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The dark side of transparency: When hiding in plain sight works [☆], [☆☆]

Tatiana Mayskaya [a], Arina Nikandrova [b,*]

[a] International College of Economics and Finance / Faculty of Economic Sciences, HSE University, 11 Pokrovsky Bulvar, Moscow, 109028, Russia

[b] Department of Economics, City, University of London, Northampton Square, London, EC1V 0HB, United Kingdom

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Abstract

A hider publicly commits to the number of seekers and then privately gets involved in a story, which may be compromising. Each seeker aims to be the first to learn and report a compromising story. The seekers learn the story privately and in continuous time. With more seekers, the hider's story gets revealed at a faster rate, but each seeker gets discouraged and ceases learning more quickly. To reduce the probability of a compromising report, the hider may optimally choose infinitely many seekers. Nevertheless, the hider unambiguously benefits from making it harder for each seeker to learn her story.

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* Corresponding author.
E-mail addresses: tmayskaya@gmail.com (T. Mayskaya), a.nikandrova@gmail.com (A. Nikandrova).
URLs: https://www.tmayskaya.com (T. Mayskaya), https://sites.google.com/view/anikandrova (A. Nikandrova).

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1. Introduction

It is a widespread belief that transparency is a key to good governance because it ensures that the public is well-informed and can hold the authorities accountable for their misdeeds.[1] Acting on the presumption that transparency necessarily leads to a well-informed public, numerous countries around the world have implemented some form of freedom of information legislation that allows the general public to access government-held data. Thus, in the UK, the Freedom of Information Act 2000 creates a public "right of access" to information held by public authorities, while the US's Accountability and Transparency Act of 2006 requires full disclosure of all entities receiving federal funds. However, since information collection requires time and effort, making information accessible does not necessarily mean that the public learns this information. In this paper, we develop this idea and show that open access to government information may backfire and hinder learning by the public.

Our model is inspired by the investigation into the COVID-19 lab-leak theory. In agreement with the US Accountability and Transparency Act, the information about the US government's funding of gain-of-function experiments at the Wuhan Institute of Virology was in the public domain. Yet, after the start of the pandemic, it took some time for the public to learn this. Eventually, the public learned the funding information from a report by a group of armchair investigators, who pieced it together from various public sources.[2] Like any other journalists, these investigators may have been motivated primarily by the desire to be the first to report a sensational story. We argue that, coupled with private learning, the drive to be first may reduce the probability that a sensational story gets reported to the public. When information is in open access, each investigator who is looking for a sensation quickly becomes pessimistic and stops looking because he thinks that, had there been anything interesting to uncover, somebody would probably have found it already. Consequently, the transparency requirement, instead of furthering public interests, benefits the government, which aims to hide the funding information to avoid being implicated in a public health emergency.

In our model, a hider, referred to as she, publicly commits to the number of seekers who have access to her. For example, the hider can be the government and the seekers can be journalists. Upon committing, the hider gets involved in a story that could be one of two types: compromising or non-compromising.[3] In the lab-leak example, the story is whether the US government funded gain-of-function experiments in Wuhan. The story becomes obsolete at some exogenous rate, which we refer to as an obsolescence rate, and which acts as a discount rate that is common to the hider and the seekers.

After the story takes place, the seekers can undertake costly learning to uncover it. Initially, they do not know the story's type but share a common prior belief that the story is compromising. Each seeker, referred to as he, has access to an information source that conclusively reveals the hider's story, together with its type, at an exogenous rate that may depend on the type of the story. The seekers' information sources are independent, conditional on the type of the hider's story. Each seeker observes neither whether other seekers are learning nor the outcomes of their

[1] According to a classic result by Holmstrom (1979), in the principal-agent model, more information about the agent's action benefits the principal. If the principal is the public and the agent is the government, then Holmstrom (1979) implies that transparency improves accountability.

[2] See the *Vanity Fair* article.

[3] The interpretation of a compromising story does not have to be literal. In our model, information is *compromising* whenever its use hurts the hider.

learning. Hence, the seekers' game is a game of *strategic experimentation* with *private learning*. Only upon learning the story can a seeker report it. He profits only from reporting a compromising story that is not obsolete and has not been reported yet; that is, the seekers' game has a *winner-takes-all* payoff structure. All reports are public.

The hider's goal is to choose the number of seekers to maximize the probability that they will fail to report a compromising story before it becomes obsolete. We interpret a higher number of seekers as lower information protection or higher transparency. In practice, a government may limit the number of people with access to classified information, or a president may choose the size of her audience by limiting the size of a press pool with direct access to her.[4]

The hider can restrict the number of seekers at no cost, but she is bound to give at least one seeker access to her. Moreover, we consider a range of parameters wherein each seeker finds at least some learning optimal. The restrictions on the hider's choice and on the parameters make perfect protection infeasible, which renders the hider's problem nontrivial. In applied settings, the hider may be able to smoothly trade off the benefit against the cost of information protection. By assuming that the cost is lexicographic – that is, the cost is infinite if the hider chooses zero seekers and is zero if she chooses any positive number of seekers – we abstract away from this trade-off, while acknowledging that technological or legal constraints may preclude perfect protection.

Given the optimal behavior of the seekers, the probability that a compromising story will be reported depends on the number of seekers with access to the hider. This result highlights the role of private learning whereby each seeker does not observe whether other seekers have already uncovered a non-compromising story. In our setting, had learning been public, the hider would have been indifferent to the number of seekers (see Section 5 for details).

The hider's optimal choice of the number of seekers is dictated by a combination of direct and indirect effects. The direct effect of a higher number of seekers is an increase in the rate at which a compromising story is revealed, keeping each seeker's learning strategy fixed. The direct effect hurts the hider. The indirect effect of a higher number of seekers operates through the change in each seeker's learning strategy. More specifically, a higher number of seekers speeds up each seeker's downwards belief updating in the absence of a compromising report, thus discouraging him from prolonged learning. The indirect effect benefits the hider.

Theorem 1 shows that for the hider, restricting access to a single seeker is optimal only if the obsolescence rate is sufficiently high. When the obsolescence rate is low, the indirect effect takes the upper hand over the direct effect, and the open access policy with an infinite number of seekers is optimal for the hider. Thus, our model predicts that a fully rational hider, whose sole objective is to avoid a compromising report, optimally chooses to endorse transparency and *hide in plain sight*.[5]

In our model, there are two crucial assumptions: first, the hider's choice of the number of seekers is public; and, second, the hider's choice cannot depend on the type of her story. If the hider's choice were not publicly observable, she would benefit from surreptitiously deviating to a lower number of seekers. If the hider could condition the number of seekers on the type of

[4] By assuming that the hider does not want a compromising story to be revealed, we introduce a sharp conflict of interests between the seekers and the hider. In Section 3.5, we discuss applications in which the hider's interests are aligned with the seekers', and she wants to maximize the probability that a compromising story is revealed.

[5] The idea of hiding in plain sight captured the imagination of many fiction writers. For example, in Edgar Allan Poe's "The Purloined Letter," an unscrupulous blackmailer leaves the stolen letter out in the open, but a careful police search fails to find it.

her story, her public choice to restrict access could signal that the hider's information is indeed sensitive, which would only attract seekers' attention.[6] To avoid signaling, in equilibrium, the informed hider might optimally choose a large audience size. We show that even in the absence of signaling, giving access to infinitely many seekers might be optimal for the hider.

The forces behind our result in Theorem 1 stem from the seekers' learning dynamics and are distinct from the discouragement effect that appears in static settings with a winner-takes-all payoff structure. In static settings, faced with numerous competitors, the seekers reduce their learning because they think that someone else is likely to beat them in the contest to learn the story.[7] In our setting, the seekers give up learning because the lack of competitors' success makes them think that there is nothing valuable to learn in the first place; had the seekers known the story's type with certainty, the open access policy would have no longer been strictly optimal.[8]

In our model, all transparency measures that are unrelated to the audience size are captured by the arrival rates of the compromising and non-compromising stories, which the hider cannot control. We interpret the story arrival rates as exogenous information protection. In practice, the arrival rates are determined by regulations that shape the format in which information must be presented. If the information is disclosed in a clear format, it is easy to interpret, which corresponds to a higher story arrival rate. In the lab-leak example, to uncover the scale of US government funding for gain-of-function experiments, the investigators had to examine the tax exemption forms filed by the non-profit organization that divvied up the US grant money. The format of this information is governed by financial disclosure regulations.

Overall, there are two types of information protection – exogenous protection through the story arrival rates and endogenous protection through the audience size. The distinction between exogenous and endogenous protection is parallel to the distinction between the opacity and the availability of information, respectively. Theorem 2 shows that these two types of information protection can be either complements or substitutes. While the substitutability of different types of protection is not surprising, the complementarity is also possible and stems from the indirect effect. The indirect effect reflects the implicit cost of endogenous protection: stronger endogenous protection – that is, a lower number of seekers – encourages prolonged learning, thus hurting the hider. Stronger exogenous protection, by reducing the duration of the seekers' learning, may alleviate the implicit cost of endogenous protection, thus incentivizing the hider to increase the endogenous protection, which makes the two types of protection complements.

Theorem 3 shows that the hider always becomes better off when exogenous protection strengthens. In light of the interpretation of exogenous protection as the opacity of the disclosed information, Theorem 3 implies that the opacity of information helps the government to hide it, which is in line with the discussion in Fox (2007), who emphasizes the importance of clarity of information for effective transparency.

Our paper makes a substantial technical contribution to the experimentation literature with private learning. The analysis of the seekers' subgame is technically challenging because the model features private learning. As a result, in addition to each seeker's own belief about the story's type, we need to keep track of other seekers' common equilibrium belief, which means

[6] The phenomenon of triggering interest by publicly censoring information is called the Streisand effect and is formally studied by Hagenbach and Koessler (2017).

[7] The discouragement effect is well-researched in the contest literature (Barut and Kovenock (1998)). Under some assumptions, the discouragement effect in symmetric contests can be so strong that the aggregate effort also decreases with the number of competitors (Fang et al. (2020)).

[8] See Section 5 for further discussion.

that the state space of each seeker's dynamic optimization problem is two-dimensional. The technical machinery that we develop allows us to analyze the range of parameters where no news is good news. In this case, the seekers' learning intensities are strategic substitutes and the unique symmetric equilibrium involves interior learning intensities. Furthermore, each seeker's equilibrium learning intensity may have a discrete jump from an interior value to full intensity (see Lemma 3) – a feature that is novel to the literature, which tends to focus on the case where no news is bad news.[9]

The rest of the paper is organized as follows. This section concludes with a review of the relevant literature. Section 2 describes the setup. Section 3 outlines the main results of the paper, provides high-level intuition and describes additional applications of the model. Section 4 contains a full analysis of the model. Section 5 discusses the assumptions and extensions of the model.

Related literature

Our paper is related to several strands of the literature. First, it belongs to a diverse literature on transparency, which studies the welfare implications of the transparency requirement in various settings. Gradwohl and Feddersen (2018) and Fehrler and Hughes (2018) show that in advisory committees, transparency may hinder information aggregation. In these models, transparency distorts the incentives of the informed party. In contrast, in our model, transparency affects the incentives of the uninformed party – the seekers.

On the technical side, our paper draws on the extensive literature on Poisson bandit-based games of learning and experimentation, initiated by Keller et al. (2005) and reviewed in Hörner and Skrzypacz (2017). More specifically, the seekers' game belongs to the growing literature on strategic experimentation with private learning efforts, payoff externalities, and only partially observable learning outcomes. Within this literature, in contrast to the competitive setting of our model, Bonatti and Hörner (2011) and Guo and Roesler (2018) study a collaboration model with observable exit but unobservable signals and effort levels. Within the competitive setting, Halac et al. (2017) study how to encourage, as opposed to discourage, learning efforts in contests. They compare different prize-sharing schemes, including the winner-takes-all contest, and allow for unobservable successes, as opposed to unobservable failures.

In our model, each seeker's learning is private and the total amount of learning depends on the number of seekers. While in the canonical strategic experimentation model of Keller et al. (2005), the total amount of experimentation is invariant to the number of experimenters, the existing literature suggests that moving from public to private learning may break this invariance. For example, Halac et al. (2017) demonstrate that the invariance featured in the public winner-takes-all contest disappears in the hidden equal-sharing contest. Another way to break the invariance is to assume that an information source does not conclusively reveal the story type. For example, Keller and Rady (2010) minimally change the framework of Keller et al. (2005) by assuming that the arrival of lump-sum payoffs is no longer fully revealing and in their setting, the total amount of experimentation depends on the number of experimenters. Earlier, Bolton and Harris (1999) demonstrate a similar point in a model where payoffs are governed by Brownian motion.

[9] Other papers with private learning in experimentation models, such as Akcigit and Liu (2016) and Cetemen and Margaria (2023), assume that unsuccessful learning makes each seeker unambiguously more pessimistic, independent of his belief about the learning intensities of other seekers. The unambiguously downward direction of belief updating ensures the existence of an equilibrium in which players never use interior learning intensities.

The paper that is closest to ours is Akcigit and Liu (2016). Similar to us, Akcigit and Liu (2016) consider a winner-takes-all competition in which learning is private and can result in either a good or a bad outcome, and players do not observe each other's bad outcomes. Players' inability or unwillingness to share bad news is central for Akcigit and Liu (2016) and for us. However, unlike us, Akcigit and Liu (2016) focus on two asymmetric players and their model is not easy to use to study the effect of changing the number of players.[10] A simpler payoff structure allows us to work with symmetric players and fully characterize the equilibrium of the seekers' subgame for an arbitrary number of players. Furthermore, Akcigit and Liu (2016) assume that the rate of outcome arrival is independent of the outcome type, which implies that no news is bad news, and players become progressively more pessimistic over time. The unambiguous direction of belief updating guarantees the existence of a simple strategy Bayesian equilibrium in which players never use interior learning intensities. In our game, the arrival rate varies with the outcome type, and so the direction of the belief updating may depend on the learning strategies of others, in which case there is no equilibrium in simple strategies. Solving the seeker's subgame for more general information technology constitutes the technical contribution of our paper.

2. The model

The game is between a hider (she) and antagonistic seekers (he). At the outset of the game, the hider publicly commits to the number of seekers, denoted by n, who have access to the hider and can learn about her. After committing to the number of seekers, the hider gets involved in a story of type $\theta \in \{0, 1\}$, where $\theta = 1$ corresponds to a compromising story and $\theta = 0$ corresponds to a non-compromising story.

The seekers know neither the hider's story nor its type θ but can undertake costly learning to uncover it. The seekers share the same prior belief that the story is compromising, $p_0 \in (0, 1)$.

Each seeker is endowed with an information source and can learn from this source in continuous time over an infinite time horizon. At each time t, seeker i chooses a learning intensity $x_t^i \in [0, 1]$. The seeker's learning reveals to him the story and its type through a Poisson process with rate $\mu_1 x_t^i$ if $\theta = 1$ and with rate $\mu_0 x_t^i$ if $\theta = 0$. Learning is associated with a flow cost $c x_t^i$. Parameters $\mu_1 > 0$, $\mu_0 > 0$ and $c > 0$ are exogenously given.[11]

Conditional on the story's type, the seekers' information sources are independent. The model features private learning: the seekers do not communicate with each other and observe neither other seekers' learning intensities nor the outcome of their learning at any given moment in time.

The story becomes obsolete through a public Poisson process with arrival rate $\rho \geq 0$. A story that has not become obsolete yet is called an up-to-date story. Given the payoff structure that we describe below, neither the hider nor the seekers care about obsolete stories, so the game effectively ends once the story becomes obsolete. Hence, the obsolescence rate ρ is equivalent to a discount rate that is common to all seekers and the hider.

Each seeker has an option to report the story, but only upon learning it. All reports are public. The payoff from reporting a story is positive, and normalized to 1, only if the story is an up-to-date compromising story that has never been reported before; otherwise, the payoff from reporting a story is negative (we do not introduce a parameter for this payoff because it plays no

[10] See Section B.2 in Online Appendix to Akcigit and Liu (2016).

[11] We allow μ_0 to be arbitrarily close to 0, which captures the possibilities that either there are hardly any non-compromising stories or that conclusive evidence of a non-compromising story is unlikely to be found. The latter possibility is relevant for the lab-leak example from the introduction.

role in the subsequent analysis). The payoff from not reporting anything is zero. Given this payoff structure, in equilibrium, upon learning an up-to-date story, each seeker always reports it if it is compromising and never reports it if it is non-compromising.[12] This asymmetry in reporting is crucial for the learning dynamics we study.

The hider's objective is to choose n to maximize the probability of avoiding the report of a compromising story before it becomes obsolete. We interpret n as the strength of information protection: higher n means lower protection.

The cost of lowering n is zero, and so it is not included in the hider's objective. We make this assumption to avoid the trivial conclusion that weak protection – that is, high n – is optimal because information protection is costly. However, we make two assumptions that preclude perfect protection. First, we assume that the hider has to give access to at least one seeker; that is, the hider can choose any $n \geq 1$. Second, we assume

Assumption 1. $\mu_1 p_0 > c$,

which guarantees that each seeker finds at least some learning optimal.

3. Main results

In this section, we preview our main results. We postpone the full equilibrium analysis until Section 4.

3.1. Optimal number of seekers

We start the preview with the case in which $\rho = 0$. This special case provides sharp intuition for the main forces that drive our results.

We focus on a symmetric equilibrium in which all seekers use the same learning strategy. Due to the winner-takes-all payoff structure, seeker i immediately stops learning when he finds out the story either through his own learning or through a public report by another seeker. In the absence of a finding, seeker i's equilibrium belief that the story is compromising, p_t, changes deterministically according to the law of motion derived from Bayes' rule:

$$\dot{p}_t = -\left(n\mu_1 - \mu_0\right) x_t^* \, p_t \left(1 - p_t\right), \tag{1}$$

where x_t^* is the equilibrium learning intensity of each seeker who has not uncovered the story yet.[13] Formula (1) shows that, over time, the seeker's equilibrium belief p_t decreases at a speed that is proportional to $n\mu_1 - \mu_0$. We refer to $n\mu_1 - \mu_0$ as the **speed of learning**, which is defined as the difference between the learning rates of compromising and non-compromising stories. Since the seekers optimally report only compromising stories, the learning rate of non-compromising stories, μ_0, does not depend on the number of seekers n.

[12] Since a compromising story is always publicly reported once one of the seekers learns it, if $\mu_0 = 0$, then all learning is essentially public. We discuss how our results change when learning is public in Section 5 on page 25.

[13] The proof for (1) is standard. By Bayes' rule, for infinitesimally small Δ, the ratio of posterior beliefs, $p_{t+\Delta}/(1 - p_{t+\Delta})$, is equal to the probability that the story is compromising and it has not been found during the time interval $[t, t + \Delta)$, $p_t(1 - n\mu_1\Delta)$, divided by the probability that the story is non-compromising and it has not been found during $[t, t + \Delta)$, $(1 - p_t)(1 - \mu_0\Delta)$.

If $\mu_0 > \mu_1$ and the hider chooses a single seeker ($n = 1$), then, by (1), the seeker becomes progressively more optimistic that the story is compromising. Then, the seeker never stops learning before he uncovers the story, and so the hider has no chance of avoiding a compromising report. To reverse the direction of belief updating, in the equilibrium, the hider chooses n greater than $\mu_0/\mu_1 > 1$. The above intuition is central in our model – the hider finds the minimal access ($n = 1$) suboptimal because she wants to discourage the seekers from prolonged learning. In fact, in Section 4, we show that the desire to discourage learning is so strong that the hider optimally chooses the open access policy ($n = +\infty$).

Case $\mu_0 > \mu_1$ provides a very simple intuition for suboptimality of $n = 1$. In contrast, the intuition for the optimality of $n = +\infty$ is sharper in the opposite case of $\mu_0 < \mu_1$. In the remainder of Section 3, unless explicitly stated otherwise, we focus on the case $\mu_0 < \mu_1$.

When $\mu_0 < \mu_1$, the choice of n cannot change the direction of belief updating, and, for any $n \geq 1$, each seeker becomes progressively more pessimistic that the story is compromising. In the absence of a finding, each seeker optimally learns with intensity 1 until he becomes sufficiently pessimistic to stop learning.[14] The belief threshold $\bar{p}$ at which each seeker gives up learning does not depend on the number of seekers n. Intuitively, once other seekers have stopped learning, seeker i behaves as if he is alone, and so the belief threshold at which he optimally stops learning is independent of the number of seekers.

Given the seekers' optimal behavior, the equilibrium probability that the hider avoids the report of a compromising story is given by

$$e^{-n\mu_1 T(\mu_1,\mu_0,c,p_0,n)}, \tag{2}$$

where

$$T(\mu_1, \mu_0, c, p_0, n) = \frac{1}{n\mu_1 - \mu_0} \ln\left(\frac{p_0(1-\bar{p})}{(1-p_0)\bar{p}}\right), \tag{3}$$

with $\bar{p}$ defined later in (16). Function T defined in (3) is the time that each seeker's belief p_t takes to reach threshold $\bar{p}$ from prior p_0; that is, T is the maximum duration of unsuccessful learning. Expression (3) for T is derived from the belief-updating process (1). Formula (2) is the probability that the exponentially distributed waiting time for a compromising report is greater than T; the rate parameter of the exponential distribution is $n\mu_1$ because each seeker learns with intensity 1, and, thus, the seekers' learning reveals the compromising story at rate $n\mu_1$.

The optimal number of seekers n maximizes (2) and balances two effects. First, n appears outside T in the power of the exponent. This occurrence of n reflects the **direct effect** whereby an increase in n increases the probability that the seekers' learning reveals a compromising story during a time interval of fixed length. Second, n creates the **indirect effect** that reflects the change in each seeker's learning strategy. This effect operates through the belief updating: an increase in n increases the speed of learning $n\mu_1 - \mu_0$, thus reducing the duration of unsuccessful learning T.

The optimal n maximizes (2), which after substituting T from (3) becomes

$$\left(\frac{p_0(1-\bar{p})}{(1-p_0)\bar{p}}\right)^{-\frac{n\mu_1}{n\mu_1-\mu_0}}. \tag{4}$$

[14] As we show in Section 4, if $\mu_0 > \mu_1$, then, on the equilibrium path, the seekers may learn with intensity lower than 1. This property of the equilibrium complicates the intuition for some of our results, which, nevertheless, hold for any relationship between μ_0 and μ_1.

In (4), only the exponent depends on n and so the maximization of (4) is equivalent to the minimization of

$$\frac{n\mu_1}{n\mu_1 - \mu_0}. \tag{5}$$

Formula (5) elucidates that our results are driven by the asymmetry in the observability of different types of learning outcomes: a finding of a compromising story is publicly observable, while a finding of a non-compromising story is only privately observable. If both types of stories were publicly observable, the learning rate of non-compromising stories would be $n\mu_0$ instead of μ_0, so that the ratio (5) would become $\frac{\mu_1}{\mu_1-\mu_0}$, making probability (4) independent of n.

As n increases, the ratio (5) increases through the numerator, which corresponds to the direct effect but decreases through the denominator, which corresponds to the indirect effect. Intuitively, the ratio (5) captures the trade-off that, as the number of seekers increases, they learn and report compromising stories faster but for a shorter time interval. Indeed, the numerator in (5) is equal to the rate at which the seekers report a compromising story while they are still learning – that is, while $t < T$. The denominator in (5) is equal to the speed of learning, which controls the duration of unsuccessful learning according to (3).

The ratio (5) decreases in n, which implies that the indirect effect always takes an upper hand over the direct effect. In other words, as the number of seekers increases, for the hider, the positive effect from the decrease in the duration of unsuccessful learning, T, is stronger than the negative effect from the increase in the rate $n\mu_1$ at which the seekers report a compromising story while they are still learning. Therefore, if there is an arbitrary exogenous upper bound, say N, on the number of seekers that the hider can choose, then the optimal n is always equal to N. For the sake of parsimony, to avoid carrying an extra parameter, we allow the hider to choose any $n \geq 1$, effectively setting $N = +\infty$. Then, the optimal n is $+\infty$, which we refer to as the open access policy.[15]

The introduction of the possibility that the story becomes obsolete, i.e., $\rho > 0$, helps to disentangle the direct and indirect effects. Intuitively, if the story quickly becomes obsolete, minimizing the probability of a compromising report at the current moment – which is captured by the direct effect – is more important for the hider than minimizing the duration of learning – which is captured by the indirect effect. In more detail, a positive obsolescence rate ρ triggers occasional exogenous termination of the seekers' unsuccessful learning, thus lowering the probability that the seekers stop learning because they became too pessimistic to continue. Hence, an increase in ρ makes the speed of the seekers' belief updating less relevant to the actual duration of learning and, thus, to the probability that the hider avoids a compromising report. Because the indirect effect operates through the seekers' belief updating, and the possibility of story obsolescence makes the belief updating less relevant to the hider, an increase in ρ weakens the power of the indirect effect. Hence, for sufficiently high ρ, the direct effect prevails, making minimal access optimal.

Theorem 1 characterizes the optimal n for an arbitrary obsolescence rate ρ and an arbitrary relationship between μ_0 and μ_1. The proof is deferred to Section 4 and Appendix A.7.

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[15] There is a subtle technical complication with taking n to $+\infty$. For any finite n, the seekers' collective learning reveals a compromising story through a Poisson process with rate $n\mu_1$. When n is $+\infty$, the collective learning process explodes as its arrival rate becomes $+\infty$. However, we show that the algebraic limit $n \to +\infty$ of the probability that none of n seeker reports a compromising story is well-defined. Hence, the optimality of $n = +\infty$ should be understood as saying that if the hider is free to choose any number of seekers n from 1 to N, for sufficiently large but finite N, she would choose $n = N$.

Theorem 1. *Under Assumption 1, there exists* $\rho^* \in (0, +\infty)$ *such that open access* ($n = +\infty$) *is optimal for* $\rho < \rho^*$*, and minimal access* ($n = 1$) *is optimal for* $\rho > \rho^*$.

The optimality of open access in Theorem 1 has an intuitive explanation. With many seekers, each individual seeker quickly becomes pessimistic and gives up learning because he thinks that, had there been anything valuable to uncover, he or another seeker would most likely have found it already. In real life, the very same mechanism can explain persistent survival of various myths that can be easily refuted. Thus, social media users are reluctant to perform elementary fact-checking because they believe that any misinformation would have already been publicly refuted. Accomplished liars and imposters successfully rely on a similar mechanism. For example, the very public lies of Frank Abagnale, Jr., whose alleged autobiography was published in 1980 and inspired the 2002 film "Catch Me If You Can," were ultimately debunked only in 2020.[16] Perhaps most astonishingly, Tutankhamun's tomb – one of the greatest archaeological miracles of the 20th century because it was found nearly intact and densely packed with invaluable items – was hiding in plain sight for centuries because people were accustomed to thinking that the Valley of the Kings had already revealed all its secrets, so they simply stopped looking.

3.2. Comparative statics

Theorem 1 introduces threshold ρ^*, which separates region $\rho \in (0, \rho^*)$, where $n = +\infty$ is optimal, from region $\rho \in (\rho^*, +\infty)$, where $n = 1$ is optimal. The value of threshold ρ^* depends on the parameters of the model – μ_1, μ_0, c and p_0. Theorem 2 characterizes the behavior of ρ^* with respect to these parameters. The proof of Theorem 2 is in Appendix A.8.

Theorem 2. *Suppose that Assumption 1 holds and let* ρ^* *be the threshold defined in Theorem 1. If* $\mu_1 = \mu_0 = \mu$*, then* ρ^* *decreases in* μ*. If* $\mu_0 < \mu_1$*, then* ρ^*

- *increases in* μ_0*, and*
- *decreases in* μ_1.

If $\mu_0 > \mu_1$*, then* ρ^*

- *decreases in* μ_0*, and*
- *decreases in* $\mu_1 \in (c/p_0, M_1)$ *and increases in* $\mu_1 \in (M_1, \mu_0)$ *for some* $M_1 \in [c/p_0, \mu_0]$.

Moreover, ρ^* *increases in the flow cost c and decreases in the prior belief* p_0.

We interpret the arrival rates of compromising and non-compromising stories, μ_1 and μ_0, as **exogenous information protection**: higher μ_1 and μ_0 means lower protection. The exogenous measures may protect compromising and non-compromising information differently, and, hence, we do not insist on μ_1 being equal to μ_0.

The comparative statics of ρ^* with respect to μ_0 and μ_1 illuminate the relationship between two types of protection: strengthening protection through decreasing the audience size n and

[16] In his 2020 book "The Greatest Hoax on Earth: Catching Truth, While We Can," Alan Logan debunks almost everything Frank Abagnale wrote in his autobiography.

strengthening protection through increasing the opacity of the disclosed information – that is, through decreasing μ_0 and μ_1. Intuitively, the two types of protection should be substitutes. Indeed, if μ_1 and μ_0 are equal, then, according to Theorem 2, an increase in exogenous protection widens the interval where open access is optimal, implying that the two types of protection are substitutes. However, the case of $\mu_1 = \mu_0$ is special. When μ_1 is not restricted to be equal to μ_0, the exogenous information protection can be controlled in various ways – by changing only μ_1, or only μ_0, or both μ_1 and μ_0 simultaneously according to some rule. Depending on how we control the exogenous information protection, it can be either a substitute for or a complement to the protection through audience size. We demonstrate the ambiguity of the relationship between the two types of protection using two extreme cases – changing μ_1 and μ_0 separately. The main goal of this exercise is to demonstrate that the complementarity of the two types of protection stems from the dynamics of the seekers' belief updating.[17]

Suppose that exogenous protection is controlled only through μ_0. Then, the comparative statics of ρ^* depend on whether $\mu_0 < \mu_1$ or $\mu_0 > \mu_1$. If $\mu_0 < \mu_1$, then following an increase in μ_0, ρ^* increases, widening the interval where open access is optimal – thus, the two types of protection are complements. If $\mu_0 > \mu_1$, then the opposite holds and the two types of protection are substitutes. The intuition for both cases can be traced back to the speed of belief updating under minimal access. This speed is equal to $\mu_1 - \mu_0$ if $\mu_0 < \mu_1$ and $\mu_0 - \mu_1$ if $\mu_0 > \mu_1$. When the belief updating speed is exogenously reduced, the interval where open access is optimal widens because opening access increases the speed, thus counteracting the exogenous change.

Now suppose that exogenous protection is controlled only through μ_1. The intuition in the previous paragraph suggests that the comparative statics of ρ^* with respect to μ_1 should mirror the comparative statics with respect to μ_0. If $\mu_0 < \mu_1$, then an increase in μ_1 increases $\mu_1 - \mu_0$, the speed of belief updating under minimal access, and so, ρ^* is expected to decrease, shortening the interval where open access is optimal. Similarly, if $\mu_0 > \mu_1$, then an increase in μ_1 is expected to increase ρ^*. However, according to Theorem 2, the comparative statics with respect to μ_1 defy the expectations in the case of $\mu_0 > \mu_1$. The reason is that the impact of μ_1 is not limited to the speed of belief updating. An increase in μ_1 increases the probability of a compromising report in any fixed-length time interval, which makes the open access policy less attractive. Consequently, ρ^* is expected to decrease as a result of an increase in μ_1. If $\mu_0 < \mu_1$, the impact of μ_1 on ρ^* through the probability of a compromising report works in the same direction as its impact through the speed of belief updating; thus, ρ^* decreases in μ_1, implying that the two types of protection are substitutes.[18] However, if $\mu_0 > \mu_1$, the two impacts pull ρ^* in opposite directions, making the comparative statics ambiguous.

Theorem 2 also asserts that threshold ρ^* increases in the flow cost of learning c and decreases in the prior belief p_0, which is intuitive. If, from the outset, the seekers find learning more attractive – either because learning is cheap, or because they believe that the hider is more likely

[17] In reality, it is hard to image protection measures that affect only μ_0 alone. However, the ambiguity of the relationship between the two types of protection in the two extreme cases, when μ_1 and μ_0 change separately, indicates that similar ambiguity would prevail in more general cases, when both μ_1 and μ_0 change simultaneously according to some rule.

[18] If $\mu_0 < \mu_1$, μ_1 also influences ρ^* through the belief threshold $\bar{p}$. An increase in μ_1 raises the benefit of learning, thus decreasing $\bar{p}$ and widening the interval of beliefs where the seekers undertake learning. A decrease in $\bar{p}$ increases the probability that the seekers' learning is terminated as a result of the story becoming obsolete, weakening the indirect effect and making the open access policy less appealing to the hider. The described impact of μ_1 on ρ^* through the belief threshold works in the same direction as its impact through the probability of a compromising report and through the speed of belief updating.

to have something compromising to hide – then the hider is more inclined to restrict access as much as possible.

The comparative statics with respect to p_0 constitutes one of the testable predictions of our model – that governments that have less trust from their citizens are less transparent. This prediction is especially interesting because the existing empirical and experimental literature focuses mainly on the causal relationship in the opposite direction and tests whether providing citizens with more information increases their trust in the government (see, for example, Grimmelikhuijsen et al. (2013), Alessandro et al. (2021)).

3.3. Policy implications

We treat the exogenous information protection parameters μ_0 and μ_1 as policy instruments. Theorem 3 shows that the hider becomes worse off when exogenous information protection weakens, either through an increase in μ_0 or through an increase in both μ_0 and μ_1 by the same amount.[19] We restrict attention to two possibilities, $n = 1$ and $n = +\infty$, because, according to Theorem 1, only such n can be optimal for the hider.

Theorem 3. *Suppose that Assumption 1 holds, the hider chooses $n = 1$ or $n = +\infty$ and the seekers behave optimally. Then, the probability that the hider avoids a compromising report weakly decreases in μ_0, and decreases when both μ_0 and μ_1 increase by the same amount.*

The proof of Theorem 3 is in Appendix A.9 and relies on the analysis in Section 4. Here, we provide intuition for Theorem 3 in the case of $\mu_0 < \mu_1$ and $n = 1$.

An increase in μ_0 decreases the speed of learning $n\mu_1 - \mu_0$ and, thus, is equivalent to the indirect effect of a decrease in n. Hence, the hider is worse off with a higher μ_0.

In contrast, when both μ_0 and μ_1 increase by the same amount, the speed of learning $\mu_1 - \mu_0$ remains unchanged. Instead, a simultaneous increase in both μ_0 and μ_1 affects the probability that the hider avoids a compromising report in two ways. First, an increase in μ_1 increases the rate at which a compromising story is revealed to a seeker, thus emulating the direct effect of an increase in n. Second, higher μ_1 decreases the belief threshold $\bar{p}$ defined below in (16), at which the seekers give up unsuccessful learning. The decrease in $\bar{p}$ induces the seekers to undertake unsuccessful learning longer. Both effects lower the probability that the hider avoids a compromising report.

Theorem 3 encapsulates the policy implications of our model. If the society aims to hold the hider accountable – which is the case if, for instance, the hider is the government – then the policy recommendation is to promote clarity of the disclosed information. In contrast, if the society has the hider's interests at heart – which is the case if the hider is a private individual – then our model advocates strong privacy protection laws (see more on this in Section 3.4).

3.4. Other applications: privacy paradox

Taken together, Theorem 1 and Theorem 3 can explain a well-documented *privacy paradox*. According to the privacy paradox, people often claim that they value privacy highly yet behave

[19] If the exogenous information protection weakens through an increase in μ_1, then the direction of the change in the hider's welfare is ambiguous. Intuitively, an increase μ_1 may make the hider better off because it increases the speed of learning. This is the familiar indirect effect that incentivizes the hider to choose the open access policy. A detailed account of the impact of μ_1 on the hider's welfare can be found in the Supplementary Material.

as if they value it very little (Norberg et al. (2007)).[20] While Theorem 1 rationalizes the hider's potentially privacy-compromising behavior, Theorem 3 predicts that the very same hider prefers strong privacy protection.[21] At the heart of this discrepancy between behavior and attitude lies the difference between the corresponding types of privacy protection. In our model, the hider's behavior affects the size of her audience, while her attitude relates to the arrival rates, which determine how easily a single seeker can find the hider's story. In practice, the hider may choose the size of her audience either through forming social connections or through online privacy settings that, for example, limit wall post access on social networking sites. In contrast, the arrival rates are controlled, for example, by government, through privacy protection laws, or by private firms, through the encryption of data by instant messaging services such as WhatsApp. Empirical evidence supports the idea that individuals draw a distinction between increasing their own visibility and facilitating the use of already disclosed information. For example, Keith et al. (2013) documents that, while online users disclose their personal and location data to social network applications, they are outraged when an application abuses their trust and makes this information easier for other users to find.[22]

3.5. Alternative objective function for the hider

In the application that we have discussed so far, in line with the model, the hider does not want a compromising story to be revealed; thus, she minimizes the probability of a compromising report. In this section, we briefly talk about alternative applications in which the hider aims to maximize the probability of such a report. These alternative applications are leading examples in Akcigit and Liu (2016).

For example, the seekers explore a new technology that can be either good or bad. If the technology is bad (good), each seeker gets conclusive evidence that the technology is bad (good) at the learning rate μ_0 (μ_1). The first seeker who discovers that the technology is good patents it and temporarily reaps monopoly profits. In contrast, no seeker who obtains evidence that the technology is bad reports it, and, hence, such evidence remains hidden from the other seekers. The hider is a benevolent social planner who aims to maximize the probability of the discovery of a good technology.

Alternatively, the seekers are mathematicians who are trying to prove a conjecture. If the conjecture is correct, each seeker independently obtains the proof at rate μ_1. However, the conjecture may be wrong, in which case the rate of finding a counterexample is μ_0. The first seeker who

[20] The existing literature tends to explain the apparent inconsistency of attitudes and behavior either through privacy calculus or through various cognitive biases (Barth and de Jong (2017), Gerber et al. (2018), Solove (2021)). According to the privacy calculus theory, individuals rationally weigh the potential costs and benefits of information disclosure. For example, an online user's laid-back behavior in relation to her privacy settings in a mobile application could be explained by the necessity to disclose information to get other benefits from this application. We provide a novel, fully rational explanation for the privacy paradox in a setting in which privacy protection is costless and information disclosure does not generate any extraneous benefits.

[21] In light of Theorem 3, it is not surprising that the public expresses dissatisfaction with existing privacy legislation. According to a Pew Research Center survey (Auxier et al. (2019)), in 2019, three quarters of Americans said that there should be more privacy regulation. Similarly, according to Ofcom (ICO and Ofcom (2020)), in 2020, more than half of adult internet users in the UK expressed support for increased regulation across social media, video sharing and instant messaging.

[22] As an example, Keith et al. (2013) considers the *Girls Around Me* app that merged Facebook and Foursquare data and layered it over Google Maps with real-time GPS location data to show a user where the nearest single women are.

finds the proof gets the acknowledgment from the academic community; however, unless the conjecture is celebrated, a counterexample may not warrant a publication and so may go unnoticed. The hider is an editor of a mathematical journal who aims to maximize the probability of a successful proof.

In the absence of story obsolescence, maximizing the probability of a report is equivalent to maximizing (5). Since expression (5) decreases in n, $n = 1$ is optimal. This stark result is driven by the indirect effect whereby $n = 1$ maximizes the duration of unsuccessful learning. Positive obsolescence rate introduces urgency in the need for a discovery which implies that at the optimum $n > 1$ (but finite).[23]

4. Full analysis

In this section, we provide the full analysis of the game between the hider and the seekers. The game will be solved backwards, starting from the seekers' game and then proceeding to characterize the hider's optimal choice of the number of seekers n.

4.1. Equilibrium in the seekers' game

Equilibrium concept

Consider seeker i. His optimal reporting strategy is trivial. Upon uncovering the hider's story, a seeker ceases to learn. He reports the story if and only if the story is compromising and up-to-date and has not been reported yet.

Let τ^i be a random time at which the **game effectively ends** for seeker i – that is, when the story becomes obsolete, when one of the other seekers reports the hider's story, or when seeker i himself discovers the story, whichever happens first. Seeker i's strategy is a deterministic learning process $\{x_t^i \mid t \geq 0\}$ that terminates at τ^i. In what follows, we refer to any seeker i for whom $t < \tau^i$ as an **active seeker**.

Prior to τ^i, seeker i updates his **subjective belief** q_t^i that the hider's story is compromising on the basis of his learning process and his belief about the learning processes of other seekers. Let $\tilde{x}_t^j$ denote seeker i's belief about the learning intensity of an active seeker j at time t and $\tilde{X}_t^{-i} = \sum_{\substack{j=1 \\ j \neq i}}^{n} \tilde{x}_t^j$ denote seeker i's belief about the aggregate intensity of all other seekers, conditional on all of them being active. Conditional on the story being compromising, all other seekers are active, and so seeker i calculates the probability that none of the seekers learns and reports the story during an infinitesimal time interval $[t, t + \Delta)$ as $1 - \mu_1(\tilde{X}_t^{-i} + x_t^i)\Delta$. Conditional on the story being non-compromising, the probability that seeker i does not learn it over time interval $[t, t + \Delta)$ is $1 - \mu_0 x_t^i \Delta$. This probability does not depend on the belief about the learning intensity of other seekers because a non-compromising story is never reported, and so seeker i does not observe when other seekers uncover a non-compromising story. By Bayes' rule, the law of motion for the subjective belief is

$$\dot{q}_t^i = \left((\mu_0 - \mu_1)x_t^i - \mu_1 \tilde{X}_t^{-i}\right) q_t^i \left(1 - q_t^i\right). \tag{6}$$

We restrict attention to symmetric equilibria in which all seekers use the same strategy $\{x_t^* \mid t \geq 0\}$. By definition, in equilibrium, each seeker i correctly anticipates the learning processes

[23] We conjecture that, if $\rho > 0$, at the optimum, $n > 1$. We prove the conjecture for the case $\mu_0 < \mu_1$. In this case, in Appendix A.7, we show that the probability of no report has a U-shaped form with respect to n, which implies the result.

of other active seekers, and so $\tilde{X}_t^{-i} = (n-1)x_t^*$. Then, by (6), the subjective belief of an active seeker i evolves according to

$$\dot{q}_t^i = \left((\mu_0 - \mu_1)x_t^i - (n-1)\mu_1 x_t^*\right) q_t^i \left(1 - q_t^i\right). \tag{7}$$

On the equilibrium path, no seeker deviates from the equilibrium strategy, and the subjective beliefs of all active seekers coincide. Denote by p_t the **common belief** of active seekers. Then, with $x_t^i = x_t^*$, (7) gives the common belief evolution (1).

We restrict attention to Markovian strategies in which, at each t, the learning intensity x_t^i depends only on the common belief p_t and on the subjective belief q_t^i.[24] Let $x^*: [0,1] \mapsto [0,1]$ be the intensity function that maps the common belief of active seekers p_t into the intensity of learning. Let $x: [0,1]^2 \mapsto [0,1]$ be the optimal strategy of seeker i that maps his subjective belief q_t and the common belief of all other active seekers p_t into the learning intensity of seeker i, assuming that all other active seekers use an intensity function x^*. Then, the intensity function x^* is a symmetric equilibrium in Markovian strategies if and only if $x^*(p) = x(p,p)$ for all $p \in [0,1]$.

Seeker i's optimization problem

At each moment, seeker i chooses learning intensity x, which maximizes his expected payoff, taking the equilibrium learning strategy of other seekers, $x^*(p)$, as given.

Let $V(q,p)$ be the value function of seeker i; that is, his expected payoff from the optimally chosen learning strategy, given subjective belief q and common belief p. Then, the Hamilton-Jacobi-Bellman (HJB) equation for function V is

$$\rho V(q,p) = \max_{x \in [0,1]} x\mathcal{L}(q,p;V) + x^*(p)\mathcal{L}^*(q,p;V), \tag{8}$$

where

$$\mathcal{L}(q,p;V) = q\mu_1 - (q\mu_1 + (1-q)\mu_0)V(q,p) + V_1(q,p)(\mu_0 - \mu_1)q(1-q) - c, \tag{9}$$

$$\mathcal{L}^*(q,p;V) = -(n-1)\mu_1 q (V(q,p) + (1-q)V_1(q,p)) + V_2(q,p)(\mu_0 - n\mu_1)p(1-p), \tag{10}$$

and V_1 and V_2 denote the derivative of V with respect to the first and second argument, respectively. Intuitively, equation (8) states that seeker i's value V is equal to the marginal change of value due to his learning, plus the marginal change of value due to other seekers' learning, discounted by ρ. Defined in (9), $\mathcal{L}(q,p;V)$ represents seeker i's marginal benefit of learning net of his marginal cost of learning. The cost part is equal to the flow learning cost c. The benefit part is equal to the expected discrete change in the payoff, which accrues when seeker i uncovers a story – $q\mu_1(1 - V(q,p))$ for a compromising story and $(1-q)\mu_0(0 - V(q,p))$ for a non-compromising story – plus the expected rate of change in the value due to seeker i's learning. Similarly, $\mathcal{L}^*(q,p;V)$ is equal to the expected discrete change in the payoff accrued when some other seeker uncovers a compromising story, $(n-1)q\mu_1(0 - V(q,p))$, plus the expected rate of change in the value due to other seekers' learning. In the terms related to the rate of change, $((\mu_0 - \mu_1)x - (n-1)\mu_1 x^*(p))q(1-q)$ and $(\mu_0 - n\mu_1)x^*(p)p(1-p)$ come from the law of motion for the subjective belief (7) and for the common belief (1), respectively.

[24] If the story never becomes obsolete – that is, $\rho = 0$ – the restriction to Markovian strategies rules out equilibria in which, at some moment, all seekers take a collective coffee break and restart learning at a later date.

The HJB equation (8) clearly shows that seeker i's optimization problem is linear in his learning intensity x and so, for every q and p, has a corner solution $x \in \{0, 1\}$. Intuitively, $x = 1$ is optimal for high q and $x = 0$ is optimal for low q because the expected benefit of learning grows with q. Hence, we guess that the best response of seeker i takes the cutoff form: $x(q, p) = 0$ for all $q < g(p)$ and $x(q, p) = 1$ for all $q > g(p)$. On the **cutoff curve** $q = g(p)$, the seeker is indifferent between learning and not learning and, hence, could optimally choose any $x(q, p) \in [0, 1]$. The main challenge is to characterize the cutoff curve $q = g(p)$.

We show that, in equilibrium, this curve takes a simple form using the *guess and verify* approach. We guess the optimal cutoff curve and calculate the expected payoff $V(q, p)$ from the resulting learning strategy for any given priors (q, p). Then, we verify that function V is the value function of seeker i's optimization problem.

The verification procedure relies on the following lemma:

Lemma 1. *A continuous function $V : [0, 1]^2 \mapsto \mathbb{R}$ is the value function for seeker i's optimization problem if*

1. *$V(0, p) = 0$ for all $p \in [0, 1]$;*
2. *V is non-negative on $[0, 1]^2$;*
3. *V is continuously differentiable everywhere on $[0, 1]^2$, except on a set $\mathcal{M} = M \cup M_0$, where M_0 is a countable set of points, and for each $(q', p') \in M$, there exists a hyperplane $H(q, p) = 0$ and a neighborhood B around (q', p') such that V is continuously differentiable on $\{(q, p) \in B : H(q, p) < 0\}$ and on $\{(q, p) \in B : H(q, p) > 0\}$; and*
4. *for all points of differentiability, V satisfies the HJB equation (8).*

Strategy $\{x_t \mid t \geq 0\}$ is optimal if and only if for every $t > 0$, it satisfies the following two conditions:

$$\mathcal{L}(q_t, p_t; V) > 0 \quad \Rightarrow \quad x_t = 1, \tag{11}$$

$$\mathcal{L}(q_t, p_t; V) < 0 \quad \Rightarrow \quad x_t = 0, \tag{12}$$

and, moreover,

$$\rho = 0,\ x^*(p_\infty) = 0,\ x_\infty = 0 \quad \Rightarrow \quad V(q_\infty, p_\infty) = 0. \tag{13}$$

Proof. See Appendix A.1. □

It turns out that, in seeker i's optimization problem, function V might not be differentiable and that additional restrictions are needed for points of non-differentiability. In Lemma 1, condition 3 encapsulates these additional restrictions and allows us to use the notion of generalized derivative and appeal to the change-of-variable formula in Theorem 3.1 in Peskir (2007).[25]

While Lemma 1 limits the class of admissible functions through condition 3, an alternative, more standard approach to limit the class of admissible functions involves viscosity solutions. However, off-the-shelf results on viscosity solutions, such as Theorem 4.11 on page 197 in Bardi

[25] Appealing to Theorem 3.1 in Peskir (2007) may seem excessive because, while in our model, all processes are piecewise continuous, Peskir's result is valid for a large class of stochastic processes, namely semimartingales with jumps of bounded variation. Therefore, our approach may prove useful in more general settings with, e.g., a Brownian component.

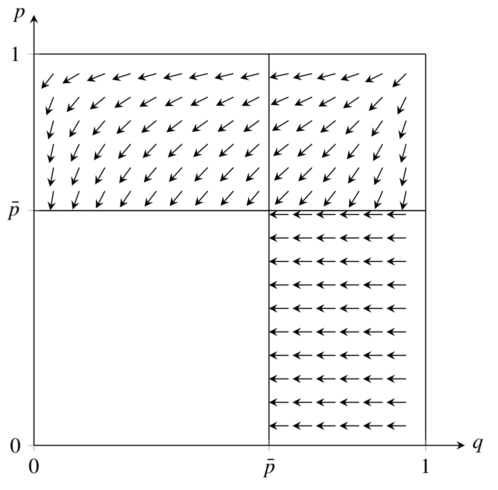

Fig. 1. Phase diagram of (1) and (7) when $\mu_0 < \mu_1$; $x^*(p)$ is defined in (14) and $x(q, p)$ is defined in (15).

and Capuzzo-Dolcetta (1997), are not directly applicable to our setting because in equilibrium, function $x^*(p)$ is discontinuous. We view Lemma 1 as a technical contribution which may prove useful in other settings with similar discontinuities.[26]

$\mu_0 < \mu_1$: growing pessimism

When $\mu_0 < \mu_1$, each active seeker who continues to learn becomes progressively more *pessimistic* that the hider's story is compromising, regardless of the learning strategy of other seekers. Formally, (7) shows that $\dot{q}_t < 0$ when $x_t = 1$, regardless of x_t^*.

We look for an equilibrium in the following form:

$$x^*(p) = \begin{cases} 1, & p > \bar{p}, \\ 0, & p < \bar{p}, \end{cases} \tag{14}$$

where all seekers learn with intensity 1 at beliefs above some threshold $\bar{p}$ and learn with intensity 0 at beliefs below that threshold. Since the seekers do not use interior intensities when they play (14), following Keller et al. (2005), we refer to (14) as an equilibrium in simple strategies. Assuming that the other seekers' behavior is described by (14), we conjecture that the best response of seeker i does not depend on belief p and has the cutoff curve $q = \bar{p}$:

$$x(q, p) = \begin{cases} 1, & q > \bar{p}, \\ 0, & q < \bar{p}. \end{cases} \tag{15}$$

Fig. 1 depicts the evolution of (q_t, p_t) when $x^*(p)$ is defined in (14) and $x(q, p)$ is defined in (15). Lemma 2 gives the optimal threshold $\bar{p}$, which is pinned down by maximizing seeker i's payoff from strategy $x(q, p)$.

[26] Escudé and Sinander (2023) face a similar discontinuity problem. In their model, the endogenous behavior of other players causes discontinuities in the player's flow payoff, while in our model, the discontinuity arises in the state evolution equation. Escudé and Sinander (2023) prove the validity of the viscosity approach in their setting; we chose a more direct method to deal with the discontinuity problem.

Lemma 2. *Suppose that $\mu_0 < \mu_1$. Then, x^* defined in (14) with*

$$\bar{p} = \frac{c}{\mu_1} \tag{16}$$

is an equilibrium. This equilibrium is the unique symmetric equilibrium in Markovian strategies.

Proof. See Appendix A.2. □

Because seeker i does not become more optimistic as a result of learning, at threshold $\bar{p}$, he must be indifferent between learning for an additional instant of time and stopping immediately. Hence, threshold $\bar{p}$ equates the flow cost of learning, c, to the expected flow benefit, given by $\bar{p}\mu_1$, the product of the instantaneous probability of uncovering a compromising story and the payoff of 1 from reporting it. Threshold $\bar{p}$ is the optimal stopping threshold for seeker i, regardless of the learning strategy of other active seekers, which is a sufficient condition for the equilibrium of the form (14) to exist.

$\mu_0 \geq n\mu_1$: growing optimism

When $\mu_0 > n\mu_1$, each active seeker who continues to learn becomes progressively more *optimistic* that the hider's story is compromising, regardless of the learning strategy of other seekers. Formally, equation (7) shows that $\dot{q}_t > 0$ when $x_t = 1$, regardless of x_t^*.

In this case, an equilibrium in simple strategies does not exist. Towards a contradiction, suppose that there exists an equilibrium in which all seekers learn with intensity 1 at any belief above some threshold and undertake no learning at any belief below this threshold. Because seeker i becomes more optimistic as a result of learning, at the threshold, seeker i must be indifferent between learning until the game ends and not learning. If other seekers do not learn, seeker i's indifference condition gives the threshold

$$\underline{p} = \frac{c\,(\rho + \mu_1)}{(\rho + \mu_0)(\mu_1 - c) + c(\rho + \mu_1)}. \tag{17}$$

However, in the conjectured equilibrium, other active seekers learn at a belief just above threshold $\underline{p}$, and so, the best response of seeker i is no learning. More specifically, if other active seekers learn, they learn with intensity 1 until the game ends because by (1), the common belief increases when $\mu_0 > n\mu_1$. Conditional on the described behavior of other active seekers, seeker i finds learning until the game ends optimal if his subjective belief is above the threshold

$$\bar{p} = \frac{c\,(\rho + n\mu_1)}{(\rho + \mu_0)(\mu_1 - c) + c(\rho + n\mu_1)}. \tag{18}$$

Threshold $\bar{p}$ reduces to threshold $\underline{p}$ when $n = 1$. When $n > 1$, threshold $\underline{p}$ is lower than threshold $\bar{p}$ because competition from other seekers, who may find a compromising story before seeker i does, reduces the expected payoff from learning. Hence, whenever $q \in \left(\underline{p}, \bar{p}\right)$, seeker i strictly prefers learning when other seekers do not learn and strictly prefers not learning when all other active seekers learn with intensity 1.

The discussion above suggests that there is no equilibrium in simple strategies. Hence, we look for an equilibrium in which, at beliefs between the thresholds $\underline{p}$ and $\bar{p}$, seekers use an interior learning intensity; that is,

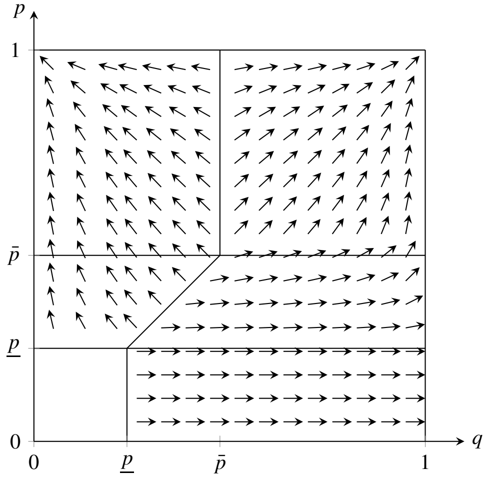

Fig. 2. Phase diagram of (1) and (7) when $\mu_0 > n\mu_1$; $x^*(p)$ is defined in (19) and $x(q,p)$ is defined in (20).

$$x^*(p) \begin{cases} = 1, & p > \bar{p}, \\ \in (0,1), & \underline{p} < p < \bar{p}, \\ = 0, & p < \underline{p}. \end{cases} \tag{19}$$

On the equilibrium path where $q = p$, the interior learning intensity $x^*(p)$ of other seekers keeps seeker i indifferent between learning and not learning on interval $(\underline{p}, \bar{p})$. Hence, for the common belief $p \in \left(\underline{p}, \bar{p}\right)$, the best response cutoff curve $q = g(p)$ for seeker i must coincide with the 45-degree line where $q = p$, as demonstrated in Fig. 2. Formally,

$$x(q,p) = \begin{cases} 1, & q > g(p), \\ 0, & q < g(p), \end{cases} \qquad g(p) = \begin{cases} \underline{p}, & p < \underline{p}, \\ p, & \underline{p} \le p \le \bar{p}, \\ \bar{p}, & p > \bar{p}. \end{cases} \tag{20}$$

This form of the best response uniquely pins down the optimal learning intensity $x^*(p)$ between the thresholds, which we derive in Lemma 3.[27]

Lemma 3. *Suppose that $\mu_0 \ge n\mu_1$. Then, x^* defined in (19) with threshold $\underline{p}$ defined in (17), threshold $\bar{p}$ defined in (18), and intermediate learning intensity*

$$x^*(p) = \frac{(1 - c/\mu_1)(p - \underline{p})}{(1 - p)(c/\mu_1 - \underline{p})}, \quad \underline{p} < p < \bar{p}, \tag{21}$$

[27] The existence of equilibrium in pure strategies (albeit with interior learning intensities) is in stark contrast with the absence of such equilibrium in the growing optimism model of Bonatti and Hörner (2017) (BH17 thereafter). In BH17, the unique symmetric equilibrium involves randomization over stopping times. The difference in the equilibrium structures arises due to the difference in players' motives. In our paper, the winner-takes-all payoff structure creates preemption fear, which is absent in BH17. Moreover, since news from others deprives seeker i from his only chance to obtain a positive payoff in the game, incentives to free-ride on learning efforts of others are muted in our model. Having preemption instead of free-riding motive lowers seeker i's incentive to backload learning, which is crucial in BH17's argument for the necessity of randomization.

is an equilibrium. This equilibrium is the unique symmetric equilibrium in Markovian strategies.

Proof. See Appendix A.3. □

Note that $x^*(p)$ from (21) belongs to [0, 1] because $\bar{p}$ from (18) is less than or equal to c/μ_1. It is intuitive that $x^*(p)$ is increasing in $p \in \left(\underline{p}, \bar{p}\right)$ and is equal to 0 at $p = \underline{p}$.

Curiously, at $p = \bar{p}$, $x^*(p)$ has a discrete jump. The size of the jump at $p = \bar{p}$ can be heuristically derived from seeker i's indifference between learning until the game ends and not learning at all at subjective belief q just below $\bar{p}$. Suppose that other seekers share seeker i's belief $p = q$; they learn with intensity x for an infinitesimally short time interval Δ and, subsequently, learn with intensity 1 until the game ends. Then, period length Δ and the initial belief q are connected through

$$\bar{p} = q + (\mu_0 - n\mu_1)xq(1-q)\Delta. \tag{22}$$

Condition (22) emerges from common belief updating equation (1) evaluated at $p = q$ and $p + \mathrm{d}p = \bar{p}$. The last condition ensures that after time interval Δ, other seekers behave as if they are on the equilibrium path and start using intensity 1 when their common belief coincides with threshold $\bar{p}$. Then, seeker i's expected payoff from learning with intensity 1 until the game ends is

$$\underbrace{(q\mu_1 - c)\Delta}_{\text{payoff in period of length }\Delta} + \underbrace{\{1 - (q\mu_1 + (1-q)\mu_0 + q(n-1)\mu_1 x + \rho)\Delta\}}_{\text{probability the game continues}} \times \underbrace{\left\{ \overbrace{\frac{q'\mu_1}{\rho + n\mu_1}}^{\substack{\text{expected payoff}\\ \text{from reporting}}} - c \overbrace{\left(\frac{q'}{\rho + n\mu_1} + \frac{1-q'}{\rho + \mu_0} \right)}^{\text{expected duration of the game}} \right\}}_{\text{continuation payoff}}, \tag{23}$$

where, by the law of motion (7),

$$q' = q + (\mu_0 - \mu_1 - (n-1)\mu_1 x)q(1-q)\Delta. \tag{24}$$

Substituting q from (22) into (23), omitting terms of order higher than Δ and equating the result to the expected payoff from not learning, which is equal to zero, we get that seeker i is indifferent between learning and not learning at belief q if and only if

$$x = \frac{(n-1)\mu_1}{\mu_0 - \mu_1}, \tag{25}$$

which coincides with (21) at $p = \bar{p}$. The difference between 1 and (25) is positive and constitutes the jump in the learning intensity at $\bar{p}$. Thus, to keep seeker i indifferent between learning until the game ends and not learning at all, the learning intensity of other seekers must have a jump as the common belief approaches $\bar{p}$.[28]

[28] Similar discontinuity in experimentation intensity also appears in Section 5 of Klein and Rady (2011) which analyzes a public experimentation game with exponential bandits of imperfectly negatively correlated types. We thank the Associate Editor for pointing this out.

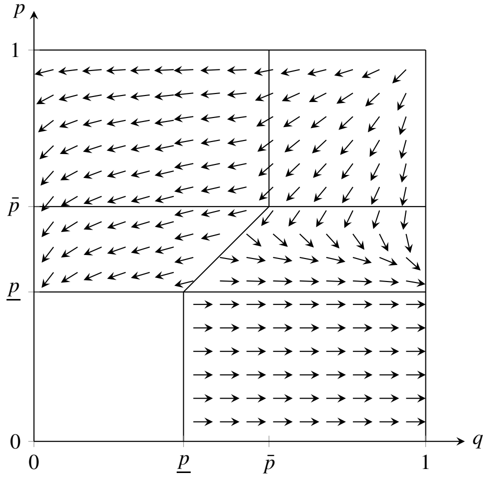

Fig. 3. Phase diagram of (1) and (7) when $\mu_1 < \mu_0 < n\mu_1$; $x^*(p)$ is defined in (19) and $x(q, p)$ is defined in (20).

$\mu_1 \le \mu_0 < n\mu_1$: mixed case

When $\mu_1 < \mu_0 < n\mu_1$, whether seeker i who learns with intensity 1 becomes progressively more optimistic or more pessimistic depends on the learning strategy of other seekers. In particular, equation (7) shows that when seeker i learns with intensity 1, $\dot{q}_t < 0$ if the intensity $x^*(p)$ is above threshold $\frac{\mu_0 - \mu_1}{(n-1)\mu_1}$ and $\dot{q}_t > 0$ if $x^*(p)$ is below that threshold.

By law of motion (1), in equilibrium, all seekers become weakly more pessimistic – that is, p weakly decreases. From the growing pessimism case, we know that with decreasing beliefs, seekers optimally learn with intensity 1 until threshold $\bar{p}$, defined in (16). Suppose that at $q = \bar{p}$, all other seekers stop learning entirely. Then, seeker i behaves as if he is alone and so would optimally learn at any $q > \underline{p}$, where $\underline{p}$ is defined in (17). In particular, seeker i strictly prefers to learn at $q = \bar{p}$ because it is greater than $\underline{p}$.

Consequently, the equilibrium in simple strategies does not exist, and we must look for an equilibrium with interior learning intensities of the form (19), with (16) replacing (18). Lemma 4 formalizes this intuition. Fig. 3 depicts the evolution of beliefs q and p in this equilibrium.[29]

Lemma 4. *Suppose that $\mu_1 \le \mu_0 < n\mu_1$. Then, x^* defined in (19) with threshold $\underline{p}$ defined in (17), threshold $\bar{p}$ defined in (16), and intermediate intensity defined in (21) is an equilibrium. This equilibrium is the unique symmetric equilibrium in Markovian strategies.*

Proof. See Appendix A.4. □

In contrast to the growing optimism case, learning intensity $x^*(p)$ does not have a jump at $p = \bar{p}$ because the common belief approaches $\bar{p}$ from above.[30]

[29] The structure of the equilibrium is very similar to the symmetric Markov perfect equilibrium in Bolton and Harris (1999), Keller et al. (2005) and Keller and Rady (2010). The similarity is not surprising: both in our model and in the public strategic experimentation games of the above-mentioned papers, the equilibrium intensity is a function of a single state variable – common belief p and a public belief, respectively.

[30] In contrast to the growing optimism case in our paper, the growing-optimism public experimentation game of Keller and Rady (2015) does not feature discontinuities in experimentation intensities. This contrast may be reconciled by

As the common belief approaches $\underline{p}$, the learning intensity $x^*(p)$ in (21) gradually diminishes to 0. The learning slows down so quickly that the common belief p does not reach threshold $\underline{p}$ in finite time, as stated in Corollary 1.

Corollary 1. *On the equilibrium path where $q = p$, starting from a prior belief above $\underline{p}$, the common belief p never reaches threshold $\underline{p}$.*

Proof. See Appendix A.5. □

Summary of the equilibrium in the seekers' game

In sum, the seekers' equilibrium behavior is characterized by learning intensity $x^*(p)$ and belief thresholds $\bar{p}$ and $\underline{p}$, described in Lemmas 2, 3 and 4. For ease of reference, we extend the definition $\underline{p}$ to ensure that this threshold is well-defined for all parameter ranges. When $\mu_0 \geq \mu_1$, threshold $\underline{p}$ is defined in (17). When $\mu_0 < \mu_1$, we define $\underline{p}$ to be equal to $\bar{p} = c/\mu_1$:

$$\underline{p} = \frac{c\,(\rho + \mu_1)}{(\rho + \max\{\mu_0, \mu_1\})(\mu_1 - c) + c(\rho + \mu_1)}. \tag{26}$$

Fig. 4 illustrates the evolution of the common belief (1) on the equilibrium path. Assumption 1 sets the lower bound of c/μ_1 on the seekers' common initial belief p_0. In the growing pessimism case, starting from p_0, the seekers learn with intensity 1 until their common belief falls to threshold $\underline{p} = \bar{p} = c/\mu_1$ where all learning ceases (see the top display in Fig. 4). In the growing optimism case, threshold $\bar{p}$, defined in (18), is less than c/μ_1; hence, the common belief drifts up from $p_0 > \bar{p}$, and so the seekers learn with intensity 1 until the game ends (see the middle display in Fig. 4). In the mixed case, the seekers start learning with intensity 1 until their common belief falls to $\bar{p} = c/\mu_1$, but they continue learning at lower intensity after $\bar{p}$ for all beliefs above $\underline{p}$ (see the bottom display in Fig. 4). As the common belief p decreases, the learning intensity $x^*(p)$ in (21) gradually diminishes to 0; by Corollary 1, p never reaches $\underline{p}$, and so no seeker ceases learning entirely before the game ends.[31]

4.2. Optimal number of seekers

The hider takes the equilibrium behavior of the seekers as given and chooses the number of seekers $n \geq 1$ to maximize the probability of avoiding the report of a compromising story, denoted by P.

Lemma 5 derives the expression for P in the growing optimism case.

Lemma 5. *Under Assumption 1, if $\mu_0 \geq n\mu_1$, then the hider avoids the report of a compromising story with probability*

$$P(\mu_1, \mu_0, c, p_0, n, \rho) = \frac{\rho}{\rho + n\mu_1}. \tag{27}$$

Proof. See Appendix A.6. □

the observation that in Keller and Rady (2015), the equilibrium intensity is nonincreasing, which means that the belief trajectory never moves from the intermediate intensity region to the full intensity region.

[31] Fig. 4 does not cover the case of $\mu_0 = n\mu_1$, in which the common belief does not change and the seekers learn with intensity 1 until the game ends. If $\mu_0 = \mu_1$ and $n > 1$, then the belief dynamics are described in the top display in Fig. 4.

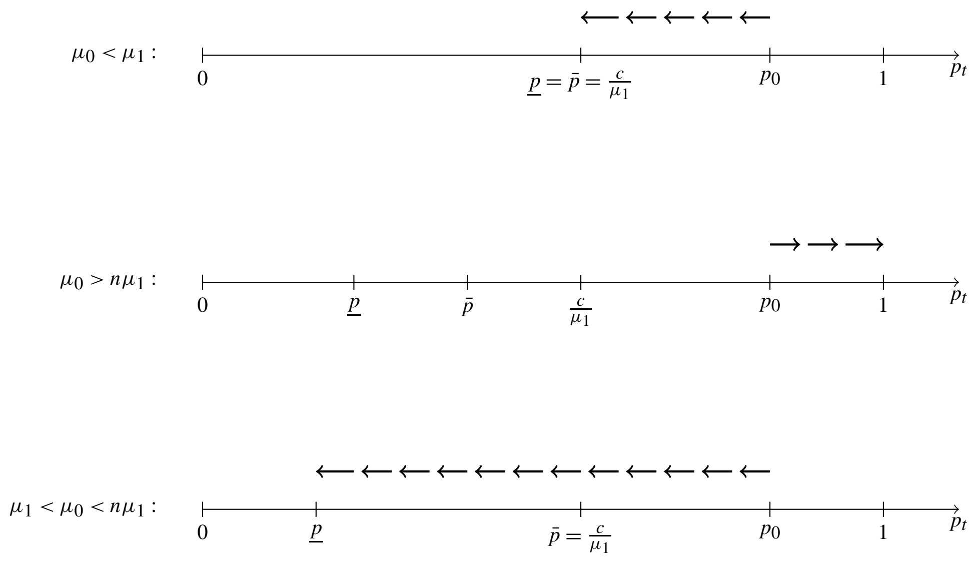

Fig. 4. Phase diagram of (1). The arrows indicate the belief trajectory that starts at p_0.

It is intuitive that P in (27) is decreasing in $1 \leq n \leq \mu_0/\mu_1$. Since, as illustrated in Fig. 4, each seeker learns with intensity 1 until his game ends, adding more seekers increases the rate at which a compromising story is revealed (direct effect) but does not affect the duration of the seekers' learning (no indirect effect).

When $\rho = 0$, P in (27) is zero. Intuitively, if the hider chooses $n \leq \mu_0/\mu_1$ and the story never becomes obsolete, the seekers never stop learning before one of them uncovers the story, and so the hider has no chance of avoiding a compromising report.

Lemma 6 derives the expression for P in the growing pessimism case.

Lemma 6. *Under Assumption 1, if $\mu_0 < \mu_1$, then the hider avoids the report of a compromising story with probability*

$$P(\mu_1, \mu_0, c, p_0, n, \rho) = \frac{\rho}{\rho + n\mu_1}\left(1 - e^{-(\rho + n\mu_1)T(\mu_1, \mu_0, c, p_0, n)}\right) + e^{-(\rho + n\mu_1)T(\mu_1, \mu_0, c, p_0, n)}, \tag{28}$$

where $T(\mu_1, \mu_0, c, p_0, n)$ is defined in (3).

Proof. See Appendix A.6. □

The hider avoids a compromising report in two cases: when the story becomes obsolete while the seekers are still learning; and when the seekers terminate unsuccessful learning while the story is still up-to-date. These cases correspond to the two terms in the right-hand side of (28). The first term in the right-hand side of (28) corresponds to interval $(\bar{p}, p_0)$, where the seekers learn with intensity 1, and is equal to the probability that the story becomes obsolete before being reported and before the common belief reaches $\bar{p}$. The second term in (28) describes the probability that the common belief reaches $\bar{p}$, where all seekers give up learning.

Equation (28) generalizes (2) to an arbitrary obsolescence rate $\rho \geq 0$. Indeed, when $\rho = 0$, the first term in (28) is equal to 0 and the second term becomes (2). Note that story obsolescence does not affect the duration of unsuccessful learning T because the stopping threshold $\bar{p}$ defined in (16) and the belief updating process (1) do not depend on ρ.

Probability P, defined in (28), depends on the number of seekers n in several ways. As discussed in the results preview on page 8, the overall effect of n can be decomposed into direct and indirect effects:

$$\frac{\partial P(\mu_1, \mu_0, c, p_0, n, \rho)}{\partial n} = Direct + Indirect. \tag{29}$$

The direct effect corresponds to the term $\rho + n\mu_1$ in (28), which is equal to the rate at which a compromising story either is found or becomes obsolete. Higher n increases the rate at which a compromising story is found, which hurts the hider. Formally, the sign of this effect is negative:

$$\begin{aligned} Direct = &-\frac{\mu_1 \rho}{(\rho + n\mu_1)^2}\left(1 - e^{-(\rho+n\mu_1)T(\mu_1,\mu_0,c,p_0,n)}\right) \\ &- e^{-(\rho+n\mu_1)T(\mu_1,\mu_0,c,p_0,n)}\left(1 - \frac{\rho}{\rho + n\mu_1}\right)\mu_1 T(\mu_1, \mu_0, c, p_0, n) < 0. \end{aligned} \tag{30}$$

The indirect effect is reflected in the speed of learning $n\mu_1 - \mu_0$ in T defined in (3). Higher n reduces the duration of unsuccessful learning T, which benefits the hider. Formally, the sign of this effect is positive:

$$Indirect = -e^{-(\rho+n\mu_1)T(\mu_1,\mu_0,c,p_0,n)} n\mu_1 \frac{\partial T(\mu_1, \mu_0, c, p_0, n)}{\partial n} > 0 \tag{31}$$

because T is decreasing in n.

In the mixed case, in which $\mu_1 \leq \mu_0 < n\mu_1$, the expression for $P(\mu_1, \mu_0, c, p_0, n, \rho)$ takes a form similar to (28). The second term, which corresponds to beliefs below $\bar{p}$, is multiplied by $P(\mu_1, \mu_0, c, \bar{p}, n, \rho)$, the probability that the hider avoids a compromising report, conditional on the common belief reaching $\bar{p}$. In the growing pessimism case, this probability is equal to 1 because the seekers stop learning at $\bar{p}$. In contrast, in the mixed case, $P(\mu_1, \mu_0, c, \bar{p}, n, \rho)$ is less than 1 because, upon reaching $\bar{p}$, the seekers continue to learn with diminishing intensity on belief interval $(\underline{p}, \bar{p})$. The general expression for $P(\mu_1, \mu_0, c, \bar{p}, n, \rho)$ is relatively complicated and relegated to Appendix A.6.

In the special case in which $\rho = 0$, the expression for $P(\mu_1, \mu_0, c, \bar{p}, n, 0)$ takes the simple form of $e^{n\mu_1 T(\mu_1,\mu_0,c,\underline{p},n)}$, where $-T(\mu_1, \mu_0, c, \underline{p}, n) > 0$ measures the cumulative intensity required to move the common belief from $\bar{p}$ to $\underline{p}$. The formula is simple because, when $\rho = 0$, $P(\mu_1, \mu_0, c, \bar{p}, n, 0)$ does not depend on the temporal allocation of the cumulative learning intensity. The overall expression for $P(\mu_1, \mu_0, c, p_0, n, 0)$ is presented in Lemma 7, which extends equation (2) to $\mu_1 \leq \mu_0 < n\mu_1$.

Lemma 7. *Under Assumption 1, if $\mu_1 \leq \mu_0 < n\mu_1$ and $\rho = 0$, then the hider avoids the report of a compromising story with probability*

$$P(\mu_1, \mu_0, c, p_0, n, 0) = e^{-n\mu_1\left(T(\mu_1,\mu_0,c,p_0,n) - T(\mu_1,\mu_0,c,\underline{p},n)\right)} \quad \text{with } \underline{p} \text{ defined in (17)}, \tag{32}$$

where $T(\mu_1, \mu_0, c, p, n)$ is defined in (3).

Proof. See Appendix A.6. □

As before, in the mixed case, the overall effect of n on $P(\mu_1, \mu_0, c, p_0, n, \rho)$ can be decomposed into direct and indirect effects, but function $P(\mu_1, \mu_0, c, \bar{p}, n, \rho)$ amends (30) and (31) with additional terms.

Theorem 1 derives the optimal number of seekers by maximizing $P(\mu_1, \mu_0, c, p_0, n, \rho)$. The direct and indirect effects of n on P work in opposite directions. As discussed in Section 3, when $\rho = 0$ and $\mu_0 < \mu_1$, the probability that the hider avoids a compromising report is increasing in n; that is, the indirect effect always takes an upper hand over the direct effect. For a positive obsolescence rate ρ, the probability P derived in Lemma 6 has a U-shape: it decreases in n for small n and increases in n for large n (the proof is in Appendix A.7). The U-shape of P leads to the bang-bang solution for the optimal number of seekers in Theorem 1.[32]

5. Discussion and concluding remarks

In our model, the hider controls the number of seekers. Each seeker is associated with one information source, and the seekers' sources are conditionally independent. Hence, by controlling the number of seekers, the hider also controls the number of information sources. In a more general model, the information sources can be conditionally correlated. For example, when all journalists talk to the same expert, their information sources are perfectly correlated – that is, they learn from the same information source. As our model assumes that the information sources are conditionally uncorrelated, it is closer to a situation in which each journalist covertly undertakes an Internet search.

In the case of private learning from perfectly correlated information sources, on the equilibrium path of a symmetric Markov equilibrium, learning is essentially public, and it is as if all seekers learn from a single source and stop learning at the same time. As the number of seekers increases, the informativeness of the source does not change. At the same time, the benefit of learning decreases in the number of seekers and is equal to $1/n$ because once the information source reveals a compromising story, the seekers toss a coin to decide who reports it and gets the payoff of 1. Hence, increasing n raises the belief threshold at which the seekers cease learning, which reduces the duration of the seekers' learning and benefits the hider. Consequently, with perfectly correlated information sources, open access – that is, $n = +\infty$ – is always optimal.

If learning is public and information sources of the seekers are conditionally independent, in a symmetric Markov equilibrium, all seekers observe when others find a non-compromising story, and it is as if all seekers learn from a single source and cease learning at the same time. Unlike the setting with perfectly correlated sources, here, the informativeness of the source increases linearly with n, and the story arrival rate is $n\mu_\theta$. At the same time, more-intense competition still reduces each seeker's benefit of learning to $1/n$. Overall, as n increases, higher informativeness of the source cancels out the reduction in the benefit of learning, making the hider indifferent to the number of seekers when $\rho = 0$ (as we argued more formally in the discussion following formula (5) in Section 3.1). When $\rho > 0$, the hider strictly prefers $n = 1$ because with lower number of seekers there is a higher chance that the story becomes obsolete before they uncover it.

In our model, the discouragement effect familiar from the contest literature does not arise. According to the discouragement effect, a higher number of seekers reduces the marginal benefit from learning, thus diminishing each seeker's incentives to learn. In a dynamic model with Poisson information sources, at every instant $[t, t + \Delta)$ before the end of the game, the seeker's

[32] In the mixed case, the proof of Theorem 1 is more direct and does not fully characterize the shape of P.

marginal benefit from learning is independent of n because it is equal to the product of the instantaneous probability of uncovering a compromising story, $q_t\mu_1\Delta$, times the payoff of 1 from reporting it. Thus, there is no discouragement effect.[33] Instead, the optimality of open access policy hinges purely on the dynamics of the seekers' belief updating. In Appendix B.2, we show that in a static analog of our model, the open access policy is also optimal, yet, in contrast to the dynamic model, there is no belief updating and the result is driven purely by the discouragement effect. Thus, even though the open access policy may be optimal both in static and in dynamic models, the forces that deliver this optimality are distinct.

An alternative way to model information protection is to allow the hider to control μ_1, the rate of arrival of a compromising story. As we briefly touched upon in Section 3.3, in addition to the direct and indirect effects extensively discussed in this paper, a change in μ_1 also affects threshold $\bar{p}$ at which the seekers cease learning. Nevertheless, the qualitative insights of our model carry over to such an alternative setting, and the hider optimally chooses protection that might be laxer than the strongest feasible.[34]

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

Appendix A. Proofs

A.1. Proof of Lemma 1

The value function describes the maximum expected payoff *conditional* on the story being up-to-date and not reported. In order to use Itô's formula (A.1), which is the key element of the proof, we need to incorporate the story *status* $\in \{irrelevant, found, \emptyset\}$ into the argument of a function. To this end, denote $W(q,p,\emptyset) = V(q,p)$; $W(q,p,found) = 1$, the payoff when seeker i finds a compromising story; $W(q,p,irrelevant) = 0$, the payoff when seeker i cannot benefit from reporting the story – that is, when either some other seeker reports a compromising story or the story becomes obsolete. We also need to introduce notions N_t^c for the Poisson process that reveals a compromising story for seeker i; N_t^{nc} for the Poisson process that reveals a non-compromising story for seeker i; N_t^p for the Poisson process that reflects public reports of other seekers; and N_t^o for the Poisson process that reveals when the story becomes obsolete. By definition, τ^i is the first time any of the four Poisson processes jumps. If no jump occurs, $\tau^i = +\infty$.

Take any initial beliefs (q_0, p_0) and any strategy $\{x_t \mid t \geq 0\}$ of seeker i. Suppose that V is continuously differentiable along the trajectory (q_t, p_t). Then, the classical Itô's formula can be applied:

[33] The absence of the discouragement effect in our model can also be seen in that the belief threshold $\underline{p}$ at which learning stops is independent of n.

[34] Details can be found in Supplementary material.

$$
\begin{aligned}
W(q_{\tau^i}, p_{\tau^i}, status_{\tau^i}) &= W(q_0, p_0, \emptyset) \\
&\quad + \int_0^{\tau^i} \frac{\partial W(q_t, p_t, \emptyset)}{\partial q} \big((\mu_0 - \mu_1)x_t - (n-1)\mu_1 x^*(p_t)\big) q_t(1-q_t)\,\mathrm{d}t \\
&+ \int_0^{\tau^i} \frac{\partial W(q_t, p_t, \emptyset)}{\partial p} (\mu_0 - n\mu_1) x^*(p_t) p_t(1-p_t)\,\mathrm{d}t + \int_0^{\tau^i} (W(0, p_t, \emptyset) - W(q_t, p_t, \emptyset))\,\mathrm{d}N_t^{nc} \\
&+ \int_0^{\tau^i} (W(1, p_t, found) - W(q_t, p_t, \emptyset))\,\mathrm{d}N_t^{c} + \int_0^{\tau^i} (W(1, p_t, irrelevant) - W(q_t, p_t, \emptyset))\,\mathrm{d}N_t^{p} \\
&+ \int_0^{\tau^i} (W(q_t, p_t, irrelevant) - W(q_t, p_t, \emptyset))\,\mathrm{d}N_t^{o} \qquad \text{(A.1)}
\end{aligned}
$$

because q_t and p_t move according to (7) and (1) as long as the story remains undiscovered. By definition of W, (A.1) is equivalent to

$$
\begin{aligned}
W(q_{\tau^i}, p_{\tau^i}, status_{\tau^i}) &= V(q_0, p_0) \\
&\quad + \int_0^{\tau^i} \frac{\partial V(q_t, p_t)}{\partial q} \big((\mu_0 - \mu_1)x_t - (n-1)\mu_1 x^*(p_t)\big) q_t(1-q_t)\,\mathrm{d}t \\
&+ \int_0^{\tau^i} \frac{\partial V(q_t, p_t)}{\partial p} (\mu_0 - n\mu_1) x^*(p_t) p_t(1-p_t)\,\mathrm{d}t + \int_0^{\tau^i} (V(0, p_t) - V(q_t, p_t))\,\mathrm{d}N_t^{nc} \\
&+ \int_0^{\tau^i} (1 - V(q_t, p_t))\,\mathrm{d}N_t^{c} + \int_0^{\tau^i} (0 - V(q_t, p_t))\,\mathrm{d}N_t^{p} + \int_0^{\tau^i} (0 - V(q_t, p_t))\,\mathrm{d}N_t^{o}. \qquad \text{(A.2)}
\end{aligned}
$$

By condition 3 of the lemma, formula (A.2) holds even if V is not continuously differentiable along the entire belief trajectory. Indeed, when the trajectory meets a point in M_0 and moves further in the belief space, it immediately leaves set M_0 because M_0 is a countable set of points. Hence, set M_0 can be ignored because V is continuous. When the trajectory (q_t, p_t) moves along set M, (A.2) holds by Theorem 3.1 in Peskir (2007), with the caveat that we write $\frac{\partial V(q,p)}{\partial q}$ and $\frac{\partial V(q,p)}{\partial p}$ for the generalized derivatives defined as

$$
\frac{\partial V(q,p)}{\partial q} := \frac{1}{2}\left(\left.\frac{\partial V(q,p)}{\partial q}\right|_{H(q,p)<0} + \left.\frac{\partial V(q,p)}{\partial q}\right|_{H(q,p)>0} \right), \qquad \text{(A.3)}
$$

$$
\frac{\partial V(q,p)}{\partial p} := \frac{1}{2}\left(\left.\frac{\partial V(q,p)}{\partial p}\right|_{H(q,p)<0} + \left.\frac{\partial V(q,p)}{\partial p}\right|_{H(q,p)>0} \right). \qquad \text{(A.4)}
$$

Taking conditional expectations and using the boundary condition 1 of the lemma, $V(0, p) = 0$, and

$$\mathbb{E}\left[\mathrm{d}N_t^{nc} \mid q_t, p_t\right] = (1-q_t)\mu_0 x_t \,\mathrm{dt}, \quad \mathbb{E}\left[\mathrm{d}N_t^{c} \mid q_t, p_t\right] = q_t \mu_1 x_t \,\mathrm{dt}, \tag{A.5}$$

$$\mathbb{E}\left[\mathrm{d}N_t^{p} \mid q_t, p_t\right] = q_t(n-1)\mu_1 x^*(p_t) \,\mathrm{dt}, \quad \mathbb{E}\left[\mathrm{d}N_t^{o} \mid q_t, p_t\right] = \rho \,\mathrm{dt}, \tag{A.6}$$

we get

$$\begin{aligned}
&\mathbb{E}\left[W(q_{\tau^i}, p_{\tau^i}, status_{\tau^i}) \mid q_0, p_0\right] = V(q_0, p_0) \\
&+ \mathbb{E}\left[\int_0^{\tau^i} \left\{ (\mathcal{L}(q_t, p_t; V) + c) x_t + \mathcal{L}^*(q_t, p_t; V) x^*(p_t) - \rho V(q_t, p_t) \right\} \mathrm{dt} \mid q_0, p_0 \right],
\end{aligned} \tag{A.7}$$

where $\mathcal{L}$ and $\mathcal{L}^*$ are given in (9) and (10), respectively, and the derivatives in $\mathcal{L}$ and $\mathcal{L}^*$ are taken in the generalized sense of (A.3) and (A.4).

The HJB equation (8) holds for all points of differentiability and, in particular, for both sides of each hyperplane of non-differentiability, $H(q, p) = 0$. Hence, (A.7) becomes

$$\begin{aligned}
V(q_0, p_0) = &\ \mathbb{E}\left[W(q_{\tau^i}, p_{\tau^i}, status_{\tau^i}) - c \int_0^{\tau^i} x_t \,\mathrm{dt} \mid q_0, p_0 \right] \\
&+ \frac{1}{2}\mathbb{E}\left[\int_0^{\tau^i} \left\{ \left(\max_{x\in[0,1]} x\mathcal{L}_-(q_t, p_t; V) \right) - x_t \mathcal{L}_-(q_t, p_t; V) \right\} \mathrm{dt} \mid q_0, p_0 \right] \\
&+ \frac{1}{2}\mathbb{E}\left[\int_0^{\tau^i} \left\{ \left(\max_{x\in[0,1]} x\mathcal{L}_+(q_t, p_t; V) \right) - x_t \mathcal{L}_+(q_t, p_t; V) \right\} \mathrm{dt} \mid q_0, p_0 \right],
\end{aligned} \tag{A.8}$$

where $\mathcal{L}_-$ and $\mathcal{L}_+$ are equal to $\mathcal{L}$ with the derivative $\frac{\partial V(q,p)}{\partial q}$ taken from the $H(q, p) < 0$ side and from the $H(q, p) > 0$ side of the hyperplane, respectively.

The second and the third terms of (A.8) are non-negative for an arbitrary strategy $\{x_t \mid t \geq 0\}$ and equal to 0 if and only if $\{x_t \mid t \geq 0\}$ satisfies conditions (11) and (12).

The first term of (A.8),

$$\mathbb{E}\left[W(q_{\tau^i}, p_{\tau^i}, status_{\tau^i}) - c \int_0^{\tau^i} x_t \,\mathrm{dt} \mid q_0, p_0 \right], \tag{A.9}$$

is greater than or equal to the expected payoff from strategy $\{x_t \mid t \geq 0\}$, with equality achieved if and only if the strategy satisfies condition (13). If seeker i's game ends with $status_{\tau^i} \in \{irrelevant, found\}$, then, by construction, $W(q_{\tau^i}, p_{\tau^i}, status_{\tau^i})$ is equal to seeker i's payoff at τ^i. Otherwise, seeker i's game ends with $status_{\tau^i} = \emptyset$, and there are two options.

First, the game could end due to seeker i finding a non-compromising story, in which case his subjective belief q jumps to 0. By the boundary condition 1 of the lemma, $W(q_{\tau^i}, p_{\tau^i}, \emptyset) = V(0, p_{\tau^i}) = 0$ is seeker i's payoff at τ^i.

Second, the game could last forever – that is, $\tau^i = +\infty$. In that case, seeker i's payoff at τ^i is 0, which is less than or equal to $V(q_{\tau^i}, p_{\tau^i})$ by condition 2 of the lemma. Hence, (A.9) is greater than or equal to the expected payoff from strategy $\{x_t \mid t \geq 0\}$. For the strategy $\{x_t \mid t \geq 0\}$ that satisfies condition (13), the equality is achieved because $V(q_{\tau^i}, p_{\tau^i}) = 0$ with probability 1 and $V(q_{\tau^i}, p_{\tau^i})$ is bounded (as it is a continuous function on a compact domain $[0, 1]^2$).

To see that $V(q_{\tau^i}, p_{\tau^i}) = 0$ with probability 1, consider four cases. First, $\rho > 0$. Then $\tau^i = +\infty$ is a zero-probability event because the time of the first jump of the Poisson process N_t^o is finite with probability 1. Second, seeker i never stops learning entirely – that is, $\int_0^{\tau^i} x_t \, \mathrm{d}t = +\infty$. This is a zero-probability event because the time of the first jump of the compound Poisson process $N_t^c + N_t^{nc}$ is finite with probability 1. In the remaining two cases, $\int_0^{\tau^i} x_t \, \mathrm{d}t < +\infty$, which implies that $x_{\tau^i} = 0$. If $x^*(p_{\tau^i}) > 0$, then by evolution of $\dot{q}_t$, (7), seeker i's subjective belief q_t converges to 0 – that is, $q_{\tau^i} = 0$ – and by the boundary condition 1 of the lemma, $V(0, p_{\tau^i}) = 0$. If $x^*(p_{\tau^i}) = 0$, then $V(q_{\tau^i}, p_{\tau^i}) = 0$ by condition (13).

In sum, $V(q_0, p_0)$ is greater than or equal to the expected payoff from strategy $\{x_t \mid t \geq 0\}$, with equality achieved if and only if the strategy satisfies conditions (11)-(13). In other words, V is the value function and $\{x_t \mid t \geq 0\}$ is the optimal strategy if and only if it satisfies conditions (11)-(13).

A.2. *Proof of Lemma 2*

A.2.1. *Construction*

Assuming that the other seekers' behavior $x^*(p)$ is described by (14), we conjecture that the best response of seeker i $x(q, p)$ is defined in (15). Then, seeker i's expected payoff function is 0 for $q \leq \bar{p}$ and satisfies the HJB equation

$$\rho V(q, p) = \mathcal{L}(q, p; V) + x^*(p)\mathcal{L}^*(q, p; V) \tag{A.10}$$

for $q > \bar{p}$. Equation (A.10) gives us the payoff function up to a univariate function, which we calculate from the continuity of the payoff function along the belief trajectory.

If $q > \bar{p} \geq p$, then $x^*(p) = 0$ and (A.10) becomes

$$\rho V(q, p) = \mathcal{L}(q, p; V). \tag{A.11}$$

The family of solutions to the differential equation (A.11) is

$$V(q, p) = \frac{q(\mu_1 - c)}{\mu_1 + \rho} - \frac{c(1-q)}{\mu_0 + \rho} + q\left(\frac{1-q}{q}\right)^{\frac{\rho+\mu_1}{\mu_1-\mu_0}} W_1(p), \tag{A.12}$$

where W_1 is some arbitrary function. Fig. 1 shows that the relevant belief trajectory leads to the line $q = \bar{p}$, at which the payoff function is equal to 0. Hence, W_1 in (A.12) can be pinned down by the continuity of the payoff function along $q = \bar{p}$:

$$V(q, p) = \frac{q(\mu_1 - c)}{\mu_1 + \rho} - \frac{c(1-q)}{\mu_0 + \rho} + \frac{q}{\bar{p}}\left(\frac{\bar{p}(1-q)}{(1-\bar{p})q}\right)^{\frac{\rho+\mu_1}{\mu_1-\mu_0}}\left(\frac{c(1-\bar{p})}{\mu_0 + \rho} - \frac{\bar{p}(\mu_1 - c)}{\mu_1 + \rho}\right). \tag{A.13}$$

Maximizing (A.13) with respect to $\bar{p}$ yields (16).

If $q > \bar{p}$ and $p > \bar{p}$, then $x^*(p) = 1$ and (A.10) becomes

$$\rho V(q, p) = \mathcal{L}(q, p; V) + \mathcal{L}^*(q, p; V). \tag{A.14}$$

The family of solutions to the differential equation (A.14) is

$$V(q,p)=\frac{q(\mu_1-c)}{n\mu_1+\rho}-\frac{c(1-q)}{\rho+\mu_0}+q\left(\frac{1-q}{q}\right)^{\frac{\rho+n\mu_1}{n\mu_1-\mu_0}}W_2\left(\frac{(1-p)q}{p(1-q)}\right), \tag{A.15}$$

where W_2 is an arbitrary function.[35]

If $p \geq q > \bar{p}$, then the relevant belief trajectory leads to the line $q=\bar{p}$, at which the payoff function is equal to 0. Hence, W_2 in (A.15) is pinned down by the continuity of the payoff function along $q=\bar{p}$:

$$V(q,p)=\frac{q(\mu_1-c)}{n\mu_1+\rho}-\frac{c(1-q)}{\mu_0+\rho}+\frac{q}{\bar{p}}\left(\frac{\bar{p}(1-q)}{(1-\bar{p})q}\right)^{\frac{\rho+n\mu_1}{n\mu_1-\mu_0}}\left(\frac{c(1-\bar{p})}{\rho+\mu_0}-\frac{\bar{p}(\mu_1-c)}{n\mu_1+\rho}\right). \tag{A.16}$$

Maximizing (A.16) with respect to $\bar{p}$ yields (16).

If $q > p > \bar{p}$, then the relevant belief trajectory leads to line $p=\bar{p}$, at which the payoff function is defined in (A.13). Hence, W_2 in (A.15) is pinned down by continuity along $p=\bar{p}$:

$$\begin{aligned}V(q,p)=&\frac{q(\mu_1-c)}{n\mu_1+\rho}-\frac{c(1-q)}{\mu_0+\rho}+\frac{q}{\bar{p}}\left(\frac{\bar{p}(1-p)}{(1-\bar{p})p}\right)^{\frac{\rho+n\mu_1}{n\mu_1-\mu_0}}\\&\times\left(\frac{(n-1)\mu_1(\mu_1-c)\bar{p}}{(n\mu_1+\rho)(\mu_1+\rho)}+\left(\frac{p(1-q)}{(1-p)q}\right)^{\frac{\rho+\mu_1}{\mu_1-\mu_0}}\left(\frac{c(1-\bar{p})}{\rho+\mu_0}-\frac{\bar{p}(\mu_1-c)}{\mu_1+\rho}\right)\right).\end{aligned} \tag{A.17}$$

A.2.2. Verification

Function $V(q,p)$ is equal to 0 for $q \leq \bar{p}$; it is defined in (A.13) for $q > \bar{p} \geq p$, in (A.16) for $p \geq q > \bar{p}$, and in (A.17) for $q > p > \bar{p}$; threshold $\bar{p}$ is defined in (16). To prove that this is the value function and that strategy (15) is optimal, it is sufficient to verify conditions in Lemma 1.

Function $V(q,p)$ is continuous by construction and can be verified to be continuously differentiable everywhere except on the line $p=\bar{p}$.[36] Hence, condition 3 of Lemma 1 holds with M_0 being the empty set and M containing all points of the line $p=\bar{p}$.

By construction, $V(q,p)$ is equal to 0 where $x=0$ – that is, for $q \leq \bar{p}$. This implies condition 1 of Lemma 1 and condition (13).

The non-negativity condition 2 of Lemma 1 holds because $V(q,p)=0$ for $q \leq \bar{p}$, $V(q,p)$ is continuously differentiable with respect to $q \geq \bar{p}$, $V(\bar{p},p)=0$, $V_1(\bar{p},p)=0$, and the second derivative is positive for all $q > \bar{p}$:

$$V_{11}(q,p)=\left(\frac{c(1-q)}{q(\mu_1-c)}\right)^{\frac{\rho+\mu_1}{\mu_1-\mu_0}}\frac{\mu_1-c}{(1-q)^2q(\mu_1-\mu_0)}>0 \quad \text{for } p<\bar{p}<q, \tag{A.18}$$

$$V_{11}(q,p)=\left(\frac{c(1-q)}{q(\mu_1-c)}\right)^{\frac{\rho+n\mu_1}{n\mu_1-\mu_0}}\frac{\mu_1-c}{(1-q)^2q(n\mu_1-\mu_0)}>0 \quad \text{for } \bar{p}<q<p, \tag{A.19}$$

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35 The argument $\frac{(1-p_t)q_t}{p_t(1-q_t)}$ of W_2 does not change with time t. This can be verified using (1) and (7). Hence, $W_2\left(\frac{(1-p)q}{p(1-q)}\right)$ is indeed constant along any belief trajectory.

36 Differentiability along $q=\bar{p}$ follows because $\bar{p}$ is optimally chosen; differentiability along the diagonal follows from direct computations.

$$V_{11}(q,p)=\left(\frac{p(1-q)}{q(1-p)}\right)^{\frac{\rho+\mu_1}{\mu_1-\mu_0}}\left(\frac{c(1-p)}{p(\mu_1-c)}\right)^{\frac{\rho+n\mu_1}{n\mu_1-\mu_0}}\frac{\mu_1-c}{(1-q)^2q(\mu_1-\mu_0)}>0 \quad \text{for } \bar{p}<p<q. \tag{A.20}$$

It remains to establish that $V(q,p)$ satisfies the HJB equation (8) and that $x(q,p)$ satisfies (11) and (12). By construction, $V(q,p)$ satisfies

$$\rho V(q,p)=x(q,p)\mathcal{L}(q,p;V)+x^*(p)\mathcal{L}^*(q,p;V), \tag{A.21}$$

which becomes the HJB equation when $x(q,p)$ satisfies (11) and (12). Conditions (11) and (12) hold because $\mathcal{L}(q,p;V)$ is negative when $x(q,p)=0$ – i.e., $q<\bar{p}$ – and positive when $x(q,p)=1$ – i.e., $q>\bar{p}$. The negativity of $\mathcal{L}(q,p;V)$ when $q<\bar{p}$ is easily verified as $\bar{p}=\frac{c}{\mu_1}$ and

$$\mathcal{L}(q,p;V)=q\mu_1-c<0 \quad \text{for } q<\frac{c}{\mu_1}. \tag{A.22}$$

To see that $\mathcal{L}(q,p;V)$ is positive for $q>\bar{p}$, note that the sign of $\mathcal{L}$ is the same as the sign of $\mathcal{L}/q$. Function $\mathcal{L}/q$ is continuous with respect to q because V is continuously differentiable with respect to q. Furthermore, $\mathcal{L}/q$ is equal to 0 at $q=\bar{p}$ and is increasing for $q>\bar{p}$ because

$$\frac{\partial}{\partial q}\left(\frac{\mathcal{L}(q,p;V)}{q}\right)=\frac{c}{q^2(\rho+\mu_0)}(\rho-L), \quad \text{where} \tag{A.23}$$

$$L=\rho\left(\frac{\bar{p}(1-p)}{p(1-\bar{p})}\right)^{\frac{\rho+\mu_0}{n\mu_1-\mu_0}}\left(\frac{p(1-q)}{q(1-p)}\right)^{\frac{\rho+\mu_0}{\mu_1-\mu_0}}<\rho, \quad \bar{p}<p<q; \tag{A.24}$$

$$L=\left(\rho-\frac{(n-1)\mu_1(\rho+\mu_0)}{n\mu_1-\mu_0}\right)\left(\frac{\bar{p}(1-q)}{q(1-\bar{p})}\right)^{\frac{\rho+\mu_0}{n\mu_1-\mu_0}}<\rho, \quad \bar{p}<q<p; \tag{A.25}$$

$$L=\rho\left(\frac{\bar{p}(1-q)}{q(1-\bar{p})}\right)^{\frac{\rho+\mu_0}{\mu_1-\mu_0}}<\rho, \quad p<\bar{p}<q. \tag{A.26}$$

A.2.3. Uniqueness

We first derive an upper bound on seeker i's equilibrium payoff $V(q,p)$. The expected payoff of each seeker is bounded above by the payoff in the seekers' game in the absence of competition, that is, when $x_t^*=0$ for all t. Indeed, while a public report of another seeker informs seeker i about the story type, it also ends the game depriving seeker i of any positive payoff. Formally, for fixed strategies of seekers, the payoff of seeker i is equal to

$$q\int_0^{+\infty}\exp\left(-\rho t-(n-1)\mu_1\int_0^t x_\tau^*\,\mathrm{d}\tau-\mu_1\int_0^t x_\tau\,\mathrm{d}\tau\right)(\mu_1-c)\,x_t\,\mathrm{d}t$$
$$-(1-q)c\int_0^{+\infty}\exp\left(-\rho t-\mu_0\int_0^t x_\tau\,\mathrm{d}\tau\right)x_t\,\mathrm{d}t, \tag{A.27}$$

which is decreasing in other seekers' learning intensity x_t^*.

In the growing pessimism case, if $x^*(p)=0$ for all p, then seeker i's value function is equal to (A.13) for $q>\bar{p}$ (with $\bar{p}$ defined in (16)) and 0 for $q<\bar{p}$, which can be verified using Lemma 1. Thus, $V(q,p)$ is bounded above by 0 for all $q<\bar{p}$ and (A.13) for all $q>\bar{p}$.

Using the derived upper bound on $V(q, p)$, we first prove that in any equilibrium, $x^*(p) = 0$ for all $p < \bar{p}$. We then show the uniqueness of equilibrium for $p > \bar{p}$.

While for $q < \bar{p}$, $V(q, p)$ is bounded above by 0, $V(q, p)$ is also trivially bounded below by 0 because seeker i can guarantee himself this payoff by undertaking no learning. Thus, $V(q, p) = 0$ for all $q < \bar{p}$.

We argue that $V(q, p) = 0$ for all $q < \bar{p}$ implies that $x(q, p) = 0$ for all $q < \bar{p}$, which, in turn, implies that, in any equilibrium, $x^*(p) = 0$ for all $p < \bar{p}$. Towards a contradiction, suppose that starting at some $q < \bar{p}$, seeker i plans to learn for nontrivial amount of time. Then, as we can see from (A.27), $V(q, p)$ increases in q: as q increases, even if seeker i does not optimally adjust his strategy, his expected payoff (A.27) increases because both integrals in (A.27) are positive. However, this contradicts that $V(q, p) = 0$ for all $q < \bar{p}$.

We now turn to the region where $p > \bar{p}$.

If $x^*(p) = 0$ for some p, then the common belief does not move and so, since we focus on equilibria in Markovian strategies, x_t^* remains equal to 0 for the duration of seeker i's game. Hence, for seeker i the relevant part of the state space shrinks to a line on which p is constant. Along this line, seeker i's value function is equal to (A.13) for $q > \bar{p}$ and 0 for $q < \bar{p}$, which can be verified using Lemma 1. Thus, $x(p, p) = 0$ only if $p < \bar{p}$ and so, in any equilibrium, $x^*(p)$ must be positive for all $p > \bar{p}$.

We now argue that the value of $x^*(p)$ for $p > \bar{p}$ can be uniquely pinned down. The argument follows backward induction, moving from lower to higher beliefs.[37]

Assume that the uniqueness of $x^*(p)$ has already been proved for all $p \leq \tilde{p}$ for some $\tilde{p} \geq \bar{p}$. In region $p \leq \tilde{p}$, the expression for V is as derived in Section A.2.1 because p is non-increasing, which means that for seeker i the relevant part of the state space shrinks to the region $p \leq \tilde{p}$. Consider p just above $\tilde{p}$. Since $x^*(p) > 0$, by (1), the belief trajectory immediately travels into the region where $p \leq \tilde{p}$. Thus, at p just above $\tilde{p}$, the optimal best response of seeker i is defined by the sign of $\mathcal{L}(q, p; V)$ evaluated at V defined as in Section A.2.1. At such V, as we showed in Section A.2.2, $\mathcal{L}(q, p; V) > 0$ for all $q > \bar{p}$. Thus, at p just above $\tilde{p}$, the unique best response of seeker i is $x(q, p) = 1$ for $q > \bar{p}$, which means that $x(p, p) = x^*(p) = 1$, in any equilibrium.

A.3. *Proof of Lemma 3*

A.3.1. *Construction*

We conjecture that the seeker i's best response to (19) is (20). Then, seeker i's expected payoff function is $V(q, p) = 0$ when $q \leq g(p)$.

If $q > \bar{p}$ and $p > \bar{p}$, then $x^*(p) = x(q, p) = 1$ and $V(q, p)$ satisfies (A.14). If $\mu_0 > n\mu_1$, then the family of solutions is given in (A.15), and any belief trajectory leads to point $q = p = 1$, where the payoff function must be bounded. The boundedness of the payoff function uniquely determines $W_2\left(\frac{(1-p)q}{p(1-q)}\right) = 0$, and (A.15) becomes

$$V(q, p) = \frac{q(\mu_1 - c)}{n\mu_1 + \rho} - \frac{c(1-q)}{\rho + \mu_0}. \tag{A.28}$$

If $\mu_0 = n\mu_1$, then q_t and p_t do not change over time and the unique solution to (A.14) is also given by (A.28). If $\bar{p}$ is defined in (18), then $V(\bar{p}, p) = 0$, and $V(q, p)$ is continuous along the line $q = \bar{p}$ for $p > \bar{p}$.

[37] A similar argument is used, for example, in Keller and Rady (2015).

If $q > \underline{p}$ and $p < \underline{p}$, then $x^*(p) = 0$, $x(q, p) = 1$ and $V(q, p)$ satisfies (A.11). If $\mu_0 > \mu_1$, then the family of solutions is given in (A.12), and any belief trajectory leads to $q = 1$, where the payoff function must be bounded. The boundedness of the payoff function uniquely determines $W_1(p) = 0$, and (A.12) becomes

$$V(q, p) = \frac{q(\mu_1 - c)}{\mu_1 + \rho} - \frac{c(1-q)}{\rho + \mu_0}. \tag{A.29}$$

If $\mu_0 = \mu_1$, then q_t and p_t do not change over time, and so the unique solution to (A.11) is also given by (A.29). If $\underline{p}$ is defined in (17), then $V\left(\underline{p}, p\right) = 0$, and $V(q, p)$ is continuous along the line $q = \underline{p}$ for $p < \underline{p}$.

If $\underline{p} < p < \bar{p}$ and $p < q$, then $x(q, p) = 1$ and $V(q, p)$ satisfies (A.10). For an arbitrary intensity $x^*(p)$, the solution to the differential equation (A.10) is not immediately obvious; hence, we use a workaround.

Any belief trajectory from a point (q_0, p_0) such that $\underline{p} < p_0 < \bar{p}$ and $p_0 < q_0$ leads to the line $p = \bar{p}$. Let T be the time at which the relevant belief trajectory reaches the line $p = \bar{p}$. Let $W(t, q_0, p_0)$ be the expected payoff at time $t \in [0, T]$, given the starting point $q = q_0$ and $p = p_0$.

The time-domain analog of the HJB equation (8) gives us the differential equation for $W(t, q_0, p_0)$:

$$c = q_t\mu_1 + W_t(t, q_0, p_0) - (q_t\mu_1 + (1-q_t)\mu_0 + q_t(n-1)\mu_1 x^*(p_t) + \rho)W(t, q_0, p_0), \tag{A.30}$$

where $W_t(t, q_0, p_0)$ denotes the derivative with respect to the time argument. The law of motion for the subjective belief (7) becomes

$$\dot{q}_t = \left(\mu_0 - \mu_1 - (n-1)\mu_1 x^*(p_t)\right) q_t (1 - q_t), \tag{A.31}$$

which allows us to express $x^*(p_t)$ in terms of $\dot{q}_t$ and q_t. Substituting the resulting expression into (A.30) yields

$$c = q_t\mu_1 + W_t(t, q_0, p_0) - \left(\rho + \mu_0 - \frac{\dot{q}_t}{1 - q_t}\right) W(t, q_0, p_0). \tag{A.32}$$

The solution to the differential equation (A.32) is

$$W(t, q_0, p_0) = e^{(\rho+\mu_0)t}(1 - q_t)\left(\frac{W(T, q_0, p_0)}{e^{(\rho+\mu_0)T}(1 - q_T)} + \int_t^T \frac{q_\tau\mu_1 - c}{e^{(\rho+\mu_0)\tau}(1 - q_\tau)}\,\mathrm{d}\tau\right). \tag{A.33}$$

The law of motion for the subjective belief (A.31) gives

$$q_t = \frac{q_0}{q_0 + (1 - q_0)e^{(n-1)\mu_1 X(t, p_0) + (\mu_1 - \mu_0)t}}, \quad t \in [0, T], \tag{A.34}$$

where

$$X(t, p_0) = \int_0^t x^*(p_\tau)\,\mathrm{d}\tau \tag{A.35}$$

is the total learning intensity on $[0,t]$, given the initial belief p_0. The law of motion for the common belief (1) gives

$$p_t = \frac{p_0}{p_0 + (1-p_0)e^{(n\mu_1-\mu_0)X(t,p_0)}}, \quad t \in [0,T]. \tag{A.36}$$

By construction, we have $p_T = \bar{p}$, and (A.36) gives us the total learning intensity on $[0,T]$:

$$X(T,p_0) = \frac{1}{\mu_0 - n\mu_1} \ln\left(\frac{\bar{p}(1-p_0)}{(1-\bar{p})p_0}\right). \tag{A.37}$$

Substituting (A.37) into (A.34) for $t = T$ gives us the subjective belief at moment T:

$$q_T = \frac{q_0}{q_0 + e^{(\mu_1-\mu_0)T}(1-q_0)\left(\frac{\bar{p}(1-p_0)}{(1-\bar{p})p_0}\right)^{\frac{(n-1)\mu_1}{\mu_0-n\mu_1}}}. \tag{A.38}$$

Continuity along the line $p = \bar{p}$, together with (A.28), implies that

$$W(T,q_0,p_0) = \frac{q_T(\mu_1-c)}{n\mu_1+\rho} - \frac{c(1-q_T)}{\rho+\mu_0}. \tag{A.39}$$

Substituting (A.39) and (A.38) into (A.33) and using (A.34) for q_τ inside the integral, we get

$$W(0,q_0,p_0) = q_0(\mu_1-c)\left(\frac{e^{-(\rho+\mu_1)T}}{\rho+n\mu_1}\left(\frac{(1-\bar{p})p_0}{\bar{p}(1-p_0)}\right)^{\frac{(n-1)\mu_1}{\mu_0-n\mu_1}} + \int_0^T e^{-(\rho+\mu_1)\tau-(n-1)\mu_1X(\tau,p_0)}\,\mathrm{d}\tau\right) - \frac{c(1-q_0)}{\rho+\mu_0}. \tag{A.40}$$

Since $V(q,p) = 0$ when $q \le g(p)$, continuity on the diagonal requires that $W(0,p_0,p_0) = 0$ for every $p_0 \in \left(\underline{p},\bar{p}\right)$. Hence, $\frac{\mathrm{d}W(0,p,p)}{\mathrm{d}p} = 0$, which allows us to pin down the equilibrium intensity $x^*(p)$.

To derive $\frac{\mathrm{d}W(0,p,p)}{\mathrm{d}p}$, note that time T is a function of p_0, and the law of motion for the common belief (1) gives

$$\frac{\mathrm{d}T(p_0)}{\mathrm{d}p_0} = -\frac{1}{(\mu_0-n\mu_1)x^*(p_0)p_0(1-p_0)}. \tag{A.41}$$

By definition (A.35),

$$X(t,p_0) = X(\Delta,p_0) + X(t-\Delta,p_\Delta) \tag{A.42}$$

for any $\Delta \in [0,t]$. As the left-hand side of (A.42) is independent of Δ, the derivative of the right-hand side of (A.42) with respect to Δ must be equal to zero. Hence, setting $\Delta \to 0$, and substituting $X_t(0,p_0) = x^*(p_0)$ and $\dot{p}_0$ from (1), we get the derivative of $X(t,p_0)$ with respect to p_0:

$$\frac{\partial X(t,p_0)}{\partial p_0} = \frac{X_t(t,p_0) - x^*(p_0)}{(\mu_0-n\mu_1)x^*(p_0)p_0(1-p_0)}. \tag{A.43}$$

Differentiating (A.40) and using (A.41), (A.43) and (A.37), we get

$$\frac{\mathrm{d}W(0,p,p)}{\mathrm{d}p} = \frac{\rho+\mu_1+(\mu_0-\mu_1-p(\mu_0-n\mu_1))x^*(p)}{(\mu_0-n\mu_1)x^*(p)p(1-p)}W(0,p,p) + \frac{c(\mu_0-\mu_1)}{p(\rho+\mu_0)(\mu_0-n\mu_1)}\left(1-\frac{(p\mu_0+\rho)(\mu_1-c)-(1-p)\mu_1(c+\rho)}{c(1-p)(\mu_0-\mu_1)x^*(p)}\right). \quad \text{(A.44)}$$

Substituting $W(0,p,p) = \frac{\mathrm{d}W(0,p,p)}{\mathrm{d}p} = 0$ into (A.44) gives us

$$x^*(p) = \frac{(p\mu_0+\rho)(\mu_1-c)-(1-p)\mu_1(c+\rho)}{c(1-p)(\mu_0-\mu_1)}, \quad \text{(A.45)}$$

which is equivalent to (21).

For any given initial beliefs (q_0, p_0) in the middle region $\underline{p} < p_0 < \bar{p}$, $q_0 > p_0$, the expected payoff $V(q_0, p_0)$ is equal to the right-hand side of (A.40). Time T is completely determined by the motion of the common belief and, therefore, does not depend on the subjective belief q_0. Hence, $V(q_0, p_0)$ is a linear function of q_0:

$$V(q,p) = qF(p) + G(p). \quad \text{(A.46)}$$

Substituting (A.46) into (A.10) gives us the differential equations for $F(p)$ and $G(p)$:

$$c + (\rho+\mu_0)G(p) = (\mu_0-n\mu_1)x^*(p)(1-p)pG'(p), \quad \text{(A.47)}$$

$$\left(\rho+\mu_1+(n-1)\mu_1x^*(p)\right)(F(p)+G(p)) - (\rho+\mu_0)G(p) - \mu_1 = (\mu_0-n\mu_1)x^*(p)(1-p)pF'(p). \quad \text{(A.48)}$$

The expected payoff along the line $p = \bar{p}$ is defined in (A.28), which gives us the boundary conditions:

$$F(\bar{p}) = \frac{c}{\rho+\mu_0} + \frac{\mu_1-c}{\rho+n\mu_1}, \quad G(\bar{p}) = -\frac{c}{\rho+\mu_0}. \quad \text{(A.49)}$$

Given the boundary condition (A.49) for $G(\bar{p})$, the unique solution to the differential equation (A.47) is

$$G(p) = -\frac{c}{\rho+\mu_0}, \quad \underline{p} < p < \bar{p}. \quad \text{(A.50)}$$

Given (A.50), the boundary condition (A.49) for $F(\bar{p})$, and the expression (18) for $\bar{p}$, we uniquely identify function $F(p)$:

$$F(p) = \frac{c}{p(\rho+\mu_0)}, \quad \underline{p} < p < \bar{p}. \quad \text{(A.51)}$$

Not surprisingly, $F(p) = -G(p)/p$ because $V(p,p) = W(0,p) = 0$ by construction of the learning intensity (A.45).[38] In sum,

$$V(q,p) = \frac{c(q-p)}{p(\rho+\mu_0)}. \quad \text{(A.52)}$$

[38] An alternative way to get the equilibrium intensity is to use condition $F(p) = -G(p)/p$. Substituting $F(p) = -G(p)/p$ and $F'(p) = G(p)/p^2 - G'(p)/p$ into (A.48) and comparing it with (A.47), we get $G(p) = \frac{\mu_1 p - c}{(\mu_0-\mu_1)(1-x^*(p))(1-p)}$. Substituting this expression for $G(p)$ into (A.47) gives us the differential equation for $x^*(p)$. The boundary condition for $x^*(\bar{p})$ comes from (A.49). It is straightforward to verify that (A.45) satisfies this boundary condition and the derived differential equation for $x^*(p)$.

A.3.2. Verification

Function $V(q,p)$ is equal to 0 for $q \le g(p)$; it is defined in (A.28) when $q > \bar{p}$ and $p > \bar{p}$, in (A.29) when $q > \underline{p}$ and $p < \underline{p}$, and in (A.52) when $\underline{p} < p < \bar{p}$ and $p < q$; thresholds $\underline{p}$ and $\bar{p}$ are defined in (17) and (18). To prove that this is the value function and that strategy (20) is optimal, it is sufficient to verify conditions in Lemma 1.

Function $V(q,p)$ is continuous by construction and continuously differentiable everywhere except for the lines $R_1 = \{(q,p):\ q = \underline{p},\ p \le \underline{p}\}$, $R_2 = \{(q,p):\ q = \bar{p},\ p \ge \bar{p}\}$, $R_3 = \{(q,p):\ q = p,\ \underline{p} \le p \le \bar{p}\}$, $R_4 = \{(q,p):\ p = \bar{p},\ q \ge \bar{p}\}$ and $R_5 = \{(q,p):\ p = \underline{p},\ q \ge \underline{p}\}$. Hence, condition 3 of Lemma 1 holds with M_0 being the set with two points, $q = p = \underline{p}$ and $q = p = \bar{p}$, and M containing all points of the lines R_1, R_2, R_3, R_4 and R_5, except for the two points from M_0.

By construction, $V(q,p)$ is equal to 0 where $x = 0$ – that is, for $q \le g(p)$. This implies condition 1 of Lemma 1 and condition (13).

The non-negativity condition 2 of Lemma 1 holds because $V(q,p)$ is continuous, equal to 0 for $q \le g(p)$ and increasing in q for $q > g(p)$.

It remains to establish that $V(q,p)$ satisfies the HJB equation (8) and $x(q,p)$ satisfies (11) and (12). By construction, $V(q,p)$ satisfies (A.21), which becomes the HJB equation when $x(q,p)$ satisfies (11) and (12). Conditions (11) and (12) hold because $\mathcal{L}(q,p;V)$ is negative when $x(q,p) = 0$ – i.e., $q < g(p)$ – and positive when $x(q,p) = 1$ – i.e., $q > g(p)$. The negativity of $\mathcal{L}(q,p;V)$ when $q < g(p)$ follows from $g(p) \le c/\mu_1$ and inequality (A.22), which holds in this case. The positivity of $\mathcal{L}(q,p;V)$ when $q > g(p)$ follows by direct computations:

$$\mathcal{L}(q,p;V) = \frac{c\left(g(p)(q-\underline{p})\rho + q(g(p)-\underline{p})\mu_1\right)}{g(p)\underline{p}(\rho+\mu_0)} > 0 \quad \text{for } q > g(p) \ge \underline{p}. \tag{A.53}$$

A.3.3. Uniqueness

The argument for the equilibrium uniqueness is similar to the one in Section A.2.3, with two differences.

The first difference is in the upper bound for $V(q,p)$. In the growing optimism case, if $x^*(p) = 0$ for all p, then seeker i's value function is equal to (A.29) for $q > \underline{p}$ (with $\underline{p}$ defined in (17)) and 0 for $q < \underline{p}$, which can be verified using Lemma 1. Thus, $V(q,p)$ is bounded above by 0 for all $q < \underline{p}$ and (A.29) for $q > \underline{p}$. Then, following the argument in Section A.2.3 yields that, in any equilibrium, $x^*(p) = 0$ for all $p < \underline{p}$ and $x^*(p) > 0$ for all $p > \underline{p}$.

The second difference is that because now the common belief is non-decreasing, in the backward induction argument, we move from higher to lower beliefs.

In any equilibrium, $x(q,p) = 1$ for sufficiently high q because $V(1,p)$ is bounded above by $(\mu_1 - c)/(\rho + \mu_1)$, the expression (A.29) evaluated at $q = 1$, and, thus, $\mathcal{L}(1,p;V) = \mu_1(1 - V(1,p)) - c > 0$. Thus, for sufficiently high p, $x^*(p) = 1$ in any equilibrium.

To argue that the value of $x^*(p)$ for any $p > \underline{p}$ can be uniquely pinned down, we assume that the uniqueness of $x^*(p)$ has already been proved for all $p \ge \tilde{p}$ for some $\tilde{p} > \underline{p}$. In region $p \ge \tilde{p}$, the expression for V is as derived in Section A.3.1 because p always increases when $x^*(p) > 0$, which means that for seeker i the relevant part of the state space shrinks to the region $p \ge \tilde{p}$. Consider p just below $\tilde{p}$. Since $x^*(p) > 0$, by (1), the belief trajectory immediately travels into the region where $p \ge \tilde{p}$. Thus, at p just below $\tilde{p}$, the optimal best response of seeker i is defined by the sign of $\mathcal{L}(q,p;V)$ evaluated at V defined as in Section A.3.1. At such V, as we showed in Section A.3.2, $\mathcal{L}(q,p;V) < 0$ for all $q < g(p)$ and $\mathcal{L}(q,p;V) > 0$ for all $q > g(p)$. Thus, at

p just below $\tilde{p}$, the unique best response of seeker i is $x(q,p)=0$ for $q<g(p)$ and $x(q,p)=1$ for $q>g(p)$.

Suppose $\tilde{p}>\bar{p}$, so that point p, which is just below $\tilde{p}$, is also greater than $\bar{p}$. Then, $p>g(p)=\bar{p}$ and, thus, $x(p,p)=1$. Thus, $x^*(p)=1$ for all $p>\bar{p}$.

Suppose $\tilde{p}\le\bar{p}$, so that at p just below $\tilde{p}$ we have $g(p)=p$. Then, for all $q<p$, the unique best response of seeker i is not to learn, and so, $V(q,p)=0$ (seeker i will never learn in the future because once p increases to $\tilde{p}$, the trajectory enters the region where we conjecture that the equilibrium is unique and, thus, we know the behavior of seeker i). By continuity of V, $V(q,p)=0$ at $q=p$. Then, since $V(p,p)=0$, $x^*(p)$ is uniquely pinned down as (A.45), as we show in Section A.3.1.

A.4. Proof of Lemma 4

A.4.1. Construction

We conjecture that the seeker i's best response to (19) is (20). His expected payoff is $V(q,p)=0$ when $q\le g(p)$; it is given by (A.29) when $q>\underline{p}$ and $p<\underline{p}$; it is given by (A.16) when $p\ge q>\bar{p}$.

If $\underline{p}<p_0<\bar{p}$ and $p_0<q_0$, then the expected payoff is given by $W(0,q_0,p_0)$, defined in (A.33). The expression (A.34) for the subjective belief simplifies the integral in (A.33) and gives

$$W(0,q_0,p_0)=(1-q_0)\left(\frac{W(T,q_0,p_0)}{e^{(\rho+\mu_0)T}(1-q_T)}-\frac{c\left(1-e^{-(\rho+\mu_0)T}\right)}{\rho+\mu_0}\right)+q_0(\mu_1-c)\int_0^T e^{-(\rho+\mu_1)\tau-(n-1)\mu_1X(\tau,p_0)}\,\mathrm{d}\tau. \tag{A.54}$$

The relevant belief trajectory leads to $q_T=1$ and $p_T=\underline{p}$ because if it approaches the diagonal $q_T=p_T$, it immediately bounces off it to the region $q>p$: the laws of motion (7) and (1) give

$$\dot{q}_t-\dot{p}_t=(\mu_0-\mu_1)(1-x^*(p_t))p_t(1-p_t)>0 \tag{A.55}$$

under the assumptions $q_t=p_t$ and $x_t=1$. Moreover, the trajectory never reaches the point $(q_T,p_T)=(1,\underline{p})$, meaning that $T=+\infty$. Taking the limit $T\to+\infty$ in (A.54) gives us

$$W(0,q_0,p_0)=q_0(\mu_1-c)\int_0^T e^{-(\rho+\mu_1)\tau-(n-1)\mu_1X(\tau,p_0)}\,\mathrm{d}\tau-\frac{c(1-q_0)}{\rho+\mu_0} \tag{A.56}$$

because

$$\lim_{T\to+\infty}e^{(\rho+\mu_0)T}(1-q_T)\overset{(A.34)}{=}\lim_{T\to+\infty}\frac{e^{(\rho+\mu_1)T}(1-q_0)}{q_0e^{-(n-1)\mu_1X(T,p_0)}+(1-q_0)e^{-(\mu_0-\mu_1)T}}=+\infty, \tag{A.57}$$

and $W(T,q_0,p_0)$ must be bounded. Differentiating (A.54) and using (A.43), we get (A.44). Continuity on the diagonal requires $W(0,p,p)=0$; hence, (A.45) must be true. For any given initial beliefs (q_0,p_0) in the middle region $\underline{p}<p_0<\bar{p}$, $q_0>p_0$, the expected payoff $V(q_0,p_0)$ is equal to the right-hand side of (A.54). Hence, $V(q_0,p_0)$ is a linear function of q_0, and $V(q,p)$ can be expressed as (A.46). Since the expression for $x^*(p)$ is the same, the differential equations for $F(p)$ and $G(p)$ are (A.47) and (A.48). The expected payoff along the line $p = \underline{p}$ is defined in (A.29), which gives us the boundary conditions:

$$F(\underline{p}) = \frac{c}{\rho + \mu_0} + \frac{\mu_1 - c}{\rho + \mu_1}, \quad G(\underline{p}) = -\frac{c}{\rho + \mu_0}. \tag{A.58}$$

Given the boundary condition (A.58) for $G(\underline{p})$, the unique solution to the differential equation (A.47) is (A.50). Given (A.50), the boundary condition (A.58) for $F(\underline{p})$, and the expression (17) for $\underline{p}$, we get (A.51). In sum, $V(q, p)$ is defined in (A.52).

If $q > p > \bar{p}$, then $V(q, p)$ satisfies (A.15), where function W_2 is pinned down by the continuity of the payoff function along $p = \bar{p}$:

$$V(q, p) = \frac{q(\mu_1 - c)}{n\mu_1 + \rho} - \frac{c(1-q)}{\mu_0 + \rho} + \frac{q}{\bar{p}} \left(\frac{\bar{p}(1-p)}{(1-\bar{p})p} \right)^{\frac{\rho + n\mu_1}{n\mu_1 - \mu_0}} \left(\frac{c(1-\bar{p})}{\rho + \mu_0} - \frac{\bar{p}(\mu_1 - c)}{n\mu_1 + \rho} \right). \tag{A.59}$$

A.4.2. Verification

Function $V(q, p)$ is equal to 0 for $q \leq g(p)$; it is defined in (A.16) when $p \geq q > \bar{p}$, in (A.59) when $q > p > \bar{p}$, in (A.29) when $q > \underline{p}$ and $p < \underline{p}$, and in (A.52) when $\underline{p} \leq p \leq \bar{p}$ and $p < q$; thresholds $\underline{p}$ and $\bar{p}$ are defined in (17) and (16). To prove that this is the value function and that strategy (20) is optimal, it is sufficient to verify conditions in Lemma 1.

Function $V(q, p)$ is continuous by construction and continuously differentiable everywhere except for the lines $N_1 = \{(q, p)\colon\ q = \underline{p},\ p \leq \underline{p}\}$, $N_2 = \{(q, p)\colon\ q = p,\ p \geq \underline{p}\}$ and $N_3 = \{(q, p)\colon\ p = \underline{p},\ q \geq \underline{p}\}$. Hence, condition 3 of Lemma 1 holds with M_0 being the set with one point, $q = p = \underline{p}$, and M containing all points of the lines N_1, N_2 and N_3, except for one point from M_0.

By construction, $V(q, p)$ is equal to 0 where $x = 0$ – that is, for $q \leq g(p)$. This implies condition 1 of Lemma 1 and condition (13).

The non-negativity condition 2 of Lemma 1 holds because $V(q, p)$ is continuous, equal to 0 for $q \leq g(p)$ and increasing in $q > g(p)$, as we now show. For $q > p$, $V(q, p)$ is defined in (A.29), (A.52) and (A.59); all three expressions are linear in q with positive slope. For $q < p$, $V(q, p)$ is defined in (A.16); the first derivative with respect to q is equal to 0 at $q = \bar{p}$; the second derivative is equal to (A.19) and, therefore, positive for all $q > \bar{p}$.

It remains to establish that $V(q, p)$ satisfies the HJB equation (8) and $x(q, p)$ satisfies (11) and (12). By construction, $V(q, p)$ satisfies (A.21), which becomes the HJB equation when $x(q, p)$ satisfies (11) and (12). Conditions (11) and (12) hold because $\mathcal{L}(q, p; V)$ is negative when $x(q, p) = 0$ – i.e., $q < g(p)$ – and positive when $x(q, p) = 1$ – i.e., $q > g(p)$. The negativity of $\mathcal{L}(q, p; V)$ when $q < g(p)$ follows from $g(p) \leq c/\mu_1$ and inequality (A.22), which holds in this case. The positivity of $\mathcal{L}(q, p; V)$ when $q > g(p)$ and $p \leq \bar{p}$ follows from inequality (A.53), which holds in this case. The positivity of $\mathcal{L}(q, p; V)$ when $q > g(p)$ and $p > \bar{p}$ follows from

$$\mathcal{L}(q, p; V) = \frac{c\left(\bar{p}(q - \underline{p})\rho + q(\bar{p} - \underline{p})\mu_1\right)}{\bar{p}\underline{p}(\rho + \mu_0)} + \frac{q\mu_1(\mu_1 - c)(n\mu_1 - \mu_0)}{(\rho + \mu_0)(\rho + n\mu_1)} \left(1 - \left(\frac{(1-p)\bar{p}}{(1-\bar{p})p} \right)^{\frac{\rho + n\mu_1}{n\mu_1 - \mu_0}} \right) > 0, \quad \text{for } q > p > \bar{p}; \tag{A.60}$$

$$\mathcal{L}(q,p;V) = \frac{q(\mu_1 - c)}{\rho+\mu_0}\left(\mu_0 - \mu_1 + \frac{\mu_1(n\mu_1-\mu_0)}{\rho+n\mu_1}\right)\left(1-\left(\frac{(1-q)\bar{p}}{(1-\bar{p})q}\right)^{\frac{\rho+n\mu_1}{n\mu_1-\mu_0}}\right) + \frac{\rho\mu_1(q-\bar{p})}{\rho+\mu_0} > 0, \quad \text{for } p > q > \bar{p}. \tag{A.61}$$

A.4.3. Uniqueness

The argument for the equilibrium uniqueness is similar to our arguments in Sections A.2.3 and A.3.3.

As in the growing optimism case in Section A.3.3, we argue that in the mixed case, $V(q, p)$ is bounded above by 0 for all $q < \underline{p}$ and (A.29) for $q > \underline{p}$. Hence, in any equilibrium, $x^*(p) = 0$ for all $p < \underline{p}$ and $x^*(p) > 0$ for all $p > \underline{p}$.

As in the growing pessimism case in Section A.2.3, in the backward induction argument for the mixed case, we move from lower to higher beliefs. However, the presence of the diagonal in the cutoff curve $g(p)$ in the mixed case makes us borrow some elements of the proof from the growing optimism case in Section A.3.3, where the cutoff curve also contains the diagonal.

To argue that the value of $x^*(p)$ for $p > \underline{p}$ can be uniquely pinned down, we assume that the uniqueness of $x^*(p)$ has already been proved for all $p \le \tilde{p}$ for some $\tilde{p} \ge \underline{p}$. Then, by the argument analogous to the argument in Section A.2.3, at p just above $\tilde{p}$ for all $q < p$, the unique best response of seeker i is $x(q, p) = 0$ for $q < g(p)$ and $x(q, p) = 1$ for $q > g(p)$. As in Section A.3.3, we consider cases $\tilde{p} < \bar{p}$ and $\tilde{p} \ge \bar{p}$ separately. If $\tilde{p} < \bar{p}$, then $V(p, p) = 0$, which implies that $x^*(p)$ is uniquely pinned down as (A.45), as we show in Section A.4.1. If $\tilde{p} \ge \bar{p}$, then $x(p, p) = x^*(p) = 1$.

A.5. Proof of Corollary 1

On the interval $p \in (\underline{p}, \bar{p})$, the evolution of belief p follows (1) with $x_t^* = x^*(p_t)$ defined in (21). Solving this differential equation yields

$$p_t = \underline{p} + \frac{\underline{p}\left(p_0 - \underline{p}\right)}{p_0\left(\exp\left(\frac{t\underline{p}(n\mu_1-\mu_0)(1-c/\mu_1)}{(c/\mu_1-\underline{p})}\right) - 1\right) + \underline{p}}, \tag{A.62}$$

which does not reach $\underline{p}$ in finite time.

A.6. Expression for $P(\mu_1, \mu_0, c, p_0, n, \rho)$ (Proof of Lemmas 5, 6 and 7)

When the story never becomes obsolete – that is, $\rho = 0$ – the probability that one seeker fails to uncover a compromising story during time interval $[0, t]$ is $\exp\left(-\mu_1 \int_0^t x^*(p_\tau)\,d\tau\right)$. Hence, the probability that none of n seekers uncovers a compromising story during time interval $[0, +\infty)$ is

$$P(\mu_1, \mu_0, c, p_0, n, 0) = \exp\left(-n\mu_1 \int_0^{+\infty} x^*(p_t)\,dt\right). \tag{A.63}$$

When the story becomes obsolete at rate $\rho > 0$, the probability that none of n seekers uncovers a compromising story during time interval $[0, t]$ and that this story becomes obsolete exactly at time t is $\exp\left(-n\mu_1 \int_0^t x^*(p_\tau)\,\mathrm{d}\tau\right) \times \exp(-\rho t)\,\rho\,\mathrm{d}t$. Hence, the hider avoids the report of a compromising story with probability

$$P(\mu_1, \mu_0, c, p_0, n, \rho) = \int_0^{+\infty} \exp\left(-n\mu_1 \int_0^t x^*(p_\tau)\,\mathrm{d}\tau - \rho t\right) \rho\,\mathrm{d}t\,. \tag{A.64}$$

When $\mu_0 \geq n\mu_1$, conditional on the story being compromising, all seekers learn with intensity 1 until either the story is reported or the story becomes obsolete. Hence, (A.63) becomes 0 and (A.64) becomes

$$P(\mu_1, \mu_0, c, p_0, n, \rho) = \int_0^{+\infty} \exp(-n\mu_1 t - \rho t)\,\rho\,\mathrm{d}t = \frac{\rho}{\rho + n\mu_1}, \tag{A.65}$$

which proves Lemma 5.

When $\mu_0 < n\mu_1$ and $\rho = 0$, the law of motion for the common belief, given in (1), allows changing the variable of integration in (A.63) from time t to belief p:

$$P(\mu_1, \mu_0, c, p_0, n, 0) = \exp\left(-n\mu_1 \int_{p_0}^{\underline{p}} \frac{1}{(\mu_0 - n\mu_1)p(1-p)}\,\mathrm{d}p\right)$$

$$= \left(\frac{(1-\underline{p})p_0}{\underline{p}(1-p_0)}\right)^{-\frac{n\mu_1}{n\mu_1 - \mu_0}} = e^{-n\mu_1\left(T(\mu_1,\mu_0,c,p_0,n) - T(\mu_1,\mu_0,c,\underline{p},n)\right)} \quad \text{with } \underline{p} \text{ defined in (26)}, \tag{A.66}$$

where $T(\mu_1, \mu_0, c, p, n)$ is defined in (3). When $\mu_1 \leq \mu_0$, (A.66) proves Lemma 7. If $\mu_0 < \mu_1$, $\underline{p} = \bar{p}$ and so $T(\mu_1, \mu_0, c, \underline{p}, n) = 0$, which proves Lemma 6 for $\rho = 0$.

When $\mu_0 < n\mu_1$ and $\rho > 0$, (A.64) becomes

$$P(\mu_1, \mu_0, c, p_0, n, \rho) = \int_0^{T(\mu_1,\mu_0,c,p_0,n)} \exp(-n\mu_1 t - \rho t)\,\rho\,\mathrm{d}t$$

$$+ \int_{T(\mu_1,\mu_0,c,p_0,n)}^{+\infty} \exp\left(-n\mu_1 T(\mu_1, \mu_0, c, p_0, n) - n\mu_1 \int_{T(\mu_1,\mu_0,c,p_0,n)}^{t} x^*(p_\tau)\,\mathrm{d}\tau - \rho t\right) \rho\,\mathrm{d}t, \tag{A.67}$$

where $T(\mu_1, \mu_0, c, p_0, n)$ is the time that the common belief takes to reach the upper threshold $\bar{p}$ from p_0. The first integral in (A.67) corresponds to the interval $(\bar{p}, p_0)$ and is equal to

$$\int_0^{T(\mu_1,\mu_0,c,p_0,n)} \exp(-n\mu_1 t - \rho t)\,\rho\,\mathrm{d}t = \frac{\rho}{\rho + n\mu_1}\left(1 - e^{-(\rho+n\mu_1)T(\mu_1,\mu_0,c,p_0,n)}\right). \tag{A.68}$$

The second integral in (A.67) corresponds to the interval $(\underline{p}, \bar{p})$ and could be rewritten as

$$e^{-(\rho+n\mu_1)T(\mu_1,\mu_0,c,p_0,n)} P(\mu_1, \mu_0, c, \bar{p}, n, \rho), \tag{A.69}$$

where

$$P(\mu_1, \mu_0, c, \bar{p}, n, \rho) = \int_{T(\mu_1,\mu_0,c,p_0,n)}^{+\infty} \exp\left(-n\mu_1 \int_{T(\mu_1,\mu_0,c,p_0,n)}^{t} x^*(p_\tau)\,\mathrm{d}\tau - \rho(t - T(\mu_1, \mu_0, c, p_0, n))\right)\rho\,\mathrm{dt} \quad \text{(A.70)}$$

describes the probability with which the hider avoids a compromising report after the common belief has reached $\bar{p}$. Combining (A.68) and (A.69), we get

$$P(\mu_1, \mu_0, c, p_0, n, \rho) = \frac{\rho}{\rho + n\mu_1}\left(1 - e^{-(\rho+n\mu_1)T(\mu_1,\mu_0,c,p_0,n)}\right) + P(\mu_1, \mu_0, c, \bar{p}, n, \rho)e^{-(\rho+n\mu_1)T(\mu_1,\mu_0,c,p_0,n)}. \quad \text{(A.71)}$$

The seekers learn with intensity 1 for beliefs in $(\bar{p}, p_0)$. The law of motion (1) gives the differential equation for p_t. Solving this equation, we get

$$p_t = \frac{p_0}{p_0 + e^{(n\mu_1-\mu_0)t}(1 - p_0)}. \quad \text{(A.72)}$$

By definition of time $T(\mu_1, \mu_0, c, p_0, n)$, p_t is equal to $\bar{p}$ at that time. Hence, (A.72) gives (3).

If $\mu_0 < \mu_1$, then interval $(\underline{p}, \bar{p})$ is degenerate, so that the seekers' learning intensity $x^*(p_t)$ is zero for all t, and expression (A.70) is equal to 1. Then, substituting $P(\mu_1, \mu_0, c, \bar{p}, n, \rho) = 1$ into (A.71) proves Lemma 6 for $\rho > 0$.

A.7. Proof of Theorem 1

We consider cases $\mu_1 > \mu_0$ and $\mu_1 \le \mu_0$ separately.

Proof for $\mu_1 > \mu_0$. Substituting T from (3) into (28), we get the expression for P:

$$P(\mu_1, \mu_0, c, p_0, n, \rho) = \frac{\rho}{\rho + n\mu_1} + \frac{n\mu_1}{\rho + n\mu_1}\left(\frac{p_0(1 - \bar{p})}{\bar{p}(1 - p_0)}\right)^{-\frac{\rho+n\mu_1}{n\mu_1-\mu_0}} \text{ with } \bar{p} \text{ defined in (16).} \quad \text{(A.73)}$$

We proceed in two steps.

STEP 1. We differentiate (A.73) with respect to $n > 1$:

$$\frac{\partial P(\mu_1, \mu_0, c, p_0, n, \rho)}{\partial n} = \frac{n\mu_1^2(\rho + \mu_0)}{(n\mu_1 - \mu_0)(\rho + n\mu_1)^2} g(y, \lambda), \quad \text{(A.74)}$$

where

$$y = \left(\frac{p_0(1 - \bar{p})}{\bar{p}(1 - p_0)}\right)^{-\frac{\rho+n\mu_1}{n\mu_1-\mu_0}}, \quad \lambda = \frac{\rho(n\mu_1 - \mu_0)}{n\mu_1(\rho + \mu_0)}, \quad \text{(A.75)}$$

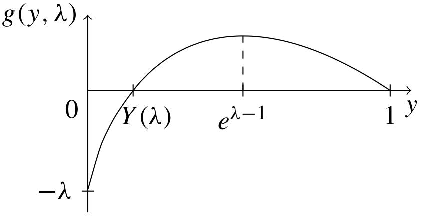
and $g(y, \lambda) = -y \ln y - (1 - y)\lambda$. Expressions (A.75) for y and λ belong to the interval $(0, 1)$ because $n\mu_1 > \mu_1 \ge \mu_0$ and the seekers start at belief p_0 no lower than $\bar{p}$ (by Assumption 1).

As a function of y, g is increasing for $0 < y < e^{\lambda-1}$ and decreasing for $y > e^{\lambda-1}$:

$$\frac{\partial g(y,\lambda)}{\partial y} = \lambda - 1 - \ln y. \tag{A.76}$$

Given that

$$\lim_{y\to 0} g(y,\lambda) = -\lambda < 0, \quad g(1,\lambda) = 0, \tag{A.77}$$



we conclude that for any $\lambda \in (0,1)$, the solution $Y(\lambda) \in (0,1)$ to $g(Y,\lambda) = 0$ exists and is unique. Moreover, $g(y,\lambda) < 0$ when $y < Y(\lambda)$ and $g(y,\lambda) > 0$ when $y > Y(\lambda)$. Hence, P increases in n if $y - Y(\lambda) > 0$ and decreases in n if $y - Y(\lambda) < 0$.

STEP 2. Define

$$h(\mu_1,\mu_0,c,p_0,n,\rho) \equiv y - Y(\lambda), \quad \text{with } y \text{ and } \lambda \text{ defined in (A.75)}, \tag{A.78}$$

so that P increases in n if $h > 0$ and decreases in n if $h < 0$. We differentiate h with respect to n:

$$\frac{\partial h(\mu_1,\mu_0,c,p_0,n,\rho)}{\partial n} = \frac{\mu_1(\rho+\mu_0)\left(-(1-\lambda)\lambda Y'(\lambda) - y\ln y\right)}{(n\mu_1-\mu_0)(\rho+n\mu_1)}, \tag{A.79}$$

with y and λ defined in (A.75).

The derivative of $Y(\lambda)$ can be found from $g(Y,\lambda) = 0$ using the implicit function theorem and substituting λ from $g(Y,\lambda) = 0$:

$$Y'(\lambda) = \frac{(1-Y)^2}{Y - 1 - \ln Y}, \tag{A.80}$$

where we write $Y = Y(\lambda)$ for short.

Substituting λ from $g(Y,\lambda) = 0$ and $Y'(\lambda)$ from (A.80) into (A.79), we get

$$\frac{\partial h(\mu_1,\mu_0,c,p_0,n,\rho)}{\partial n} = \frac{\mu_1(\rho+\mu_0)}{(n\mu_1-\mu_0)(\rho+n\mu_1)}\left(\left(\frac{r_1(Y)}{r_2(Y)} + 1\right) Y\ln Y - y\ln y\right), \tag{A.81}$$

where

$$r_1(Y) = 2(1-Y) + (1+Y)\ln Y, \quad r_2(Y) = Y - 1 - \ln Y. \tag{A.82}$$

Function $r_1(Y)$ is negative for all $Y \in (0,1)$ because $r_1''(Y) = -\frac{1-Y}{Y^2} < 0$, $r_1'(1) = r_1(1) = 0$. Function $r_2(Y)$ is positive for all $Y \in (0,1)$ because $r_2'(Y) = -\frac{1-Y}{Y} < 0$, $r_2(1) = 0$. Equation (A.81) implies that at $h = 0$, where $Y = y$ the derivative of h w.r.t. n is positive:

$$\left.\frac{\partial h(\mu_1,\mu_0,c,p_0,n,\rho)}{\partial n}\right|_{h=0} > 0. \tag{A.83}$$

Hence, as a function of n, h crosses 0 from below (if ever).

Consequently, there exists $n^* \geq 1$ (possibly infinite) such that for all $n < n^*$, $h < 0$ and, thus, P decreases; and for all $n > n^*$, $h > 0$ and, thus, P increases. Hence, as claimed on page 25, P has a U-shape and reaches its maximum at either $n = 1$ or $n = +\infty$.

Substituting $n = 1$ and $n = +\infty$ into (A.73), we get that the minimal access ($n = 1$) yields

$$P(\mu_1, \mu_0, c, p_0, 1, \rho) = \frac{\rho}{\rho + \mu_1} + \frac{\mu_1}{\rho + \mu_1} \left(\frac{\bar{p}(1 - p_0)}{p_0(1 - \bar{p})} \right)^{\frac{\rho + \mu_1}{\mu_1 - \mu_0}} \quad \text{with } \bar{p} \text{ defined in (16)}, \tag{A.84}$$

while open access ($n = +\infty$) yields

$$P(\mu_1, \mu_0, c, p_0, +\infty, \rho) = \frac{\bar{p}(1 - p_0)}{p_0(1 - \bar{p})} \quad \text{with } \bar{p} \text{ defined in (16)}. \tag{A.85}$$

If (A.84) is higher than (A.85), then $n = 1$ is optimal; if (A.85) is higher, then $n = +\infty$ is optimal.

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The derivative of the difference $P(\mu_1, \mu_0, c, p_0, +\infty, \rho) - P(\mu_1, \mu_0, c, p_0, 1, \rho)$ with respect to $\rho > 0$ is:

$$\frac{\partial}{\partial \rho} \left(P(\mu_1, \mu_0, c, p_0, +\infty, \rho) - P(\mu_1, \mu_0, c, p_0, 1, \rho) \right) = \frac{\mu_1 \left(y(1 - \ln y) - 1 \right)}{(\rho + \mu_1)^2}, \tag{A.86}$$

where $y = z^{\frac{\rho + \mu_1}{\mu_1 - \mu_0}}$ and $z = \frac{\bar{p}(1 - p_0)}{p_0(1 - \bar{p})}$. By Assumption 1, $p_0 > \bar{p}$ and, thus, z and y both belong to $(0, 1)$. Then, expression (A.86) is negative because function $y(1 - \ln y) - 1$ is negative for all $y \in (0, 1)$. At $\rho = 0$, the difference $P(\mu_1, \mu_0, c, p_0, +\infty, \rho) - P(\mu_1, \mu_0, c, p_0, 1, \rho)$ is positive:

$$P(\mu_1, \mu_0, c, p_0, +\infty, 0) - P(\mu_1, \mu_0, c, p_0, 1, 0) = z - z^{\frac{\rho + \mu_1}{\mu_1 - \mu_0}} > 0. \tag{A.87}$$

At $\rho = +\infty$, the difference $P(\mu_1, \mu_0, c, p_0, +\infty, \rho) - P(\mu_1, \mu_0, c, p_0, 1, \rho)$ is negative:

$$P(\mu_1, \mu_0, c, p_0, +\infty, +\infty) - P(\mu_1, \mu_0, c, p_0, 1, +\infty) = z - 1 < 0. \tag{A.88}$$

Hence, there exists ρ^* such that open access is optimal for $\rho < \rho^*$ and the minimal access is optimal for $\rho > \rho^*$.

Proof for $\mu_1 \le \mu_0$. For $1 \le n \le \mu_0/\mu_1$, the expression for P is given in (27). Substituting T from (3) into (A.71), we get the expression for P when $n > \mu_0/\mu_1$:

$$P(\mu_1, \mu_0, c, p_0, n, \rho) = \frac{\rho}{\rho + n\mu_1} + \left(P(\mu_1, \mu_0, c, \bar{p}, n, \rho) - \frac{\rho}{\rho + n\mu_1} \right) \times \left(\frac{p_0(1 - \bar{p})}{\bar{p}(1 - p_0)} \right)^{-\frac{\rho + n\mu_1}{n\mu_1 - \mu_0}} \quad \text{with } \bar{p} \text{ defined in (16)}. \tag{A.89}$$

Expression (A.89) includes function $P(\mu_1, \mu_0, c, \bar{p}, n, \rho)$; for expositional convenience, we explore properties of this function separately in Lemma 11 in Appendix B.1.

To prove that $P(\mu_1, \mu_0, c, p_0, n, \rho)$ reaches its maximum at either $n = 1$ or $n = +\infty$, we need to show that for all $n > 1$,

$$\max \{ P(\mu_1, \mu_0, c, p_0, 1, \rho), P(\mu_1, \mu_0, c, p_0, +\infty, \rho) \} > P(\mu_1, \mu_0, c, p_0, n, \rho), \tag{A.90}$$

where $P(\mu_1, \mu_0, c, p_0, +\infty, \rho) \equiv \lim_{n \to +\infty} P(\mu_1, \mu_0, c, p_0, n, \rho)$.

By (27),

$$P(\mu_1, \mu_0, c, p_0, 1, \rho) = \frac{\rho}{\rho + \mu_1}. \tag{A.91}$$

By (A.89) and by (B.1) in Lemma 11,

$$P(\mu_1, \mu_0, c, p_0, +\infty, \rho) = \frac{\rho + \mu_1}{\rho + \mu_0} \frac{\bar{p}(1 - p_0)}{p_0(1 - \bar{p})} \quad \text{with } \bar{p} \text{ defined in (16).} \tag{A.92}$$

It is immediate to see that $P(\mu_1, \mu_0, c, p_0, n, \rho)$ is decreasing in $1 \leq n \leq \mu_0/\mu_1$. Hence, (A.90) holds for all $1 < n \leq \mu_0/\mu_1$. Consider $n > \mu_0/\mu_1$. Expressions (A.89), (A.91) and (A.92) allow us to rewrite condition (A.90) as

$$F\left(\mu_1, \mu_0, c, \frac{\bar{p}(1 - p_0)}{p_0(1 - \bar{p})}, n, \rho\right) > 0, \tag{A.93}$$

where

$$F(\mu_1, \mu_0, c, y, n, \rho) = \max\left\{\frac{\rho + \mu_1}{\rho + \mu_0} y, \frac{\rho}{\rho + \mu_1}\right\} - \frac{\rho}{\rho + n\mu_1} - \left(P(\mu_1, \mu_0, c, \bar{p}, n, \rho) - \frac{\rho}{\rho + n\mu_1}\right) y^{\frac{\rho + n\mu_1}{n\mu_1 - \mu_0}}. \tag{A.94}$$

Argument y belongs to $[0, 1]$ because by Assumption 1, the seekers start at belief p_0 no lower than $\bar{p}$. Define point

$$y^* \equiv \frac{\rho(\rho + \mu_0)}{(\rho + \mu_1)^2}, \tag{A.95}$$

where the arguments in the maximum in (A.94) are equal. Then, if $y < y^*$, function F is decreasing in y:

$$\frac{\partial F(\mu_1, \mu_0, c, y, n, \rho)}{\partial y} = -\frac{\rho + n\mu_1}{n\mu_1 - \mu_0} \left(P(\mu_1, \mu_0, c, \bar{p}, n, \rho) - \frac{\rho}{\rho + n\mu_1}\right) y^{\frac{\rho + n\mu_1}{n\mu_1 - \mu_0}} < 0, \tag{A.96}$$

and if $y > y^*$, function F is concave in y:

$$\frac{\partial^2 F(\mu_1, \mu_0, c, y, n, \rho)}{\partial y^2} = -\frac{(\rho + \mu_0)(\rho + n\mu_1)}{y^2(n\mu_1 - \mu_0)^2} \times \left(P(\mu_1, \mu_0, c, \bar{p}, n, \rho) - \frac{\rho}{\rho + n\mu_1}\right) y^{\frac{\rho + n\mu_1}{n\mu_1 - \mu_0}} < 0 \tag{A.97}$$

because, by (B.2) in Lemma 11, $P(\mu_1, \mu_0, c, \bar{p}, n, \rho) > \frac{\rho}{\rho + n\mu_1}$. Hence, on $y \in [0, 1]$, F achieves minimum at $y = 1$ if $y^* \geq 1$, and at either $y = 1$ or $y = y^*$ if $y^* < 1$.

Substituting $y = 1$ into (A.94) gives

$$F(\mu_1, \mu_0, c, 1, n, \rho) = \max\left\{\frac{\rho + \mu_1}{\rho + \mu_0}, \frac{\rho}{\rho + \mu_1}\right\} - P(\mu_1, \mu_0, c, \bar{p}, n, \rho) \geq \frac{\rho + \mu_1}{\rho + \mu_0} - P(\mu_1, \mu_0, c, \bar{p}, n, \rho). \tag{A.98}$$

By (B.2) in Lemma 11, the right-hand side in (A.98) and, hence, also $F(\mu_1, \mu_0, c, 1, n, \rho)$ are positive. Substituting $y = y^*$ into (A.94) gives

$$F\left(\mu_1, \mu_0, c, y^*, n, \rho\right) = \frac{\rho}{\rho+\mu_1} - \frac{\rho}{\rho+n\mu_1} - \left(P(\mu_1, \mu_0, c, \bar{p}, n, \rho) - \frac{\rho}{\rho+n\mu_1}\right)\left(\frac{\rho(\rho+\mu_0)}{(\rho+\mu_1)^2}\right)^{\frac{\rho+n\mu_1}{n\mu_1-\mu_0}}. \quad \text{(A.99)}$$

By (B.4) in Lemma 11, $F(\mu_1, \mu_0, c, y^*, n, \rho)$ defined in (A.99) is positive because condition (B.3) is equivalent to $y^* < 1$. Hence, $F(\mu_1, \mu_0, c, y, n, \rho) > 0$ for all $y \in [0, 1]$, which proves that (A.93) and, consequently, (A.90) hold.

Hence, $P(\mu_1, \mu_0, c, p_0, n, \rho)$ reaches its maximum at either $n = 1$ or $n = +\infty$. If $P(\mu_1, \mu_0, c, p_0, 1, \rho)$ given in (A.91) is higher than $P(\mu_1, \mu_0, c, p_0, +\infty, \rho)$ given in (A.92), then $n = 1$ is optimal; if $P(\mu_1, \mu_0, c, p_0, +\infty, \rho)$ is higher, then $n = +\infty$ is optimal.

COMPARATIVE STATICS WITH RESPECT TO ρ.

The difference $P(\mu_1, \mu_0, c, p_0, +\infty, \rho) - P(\mu_1, \mu_0, c, p_0, 1, \rho)$ is positive if and only if

$$z \equiv \frac{\bar{p}(1-p_0)}{p_0(1-\bar{p})} > \frac{\rho(\rho+\mu_0)}{(\rho+\mu_1)^2} \equiv g(\rho). \quad \text{(A.100)}$$

By Assumption 1, $p_0 > \bar{p}$ and, thus, z belongs to $(0, 1)$. At $\rho = 0$, open access is optimal because $g(0) = 0 < z$. At $\rho = +\infty$, the minimal access is optimal because $g(+\infty) = 1 > z$. If $2\mu_1 \geq \mu_0$, then function $g(\rho)$ is increasing for $\rho \in (0, +\infty)$; if $2\mu_1 < \mu_0$, then function $g(\rho)$ is increasing for $\rho \in \left(0, \frac{\mu_0\mu_1}{\mu_0-2\mu_1}\right)$ and decreasing for $\rho \in \left(\frac{\mu_0\mu_1}{\mu_0-2\mu_1}, +\infty\right)$:

$$g'(\rho) = \frac{\mu_0\mu_1 + \rho(2\mu_1-\mu_0)}{(\rho+\mu_1)^3}. \quad \text{(A.101)}$$

Hence, there exists $\rho^* > 0$ such that $g(\rho) < z$ for $\rho < \rho^*$ and $g(\rho) > z$ for $\rho > \rho^*$.

A.8. *Proof of Theorem 2*

By Theorem 1, $\rho^*(\mu_1, \mu_0, c, p_0)$ is defined as the unique solution to

$$F(\mu_1, \mu_0, c, p_0, \rho) = 0, \quad \text{(A.102)}$$

where we define

$$F(\mu_1, \mu_0, c, p_0, \rho) = P(\mu_1, \mu_0, c, p_0, +\infty, \rho) - P(\mu_1, \mu_0, c, p_0, 1, \rho). \quad \text{(A.103)}$$

By the implicit function theorem, for any variable $x \in \{\mu_1, \mu_0, c, p_0\}$,

$$\frac{\partial \rho^*(\mu_1, \mu_0, c, p_0)}{\partial x} = -\frac{\partial F(\mu_1, \mu_0, c, p_0, \rho)}{\partial x} \Bigg/ \frac{\partial F(\mu_1, \mu_0, c, p_0, \rho)}{\partial \rho}\Bigg|_{\rho=\rho^*(\mu_1,\mu_0,c,p_0)}. \quad \text{(A.104)}$$

We consider cases $\mu_1 > \mu_0$ and $\mu_1 \leq \mu_0$ separately.

Proof for $\mu_1 > \mu_0$. Expression for $P(\mu_1, \mu_0, c, p_0, 1, \rho)$ is given in (A.84), expression for $P(\mu_1, \mu_0, c, p_0, +\infty, \rho)$ is given in (A.85). Hence,

$$F(\mu_1, \mu_0, c, p_0, \rho) = \frac{\bar{p}(1-p_0)}{p_0(1-\bar{p})} - \frac{\rho}{\rho+\mu_1} - \frac{\mu_1}{\rho+\mu_1}\left(\frac{\bar{p}(1-p_0)}{p_0(1-\bar{p})}\right)^{\frac{\rho+\mu_1}{\mu_1-\mu_0}}$$

with $\bar{p}$ defined in (16). (A.105)

The derivative of F with respect to ρ is calculated in (A.86) and is shown to be negative. Hence, by (A.104), the sign of $\partial\rho^*(\mu_1, \mu_0, c, p_0)/\partial x$ coincides with the sign of $\partial F(\mu_1, \mu_0, c, p_0, \rho)/\partial x$ at point $\rho = \rho^*(\mu_1, \mu_0, c, p_0)$.

Denote

$$y \equiv \left(\frac{\bar{p}(1-p_0)}{p_0(1-\bar{p})}\right)^{\frac{\rho+\mu_1}{\mu_1-\mu_0}} \quad \text{with } \bar{p} \text{ defined in (16).} \tag{A.106}$$

By Assumption 1, $p_0 > \bar{p}$ and, thus, $y < 1$.

Differentiating (A.105) with respect to p_0 yields

$$\frac{\partial F(\mu_1, \mu_0, c, p_0, \rho)}{\partial p_0} = -\frac{\mu_1 \bar{p}\,(1-\mu_0/\mu_1 - y)}{(1-\bar{p})p_0^2(\mu_1-\mu_0)} \tag{A.107}$$

Lemma 8. *If ρ is equal to $\rho^*(\mu_1, \mu_0, c, p_0)$, $1-\mu_0/\mu_1 - y > 0$, where y is defined in (A.106).*

Proof. $1-\mu_0/\mu_1 - y > 0$ evaluated at $\rho = \rho^*(\mu_1, \mu_0, c, p_0)$ is equivalent to

$$\rho^*(\mu_1, \mu_0, c, p_0) - \hat{\rho}(\mu_1, \mu_0, c, p_0) > 0, \tag{A.108}$$

where

$$\hat{\rho}(\mu_1, \mu_0, c, p_0) = (\mu_1-\mu_0)\ln\left(1-\frac{\mu_0}{\mu_1}\right) \Big/ \ln\left(\frac{\bar{p}(1-p_0)}{p_0(1-\bar{p})}\right) - \mu_0. \tag{A.109}$$

Hence, it is sufficient to prove (A.108).

Function $\hat{\rho}$ is decreasing in p_0 because

$$\frac{\partial\hat{\rho}(\mu_1, \mu_0, c, p_0)}{\partial p_0} = (\mu_1-\mu_0)\ln\left(1-\frac{\mu_0}{\mu_1}\right) \Big/ \left(p_0(1-p_0)\left(\ln\frac{\bar{p}(1-p_0)}{p_0(1-\bar{p})}\right)^2\right) < 0. \tag{A.110}$$

By (A.107), the sign of $\partial\rho^*(\mu_1, \mu_0, c, p_0)/\partial p_0$ coincides with the sign of $-(1-\mu_0/\mu_1 - y)$; and the sign of $1-\mu_0/\mu_1 - y$ coincides with the sign of the left-hand side of (A.108). Hence, if the left-hand side of (A.108) is negative, its derivative $\partial\rho^*(\mu_1, \mu_0, c, p_0)/\partial p_0 - \partial\hat{\rho}(\mu_1, \mu_0, c, p_0)/\partial p_0$ is positive. Thus, to prove (A.108), it is sufficient to prove that (A.108) holds for the lowest admissible value of p_0. By Assumption 1, the lowest admissible value of p_0 is $\bar{p}$.

As p_0 approaches $\bar{p}$, the sign of the left-hand side of (A.108) is not immediately clear because both ρ^* and $\hat{\rho}$ approach $+\infty$, and so, additional analysis is required. Since $\rho = \rho^*(\mu_1, \mu_0, c, p_0)$ solves $F(\mu_1, \mu_0, c, p_0, \rho) = 0$ and function $F(\mu_1, \mu_0, c, p_0, \rho)$ is decreasing in ρ, to show that (A.108) holds as p_0 approaches $\bar{p}$, it is sufficient to show that $F(\mu_1, \mu_0, c, p_0, \rho)$ is positive at point $\rho = \hat{\rho}(\mu_1, \mu_0, c, p_0)$ as p_0 approaches $\bar{p}$. The limit $p_0 \to \bar{p}$ of $F(\mu_1, \mu_0, c, p_0, \rho)$ at point $\rho = \hat{\rho}(\mu_1, \mu_0, c, p_0)$ is 0, and so, to determine whether F approaches 0 from below or from above, we divide F by $p_0 - \bar{p}$. The limit $p_0 \to \bar{p}$ of $F(\mu_1, \mu_0, c, p_0, \rho)/(p_0 - \bar{p})$ at point $\rho = \hat{\rho}(\mu_1, \mu_0, c, p_0)$ is equal to

$$-\frac{\mu_1}{(1-\bar{p})\bar{p}(\mu_1-\mu_0)\ln(1-\mu_0/\mu_1)}\left(\frac{\mu_0}{\mu_1} + \left(1-\frac{\mu_0}{\mu_1}\right)\ln\left(1-\frac{\mu_0}{\mu_1}\right)\right), \tag{A.111}$$

which is positive because $x + (1-x)\ln(1-x) > 0$ for all $x \in (0, 1)$. Hence, F approaches 0 from above, and so, (A.108) holds as p_0 approaches $\bar{p}$. □

Lemma 8 implies that (A.107) is negative, and so, $\partial\rho^*(\mu_1, \mu_0, c, p_0)/\partial p_0$ is negative.

Differentiating (A.105) with respect to c yields

$$\frac{\partial F(\mu_1, \mu_0, c, p_0, \rho)}{\partial c} = \frac{(1-p_0)\,(1-\mu_0/\mu_1 - y)}{(1-\bar{p})^2 p_0(\mu_1-\mu_0)}, \tag{A.112}$$

which is positive by Lemma 8, and so, $\partial\rho^*(\mu_1, \mu_0, c, p_0)/\partial c$ is positive.

Differentiating (A.105) with respect to μ_0 yields

$$\frac{\partial F(\mu_1, \mu_0, c, p_0, \rho)}{\partial \mu_0} = -\frac{\mu_1 y \ln(y)}{(\rho+\mu_1)(\mu_1-\mu_0)} > 0, \tag{A.113}$$

and so, $\partial\rho^*(\mu_1, \mu_0, c, p_0)/\partial \mu_0$ is positive.

Differentiating (A.105) with respect to μ_1 yields

$$\begin{aligned}\frac{\partial F(\mu_1, \mu_0, c, p_0, \rho)}{\partial \mu_1} &= \frac{\zeta(\mu_1, \mu_0, c, p_0, \rho)}{(\mu_1-\mu_0)^2(\rho+\mu_1)p_0(1-\bar{p})\mu_1} \\ &+ \left(\frac{\rho}{\mu_1(\rho+\mu_1)} - \frac{\rho+\mu_1}{(1-\bar{p})\mu_1(\mu_1-\mu_0)} - \frac{(\rho+\mu_0)\ln(y)}{(\rho+\mu_1)(\mu_1-\mu_0)}\right) F(\mu_1, \mu_0, c, p_0, \rho),\end{aligned} \tag{A.114}$$

where

$$\begin{aligned}\zeta(\mu_1, \mu_0, c, p_0, \rho) &= \left(\frac{\mu_1-\mu_0}{1-\bar{p}} + \mu_1 \ln\left(\frac{\bar{p}(1-p_0)}{p_0(1-\bar{p})}\right)\right)(\rho+\mu_0) \\ &\times (\bar{p}\mu_1(1-p_0) - (p_0-\bar{p})\rho) - \rho\bar{p}(\mu_1-\mu_0)^2.\end{aligned} \tag{A.115}$$

By (A.102), the second term in (A.114) is 0 at point $\rho = \rho^*(\mu_1, \mu_0, c, p_0)$. Hence, the sign of $\partial\rho^*(\mu_1, \mu_0, c, p_0)/\partial\mu_1$ coincides with the sign of $\zeta(\mu_1, \mu_0, c, p_0, \rho)$ at point $\rho = \rho^*(\mu_1, \mu_0, c, p_0)$.

To determine the sign of $\zeta(\mu_1, \mu_0, c, p_0, \rho)$ at point $\rho = \rho^*(\mu_1, \mu_0, c, p_0)$, we need two additional lemmas.

Lemma 9. *If ρ is equal to $\rho^*(\mu_1, \mu_0, c, p_0)$, then $\bar{p}\mu_1(1-p_0) > (p_0-\bar{p})\rho$.*

Proof. Inequality $\bar{p}\mu_1(1-p_0) > (p_0-\bar{p})\rho^*(\mu_1, \mu_0, c, p_0)$ is equivalent to

$$\rho^*(\mu_1, \mu_0, c, p_0) < \hat{\rho}(\mu_1, \mu_0, c, p_0) \equiv \frac{\bar{p}\mu_1(1-p_0)}{p_0-\bar{p}}. \tag{A.116}$$

Since $\rho = \rho^*(\mu_1, \mu_0, c, p_0)$ solves $F(\mu_1, \mu_0, c, p_0, \rho) = 0$ and function $F(\mu_1, \mu_0, c, p_0, \rho)$ is decreasing in ρ, to show that (A.116), it is sufficient to show that $F(\mu_1, \mu_0, c, p_0, \rho)$ is negative at point $\rho = \hat{\rho}(\mu_1, \mu_0, c, p_0)$.

By definition (A.105),

$$\begin{aligned}F(\mu_1, \mu_0, c, p_0, \rho) &= \frac{\bar{p}(1-p_0)}{p_0(1-\bar{p})} - \frac{\rho}{\rho+\mu_1} - \frac{\mu_1}{\rho+\mu_1}y < \frac{\bar{p}(1-p_0)}{p_0(1-\bar{p})} - \frac{\rho}{\rho+\mu_1} \\ &= \frac{(p_0-\bar{p})\left(\hat{\rho}(\mu_1, \mu_0, c, p_0) - \rho\right)}{p_0(1-\bar{p})(\rho+\mu_1)},\end{aligned} \tag{A.117}$$

where the inequality holds because $y > 0$. Hence, $F(\mu_1, \mu_0, c, p_0, \rho) < 0$ at $\rho = \hat{\rho}(\mu_1, \mu_0, c, p_0)$. □

Lemma 10. *If* ρ *is equal to* $\rho^*(\mu_1, \mu_0, c, p_0)$*, then* $\bar{p}\mu_0(1-p_0) < (p_0 - \bar{p})\rho$.

Proof. Rearranging and using definition (A.105),

$$(p_0 - \bar{p})\rho - \bar{p}\mu_0(1-p_0) = \bar{p}\mu_1(1-p_0)\left(1 - \frac{\mu_0}{\mu_1} - y\right) - (1-\bar{p})p_0(\rho+\mu_1)F(\mu_1, \mu_0, c, p_0, \rho). \quad \text{(A.118)}$$

By (A.102) and Lemma 8, the right-hand side of (A.118) is positive at point $\rho = \rho^*(\mu_1, \mu_0, c, p_0)$. □

Now we are ready to determine the sign of $\zeta(\mu_1, \mu_0, c, p_0, \rho)$ at point $\rho = \rho^*(\mu_1, \mu_0, c, p_0)$. If

$$\frac{\mu_1 - \mu_0}{1-\bar{p}} + \mu_1 \ln\left(\frac{\bar{p}(1-p_0)}{p_0(1-\bar{p})}\right) \quad \text{(A.119)}$$

is non-positive, then $\zeta(\mu_1, \mu_0, c, p_0, \rho)$ at point $\rho = \rho^*(\mu_1, \mu_0, c, p_0)$ is negative by Lemma 9. To determine the sign of $\zeta(\mu_1, \mu_0, c, p_0, \rho)$ when (A.119) is positive, we rewrite (A.115) as

$$\begin{aligned}\zeta(\mu_1, \mu_0, c, p_0, \rho) &= -\bar{p}\mu_1(1-p_0)(\mu_1-\mu_0)^2\left(-\frac{\mu_0}{\mu_1} - \ln\left(1-\frac{\mu_0}{\mu_1}\right) + \ln\left(\frac{1-\mu_0/\mu_1}{y}\right)\right) \\ &\quad - \left(\frac{\bar{p}(\mu_1-\mu_0)^2}{1-\bar{p}} + \left(\frac{\mu_1-\mu_0}{1-\bar{p}} + \mu_1 \ln\frac{\bar{p}(1-p_0)}{p_0(1-\bar{p})}\right)(\rho+\mu_0)\right) \\ &\quad \times ((p_0-\bar{p})\rho - \bar{p}\mu_0(1-p_0)). \quad \text{(A.120)}\end{aligned}$$

If (A.119) is positive, then (A.120) at point $\rho = \rho^*(\mu_1, \mu_0, c, p_0)$ is negative because $-x - \ln(1-x) > 0$ for all $x \in (0,1)$, $\ln\left(\frac{1-\mu_0/\mu_1}{y}\right) > 0$ by Lemma 8 and $(p_0 - \bar{p})\rho - \bar{p}\mu_0(1-p_0) > 0$ by Lemma 10. Consequently, $\zeta(\mu_1, \mu_0, c, p_0, \rho)$ is always negative at point $\rho = \rho^*(\mu_1, \mu_0, c, p_0)$, and so, $\partial\rho^*(\mu_1, \mu_0, c, p_0)/\partial\mu_1$ is negative.

Proof for $\mu_1 \le \mu_0$. Expression for $P(\mu_1, \mu_0, c, p_0, 1, \rho)$ is given in (A.91), expression for $P(\mu_1, \mu_0, c, p_0, +\infty, \rho)$ is given in (A.92). Hence,

$$F(\mu_1, \mu_0, c, p_0, \rho) = \frac{\rho+\mu_1}{\rho+\mu_0}\frac{\bar{p}(1-p_0)}{p_0(1-\bar{p})} - \frac{\rho}{\rho+\mu_1} \quad \text{with } \bar{p} \text{ defined in (16).} \quad \text{(A.121)}$$

The derivative of F with respect to ρ is equal to

$$\frac{\partial F(\mu_1, \mu_0, c, p_0, \rho)}{\partial\rho} = -\frac{F(\mu_1, \mu_0, c, p_0, \rho)}{\rho+\mu_0} - \frac{p_0-\bar{p}}{(1-\bar{p})p_0(\rho+\mu_0)} - \frac{\mu_1(\mu_0-\mu_1)}{(\rho+\mu_0)(\rho+\mu_1)^2}, \quad \text{(A.122)}$$

which is negative at point $\rho = \rho^*(\mu_1, \mu_0, c, p_0)$ where $F(\mu_1, \mu_0, c, p_0, \rho) = 0$. Hence, by (A.104), the sign of $\partial\rho^*(\mu_1, \mu_0, c, p_0)/\partial x$ coincides with the sign of $\partial F(\mu_1, \mu_0, c, p_0, \rho)/\partial x$ at point $\rho = \rho^*(\mu_1, \mu_0, c, p_0)$.

Differentiating (A.121) with respect to p_0 yields

$$\frac{\partial F(\mu_1, \mu_0, c, p_0, \rho)}{\partial p_0} = -\frac{\bar{p}(\rho+\mu_1)}{(1-\bar{p})p_0^2(\rho+\mu_0)} < 0, \quad \text{(A.123)}$$

and so, $\partial \rho^*(\mu_1, \mu_0, c, p_0)/\partial p_0$ is negative.

Differentiating (A.121) with respect to c yields

$$\frac{\partial F(\mu_1, \mu_0, c, p_0, \rho)}{\partial c} = \frac{(1-p_0)(\rho+\mu_1)}{(1-\bar{p})^2 p_0 \mu_1 (\rho+\mu_0)} > 0, \tag{A.124}$$

and so, $\partial \rho^*(\mu_1, \mu_0, c, p_0)/\partial c$ is positive.

If $\mu_1 = \mu_0 = \mu$, then (A.102) has the explicit solution

$$\rho^*(\mu, \mu, c, p_0) = \frac{\bar{p}(1-p_0)\mu}{p_0 - \bar{p}} = \frac{c(1-p_0)}{p_0 - c/\mu}. \tag{A.125}$$

It is clear that $\rho^*(\mu, \mu, c, p_0)$ is decreasing in μ.

Differentiating (A.121) with respect to μ_0 yields

$$\frac{\partial F(\mu_1, \mu_0, c, p_0, \rho)}{\partial \mu_0} = -\frac{\bar{p}(1-p_0)(\rho+\mu_1)}{(1-\bar{p})p_0(\rho+\mu_0)^2} < 0, \tag{A.126}$$

and so, $\partial \rho^*(\mu_1, \mu_0, c, p_0)/\partial \mu_0$ is negative.

Differentiating (A.121) with respect to μ_1 yields

$$\frac{\partial F(\mu_1, \mu_0, c, p_0, \rho)}{\partial \mu_1} = -\frac{F(\mu_1, \mu_0, c, p_0, \rho)}{\rho+\mu_1} + \frac{\bar{p}(1-p_0)(2(1-\bar{p})\mu_1 - \mu_1 - \rho)}{p_0(1-\bar{p})^2 \mu_1 (\rho+\mu_0)} \tag{A.127}$$

By (A.102), the first term in (A.127) is 0 at point $\rho = \rho^*(\mu_1, \mu_0, c, p_0)$. Hence, the sign of $\partial \rho^*(\mu_1, \mu_0, c, p_0)/\partial \mu_1$ coincides with the sign of

$$\hat{\rho}(\mu_1, \mu_0, c, p_0) - \rho^*(\mu_1, \mu_0, c, p_0), \tag{A.128}$$

where we define

$$\hat{\rho}(\mu_1, \mu_0, c, p_0) = 2(1-\bar{p})\mu_1 - \mu_1 = \mu_1 - 2c. \tag{A.129}$$

If (A.128) is equal to 0, then $\partial \rho^*(\mu_1, \mu_0, c, p_0)/\partial \mu_1 = 0$ by (A.127), and so, as a function of μ_1, (A.128) is increasing in μ_1. Thus, if function (A.128) crosses 0, it does so from below, which means that there exists M_1 such that (A.128) is negative for $\mu_1 < M_1$ and positive for $\mu_1 > M_1$.

A.9. Proof of Theorem 3

To prove the theorem, we need to show that $\frac{\partial P(\mu_1,\mu_0,c,p_0,1,\rho)}{\partial \mu_0} \le 0$ and $\frac{\mathrm{d} P(\mu_0+\delta,\mu_0,c,p_0,1,\rho)}{\mathrm{d}\mu_0} < 0$ for any fixed δ such that $\mu_1 = \mu_0 + \delta$ satisfies Assumption 1.

We consider cases $\mu_1 > \mu_0$ and $\mu_1 \le \mu_0$ separately.

Proof for $\mu_1 > \mu_0$. If $n = 1$, then P is given in (A.84). Let $y < 1$ be defined in (A.106). Then,

$$\frac{\partial P(\mu_1, \mu_0, c, p_0, 1, \rho)}{\partial \mu_0} = \frac{\mu_1 y \ln(y)}{(\rho+\mu_1)(\mu_1 - \mu_0)} < 0, \tag{A.130}$$

and

$$\frac{\mathrm{d} P(\mu_0+\delta, \mu_0, c, p_0, 1, \rho)}{\mathrm{d}\mu_0} = -\frac{\rho(1-y) - \mu_1 y \ln(y)}{(\rho+\mu_1)^2} - \frac{y}{(1-\bar{p})(\mu_1 - \mu_0)} < 0$$

with $\mu_1 = \mu_0 + \delta$ and $\bar{p}$ defined in (16), (A.131) where inequalities follow because $0 < y < 1$.

If $n = +\infty$, then P is given in (A.85). Then,

$$\frac{\partial P(\mu_1, \mu_0, c, p_0, +\infty, \rho)}{\partial \mu_0} = 0, \tag{A.132}$$

and

$$\frac{\mathrm{d}\, P(\mu_0 + \delta, \mu_0, c, p_0, +\infty, \rho)}{\mathrm{d}\, \mu_0} = -\frac{c\left(1 - \hat{p}_0\right)}{p_0\left(\mu_0 + \delta - c\right)^2} < 0. \tag{A.133}$$

Proof for $\mu_1 \leq \mu_0$. If $n = 1$, then P is given in (A.91). Then,

$$\frac{\partial P(\mu_1, \mu_0, c, p_0, 1, \rho)}{\partial \mu_0} = 0, \tag{A.134}$$

and

$$\frac{\mathrm{d}\, P(\mu_0 + \delta, \mu_0, c, p_0, 1, \rho)}{\mathrm{d}\, \mu_0} = -\frac{\rho}{(\rho + \mu_0 + \delta)^2} < 0. \tag{A.135}$$

If $n = +\infty$, then P is given in (A.92). Then,

$$\frac{\partial P(\mu_1, \mu_0, c, p_0, +\infty, \rho)}{\partial \mu_0} = -\frac{\rho + \mu_1}{(\rho + \mu_0)^2} \frac{\bar{p}(1 - p_0)}{p_0(1 - \bar{p})} < 0 \quad \text{with } \bar{p} \text{ defined in (16)}, \tag{A.136}$$

and

$$\frac{\mathrm{d}\, P(\mu_0 + \delta, \mu_0, c, p_0, +\infty, \rho)}{\mathrm{d}\, \mu_0} = -\frac{(\rho + \mu_1)^2 + (\mu_0 - \mu_1)(c + \rho)}{\mu_1(\rho + \mu_0)^2} \frac{\bar{p}(1 - p_0)}{p_0(1 - \bar{p})^2} < 0$$
$$\text{with } \mu_1 = \mu_0 + \delta \text{ and } \bar{p} \text{ defined in (16)}. \tag{A.137}$$

Appendix B. Supplementary material

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.jet.2023.105699.

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