

City Research Online

City, University of London Institutional Repository

Citation: Kolakkattil, R., Tsavdaridis, K. D. & Sanjeev, A. J. (2023). A state-of-the-art review of progressive collapse research and guidelines for single-layer lattice shell structures. Structures, 56, 104945. doi: 10.1016/j.istruc.2023.104945

This is the accepted version of the paper.

This version of the publication may differ from the final published version.

Permanent repository link: https://openaccess.city.ac.uk/id/eprint/31044/

Link to published version: https://doi.org/10.1016/j.istruc.2023.104945

Copyright: City Research Online aims to make research outputs of City, University of London available to a wider audience. Copyright and Moral Rights remain with the author(s) and/or copyright holders. URLs from City Research Online may be freely distributed and linked to.

Reuse: Copies of full items can be used for personal research or study, educational, or not-for-profit purposes without prior permission or charge. Provided that the authors, title and full bibliographic details are credited, a hyperlink and/or URL is given for the original metadata page and the content is not changed in any way.

City Research Online: <http://openaccess.city.ac.uk/>publications@city.ac.uk

A state-of-the-art review of progressive collapse research and guidelines for single-layer lattice shell structures

Ranjith Kolakkattil^{a,∗}, Konstantinos Daniel Tsavdaridis^b, Arul Jayachandran Sanjeevi^a

^aDepartment of Civil Engineering, Indian Institute of Technology Madras, Chennai, 600036, Tamil Nadu, India

^bDepartment of Engineering, School of Science and Technology, City, University of London, London, EC1V 0HB, United Kingdom

Abstract

The importance of progressive collapse analysis of structures has increased significantly due to various structural collapses. Single-layer lattice shell structures are susceptible to progressive collapse, as the failure or instability of a localised region can result in a global collapse due to their lightness and wide span. The geometry of these structures plays a crucial role in the load resistance, and any damage that alters the overall geometry can initiate different instabilities, which may also result in progressive collapse. Therefore, the possibility of progressive collapse is high among these lighter structural forms. However, research on the progressive collapse of single-layer lattice shell structures is limited compared to frame structures, highlighting the need for further investigation. Therefore, this manuscript aims to provide a comprehensive review of research conducted on the progressive collapse of single-layer lattice shell structures, focusing on experimental, numerical, and theoretical investigations. Earlier studies have explored the propagation of local instabilities leading to overall failure. More recent research has primarily focused on two approaches: (1) alternate path analysis, which examines the structure's response after removing a member, and (2) substructure analysis, which aims to identify collapsepreventive mechanisms. Based on these analyses, various methods to enhance progressive collapse resistance have been discussed, including specific local resistance, alternate load paths, compartmentalisation, and tie forces. This review also highlights the limitations present in the current literature and suggests future research directions, which helps to develop design guidelines that can effectively increase the progressive collapse resistance of single-layer lattice shell structures, making them more reliable for various structural applications.

Keywords: Single-layer lattice shells, Progressive collapse, Alternate path method, Specific local resistance, Structural robustness, Kiewitt dome, Cylindrical shell, Snap-through buckling, Review

1. Introduction

Civil structures are generally designed to resist the anticipated loads they will experience over their operational lifespan. However, there are instances where structures may face extreme or accidental loads that exceed their design parameters. These loads can trigger local failures within the structure, eventually leading to progressive collapse. Progressive collapse refers to the scenario where the failure of a single element or a group of elements initiates further failures, ultimately resulting in the partial or total collapse of the structure [1]. A structural collapse can be classified as progressive when the initial failure is local, the spread of the failure is from a local failure, and the final collapse is disproportionate to the initial local failure [2]. The collapse of a structure occurs when the system fails to attain a new equilibrium following a local failure. The partial collapse of Ronan Point Apartment building, Murrah Federal Office Building, and the failure of the World Trade Centre twin towers has led to extensive research on the field of progressive collapse.

A spatial structure is a type of structural system in which linear elements are connected in three-dimensional space to resist external loads [3]. The notable advantage of spatial structures, particularly those with curved surfaces, lies in the influence of their geometry on load resistance [4]. Due to this advantage, spatial structures primarily experience axial internal forces in the members, allowing for efficient utilisation of the cross-section to withstand external forces. These structures are lightweight and capable of spanning large areas with minimal intermittent support. The lightness and wide span of spatial structures can give rise to various instabilities and potential failures. The stability and load resistance of these shell structures are influenced by their form, configuration, and connections, which constitute the structural system [3]. The'form' explains the shape of the overall structure, and the 'configuration' or the internal structure gives the arrangement by which members are connected on the surface or multiple surfaces. Depending on the requirements, the configuration can be single- or multi-layered. Many space structures are formed by the spatial arrangement of linear members with the help of suitable connections, where the behaviour of the structure varies with the type of connection (rigid, semi-rigid, or pin) (Figure 1). Different types of instabilities and local failures can cause the progressive collapse of spatial structures, which is influenced by the three given factors.

Preprint submitted to Structures July 31, 2023

[∗]Corresponding Author

Email address: ranjithk@alumni.iitm.ac.in (Ranjith Kolakkattil)

Figure 1: The structural system of a spatial structure is influenced by three factors: form, configuration, and connections. These factors play a crucial role in determining the structural stability and resistance to the progressive collapse of any spatial structure.

Higher redundancy helps to increase the progressive collapse resistance of different structures [5, 6]. Although lattice structures have higher redundancy in the surface of revolution, they are vulnerable to progressive collapse because of the spatial arrangement of interconnected members and the lower redundancy and stiffness in the plane perpendicular to the 'surface of revolution' (Figure 2). 'Surface of revolution' or 'form' is the geometrical surface on which the members are interconnected. For example, the surface of revolution for the dome in Figure 2 is a sphere. The availability of vertical redundant forceresisting systems is lower for spatial structures compared to frame structures, resulting in higher chance of progressive collapse in the former compared to the latter. In contrast to framed structures, local failures in spatial structures, such as uneven snow load distribution, often lead to the complete collapse of the structure. As an example, the failure of the Bucharest dome was attributed to an unsymmetrical distribution of snow load, which was only 30% of the intended design load [7]. Similarly, the CW post auditorium experienced snap-through buckling and collapse of the entire dome structure due to uneven snow load distribution caused by wind [8]. Furthermore, design errors led to compression members being overloaded by snow load, resulting in member buckling and the subsequent progressive collapse of the Hartford Civic Center Coliseum [9]. These incidents highlight the

Figure 2: Comparison of the progressive collapse resistance of (a) framed structure and (b) single-layer lattice shell structure: The impact of a local failure is higher in single-layer lattice shells due to the spatial arrangement of the members and the lack of vertical redundant force-resisting systems. Unlike framed structures, where the non-structural members (such as infill walls and floor slabs) contribute to the resistance to collapse, glazing or covering panels in single-layer lattice structures have a lower impact on the progressive collapse resistance.

significance of preventing the propagation of local failures to avoid progressive collapse.

The utilisation of lightweight materials, probabilistic design approaches, and modern construction techniques have led to structures with reduced reserve capacity. For costeffective designs, structural engineers often opt for slender sections with limited reserve capacity. This poses challenges when structures are subjected to abnormal loads such as sabotage, explosions, and natural hazards. The reduced reserve capacity and increased likelihood of abnormal load occurrences contribute to a higher risk of progressive collapse in newly constructed spatial structures.

Achieving maximum resistance to abnormal loads in every structure is not economically feasible or practically quantifiable. Therefore, there is a trade-off between economy and safety in the design of structures for abnormal loading scenarios. While it may not be possible to prevent local collapse under every abnormal load, appropriate design measures can limit the propagation of the collapse. Therefore, it is crucial to conduct progressive collapse analysis for lightweight structures, such as single-layer spatial structures, to ensure their resilience and safety.

Single-layer spatial structures (Figure 3) have lower redundancy and stiffness compared to multi-layer spatial structures, making them more susceptible to progressive collapse. The availability of vertical redundant force-resisting systems (i.e., vertical elements)

Figure 3: Single-layer reticulated shells: (a) Cylindrical shell (b) Dome. The probability of various instabilities leading to progressive collapse is greater in these single-layer spatial structures than in multi-layer spatial structures.

in space grids and multi-layer spatial structures increases their ability to resist progressive collapse compared to single-layer spatial structures. Single-layer spatial structures are more dependent on the 'form' to resist the load, and any change in the form can increase the member stresses, reducing the efficiency of similar structures. Therefore, single-layer spatial structures are more prone to progressive collapse than multi-layer spatial structures.

It is important to note that local collapse in single-layer reticulated shell structures can occur in various ways. A member can fail due to yielding, rupture, or buckling. When tension members yield, they can still carry additional load due to the strain-hardening of the material. However, buckled members lose their load-carrying capacity, leading to a transfer of forces to adjacent members. Moreover, the buckling load capacity of members is typically lower than their yield strength. Slender members in spatial structures can experience different types of instabilities when subjected to external loads [10–12], which can propagate and result in the failure of the entire structure [13–15]. Additionally, spatial structures can suffer from structural damage caused by planning, design, and construction errors. Consequently, the likelihood of local failure and propagation is higher among single-layer reticulated shells.

The research conducted on the progressive collapse of single-layer lattice shells is not as extensive as that on framed structures and space grids. It is essential to review the existing literature on progressive collapse to identify the research gaps and scope for further investigation. While review articles comprehensively discuss progressive collapse studies on framed structures [2, 16–18], there is a lack of similar extensive reviews for reticulated spatial structures. The available research on the progressive collapse of these structures provides valuable insights into the current progress and the areas that require further investigation. Therefore, this paper aims to present a comprehensive overview of the existing numerical, experimental, and analytical studies conducted on progressive collapse in single-layer spatial structures. It also highlights the potential for future research to enhance collapse resistance based on the existing literature.

2. History of progressive collapse analysis and design guidelines

2.1. Progressive collapse and disproportionate collapse

An initial failure that spreads to other members, resulting in the failure of a significant portion of the structural system, is referred to as 'progressive collapse' [19–22]. Conversely, when the magnitude of failure is much larger than the initial damage, it is termed 'disproportionate collapse' [22, 23]. Disproportionate collapse pertains to the size of the collapse rather than its propagation [18]. While progressive collapse can be either disproportionate or proportionate, the disproportionate nature of progressive collapse is of greater concern [5].

2.2. Major progressive collapses and the earlier research

All theoretical knowledge on the progressive collapse of structures is based on research conducted on framed structures [2, 18, 24]. Previously, knowledge and guidelines on the progressive collapse of framed structures were limited due to the lack of experimental and numerical investigations. The failure of important buildings [7, 25–27] has led to extensive research on this topic (Figure 4). The design guidelines were prepared based on examining these failures and the latest research findings [1, 6, 28–32]. The advancement in Finite Element (FE) analysis softwares helped to investigate the progressive collapse of structures in detail by considering different parameters [33–35]. The primary objective of the investigations was to prevent progressive collapse from local failures (due to gas explosions, bomb explosions, vehicle collisions, etc.) so that damage to the building and the public could be avoided or minimised. The impact of these events is very high, even though the probability of their occurrence is lower. Therefore, the need to improve the progressive collapse resistance of critical buildings arises, and the designers should make sure that they have adequate progressive collapse resistance.

After the partial collapse of the Ronan Point building, Lewicki et al. indicated that it might not be economical to prevent the local collapse from the extreme loading environment, even though it is possible to prevent the progressive collapse [37]. The intensity of the external load that triggers the local collapse cannot be calculated, and it will be uneconomical to design structures to prevent local collapse. Therefore, the primary aim for the progressive collapse design of structures should be to prevent the spreading of the local collapse leading to the entire collapse of the structure so that large casualties can be avoided. By taking these things into consideration, Leyendecker et al. suggested methods to reduce the risk of progressive collapse, which include direct design strategies such

Figure 4: History of major progressive collapses and design guidelines or standards recommending methods to improve the progressive collapse resistance. The data is adapted from Wang et al. [36], Kiakojouri et al. [2], and Adam et al. [18]

as the alternate load path method and the specific load resistance method [38]. These methods are still the basic foundation upon which progressive collapse design methods are formulated for any structure [1, 39, 40], whether it is a framed or a spatial structure.

The earlier studies were based on static analyses of the structures. The actual stresses and strains generated in members due to the dynamic load effect will be greater than those obtained from the static analysis. The experimental and numerical analysis found that the dynamic analysis can predict the failure load for which the static analysis may provide a safe response [41]. Later, the dynamic effects of abnormal loads were studied by various researchers [42–46]. Many researchers conducted a detailed review of the literature on the progressive collapse of framed structures [2, 18, 24, 47]. The review suggests that the arch effect of the beam, bending of the beam, catenary action of the beam and slabs, the effect of non-structural elements, and the Vierendeel action are the methods by which progressive collapse is prevented in framed structures when a column fails [18]. Catenary action, which significantly improves progressive collapse resistance, has been studied by many [48–50]. Based on the analysis of structures, different guidelines were created to increase the progressive collapse resistance of important structures.

2.3. Design guidelines

Building codes and standards must consider the effect of accidental loads to prevent progressive collapse. However, it is difficult to quantify the design requirements for structures because of the wide variety of structural types and the difficulty in identifying the intensity of collapse-triggering events. Therefore, the design guidelines suggest standard methods that can be adopted for improving structural robustness [51]. In general, provisions to improve continuity and ductility enhance the progressive collapse resistance, similar to the earthquake-resistant design [52, 53].

After the Ronan Point collapse (1968), the UK building guidelines insisted on preventing the progressive collapse of the building by considering redundant members, sufficient strength, and tying building elements [54]. Later, guidelines from the United States insisted on 'general structural integrity' for the building to resist progressive collapse, where structural integrity is achieved by continuity, redundancy, and ductility [55].

The partial collapse of the Murrah Federal Building and the collapse of the twin towers in the United States led to a significant revision of the design guidelines in the country [56– 58]. Guidelines for federal and defence buildings were formulated in response to these incidents [6, 31, 39, 59]. ASCE 7-98 [57] underwent revisions in 2002 [58] and 2017 [1], highlighting the need for structural robustness to enhance resistance against progressive collapse.

The National Building Code of Canada addresses the requirement of robustness in structures [28, 29]. The Australian code discusses the need for continuity and ductility to improve progressive collapse resistance [60]. In China, earlier guidelines focused on preventing global collapse resulting from local failure [61]. Subsequently, a specific code for the anti-collapse design of building structures was developed, which outlines design methods to prevent collapse [62]. It should be noted that only a limited number of guidelines provide a comprehensive overview of preventive measures for progressive collapse.

The ASCE Standard 7-16 [1] emphasises the need to ensure structural safety against extreme loads resulting from accidents, sabotage, and misuse. It discusses the design approaches proposed by Ellingwood and Leyendecker, namely the 'direct design method' and the 'indirect design method' [63]. The direct design methods, which include the Alternate Path Method (APM) and the Specific Local Resistance Method (SLRM), address progressive collapse resistance during the design phase. APM prevents global collapse by incorporating alternate load paths, while SLRM ensures sufficient strength in critical regions to prevent progressive collapse. On the other hand, the indirect method addresses progressive collapse resistance implicitly by considering minimum strength, ductility, and continuity requirements. The ASCE Standard 7 provides guidelines for maintaining structural integrity, highlighting the importance of ductility, large deformations, and high energy absorption of connections under abnormal loads. However, these requirements are not quantified in ASCE Standard 7.

The National Earthquake Hazard Reduction Program (NEHRP) provides seismic regulations for structures [64], which include provisions for designing against progressive collapse. NEHRP recommends the construction of redundant structures to establish alternate load paths that can prevent collapse due to lateral forces. In framed structures, the inclusion of moment-resisting joints in seismic force-resisting systems enhances both vertical load resistance and progressive collapse resistance by providing structural redundancy.

When determining the likelihood of progressive collapse, FEMA-426 [40] emphasises the importance of considering local components as well as the overall stability behaviours of the building. The document also provides a summary of the correlation between progressive collapse and various characteristics of structural systems, including load path, redundancy, ductility, and tying.

The General Services Administration (GSA) created design guidelines [39] to address the progressive collapse in planning, design, and construction of new buildings as well as renovation projects. According to the GSA, structures can be evaluated for their potential for progressive collapse using either linear or nonlinear procedures. The linear procedure involves a simplified approach using static or dynamic linear-elastic FE analysis. On the other hand, the nonlinear procedure is a more advanced approach that utilises static or dynamic elasto-plastic FE analysis, considering material and geometric nonlinearity. The GSA recommends a threat-independent methodology to mitigate progressive collapse, which focuses on preventing global collapse resulting from local failure, regardless of the

specific abnormal load. Redundancy plays a crucial role in reducing the likelihood of failure propagation by providing alternative load paths and multiple yielding locations in the event of localised damage.

Three methods to improve the progressive collapse resistance are discussed in the United States Department of Defence's Unified Facilities Criteria (UFC) [31]. These methods are the tie-force method, the alternate load path method, and the Enhanced Local Resistance (ELR) method. In the tie-force method, structural components are mechanically tied together to increase continuity, ductility, and alternate load paths. The alternate path method is used when tie strength cannot be achieved in vertical elements. The structure is examined to see if the spread of local collapse, created by removing one member at a time, can be avoided. In the ELR method, ductile failure should occur in the structure when the column or wall is loaded laterally. Therefore, the column or wall should not fail in shear before flexural failure and should satisfy the ELR criteria.

NIST – 2007 [59] summarises all the codes and standards and proposes practices for reducing the progressive collapse potential for new and existing facilities. A detailed review of the direct method (specific local resistance and alternate path method) and the indirect method (tie force enhancement) is provided by NIST.

The analysis of design codes and guidelines indicates that there is no standard approach to enhance the resistance to progressive collapse in structures. Various methods can be employed depending on the specific structure and the desired limitations on local collapse. These methods may include the use of ties, implementation of alternate load paths, and enhancing local resistance, among others. The selection of appropriate measures should be tailored to the unique features and requirements of each structure.

2.4. Alternate path method

The alternate path method is the standard analysis technique recommended by various guidelines to examine the progressive collapse resistance, in which the ability of the structure to withstand the load after the removal of a member is analysed [1, 6, 31]. This method was extensively adopted for framed structures. Later, it was used to assess the progressive collapse potential of other structures, such as spatial structures, where the sensitivity of the structure towards the removal of each member could be monitored. Static analysis was adopted during the earlier days. The use of dynamic analysis was suggested by many researchers and design guidelines, as static analysis cannot capture the actual dynamic effect of the sudden removal of a member [41, 45, 46]. Static and dynamic analysis based on the alternate path method has gained popularity among designers and researchers for estimating the progressive collapse resistance of framed structures and long-span spatial structures.

GSA discusses the detailed procedure to be adopted in the alternate path method [39].

In this method, where the progressive collapse resistance of a framed structure is studied by removing one column, the collapse resistance is examined when the following load combination is applied to the structure.

For static analysis,

$$
W = 2(DL + 0.25LL)
$$
 (1)

For dynamic analysis,

$$
W = DL + 0.25LL
$$
 (2)

where W is the analysis load, DL is the dead load, and LL is the live load.

Acceptance criteria for structural components are determined based on the Demand Capacity Ratio (DCR) (Equation 3).

$$
DCR = \frac{Q_{UD}}{Q_{CE}}\tag{3}
$$

Here, Q_{UD} is acting force and Q_{CE} is expected ultimate capacity.

The value of DCR should be less than 2 for typical structures (simple structural layout) and 1.5 for atypical structures (structural layout with distinguishing features such as vertical discontinuities, layout irregularities, and closely spaced columns). This method is similar to the one used in FEMA-273 [65] and FEMA-356 [66].

The evaluation of progressive collapse potential commences with a linear static analysis once the designated column has been removed. An element is treated as a failed member if the DCR based on shear force exceeds the limit specified or a mechanism is formed after exceeding the DCR values based on the flexure. The failed member should be removed from the structure, and the corresponding live and dead loads should be redistributed to the adjacent members. After that, the analysis should continue until the DCR value of the remaining elements in the structure is below the limit. The structure has a high progressive collapse potential if the DCR values exceed the specified limit. In dynamic analysis, the vertical element should be removed such that it should be done within one-tenth of the period associated with the structural response mode of the building. The members whose DCR value is exceeded should be redesigned to reduce the chance of progressive collapse. The redesigned structure should be analysed again to ensure the potential for progressive collapse is reduced.

2.5. Methods to improve the progressive collapse resistance

There is no universal method by which the design for progressive collapse can be applied to every structure. All the guidelines recommend different methods derived from major progressive collapses and many research findings [6, 20, 31]. Increasing the structure's robustness can reduce the chance of progressive collapse, where robustness refers to the structure's ability to prevent a global collapse from a local collapse [67]. Increased strength, ductility, continuity, stability, and redundancy are methods that help to improve the progressive collapse resistance of a structure [2, 31, 39, 59].

More specifically, the design strategy for progressive collapse can be either preventing local collapse or designing for local failure [68]. To prevent local failure, 'specific local resistance' or 'non-structural protective methods' can be adopted, which do not increase the structural robustness of the system. For designing for the local failure, 'alternate load paths' or 'isolation by compartmentalisation' methods can be adopted, which increase the robustness of the structure by limiting the failure to a specified region [23, 67]. The safety of key elements also needs to be ensured so that the local collapse can be restricted. The prevention of local failure is not in the hands of structural designers, as the intensity, location, and spread of local failure cannot be anticipated. On the other hand, the designer can ensure the prevention of spreading local failure by providing alternate load paths, compartmentalisation, and designing for critical elements. Therefore, designing to restrict the propagation of local collapse is a more suitable approach for structures that are important and prone to progressive collapse, which ensures high safety at a reasonable increase in construction cost and is less dependent on the uncertainty of external threats.

In summary, the progressive collapse prevention methods can be broadly divided as follows:

- Alternate load path or Redundancy
- Local resistance or key element design
- Interconnection or continuity (Tying force)
- Isolation by compartmentalisation (Segmentation)

Redundancy is essential for improving structural robustness by providing different load paths (Figure 5(a)), so that the system can resist local damage without significant progression. The alternate path method is the most popular way of accessing progressive collapse resistance, and most of the existing design guidelines discuss it [1, 6, 29, 31, 32, 62, 69, 70]. This method accesses the ability of the structure to redistribute the forces after the local failure. The progressive collapse resistance based on the alternate path method can be accessed through static analysis, dynamic analysis, or static analysis by considering the dynamic effect through an amplification factor [39].

Local resistance or key element design method known as the specific local resistance, proposed by Ellingwood and Leyendecker [63], and later incorporated by ASCE/SEI 7 [58]. Design codes such as EN 1991-1-7 [69], UFC 4-023-03 [31], GSA 2013[39], CECS

Figure 5: Methods to increase the resistance against progressive collapse are categorised as (a) Alternate load path or Redundancy, (b) Local resistance or key element design, (c) Interconnection or continuity (Tying force), and (d) Isolation by compartmentalisation (Segmentation)

392:2014 [62], and ASCE/SEI 7-16 [1] discuss the key element design method. This method can be helpful when there is no way in which alternate paths can be provided to the structure. The key elements are identified in a structure, and their strength is enhanced so that local failure of the critical elements will not cause progressive collapse (Figure 5(b)).

Interconnections or tying methods provide a minimum level of continuity and ductility (Figure 5(c)). Tying methods are mentioned in design guidelines such as ASCE/SEI 7–16 [1], UFC 4-023-03 [31], IBC 2010 [56], and EN 1991-1-7 [69].

Another method to improve the progressive collapse is through segmentation of the structural system so that failure is not transferred from one location to the other (Figure 5(d)). Isolation by compartmentalisation in structures is provided by either stopping or strengthening the continuity across different regions or compartments such that the progression of the collapse can be prevented [67]. In some cases, such as reticulated shells, the continuity (or alternate paths) could trigger progressive collapse, if the resulting alternate paths after the local collapse lack the strength to resist the inertial effects due to dynamic forces. If an element or a local region of a reticulated structure fails with a significant dynamic effect, it can trigger overload or stress concentration in adjacent elements. This increase in stress or load distribution in neighbouring elements may lead to a chain reaction of failures, resulting in the progressive collapse of the structure. Therefore, discontinuity can prevent the propagation of local failure in similar situations. A good example is the snap-through buckling originating in a local region and propagating to the entire structure. The dynamic effect of initial buckling could trigger further node buckling if sufficient strength is unavailable for the surrounding region. Segmentation or compartmentalisation is an efficient method to prevent progressive collapse if the alternate path method cannot provide sufficient strength to resist progressive collapse. Segmentation can be achieved by either less continuity or high strength of the configuration near the local failure [67].

3. Single-layer lattice shells

Research on the progressive collapse of spatial structures commenced at a later stage compared to framed structures, primarily due to the relatively lower occurrence of progressive collapse incidents in spatial structures when compared to framed structures (Figure 4). Two notable instances of collapse in reticulated shells occurred in the form of the Bucharest dome failure in 1963 and the Hartford Civic Centre collapse in 1978. Both incidents occurred due to the presence of an unsymmetrical distribution of snow load, leading to structural failure. Initially, localised failure was triggered by the uneven load distribution, which ultimately led to the collapse of the entire structure [7–9, 71]. Other progressive collapse incidents include the roof failure of Kemper Arena in 1979, the collapse of the Bad Reichenhall Ice Rink roof in 2006, the roof collapse of the Katowice Fair building in 2006, and the partial collapse of the Twente stadium roof during its construction in 2011. In each case, the failure originated in a small region and progressively spread to a larger area [25, 72–74]. The lightweight nature and absence of intermittent supports in spatial structures contribute to an increased susceptibility to progressive collapse. Singlelayer reticulated shell structures are more sensitive to progressive collapse due to their lower redundancy and stiffness compared to other multi-layer spatial structures, such as space grids.

The fundamental elements of spatial structures, namely form, configuration, and joint system, significantly impact the probability of progressive collapse. The progressive collapse in single-layer reticulated shell structures can be attributed to a combination of internal factors, including substandard design, insufficient member quality, and inadequate connections, as well as external factors, like the application of extreme or asymmetric loads. Single-layer reticulated shell structures, characterised by slender members and lightweight design, have a higher probability of experiencing progressive collapse due to different instabilities arising from given factors. As the occurrence of progressive collapse is heavily influenced by various types of instabilities and the level of connection rigidity, the subsequent sections will delve into the examination of different instabilities, the dynamic implications associated with these instabilities, and the impact of joint rigidity.

3.1. Instabilities in single-layer lattice structures

The likelihood of encountering various instabilities that can lead to progressive collapse is significantly elevated for single-layer spatial structures in comparison to other structures. Earlier studies have found that instabilities can cause catastrophic failures in similar structures [75]. There are different types of local instability modes that can arise in spatial structures, such as node instability, member instability, line instability, ring instability, torsional instability, and general instability [12]. Local instabilities such as member instability and node instability can spread and cause the failure of the entire structure (Figure 6). Yan et al. coined the term 'progressive instability' (Figure 7) to describe the phenomenon where member buckling spreads and ultimately leads to the collapse of the entire structure based on the measurement of deflections at the middle and end portion of a member [14].

The instabilities can be divided into bifurcation point instability and limit point instability [76]. Bifurcation instability occurs when the structure transitions from one equilibrium path to another after reaching the critical load, resulting in bifurcation of its behaviour (Figure $8(a)$). When the compressive force increases, the member or structure that initially deflects in the direction of the applied loads abruptly changes its direction of deflection. The deflection path that occurs before bifurcation is referred to as the primary path, while the deflection path that occurs after bifurcation is the secondary or post-buckling path.

Figure 6: Major instability modes that can arise in single-layer spatial structures are (a) Member instability and (b) Node instability

Figure 7: Progressive instability arising from the member buckling. In the initial stages, the occurrence of member instability within the structure gives rise to the subsequent buckling of additional members. This progressive phenomenon ultimately leads to the overall failure of the structure [14]. *P^m* is the load at which member buckling occurs, and P_s is the load at which the overall buckling of the structure occurs.

Figure 8: Type of instabilities: (a) Bifurcation instability and (b) Limit point instability

Bifurcation analysis with symmetrical load is not sufficient for the design of single-layer spatial structures such as barrel vaults. Unsymmetrical loading predominantly triggers instabilities [12]. Limit point instability occurs when the system loses its stability at a specific point or load level where the buckling mode aligns with its initial mode of deformation. In limit point buckling, the structure reaches the maximum load-carrying capacity before undergoing instability and failure, and it experiences an unstable response and collapses once the limit point is exceeded (Figure 8(b)). Once the limit point is reached, the system can snap through from one equilibrium position to another equilibrium position, which is known as snap-through buckling. Therefore, different instabilities in single-layer shell structures can spread and cause progressive collapse.

3.2. Dynamic effects of snap-through buckling

Snap-through buckling is a type of structural instability where the structure suddenly transitions from one stable configuration to another through an unstable configuration. It occurs when the applied loads exceed a critical load, causing the system to undergo a sudden and significant change in its shape. In snap-through buckling, the structure initially deforms smoothly under increasing loads until it reaches a critical load (Figure 9). At this load, the structure quickly transitions to a different state due to the presence of an unstable state (i.e., As the path AB in Figure 9 represents an unstable trajectory, the structure will undergo a 'snap-through' motion from Point A to Point C) [77]. Snap-through buckling can lead to substantial displacements or changes in the structural response of the system. These abrupt changes can cause a dynamic response in the structure, resulting in further displacements (Figure 10). Additionally, snap-through buckling involves releasing poten-

Figure 9: Snap-through buckling in a structure. The structure will undergo a snap-through from point A to C as path AB denotes an unstable trajectory.

tial energy stored in the structure as it transitions from one stable state to another state. This energy release can generate additional dynamic effects, leading to further deformations in the structure.

Previous research has focused on analysing the dynamic consequences of snap-through buckling. Lenza [75] investigated the dynamic implications of nodal snap-through phenomena in vault structures. The substantial inertial forces generated during nodal snapthrough play a crucial role in the propagation of failure, ultimately leading to structural collapse. Gioncu [12] discussed the danger of propagation of node instability and member instability due to the strong dynamic effect associated with them. Abedi has conducted research on the dynamic behaviour of nodal snap-through buckling and its propagation in braced domes [46, 78]. They adopted a procedure where the dynamic effect of snapthrough in braced domes is represented by assigning suitable initial velocities to the nodes experiencing snap-through. The study found that the dynamic propagation of nodal snapthrough events can lead to the collapse of the whole structure. Mohammadi et al. [76] discussed the requirement of dynamic instability analysis for structures such as singlelayer vaults. During certain instabilities, such as snap-through buckling, a significant release of kinetic energy occurs at a localized region within the structure. This release of energy possesses dynamic characteristics and has the potential to result in overall collapse. Therefore, relying solely on static collapse analysis is insufficient in order to evaluate the realistic response of similar structures. It is necessary to complete it with dynamic snapthrough analysis to accurately evaluate the dynamic behaviour and potential snap-through buckling.

The investigations indicate that the dynamic effects linked with the snap-through are often characterised by the release of significant amounts of energy. This energy release can lead to the partial or complete progressive collapse of the structure. Therefore, dy-

Figure 10: The effect of snap-through buckling in a single-layer reticulated dome. The initial occurrence of snap-through instigates a dynamic effect that leads to the spread of snap-through and ultimately results in the failure of the entire structure.

namic analysis is necessary to accurately examine the snap-through instability occurring in single-layered structures.

3.3. Effect of joint rigidity

The rigidity of the connection plays a major role in the propagation of local instability or local failures, resulting in progressive collapse. The earlier research focused on the effect of joint rigidity on the stability of structures. Later, the effect of joint rigidity on the progressive collapse was studied through alternate path analysis.

3.3.1. Effect of joint rigidity on stability

The bending stiffness of a connection can significantly influence the overall stability and failure mode of the structure [79]. The presence of full rigidity in the joints can eliminate or mitigate snap-through instability [80]. The experimental test on a singlelayer dome structure revealed that snap-through buckling was prevented due to the high rigidity of the joints. However, a single-layer latticed dome with a semi-rigid joint of a bolt-ball system can be a suitable solution for space structures with small to medium spans, as similar joints can ensure high load capacity [81].

In a study by considering double elements and imperfect members, the authors revealed that considering joint stiffness and member imperfection is crucial as they have a significant impact on the buckling capacity of latticed shell structures. These factors should be taken into account to ensure accurate analysis and design of such structures [82].

The stability study on the aluminium alloy single-layer dome structure with Temcor joints found that the semi-rigidity of the joints could reduce global stability [83]. The experimental and numerical study of the single-layer reticulated shell with aluminium alloy gusset joints found that the rigid joints enhance their overall stiffness. However, the semirigid stiffness of the joints can potentially compromise the buckling behaviour of similar structures [84–86].

The impact of joint size on the overall rigidity and load-carrying capacity of reticular shells is substantial. As the joint size decreases, the limit load gradually decreases, indicating an approximately linear relationship between the limit load and joint size [87]. Design optimisation of dome structures by simultaneously considering member section and joint rigidity found that the stability of the optimal dome with flexible joints is nearly equivalent to that of the optimal dome with rigid joints. However, a notable reduction in joint steel consumption is observed in the case of flexible joints during the study [88].

3.3.2. Effect of joint rigidity through alternate path analysis

All the progressive collapse investigations on single-layer shell structures based on the alternate load path had considered the rigid joints between the members, which are discussed in sections 5 and 6. However, two studies considered the effect of joint rigidity in the progressive collapse resistance of single-layer spatial structures [89, 90].

Xu et al. [89] studied the progressive collapse of single-layer lattice domes with assembled hub joints. Domes with welded joints exhibit a critical progressive collapse resistance that is 5% to 22% higher compared to domes with semi-rigid joints. In the experimental and numerical research with the single-layer reticulated cylindrical shell, the effect of joint rigidity on progressive collapse resistance was captured based on the connection failure [90]. Based on a comprehensive parametric study considering different joint stiffness, it was observed that cylindrical shells with semi-rigid joints exhibited improved ductility, while cylindrical shells with rigid joints demonstrated higher resistance to progressive collapse.

The studies on the rigidity of joints have revealed the significant influence of connection flexibility on instability and its propagation within single-layer spatial structures. When a local failure occurs, rigid connections play a crucial role in effectively redistributing the loads to alternate load paths, thereby preventing progressive collapse. Conversely, flexible connections may result in inadequate load redistribution, leading to collapse propagation. Therefore, the consideration of connection rigidity holds great significance during the design phase of single-layer spatial structures.

4. Methods of progressive collapse investigations

The investigation of instability propagation in single-layer spatial structures was conducted in the early stages. Subsequently, the application of alternate path analysis to assess structural robustness was introduced, similar to frame structures. In recent studies, the analysis of progressive collapse in single-layer spatial structures predominantly relies on alternate path analysis and substructure analysis.

Similar to any field of study, the investigations of the progressive collapse of singlelayer spatial structures can be further classified into experimental, numerical, and analytical research. Experimental investigations are scarce, as it is challenging to create a testing model and examine progressive collapse. Therefore, most of the studies were conducted using numerical analysis. Since large-span and large-scale models are not feasible in most cases, parametric studies are conducted with the help of FE packages, after verifying FE models with the very few experimental results available. The accuracy of the analytical methods is debatable, as they involve many assumptions and simplifications. Consequently, research based on analytical methods to study progressive collapse is minimal compared to experimental and numerical methods. The potential for progressive collapse is measured based on the alternate path method, where the behaviour of the structure is monitored after removing an element. A significant amount of studies are based on alternate path methods. Substructure studies can be utilised to identify the collapse-resisting mechanism. The following sections provide a detailed analysis of all the methods adopted to investigate the progressive collapse of single-layer reticulated shells.

5. Experimental investigations

Experimental research contributed to the expansion of knowledge on the progressive collapse of reticulated shells and significantly improved the reliability of numerical analysis based on FE packages. Experimental studies focusing on the progressive collapse of single-layer spatial structures can be classified into two categories: small-scale model tests based on the alternate path method and substructure tests aimed at identifying the collapse resistance mechanism.

5.1. Experiments based on alternate path method

The experimental research to identify the progressive collapse resistance or the critical elements is primarily based on the alternate path method. An element is removed from the shell structure to examine whether the element failure can cause the progressive collapse of the structure. If the element failure results in the progressive collapse of the structure, the removed member is a critical member. However, experiments on full-scale models of single-layer spatial structures are challenging to conduct due to many reasons, such as difficulty in the fabrication and erection of full-scale models, application of loading to the shell surface, accurate measurement of imperfections, creating a local collapse by removing a member or a connection, and capturing the dynamic response accurately after the local collapse. Despite the difficulty in conducting experiments on full-scale models, some research groups conducted experimental investigations by constructing shell structures with shorter spans and scale-down models. The studies were conducted based on removing a member or multiple members after applying the service load. The critical steps involved in an experiment are creating the model and measurement devices (experimental setup), simulating a local failure, and recording the responses.

5.1.1. Experimental Model

Constructing the experimental model of the single-layer reticulated shells is the first and most challenging step. Many experiments used scaled-down models with a span of less than 5 metres, due to the difficulty in the construction of large-span shell structures. The properties of the experimental models of the single-layer reticulated shell structures fabricated by different research groups are given in Table 1. The Kiewitt-6 dome configuration, which is widely recognised as the standard configuration for single-layer reticulated domes, was predominantly used as the basis for the test models in the majority of the experiments conducted. The triangulation in the Kiewitt dome makes it more efficient compared to other dome configurations [91]. All dome configurations adopted rigid connections, whereas the experiment on the single-layer cylindrical shell by Xu et al. [90] used semi-rigid connections (Assembled Hub joints). The rigid connections were established by welding the members to the spherical nodes. The data presented in Table 1 indicate that a considerable number of investigations were carried out on single-layer domes in comparison to cylindrical shells.

5.1.2. Load application

Apart from the construction of models, the limited availability of experimental investigations on reticulated shell structures can be ascribed to the difficulties encountered in load application. The complex task of applying the required load to multiple nodes of a structure arises from the variability in the heights of each node. Previous experimental investigations on single-layer domes primarily employed apex load application techniques within testing laboratories, utilising standard loading apparatuses [78, 80]. However, these methods were found to be inadequate for progressive collapse analysis, as they fail to provide the necessary uniform load distribution required to accurately represent gravity loading. As a result, hanging weights on the nodes were used in experimental studies to examine progressive collapse [90, 92–98]. The experimental models apply a hanging load on the joints to simulate the combined effects of dead and live loads, including scenarios such as snow load accumulation. Three methods were generally adopted for load application in the experiments (Figure 11). The first method (Figure $11(a)$) is the preferred choice because the rotation of the spherical nodes can be minimised due to the eccentricity while applying the weights (Figure 11(b)). However, sufficient care should be taken to reduce the eccentricity in the second method (Figure 12). The magnitude of the load to be applied was determined from the elasto-plastic buckling analysis using the FE analysis software [33, 34]. Certain experimental studies employed a multi-stage load application technique, progressing from the interior towards the exterior of the shell structure, to minimise the destabilising effects during the loading phase [90, 96].

5.1.3. Response measurement

Most of the researchers were able to measure the geometric imperfections of the shell structures with the help of total stations [93, 96], which was useful in developing accurate numerical models. The experiments were conducted in two steps. Initial static testing with load increment, followed by dynamic testing with local collapse [90]. The displacement of the joints during and after the local collapse was captured by a non-contact acquisition system, which consists of high-speed cameras located near the model and observation targets at the member connections [90, 92, 93]. Strain gauges were used to capture the strain in critical members, which will help to calculate the stresses in the remaining members after the removal of the failed member. Most of the experiments adopted two strain gauges at

Sl _{No}	Experimen- tal study	Objective of the study	Type of configu- ration	Dimensions of the shell structure	Properties of Members	Properties of Connections	Support condition
$\overline{1}$	Xu et al. (2017) [92]	To comprehend the process of internal force redistribution during the progressive collapse of domes	Kiewitt lamella dome and Geodesic domes	$Span - 4 m$, Rise-to-span ratio - 1/5	Aluminium alloy 5052 Tubular cross-section. Ribs and rings $-9X1.5$ mm, Diagonals - 8 X 1.5 mm	Hollow ball joints with diameter of 50 mm and thickness of 2.5 mm	Pinned
\overline{c}	Zhao et al. (2017) [93]	Comparing the progressive collapse resistance of single-layer lattice dome under different load intensity	Kiewitt-6 dome	Span - 4.2 m, Rise-to-span ratio - $1/7$	DIN2391 St.35 steel pipes. Meridian members and outermost diagonal members - 20 X 1 mm, and other members - $16X$ 1 mm	Solid steel ball with diameter of 60 mm and a vertical hole in steel ball with diameter of 16 mm	Pinned
3	Tian et al. (2019a) $[94]$	The effect of rise-to-span ratio on progressive collapse resistance	Kiewitt-6 dome	Span - 2.4 m, Rise-to-span ratio - 1/8 and 1/6	Q235 Tubular cross-sections of $10X1$ mm	Hollow ball joints of diameter 40 mm and thickness of 4 mm	Fixed
4	Tian et al. (2019b) [95]	Comparison of progressive collapse resistance of two different single-layer domes	Kiewitt-6 dome and Three-way diagonal grid dome	$Span - 2.4 m$, Rise-to-span $ratio - 1/8$	Q235 Tubular cross-section of 10 X 1 mm	Hollow ball joints of diameter of 40 mm and thickness of 4 mm	Fixed
5	Tian et al. (2021a) $[96]$	Progressive collapse resistance of Kiewitt dome subjected to non-uniform loading	Kiewitt-6 dome	Span - 2.4 m, Rise-to-span ratio - 1/8	Q235 Steel Tubular cross- sections of 10 X 1 _{mm}	Hollow spherical joint (Dimension not specified)	Fixed
6	Xu et al. (2021) [90]	The effect of joint stiffness on the progressive collapse resistance of single-layer cylindrical lattice shells	Three-way single-layer cylindrical latticed shell	Length - 6 m, Span - 4.5 m, Rise-to-span ratio - 1/4	S235JR rectan- gular steel tubes of cross-section 30 X 15 X 1.5 mm	Type-II Assem- bled Hub (AH) joints	Hinge support on longitudi- nal edges
7	Tian et al. (2021b) [97]	Progressive collapse resistance of partial double- layer Kiewitt dome	Kiewitt-6 (Partial double-layer)	Span - 2.4 m, Rise-to-span ratio - 1/8	Q235 Steel Tubular cross- sections of 10 X 1 _{mm}	Hollow ball joints with diameter of 40 mm and thickness of 4 mm	Fixed
8	Tian et al. (2021c) [98]	Comparison of progressive collapse resistance with successive and simultaneous removal of members	Kiewitt-6 dome	Span - 2.4 m, Rise-to-span ratio - 1/8	Q235 Steel Tubular cross- sections of 10 X 1 mm	Hollow ball joints with diameter of 40 mm and thickness of 4 mm	Fixed

Table 1: The characteristics of single-layer lattice shell structures employed in experimental studies focusing on progressive collapse analysis using the alternate path method

Figure 11: (a) Three loading methods were adopted in different experiments: (a) Iron rods were attached to the spherical nodes through a hole, and loads were attached as hanging weights [93] (b) A rode was welded on the top of the spherical node, and weights were attached in the rode above the spherical ball [92] (c) A U-shaped plate was welded to the bottom of the spherical joint, and load blocks were attached with the help of a spring buckle [95]. Figures (a) and (c) are reproduced with permission.

Figure 12: Method of load application: (a) Stabilising and (b) destabilising effects: It is preferable to hang the weights below the spherical node to reduce the instability effect due to the eccentricity of load application, which can cause the rotation of the spherical node and torsional effect on members.

the centre of the members (one at the top and another at the bottom) [90, 97], which were able to capture the bending stresses as well. However, Zhao et al. [93] used three strain gauges for each member for which strain needed to be measured.

The difficulty of capturing accurate responses during the progressive collapse experiment is high for shell structures, due to the spatial arrangement and the dynamic response of members and joints. One of the experiments failed to capture the responses due to the failure of the displacement measurement system, and the results based on the numerical analysis were used to identify the failure mechanism for the geodesic dome [92]. Therefore, it is crucial to account for the significant deflections exhibited by members and joints during dynamic responses to comprehensively capture the progressive collapse mechanism in experiments conducted on single-layer shell structures.

5.1.4. Member failure mechanism

The experimental studies were based on the alternate path method, where a member is removed, and the response of the structure is analysed based on how the geometry was able to transfer the loads to the supports through the alternate load paths. The major challenge was to introduce a local collapse to the structure during this phase. The local collapse should be such that it can trigger the failure of a member or connection in single-layer reticulated structures, and the same failure should be able to be modelled in numerical analysis. Researchers adopted a local failure creation method where a 'member-breaking device' was introduced to the members for which the local failure was planned during the experiment (Figure 13). The member was cut before the experiment and connected with the help of a member-breaking device so that the member would act as a single element

Figure 13: Member-breaking devices used in the experiments to create local failure: (a) Xu et al. [92] (b) Zhao et al. [93] (c) Tian et al. [94] (d) Xu et al. [90]. All the experiments used similar member-breaking devices. Figures reproduced with permission.

without any damage before the experiment [92–98]. During the experiment, these devices were triggered so that the connection between the two parts of the members was broken, resulting in an artificial member failure.

5.1.5. Selection criteria for the members to create local collapse

The selection of members to create local failure is critical in determining the progression of collapse. The selection of the most sensitive members will help to examine the progressive collapse resistance of the shell structures more effectively. In many experiments, 'failure members' were selected from the FE analysis before the experiment (Figure 14). Xu et al. [92] selected 14 initial failure members in the sensitivity analysis. During progressive collapse analysis, three members were selected as initial failure members in the Kiewitt Lamella dome, and one member was selected in the geodesic dome. The experiment by Zhao et al. [93] removed the meridian member (radial member) in both dome configurations. The precise reason for selecting these members was not stated in both cases. In the experiments conducted by Tian et al. [94, 95, 98], four members (two radial and two diagonal members) were removed from each dome, where the reason for selecting these members was clearly specified. As per the authors, the critical member can be located in the second ring for similar dome configurations [99]. Additionally, three load paths were available in the dome, where six radial members had a significant role in transferring the load to the supports. Therefore, one radial member from the second ring was removed first. Then the adjacent diagonal member was removed to detach the load path completely. Tian et al. [96] selected critical members from the region with higher load application for finding the effect of unsymmetrical load distribution on the progressive collapse analysis of a single-layer Kiewitt-6 dome. The radial and adjacent diagonal members were selected as failure members, similar to the investigations conducted by the same group. In the experiment to analyse the progressive collapse resistance of a single-layer cylindrical vault, Xu et al. [90] determined the initial failure joint based on the component removal method, where the joint for which the lowest critical load was selected based on numerical analysis. Tian et al. [97] removed two radial members (one above and one below in the partial double-layer) and adjacent diagonal members to determine the change in progressive collapse resistance with a partial double-layer Kiewitt-6 dome. In a similar way, a total of nine members were removed from the partial double-layer dome.

The investigations clearly show that the initial experiments did not consider the most sensitive members or explain why a member was chosen to cause local collapse. As local failure can occur in any location or any member, the researchers have the freedom to choose the member for creating local failure. However, numerical simulations can be employed to determine the most sensitive members in single-layer reticulated shell structures. By identifying these critical members, experimental investigations can be conducted specifically targeting their failure, thereby facilitating the study of progressive collapse resistance and collapse mechanisms. Experiments in recent years have been able to identify the most critical members for progressive collapse.

5.1.6. Key findings from alternate path method

The response of the structure to the progressive collapse experiment based on the alternate path method differed based on the type of configuration, loading, span-to-rise ratio, and initial member adopted for creating local failure.

There are two experimental studies comparing the progressive collapse mechanism of different configurations [92, 95]. Xu et al. [92] compared the progressive collapse resistance of the Kiewitt lamella dome and the geodesic dome (Figure 15). The Kiewitt lamella dome exhibited snap-through buckling after the removal of the third member, resulting in the propagation of local collapse. However, progressive collapse was initiated due to snap-through buckling after the removal of one member from the geodesic dome. Stiffness degradation was much faster in the geodesic dome than in the Kiewitt lamella dome.

Figure 14: Selection of members for creating local failure in different lattice shell structures: (a) Kiwitt lamella dome in Xu et al. (2017) [92] (b) Geodesic dome in Xu et al. (2017) [92] (c) Kiewitt dome in Zhao et al. (2017) [93] (d) Cylindrical shell in Xu et al. (2021) [90] (e) Kiewitt dome in Tian et al. (2019a) [94], Tian et al. (2019b)[95], Tian et al. (2021a) [96], Tian et al. (2021b) [97], and Tian et al. (2021c) [98]. For (a), (b), and (c), a partial representation of the dome is represented due to the symmetrical nature of the structure.

Figure 15: (a) Kiewitt lamella dome and (b) Geodesic dome. A comparison of the progressive collapse resistance of both structures based on experiments revealed that the resistance to progressive collapse is higher for the Kiewitt lamella dome even though both structures had triangulated geometry [92]

The experiment revealed that the progressive collapse resistance of the Kiewitt lamella dome is higher than that of the geodesic dome, although both have triangulated unit cells. However, the investigations did not compare the total weight or the total construction cost of both structures. The comparison of those parameters, along with progressive collapse resistance, will provide a clear picture of the advantage of adopting the Kiewitt lamella dome over the geodesic dome.

In the second experiment to investigate the variation in the progressive collapse mechanism with different dome configurations (Kiewitt dome and three-way diagonal grid dome), the collapse mode was found to be similar in both cases [95]. The snap-through buckling resulted in the complete overturn of both dome structures. The Kiewitt dome depended on the arch (compression) action, whereas the three-way diagonal grid dome relied on the beam (flexural) action to resist progressive collapse.

The experiment—to identify the influence of the magnitude of loading on the progressive collapse resistance of the Kiewitt dome—found that the local failure did not cause the progressive collapse of the dome structure with a low intensity of loading [93]. However, snap-through buckling was observed with a high intensity of loading. The arch effect was the primary load-carrying mechanism for both cases. The study suggests the importance of local resistance near the failed member to prevent the propagation of the collapse. As it is obvious that a higher magnitude of load enhances progressive collapse, including further parameters such as varying the rise-to-span ratio or the support condition might have helped to extract more results from the experiment. However, the study helped to conduct extensive numerical analysis considering different parameters.

In an experiment examining the impact of the rise-to-span ratio on progressive collapse resistance [94], it was observed that the Kiewitt-6 dome exhibited greater resistance to progressive collapse when possessing a higher rise-to-span ratio. Domes primarily relied on arch action to mitigate progressive collapse, and the effectiveness of the arch effect was more pronounced in domes with higher rise-to-span ratios.

Past instances of dome structure failures caused by the accumulation of snow loads have highlighted the significance of considering non-uniform load distribution during the design of similar structures [7, 8, 71]. In an experimental investigation examining the effect of non-uniform loading, it was observed that dome structures subjected to nonuniform loads had a higher probability of experiencing progressive collapse compared to those subjected to uniform loads [96].

There was only one progressive collapse experiment on the single-layer reticulated cylindrical shell [90], where a connection failure was created to examine progressive collapse for the first time. The removal of the connection resulted in the downward deflection of adjacent nodes, resulting in 'node buckling.' The arch effect was lost due to the discontinuity in member connections. As a result, the instability propagated longitudinally to cause further collapse. In a similar experiment on the Kiewitt dome structure, the simultaneous removal of three members resulted in progressive collapse compared to successive removal of the same members [98].

The progressive collapse resistance of the partial double-layer Kiewitt dome was higher compared to the single-layer structure [97]. Although the upper radial members buckled, the lower radial members helped to transfer the load by acting as an alternate load path, which resulted in high progressive collapse resistance in the partial double-layer Kiewitt dome compared to the single-layer Kiewitt dome. This investigation stands out as the sole study that employs the alternate path method, or redundancy, to enhance the progressive collapse resistance of single-layer lattice structures.

All the experimental investigations of single-layer dome structures revealed that the 'node buckling' adjacent to the failed member is the prime reason for the propagation of local collapse. The inability to prevent the spread of the local collapse through either alternate load paths or the local resistance (arch and bending effect) results in the progressive collapse of the structure. The affected area of the local collapse (simultaneous failure of multiple members) also influences the chance of progressive collapse from node buckling. However, the investigations were conducted based on the hanging load on joints, which amplifies the initiation of node buckling and propagation in these models, which may not resemble the actual structures. The tying effect of glazing panels also comes into play in preventing node buckling. Therefore, experiments on large-span single-layer lattice shells can provide a clear picture of how the structure will react to similar local failure.

The failure of a single member can initiate the progressive collapse of lattice struc-

tures, such as geodesic domes. For the Kiewitt dome, removing multiple members may be required to trigger progressive collapse. Single-layer Kiewitt domes were used as the test specimen for many experiments, as they are the most efficient and commonly adopted single-layer dome structure. The results state that the progressive collapse of the singlelayer dome depends on the type of configuration, rise-to-span ratio, magnitude of loading, extent of local collapse, and asymmetry of loading. Based on the experimental results, specific methods have been proposed to improve the progressive collapse resistance of the shell structures, such as partial double-layer domes, increasing the rise of the dome, and double-layer arrangement of critical members. Experimental investigations on the progressive collapse of single-layer reticulated shell structures are in their initial stages. Further studies need to be conducted to identify the precise method by which the progressive collapse resistance of different configurations can be improved. The limitations and possible future directions of the experimental research are discussed in the final section (Section 9) of this manuscript.

5.1.7. Experiments considering earthquake effect

There are few experimental studies on single-layer shell structures to examine the earthquake effect, where the progressive collapse was discussed [100–107]. However, these experiments did not conclusively establish progressive collapse as the primary cause for the failure of the entire structure. While the members did exhibit buckling under seismic excitation, there is insufficient evidence from the research to categorise the entire failure as a progressive collapse, as the seismic excitation persisted even after the initial collapse. The final collapse will be the result of multiple local collapses in different locations, which may happen simultaneously. Therefore, a shake table test is not ideal for studying the progressive collapse of reticulated structures, as similar structures consist of many elements where the chance of multiple local failures prevents accurate monitoring of the system during global failure. Although the scaled-down model could capture the seismic behaviour of the shell structures, the modelling does not provide the actual behaviour of a real-world structure. The behaviour of the connections and the members in the model used for the test will differ from that of an actual structure.

5.2. Experiments based on substructures

Researchers have explored the concept of employing substructures to examine the mechanisms of progressive collapse resistance due to the challenges associated with conducting experiments using full-scale or scaled-down models. Similar studies were common for framed structures, where a portion of the structure was constructed to investigate the progressive collapse [108–111]. Several experimental studies have been conducted using substructures of space grid structures, where the results are also applicable to singlelayer reticulated shell structures.

One experiment investigated the anti-collapse mechanism of spatial grid structures by examining substructures [112] based on two commonly adopted primary unit cells: triangular and rectangular (Figure 16). Experiments were carried out on a series of eight full-scale specimens to investigate the anti-collapse mechanism, considering factors such as member inclination and various modes of member failure (Figure 17). The test results demonstrated that the anti-collapse mechanism is influenced by both the inclination of the members and the end restraint. In specimens with zero inclination, the resistance against external loads was primarily provided by beam action when the displacement was below 50 mm. However, once the displacement of the centre exceeded 150 mm, the catenary action became the dominant resistance mechanism. Conversely, in test specimens with a 30-degree inclination, the arch action played a significant role in resisting external loads. Structures featuring triangulated grids exhibited superior collapse resistance compared to structures employing rectangular grids. During the experimental setup, the ends of the members in a substructure were rigidly connected using welding for simplification, which may not resemble the actual reticulated shells. Therefore, additional analysis that takes into account the partial end rigidity of the members is necessary to validate the findings obtained.

Typical failure mechanisms were studied by conducting experiments on the substructures of spatial grid structures by Wei et al. [113]. Four substructure specimens, based on triangulated spatial structures, were used in the study: (a) a six-member substructure with zero inclination, (b) a six-member substructure with 30-degree inclination, (c) a substructure with zero inclination and one member removed, and (d) a substructure with a 30-degree inclination with one member removed. The test results for those substructures showed two kinds of failures: strength and stability. The substructure with zero inclination showed strength failure, and the substructure with a 30-degree inclination showed stability failure. The strength and stability of the substructure were higher for the members with inclination. However, the failure occurred without warning. The authors mentioned the importance of increasing redundancy to avoid stability failure. Even though increasing the redundancy will increase the buckling load, the type of failure will remain as stability failure. In addition, fixed supports for the substructures in the experiment do not provide the actual behaviour of members in spatial structures.

In another experiment with substructures of triangular cells and rectangular cells with and without inclination, the members were fitted with short steel pipes to increase the strength [114]. The welding of the steel pipe portion had a negligible effect when the substructure failed due to instability. The welded pipes help to reduce the effective length, which increases the buckling strength of the individual members. The substructures with zero inclination failed due to the fracture near the short welded steel pipes, and the reinforcement reduced the load capacity of the members. Therefore, the experimental study

Figure 16: Basic unit cells found in commonly adopted single-layer lattice shell structures are (a) triangular cells and (b) rectangular cells. For large-span single-layer ribbed domes, the unit cells exhibit a shape that closely resembles a rectangle. Therefore, conducting experiments on substructures derived from these unit cells can explain the collapse resistance mechanism for single-layer lattice structures, including cylindrical shells and domes, in a manner similar to that of space grids.

 $(a) T-0-I$

(b) $T-O-D$

 (c) R-0-I

 (d) R-0-D

Figure 17: Failure pattern observed in the substructure experiment to investigate the anti-collapse mechanism [112]. T and R in the nomenclature denote triangular and rectangular substructures, respectively. '0' represents the angle of inclination of the members. 'I' denotes the substructure without defects, and 'D' denotes the substructure with defects (i.e., a member was removed to simulate local failure). The figure represents only the substructures with zero inclination. Images reproduced with permission.

proves that similar reinforcement does not increase the progressive collapse resistance of the substructure, as welded reinforcement reduces the deformation capacity of the member. The deformation capacity is essential to reduce the impact of local collapse by dissipating the energy. Based on the findings from the experimental results, the authors proposed and verified methods to improve the progressive collapse resistance with the help of FE analysis. However, if they had conducted the experiment with the proposed arrangement, in addition to the first set of experiments, that could have strengthened the validity of the proposed methods. Another drawback was the fixity of supports, which was observed in the other two investigations as well [112, 113].

As the three investigations on substructure were based on fixed-end consideration [112–114], it is difficult to generalise the findings for spatial structures. An experimental set-up based on semi-rigid supports can predict the collapse behaviour more accurately compared to rigid supports. Therefore, further experimental investigations considering actual support conditions must be conducted to validate the results for reticulated shell structures. Additionally, these substructure investigations do not consider the dynamic effects that arise due to a local collapse.

6. Numerical investigations

Numerical studies are widely adopted for studying the progressive collapse of shell structures, as experimental investigations are difficult, expensive, and time-consuming. Advancements in programming and software development enabled high-performance structural software programmes to analyse the data accurately and quickly [33–35]. In fact, every paper published on the progressive collapse of shell structures in recent years had results based on numerical analysis, and the number of investigations based on numerical analysis exceeds the number of investigations based on experiments (considering research published in journals). The analysis of lattice shell structures involves many parameters, such as type of shell, rise-to-span ratio, support condition, loading, connection rigidity, cross-section of members, and location of local failure. Analysing the effect of all these parameters with the help of multiple experiments is really difficult and expensive. In these situations, numerical studies come in handy. Several parametric studies can be carried out using numerical studies after validating the numerical modelling with the help of an experiment. The cost involved in conducting numerical studies is very low compared to that of conducting an experiment. Therefore, many academic and industry researchers rely on numerical modelling to investigate the progressive collapse of lattice shell structures.

The finite element method, the discrete element method, the applied element method, and the cohesive element method are the major numerical techniques used to study the progressive collapse of structures [18]. Among these, finite element and discrete element methods are used by various researchers to study the progressive collapse behaviour of lattice shell structures. This section provides a detailed description of the research based on numerical methods to analyse the progressive collapse of single-layer reticulated shell structures.

6.1. Finite Element Method (FEM)

The Finite Element Method is popular among researchers for studying the progressive collapse analysis of lattice structures. FE packages can choose whether the study requires linear or nonlinear analysis, two-dimensional or three-dimensional models, static or dynamic analysis, implicit or explicit calculation, and element type (beam, shell, or solid elements) based on the precision required. Abaqus [33], Ansys [34], and LS-DYNA [35] are the common software packages used to analyse the progressive collapse resistance of reticulated shells. In FE packages, the entire structure, a substructure, or a member can be modelled and analysed. Most of the studies used beam elements while modelling the entire shell structure [92, 93, 95, 115, 116]. For member and substructure modelling, shell or solid elements were used based on the requirements [112]. Few studies used a multi-scale model, with members represented as a combination of linear and shell elements [113]. The regions where the behaviour must be accurately studied were represented by shell or beam elements, while the remaining regions were represented by linear elements in a multi-scale model.

Research employing FEM for investigations can be classified into two primary approaches: the alternate path method and substructure analysis. The alternate path method predominantly employed linear elements, while substructure analysis relied on shell elements.

6.1.1. Alternate path method

The majority of FE investigations on single-layer spatial structures employ the alternate path method to analyse progressive collapse, similar to experimental studies. This approach offers the advantage of manipulating various parameters during FE analysis, thereby assessing their impact on progressive collapse.

The single-layer and double-layer reticulated dome were modelled to compare the progressive collapse potential based on the alternate path method [115]. The results indicated that the sensitivity index of the members and nodes of the single-layer reticulated domes was much higher than that of the double-layer reticulated domes.

Many investigations used FE packages to conduct extensive parametric analysis after validating the data obtained from the experiments. One such example is the research by Zhao et al. [93], where the removal of the member was achieved by reducing Young's modulus based on the user subroutine. FE analysis could capture all the behaviours observed in the experiments except the fracture, which implies the failure of the commonly

adopted fracture criteria in predicting the actual failure of the members. The effect of parameters such as strain rate, fracture, damage, and stress triaxiality on the progressive collapse of lattice shells can be studied with the help of numerical models [117].

The comparison of progressive collapse resistance of different single-layer lattice dome structures indicated higher progressive collapse resistance of the Kiewitt dome compared to the geodesic dome [92]. Similarly, another study showed the higher progressive collapse resistance of the Kiewitt-6 dome compared to the three-way diagonal grid dome [95]. Both studies would be more informative if the authors compared the total material utilisation or the cost of the structures along with the resistance to progressive collapse.

The main challenge in reticulated shell structures is finding the critical members of progressive collapse. The first ten eigenvalue buckling modes and the non-linear buckling mode together will help to identify the most sensitive elements in a spatial structure [99]. The applicability of this method was demonstrated with the help of the roof model of the 'Universiade Sports Centre' created using FE package, where identified critical members were removed individually in the progressive collapse analysis after considering the construction effects. Similarly, Yan et al. [118] conducted non-linear dynamic alternate path analysis on four types of single-layer reticulated domes to identify the distribution of critical members. The results showed that the 'node buckling' adjacent to the removed member triggers the progressive collapse. The numerical results helped the authors define the criticality index based on the node buckling. However, the study was unable to find the exact collapse load during the numerical analysis, as the load increment was limited to $0.25 \ kN/m^2$. In another study, Zhang et al. [119] performed a sensitivity analysis of the Kiewitt-Lamella dome based on the removal of members, considering different parame-Kiewitt-Lamella dome based on the removal of members, considering different parameters.

Tian et al. [120] used the quantitative evaluation index called 'Collapse Margin Ratio' (CMR) to assess the progressive collapse resistance of single-layer spatial structures based on IDA (Incremental Dynamic Analysis) where the importance of considering initial geometric imperfections based on the consistent mode imperfection method has been identified.

Another study focused on investigating the impact of the rise-to-span ratio on the progressive collapse resistance of Kiewitt-6 domes by dynamic implicit analysis, based on the 'element birth-and-death method' to simulate the initial failure of the members [94]. Extensive numerical analyses with various rise-to-span ratios revealed that the progressive collapse resistance of the dome increases as the rise of the structure increases. Because steel consumption increases with the rise-to-span ratio, the optimum rise-to-span ratio was determined to be 1/5 considering both progressive collapse resistance and steel consumption.

Many investigations on progressive collapse focused on single-layer dome configura-

tions featuring rigid connections. However, Xu et al. [89] studied the progressive collapse of single-layer lattice domes with assembled hub joints. The influence of the rigidity of joints, grid forms, and initial failure types on the progressive collapse resistance was examined based on numerical analysis. The results showed that the dome configurations with rectangular grid patterns had lower progressive collapse resistance than those with triangular grid patterns. In addition, two modes of failure propagation were observed during the numerical analysis: radial and circumferential propagation.

The investigations on framed structures revealed that alternate paths or redundancy could increase the progressive collapse resistance [2]. However, similar research with the effect of alternate paths improving the progressive collapse is rare for single-layer reticulated shell structures. The progressive collapse investigation of partial double-layer latticed domes involved numerical simulations using the implicit time integration solver in Abaqus. The study demonstrated that the partial double-layer Kiewitt-6 dome exhibited superior progressive collapse resistance compared to the single-layer Kiewitt dome. This enhanced resistance can be attributed to the availability of additional alternate load paths in the partial double-layer configuration [97].

The use of the multi-degree-of-freedom system is common in the dynamic analysis of structures. Similarly, the progressive collapse behaviour of single-layer spatial grid structures can also be examined with the help of a multi-degree-of-freedom system [121]. In contrast to other investigations, the authors of this study defined collapse as a condition where the vertical displacement surpasses 1/50 of the span or when the overall stiffness decreases by 20%. The progressive collapse analysis revealed that the middle portion between the apex and the support of the dome (the third and fourth rings) contains the most sensitive members, and the support position has the most negligible effect. The study illustrated the importance of considering specific dynamic behaviour in the progressive collapse of single-layer shell structures.

Up until that point, each investigation had taken into account the effects of a uniform gravity load when analysing progressive collapse. Nevertheless, a progressive collapse analysis of single-layer latticed domes under non-uniform snow loading, incorporating the influence of varying degrees of non-uniformity and rise-to-span ratios, unveiled new findings. It was observed that single-layer domes exhibited a higher susceptibility to progressive collapse when subjected to non-uniform load distributions along the ring direction as opposed to the radial direction [96]. This study underlines the importance of considering the load distribution characteristics while assessing the progressive collapse potential of single-layer latticed domes. Similar to earlier investigations on the rise-to-span ratio [94], lower depth reduces the resistance of single-layer lattice domes subjected to non-uniform loading.

Single-layer cylindrical shells have received limited attention in comparison to lattice

Figure 18: The spring model adopted in the numerical analysis of single-layer cylindrical lattice shells with semi-rigid joints [90]. Five nonlinear springs were utilised to simulate each connection between the hub and rectangular tube. The image reproduced with permission.

domes, with only a few investigations conducted on them. The progressive collapse analysis of single-layer cylindrical lattice shells with Assembled Hub (AH) joints revealed the effect of joint rigidity on progressive collapse [90]. For the first time, the authors incorporated connection failure into their investigation of progressive collapse in cylindrical shells. Each connection between the hub and the member was simulated with five nonlinear spring elements (Figure 18). Based on a comprehensive parametric study of singlelayer lattice cylindrical shells with varying joint stiffness, it was observed that shells with semi-rigid joints exhibited improved ductility, while shells with rigid joints demonstrated higher resistance to progressive collapse.

The FE analyses conducted on single-layer reticulated shell structures unveiled the significant impact of several factors on the resistance to progressive collapse, which includes the unit cell of configuration, rise-to-span ratio, non-uniform loading, and the rigidity of joints. In addition, FE analysis was employed to develop and validate various techniques aimed at enhancing the structural resistance against progressive collapse. These methodologies are comprehensively discussed in section 8, providing insights into their effectiveness and applicability.

Figure 19: Single member analysis conducted by Tian et al. [95]: (a) a restricted member (the horizontal movement was prevented at both ends) and (b) a released member (the horizontal movement was released at one end)

6.1.2. Substructure analysis

Similar to the experimental investigations, the substructure analysis was conducted to identify the progressive collapse resistive mechanism using FE packages.

A single-member model was simulated to identify the stress distribution and the anticollapse mechanism of the substructure [112]. The 'linear four-sided shell element' (S4R) in Abaqus [33] was adopted for modelling the member. Two types of boundary conditions and member inclinations (zero and thirty degrees) were adopted for the member (Figure 19). The anti-collapse mechanism varied with the inclination of the members. Members with lower inclination primarily relied on the beam and catenary mechanism, whereas those with larger inclination depended on the compression mechanism. Although this single-member analysis was verified with the experimental results, only a full sub-structure analysis would be able to capture the actual behaviour of the structure, as the assumed boundary conditions do not resemble the real structures. Additionally, single-member analysis cannot replicate the dynamic effect due to local collapse.

The complete numerical model based on linear elements may not provide the local effects in members. However, complete numerical modelling based on shell elements or solid elements may not be efficient. In similar situations, a combination of linear elements and shell elements can provide a better understanding of the progressive collapse mechanism based on substructure analysis. Wei et al. [113] proposed methods to improve the progressive collapse resistance of single-layer shells based on simulation with multi-scale technology, to increase the accuracy and efficiency of the numerical analysis (Figure 20).

Figure 20: The multi-scale method adopted for studying the progressive collapse resistance [113]: (a) Shell element and (b) multi-scale element as a combination of shell and linear elements. The image reproduced with permission.

The analysis of stress and displacement near the ends of the members has greater significance due to the increased probability of failure in these specific regions. As a result, 'S4R' shell elements were used to model two ends of a member with a length equal to twice the diameter of the member, and the 'B31' linear element was used to create the remaining members. Additionally, the member connections were modelled as a rigid body to further reduce computational resources. The local stress was displayed more accurately in the multi-scale model, which was helpful in failure analysis.

FE analysis can be used to obtain methods for increasing progressive collapse resistance. Tian et al. [114] discussed optimisation methods to resist the progressive collapse of long-span single-layer spatial grid structures by improving strength and stability. The shell element 'S4R' was adopted to model the substructure. The authors proposed a doublelayer member arrangement to resist buckling failure and embedded two short steel pipes at the ends to resist strength failure.

The three investigations collectively demonstrate that substructure analysis, utilising FE packages, predominantly employed shell elements to identify the mechanism of progressive collapse resistance. Analysis based on shell elements offers enhanced clarity in capturing local effects within members compared to beam elements.

6.1.3. Investigations considering blast loading

All FE analyses were based on the alternate path method and substructure analysis. However, few investigations considered the effect of blast loading [116, 122]. The FE model of single-layer reticulated domes subjected to interior blast loading indicated that single-layer reticulated domes under initial stress could collapse with a lower dynamic effect [116]. In another study, numerical modelling of the Kiewitt dome subjected to blast loads considered the effect of explosive weight, stand-off distance, explosion height, and rise-to-span ratio [122]. The cost-effectiveness of protective measures, such as bollards, was assessed by the authors through the application of sensitivity analysis, reliability analysis, and risk assessment.

Extensive FE analyses can help to identify and categorise events occurring during progressive collapse. In this regard, a three-stage collapse mechanism was developed by employing 'fiber-beam elements' that incorporate the theory of damage accumulation [123]. The three stages are: (1) A limited number of members enter the plastic stage while the majority remain in the elastic stage; (2) Fracture occurs in a few members due to the accumulation of damage; (3) Local collapse propagates to a larger area, leading to progressive collapse. However, it should be noted that differentiating between the first two stages can be challenging due to the spread of local collapse, resulting in the plastic deformation of members throughout the collapse process.

6.2. Discrete Element Method (DEM)

Due to the many disadvantages of the FE method, such as convergence difficulties and calculation speed, few studies have adopted the Discrete Element Method (DEM) to investigate the progressive collapse of single-layer lattice shell structures. The DEM is frequently used to analyse discontinuous materials with large displacements [124]. Many progressive collapse studies on framed structures adopted DEM, where the computational time was found to be higher during the earlier days. After the advancement in the computational field, DEM has been widely applied in the progressive collapse analysis of framed and spatial structures [101, 102].

Jihong and Nian [101] used the DEM to simulate the progressive collapse in singlelayer reticulated domes. DEM discretises the members into a finite number of simple, rigid shapes connected through the springs (Figure 21). The proposed model could capture the plasticity development and the fracture of the sections, which was verified with the help of the results of the dome model based on the shake table analysis. Compared to other numerical methods, the DEM has the advantage of overcoming problems such as strong nonlinearity, discontinuity, and the need for a higher number of iterations.

Xu et al. [102] proposed an algorithm based on the MDEM (Member Discrete Element Method), which was used to simulate the progressive collapse of single-layer reticulated domes with multi-support excitation. MDEM had fewer disadvantages compared to conventional DEM, such as low accuracy. In spite of the many advantages of MDEM compared to FEM, there is further scope to improve material constitutive models, fractures, and contact impacts.

Figure 21: The fibre model of the DEM used by JIhong and Nian to study the progressive collapse [101]. Members were discretised into a finite number of rigid shapes connected through springs. The image reproduced with permission.

Later, a method combining FEM and DEM was introduced to overcome the disadvantages of both of the methods [125]. The region with small deformation was simulated using FEM, while the region with significant deformation or failure was modelled using DEM. The authors highlighted the high precision and efficiency of the method with the help of the Kiewitt dome model.

The predominant approach in numerical investigations of single-layer lattice shells is FE analysis, while only a limited number of studies utilise discrete element methods. While the results obtained from numerical analyses are considered reliable, there is still room for enhancing the dependability of these analyses by minimising assumptions and simplifications. It is worth noting that the application of the Applied Element Method (AEM) [126], which offers a distinct perspective, has yet to be utilised in progressive collapse analysis of reticulated shells. Alternative methods like AEM could provide valuable insights beyond the current findings.

7. Analytical investigations

Progressive collapse analysis of structures does not often utilise analytical studies, which rely on various assumptions and simplifications and may only partially capture the complexity of the problem involved. However, it can be a powerful tool for understanding the behaviour of complex systems. For example, a single degree of freedom system was adopted for the progressive collapse analysis of reinforced concrete framed structure based on incremental dynamic analysis [17]. In earlier days, analytical methods were widely adopted to study the behaviour of structures such as lattice shell structures [127]. Nowadays, analytical research acts as a complementary approach for designing experimental and numerical studies of similar structures. A few theoretical studies were conducted to identify the progressive collapse resistance of reticulated shells [98, 99, 118].

In an analytical study conducted by Tian et al. [99], an evaluation method was introduced to identify critical members in spatial structures. This method considered multiple responses such as load capacity, natural frequency, strain energy, member stress, and node displacement. The progressive collapse analysis was performed using finite element software by removing the selected critical members identified through the multi-response analysis.

The critical members in single-layer reticulated domes, including the Kiewitt, ribbed, Schwedler, and lamella domes, can be identified using a criticality index [118]. The progressive collapse of these reticulated structures was initiated due to the snap-through buckling of the nodes located at the end of the removed member. Therefore, an index (Equation 4) was proposed to assess the criticality of a member, which considered factors such as the applied load on the node, the stiffness of the connected members, the boundary condition of the connecting members, and the angle of the gap resulting from the removal of the member. The results obtained through this approach aligned with those derived from the alternate path analysis.

$$
C = \frac{f_P}{\frac{f_K}{f_G} \cdot f_G \cdot [\Delta]} = \frac{4\pi^2 P \cdot \cos^2(\alpha)}{[\Delta] \cdot (2\pi - \beta)^2 \cdot \sum \left(\frac{EA^i}{l^i} \sin^2(\theta^i) + \frac{3EI^i}{(l^i)^3} \cos^2(\theta^i)\right)}
$$
(4)

Here,

C = dimensionless criticality index

 f_P = factor measuring the applied load on the node

 f_K = factor measuring the stiffness

 f_R = factor finding the influence of out-of-plane lateral restraint at the end of the member

 f_G = factor measuring the influence of changing the in-plane geometry due to the member removal

 $[\Delta]$ = admissible displacement in the direction of node-buckling

 $P =$ load acting on the node

 α = angle between the load direction and the node-buckling direction

 β = the gap angle after the removal of the member

 $E =$ modulus of elasticity

A = cross-sectional area

 $I =$ second moment of area of the cross-section

 $l =$ member length

 θ = angle of tilt

The derived equation in the study was formulated under certain assumptions, including simplified assumptions about the boundary conditions of members and lateral restraints. While the equation successfully identified the most critical members in the structure, it fell short in accurately determining the collapse resistance, as the criticality index only focused on one joint at a time. However, the study proposed potential methods to enhance the progressive collapse resistance of single-layer dome structures based on the derived criticality index. These methods included increasing the rise-to-span ratio and incorporating a double layer in critical areas. The authors noted that stress distribution derived from the static analysis offered more precise results for identifying critical members compared to mode shapes obtained from eigenvalue buckling analysis. Nevertheless, member selection based on the criticality index was considered more reliable and accurate than static analysis.

Theoretical analysis was employed to illustrate that the possibility of progressive collapse in the Kiewitt dome was higher when multiple members were removed simultaneously as opposed to when members were removed successively [98]. The comparison was conducted by estimating the displacement variation of the joint using the applied force and joint stiffness. The stiffness of the joint was determined through the application of the principle of virtual work, incorporating several assumptions to transform the load transfer modes into an arch and subsequently into equal sections of an arc.

Considering the limited number of analytical studies on the progressive collapse of reticulated shell structures, there is a significant opportunity for further investigation employing similar approaches. Theoretical studies can provide valuable insights into the underlying mechanisms of progressive collapse, facilitating the development of procedures and guidelines to enhance the progressive collapse resistance of single-layer reticulated shells.

8. Approaches for enhancing resistance to progressive collapse in spatial structures

Various methods have been proposed to mitigate progressive collapse in structures. Among them, protecting key elements and implementing the alternate path method are commonly adopted strategies in spatial structures.

8.1. Local resistance

Spatial structures exhibit susceptibility to instabilities and local failures, which can propagate and ultimately lead to progressive collapse. Among the various failure modes, compression members are particularly prone to causing instabilities in these structures. Hanaor et al. introduced a technique aimed at enhancing the ductility and load-carrying capacity of space trusses [128], which holds potential for application in single-layer spatial structures as well. By strategically under-designing tension members and over-designing compression members, the ductility of structures can be effectively managed. Additional design strategies, such as implementing Force Limiting Devices (FLD) and strengthening critical members to enhance their strength and stability, further contribute to improving local resistance against progressive collapse.

8.1.1. Force Limiting Devices (FLD)

The FLDs are used to prevent the sudden failure of compression members by modifying the brittle post-buckling behaviour of compression members into an elastic-perfectly plastic post-buckling behaviour. FLDs attached to the critical members help to provide artificial ductility, which will help to increase the load-carrying capacity and improve the robustness of the structure [129]. Parke designed a truss member to act as an FLD [130]. The FLD is constructed using an outer tube, an inner tube, and four intermediate strips (Figure 22).

The utilisation of FLDs in the most heavily strained top chord members of a space truss can significantly improve the overall behaviour and strength of the structure [131]. The

Figure 22: A member with FLD where the inner and outer tubes are in compression. The middle strips are in tension [130].

study identified an increase in the performance of the structure with more FLDs. However, the practicality of utilising FLDs on a larger scale is highly unlikely due to their high cost, which restricts their application to only a limited number of members. In a different study, an innovative force-limiting device named the 'Accordion Force Limiting Device', which combines buckling restrained braces and accordion metallic damper, was employed on critical compression members of space structures to prevent buckling [132]. The study also examined the impact of the gap size between the core and encasing, as well as the core material, through nonlinear FE analyses.

A newly introduced Buckling Controlled Member (BCM) demonstrated promising capabilities to enhance the ductility and delay the brittle buckling of members in space structures [133]. The BCM comprising four components—-the encasing, joints, core, and adjustable nuts—-aims to provide the necessary structural resilience. Extensive finite element analysis revealed that the arrangement of inner elements emerged as the primary factor influencing ductility and effectively delaying member buckling.

While previous studies have primarily employed force-limiting devices to enhance the performance of double-layer structures, it is worth noting that similar devices can also be utilised in single-layer spatial structures. By incorporating such devices, member buckling can be mitigated, thereby reducing the likelihood of local failures.

8.1.2. Strength and stability enhancement of members

The FE investigation based on multi-scale technology proposed methods to increase the progressive collapse resistance [113]. During the analysis of a single member, fractures were observed at the ends of the member. This indicated that the remaining portion of the member was underutilised, highlighting the critical nature of the member ends. To mitigate this local failure, a kinked steel pipe was welded to the end of the member, which significantly improved both the bearing capacity and deformation characteristics of the

Figure 23: Kinked steel pipe reinforcement for increasing the local resistance [113]. The image reproduced with permission.

Figure 24: Double-layer member arrangement for increasing the stability [114]. The image reproduced with permission.

structure (Figure 23).

Tian et al. explored optimisation methods aimed at enhancing the strength and stability of long-span single-layer spatial grid structures to mitigate progressive collapse [114]. The authors proposed a double-layer member arrangement as a means to counteract buckling failure and enhance member stability. This arrangement effectively transformed sudden buckling failure into a more desirable ductile failure mode (Figure 24). To mitigate strength failure, members were optimised by incorporating two short steel pipes at the ends. These pipes were designed to be unconnected and capable of sliding inside the main pipe, providing an effective means to enhance the structural strength of the members (Figure 25). The numerical analysis demonstrated that both of these methods effectively enhanced the resistance to progressive collapse. However, it is important to note that the force direction within the same member can change based on the external load. This means that both of the proposed methods would need to be applied to a single member. Further clarification and modification are required to ensure a clear understanding of the applicability and effectiveness of these methods.

8.2. Alternate path method

In single-layer spatial structures, the creation of alternate paths can be achieved through the adoption of additional members within the dome plane or the implementation of a partial double-layer structure. Existing literature has proposed reinforcing the structure with

Figure 25: Unbounded member arrangement for increasing strength [114]. The image reproduced with permission.

additional members in a different plane as a means to create a partial double-layer configuration to enhance the availability of alternate paths [97, 113].

Eigenvalue and nonlinear buckling analyses can be employed to determine the optimal locations for additional members [113]. The inclusion of extra members within the buckling-prone areas can improve the load capacity of the overall structure and enhance its resistance to progressive collapse. Through the analysis, it was identified that the radial rib members and latitudinal members in the middle region played critical roles. Consequently, extra members were strategically arranged for these members to form a truss system, effectively increasing the ultimate loads of Kiewitt and geodesic domes by a factor of 2.2 and 4.5, respectively. Progressive collapse analysis, utilising the alternate path method, revealed that these configurations with increased alternate load paths exhibited high resistance to progressive collapse. The authors coined the term 'global reinforcement' to describe the partial double-layer arrangement (Figure 26(a)).

The effectiveness of another partial double-layer Kiewitt dome was evaluated through a combination of experimental and numerical studies [97]. The results demonstrated that the partial double-layer configuration (Figure 26(b)) exhibited higher resistance to progressive collapse compared to the single-layer dome. Building upon the experimental findings, FE analysis, and theoretical studies, the authors introduced a novel cable-stiffened partial double-layer latticed dome (Figure 27). This innovative design exhibited exceptional resistance to progressive collapse compared to single-layer and partial double-layer Kiewitt-6 domes.

8.3. Effect of rise-to-span ratio

In addition to local resistance and alternate path enhancement, increasing the rise of the dome structure can influence the progressive collapse resistance.

Experimental studies conducted on the Kiewitt-6 dome highlighted an enhanced resistance to progressive collapse in dome structures with a high rise-to-span ratio [94]. Extensive numerical analyses, considering various rise-to-span ratios, further supported these findings by demonstrating that the progressive collapse resistance of the dome improves as the rise of the structure increases. The arch action is the dominant mechanism

Figure 26: A partial double-layer arrangement was adopted to enhance the resistance to the progressive collapse of Kiewitt-6 dome: (a) double-layer member arrangement is provided along radial members and latitudinal members in the middle region [113], and (b) double-layer member arrangement is provided along radial members [97]. The double-layer member arrangement is highlighted using red colour to indicate its location in the figure.

Figure 27: The FE model of the novel cable-reinforced partial double-layer Kiewitt-6 dome demonstrates superior resistance to progressive collapse compared to traditional single-layer and partial double-layer Kiewitt-6 domes [97]. The image reproduced with permission.

contributing to this improved resistance, which is more pronounced in dome structures with higher rise-to-span ratios. It is important to note, however, that as the rise-to-span ratio of the structure increases, there is a corresponding increase in steel consumption.

9. Limitations in the existing literatures and future perspectives

The research conducted on the progressive collapse of single-layer reticulated shells primarily centred around the application of the alternate path method, wherein the behaviour of the structure was examined by selectively removing members. The primary objectives of these investigations were to identify the progressive collapse mechanisms, determine critical members within the structure, and propose strategies to enhance the progressive collapse resistance of shell structures. Through a comprehensive review of experimental, numerical, and analytical studies on single-layer reticulated shell structures, few limitations within the existing literature were identified, emphasising the need for further research in this field.

1. *Need for experimental investigations on full-scale models:* All experiments so far have been conducted on scaled-down models of single-layer reticulated shell structures. While these models have provided valuable insights, conducting full-scale experiments on actual shell structures can yield more reliable results. The dimensions of members and connections in full-scale shell structures differ, and the size effect of these elements can significantly influence the progressive collapse (Figure 28). The experiments conducted on scaled-down models employed hanging loads, which may not accurately represent the true loading conditions in structures. The amplified oscillating effect of hanging loads can decrease the progressive collapse resistance. Node buckling in scale-down models is primarily driven by the hanging

Figure 28: The comparison of an actual lattice shell structure and the experiment model: (a) A singlelayer lattice shell employed for a railway station platform, and (b) a Kiewitt dome model utilised for an experiment [98] (the image (b) is reproduced with permission). The experimental model has limitations in accurately replicating the progressive collapse behaviour observed in real structures. This is due to the differences arising from factors such as variations in load application, the size effect, and the occurrence of local collapse.

loads on the joints. However, in actual structures where the loads are distributed across the structure, the possibility of node buckling may be reduced, or the rate at which node snap-through buckling occurs may be slowed down. Another important parameter is the effect of glazing panels (roofing sheets). The influence of glazing panels on the progressive collapse resistance in single-layer reticulated shell structures has not been accounted for in previous experiments. Glazing panels or the covering material have a 'tying effect' that can restrict the propagation of progressive collapse. Conducting experiments on real-world structures or fullscale models can yield more precise results. The progressive collapse experiment can be conducted using a fabricated large-span shell structure for experiment or a temporary structure constructed for purposes such as exhibitions or storage. Similar experiments have been carried out previously for framed structures[134–136]. The observations based on experiments with large-scale structures are really helpful for creating 'threat-dependent design methods' for lattice shell structures.

2. *Applicability of member-breaking devices in experiments:* All conducted experiments utilised member-breaking devices to induce local failures by initially breaking and then recombining the members with the assistance of these devices. However, these devices cannot accurately replicate the dynamic effects of local failures in lattice shell structures as the members are broken before the experiment. Therefore, it is necessary to validate the use of member-breaking devices by incorporating sudden member breakage after applying loads to the structure, in order to simulate the dynamic effects of local failures accurately. Furthermore, the location of the failure within the members is also a crucial factor when conducting experiments on small-scale lattice structures.

- 3. *Relevance of substructure experiments with rigid supports:* Limited investigations have utilised substructures to analyse the anti-collapse mechanism in spatial structures, similar to frame structures. However, the use of rigid supports in the substructure to analyse the spatial structure is debatable, as the spatial arrangement of members provides only partial fixity of members, except for those directly connected to the supports. Therefore, substructure analysis should consider the relative flexibility of member supports in the substructure to achieve a more realistic understanding of the collapse prevention mechanism occurring in lattice shell structures. Furthermore, it is worth exploring the effect of member connection rigidity by employing semi-rigid connections. Existing experiments and numerical methods have predominantly employed rigid connections between members in the substructure, thus indicating the need of further investigation on the effects of connection rigidity.
- 4. *Numerical investigations:* Numerical analysis plays a critical role in conducting comprehensive parametric studies by validating the model with experimental results. The majority of studies on the progressive collapse of reticulated shells have employed the FEM, with only a few adopting the Discrete Element Method. However, there are opportunities to explore alternative numerical techniques, such as the Applied Element Method, for assessing the progressive collapse resistance of lattice shells, similar to framed structures. While most numerical models of shell structures have utilised linear elements, there is a need for investigations involving large-scale models using shell elements or solid elements to capture more detailed information about the progressive collapse mechanism. Such efforts will contribute to the development of local collapse-resistant designs. The majority of FE model studies have employed the alternate path method to analyse progressive collapse. However, numerous possibilities exist for exploring 'threat-dependent analysis' based on numerical models by considering the intensity of local collapse. Therefore, conducting numerical investigations using shell or solid elements will enable a more accurate identification of the structural response when a local failure occurs, which is crucial for designing structures based on threat dependent method. Additionally, the FE method can also be utilised to examine the influence of secondary elements, such as coverings or glazing panels, on progressive collapse prevention. It is worth noting that many of the limitations mentioned for experimental studies also apply to numerical investigations.
- 5. *Analytical investigations:* Limited analytical investigations have been conducted

on the progressive collapse of reticulated shell structures. In addition to discrete analysis, employing a continuum approach or an equivalent shell analogy method [127, 137] can provide valuable insights into the progressive collapse resistance of lattice structures. The analytical approach facilitates a deeper understanding of the fundamental concepts underlying the collapse mechanism, thereby enabling improvements in the progressive collapse resistance of structures. Moreover, analytical studies contribute to the development of design guidelines and serve as a means to educate structural designers on progressive collapse, reducing the necessity for extensive analysis of every structure.

- 6. *Different types of shell configurations:* Apart from the Kiewitt configurations, there are several other single-layer configurations that can be used, including Shwedler, sunflower, lamella, diamatic, geodesic, and honeycomb domes. Comparing the progressive collapse resistance of these structures can help to identify the optimal configuration for different scenarios. Some of these configurations consist of nontriangulated unit cells, and studying their impact on damage propagation can provide valuable insights into the collapse resistance mechanism. Similarly, attention is required for single-layer cylindrical shells with different member arrangements. Single-layer cylindrical shells exhibit a higher vulnerability to progressive collapse due to their zero curvature in one direction, which restricts the effectiveness of the compression mechanism (arch effect). Additionally, the effect of 'form' can be explored by considering different spatial structures, such as hyperboloid paraboloids and free-form structures.
- 7. *Effect of connection failure or multiple member failure:* The alternate path method, primarily focused on column removal, has proven effective for frame structures. However, when it comes to single-layer spatial structures like the Kiewitt dome, relying exclusively on the alternate path method by removing a single member may not be adequate. Previous investigations on the Kiewitt dome have consistently highlighted this limitation. In spite of that, most studies have examined the progressive collapse resistance and mechanism of reticulated shell structures by analysing the failure of one member at a time. It is important to recognise that local failure can manifest in various ways within these structures. It can result from the simultaneous failure of multiple members or from the failure of a connection. Xu et al. [90] have explored the influence of connection removal in single-layer cylindrical lattice shells through experimental analysis. However, apart from this particular research, the possibility of progressive collapse due to connection failure has yet to be extensively discussed for single-layer lattice domes or cylindrical shells. Moreover, local failure can also occur due to the collapse of supports under conditions like vehicle collisions or explosions. Therefore, relying solely on the alternate

path analysis based on single-member removal may not yield practical insights into the progressive collapse behaviour of spatial structures. To comprehensively assess the progressive collapse resistance, it is essential to consider numerous scenarios, including the failure of critical elements, combinations of elements, and supports within single-layer lattice shell structures. By incorporating these factors, the concept of progressive collapse analysis based on alternate methods can be refined for similar structures.

- 8. *Construction Material:* Significant research has been dedicated to reticulated shell structures constructed with steel. However, it is important to acknowledge that shell structures are now being constructed using a wide range of materials, including aluminium and wood. The mechanical response of wood-based structures differs significantly from that of steel structures due to the brittle nature of wood. Hence, there is a need for extensive experimental and numerical studies to explore the progressive collapse resistance of reticulated shell structures constructed with different materials.
- 9. *Economy of the structure:* The present investigations on improving progressive collapse resistance in reticulated shell structures did not consider economic viability. The trade-off between increased strength and progressive collapse resistance against the total costs of construction needs a detailed evaluation. A thorough analysis of these aspects would provide valuable insights for designers and fabricators, enabling them to enhance progressive collapse resistance while minimising the impact on the overall cost of the structure. It is crucial to conduct a detailed examination of design methods, including alternate load paths and specific local resistance, to identify the most economically viable approach for single-layer reticulated shell structures. Furthermore, studies should aim to optimise the 'progressive collapse resistance design' by selecting the most effective combination of methods for each type of shell configuration.
- 10. *Comparison of progressive collapse design with earthquake-resistant design:* The influence of earthquake-resistant design on the variation of progressive collapse resistance in reticulated shells needs to be studied. While seismic analysis primarily considers loading in the horizontal direction, progressive collapse analysis predominantly concentrates on vertical loading. Despite their distinct objectives, seismic design, through enhancing structural ductility, has the potential to improve the progressive collapse resistance of shell structures. Currently, there is a lack of comprehensive studies exploring the specific influence of seismic design on progressive collapse design in reticulated shell structures. Therefore, a comparative study can be conducted using experimental and numerical analyses to evaluate the relationship between seismic design and progressive collapse design, thereby filling this research

gap.

- 11. *The effect of design and construction errors:* Design and construction errors, including connection defects (such as welding and bolting) and imperfections in nodes and members, can affect the progressive collapse resistance of spatial structures. Singlelayer reticulated shells are extremely sensitive to imperfections, as even minor imperfections can lower their progressive collapse resistance. It is crucial to investigate the potential for improving the progressive collapse of shell structures with design and construction errors. This exploration would help to identify strategies to mitigate the detrimental effects of existing design errors and improve the overall collapse resistance.
- 12. *Progressive collapse during the construction stage:* A few incidents of progressive collapse of spatial structures happened during the construction stage [138, 139]. The chance of progressive collapse of single-layer reticulated shell structures during construction stage is high, due to the unsymmetrical loading and lack of proper support that can arise during this stage. These factors received little attention from the researchers. The progressive collapse mechanism will also differ during construction, as the structure has not yet completed its full geometrical form. Therefore, the possibility and preventive measures of progressive collapse during the construction stage should be investigated through experiments or numerical methods by considering various construction effects such as unsymmetrical loading, temporary supports, and the partial geometry of the shell structure.
- 13. *Threat-dependent investigations:* While current guidelines predominantly advocate for threat-independent methods, it is crucial to recognise that a significant number of failures occur as a result of loads that impact localised regions with substantial dynamic effect. Therefore, it is critical to conduct threat-dependent analyses for reticulated shell structures. Enhancing specific local resistance can be achieved by implementing a two-layer arrangement of members, as demonstrated by Tian et al. [95]. However, further investigations are necessary to explore additional techniques for improving local resistance. Valuable insights can be gained on the required extent of enhancing specific local resistance or alternate paths by examining the variation in the spread of progressive collapse based on the intensity of local failure. This knowledge will contribute to improving the overall robustness of reticulated shell structures.
- 14. *Additional methods to increase progressive collapse resistance:* There are various methods by which the collapse resistance of shell structures can be improved. Only a few studies had proposed methods to improve the resistance of single-layer reticulated shells, such as adopting two-layer members [114] and partial double-layer domes [97]. There is further scope to find numerous methods by which the pro-

gressive collapse resistance of reticulated shells can be improved. Segmentation or compartmentalisation [67], tying different elements together, inventing new connections, and improving the local resistance of the elements are methods by which the progressive collapse resistance can be improved. The ability of these methods to increase the progressive collapse of single-layer reticulated shells needs to be investigated. In addition, the combination of different methods, such as the alternate load path method and the specific local resistance method, can be used to improve the progressive collapse resistance, which will help find the most suitable design methods.

- 15. *Effect of high-impact loading:* Significant number of investigations on progressive collapse are based on single-member removal, where the dynamic effect will be minimal. Studies with high-impact loading will help to understand the need for the energy-absorbing ability of single-layer reticulated shell structures so that progressive collapse can be prevented. A detailed dynamic analysis with the help of an energy-based approach will aid in determining the effect of high-impact loading on the structure and the progressive collapse preventive mechanisms.
- 16. *The snap-through buckling and progressive collapse:* The exclusive dependence on the alternate path method for progressive collapse analysis was found to be inadequate for lightweight structures like single-layer lattice structures, given the potential occurrence of various instabilities leading to significant dynamic effects. Snapthrough collapse, a notable failure mode in single-layer reticulated shell structures, involves considerable dynamic effects resulting from instability propagation, ultimately leading to the collapse of the entire structure. The conventional alternate path analysis, focused on member collapse, fails to capture the instability effects of snap-through buckling. Consequently, the research community must devote more attention to the likelihood of snap-through buckling causing the progressive collapse in reticulated structures by considering the amplified dynamic effects within the alternate path method.
- 17. *Assessment of progressive collapse potential in existing reticulated structures:* The potential for progressive collapse in existing single-layer reticulated structures has not been extensively investigated. It is crucial to explore strategies that can enhance the collapse resistance and prolong the service life of these structures. Alternative path method, specific local resistance, segmentation, and ties can be explored as viable options for accomplishing this objective. A combination of the above methods can be adopted for the existing structures based on the analysis and the importance of the structure.
- 18. *Requirement of design guidelines specific to spatial structures:* The implementation of progressive collapse-resistant design is a relatively recent requirement, and

the development of design specifications is still in its early stages, even for framed structures. IS 800-2007 [140], for example, makes no explicit mention of progressive collapse or structural robustness. The detailed guidelines for designing for progressive collapse are not updated based on recent research in any country. Therefore, there is a need for specialised design guidelines to address progressive collapse analysis and design of reticulated structures, given the significant impact of geometry on load-bearing in such structures. The simplified design guidelines will help the designers to examine the progressive collapse resistance of the buildings without detailed analysis.

19. *Effect of fire on progressive collapse:* Fire-induced local failures can initiate progressive collapse in single-layer shell structures, similar to steel framed structures [141, 142]. The dynamic effect of local failure will be lower during fire accidents, which can generate new progressive collapse mechanisms in spatial structures. Therefore, a detailed analysis of fire-induced progressive collapse needs to be conducted for single-layer reticulated shell structures.

10. Concluding remarks

Single-layer spatial structures, characterised by their lightweight, large span and the influence of geometry on load resistance, indicate a high potential for progressive collapse arising from various local instabilities. While progressive collapse investigations have primarily focused on frame structures, there exists considerable scope for creating design guidelines to improve the robustness of single-layer spatial structures through extensive studies. Therefore, this article offers a comprehensive review of research conducted in the field of progressive collapse analysis and design of single-layer reticulated shell structures, providing key findings and future directions for additional exploration.

Previous studies have explored the influence of various instabilities, including member and snap-through buckling, on the global behaviour of single-layer spatial structures. The dynamic effects resulting from these instabilities have highlighted the importance of including dynamic analysis in similar structural systems. Furthermore, the presence of complete rigidity between the members contributes to structural robustness by enabling efficient load transfer during instances of local instability.

After the widespread adoption of the alternate path method in frame structure analysis, progressive collapse analysis of spatial structures also incorporated this approach. Here, the structural response is investigated by selectively removing an element to assess its influence on the overall behaviour of the structure.

Experiments conducted on reduced-scale models, utilising suspended loads and a memberbreaking device, have provided valuable insights into the progressive collapse behaviour of

single-layer spatial structures using the alternate path method. The extensive experimental and numerical investigations established that node buckling near the removed member can initiate progressive collapse. Substructure analyses employing rigidly supported members have examined the anti-collapse mechanism, revealing the significance of compression, flexural, and catenary action in effectively resisting progressive collapse.

Numerical investigations primarily utilised FEM to assess the progressive collapse potential of the structure. Linear, shell, and a combination of linear and shell elements were employed to accurately model the behaviour of the structure. Linear elements were primarily used for alternate path analysis, while shell elements were applied for substructure analysis. The numerical studies have revealed the influence of various factors, including configuration type, rise-to-span ratio, non-uniform loading, connection rigidity, and extent of failure, on the potential for progressive collapse. Analytical methods to investigate progressive collapse resistance in shell structures are limited due to the dependency on numerous assumptions and simplifications, which can result in inaccurate solutions.

The investigations have identified various methods to enhance progressive collapse resistance, such as improving local resistance through the use of force-limiting devices, enhancing stability and strength through local reinforcement, and increasing the alternate path by incorporating partial double-layer member arrangements and cable reinforcement. Moreover, increasing the rise-to-span ratio and employing rigid joints can also enhance the resistance to progressive collapse.

The studies have highlighted several limitations and scope for further research in the field, including the need for experimental investigations using full-scale models to account for size, roofing sheet effects, and realistic loading conditions. It has been found that single-member removal analysis alone is insufficient for lattice structures, and the removal of connections or multiple members is necessary, particularly for triangulated configurations like the Kiewitt dome. Existing numerical and experimental studies have revealed the need for threat-dependent analysis in spatial structures, which is crucial for identifying the most effective methods to enhance progressive collapse resistance in similar structures. Moreover, it is essential to explore new design methods that consider a combination of alternate paths, specific local resistance, tying force, and compartmentalisation techniques. This combination approach not only improves progressive collapse resistance but also optimises the overall design.

There exists significant potential for further research on the progressive collapse of single-layer reticulated shell structures, considering the various factors discussed in the literature. The application of diverse research methodologies can be extensively employed to examine the mechanisms underlying progressive collapse and explore methods for enhancing collapse resistance. The outcomes of these investigations will facilitate the development of specific design guidelines tailored to single-layer spatial structures. Consequently, designers will be empowered to select the most effective strategies for improving progressive collapse resistance, considering the unique characteristics of each shell configuration.

References

- [1] American Society of Civil Engineers (ASCE), Minimum design loads and associated criteria for buildings and other structures (ASCE/SEI 7 - 17), Structural Engineering Institute of the ASCE, 2017.
- [2] F. Kiakojouri, V. De Biagi, B. Chiaia, M. R. Sheidaii, Progressive collapse of framed building structures: Current knowledge and future prospects, Engineering Structures 206 (2020) 110061.
- [3] Y. Tsuboi, Analysis, design and realization of space frames: a state-of-the-art report, Bulletin of the International Association of Shell and Spatial Structures (IASS), 1984.
- [4] Z. Makowski, History and development of various types of braced barrel vaults and review of recent achievements all over the world, in: Z. Makowski (Ed.), Analysis, design, and construction of braced barrel vaults, Granada Publishing Ltd., 1985, pp. 1–36.
- [5] R. S. Nair, Progressive collapse basics, Modern steel construction 44 (3) (2004) 37–44.
- [6] GSA (General Services Administration), Alternate path analysis and design guidelines for progressive collapse resistance, General Services Administration, 2013.
- [7] D. T. Wright, Membrane forces and buckling in reticulated shells, Journal of the structural division 91 (1) (1965) 173–201. doi:10.1061/JSDEAG.0001205.
- [8] M. Levy, M. Salvadori, Why buildings fall down: How structures fail, WW Norton & Company, 2002.
- [9] R. Martin, N. J. Delatte, Another look at hartford civic center coliseum collapse, Journal of Performance of Constructed facilities 15 (1) (2001) 31–36.
- [10] V. Gioncu, P. Lenza, Propagation of local buckling in reticulated shells., in: Space structures 4, 1993, pp. 147–155.
- [11] WG 8 IASS, Guide to buckling load evaluation of metal reticulated roof structures (draft), Report of Activities of IASS WG 8, 2014.
- [12] V. Gioncu, Buckling of reticulated shells: state-of-the-art, International Journal of Space Structures 10 (1) (1995) 1–46. doi:10.1177/026635119501000101.
- [13] F. Fan, J. Yan, Z. Cao, Elasto-plastic stability of single-layer reticulated domes with initial curvature of members, Thin-Walled Structures 60 (2012) 239–246. doi:10.1016/j.tws.2012.01.012.
- [14] J. Yan, F. Qin, Z. Cao, F. Fan, Y. Mo, Mechanism of coupled instability of single-layer reticulated domes, Engineering Structures 114 (2016) 158–170. doi:10.1016/j.engstruct.2016.02.005.
- [15] F. Fan, J. Yan, Z. Cao, Stability of reticulated shells considering member buckling, Journal of Constructional Steel Research 77 (2012) 32–42.
- [16] Q. S. Wang, X. Y. Lu, S. Y. Chen, Regular hexagonal grid single-layer cylindrical latticed shell parametric design, in: Applied Mechanics and Materials, Vol. 518, Trans Tech Publ, 2014, pp. 231–235.
- [17] K. Qian, B. Li, Quantification of slab influences on the dynamic performance of rc frames against progressive collapse, Journal of Performance of Constructed Facilities 29 (1) (2015) 04014029.
- [18] J. M. Adam, F. Parisi, J. Sagaseta, X. Lu, Research and practice on progressive collapse and robustness of building structures in the 21st century, Engineering Structures 173 (2018) 122–149.
- [19] J. L. Gross, W. McGuire, Progressive collapse resistant design, Journal of Structural engineering 109 (1) (1983) 1–15.
- [20] B. R. Ellingwood, Mitigating risk from abnormal loads and progressive collapse, Journal of Performance of Constructed Facilities 20 (4) (2006) 315–323.
- [21] D. E. Allen, W. Schriever, Progressive collapse, abnormal loads, and building codes, in: Structural failures: Modes, causes, responsibilities, ASCE, 1972, pp. 21–47.
- [22] J. Agarwal, J. England, Recent developments in robustness and relation with risk, Proceedings of the Institution of Civil Engineers-Structures and Buildings 161 (4) (2008) 183–188.
- [23] U. Starossek, M. Haberland, Disproportionate collapse: terminology and procedures, Journal of performance of constructed facilities 24 (6) (2010) 519–528.
- [24] K. Qian, B. Li, Research advances in design of structures to resist progressive collapse, Journal of Performance of Constructed Facilities 29 (5) (2015) B4014007.
- [25] A. Biegus, K. Rykaluk, Collapse of katowice fair building, Engineering Failure Analysis 16 (5) (2009) 1643–1654.
- [26] L. Zetlin, Report of the engineering investigation concerning the causes of the collapse of the hartford coliseum space truss roof on january 18 (1978).
- [27] F. Fu, G. Parke, Assessment of the progressive collapse resistance of double-layer grid space structures using implicit and explicit methods, International Journal of Steel Structures 18 (3) (2018) 831–842.
- [28] National Research Council Canada, National building code of Canada, Associate Committee on the National Building Code, National Research Council, 1975.
- [29] National Research Council Canada, National building code of Canada, Associate Committee on the National Building Code, National Research Council, 1995.
- [30] A. Committee, Building code requirements for structural concrete (ACI 318-08) and commentary, American Concrete Institute, 2008.
- [31] Department of Defense, UFC 4-023-03: Unified Facilites Criteria (UFC)–design of buildings to resist progressive collapse, Tech. rep., Department of Defense (2016).
- [32] B. Meacham, Ultimate health & safety (UHS) quantification: individual and societal risk quantification for use in National Construction Code (NCC), The Australian Building Codes Board, Australia, 2016.
- [33] Abaqus CAE, ABAQUS/CAE user's guide (Abaqus 6.11), Dassault Systemes Simulia Corporation, Providence, RI, USA, 2011.
- [34] T. Stolarski, Y. Nakasone, S. Yoshimoto, Engineering analysis with ANSYS software, Butterworth-Heinemann, 2018.
- [35] J. O. Hallquist, LS-DYNA KEYWORD USER'S MANUAL Volume I, Livermore Software Technology Corporation, 2009.
- [36] H. Wang, A. Zhang, Y. Li, W. Yan, A review on progressive collapse of building structures, The Open Civil Engineering Journal 8 (1) (2014).
- [37] B. Lewicki, S. Olesen, Limiting the possibility of progressive collapse, Building Research and Practice 2 (1) (1974) 10–13.
- [38] E. V. Leyendecker, B. R. Ellingwood, Design to reduce the risk of progressive collapse, Building Science Series 98 (1977).
- [39] GSA (General Services Administration), Progressive collapse analysis and design guidelines for new federal office buildings and major modernization projects, General Services Administration, 2003.
- [40] Department of Homeland Security. Federal Emergency Management Agency, Reference Manual To Mitigate Potential Terrorist Attacks Against Buildings (FEMA 426), Government Printing Office, 2003.
- [41] A. Pretlove, M. Ramsden, A. Atkins, Dynamic effects in progressive failure of structures, International Journal of Impact Engineering 11 (4) (1991) 539–546.
- [42] R. B. Malla, B. B. Nalluri, Dynamic effects of member failure on response of trusstype space structures, Journal of Spacecraft and Rockets 32 (3) (1995) 545–551.
- [43] G. Kaewkulchai, E. B. Williamson, Beam element formulation and solution procedure for dynamic progressive collapse analysis, Computers & Structures 82 (7-8) (2004) 639–651.
- [44] S. Marjanishvili, E. Agnew, Comparison of various procedures for progressive collapse analysis, Journal of performance of constructed facilities 20 (4) (2006) 365– 374.
- [45] F. Casciati, L. Faravelli, Progressive failure for seismic reliability analysis, Engineering Structures 6 (2) (1984) 97–103.
- [46] K. Abedi, G. Parke, Progressive collapse of single-layer braced domes, International Journal of Space Structures 11 (3) (1996) 291–306.
- [47] I. M. Alshaikh, B. A. Bakar, E. A. Alwesabi, H. M. Akil, Experimental investigation of the progressive collapse of reinforced concrete structures: An overview, Structures 25 (2020) 881–900.
- [48] J. Kim, D. An, Evaluation of progressive collapse potential of steel moment frames considering catenary action, The structural design of tall and special buildings 18 (4) (2009) 455–465.
- [49] H. Xiao, B. Hedegaard, Flexural, compressive arch, and catenary mechanisms in pseudostatic progressive collapse analysis, Journal of Performance of Constructed Facilities 32 (1) (2018) 04017115.
- [50] J. Yu, K. H. Tan, Special detailing techniques to improve structural resistance against progressive collapse, Journal of Structural Engineering 140 (3) (2014) 04013077.
- [51] U. Starossek, M. Haberland, Approaches to measures of structural robustness, Structure and Infrastructure Engineering 7 (7-8) (2011) 625–631.
- [52] C. Gurley, Progressive collapse and earthquake resistance, Practice Periodical on Structural Design and Construction 13 (1) (2008) 19–23.
- [53] J. R. Hayes Jr, S. C. Woodson, R. G. Pekelnicky, C. D. Poland, W. G. Corley, M. Sozen, Can strengthening for earthquake improve blast and progressive collapse resistance?, Journal of Structural Engineering 131 (8) (2005) 1157–1177.
- [54] Minister of Housing and Local Government, The Building (Fifth Amendment) Regulations 1970, The Building (Fifth Amendment) Regulations 1970. Statutory Instruments of the UK, S.I. 26 1970, No. 109, 1970.
- [55] Federal Housing Administration (FHA), Provisions to prevent progressive collapse, US Department of Housing and Urban Development (HUD), Federal Housing Administration (FHA). Provisions, 1971.
- [56] International Code Council, International building code (IBC 2009), International Code Council, 2009.
- [57] American Society of Civil Engineers (ASCE), Minimum design loads and associated criteria for buildings and other structures (ASCE/SEI 7 - 98), Structural Engineering Institute of the ASCE, 1998.
- [58] American Society of Civil Engineers (ASCE), Minimum design loads and associated criteria for buildings and other structures (ASCE/SEI 7 - 02), Structural Engineering Institute of the ASCE, 2002.
- [59] B. Ellingwood, R. Smilowitz, D. Dusenberry, D. Duthinh, H. Lew, N. Carino, Best practices for reducing the potential for progressive collapse in buildings (2007).
- [60] Standards Australia International, AS/NZS 1170.0 2002, Structural Design Actions: General Principles, Standards Australia International, 2002.
- [61] MOHURD, GB50068: Unified standard for reliability design of building structures, Ministry of Housing and Urban-Rural Construction of the People's Republic of China, Haidian District, Beijing, China, 2001.
- [62] China Association for Engineering Construction Standardization (CECS), CECS 392: 2014, Code for anti-collapse design of building structures, Beijing (China), 2014.
- [63] B. Ellingwood, E. Leyendecker, Approaches for design against progressive collapse, Journal of the Structural Division 104 (3) (1978) 413–423.
- [64] Council, Building Seismic Safety, NEHRP Recommended provisions for seismic regulations for new buildings and other structures, Part 1: Provisions (FEMA 368) & Part 2: Commentary (FEMA 369), Council, Building Seismic Safety, Washington DC, 2001.
- [65] Federal Emergency Management Agency, NEHRP guidelines for the seismic rehabilitation of buildings (FEMA 273), FEMA, 1997.
- [66] Federal Emergency Management Agency, Prestandard and commentary for the seismic rehabilitation of buildings (FEMA 356), Federal Emergency Management Agency, Washington, DC, 2000.
- [67] U. Starossek, Progressive collapse of structures: Nomenclature and procedures, Structural engineering international 16 (2) (2006) 113–117.
- [68] U. Starossek, Typology of progressive collapse, Engineering structures 29 (9) (2007) 2302–2307.
- [69] CEN (European Committee for Standardization), Eurocode 1: Actions on structures-Part 1-7: General actions-Accidental actions (EN 1991-1-7), CEN, 2006.
- [70] Department of Communities and Local Government, Building regulations 2000, Approved document F1: Means of ventilation (2010).
- [71] E. A. Smith, H. I. Epstein, Hartford coliseum roof collapse: structural collapse sequence and lessons learned, Civil Engineering—ASCE 50 (4) (1980) 59–62.
- [72] Task Committee on Latticed Structures under Extreme Dynamic Loads of the Committee on Special Structures of the Committee on Metals of the Structural Div., Dynamic considerations in latticed structures, Journal of Structural Engineering 110 (10) (1984) 2547–2550.
- [73] J. Munch-Andersen, P. Dietsch, Robustness of large-span timber roof structures—two examples, Engineering Structures 33 (11) (2011) 3113–3117.
- [74] K. Terwel, W. Boot, M. Nelisse, Structural unsafety revealed by failure databases, Proceedings of the Institution of Civil Engineers-Forensic Engineering 167 (1) (2014) 16–26.
- [75] P. Lenza, Instability of single layer doubly curved vaults, International Journal of Space Structures 7 (4) (1992) 253–264. doi:10.1177/026635119200700403.
- [76] M. Mohammadi, K. Abedi, N. Taghizadieh, Stability analysis of single-layer barrel vault space structures, International Journal of Space Structures 27 (4) (2012) 203– 218. doi:10.1260/0266-3511.27.4.203.
- [77] W. F. Chen, E. M. Lui, Structural stability: theory and implementation, Elscvier Science Publishing Co., 1987.
- [78] K. Abedi, G. Parke, Experimental study of dynamic propagation of local snapthrough in single-layer braced domes, International Journal of Space Structures 16 (2) (2001) 125–136.
- [79] A. El-Sheikh, Numerical analysis of space trusses with flexible member-end joints, International Journal of Space Structures 8 (3) (1993) 189–197.
- [80] A. López, I. Puente, M. A. Serna, Numerical model and experimental tests on single-layer latticed domes with semi-rigid joints, Computers & structures 85 (7- 8) (2007) 360–374. doi:10.1016/j.compstruc.2006.11.025.
- [81] H. Ma, F. Fan, S. Shen, Numerical parametric investigation of single-layer latticed domes with semi-rigid joints, Journal of the International Association for Shell and Spatial Structures 49 (2) (2008) 99–110.
- [82] Z. Zhao, Z. Chen, X. Yan, H. Xu, B. Zhao, Simplified numerical method for latticed shells that considers member geometric imperfection and semi-rigid joints, Advances in Structural Engineering 19 (4) (2016) 689–702.
- [83] H. Liu, Y. Ding, Z. Chen, Static stability behavior of aluminum alloy single-layer spherical latticed shell structure with temcor joints, Thin-Walled Structures 120 (2017) 355–365.
- [84] Z. Xiong, X. Guo, Y. Luo, S. Zhu, Y. Liu, Experimental and numerical studies on single-layer reticulated shells with aluminium alloy gusset joints, Thin-Walled Structures 118 (2017) 124–136.
- [85] Z. Xiong, S. Zhu, X. Zou, S. Guo, Y. Qiu, L. Li, Elasto-plastic buckling behaviour of aluminium alloy single-layer cylindrical reticulated shells with gusset joints, Engineering Structures 242 (2021) 112562.
- [86] H. Liu, Q. Zang, Z. Chen, Stability of single-layer reticulated shells with aag joints, Thin-Walled Structures 180 (2022) 109675.
- [87] H.-j. Li, Y. Taniguchi, Effect of joint stiffness and size on stability of three-way single-layer cylindrical reticular shell, International Journal of Space Structures 35 (3) (2020) 90–107.
- [88] J. Ye, M. Lu, Design optimization of domes against instability considering joint stiffness, Journal of Constructional Steel Research 169 (2020) 105757.
- [89] Y. Xu, X. Zhang, Q. Han, Progressive collapse analysis of single-layer latticed domes with assembled hub joints, in: Proceedings of IASS Annual Symposia, Vol. 18, International Association for Shell and Spatial Structures (IASS), 2020, pp. 1–10.
- [90] Y. Xu, X. Zhang, Q. Han, Research on the progressive collapse resistance of singlelayer cylindrical latticed shells with AH joints, Thin-Walled Structures 158 (2021) 107178.
- [91] R. Kolakkattil, A. Jayachandran, Global stability behaviour of single-layer reticulated domes created using a new nomenclature, Journal of the International Association for Shell and Spatial Structures 63 (1) (2022) 31–48. doi:10.20898/j.iass.2021.012.
- [92] Y. Xu, Q.-H. Han, G. Parke, Y.-M. Liu, Experimental study and numerical simulation of the progressive collapse resistance of single-layer latticed domes, Journal of Structural Engineering 143 (9) (2017) 04017121.
- [93] X. Zhao, S. Yan, Y. Chen, Comparison of progressive collapse resistance of singlelayer latticed domes under different loadings, Journal of Constructional Steel Research 129 (2017) 204–214.
- [94] L.-m. Tian, Q.-b. Li, W.-h. Zhong, J.-p. Wei, Effects of the rise-to-span ratio on the progressive collapse resistance of kiewitt-6 single-layer latticed domes, Engineering Failure Analysis 106 (2019) 104158.
- [95] L.-m. Tian, X.-n. Nie, W.-h. Zhong, J.-p. Wei, Comparison of the progressive collapse resistances of different single-layer latticed domes, Journal of Constructional Steel Research 162 (2019) 105697.
- [96] L.-m. Tian, J.-x. He, C.-b. Zhang, R. Bai, Progressive collapse resistance of singlelayer latticed domes subjected to non-uniform snow loads, Journal of Constructional Steel Research 176 (2021) 106433.
- [97] L.-m. Tian, C. Bai, W.-h. Zhong, Experimental study and numerical simulation of partial double-layer latticed domes against progressive collapse in member-removal scenarios, in: Structures, Vol. 29, Elsevier, 2021, pp. 79–91.
- [98] L.-m. Tian, D.-y. Li, W. Li, Y.-f. Kou, Progressive collapse resistance of singlelayer latticed domes under different failure conditions at the same joint, Journal of Building Engineering 36 (2021) 102132.
- [99] L.-m. Tian, J. Wei, J.-p. Hao, X. Wang, Dynamic analysis method for the progressive collapse of long-span spatial grid structures, Steel and Composite Structures 23 (4) (2017) 435–444.
- [100] Z. Nanhai, Y. Jihong, Structural vulnerability of a single-layer dome based on its form, Journal of Engineering Mechanics 140 (1) (2014) 112–127.
- [101] Y. Jihong, Q. Nian, Progressive collapse simulation based on dem for single-layer reticulated domes, Journal of Constructional Steel Research 128 (2017) 721–731.
- [102] L. Xu, J. Ye, Dem algorithm for progressive collapse simulation of single-layer reticulated domes under multi-support excitation, Journal of earthquake engineering 23 (1) (2019) 18–45.
- [103] G.-b. Nie, C.-x. Zhang, X.-d. Zhi, J.-w. Dai, Collapse of the single layered cylinder shell with model experimental study, Archives of Civil and Mechanical Engineering 19 (2019) 883–897.
- [104] Z.-w. Shan, H.-h. Ma, Z.-w. Yu, F. Fan, Dynamic failure mechanism of singlelayer reticulated (SLR) shells with bolt-column (BC) joint, Journal of Constructional Steel Research 169 (2020) 106042.
- [105] G.-b. Nie, C.-x. Zhang, D.-f. Li, Q. Chen, Z.-y. Wang, Collapse of the spatial double-layer cylinder shell by experimental study, Engineering Structures 245 (2021) 112862.
- [106] G.-b. Nie, C.-x. Zhang, Z.-y. Wang, W.-d. Xu, Y.-j. Shi, Experimental study of collapse behavior of single-layer cylinder shells with infilled walls, Structures 36 (2022) 793–804.
- [107] J. Wu, J. Zheng, G. Sun, M. Feng, Shaking table test of the single-layer aluminum alloy cylindrical reticulated shell, Thin-Walled Structures 172 (2022) 108866.
- [108] K. Qian, B. Li, Performance of three-dimensional reinforced concrete beam-column substructures under loss of a corner column scenario, Journal of structural engineering 139 (4) (2013) 584–594.
- [109] Q. Kai, B. Li, Dynamic performance of rc beam-column substructures under the scenario of the loss of a corner column—experimental results, Engineering Structures 42 (2012) 154–167.
- [110] H. Jian, S. Li, L. Huanhuan, Testing and analysis on progressive collapse-resistance behavior of rc frame substructures under a side column removal scenario, Journal of Performance of Constructed Facilities 30 (5) (2016) 04016022.
- [111] N. S. Lim, K. Tan, C. Lee, Experimental studies of 3d rc substructures under exterior and corner column removal scenarios, Engineering Structures 150 (2017) 409– 427.
- [112] L. Tian, J. Wei, J. Hao, Anti-progressive collapse mechanism of long-span singlelayer spatial grid structures, Journal of Constructional Steel Research 144 (2018) 270–282.
- [113] J.-p. Wei, L.-m. Tian, J.-p. Hao, Improving the progressive collapse resistance of long-span single-layer spatial grid structures, Construction and Building Materials 171 (2018) 96–108.
- [114] L.-m. Tian, J.-p. Wei, J.-p. Hao, Optimisation of long-span single-layer spatial grid structures to resist progressive collapse, Engineering Structures 188 (2019) 394– 405.
- [115] Q. Han, M. Liu, Y. Lu, C. Wang, Progressive collapse analysis of large-span reticulated domes, International Journal of Steel Structures 15 (2) (2015) 261–269.
- [116] J. Ma, F. Fan, C. Wu, X. Zhi, Counter-intuitive collapse of single-layer reticulated domes subject to interior blast loading, Thin-Walled Structures 96 (2015) 130–138.
- [117] J.-p. Wei, L.-m. Tian, J.-p. Hao, Parameter analysis of progressive collapse simulation of long-span spatial grid structures, International Journal of Steel Structures 19 (2019) 1718–1731.
- [118] S. Yan, X. Zhao, K. J. Rasmussen, H. Zhang, Identification of critical members for progressive collapse analysis of single-layer latticed domes, Engineering Structures 188 (2019) 111–120.
- [119] Q. Zhang, W. Huang, Y. Xu, J. Cai, F. Wang, J. Feng, Analysis of key elements of single-layer dome structures against progressive collapse, Steel and Composite Structures 42 (2) (2022) 257–264.
- [120] L.-M. Tian, J.-P. Wei, J.-P. Hao, X. Wang, Method for evaluating the progressive collapse resistance of long-span single-layer spatial grid structures, Adv. Steel Constr 15 (1) (2019) 109–115.
- [121] L.-m. Tian, J.-p. Wei, R. Bai, X.-c. Zhang, Dynamic behavior of progressive collapse of long-span single-layer spatial grid structures, Journal of performance of constructed facilities 35 (2) (2021) 04021002.
- [122] X. Zhi, M. Stewart, Damage and risk assessment for single-layer reticulated domes subject to explosive blast loads, International Journal of Structural Stability and Dynamics 17 (09) (2017) 1750108.
- [123] H. Zhou, Y. Zhang, F. Fu, J. Wu, Progressive collapse analysis of reticulated shell structure under severe earthquake loading considering the damage accumulation effect, Journal of Performance of Constructed Facilities 32 (2) (2018) 04018004.
- [124] Z. Lu, X. He, Y. Zhou, Discrete element method-based collapse simulation, validation and application to frame structures, Structure and Infrastructure Engineering 14 (5) (2018) 538–549.
- [125] Y. Jihong, Q. Nian, Combination of dem/fem for progressive collapse simulation of domes under earthquake action, International Journal of Steel Structures 18 (2018) 305–316.
- [126] K. Meguro, H. Tagel-Din, Applied element method for structural analysis theory and application for linear materials, Doboku Gakkai Ronbunshu 2000 (647) (2000) 31–45.
- [127] S. Kato, T. Yamashita, Evaluation of elasto-plastic buckling strength of two-way grid shells using continuum analogy, International Journal of Space Structures 17 (4) (2002) 249–261.
- [128] A. Hanaor, C. Marsh, G. A. Parke, Modification of behavior of double-layer grids: overview, Journal of Structural Engineering 115 (5) (1989) 1021–1037.
- [129] L. C. Schmidt, A. Hanaor, Force limiting devices in space trusses, Journal of the Structural Division 105 (5) (1979) 939–951.
- [130] G. A. R. Parke, THE BEHAVIOUR OF SPACE TRUSSES INCORPORATING NOVEL COMPRESSION MEMBERS, PhD Thesis, University of Surrey, 1989.
- [131] A. El-Sheikh, Effect of force limiting devices on behaviour of space trusses, Engineering structures 21 (1) (1999) 34–44.
- [132] M. Poursharifi, K. Abedi, M. Chenaghloua, 05.32: Experimental and numerical study on the collapse behavior of an all-steel accordion force limiting device, Ce/papers 1 (2-3) (2017) 1315–1324.
- [133] M. Kheirollahi, M. R. Chenaghlou, K. Abedi, Buckling-controlled member for improving the ductile behavior of double-layer latticed space structures, International Journal of Space Structures (2023) 09560599231161447.
- [134] B. I. Song, H. Sezen, Experimental and analytical progressive collapse assessment of a steel frame building, Engineering structures 56 (2013) 664–672.
- [135] M. Sasani, S. Sagiroglu, Progressive collapse resistance of hotel san diego, Journal of structural engineering 134 (3) (2008) 478–488.
- [136] M. Sasani, Response of a reinforced concrete infilled-frame structure to removal of two adjacent columns, Engineering Structures 30 (9) (2008) 2478–2491.
- [137] J. Sumec, General stability analysis of lattice shells by continuum modeling, International Journal of Space Structures 7 (4) (1992) 275–283.
- [138] E. K. Kabando, J. Gong, An overview of long-span spatial grid structures failure case studies, Asian Journal of Civil Engineering 20 (2019) 1137–1152.
- [139] H. Borsje, B. Renier, H. Burggraaf, Collapse of the roof of a football stadium, in: IABSE Symposium: Engineering for Progress, Nature and People, Madrid, Spain, 3-5 September 2014, 2014, pp. 1055–1062.
- [140] Bureau of Indian Standard, IS 800 2007: General construction in steel-code of practice, BIS, New Delhi, India, 2007.
- [141] R. Sun, Z. Huang, I. W. Burgess, Progressive collapse analysis of steel structures under fire conditions, Engineering structures 34 (2012) 400–413.
- [142] G.-L. F. Porcari, E. Zalok, W. Mekky, Fire induced progressive collapse of steel building structures: A review of the mechanisms, Engineering Structures 82 (2015) 261–267.