected.

3) Changes in Volatility. Structural Approach Models predict that the higher the volatility of the assets, the higher the probability that the signalling variable will hit the default barrier. We calculate monthly standard deviations from daily stock market returns for each country (indexes are expressed in dollars) as a proxy of the volatility of the value of the country.

Common behaviour across stock returns is visible across volatilities. We will decompose the stock volatility of each country into a "Systematic Volatility", arising from the interdependence across countries, and a "Country-Specific Volatility", that reflects the reaction to domestic events. Using principal component analysis, we find that the first principal component can explain about 52% of the total variance of the changes in volatility for all the countries. Such a component will be our estimate for the Systematic Volatility. To estimate the Country-Specific Volatility factor, we regress the individual country stock volatility on the Systematic Volatility. The residuals of those regressions will be considered an estimate for the Country-Specific Volatility.

4) Stock returns. The importance of stock returns on credit spreads at the aggregate level, has been discussed extensively in the literature (see for example, Campbell and Ammer (1993), Fama and French (1993)). This variable can be seen as an indicator of the financial market conditions in the country. Alternatively, an increase in the stock

market means an increase in tax revenues and therefore the ability of the government to pay its debt.

Stock returns are likely to capture domestic and global shocks. A principal component analysis of this variable shows a high interdependence across countries and the presence of one Systematic Factor (1st Principal Component) explaining the 63.50% of the total variability of the four markets[16]. Exploring the determinants of such a systematic factor, we find that about 48% of the total variance of such a factor can be explained by S&P500 stock market returns, changes in the slope of the yield curve and the Systematic Volatility component obtained above. Therefore, in order to make the analysis more transparent and reduce multicollinearity between the variables, we will use the following decomposition of stock returns as explanatory variables: 1) a Country-Specific Factor affecting the country stock market. This factor is approximated by the residuals of the regression of stock market returns on the Systematic Factor; 2) identifiable variables that explain the systematic factor: S&P 500 returns, changes in the slope of the yield curve and Systematic Volatility; and 3) an Unexplained Systematic Factor that has not been explained by the previous variables.

4.7. Why do Credit Risk evolve together across Countries? The Empirical Results

A fundamental issue of debate among investors is the extent to which the credit risks of these countries evolve together. Do investors differentiate credit risk between different countries? In the previous section, a correlation analysis across the countries' distance-to-default indicated that creditworthiness moves very closely for the four nations; hence there must be important common determinants. It should not be surprising to find co-movements across the credit risk of different issuers, since the distance-to-default is likely to be affected by aggregate variables such as the risk free interest rate, market volatility and general condition of the global economy. The interesting point is to what extent the interdependence across markets is relevant to determining credit risk, and also whether we can explain all the common factors that underlie such interdependence.

Next we investigate how well the variables explained in the previous section can explain changes in the distance-to-default. We should point out that the original database consisted of a wider set of variables than that discussed above, including several lags for each variable[17]. The final model, below, was selected by applying the General-to-Specific Approach (see Hendry and Doornik, 2001) for each country:

$$\begin{aligned}
&\Delta(\text{Distance-to-default})_t = \alpha + \gamma_1(\text{R.S\&P500})_t + \gamma_2\Delta(\text{Slope.Treasury})_t + \gamma_3\Delta(\text{Sentiment})_t + \\
&\gamma_4\Delta(\text{SMkt.Country.Specific.Factor})_t + \gamma_5\Delta(\text{Systematic.Volatility})_t + \\
&\gamma_6\Delta(\text{SMkt.Unexplained.Systematic.Factor})_t + \gamma_7\Delta(\text{Country.Specific.Volatility})_t + \\
&\gamma_8\Delta\log(\text{Reserves})_t + \gamma_9\text{Inflation}_t + \gamma_{10}\Delta\text{Oil}_t + \varepsilon_t \\
&\qquad\qquad\qquad\qquad\qquad \varepsilon_t \sim N(0,1)
\end{aligned}$$

where the description of the variables is as follows:

$(\text{S\&P500.Returns})_t$ = return of the US stock index, S&P500.

$\Delta(\text{Slope.Treasury})_t$ = changes in the slope of the US Treasury Yield Curve.

$\Delta(\text{Sentiment})_t$ = changes in Market Sentiment, measured as a discount index of country funds.

$\Delta(\text{SMkt.Country.Specific.Factor})_t$ = changes in the Country-Specific Factor affecting stock market returns.

$\Delta(\text{Systematic.Volatility})_t$ = the Systematic Component that affects the volatility changes of each country.

$\Delta(\text{SMkt.Unexplained.Systematic.Factor})_t$ = the Systematic Component affecting stock market returns of the four countries which has not been explained by S&P500 returns, Systematic Volatility and Risk Free Term Structure.

$\Delta(\text{Country.Specific.Volatility})_t$ = changes in Country-Specific Volatility

$\Delta\log(\text{Reserves})_t$ = variations of the level of international reserves.

Inflation_t = the country's rate of inflation.

ΔOil_t = Brent Oil price variations.

[16] See Appendix A for a correlation analysis of the stock returns.

[17] Some of the variables include US industrial production, total external debt in percent of GDP, exports/Industrial production, exchange rate.

Table 4.5. presents the results calculated using OLS and several lags of each explanatory variable. Whenever necessary, standard errors were corrected for heteroskedasticity using the White method. We omitted those variables which were not significant.

All the regressions show the expected sign for all the coefficients. The adjusted R-squared ranges from 60.4% for Venezuela up to 81.1% for Brazil. According to the diagnostic statistics, the hypothesis of normality of the residuals cannot be rejected. The Durbin Watson statistics tell us that there is no evidence of autocorrelation in any of the cases. Also, using the CUSUM test and the CUSUM of squares test we did not find evidence of instability in the parameters (see *Figure 4.6.* for the CUSUM of squares test). Hence the models seem well specified.

The results suggest the following:

1. The return of the S&P 500 (expressed in dollar terms) is highly significant and positive for all four countries. A simple regression of the distance-to-default on this variable accounts for between 21% and 26% of the total variance for each country. The dependence of emerging markets on the US economy has been documented by Arora and Cerisola (2001) and Calvo et al (1993). They argue that the increase of capital flows to developing markets and the structural reforms on U.S. monetary policy during the 1990s, have had a cost on the availability of funds and creditworthiness of developing countries. Here an increase in return of one percent in the S&P 500 produces a similar impact of about 2.50% on Argentina, Brazil and Mexico. The impact on Venezuela is more significant: 3.78%.

2. The common component of volatility (Systematic Volatility) is negative and significant in all cases. Coefficients vary within a very small range, between –0.058 and –0.040, suggesting a similar impact on the distance-to-default across countries. Running univariate regressions, we find that this variable is highly explicative. It

explains about 18% of the total variance of the distance-to-default of Argentina, Brazil and Mexico, and around 11% of Venezuela's[18].

3. The <u>slope of the yield curve</u> is significant and positive,[19] though its contribution to the total variance is very little. The positive coefficient is consistent with the conclusions of Duffee (1998) and Mansi, and Maxwell (1999) for companies. Following Collin-Dufresne et al (2002), an increase in the slope of the Treasury curve should raise the expected future short rate, increasing the distance-to-default. Contrasting other results, changes in the level of the yield curve, though positive, is not significant (therefore the variable was eliminated from the regression).

4. Another important variable that explains a high proportion of the total variance of the distance-to-default is the <u>Unexplained Systematic Factor</u> that affects stock markets. According to the R^2 statistic, this variable explains about 23% of the distance-to-default of each country.

5. Another interesting finding relates to changes in <u>market sentiment</u>. This coefficient is always significant and negative for all the countries, apart from Venezuela (whose coefficient is negative but not significant). There is also evidence of persistent sentiment in the case of Argentina and Brazil. In the case of Venezuela, the amount of its Brady debt is significantly smaller that for the other countries, so it is plausible that such a debt is traded by institutional investors rather than by noise investors.

6. The importance of the country-specific factor affecting stock market returns (<u>SMkt Country-Specific Factor</u>) is more relevant for Argentina and Mexico. The coefficients

[18] Stock market volatility has commonly used in the literature as a variable that measures turbulence in the markets or market sentiment. A scatter plot between the Systematic Volatility term and our market sentiment variable reveals no linear relationship between these two variables, eliminating the possibility of multicollinearity in the regression.

[19] Similar results are obtained when instead of using the second component of the yield curve, a new variable defined as the difference between the 30-year and 3-months Treasury Bill is calculated. Such a variable is interpreted as the slope of the yield curve and its correlation coefficient with the second component is approximately 0.93.

are 1.223 and 1.545 respectively, whereas for Venezuela the impact of this variable is much less, its coefficient being 0.556.

7. The level of reserves is only significant for Argentina and Mexico. Their coefficient is positive as expected. A high level of reserves will increase the ability of the country to pay its debt and therefore will increase its creditworthiness.

Overall, the results show that S&P 500 returns have very significant effects on the creditworthiness of emerging markets. Other systematic factors such as stock market volatility or an unexplained systematic factor seem to be much more relevant than country-specific factors in explaining changes in the distance-to-default. Applying principal component analysis on the residuals of these regressions, we find that there is still a systematic factor explaining about 60% of the remaining variance. Having accounted for systematic factors coming through the stock market, this new systematic factor should be purely related to the bond market.

4.8. Conclusions and Implications

Using an extended structural model, we extracted a measure of the creditworthiness of four emerging economies implicit in Brady bond prices. Such a measure is associated with the distance to default and provides a continuous indicator of the perception of credit risk across time. We assessed the importance of country-specific factors and global variables in influencing this variable. The main findings of this analysis can be summarised as follows.

- We were able to explain about 80% of the total variance of the distance-to-default for Argentina, Brazil and Mexico. For the case of Venezuela we explained around 60%.

- The driving sources of credit risk of these emerging markets can be split into three elements. One element, which is the least relevant, is the result of shocks through country-specific fundamentals. These shocks represent about 10% of the total explained variance of the distance-to-default of Argentina, Brazil and Venezuela, but they represent 25% in the case of Mexico. The second element is the result of US variables, such as stock market returns and the slope of the treasury bond curve. Such variables contribute about 30% of the total explained variance. The third and most important element is the contribution of common factors such as an unexplained systematic component of the stock markets, a systematic volatility component and market sentiment. These variables represent around 50% of the total explained variability.

- Consistent with the literature on companies, interest rates play a role, with the slope of the yield curve being significant. However their contribution is not very important.

In summary, we found that the distance-to-default is largely driven by systematic factors; therefore bond investors view countries' credit risk as non-diversifiable. The fact that common factors or contagion variables may be very important in determining the creditworthiness of a country has implications for pricing and risk management of bond portfolios. Structural models do not consider such contagion effects, yet it seems that these effects may be the most relevant to model. This goes along with the findings of Collin-Dufresne et al (2001b) that credit spread changes are principally driven by systematic factors. Therefore further research on pricing models that takes into account this empirical evidence needs to be done.

One shortcoming of our regression analysis is that we have fitted only one set of parameters for the whole sample. Though stability tests indicate reasonable robustness in our results, it is very likely that correlations between variables change over time. A larger sample period and possibly different econometric technique are needed to explore this changes in parameters across time. Also a larger sample of countries from different regions would be useful to study, to see if changes in the distance-to-default are determined by similar factors. The search for the best proxies for global factors and country-specific fundamentals is a complicated task. Other model specifications or proxies for determining creditworthiness should also be able to be tested.

Furthermore, other variables such as liquidity, left out of this study, are worth exploring in future research. It is very likely that liquidity plays an important role in explaining contagion or systematic movements across countries, which we were unable to explain.

Finally, a large literature on the determinants of credit spreads on corporate bonds and sovereign bonds has been produced. Though intuitively both measures, credit spreads and distance-to-default, should move in opposite directions, there are some other factors such as maturity, collateral, or different coupons affecting the size of credit spreads. Further research is needed to analyse the relationship between credit spreads and the distance-to-default.

Table 4.1. Characteristics of Brady Par Bonds

Country	Issue Date	Principal Amount (US.Bin)	Semi-Annual Coupon	Final Maturity	Collateral/Interest Guarantees**
Argentina	Apr-93	14.9	Step- up*	Apr-23	Z-C/12 months
Brazil	Apr-94	8.4	Step- up*	Apr-24	Z-C/12 months
Mexico	Mar-90	22.6	6.25%	Dec-19	Z-C/18 months
Venezuela	Dec-90	6.7	6.75%	Mar-20	Z-C/14 months

* In the case of Argentina and Brazil the first coupon is 4% but this increases periodically up to 6% in year seven.

** Z-C means that the principal is collateralised by zero-coupon U.S. Treasury bonds.

Table 4.2. Summary Statistics of Monthly Brady Prices and their Returns

	Mean	Std Dev	Minimum	Maximum	Skewness	Kurtosis	Autocorrelation
Panel A: Prices							
Argentina	62.067	9.684	39.000	76.750	-0.522	2.313	0.931
Brazil	59.284	10.396	36.125	75.500	-0.575	2.281	0.938
Mexico	74.954	11.315	46.906	96.370	-0.340	2.421	0.942
Venezuela	67.390	12.368	42.875	88.250	-0.360	2.123	0.940
Panel B: Returns							
Argentina	0.001	0.057	-0.216	0.117	-0.832	4.532	-0.128
Brazil	0.005	0.054	-0.208	0.109	-0.910	4.738	-0.062
Mexico	0.004	0.043	-0.155	0.125	-0.901	5.337	0.020
Venezuela	0.005	0.057	-0.253	0.116	-1.102	6.384	-0.128

Panel C: Correlations of the returns

	Argentina	Brazil	Mexico	Venezuela
Argentina	1			
Brazil	0.755	1		
Mexico	0.716	0.717	1	
Venezuela	0.701	0.662	0.673	1

Data is obtained from Bloomberg and Datastream. The sample corresponds to monthly observations of prices during the period April 1994-October 2001. Returns are calculated as the difference of natural logs of prices. The Autocorrelation coefficient corresponds to the first order serially correlated coefficient.

Table 4.3. Estimation Results and Diagnostic Tests

Panel A: Estimation Results for the parameters of the CEJ model

Country	Hazard Rate		Latent Variable	Log Likelihood function
	$a_r\,(x10^3)$	$b_r\,(x10^3)$	α_x	*QML*
Argentina	0.00000	0.00114	0.18709* (4.136)	176.42
Brazil	0.00010	0.00172	0.23020** (8.815)	182.73
México	0.00087	0.00413	0.35632** (20.900)	186.88
Venezuela	0.00000	0.00003	0.27274** (14.135)	169.18

The figures in brackets correspond to the likelihood ratio statistics (LR) that test the significance of the parameter.
* and ** mean that parameters are significant at 95% and 99% confidence level, respectively.
a_r and b_r are the parameters of the hazard rate defined as $\lambda = a_r + b_r \cdot r_t$.

α_x is the drift of the latent variable X_t and has been estimated using the specification of the following transition equation in the Kalman Filter:

$$Y_{t|t-1} = Y_{t-1} + \left(\alpha_x - \frac{\sigma_x^2}{2}\right)\frac{1}{12} + \sigma_x\sqrt{1/12}\eta_t, \quad \text{where } Y_t = \ln(X_t / X_\ell),$$

The critical value for the significance of α_x at 95% is $\chi^2_{95\%}(1) = 3.84$.
Variables are standardised by setting $\sigma_x \equiv 1$.

Panel B: Diagnostic Tests

	Normality Test Jarque-Bera	Autocorrelation Q-stat (k=1)	Heteroscedasticity
Argentina	391.6993 (0.000)	1.0016 (0.317)	26.9976 (0.559)
Brazil	431.5358 (0.000)	0.9265 (0.336)	30.7267 (0.385)
Mexico	204.1265 (0.000)	0.2993 (0.584)	13.2891 (0.996)
Venezuela	371.0545 (0.000)	0.2213 (0.638)	22.0513 (0.852)

p-values are shown in brackets.

The standard residuals are defined as: $\tilde{v}_t = v_t / \sqrt{f_t}$, where $v_t = B_t - B_{t|t-1}$
For the description of the heteroscedasticity test see Harvey (pag 259).

Table 4.4. Summary Statistics of the Distance-to-Default

	Mean	Std Dev	Minimum	Maximum	Skewness	Kurtosis	Autocorrelation
Panel A: Levels							
Argentina	3.518	0.871	1.596	5.644	0.335	2.648	0.910
Brazil	3.094	0.826	1.743	5.116	0.698	2.485	0.927
Mexico	2.989	0.678	1.462	4.067	-0.483	2.008	0.903
Venezuela	2.632	0.944	1.246	5.087	0.833	2.611	0.945
Panel B: Differences							
Argentina	-0.028	0.301	-1.675	0.341	-2.354	12.450	0.106
Brazil	-0.007	0.296	-1.625	0.525	-2.285	12.694	0.100
Mexico	0.000	0.297	-1.378	0.429	-1.949	9.125	0.060
Venezuela	0.001	0.300	-1.695	0.480	-2.089	12.644	0.054

Panel C: Correlations of the differences of the Distance-to-Default

	Argentina	Brazil	Mexico	Venezuela
Argentina	1			
Brazil	0.845	1		
Mexico	0.766	0.747	1	
Venezuela	0.768	0.744	0.704	1

Panel D: Loadings of the PCA of the variations of the Distance-to-Default

	Comp 1	Comp 2	Comp 3	Comp 4
Eigenvalue	3.289	0.296	0.263	0.152
Variance Prop.	0.822	0.074	0.066	0.038
Cumulative Prop.	0.822	0.896	0.962	1.000

Panel E: Eigenvectors

Variable	Vector 1	Vector 2	Vector 3	Vector 4
Argentina	0.515	0.007	-0.382	0.768
Brazil	0.508	-0.027	-0.585	-0.632
Mexico	0.488	-0.696	0.523	-0.061
Venezuela	0.488	0.717	0.489	-0.091

The distance-to-default has been estimated by fitting the CEJ model in the period April 1994-October 2001.

Table 4.5. Determinants of the Distance-to-Default

Variable	Argentina	Brazil	Mexico	Venezuela
Intercept	-0.061 (-3.427)	-0.016 (-1.042)	-0.022 (-1.286)	-0.040 (-1.737)
$(S\,\&\,P500.Returns)_t$	2.899 (7.364)	2.223 (6.359)	2.390 (6.777)	3.379 (5.807)
$(S\,\&\,P500.Returns)_{t-2}$	0.889 (2.380)			
$\Delta(Slope.Treasury)_t$	0.041 (2.902)	0.043 (3.228)	0.039 (2.212)	0.056 (3.417)
$\Delta(Slope.Treasury)_{t-1}$		-0.027 (-2.096)	-0.037 (-3.056)	
$\Delta(Sentiment)_t$	-0.017 (-3.082)	-0.019 (-4.754)	-0.010 (-2.226)	
$\Delta(Sentiment)_{t-1}$		-0.015 (-3.975)		
$\Delta(Sentiment)_{t-2}$	0.014 (3.372)			
Δ (SMkt.Country. Specific.Factor)$_t$	1.223 (4.266)	0.961 (4.464)	1.545 (4.849)	0.556 (2.589)
Δ (SMkt.Country. Specific.Factor)$_{t-2}$				0.458 (2.119)
$\Delta(Systematic.Volatility)_t$	-0.040 (-2.682)	-0.058 (-4.960)	-0.053 (-4.107)	-0.048 (-2.900)
Δ (SMkt.Unexplained. Systematic.Factor)$_t$	0.117 (7.800)	0.132 (10.456)	0.116 (9.668)	0.102 (5.947)
$\Delta OilP_t$			0.016 (2.204)	0.019 (2.327)
$\Delta\log(Reserves)_t$	0.742 (2.800)		0.323 (3.565)	
$Dummy_{Oct\text{-}97}$	-0.465 (-4.321)	-0.512 (-3.341)	-0.250 (-2.775)	
R-squared	0.790	0.811	0.786	0.604
Adjusted R-squared	0.760	0.790	0.759	0.570
S.E. of regression	0.147	0.136	0.146	0.196
Durbin-Watson stat	1.788	1.737	1.977	1.956
F-statistic	26.296	37.774	28.67	17.671

The set of data is composed of monthly observations from the period April 1994 to October 2001. Regressions are run by using Ordinary Least Squares, where the dependent variable is the first difference of the distance to default. Whenever necessary, standard errors are adjusted for heteroskedasticity. The numbers in parenthesis correspond to the t-statistics.

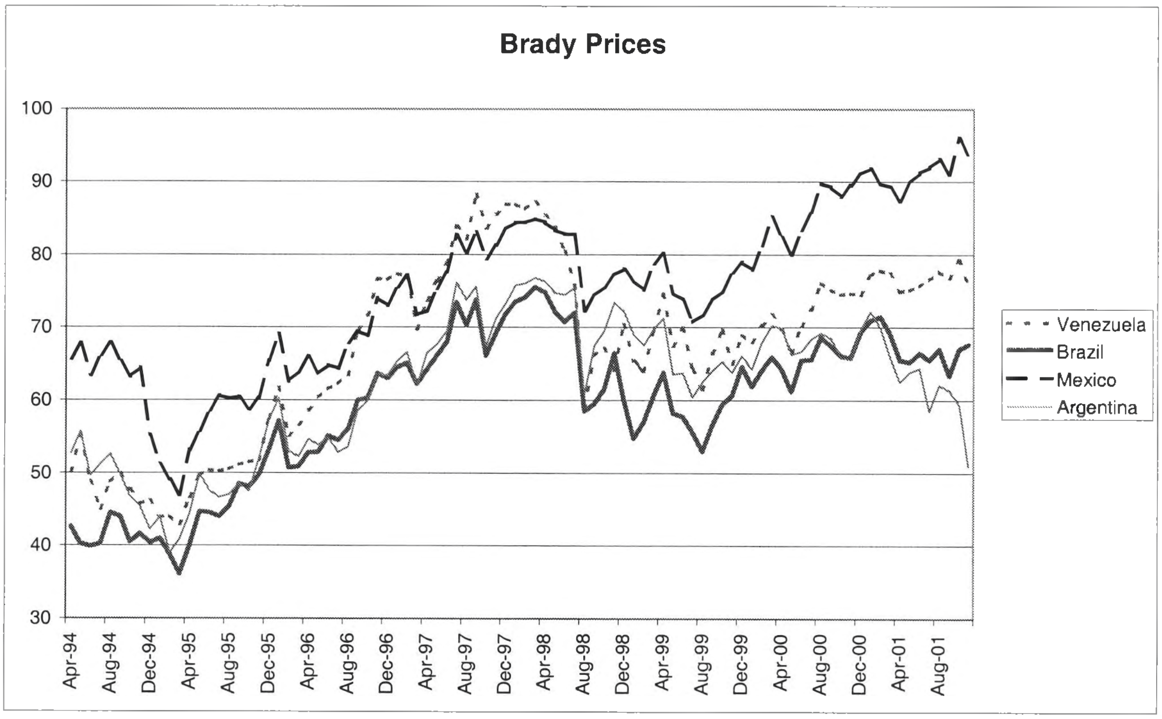

Figure 4.1. Market Prices of Brady Bonds

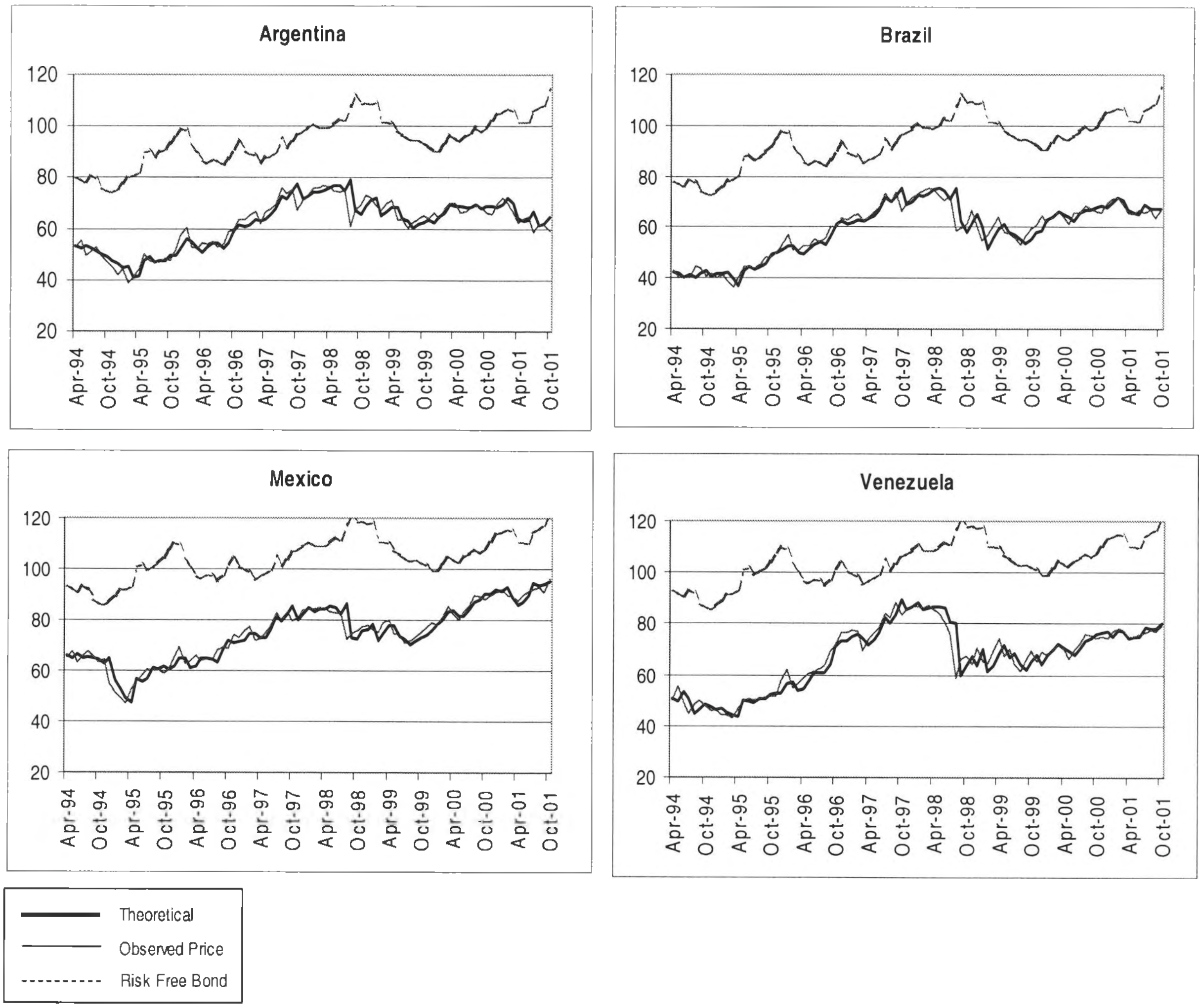

*Figure 4.2. Observed Prices vs. Theoretical Prices (one-step-ahead fitted values)
The upper series in each plot corresponds to a Theoretical Risk-Free bond with the same maturity and coupon as the risky bond. Prices are in dollars.*

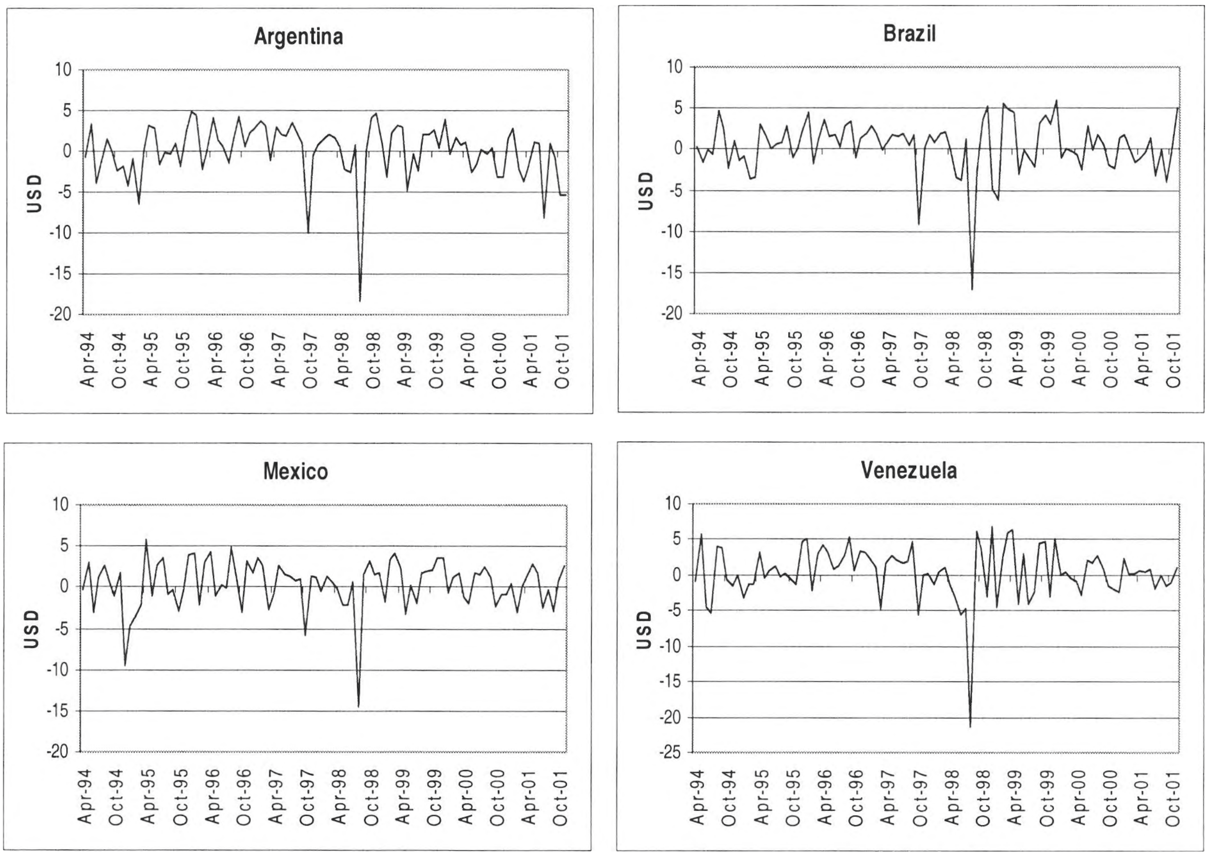

Figure 4.3. One- step-ahead Residuals

$v_t = Bt - B_{t|t-1}$

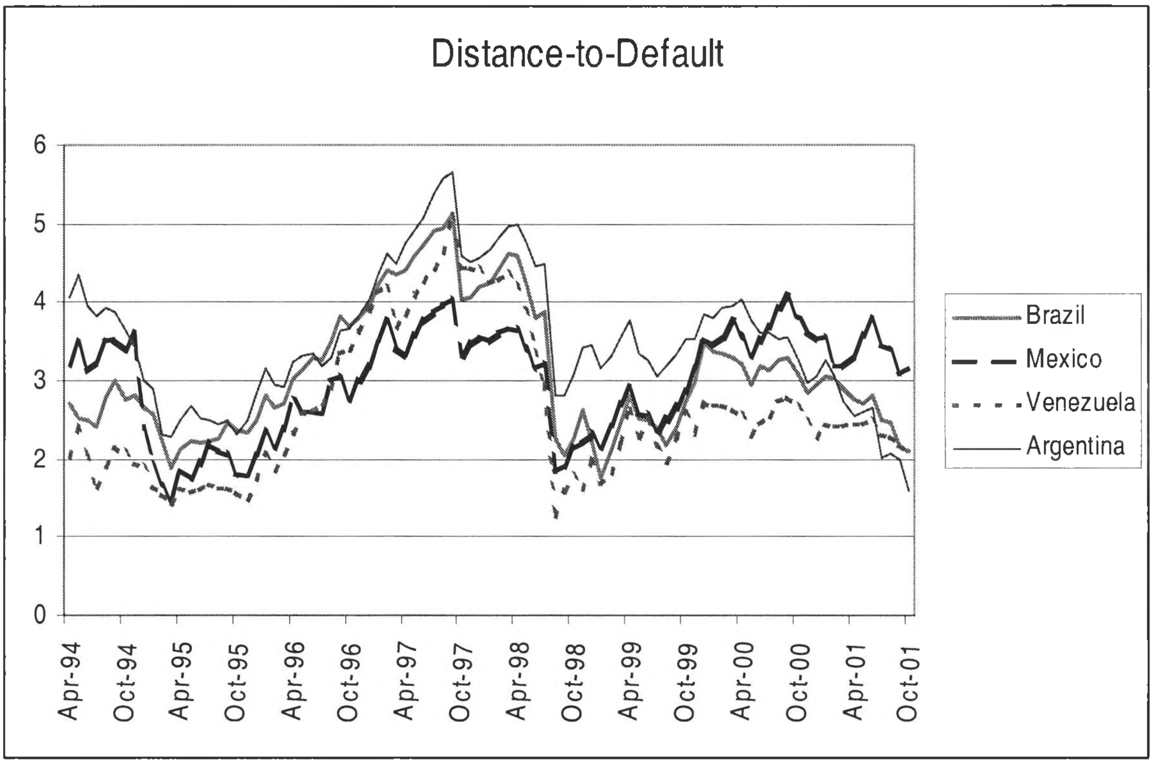

Figure 4.4. The Distance-to-Default implied by the CEJ Model

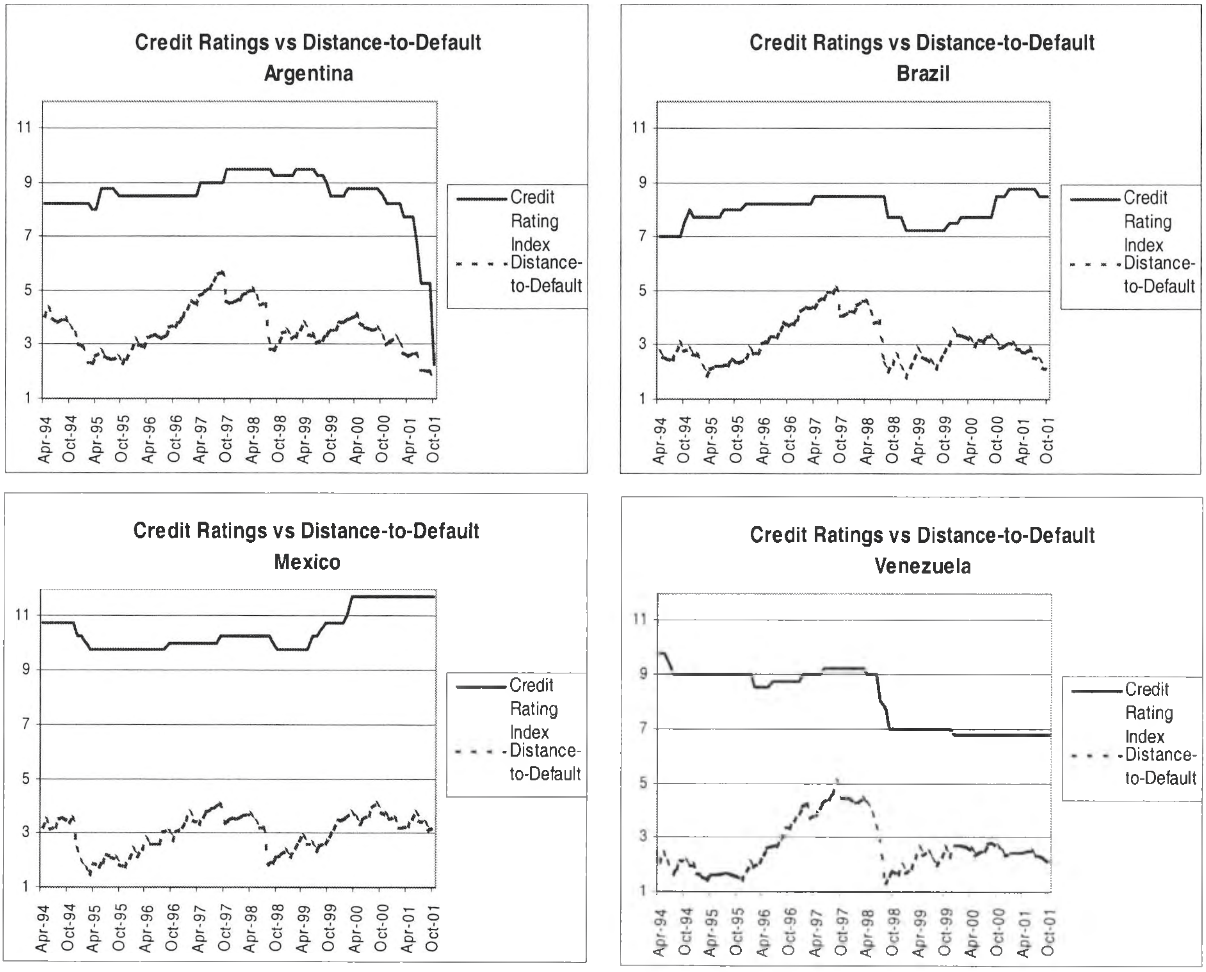

Figure 4.5. Credit Rating Index vs. the Distance-to-Default
The original categorical credit ratings from Standard and Poor's and Moody's have been transformed into a numerical scale and averaged in order to produce a Credit Rating Index. The larger the index the higher the credit quality of the country.

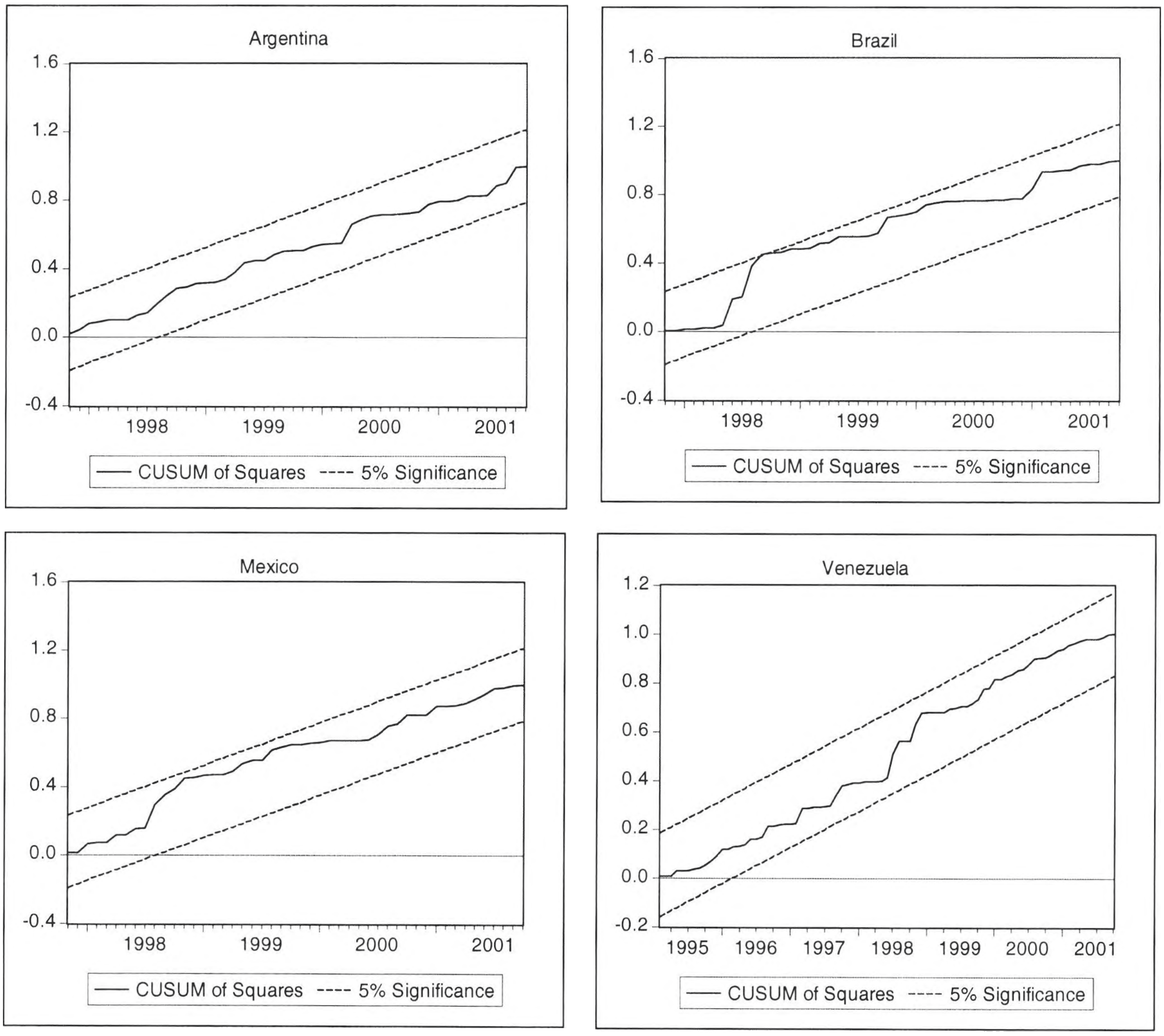

Figure 4.6. Stability Tests
Based on the cumulative sum of squared residuals statistics (CUSUM of Squares).
Confidence bands for a 95% percent level of significance.

Appendix A: Correlation Analysis of the Stock Market Returns

Table A.4.1. shows the correlation coefficients between the stock market indexes of all the countries and the distance-to-default in the period April 1994-October 2001. There are several features to observe from the table: *Panel A* shows an important interdependence between stock market returns and the variations of the distance-to-default for each country. The correlation coefficient goes from 0.519 for Venezuela up to 0.842 for Mexico. Stock returns also maintain an important relationship with the distance-to-default of other countries. The distance-to-default of Venezuela seems relatively more affected by movements in other markets (correlation coefficients go from 0.522 to 0.604) than by movements in its own stock market (0.519).

Panel B in *Table A.4.1.* shows that stock returns, particularly from Argentina, Brazil and Mexico are closely related. Their correlation coefficients vary between 0.666 and 0.690. The high correlation coefficients across the countries' distance-to-default observed in *Panel C* of *Table 4.4.* (coefficients varied between 0.704 and 0.845) may be explained by this high interdependence across stock markets. If movements in the bond market can be explained by movements in the stock market according to *Panel B* in *Table A.4.1.*, then part of the co-movements in the distance-to-default could be attributed to the interdependence of stock markets. However, this does not seem to be the whole story: the cross correlations of the distance-to-default of the four countries are higher than the cross correlations of stock returns. In the case of Venezuela for example, the correlation of its stock market with the rest of the countries varies between 0.278 and 0.350, whereas the correlation of the Venezuelan distance-to-default with the other economies goes from 0.704 to 0.768.

The above analysis suggests that apart from credit contagion due to the interdependence of stock markets, there may be additional transmission mechanisms of credit risk among bond markets. A possible source of these additional co-movements could be attributed to herd behaviour or changes in market sentiment in the bond market, specially during periods of crisis.

In order to differentiate between country-specific factors and common factors affecting the distance-to-default, we carry out a Principal Component Analysis (PCA) on the Stock Market returns. Results are shown in *Table A.4.2*. In panel A, the eigenvalues of the first two components are quite large: 2.54 and 0.806 respectively. The first component represents the 63.50% of the total variability of the four markets, whereas the second component contributes in 20.1% to the total variance. The coefficients of the first normalised eigenvector in *Panel B* indicate that the first component is a common factor that affects the four countries systematically. Argentina, Brazil and Mexico contribute approximately in the same proportion, while the influence of Venezuela on the first factor is slightly less. The Venezuelan stock market seems to behave rather differently and this is confirmed by the coefficients of the second eigenvector. The coefficient for Venezuela in the second vector is highly positive (0.934), whereas for the other countries' coefficients are small and negative (between –0.286 and –0.146).

Having identified that the major source of dependence across market returns comes from one single common factor, in the analysis we use a decomposition of stock market returns into two factors: 1) a systematic factor approximated by the first principal component and 2) a country-specific factor approximated by the residuals of a regression of the stock returns on the first principal component.

On regressing the first principal component of the distance-to-default on the first component of the stock returns, we find that the latter can explain 69.82% of the variance of the distance-to-default according to the goodness of fit parameter R^2. This suggests that most of the dynamics of the common factor that drives the distance-to-default of the four countries are due to the same common factor that drives the stock markets.

Table A.4.1. Correlations between variations in the Distance-to-Default and Stock Market Returns

		Distance to Default				Stock Market Returns			
		Argentina	Brazil	Mexico	Venezuela	Argentina	Brazil	Mexico	Venezuela
		Panel A				**Panel B**			
Stock Market Returns	Argentina	**0.773**	0.685	0.641	0.604	1			
	Brazil	0.642	**0.808**	0.627	0.522	0.666	1		
	Mexico	0.631	0.627	**0.842**	0.609	0.690	0.669	1	
	Venezuela	0.337	0.332	0.226	**0.519**	0.350	0.318	0.278	1

Table A.4.2. Principal Component Analysis of the Market Returns

Panel A: Loadings

	Comp 1	Comp 2	Comp 3	Comp 4
Eigenvalue	2.540	0.806	0.350	0.305
Variance Prop.	0.635	0.201	0.087	0.076
Cumulative Prop.	0.635	0.836	0.924	1.000

Panel B: Eigenvectors

Variable	Vector 1	Vector 2	Vector 3	Vector 4
Argentina	0.549	-0.154	0.479	0.668
Brazil	0.541	-0.146	-0.821	0.111
Mexico	0.539	-0.286	0.308	-0.730
Venezuela	0.340	0.934	0.045	-0.097

The sample corresponds to monthly observations of prices during the period April 1994-October 2001. The stock market returns correspond to the variables expressed in dollars.

Chapter 5.

A Systematic Comparison of Two Approaches to Measuring Credit Risk: CreditMetrics versus CreditRisk+

Abstract of Chapter 5

The objective of this chapter is to compare two approaches to modelling Credit-Value-at-Risk: CreditMetrics and CreditRisk+. This is important for regulators and for risk managers who are concerned with allocating capital efficiently. The few studies already available on this subject focus narrowly on the risk of default. This chapter incorporates both the risk of default and the risk which arises from changes in credit ratings (migration risk).

The chapter builds on the work done by Koyluoglu and Hickman (1998), but we make a significant extension by assessing the impact of migration risk on credit-risk. We make very careful comparison of Credit-Value-at-Risk for the two models using Monte Carlo techniques on standardised portfolios of bonds.

The conclusion is that for regulators, which model is used matters very little. This is because regulators are concerned with extreme values, and loss distributions of both models capture information about defaults at very high confidence levels. However, for internal purposes, where rating migrations matter more than default, CreditMetrics can generate higher estimates of risk.

5.1. Introduction

In recent years, Credit Risk Modelling has become a topic of active research. Progress in the area is the result of several factors; such as the success of Credit Derivatives, and the concern of banking authorities and risk managers to quantify capital adequacy requirements and economic capital.

In the academic literature and within the banking industry, there are two credit portfolio models which have become popular: CreditMetrics of J.P. Morgan (1997) and CreditRisk+ of Credit Suisse Financial Products (1997). At first sight, the models are very different, as they are based on different definitions of credit risk. On the one hand CreditRisk+ is a Default Model. Under this approach credit risk is the risk that a security's borrower defaults on the promised obligation. Therefore, only borrowers' defaults can cause losses in the portfolio. On the other hand, CreditMetrics is a Rating Migration Model. This approach defines credit risk as the risk that the security holder does not obtain the expected value of the security due to the deterioration of the borrower's credit quality. Therefore in CreditMetrics, it is not only default which can cause losses, but also a downgrading in the credit quality of a borrower.

A few studies have already examined the differences between these two models: see for example Gordy (2000), Koyluoglu and Hickman (1998) and Finger (1999). They conclude that CreditMetrics and CreditRisk+ are conceptually very similar. However, these papers examine only the default component of credit risk and fail to incorporate changes in credit ratings as another source of credit losses. The objective of this chapter is to compare systematically the Credit-Value-at-Risk (CVaR) for fixed income portfolios, as estimated by CreditMetrics and CreditRisk+. This measure is important both for regulators (who need to calculate capital adequacy requirements) and for risk managers (who want to allocate capital efficiently).

In this chapter we extend the analysis carried out by Koyluoglu and Hickman (1998). They formulate CreditRisk+ and a restricted version of CreditMetrics (which considers that only default can cause losses in the portfolio) under a common mathematical framework. This framework allows comparison of the default distributions of both models under equivalent parameters. We extend this analysis in two respects: First by comparing CreditRisk+ and the full version of CreditMetrics which considers migration risk. Second, by setting up a common mathematical framework to compare the loss distributions. Loss distributions are the main output of any credit risk portfolio model, as they allow estimation of the CVaR and examination of the impact on capital requirements.

We use Monte Carlo techniques to implement the new mathematical formulation of both models in two simulated bond portfolios, one with high credit quality and the other with low credit quality. We then examine the sensitivity of CVaR to changes in parameters. The analysis is restricted to a one-year time horizon.

The differences in CVaR between the models can be attributed to three sources: 1) the omission of migration risk in CreditRisk+; 2) the shape of the default distribution of each models; and 3) the definition of "credit exposure" in CreditRisk+. We conclude that for both types of portfolio (low- and high-quality), most of the differences in CVaR between the models are due to the underlying assumptions of the distribution of default. However, for high-quality portfolios and low confidence intervals of CVaR, the omission of migration risk is also significant in explaining the differences between the models.

This chapter contributes to the existing literature because it provides a comparison between a Default Model and a Credit Rating Model. We also assess the impact of migration risk on CVaR and identify portfolios for which migration risk is relevant to determine the differences of CVaR between the models.

For practitioners, the conclusions of this chapter have important implications: 1) For the calculation of capital requirements, choosing between CreditMetrics or CreditRisk+ seems to be irrelevant. At the extreme tails of the loss distribution, information about default is captured by either of the two models. 2) For internal purposes, such as estimation of reserves, where rating migrations matter more than default, CreditMetrics may be a better approach.

The chapter is structured as follows. Section 5.2 briefly describes the conceptual frameworks of CreditMetrics and CreditRisk+. Section 5.3 presents a revision of the literature on the comparison between these two models. In Section 5.4 we extend the analysis of Koyluoglu and Hickman and derive a common mathematical framework for both models. This formulation allows us to parameterise the models in a convenient way so that we can perform a valid comparison between their loss distributions. In Section 5.5, we implement the models in two types of simulated portfolios using Monte Carlo techniques. In Section 5.6, we analyse the sources of the differences of CVaR between the models. Section 5.7 gives the conclusions.

5.2. Description of the Models

5.2.1. CreditMetrics

The idea of the CreditMetrics model is an application of the Structural Approach originally proposed by Merton (1974). Under this setting the firm value drives all the dynamics of default. Default can occur only at debt maturity and when the firm's asset value falls below a specific threshold, which is the face value of the firm's liabilities. If the firm's asset returns follow a normal distribution then the probability of default can be expressed in terms of a normal variable falling below a specific threshold.

In CreditMetrics, Merton's model is extended by assuming that the asset returns of the firm also drive changes to other credit ratings. Therefore, to calculate CVaR, the first step in CreditMetrics consists of determining the credit rating of the firm at the end of the time horizon. To do this, a set of thresholds in the normal distribution must be provided to map asset returns into credit ratings.

For each possible credit rating at the end of the time horizon, the debt is priced by discounting the remaining cash flows at the corresponding interest rates[1].

In order to integrate all the individual exposures in the portfolio, credit correlations between borrowers are approximated by their asset correlations. The distribution of the portfolio value can then be constructed using Monte Carlo techniques[2]. CVaR is calculated as the difference between the expected and unexpected value (usually the 95% or 99% quantile) of the loss distribution.

5.2.2. CreditRisk+

CreditRisk+ can be seen as an application of the Reduced Form Approach for credit pricing. Borrowers are grouped within sectors or sub-portfolios, each of which has a mean default rate $\bar{p}_k$ and a default rate volatility σ_k [3]. The default rate or intensity parameter $p_i(x_k)$ for sector k is assumed to be driven by an unknown economic factor X_k, which follows a Gamma Distribution. Therefore, conditional on one realisation of the economic factor X_k, borrowers within the same sector k are independent.

In addition, within each sub-portfolio, borrowers are classified into bands according to their credit exposure. In each band, the size of each credit exposure is adjusted, so

[1] The risky term structure is calculated using a deterministic risk-free yield curve and credit spreads related to each credit rating. Due to the fact that CreditMetrics generates estimates for the mark-to-market value of the debt at the end of the time-horizon, this model is also known as a Mark-to-Market Model (MTM).

[2] Monte Carlo techniques are needed to simulate the possible realisations of the asset returns and consequently the credit quality of the borrower at the end of the time horizon.

each band is characterised by a common exposure V_i. Therefore, the default rate $p_i(x_k)$ is given by:

$$p_i(X_k) = \frac{\varepsilon_i}{V_i}\frac{X_k}{p_k} \sim \text{Gamma}(\alpha,\beta) \qquad (5.1)$$

where ε_i stands for the expected loss in band i, $\alpha = \frac{p_k^2}{\sigma_k^2}$ and $\beta = \frac{\sigma_k^2}{p_k}$.

For homogeneous sub-portfolios[4] and given a realisation of the economic factor, the default distribution follows a Poisson distribution. To obtain the unconditional default distribution for the sub-portfolio, we should account for all possible realisations of the economic factor. This involves estimating the convolution of the Poisson distribution with the Gamma distribution. This distribution produces a closed-form analytical function for the loss distribution.

In contrast with CreditMetrics, the value of the debt is not modelled directly in CreditRisk+, so in the event of default the debt holder incurs a loss equal to the amount of debt less the recovery rate.

5.3. Literature Review on the Comparison of Models

There is only a small literature on the comparison of credit risk models. Gordy (2000) substitutes the distributional assumptions of CreditMetrics into the mathematical structure of CreditRisk+ and then repeats the process the other way around. He shows that the structures of both models are similar when CreditMetrics is restricted to measuring only default risk. Gordy also carries out an empirical exercise and simulates portfolios by assuming a single systematic economic factor and four credit-types of assets. Models are calibrated so that they yield the same unconditional expected default rate and default correlation. He concludes that: 1) there are no

[3] CreditRisk+ suggests obtaining the unconditional default rate ($\bar{p}_k$) and default volatility (σ_k) from rating agencies.

dramatic differences in CVaR between the restricted form of CreditMetrics and CreditRisk+; 2) on average both models behave similarly for low values of default volatility; and 3) CreditRisk+ is more responsive to the credit quality of the portfolio.

Finger (1999) compares CreditRisk+ and also a restricted version of CreditMetrics using two types of bond portfolios: low and high credit quality portfolios. Within each portfolio, issuers are assumed to be homogeneous and their credit quality is driven by a single economic factor. Large discrepancies between the models occur when the portfolio is composed of high-quality bonds. The extreme tails of the default distributions generated by the models are very different. Finger concludes that when the asset correlation coefficient of CreditMetrics and the default volatility of CreditRisk+ are parameterised in a consistent way, both models produce similar distributions of default. However, in practice discrepancies can arise due to inconsistent parameters between the models, or technical implementations.

Koyluoglu and Hickman (1998) analyse the theoretical similarities between CreditMetrics and CreditRisk+. They assume a simple framework: the Vasicek representation of asset returns, fixed recovery rates and homogeneous portfolios. To generate the distribution of losses, they identify three common elements in the mathematical structure of the models. 1) The estimation of Default Rates. In both models default rates are driven directly or indirectly by stochastic economic factors. Thus, for each state of the economy a conditional default rate can be generated for each borrower. Therefore, default rates are also random variables and their probability distributions depend on stochastic movements of the economic factors. 2) The estimation of the Conditional Distribution of Portfolio Default Rate. For each state of the economy, the conditional default rate distribution of a homogeneous sub-portfolio is estimated as if individual borrowers defaulted independently. This is because all the joint behaviour of borrowers has already been considered when calculating the conditional default rates. 3) Aggregation or the estimation of the Unconditional

[4] A portfolio is homogeneous if borrowers have similar credit ratings and size exposures.

Distribution of Portfolio Defaults. This distribution is obtained by aggregating homogeneous sub-portfolios[5] across all possible realisations of the economic factor.

The above common set up of the models allows Koyluoglu and Hickman to examine their similarities in a structural way. They perform comparisons between the distributions of default rather than between the loss distributions, arguing that for very large portfolios both distributions are very similar. Empirically the differences between the default distributions do not seem to be significant when parameters have been set up consistently. They conclude that CreditMetrics and CreditRisk+ are conceptually based on the same philosophy. But differences between the models can arise due to aggregation techniques and the estimation of parameters.

Although some of the assumptions used by Koyluoglu and Hickman (1998) or Finger (1999) might be considered unrealistic, they are commonly used by practitioners. For instance, practitioners often assume that the determinants of credit losses are independent. The recovery rate in most cases is a deterministic parameter. Borrowers within the same specific risk sectors are assumed to be the same statistically. Practitioners also often assume that model parameters are stable. Though none of these assumptions seems to be true empirically, lack of data generally limits the use of more sophisticated assumptions.

The above academic analyses contrast with the project on Credit Risk Modelling led by the Institute of International Finance and The International Swaps and Derivatives Association (IIF/ISDA 2000), which was carried out by practitioners. The objective of this project was to understand the performance of credit risk models[6] used in 25 banks from 10 countries with different sizes and specialities. In this study, risk managers were given standard portfolios and inputs and asked to report CVaR. Practitioners were also asked to run models through a variety of implementations and scenarios.

[5] Each sub-portfolio is affected by one economic factor.

[6] The examined models were: CreditMetrics (J.P.Morgan), CreditPortfolioView (McKinsey), CreditRisk+ (Credit Suisse Financial Products), Portfolio Manager (KMV) and 11 proprietary internal models.

The objective was to determine whether models used by practitioners were directionally consistent (model outputs moved in the same direction), when given similar key inputs. In the end, the exercise led to different outcomes and to no very clear conclusions about what the sizes and sources of the differences between the models, as implemented, were. We should point out that no efforts were made to parameterise the models so that they yielded consistent results. This might explain the wide range in the results.

The IIF/ISDA study concludes that there is consistency among the results obtained from using the same type of model. For instance, CreditMetrics results resemble other mark-to-market approaches. Credit Risk+ gives the highest estimates of CVaR (in relation to internal models and CreditMetrics). The explanation lies in the correlation assumption: whereas CreditMetrics and other mark-to-market models were run with equivalent correlation coefficients, CreditRisk+ was fed with a more conservative parameter. Hence it is concluded that the calculations of the correlation coefficient play an important role in the generation of discrepancies. Finally IIF/ISDA conclude that differences between models must be attributed to model inputs, pre-processing of data, valuation and different implementations.

In summary, the literature suggests that CreditMetrics and CreditRisk+ are both conceptually and practically very similar, provided that they are fed with proper parameters. However, the IIF/ISDA study seems to indicate that discrepancies in implementation are common.

5.4. A Common Framework for CreditMetrics and CreditRisk+

To develop a common framework for both models, we will make the following assumptions: 1) There are three credit states or ratings defined under the standard version of CreditMetrics: A, B and D, where A represents the highest credit quality, B is an intermediate state and D represents the default state. For CreditRisk+ and the restricted version of CreditMetrics, the credit quality of a borrower is restricted to only two states: D and ND, where ND represents the no-default state. 2) The recovery rate is fixed. 3) The time horizon is one single-period. 4) Portfolios are formed of N equally-rated bonds, with the same exposure size, the same time to maturity, and affected by a single economic or systematic factor.

Following Koyluoglu and Hickman's approach (1998), we derive the distribution of losses by identifying three components in the distribution of losses under each model:

1. Default Rates and Migration Rates. They vary across time and according to realisations of the systematic factor. Borrowers within a portfolio are related to the extent to which their migration rates vary together through different states of the economy.
2. Conditional Distributions of Portfolio Default Rate and Portfolio Migration Rates. For each realisation of the systematic or economic factor, the conditional distributions in a homogeneous portfolio can be calculated as if borrowers are independent. This is because all the joint-behaviour has been accounted for when default rates and migration rates were calculated.
3. Aggregation. The individual conditional distributions under each realisation of the systematic factor are aggregated. This distribution is the convolution of the unconditional distribution with the distribution of the systematic factor.

5.4.1.Default Rates and Migration Rates

CreditMetrics and CreditRisk+ implicitly or explicitly assume that there is a systematic factor driving the credit quality of the borrower. Below we will derive transformations between such a systematic factor and the default rates and migration rates for each model.

CreditMetrics

As explained in Section 5.2, default is driven by the asset returns of the firm. We can see asset returns for the ith firm as driven by a set a set of k normally distributed orthogonal systematic factors:

$$r_i = b_{i,1}X_1 + b_{i,2}X_2 + ... + \sqrt{1-\sum_k b_{i,k}{}^2}\,\varepsilon_i, \qquad \varepsilon_i \sim N(0,1) \qquad (5.2)$$

where $b_{i,k}$ are the factor loadings, X_k represents the k-th systematic factor and ε_i are movements specific to each firm.

When the portfolio is composed of borrowers who are affected by a single economic factor and have similar size exposures and credit ratings, the systematic factors in equation 5.2 can be represented by a single variable X~N(0,1) as follows:

$$r_i = \sqrt{\rho}X + \sqrt{1-\rho}\varepsilon_i \qquad (5.3)$$

where $\rho = \sum_k b_k^2$ is the correlation of the borrowers´ assets in the portfolio.

According to CreditMetrics, default occurs when $r_i \leq c_1$, where the area below c_1 under the normal distribution represents the unconditional default rate, i.e., $\Phi(c_1) = p_D$. Using the above representation of the returns, Vasicek (1987) derives a functional form for the default process conditioned on the realisations of the systematic factor, as follows:

$$p_{D|X}^{CM} = \Phi\left[\frac{c_1 - \sqrt{\rho}x}{\sqrt{1-\rho}}\right] \qquad (5.4)$$

Migration rates to ratings A and B can be obtained similarly (see *Appendix A* for the full derivation):

$$p_{B|X}^{CM} = \Phi\left[\frac{c_2 - \sqrt{\rho}x}{\sqrt{1-\rho}}\right] - \Phi\left[\frac{c_1 - \sqrt{\rho}x}{\sqrt{1-\rho}}\right] \qquad (5.5)$$

$$\text{and } p_{A|X}^{CM} = 1 - p_{B|X}^{CM} - p_{D|X}^{CM} \qquad (5.6)$$

where $c_2 = \Phi^{-1}(\bar{p}_B + \bar{p}_D)$ and $\bar{p}_B$ is the unconditional migration rate to credit rating B[7].

CreditRisk+

CreditRisk+ assumes that default is driven by a systematic factor which is Gamma-distributed (see formula 5.1). As a result the default rate is also Gamma distributed. In order to make the comparison of models consistent, Koyluoglu and Hickman (1998) suggest replacing the gamma systematic factor by a normally distributed factor. To preserve the gamma distribution of the default rate, they transform the functional form between the systematic factor and the default rate. The transformation function consists of all points (χ, ξ) that satisfy:

$$\int_0^{\xi} \Gamma(\bar{p}_D; \alpha, \beta) dp = \int_{\chi}^{\infty} \varphi(x) dx \qquad (5.7)$$

where $\Gamma(z;\alpha,\beta)$ is the Gamma density function with parameters α and β, and $\varphi(x)$ is the Normal density function.

Then the conditional rate of default resulting from the transformation in 5.7 is given by:

$$p_{D|X}^{CR} = \Psi^{-1}(1 - \Phi(x); \alpha, \beta) \qquad (5.8)$$

where $\Psi(z;\alpha,\beta)$ is the Gamma cumulative distribution[8].

Thus the no-default rate, given a realisation of the economic factor, is:

$$p_{ND|X}^{CR} = 1 - p_{D|X}^{CR} \qquad (5.9)$$

The effect of realisations of the systematic factor on migration rates and default rates are illustrated in *Appendix A*.

[7] The parameter $\bar{p}_D$ as well as other migration rates $\bar{p}_A$ and $\bar{p}_B$ and their respective volatilities, can usually be obtained from rating agencies.

[8] Recall that the parameters α and β depend on the mean and standard deviation of the default rate ($\bar{p}_D$ and σ_D). Both parameters can usually be obtained from rating agencies.

5.4.2 Conditional Distributions of Portfolio Default Rate and Portfolio Migration Rates

Given a realisation of the economic factor, borrowers in the portfolio are independent. This is because all the correlation between borrowers has been captured through their relationship with the economic factor. Under fixed default and migration rates and a homogeneous portfolio[9], CreditMetrics implicitly assumes a Multinomial distribution to describe the number of individuals in each credit rating at the end of the time horizon. CreditMetrics uses Monte Carlo techniques to simulate the changes in the credit quality of the borrowers in the portfolio. At the end of the period, an individual can migrate to any of the three credit states [A, B, D] with probabilities $[p^{CM}_{A|x}, p^{CM}_{B|x}, p^{CM}_{D|x}]$ respectively. If we sample N individuals independently and $[N^{CM}_{D|x}, N^{CM}_{B|x}, N^{CM}_{A|x}]$ are the number of defaults, the number of B-rated borrowers and the number of A-rated borrowers in the portfolio at the end of the time-horizon, then this vector will follow a Multinomial Distribution,

$$\begin{bmatrix} N^{CM}_{B|x} \\ N^{CM}_{D|x} \end{bmatrix} \approx \text{Multinomial}(p^{CM}_{B|x}, p^{CM}_{D|x}, N) \qquad (5.10)$$

and $N^{CM}_{A|x} = N - N^{CM}_{B|x} - N^{CM}_{D|x}$, where N is the size of the portfolio.

In CreditRisk+, the Binominal distribution is approximated by a Poisson distribution. Asymptotic properties of the Binomial Distribution state that when the rate of default $p^{CR}_{D|x}$ is small and the number of bonds in the portfolio is large (N), the number of defaults in the portfolio is approximately Poisson-distributed with parameter $\lambda = (p^{CR}_{D|x}) \cdot N$:

$$N^{CR}_{D|x} \sim \text{Poisson}(\lambda) \qquad (5.11)$$

[9] All individuals in the portfolio have the same unconditional default rate and migration rates, since they are homogeneous.

Thus, the number of no-defaults in the portfolio is $N^{CR}_{ND|x} = N - N^{CR}_{D|x}$.

Observe that when the Multinomial distribution is restricted to only two states (default and no-default), this becomes a Binomial distribution with parameters $p^{CM}_{D|x}$ and N. As the limit distribution of a Binomial distribution is a Poisson distribution, a restricted version of CreditMetrics and the standard version of CreditRisk+ will asymptotically yield similar shapes of the default distribution.

5.4.3. Aggregation

We aggregate the conditional distributions of the portfolio under all possible realisations of the systematic factor. In CreditMetrics, the conditional distribution of the number of individuals in each credit rating is conditioned on a normally distributed economic factor. Therefore the unconditional distribution of the number of individuals in each credit rating is given by the following convolution integral[10]:

$$P_{CM}(N^{CM}_D, N^{CM}_B, N^{CM}_A) = \int_X \text{Multinomial}(N, p_{D|x}, p_{B|x})\varphi(x)dx \qquad (5.12)$$

Likewise in CreditRisk+, the conditional distribution of portfolio default rate is conditioned upon a gamma default rate. Therefore the unconditional distribution of the number of defaults is given by[11]:

$$P_{CR}(N^{CR}_D) = \int_p \text{Poisson}(N, p^{CR}_{D|x})\Gamma(p^{CR}_{D|x}, \alpha, \beta)dp \qquad (5.13)$$

Finally, to calculate the distribution of losses, the information about the number of individuals in each credit rating and the size of the exposures should be combined. For CreditRisk+ the distribution of losses should resemble equation 5.13. Only the random variable N^{CR}_D needs to be scaled up by the size of the exposures under default. CreditMetrics produces a distribution of the portfolio value rather than a

[10] CreditMetrics generates this mixed distribution using Monte Carlo simulations, rather than a closed-form distribution.
[11] This mixed distribution yields a closed-form function, which is the Negative Binomial Distribution.

distribution of losses. For comparison of CVaR figures, we will transform the distribution of portfolio values of CreditMetrics into a distribution of losses.

5.4.4. Consistent Parameterisation of the Models

CreditMetrics can be seen as an extension of CreditRisk+ in which other credit ratings apart from default are considered. Default distributions should then be considered as the link between the models and they should be parameterised consistently. According to Koyluoglu and Hickman, consistency means that means and standard deviations of the default distributions are the same across the models. The mean of the default rate ($\bar{p}_D$) is an input to both models, so the parameter is the same for both models. However, the standard deviation of the default rate (σ_D) is an input just for CreditRisk+. For CreditMetrics the default rate volatility for CreditMetrics can be expressed as a function of $\bar{p}_D$ and ρ :

$$\sigma_{CM}^2 = \int_{-\infty}^{+\infty} \left(\Phi\left[\frac{\Phi^{-1}(\bar{p}_D) - \sqrt{\rho}x}{\sqrt{1-\rho}} \right] - \bar{p}_D \right)^2 \varphi(x)dx \qquad (5.14)$$

If we set up $\sigma_D \equiv \sigma_{CM}^2$, then the equation above establishes a relationship between the asset correlation parameter of CreditMetrics (ρ) and the variance of CreditRisk+ (σ_D). This relationship is plotted in *Figure 5.1*. From the plot, the higher the asset correlation or default correlation, the higher the volatility of the default rate. Intuitively, if asset returns are highly correlated then the default of one borrower is likely to be followed by the default of another. In other words, if borrowers' asset values are highly correlated through the effect of the same economic factor, their default rates will move together, causing high volatility levels.

5.5. Implementation of the Models

We implement the models in two stages using Monte Carlo techniques. First, we calculate default rates and migration rates given a realisation of the economic factor. We then generate the probability distributions of the default rate and migration rates by simulating 1,000,000 realisations of the economic factor. In the second stage, we combine the number of individuals in each credit rating and its individual losses to generate the loss portfolio distribution and CVaR. We simulate two types of bond portfolios: a Low-Credit Quality portfolio (LQ) and a High-Credit Quality portfolio (HQ). The LQ portfolio consists of bonds whose issuers or borrowers have credit quality "B", whereas the HQ portfolio consists of bonds whose borrowers have "A" credit quality. The migration rates for credit ratings A and B appear in *Table 5.1*[12]. Each bond has two years to maturity and a face value of $1. Each portfolio has been designed with 10,000 borrowers.

To gain some insights into the performance of the models we run several Monte Carlo exercises to test the sensitivity of CVaR to different values of the correlation parameter[13] (ρ=0.05, 0.15, 0.25, 0.35, 0.45) and confidence levels ((1-α)%=90, 93, 95, 97, 99, 99.9%). The time horizon for CVaR is one year.

5.5.1. Generation of the Default Rate Distribution and Migration Rates Distributions

We use Monte Carlo techniques to simulate realisations of the systematic factor and construct the default distribution and migration distributions for CreditMetrics. The mean and volatility of the default distribution of CreditMetrics are used as input

[12] Transition Probabilities are consistent with data released by Moody's.
[13] This range of the correlation parameter has been chosen considering that the one-year default volatilities estimated by Moody's over the period 1920-1998 is less than 5% for all the credit ratings. See Moody's Investors Sevice (1999).

parameters to construct the default distribution for CreditRisk+. The description of the Monte Carlo Simulations are given in *Appendix B* (Sections B.1 and B.2)

Figure 5.2. shows the distributions of the default rate for CreditMetrics, for different values of the correlation parameter[14]. Observe that high correlations (rho=0.45) are associated with long and fat tails. Note that the distributions of the HQ portfolios are shifted to the left, which indicates a higher probability of getting low default rates than for LQ portfolios.

In *Table 5.2.* we report some descriptive statistics for the distributions of both types of portfolios and different levels of asset correlation (0.05 to 0.45). There are several features to observe:

The higher the correlation, the higher the standard deviation, skewness and kurtosis. The low quality (LQ) portfolio exhibits significantly higher levels of standard deviation than the high quality (HQ) portfolio. For both models, skewness and kurtosis of HQ portfolios are more sensitive to changes in the correlation parameter than those of LQ portfolios. For example, in CreditMetrics a change of 28% in the correlation level (from ρ =0.35 to ρ =0.45) produces a change of 10% in the kurtosis of the LQ portfolio (from 11.47% to 12.65%), whereas this figure is 27% (from 68.95% to 87.58%) for the HQ portfolio.

CreditRisk+ is more sensitive to changes in the correlation parameter than CreditMetrics. For instance, for an equivalent change of 28% in the correlation coefficient (from ρ =0.35 to ρ =0.45), the change in kurtosis for CreditRisk+ for LQ portfolio is 32% (from 13.76% to 18.14%), whereas for CreditMetrics this figure is 10%.

[14] Plots look very similar for CreditRisk+.

For HQ portfolios, CreditMetrics distributions are dramatically more leptokurtic (coefficient is larger than 3) than those for CreditRisk+. Only for LQ portfolios and high correlations does CreditRisk+ produce higher estimates than CreditMetrics. Therefore, for HQ portfolios, and for LQ portfolios which are poorly correlated, CreditMetrics forecasts larger credit losses and capital requirements due to default than CreditRisk+. This implies that more capital would be required by regulators if migrations to other non-default states were considered.

Looking at the distributions of default, we conclude that although the two models can be parameterised to yield the same mean and standard deviation of the default distribution, the differences between higher moments suggest that CVaR figures (of credit losses) may differ significantly. This difference is even more pronounced for HQ portfolios than for LQ portfolios.

5.5.2. Generation of the Distribution of Losses

To calculate the distribution of losses, for each realisation of the economic factor[15] we compute the number of individuals in the portfolio that fall into each credit state at the end of the period. These numbers can be found by inverting the integrals in equations 5.12 and 5.13 and using Monte Carlo techniques. The number of individuals in each rating class is combined with the size of the exposures to produce an estimate for the losses in the portfolio.

We need to explain briefly how the loss function is defined for each of the models: In the standard version of CreditMetrics, the mark-to-market value of a bond at the end of the time horizon is calculated by discounting the remaining cashflows and using the term-structure associated with the credit rating of the borrower at the end of period. Let $P_{t+1,J}$ be the mark-to-market price of the bond associated with credit rating J (J=A, B, D). Therefore the mark- to market value of a bond (P_{t+1}) at the end of the time horizon is:

$$P_{t+1} = \sum_J P_{t+1,J} I_{(J)}\ , \quad J{=}A, B, D$$

Where $I_{(J)}$ is an indicator function with value 1 when the borrower has been rated with credit quality J at the end of the period, and 0 otherwise.

Define the credit loss of a bond for the standard version of CreditMetrics as the difference between the expected value of the bond and its value at the end of the time horizon[16]:

Standard Version of CreditMetrics(CM3)

$$L_{t+1}^{CM3} = \sum_J \left(E_t(P_{t+1}) - P_{t+1,J}\right) I_{(J)} \qquad J{=}A, B, D \qquad (5.15)$$

where $E_t(P_{t+1})$ is the expected value of the bond, which is equal to $\sum_{J=A,B,D} p_J P_{t+1,J}$, and p_J is the probability of migration to rating J (=A, B, D). Also remember that the value of the bond in case of default is defined as the recovery rate[17], which in this case is assumed fixed.

Likewise, for the restricted or default version of CreditMetrics, we define losses as:

Restricted Version of CreditMetrics(CM2)

$$L_{t+1}^{CM2} = \sum_J \left(E_t(P_{t+1}) - P_{t+1,J}\right) I_{(J)} \qquad J{=}D, ND \qquad (5.16)$$

where ND is the no-default state and $P_{t+1,ND} = \left(\frac{\sum_{J=A,B} p_J P_{t+1,J}}{\sum_{J=A,B} p_J}\right)$ is the value of a non-defaulted bond.

[15] Recall that a realisation of the economic factor generates a set of conditional probabilities of migration, which are needed to estimate the probabilities of number of borrowers in each credit rating at the end of the period.

[16] This definition of losses is consistent with the definition of mark-to-market value of a bond, in the sense that the CVaR of both distributions (portfolio value and losses) would yield the same results.

For CreditRisk+, credit losses occur only when default occurs:

$$L_{t+1}^{CR+} = (\text{Exposure})I_{(D)}$$

where different definitions of "Exposure" have been used in the literature and among practitioners[18].

In order to carry out comparisons between the models, we consider three versions of CreditRisk+. They differ only in the definition of exposure:

Non-default-Value Version of CreditRisk+ (CR+1)

$$L_{t+1}^{CR+1} = (P_{t+1,ND} - P_{t+1,D})I_{(D)} \qquad (5.17)$$

Book-Value Version of CreditRisk+ (CR+2)

$$L_{t+1}^{CR+2} = (BV_t - P_{t+1,D})I_{(D)} \qquad (5.18)$$

Expected-Value Version of CreditRisk+ (CR+3)

$$L_{t+1}^{CR+3} = (E_t(P_{t+1}) - P_{t+1,D})I_{(D)} \qquad (5.19)$$

where "BV" is the book value of the bond and is the original version of the model.

The steps to generate the loss distribution using Monte Carlo methods are given in *Appendix B* (Sections *B.3* and *B.4*). *Table 5.3.* shows some descriptive statistics of the loss distribution under the standard version of CreditMetrics and the Book Value version of CreditRisk+. The latter model produces similar levels of skewness and kurtosis for any version.

Comparing *Tables 5.2.* with *5.3.*, we can observe that the properties of the distribution of losses inherit the properties of the default distribution. Therefore we can anticipate that the default distribution will have a significant effect on the CVaR of the portfolio. Observe that the largest differences between skewness and kurtosis for the two distributions are those for the High Quality portfolio, under the version of CreditMetrics. This is consistent with the belief that the tail of the distribution of losses,

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[17] A recovery rate of 35% for the LQ portfolio and 45% for the HQ portfolio are assumed.

under CreditMetrics, incorporates information not only about default but also about downgrading in the portfolio.

5.6. Analysis of the Differences in CVaR between the Models

We attribute the differences in CVaR between the two models to three factors: a) the omission of migration risk in CreditRisk+; b) the shape of the tails of the default distributions of each model; and c) the definition of credit exposure in CreditRisk+. In Section 5.6.1., we examine the individual impact of these three factors in the discrepancies of CVaR. In Section 5.6.2., we put these three factors together and analyse their global impact on CVaR.

5.6.1. Effect of Individual Factors that explain the Differences in CVaR

a) The Effect of Migration Risk

Consider the two versions of CreditMetrics: the standard version and its restricted version. The difference between the two versions lies in the definition of credit loss. The restricted version of CreditMetrics aggregates information from the two non-default states (A and B), leaving out the effect of migration risk. In contrast, the full version of CreditMetrics considers both states individually (equations 5.15 and 5.16 respectively), taking into account migration risk.

In order to quantify the differences between the two versions, we compute the ratio of CVaRs, for a given confidence level. This ratio (CVaR of the restricted version of CreditMetrics divided by the CVaR of the standard version) will be referred to as the

[18] Recall that "exposure" has been defined as the amount owed minus the recovery rate (which is the actual value of the debt under default).

"Effect of Migration Risk". As the CVaR for the standard version is always higher than the CVaR for the restricted version, this ratio is bounded by one.

A summary of the effect of migration risk for a range of correlations and confidence levels of CVaR is reported in *Table 5.4*. Observe that for the LQ portfolio, most of the ratios are close to one. The omission of non-default credit states or migration risk is practically irrelevant. At high confidence levels, the coefficients are one. This indicates that for the LQ portfolio the information contained in the tails of the loss distributions for both versions is the same, provided that model inputs are the same except for the number of credit states. Therefore, it seems that for LQ portfolios, information about losses accumulated in the tails of the loss distribution comes mainly from defaults in the portfolio, and not from downgradings to other non-default states.

The omission of migration risk in the restricted version is more relevant for the HQ portfolio. From *Table 5.4*, figures are significantly less than one for a given correlation coefficient. For a given low confidence level (90%, 93%, 95%), the higher the correlation levels the more the information that is omitted. For example, for ρ=0.45 and for CVaRs at 90% confidence, the ratio is 0.908, whereas this number is 0.954 when ρ=0.05. In percentage terms those numbers are equal to -9.2% (=0.908-1) and –4.6% (=0.954-1) respectively. Therefore, the default version of CreditMetrics omits up to 9.2% of information about downgrades. Intuitively, higher correlations are associated with higher levels of volatilities of the migration rates. Therefore, more downgrades and losses are expected to take place.

Also, for the HQ portfolio and high confidence levels, ratios are closer to one. This suggests that in the very extreme tails, distributions contain more information about losses generated by defaults than by downgrading events. At the 99.9% confidence level, the largest omission of information is only for 3% (=0.97-1). For high correlation levels, ratios are even closer to one. This is because high volatility of the default rate causes more defaults in the portfolio. Therefore, both versions of credit risk should be more alike in the tails.

b) The Effect of the Distribution of Default

Consider the restricted version of CreditMetrics and the first version of CreditRisk+ (its loss function is given by 5.17 and denoted by CR+1). The definitions of credit losses for the two models (5.16 vs 5.17) are algebraically the same, except for an additive constant. This constant is irrelevant for CVaR calculations. As each model uses its own distributional assumptions, the differences in CVaR result from the discrepancies between their distributions of default. In order to quantify such discrepancies, for a given confidence level, we compute the ratio of CVaRs (CVaR of CR+1 divided by the CVaR of the restricted version of CreditMetrics). This ratio will be referred as the "Effect of the Distribution of Default".

A summary of the discrepancies in CVaR due to the distributions of default for a range of correlations and confidence levels is reported in *Table 5.5*. Note that for the LQ portfolio, most of the ratios are less than one. This means that the distribution of default produced by CreditMetrics is thicker and longer. As a consequence, CreditMetrics forecasts higher CVaR numbers and capital requirements than CreditRisk+ due to default. Only for high confidence intervals (99.9%) and high correlation levels (0.25-0.45), does CreditRisk+ produce higher CVaRs than CreditMetrics. These conclusions are consistent with the results obtained in Section 5.5.1.

For the HQ portfolio, differences in CVaR for the two models are more dramatic. For high confidence intervals (99.9%), CreditRisk+ produces CVaR up to 17% (=0.827-1) lower than CreditMetrics. This is because the tail of the default distribution of CreditMetrics is thicker. For low confidence intervals, the opposite occurs: CreditRisk+ produces CVaRs up to 26.3% (=1.263-1) higher than CreditMetrics.

c) The effect of the Exposure

Finally, consider the differences between the three versions of CreditRisk+ equations 5.17, 5.18 and 5.19 respectively). The loss functions are algebraically the same except for the definition of credit exposure. Hence the difference of CVaRs between any two versions should be a multiplicative constant. This constant is equal to the ratio of the exposures. Therefore, CVaR ratios between CR+2 and CR+1 or between CR+3 and CR+1 should be interpreted as the "Effect of the Exposure". Contrary to other effects, the effect of the exposure is theoretically a fixed number[19] , as the exposures for each model are calculated exogenously, under CreditRisk+. The discrepancies of CVaR due to differences in the definition of "exposure" are shown in *Table 5.6.*

5.6.2. Global Effect of the Factors that explain the Differences in CVaR

In this section we examine the interaction of the three factors that explain the discrepancies in CVaR. *Tables 5.7* and *5.8* illustrate the differences of CVaR between the Book-Value version of CreditRisk+ (CR+2) and the standard version of CreditMetrics[20], for the low quality and high quality portfolios respectively.

In each table, the first line of each block represents the differences of CVaRs in percentage terms. The second line corresponds to the simple ratio of CVaR figures. The third line represents the effect of the exposure. The fourth line indicates the effect of the default distribution. Finally, the last line corresponds to the effect of migration risk. Observe that numbers in the second line are the arithmetic products of the numbers in the following three lines. This means that the variation in CVaR can be decomposed into those three factors.

[19] Some differences can arise due to calculation error.

[20] Similar analysis can be done using the third version of CreditRisk+ (CR+3). Results are found in Appendix C.

From the LQ portfolio in *Table 5.7.*, we can conclude the following:

Differences of CVaR vary between –5.43% and 16.18%. The most dramatic differences correspond to when there are high correlations (0.35-0.45) and high confidence intervals (99%, 99.9%). Observe the joint effect of the three components that explain CVaR. The effect on the overall difference of omitting migration risk is practically nil. These numbers are close to one, so they do not make any contribution. The most relevant effect is that of the distribution of default.

In most cases, the effect of the exposure reduces the discrepancies due to the effect of the default distribution. The negative effect of the distribution of default (numbers less than one) is offset partially by the positive effect of the exposure (numbers bigger than one). However, for a very large confidence interval, 99.9%, and correlation levels, ρ=0.25, 0.35, 0.45, both effects are positive, and therefore differences in CVaR become larger.

From the HQ portfolio in *Table 5.8.*, we can conclude the following:

The differences of CVaR are wider than those for the LQ portfolio. They vary between –18.96% and 17.73%. Considering the overall effect of the three factors, we can say that CreditRisk+ generally gives higher estimates than CreditMetrics at low confidence levels and lower estimates than CreditMetrics at high confidence levels. These differences are mainly due to the discrepancies in the distribution of default. At low confidence intervals (90, 93, 95%) CreditRisk+ produces higher CVaRs estimates due to default risk (numbers are much larger than one). This positive effect is slightly offset by the fact that CreditMetrics measures migration risk. For low confidence intervals, the effect of omitting rating changes is less than one. For high confidence intervals, the reverse occurs. Overall, CreditRisk+ underestimates CreditMetrics. This is due to important underestimation of the default risk with respect to CreditMetrics. In addition, CreditRisk+ does not account for migration risk, so differences become larger.

The difference between models due to the definition of exposure in CreditRisk+ is not significant. This difference accounts for only 1%(=1.007-1) of the overall difference. Perhaps a debt with longer maturity would produce a more significant effect.

To summarise, we find that most of the discrepancies between the models are due to the differences in their probabilities of default. The omission of migration risk is relevant only for high-quality (HQ) portfolios and low confidence levels. Roughly speaking, if we assume that there are no discrepancies in the distributions of default, then the use of a two-credit-state model instead of a three-credit-state model will miss out up to 3% of information about downgrading at very extreme confidence levels (99.9%). This figure is 9.2% for lower confidence levels (90%).

5.7. Conclusions and Implications

Having assumed homogeneous portfolios and a single systematic factor, we carried out a structural comparison between CreditRisk+ and the standard version of CreditMetrics. We made an extension of the Koyluoglu and Hickman (1998) framework and identified common components to derive the distribution of losses. Likewise, we derived consistent parameters, in order to make the models comparable.

We find differences in Credit Value-at-Risk of up to 19% between the models. The model that forecasts higher values is not always the same: results depend on the quality of the portfolio, parameter values and confidence levels.

Three particular factors might explain the differences between the models: a) the omission of migration risk in CreditRisk+, b) the shape of the probability of default in the two models, and c) the definition of credit exposure in CreditRisk+. In general, the differences in the shape of the default distributions generated by the two models explain most of the differences of CVaR. The omission of migration risk is significant only for high-quality portfolios and low confidence levels (lower than 90%). If we assume that no discrepancies exist in the distribution of default, then CreditRisk+ estimates lower CVaR than CreditMetrics, by up to 10%.

For low-quality portfolios, the two models behave similarly for all except extreme confidence intervals (larger than 99%). In these extreme cases CreditRisk+ estimates CVaRs up to 16% higher than CreditMetrics. For high-quality portfolios, differences are more dramatic. CreditRisk+ again estimates higher CVaRs than CreditMetrics, except for high confidence intervals. In all these results the main driver of the differences between the models is the shape of the distribution of default. These results are important since the use of high confidence levels in the measurement of credit risk is a common practice in the banking community. High confidence levels often compensate for the inability to test the reliability of the models. Hence capital requirements based on high confidence intervals seem to depend highly on which model is chosen.

The implications of the above results for risk management are quite clear. For low-quality portfolios we may forget about CreditMetrics, since migration risk accounts for very little of the overall CVaR of the portfolio. In this case, CreditRisk+ is a faster and less expensive approach for calculating capital requirements. On the other hand, if our purpose is to estimate reserves of capital, then CreditMetrics may provide more accuracy about the sources of losses.

In answering the question about which is the better model to implement in banks, it is necessary to take into account the following considerations:

- The type of credit risk that is to be measured. In solving this paradigm, it might be helpful to think of the objective of the measurement. In the calculation of capital requirements, the main interest is in the very extreme values of the distributions. Therefore, as was mentioned, default models should be sufficient to estimate the overall picture of the losses. However, if the model is needed to estimate reserves or provisions for credit risk, CreditMetrics would give more information about the size of the reserves needed for non-defaulted loans.

- The costs and reliability of the inputs. In particular CreditMetrics requires large amount of data. Whereas CreditRisk+ is cheaper and easier to implement.

- The overall environment of the risk management process, i.e., how often the model is to be revised, the control processes, the ability of the managerial team to understand inputs and outputs of the model, etc. Good technical knowledge and skills are required to interpret outputs from both models. However, CreditMetrics seems to be more demanding in the administration and control of inputs.

Further research is needed into finding what types of portfolios or parameters produce bigger discrepancies. Stress-testing some parameters, such as the unconditional rate of default, the asset correlation coefficient, the recovery rate or interest rate could give more insights about the sensitivity of results. Likewise, it would be useful to analyse the impact of specific grading systems on the overall performance of the models. For example, does the number of grades in the rating system matter? What is the impact of ranking borrowers in one credit class or in another? This analysis would provide a better understanding of the vulnerability of the models to other inputs. This is particularly important in credit risk, because the quality of data is usually the most important restriction in the implementation of models.

In this exercise the omission of migration risk in CreditRisk+ seemed to have been marginal. However, two broad credit states (default and no-default) might not represent accurately the riskiness of the portfolio for multi-year term loans. Also, it would be worth exploring the effect of the time horizon, as this is likely to influence the results. Ratings migrations will perhaps make more difference at longer horizons.

Finally, there are some other caveats to discuss about the models. For instance, neither of these models has been exactly designed to capture economic cycles. Although implicitly there are some economic factors that drive the credit quality of the borrowers, transition matrices, standard deviations and correlations are usually calculated from historical data across many credit cycles. In this sense, both models have been criticised, since it is empirically clear that parameters depend on business cycles. Hence credit losses might be overestimated in recession periods and underestimated in boom periods (Nickell and Perraudin and Varotto (2000)). It remains to balance the benefits of the simplicity of the assumptions against the reliability of the outputs. However, it would be important to investigate the sensitivity of these models to transition matrices that reflect different periods of the business cycle.

Table 5.1. Unconditional Transition Probabilities

Transition Probabilities		
Rating	Low Quality	High Quality
A	0.88	89.45
B	92.06	9.55
D (default)	7.06	1.00

Table 5.2. Statistics of the Distributions of Default for CreditMetrics and CreditRisk+

STATISTICS OF THE DISTRIBUTION OF DEFAULT						
			CreditMetrics		CreditRisk+	
Correlation	Mean	Stand Dev.	Skewness	Kurtosis	Skewness	Kurtosis
LOW QUALITY PORTFOLIO (LQ)						
0.05	7.06%	3.10%	0.997	4.531	0.878	4.150
0.15	7.06%	5.66%	1.733	7.394	1.600	6.821
0.25	7.06%	7.67%	2.211	9.735	2.167	10.009
0.35	7.06%	9.51%	2.556	11.472	2.686	13.756
0.45	7.06%	11.29%	2.811	12.652	3.188	18.138
HIGH QUALITY PORTFOLIO (HQ)						
0.05	1.00%	0.64%	1.728	8.183	1.273	5.419
0.15	1.00%	1.26%	3.524	24.714	2.509	12.391
0.25	1.00%	1.84%	5.081	46.651	3.663	22.985
0.35	1.00%	2.45%	6.424	68.945	4.870	38.205
0.45	1.00%	3.10%	7.522	87.575	6.171	59.430

Means and Standard Deviations are the same for both models.

Table 5.3. Statistics of the Loss Distributions for CreditMetrics and CreditRisk+

STATISTICS OF THE DISTRIBUTION OF LOSSES				
	CreditMetrics		CreditRisk+	
Correlation	Skewness	Kurtosis	Skewness	Kurtosis
LOW QUALITY PORTFOLIO				
0.05	0.991	4.513	0.879	4.155
0.15	1.729	7.377	1.600	6.822
0.25	2.207	9.713	2.167	10.009
0.35	2.553	11.455	2.685	13.737
0.45	2.810	12.640	3.188	18.140
HIGH QUALITY PORTFOLIO				
0.05	1.669	7.849	1.272	5.409
0.15	3.401	23.214	2.513	12.428
0.25	4.904	43.697	3.664	22.990
0.35	6.209	64.742	4.871	38.245
0.45	7.286	82.665	6.173	59.486

This version of CreditMetrics corresponds to the standard version. The version of CR+ corresponds to the Book Value version.

Table 5.4. The Effect of Migration Risk on CreditMetrics

Differences of CVaR: Default version of CreditMetrics vs MTM version of CreditMetrics						
	Confidence Levels					
Correlation	0.9	0.93	0.95	0.97	0.99	0.999
LOW QUALITY PORTFOLIO						
0.05	0.998	0.999	1.000	1.000	1.000	1.000
0.15	1.000	1.000	1.000	1.000	0.999	1.000
0.25	1.000	0.999	1.000	1.000	1.000	1.000
0.35	0.999	1.000	1.000	1.000	1.000	1.000
0.45	0.999	0.999	1.000	1.000	1.000	1.000
HIGH QUALITY PORTFOLIO						
0.05	0.954	0.957	0.960	0.962	0.967	0.970
0.15	0.953	0.955	0.961	0.965	0.969	0.979
0.25	0.944	0.952	0.958	0.964	0.973	0.983
0.35	0.932	0.946	0.954	0.963	0.976	0.987
0.45	0.908	0.934	0.946	0.961	0.977	0.991

The table compares the CVaR generated by the default version of CreditMetrics with its standard version.

Table 5.5. The Effect of the Distributions of Default of CreditRisk+ and CreditMetrics

Differences of CVaR: CR+1 vs Default version of CreditMetrics						
	Confidence Intervals					
Correlation	0.9	0.93	0.95	0.97	0.99	0.999
LOW QUALITY PORTFOLIO						
0.05	1.002	0.997	0.988	0.982	0.973	0.963
0.15	1.009	0.998	0.991	0.981	0.972	0.972
0.25	1.005	0.992	0.983	0.975	0.974	1.013
0.35	0.990	0.976	0.968	0.962	0.977	1.079
0.45	0.966	0.948	0.940	0.940	0.974	1.154
HIGH QUALITY PORTFOLIO						
0.05	1.042	1.020	1.001	0.972	0.929	0.854
0.15	1.159	1.115	1.080	1.034	0.951	0.827
0.25	1.239	1.193	1.153	1.094	0.991	0.838
0.35	1.263	1.236	1.196	1.138	1.018	0.866
0.45	1.217	1.231	1.215	1.157	1.036	0.892

The table compares the CVaR generated by the default version of CreditMetrics with a version of CreditRisk+ (CR+1) which defines credit losses according to equation 5.17.

Table 5.6. The Effect of the Exposure in CreditRisk+

Differences of CVaR: CR+2 and CR+3 vs CR+1		
	CR+2/CR+1	CR+3/CR+1
LOW QUALITY PORTFOLIO		
	1.007	0.929
HIGH QUALITY PORTFOLIO		
	1.001	0.990

The table compares the three versions of CreditRisk+ (CR+1, CR+2, CR+3) which defines credit losses according to equation 5.17, 5.18 and 5.19.

Table 5.7. Differences of CVaR between CR+2 and CreditMetrics Low Quality Portfolio.

Comparison of Distributions of Losses: CR+2 versus CreditMetrics (MTM) LOW QUALITY PORTFOLIO							
correlation	quantiles	90%	93%	95%	97%	99%	99.90%
	%Variation in CVaR	**0.72%**	**0.35%**	**-0.38%**	**-1.10%**	**-2.16%**	**-3.06%**
0.05	Variation in CVaR	1.007	1.003	0.996	0.989	0.978	0.969
	Effect of Exposure	1.008	1.007	1.008	1.007	1.006	1.007
	Effect of Dist.of Default	1.002	0.997	0.988	0.982	0.973	0.963
	Effect of Migration Risk	0.998	0.999	1.000	1.000	1.000	1.000
	%Variation in CVaR	**1.57%**	**0.52%**	**-0.24%**	**-1.25%**	**-2.23%**	**-2.06%**
0.15	Variation in CVaR	1.016	1.005	0.998	0.987	0.978	0.979
	Effect of Exposure	1.006	1.007	1.007	1.007	1.007	1.007
	Effect of Dist.of Default	1.009	0.998	0.991	0.981	0.972	0.972
	Effect of Migration Risk	1.000	1.000	1 000	1.000	0.999	1.000
	%Variation in CVaR	**1.15%**	**-0.14%**	**-1.02%**	**-1.83%**	**-1.96%**	**1.94%**
0.25	Variation in CVaR	1.011	0.999	0.990	0.982	0.980	1.019
	Effect of Exposure	1.007	1.007	1.007	1.007	1.007	1.006
	Effect of Dist.of Default	1.005	0.992	0.983	0.975	0.974	1.013
	Effect of Migration Risk	1.000	0.999	1.000	1.000	1.000	1.000
	%Variation in CVaR	**-0.38%**	**-1.70%**	**-2.60%**	**-3.19%**	**-1.65%**	**8.60%**
0.35	Variation in CVaR	0.996	0.983	0.974	0.968	0.983	1.086
	Effect of Exposure	1.007	1.007	1.007	1.007	1.007	1.007
	Effect of Dist.of Default	0.990	0.976	0.968	0.962	0.977	1.079
	Effect of Migration Risk	0.999	1.000	1.000	1.000	1.000	1.000
	%Variation in CVaR	**-2.82%**	**-4.54%**	**-5.43%**	**-5.43%**	**-1.99%**	**16.18%**
0.45	Variation in CVaR	0.972	0.955	0.946	0.946	0.980	1.162
	Effect of Exposure	1.007	1.007	1.007	1.007	1.007	1.007
	Effect of Dist.of Default	0.966	0.948	0.940	0.940	0.974	1.154
	Effect of Migration Risk	0.999	0.999	1.000	1.000	1.000	1.000

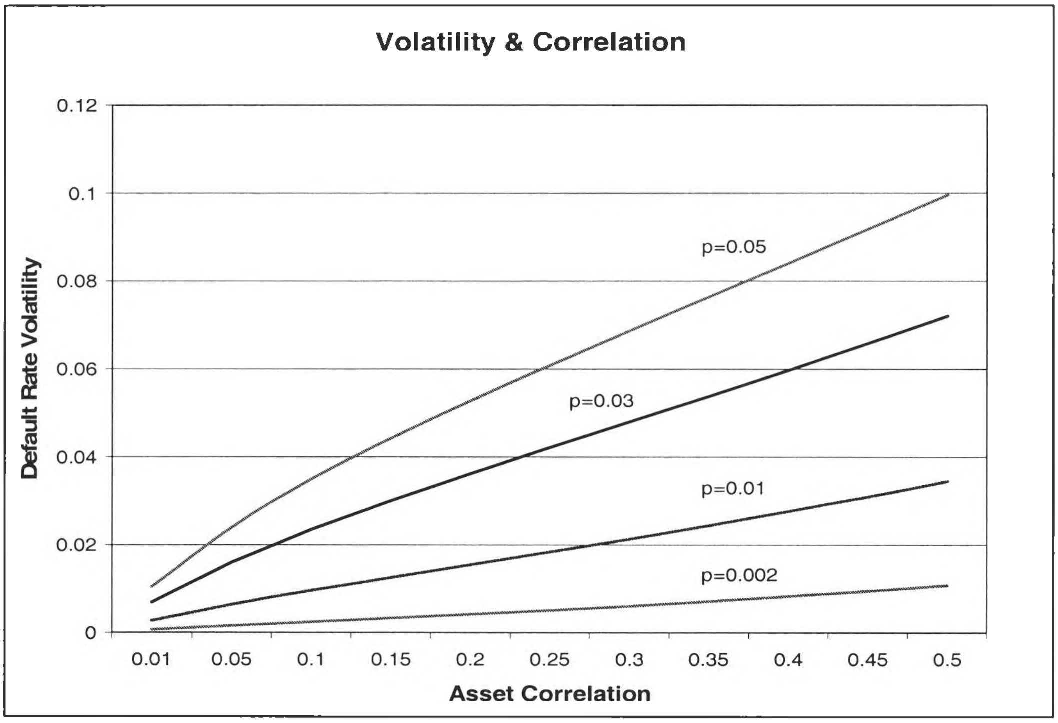
The table compares the CVaR generated for the Low Quality Portfolio by the Book-Value version of CreditRisk+ (CR+2) with the standard version or mark-to-market version of CreditMetrics(MTM).

Table 5.8. Differences of CVaR between CR+2 and CreditMetrics High Quality Portfolio

Comparison of Distributions of Losses: CR+2 versus CreditMetrics (MTM)
HIGH QUALITY PORTFOLIO

correlation	quantiles	90%	93%	95%	97%	99%	99.90%
	%Variation in CVaR	**-0.05%**	**-2.06%**	**-3.91%**	**-5.84%**	**-10.06%**	**-17.11%**
0.05	Variation in CVaR	1.000	0.979	0.961	0.942	0.899	0.829
	Effect of Exposure	1.005	1.004	1.001	1.007	1.002	1.000
	Effect of Dist.of Default	1.042	1.020	1.001	0.972	0.929	0.854
	Effect of Migration Risk	0.954	0.957	0.960	0.962	0.967	0.970
	%Variation in CVaR	**10.46%**	**6.77%**	**3.94%**	**-0.13%**	**-7.70%**	**-18.96%**
0.15	Variation in CVaR	1.105	1.068	1.039	0.999	0.923	0.810
	Effect of Exposure	1.000	1.003	1.002	1.002	1.001	1.001
	Effect of Dist.of Default	1.159	1.115	1.080	1.034	0.951	0.827
	Effect of Migration Risk	0.953	0.955	0.961	0.965	0.969	0.979
	%Variation in CVaR	**17.08%**	**13.57%**	**10.44%**	**5.68%**	**-3.52%**	**-17.46%**
0.25	Variation in CVaR	1.171	1.136	1.104	1.057	0.965	0.825
	Effect of Exposure	1.001	1.000	1.000	1.002	1.001	1.001
	Effect of Dist.of Default	1.239	1.193	1.153	1.094	0.991	0.838
	Effect of Migration Risk	0.944	0.952	0.958	0.964	0.973	0.983
	%Variation in CVaR	**17.73%**	**16.95%**	**14.28%**	**9.69%**	**-0.51%**	**-14.47%**
0.35	Variation in CVaR	1.177	1.169	1.143	1.097	0.995	0.855
	Effect of Exposure	1.000	1.000	1.002	1.001	1.001	1.001
	Effect of Dist.of Default	1.263	1.236	1.196	1.138	1.018	0.866
	Effect of Migration Risk	0.932	0.946	0.954	0.963	0.976	0.987
	%Variation in CVaR	**11.22%**	**15.06%**	**15.13%**	**11.27%**	**1.24%**	**-11.52%**
0.45	Variation in CVaR	1.112	1.151	1.151	1.113	1.012	0.885
	Effect of Exposure	1.006	1.000	1.001	1.001	1.001	1.001
	Effect of Dist.of Default	1.217	1.231	1.215	1.157	1.036	0.892
	Effect of Migration Risk	0.908	0.934	0.946	0.961	0.977	0.991

The table compares the CVaR generated for the High Quality Portfolio by the Book-Value version of CreditRisk+ (CR+2) with the standard version or mark-to-market version of CreditMetrics (MTM).



p is the unconditional probability of default.

Figure 5.1. Consistent Parameters for CreditMetrics and CreditRisk+

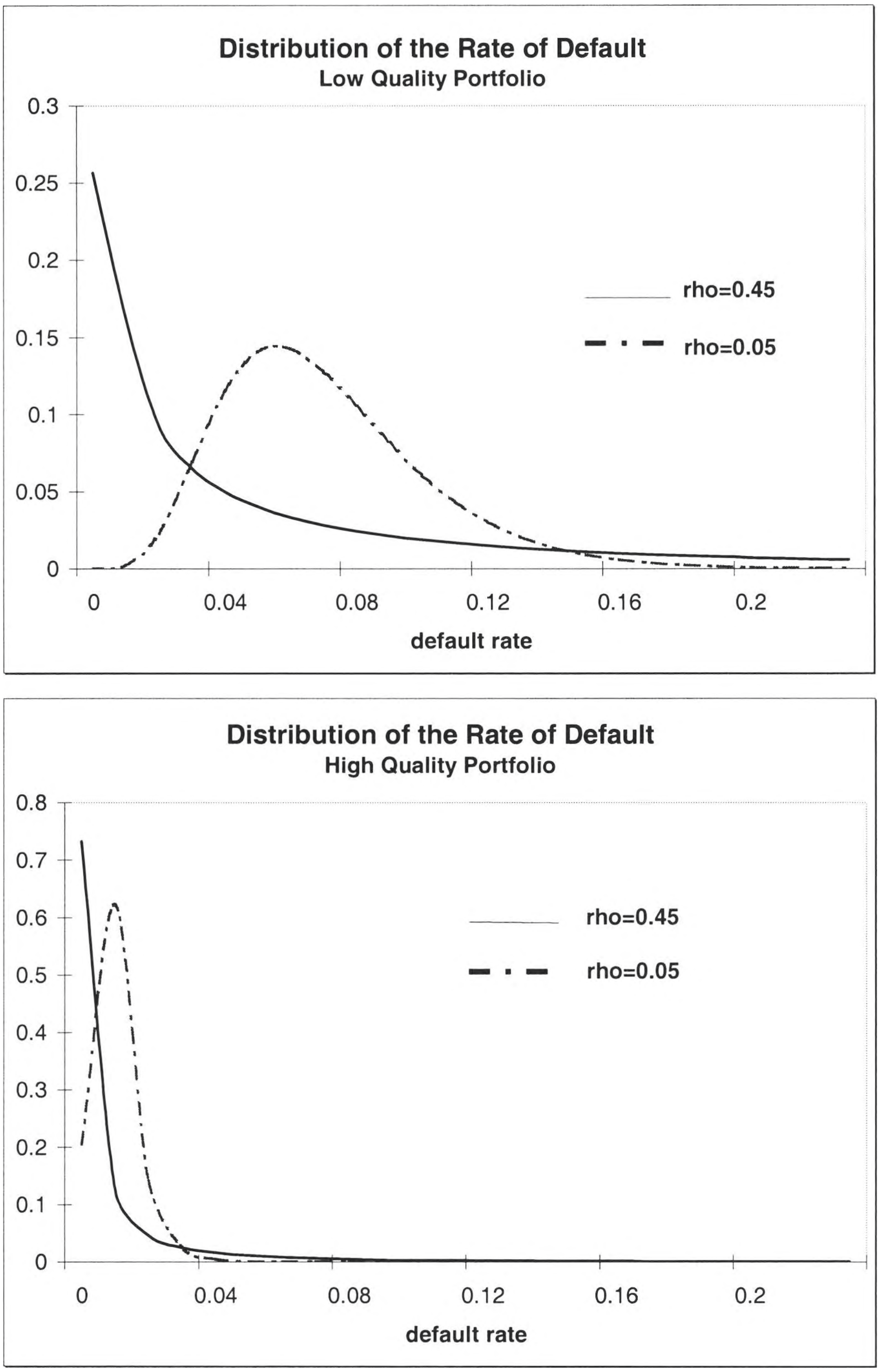


Figure 5.2. Default Rate Distributions for CreditMetrics. Low-Quality (LQ) and High-Quality(HQ) Portfolios

Appendix A: Derivation of the Migration Rates for CreditMetrics

Assume a credit rating system composes of three states: A, B and D, where A represents the highest credit quality, B represents and intermediate state and D represents the state of default. The transition probabilities or migration rates are determined by the vector $[\bar{p}_A,\ \bar{p}_B,\ \bar{p}_D]$.

Assume that the firm's returns follow a Vasicek's representation:

$$r_i = \sqrt{\rho}X + \sqrt{1-\rho}\varepsilon_i$$

where X and ε_i represent the systematic and non-systematic factors of the firm respectively, and both are normally distributed. According to CreditMetrics, for very large portfolios, the economic factor drives the credit quality of the firms or borrowers in the portfolio.

In CreditMetrics, default occurs with probability $\bar{p}_D$ when the firm's returns fall below a threshold c_1 in the standard Normal distribution. See *Figure A.5.1.*

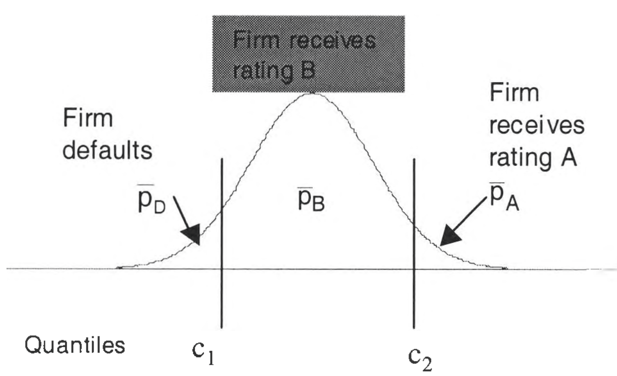


Figure A.5.1. Distribution of the Credit Quality of the Firm

Let $p^{CM}_{D|x}$ be the conditional default rate for CreditMetrics, $c_1 = \Phi^{-1}(\bar{p}_D)$, where $\bar{p}_D$ is the unconditional default rate and $\Phi(s)$ is the normal cumulative density function.

Then the default rate is the probability that asset returns fall below a specific quantile c_1, given a specific value of the economic or systematic factor X. This rate can be calculated as follows:

$$p_{D|x}^{CM} = P(r \leq c_1|x) = P\left(\sqrt{\rho}x + \sqrt{1-\rho}\varepsilon \leq c_1|x\right) = P\left(\varepsilon \leq \frac{c_1 - \sqrt{\rho}x}{\sqrt{1-\rho}}|x)\right) = \Phi\left(\frac{c_1 - \sqrt{\rho}x}{\sqrt{1-\rho}}\right)$$

The density function for the default rate can be estimated as follows:

$$P(p_D^{CM} \leq p) = P\left(\Phi\left(\frac{c_1 - \sqrt{\rho}x}{\sqrt{1-\rho}}\right) \leq p\right) = P\left(x \leq \frac{c_1 - \sqrt{1-\rho}\Phi^{-1}(p)}{\sqrt{\rho}}\right) = \Phi\left(\frac{c_1 - \sqrt{1-\rho}\Phi^{-1}(p)}{\sqrt{\rho}}\right)$$

$$f(p) = \frac{dP}{dp} = \frac{\sqrt{1-\rho}\varphi\left(\dfrac{c_1 - \sqrt{1-\rho}\Phi^{-1}(p)}{\sqrt{\rho}}\right)}{\sqrt{\rho}\varphi\left(\Phi^{-1}(p)\right)}$$

where $\varphi(z)$ is the standardised normal density function.

According to Koyluoglu and Hickman (1998), the default rate volatility can be expressed in terms of ρ and $\overline{p}_D$, using the definition of variance as follows:

$$\sigma_D{}^2 = \int_{-\infty}^{\infty}(p_D^{CM}|_x - \overline{p}_D)^2\varphi(x)dx = \int_{-\infty}^{\infty}\left(\Phi\left[\frac{\phi^{-1}(\overline{p}_D^{CM}) - \sqrt{\rho}x}{\sqrt{1-\rho}}\right] - \overline{p}_D\right)^2\varphi(x)dx$$

The <u>migration rate to the credit state B</u> can be derived in a similar way. By assuming that migration to rating B occurs when returns fall between the thresholds c_1 and c_2 (see *Figure A.5.1.*), where $c_2 = \Phi^{-1}(\overline{p}_D + \overline{p}_B)$, then the migration rate to state B given a specific value of the background factor is:

$$p_{B|x}^{CM}P(c_1 \leq r \leq c_2|x) = P(r < c_2|x) - P(r < c_1|x) = \Phi\left(\frac{c_2 - \sqrt{\rho}x}{\sqrt{1-\rho}}\right) - \Phi\left(\frac{c_1 - \sqrt{\rho}x}{\sqrt{1-\rho}}\right)$$

Finally, the migration rate to the credit state A can be derived using the fact that the sum of the transition probabilities should equal one. Therefore:

$$p_{A|x}^{CM} = 1 - p_{B|x}^{CM} - p_{D|x}^{CM}$$

The effect of the systematic factor on the determination of the default rate and migration rates can be illustrated in *Figure A.5.2*. The realisations of the economy are modelled by a normal random variable, which is the distribution at the top of the diagram. The five transformations that we have derived (equations 5,4, 5.5, 5.6, 5.8 and 5.9) map realisations of the economic factor into the default rates and migration rates.

Assume a borrower with intermediate credit quality B at the beginning of the period. In *Figure A.5.2.*, assume a realisation of the economic factor represented by an observation from the left-tail of the normal distribution Xa. A small value Xa indicates an economy in recession, which leads to more defaults in the portfolio, or equivalently, to high values of the borrower's default rate. In CreditMetrics and CreditRisk+ such default rates are represented by $p_{D|x_a}$ and $p'_{D|x_a}$ respectively. In *Figure A.5.2.*, the realisation of Xa generates a high value of $p_{D|x_a}$ in CreditMetrics, which means a point at the right-tail of the Default Rate Distribution. The effect on CreditRisk is similar.

The realisation of the economy Xa, also affects the migration rates to states A and B. In the diagram for CreditMetrics, only the distributions for the default rate and migration rate to rating B are plotted. When the economy is in recession (i.e. for small values of Xa), the probability that a borrower suffers a downgrading in his credit rating increases. Therefore, the probability that a B-rated borrower keeps the same rating at the end of the period is small, as his credit quality is likely to deteriorate. Therefore, a low value of the economic factor implies low values of migration rates to ratings A or B, i.e., $p_{A|x_a}$ and $p_{B|x_a}$ respectively. In the figure, $p_{B|x_a}$ is located close to zero, representing a low value for this variable. The reverse is true for a high value of the economic factor x_b.

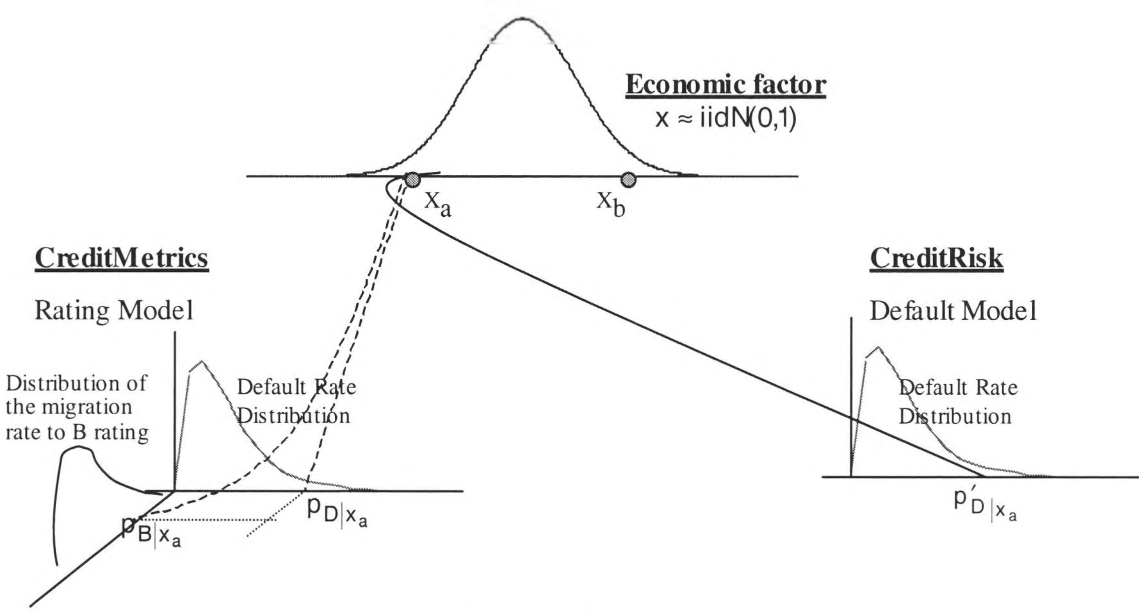


Figure A.5.2. Effect of the Economic Factor on Migration Rates

To generate the distributions of default and the distributions for the migration rates in *Figure A.5.2.*, we need several realisations of the economic factor. According to CreditRisk+, the distribution of the default rate follows a Gamma distribution, whereas that for CreditMetrics this distribution is as equation f(p) above.

Appendix B: Monte Carlo Simulations

B.1 Simulations for the generation of the Distributions of the Default Rate and Migration Rate for CreditMetrics

The inputs required to generate the default distribution of CreditMetrics are the unconditional default rate and migration rates, i.e., $\bar{p}_D$, $\bar{p}_B$ and $\bar{p}_A$, and the correlation between the borrowers in the portfolio ρ. The Monte Carlo simulations involve the following steps:

1. Simulate a standard Normal variable, which represents the economic factor of the portfolio. In order to get reliable results we need a large number of simulations. Faure quasi-random sequence numbers are used to generate random numbers in the interval [0,1]. Application of the inverse of the Normal distribution provides us with a standard normal distribution[21].
2. For each realisation of the economic factor, calculate the default rate $p_{D|X}^{CM}$ according to equation 5.4.
3. Repeat the process (1 and 2) 1,000,000 times (the number of realisations of the economic factor).
4. Calculate the mean, standard deviation, skewness, and kurtosis of the sample.
5. Calculate α and β for CreditRisk+ using the estimated mean and standard deviation of CreditMetrics.

This procedure is repeated for all the combinations of the vector of transition probabilities [$\bar{p}_A$, $\bar{p}_B$, $\bar{p}_D$] and the correlation coefficient. See *Table 5.1.* for the values of the vector of transition probabilities. Use the following correlation coefficients ρ=0.05, 0.15, 0.25, 0.35, 0.45.

[21] Moro (1995) derivation is used to approximate the inverse cumulative Normal distribution.

B.2 Simulations for the generation of the Distribution of the Default Rate under CreditRisk+

The inputs required to generate the default distribution for CreditRisk+ are α and β as estimated by using CreditMetrics.

1. As for CreditMetrics, simulate a standard Normal variable, which represents the economic factor.

2. For a realisation of the economic factor calculate the default rate according to equation 5.8.[22]

3. Repeat the process (1 and 2) 1,000,000 times (number of realisations of the economic factor).

4. Calculate skewness and kurtosis of the empirical distribution of the default rate.

B.3 Simulations for the generation of the Distribution of Losses under CreditMetrics

As inputs for this process we use: a) the above generated samples of the rates of migration and default, generated under CreditMetrics; b) a set of transition probabilities (they determine the type of portfolio); and c) a specific value of the correlation coefficient ρ.

1. For each set of transition rates, calculate the conditional number of individuals that fall into each rating category. These variables follow a multinomial distribution. Therefore, we apply the Kemp and Kemp (1987) algorithm to simulate multinomial random numbers:

1.1 Simulate the number of defaults $N^{CM}_{D|x}$ as a binomial with parameters

$(N=10{,}000, p^{CM}_{D|x})$.

[22] The calculation of this equation may not be straightforward. Instead, solve equation 5.7 using numerical integration. For a given value of χ, find ξ, such that both integrals are equal. To approximate the cumulative distribution of a variable $X \sim \Gamma(\alpha,\beta)$, it is more convenient to approximate the integral when $Y \sim \Gamma(\alpha,1)$ and use the fact that $X = \beta Y \sim \Gamma(\alpha,\beta)$.

1.2 Then simulate N_B^{CM} as a binomial $(N-N_{D|x}^{CM}, \frac{p_{B|x}^{CM}}{1-p_{D|x}^{CM}})$.

1.3 Calculate $N_{A|x}^{CM}=N-N_{B|x}^{CM}-N_{D|x}^{CM}$

2. Obtain the portfolio loss by adding the individual losses of each bond according to the following equation:

$$PL_{t+1|x}^{CM3}=\sum_J (E_t(P_{t+1})-P_{t+1,J})N_{J|x}^{CM} \qquad J=A,B,$$

Also calculate the portfolio loss for the restricted version of CreditMetrics:

$$PL_{t+1|x}^{CM2}=\sum_J (E_t(P_{t+1})-P_{t+1,J})N_{J|x}^{CM} \qquad J=D,\ ND$$

3. In order to obtain the unconditional distribution of losses, repeat steps 1 and 2 for each set of transition rates $[p_{D|x}^{CM}, p_{B|x}^{CM}, p_{A|x}^{CM}]$.

4. Calculate descriptive statistics, including CVaR at the confidence levels: 90%, 93%, 95%, 97%, 99% and 99.9%.

5. Repeat 1-4 for all possible samples, i.e., combinations of different portfolios qualities (LQ and HQ) and levels of the correlation coefficient ρ (=0.05, 0.15, 0.25, 0.35, 0.45).

B.4 Simulations for the generation of the Distribution of Losses under CreditRisk+

For CreditRisk+, the number of defaults is simulated in a similar way, except for steps 1 and 2, which are as follows:

1. Calculate the conditional number of defaults in the portfolio $N_{D|x}^{CR}$ by simulating a Poisson variable with parameter $\lambda=N^{*}p_{D|x}^{CR}$.

2. Obtain the portfolio loss using ND, and the size of the exposures under each scenario.

$$PL_{t+1|x}^{CR+1}=(P_{t+1,ND}-P_{t+1,D})N_{D|x}^{CR}$$

Also calculate the portfolio loss for the other two versions of CreditRisk+, using equations 5.18 and 5.19.

Appendix C: Differences of CVaR between CR+3 and CreditMetrics

Panel C.5.1. Low Quality Portfolio

Comparison of Distributions of Losses: CR+3 versus CreditMetrics (MTM)
LOW QUALITY PORTFOLIO

correlation	quantiles	90%	93%	95%	97%	99%	99.90%
	%Variation in CVaR	**-7.03%**	**-7.50%**	**-8.08%**	**-8.76%**	**-9.70%**	**-10.49%**
0.05	Variation in CVaR	0.930	0.925	0.919	0.912	0.903	0.895
	Effect of Exposure	0.930	0.929	0.930	0.929	0.929	0.930
	Effect of Dist.of Default	1.002	0.997	0.988	0.982	0.973	0.963
	Effect of Migration Risk	0.998	0.999	1.000	1.000	1.000	1.000
	%Variation in CVaR	**-6.30%**	**-7.22%**	**-7.94%**	**-8.87%**	**-9.73%**	**-9.60%**
0.15	Variation in CVaR	0.937	0.928	0.921	0.911	0.903	0.904
	Effect of Exposure	0.928	0.929	0.929	0.929	0.930	0.930
	Effect of Dist.of Default	1.009	0.998	0.991	0.981	0.972	0.972
	Effect of Migration Risk	1.000	1.000	1.000	1.000	0.999	1.000
	%Variation in CVaR	**-6.68%**	**-7.85%**	**-8.67%**	**-9.38%**	**-9.50%**	**-5.90%**
0.25	Variation in CVaR	0.933	0.921	0.913	0.906	0.905	0.941
	Effect of Exposure	0.929	0.929	0.929	0.929	0.929	0.929
	Effect of Dist.of Default	1.005	0.992	0.983	0.975	0.974	1.013
	Effect of Migration Risk	1.000	0.999	1.000	1.000	1.000	1.000
	%Variation in CVaR	**-8.06%**	**-9.29%**	**-10.08%**	**-10.65%**	**-9.21%**	**0.26%**
0.35	Variation in CVaR	0.919	0.907	0.899	0.893	0.908	1.003
	Effect of Exposure	0.929	0.929	0.929	0.929	0.929	0.930
	Effect of Dist.of Default	0.990	0.976	0.968	0.962	0.977	1.079
	Effect of Migration Risk	0.999	1.000	1.000	1.000	1.000	1.000
	%Variation in CVaR	**-10.27%**	**-11.91%**	**-12.71%**	**-12.69%**	**-9.52%**	**7.24%**
0.45	Variation in CVaR	0.897	0.881	0.873	0.873	0.905	1.072
	Effect of Exposure	0.930	0.929	0.929	0.929	0.929	0.929
	Effect of Dist.of Default	0.966	0.948	0.940	0.940	0.974	1.154
	Effect of Migration Risk	0.999	0.999	1.000	1.000	1.000	1.000

Panel C.5.2. High Quality Portfolio

Comparison of Distributions of Losses: CR+3 versus CreditMetrics (MTM)
HIGH QUALITY PORTFOLIO

correlation	quantiles	90%	93%	95%	97%	99%	99.90%
	%Variation in CVaR	**-1.40%**	**-3.36%**	**-4.75%**	**-7.09%**	**-11.09%**	**-18.08%**
0.05	Variation in CVaR	0.986	0.966	0.953	0.929	0.889	0.819
	Effect of Exposure	0.992	0.991	0.992	0.994	0.990	0.988
	Effect of Dist.of Default	1.042	1.020	1.001	0.972	0.929	0.854
	Effect of Migration Risk	0.954	0.957	0.960	0.962	0.967	0.970
	%Variation in CVaR	**9.11%**	**5.48%**	**2.73%**	**-1.17%**	**-8.71%**	**-19.91%**
0.15	Variation in CVaR	1.091	1.055	1.027	0.988	0.913	0.801
	Effect of Exposure	0.988	0.990	0.990	0.991	0.990	0.989
	Effect of Dist.of Default	1.159	1.115	1.080	1.034	0.951	0.827
	Effect of Migration Risk	0.953	0.955	0.961	0.965	0.969	0.979
	%Variation in CVaR	**15.67%**	**12.41%**	**9.29%**	**4.46%**	**-4.60%**	**-18.37%**
0.25	Variation in CVaR	1.157	1.124	1.093	1.045	0.954	0.816
	Effect of Exposure	0.989	0.990	0.990	0.990	0.990	0.990
	Effect of Dist.of Default	1.239	1.193	1.153	1.094	0.991	0.838
	Effect of Migration Risk	0.944	0.952	0.958	0.964	0.973	0.983
	%Variation in CVaR	**16.57%**	**15.54%**	**13.05%**	**8.54%**	**-1.60%**	**-15.35%**
0.35	Variation in CVaR	1.166	1.155	1.131	1.085	0.984	0.847
	Effect of Exposure	0.990	0.988	0.991	0.991	0.990	0.990
	Effect of Dist.of Default	1.263	1.236	1.196	1.138	1.018	0.866
	Effect of Migration Risk	0.932	0.946	0.954	0.963	0.976	0.987
	%Variation in CVaR	**9.85%**	**13.69%**	**13.93%**	**10.01%**	**0.13%**	**-12.45%**
0.45	Variation in CVaR	1.099	1.137	1.139	1.100	1.001	0.875
	Effect of Exposure	0.994	0.988	0.991	0.990	0.990	0.991
	Effect of Dist.of Default	1.217	1.231	1.215	1.157	1.036	0.892
	Effect of Migration Risk	0.908	0.934	0.946	0.961	0.977	0.991

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