FOREIGN CURRENCY OPTIONS: AN EMPIRICAL ANALYSIS

Submitted by

Thomas Khong Fock Phung

for the degree of

Doctor of Philosophy

of City University, London

Department of Banking and Finance

June 1998

Table of Contents

List of Tables

List of Figures

Acknowledgements

I would like to express my sincere thanks and gratitude to my supervisor Professor Gordon Gemmill for his advice and guidance throughout the Ph.D. process. In particular, I would like to thank him for many helpful comments and suggestions which I received during this research project.

I would also like to thank Professor Dean Paxson and Dr. David Newton from Manchester Business School in which I started my first year research, also to Dr David Edelshain from EMBA for his encouragement and helpful suggestions and the CUBS-EMBA programme supports for my two-years' research studentship. I am grateful to Philadelphia Stock Exchange and NatWest Markets for providing the data and many thanks to Ms Helen Lewis who had made the PHLX intra-day traded data available for my research. My special thanks to Dr Keith Garbutt at Midland Global Markets for supplying the CME data for my early field project.

Finally, I would like to thank my wife - Yi Mei for her understanding and complete support.

Declaration

Abstract of the Thesis

The objective of this thesis is to study the pricing of foreign currency options by using transaction data. It is to investigate empirically the hypothesis of option markets efficiency on the Philadelphia Stock Exchange (PHLX) between August 1987 and October 1994. It uses daily trade-to-trade transaction for the studies on (1) European put-call parity, (2) American put-call pricing relationship, (3) the early-exercise premium of the American options, (4) the volatility smile and risk-neutral distribution of the American options, (5) models for predicting option prices with yesterday's market volatility smile, and (6) the difference between volatility smiles on the over-the-counter (OTC) and PHLX markets.

The first three tests employ both non-transaction-cost and full-transaction-cost scenarios. The put-call party does not hold and the results provide evidence of arbitrage profit opportunities for investors (i.e., an investor might be able to earn risk-free profit). It is not possible to reject (with confidence) the hypothesis that PHLX currency option market is inefficient.

Deutsche Mark options are the most heavily traded options on the PHLX and they have therefore been used for sections (4) to (6). The fourth test confirms that the volatility smiles of calls and puts are asymmetrical, both calls and puts are skewed from the right to the left. The smile has an average positive (+1.2%) volatility skewness. The calls also have higher volatility than the puts in general. However, the OTC options in section (6) give a different result. The OTC and exchange markets have different volatility smiles and the OTC options' volatility smile is negatively skewed.

Although the observed options' volatilities have a smile, the predicting of tomorrow's option prices in the section (5) with today's market volatility smile gives larger errors than assuming no smile at all. The PHLX options in section (6) have an irregular wave-shaped volatility smile across strike prices and observed period. It reflects that prediction of option's volatility required a more powerful deterministic volatility function. In section (6), PHLX options have higher volatility than OTC options. Moreover, both PHLX and OTC markets have different volatility skewness, it allows investors with low transaction costs to obtain risk-free arbitrages.

In summary, the six tests have showed that the options' markets are not perfectly efficient.

List of Symbols

Notation for the Equations, Formulae and Tables

C	Price of a call option
P	Price of a put option
r	Domestic Dollar (US$) risk-free interest rate.
R	Foreign currency risk-free interest rate.
t	Option time period to expiry (T_1 - T_0), where T_1 is the date of expiration and T_0 is the date when option is contracted.
σ	Standard deviation of the foreign exchange spot rate.
e	Returns e raised to the power of number. For example, the approximate of e is [$e^1 = 2.718282$], where 1 is the exponent applied to base e and [$e^{ln(3)} = 3$].
S	Foreign exchange spot rate when option is contracted.
S^*	Foreign exchange spot rate at expiration.
X	Strike price of the currency option
F	Future price of the Foreign Exchange ($Se^{(r-R)t}$), where $(r-R)$ *is the* carrying cost of the underlying assets.
$N(.)$	Cumulative normal distribution function.
$n(.)$	Normal density function, where $n(x) = \dfrac{e^{\left(\frac{-x^2}{2}\right)}}{\sqrt{2\pi}}$.
X_1 & X_2	Strike price of the call and put options, where $X_1 < X_2$.
$C(X_2)_p$	Buying of the a call option at strike price (X_2).
$C(X_2)_w$	Selling of the call option at strike price (X_2).
$P(X_2)_p$	Buying of the put options at strike price (X_2).
$P(X_2)_w$	Selling of the put option at strike price (X_2).
+ve	Cash inflow from selling an option.
-ve	Cash outflow from buying an option.
t	Critical values of the t-distribution $\left(x\sqrt{s} / u\right)$, where x is the mean of the sample, u *is the* standard derivation of sample and s is the sample size

Chapter 1: Introduction and General Review

In recent years we have seen an increase in the trading volume of the currency options, both for exchange-listed and over-the-counter markets. The increase in international trade and in the importance of multi-national corporations have generated many currency transactions and exposures. The traditional method of hedging with forward contracts is widely applied, but there is also increasing trade in currency options. Since 1982 the presence of exchange-listed currency options has allowed investors to hedge their positions with standardised contracts.

The exchanges and banks have developed a variety of new financial instruments to give customers the option to buy or sell foreign currencies. Exchanges in Amsterdam, Montreal, and Philadelphia started trading in standardised options on foreign currencies in 1982. Banks also responded by resurrecting an old practice of writing tailor-made foreign currency options for their customers.

1.1 Objective of the Study

Most academic studies (reference later in the thesis) have shown that currency options are priced efficiently. However, they typically used small sets of data on closing prices and even done in the 1980s.

This study uses transactions data from the Philadelphia Stock Exchange. Its purpose is to answer the following questions:

a) Do European put and call options exhibit the expected parity relationship?

b) Is the put and call pricing relationships for American options as expected?

c) Is there an early-exercise premium in American options and is it of the expected size?

d) What do the options indicate about implied asset distributions?

e) How do the forecasting performance of difference model for currency options compare?

f) Are the implied distributions for OTC options and exchange-traded options similar?

1.2 Outline and Limitation of the Studies

There are six separate empirical analyses on currency options in this thesis. Each chapter is relatively self contained.

In Chapter 2, the study examines the arbitrage relationships between the European-style put and call option prices. The option prices are intra-day trades on the Philadelphia Stock Exchange (PHLX) from August 27 1987 to October 18 1994. It tests the frequency of violations of the put-call parity and box-arbitrage conditions. Approximately 94% of the selected put-call pairs violate parity and more than 80% of the selected box-groups violate the box-conditions. These violations represent risk-free arbitrage opportunities. After accounting for transactions costs, the arbitrage opportunities fall to 27% and 10% for put-call pairs and box-groups respectively. These significant numbers of arbitrages seem to indicate inefficient pricing of the options. Moreover there is no systematic decline in arbitrage over time as the market matures.

Chapter 3 extends the analysis to examine the pricing efficiency on the intra-day American currency options. The put-price in the traded put-call pair is used to estimate the call-price and compares it with the traded call-price. Almost all **traded** call prices are different from the **estimated** call prices. If the traded put options are correctly priced, then the traded call options are mispriced. When transactions costs are incorporated, there is still about 20% of risk-free arbitrage opportunities in the observed currencies. This result is robust to changes in: the method of pairing and to observation time and spot price. Dividing the sample into two sub-periods, only Australian Dollar and Swiss Franc systematically improved in pricing efficiency over time. The Deutsche Mark in the second period represents a risk-free arbitrage profit of an average $2,500 (100 trades with mispricing of above $50) per trading day. The results in the regression analysis show that the moneyness has significant positive (over-priced) effect on in-the-money British Pound call options and also has negative (under-priced) effect on the in-the-money Canadian Dollar, Deutsche Mark and Japanese Yen call options. Moreover, it also shows that time-to-expiration has significant positive (over-priced) effects on long-expiration's Canadian Dollar and Deutsche Mark options and has negative (under-priced) effects on long-expiration's British Pound, French Franc and Japanese Yen options.

Chapter 4 examines the early-exercise premium of the American options. The analysis of early-exercise premium requires the matching of European and American options. The numbers of traded transactions of the European-style options are lessees than the transactions of the American-style options. This has restricted the tests on the early-exercise premium to the maximum matching numbers of European options to the American options. The American-style option is similar to the European-style option and, in addition, it gives the buyer the right to exercise before the expiration. This early-exercise feature has an extra cost, (called the early-exercise premium), therefore, the American-style option should cost at least as much or more than the European-style option. The study also compares the option prices between the American and European-style currency options traded at the Philadelphia Stock Exchange. This process involves a complicated pairing procedure for the two types of options. The results show that more than 60% of American puts and 20% of American calls are priced below the equivalent European options. The under-pricing of American-style options violates market efficiency. It suggests that market-makers are able to make risk-free profit by buying relatively under-priced American options and writing relatively over-priced European options. When transactions costs are accounted for, the risk-free arbitrage opportunities are reduced to 23% for puts and 1% for calls in the samples. Dividing the samples into two sub-periods, the results show that only the Australian Dollar and Canadian Dollar have systematically improved in pricing over time. The put options of Deutsche Mark, British Pound and Swiss Franc have worsened over time. The Deutsche Mark puts in the second sub-period offered a risk-free arbitrage profit of more than $5,000 per trading day. The results of a regression analysis show that mispricing is sensitive to time-to-expiration, however, the put options have negative (under-priced) effect to time-to-expiration, i.e., the long-expiration puts tend to be under-priced relative to the long-expiration calls. In summary, the early-exercise premium tests show that the assumption that American options are more expensive than European options does not hold and this provides a risk-free arbitrage opportunity.

The earlier three chapters show that mispricing exists in both European and American options, moreover, the American options are more volatile and their options' prices are below European equivalents. Chapter 5 examines volatility smiles and risk-neutral probability distributions of the American-style Deutsche Mark options traded on Philadelphia Stock Exchange. The volatility smile is asymmetrical, particularly in the later period of observation. It is skewed upward for moneyness [F/X] greater than 1. The volatility smiles from calls and puts both imply higher volatility on the right-hand side of the moneyness. The smile skewness has a mean of +1.2% over 1,763 observations. The skewness changes from generally negative before 1991 to generally positive after 1991. The implied distribution is recovered from the volatility smile with a modified version of the Shimko's (1993) method. The quarterly averages for implied distributions derived from calls versus puts have different probabilities across strike prices. This is consistent with the deviation which was found in the earlier chapters on the put/call pricing relationship (in Chapter 3). The results on implied distributions show skewness that corresponds to the smile skewness, but the kurtosis which is around 3 (i.e., not a leptokurtic distribution). The differences between implied distributions for calls and puts indicates that the level of mispricing changes across the moneyness.

In Chapter 6, the study examines the stability of the volatility smile for Deutsche Mark options on the Philadelphia Stock Exchange (PHLX), assuming different pricing models. The objective is to price tomorrow's option with today's market volatility smile by using four alternative option pricing models. The analysis requires the separation of the traded options into "bid" and "ask" trades. However, the records do not contain the "bid/ask" details, i.e., whether it is a "bid" or "ask" trade. An assumption has been used to determine each trade as a "bid' or "ask" by matching the trade prices with the bid/ask quotes. Sample period, number of observations and the trades are separated into two sub-groups, the "bid" and "ask" contracts, in order to ascertain whether each of the sub-groups performs better than the overall "average" volatility. There are three choices of volatility assumption used for the pricing, i.e., (i) constant volatility, (ii) volatility with smile effect and (iii) adjusted volatility with smile effect. The results show that the modified Hull and White (1987) stochastic volatility model and the Barone-Adesi and Whaley (1987) model provides better estimates than the other two option models (European and CEV models). They have smallest root-mean-square-error with the constant-volatility assumptions for "ask" and "bid" trades with overall "average" volatility and "bid" trades with "bid" volatility, while only "ask" trades with "ask" volatility for adjusted volatility-with-smile effect assumption. The volatility-with-smile assumption does not perform better than the constant-volatility assumption. The results show that implied volatility is superior to historical volatility. However, models which allow for a smile (deterministic volatility models) are only marginally better than the simple Garman-Kohlhagen (1983) model.

In Chapter 7 the price of OTC and exchange-traded options are compared. The information from the OTC options is limited due to the methods of collection. The data are quotes of risk-reversal spreads, strangle spreads, and at-the-spot volatility with the spot rate collected at London's mid-day. An quadratic approximation is used to recover the OTC option's volatility across strike prices. The PHLX options are daily transaction options, an quadratic approximation is also used to recover the PHLX option's volatility of one-month expiration. Both options are of the same currency pair, but are traded at different locations: OTC is in London and PHLX is in United States. However, both markets should have similar expectations on the volatility smiles, because they are of the same underlying currency pair and closely linked and traded at approximately the same time. The results show that the PHLX has higher volatility for the out-of-the-money calls and the OTC market has higher volatility for the out-of-the-money puts, i.e., PHLX calls are more expensive than the OTC calls, while the OTC puts are more expensive than the PHLX puts. The markets have differences in volatility skewness: PHLX has an average of small positive (+0.01%) skewness compared with OTC which has an average of negative (-1.47%) skewness. The implied distributions of both OTC and PHLX are leptokurtic with the kurtosis significantly larger than 3 (except for Mar-94 of PHLX). The differences in the volatility smiles have shown that the PHLX trades are Deutsche Mark biased, whereas OTC trades are Dollar biased during my observed period. Due to limited data points for OTC trades, the OTC smiles are constant over the observed period. PHLX smiles and distributions are not consistent over the observed period.

The final chapter draws together the main conclusions derived from the previous analysis and gives some suggestions for further research.

1.3 General Review of the Literature on Currency Option Pricing

This section outlines briefly the literature on options which is relevant to currencies. The specific literature which has direct relevance with the empirical work in this thesis is explained in each respective chapter.

1.3.1 Introduction of the Philadelphia Stock Exchange

The Philadelphia Stock Exchange (PHLX) was the first exchange in the world to introduce exchange-listed options on currencies. Thereafter, the opportunities to hedge in foreign currencies was not restricted to a small number of institutions who were able to make substantial investments and take substantial risks. The exchange-traded currency options supply versatile and well-leveraged hedging instruments with limited risk. However, these hedging instrument have a cost called option's premium.

The objective of the PHLX is to provide a continuously active and liquid market on put and call currency options. It enables option buyers and writers to liquidate existing option positions by off-setting transactions prior to expiration. The exchange's first American-style options on Dollar per Sterling started from December 12 1982. The European-style options started from August 27 1987 while the cross-rate Deutsche Mark/Japanese Yen options started in 1991 and Deutsche Mark/Sterling options started in 1992.

1.3.2 Currency Options' Models

Before reviewing the literature on currency options, we will review the research and development of the pricing models for options in general. Since the work of Black and Scholes (1973), there have been many extensive works on option pricing. The first attempt to obtain a foreign currency option price was by Feiger and Jacquillat (1979). They developed the pricing of a currency option bond, however, they did not provide a simple closed-form solution. Stulz (1982) continued with the currency option bond pricing, however with his approach to default-risk it was not easy to grasp fundamentals of currency option pricing.

The European currency option pricing models are modified versions of Black and Scholes (1973), Merton (1973a) and Black (1976) models. Three separate papers by Biger and Hull (1983), Garman and Kohlhagen (1983) and Giddy (1983a) covered the option pricing model in details. Moreover, Grabbe (1983) extended the pricing model to account for two relevant stochastic interest rates using Merton (1973a) approach on incorporating the stochastic interest rates into the option pricing model.

The later work by Cox, Ingersoll and Ross (1985) developed the discount bond option pricing model for currency options. Yang (1985) examined the pricing of European currency options with the development of pricing model follows the risk-neutral approach of Cox and Ross (1976) with the mix application of the Merton's (1973a) model.

The pricing of American options has been explained by Gemmill (1986), this is due to its additional early-exercise feature. It has significant value to the options and pricing these options with simple European model may result in a large error. He suggested that the binomial pricing approach for options with early-exercise values. The binomial approach from Cox, Ross and Rubinstein's (1979) method can be used. An analytic solution for the American option pricing model was developed by MacMillan (1986) and Barone-Adesi and Whaley (1987) and the model was used for pricing American dividend yield equity and currency options with the accountability of early-exercise premium.

The above standard currency option pricing models are widely used with various assumptions on volatility and both interest rates. The recent developments on option model are more concentrate for the exotic models.

1.3.3 Currency Options Empirical Studies

The study by Black (1975) explained the equity options in detail. Figlewski (1989a) extended the theories of option valuation with the Black and Scholes' model, using the less sophisticated mathematics to demonstrate the consistency of the Black and Scholes' formula. On currency options, Gendreau (1984) has provided an outline of development on the currency option markets and details of the currency options in the PHLX. Pasmantier (1992) extended this information to currency options in CME futures market and over-the-counter markets. Hopper (1995) further explained in detail the use of currency options: its function, types of risks and exposure related to the options.

The earliest empirical study on currency options was conducted by Gadkari (1986). His results showed that the European model was easier to use for calculating, however, it mispriced the American options due to the early-exercise feature.

Adams and Wyatt (1987a and 1989) examined prices of the American and European options. They tested American options' premiums and showed that the European model was mispricing American options due to the early-exercise feature. Adams and Wyatt (1987b) later examined the biases in option prices with PHLX options by applying Garman and Kohlhagen's (1983) and Grabbe's (1983) models. After adjusting the interest rate, the model tends to overprice the call options.

Bodurtha, Jr. and Courtadon (1987a) examined the PHLX options with an American option model. They found that out-of-the-money options were under-priced relative to at-the-money and in-the-money options. Their results suggested that a mixed-jump-diffusion process model can be used to correct the mispricing on options' valuation. Bodurtha, Jr., and Courtadon (1987b) later compared foreign currency options between spot and futures markets. Their results showed that there were pricing differences on the options' valuation. Borensztein and Dooley (1987) also examined currency options traded on PHLX by using the database of Bodurtha and Courtadon (1986). The results showed the option prices were consistent with information of exchange rate expectations.

The mispricing raised concern on the hedging process. Hull and White (1987b) provided ways to hedge risk for currency option sellers. The results suggested that short maturity options priced with constant volatility should use a mix of delta plus gamma hedges. Eisenberg (1993) later examined the second-order derivative to Garman and Kohlhagen's (1983) model and the cross-currency model of a derivative, the results suggested that the interest rate hedges and gamma hedges for the foreign currency option markets are invariant to book-currency, however, hedging an illiquid cross-currency option with no mixed gamma risk was not invariant to book-currency.

The area of implied volatility was first studied by Latane and Rendleman (1976). On currency options, Heaton (1986) tested the options' market efficiency by examining the options' implied volatility functions. His results showed that the implied volatility reduced noise for forecasting the option prices, hence performance better than the historical volatility. Shastri and Wethyavivorn (1987) later examined the currency option prices with four valuation models. The results showed that only options' implied volatilities obtained from the Cox and Ross's (1976) pure diffusion model were consistent with the function of exchange rate across the strike prices.

Taylor and Xu (1994c) examined the implied volatility of S&P-500 future options when option prices and the variance were correlated. They used the square-root volatility model which predicted a smaller magnitude of smiles and skewness than the empirical results. Taylor and Xu (1995) showed that the informational efficiency of the PHLX currency option market by using the ARCH approach. They found that implied volatility contained incremental informational relative to standard ARCH specifications than the information derived from past return.

Guo (1996a) extended the test on information content with the implied stochastic volatility on PHLX Dollar/Yen currency options. The results showed that Hull and White's (1987a) model was better than Heston's (1993) model and was also slightly better then the Black and Scholes' (1973) model. Bates (1996a and 1996b) applied stochastic volatility and jump-diffusion approaches to Deutsche Mark futures options. The jump-diffusion approach allows for the maturity effects on skewness and excess kurtosis of the option prices. The results showed that jump-diffusion model's performed was better than standard stochastic volatility model.

Other areas of currency options were examined by Briys and Crouhy (1988) on the creation and pricing of the hybrid currency options by banks and financial institutions. Melino and Turnbull (1995) examined the long-term currency options, i.e., options with expiration more than 12 months. They concluded that constant volatility performed poorly versus the stochastic volatility approach over longer expiration options.

The above literature shows that the options' markets are efficient and levels of mispricing are below transaction costs. Moreover, the implied volatility of options have incremental informational of future expectation for underlying assets. However, the data were mostly day-end prices or bid/ask quotes; only a few used trade-to-trade options prices, which would provided stronger support for the market efficiency test.

1.3.4 Selected Papers on Option Pricing for Other Assets

In other options, Ogden and Tucker (1987) conducted an empirical efficiency test on the synchronous transaction data of American futures options traded on CME. The results showed that options had 3.1% of violation for lower put-call parity bound, however, then were less than 0.7% of arbitrage opportunities after accounting for the transaction costs. Jorion (1995) later examined the information content and predictive power of implied volatility of CME currency future options. The results showed that the implied volatility appeared to be biased as a forecasts.

On deterministic volatility functions, Dupire (1993) developed a stochastic volatility term structure model which provided preference-free exotic option prices. He showed that the market practitioners were looking for arbitrage pricing and not the equilibrium pricing, which was sensitive to preferences and expectations. Paxson (1994) examined the implied smile effect on LIFFE equity options. The results indicated that the volatility levels vary across strike prices. In order to price options more accurately, the stochastic volatility function must be incorporated with pricing models to adjust the volatility.

Stein and Stein (1991) derived the stochastic volatility option model with an analytic approach, however, the model assumed non-correlation between volatility and spot rate. It relaxed the accountability of skewness effect arise from the correlation. Heston (1993) derived a closed-form stochastic volatility solution for pricing bond and currency options, which accounted for the risk-premium whereas Hull and White (1987a) assumed it was zero. He concluded that the model could overcome all types of bias to option prices. However, Guo (1996a) found that Hull and White (1987a) model performs better on the PHLX options.

Amin and Ng (1993) extended the works of Rubinsten (1976) and Brennan (1979) to derive an option model accounted for systematic stochastic volatility. However, this jump-diffusion formula is significant different from Merton's (1976) model. it set higher prices for options at- and in-the-money and set lower prices for options out-of-the-money.

The above relevant literature suggested that correct volatility is the key variable for an accurate option price. The deterministic volatility functions can be too costly for the standard currency options. However, it is one of the accurate pricing model.

1.3.5 The Implied Probability Distribution

The initial studies on the risk-neutral distribution, made by Breeden and Litzenberger (1987), were applied by Shimko (1993), using the second derivative with respect to the strike price. Other approach with two-log-normal distributions was examined by Bahra (1996) on future asset prices. Mizrach (1996) applies a mixture of log-normal distributions by using a Monte Carlo option model approach to recover the implied distribution of British Pound and French Franc options traded on PHLX during the ERM crisis. A separate tests on the ERM crisis was made by Malz (1996), who examined the cross-currency OTC options with the implied distribution recovered from the risk-reversal premium, strangle spread, at-the-money volatility and spot rate using the mixture of log-normal distribution.

Garcia, Sherrick and Tirupattur (1996) examined (CBOT) options on markets by applying the implied distribution recovered from mixture of log-normal distributions. Melick and Thomas (1997) examined crude oil options on futures with the implied distribution recovered from the option prices, and their recovering methods had provided better explanations on the unsettled, i.e., during the Persian Gulf crisis, than the standard techniques.

This implied distribution technique is used in the later chapters in order to provide the distribution on the volatility smile and volatility skewness.

1.3.6 The Implied Volatility

Volatility is the term to be used in finance to denote the standard deviation of returns on an asset. The only unknown in the Black and Scholes option pricing model is the volatility, so that trading in options is effectively trading in volatility. It is possible to solve iteratively in order to find the volatility which equates model and market prices, which is called implied volatility. There are many methods to impute the implied volatility.

The early imputation of a single weighted implied volatility by Latané and Rendleman (1976) used the partial derivative of call price with respect to volatility to give weight. Chiras and Manaster (1978) used elasticity of call with respect to volatility for their weights. Macbeth and Merville (1979) examined the option volatility with Black and Scholes (1973) model, finding that the implied variances were different in general across strike prices and time-to-expiration. A better method were developed by Koehler and Manaster (1982) to estimated the implied volatility from option prices with the Black and Scholes (1973) model. Finucane (1989) later developed a linear weighting function to calculate the implied volatility from Black and Scholes (1983) call and put options. The simple methods by Brenner and Subrahmanyam (1988) and Feinstein (1988) are almost linear in volatility when option is at-the-money, however, the accuracy for options away-from-the-money, therefore less good. This method was further improved by Chance (1993) using a quadratic approach in respect of the changes in volatility which corrected the errors from both the tails.

Lai, Lee and Tucker (1992) applied the inverse of standard cumulative normal distribution to develop a new method for calculating implied volatility. This method was further developed by Corrado and Miller, Jr. (1996a, 1996b, and 1996c) with a simple quadratic approximation on the Black and Scholes' (1973) formula for the calculation of implied volatility which was accurate for options across a wide range of strike prices.

In the later chapters, the method for estimating implied volatility is explained in detail where we compute the implied volatilities from market prices.

1.3.7 Summary

Early research tended to show that the option market was efficient, however, most studies were conducted with day-end prices and bid/ask quotes. The literature showed that volatility is the key element for the option price. Therefore our study will be focused on the volatility of the put and call prices. Moreover, the increase in volume of volatility trading, the risk-reversal spread and volatility skewness suggests volatility changes systematically and that distributions are not lognormal.

1.4 Appendix: The Options Terminology

Although the nomenclature of the option trading has been discussed earlier, a brief review may be helpful as followings:

A call option - An option to purchase a stated number of units of the underlying currency at a specific price per unit during a specific period of time.

A put option - An option to sell a stated number of units of the underlying currency at a specific price per unit during a specific period of time.

Option buyer - The party who obtains the right, by paying a premium, that is conveyed by an option: The right - but not an obligation - to buy the currency if the option is a call or to sell the currency if the option is a put. The option buyer is also known as the option holder.

Option seller - The party who is obligated to perform if an option is exercised: To sell the currency at a stated prices if a call is exercised or to buy the currency at a stated price if a put is exercised. The option seller is also known as the option writer.

Strike price - The price at which the option buyer (holder) has the right to purchase or sell the underlying currency. The strike price is also known as the exercise price.

Expiration month - The expiration months for options on currency are March, June, September, December plus an additional near two months. In addition, Long-Term options with 18 to 24 month expirations are available for June and December[1]. At any given time, trading is conducted in options which expire in 1, 2, 3, 6, 9, 12, 18, 24 months.

Expiration date - The last date on which an option may be exercised: In the case of regular or mid-month currency options, the Friday before the third Wednesday of the expiration month. The month-end options which expire on the last Friday of the month in the three near term expiration months.

Option price - The option price is, in effect, the price of an option - the sum of the money that the buyer of an option pays when purchasing an option and that the writer of an option receives when selling an option. The option price is also known as the option premium.

American option - Option holder may exercise their options on any date prior to the expiration date.

European option - Option holder is restricted from exercising until the expiration date.

Intrinsic value - An option is said to have intrinsic value if and to the extent that the option would currently be profitable[2] to exercise. In the case of a call, if the spot price of the underlying currency is above the option's strike price. Or, in the case of a put, if the spot price is below the option's strike price. An option with intrinsic value is said to be in-the-money.

At-the-money - An option whose strike price is the same - or near the same - as the spot price.

---

[1] Initially listed 18 and 24 month expirations on four currencies. PHLX has information on the Long-Term options for 30 and 36 month expirations.

[2] For the purposes of determining intrinsic value, the option premium and transaction costs are excluded. They are essential in determining the ultimate profitability of an option transaction.

Out-of-the-money - A call whose strike price is above the current spot price of the underlying currency or a put whose strike price is below the current spot price of the underlying currency. Out-of-the-money option has no intrinsic value.

In-the-money - A call whose strike price is below the current spot price of the underlying currency or a put whose strike price is above the current spot price of the underlying currency. In-the-money option has intrinsic value.

Chapter 2: Put-Call Parity on European Currency Options

2.1. Introduction

In a perfect world, a European call option can be replicated with a European put option and a forward contract. This replication is known as the put-call parity relationship. When prices violate put-call parity, an arbitrage opportunity exists.

The purpose of this chapter is to examine the pricing relationships between put and call options on currency traded at the Philadelphia Stock Exchange (PHLX). The study investigates both put-call parity and box-arbitrage conditions on intra-day transaction data. These conditions have the advantage that they allow examination of market efficiency without using a particular option pricing model. There have been many earlier studies of these arbitrages, but they mostly used equity-index options: only a few used currency options. In addition, most of their data were either day-end prices or non-traded bid-ask quotations.

A test for market efficiency is to see if violations exist on an ex-ante basis. Such a test would be from the viewpoint of a trader who observed a deviation then tried to exploit it at the next available opportunity. This study (on the daily trade-to-trade data) makes such an ex-ante test.

The chapter has the following structure. It begins with a review of previous research on put-call parity and box-arbitrage conditions (in section 2.2). The data base is described (in section 2.3) and methodology is explained (in section 2.4). The results of put-call parity and box-arbitrages conditions are in sections 2.5 and section 2.6 respectively. The conclusion is in section 2.7.

2.2. **Previous Research**

There are many previous studies on the pricing of put and call options, especially on European-style options, i.e., options whose only possible exercise is at expiration. These options have deterministic relationships between put and call prices on the same underlying asset.

2.2.1 Previous Research on Put-Call Parity

A very early study was conducted by Boness (1964) on over-the-counter equity options. His results showed that the prices of call options were high relative to put options, due (he said) to higher volume traded on the call options. The assumption of higher call option prices remained until Stoll (1969) conducted a study with put and call options prices of 25 companies and derived the put-call parity condition. He used the no-arbitrage principle to price call (put) options relative to put (call) options. His theory was supported by time-series and cross-section regression analysis. This parity only applies to European-style options and was clarified by Stoll (1973) after comments from Merton (1973b). Following Merton's (1973b) suggestion, Gould and Galai (1974) extended Stoll's work and demonstrated some violations for equity options, however these were eliminated after taking into account transactions costs, i.e., put-call parity held.

Two of the earliest put-call parity studies on currency options were conducted by Goodman, Ross and Schmitt (1985) and Shastri and Tandon (1985). Both studies used daily closing price of the PHLX. Their data were obtained from the Wall Street Journal for period January 1983 to October 1983. Goodman, Ross and Schmitt (1985) compared the American put and call option prices with theoretical values by using the Garman-Kohlhagen (1983) option pricing model. Their results showed violation of parity, but violations were eliminated after taking into account transactions costs. Shastri and Tandon (1985) used a PHLX sample of options on the British Pound, Japanese Yen, Deutsche Mark, and Swiss Franc. They found approximately 39% of options mispriced by more than $50 per contract. However, their data had a problem of synchronisation, because end of day traded options were used. Bodurtha and Courtadon (1986) later conducted a synchronised test on transactions data for PHLX American-style currency options from February 28 1983 to September 14 1984. Their tests were based on the transaction-cost-adjusted early exercise and put-call-parity pricing boundaries. Their study showed that the market was efficient if the tests used simultaneous prices and corrected for transactions costs.

In summary, the above studies suggest that it is important (i) to use synchronised data (for spot rates and option prices) when testing for market efficiency and (ii) to account for transactions costs. Although Bodurtha and Courtadon (1986) showed that the currency options market was efficient, their data were for American-style rather than European-style options. This study applies put-call parity to trade-to-trade data on European-style options, and so is able to produce a much stronger test of market efficiency.

2.2.2 Previous Research on Box-arbitrage Conditions

The box-arbitrage condition has also attracted much attention. Shastri and Tandon (1985) tested the PHLX data with this condition. Their results showed that the violation of the box-condition was smaller than for put-call parity: 25% of the box-arbitrage groups gave profit opportunities in excess of $100 per contract. The box-arbitrage condition also has less arbitrage opportunities than put-call parity because of higher transaction costs (as it requires two put-call option pairs). Chance (1987) showed that S&P-100 index options violated put-call parity but not the box-arbitrage condition, because an over-priced option pair in the box was matched by an under-priced option pair. Ronn and Ronn (1989) applied a high transaction cost to set-up the box-arbitrage condition. They tested the lower box-arbitrage condition bound on the option-spread position using the Berkeley Options Data Base for Chicago Board Options Exchange prices. They found that arbitrage opportunities (784 positions over sample period from 1977 to 1984) only existed for lower transaction-cost agents who could implement strategies cheaply.

2.3. Data and Sample Selection

The purpose of this section is to explain the data and the sample selection criteria. Sample selection is very important in this study because we use intra-day traded data and we need to ensure this selection will provide a synchronised data set for puts and calls.

2.3.1 Data

The Philadelphia Stock Exchange (PHLX) started currency options trading with American-style options (British Pound) on December 12 1982. The exchange currently has eight dollar-based options (both American- [except for ECU] and European- styles); they are British Pound, Australia Dollar, Canadian Dollar, Deutsche Mark, Swiss Franc, French Franc, Japanese Yen, and European Currency Unit [ECU]. The cross-rate options are Deutsche Mark / Japanese Yen, and British Pound / Deutsche Mark. The first European style options started trading on August 28 1987.

The data base consists of trade-by-trade records of all transactions for European options from August 28 1987 to October 18 1994. It contains the following information; the trade date, the style (call or put, European or American) and currency, expiration month, strike price, time-of-trade (to the minute), number of contracts traded, traded price, ask and bid prices, and the simultaneous spot exchange rate. It records the underlying inter-bank spot exchange rate (Telerate) at the exact clock time for every trade. The data allow complete synchronisation of option prices and underlying exchange rates.

2.3.2 Sample Selection

This study selected options from seven different currencies between August 28 1987 and October 18 1994. They are Australian Dollar, Canadian Dollar, Deutsche Mark, French Franc, Japanese Yen, British Pound, and Swiss Franc. The options are "mid-month options" for which the contracts expire on the Friday[3] before the third Wednesday of the contract expiry month.

[3] Prior 1995, the contract expired on the Saturday before the third Wednesday of the contract expiry month. However, there was no trading on Saturday and expiring option classes cease trading on the day before the expiry.

Table 2.1 shows the currency of denomination, the contract size and the minimum required premium change for each currency. This table is used in the later sections to convert the mispriced values into US dollars per contract.

Table 2.1: Options Contract Details

Products	Base Currency	Underlying Currency	Contract Size of Underlying	Minimum Premium Change
Australian $	US$	A$	50,000	0.(00)01 = \$5.000
British Pound	US$	GBP	31,250	0.(00)01 = \$3.125
Canadian $	US$	C$	50,000	0.(00)01 = \$5.000
Deutsche Mark	US$	DM	62,500	0.(00)01 = \$6.250
French Franc	US$	FFr	250,000	0.(000)02 = \$5.000
Japanese Yen	US$	Yen	6,250,000	0.(0000)01 = \$6.250
Swiss Franc	US$	SFr	62,500	0.(00)01 = \$6.250

The total traded volumes and numbers of the trades for European-style options are in Table 2.2. Panel A in Table 2.2 shows the whole sample while Panels B and C show the two sub-periods. The first sub-period is from August 28 1987 till December 31 1990 and the second sub-period from January 1 1991 until October 18 1994.

There are 5,571,851 calls and 4,623,243 puts in the data base, i.e., a total of 10,195,094 options. Of these, only 1,080,067 options were traded in the first sub-period while 9,115,027 options were traded in the second sub-period.

In the first sub-period, the most active contracts (by number of options) were for Australian Dollars (319,963) and Swiss Franc (263,508). In the second sub-period of the sample, the French Franc was by far the most active contract (6,396,536), followed by the Deutsche Mark (1,885,624).

There is a contrast between number of contracts and number of trades. In the second sub-period the Deutsche Mark was the most active currency by number of trades, but there were some very large trades in the French Franc which resulted is its high volume[4].

[4] These trades were probably related to arises in the European Monetary System in September 1992.

Table 2.2: Total Volume and Transactions of the European Style Option

Panel A: Whole Period

Option's Currency	Start Date	End Date	Call (V)	Put (V)	Call (T)	Put (T)	Call (V/T)	Put (V/T)
Australian Dollar	Aug-28-87	Oct-18-94	204,413	208,847	1,917	1,718	107	122
British Pound	Aug-28-87	Oct-18-94	155,017	113,172	3,832	3,372	40	34
Canadian Dollar	Aug-28-87	Oct-18-94	144,096	133,954	3,501	3,576	41	37
Deutsche Mark	Sep-02-87	Oct-18-94	1,192,185	749,539	10,716	11,893	111	63
French Franc	Aug-31-87	Oct-18-94	3,461,467	2,955,402	2,812	2,382	1,231	1,241
Japanese Yen	Sep-11-87	Oct-18-94	120,311	217,971	2,823	2,757	43	79
Swiss Franc	Sep-02-87	Oct-18-94	294,362	244,358	5,985	5,295	49	46
		Total	5,571,851	4,623,243	31,586	30,993		

Panel B: First Sub-period (August 27 1987 to December 31 1990)

Option's Currency	Start Date	End Date	Call (V)	Put (V)	Call (T)	Put (T)	Call (V/T)	Put (V/T)
Australian Dollar	Aug-28-87	Dec-31-90	159,360	160,603	1,152	1,095	138	147
British Pound	Aug-28-87	Dec-31-90	41,342	37,750	827	714	50	53
Canadian Dollar	Aug-28-87	Dec-31-90	80,416	82,838	2,022	2,060	40	40
Deutsche Mark	Sep-02-87	Dec-31-90	21,274	34,826	562	514	38	68
French Franc	Aug-31-87	Dec-31-90	1,022	19,311	36	69	28	280
Japanese Yen	Sep-11-87	Dec-31-90	27,075	150,742	490	991	55	152
Swiss Franc	Sep-02-87	Dec-31-90	138,410	125,098	1,368	1,152	101	109
		Total	468,899	611,168	6,457	6,595		

Panel C: Second Sub-period (January 1 1991 to October 18 1994)

Option's Currency	Start Date	End Date	Call (V)	Put (V)	Call (T)	Put (T)	Call (V/T)	Put (V/T)
Australian Dollar	Jan-01-91	Oct-18-94	45,053	48,244	765	623	59	77
British Pound	Jan-01-91	Oct-18-94	113,675	75,422	3,005	2,658	38	28
Canadian Dollar	Jan-01-91	Oct-18-94	63,680	51,116	1,479	1,516	43	34
Deutsche Mark	Jan-01-91	Oct-18-94	1,170,911	714,713	10,154	11,379	115	63
French Franc	Jan-01-91	Oct-18-94	3,460,445	2,936,091	2,776	2,313	1,247	1,269
Japanese Yen	Jan-01-91	Oct-18-94	93,236	67,229	2,333	1,766	40	38
Swiss Franc	Jan-01-91	Oct-18-94	155,952	119,260	4,617	4,143	34	29
		Total	5,102,952	4,012,075	25,129	24,398		

Key: (V) is volume, i.e., number of option contracts.
(T) is transactions, i.e., number of trades.
(V/T) is volume per transaction.

It is extremely unlikely that the put and call options are traded at exactly the same time and with the same spot rate. Therefore sample selection is important. The aim is to obtain a sample in which all the options in an arbitrage position are traded within a given period of time and within a given range of spot rates. For selection in any group, put-call pairs must be traded on the same day with the same strike price and with the same time-to-expiration. We then want to choose put-call pairs which are as close as possible in time of trade and currency rate. Four different samples have been selected, based on (i) the changes in spot rate between put and call trades and (ii) the changes in time-of-trade between put and call trades. The selection criteria are summarised in Table 2.3. Group A is the most-restricted set, allowing no change in spot rate and a 30-minute window. Group B has at most a 5 basis-point change in spot rate with a 30-minute window, Group C at most a 10 basis-point change in spot rate and 60-minute window, while Group D includes all pairs on any day, which is the least-restricted set.

Table 2.3: Selection Criteria

Options' Variables	Group A	Group B	Group C	Group D
Trade Date	Same	Same	Same	Same
Expiration	Same	Same	Same	Same
Strike Price	Same	Same	Same	Same
Change in Spot Rate	Zero	5 pb*	10 bp**	>10bp**
Change in Trade Time	30-min	30-min	60-min	>60-min

Key: *FFr is half a basis-point while Yen is one twentieth of a basis-point.
**FFr is 1 basis-point while Yen is one tenth of a basis-point.

Table 2.4 shows the number of non-repeating (i.e., not double counted) put-call pairs in each of the selections from the sample period. Group D has the largest number of matches and it represents 9.8% (2,924) of the total options traded during the selected period. Groups C and B, with more control over both changes in time-of-trade and changes in spot rate, fall to 6.0% (1,776) and 5.2% (1,531) respectively. Group A, with the most-restricted control, is only 2.6% (782) of total trades. In summary, sample selection is critical: it ensures that the data are synchronised for use in the ex-ante tests.

Table 2.4: Selected Put-Call Pairs of Trades

Products	Group A	Group B	Group C	Group D
Australian Dollar	55	65	71	90
British Pound	38	96	119	232
Canadian Dollar	121	175	201	289
Deutsche Mark	308	638	739	1,398
French Franc	78	168	201	290
Japanese Yen	64	122	144	195
Swiss Franc	118	267	301	430
Total	782	1,531	1,776	2,924
% of Total Sample	2.63%	5.15%	5.97%	9.83%

Key: Group A - Same Date; Expiry; Strike Price, Spot Rate, Trading Time 30mins gap
Group B - Same Date; Expiry; Strike Price, Spot Rate 5bp* gap, Trading Time 30mins gap
Group C - Same Date; Expiry; Strike Price, Spot Rate 10bp** gap, Trading Time 60mins gap
Group D - Same Date; Expiry; Strike Price, Spot Rate >10bp**, Trade Time >60mins gap

2.3.3 Interest Rates

The domestic and foreign interest rates for each currency are the London Euro-currency deposit interest rates of cross currencies. These have been obtained from Datastream as 1 day, 1 week, 1 month, 3 months, 6 months, and 1 year rates. Rates for intermediate dates have been linearly interpolated.

2.4. Theorv, Methodology and Transactions Costs

The purpose of this section is to explain theories of put-call parity and box-arbitrage condition.

2.4.1 Put-Call Parity Theory

In order to prove that put-call parity will hold in market equilibrium, Tables 2.5 and 2.6 illustrate possible arbitrages. In Table 2.5, portfolio 1 consists of buying a put option, buying a forward contract and borrowing the present value of the option's strike price. Portfolio 2 is simply buying a call option. At expiration, the future spot rate (S*) can be at any of three positions relative to the strike price (X). The net cash flow of portfolio 1 is equal to that of the call option at any of these future spot rates. Therefore, a call option can be replicated by portfolio 1. In Table 2.6, portfolio 4 is a put option and it can be replicated by portfolio 3, i.e., buying a call option, selling a forward contract, and lending the present value of a sum equal to the call's strike price. Payouts are the same under all spot rates.

From Tables 2.5 and 2.6, the put-call parity relations can be written as follow.

$$C_{Euro} = P_{Euro} + Se^{-Rt} - Xe^{-rt} \tag{2.1}$$

$$P_{Euro} = C_{Euro} - Se^{-Rt} + Xe^{-rt} \tag{2.2}$$

where C_{Euro} and P_{Euro} are call and put prices of the European options respectively. R and r are the foreign risk-free interest rate and domestic ($) risk-free interest rate respectively, t is time-to-expiration, S is spot exchange rate ($/currency) and X is strike price ($/currency). $F = Se^{(r-R)t}$, therefore, $Se^{-Rt} = Fe^{-rt}$.

Table 2.5: Option Replication - Strategy A

Strategy A	Initial Value	Value At Expiration		
Portfolio 1:		S* < X	S* = X	S* > X
Buy a Put	P	X - S*	0	0
Buy a Forward	Fe^{-rt}	S*	S* or X	S*
Amount Borrowed	$-Xe^{-rt}$	-X	-X	-X
Net Cash Flow	$P + Fe^{-rt} - Xe^{-rt}$	0	0	S* - X
Portfolio 2: Buy a Call	C	0	0	S* - X

Table 2.6: Option Replication - Strategy B

Strategy B	Initial Value	Value At Expiration		
Portfolio 3:		$S^* < X$	$S^* = X$	$S^* > X$
Buy a Call	C	0	0	S^* - X
Short on Forward	$-Fe^{-rt}$	$-S^*$	$-S^*$ or -X	$-S^*$
Amount Lend	Xe^{-rt}	X	X	X
Net Cash Flow	$C - Fe^{-rt} + Xe^{-rt}$	$X - S^*$	0	0
Portfolio 4 : Buy a Put	P	$X - S^*$	0	0

2.4.2 Option Boundary Condition

In theory, options are priced above the lower bound, i.e., a call (put) option price must always be equal to or above its intrinsic value. The intrinsic values of European currency call and put prices are $[Se^{-Rt} - Xe^{-rt}]$ and $[Xe^{-rt} - Se^{-Rt}]$ respectively. When an option price falls below its intrinsic value, put-call parity will imply a negative value for the relevant put or call option. In practice, options are sometimes traded below their intrinsic values (possibly due to transactions costs). In this study, any out-of-bound options have been excluded in the estimation of parities.

Table 2.7 shows the percentage of selected put-call pairs which traded below their intrinsic values. 12.3% (384) of selected call and 1.6% (49) of selected put transactions were outside the boundary and so were excluded.

Table 2.7: Percentage of Out-of-Bound Options in Selected Put-call Pairs

Currency	Out-of-bound		Total	Percentage of Total	
Options'	Calls	Puts	Call/Put	Call	Put
Australian Dollar	20	2	92	21.74%	2.17%
British Pound	25	0	245	10.20%	0.00%
Canadian Dollar	91	1	312	29.17%	0.32%
Deutsche Mark	96	10	1,509	6.36%	0.66%
French Franc	139	3	308	45.13%	0.97%
Japanese Yen	2	14	203	0.99%	6.90%
Swiss Franc	11	19	462	2.38%	4.11%
Total	384	49	3,131	12.26%	1.56%

Table 2.8 shows numbers of box-arbitrage groups available for the observed put-call pairs. The sample is much smaller than for put-call parity because the box-arbitrage condition requires two put-call pairs on the same underlying asset with the same expiration. For example, in Group D (the set with most relaxed selection) there are only 400 trades, whereas for put/call parity (see right hand column of Table 2.4) there were 2,924 trades. The other groups have 148 trades for group (C), 119 trades for group (B) and 48 trades for group (A).

Table 2.8: Numbers of Box-groups

Products	Group A	Group B	Group C	Group D
Australian Dollar	1	3	4	5
British Pound	0	3	5	10
Canadian Dollar	10	16	17	22
Deutsche Mark	30	77	95	300
French Franc	2	7	10	18
Japanese Yen	2	1	2	4
Swiss Franc	3	12	15	41
Total	48	119	148	400

Key: For Groups Description see Table 2.4

2.4.3 Replication of Put and Call Prices

For put-call pairs with the same spot rates (group A), one can apply simple put-call parity as in Equations (2.1) and (2.2). A replication process is required for put-call pairs with different spot rates (groups B, C & D), i.e., an adjustment to account for the differences between call spot rate [S_c] and put spot rate [S_p]. The adjustment uses the delta of the call (put) option. In order to estimate delta, the Garman and Kohlhagen (1983) European currency option pricing model[5] is used. The call and put option pricing models are given in Equations (2.3) and (2.4).

$$C_{GK} = Se^{-Rt}N(d_1) - Xe^{-rt}N(d_2) \tag{2.3}$$

$$P_{GK} = Se^{-Rt}\left[N(d_1) - 1\right] - Xe^{-rt}\left[N(d_2) - 1\right] \tag{2.4}$$

where C_{GK} and P_{GK} indicate call and put price of Garman and Kohlhagen (1983) model respectively.

[5] See Appendix-A for full details of the Garman and Kohlhagen (1983) option pricing model.

The implied volatility[6] of the option price is found from the market price and the delta is taken at that volatility. The deltas for calls [ΔC_{GK}] and puts [ΔP_{GK}] are defined in Equations (2.5) and (2.6) respectively. The put-call parity equations are then modified for estimating adjusted call and put option prices as in Equations (2.7) and (2.8) respectively.

$$\Delta C_{GK} = e^{-Rt} N(d_1) \tag{2.5}$$

$$\Delta P_{GK} = 1 - \Delta C_{GK} = 1 - e^{-Rt} N(d_1) \tag{2.6}$$

$$C = Se^{-Rt} - Xe^{-rt} + P + \Delta P_{GK}(\Delta S) \tag{2.7}$$

$$P = Xe^{-rt} - Se^{-Rt} + C + \Delta C_{GK}(\Delta S) \tag{2.8}$$

where ΔS is the change in spot price and ΔC_{GK} and ΔP_{GK} are delta-call and delta-put respectively.

2.4.4 Box-arbitrage Conditions

The box-arbitrage condition consists of a call bull (bear) spread and put bear (bull) spread. In market equilibrium, the cost or surplus of the box-condition should be equal to the present value of the difference between the two strike prices.

In Table 2.9 (box lending) portfolio A consists of a call bull spread and a put bear spread, and portfolio B is an amount of differences between the two strike prices which is invested at the risk-free rate. At expiration, the expected future spot rate [S^*] is considered at three different positions and net cash flow is the same for all three, so we have:

$$[C(X_1)_p - C(X_2)_w - P(X_1)_w + P(X_2)_p] = (X_2 - X_1)e^{-rt} \tag{2.9}$$

[6] See Appendix-E for full details on the method to estimate the implied volatility.

Table 2.9: Box Trading Strategy 1 - Box Lending Strategy

Strategy 1		Value at Expiration		
Portfolio A:	Initial Value	$S^* < X_1$	$X_1 < S^* < X_2$	$X_2 < S^*$
Buy $C(X_1)$	$-C(X_1)$	-	$S^* - X_1$	$S^* - X_1$
Buy $P(X_2)$	$-P(X_2)$	$X_2 - S^*$	$X_2 - S^*$	-
Sell $C(X_2)$	$C(X_2)$	-	-	$X_2 - S^*$
Sell $P(X_1)$	$P(X_1)$	$S^* - X_1$	-	-
Net Cash Flow	$[C(X_2)-P(X_2)+P(X_1)-C(X_1)]$	$X_2 - X_1$	$X_2 - X_1$	$X_2 - X_1$
Portfolio B: Lend (X_2-X_1)	$(X_2-X_1)e^{-rt}$	$X_2 - X_1$	$X_2 - X_1$	$X_2 - X_1$

Note: the strike prices $X_1 < X_2$

In Table 2.10 (box borrowing) portfolio C consists of a put bull spread and a call bear spread, and portfolio D is an amount of differences between two strike prices which has been borrowed. As in strategy 1, at expiration both portfolios receive the same net cash flow at any expected future spot rate [S^*] position. In market equilibrium, the box-arbitrage condition is therefore:

$$[C(X_2)_p - C(X_1)_w - P(X_2)_w + P(X_1)_p] = (X_1 - X_2)e^{-rt} \qquad (2.10)$$

Table 2.10: Box Trading Strategy 2 - Box Borrowing Strategy

Strategy 2		Value at Expiration		
Portfolio C:	Initial Value	$S^* < X_1$	$X_1 < S^* < X_2$	$X_2 < S^*$
Buy $C(X_2)$	$-C(X_2)$	-	-	$S^* - X_2$
Buy $P(X_1)$	$-P(X_1)$	$X_1 - S^*$	-	-
Sell $C(X_1)$	$C(X_1)$	-	$X_1 - S^*$	$X_1 - S^*$
Sell $P(X_2)$	$P(X_2)$	$S^* - X_2$	$S^* - X_2$	-
Net Cash Flow	$[C(X_1)-P(X_1)+P(X_2)-C(X_2)]$	$X_1 - X_2$	$X_1 - X_2$	$X_1 - X_2$
Portfolio D: Borrow (X_2-X_1)	$(X_1-X_2)e^{-rt}$	$X_1 - X_2$	$X_1 - X_2$	$X_1 - X_2$

Note: the strike prices $X_1 < X_2$

Another way of examining a box arbitrage is to treat it as two put-call-parity pairs at different strike prices. Equation (2.11) shows put-call parity with strike price X_1 and the corresponding relationships with a higher strike price X_2 is in Equation (2.12). In market equilibrium, both Equations should be equal to zero, therefore, Equation (2.12) is equal to Equation (2.11), from which Equation (2.13) is derived (the same as Equation (2.9)):

$$C(X_1) - P(X_1) - Se^{-Rt} + X_1 e^{-rt} = 0 \qquad (2.11)$$

$$C(X_2) - P(X_2) - Se^{-Rt} + X_2e^{-rt} = 0 \tag{2.12}$$

$$C(X_1) - P(X_1) - C(X_2) + P(X_2) = (X_2 - X_1)e^{-rt} \tag{2.13}$$

As Chance (1987) has mentioned, it is possible that options violate put-call parity but not the box-condition. Let us assume put prices of X_1 and X_2 are mispriced by ±50 basis-points, then Equations (2.11) and (2.12) will not hold, but Equation (2.13) will hold: the reason is that it nets off the exact mispriced values. Therefore, risk-free arbitrage opportunities are less frequent from the box-condition than from the put-call parity condition.

In equilibrium, with an assumption of zero transactions costs, the box-condition holds for Equations (2.9) and (2.10), i.e., the RHS is equal to LHS. When the RHS is not equal to LHS, it violates the condition and suggests that one or more of the options may be mispriced.

$$Arbitrage \Rightarrow (X_2 - X_1)e^{-rt} - [C(X_1)_p - C(X_2)_w - P(X_1)_w + P(X_2)_p] \Rightarrow +ve \tag{2.14}$$

$$Arbitrage \Rightarrow (X_1 - X_2)e^{-rt} - [C(X_2)_p - C(X_1)_w - P(X_2)_w + P(X_1)_p] \Rightarrow +ve \tag{2.15}$$

The arbitrage's value derived from Equation (2.14) [call bull spread and put bear spread] may not always be positive, due to the level of mispricing between calls and puts in box-groups. Equation (2.15) [put bull spread and call bear spread], is applied when an arbitrage value of Equation (2.14) is negative. Although Equation (2.15) requires an additional margin cost, it can lead to the same arbitrage amount.

2.4.5 Transactions Costs

Transactions costs for PHLX currency options are given in Table 2.11. Panel A shows the variable cost charged per contract by the Exchange and Panel B shows the fixed costs incurred by PHLX traders. The information has been obtained from the PHLX[7]. For an option customer in London, the commission[8] for trading PHLX

[7] Information provided by PHLX European office in London.
[8] Quoted by Philip Alexander Securities and Futures Limited, London office, and also published in the Financial Times.

options is US$32 (maximum) per contract (round-turn, i.e. inclusive of buying and exercising the options). This $32 commission is for an "outsider": costs are less for a member. When trades are for 10 or more contracts, the transaction cost is reduced to US$25 per contract. The estimated transaction costs in Table 2.11 are less than $32 for a put/call parity arbitrage. However, to be conservative, transaction costs of $50 per put-call pair (round-turn) contract are assumed in this study.

Table 2.11: Variable Transaction Costs and Additional Fixed Free

Panel A: The Variable Transaction Costs	
Transaction costs per contract (dollars based & cross rate options)	
Option Comparison Charge	
Firm (Proprietary and Customer Executions)	$ 0.05
Registered Option Trader	$ 0.05
Specialist Trader	$ 0.00
Option Transaction Charge	
Customer Executions	$ 0.28
Firm (Proprietary Executions)	$ 0.23
Registered Option Trader and Specialist	$ 0.07
Option Floor Brokerage Assessment	
(5% of net floor brokerage income)	
Floor Broker Transaction Fee	
(executing transactions for their own member firm)	$ 0.05

Panel B: The Additional Fee (Fixed Costs)	
Membership Dues or Foreign Currency User Fees	$ 1,000.00 semi-annually
Application Fee	$ 200.00
Initiation Fee	$ 1,500.00
Transfer Fee	$ 300.00
Trading Post / Booth	$ 375.00 quarterly
Floor Facility Fees	$ 187.50 quarterly
Direct Wire to Floor	$ 60.00 quarterly
Telephone System Line Extensions	$ 22.50 monthly / per line
Execution Services / Communication Charge	$ 200.00 monthly
Stock Execution Machine	$ 250.00 monthly
Equity, Option or FCO Transmission Charge	$ 500.00 monthly
Quotron Equipment	$ 225.00 monthly
Instinet, Reuters Equipment	cost pass thru

2.5. Empirical Results of Put-Call Parity Tests

The first sub-section presents the tests of parity, without transactions costs. In the second sub-section, transactions costs are introduced.

2.5.1 Put-Call Parity Tests Without Transactions Costs

The results are divided into groups A - D, with group A having the most restrictive selection of the sample (see section 2.3.1 for full details of selection criteria).

The results in Table 2.12 show that approximately 95% of call options, in all groups and for all currencies, violate put-call parity. The British Pound has the highest frequency of violation: in the most-restricted groups A and B, the Pound call options violate parity. As the control of sample selection is relaxed (in groups C and D), the violation of parity for the British Pound still remains at 99%.

In Table 2.12, three other currency options (Canadian Dollar, Deutsche Mark, and Swiss Franc) behave in the same was as the British Pound. As expected, they have less violations in the least-restricted group (D) as compared to the most-restricted group (A), but still more than 90%. This shows that the choice of sample (by time-of-trade between put-call pairs) does not greatly affect the frequency of violation of parity.

Table 2.13 shows the results of t-tests of significance of mispricing. In Table 2.13, [+*ve*] indicates over-priced while [-*ve*] indicates under-priced. The means of mispriced values [*x*] are presented in basis-points for all currency options except French Franc, for which it is one-tenth of a basis-point, and Japanese Yen, for which it is one-hundredth of a basis-point. Underlined values indicate significance at the 95% level. In the whole period (Panel A in Table 2.13), there are 3 out 7 currencies which have significant violation for the most-restricted group (A), while the least-restricted group (D) has 5 out 7 currencies with significant violation.

In order to see whether mispricing changes over time, the sample is divided into two sub-periods. The first sub-period is from August 28 1987 to December 31 1990, and the second is from January 1 1991 to October 18 1994.

Table 2.12: Summary of the Put-Call Parity Violation

Panel A: Distribution Table for Call Options - Group A

Options' Pricing	A$ Call	C$ Call	DM Call	FFr Call	SFr Call	GBP Call	Yen Call	Total Call	% Call
< $0	35	69	173	45	57	20	25	424	
$0	6	2	14	8	6	0	7	43	5%
> $0	14	50	121	25	55	18	32	315	
Total	55	121	308	78	118	38	64	782	
<> $0	89%	98%	95%	90%	95%	100%	89%	95%	

Panel B: Distribution Table for Call Options - Group B

Options' Pricing	A$ Call	C$ Call	DM Call	FFr Call	SFr Call	GBP Call	Yen Call	Total Call	% Call
< $0	44	99	331	109	126	53	52	814	
$0	6	5	35	9	15	0	9	79	5%
> $0	15	71	272	50	126	43	61	638	
Total	65	175	638	168	267	96	122	1531	
<> $0	91%	97%	95%	95%	94%	100%	93%	95%	

Panel C: Distribution Table for Call Options - Group C

Options' Pricing	A$ Call	C$ Call	DM Call	FFr Call	SFr Call	GBP Call	Yen Call	Total Call	% Call
< $0	45	117	382	130	142	68	53	937	
$0	7	6	43	9	18	1	9	93	5%
> $0	19	78	314	62	141	50	82	746	
Total	71	201	739	201	301	119	144	1776	
<> $0	90%	97%	94%	96%	94%	99%	94%	95%	

Panel D: Distribution Table for Call Options - Group D

Options' Pricing	A$ Call	C$ Call	DM Call	FFr Call	SFr Call	GBP Call	Yen Call	Total Call	% Call
< $0	53	178	740	174	213	140	81	1579	
$0	7	11	88	14	25	2	12	159	5%
> $0	30	100	570	102	192	90	102	1186	
Total	90	289	1398	290	430	232	195	2924	
<> $0	92%	96%	94%	95%	94%	99%	94%	95%	

Key: Refer to Table 2.4 for Groups' Description
$0 is ranging from -$0.50 to $0.50 per option contract

Despite their high frequency of mispricing, Table 2.13 shows that the British Pound options are not significantly mispriced on average at the 95% confidence level for any groups (A, B, C, & D) or sub-periods. Other currencies are significantly mispriced, for at least one of the four groups in one period. In all 28 tests (i.e., 7 currencies in 4 groups), 16 out of 28 are significant, which is more than 50% of all currencies. The results in the two sub-periods (Panels B and C in Table 2.13) show that all currencies except Australian Dollar and British Pound are significantly mispriced in the second sub-period: 11 out of 28 tests show significance in the second sub-period. These results indicate that options did not become more accurately priced in the second, more active, sub-period.

In summary, Table 2.12 shows that an average of 95% of the call options violate put-call parity. Furthermore, the t-tests in Table 2.13 indicate that more than half of the mispriced means are significant and occurred in both sub-periods. These mispriced values are converted into dollars and subjected to further analysis in the next sub section.

Table 2.13: Mispriced Means and t-tests of the Call Options

Panel A: Whole Period

Group	A			B			C			D		
Products	x	t	s	x	t	s	x	t	s	x	t	s
A$	-3.35	-2.62	55	-4.03	-3.59	65	-3.28	-2.95	71	-2.09	-1.98	90
GBP	-6.58	-1.43	38	-1.06	-0.88	96	-2.64	-1.30	119	8.42	0.71	232
C$	10.42	2.34	121	9.30	2.50	175	6.21	2.27	201	3.22	1.65	289
DM	9.07	3.53	308	4.11	3.23	638	3.52	3.13	739	1.39	2.01	1398
FFr	-3.87	-1.07	78	-8.65	-3.99	168	-8.15	-4.02	201	-5.27	-3.32	290
Yen	-8.41	-1.41	64	-5.95	-1.63	122	-3.70	-1.18	144	-7.02	-2.43	195
SFr	-2.43	-1.39	118	-1.64	-1.85	267	-1.41	-1.75	301	-1.69	-2.53	430

Panel B: First Sub-period

Group	A			B			C			D		
Products	x	t	s	x	t	s	x	t	s	x	t	s
A$	-4.56	-3.15	44	-5.20	-4.25	54	-4.69	-3.94	58	-3.34	-3.07	74
GBP	-4.80	-0.53	5	-1.07	-0.18	18	-0.76	-0.15	21	-1.78	-0.46	37
C$	9.86	1.81	84	8.77	1.89	126	9.74	1.76	148	2.56	1.13	211
DM	-0.63	-0.50	16	-0.29	-0.34	35	-0.22	-0.25	36	0.28	0.31	40
FFr	---	---	---	-3.33	-0.43	3	-3.33	-0.43	3	-3.33	-0.43	3
Yen	-7.64	-1.30	33	-4.30	-0.94	47	-0.34	-0.10	64	-0.14	-0.05	78
SFr	-0.07	-0.45	45	0.92	1.24	92	0.81	1.13	96	1.06	1.34	112

Panel C: Second Sub-period

Group	A			B			C			D		
Products	x	t	s	x	t	s	x	t	s	x	t	s
A$	1.73	0.79	11	1.72	0.79	11	3.00	1.29	13	4.13	1.70	16
GBP	-6.85	-1.33	33	-2.42	-0.89	78	-3.04	-1.38	98	10.36	0.74	195
C$	11.70	1.50	37	10.65	1.79	49	4.95	1.57	53	5.00	1.29	78
DM	9.61	3.54	292	4.36	3.25	603	3.71	3.14	703	1.42	2.00	1358
FFr	-3.87	-1.07	78	-8.75	-3.96	165	-8.23	-4.00	198	-5.29	-3.31	287
Yen	-9.23	-0.86	31	-6.99	-1.34	75	-6.39	-1.30	80	-11.60	-2.69	117
SFr	-3.89	-1.42	73	-2.98	-2.33	175	-2.45	-2.17	205	-2.66	-3.11	318

Key: Refer to Table 2.4 for Groups' Description

- [t] is the critic-value of the t-test
- [s] is the sample size
- [$+ve$] indicates over-priced call value.
- [$-ve$] indicates under-priced call value.
- [x] is the means of mispriced values and presented in basis-points for all currency options except French Franc, for which it is one-tenth of a basis-point, and Japanese Yen, for which it is one-hundredth of a basis-point

2.5.2 Put-Call Parity Tests With the Transactions Costs

In practice, each trade involves an amount of transaction cost, so an assumed $50 has been applied for each round-trip. The mispriced basis-points are converted into dollars per contract and re-arranged in the distribution tables below (Tables 2.14, 2.15 and 2.16). In each table, the results show the mispriced put-call pairs for four groups.

Table 2.14: Distribution of the Whole Period of the Mispriced Call Options

Panel A: Distribution Table for Group A

Options' Pricing	A$ Call	C$ Call	DM Call	FFr Call	SFr Call	GBP Call	Yen Call	Total Call	% Call
< -$50	9	18	45	12	25	8	9	126	
-$50 to $50	43	82	220	57	81	25	44	552	71%
>$50	3	21	43	9	12	5	11	104	
Total	55	121	308	78	118	38	64	782	
-$50<> $50	22%	32%	29%	27%	31%	34%	31%	29%	

Panel B: Distribution Table for Group B

Options' Pricing	A$ Call	C$ Call	DM Call	FFr Call	SFr Call	GBP Call	Yen Call	Total Call	% Call
< -$50	13	21	82	39	48	20	21	244	
-$50 to $50	49	124	483	114	193	57	78	1098	72%
>$50	3	30	73	15	26	19	23	189	
Total	65	175	638	168	267	96	122	1531	
-$50<> $50	25%	29%	24%	32%	28%	41%	36%	28%	

Panel C: Distribution Table for Group C

Options' Pricing	A$ Call	C$ Call	DM Call	FFr Call	SFr Call	GBP Call	Yen Call	Total Call	% Call
< -$50	13	23	94	49	54	22	21	276	
-$50 to $50	53	145	566	132	219	77	87	1279	72%
>$50	5	33	79	20	28	20	36	221	
Total	71	201	739	201	301	119	144	1776	
-$50<> $50	25%	28%	23%	34%	27%	35%	40%	28%	

Panel D: Distribution Table for Group D

Options' Pricing	A$ Call	C$ Call	DM Call	FFr Call	SFr Call	GBP Call	Yen Call	Total Call	% Call
< -$50	16	50	204	59	81	42	39	491	
-$50 to $50	65	197	1053	198	299	153	114	2079	71%
>$50	9	42	141	33	50	37	42	354	
Total	90	289	1398	290	430	232	195	2924	
-$50<> $50	28%	32%	25%	32%	30%	34%	42%	29%	

Key: Refer to Table 2.4 for Groups' Description

In Table 2.14, the most-restricted group (A) has across all currencies (right of table), 29% of all call options mispriced by more than $50 per contract. The less-restricted groups B and C have 28% each, and group D has 29% of mispriced values exceeding $50 per contract. Most of the currencies are affected by the sample selection: arbitrage opportunities increase as the time-of-trade is relaxed. On average across all groups, Japanese Yen call options have the highest risk-free arbitrage opportunities: more than 30%. Although the t-tests (see Table 2.13) show that British Pound call options are not significantly mispriced, the number of British Pound transactions mispriced above $50 per contract is larger than for Swiss Franc call options. Deutsche Mark call options have the highest number of trades (by number of trades, not by number of options) on the PHLX and so are the most liquid. They have risk-free arbitrage opportunities of more than 23%. The Australian Dollar call options have the least arbitrage opportunities, an average of less than 25% in group D and only 9% in group A.

Table 2.15: Distribution in First Sub-period of the Mispriced Call Options

Panel A: Distribution Table for Group A

Options' Pricing	A$ Call	C$ Call	DM Call	FFr Call	SFr Call	GBP Call	Yen Call	Total Call	% Call
<-$50	9	11	0	0	7	1	6	34	
-$50 to $50	33	59	15	0	31	3	21	162	71%
>$50	2	14	1	0	7	1	6	31	
Total	44	84	16	0	45	5	33	227	
-$50<> $50	25%	30%	6%	0%	31%	40%	36%	29%	

Panel B: Distribution Table for Group B

Options' Pricing	A$ Call	C$ Call	DM Call	FFr Call	SFr Call	GBP Call	Yen Call	Total Call	% Call
<-$50	13	14	2	0	11	6	7	53	
-$50 to $50	39	93	31	3	65	6	29	266	71%
>$50	2	19	2	0	16	6	11	56	
Total	54	126	35	3	92	18	47	375	
-$50<> $50	28%	26%	11%	0%	29%	67%	38%	29%	

Panel C: Distribution Table for Group C

Options' Pricing	A$ Call	C$ Call	DM Call	FFr Call	SFr Call	GBP Call	Yen Call	Total Call	% Call
<-$50	13	16	2	0	11	6	7	55	
-$50 to $50	42	110	32	3	69	8	33	297	70%
>$50	3	22	2	0	16	7	24	74	
Total	58	148	36	3	96	21	64	426	
-$50<> $50	28%	26%	11%	0%	28%	62%	48%	30%	

Panel D: Distribution Table for Group D

Options' Pricing	A$ Call	C$ Call	DM Call	FFr Call	SFr Call	GBP Call	Yen Call	Total Call	% Call
<-$50	16	34	2	0	15	8	8	83	
-$50 to $50	53	149	34	3	75	19	40	373	67%
>$50	5	28	4	0	22	10	30	99	
Total	74	211	40	3	112	37	78	555	
-$50<> $50	28%	29%	15%	0%	33%	49%	49%	33%	

Key: Refer to Table 2.4 for Groups' Description

Table 2.16: Distribution in Second Sub-period of the Mispriced Call Options

Panel A: Distribution Table for Group A

Options' Pricing	A$ Call	C$ Call	DM Call	FFr Call	SFr Call	GBP Call	Yen Call	Total Call	% Call
<-$50	0	7	44	12	18	7	3	91	
-$50 to $50	10	23	205	57	50	22	23	390	70%
>$50	1	7	43	9	5	4	5	74	
Total	11	37	292	78	73	33	31	555	
-$50<>$50	9%	38%	30%	27%	32%	33%	26%	30%	

Panel B: Distribution Table for Group B

Options' Pricing	A$ Call	C$ Call	DM Call	FFr Call	SFr Call	GBP Call	Yen Call	Total Call	% Call
<-$50	0	7	80	39	37	14	14	191	
-$50 to $50	10	31	452	111	128	51	49	832	72%
>$50	1	11	71	15	10	13	12	133	
Total	11	49	603	165	175	78	75	1156	
-$50<>$50	9%	37%	25%	33%	27%	35%	35%	28%	

Panel C: Distribution Table for Group C

Options' Pricing	A$ Call	C$ Call	DM Call	FFr Call	SFr Call	GBP Call	Yen Call	Total Call	% Call
<-$50	0	7	92	49	43	16	14	221	
-$50 to $50	11	35	534	129	150	69	54	982	73%
>$50	2	11	77	20	12	13	12	147	
Total	13	53	703	198	205	98	80	1350	
-$50<>$50	15%	34%	24%	35%	27%	30%	33%	27%	

Panel D: Distribution Table for Group D

Options' Pricing	A$ Call	C$ Call	DM Call	FFr Call	SFr Call	GBP Call	Yen Call	Total Call	% Call
<-$50	0	16	202	59	66	34	31	408	
-$50 to $50	12	48	1019	195	224	134	74	1706	72%
>$50	4	14	137	33	28	27	12	255	
Total	16	78	1358	287	318	195	117	2369	
-$50<>$50	25%	38%	25%	32%	30%	31%	37%	28%	

Key: Refer to Table 2.4 for Groups' Description

Tables 2.15 and 2.16 give results for the two sub-periods. Both sub-periods show risk-free arbitrage opportunities of about 30%. The ranking of mispriced options in the two sub-periods has slight changes. In Table 2.15, the top three options by mispricing remain the same as in Table 2.14. However, French Franc options have the least trades in first sub-period. Canadian Dollar options have most transactions (more than one-third of total) in the first period (Table 2.15) and have more than 26% of call options mispriced by more than $50 per contract. In the second sub-period (Table 2.16), Canadian Dollar options have small numbers of transactions as compared to other options, but have more than 35% of call options mispriced by more than $50 per contract. Although Deutsche Mark options are ranked the second-least mispriced in the whole sample and the two sub-periods, they have the most transactions (more than 50% of total) in the second sub-period and have an average of 35% transaction mispriced by more than $50 per contract.

Over the two sub-periods, only the Australian Dollar, British Pound, and Japanese Yen options are more accurately priced in the later period. Swiss franc options show inconsistent results (in most-restricted group (A) in later period). The other three currency options (Canadian Dollar, Deutsche Mark, and French Franc) have higher levels of mispricing in the later period.

In summary, after taking account of transactions costs, the results still show approximately 30% of risk-free arbitrage opportunities in the call options. The selection criteria do not affect these opportunities (except with respect to transaction volume). Options in the second sub-period are as mispriced as in the first sub-period.

2.6. Empirical Results of Box-arbitrage Condition Tests

Deutsche Mark options have the largest sample for boxes and represent more than half of the total set of data samples. The Canadian Dollar and Swiss Franc options are the next most frequent boxes in the four groups. Australian Dollar, British Pound and Japanese Yen options have the least boxes in four groups.

We begin under the assumption of no transactions costs. Table 2.17 shows that there are possible arbitrage transactions in each of the groups. The most-restricted group (A) has 83% violation while the least-restricted group (D) has 97% violation, hence violations are affected by sample selection. The violations increase as the control of time-of-trade is relaxed, i.e., group D has the most arbitrage opportunities.

Table 2.17: Number of Mispriced Box-arbitrage Groups

Panel A: Distribution Table for Group A

Options'	A$		C$		DM		FFr		SFr		GBP		Yen			
Portfolio	Lend	Borrow	Lend	Borrow	Lend	Borrow	Lend	Borrow	Lend	Borrow	Lend	Borrow	Lend	Borrow	Total	%
$0			3		4								1		8	17%
>$0	1	0	5	2	13	13	1	1	1	2	0	0	0	1	40	83%
Total	1	0	8	2	17	13	1	1	1	2	0	0	1	1	48	

Panel B: Distribution Table for Group B

Options'	A$		C$		DM		FFr		SFr		GBP		Yen			
Portfolio	Lend	Borrow	Lend	Borrow	Lend	Borrow	Lend	Borrow	Lend	Borrow	Lend	Borrow	Lend	Borrow	Total	%
$0	1		5		7								1		14	12%
>$0	2	0	7	4	37	33	1	6	7	5	3	0	0	0	105	88%
Total	3	0	12	4	44	33	1	6	7	5	3	0	1	0	119	

Panel C: Distribution Table for Group C

Options'	A$		C$		DM		FFr		SFr		GBP		Yen			
Portfolio	Lend	Borrow	Lend	Borrow	Lend	Borrow	Lend	Borrow	Lend	Borrow	Lend	Borrow	Lend	Borrow	Total	%
$0	1		5		7								1		14	9%
>$0	3	0	8	4	51	37	2	8	7	8	3	2	0	1	134	91%
Total	4	0	13	4	58	37	2	8	7	8	3	2	1	1	148	

Panel D: Distribution Table for Group D

Options'	A$		C$		DM		FFr		SFr		GBP		Yen			
Portfolio	Lend	Borrow	Lend	Borrow	Lend	Borrow	Lend	Borrow	Lend	Borrow	Lend	Borrow	Lend	Borrow	Total	%
$0	1		5		6								1		13	3%
>$0	4	0	13	4	172	122	8	10	27	14	6	4	1	2	387	97%
Total	5	0	18	4	178	122	8	10	27	14	6	4	2	2	400	

Key: Refer to Table 2.4 for Groups' Description

Each box-arbitrage consists of four simultaneously traded options, i.e., buying two options and selling another two options. The transaction cost is approximately four times the costs of each option's trade, i.e., 4 x $32 = $128. The transaction cost is therefore (conservatively) assumed to be $150 for each box. The box values are converted into dollar terms and are presented in four distributions (Panels A, B, C and D in Table 2.18). Any mispriced value in a box which is above $150 represents a risk-free arbitrage opportunity. The accumulated percentage shown in Table 2.20 is the combination of both portfolios.

After taking transactions costs into account, the arbitrage opportunities fall to 24% in Panel D, the least-restricted group (D) and 10% for the most-restricted group in Panel A. But group C, which has up to 60 minutes gap, also has 24% arbitrage opportunities. Group B has 21% of risk-free arbitrage opportunities and is mainly dominated by the Deutsche Mark options. Deutsche Mark options have higher arbitrage percentages than the average for all groups.

The box-arbitrage condition shows that for all currencies the risk-free arbitrage opportunities of group A are 10%, with 21% for group B and 24% for groups C and D. These box-arbitrage results therefore confirm the violations found in the put-call parity tests.

Table 2.18: Distribution Table for Box-arbitrage Condition

Panel A : Box-arbitrage Group A

Number of Box-Condition (Showing Profit in US$ per Contract)

Options'	A$		C$		DM		FFr		SFr		GBP		Yen			Cum
Portfolio	Lend	orro	Lend	orro	Lend	Borro	Lend	orro	Lend	orro	Lend	Borro	Lend	orro	Total	%
$0			3		4								1		8	
<$50			4	2	8	9	1	1	1	2					28	83%
<$100			1		1	2								1	5	25%
<$150					1	1									2	15%
<$200						1									1	10%
<$250					2										2	8%
>$250	1				1										2	4%
Total	1	0	8	2	17	13	1	1	1	2	0	0	1	1	48	0%

Panel B: Box-arbitrage Group B

Number of Box-Condition (Showing Profit in US$ per Contract)

Options'	A$		C$		DM		FFr		SFr		GBP		Yen			Cum
Portfolio	Lend	orro	Lend	orro	Lend	Borro	Lend	orro	Lend	orro	Lend	Borro	Lend	orro	Total	%
$0	1		5		7								1		14	
<$50			5	4	19	15	1	3	5	3	2				57	88%
<$100			2		4	9		2		1	1				19	40%
<$150					2			1	1						4	24%
<$200	1				3	4									8	21%
<$250					3	2									5	14%
>$250	1				6	3			1	1					12	10%
Total	3	0	12	4	44	33	1	6	7	5	3	0	1	0	119	0%

Panel C: Box-arbitrage Group C

Number of Box-Condition (Showing Profit in US$ per Contract)

Options'	A$		C$		DM		FFr		SFr		GBP		Yen			Cum
Portfolio	Lend	orro	Lend	orro	Lend	Borro	Lend	orro	Lend	orro	Lend	Borro	Lend	orro	Total	%
$0	1		5		7								1		14	
<$50			6	4	23	16	1	4	5	3	2	2		1	67	91%
<$100			2		7	11		2		3	1				26	45%
<$150					3	1		1	1						6	28%
<$200	1				5	3	1	1							11	24%
<$250					3	1									4	16%
>$250	2				10	5			1	2					20	14%
Total	4	0	13	4	58	37	2	8	7	8	3	2	1	1	148	0%

Panel D: Box-arbitrage Group D

Number of Box-Condition (Showing Profit in US$ per Contract)

Options'	A$		C$		DM		FFr		SFr		GBP		Yen			Cum
Portfolio	Lend	orro	Lend	orro	Lend	Borro	Lend	orro	Lend	orro	Lend	Borro	Lend	orro	Total	%
$0	1		5		6								1		13	
<$50			8	4	69	46	6	6	16	6	2	1		2	166	97%
<$100	1		4		36	30	1	2	5	5		1	1		86	55%
<$150			1		18	15		1	3	1	1	1			41	34%
<$200	1				15	11	1	1		1	1				31	24%
<$250					7	7									14	16%
>$250	2				27	13			3	1	2	1			49	12%
Total	5	0	18	4	178	122	8	10	27	14	6	4	2	2	400	0%

Key: Refer to Table 2.4 for Groups' Description

2.7. Conclusions on Parity and Box Tests

This study has used a trade-by-trade data set, which is quite different from most previous studies (which used daily closing prices). The sample period is from August 28 1987 to October 18 1994 and has total traded options of 5,571,851 calls and 4,623,243 puts. The results show that many options violate put-call parity. Approximately 95% of put-call pairs do not accord with put-call parity, of which half are on average significant at the 95% confidence level. These mispriced values can be converted into dollars per contract, so that transactions costs may be applied. The result after deducting transactions costs (for all four groups) is still more than 25% risk-free arbitrage opportunities.

The box-arbitrage condition confirms the put-call parity test. The results suggest that more than 80% of the boxes violate the arbitrage-condition. However, after an assumption of $150 for transactions costs, the risk-free arbitrage opportunities fall to 10%.

Group A of the put-call parity test does not use any options model. The results show the same trend as other groups which use delta adjustment for changes in the spot rate. Choice of sample does not seem to be critical to the results.

The analysis of two sub-periods has shown that there are no systematic changes in mispricing over time. Deutsche Mark, Canadian Dollar and French Franc options have larger mispricing in the second sub-period, while other options (Australian Dollar, British Pound and Japanese Yen) have more efficient pricing in the second sub-period. Swiss Franc call options have higher mispriced values in group A of the second sub-period while other groups (B, C, D) have more efficient pricing in the second sub-period.

The interesting question is why such a high level of mispricing continues over the time? The first possible reason could be that traders do not understand put-call parity. This is very unlikely, because most traders are aware of parity and some apply it in their trading strategies. The second possible reason could be that the set-up costs to monitor and trade these mispricing opportunities are higher than the profit opportunities[9]. This also seems unlikely given a potential profit of more than $100,000

[9] Since Deutsche Mark options have the highest transaction volume, in the sample group A of the later sub period, I have 30% of 292 transactions are profitable. Over the period of less than four years, that represents average 2 trades per month, the average contracts per transaction (see Table 3.1)

per month on Deutsche Mark options alone. The third possible reason could be more (and new) traders have been attracted to the market in the later period as the volume has risen. A different trading and hedging strategy may be used by these traders that maintains the mispriced values in the later period.

This study has shown that more than 25% mispricing appeared in the European-style options. This has not been satisfactorily explained. The next chapter extends the methodology to cover the American-style options for the same sample period.

---

is 112. The mispricing contract is about $100 from the sample, i.e., $50 profit per contract, that will represent an average profit of $112,000 per month (2 @ 112 x $50).

2.8 Appendix: Example of Box-Condition and Arbitrage Opportunities in Put-Call Parity Relationship.

2.8.1 Example of Box-Condition

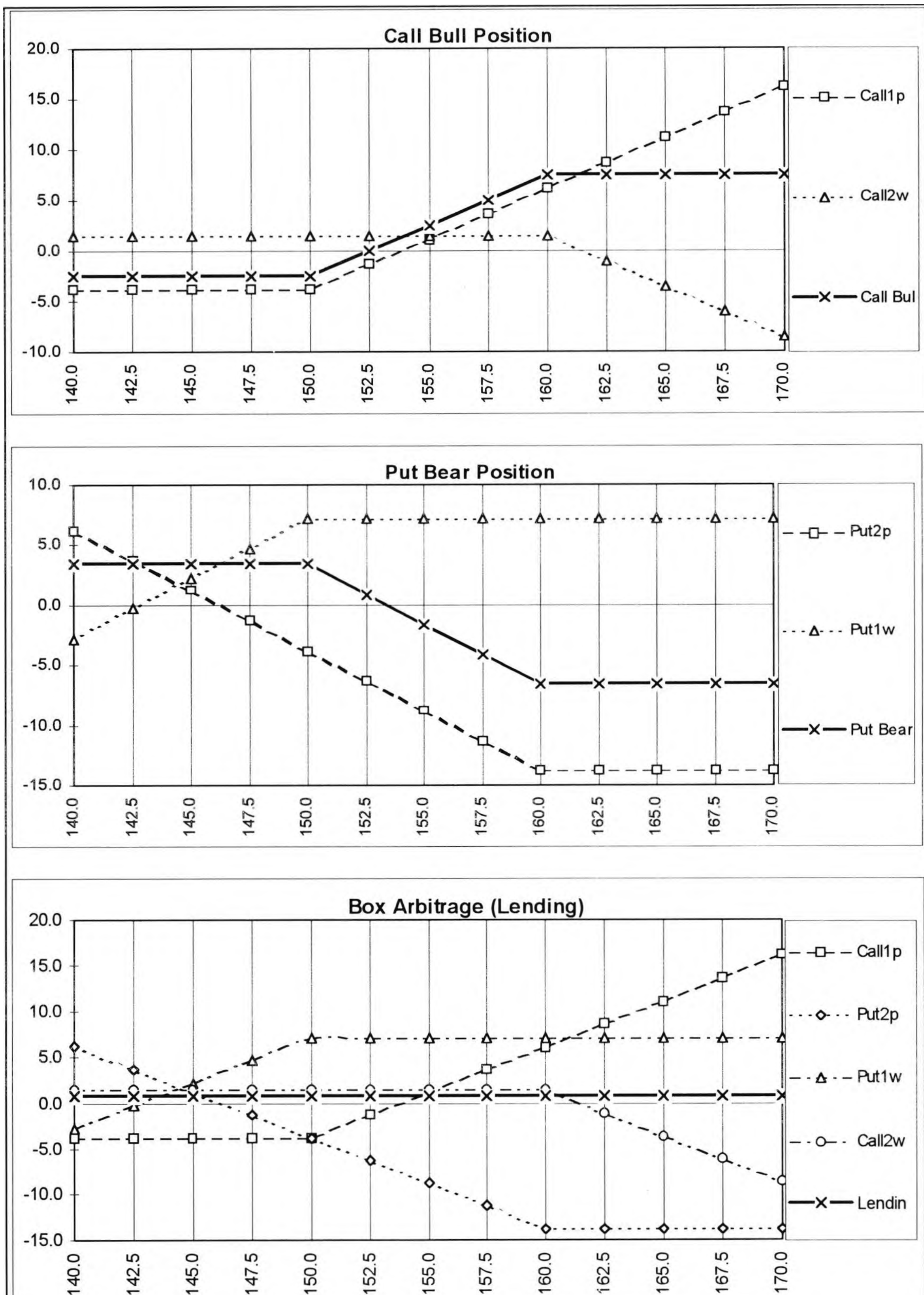
Table 2.19 gives an example of put and call option prices for two strike prices. The two strike prices are Dollar per Sterling \$1.50 and \$1.60. The table values are shown in US cents. The call and put prices of first strike price are 3.8 cents and 7.2 cents respectively. For the second strike price, call and put prices are 1.4 cents and 13.8 cents respectively.

Table 2.19: Examples of Options Pricing

	Strike Price	Spot Rate	Sigma	Option's Period	Interest Rates		Option's Prices	
					Domestic	Foreign	Call	Put
X_1	150	150	10.0%	1.0	10.0%	12.5%	3.8	7.2
X_2	160	150	10.0%	1.0	10.0%	12.5%	1.4	13.8

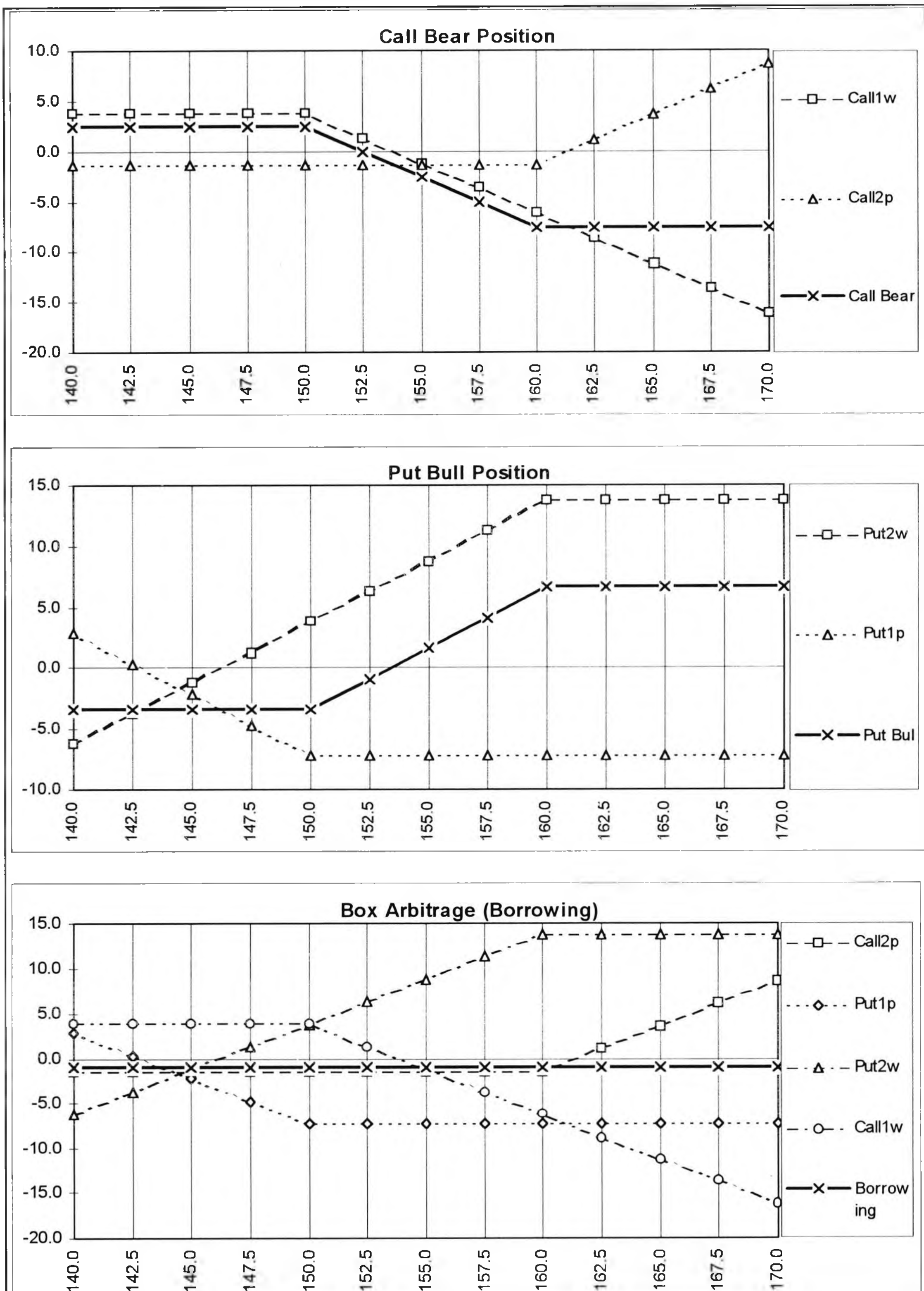
In Figure 2.1, using information of Table 2.19 we have three pay-off charts: a call bull spread at the top, a put bull spread at the middle, and the combined spreads at the bottom. Both the combined spreads give a box-condition lending position shown in the bottom pay-off chart. This bottom pay-off chart is consistent with Table 2.9 and Equation (2.9). In Figure 2.2, we have a put bull spread in the top pay-off chart and a call bear spread in the middle pay-off chart. With both spreads combined, it gives a box-condition borrowing position shown in the bottom pay-off chart. This bottom pay-off chart is consistent with Table 2.10 and Equation (2.10).

Figure 2.1: Example of the Box-Condition - Lending Strategy



Key: Call1p is purchase a Call option at Strike Price (X_1)
Call2w is sell a Call option at Strike Price (X_2)
Put1w is sell a Put option at Strike Price (X_1)
Put2p is purchase a Put option at Strike Price (X_2)
Lending is the Net Cash Flow position or $(X_2-X_1)e^{-rt}$

Figure 2.2: Example of the Box-Condition - Borrowing Strategy



Key: Call2p is purchase a Call option at Strike Price (X_2)
Call1w is sell a Call option at Strike Price (X_1)
Put2w is sell a Put option at Strike Price (X_2)
Put1p is purchase a Put option at Strike Price (X_1)
Borrowing is the negative Net Cash Flow position or $(X_1-X_2)e^{-rt}$

2.8.2 Arbitrage Opportunity in the Put-Call Parity Relationship

When option prices violate the put-call parity relationship, we can make a risk-free profit (assuming zero transaction cost) on zero investment. That is, by writing the relatively overpriced option and using proceeds to buy the relatively under-priced option, together with an appropriate position in the forward contract and borrowing or lending. The remaining proceeds will be risk-free profit, since the portfolio will require no cash outflow (or inflow) on the expiration date of the options. In Table 2.20, we have examples of four mispriced call options. (Note: these are just examples, not actual data from the PHLX). There are three in-, at-, and out-of- the-money three-month US$/GBP options in panel A; the values are in hundredths of US cents. The spot rate is 150c/£, with annualised risk-free domestic and foreign interest rates of 8% and 10% respectively. The call prices are estimated from traded put prices with the put-call parity relationship.

In panel B of Table 2.20, Example (1), we can invest in two three-month options. Put and call are available at 3.306c and 3.300c respectively. Since buying the present value of forward contract [Se^{-Rt}] costs 146.296c and the present value of borrowing [Xe^{-rt}] costs 147.030c, with put selling at 3.306c we know from the put-call relationship that the call should be worth only 2.573c. Since the call is overpriced relative to put, we can (a) write one ATM 150c call for 3.000c; (b) buy one ATM 150c put for 3.306c; (c) buy one foreign currency at 150c and earn 10% p.a. for three months; and (d) borrow 150c at 8% p.a. to be paid back in three months. There is an immediate gain of 0.427c, representing the extent of relative overpriced call. On expiration date, no further gain or loss results. Further examples of mispriced call options are given in Panels C, D and E.

Table 2.20: Examples of Arbitrage Opportunity in Put-Call Parity Relationship

Panel A - Call Prices Estimated from Traded Puts with Put-Call Parity

Strike Price	Spot Rate	Domestic Interest	Foreign Interest	Time to Expiration	Traded Put	Estimated Call
145.00	150.00	8%	10%	0.25	1.260	5.428
150.00	150.00	8%	10%	0.25	3.306	2.573
155.00	150.00	8%	10%	0.25	6.602	0.968

Panel B - Example (1):

Market selling ATM Call 150.00 @ 3.000, i.e., Call is overpriced by 0.427

a. Write one ATM Call 150.00 @ 3.000 [+call]

b. Buy one ATM Put 150.00 @ 3.306 [-put]

c. Buy one FX 150.00 and earn 10% p.a. for three months [-Se-Rt]

d. Borrow one 150.00 @ 8% p.a. to be paid back in three months [+Xe-rt]

	(a) +call	(b) -put	(c) -Se-Rt	(d) +Xe-rt	Net(inflow)
The net cash flow for above four steps	3.000	-3.306	-146.296	147.030	0.427

Panel C - Example (2):

Market selling ATM Call 150.00 @ 2.000, i.e., Call is underprice by 0.573

a. Buy one ATM Call 150.00 @ 2.000 [-call]

b. Write one ATM Put 150.00 @ 3.306 [+put]

c. Short Sell one FX 150.00 @ 10% p.a. to matured in three months [+Se-Rt]

d. Lend one 150.00 @ 8% p.a. to be recieved back in three months [-Xe-rt]

	(a) -call	(b) +put	(c) +Se-Rt	(d) -Xe-rt	Net(inflow)
The net cash flow for above four steps	-2.000	3.306	146.296	-147.030	0.573

Panel D - Example (3):

Market selling ITM Call 145.00 @ 5.600, i.e., Call is overpriced by 0.172

a. Write one ITM Call 145.00 @ 5.600 [+call]

b. Buy one OTM Put 145.00 @ 1.260 [-put]

c. Buy one FX 150.00 and earn 10% p.a. for three months [-Se-Rt]

d. Borrow one 150.00 @ 8% p.a. to be paid back in three months [+Xe-rt]

	(a) +call	(b) -put	(c) -Se-Rt	(d) +Xe-rt	Net(inflow)
The net cash flow for above four steps	5.600	-1.260	-146.296	142.129	0.172

Panel E - Example (4):

Market selling OTM Call 155.00 @ 0.500, i.e., Call is underprice by 0.468

a. Buy one OTM Call 155.00 @ 0.500 [-call]

b. Write one ITM Put 155.00 @ 6.602 [+put]

c. Short Sell one FX 150.00 @ 10% p.a. to matured in three months [+Se-Rt]

d. Lend one 150.00 @ 8% p.a. to be recieved back in three months [-Xe-rt]

	(a) -call	(b) +put	(c) +Se-Rt	(d) -Xe-rt	Net(inflow)
The net cash flow for above four steps	-0.500	6.602	146.296	-151.931	0.468

NB: The values are in US cents

Chapter 3: The American Spot Currency Options Pricing Efficiency

3.1. Introduction

The purpose of this chapter is to examine the pricing efficiency of the American-style options. The PHLX started to trade such options in 1982, five years earlier than the European-style options. The previous chapter has shown that the latter have been regularly mispriced, (approximately 27% of risk-free arbitrage opportunities in the put-call pairs). The longer trading experience might be expected to result in more efficient prices for the American options.

In theory, put-call parity does not hold for American options. A different approach to testing efficiency has therefore been used which compares the implied volatilities of put and call options. Put and call options with the same strike price should reflect the same distribution of future spot rates, hence have the same implied volatility.

The chapter is structured as follows. We review some of the previous research (in section 3.2), the database (in section 3.3) and the methodology to be used (in section 3.4). The results are in section 3.5 and section 3.6 draws together the conclusions.

3.2. Previous Research

There have been many previous studies on both spot [PHLX] and future [Chicago Mercantile Exchange (CME)] currency options. Shastri and Tandon (1985) conducted a market efficiency test on American options traded at PHLX. They examined the rational pricing boundaries on closing prices and found that there was frequent mispricing in excess of $50 per contract. However, when Bodurtha and Courtadon (1986) followed-up with a test for the period February 28 1983 to September 14 1984, they found that the market was efficient when the tests used simultaneous prices and also took account of the transactions costs.

An implied volatility test was suggested by Whaley (1982) for stock options and applied in Whaley (1986) to test the market efficiency of American futures options. He used the traded S&P 500 futures options from the CME. The results indicated that 75% of the estimated volatilities for call options were not equal to volatilities for put options. Johnson (1986) conducted a tested on spot currency options, with an hypothesis that the call and put volatilities were equal on average across time for each currency. He used daily closing prices of American-style currency options on the PHLX obtained from the Wall Street Journal from December 10 1982 until February 24 1984 and applied a European option-pricing model. He concluded the market was efficient. However, his data were not paired with the same underlying spot rate and were day-end prices.

Both Whaley (1986) and Johnson (1986) suffered from data synchronicity problems. The present study uses trade-to-trade transaction data to test for market efficiency and so overcomes the data synchronicity problem. In addition, the approach to comparing the volatilities is different from Whaley's (1986) and Johnson's (1986).

3.3. Data and Sample Selection

3.3.1 Data

The Philadelphia Stock Exchange (PHLX) data have been explained in chapter 2 (section 2.3.1). The basic details about contracts were given in Table 2.1 of Chapter 2. This information is later used to convert the levels of mispricing into US dollars per contract.

3.3.2 Sample Selection

The same procedure was used as for the European options, which was described in Chapter 2 and the same period was used: August 28 1987 to October 18 1994. Table 3.1 shows the total volumes and numbers of the transactions for the American-style options traded.

Table 3.1: Total Volume and Transactions of the American-style Options

American Options	Start Date	End Date	Volume (V)		Transaction (T)	
			Puts	Calls	Puts	Calls
Australian Dollar	Aug-28-87	Oct-18-94	839,574	661,302	15,089	14,281
British Pound	Aug-28-87	Oct-18-94	2,252,892	2,458,568	49,193	49,377
Canadian Dollar	Aug-28-87	Oct-18-94	871,135	763,592	30,174	20,440
Deutsche Mark	Aug-28-87	Oct-18-94	17,664,394	17,974,050	202,489	170,906
French Franc	Aug-28-87	Oct-18-94	293,047	429,214	2,723	2,078
Japanese Yen	Aug-28-87	Oct-18-94	7,214,425	7,394,011	107,071	104,179
Swiss Franc	Aug-28-87	Oct-18-94	2,012,054	2,303,852	52,871	48,432
		Total / Average	31,147,521	31,984,589	459,610	409,693

Key: V is volume (number of option contracts)
T is transactions (number of trades)

There are 31,984,589 calls and 31,147,521 puts in the database, i.e., a total of 63,132,110 options in 869,303 trades. In the sample, the most active contracts (by number of options and by number of trades) were for Deutsche Marks (25,638,444 options in 373,395 trades) and Japanese Yen (14,608,436 options in 211,250 trades).

The selection criteria are summarised in Table 3.2. They are the same as in earlier chapter (see section 2.3.2), except that an additional group A, with zero-minute gap on the time of trade and zero gap on the level of currency rate, has been added.

Table 3.2: Sample Selection Criteria

Options' Variables	Group A	Group B	Group C	Group D	Group E
Trade Date	Same	Same	Same	Same	Same
Expiration	Same	Same	Same	Same	Same
Strike Price	Same	Same	Same	Same	Same
Change in Spot Rate	Zero	Zero	Zero	Zero	Zero
Change in Trade Time	Zero-min	10-min	30-min	60-min	>60-min

Table 3.3 shows the numbers of non-repeating (i.e., not double counted) put-call pairs in the sample selections.

Table 3.3: Selected Options Pairs of Trades

American Options Selection (Mins)	Group A (Zero)	Group B (10)	Group C (30)	Group D (60)	Group E (Same Day)
Australian Dollar	263	849	903	940	1,053
British Pound	406	1,456	1,631	1,738	2,040
Canadian Dollar	136	917	1,095	1,225	1,463
Deutsche Mark	2,741	9,191	10,947	12,266	16,140
French Franc	34	83	83	83	85
Japanese Yen	1,658	5,650	6,135	6,509	7,567
Swiss Franc	369	1,456	1,649	1,743	2,057
Total Sample	5,607	19,602	22,443	24,504	30,405
% of Observation	1.37%	4.79%	5.48%	5.98%	7.42%

Key: Group A - Same Trade Date; Expiry; Strike Price, Spot Rate, Trading Time is Zero-min gap.
Group B - Same Trade Date; Expiry; Strike Price, Spot Rate, Trading Time is 10-min gap.
Group C - Same Trade Date; Expiry; Strike Price, Spot Rate, Trading Time is 30-min gap.
Group D - Same Trade Date; Expiry; Strike Price, Spot Rate, Trading Time is 60-min gap.
Group E - Same Trade Date; Expiry; Strike Price, Spot Rate, Trading Time is >60-min gap.

Two filtering steps are applied: (i) all put-call pairs within the last 5 days to expiration have been removed because the implied volatility may behave erratically; (ii) options that violate both American and European boundary conditions have been omitted, since they violate the rational pricing bound and have zero implied volatility. Table 3.4 gives a detailed view of the impact of these conditions on the largest sub-group - Group E. It indicates a loss of about 5.8% of the original put-call pairs from the lower bound test and about 5.9% from the 5-day requirement.

Table 3.4: Selected Option-Pairs of Trades in Group E

American Options Selection	Excluding last 5 Trading Days	Out-of-the-Boundary (Both Amex & Euro)	Calls	Puts
Australian Dollar	943	919	916	889
British Pound	1,749	1,722	1,719	1,686
Canadian Dollar	1,346	1,338	1,338	1,276
Deutsche Mark	12,373	12,245	12,160	12,167
French Franc	76	76	76	76
Japanese Yen	5,926	5,926	5,552	5,548
Swiss Franc	1,565	1,545	1,516	1,540
Total Sample	23,978	23,771	23,277	23,182
% of Observation	5.86%	5.80%	5.68%	5.66%

Key: Refer to Table 3.3 for Groups' Description
'Calls' is the maximum number of calls available in Group E
'Puts' is the maximum number of puts available in Group E

Table 3.5 shows the final sample of put-call pairs. The groups range in size from 23,277 (5.7%) of the original data (same day trading, group E) to 4,525 (1.1%) of the original data (no time delay, group A). All groups contain pairs of put and call options struck at the same spot rate.

Table 3.5: Number of Options in the Selected Put-Call Pairs

American Options Selection (Min)	Group A (zero)	Group B (10-min)	Group C (30-min)	Group D (60-min)	Group E (Same Day)
Australian Dollar	239	758	798	826	916
British Pound	360	1,242	1,378	1,467	1,719
Canadian Dollar	125	847	1,004	1,121	1,338
Deutsche Mark	2,156	7,018	8,297	9,312	12,160
French Franc	29	74	74	74	76
Japanese Yen	1,324	4,276	4,617	4,854	5,552
Swiss Franc	292	1,119	1,245	1,308	1,516
Total Sample	4,525	15,334	17,413	18,962	23,277
% of Observation	1.10%	3.74%	4.25%	4.63%	5.68%

Key: Refer to Table 3.3 for Groups' Description

3.3.3 Interest Rates

The domestic and foreign interest rates for each currency are the London Euro-currency deposit interest rates of cross currencies. These have been obtained from DataStream as 1 day, 1 week, 1 month, 3 months, 6 months and 1 year. Rates have been interpolated for exact option maturities.

3.4. Theory, Methodology and Transactions Costs

3.4.1 American Put-Call Parity Theory

European put and call option prices have a deterministic relationship in their pricing, but American-style options are different due to the early-exercise feature. The put-call parity of European-style options [see Stoll (1969)] has been shown in Equations (2.1) and (2.2) in Chapter 2. The early comments from Merton (1973b) and Stoll (1973) confirmed that Equations (2.1) and (2.2) did not apply to the American-style options. Stoll and Whaley (1986) suggested that the relationships which linked the European and American option prices were the inequalities given in Equations (3.1) and (3.2).

$$S - X \leq C_{Amex} - P_{Amex} \leq Se^{-Rt} - Xe^{-rt}\text{, when } R \geq r \tag{3.1}$$

$$Se^{-Rt} - X \leq C_{Amex} - P_{Amex} \leq S - Xe^{-rt}\text{, when } R < r \tag{3.2}$$

where C_{Amex} and P_{Amex} are call and put prices of the American options respectively, R and r are the foreign currency risk-free interest rate and domestic (\$) risk-free interest rate respectively, t is time-to-expiration, S is spot exchange rate (\$/currency), and X is strike price (\$/currency).

However, both put and call options of American-style as well as European-style should have the same expectation on the future spot rate, therefore, options of the same strike price, expiration and underlying spot rate must have the same volatility.

3.4.2 Methodology for Comparison

We can test the put-call pairs from selected samples by comparing the pairs' implied volatilities, i.e., the call should have the same implied volatility as the traded put. The American option-pricing model of Barone-Adesi and Whaley (1987) is used to find the implied volatilities[10]. This American model[11] does not have the biases which exist in the European model. The American call and put option pricing models are as Equations (3.3 and 3.4). We can derive the American put-call parity[12] as in Equations

[10] See Appendix-E for full detail on the method to estimate the implied volatility.

[11] See Whaley (1986), he has demonstrated that when early-exercise premium of the American call options on a dividend-paying stock is accounted for in the valuation model, the strike price and time-to-expiration biases which been document for the European model disappear.

[12] For proof, please see the numeric examples in section 3.4.3.

(3.5 and 3.6) by rewriting Equations (3.3 and 3.4) with the early-exercise features. However, this version of put-call parity can only be applied when the critical spot price of the call is greater than the observed spot price and the critical spot price of the put is smaller than the observed spot price.

$$C_{BW} = C_{GK} + A_2(S/S_i)^{q_2} \qquad when\ S < S_i \qquad \} \qquad (3.3)$$
$$C_{BW} = S - X \qquad when\ S \geq S_i \qquad \}$$

$$P_{BW} = P_{GK} + A_1(S/S_i)^{q_1} \qquad when\ S > S_i \qquad \} \qquad (3.4)$$
$$P_{BW} = X - S \qquad when\ S \leq S_i \qquad \}$$

$$C_{BW} = P_{BW} + Se^{-Rt} - Xe^{-rt} + A_2(S/S_i^C)^{q_2} - A_1(S/S_i^P)^{q_1} \qquad (3.5)$$

$$P_{BW} = C_{BW} + Xe^{-rt} - Se^{-Rt} + A_1(S/S_i^P)^{q_1} - A_2(S/S_i^C)^{q_2} \qquad (3.6)$$

where S_i is the critical spot of the call and put options, BW indicates call or put prices of Barone-Adesi and Whaley (1987) model and GK indicates call or put price of Garman and Kohlhagen (1983) model.

The put option prices in put-call pairs must be equal to or greater than the intrinsic value. The intrinsic values of American options are $[P_{BW} \geq (X - S)]$.

The sampled put-call pairs (groups A to E) have same spot rates. These do not require further adjustment before the comparison. In order to ascertain the level of mispricing in dollar terms (instead of the implied volatility [percentage]), the call price is estimated with the implied volatility of the traded put, by using the Barone-Adesi and Whaley (1987) option-pricing model. The estimated call price is then compared with the observed call price in the put-call pair. The difference in price is then converted into dollars per contract via the information from Table 2.1.

3.4.3 Regression Test on the Mispriced Options

It is hypothesised that mispricing is related to moneyness (F/X), the interest-rate differential (r-R) and time-to-expiration (t). This is tested with a regression of the form shown in Equation (3.7):

$$C_{Amex} - C_{Estimated} = a + b(F/X) + c(r - R) + d(t) + e \qquad (3.7)$$

where C_{Amex} is the observed American call price and $C_{Estimated}$ is the estimated American option call from American put price of the option-pair. (F/X) is the Moneyness, (r-R) is the interest rate differential and (t) is the time-to-expiration.

3.4.4 Transactions Costs

The typical PHLX transaction variable costs and fixed costs have been mentioned in Chapter 2 (section 2.4.5). The details of variable and fixed costs were listed in Table 2.12. The transactions costs estimated from the PHLX information is less than \$25, however, to be conservative, \$50 per round-turn is used for this study.

3.5. Empirical Results

The results are divided into two sub-sections: the first examines the option pricing without transactions costs and the second takes account of transactions costs.

3.5.1 Implied Volatility Test

The results (without transactions costs) in Table 3.6 show that almost 100% of call and put prices in put-call pairs have different implied volatilities, i.e., in 3 decimal places (0.1%). This violates the methodology assumption, i.e., put and call options with common variables[13] should have the same implied volatility on their expected future spot rates. If the traded puts are correctly priced, then the traded calls are mispriced[14]. We define "Zero" as mispricing of less than ±\$0.50 per contract. Table 3.6 shows numbers of mispriced call options in put-call pairs by positive, zero or negative mispricing. It shows that more than 97% of put-call pairs had different volatilities. Group (A), the most restricted sample (see Panel A in Table 3.6), has approximately the same level of mispricing as the other four groups (B, C, D and E), so the choice of sample is not critical for the results. The two sub-periods'[15] results are given in Tables 3.7 and 3.8. Violations are high in both sub-periods, with above 97% of options mispriced (by more than \$0.50 per contract) on average for all groups.

T-tests are carried out to check for the significance of average mispricing. Panel A of Table 3.9 shows that mispricing of call options is significant at the 95% level for all currencies in all groups, except the British Pound in group A. The extension to two sub-periods [see Panels B and C of Table 3.10] indicates that only Australian Dollar and Swiss Franc calls are more accurately priced in the second sub-period while other currency options are not systematically more or less accurately priced over time. However, the British Pound and French Franc calls show significant mispricing only in the second sub-period.

---

[13] The options of same currency traded on same day with the same strike price, spot exchange rate and expiration.

[14] Please refer to section 3.4.3 for the procedure in order to ascertain the relatively over- and under-priced call options.

[15] The first period is from August 28 1987 to December 31 1990, the second period is from January 1 1991 to October 18 1994.

Table 3.6: Number of Mispriced **Call** Options in Put-Call Pairs - **Whole Period**

Panel A: Distribution Table for Group A

Options' Pricing	A$ Call	C$ Call	DM Call	FFr Call	SFr Call	GBP Call	Yen Call	Total Call
< $0	150	93	1,168	19	129	179	556	2,294
$0	3	0	40	1	4	4	18	70
> $0	86	32	948	9	159	177	750	2,161
Total	239	125	2,156	29	292	360	1,324	4,525
<> $0	99%	100%	98%	97%	99%	99%	99%	98%

Panel B: Distribution Table for Group B

Options' Pricing	A$ Call	C$ Call	DM Call	FFr Call	SFr Call	GBP Call	Yen Call	Total Call
< $0	460	611	3,704	43	495	680	1,765	7,758
$0	10	18	88	1	14	15	57	203
> $0	288	218	3,226	30	610	547	2,454	7,373
Total	758	847	7,018	74	1,119	1,242	4,276	15,334
<> $0	99%	98%	99%	99%	99%	99%	99%	99%

Panel C: Distribution Table for Group C

Options' Pricing	A$ Call	C$ Call	DM Call	FFr Call	SFr Call	GBP Call	Yen Call	Total Call
< $0	490	727	4,320	43	546	765	1,914	8,805
$0	10	18	105	1	16	16	61	227
> $0	298	259	3,872	30	683	597	2,642	8,381
Total	798	1,004	8,297	74	1,245	1,378	4,617	17,413
<> $0	99%	98%	99%	99%	99%	99%	99%	99%

Panel D: Distribution Table for Group D

Options' Pricing	A$ Call	C$ Call	DM Call	FFr Call	SFr Call	GBP Call	Yen Call	Total Call
< $0	504	808	4,820	43	573	815	2,013	9,576
$0	10	20	124	1	18	17	63	253
> $0	312	293	4,368	30	717	635	2,778	9,133
Total	826	1,121	9,312	74	1,308	1,467	4,854	18,962
<> $0	99%	98%	99%	99%	99%	99%	99%	99%

Panel E: Distribution Table for Group E

Options' Pricing	A$ Call	C$ Call	DM Call	FFr Call	SFr Call	GBP Call	Yen Call	Total Call
< $0	572	960	6,318	45	663	986	2,321	11,865
$0	11	21	170	1	20	23	78	324
> $0	333	357	5,672	30	833	710	3,153	11,088
Total	916	1,338	12,160	76	1,516	1,719	5,552	23,277
<> $0	99%	98%	99%	99%	99%	99%	99%	99%

Key: Refer to Table 3.3 for Groups' Description
$0 is ranging from -$0.50 to $0.50 per option contract

Table 3.7: Number of Mispriced **Call** Options in Put-Call Pairs - **First Sub-period**

Panel A: Distribution Table for Group A

Options' Pricing	A$ Call	C$ Call	DM Call	FFr Call	SFr Call	GBP Call	Yen Call	Total Call
< $0	107	78	412	8	93	111	388	1,197
$0	3	0	16	1	4	2	16	42
> $0	39	27	454	8	36	116	553	1,233
Total	149	105	882	17	133	229	957	2,472
<> $0	98%	100%	98%	94%	97%	99%	98%	98%

Panel B: Distribution Table for Group B

Options' Pricing	A$ Call	C$ Call	DM Call	FFr Call	SFr Call	GBP Call	Yen Call	Total Call
< $0	332	498	1,372	16	336	375	1,200	4,129
$0	5	16	41	1	8	10	43	124
> $0	142	167	1,574	14	450	353	1,658	4,358
Total	479	681	2,987	31	794	738	2,901	8,611
<> $0	99%	98%	99%	97%	99%	99%	99%	99%

Panel C: Distribution Table for Group C

Options' Pricing	A$ Call	C$ Call	DM Call	FFr Call	SFr Call	GBP Call	Yen Call	Total Call
< $0	356	604	1,661	16	372	412	1,327	4,748
$0	5	16	49	1	10	11	47	139
> $0	151	204	1,949	14	504	380	1,820	5,022
Total	512	824	3,659	31	886	803	3,194	9,909
<> $0	99%	98%	99%	97%	99%	99%	99%	99%

Panel D: Distribution Table for Group D

Options' Pricing	A$ Call	C$ Call	DM Call	FFr Call	SFr Call	GBP Call	Yen Call	Total Call
< $0	366	677	1,855	16	389	440	1,403	5,146
$0	5	18	61	1	10	11	49	155
> $0	158	234	2,244	14	526	407	1,936	5,519
Total	529	929	4,160	31	925	858	3,388	10,820
<> $0	99%	98%	99%	97%	99%	99%	99%	99%

Panel E: Distribution Table for Group E

Options' Pricing	A$ Call	C$ Call	DM Call	FFr Call	SFr Call	GBP Call	Yen Call	Total Call
< $0	406	807	2,370	16	452	516	1,650	6,217
$0	6	19	77	1	11	16	58	188
> $0	168	285	2,869	14	615	448	2,228	6,627
Total	580	1,111	5,316	31	1,078	980	3,936	13,032
<> $0	99%	98%	99%	97%	99%	98%	99%	99%

Key: Refer to Table 3.3 for Groups' Description
$0 is ranging from -$0.50 to $0.50 per option contract

Table 3.8: Number of Mispriced **Call** Options in Put-Call Pairs - **Second Sub-period**

Panel A: Distribution Table for Group A

Options' Pricing	A$ Call	C$ Call	DM Call	FFr Call	SFr Call	GBP Call	Yen Call	Total Call
< $0	43	15	756	11	36	67	168	1,096
$0	0	0	24	0	0	2	2	28
> $0	47	5	494	1	35	61	197	840
Total	90	20	1,274	12	71	130	367	1,964
<> $0	100%	100%	98%	100%	100%	98%	99%	99%

Panel B: Distribution Table for Group B

Options' Pricing	A$ Call	C$ Call	DM Call	FFr Call	SFr Call	GBP Call	Yen Call	Total Call
< $0	128	113	2,332	27	159	305	565	3,629
$0	5	2	47	0	6	5	14	79
> $0	146	51	1,652	16	160	194	796	3,015
Total	279	166	4,031	43	325	504	1,375	6,723
<> $0	98%	99%	99%	100%	98%	99%	99%	99%

Panel C: Distribution Table for Group C

Options' Pricing	A$ Call	C$ Call	DM Call	FFr Call	SFr Call	GBP Call	Yen Call	Total Call
< $0	134	123	2,659	27	174	353	587	4,057
$0	5	2	56	0	6	5	14	88
> $0	147	55	1,923	16	178	217	822	3,358
Total	286	180	4,638	43	358	575	1,423	7,503
<> $0	98%	99%	99%	100%	98%	99%	99%	99%

Panel D: Distribution Table for Group D

Options' Pricing	A$ Call	C$ Call	DM Call	FFr Call	SFr Call	GBP Call	Yen Call	Total Call
< $0	138	131	2,965	27	184	375	610	4,430
$0	5	2	63	0	8	6	14	98
> $0	154	59	2,124	16	191	228	842	3,614
Total	297	192	5,152	43	383	609	1,466	8,142
<> $0	98%	99%	99%	100%	98%	99%	99%	99%

Panel E: Distribution Table for Group E

Options' Pricing	A$ Call	C$ Call	DM Call	FFr Call	SFr Call	GBP Call	Yen Call	Total Call
< $0	166	153	3,948	29	211	470	671	5,648
$0	5	2	93	0	9	7	20	136
> $0	165	72	2,803	16	218	262	925	4,461
Total	336	227	6,844	45	438	739	1,616	10,245
<> $0	99%	99%	99%	100%	98%	99%	99%	99%

Key: Refer to Table 3.3 for Groups' Description
$0 is ranging from -$0.50 to $0.50 per option contract

Table 3.9: The t-distribution for Mispriced **Call** Options in Put-Call Pairs

Panel A: Whole Period

Group	A			B			C			D			E		
Products	x	t	s	x	t	s	x	t	s	x	t	s	x	t	s
A$	-3.14	-3.23	239	-3.17	-5.20	758	-3.76	-5.95	798	-3.85	-5.85	826	-4.72	-6.73	916
GBP	-2.34	-1.74	360	-4.72	-4.05	1,242	-7.28	-5.04	1,378	-6.00	-4.08	1,467	-11.17	-6.91	1,723
C$	-3.63	-6.11	125	-3.34	-14.82	847	-3.35	-16.44	1,004	-3.34	-17.57	1,121	-3.37	-19.37	1,338
DM	-0.77	-4.59	2,156	-0.46	-3.41	7,018	-0.39	-3.09	8,297	-0.38	-3.10	9,312	-0.48	-4.60	12,160
FFr	-10.95	-2.48	29	-6.45	-2.74	74	-6.45	-2.47	74	-6.45	-2.47	74	-9.83	-2.79	76
Yen	-1.80	-2.69	1,324	-2.56	-6.05	4,276	-2.66	-6.44	4,617	-2.65	-6.55	4,885	-2.60	-7.09	5,552
SFr	1.74	2.60	292	1.63	4.72	1,119	1.70	5.13	1,245	1.75	5.41	1,308	1.56	4.94	1,516

Panel B: First Sub-period

Group	A			B			C			D			E		
Products	x	t	s	x	t	s	x	t	s	x	t	s	x	t	s
A$	-5.54	-3.79	149	-5.33	-5.93	479	-5.93	-6.52	512	-6.11	-6.41	529	-6.44	-6.73	580
GBP	-1.47	-1.00	230	-0.69	-0.59	738	-0.21	-0.17	803	1.67	1.13	858	0.33	0.24	980
C$	-3.57	-5.20	105	-3.19	-14.44	681	-3.26	-16.12	824	-3.26	-17.23	929	-3.32	-18.92	1,111
DM	0.02	0.08	882	0.63	3.82	2,987	0.79	4.93	3,659	0.90	6.02	4,160	0.86	6.84	5,316
FFr	-2.00	-0.51	17	-1.75	-0.68	31	-1.75	-0.68	31	-1.75	-0.68	31	-1.75	-0.68	31
Yen	-1.03	-1.65	957	-1.97	-4.39	2,901	-2.11	-4.77	3,194	-2.11	-4.81	3,388	-2.19	-5.48	3,936
SFr	2.62	3.29	221	2.28	5.72	794	2.42	6.24	886	2.50	6.63	925	2.16	5.60	1,078

Panel C: Second Sub-period

Group	A			B			C			D			E		
Products	x	t	s	x	t	s	x	t	s	x	t	s	x	t	s
A$	0.84	1.15	90	0.54	1.02	279	0.12	0.19	286	0.17	0.28	297	-1.75	-1.85	336
GBP	-3.89	-1.45	130	-10.62	-4.67	504	-17.15	-5.79	575	-16.81	-5.96	609	-26.42	-8.23	739
C$	-3.90	-4.44	20	-3.95	-5.58	166	-3.78	-5.71	180	-3.74	-5.95	192	-3.60	-6.44	227
DM	-1.32	-5.85	1,274	-1.27	-6.34	4,031	-1.33	-7.01	4,638	-1.42	-7.67	5,152	-1.53	-9.74	6,844
FFr	-23.64	-2.94	12	-9.84	-2.43	43	-9.84	-2.43	43	-9.84	-2.43	43	-15.39	-2.77	45
Yen	-3.82	-2.14	367	-3.81	-4.17	1,375	-3.89	-4.34	1,423	-3.90	-4.45	1,466	-3.59	-4.51	1,616
SFr	-0.99	-0.85	71	0.03	0.04	325	-0.06	-0.10	359	-0.05	-0.09	383	0.09	0.16	438

Key: Refer to Table 3.3 for Groups' Description

- [t] is the critic-value of the t-test
- [s] is the sample size
- [$+ve$] indicates over-pricing.
- [$-ve$] indicates under-pricing.
- [x] is the means of mispriced values and presented in basis-points for all currency options except French Franc, for which it is one-tenth of a basis-point, and Japanese Yen, for which it is one-hundredth of a basis-point

3.5.2 Arbitrage Opportunities

In order to be realistic about the arbitrage opportunities, we must account for transactions costs. $50 is used as the transaction cost[16] for each put-call lot[17]. The transactions costs reduce the arbitrage opportunities for call options in the implied volatility test to an average of 20% (by frequency) for all five groups [by comparing the results in Tables 3.6 and 3.10]. This result is similar across groups, (Groups A, B, C, D and E), so sample selection is not critical.

For the two sub-periods (Tables 3.11 and 3.12), the results show that the Australian Dollar and Swiss Franc improve in pricing (arbitrage frequency falls) while other currencies become worse. The British Pound and French Franc calls have the highest arbitrage opportunities in the second sub-period with 40% and 31% respectively [see panel E of Table 3.12]. But the Deutsche Mark has the largest number of mispriced call options traded in both sub-periods. Although the Deutsche Mark call has only 18% of all arbitrage opportunities in group E [see panel E of Table 3.12], that represents approximately 1 trade[18] on each trading day over the second sub-period. In Table 3.1, average numbers of Deutsche Mark call contracts per trade are approximately 105 per day, with an average of $25 mispricing over transactions costs. This indicates a risk-free profit of $2,500 per trading day for one trade of Deutsche Mark call options. The Japanese Yen has an equal mispricing percentage over the two sub-periods [see Tables 3.11 and 3.12]. Although the numbers of Japanese Yen call transactions fall in the second sub-period, there is no systematic improvement in pricing. The Japanese Yen call has an arbitrage opportunity of 3 trades in every fortnight[19]. The average numbers of contracts per trade are 70 [see Table 3.1] above transaction cost per contract. The resulting risk-free profit is approximately $5,250 per fortnight.

[16] See earlier section 3.4.3 for arbitrage example and section 3.4.4 for actual transaction cost per lot.
[17] Each option trade can has more than 1 lot (contract), an average of 10 or more lots are traded in each trade. A put-call lot consists of the round-turn of a call and put contract.
[18] See Panel E in Table 3.13, Deutsche Mark has 1,236 trades (701 + 531) mispricing above US$50, with 260 trading days per year, the second sub period represents 3 years and 10.5 month, approximate 1,010 trading days, that will give us an approximate of 1 trade in every trading days during the second sub period.
[19] See Penal E in Table 3.13 for the Yen call column and used the calculation method of the Deutsche Mark.

Table 3.10: Number of Mispriced **Call** Options in Put-Call Pairs - **Whole Period**

Panel A: Distribution Table for Group A

Options' Pricing	A$ Call	C$ Call	DM Call	FFr Call	SFr Call	GBP Call	Yen Call	Total Call
< -$50	39	10	200	6	36	61	132	484
-$50 to $50	183	109	1,798	22	209	250	1,046	3,617
> $50	17	6	158	1	47	49	146	424
Total	239	125	2,156	29	292	360	1,324	4,525
-$50<>$50	23%	13%	17%	24%	28%	31%	21%	20%

Panel B: Distribution Table for Group B

Options' Pricing	A$ Call	C$ Call	DM Call	FFr Call	SFr Call	GBP Call	Yen Call	Total Call
< -$50	121	80	615	12	104	247	432	1,611
-$50 to $50	576	748	5,810	57	840	818	3,334	12,183
> $50	61	19	593	5	175	177	510	1,540
Total	758	847	7,018	74	1,119	1,242	4,276	15,334
-$50<>$50	24%	12%	17%	23%	25%	34%	22%	21%

Panel C: Distribution Table for Group C

Options' Pricing	A$ Call	C$ Call	DM Call	FFr Call	SFr Call	GBP Call	Yen Call	Total Call
< -$50	134	106	736	12	117	290	477	1,872
-$50 to $50	601	877	6,850	57	936	892	3,583	13,796
> $50	63	21	711	5	192	196	557	1,745
Total	798	1,004	8,297	74	1,245	1,378	4,617	17,413
-$50<>$50	25%	13%	17%	23%	25%	35%	22%	21%

Panel D: Distribution Table for Group D

Options' Pricing	A$ Call	C$ Call	DM Call	FFr Call	SFr Call	GBP Call	Yen Call	Total Call
< -$50	139	125	809	12	120	316	502	2,023
-$50 to $50	620	975	7,710	57	981	935	3,763	15,041
> $50	67	21	793	5	207	216	589	1,898
Total	826	1,121	9,312	74	1,308	1,467	4,854	18,962
-$50<>$50	25%	13%	17%	23%	25%	36%	22%	21%

Panel E: Distribution Table for Group E

Options' Pricing	A$ Call	C$ Call	DM Call	FFr Call	SFr Call	GBP Call	Yen Call	Total Call
< -$50	168	162	1,041	14	141	397	587	2,510
-$50 to $50	674	1,149	10,160	57	1,136	1,074	4,315	18,565
> $50	74	27	959	5	239	248	650	2,202
Total	916	1,338	12,160	76	1,516	1,719	5,552	23,277
-$50<>$50	26%	14%	16%	25%	25%	38%	22%	20%

Key: Refer to Table 3.3 for Groups' Description

Table 3.11: Number of Mispriced **Call** Options in Put-Call Pairs - **First Sub-period**

Panel A: Distribution Table for Group A

Options' Pricing	A$ Call	C$ Call	DM Call	FFr Call	SFr Call	GBP Call	Yen Call	Total Call
< -$50	34	10	69	2	24	39	89	267
-$50 to $50	104	89	745	14	76	157	759	1,944
> $50	11	6	68	1	33	33	109	261
Total	149	105	882	17	133	229	957	2,472
-$50<>$50	30%	15%	16%	18%	43%	31%	21%	21%

Panel B: Distribution Table for Group B

Options' Pricing	A$ Call	C$ Call	DM Call	FFr Call	SFr Call	GBP Call	Yen Call	Total Call
< -$50	107	63	203	4	71	143	280	871
-$50 to $50	331	604	2,538	26	585	477	2,265	6,826
> $50	41	14	246	1	138	118	356	914
Total	479	681	2,987	31	794	738	2,901	8,611
-$50<>$50	31%	11%	15%	16%	26%	35%	22%	21%

Panel C: Distribution Table for Group C

Options' Pricing	A$ Call	C$ Call	DM Call	FFr Call	SFr Call	GBP Call	Yen Call	Total Call
< -$50	119	88	260	4	81	154	314	1,020
-$50 to $50	350	720	3,091	26	653	523	2,482	7,845
> $50	43	16	308	1	152	126	398	1,044
Total	512	824	3,659	31	886	803	3,194	9,909
-$50<>$50	32%	13%	16%	16%	26%	35%	22%	21%

Panel D: Distribution Table for Group D

Options' Pricing	A$ Call	C$ Call	DM Call	FFr Call	SFr Call	GBP Call	Yen Call	Total Call
< -$50	122	106	275	4	81	170	333	1,091
-$50 to $50	363	807	3,536	26	681	546	2,629	8,588
> $50	44	16	349	1	163	142	426	1,141
Total	529	929	4,160	31	925	858	3,388	10,820
-$50<>$50	31%	13%	15%	16%	26%	36%	22%	21%

Panel E: Distribution Table for Group E

Options' Pricing	A$ Call	C$ Call	DM Call	FFr Call	SFr Call	GBP Call	Yen Call	Total Call
< -$50	140	138	340	4	96	194	396	1,308
-$50 to $50	390	952	4,552	26	796	627	3,062	10,405
> $50	50	21	424	1	186	159	478	1,319
Total	580	1,111	5,316	31	1,078	980	3,936	13,032
-$50<>$50	33%	14%	14%	16%	26%	36%	22%	20%

Key: Refer to Table 3.3 for Groups' Description

Table 3.12: Number of Mispriced **Call** Options in Put-Call Pairs - **Second Sub-period**

Panel A: Distribution Table for in Group A

Options' Pricing	A$ Call	C$ Call	DM Call	FFr Call	SFr Call	GBP Call	Yen Call	Total Call
< -$50	5	0	131	4	12	22	43	217
-$50 to $50	79	20	1,053	8	54	92	287	1,593
> $50	6	0	90	0	5	16	37	154
Total	90	20	1,274	12	71	130	367	1,964
-$50<>$50	12%	0%	17%	33%	24%	29%	22%	19%

Panel B: Distribution Table for Group B

Options' Pricing	A$ Call	C$ Call	DM Call	FFr Call	SFr Call	GBP Call	Yen Call	Total Call
< -$50	14	17	412	8	33	104	152	740
-$50 to $50	245	144	3,272	31	255	341	1,069	5,357
> $50	20	5	347	4	37	59	154	626
Total	279	166	4,031	43	325	504	1,375	6,723
-$50<>$50	12%	13%	19%	28%	22%	32%	22%	20%

Panel C: Distribution Table for Group C

Options' Pricing	A$ Call	C$ Call	DM Call	FFr Call	SFr Call	GBP Call	Yen Call	Total Call
< -$50	15	18	476	8	36	136	163	852
-$50 to $50	251	157	3,759	31	282	369	1,101	5,950
> $50	20	5	403	4	40	70	159	701
Total	286	180	4,638	43	358	575	1,423	7,503
-$50<>$50	12%	13%	19%	28%	21%	36%	23%	21%

Panel D: Distribution Table for Group D

Options' Pricing	A$ Call	C$ Call	DM Call	FFr Call	SFr Call	GBP Call	Yen Call	Total Call
< -$50	17	19	534	8	39	146	169	932
-$50 to $50	257	168	4,174	31	300	389	1,134	6,453
> $50	23	5	444	4	44	74	163	757
Total	297	192	5,152	43	383	609	1,466	8,142
-$50<>$50	13%	13%	19%	28%	22%	36%	23%	21%

Panel E: Distribution Table for Group E

Options' Pricing	A$ Call	C$ Call	DM Call	FFr Call	SFr Call	GBP Call	Yen Call	Total Call
< -$50	28	24	701	10	45	203	191	1,202
-$50 to $50	284	197	5,608	31	340	447	1,253	8,160
> $50	24	6	535	4	53	89	172	883
Total	336	227	6,844	45	438	739	1,616	10,245
-$50<>$50	15%	13%	18%	31%	22%	40%	22%	20%

Key: Refer to Table 3.3 for Groups' Description

The results show that sample selection has little effect on deviations, so the largest sample group (E) is used to test whether mispricing is related to moneyness, interest rates and maturity. The results for the whole period are given in Table 3.13.

Except for the Australian Dollar and French Franc, all other calls in different currencies are significantly mispriced at the 1% level by the F test. The t-statistics (bottom six rows of Table 3.13) show that moneyness, interest-rate differential and time-to-expiration all have significant effects on mispricing for the other currencies.

For the whole period of observation (see Table 3.13), moneyness has a significant negative effect on Japanese Yen calls while it has significant positive effect on British Pound calls. It follows that in-the-money Japanese Yen calls tend to be under-priced relative to out-of-the-money puts, while in-the-money British Pound calls tend to be over-priced relative to out-of-the-money puts.

The interest-rate differential (domestic minus foreign) has a significant positive effect on British Pound, Canadian Dollar, Deutsche Mark, Japanese Yen and Swiss Franc calls. Hence when the dollar interest rate is higher than foreign interest rates, the calls tend to be over-priced relative to the puts.

Time-to-expiration has a significant positive effect on the Canadian Dollar and Deutsche Mark calls while it has a negative effect on British Pound, French Franc and Japanese Yen calls. Hence long-expiration Canadian Dollar and Deutsche Mark calls tend to be over-priced relative to puts, while long-expiration British Pound, French Franc and Japanese Yen calls tend to be under-priced relative to puts.

The regression results for the two sub-periods [see Tables 3.14 and 3.15] confirm that there is no relationship for the Australian Dollar and French Franc. Among the other currencies, only the Swiss Franc does not show a significant relationship in the second sub-period. All other currencies show significant relationship in both sub-periods.

Table 3.13 - Regression Test for Mispriced Values versus Moneyness, Interest Rates and Expiration - **Whole Period**

Regression Statistics	*A$*	*GBP*	*C$*	*DM*	*FFr*	*Yen*	*SFr*
R Square	0.005	0.035	0.074	0.014	0.091	0.090	0.011
Adjusted R Square	0.002	0.034	0.072	0.014	0.054	0.090	0.009
Standard Error	106	206	31	72	75	163	77
Observations	916	1,719	1,338	12,160	76	5,552	1,516
ANOVA	*A$*	*GBP*	*C$*	*DM*	*FFr*	*Yen*	*SFr*
Regression F	2	21	36	59	2	184	6
Significance F	17.6%	0.0%	0.0%	0.0%	7.3%	0.0%	0.1%
	A$	*GBP*	*C$*	*DM*	*FFr*	*Yen*	*SFr*
Coefficients - Intercept	86	-1,987	74	53	-466	2,641	12
(t-Stat - Intercept)	0.41	<u>-6.86</u>	0.92	1.40	-1.26	<u>22.95</u>	0.10
Coefficients - Moneyness	-97	1,964	-81	-55	470	-2,628	-6
(t-Stat - Moneyness)	-0.46	<u>6.73</u>	-1.00	-1.46	1.28	<u>-23.07</u>	-0.05
Coefficients - Interest Rate[r-R]	327	729	546	229	314	270	256
(t-Stat - Interest Rates[r-R])	1.52	<u>2.86</u>	<u>8.73</u>	<u>11.89</u>	1.33	<u>2.71</u>	<u>3.77</u>
Coefficients - Time-to-expiration	28	-131	30	25	-128	-55	4
(t-Stat - Time-to-expiration)	0.84	<u>-2.96</u>	<u>2.80</u>	<u>4.18</u>	<u>-2.54</u>	<u>-4.39</u>	0.30

Note: <u>Underlined</u> t-Stat results indicate significance at 95% confidence level.

Table 3.14 - Regression Test for Mispriced Values versus Moneyness, Interest Rates and Expiration - **First Sub-period**

Regression Statistics	*A$*	*GBP*	*C$*	*DM*	*FFr*	*Yen*	*SFr*
R Square	0.004	0.048	0.090	0.002	0.158	0.088	0.012
Adjusted R Square	-0.002	0.045	0.087	0.001	0.064	0.088	0.009
Standard Error	115	131	28	58	35	150	79
Observations	580	980	1,111	5,316	31	3,936	1,078
ANOVA	*A$*	*GBP*	*C$*	*DM*	*FFr*	*Yen*	*SFr*
Regression F	1	16	36	3	2	127	4
Significance F	56.5%	0.0%	0.0%	1.7%	19.3%	0.0%	0.6%
	A$	*GBP*	*C$*	*DM*	*FFr*	*Yen*	*SFr*
Coefficients - Intercept	-107	496	276	59	743	2,368	-24
(t-Stat - Intercept)	-0.37	1.71	<u>2.82</u>	1.26	1.37	<u>19.17</u>	-0.17
Coefficients - Moneyness	60	-483	-283	-56	-737	-2,357	27
(t-Stat - Moneyness)	0.21	-1.66	<u>-2.89</u>	-1.21	-1.35	<u>-19.29</u>	0.19
Coefficients - Interest Rate[r-R]	-171	650	510	127	-248	227	346
(t-Stat - Interest Rates[r-R])	-0.54	<u>3.08</u>	<u>8.41</u>	<u>2.58</u>	-0.35	1.55	<u>3.03</u>
Coefficients - Time-to-expiration	21	148	60	4	20	-45	11
(t-Stat - Time-to-expiration)	0.43	<u>3.50</u>	<u>4.41</u>	0.60	0.34	<u>-3.18</u>	0.84

Note: <u>Underlined</u> t-Stat results indicate significance at 95% confidence level.

Table 3.15 - Regression Test for Mispriced Values versus Moneyness, Interest Rates and Expiration - **Second Sub-period**

Regression Statistics	*A$*	*GBP*	*C$*	*DM*	*FFr*	*Yen*	*SFr*
R Square	0.008	0.063	0.079	0.007	0.095	0.099	0.005
Adjusted R Square	-0.001	0.060	0.067	0.006	0.029	0.098	-0.002
Standard Error	87	265	41	81	92	190	72
Observations	336	739	227	6,844	45	1,616	438
ANOVA	*A$*	*GBP*	*C$*	*DM*	*FFr*	*Yen*	*SFr*
Regression F	1	17	6	15	1	59	1
Significance F	46.0%	0.0%	0.0%	0.0%	24.7%	0.0%	55.0%
	A$	*GBP*	*C$*	*DM*	*FFr*	*Yen*	*SFr*
Coefficients - Intercept	262	-3,234	115	125	-489	3,399	-101
(t-Stat - Intercept)	0.66	<u>-6.47</u>	0.36	<u>2.08</u>	-0.98	<u>12.98</u>	-0.46
Coefficients - Moneyness	-259	3,087	-115	-128	474	-3,378	101
(t-Stat - Moneyness)	-0.65	<u>6.11</u>	-0.35	<u>-2.12</u>	0.95	<u>-13.02</u>	0.45
Coefficients - Interest Rate[r-R]	392	-1,060	962	248	53	174	-152
(t-Stat - Interest Rates[r-R])	0.99	-1.89	<u>3.74</u>	<u>3.82</u>	0.14	0.44	-0.66
Coefficients - Time-to-expiration	34	-238	17	47	-153	-73	-28
(t-Stat - Time-to-expiration)	0.81	<u>-2.96</u>	0.56	<u>4.70</u>	<u>-2.03</u>	<u>-2.72</u>	-1.06

Note: <u>Underlined</u> t-Stat results indicate significance at 95% confidence level.

3.6. Conclusions

This study has used trade-by-trade data on options to examine the American put-call pricing relationship. It is quite different from previous research (which used daily closing prices) having a sample of up to 23,277 put-call pairs, which is drawn from trades on over 63 million options. The results show that almost 100% of all **traded call** prices are not equal (by more than $0.50 per contract) to our **estimated call** prices (the latter being computed from the traded put in a put-call pair, using an American option-pricing model). When transactions costs of $50 are applied, the proportion of deviant trades still remains at about 20% for all currencies in all sample groups.

Moneyness, interest rates and time-to-expiration have significant effects on level of mispricing for calls, but not in a consistent way across currencies. At the 95% confidence level, moneyness, interest rates and time-to-expiration have significant effects on British Pound and Japanese Yen calls (no significant effects on Australian Dollar and French Franc), interest rates and time-to-expiration have significant effects on Canadian Dollar and Deutsche Mark calls, and time-to-expiration has a significant effect on Swiss Franc calls.

The rather small variation in the results between different samples shows that the sampling criteria have very little effect on the results. The t-test for the two sub-periods show that only Australian Dollar and Swiss Franc calls improve in pricing over time. Calls in all other currencies have worsened and Deutsche Mark calls have shifted from being relatively over-priced to being relatively under-priced. After taking account of transactions costs, the numbers of mispriced call options and risk-free arbitrage opportunities remain high. The results on Deutsche Mark calls in the second sub-period indicate an approximate risk-free profit of $2,500 per trading day. Other currencies show smaller amounts. The Japanese Yen calls have an approximate risk-free profit of $5,250 per fortnight.

Why does put-call mispricing continue over time? Some possible reasons have been mentioned in Chapter 2, namely ignorance, an influx of new traders or new entry due to high fixed costs. Another possible reason could be the use of a European model by traders to price American options. In the next chapter, we are going to analyse the

American early-exercise premium, which may contribute to the mispricing of the put-call pairs in this chapter.

3.7. Appendix: Arbitrage Opportunities for Call Options in the Lower Boundary Condition

The lower boundary restriction for call options is $[C = P + Se^{-Rt} - Xe^{-rt}]$. When call option prices violate the lower boundary restriction, we can make a certain profit (assuming zero transaction cost) on zero investment. It involves writing relatively over-priced options and using the proceeds to buy relatively under-priced options, together with an appropriate position in the forward contract for borrowing or lending. The remaining proceeds would be the a risk-free profit, since the portfolio would require no cash outflow (or inflow) on the expiration date of the options. Table 3.15 gives four examples of mispriced call options. There are three in-, at-, and out-of-the-money for three-month US$/GBP call options in panel A, the values are in one-hundredth of a US cent. The spot rate is 150c (US cents), with annualised risk-free domestic and foreign interest rates of 8% and 10% respectively. The call prices are estimated from the implied volatility of traded put prices with the American option-pricing model.

In panel B of Table 3.16, for instance in example (1), we can invest in two three-month 150c options. Puts and calls are available at 3.307 and 3.000 respectively. With the put selling at 3.307, we know (from the implied volatility of put price and American call option-pricing model) that the call should be worth 2.648, and the 'fair' call early exercise premium is 0.075. The buying forward contract $[Se^{-Rt}]$ @ 150c and borrowing $[Xe^{-rt}]$ @ 150c for three months are 146.296 and 147.030 respectively. Since the call is then over-priced relative to put, we can (a) write one ATM 150c call for 3.000; (b) buy one ATM 150c Put for 3.307; (c) buy one foreign currency at 150c and earn 10% p.a. for three months; (d) borrow 150c at 8% p.a. to be paid back in three months; and (e) addition provision of the call early exercise premium of 0.075. It nets us 0.352 immediately representing the extent of relative overpricing of the calls.

During the option exercisable period, if the call has not been exercised earlier, the entire position is held until the expiration date and the early exercise premium is invested. At expiration, the position is closed out with no further gain or loss, and we still have the early exercise premium with its accumulated interest. If the call is exercised before the expiration date, then the foreign currency is deliver and the strike amount received. Using the strike amount to pay back the borrowing, if the interest rates are favourable, it left over with a positive amount. In addition, the portfolio still balances with a long put and the early exercise premium with its accumulated interest. The Panels C, D and E are other examples of mispriced of call options.

Table 3.16: Example of Arbitrage Opportunity on Mispriced Call Options

Panel A - Call Prices are estimated from implied volatility of the traded Puts.

Strike Price	Spot Rate	Domestic Interest	Foreign Interest	Time to Expiration	Traded Put	Estimated Call	Early Ex Premium
145.000	150.000	8%	10%	0.25	1.260	5.635	0.208
150.000	150.000	8%	10%	0.25	3.307	2.648	0.075
155.000	150.000	8%	10%	0.25	6.602	0.996	0.028

Panel B - Example (1):

Market selling ATM Call 150.00 @ 3.000, i.e., Call is overpriced by 0.352 [3.000 - 2.648]
a. Write one ATM Call 150.00 @ 3.000 [+call]
b. Buy one ATM Buy 150.00 @ 3.307 [-put]
c. Buy one FX 150.00 and earn 10% p.a. for three months [-Se-Rt]
d. Borrow one 150.00 @ 8% p.a. to be paid within three months [+Xe-rt]
e. Addition provision on Early Exercise Premium of Call @ 0.075 [-Early Ex]

The net cash flow	(a) +call	(b) -put	(c) -Se-Rt	(d) +Xe-rt	(e) -E. Ex	Net(inflow)
for above four steps	3.000	-3.307	-146.296	147.030	-0.075	0.352

Panel C - Example (2):

Market selling ATM Call 150.00 @ 2.000, i.e., Call is underpriced by 0.648 [2.648 - 2.000]
a. Buy one ATM Call 150.00 @ 2.000 [-call]
b. Write one ATM Put 150.00 @ 3.307 [+put]
c. Short Sell one FX 150.00 @ 10% p.a. to matured in three months [+Se-Rt]
d. Lend one 150.00 @ 8% p.a. to be received back in three months [-Xe-rt]
e. Saving from the Early Exercise Premium of Call @ 0.075 [+Early Ex]

The net cash flow	(a) -call	(b) +put	(c) +Se-Rt	(d) -Xe-rt	(e) +E. Ex	Net(inflow)
for above four steps	-2.000	3.307	146.296	-147.030	0.075	0.648

Panel D - Example (3):

Market selling ITM Call 145.00 @ 6.000, i.e., Call is overpriced by 0.365 [6.00 - 5.635]
a. Write one ITM Call 145.00 @ 6.000 [+call]
b. Buy one OTM Put 145.00 @ 1.260 [-put]
c. Buy one FX 150.00 and earn 10% p.a. for three months [-Se-Rt]
d. Borrow one 150.00 @ 8% p.a. to be paid back in three months [+Xe-rt]
e. Addition provision on Early Exercise Premium of Call @ 0.208 [-Early Ex]

The net cash flow	(a) +call	(b) -put	(c) -Se-Rt	(d) +Xe-rt	(e) -E. Ex	Net(inflow)
for above four steps	6.000	-1.260	-146.296	142.129	-0.208	0.365

Panel E - Example (4):

Market selling OTM Call 155.00 @ 0.500, i.e., Call is underpriced by 0.496 [0.996 - 0.500]
a. Buy one OTM Call 155.00 @ 0.500 [-call]
b. Write one ITM Put 155.00 @ 6.602 [+put]
c. Short Sell one FX 150.00 @ 10% p.a. to matured in three months [+Se-Rt]
d. Lend one 150.00 @ 8% p.a. to be received back in three months [-Xe-rt]
e. Saving from the Early Exercise Premium of Call @ 0.028 [+Early Ex]

The net cash flow	(a) -call	(b) +put	(c) +Se-Rt	(d) -Xe-rt	(e) +E. Ex	Net(inflow)
for above four steps	-0.500	6.602	146.296	-151.931	0.028	0.496

Chapter 4: The Early-Exercise Premium of American Currency Options

4.1. Introduction

The purpose of this study is to examine the early-exercise premium of the American currency options traded on the Philadelphia Stock Exchange (PHLX). The two earlier chapters on the European-style and American-style options have shown that mispriced values are significant and that risk-free arbitrages exist even after allowing for transactions costs. This study aims to ascertain how large is the early-exercise premium and whether it contributes to the mispricing of the currency options.

An American-style option is similar to a European-style option, but it allows the buyer to exercise it early. Its extra cost is known as the early-exercise premium. When both styles of option have the same strike price and time-to-expiration, the American-style option must cost at least as much or more than the European-style option. When the prices violate this assumption, an arbitrage opportunity is available.

In the next few sections, we review some of the previous research (in section 4.2), the database (in section 4.3) and the methodology to be used (in section 4.4). The results are in section 4.5 and the conclusion is in section 4.6.

4.2. Previous Research

Merton (1973a) first highlighted that American option prices must always be equal to or higher than European option prices. This feature has attracted many studies since then.

For currency options, Fabozzi, Hauser, and Yaari (1990) examined the early-exercise premium on daily closing prices traded on the PHLX from August 1983 to December 1984. They compared the traded American option prices with a European options model and not directly with European option prices. Their results showed that the put options were not priced according to the model, with some biases for moneyness and expiration, however the call options were consistent with the model. Jorion and Stoughton (1989) compared the closing prices of American options traded at the PHLX with the European options traded at the Chicago Board Options Exchange (CBOE). Their results showed that the average early-exercise premium of the calls was larger than for the puts. Both premiums were positive, as expected. Hilliard and Tucker (1991) then examined European and American currency options traded at the PHLX using intra-day data for the 20-month period from September 1987 to April 1989. Only 18 American calls and 7 American puts out of 5,895 paired options were priced below European values. When they accounted for transactions costs, the risk-free arbitrage opportunity was zero for both calls and puts. Their results showed that the average early-exercise premium was +2.17% for the calls, and +1.38% for the puts, however, the change of spot rate between American and European options (±0.5%) was not accounted for in the option prices.

The current study is different from both Fabozzi, Hauser, and Yaari (1990), and Jorion and Stoughton (1989). Although the early sample period overlaps with Hilliard and Tucker (1991), we use a different methodology.

4.3. Data and Sample Selection

4.3.1 Data

The PHLX data have been explained in Chapter 2 (section 2.3.1). The basic details about contracts were given in Table 2.1 of Chapter 2. This information is later used to convert the levels of mispricing into US dollars per contract.

4.3.2 Sample Selection

The same procedure was used as for the European and American options, which were described in Chapters 2 and 3, using the same period: August 28 1987 to October 18 1994. Table 4.1 (mentioned earlier in Chapter 3) shows the total volumes and numbers of the transactions for the American-style options traded: Panel A covers the whole sample period, while Panels B and C are the two sub-periods.

Table 4.1: Total Volume and Transactions of the American-style Options

Panel A: Whole Observation								
American Options	Start Date	End Date	Volume (V)		Transaction (T)		V / T	
			Puts	Calls	Puts	Calls	Puts	Calls
Australian Dollar	Aug-28-87	Oct-18-94	839,574	661,302	15,089	14,281	55	46
British Pound	Aug-28-87	Oct-18-94	2,252,892	2,458,568	49,193	49,377	45	49
Canadian Dollar	Aug-28-87	Oct-18-94	871,135	763,592	30,174	20,440	28	37
Deutsche Mark	Aug-28-87	Oct-18-94	17,664,394	17,974,050	202,489	170,906	87	105
French Franc	Aug-28-87	Oct-18-94	293,047	429,214	2,723	2,078	107	206
Japanese Yen	Aug-28-87	Oct-18-94	7,214,425	7,394,011	107,071	104,179	67	70
Swiss Franc	Aug-28-87	Oct-18-94	2,012,054	2,303,852	52,871	48,432	38	47
		Total / Average	31,147,521	31,984,589	459,610	409,693	67	78
Panel B: First-Half Period								
American Options	Start Date	End Date	Volume (V)		Transaction (T)		V / T	
			Puts	Calls	Puts	Calls	Puts	Calls
Australian Dollar	Aug-28-87	Dec-31-90	618,442	444,998	10,116	9,417	61	47
British Pound	Aug-28-87	Dec-31-90	1,326,407	1,529,935	21,563	23,586	62	65
Canadian Dollar	Aug-28-87	Dec-31-90	543,230	512,206	20,120	14,684	27	35
Deutsche Mark	Aug-28-87	Dec-31-90	6,636,952	8,004,810	69,495	84,454	96	95
French Franc	Aug-28-87	Dec-31-90	46,535	305,223	450	1,090	103	280
Japanese Yen	Aug-28-87	Dec-31-90	4,758,731	5,134,509	62,506	68,961	76	74
Swiss Franc	Aug-28-87	Dec-31-90	1,313,824	1,733,737	23,272	28,269	56	61
		Total / Average	15,244,121	17,665,418	207,522	230,461	73	76
Panel C: Second-Half Period								
American Options	Start Date	End Date	Volume (V)		Transaction (T)		V / T	
			Puts	Calls	Puts	Calls	Puts	Calls
Australian Dollar	Jan-01-91	Oct-18-94	221,132	216,304	4,973	4,864	44	44
British Pound	Jan-01-91	Oct-18-94	926,485	928,633	27,630	25,791	34	36
Canadian Dollar	Jan-01-91	Oct-18-94	327,905	251,386	10,054	5,756	33	44
Deutsche Mark	Jan-01-91	Oct-18-94	11,027,442	9,969,240	132,994	86,452	83	115
French Franc	Jan-01-91	Oct-18-94	246,512	123,991	2,273	988	108	125
Japanese Yen	Jan-01-91	Oct-18-94	2,455,694	2,259,502	44,565	35,218	55	64
Swiss Franc	Jan-01-91	Oct-18-94	698,230	570,115	29,599	20,163	24	28
		Total / Average	15,903,400	14,319,171	252,088	179,232	63	79

Key: V is volume (number of option contracts)
T is transactions (number of trades)
V/T is number of contract per trades

Table 4.2 (mentioned earlier in Chapter 2) shows the total volumes and numbers of the transactions for the European-style options traded: Panel A covers the whole sample period, while Panels B and C are the two sub-periods.

Table 4.2: Total Volume and Transactions of the European-style Options

European Options	Start Date	End Date	Volume (V)		Transaction (T)		V / T	
			Puts	Calls	Puts	Calls	Puts	Calls
Panel A: Whole Observation								
Australian Dollar	Aug-28-87	Oct-18-94	204,413	208,847	1,917	1,718	107	122
British Pound	Aug-28-87	Oct-18-94	155,017	113,172	3,832	3,372	40	34
Canadian Dollar	Aug-28-87	Oct-18-94	144,096	133,954	3,501	3,576	41	37
Deutsche Mark	Sep-02-87	Oct-18-94	1,192,185	749,539	10,716	11,893	111	63
French Franc	Aug-31-87	Oct-18-94	3,461,467	2,955,402	2,812	2,382	1,231	1,241
Japanese Yen	Sep-11-87	Oct-18-94	120,311	217,971	2,823	2,757	43	79
Swiss Franc	Sep-02-87	Oct-18-94	294,362	244,358	5,985	5,295	49	46
Total / Average			5,571,851	4,623,243	31,586	30,993	176	149
Panel B: First-Half Period								
Australian Dollar	Aug-28-87	Dec-31-90	159,360	160,603	1,152	1,095	138	147
British Pound	Aug-28-87	Dec-31-90	41,342	37,750	827	714	50	53
Canadian Dollar	Aug-28-87	Dec-31-90	80,416	82,838	2,022	2,060	40	40
Deutsche Mark	Sep-02-87	Dec-31-90	21,274	34,826	562	514	38	68
French Franc	Aug-31-87	Dec-31-90	1,022	19,311	36	69	28	280
Japanese Yen	Sep-11-87	Dec-31-90	27,075	150,742	490	991	55	152
Swiss Franc	Sep-02-87	Dec-31-90	138,410	125,098	1,368	1,152	101	109
Total / Average			468,899	611,168	6,457	6,595	72	92
Panel C: Second-Half Period								
Australian Dollar	Jan-01-91	Oct-18-94	45,053	48,244	765	623	59	77
British Pound	Jan-01-91	Oct-18-94	113,675	75,422	3,005	2,658	38	28
Canadian Dollar	Jan-01-91	Oct-18-94	63,680	51,116	1,479	1,516	43	34
Deutsche Mark	Jan-01-91	Oct-18-94	1,170,911	714,713	10,154	11,379	115	63
French Franc	Jan-01-91	Oct-18-94	3,460,445	2,936,091	2,776	2,313	1,247	1,269
Japanese Yen	Jan-01-91	Oct-18-94	93,236	67,229	2,333	1,766	40	38
Swiss Franc	Jan-01-91	Oct-18-94	155,952	119,260	4,617	4,143	34	29
Total / Average			5,102,952	4,012,075	25,129	24,398	203	164

Key: V is volume (number of option contracts)
T is transactions (number of trades)
V/T is number of contract per trades

The selection procedure involves pairing European and American calls, and European and American puts. Each call or put option-pair is matched according to five variables: trade-date, expiration period, strike price, spot rate and time-of-trade. No single option is repeated in any match. Table 4.3 shows the maximum number of non-repeating (i.e., not double counted) call and put option-pairs (10,116 calls and 12,143 puts) before eliminating the out-of-the boundary option-pairs. In table 4.3, there are 1,406 call pairs with zero change in spot rate and 1,641 put pairs with zero change in spot rate. Allowing spot rates to change, there are 8,710 call pairs and 10,502 put pairs.

Table 4.3: Number of Call and Put Option-Pairs available for Selection

American/European Option Pairs	Change in Spot Zero		Change in Spot +/- 0		Total Option Pairs	
Types	Calls	Puts	Calls	Puts	Calls	Puts
Australian Dollar	78	44	361	327	439	371
British Pound	137	90	1,235	989	1,372	1,079
Canadian Dollar	176	239	670	835	846	1,074
Deutsche Mark	699	1,045	4,590	6,296	5,289	7,341
French Franc	6	5	51	106	57	111
Japanese Yen	219	127	952	987	1,171	1,114
Swiss Franc	91	91	851	962	942	1,053
Total	1,406	1,641	8,710	10,502	10,116	12,143

Key: Zero is no change in spot rate between American and European options
+/- 0 is American and European options have different spot rates

Table 4.4 shows that 903 calls and 535 puts of the option-pairs violate the American and European option boundary condition. We eliminate all call and put option-pairs that violate both the American and European boundaries because they have zero implied volatility.

Table 4.4: Number of Out of Boundary Call and Put Option-Pairs

Option-Pairs' Selection	Out-of-Boundary (Calls)	Out-of-Boundary (Puts)
Australian Dollar	36	23
British Pound	158	43
Canadian Dollar	186	90
Deutsche Mark	285	250
French Franc	5	13
Japanese Yen	140	76
Swiss Franc	93	40
Total Option-Pairs	903	535

It is extremely unlikely that put or call options of both the American and the European styles are traded at exactly the same time and spot rate. Therefore, sample selection is important. The aim is to obtain a sample in which all of the options in an arbitrage position are traded within a given period of time and within a given range of spot rates. The sample selection criteria are summarised in Table 4.5. The criteria are similar to those used in Chapter 3, with the exception of change in spot rate and time of trade. Five different samples have been selected, based on (i) the change in spot rate between the American and European trades and (ii) the elapsed time between the American and European trades.

Table 4.5: Selection Criteria

Options' Variables	Group A	Group B	Group C	Group D	Group E
Trade Date	Same	Same	Same	Same	Same
Expiration	Same	Same	Same	Same	Same
Strike Price	Same	Same	Same	Same	Same
Change in Spot Rate	10bp	25bp	50bp	100bp	>100bp
Change in Trade Time	10-min	10-min	30-min	60-min	>60-min

Key: Group A - Same Date; Expiry; Strike Price, Spot Rate is 10bp gap, Trading Time is 10-min gap
Group B - Same Date; Expiry; Strike Price, Spot Rate is 25bp gap, Trading Time is 10-min gap
Group C - Same Date; Expiry; Strike Price, Spot Rate is 50bp gap, Trading Time is 30-min gap
Group D - Same Date; Expiry; Strike Price, Spot Rate is 100bp gap, Trading Time is 60-min gap
Group E - Same Date; Expiry; Strike Price, Spot Rate >100bp gap, Trading Time >60-min gap

Table 4.6 shows the number of call observations and Table 4.7 shows the number of put observations in the sample after excluding the options violating the out-of-the-boundary conditions. For call option-pairs (see Table 4.6), we have 1,255 in group A (most restricted) and 9,213 in group E (least restricted). For put option-pairs (see Table 4.7) we have 1,629 in group A (most restricted) and 11,614 in group E (least restricted). In these selected option-pairs, Deutsche Mark options comprise more than 50% of all paired observations (puts and calls).

Table 4.6: Selected Call Option Pairs of Trades

American/European Change in Spot Change in Time	Group A (10 bp) (10 min)	Group B (25 bp) (10 min)	Group C (50 bp) (30 min)	Group D (100 bp) (60 min)	Group E (>100 bp) (>60 min)
Australian Dollar	45	45	73	100	403
British Pound	145	158	270	393	1,214
Canadian Dollar	114	115	199	286	660
Deutsche Mark	723	729	1,376	2,013	5,004
French Franc	7	9	12	14	52
Japanese Yen	149	151	272	394	1,031
Swiss Franc	72	74	168	294	849
Total Sample	1,255	1,281	2,370	3,494	9,213

Key: Refer to Table 4.5 for Groups' Descriptions

Table 4.7: Selected Put Option Pairs of Trades

American/European Change in Spot Change in Time	Group A (10 bp) (10 min)	Group B (25 bp) (10 min)	Group C (50 bp) (30 min)	Group D (100 bp) (60 min)	Group E (>100 bp) (>60 min)
Australian Dollar	8	8	29	50	348
British Pound	112	120	220	328	1,036
Canadian Dollar	192	193	335	466	984
Deutsche Mark	1,038	1,053	2,000	3,024	7,097
French Franc	4	4	12	23	98
Japanese Yen	141	150	255	377	1,038
Swiss Franc	134	139	247	341	1,013
Total Sample	1,629	1,667	3,098	4,609	11,614

Key: Refer to Table 4.5 for Groups' Descriptions

4.3.3 Interest Rates

The domestic and foreign interest rates for each currency are the London Euro-currency deposit interest rates of cross currencies. These have been obtained from Datastream as 1 day, 1 week, 1 month, 3 months, 6 months, and 1 year. Options that are expiring less than the fixed period of the interest rate use interpolated rates.

4.4. Theory, Methodology and Transactions Costs

4.4.1 Option Pricing Models used for the Analysis

In order to ascertain the values of early-exercise premiums of the American options, two option pricing models are used. The European-style options use the Garman and Kohlhagen (1983) option pricing model[20]. The call and put options pricing models are in Equations (2.3) and (2.4) respectively (see Chapter 2). The American-style options use the Barone-Adesi and Whaley (1987) (BW) analytical option-pricing model[21], which accounts for the early exercise feature. The BW call and put options pricing models are in Equations (3.3) and (3.4) respectively (see Chapter 3). The early-exercise premium can be computed with Equations (3.3) and (3.4) (see Chapter 3). The critical spot rates in the put and call option-pricing models will determine the early-exercise premium.

4.4.2 Method to Derive the Early-exercise Premium

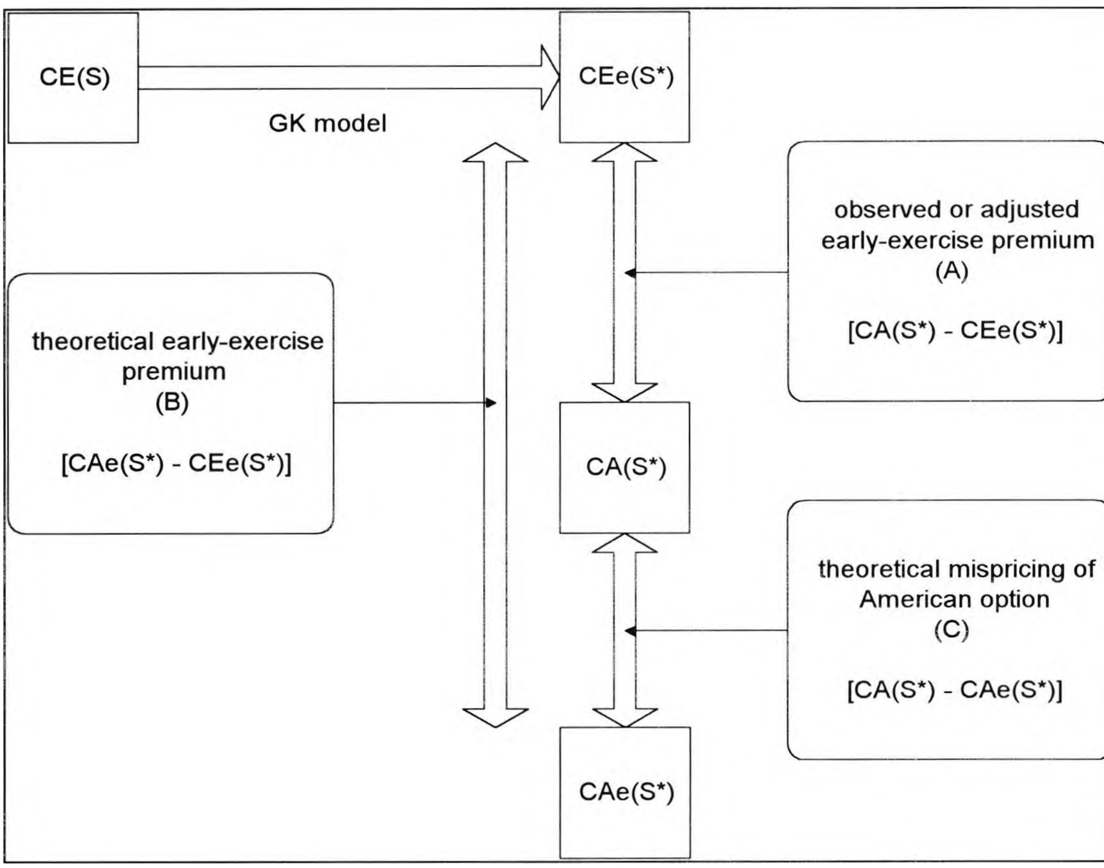
The test involves comparing option prices of the American and the European styles. Two comparisons can be made. The first (A) is the **observed** early-exercise premium between American and European options. The second (B) is the **theoretical** early-exercise premium for the same pair of options. When the European options are not traded with the same spot rate as the American options, **an adjustment is made to the European option price to account for the difference in the spot rate**. This is done with the Garman and Kohlhagen (1983) model and the implied volatility. The difference between **traded** American price and traded or adjusted European price is the observed early-exercise premium (see Figure 4.1, the comparison of (A) = [CA(s*) - CEe(s*)]). The **theoretical** early-exercise premium is computed using the Barone-Adesi and Whaley (1987) American option-pricing model at the implied volatility of the European options. The expected early-exercise premium is estimated from the adjusted European price, which is the difference between theoretical American price and the adjusted European price (see Figure 4.1, the comparison of (B) = [CAe(S*) - CEe(S*)]. The difference between traded American price and the theoretical American

[20] See Appendix-A for full details of the Garman and Kohlhagen (1983) option pricing model.
[21] See Appendix-D for full details of the Barone-Adesi and Whaley (1987) option pricing model.

prices is the theoretical mispricing of the American option (see Figure 4.1, the comparison of (C) = [CA(S*) -CAe(S*)]).

In order to estimate the apparent level of mispricing, the observed early-exercise premium (A) is compared with the theoretical early-exercise premium (B).

Figure 4.1: Procedure to Derive the Early-exercise Premium



Key: CE(S) is the observed European Call at Spot rate (1)
CEe(S*) is the adjusted European Call at observed Spot rate (2)
CA(S*) is the observed American Call at Spot rate (2)
CAe(S*) is the estimated American Call from European Call at observed Spot rate (2)

In the option-pairs, the American and European option prices must be equal to or greater than their intrinsic values. All option-pairs with either option price less than its intrinsic value are excluded from the test. The intrinsic values of the American and European options are in Equations (4.1) and (4.2) respectively.

$$\left.\begin{array}{l}\left[P_{BW} \geq (X - S)\right] \text{ for the American Put option} \\ \left[C_{BW} \geq (S - X)\right] \text{ for the American Call option}\end{array}\right\} \quad (4.1)$$

$$[P_{GK} \geq (Xe^{-rt} - Se^{-Rt})] \text{ for the European Put option} \quad \} \qquad (4.2)$$

$$[C_{GK} \geq (Se^{-Rt} - Xe^{-rt})] \text{ for the European Call option} \quad \}$$

4.4.3 Arbitrage Opportunities for Relatively Under-Priced American Options

When an American option is priced lower than a European option, we can make a risk-free profit (assuming zero transactions costs) with zero investment, i.e., by writing the relatively over-priced European option and using the proceeds to buy the relatively under-priced American option. The remaining proceeds would be a sure profit, since the portfolio would require no cash outflow (or inflow) on the expiration date of the options.

4.4.4 Transactions Costs

The typical PHLX transaction variable costs and fixed costs have been mentioned in earlier Chapter (section 2.4.5). The details of variable and fixed costs were showed in Table 2.13 of Chapter 2. The transactions costs estimated from the PHLX cost information is less than $25, however, to be conservative, $50 per round-turn is used for this study.

4.5. **Empirical Results**

There are four sections in the results. The first two sections (4.5.1 and 4.5.2) examine how many American options were priced below European equivalents and by how much, first without and then with transactions costs. The third section (4.5.3) compares the **observed** early-exercise premium with its **theoretical** value. Finally, section 4.5.4 attempts to relate mispricing of the early-exercise premium to moneyness and other variables.

4.5.1 Arbitrage Opportunities Without Transactions Costs

The test results [see Table 4.8 for puts and Table 4.9 for calls, without transactions costs] show that approximately 60% of the American puts and 20% of the American calls are priced below European options with the same strike and expiration. This violates rational pricing, as the American option prices should be equal to or greater than the European option prices. It is a very surprising result.

Table 4.8: Number of Observed American **Put** Options Traded Below European Values - **Whole Period**

Currency	A$	GBP	C$	DM	FFr	Yen	SFr	Total
Panel A: Group A Option-Paris (Put)								
Total	8	112	192	1,038	4	141	134	1,629
<$0	6	76	97	685	4	34	81	983
<$0(%)	75%	68%	51%	66%	100%	24%	60%	60%
Panel B: Group B Option-Paris (Put)								
Total	8	120	193	1,053	4	150	139	1,667
<$0	6	83	98	685	4	39	86	1,011
<$0(%)	75%	69%	51%	66%	100%	26%	62%	61%
Panel C: Group C Option-Paris (Put)								
Total	29	220	335	2,000	12	255	247	3,098
<$0	25	158	207	1,437	11	74	162	2,074
<$0(%)	86%	72%	62%	72%	92%	29%	66%	67%
Panel D: Group D Option-Paris (Put)								
Total	50	328	466	3,023	23	377	341	4,608
<$0	44	249	310	2,246	19	116	237	3,221
<$0(%)	88%	76%	67%	74%	83%	31%	70%	70%
Panel E: Group E Option-Paris (Put)								
Total	348	1,036	984	7,097	98	1,038	1,013	11,614
<$0	272	846	742	5,517	85	322	678	8,462
<$0(%)	78%	82%	75%	78%	87%	31%	67%	73%

Key: Refer to Table 4.5 for Group's Description
<$0 is refers to amount less than -$0.50 per option contract

Before discussing the results, it may help to explain the structure of Tables 4.8, 4.9, and other tables with similar structure in later sections, i.e., Tables 4.10, 4.11, 4.12, 4.13, 4.16, 4.17, 4.18, 4.19, 4.20 and 4.21. Each column is a particular foreign currency and each panel is a sample group. For each group, "Total" is the total number of transactions, the number of transactions traded below the European option level is indicated as "<$0" and the percentage of the under-priced transactions traded is indicated as "<$0(%)".

Table 4.9: Number of Observed American **Call** Options Traded Below European Values - **Whole Period**

Currency	A$	GBP	C$	DM	FFr	Yen	SFr	Total
Panel A: Group A Option-Paris (Call)								
Total	45	145	114	723	7	149	72	1,255
<$0	6	12	33	134	2	51	16	254
<$0(%)	13%	8%	29%	19%	29%	34%	22%	20%
Panel B: Group B Option-Paris (Call)								
Total	45	158	115	729	9	151	74	1,271
<$0	6	13	33	135	2	53	16	258
<$0(%)	13%	8%	31%	19%	22%	35%	22%	20%
Panel C: Group C Option-Paris (Call)								
Total	73	270	199	1,376	12	272	168	2,370
<$0	15	29	45	255	3	18	46	511
<$0(%)	21%	11%	23%	19%	25%	43%	27%	22%
Panel D: Group D Option-Paris (Call)								
Total	100	393	286	2,013	14	394	269	3,469
<$0	17	45	59	361	3	173	76	734
<$0(%)	17%	11%	21%	18%	21%	44%	28%	21%
Panel E: Group E Option-Paris (Call)								
Total	403	1,214	660	5,004	52	1,031	849	9,213
<$0	60	137	113	805	5	505	222	1,847
<$0(%)	15%	11%	17%	16%	10%	49%	26%	20%

Key: Refer to Table 4.5 for Group's Description
<$0 is refers to amount less than -$0.50 per option contract

Panel A of Tables 4.8 and 4.9 give results for the most restricted sample (Group A), while Panel E in Tables 4.8 and 4.9 give results for the least restricted sample (Group E). Across all five groups, there is only slight change in the results. The number of trades increases as the controls on time-of-trade and spot rates are relaxed, but the sample selection criteria have very little effect on the results.

All of the currencies give similar results, with 60% to 70% of puts under-priced (except for the Japanese Yen, for which 30% of puts are under-priced). Apart from the Japanese Yen, all other currencies have greater mispricing of puts than of calls, for which the frequency is about 25%. The Deutsche Mark is the most frequently traded option: it had more than 75% of its American puts under-priced relative to the European puts [see Panel E in Table 4.8], i.e., 5,517 out of 7,097 trades and 16% of its American calls under-priced (Panel E, Table 4.9), i.e., 805 out of 5,004 trades.

In the two sub-periods (Tables 4.10 and 4.12 for puts and Tables 4.11 and 4.13 for calls), there are more relatively under-priced calls in the first sub-period [see Tables 4.10 and 4.11], and more relatively under-priced puts in the second sub-period [see Tables 4.12 and 4.13]. Both of the tables show that the Australian Dollar and Canadian Dollar have less anomalies in the second sub-period. All other (five) currencies show slightly less mispriced values of American calls, but under-priced American puts remains frequent.

Table 4.10: Number of Observed American **Put** Options Treaded Below European Values - **First Sub-period**

Currency	A$	GBP	C$	DM	FFr	Yen	SFr	Total
Panel A: Group A Option-Paris (Put)								
Total	6	31	162	29	0	70	26	324
<$0	6	24	87	11	0	9	10	147
<$0(%)	100%	77%	54%	38%	0%	13%	38%	45%
Panel B: Group B Option-Paris (Put)								
Total	6	33	163	29	0	72	26	329
<$0	6	25	88	11	0	11	10	151
<$0(%)	1005	76%	54%	38%	0%	15%	38%	46%
Panel C: Group C Option-Paris (Put)								
Total	19	64	285	53	0	117	27	565
<$0	18	44	182	17	0	19	11	291
<$0(%)	95%	69%	64%	32%	0%	16%	41%	52%
Panel D: Group D Option-Paris (Put)								
Total	33	89	286	84	0	159	27	778
<$0	31	65	262	33	0	24	11	426
<$0(%)	94%	73%	68%	39%	0%	15%	41%	55%
Panel E: Group E Option-Paris (Put)								
Total	236	237	766	177	0	396	34	1,846
<$0	190	189	583	73	0	69	16	1,120
<$0(%)	81%	80%	76%	41%	0%	17%	47%	61%

Key: Refer to Table 4.5 for Group's Description
<$0 is refers to amount less than -$0.50 per option contract

Table 4.11: Number of Observed American **Call** Options Traded Below European Values - **First Sub-period**

Currency	A$	GBP	C$	DM	FFr	Yen	SFr	Total
Panel A: Group A Option-Paris (Call)								
Total	29	13	88	42	0	67	13	252
<$0	4	2	30	8	0	27	2	73
<$0(%)	14%	15%	34%	19%	0%	40%	15%	29%
Panel B: Group B Option-Paris (Call)								
Total	29	20	89	42	0	67	13	260
<$0	4	3	30	8	0	27	2	74
<$0(%)	14%	15%	34%	19%	0%	40%	15%	28%
Panel C: Group C Option-Paris (Call)								
Total	54	53	156	64	0	100	33	460
<$0	12	6	39	11	0	53	7	128
<$0(%)	22%	11%	25%	17%	0%	53%	21%	28%
Panel D: Group D Option-Paris (Call)								
Total	76	86	223	92	0	136	42	655
<$0	14	7	51	22	0	69	7	170
<$0(%)	18%	8%	23%	24%	0%	51%	17%	26%
Panel E: Group E Option-Paris (Call)								
Total	290	237	491	170	1	306	82	1,577
<$0	43	27	86	44	1	186	16	403
<$0(%)	15%	11%	18%	26%	100%	61%	20%	26%

Key: Refer to Table 4.5 for Group's Description
<$0 is refers to amount less than -$0.50 per option contract

Table 4.12: Number of Observed American **Put** Options Traded Below European Values - **Second Sub-period**

Currency	A$	GBP	C$	DM	FFr	Yen	SFr	Total
Panel A: Group A Option-Paris (Put)								
Total	2	127	30	1,009	4	71	108	1,251
<$0	0	52	10	674	4	25	71	836
<$0(%)	0%	41%	33%	67%	100%	35%	66%	62%
Panel B: Group B Option-Paris (Put)								
Total	2	87	30	1,034	4	78	113	1,348
<$0	0	58	10	684	4	28	76	860
<$0(%)	0%	67%	33%	66%	100%	36%	67%	64%
Panel C: Group C Option-Paris (Put)								
Total	10	156	50	1,947	12	138	220	2,533
<$0	7	114	25	1,420	11	55	151	1,783
<$0(%)	70%	73%	50%	73%	92%	40%	69%	70%
Panel D: Group D Option-Paris (Put)								
Total	17	239	80	2,939	23	218	314	3,830
<$0	13	184	48	2,213	19	92	226	2,795
<$0(%)	76%	77%	60%	75%	83%	42%	72%	73%
Panel E: Group E Option-Paris (Put)								
Total	112	799	218	6,920	98	642	979	9,768
<$0	82	657	159	5,444	85	253	662	7,342
<$0(%)	73%	82%	73%	79%	87%	39%	68%	75%

Key: Refer to Table 4.5 for Group's Description
<$0 is refers to amount less than -$0.50 per option contract

Table 4.13: Number of Observed American **Call** Options Traded Below European Values - **Second Sub-period**

Currency	A$	GBP	C$	DM	FFr	Yen	SFr	Total
Panel A: Group A Option-Paris (Call)								
Total	16	130	26	681	7	82	59	1,001
<$0	2	10	3	126	2	24	14	181
<$0(%)	13%	8%	12%	19%	29%	29%	24%	18%
Panel B: Group B Option-Paris (Call)								
Total	16	138	26	687	9	84	61	1,018
<$0	2	10	3	127	2	26	14	184
<$0(%)	13%	7%	13%	18%	22%	31%	23%	18%
Panel C: Group C Option-Paris (Call)								
Total	19	217	43	1,312	12	172	135	1,910
<$0	3	23	6	244	3	65	39	564
<$0(%)	16%	11%	14%	19%	25%	38%	29%	20%
Panel D: Group D Option-Paris (Call)								
Total	24	307	63	1,921	14	258	227	2,814
<$0	3	28	8	339	3	104	69	564
<$0(%)	13%	12%	13%	18%	21%	40%	30%	20%
Panel E: Group E Option-Paris (Call)								
Total	113	977	169	4,834	44	725	767	7,629
<$0	17	110	27	761	4	319	206	1,444
<$0(%)	15%	11%	16%	16%	9%	44%	27%	19%

Key: Refer to Table 4.5 for Group's Description
<$0 is refers to amount less than -$0.50 per option contract

The t-tests for average mispricing of puts and calls are in Tables 4.14 and 4.15 respectively. [*x*] indicates the mean option premium relative to the European option; [*t*] indicates the critical value; the under-lined results are significant at 95% confidence level. Each table is divided into columns by group and rows by foreign currency.

Table 4.14: The t-distribution of Observed American **Puts** Traded Below European Values

Group	A			B			C			D			E		
Products	*x*	*t*	*s*	*x*	*t*	*s*	*x*	*t*	*s*	*x*	*t*	*s*	*x*	*t*	*s*
Panel A: Whole Period															
A$	-10	-2.29	8	-10	-2.29	8	-16	-4.16	29	-17	-5.97	50	-14	-12.49	348
GBP	-12	-3.20	112	-12	-3.40	120	-22	-4.96	220	-23	-7.36	328	-30	-17.68	1,036
C$	-6	-6.04	192	-6	-6.15	193	-11	-9.44	335	-12	-11.67	466	-14	-18.99	984
DM	-8	-16.77	1,038	-8	-16.86	1,053	-9	-26.61	2,000	-10	-33.87	3,023	-12	-52.95	7,097
FFr	-20	-1.82	4	-20	-1.82	4	-44	-2.88	12	-42	-4.02	23	-44	-10.09	98
Yen	13	4.59	141	12	4.54	150	10	6.20	255	10	7.43	377	9	12.89	1,038
SFr	-4	-6.05	134	-4	-6.27	139	-5	-8.37	247	-5	-10.54	341	-5	-14.10	1,013
Panel B: First Sub-period															
A$	-15	-3.15	6	-15	-3.15	6	-19	-4.42	19	-21	-6.15	33	-17	-11.01	236
GBP	-10	-4.64	31	-9	-4.44	33	-17	-4.27	64	-19	-4.90	89	-27	-10.91	237
C$	-7	-5.83	162	-7	-5.95	163	-12	-8.99	285	-14	-11.09	386	-16	-17.56	766
DM	1	1.41	29	1	1.41	29	1	1.96	53	0	1.04	84	0	1.15	177
FFr	-	-	0	-	-	0	-	-	0	-	-	0	-	-	0
Yen	24	4.55	70	23	4.49	72	21	6.20	117	20	7.12	159	16	10.79	396
SFr	-1	-1.75	26	-1	-1.75	26	-1	-2.06	27	-1	-2.06	27	-1	-2.09	34
Panel C: Second Sub-period															
A$	-	-	2	-	-	2	-9	-1.29	10	-8	-1.93	17	-9	-6.69	112
GBP	-12	-2.46	81	-13	-2.68	87	-24	-3.99	156	-25	-6.06	239	-30	-14.87	799
C$	-3	-1.64	30	-3	-1.64	30	-5	-3.43	50	-6	-4.17	80	-9	-7.65	218
DM	-8	-16.92	1,009	-8	-17.00	1,024	-9	-26.84	1,947	-10	-34.15	2,939	-12	-53.33	6,920
FFr	-20	-1.82	4	-20	-1.82	4	-44	-2.88	12	-42	-4.02	23	-44	-10.09	98
Yen	2	1.93	71	2	1.94	78	2	2.46	138	3	3.46	218	4	8.29	642
SFr	-5	-5.94	108	-5	6.17	113	-5	-8.25	220	-5	-10.47	314	-5	-14.04	979

Key: Refer to Table 4.5 for Groups' Description
x is the mean (average early-exercise premium)
t is the critical value of t-distribution at 95% confidence level
s is the sample size
under-lined result is significant at 95% confidence level

The put results [*x*] for the whole period in Table 4.14 show that Japanese Yen puts are significantly over-priced, but all puts in other currencies are significantly under-priced. For the first sub-period in Panel B Deutsche Mark puts are relatively correctly priced, however there are less Deutsche Mark trades in that period than in the second sub-period (Panel C). For the second sub-period, Panel C shows that all American puts except those for the Japanese Yen are under-priced as compared to European puts. Canadian Dollar and French Franc puts are significantly mispriced in groups C, D and E, and the Australian Dollar is significant in group E. These three currencies improve in mispricing over time. The other four currencies become worse in the second sub-period.

Table 4.15: The t-distribution of Observed American **Calls** Traded Below European Values

Panel A: Whole Period

Group	A			B			C			D			E		
Products	*x*	*t*	*s*	*x*	*t*	*s*	*x*	*t*	*s*	*x*	*t*	*s*	*x*	*t*	*s*
A$	14	5.78	45	14	5.78	45	15	5.96	73	15	7.19	100	15	10.24	403
GBP	99	6.24	145	97	6.47	158	69	7.60	270	57	8.82	393	39	16.33	1,214
C$	3	5.43	114	3	5.46	115	6	8.09	199	6	10.75	286	8	10.87	660
DM	7	11.15	723	7	11.27	729	7	17.02	1,376	7	21.95	2,013	10	41.20	5,004
FFr	75	2.32	7	61	2.30	9	48	2.29	12	58	2.61	14	51	5.29	52
Yen	-2	-1.95	149	-2	-2.30	151	-2	-2.92	272	-2	-3.26	394	-2	-5.15	1,031
SFr	4	3.43	72	4	3.75	74	3	5.17	168	3	6.21	269	4	12.42	849

Panel B: First Sub-period

Group	A			B			C			D			E		
Products	*x*	*t*	*s*	*x*	*t*	*s*	*x*	*t*	*s*	*x*	*t*	*s*	*x*	*t*	*s*
A$	14	4.63	29	14	4.63	29	16	5.05	54	15	6.07	76	16	8.24	290
GBP	27	2.41	15	21	2.40	20	40	5.55	53	37	7.68	86	33	11.72	237
C$	3	4.46	88	3	4.49	89	6	8.29	156	6	10.36	223	8	8.92	491
DM	3	3.74	42	3	3.74	42	2	4.54	64	2	4.89	92	3	6.21	170
FFr	-	-	0	-	-	0	-	-	0	-	-	0	-	-	1
Yen	-5	-2.78	67	-5	-2.78	67	-6	-4.40	100	-6	-4.68	136	-6	-7.81	306
SFr	8	2.82	13	8	2.82	13	6	3.85	33	6	4.67	42	6	6.65	82

Panel C: Second Sub-period

Group	A			B			C			D			E		
Products	*x*	*t*	*s*	*x*	*t*	*s*	*x*	*t*	*s*	*x*	*t*	*s*	*x*	*t*	*s*
A$	15	3.35	16	15	3.35	16	13	3.26	19	14	4.03	24	13	7.64	113
GBP	107	6.13	130	108	6.39	138	76	6.85	217	62	7.67	307	40	13.98	977
C$	3	3.77	26	3	3.77	26	7	2.93	43	7	4.28	63	7	6.94	169
DM	7	10.95	681	7	11.07	687	7	16.82	1,312	8	21.78	1,921	10	41.02	4,834
FFr	75	2.32	7	61	2.30	9	48	2.29	12	58	2.61	14	49	5.10	51
Yen	1	0.87	82	1	0.70	84	1	1.36	172	0	1.10	258	0	-0.03	725
SFr	3	2.41	59	3	2.79	61	2	3.80	135	2	4.78	227	4	11.17	767

Key: Refer Table 4.5 for Groups' Description
x is the mean (average early-exercise premium)
t is the critical value of t-distribution at 95% confidence level
s is the sample size
under-lined result is significant at 95% confidence level

The call results [*x*] in Table 4.15 show that American call options on Japanese Yen are under-priced relative to their European equivalents. For the first sub-period (Panel B), all five groups are frequently under-priced relative to second sub-period (see Panel C). Panel C shows that American call options were relatively efficiently priced in the second sub-period. The Japanese Yen calls (American) improve their degree of under-pricing in the second sub-period.

The American early-exercise premium tests suggest that Japanese Yen calls and all other currencies' puts are under-priced relative to European. This provides risk-free opportunities for the market-makers to arbitrage by buying the relative cheap American puts and writing the relative expensive European puts.

4.5.2 Arbitrage Opportunities After Transactions Costs

An amount of $50 is used as transaction cost[22] for each option-pair lot[23]. Accounting for transactions costs reduces the risk-free arbitrage opportunities to about 30% for puts [see Table 4.16] and only 2% for calls [see Table 4.17]. In each section of the table, [<-$50] now represents the number of American trades under-priced by more than $50.

Table 4.16: Number of Observed American **Put** Options Traded Below European Values - **Whole Period**

Currency	A$	GBP	C$	DM	FFr	Yen	SFr	Total
Panel A: Group A Option-Paris (Put)								
Total	8	112	192	1,038	4	141	134	1,629
<-$50	3	19	37	296	1	1	29	386
<-$50(%)	38%	17%	19%	29%	25%	1%	22%	24%
Panel B: Group B Option-Paris (Put)								
Total	8	120	193	1053	4	150	139	1,667
<-$50	3	21	38	302	1	1	30	396
<-$50(%)	38%	18%	20%	29%	25%	1%	22%	24%
Panel C: Group C Option-Paris (Put)								
Total	29	220	335	2,000	12	255	247	3,098
<-$50	10	70	94	693	6	6	59	938
<-$50(%)	34%	32%	28%	35%	50%	2%	24%	30%
Panel D: Group D Option-Paris (Put)								
Total	50	328	466	3,023	23	377	341	4,608
<-$50	22	123	145	1,123	12	11	89	1,525
<-$50(%)	44%	38%	31%	37%	52%	3%	26%	33%
Panel E: Group E Option-Paris (Put)								
Total	348	1,036	984	7,097	98	1,038	1,013	11,614
<-$50	155	509	379	2,994	64	26	255	4,382
<-$50(%)	45%	49%	39%	42%	65%	3%	25%	37%

Key: Refer to Table 4.5 for Group's Description

[22] See earlier section 4.4.4. for arbitrage example and section 4.4.5 for actual transaction cost per lot.

[23] Each option trade can has more than 1 lot (contract), an average of 10 or more lots are traded in each trade. An option-pair lot consists of the round-turn of both the American and European calls (puts) contract.

Table 4.17: Number of Observed American **Call** Options Traded Below European Values - **Whole Period**

Currency	A$	GBP	C$	DM	FFr	Yen	SFr	Total
Panel A: Group A Option-Paris (Call)								
Total	45	145	114	723	7	149	71	1,255
<$50	0	0	0	5	0	11	1	17
<$50(%)	0%	0%	0%	1%	0%	7%	1%	1%
Panel B: Group B Option-Paris (Call)								
Total	45	158	105	729	9	151	74	1,271
<$50	0	0	0	6	0	12	1	19
<$50(%)	0%	0%	0%	1%	0%	8%	1%	1%
Panel C: Group C Option-Paris (Call)								
Total	73	270	199	1,376	12	272	168	2,370
<$50	0	2	0	16	0	27	5	50
<$50(%)	0	1%	0%	1%	0%	10%	3%	2%
Panel D: Group D Option-Paris (Call)								
Total	100	393	286	2,013	14	394	269	3,469
<$50	1	4	0	21	0	46	10	82
<$50(%)	1%	1%	0%	1%	0%	12%	4%	2%
Panel E: Group E Option-Paris (Call)								
Total	403	1,214	660	5,004	52	1,031	849	9,213
<$50	10	24	4	63	0	154	42	297
<$50(%)	2%	2%	1%	1%	0%	15%	5%	3%

Key: Refer to Table 4.5 for Group's Description

The results are divided into two sub-periods as before. Transactions costs have a big impact in the first sub-period. Only the Japanese Yen call now has frequent risk-free arbitrage opportunities. The Japanese Yen calls give arbitrage opportunities ranging from 13% in group (A) to 24% in group (E). The French Franc calls have zero opportunity for all groups, while Deutsche Mark and Canadian Dollar calls are only profitable in group E. In the put columns, the Australian Dollar puts have about 50% risk-free arbitrage opportunities and the British Pound and Canadian Dollar puts have more than 20% risk-free arbitrage opportunities. However, the Swiss Franc puts have zero arbitrage opportunity.

Table 4.18: Number of Observed American **Put** Options Traded Below European Values - **First Sub-period**

Currency	A$	GBP	C$	DM	FFr	Yen	SFr	Total
Panel A: Group A Option-Paris (Put)								
Total	6	31	162	29	0	70	26	324
<$50	3	8	34	0	0	0	0	45
<$50(%)	50%	26%	21%	0%	0%	0%	0%	4%
Panel B: Group B Option-Paris (Put)								
Total	6	33	163	29	0	72	26	329
<$50	3	8	35	0	0	0	0	46
<$50(%)	50%	24%	21%	0%	0%	0%	0%	14%
Panel C: Group C Option-Paris (Put)								
Total	19	64	285	53	0	117	27	565
<$50	9	21	85	0	0	0	0	115
<$50(%)	47%	33%	30%	0%	0%	0%	0%	20%
Panel D: Group D Option-Paris (Put)								
Total	33	89	386	84	0	159	27	778
<$50	20	30	130	1	0	0	0	181
<$50(%)	61%	34%	34%	1%	0%	0%	0%	23%
Panel E: Group E Option-Paris (Put)								
Total	236	237	766	177	0	396	34	1,846
<$50	118	119	311	3	0	2	0	553
<$50(%)	50%	50%	41%	2%	0%	1%	0%	30%

Key: Refer to Table 4.5 for Group's Description

Table 4.19: Number of Observed American **Call** Options Traded Below European Values - **First Sub-period**

Currency	A$	GBP	C$	DM	FFr	Yen	SFr	Total
Panel A: Group A Option-Paris (Call)								
Total	29	13	88	42	0	67	13	252
<$50	0	0	0	0	0	9	1	10
<$50(%)	0%	0%	0%	0%	0%	13%	8%	4%
Panel B: Group B Option-Paris (Call)								
Total	29	20	89	42	0	67	13	260
<$50	0	0	0	0	0	9	1	10
<$50(%)	0%	0%	0%	0%	0%	13%	8%	4%
Panel C: Group C Option-Paris (Call)								
Total	54	53	156	64	0	100	33	460
<$50	0	0	0	0	0	20	2	22
<$50(%)	0%	0%	0%	0%	0%	20%	6%	5%
Panel D: Group D Option-Paris (Call)								
Total	76	86	223	92	0	136	42	655
<$50	1	0	0	0	0	27	2	30
<$50(%)	1%	0%	0%	0%	0%	20%	5%	5%
Panel E: Group E Option-Paris (Call)								
Total	290	237	491	170	1	306	82	1,577
<$50	8	7	4	1	0	74	3	97
<$50(%)	3%	3%	1%	1%	0%	24%	4%	6%

Key: Refer to Table 4.5 for Group's Description

In the second sub-period [see Tables 4.20 and 4.21], the under-pricing of the American options is different from first period. Three major currencies have more puts under-priced relative to the European puts: they are Deutsche Mark, British Pound and Swiss Franc. The Deutsche Mark ranges from 29% in group (A) to 43% in group (E), British Pound ranges from 8% in group (A) to 48% in group (E) and the Swiss Franc puts have about 26% risk-free arbitrage opportunity for all five groups. The French Franc and Canadian Dollar have zero arbitrage for the call options in all five groups.

Table 4.20: Number of Observed American **Put** Options Traded Below European Values - **Second Sub-period**

Currency	A$	GBP	C$	DM	FFr	Yen	SFr	Total
Panel A: Group A Option-Paris (Put)								
Total	2	127	30	1,009	4	71	108	1,361
<$50	0	11	3	296	1	1	29	341
<$50(%)	0%	9%	10%	29%	25%	1%	27%	25%
Panel B: Group B Option-Paris (Put)								
Total	2	87	30	1,034	4	78	113	1,348
<$50	0	13	3	302	1	1	30	350
<$50(%)	0%	15%	10%	29%	25%	1%	27%	26%
Panel C: Group C Option-Paris (Put)								
Total	10	156	50	1,947	12	138	220	2,533
<$50	1	49	9	693	6	6	59	823
<$50(%)	10%	31%	18%	36%	50%	4%	27%	32%
Panel D: Group D Option-Paris (Put)								
Total	17	239	80	2,939	23	218	314	3,830
<$50	2	93	15	1,122	12	11	89	1,344
<$50(%)	12%	39%	19%	38%	52%	5%	28%	35%
Panel E: Group E Option-Paris (Put)								
Total	112	799	218	6,920	98	642	979	9,768
<$50	37	390	68	2,991	64	24	255	3,829
<$50(%)	33%	49%	31%	43%	65%	4%	26%	39%

Key: Refer to Table 4.5 for Group's Description

Deutsche Mark puts have the highest number of arbitrage trades [see Panel E of Table 4.20], 2,991 out of 6,920 trades in the second sub-period of approximately 986 trading days, i.e., equal to 3 trades in every trading day. From Tables 4.1 and 4.2, the average number of contracts per trade is approximately 83 for American puts and 115 for European puts [see Panel C of both tables]. With average contract under-priced by $75, i.e., $25 arbitrage profit, there is an approximate $1.6 million, risk-free arbitrage profit per year [3 trades per trading day with $25 profit per contract, each trade consisting of an average 83 contracts and 260 trading days per year].

Table 4.21: Number of Observed American **Call** Options Traded Below European Values - **Second Sub-period**

Currency	A$	GBP	C$	DM	FFr	Yen	SFr	Total
Panel A: Group A Option-Paris (Call)								
Total	16	130	26	681	7	82	59	1,001
<$50	0	0	0	5	0	2	0	7
<$50(%)	0%	0%	0%	1%	0%	2%	0%	1%
Panel B: Group B Option-Paris (Call)								
Total	16	138	23	687	9	84	61	1,018
<$50	0	0	0	6	0	3	0	9
<$50(%)	0%	0%	0%	1%	0%	4%	0%	1%
Panel C: Group C Option-Paris (Call)								
Total	19	217	43	1,321	12	172	135	1,910
<$50	0	2	0	16	0	7	3	28
<$50(%)	0%	1%	0%	1%	0%	4%	2%	1%
Panel D: Group D Option-Paris (Call)								
Total	24	307	63	1,921	14	258	227	2,814
<$50	0	4	0	21	0	19	8	52
<$50(%)	0	1%	0%	1%	0%	7%	4%	2%
Panel E: Group E Option-Paris (Call)								
Total	113	977	169	4,834	44	725	767	7,629
<$50	2	17	0	62	0	80	39	200
<$50(%)	2%	2%	0%	1%	0%	11%	5%	3%

Key: Refer to Table 4.5 for Group's Description

4.5.3 Theoretical Versus Observed Early-exercise Premiums

In this section, the observed early-exercise premium is compared with the theoretical American early-exercise premium. The deviations of the early-exercise premiums are showed in Tables 4.22, 4.23. Summarise the finding are showed in Appendix at the end of this chapter (Tables 4.26, 4.27, 4.28, 4.29, 4.30 and 4.31 cover the whole period and two sub-periods with all trades, in-the-money trades, at-the-money trades and out-of-the-money trades).

The call deviations [see Table 4.22] become larger when the sample criteria relaxed. The deviations move from positive (observed premium below the expected) in group (A) to negative (observed premium above the expected) in group (E). In the two sub-periods, groups (D and E) have larger deviations in the second sub-period than in the first. The results for in-the-money, at-the-money and out-of-the-money trades are different from all trades. In-the-money trades (see Table 4.30) have the smallest deviation while at-the-money trades (see Table 4.28) have similar deviation for both sub-periods.

The put deviations [see Table 4.23] are similar to those for calls, with larger deviations when the sampling criteria are relaxed. The deviation is negative in group (A) but is positive in group (E). Larger deviations occur in the second than in the first sub-period. The results for out-of-the-money, at-the-money and in-the-money trades [see Tables 4.27, 4.29 and 4.31] are similar for all groups with larger deviations in the second sub-period.

The premium deviation shows that the observed premiums for puts are lower than the expected. (Equivalently, the call premiums are higher than expected.) Below are the detailed analyses of the premium deviation for all trades.

Panel A of Table 4.22 shows results for calls for whole period. Deviations in group (A) are less frequent than in group (E). Group (A) for the whole period has an observed premium of $571 whereas the expected premium is $558. Hence the observed value is 3% below the expected. By contrast, Group (E) has an observed premium of $441 and an expected premium of $246. Hence the observed value is 79% above the expected. Except for the British Pound and Japanese Yen, other currencies have negative deviations, which indicate that observed premiums are higher than expected premiums. Table 4.22, Panels B and C for first and second sub-periods, show that group (E) of second sub-period (-160%) has higher deviation than first sub-period (-41%).

Table 4.22: Deviation of **Calls'** Early-exercise Premium - Total ($ per Contract)

Panel A: Whole Period

	Group (A)			Group (B)			Group (C)			Group (D)			Group (E)		
FX	Obs	Exp	(E-O)	Obs	Exp	(E-O)	Obs	Exp	(E-O)	Obs	Exp	(E-O)	Obs	Exp	(E-O)
A$	35	21	-14	35	21	-14	46	19	-27	59	20	-39	75	19	-56
GBP	308	448	140	302	417	115	216	256	40	177	181	4	121	149	28
C$	17	7	-10	17	7	-10	29	30	1	30	23	-7	39	16	-23
DM	43	13	-30	43	13	-30	42	11	-31	46	15	-31	60	14	-46
FFr	156	91	-65	127	71	-56	101	57	-44	130	59	-71	128	41	-87
Yen	-11	1	12	-11	1	12	-11	1	12	-10	1	11	-10	1	11
SFr	23	7	-16	25	7	-18	19	5	-14	18	4	-14	28	6	-22
Total	571	588	3%	538	537	0%	442	379	-17%	450	303	-49%	441	246	-79%

Panel B: First Sub-period

	Group (A)			Group (B)			Group (C)			Group (D)			Group (E)		
FX	Obs	Exp	(E-O)	Obs	Exp	(E-O)	Obs	Exp	(E-O)	Obs	Exp	(E-O)	Obs	Exp	(E-O)
A$	51	28	-23	51	28	-23	58	23	-35	71	24	-47	85	23	-62
GBP	83	38	-45	65	30	-35	124	42	-82	117	35	-82	104	31	-73
C$	17	8	-9	17	8	-9	27	37	10	29	28	-1	40	20	-20
DM	16	12	-4	16	12	-4	14	8	-6	13	7	-6	16	5	-11
FFr	148	190	42	148	190	42	148	190	42	148	190	42	148	190	42
Yen	-29	0	29	-29	0	29	-38	0	38	-35	0	35	-35	1	36
SFr	52	22	-30	52	22	-30	36	15	-21	36	15	-21	37	10	-27
Total	338	298	-13%	320	290	-10%	369	315	-17%	379	299	-27%	395	280	-41%

Panel C: Second Sub-period

	Group (A)			Group (B)			Group (C)			Group (D)			Group (E)		
FX	Obs	Exp	(E-O)	Obs	Exp	(E-O)	Obs	Exp	(E-O)	Obs	Exp	(E-O)	Obs	Exp	(E-O)
A$	6	7	1	6	7	1	11	7	-4	26	8	-18	49	8	-41
GBP	334	495	161	336	473	137	239	309	70	193	222	29	125	90	-35
C$	16	5	-11	16	5	-11	33	7	-26	35	6	-29	37	6	-31
DM	45	13	-32	45	13	-32	43	11	-32	48	15	-33	61	15	-46
FFr	157	75	-82	125	56	-69	97	45	-52	128	49	-79	128	38	-90
Yen	4	2	-2	3	2	-1	4	1	-3	3	1	-2	0	1	1
SFr	17	4	-13	20	4	-16	15	3	-12	15	3	-12	27	6	-21
Total	579	601	4%	551	560	2%	442	383	-15%	448	304	-47%	427	164	-160%

Key: Value is shown in US$ per option contract
Obs is the observed mean of early-exercise premium
Exp is the expected mean of early-exercise premium
(E-O) is the difference between expected and observed means, and (E-O)% is |(E-O)/E|

The puts are shown in Table 4.23. Except for the Japanese Yen, all other currencies have negative observed early-exercise premium. The deviations of puts are more frequent than for calls, with the second sub-period having higher frequency than the first sub-period for all groups. Group (A) has less frequent deviations than group (E) for both periods. In Panel A, group (A) for whole period has an observed premium of -\$158 whereas the expected premium is \$23. Hence the observed value is 787% below the expected. Group (E) for whole period has an observed premium of -\$399 and an expected premium of \$17. Hence the observed value is 2,447% below the expected.

Table 4.23: Deviation of **Puts**' Early-exercise Premium - Total (\$ per Contract)

Panel A: Whole Period

	Group (A)			Group (B)			Group (C)			Group (D)			Group (E)		
FX	Obs	Exp	(E-O)	Obs	Exp	(E-O)	Obs	Exp	(E-O)	Obs	Exp	(E-O)	Obs	Exp	(E-O)
A$	-52	0	52	-52	0	52	-79	0	79	-84	1	85	-70	0	70
GBP	-31	0	31	-36	0	36	-68	0	68	-73	0	73	-93	0	93
C$	-31	0	31	-32	0	32	-54	0	54	-62	0	62	-72	0	72
DM	-48	0	48	-48	0	48	-57	0	57	-63	0	63	-76	1	77
FFr	-49	0	49	-49	0	49	-111	0	111	-104	0	104	-111	0	111
Yen	79	22	-57	74	21	-53	65	17	-48	64	20	-44	53	16	-37
SFr	-26	1	27	-26	1	27	-29	0	29	-32	0	32	-30	0	30
Total	-158	23	787%	-169	22	868%	-333	17	2059%	-354	21	1786%	-399	17	2447%

Panel B: First Sub-period

	Group (A)			Group (B)			Group (C)			Group (D)			Group (E)		
FX	Obs	Exp	(E-O)	Obs	Exp	(E-O)	Obs	Exp	(E-O)	Obs	Exp	(E-O)	Obs	Exp	(E-O)
A$	-74	0	74	-74	0	74	-97	0	97	-107	2	109	-85	1	86
GBP	-32	0	32	-29	0	29	-52	0	52	-61	0	61	-84	0	84
C$	-34	0	34	-35	0	35	-59	0	59	-69	0	69	-80	0	80
DM	5	2	-3	5	2	-3	5	2	-3	2	1	-1	3	1	-2
FFr			0			0			0			0			0
Yen	147	42	-105	142	41	-101	130	35	-95	129	44	-85	100	35	-65
SFr	-7	3	10	-7	3	10	-8	2	10	-8	2	10	-7	2	9
Total	5	47	89%	2	46	96%	-81	39	308%	-114	49	333%	-153	39	492%

Panel C: Second Sub-period

	Group (A)			Group (B)			Group (C)			Group (D)			Group (E)		
FX	Obs	Exp	(E-O)	Obs	Exp	(E-O)	Obs	Exp	(E-O)	Obs	Exp	(E-O)	Obs	Exp	(E-O)
A$	15	0	-15	15	0	-15	-46	0	46	-40	0	40	-42	0	42
GBP	-31	0	31	-39	0	39	-74	0	74	-78	0	78	-95	0	95
C$	-17	0	17	-17	0	17	-26	0	26	-30	0	30	-45	0	45
DM	-49	0	49	-49	0	49	-59	0	59	-65	0	65	-78	1	79
FFr	-49	0	49	-49	0	49	-111	0	111	-104	0	104	-111	0	111
Yen	12	3	-9	11	3	-8	11	3	-8	20	3	-17	25	4	-21
SFr	-30	0	30	-31	0	31	-32	0	32	-34	0	34	-31	0	31
Total	-149	3	5067%	-159	3	5400%	-337	3	11333%	-331	3	11133%	-377	5	7640%

Key: Value is shown in US\$ per option contract
Obs is the observed mean of early-exercise premium
Exp is the expected mean of early-exercise premium
(E-O) is the difference between expected and observed means, and (E-O)% is [(E-O)/E]
Blank indicates there is no trade for that period

4.5.4 Factors Affecting Mispricing of American Options

The theoretical value is estimated from the observed European price using the BW model. A test (see Equation 4.3) of mispricing versus time-to-expiration, moneyness, domestic and foreign interest rates has been conducted with the sample from group (E). (Earlier results show that the sample selection has little effect across the groups so we shown the largest sample). The regression results in Tables 4.24 and 4.25 show that the independent variables: time-to-expiration, moneyness[24], (domestic - foreign) interest rates - have significant relationships to mispricing of American options. However, the coefficients vary across currencies.

$$C_{Amex} - C_{Estimated} = a + b(F/X) + c(r - R) + d(t) + e \qquad (4.3)$$

where C_{Amex} is the observed American option price and $C_{Estimated}$ is the estimated American option price from European option of the option-pair (F/X) is the Moneyness, (r-R) is the interest rate differential and (t) is the time-to-expiration.

The t-test results for **puts** at the base of Table 4.24 show that all variables (except time-to-expiration) have significant positive (over-priced) effects on all currencies. In-the-money puts tend to be over-priced relative to the model puts. Puts at lower foreign interest rate tend to be over-priced relative to model puts. Time-to-expiration has significant negative (under-pricing) effect on all currencies. The long-expiration puts tend to be under-priced relative to the model puts.

[24] Moneyness for Call option is Future Spot Rate / Strike Price, i.e., (F/X)

Table 4.24: Regression Test for Level of Mispricing as a Function of Moneyness, Interest Rates and Expiration - **Put** Options

Regression Statistics	A$	GBP	C$	DM	FFr	Yen	SFr
R Square	0.427	0.584	0.212	0.462	0.599	0.174	0.145
Adjusted R Square	0.422	0.583	0.210	0.461	0.587	0.172	0.143
Standard Error	17	35	38	16	28	38	20
Observations	338	1,063	1,065	7,289	108	1,073	1,043
ANOVA	A$	GBP	C$	DM	FFr	Yen	SFr
Regression - F	83	495	95	2,083	52	75	59
Significance F	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
	A$	GBP	C$	DM	FFr	Yen	SFr
Coefficients - Intercept	-11	-557	-789	-207	-618	-341	-164
(t-stat - Intercept)	-1.47	<u>-14.69</u>	<u>-11.55</u>	<u>-26.83</u>	<u>-4.90</u>	<u>-5.69</u>	<u>-7.02</u>
Coefficients - Moneyness	20	591	796	216	655	347	169
(t-stat - Moneyness)	<u>2.84</u>	<u>15.69</u>	<u>11.67</u>	<u>28.44</u>	<u>5.14</u>	<u>5.86</u>	<u>7.32</u>
Coefficients - Interest Rates[r-R]	255	734	316	246	788	440	250
(t-stat - Interest Rates[r-R])	<u>5.04</u>	<u>14.76</u>	<u>3.67</u>	<u>22.39</u>	<u>5.87</u>	<u>7.10</u>	<u>6.47</u>
Coefficients - Time-to-expiration	-78	-311	-62	-96	-299	-110	-58
(t-stat - Time-to-expiration)	<u>-15.17</u>	<u>-34.83</u>	<u>-10.30</u>	<u>-69.24</u>	<u>-11.50</u>	<u>-14.30</u>	<u>-9.96</u>

Note: <u>Underlined t-Stat results</u> indicate significance at 95% confidence level.

The t-test results for **calls** at the base of Table 4.25 show that all variables are significant for all currencies. Moneyness has a significant negative effect on Australian Dollar, Deutsche Mark, Japanese Yen and Swiss Franc calls. These in-the-money calls tend to be under-priced relative to model calls. The foreign interest rate has a significant negative effect on Deutsche Mark, Japanese Yen and Swiss Franc calls. Calls with higher foreign interest rates tend to be under-priced relative to model calls. Time-to-expiration has significant negative effect on British Pound, Canadian Dollar and Japanese Yen calls while it has a significant positive effect on Deutsche Mark and French Franc calls. The long-expiration British Pound, Canadian Dollar and Japanese Yen calls tend to be under-priced relative to model calls, while long-expiration calls on Deutsche Mark and French Franc tend to be over-priced relative to model calls.

Table 4.25: Regression Test for Level of Mispricing as a Function of Moneyness, Interest Rates and Expiration - **Call** Options

Regression Statistics	A$	GBP	C$	DM	FFr	Yen	SFr
R Square	0.088	0.008	0.013	0.067	0.245	0.204	0.042
Adjusted R Square	0.081	0.006	0.009	0.066	0.201	0.201	0.039
Standard Error	252	249	779	26	52	10	21
Observations	433	1,355	833	5,213	56	1,049	899
ANOVA	A$	GBP	C$	DM	FFr	Yen	SFr
Regression - F	14	4	4	125	6	89	13
Significance F	0.0%	1.2%	1.3%	0.0%	0.2%	0.0%	0.0%
	A$	GBP	C$	DM	FFr	Yen	SFr
Coefficients - Intercept	740	516	1,579	134	120	57	73
(t-stat - Intercept)	<u>6.02</u>	1.82	0.97	<u>8.17</u>	0.47	<u>3.54</u>	<u>2.57</u>
Coefficients - Moneyness	-753	-496	-1,536	-140	-140	-56	-76
(t-stat - Moneyness)	<u>-6.40</u>	-1.75	-0.95	<u>-8.52</u>	-0.55	<u>-3.50</u>	<u>-2.69</u>
Coefficients - Interest Rates[r-R]	60	-36	734	-177	-588	-238	-151
(t-stat - Interest Rates[r-R])	0.10	-0.11	0.43	<u>-8.83</u>	-1.82	<u>-12.51</u>	<u>-3.70</u>
Coefficients - Time-to-expiration	67	-135	-672	36	93	-24	26
(t-stat - Time-to-expiration)	0.83	<u>-3.23</u>	<u>-3.28</u>	<u>9.28</u>	<u>2.47</u>	<u>-6.98</u>	<u>3.45</u>

Note: Underlined t-Stat results indicate significance at 95% confidence level.

4.6. Conclusions

This study uses matched trade-by-trade data from August 28 1978 to October 18 1994 for American and European currency options. The sample has 9,213 call pairs and 11,614 put pairs. It is quite different from previous studies which used only the daily closing prices (with the exception of Hilliard and Tucker (1991) who also used intra-day data). The results show that the American options are often priced lower than the European options. Approximately 60% of the American puts and 20% of the American calls are priced lower than equivalent European options. When transactions costs of $50 are applied, the number of arbitrage opportunities on still remains about 30% for puts but falls to only 2% for calls.

The early sample period overlaps with Hilliard and Tucker (1991), but has difference results due to our sample selection criteria and adjustment to the European price when its spot rate is different from American. The results from different samples show that the selection criteria have only a small effect on the results. The two sub-periods show that the American call options (except French Franc and Swiss Franc) have become less mispriced over time, however the American puts have worsened over time. The Deutsche Mark put prices have shifted from "positive" early-exercise premium to "negative" early-exercise premium[25] as compared to the European put prices. The other put options (except the Japanese Yen) have become cheaper over time. Although taking account of transactions costs reduces the number of risk-free arbitrage opportunities, they are still high. The results on Deutsche Mark puts in the second sub-period indicate a risk-free arbitrage profit of more than $5,000[26] per trading day.

Apart from the observed values, the results are compared with theoretical American prices which confirm a very high level of deviation from the theoretical values. In general, the observed puts are priced lower than expected while the observed calls are priced higher than the expected. The regression of mispricing vs moneyness shows that in-the-money puts are priced higher than model puts, while the calls are priced lower than model calls. The lower the foreign interest rate, the higher

---

[25] The "negative" early-exercise premium means the price of the American option is lower than the European option, while "positive" early-exercise premium means the price of the American option is above the European option.
[26] Please refer to section 4.5.2

the observed over pricing for both calls and puts. Longer-term puts are priced lower than model puts. The longer-term calls (except Deutsche Mark and French Franc) are also priced lower than the model calls (except for the Deutsche Mark and French Franc).

This study shows that approximately 30% of the American put options are priced below equivalent European puts. Why does the high mispricing of American options continue over the time? The first possible reason could be that the data on puts (as collected) are wrong. However, we have reconfirmed with PHLX that there is no mistake in the data set. A second reason could be that investors are willing to pay a large liquidity premium for the American options, which trade more frequently than the European options. This seems unlikely, as the European options do trade regularly. A third reason could be our use of one-daily interest rates. However the effect would have to be large to generate the level of observed mispricing. In sum, we cannot explain these result, except as an "anomaly".

4.7. Appendix: Other Findings on Theoretical versus Observed Early-exercise Premiums

The results of deviation may be categorised into out-of-the-money, at-the-money and in-the-money. Tables 4.26 and 4.27 show the results for the out-of-the-money calls and puts respectively.

In Table 4.26, the calls show higher deviation in the first sub-period (Panel B) than second sub-period (Panel C). Group (E) for the first sub-period has an observed premium of $251 whereas the expected premium is $50. Hence the observed value is 402% above the expected. Group (E) for the second sub-period has an observed premium of $465 whereas the expected premium is $179. Hence the observed value is 160% above the expected.

Table 4.26: Deviation of **Calls**' Early-exercise Premium
- Out-of-the-money ($ per Contract)

Panel A: Whole Period

	Group (A)			Group (B)			Group (C)			Group (D)			Group (E)		
FX	Obs	Exp	(E-O)	Obs	Exp	(E-O)	Obs	Exp	(E-O)	Obs	Exp	(E-O)	Obs	Exp	(E-O)
A$	19	14	-5	19	14	-5	46	16	-30	52	17	-35	70	16	-54
GBP	435	706	271	430	668	238	284	392	108	222	269	47	128	92	-36
C$	31	9	-22	31	9	-22	44	8	-36	52	13	-39	57	11	-46
DM	53	12	-41	52	12	-40	50	10	-40	53	15	-38	65	13	-52
FFr	259	131	-128	259	131	-128	154	83	-71	133	70	-63	127	32	-95
Yen	3	1	-2	3	1	-2	1	1	0	2	1	-1	0	1	1
SFr	27	6	-21	27	6	-21	18	3	-15	22	3	-19	33	4	-29
Total	827	879	6%	821	841	2%	597	513	-16%	536	388	-38%	480	169	-184%

Panel B: First Sub-period

	Group (A)			Group (B)			Group (C)			Group (D)			Group (E)		
FX	Obs	Exp	(E-O)	Obs	Exp	(E-O)	Obs	Exp	(E-O)	Obs	Exp	(E-O)	Obs	Exp	(E-O)
A$	23	16	-7	23	16	-7	59	19	-40	63	20	-43	82	19	-63
GBP	53	10	-43	24	4	-20	81	12	-69	87	12	-75	81	11	-70
C$	27	9	-18	27	9	-18	40	14	-26	47	13	-34	58	12	-46
DM	14	7	-7	14	7	-7	14	5	-9	13	5	-8	16	4	-12
FFr			0			0			0			0			0
Yen	-8	0	8	-8	0	8	-19	0	19	-12	0	12	-16	1	17
SFr			0			0	22	2	-20	26	3	-23	30	3	-27
Total	109	42	-160%	80	36	-122%	197	52	-279%	224	53	-323%	251	50	-402%

Panel C: Second Sub-period

	Group (A)			Group (B)			Group (C)			Group (D)			Group (E)		
FX	Obs	Exp	(E-O)	Obs	Exp	(E-O)	Obs	Exp	(E-O)	Obs	Exp	(E-O)	Obs	Exp	(E-O)
A$	12	10	-2	12	10	-2	15	10	-5	25	10	-15	38	8	-30
GBP	491	808	317	488	767	279	342	502	160	259	340	81	139	111	-28
C$	41	10	-31	41	10	-31	76	15	-61	74	13	-61	55	9	-46
DM	55	12	-43	55	12	-43	52	10	-42	55	16	-39	67	14	-53
FFr	259	131	-128	259	131	-128	154	83	-71	133	70	-63	127	32	-95
Yen	13	2	-11	12	2	-10	11	1	-10	9	1	-8	6	1	-5
SFr	27	6	-21	27	6	-21	18	4	-14	22	4	-18	33	4	-29
Total	898	979	8%	894	938	5%	668	625	-7%	577	454	-27%	465	179	-160%

Key: Value is shown in US$ per option contract
Obs is the observed mean of early-exercise premium
Exp is the expected mean of early-exercise premium
(E-O) is the difference between expected and observed means and (E-O)% is | (E-O)/E |
Blank indicates there is no trade for that period

Table 4.27 shows the deviations for out-of-the-money puts to be higher in the second sub-period (Panel C) than in the first sub-period (Panel B) for all groups. In the second sub-period, except for Australia Dollar and Japanese Yen, all other currencies have observed values below the expected, the deviation for the observed values in groups (A, B and D) are more than 5,000% below the expected values while the observed values of groups (C and E) are more than 10,000% below the expected.

Table 4.27: Deviation of **Puts**' Early-exercise Premium
- Out-of-the-money ($ per Contract)

Panel A: Whole Period

	Group (A)			Group (B)			Group (C)			Group (D)			Group (E)		
FX	Obs	Exp	(E-O)	Obs	Exp	(E-O)	Obs	Exp	(E-O)	Obs	Exp	(E-O)	Obs	Exp	(E-O)
A$	-36	0	36	-36	0	36	-45	0	45	-63	2	65	-59	1	60
GBP	-28	0	28	-29	0	29	-50	0	50	-58	0	58	-71	0	71
C$	-14	0	14	-14	0	14	-32	0	32	-30	0	30	-39	0	39
DM	-29	0	29	-29	0	29	-41	0	41	-43	0	43	-47	0	47
FFr	-26	0	26	-26	0	26	-72	0	72	-103	0	103	-133	0	133
Yen	31	7	-24	29	6	-23	44	10	-34	46	14	-32	83	52	-31
SFr	-23	1	24	-24	1	25	-28	1	29	-28	0	28	-20	0	20
Total	-125	8	1663%	-129	7	1943%	-224	11	2136%	-279	16	1844%	-286	53	640%

Panel B: First Sub-period

	Group (A)			Group (B)			Group (C)			Group (D)			Group (E)		
FX	Obs	Exp	(E-O)	Obs	Exp	(E-O)	Obs	Exp	(E-O)	Obs	Exp	(E-O)	Obs	Exp	(E-O)
A$	-62	0	62	-62	0	62	-66	0	66	-88	3	91	-75	1	76
GBP	-21	0	21	-21	0	21	-41	0	41	-44	0	44	-66	0	66
C$	-10	0	10	-10	0	10	-32	0	32	-31	0	31	-38	0	38
DM	9	2	-7	9	2	-7	9	2	-7	8	2	-6	9	1	-8
FFr			0			0			0			0			0
Yen	53	11	-42	53	11	-42	85	19	-66	77	31	-46	59	20	-39
SFr	-24	0	24	-24	0	24	-19	0	19	-23	0	23	-28	0	28
Total	-55	13	523%	-55	13	523%	-64	21	405%	-101	36	381%	-139	22	732%

Panel C: Second Sub-period

	Group (A)			Group (B)			Group (C)			Group (D)			Group (E)		
FX	Obs	Exp	(E-O)	Obs	Exp	(E-O)	Obs	Exp	(E-O)	Obs	Exp	(E-O)	Obs	Exp	(E-O)
A$	15	0	-15	15	0	-15	-5	0	5	-11	0	11	-29	0	29
GBP	-31	0	31	-32	0	32	-54	0	54	-64	0	64	-73	0	73
C$	-34	0	34	-34	0	34	-36	0	36	-27	0	27	-41	0	41
DM	-31	0	31	-31	0	31	-43	0	43	-45	0	45	-49	0	49
FFr	-26	0	26	-26	0	26	-72	0	72	-103	0	103	-133	0	133
Yen	12	2	-10	11	2	-9	10	2	-8	25	3	-22	27	3	-24
SFr	-29	0	29	-30	0	30	-31	0	31	-31	0	31	-23	0	23
Total	-124	2	6300%	-127	2	6450%	-231	2	11650%	-256	3	8633%	-321	3	10800%

Key: Value is shown in US$ per option contract
Obs is the observed mean of early-exercise premium
Exp is the expected mean of early-exercise premium
(E-O) is the difference between expected and observed means, and (E-O)% is | (E-O)/E |
Blank indicates there is no trade for that period

For at-the-money options, the results are in Table 4.28 and 4.29 for calls and puts respectively. The first sub-period at-the-money calls [see Panel B of Table 4.28] have less frequent deviations than was found for out-of-the-money calls [see earlier Table 4.26] while over the whole period [see Panel A] and in the second sub-period [see Panel C] at-the-money calls have higher frequency of deviation than out-of-the-money calls. However, the deviation is still lower than for the at-the-money puts [see Table 4.29].

Table 4.28: Deviation of **Calls**' Early-exercise Premium
- At-the-money ($ per Contract)

Panel A: Whole Period

FX	Group (A)			Group (B)			Group (C)			Group (D)			Group (E)		
	Obs	Exp	(E-O)	Obs	Exp	(E-O)	Obs	Exp	(E-O)	Obs	Exp	(E-O)	Obs	Exp	(E-O)
A$	20	9	-11	20	9	-11	20	7	-13	62	14	-48	85	19	-66
GBP	86	43	-43	89	40	-49	80	29	-51	71	23	-48	79	21	-58
C$	14	7	-7	14	7	-7	23	8	-15	23	7	-16	35	10	-25
DM	19	5	-14	21	5	-16	25	5	-20	28	6	-22	38	10	-28
FFr	-10	1	11	9	1	-8	9	1	-8	122	27	-95	112	27	-85
Yen	-17	1	18	-15	1	16	-14	1	15	-12	1	13	-14	1	15
SFr	13	3	-10	19	3	-16	12	2	-10	12	3	-9	28	4	-24
Total	125	69	-81%	157	66	-138%	155	53	-192%	306	81	-278%	363	92	-295%

Panel B: First Sub-period

FX	Group (A)			Group (B)			Group (C)			Group (D)			Group (E)		
	Obs	Exp	(E-O)	Obs	Exp	(E-O)	Obs	Exp	(E-O)	Obs	Exp	(E-O)	Obs	Exp	(E-O)
A$	46	16	-30	46	16	-30	30	9	-21	70	16	-54	98	24	-74
GBP	147	74	-73	52	26	-26	214	60	-154	150	37	-113	103	24	-79
C$	16	8	-8	16	8	-8	24	9	-15	23	8	-15	36	12	-24
DM	-13	0	13	-13	0	13	-11	1	12	-2	1	3	9	1	-8
FFr			0			0			0			0			0
Yen	-35	0	35	-35	0	35	-36	0	36	-34	0	34	-36	1	37
SFr	0	8	8	0	8	8	-4	5	9	13	6	-7	34	7	-27
Total	161	106	-52%	66	58	-14%	217	84	-158%	220	68	-224%	244	69	-254%

Panel C: Second Sub-period

FX	Group (A)			Group (B)			Group (C)			Group (D)			Group (E)		
	Obs	Exp	(E-O)	Obs	Exp	(E-O)	Obs	Exp	(E-O)	Obs	Exp	(E-O)	Obs	Exp	(E-O)
A$	-6	2	8	-6	2	8	-6	2	8	35	6	-29	59	8	-51
GBP	84	42	-42	93	41	-52	64	26	-38	58	21	-37	73	20	-53
C$	8	4	-4	8	4	-4	20	5	-15	22	4	-18	32	6	-26
DM	19	5	-14	22	5	-17	25	5	-20	29	6	-23	39	10	-29
FFr	-10	1	11	9	1	-8	9	1	-8	122	27	-95	112	25	-87
Yen	-1	1	2	-2	1	3	-3	1	4	-2	1	3	-5	1	6
SFr	14	3	-11	21	3	-18	14	2	-12	12	2	-10	20	5	-15
Total	108	58	-86%	145	57	-154%	123	42	-193%	276	67	-312%	330	75	-340%

Key: Value is shown in US$ per option contract
Obs is the observed mean of early-exercise premium
Exp is the expected mean of early-exercise premium
(E-O) is the difference between expected and observed means, and (E-O)% is | (E-O)/E |
Blank indicates there is no trade for that period

The at-the-money puts [see Table 4.29] have higher deviations than the equivalent calls with more frequent deviations in the second sub-period. The puts results show that all currencies (except Japanese Yen) have observed early-exercise premium below the expected premium. This is similar to the results for out-of-the-money puts [see Table 4.27 earlier]; the deviation are more than 1,000% in second sub-period. Group (C) for second sub-period has an observed premium of -$299 whereas the expected premium is $3. Group (D) for second sub-period has an observed premium of -$284 whereas the expected premium is $2.

Table 4.29: Deviation of **Puts'** Early-exercise Premium
- **At-the-money** ($ per Contract)

Panel A: Whole Period

	Group (A)			Group (B)			Group (C)			Group (D)			Group (E)		
FX	Obs	Exp	(E-O)	Obs	Exp	(E-O)	Obs	Exp	(E-O)	Obs	Exp	(E-O)	Obs	Exp	(E-O)
A$	-98	0	98	-98	0	98	-128	0	128	-115	0	115	-83	0	83
GBP	-19	0	19	-19	0	19	-41	0	41	-62	0	62	-95	0	95
C$	-20	0	20	-20	0	20	-39	0	39	-48	0	48	-57	0	57
DM	-34	0	34	-33	0	33	-41	0	41	-47	0	47	-60	0	60
FFr	-73	0	73	-73	0	73	-180	0	180	-122	0	122	-108	0	108
Yen	158	22	-136	154	22	-132	98	15	-83	91	16	-75	81	22	-59
SFr	-24	0	24	-24	0	24	-19	0	19	-23	0	23	-28	0	28
Total	-110	22	600%	-113	22	614%	-350	15	2433%	-326	16	2138%	-350	22	1691%

Panel B: First Sub-period

	Group (A)			Group (B)			Group (C)			Group (D)			Group (E)		
FX	Obs	Exp	(E-O)	Obs	Exp	(E-O)	Obs	Exp	(E-O)	Obs	Exp	(E-O)	Obs	Exp	(E-O)
A$	-98	0	98	-98	0	98	-154	0	154	-167	0	167	-99	0	99
GBP	-43	0	43	-43	0	43	-80	0	80	-122	0	122	-118	0	118
C$	-23	0	23	-23	0	23	-42	0	42	-52	0	52	-63	0	63
DM	-13	0	13	-13	0	13	-14	0	14	-17	0	17	-13	0	13
FFr			0			0			0			0			0
Yen	261	36	-225	250	34	-216	197	29	-168	243	40	-203	186	52	-134
SFr			0			0			0			0	-21	0	21
Total	84	36	-133%	73	34	-115%	-93	29	421%	-115	40	388%	-128	52	346%

Panel C: Second Sub-period

	Group (A)			Group (B)			Group (C)			Group (D)			Group (E)		
FX	Obs	Exp	(E-O)	Obs	Exp	(E-O)	Obs	Exp	(E-O)	Obs	Exp	(E-O)	Obs	Exp	(E-O)
A$			0			0	-26	0	26	-24	0	24	-44	0	44
GBP	-5	0	5	-5	0	5	-25	0	25	-43	0	43	-93	0	93
C$	1	0	-1	1	0	-1	-23	0	23	-32	0	32	-37	0	37
DM	-34	0	34	-33	0	33	-41	0	41	-47	0	47	-61	0	61
FFr	-73	0	73	-73	0	73	-180	0	180	-122	0	122	-108	0	108
Yen	20	4	-16	20	4	-16	15	3	-12	7	2	-5	19	4	-15
SFr	-24	0	24	-20	0	20	-19	0	19	-23	0	23	-28	0	28
Total	-115	4	2975%	-110	4	2850%	-299	3	10067%	-284	2	14300%	-352	4	8900%

Key: Value is shown in US$ per option contract
- Obs is the observed mean of early-exercise premium
- Exp is the expected mean of early-exercise premium
- (E-O) is the difference between expected and observed means, and (E-O)% is | (E-O)/E |
- Blank indicates there is no trade for that period

The at-the-money results are in Tables 4.30 and 4.31 for calls and puts respectively. The calls have higher deviations in the second sub-period [see Panel C of Table 4.30], however the Japanese Yen calls have observed early-exercise premium below the expected for both sub-periods and across all groups. Group (E) for first sub-period has an observed premium of $409 whereas the expected premium is $431. Hence the observed value is 5% below the expected. By construct, group (E) for second sub-period has an observed premium of $441 whereas the expected premium is $264. Hence the observed value is 67% above the expected. The deviations of the in-the-money calls are lower than for equivalent puts [see Table 4.31].

Table 4.30: Deviation of **Calls**' Early-exercise Premium
- **In-the-money** ($ per Contract)

Panel A: Whole Period

	Group (A)			Group (B)			Group (C)			Group (D)			Group (E)		
FX	Obs	Exp	(E-O)	Obs	Exp	(E-O)	Obs	Exp	(E-O)	Obs	Exp	(E-O)	Obs	Exp	(E-O)
A$	89	49	-40	89	49	-40	66	36	-30	73	32	-41	82	30	-52
GBP	169	106	-63	162	99	-63	177	108	-69	161	89	-72	149	103	-46
C$	9	7	-2	9	7	-2	17	132	115	18	96	78	16	56	40
DM	47	43	-4	46	43	-3	36	33	-3	43	32	-11	61	31	-30
FFr	167	122	-45	167	122	-45	136	89	-47	136	89	-47	155	120	-35
Yen	-35	2	37	-35	2	37	-39	2	41	-47	1	48	-36	2	38
SFr	33	13	-20	33	13	-20	29	11	-18	20	9	-11	24	15	-9
Total	479	342	-40%	471	335	-41%	422	411	-3%	404	348	-16%	451	357	-26%

Panel B: First Sub-period

	Group (A)			Group (B)			Group (C)			Group (D)			Group (E)		
FX	Obs	Exp	(E-O)	Obs	Exp	(E-O)	Obs	Exp	(E-O)	Obs	Exp	(E-O)	Obs	Exp	(E-O)
A$	109	60	-49	109	60	-49	77	43	-34	88	39	-49	85	40	-45
GBP	175	128	-47	112	80	-32	207	124	-83	167	85	-82	171	94	-77
C$	10	7	-3	10	7	-3	19	141	122	19	103	84	18	62	44
DM	35	33	-2	35	33	-2	31	30	-1	31	30	-1	34	27	-7
FFr	148	190	42	148	190	42	148	190	42	148	190	42	148	190	42
Yen	-59	0	59	-59	0	59	-77	1	78	-86	1	87	-91	1	92
SFr	68	26	-42	68	26	-42	48	20	-28	46	18	-28	44	17	-27
Total	486	444	-9%	423	396	-7%	453	549	17%	413	466	11%	409	431	5%

Panel C: Second Sub-period

	Group (A)			Group (B)			Group (C)			Group (D)			Group (E)		
FX	Obs	Exp	(E-O)	Obs	Exp	(E-O)	Obs	Exp	(E-O)	Obs	Exp	(E-O)	Obs	Exp	(E-O)
A$	8	6	-2	8	6	-2	18	5	-13	18	5	-13	75	5	-70
GBP	168	103	-65	172	103	-69	168	102	-66	159	90	-69	142	105	-37
C$	-10	0	10	-10	0	10	-6	0	6	1	0	-1	0	0	0
DM	48	45	-3	48	44	-4	36	33	-3	44	32	-12	61	31	-30
FFr	186	53	-133	186	53	-133	130	39	-91	130	39	-91	156	106	-50
Yen	-3	4	7	-3	4	7	2	3	1	-9	2	11	-12	2	14
SFr	-1	1	2	-1	1	2	9	2	-7	2	2	0	19	15	-4
Total	396	212	-87%	400	211	-90%	357	184	-94%	345	170	-103%	441	264	-67%

Key: Value is shown in US$ per option contract
- Obs is the observed mean of early-exercise premium
- Exp is the expected mean of early-exercise premium
- (E-O) is the difference between expected and observed means, and (E-O)% is | (E-O)/E |
- Blank indicates there is no trade for that period

The put results in Table 4.31 show higher deviations than for the calls and the second sub-period gives deviations above 1,000%. Although there are less trades for some currencies in groups (A and B) on both sub-periods, only Japanese Yen has observed early-exercise premium above the expected.

Table 4.31: Deviation of **Puts**' Early-exercise Premium
- In-the-money ($ per Contract)

Panel A: Whole Period

	Group (A)			Group (B)			Group (C)			Group (D)			Group (E)		
FX	Obs	Exp	(E-O)	Obs	Exp	(E-O)	Obs	Exp	(E-O)	Obs	Exp	(E-O)	Obs	Exp	(E-O)
A$			0			0	-237	0	237	-125	0	125	-91	0	91
GBP	-67	0	67	-93	0	93	-167	0	167	147	0	-147	159	0	-159
C$	-60	0	60	-62	0	62	-93	0	93	-117	0	117	-144	0	144
DM	-109	0	109	-108	0	108	-119	1	120	-130	1	131	-159	3	162
FFr			0			0	-92	0	92	-69	0	69	-105	0	105
Yen	145	126	-19	132	117	-15	119	93	-26	99	69	-30	39	9	-30
SFr	-51	0	51	-51	0	51	-64	0	64	-69	0	69	-78	0	78
Total	-142	126	213%	-182	117	256%	-653	94	795%	-264	70	477%	-379	12	3258%

Panel B: First Sub-period

	Group (A)			Group (B)			Group (C)			Group (D)			Group (E)		
FX	Obs	Exp	(E-O)	Obs	Exp	(E-O)	Obs	Exp	(E-O)	Obs	Exp	(E-O)	Obs	Exp	(E-O)
A$			0			0	-314	0	314	-105	0	105	-96	0	96
GBP	-63	0	63	-47	0	47	-65	0	65	-62	0	62	-248	0	248
C$	-70	0	70	-73	0	73	-107	0	107	-130	0	130	-157	1	158
DM	-9	0	9	-9	0	9	-4	0	4	-18	0	18	-18	0	18
FFr			0			0			0			0			0
Yen	323	267	-56	271	229	-42	211	158	-53	160	125	-35	142	92	-50
SFr			0			0			0			0			0
Total	181	267	32%	142	229	38%	-279	158	277%	-155	125	224%	-377	93	505%

Panel C: Second Sub-period

	Group (A)			Group (B)			Group (C)			Group (D)			Group (E)		
FX	Obs	Exp	(E-O)	Obs	Exp	(E-O)	Obs	Exp	(E-O)	Obs	Exp	(E-O)	Obs	Exp	(E-O)
A$			0			0	-198	0	198	-159	0	159	-82	0	82
GBP	-69	0	69	-112	0	112	-206	0	206	-182	0	182	-167	0	167
C$	-18	0	18	-18	0	18	-20	0	20	-30	0	30	-75	0	75
DM	-109	0	109	-109	0	109	-119	1	120	-131	1	132	-160	3	163
FFr			0			0	-92	0	92	-69	0	69	-105	0	105
Yen	-7	4	11	-7	4	11	-8	4	12	35	10	-25	20	7	-13
SFr	-51	0	51	-51	0	51	-64	0	64	-69	0	69	-78	0	78
Total	-254	4	6450%	-297	4	7525%	-707	5	14240%	-605	11	5600%	-647	10	6570%

Key: Value is shown in US$ per option contract
Obs is the observed mean of early-exercise premium
Exp is the expected mean of early-exercise premium
(E-O) is the difference between expected and observed means, and (E-O)% is | (E-O)/E |
Blank indicates there is no trade for that period

In sum, the above six tables show that deviations are high for both calls and puts, and across all strike prices. The puts have higher frequency of deviation because the observed American prices are below traded European values. Moreover, the second sub-period shows higher deviations than the first sub-period.

Chapter 5: The Volatility Smiles and Implied Distributions for Deutsche Mark Options

5.1. Introduction

The implied volatility strongly depends on the maturity and strike price of an option. The dependence of implied volatility on the strike, for a given maturity, is known as the volatility smile.

The purpose of this chapter is to examine the behaviour over time of the volatility smile of the Deutsche Mark options traded on the Philadelphia Stock Exchange (PHLX). We selected Deutsche Mark options because they have the highest traded volume and also the greatest degree of mispricing found in earlier chapters. For example, traders tend to price certain calls higher than puts in one range of strike prices and do the opposite for another range of strike prices. This chapter examines both the smile and the implied asset distribution in order to see whether they can help to explain these earlier results.

In next few sections, we review some of the previous research (in section 5.2), the database (in section 5.3) and the methodology to be used (in section 5.4). The results are in section 5.5 and conclusion is in section 5.6.

5.2. Previous Research

Early empirical studies found the existence of a volatility smile for PHLX currency options [Taylor and Xu (1994a)] and for European Options Exchange stock index options [Heynen (1994)]. They found little evidence of asymmetry in the volatility smiles. However, there was a smile, which is inconsistent with the Black and Scholes (1973) assumption of constant volatility for all strike prices and maturities.

The asymmetry (skewness) of the volatility smile in currency options is discussed by Hicks (1992), Murphy (1994) and McCauley and Melick (1996) and the latter also review a risk-reversal trading strategy which is based on this. Empirical studies have found skewness of the volatility smile in S&P-500 [Bates (1991)] and FTSE-100 Index options [Gemmill (1996)]. These bullish and bearish trading patterns have been used to examine whether skewness of options is useful in forecasting the prices of underlying assets.

The implied probability distribution recovered from the volatility smile is another way to reveal the information implicit in the option prices. The theory by Breeden and Litzenberger (1987) shows that the second derivative of the call price with respect to the strike price can be used to impute the cumulative probability distribution. Shimko (1993) extended the technique to recover the probability from the volatility smile rather than from option prices. Empirical studies on OTC currency options by Malz (1996), on soybean-future options by Garcia, Sherrick and Tirupattur (1996) and on S&P-500 index options by Jackwerth and Rubinstein (1996), all suggest that the implied distributions provide information useful in forecasting future asset prices. However, a study by Dumas, Fleming and Whaley (1996) questions their usefulness.

This study uses data on individual trades which are more reliable than daily closing prices or quotations. It can provide a robust contribution to understanding the behaviour of the volatility skewness.

5.3. Data and Sample Selection

5.3.1 Data

The PHLX data have been explained in Chapter 2 (section 2.3.1). The basic details about contracts were given in Table 2.1 of Chapter 2. This information is later used to convert the levels of mispricing into US dollars per contract.

5.3.2 Sample Selection

This study has selected the standard[27] Dollar/Deutsche Mark options from August 28 1987 to October 18 1994, i.e. a period of more than 7 years. They were chosen because of the high numbers of transactions traded across a wide range of strike prices.

Table 5.1 shows the total volumes and numbers of transactions traded for the observed period. It covers the whole period of observation, and two sub-periods. Panel A shows the traded American-style options, the numbers of transactions for puts and calls are 202,489 and 170,906 respectively. Panel B shows the traded European-style options, the numbers of transactions for puts and calls are 11,893 and 10,716 respectively.

Table 5.1: Total Volume and Transactions of American and European Options

Panel A: The American Options								
Deutsche Mark Options	Start Date	End Date	Volume (V)		Transaction (T)		V / T	
			Puts	Calls	Puts	Calls	Puts	Calls
Whole Observation	Aug-28-87	Oct-18-94	17,664,394	17,974,050	202,489	170,906	87	105
First-Half Period	Aug-28-87	Dec-31-90	6,636,952	8,004,810	69,495	84,454	96	95
Second-Half Period	Jan-01-91	Oct-18-94	11,027,442	9,969,240	132,994	86,452	83	115
Panel B: The European Options								
Deutsche Mark Options	Start Date	End Date	Volume (V)		Transaction (T)		V / T	
			Puts	Calls	Puts	Calls	Puts	Calls
Whole Observation	Sep-02-87	Oct-18-94	749,539	1,192,185	11,893	10,716	63	111
First-Half Period	Sep-02-87	Dec-31-90	34,826	21,274	514	562	68	38
Second-Half Period	Jan-01-91	Oct-18-94	714,713	1,170,911	11,379	10,154	63	115

Key: V is volume (number of option contracts)
T is transactions (number of trades)
V/T is the number of contacts per trade

[27] The standard "mid-month option" expire on Friday before the third Wednesday of the option's expiry month.

The selection procedure involves all traded European and American options. Table 5.2 gives the maximum number of trades (on call and put options) available after eliminating both the out-of-boundary options and those expiring within a week. The European options are less than 6% of the American options, i.e., 18,609 out of 320,024 options.

Table 5.2: Number of Traded Transactions of Options available from Selection

Options' Style Selection	American Call	American Put	European Call	European Put
Deutsche Mark	144,133	175,891	8,350	10,259

Table 5.3 gives numbers of transactions which violate the American and European boundary[28] and numbers of transaction with options traded during the last week[29]. It shows that 5,330 (2.9%) of all calls and 5,428 (2.5%) of all puts were outside the boundary. It also shows that 23,809 (13.1%) of all calls and 22,804 (10.6%) of all puts were dropped as they were in the last week to maturity.

Table 5.3: Number of Traded Transactions Violated American and European Boundary & within a Week to Expiration

Options' Style Selection	Out-of-boundary Call	Out-of-boundary Put	Expiration (<5Days) Call	Expiration (<5Days) Put
American-Style	4,049	4,833	22,724	21,765
European-Style	1,281	595	1,085	1,039
Total Options	5,330	5,428	23,809	22,804

Table 5.4 divides the samples into maturity groups, Panels A and B for American and European options respectively. Most options are 8-30 days from expiration, i.e., 125,203 (39%) of American options and 6,909 (37%) of European options. Options traded beyond 180 days are less than 10% of the total, i.e., 12,208 (4%) of American options and 1,327 (7%) of European options. The American options within one-month to expiration have the most trades and also a wide range of strike prices. We need high volume across strike prices to impute the smile, therefore the one-month expiration options are selected for this study.

[28] An option which has violated the American or European boundary, the rational pricing bound, has a zero implied volatility.

[29] Options that traded within the last week of expiration may have unexpectedly high volatility, therefore they are also eliminated from the sample selection.

Table 5.4: Number of Traded Transactions for American and European Options by Expiration

Panel A: The American Options						
Amex DM Options Expiration Aug-28-87 to Oct-18-94	Between 8-30 Days	Between 31-60 Days	Between 61-90 Days	Between 91-180 Days	Between 181-270 Days	Between 271-360 Days
Call - Whole Period	60,948	45,935	18,806	14,433	2,850	1,161
Put - Whole Period	64,255	52,864	24,752	25,823	6,556	1,641
Calls - 1st Half	27,736	24,552	10,410	7,748	1,345	597
Calls - 2nd Half	33,212	21,383	8,396	6,685	1,505	564
Puts- 1st Half	23,516	20,879	7,952	6,121	1,386	536
Puts - 2nd Half	40,739	31,985	16,800	19,702	5,170	1,105
Total Sample	125,203	98,799	43,558	40,256	9,406	2,802

Panel B: The European Options						
Euro DM Options Expiration Sep-02-87 to Oct-18-94	Between 8-30 Days	Between 31-60 Days	Between 61-90 Days	Between 91-180 Days	Between 181-270 Days	Between 271-360 Days
Call - Whole Period	3,380	2,675	950	874	241	230
Put - Whole Period	3,529	2,923	1,428	1,523	497	359
Calls - 1st Half	106	105	67	67	55	49
Calls - 2nd Half	3,274	2,570	883	807	186	181
Puts- 1st Half	148	127	60	73	33	32
Puts - 2nd Half	3,381	2,796	1,368	1,450	464	327
Total Sample	6,909	5,598	2,378	2,397	738	589

Table 5.5 shows the number of transactions by quarter for options within one-month to expiration. It has about 39% of the total traded American-style options transactions (i.e., 125,203 out of 320,024 transactions on American puts and calls).

Table 5.5: The Number of Traded Transactions on American (8-30 Days) Options

Amex DM Options Fr Aug-28-87 to Oct-18-94	Numbers of Transaction Traded with Expiration between 8-30 Days							
	Year 1987	Year 1988	Year 1989	Year 1990	Year 1991	Year 1992	Year 1993	Year 1994
Calls - Q1		1,623	2,230	2,377	3,173	2,534	2,415	1,179
Calls - Q2		784	2,656	1,855	3,417	2,083	2,059	941
Calls - Q3	674	892	2,723	2,122	3,233	2,492	1,737	1,265
Calls - Q4	2,706	1,868	3,166	2,060	2,733	2,419	1,430	102
Puts - Q1		1,444	2,119	2,717	4,372	3,219	2,776	1,445
Puts - Q2		1,060	1,893	1,747	4,122	2,396	2,556	1,141
Puts - Q3	398	781	2,146	1,874	3,174	3,251	2,305	1,465
Puts - Q4	1,566	1,463	2,534	1,774	2,626	3,694	1,960	237
Total Sample	5,344	9,915	19,467	16,526	26,850	22,088	17,238	7,775

The volatility skewness is measured from out-of-the-money options for both calls and puts. Options traded at $\pm$ 2% from the forward price are used. These options are traded in a month prior to the expiration month, i.e., January traded options that are to expire in February and so on. Table 5.6 shows the number of transactions for the calls and puts and the number of trading days. We have 1,763 out of total 1,771 trading days available for the study. The difference of 8 is due to puts and calls being thinly traded on those days.

Table 5.6: Number of Observations on American Options for the Skewness Test

American-style Deutsche Mark Options	Number of Transaction traded on the month prior the expiration month.		Number of Trading Days	Number of Rejected Days*	Net Observed Sample
	Calls	Puts			
Aug-28-87 to Oct-18-94	66,787	71,582	1,771	8	1,763

NB: * A day was excluded when the puts and calls have not traded with minimum three options (in-the-money, at-the-money and out-of-the-money) in a same day.

5.3.3 Interest Rates and Foreign Exchange Rates

The domestic and foreign interest rates for each currency are the London Euro-currency deposit interest rates. These have been obtained from Datastream as 1 day, 1 week, 1 month, 3 months, 6 months, and 1 year. Rates have been interpolated to match option maturities. The one-month forward US$/DM foreign exchange rate is obtained from Datastream.

5.4. Theory, Methodology and Transactions Costs

5.4.1 The Option Pricing Models

The Garman and Kohlhagen (1983) option pricing model[30] is used for the European options [the call model is Equation (2.3) and the put model is Equation (2.4)]. The Barone-Adesi and Whaley (1987) option pricing model[31] [see Equations (3.3) and (3.4)] is used for the American options. It accounts for the value of early-exercise.

5.4.2 Methodology for Calculating the Implied Volatility

The method to estimate[32] the implied volatility is similar to that used in earlier sections (see Chapter 4).

5.4.3 Smoothing the Smile

The implied volatilities of the Deutsche Mark options have been classified according to their maturity [see Table 5.4]. The options are arranged in the order of strike price, i.e., the moneyness [F/X] for both the calls and puts. The implied volatilities at each moneyness (rounded up to 2 nearest basis-points) are the average implied volatility for each quarter.

In order to characterise the shape of the smile, it is smoothed on each day by fitting a quadratic equation [see Equation (5.1)] of the form:

$$\sigma \cong A_0 + A_1 X + A_2 X^2 \tag{5.1}$$

where σ is the implied volatility of the option price, t is the time-to-expiration, A_0, A_1 and A_2 are constants and X is the option's strike price

[30] See Appendix-A for full details of the Garman and Kohlhagen (1983) option pricing model.
[31] See Appendix-D for full details of the Barone-Adesi and Whaley (1987) option pricing model.
[32] See Appendix-E for full details on the method to estimate the implied volatility.

5.4.4 Methodology for Calculating the Smile Skewness

The skew phenomenon exists when the volatility of a [F + *x%*] call differs from the price of a [F - *x%*] put, (where F is the forward price), i.e., if a 'risk-reversal' has value. In order to test for smile skewness, we use (F + 2%) calls and (F - 2%) puts. On every day, linear interpolation is used to estimate the implied volatility at the required moneyness. The formula to estimate the smile skewness is Equation (5.2). This method is similar to that of Gemmill (1996), who tested options on the FTSE-100 index. In order to avoid any put/call bias in the smile, the daily difference between implied volatilities for at-the-money puts and calls is taken into account. The smile skewness calculated in this way gives a simple overview of shape of the smile.

$$Smile\ \ Skewness = \left[\frac{\left(\sigma_{Call(F+2\%)} + \Delta\sigma_{ATM} - \sigma_{Put(F-2\%)} \right)}{\sigma_{Call(F+2\%)}} \right] * 100\% \qquad (5.2)$$

where $\sigma_{Call(F+2\%)}$ and $\sigma_{Put(F-2\%)}$ are the implied volatilities of both the call and put out-of-the-money options, $F \pm 2\%$ indicates options are 2% away from the forward price. $\Delta\sigma_{ATM}$ is the difference in implied volatility of at-the-money puts from at-the-money calls.

5.4.5 Methodology for Estimating the Implied Probability Distribution

The implied probability distribution is recovered with a modified version of the Shimko's (1993) method. He uses European options, whereas our sample is of American-style options. The second derivative [see Equations (2.3), (2.4), (3.3) and (3.4)] of an American call price with respect to strike price of the option is same as for a European call.

The steps involve smoothing the observed option implied volatilities by multiplying the volatility with the square root of the time to maturity $\left[V = \sigma\sqrt{t} \right]$ for each moneyness [F/X]. Then [V] is regressed on the moneyness and moneyness squared, Equation (5.3). The cumulative probability distribution for the asset can then be estimated with Equation (5.4) which inverts the smile into a distribution. This creates a cumulative probability formula which can be numerically differentiated to find the marginal probability for each particular range of moneyness.

$$V = \sigma\sqrt{t} \cong A_0 + A_1\left(\frac{F}{X}\right) + A_2\left(\frac{F}{X}\right)^2 \tag{5.3}$$

$$F\left(S \middle| S = \frac{X}{F}\right) = 1 - N(d_2) + n(d_2)\left(A_1\left(\frac{X}{F}\right) + 2A_2\left(\frac{X}{F}\right)^2\right) \tag{5.4}$$

where $d_2 = \dfrac{\left(\ln\left(\frac{F}{X}\right) - \frac{V^2}{2}\right)}{V}$, and $n(d_2) = \dfrac{e^{\left(\frac{-(d_2)^2}{2}\right)}}{\sqrt{2\pi}}$

F is the forward price, $V = \sigma\sqrt{t}$. X is strike price and t is the time-to-expiration of the options. N(.) is the cumulative normal distribution function and n(.) is normal density function.

The range of traded moneyness (31 observations, i.e., ±15% of forward price at 1% interval) covers approximately 99.99% of the implied distribution. Jackwerth and Rubinstein (1996) have suggested that 8 option prices would have enough information to generate the general shape of the implied distribution, therefore, our data with 31 observations should be able to generate a reliable shape of implied distribution. The skewness [see Equation (5.5)] and kurtosis [see Equation (5.6)] of the implied distribution are estimated to summarise the shape.

A coefficient of variation [q] [see Equation (5.7)] and the implied return volatility [σ'] [see Equation (5.8)] can also be estimated for the implied distribution. The skewness [l_3] [see Equation (5.9)] and kurtosis [l_4] [see Equation (5.10)] of the log-normal distribution are also calculated for comparative purposes

$$P_1 = E[\ f_1 * m_1, f_2 * m_2, \ldots, f_{n-1} * m_{n-1}, f_n * m_n]$$

$$P_2 = E\left[f_1 * (m_1 - P_1)^2, f_2 * (m_2 - P_1)^2, \ldots, f_{n-1} * (m_{n-1} - P_1)^2, f_n * (m_n - P_1)^2\right]$$

$$P_3 = \frac{E\left[f_1 *(m_1 - P_1)^3, f_2 *(m_2 - P_1)^3, \ldots, f_{n-1} *(m_{n-1} - P_1)^3, f_n *(m_n - P_1)^3\right]}{\sqrt{(P_2)^3}} \tag{5.5}$$

$$P_4 = \frac{E\left[f_1 *(m_1 - P_1)^4, f_2 *(m_2 - P_1)^4, \ldots, f_{n-1} *(m_{n-1} - P_1)^4, f_n *(m_n - P_1)^4\right]}{\sqrt{(P_2)^4}} \tag{5.6}$$

where f_1 is marginal probability in the range of moneyness [m_1]. To calculate the Skewness [P_3] and Kurtosis [P_4] of the implied distribution, the mean [P_1] and variance [P_2] of the implied distribution must be obtained first.

$$q = \left(\sqrt{P_2}\right) / P_1 \tag{5.7}$$

$$\sigma' = \sqrt{\ln\left(1+q^2\right)/t} \tag{5.8}$$

$$l_3 = 3q + q^3 \tag{5.9}$$

$$l_4 = 3 + 16q^2 + 15q^4 + 6q^6 + q^8 \tag{5.10}$$

where [q] is the coefficient of variation, [σ'] is the implied return volatility, (l_3) the skewness and [l_4] the kurtosis of the log-normal distribution.

5.4.6 Transactions Costs

The typical PHLX transaction variable costs and fixed costs have been mentioned in earlier Chapter (section 2.4.5). The details of variable and fixed costs were showed in Table 2.13 of Chapter 2. The transactions costs estimated from the PHLX cost information is less than \$25, however, to be conservative, \$50 per round-turn is used for this study.

5.5. Empirical Results

The results are divided into three sub-sections. The first sub-section analyses the general shape of the smiles and is followed by the skewness of the smile in the second sub-section. The third sub-section examines the distributions implied from the smiles.

5.5.1 Implied Volatility Smiles

Calls and puts have been examined separately. Figure 5.1 gives the fitted volatility smiles for the call options. Panels A, B and C show results for the whole period, first sub-period and second sub-period respectively. The six curves[33] in each panel represent different maturities ranging from 8 to 360 days. Each curve is the result of fitting a quadratic function to the average volatilities across the relevant sample period. In all three Panels, the shortest maturity options show the steepest smiles. Options with expiration more than 90 days are generally flatter and options with expiration more than 270 days have a concave shape of smile.

The first sub-period calls (see Panel B of Figure 5.1) show a little asymmetry for the one, two and three month options (higher volatility out-of-the-money than in-the-money). The four-to-six-month and seven-to-nine-month curves show symmetry while longest expiration curve has a hump-shaped instead of U-shaped curve. In the second sub-period (see Panel C of Figure 5.1), the smile shows some asymmetry, being steeper in-the-money. In all cases, the minimum volatility in the smile is close to the forward price, i.e., at F/X = 1.

Similar smiles are obtained for puts [see Figure 5.2], except that they are more symmetric. For example, they do not show asymmetry in the second sub-period [c.f. calls]. The results presented here are similar to those of Taylor and Xu (1994a), with one exception: they found greater asymmetry for puts than calls. The difference in smile skewness between Taylor and Xu (1994a) and our results may have been due to the our data sources. Taylor and Xu (1994a) used daily closing prices. Our data are intra-day trades and therefore the results are somewhat more reliable.

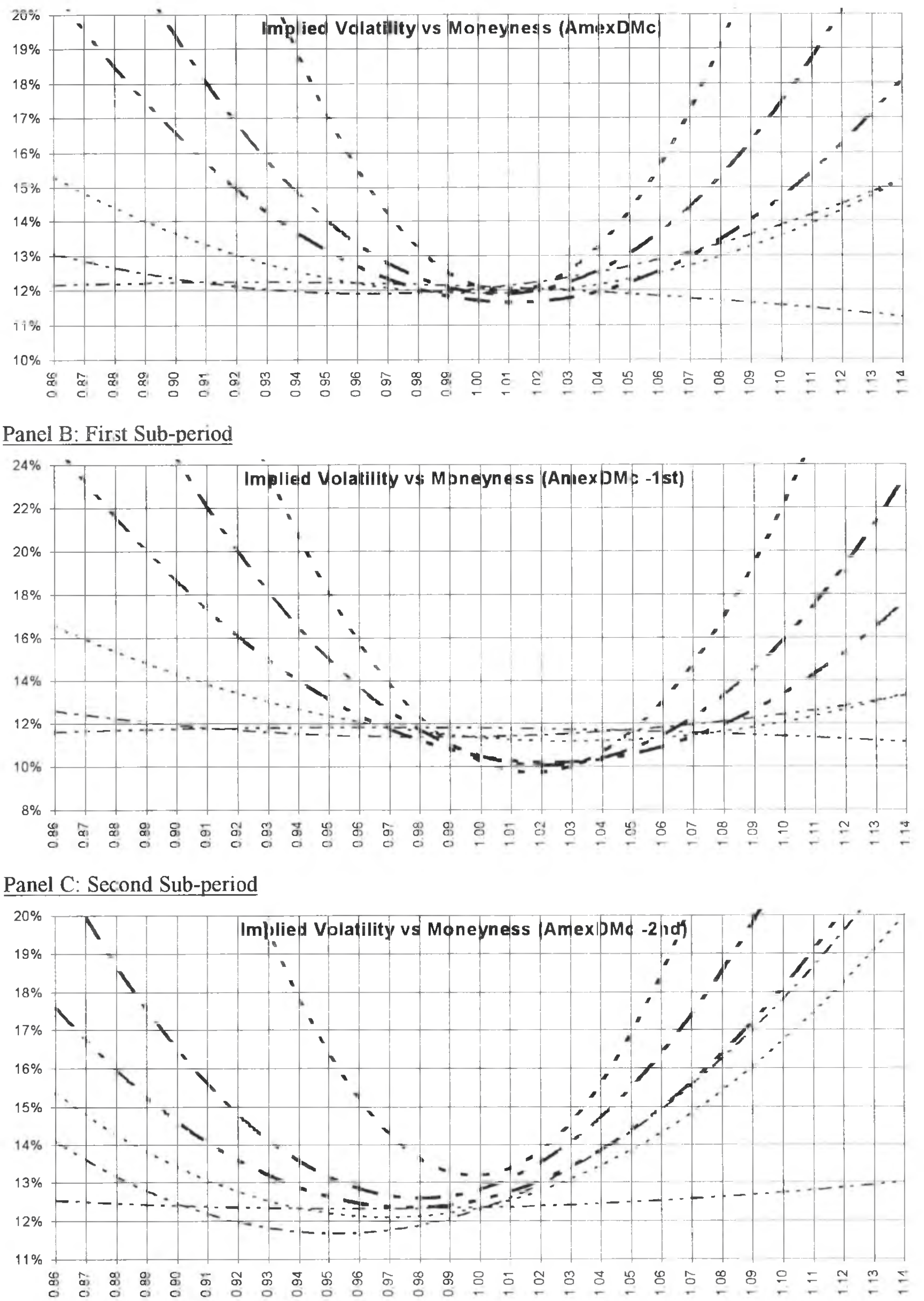
[33] A Volatility Fitted (2 polynomial fit).

Figure 5.1: Implied Volatility Smiles of the Deutsche Mark Call Options

Panel A: Whole Period

Panel B: First Sub-period

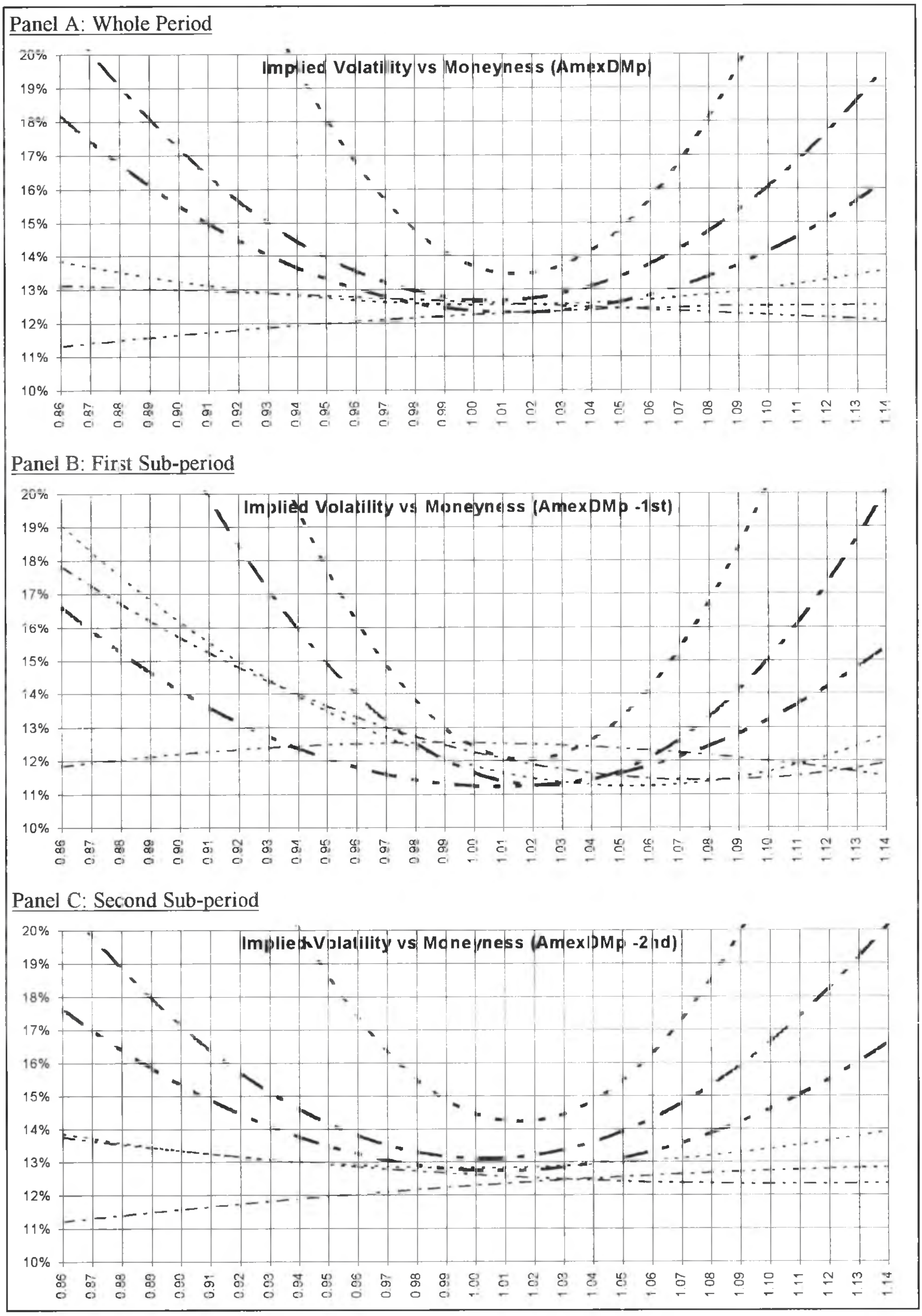
Panel C: Second Sub-period



NB: X-Axis is [F/X] Moneyness for Call, Y-Axis is Implied Volatility

-----	8-30 Days to Expiration	-----	91-180 Days to Expiration
-.-.-	31-60 Days to Expiration	-.-.-	181-270 Days to Expiration
-..-.-	61-90 Days to Expiration	-..-.-	271-360 Days to Expiration

Figure 5.2: Implied Volatility Smiles of The Deutsche Mark Put Options



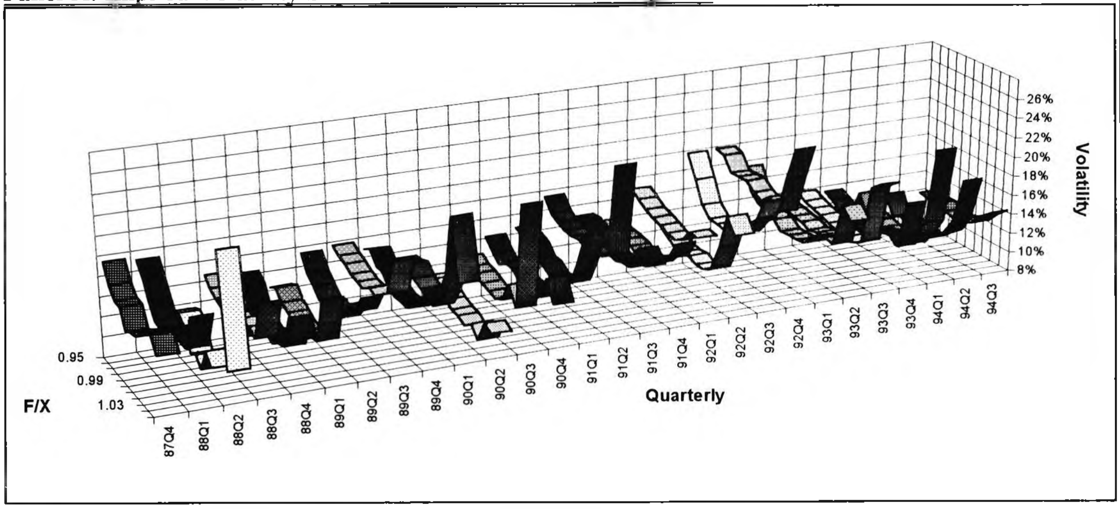
NB: X-Axis is [F/X] Moneyness for Put, Y-Axis is Implied Volatility

-----	8-30 Days to Expiration	-----	91-180 Days to Expiration
-.-.-	31-60 Days to Expiration	-.-.-	181-270 Days to Expiration
-..-.-	61-90 Days to Expiration	-..-.-	271-360 Days to Expiration

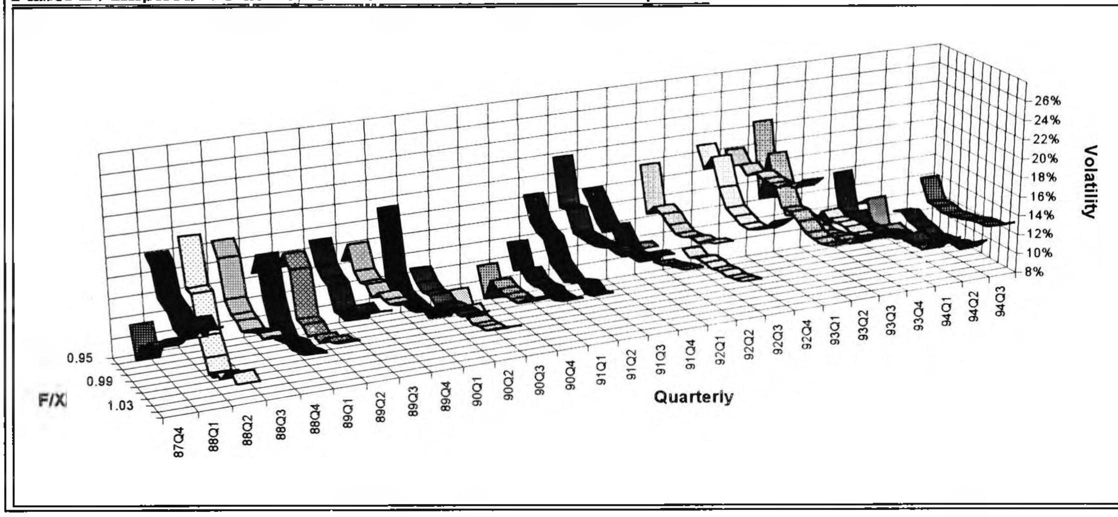
Figure 5.3 plots 3D-Strip graphs of each quarter's average implied volatility versus moneyness. Panels A and B are for calls and puts respectively. They are the smiles with one-month to expiration, (i.e., 8-30 day options). The third quarter of 1987 and last quarter of 1994 are excluded because of the low number of observations.

Figure 5.3: Quarterly Average of Volatility Smile

Panel A: Implied Volatility Smile of Deutsche Mark Call Options



Panel B: Implied Volatility Smile of Deutsche Mark Put Options

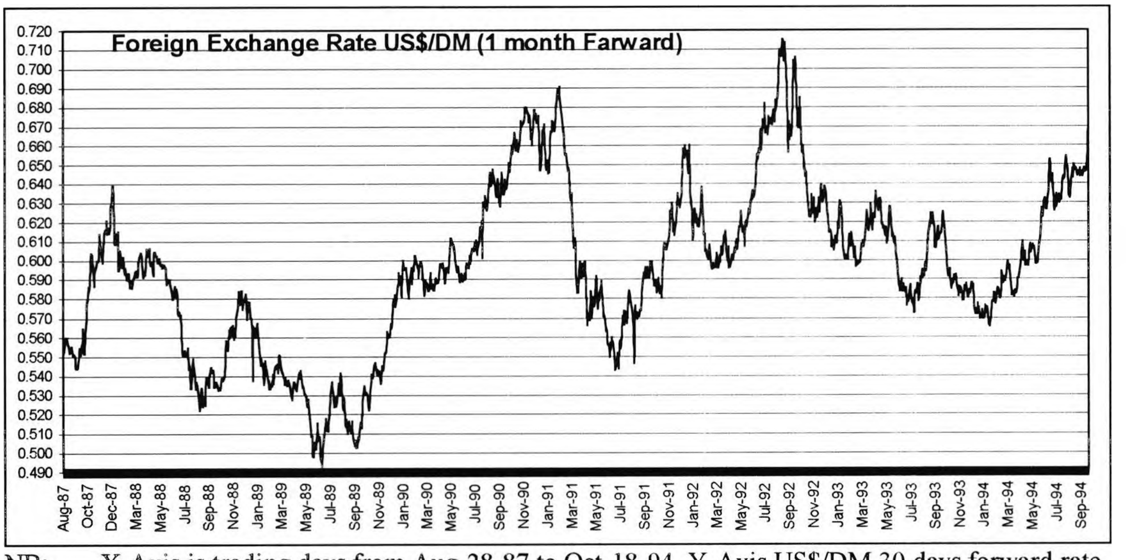


NB: [*F/X*]is Moneyness for Call and Put

It can be seen that the smile changes quite frequently, out-of-the-money options (calls and puts) are priced at different implied volatilities from at-the-money options in every quarter. The put smile is steeper on the LHS and call smile is steeper on the RHS, which is consistent as the LHS is in-the-money for puts and RHS is in-the-money for calls.

Figure 5.4 shows that the \$/DM exchange rate was volatile over the sample period. In the next section we calculate a measure of the skewness of the smile and try to relate it (informally) to change in the exchange rate.

Figure 5.4: Daily One-month US\$/DM Forward Exchange Rates



NB: X-Axis is trading days from Aug-28-87 to Oct-18-94, Y-Axis US\$/DM 30 days forward rate.

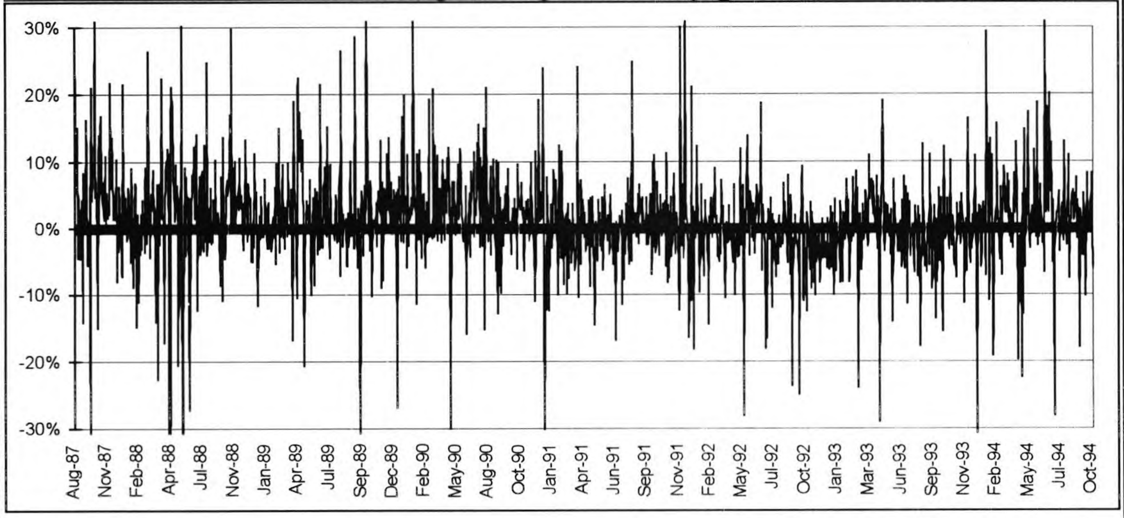
5.5.2 Smile Skewness

According to Bates (1991), market expectations of the asset price may possibly be obtained by measuring the smile skewness [see Equation (5.2)]. This reveals "bullish" and "bearish" periods. Figure 5.5 shows the smile skewness of Deutsche Mark options. The results in Panel A show that the daily smile skewness varied between ±30%. It suggests that market expectations on the asset price distribution change frequently over time and are measured with noise. The use of a moving average (MA) might help to reduce the noise. In Panel B, MA (20) is plotted, but skewness still shows quite high variation over time. The general trend of smile skewness is given in Panel C by fitting a second order polynomial. It suggests that skewness became less evident over time from August 1987 to October 1994.

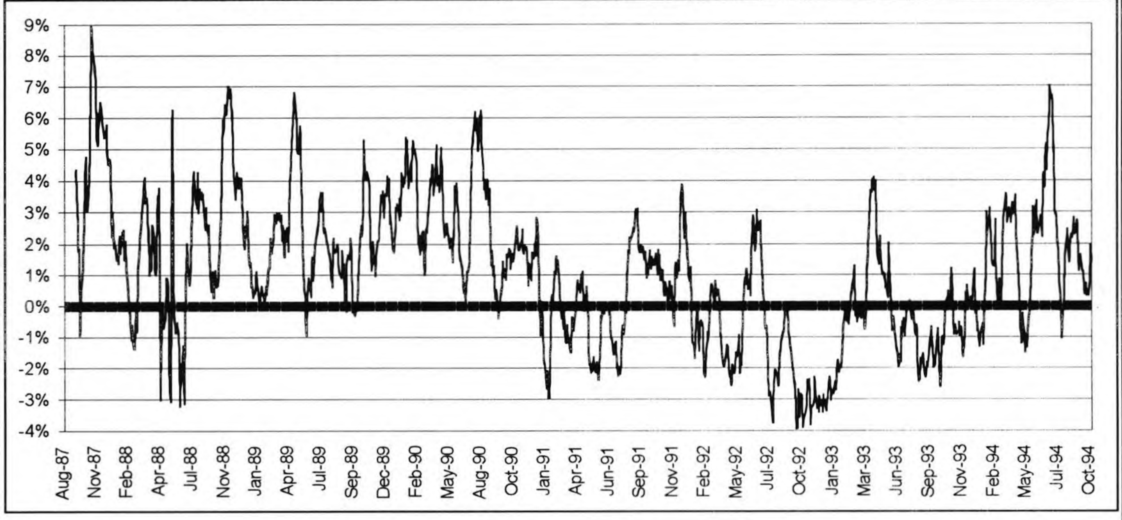
By comparing the trend of the skewness (Panel C of Figure 5.5) with the trend of the underlying exchange rate (Figure 5.4), there is weak evidence that a low $/DM rate was associated with high level of skewness. This suggests that before 1993, the options market may have expected the Deutsche Mark to depreciate against the Dollar and then after 1993 to appreciate against the Dollar.

Figure 5.5: Skewness of the Implied Volatility Smiles

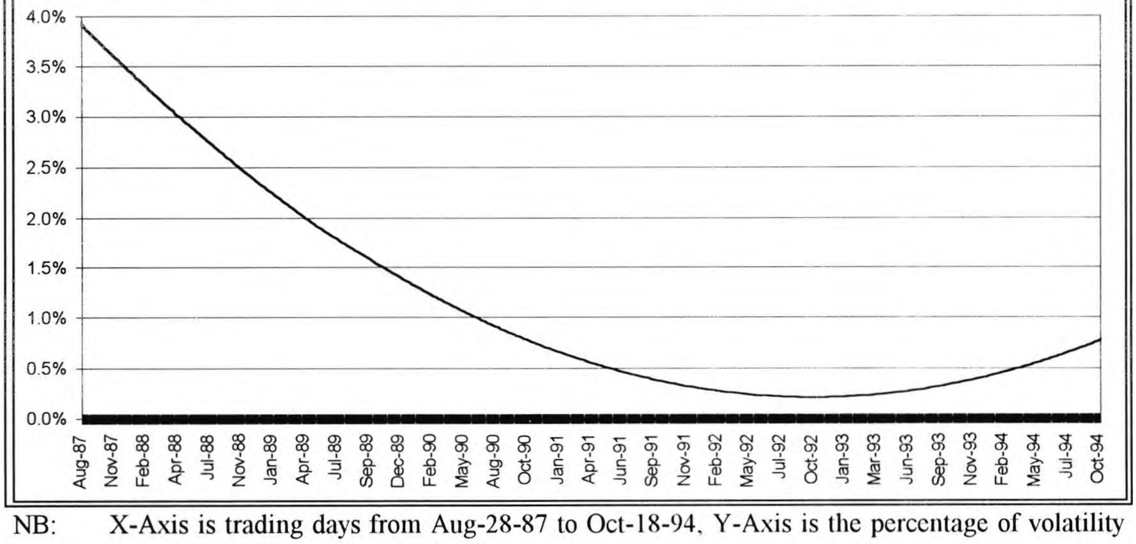
Panel A: Skewness of Deutsche Mark Options Implied Volatility @ 2% out-of-the-money Options



Panel B: Moving Average (20) on Skewness of DM Options Implied Volatility @ ± 2% of Moneyness



Panel C: 2 Polynomial Fit on Skewness of DM Options' Implied Volatility @ ± 2% of Moneyness



NB: X-Axis is trading days from Aug-28-87 to Oct-18-94, Y-Axis is the percentage of volatility skewness, see Equation (5.2) for detail.

5.5.3 Implied Distributions from the Volatility Smiles

Another way to interpret the options' prices is to derive the implied distribution. This is easily recovered [with Shimko's (1993) method] from the volatility smile. The results in Table 5.7 indicate that the skewness is positive for both call and put prices, and the kurtosis is measured at 3, which is very close to the value for a log-normal distribution (see the right-hand column of Table 5.7). The mean of the implied probability distribution is at the forward rate, i.e., around F/X = 1.0. The implied distribution is close to a log-normal and so we expect the marginal probabilities at each strike price for calls and puts to be similar.

In Figure 5.6 (which is quarterly), Panel A is the call implied distribution, Panel B is the put implied distribution, and Panel C is the difference between the two implied distributions. It shows that call and put distributions are different, but in a way which changes from quarter to quarter. The call (put) options traded before 1992 tend to have higher (lower) marginal distribution near-the-forward price relative to the put (call) options near-the-forward and have lower (higher) marginal distribution away-from-the-forward. There is a shift in the average pricing after the third quarter of 1991: there after the put (call) options trend to have higher (lower) marginal distributions at the forward price relative to the call (put) options near-the-forward and lower (higher) marginal distributions away-from-the-forward.

Figure 5.7 shows implied distributions for four particular quarters. These four quarters are selected because of the large differences in distributions' kurtosis, skewness and variance (see the right-columns of Table 5.7). Panels A and B show the difference between calls and puts across strike prices for 88Q2 and 91Q3 respectively. Panel B shows a greater variation than Panel A: a wider deviation from lognormality of the call relative to the put. The difference in the kurtosis is approximately 0.5 and the put has a fatter distribution with positive skewness of 1.5 [see second column of Table 5.7]. Panel C (92Q2) is selected to demonstrate the expected implied probability distribution for the call and put option prices of same underlying asset. Out of 28 quarters in the sample, only 92Q2 has very little variation between the call and put. Panel D plots the probability distribution for 94Q2. It shows a shift in the distribution between the call and put. It also shows a change in mispricing between the calls and puts in this later period as compared to earlier periods.

The results from the implied distributions (see Table 5.7) suggests that only a small positive smile skewness exists and the distribution is close to lognormal (see third and last columns of Table 5.7), with no kurtosis. By contrast, the volatility smiles of Figures 5.1 and 5.2 indicate some kurtosis and Figure 5.5 suggests skewness (from time to time). The smiles are also consistent with the level of mispricing of calls and puts found in Chapter 3. Both calls and puts of the same underlying currency should have the same distribution, but they do not (see Figure 5.7).

Table 5.7: Kurtosis and Skewness of Calls' and Puts' Implied Distributions

Panel A: The Results of Call's Implied Distribution

	Observed						Lognormal	
Call	$Skewness_D$	$Kurtosis_D$	Variance	Mean	Coeff of Var	Imp Ret Vol	xSkewness	xKurtosis
Q4-87	0.1263	2.9971	0.1292%	1.0061	3.5721%	11.77%	0.1072	3.0204
Q1-88	0.1272	3.0031	0.1236%	1.0061	3.4940%	11.51%	0.1049	3.0196
Q2-88	0.0745	3.0097	0.0547%	1.0055	2.3268%	7.67%	0.0698	3.0087
Q3-88	0.1455	3.0044	0.1271%	1.0061	3.5440%	11.67%	0.1064	3.0201
Q4-88	0.1066	3.0163	0.0923%	1.0059	3.0201%	9.95%	0.0906	3.0146
Q1-89	0.1577	3.0122	0.1237%	1.0061	3.4960%	11.52%	0.1049	3.0196
Q2-89	0.1097	3.0138	0.1016%	1.0060	3.1685%	10.44%	0.0951	3.0161
Q3-89	0.1219	3.0067	0.1181%	1.0061	3.4153%	11.25%	0.1025	3.0187
Q4-89	0.1265	3.0022	0.1243%	1.0061	3.5038%	11.54%	0.1052	3.0197
Q1-90	0.0868	3.0127	0.0761%	1.0057	2.7422%	9.03%	0.0823	3.0120
Q2-90	0.0719	3.0090	0.0527%	1.0055	2.2823%	7.52%	0.0685	3.0083
Q3-90	0.0875	3.0133	0.0691%	1.0057	2.6138%	8.61%	0.0784	3.0109
Q4-90	0.0916	3.0136	0.0817%	1.0058	2.8413%	9.36%	0.0853	3.0129
Q1-91	0.1266	3.0056	0.1208%	1.0061	3.4548%	11.38%	0.1037	3.0191
Q2-91	0.1229	2.9986	0.1270%	1.0061	3.5426%	11.67%	0.1063	3.0201
Q3-91	0.1053	3.0121	0.1022%	1.0060	3.1780%	10.47%	0.0954	3.0162
Q4-91	0.0959	3.0141	0.0875%	1.0059	2.9404%	9.69%	0.0882	3.0138
Q1-92	0.1292	2.9963	0.1306%	1.0061	3.5918%	11.83%	0.1078	3.0207
Q2-92	0.0935	3.0150	0.0744%	1.0057	2.7129%	8.94%	0.0814	3.0118
Q3-92	0.1762	2.9547	0.1648%	1.0057	4.0366%	13.30%	0.1212	3.0261
Q4-92	0.2165	2.9171	0.1877%	1.0051	4.3102%	14.20%	0.1294	3.0298
Q1-93	0.1323	2.9977	0.1299%	1.0061	3.5822%	11.80%	0.1075	3.0206
Q2-93	0.0982	3.0148	0.0875%	1.0059	2.9406%	9.69%	0.0882	3.0138
Q3-93	0.1369	3.0129	0.1161%	1.0061	3.3869%	11.16%	0.1016	3.0184
Q4-93	0.1207	3.0133	0.1087%	1.0060	3.2776%	10.80%	0.0984	3.0172
Q1-94	0.1198	3.0206	0.0936%	1.0059	3.0408%	10.02%	0.0913	3.0148
Q2-94	0.1269	3.0135	0.1114%	1.0060	3.3174%	10.93%	0.0996	3.0176
Q3-94	0.1013	3.0149	0.0911%	1.0059	3.0000%	9.88%	0.0900	3.0144

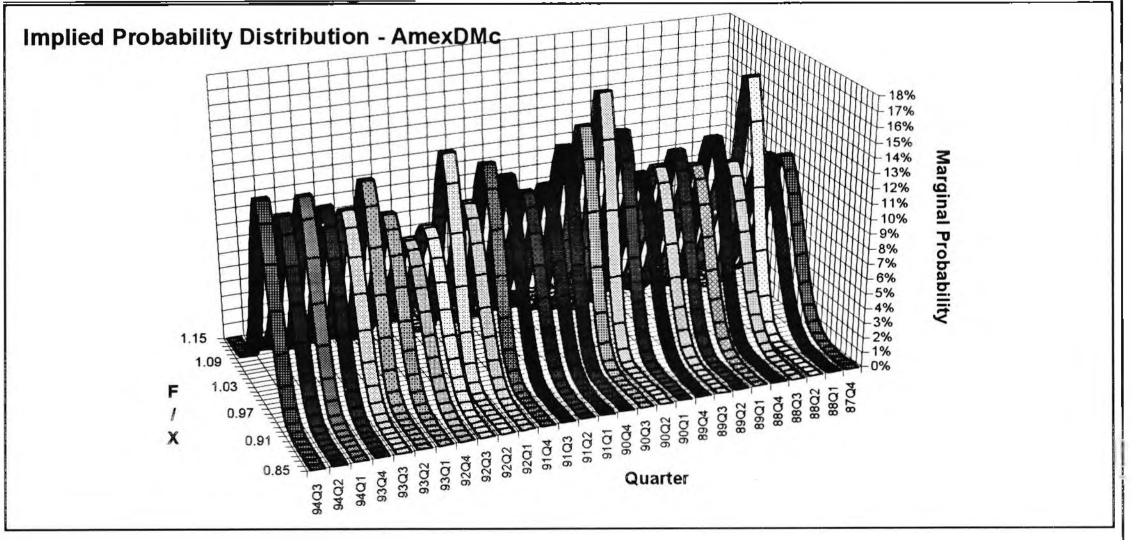
Panel B: The Results of Puts' Implied Distribution

	Observed						Lognormal	
Put	$Skewness_D$	$Kurtosis_D$	Variance	Mean	Coeff of Var	Imp Ret Vol	xSkewness	xKurtosis
Q4-87	0.1381	2.9923	0.1356%	1.0061	3.6604%	12.06%	0.1099	3.0215
Q1-88	0.1826	2.9487	0.1687%	1.0057	4.0842%	13.45%	0.1226	3.0267
Q2-88	0.2829	2.8981	0.2014%	1.0042	4.4691%	14.72%	0.1342	3.0320
Q3-88	0.1820	2.9527	0.1664%	1.0057	4.0558%	13.36%	0.1217	3.0264
Q4-88	0.1189	3.0059	0.1181%	1.0061	3.4151%	11.25%	0.1025	3.0187
Q1-89	0.2012	2.9413	0.1742%	1.0055	4.1507%	13.67%	0.1246	3.0276
Q2-89	0.1557	2.9763	0.1496%	1.0060	3.8449%	12.66%	0.1154	3.0237
Q3-89	0.1067	3.0107	0.1061%	1.0060	3.2373%	10.66%	0.0972	3.0168
Q4-89	0.9124	2.7443	0.3365%	0.9884	5.8692%	19.32%	0.1763	3.0553
Q1-90	0.1031	3.0148	0.0935%	1.0059	3.0397%	10.01%	0.0912	3.0148
Q2-90	0.1155	3.0173	0.0979%	1.0059	3.1096%	10.24%	0.0933	3.0155
Q3-90	0.1866	2.9507	0.1679%	1.0057	4.0749%	13.42%	0.1223	3.0266
Q4-90	0.2092	2.9302	0.1805%	1.0053	4.2260%	13.92%	0.1269	3.0286
Q1-91	0.2349	2.9130	0.1909%	1.0049	4.3478%	14.32%	0.1305	3.0303
Q2-91	0.2278	2.9103	0.1917%	1.0049	4.3569%	14.35%	0.1308	3.0304
Q3-91	1.4662	2.5226	0.7684%	0.9412	9.3139%	30.62%	0.2802	3.1399
Q4-91	0.0822	3.0115	0.0741%	1.0057	2.7074%	8.92%	0.0812	3.0117
Q1-92	0.1142	3.0013	0.1221%	1.0061	3.4736%	11.44%	0.1043	3.0193
Q2-92	0.0786	3.0108	0.0619%	1.0056	2.4741%	8.15%	0.0742	3.0098
Q3-92	0.2557	2.8918	0.2023%	1.0044	4.4778%	14.75%	0.1344	3.0321
Q4-92	0.1492	2.9587	0.1602%	1.0059	3.9786%	13.10%	0.1194	3.0254
Q1-93	0.2469	2.8976	0.1990%	1.0045	4.4407%	14.63%	0.1333	3.0316
Q2-93	0.0959	3.0140	0.0875%	1.0059	2.9404%	9.69%	0.0882	3.0138
Q3-93	0.0811	3.0114	0.0690%	1.0057	2.6125%	8.61%	0.0784	3.0109
Q4-93	0.1038	3.0146	0.0947%	1.0059	3.0595%	10.08%	0.0918	3.0150
Q1-94	0.0734	3.0094	0.0547%	1.0055	2.3258%	7.66%	0.0698	3.0087
Q2-94	0.0707	3.0087	0.0515%	1.0055	2.2566%	7.43%	0.0677	3.0082
Q3-94	0.1273	3.0037	0.1229%	1.0061	3.4842%	11.48%	0.1046	3.0194

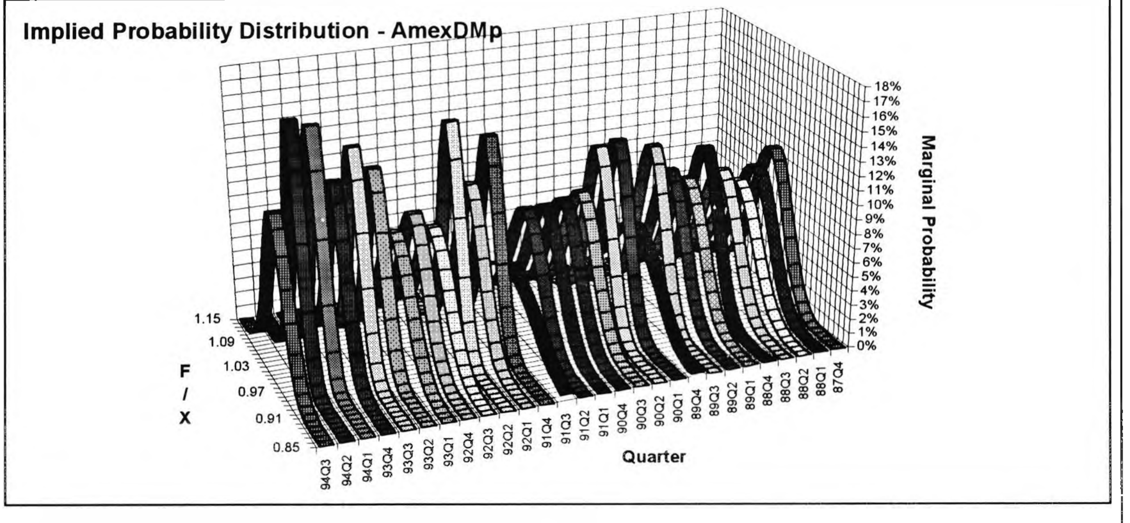
NB:
- $Skewness_D$ is the skewness of the Implied Distribution.
- $Kurtosis_D$ is the kurtosis of the Implied Distribution.
- Variance is the variance of the Implied Distribution.
- Mean is the mean of the Implied Distribution.
- Coeff of Var is the coefficient of variation for the Implied Distribution.
- Imp Vol is the implied return volatility for the Implied Distribution.
- xSkewness is skewness of the log-normal Distribution.
- xKurtosis is kurtosis of the log-normal Distribution.

Figure 5.6: Implied Distribution of the Call and Put Options

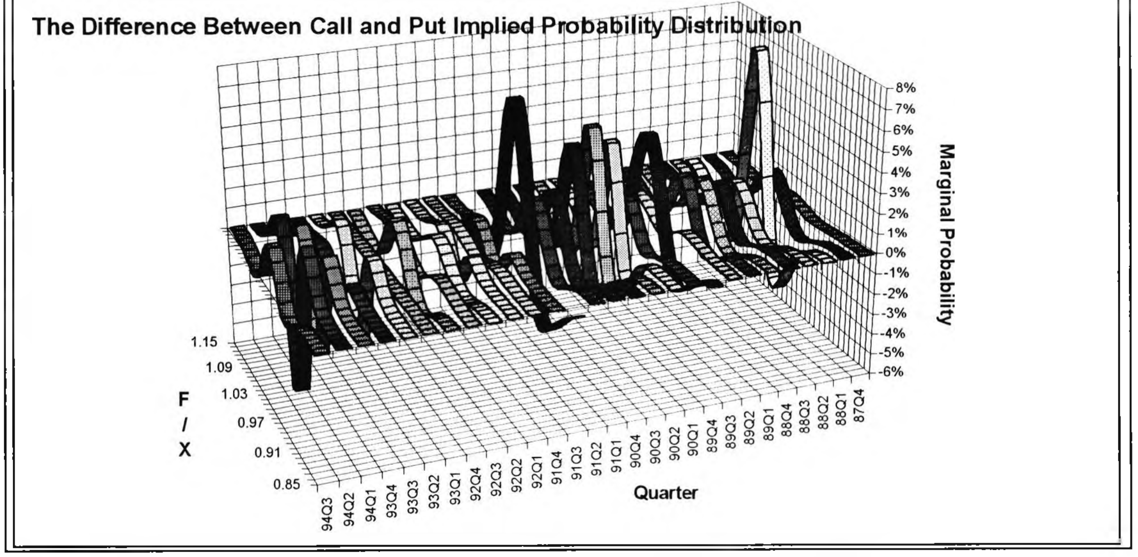
Panel A: Deutsche Mark Call Options



Panel B: Deutsche Mark Put Options

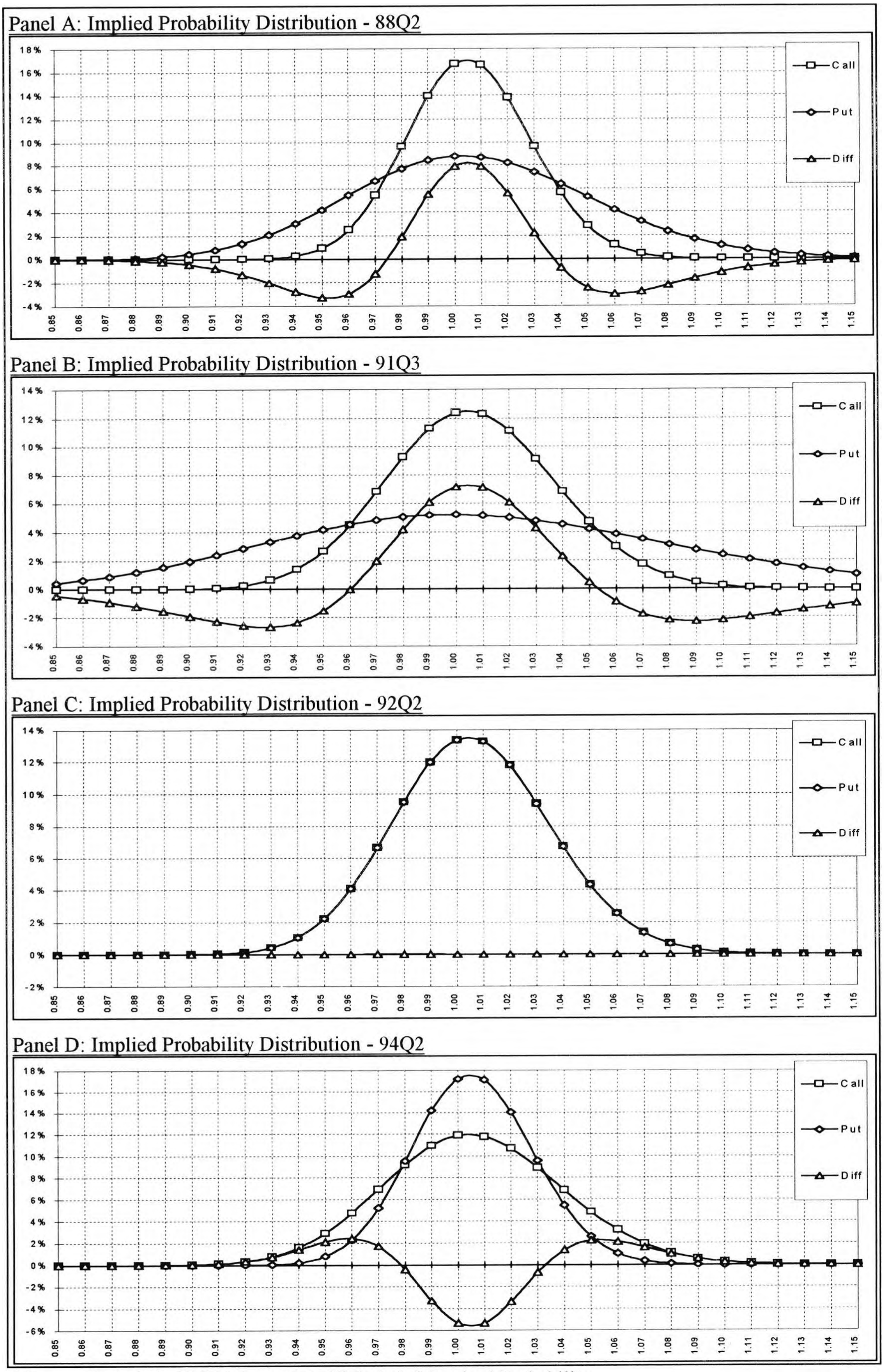


Panel C: Difference Between Call and Put Options



NB: [F/X] is Moneyness for Call and Put, i.e., Forward price / Strike Price

Figure 5.7: Selected Implied Distributions of the Call and Put Options



NB: X-Axis is [F/X] Moneyness, Y-Axis is Marginal Probability

5.6. Conclusions

This chapter uses 125,203 DM/$ options in order to study smiles and implied distributions. We find that: (i) the minimum volatility is close to the forward rate in all calls; (ii) the volatility smile is slightly asymmetric for calls with average volatility higher when in-the-money; (iii) the smile steeper as maturity approaches (as expected [see Bates (1996)]); and (iv) puts and calls show different marginal distributions, which is consistent with the violations of put-call parity found in Chapter 3.

The results for the implied distributions show that the probability distribution has positive skewness but kurtosis is about 3, i.e., it is not leptokurtic. Both panels of Table 5.7 show that the kurtosis, skewness and variances of call and put differ in all quarters. However, it is difficult to appreciate the results visually. The selected three particular quarters in Figure 5.7 show the difference in the put and call marginal distributions across strike prices. Before 91Q1 calls around the forward price have higher marginal distribution (are overpriced) relative to puts, however, after 1991, the calls around forward price have lower marginal distribution (are under-priced) relative to puts.

The results from the implied distributions indicate small significant smile skewness and kurtosis, but they also demonstrate how much easier it is to review the distribution's skewness and kurtosis from the volatility smile. Going from smiles to implied distributions has rather small utility (from a research viewpoint).

Chapter 6: Alternative Option Pricing Models with Volatility Smiles

6.1. Introduction

This study performs tests of four alternative models for pricing tomorrow's options with three different volatility assumptions. The models are: the European option model, the American option model, the Hull and White stochastic volatility option model and the square-root constant elasticity of variance (CEV) model. The volatility assumptions are constant volatility, volatility with smile effect and adjusted volatility with smile effect. The objective is to discuss which choice of volatility assumption and model works best from the 12 alternatives considered

In order to examine the stability of the volatility smile, the implied volatilities are imputed from the option prices with each of the option models over a range of strike prices. In this way an initial volatility smile is defined for each of the three volatility assumptions. Each model is then used with the three alternative volatility assumptions to price tomorrow's options. Models are compared on the basis of forecasting performance.

In the next few sections, some of the previous studies (in section 6.2) are reviewed; the database is explained (in section 6.3) and the methodology is outlined (in section 6.4). The results are in section 6.5 and the conclusions in section 6.6.

6.2. Previous Research

When the Black and Scholes (1983) model is used to imply volatilities from market option prices, the implied volatilities vary systematically across strike prices and time-to-expiration (as shown in Chapter 5). However, it is not clear whether taking account of the smile has forecasting value. Dumas, Fleming and Whaley (1996) find little value in assuming a smile when forecasting US index options, whereas (for example) Kamiyama (1997) finds that it is useful when forecasting Nikkei index options.

There have been several papers which empirically examine models based upon arbitrary distributions. A recent one is by Corrado and Su (1997) on S&P-500 index options. They compare the Jarrow and Rudd (1982) and Black and Scholes (1973) option models, using the previous day's implied volatility. The Jarrow and Rudd (1982) formula accounts for the skewness and kurtosis of the asset distribution. The data are the mid-points of CBOE dealers' bid-ask price quotations. The results show that the Jarrow and Rudd (1982) model has smaller pricing errors than the Black and Scholes (1973) model.

The deterministic volatility approach (i.e., implied volatility functions and an implied binomial tree) is examined by Dupire (1994), Derman and Kani (1994), Rubinstein (1994), Chriss (1996) and Derman, Kani and Zou (1996). They assume that the variation of volatility across strike price and time-to-expiration is driven by the fact that the volatility rate of asset return varies with the level of asset price and time. However, when Dumas, Fleming and Whaley (1996) indirectly tested the deterministic volatility function option model with S&P-500 index options, their results showed that asset hedging with a simple model is better (i.e., Black and Scholes) than with the deterministic models.

There have been a few studies of a stochastic-volatility model using currency data. Melino and Turnbull (1990) examine the PHLX Dollar/Canadian Dollar option prices using a stochastic volatility model to estimated daily implied volatilities. The results show that the pricing error from the stochastic volatility model is smaller than the Black and Scholes (1973) model using constant volatility. Heston (1993) tested the currency options with stochastic volatility model which took with account of the risk-premium, whereas Hull and White (1987a) assumed the premium to be zero. Guo

(1996a and 1996b) tested Heston's (1993) model with the PHLX Dollar/Yen options and found that Hull and White's (1987a) model performed better than Heston's (1993) model and also slightly better than Garman and Kohlhagen's (1983) model. Bates (1996a and 1996b) tested the stochastic volatility model with jump-diffusion and the results indicated that it performed better than standard stochastic volatility model.

In this study, we test the four alternative (simple) models using yesterday's market price. The approach is similar to Corrado and Su (1997), but we account for the volatility with smile effect and extend the test to four models.

6.3. <u>Data and Sample Selection</u>

6.3.1 Data

The PHLX data have been explained in Chapter 2 (section 2.3.1). The basic details about contracts were given in Table 2.1 of Chapter 2. This information is later used to convert the levels of mispricing into US dollars per contract.

6.3.2 Sample Selection

This study uses the Dollar/Deutsche Mark American options traded on the PHLX from January 3 1994 till September 9 1994, i.e., a period of more than 8 months. The American Deutsche Mark options were selected because of the high number of daily trades and the wide spread of strike prices enables us to generate a good approximation for the smile. The sample consists of about 40% of the total trades in all currencies at the PHLX. These are standard "mid-month" options that expire on the Friday before the third Wednesday of the contract expiry month.

Options with less than a week to expiration are excluded, because of unstable pricing [as also observed by Taylor and Xu (1994a and 1994b)]. However, options with more than six weeks time-to-expiration are thinly traded. The chosen options range is therefore from 1 week to 6 weeks in time-to-expiration. We monitor the estimated option prices over a 9-month period, i.e., January traded options with February's expiration, February traded options with March's expiration, and so on.

The sample has strike prices over the range from 3% out-of-the-money to 2% in-the-money for each trading day. There is a total number of 5,708 observed trades, i.e., approximately 30 trades per day.

Unfortunately, an individual trade does not indicate whether it is a "bid" or "ask". However, transaction records also include a bid/ask quote at the time of trade. The trades were therefore separated into sales and purchases by comparison of the option price with the bid/ask quotes: the closer of two determined a trade as a "bid" (sale) or "ask" (purchase). The purpose of this separation of trades is to ascertain whether the observed "bid" or "ask" volatility performs better than the overall "average" volatility for the next day's option prices. Hence to the 12 alternatives are now added 3 volatility alternatives, giving 36 model results in total.

6.3.3 Interest Rates

The domestic and foreign interest rates for each currency are the London Euro-currency deposit interest rates. These have been obtained from Datastream as 1 day, 1 week, 1 month, 3 months, 6 months, and 1 year. The rates have been interpolated to match option maturities.

6.4. Theory and Methodology

This study tests the stability of the volatility smile for pricing tomorrow's options with the use of four option pricing models: they are the European Model[34], American Model[35], Stochastic Volatility Model[36] and Square-Root Constant Elasticity of Variance (CEV) Model[37]. The implied volatilities of the options prices are imputed with each of the modified option pricing models.

6.4.1 Alternative Option Pricing Models

6.4.1.1 European Model

The Garman and Kohlhagen (1983) model[38] (EU) is used for the European currency option pricing model. The call option pricing model is in Equation (2.3).

6. 4.1.2 American Model

The Barone-Adesi and Whaley (1987) model (BW) is used as the American currency option pricing model. It has the early-exercise premium feature that can be used to test and price the American-style options. The call option pricing model is in Equation (3.3). In order to estimate the early-exercise premium, it requires the critical spot rates[39] for call.

6.4.1.3 Stochastic Volatility Model

The original Hull and White (1987a) stochastic volatility option pricing model[40] (HW) was developed for European options. This model allows for a changing volatility. Guo's (1996a) empirical work on European PHLX Dollar/Yen options shows that it performs better than the Heston (1993) model. It can be modified with the early-exercise feature to price the American options. This approach has been

---

[34] It is the Garman and Kohlhagen (1983) Option Pricing Model for European currency options.

[35] It is the Barone-Adesi and Whaley (1987) Option Pricing Model for American currency option with early-exercise premium.

[36] It is the modified version of Hull and White (1987a) Stochastic Volatility for European Options with early-exercise premium.

[37] It is the close form solution in Beckers (1980) for Square-Root Constant Elasticity of Variance Option Pricing Model with adjusted for the early-exercise premium.

[38] See Appendix-A for full details of the Black and Scholes (1973) option pricing model.

[39] See Appendix-D for full details of the Barone-Adesi and Whaley (1987) option pricing model.

[40] See Appendix-C for full details of the Hull and White (1987a) option pricing model.

mentioned by Abken and Nandi (1996) and applied in Guo (1996b) to the predictive power of implied volatility for currency options. The closed form solution to the modified call option pricing model is Equation (6.1).

$$C_{SV} = C_{GK}(\overline{V}) + \frac{d^2C}{d\overline{V}^2}\frac{Var(\overline{V})}{2!} + \frac{d^3C}{d\overline{V}^3}\frac{Skew(\overline{V})}{3!} + A_2(S/S_1)^{q_2} \tag{6.1}$$

whereas C_{SV} is the call price of the stochastic-volatility option.

$C_{GK}(V)$ is the standard European call option (see Chapter 2)

$$\frac{d^2C}{d\overline{V}^2} = \frac{S\sqrt{t}\left[n(d_1)(d_1d_2 - 1)\right]}{4\sigma^3}$$

$$\frac{d^3C}{d\overline{V}^3} = \frac{S\sqrt{t}n(d_1)\left[(d_1d_2 - 1)(d_1d_2 - 3) - (d_1^2 + d_2^2)\right]}{8\sigma^5}$$

$$Var(\overline{V}) = \left[\frac{2\sigma^4(e^k - k - 1)}{k^2} - \sigma^4\right]$$

$$Skew(\overline{V}) = \sigma^6\left[\frac{e^{3k} - (9 + 18k)e^k + (8 + 24k + 18k^2 + 6k^3)}{3k^3}\right]$$

$$d_1 = \frac{\ln(S/X) + (r - R + \sigma^2/2)t}{\sigma\sqrt{t}}$$

$$d_2 = d_1 - \sigma\sqrt{t}, \quad n(d_1) = \frac{e^{-\frac{(d_1)^2}{2}}}{\sqrt{2\pi}}, \quad k = t$$

6.4.1.4 <u>Square-root CEV Model</u>

A closed-form solution of the Square-root CEV model[41] (CV) is used for the test, i.e., when $\sigma = \sigma_1\sqrt{S}$, where σ is the foreign exchange return standard deviation, S is the foreign exchange spot price and σ_1 is constant. Beckers (1980) derived this formal solution. The Square-root CEV model is modified to allow for pricing the American options. The modified pricing model for American call option has included the early-exercise premium feature as in Equation (6.2). This is done by using the

[41] See Appendix-B for full details of the Beckers (1980) Square-root CEV option pricing model.

Barone-Adesi and Whaley (1987) quadratic approximate method of $A_s\left(S/S_i\right)^{q_2}$ (see Chapter 3).

$$C_{Sqr} = Se^{-Rt}N\left(q\{4\}\right) - Xe^{-rt}N\left(q\{0\}\right) + A_2\left(S/S_i\right)^{q_2} \tag{6.2}$$

6.4.2 Estimation of Implied Volatility for the Alternative Option Models

The implied volatilities of option prices for the pricing models have been estimated for the range of strike prices from 97% to 102% of moneyness [F/X]. The implied volatility imputing procedure is the method of bisection[42]. It is used because it is not sensitive to the initial volatility estimate. It starts by choosing a low estimate for volatility [σ_L] corresponding to a low option value [C_L] and a high estimated for volatility [σ_H] corresponding to a high option value [C_H]. The formula for the new estimate [σ_N] is shown in Equation (6.3), where [C] is the observed market price. When the option value corresponding to the interpolated estimate for volatility is below the observed market price, it replaces the low volatility estimate with the interpolated estimate and repeats the calculation. If the estimate for option value is above the observed market price, it replaces the high volatility estimate with the interpolated estimate and repeat the calculation. When the option value corresponding to the estimate for volatility equal to the observed market price, the procedure has arrived at the implied volatility for the observed market price.

$$\sigma_N = \sigma_L + \frac{\left(C - C_L\right)\left(\sigma_H - \sigma_L\right)}{\left(C_H - C_L\right)} \tag{6.3}$$

6.4.3 Estimating a Smooth Smile from the Implied Volatilities

The volatility smile is an approximation of all volatilities across the strike prices. The least-square fit regression [σ_R] is run over the range of volatilities across the strike prices. The formula is in Equation (6.4). The quadratic approximation uses

[42] This is method is mentioned in Kritzman (1991) along with other method to impute the implied volatility from the option prices.

only the first three values of Equation (6.4) for the curve. The observed volatility function as a quadratic curve, $\left[\hat{\sigma}_{F/X}\right]$ is expressed in Equation (6.5).

$$\sigma_R = A_0 + A_1\left(\frac{F}{X}\right) + A_2\left(\frac{F}{X}\right)^2 + A_3\left(\frac{F}{X}\right)^3 + \ldots + A_n\left(\frac{F}{X}\right)^n + e_t \qquad (6.4)$$

$$\hat{\sigma}_{\frac{F}{X}} = A_0 + A_1\left(\frac{F}{X}\right) + A_2\left(\frac{F}{X}\right)^2 \qquad (6.5)$$

where F is forward price, X is strike price and A_0, A_1, and A_2 are constant and e_t is the error term.

6.4.4 Estimation of Tomorrow's Option Volatility

The option prices of tomorrow are forecast with three choices of volatility assumptions on the four option pricing models (i.e., 12 alternative forecasts). The choices of volatility and option models are listed below. The observed volatilities on today's market values are fitted with a quadratic approximation, the least-square fit of the volatility with smile effect. Panel A in Figure 6.1 gives an example of the observed smile (U-shaped curve).

The models:	(1) European option pricing model (EU) (2) American option pricing model (BW) (3) Stochastic Volatility option pricing model (HW) (4) Square-root CEV option pricing model (CV)
The choices:	(a) Constant volatility. (b) Volatility with smile effect. (c) Adjusted volatility with smile effect.

The first choice (a) is the constant volatility. This method uses only the implied volatility at [F/X] equal to 1. This volatility is estimated (as are others) from Equation (6.5). Panel B in Figure 6.1 gives an example of an estimated straight line.

The second choice (b) is the volatility with smile effect. It is very similar to the first choice except it allows the moneyness to vary according to the strike prices of the options. The formula to calculate the volatility with smile effect is in Equation (6.6). Panel C in Figure 6.1 gives an example of the estimated quadratic curve cut through all of today's volatilities (U-shaped curve).

$$\sigma_{\frac{F}{X}|t+1} \cong \sigma_{\frac{F}{X}|t} = \hat{\sigma}_{\left(\frac{F}{X}\right)} = A_{0|t} + A_{1|t}\left(\frac{F}{X}\right) + A_{2|t}\left(\frac{F}{X}\right)^2 \tag{6.6}$$

The third choice (c) is an adjusted volatility with smile effect. It assumes that volatility changes with time-to-expiration, i.e., it allows for the difference of one day. In order to estimate the movement of volatility in relation to maturity, the observed months of February and March 1994 are selected. Figure 6.2 shows the change of at-the-money volatility for February and March 1994 and the average of the two months. The estimated equation indicates an upward movement of volatility as time to maturity declines. The formula to estimate the adjusted volatility with smile effect across the strike prices is Equation (6.7). It accounts for the change in at-the-money volatility from today to tomorrow, but maintains the smile effect. Panel D in Figure 6.1 gives an example of a forecast volatility curve which shifts upward.

$$\sigma_{\frac{F}{X}|t+1} \cong \sigma_{\frac{F}{X}|t} + \left[\sigma_{ATM|t+1}\left(\sqrt{\frac{t}{t+1}} - 1\right)\right] \tag{6.7}$$

$$\textit{where } \sigma_{\frac{F}{X}|t} = \hat{\sigma}_{\left(\frac{F}{X}\right)} = A_{0|t} + A_{1|t}\left(\frac{F}{X}\right) + A_{2|t}\left(\frac{F}{X}\right)^2$$

$$\textit{and } \sigma_{ATM|t+1} = \hat{\sigma}_{\left(\frac{F}{X}\right)} = A_{0|t} + A_{1|t}\left(\frac{1}{1}\right) + A_{2|t}\left(\frac{1}{1}\right)^2$$

6.4.5 Method for Measuring the Performance

The estimated prices of tomorrow's options are divided into three sets, according to whether they are "bid", "ask" or "average" estimates. In order to price tomorrow's options, the "bid" volatility with smile effect is used for the "bid" trades, the "ask" volatility with smile effect is used for the "ask" trades, while the overall "average" volatility with smile effect is applied to all trades. The estimated model prices are compared with tomorrow's market prices in each case.

The results are separated into four groups, they are the Ask/Ask group, i.e., the "ask" trades with the "ask" volatility, the Ask/Ave group, i.e., the "ask" trades with the "average" volatility, the Bid/Ave group, i.e. the "bid" trades with the "average" volatility and the Bid/Bid group, i.e., the "bid" trades with the "bid" volatility. When only "bid" or "ask" trades are available from today volatility smile, only "bid" or "ask" options of tomorrow are estimated, however all options of tomorrow will be estimated with the overall "average" volatility smile of today.

The model performance is measured by the Mean Error [ME], Mean Absolute Error [MAE] and Root Mean Square Error [$RMSE$] as defined in Equations (6.8), (6.9) and (6.10), where [MP] is the market price and [TP] is the estimated price.

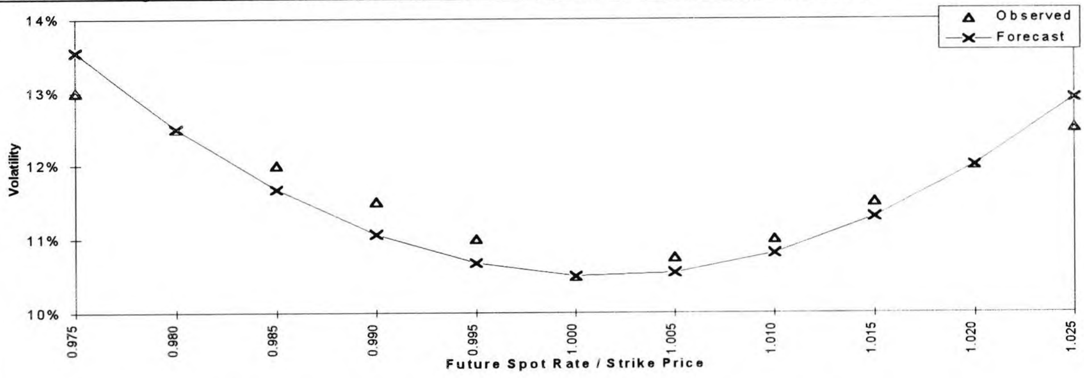
$$ME = \frac{1}{N}\sum_{t=1}^{N}\left(MP_{t+1} - TP_{t+1}\right) \tag{6.8}$$

$$MAE = \frac{1}{N}\sum_{t=1}^{N}\left|MP_{t+1} - TP_{t+1}\right| \tag{6.9}$$

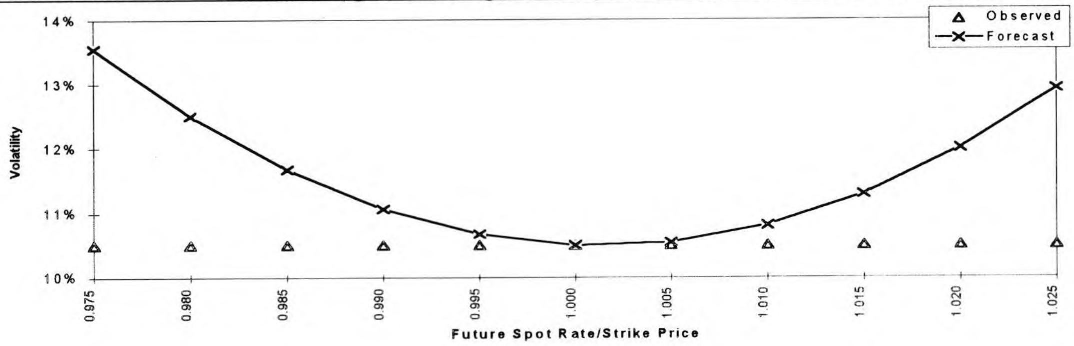
$$RMSE = \sqrt{\frac{1}{N}\sum_{t=1}^{N}\left(MP_{t+1} - TP_{t+1}\right)} \tag{6.10}$$

Figure 6.1: Volatility Smiles of Three Different Estimations

Panel A: Today's Market Volatilities (Δ) with a Quadratic Approximation Fit (X)

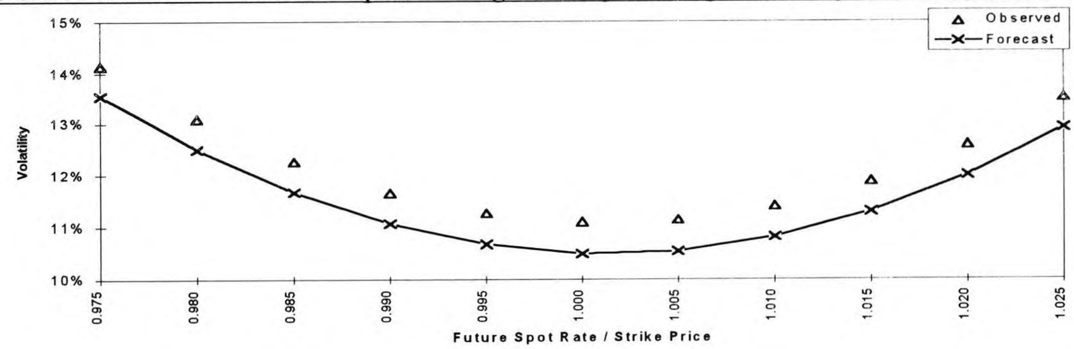


Panel B: Forecast of Tomorrow's Option Pricing Volatility at Constant ATM Volatility of the Smile



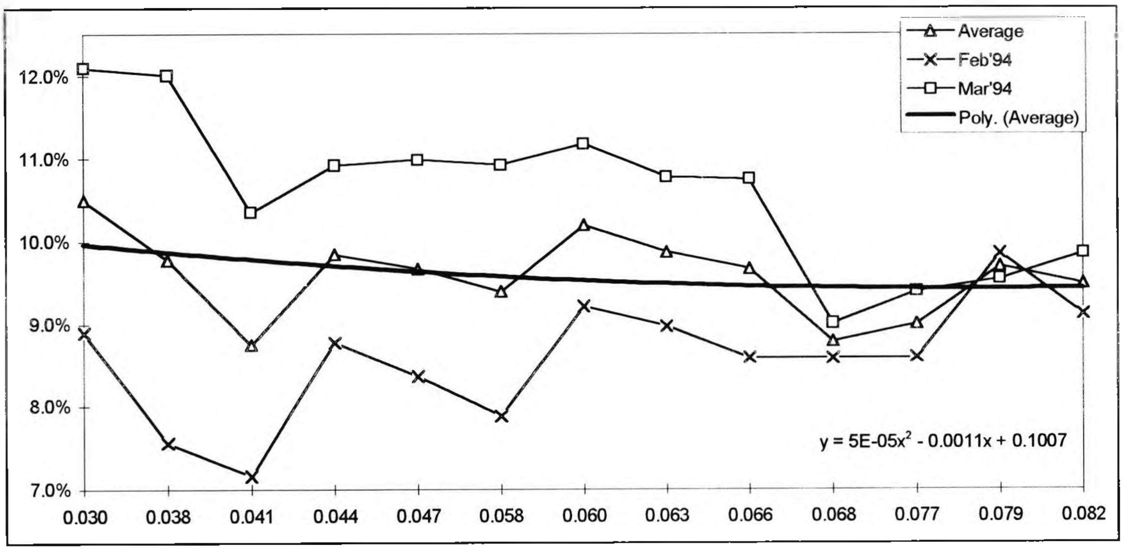
Panel C: Forecast of Tomorrow's Option Pricing Volatility with Today's Volatility Smile

Panel D: Forecast of Tomorrow's Option Pricing Volatility with Adjusted Today's Volatility Smile



NB: X-Shaped (X) indicates the fit Volatility Smile in Panels B, C and D.

Figure 6.2: At-the-moneyness Volatilities for February and March 1994



NB: X-Axis is Time-to-expiration
Y-Axis is Options' Implied Volatility

6.5. Empirical Results

The results are the option prices estimated with four alternative option models and the three assumptions on choice of volatility. The comparison of the estimated option prices with the market option prices are presented according to the three choices of volatility assumption, [i.e., (a) constant volatility, (b) volatility with smile effect, and (c) adjusted volatility with smile effect]. The results are in four separate sub-groups[43], i.e., Ask/Ask, Ask/Ave, Bid/Ave and Bid/Bid.

6.5.1 The Constant Volatility

The results estimated with the constant volatility assumption are shown in Table 6.1. The values in the table are in basis-points and each basis-point represents US$6.25 per option contract[44]. Negative values in the mean-error column indicate that the estimated model values are higher than market values. Panel A presents the results of individual entries for each observed strike price, while Panel B (the bottom four rows) shows the average across the observed strike prices.

Panel B in the Table 6.1 shows that the stochastic volatility model (HW) and the American model (BW) perform better in pricing the options than the other two option models. There is a clear rank order for ME, MAE and RMSE which is the same whether bid-, ask- or average-price trades are used. The stochastic volatility model (HW) and American model (BW) have smaller root-mean-square errors for all of the sub-groups, compared with the European model (EU) and CEV square-root model (CV).

The analysis of individual group (see Panel A) shows that the stochastic volatility model (HW) and American model (BW) give smaller errors on average across all strike prices for all sub-groups, however, the CEV square-root model (CV) is better for low F/X options in Ask/Ask and Ask/Ave sub-groups.

---

[43] Please refer to the earlier section 6.4.5 for the explanation on the sub-groups

[44] Each $/DM contract in PHLX consists of DM62,500, therefore, US$0.0001 * 62,500 => US$6.25.

Table 6.1: Results of Market Prices vs. Estimated Prices with Constant Volatility

Panel A: The Individual Entries for each Observed Strike Price.

Option Model:		EU	BW	HW	CV	EU	BW	HW	CV	EU	BW	HW	CV
Choice 1		Mean Error				Mean Absolute Error				Root Mean Square Error			
Ask/Ask	0.97	4.92	3.26	3.17	0.40	4.92	3.26	3.18	2.48	5.97	4.30	4.23	3.20
(F/X)	0.98	4.59	2.64	2.60	1.87	5.29	4.14	4.13	4.26	6.82	5.58	5.56	5.53
	0.99	5.62	3.34	3.35	3.78	6.19	4.60	4.60	5.02	7.72	5.98	5.99	6.47
	1.00	5.11	2.56	2.59	3.28	6.32	4.70	4.71	5.33	7.71	6.00	6.01	6.59
	1.01	3.56	1.17	1.18	1.34	4.88	3.37	3.37	4.06	6.09	4.30	4.30	5.06
	1.02	2.59	0.73	0.71	-0.67	4.59	3.48	3.48	4.64	5.48	4.29	4.29	5.60
Ask/Ave	0.97	4.16	2.45	2.36	-0.90	4.29	2.79	2.72	2.14	4.69	3.31	3.26	2.78
(F/X)	0.98	4.43	2.44	2.40	1.41	4.98	3.83	3.81	3.94	6.10	4.94	4.92	5.19
	0.99	4.95	2.74	2.76	2.97	5.59	4.01	4.01	4.50	6.73	5.05	5.05	5.92
	1.00	4.84	2.09	2.13	2.90	5.85	4.19	4.18	4.95	7.24	5.47	5.46	6.35
	1.01	3.43	0.96	0.96	1.28	4.86	3.39	3.39	3.80	6.03	4.53	4.53	5.48
	1.02	2.75	0.50	0.48	-1.09	3.94	3.05	3.06	4.55	5.13	4.04	4.05	5.73
Bid/Ave	0.97	2.14	0.97	0.87	-3.15	2.89	2.63	2.60	4.95	3.56	3.10	3.08	6.02
(F/X)	0.98	2.52	0.96	0.91	-0.15	3.97	3.34	3.34	3.99	5.17	4.50	4.50	5.36
	0.99	2.20	0.35	0.36	0.37	4.08	3.60	3.60	4.38	5.41	5.05	5.03	6.18
	1.00	1.22	-0.81	-0.79	0.05	4.60	4.13	4.13	5.20	5.75	5.34	5.34	6.93
	1.01	0.37	-1.34	-1.32	-1.29	4.15	4.06	4.03	5.02	5.38	5.23	5.22	6.24
	1.02	1.53	-0.84	-0.87	-2.20	4.89	4.61	4.61	5.75	5.85	5.60	5.59	7.60
Bid/Bid	0.97	1.91	1.13	1.03	-2.87	2.74	2.64	2.61	4.86	3.25	3.19	3.17	5.75
(F/X)	0.98	2.40	0.90	0.85	-0.23	3.42	2.99	3.00	3.72	4.54	4.03	4.03	4.82
	0.99	2.01	0.43	0.42	0.58	4.28	3.75	3.74	4.64	5.55	5.23	5.23	6.18
	1.00	2.63	0.71	0.72	1.73	4.83	4.24	4.24	5.75	6.12	5.42	5.42	7.50
	1.01	0.95	-0.42	-0.41	-0.16	4.21	3.75	3.74	4.88	5.37	4.82	4.81	6.82
	1.02	2.16	0.33	0.30	-0.76	4.43	3.92	3.91	5.26	5.49	4.83	4.79	6.74

Panel B: The Average across all Strike Prices for each of the group.

Option Model:	EU	BW	HW	CV	EU	BW	HW	CV	EU	BW	HW	CV
Choice 1	Mean Error				Mean Absolute Error				Root Mean Square Error			
Ask/Ask	4.87	2.57	2.57	2.69	5.82	4.30	4.30	4.77	7.25	5.61	5.61	6.07
Ask/Ave	4.50	2.12	2.13	2.12	5.36	3.86	3.85	4.36	6.59	5.01	5.00	5.79
Bid/Ave	1.73	-0.07	-0.08	-0.45	4.16	3.72	3.72	4.72	5.35	4.95	4.94	6.31
Bid/Bid	2.13	0.54	0.52	0.34	4.16	3.67	3.67	4.87	5.38	4.87	4.87	6.45

NB: Bid is the Bid Trades.
Ask is the Ask Trades
Ave is Average (Bid + Ask) of Trades.
[F/X] is the Forward price / Strike Price.
EU is the European Garman and Kohlhagen (1983) Option Model.
BW is the American Barone-Adesi and Whaley (1987) Option Model.
HW is the Hull and White (1987a) Stochastic Volatility Option Model.
CV is the Beckers (1980) CEV Square-root Option Model.

6.5.2 Volatility with the Smile Effect

This approach uses the volatility with the smile effect assumption, i.e., the volatility varies across strike prices. The results in Table 6.2 have the same table-structure as Table 6.1 (individual strike price results at the top in Panel A and the average below in Panel B). In general, Panel B shows that assuming a smile gives larger root-mean-square-errors compared with the earlier constant volatility assumption. However, the "Ask/Ask" group in the mean-absolute-error and mean-error, "Ask/Ave" and "Bid/Bid" groups have smaller mean errors than worse found with constant volatility. As before, the stochastic volatility model (HW) and the American model (BW) perform better in pricing compared with the other two option models. The average root-mean-square-errors are approximately half a basis-point larger than for the earlier results of the constant volatility. However, the mean-error for the European model is smaller in the "Bid/Ave" group and stochastic volatility model and for the American model in the "Bid/Bid", "Ask/Ask" and "Ask/Ave" groups compared with the results of the earlier constant volatility assumption. The "Ask/Ask" group in the mean-absolute-error is approximately one quarter basis-point smaller than for the earlier constant volatility.

The individual analysis of each group (see Panel A of Table 6.2) across moneyness shows that the estimated values have small errors for options away-from-the-money but the root-mean-square-errors are still larger than for the constant volatility results. Thus, taking account of the smile (volatility with smile effect) does not appear to perform better in pricing tomorrow's options than just assuming constant volatility. This confirms for currency options result which was found by Dumas, Fleming and Whaley (1996) for index options.

Table 6.2: Results of Market Prices vs. Estimated Prices with Volatility Smile

Panel A: The Individual Entries for each Observed Strike Price.

Option Model:		EU	BW	HW	CV	EU	BW	HW	CV	EU	BW	HW	CV
Choice 2		Mean Error				Mean Absolute Error				Root Mean Square Error			
Ask/Ask	0.97	0.78	-0.39	-0.40	-0.58	2.60	2.11	2.11	2.38	3.69	3.35	3.35	3.52
(F/X)	0.98	2.39	0.99	1.00	1.55	4.82	4.34	4.34	4.85	6.17	6.02	6.02	6.55
	0.99	3.36	1.53	1.55	2.37	4.48	3.29	3.29	3.98	5.75	4.32	4.33	5.43
	1.00	4.77	2.25	2.28	3.01	6.34	4.68	4.69	5.17	7.74	6.09	6.10	6.57
	1.01	4.97	1.97	2.00	2.85	6.75	5.26	5.26	6.04	8.50	6.76	6.76	7.47
	1.02	2.01	-0.68	-0.66	0.63	6.47	5.65	5.64	6.10	7.79	7.63	7.62	8.10
Ask/Ave	0.97	-1.29	-2.57	-2.57	-2.58	4.11	4.10	4.10	3.86	5.15	5.58	5.57	5.42
(F/X)	0.98	2.79	1.36	1.39	1.87	4.64	4.16	4.14	4.28	5.94	5.40	5.39	5.73
	0.99	3.39	1.64	1.65	2.21	4.61	3.66	3.67	4.30	5.81	4.69	4.70	5.82
	1.00	4.70	1.97	2.02	2.86	5.71	4.13	4.10	4.88	7.15	5.46	5.40	6.21
	1.01	3.52	0.48	0.49	1.59	5.94	4.85	4.85	5.22	7.34	6.13	6.13	6.71
	1.02	3.79	0.58	0.62	2.23	7.07	6.52	6.52	6.85	9.06	8.00	8.01	8.63
Bid/Ave	0.97	-0.96	-1.79	-1.80	-1.85	3.69	4.01	4.00	4.34	4.86	5.33	5.31	5.40
(F/X)	0.98	-0.81	-2.02	-2.02	-1.58	3.61	3.84	3.83	3.97	4.76	5.02	5.02	5.40
	0.99	0.76	-0.67	-0.67	-0.24	3.71	3.71	3.71	4.31	5.12	5.13	5.13	6.05
	1.00	1.15	-0.84	-0.82	0.09	4.53	4.08	4.08	5.12	5.59	5.29	5.29	6.87
	1.01	0.45	-1.33	-1.25	-0.36	5.85	4.99	5.00	5.94	9.71	6.49	6.48	7.67
	1.02	1.32	-2.04	-2.01	-1.06	6.68	5.93	5.91	6.58	8.24	7.52	7.50	8.58
Bid/Bid	0.97	-0.54	-1.06	-1.07	-1.07	3.28	3.33	3.33	4.03	4.50	4.51	4.51	5.01
(F/X)	0.98	-0.16	-1.19	-1.19	-0.82	3.78	3.66	3.65	3.90	5.02	4.92	4.92	5.32
	0.99	1.49	0.27	0.28	0.74	4.24	3.76	3.76	4.77	5.90	5.46	5.47	6.85
	1.00	2.50	0.62	0.63	1.71	4.86	4.32	4.33	5.80	6.09	5.49	5.49	7.55
	1.01	0.53	-1.19	-1.17	-0.08	4.95	4.55	4.53	5.16	6.28	5.89	5.87	6.95
	1.02	1.36	-1.17	-1.16	0.17	5.32	4.90	4.87	5.98	6.67	6.13	6.10	7.25

Panel B: The Average across all Strike Prices for each of the group.

Option Model:	EU	BW	HW	CV	EU	BW	HW	CV	EU	BW	HW	CV
Choice 2	Mean Error				Mean Absolute Error				Root Mean Square Error			
Ask/Ask	3.79	1.61	1.63	2.36	5.49	4.23	4.24	4.81	6.99	5.70	5.71	6.36
Ask/Ave	3.56	1.29	1.32	2.06	5.26	4.20	4.19	4.71	6.67	5.49	5.47	6.20
Bid/Ave	0.38	-1.26	-1.24	-0.63	4.34	4.14	4.14	4.78	6.12	5.50	5.49	6.48
Bid/Bid	1.17	-0.30	-0.29	0.44	4.42	4.05	4.04	4.96	5.80	5.40	5.40	6.72

NB: See Table 6.1 for Descriptions

6.5.3 Adjusted Volatility with the Smile Effect

The adjusted volatility with the smile effect assumption (like the previous one, but allowing one day's time effect) is now applied. The results in Table 6.3 have the same table-structure as Table 6.1. In the average results (see Panel B in Table 6.3), this adjusted volatility smile gives smaller errors than the simple volatility smile. The "Ask/Ask" group has smaller root-mean-square-errors and mean-absolute-errors as compared with the earlier two volatility assumptions.

The stochastic volatility model (HW) and the American model (BW) are still better than the other two option models in term of root-mean-square-errors for all four groups. However, the European model (EU) is best for mean-error in the "Bid/Bid" and "Bid/Ave" groups.

The analysis for the individual sub-groups (see Panel A of Table 6.3) shows that the root-mean-square-errors for out-of-the-money options are smaller than for in-the-money options. All models have smaller mean-errors for options near-the-money in the "Ask/Ask" and "Ask/Ave" groups, but worse in the "Bid/Bid" and "Bid/Ave" groups.

Table 6.3: Results of Market Prices vs. Estimated Prices with Adjusted Volatility Smile

Panel A: The Individual Entries for each Observed Strike Price.

Option Model: Choice 3		EU	BW	HW	CV	EU	BW	HW	CV	EU	BW	HW	CV
		Mean Error				Mean Absolute Error				Root Mean Square Error			
Ask/Ask (F/X)	0.97	-0.17	-1.39	-1.40	-1.37	2.83	2.75	2.75	2.86	3.80	3.73	3.73	3.85
	0.98	1.22	-0.25	-0.23	0.39	4.57	4.25	4.25	4.62	5.90	5.99	5.99	6.44
	0.99	1.85	-0.05	-0.03	0.82	4.09	3.23	3.24	3.82	5.23	4.17	4.17	5.14
	1.00	3.09	0.50	0.53	1.28	5.55	4.26	4.26	4.65	6.97	5.72	5.72	6.07
	1.01	3.47	0.44	0.46	1.35	6.26	5.09	5.09	5.84	7.89	6.52	6.51	7.11
	1.02	0.78	-1.95	-1.92	-0.51	6.29	5.90	5.89	6.17	7.92	8.10	8.10	8.42
Ask/Ave (F/X)	0.97	-2.24	-3.57	-3.57	-3.42	4.41	4.68	4.68	4.34	5.59	6.17	6.16	5.92
	0.98	1.70	0.20	0.23	0.78	4.44	4.12	4.10	4.21	5.57	5.25	5.23	5.51
	0.99	1.86	0.04	0.04	0.65	4.00	3.38	3.38	3.94	5.22	4.44	4.45	5.51
	1.00	3.03	0.24	0.29	1.16	5.13	3.98	3.96	4.38	6.44	5.24	5.16	5.77
	1.01	2.05	-1.04	-1.02	0.13	5.60	4.88	4.88	5.10	6.96	6.26	6.26	6.60
	1.02	2.73	-0.52	-0.48	1.27	7.05	6.73	6.73	6.92	8.87	8.15	8.15	8.63
Bid/Ave (F/X)	0.97	-1.87	-2.74	-2.74	-2.55	4.00	4.43	4.42	4.75	5.25	5.82	5.81	5.86
	0.98	-2.02	-3.29	-3.29	-2.77	4.25	4.69	4.69	4.58	5.32	5.78	5.78	6.05
	0.99	-0.81	-2.31	-2.30	-1.81	3.95	4.31	4.31	4.67	5.34	5.68	5.68	6.49
	1.00	-0.58	-2.62	-2.60	-1.63	4.62	4.66	4.65	5.40	5.75	5.94	5.94	7.22
	1.01	-1.22	-3.04	-2.96	-2.00	6.09	5.59	5.58	6.41	9.87	7.05	7.02	8.01
	1.02	0.20	-3.20	-3.17	-2.17	6.69	6.34	6.31	6.86	8.38	8.06	8.04	8.98
Bid/Bid (F/X)	0.97	-1.52	-2.06	-2.07	-1.76	3.43	3.62	3.62	4.28	4.85	4.95	4.95	5.38
	0.98	-1.44	-2.52	-2.51	-2.05	4.27	4.38	4.38	4.31	5.45	5.57	5.56	5.83
	0.99	-0.20	-1.48	-1.47	-0.95	4.24	4.11	4.11	4.94	5.93	5.80	5.80	7.11
	1.00	0.72	-1.21	-1.20	-0.06	4.77	4.58	4.58	5.88	5.84	5.72	5.71	7.59
	1.01	-1.23	-2.98	-2.97	-1.80	5.21	5.06	5.04	5.60	6.54	6.57	6.55	7.45
	1.02	0.18	-2.38	-2.37	-0.99	5.33	5.33	5.29	6.10	6.78	6.61	6.58	7.55

Panel B: The Average across all Strike Prices for each of the group.

Option Model: Choice 3	EU	BW	HW	CV	EU	BW	HW	CV	EU	BW	HW	CV
	Mean Error				Mean Absolute Error				Root Mean Square Error			
Ask/Ask	2.30	0.06	0.08	0.85	5.00	4.07	4.07	4.55	6.45	5.52	5.52	6.06
Ask/Ave	2.10	-0.23	-0.20	0.59	4.83	4.11	4.10	4.44	6.17	5.39	5.37	5.94
Bid/Ave	-1.10	-2.78	-2.76	-2.07	4.62	4.77	4.76	5.19	6.37	6.11	6.11	6.92
Bid/Bid	-0.39	-1.91	-1.90	-1.08	4.54	4.47	4.46	5.20	5.88	5.83	5.82	6.98

NB: See Table 6.1 for Descriptions

6.5.4 Overall Summary of the Three Volatility Assumptions

The overall performance in estimating tomorrow's option prices shows that the volatility with smile effect offers very little improvement over the assumption of constant volatility. Adjusting the smile for time decay leads to smaller errors than for the simple volatility with smile effect, but is still not better than constant volatility.

Stochastic Volatility (HW) and American (BW) models perform better than the European (EU) and CEV Square-root (CV) models in term of root-mean-square-error. When comparing the volatility assumptions, the constant volatility performed best in three out of four groups, while the adjusted volatility with smile effect performed best in the "Ask/Ask" group.

The mean-absolute-error column shows the same ranking in performance as the root-mean-square-error. However, the mean-error column (see Table 6.1) shows that the "Bid/Ave" group is best with constant volatility, while the "Bid/Bid" group has smallest error with volatility-with-smile effect and the "Ask/Ask" and "Ask/Ave" groups has smallest error with the adjusted-volatility-with-smile effect.

6.6. Conclusions

This study used the Deutsche Mark options traded at PHLX from January 3 1994 to September 9 1994. There were 5,708 observed trades over this 9 months period. The results show that for forecasting the next day's price constant volatility performs best when used with Stochastic Volatility (HW) and American (BW) models.

All forecasts give close predictions of tomorrow's option prices, because the market volatility used is only different by one day. However, the estimated errors remain on average above 5bp[45], i.e., approximately $30 per option contract.

The Stochastic Volatility (HW) and American (BW) models provide very similar results for all three volatility assumptions and for the four groups. These two models forecast better than the European (EU) and CEV Square-root (CV) models. We can reject the earlier assumption that the market uses the European (EU) model to price the American options. The CEV Square-root (CV) model is also not performing better than the Stochastic Volatility (HW) and American (BW) models. However, it provides the best prediction for out-of-the-money Ask/Ask and Ask/Ave groups in the mean-errors. We would expect it to provide a closer estimate in one of the mix groups and volatility assumptions, but it does not.

The root-mean-square-error results show that constant volatility provides best estimated prices for the "Ask/Ave", "Bid/Ave" and "Bid/Bid" groups while the adjusted volatility with smile effect provides best estimated prices for the "Ask/Ask" group. The volatility with smile effect only provides better estimated prices for the "Bid/Ave" and "Bid/Bid" groups. This indicates that pricing with a volatility smile requires a more flexible function. The adjusted volatility smile performs better than the volatility smile in two of the four groups. A different adjusted volatility smile might be able to account for the remain two groups.

The test main conclusion is that simple, constant volatility is easy and provides an accurate method to estimate tomorrow's option prices. Dumas, Fleming and Whaley (1996) have shown that this simple model (constant volatility) performs best for their hedge ratios with index options. Our results show for currency options that the simple model performs best for predicting tomorrow's prices. It is not necessary to assume a smile.

---

[45] Option price is quoted in basis-point (bp), for DM option, each bp is equal to US$6.25.

The errors from using a smile model reflect the instability of the volatility smile. The earlier results in Chapter 5 show that options $\pm$ 2% away-from-the-money had large variation in volatility. This raises the question of options beyond the $\pm$ 2% range. They would be expected to be more volatile and thinly traded. Further research for options beyond this range would be helpful.

Chapter 7: A Comparison of Smiles of the Over-The-Counter and Philadelphia Stock Exchange Options

7.1. Introduction

The purpose of this study is to compare the volatilities traded on the Over-The-Counter (OTC) options with the PHLX options. This will give an indication of whether results for the PHLX are likely to carry over to the (much larger) OTC market. It should test the hypothesis that market structure is not significant in the pricing of currency options. The period of study is from September 13 1993 to October 18 1994 for both markets on Deutsche Mark options.

The OTC data are provided by NatWest Markets and they are mid-day quotations with one-month expiration. In order to match the PHLX options with the OTC options, a quadratic approximation is used to interpolate a range of traded PHLX options' implied volatility to an exact one-month expiration. The volatility smile of the OTC options is recovered from the implied volatilities of risk reversals and strangles. The objective is to compare the volatility smiles of the OTC and PHLX markets. When the OTC volatility smile is different from PHLX volatility smile, it suggests that traders in the two markets have different expectations or that arbitrage is not being done.

In the next few sections, we review some of the previous studies (in section 7.2); the database (in section 7.3) and the methodology to be used (in section 7.4). The results are in section 7.5 and conclusion in section 7.6.

7.2. Previous Research

OTC currency options[46] are widely traded by financial institutions around the world. Recent studies on volatility trades examine the skewness and kurtosis of asset distributions. For example, McCauley and Melick (1996) examined risk-reversals on OTC options using data from NatWest Markets, London. They recovered the probability distribution function from the posted volatility in order to show the relationship between the price of a risk-reversal and the skewness of the distribution. Malz (1996) established the probability distribution function from the prices of cross-rate (Deutsche Mark/Sterling) OTC options' risk-reversals and strangles during 1992, the period of pre- and post- ERM crisis. The daily implied volatility smiles suggested that Sterling did not face a high devaluation risk until 3 days before the currency left the ERM.

Our approach to estimating the volatility is similar to Malz's (1996), but uses a different method to impute the implied distribution. This implied distribution allows us to calculate the statistical skewness and kurtosis of the implied distributions for both OTC and PHLX markets and so make comparisons for the same days.

[46] For general information on OTC options, Hicks (1990) explains the development of OTC trades in the 1990s while Thanassoulas (1992) covers the products type and transactions in the OTC markets.

7.3. Data and Sample Selection

7.3.1 Data

The PHLX data have been explained in Chapter 2 (section 2.3.1). The basic details about contracts were given in Table 2.1 of Chapter 2. This information is later used to convert the levels of mispricing into US dollars per contract.

The OTC data have been provided by the NatWest Markets, London. These data are collected daily at noon in London and begin on December 1 1989. They are quotations at noon instead of traded contracts. They comprise the date, spot rate, one-month at-the-money volatility, one-month 25-delta[47] risk-reversal premium[48] for Deutsche Mark calls, and the one-month 25-delta strangle call spread[49]. A complete collection of the spot rate, at-the-money volatility, risk-reversal and 25-delta call spread only started from September 13 1993.

7.3.2 Sample Selection

The sample period is limited to thirteen months, i.e., a period from September 13 1993 until October 18 1994. This is due to two restrictions, (a) the start date of complete information on the OTC data is September 13 1993 and (b) the data from the PHLX end on October 18 1994. During this period, the total number of trading days for the OTC and PHLX are 285 and 278 respectively. The difference of 7 trading days is due to the national holidays in the United States.

The American Deutsche Mark options from the PHLX have been selected rather than the European options, because interpolation requires daily options with three different expirations and three different strike prices. These are the standard "mid-month" options which expire on the Friday before the third Wednesday of the month. There are 11,765 calls and 17,817 puts on the PHLX during the study period, with expiration ranging from 2 days to 90 days. On each trading day, there are on average 42 trades for calls and 62 trades for puts. On some trading days, options with

[47] Please see section 7.4.2 for explanation for the 25-delta calls and puts.
[48] It is the volatility of the one-month X_{25} call minus the volatility of the one-month X_{75} put.
[49] It is the different between the average of both call and put one-month out-of-the-money volatilities and the one-month at-the-money volatility.

expiration less than one week are used because the options with longer expiration are not traded.

7.3.3 Interest Rates and Foreign Exchange Rates

The domestic and foreign interest rates for each currency are the London Euro-currency deposit interest rates. These have been obtained from Datastream as 1 day, 1 week, 1 month, 3 months, 6 months, and 1 year. Rates have been interpolated to match option maturities. The one-month forward US$/DM foreign exchange rate is obtained from Datastream.

7.4. **Theory, Methodology and Transactions Costs**

In order to compare the OTC and PHLX options, an exact match must be established, i.e., the currency pair and expiration of options traded. The sub-sections below explain the necessary procedure to establish the matching. The methods on recovering the volatility smiles and implied distributions are explained in the later sub-sections.

7.4.1 Matching of the PHLX Data with the OTC Data

It is much easier to match PHLX data to OTC data than vice versa, due to the limited information available from the OTC source. The matching procedure involves two critical elements: the expiration, and the quotation of the options. The comparison uses the implied volatilities of both the OTC and PHLX options rather that their prices. The PHLX options traded on the exchange have various expiration periods, i.e., it is not possible to have an exact one-month expiration for each trading day. The options have expiration ranging from less than one month to approximately three months. A quadratic approximation technique is applied to estimate the implied volatility for an exact one-month expiration. This procedure is explained in the following sub-section (see section 7.4.3).

The PHLX options are quoted in Dollars per Deutsche Mark while the OTC options are quoted in Deutsche Marks per Dollar[50]. To allow comparison, the standard moneyness [X/F] is used, where [F] is forward price and [X] is the strike price of options. The procedure for converting both the OTC and PHLX quotes is in Equation (7.1).

$$Moneyness_{\frac{X}{F}} = \left[\frac{X_{DM/\$}}{F_{DM/\$}}\right]_{OTC} = \left[\frac{X_{\$/DM}}{F_{\$/DM}}\right]_{PHLX} \qquad (7.1)$$

7.4.2 Estimation of Implied Volatilities and Strike Prices from the OTC Options

With a lognormal distribution, the delta of an at-the-forward option would be 50%. In OTC dealing, the strike prices are often chosen such that the option's delta is equal to 25% or 30%. For a "25-delta" call, the strike price is calculated by setting the

[50] Although the OTC options are quoted in Deutsche Mark per Dollar, the calls give the holder the right to buy Deutsche Mark and the puts give holder the right to sell Deutsche Mark. This has been confirmed by the NatWest Markets.

call's delta equal to 25% and solving Garman and Kohlhagen's (1983) model for the strike price. This property of the delta has created a convenient metric for moneyness. It is expressed in Equation (7.2), i.e., given lognormality of the asset, for a "25-delta" call [$X_{25\Delta}$] and a "25-delta" put [$X_{75\Delta}$] with the same maturity and the same implied volatility, the strike prices of the two options are then an equal percentage distance from the forward price [F].

$$\frac{X_{25\Delta}}{F} = \frac{F}{X_{75\Delta}} \tag{7.2}$$

e.g. $$\frac{1.5000}{1.6000} = \frac{1.6000}{1.7067}$$

For example, for the DM/$ rate, if $F = 1.6000$ and $X_{25\Delta} = 1.5000$, then $X_{75\Delta} = 1.7067$.

The implied volatilities for "25-delta" call [V_{25}] and "25-delta" put [V_{75}] are not directly available but may be recovered from the information provided on at-the-forward volatility [V_{50}], "25-delta" call risk-reversal spread and "25-delta" call strangle spread. The risk-reversal spread [V_R] is the difference between two equally out-of-the-money (*+ x% of call and - x% of put*) options' volatilities: Equation (7.3). The strangle spread [V_S] is the difference between the average of both out-of-the-money options' volatilities and at-the-money volatility: Equation (7.4). Manipulating Equations (7.3) and (7.4) leads to formulae for "25-delta" call [V_{25}] and "25-delta" put [V_{75}], Equations (7.5) and (7.6) respectively. The approach is similar to that of Malz (1996).

$$\textit{Risk Reversal spread} \Rightarrow \quad V_R = V_{75} - V_{25} \tag{7.3}$$

$$\textit{Strangle spread} \Rightarrow \quad V_S = \left(\frac{V_{25} + V_{75}}{2}\right) - V_{50} \tag{7.4}$$

$$\textit{Therefore } V_{25} = V_{75} - V_R \quad \textit{and} \quad V_{75} = 2\left(V_S + V_{50}\right) - V_{25}$$

$$V_{75} = \left(V_S + V_{50}\right) + \frac{V_R}{2} \tag{7.5}$$

$$V_{25} = \left(V_S + V_{50}\right) - \frac{V_R}{2} \tag{7.6}$$

The standard European currency option pricing model is the Garman and Kohlhagen (1983) model in Equations (2.3) and (2.4) [see Chapter 2]. The first derivative with respect to the change of underlying spot price is known as the delta-call [ΔC_{GK}] for call option and the delta-put [ΔP_{GK}] for put option, [see Equations (2.5) and (2.6) of Chapter 2].

The strike prices for the call (put) at "25-delta" call ("75-delta" put) and "75-delta" call ("25-delta" put) can be solved with the formulae ΔC_{GK} and ΔP_{GK}, by using a refined quadratic approximation for the expansion of the normal distribution function [$N(d_1)$] in Equation (7.7). The formulae are substituted by the quadratic approximation. The formulae are rewritten to solve for the strike prices of the call [X_C] and put [X_P] in Equations (7.8) and (7.9) respectively. The left hand side is substituted with the given delta, i.e., 25% and 75%, while the right hand side is substituted with the given variables of [r, R, t, S, V_{25} and V_{75}], then [X_C] and [X_P] are found.

$$N\left(d_1\right) \cong \frac{1}{2} + \frac{1}{\sqrt{2\pi}}\left(d_1 - \frac{d_1^3}{6} + \frac{d_1^5}{40} + \ldots\ldots\right) \tag{7.7}$$

$$\textit{therefore } \Delta C_{GK} \cong e^{-Rt}\left(\frac{1}{2} + \frac{d_1}{\sqrt{2\pi}}\right) \textit{ and } \Delta P_{GK} \cong 1 - e^{-Rt}\left(\frac{1}{2} + \frac{d_1}{\sqrt{2\pi}}\right)$$

$$X_C \cong \frac{F}{e^{\left(\frac{\Delta C_{GK}\sigma\sqrt{t}\sqrt{2\pi}}{e^{-Rt}} - \frac{\sigma\sqrt{t}\sqrt{2\pi}}{2} - \frac{\sigma^2 t}{2}\right)}} \tag{7.8}$$

$$X_P \cong \frac{F}{e^{\left(\frac{\Delta P_{GK}\sigma\sqrt{t}\sqrt{2\pi}}{e^{-Rt}} + \frac{\sigma\sqrt{t}\sqrt{2\pi}}{2} - \frac{\sigma^2 t}{2}\right)}} \tag{7.9}$$

The calculated strike prices are used to work-out the moneyness [X/F], together with the recovered volatilities. On each trading day, the strike prices have slight variation in moneyness, i.e., within the range of 2% to 3% out-of-the-money: this is due to changes in the delta as volatility changes. In order to have a range of volatilities [σ_R]over the observed moneyness, a quadratic approximation [see Equation (7.10)] is used to estimate a range of volatilities for strike prices of -3% and +3% from the forward price. The quadratic approximation has only three values of Equation (7.10) for the curve. The observed volatility function as a quadratic curve, $\hat{\sigma}_{X/F}]$ can be expressed as Equation (7.11).

$$\sigma_R = A_0 + A_1\left(\frac{X}{F}\right) + A_2\left(\frac{X}{F}\right)^2 + A_3\left(\frac{X}{F}\right)^3 + \ldots + A_n\left(\frac{X}{F}\right)^n + e_t \qquad (7.10)$$

$$\hat{\sigma}_{\frac{X}{F}} = A_0 + A_1\left(\frac{X}{F}\right) + A_2\left(\frac{X}{F}\right)^2 \qquad (7.11)$$

where F is forward price, X is strike price and A_0, A_1, and A_2 are constant and e_t is the error term.

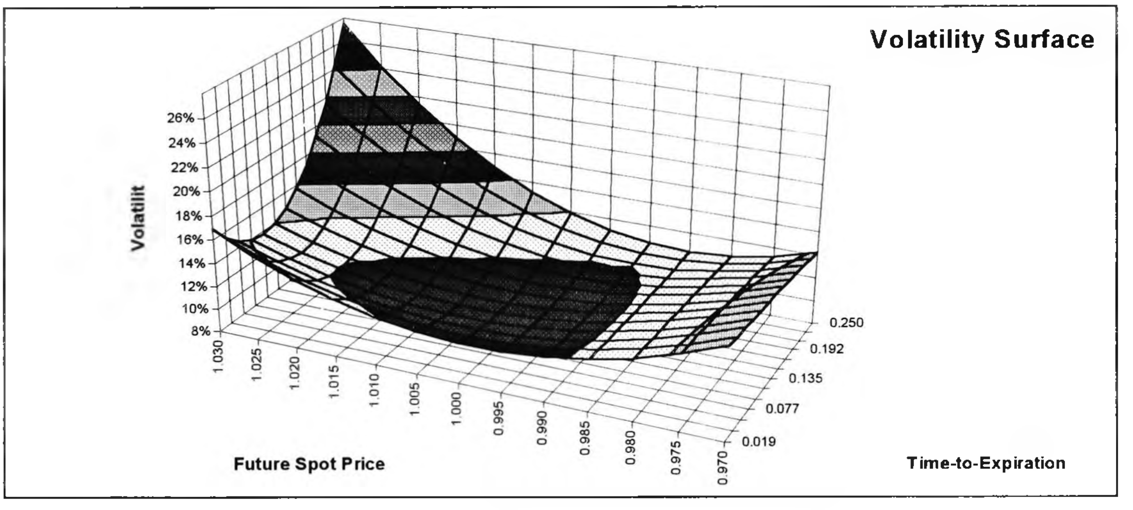
7.4.3 <u>Estimation of Implied Volatilities from the PHLX Options</u>

The implied volatilities of the PHLX options are imputed with the Barone-Adesi and Whaley (1987) option pricing model[51] [see Equations (3.3) and (3.4) of Chapter 3]. They are then used to approximate the volatility which is exactly one-month to expiration. It needs three different maturities of options traded on the same day to do this. Each group of options has expiration ranging from 2 days to 90 days. The strike prices in each must include out-of-the-money, near-the-money, and in-the-money options. On each trading day, if three strikes/maturities of options are not available, a minimum of two sets of options must be obtained and linear interpolation is used to generate the third set of options before applying the quadratic approximation. The volatility surface is interpolated from the 9 different volatilities across a range of

[51] See Appendix-D for full detail of the Barone-Adesi and Whaley (1987) option pricing model.

strikes/expirations, using a quadratic approximation, Equation (7.11), Figure 7.1 gives an example of the volatility surface on a particular day. This Equation (7.11) can then be used to estimate volatility at any strike/expiration within the range of the volatility surface for any trading day.

Figure 7.1: Example of Volatility Surface with Three Strikes/Expiration of Option Prices on a Traded day



7.4.4 Estimation of the Smile Skewness

The smile skewness is calculated using volatilities for option which are 2% on either side of the forward price (as in Chapter 5). For the PHLX options, the formula is modified to account for the potential difference between put and call volatilities at-the-money. The volatility skewness formula is Equation (7.12), (with $\Delta\sigma_{ATM} = 0$ for the OTC options).

$$Skewness^{Smile}_{PHLX} = \left[\frac{\left(\sigma_{Call(F+2\%)} + \Delta\sigma_{ATM} - \sigma_{Put(F-2\%)}\right)}{\sigma_{Call(F+2\%)}}\right] * 100\% \qquad (7.12)$$

where $\sigma_{Call(F+2\%)}$ and $\sigma_{Put(F-2\%)}$ are the implied volatilities of both the call and put out-of-the-money options respectively. $F\pm 2\%$ indicates the options are 2% on either side of the forward price. $\Delta\sigma_{ATM} = \left(\sigma_{Put(ATM)} - \sigma_{Call(ATM)}\right)$, $\sigma_{Put_{(ATM)}}$ and $\sigma_{Call_{(ATM)}}$ are both the at-the-forward options' implied volatilities for the put and call options respectively

7.4.5 Estimation of Implied Distribution from the Volatility Smile

The implied distribution is estimated from the volatility smile of the option prices as explained in Chapter 5.

7.4.6 Arbitrage Opportunities between OTC and PHLX Options

Options with same strike price[52] and maturity of the same underlying asset should have the same implied volatility. When their volatilities are different, it suggests that either one or both of the options is mispriced. In order to ascertain the arbitrage opportunities in the differences between volatilities, a 4% difference in the volatility is needed for the option prices to have approximately 10 basis-points spread, i.e., US$62.50 per contract (1 basis-point in the price represents US$6.25 per contract[53]). Hence a 4% volatility difference between OTC and PHLX options will be used to test for arbitrage. For an example on the arbitrage opportunities, please refer to Appendix (section 7.7) at the end of this chapter.

7.4.7 Transactions Costs

The typical PHLX transaction variable costs and fixed costs are given in Chapter 2 (section 2.4.5). The details of variable and fixed costs are showed in Table 2.12 of Chapter 2. The transaction cost estimated from the PHLX cost information is less than $25, however, to be conservative, $50 per round-turn[54] contract is used for this study.

---

[52] The OTC option's strike price varies in according to the priced option's volatility.
[53] See section 2.3.1 for the premium charges on the change in option value. The lot size refers to one PHLX contract of DM62,500.
[54] A round-turn contract presents cost of buying and selling of two options contracts.

7.5. **Empirical Results**

The comparative results for the OTC and PHLX volatilities are presented in this section. We compare both the volatility smiles and volatility skewness. We then analyse the behaviour of the implied distributions recovered from the volatility smiles.

7.5.1 The Volatility Smiles

Figure 7.2 shows the average smiles of OTC and PHLX options. They are asymmetric, with higher volatility on one side of the trades: the OTC has higher volatility (approximately 1.0% on average over the observed period) for out-of-the-money puts, while the PHLX has higher volatility (approximate 0.5% on average over the observed period) for out-of-the-money calls. The daily differences between the OTC and PHLX volatilities show that arbitrage opportunities may exist.

Table 7.1 shows the results by period for options at different levels of moneyness. The volatility smiles of the OTC and PHLX options have slight differences. Panel A shows the results of individual months and Panel B shows the average of the 278 trading days. It shows that the out-of-the-money OTC puts are more expensive relative to PHLX puts and the out-of-the-money PHLX calls are more expensive relative to OTC puts, (as already seen in Figure 7.2).

Figure 7.2: Average of Daily OTC and PHLX Volatilities

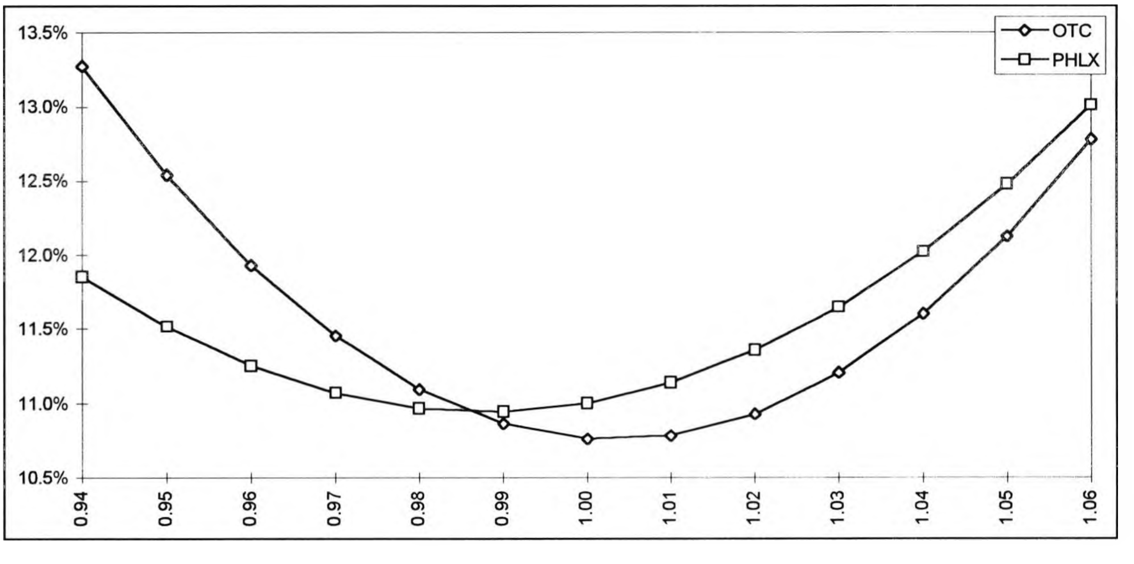


Table 7.1 - Volatilities for PHLX and OTC options, by Period and Moneyness

Panel A: Monthly Average

Volatility	Out-of-the-money Puts						ATM	Out-of-the-money Calls					
Strike Prices	-6%	-5%	-4%	-3%	-2%	-1%	Forward	+1%	+2%	+3%	+4%	+5%	+6%
Sep-93 (OTC)	14.89%	14.31%	13.84%	13.46%	13.20%	13.03%	12.97%	13.00%	13.15%	13.39%	13.74%	14.19%	14.74%
Sep-93 (PHLX)	15.61%	14.86%	14.24%	13.74%	13.38%	13.16%	13.06%	13.10%	13.26%	13.56%	13.99%	14.55%	15.25%
Difference (O-P)	-0.73%	-0.55%	-0.40%	-0.28%	-0.19%	-0.13%	-0.09%	-0.09%	-0.12%	-0.17%	-0.26%	-0.37%	-0.51%
Oct-93 (OTC)	14.60%	13.87%	13.27%	12.80%	12.46%	12.26%	12.19%	12.26%	12.46%	12.79%	13.26%	13.85%	14.58%
Oct-93 (PHLX)	12.22%	12.24%	12.26%	12.28%	12.31%	12.35%	12.40%	12.45%	12.50%	12.57%	12.64%	12.71%	12.79%
Difference (O-P)	2.38%	1.63%	1.01%	0.52%	0.15%	-0.09%	-0.20%	-0.19%	-0.05%	0.22%	0.62%	1.14%	1.79%
Nov-93 (OTC)	13.00%	12.34%	11.80%	11.38%	11.10%	10.94%	10.90%	11.00%	11.22%	11.56%	12.03%	12.63%	13.36%
Nov-93 (PHLX)	10.96%	11.00%	11.03%	11.06%	11.09%	11.10%	11.12%	11.13%	11.13%	11.13%	11.12%	11.11%	11.09%
Difference (O-P)	2.04%	1.34%	0.76%	0.32%	0.01%	-0.17%	-0.21%	-0.13%	0.09%	0.43%	0.91%	1.52%	2.27%
Dec-93 (OTC)	12.50%	11.88%	11.38%	11.00%	10.73%	10.57%	10.53%	10.60%	10.78%	11.09%	11.50%	12.03%	12.67%
Dec-93 (PHLX)	12.53%	11.96%	11.52%	11.18%	10.95%	10.84%	10.84%	10.96%	11.18%	11.52%	11.97%	12.53%	13.20%
Difference (O-P)	-0.03%	-0.08%	-0.13%	-0.18%	-0.23%	-0.27%	-0.32%	-0.36%	-0.40%	-0.43%	-0.47%	-0.50%	-0.53%
Jan-94 (OTC)	12.10%	11.32%	10.68%	10.19%	9.84%	9.62%	9.56%	9.63%	9.84%	10.20%	10.69%	11.33%	12.11%
Jan-94 (PHLX)	11.07%	10.71%	10.41%	10.18%	10.02%	9.92%	9.88%	9.92%	10.02%	10.18%	10.41%	10.71%	11.08%
Difference (O-P)	1.02%	0.61%	0.27%	0.01%	-0.18%	-0.29%	-0.33%	-0.29%	-0.18%	0.01%	0.28%	0.62%	1.04%
Feb-94 (OTC)	12.66%	11.87%	11.22%	10.71%	10.34%	10.12%	10.04%	10.10%	10.30%	10.65%	11.13%	11.76%	12.53%
Feb-94 (PHLX)	9.84%	9.91%	9.98%	10.04%	10.10%	10.16%	10.22%	10.27%	10.32%	10.37%	10.41%	10.46%	10.50%
Difference (O-P)	2.81%	1.95%	1.24%	0.67%	0.24%	-0.04%	-0.18%	-0.17%	-0.02%	0.28%	0.72%	1.30%	2.03%
Mar-94 (OTC)	13.50%	12.72%	12.06%	11.53%	11.13%	10.86%	10.71%	10.69%	10.80%	11.04%	11.41%	11.90%	12.52%
Mar-94 (PHLX)	12.76%	12.06%	11.52%	11.13%	10.91%	10.84%	10.94%	11.19%	11.61%	12.18%	12.92%	13.81%	14.87%
Difference (O-P)	0.74%	0.66%	0.54%	0.40%	0.22%	0.01%	-0.23%	-0.50%	-0.81%	-1.14%	-1.51%	-1.91%	-2.34%
Apr-94 (OTC)	12.70%	11.78%	11.03%	10.43%	9.98%	9.70%	9.57%	9.60%	9.79%	10.14%	10.65%	11.31%	12.13%
Apr-94 (PHLX)	11.09%	10.62%	10.26%	9.99%	9.83%	9.77%	9.81%	9.95%	10.19%	10.54%	10.99%	11.54%	12.19%
Difference (O-P)	1.60%	1.16%	0.77%	0.43%	0.16%	-0.07%	-0.23%	-0.34%	-0.40%	-0.40%	-0.34%	-0.23%	-0.06%
May-94 (OTC)	12.56%	11.66%	10.91%	10.33%	9.90%	9.64%	9.54%	9.59%	9.81%	10.18%	10.72%	11.41%	12.27%
May-94 (PHLX)	10.02%	9.90%	9.80%	9.75%	9.72%	9.73%	9.78%	9.86%	9.97%	10.12%	10.30%	10.52%	10.77%
Difference (O-P)	2.54%	1.76%	1.11%	0.58%	0.18%	-0.09%	-0.24%	-0.27%	-0.17%	0.06%	0.41%	0.89%	1.50%
Jun-94 (OTC)	13.04%	12.08%	11.29%	10.65%	10.18%	9.87%	9.72%	9.73%	9.91%	10.24%	10.74%	11.40%	12.22%
Jun-94 (PHLX)	11.06%	10.60%	10.27%	10.06%	9.96%	9.99%	10.14%	10.41%	10.81%	11.32%	11.95%	12.70%	13.58%
Difference (O-P)	1.98%	1.48%	1.02%	0.60%	0.22%	-0.12%	-0.42%	-0.68%	-0.90%	-1.08%	-1.21%	-1.31%	-1.36%
Jul-94 (OTC)	14.33%	13.63%	13.03%	12.55%	12.18%	11.92%	11.78%	11.74%	11.82%	12.01%	12.31%	12.72%	13.24%
Jul-94 (PHLX)	12.56%	12.33%	12.15%	12.03%	11.96%	11.96%	12.02%	12.13%	12.31%	12.54%	12.83%	13.18%	13.59%
Difference (O-P)	1.77%	1.30%	0.89%	0.53%	0.22%	-0.04%	-0.24%	-0.39%	-0.49%	-0.53%	-0.52%	-0.46%	-0.35%
Aug-94 (OTC)	13.65%	13.04%	12.52%	12.10%	11.77%	11.53%	11.38%	11.33%	11.37%	11.51%	11.73%	12.05%	12.46%
Aug-94 (PHLX)	13.11%	12.45%	11.94%	11.60%	11.42%	11.40%	11.54%	11.84%	12.30%	12.93%	13.71%	14.66%	15.77%
Difference (O-P)	0.53%	0.59%	0.58%	0.50%	0.35%	0.13%	-0.15%	-0.51%	-0.93%	-1.42%	-1.98%	-2.61%	-3.31%
Sep-94 (OTC)	13.70%	13.15%	12.68%	12.30%	12.00%	11.78%	11.65%	11.59%	11.63%	11.74%	11.94%	12.22%	12.58%
Sep-94 (PHLX)	12.44%	12.05%	11.78%	11.61%	11.56%	11.61%	11.77%	12.04%	12.41%	12.90%	13.50%	14.20%	15.01%
Difference (O-P)	1.26%	1.09%	0.90%	0.68%	0.44%	0.17%	-0.12%	-0.44%	-0.79%	-1.16%	-1.56%	-1.98%	-2.43%
Oct-94 (OTC)	13.28%	12.63%	12.09%	11.65%	11.30%	11.05%	10.91%	10.86%	10.91%	11.05%	11.30%	11.65%	12.09%
Oct-94 (PHLX)	11.41%	11.24%	11.12%	11.04%	11.00%	11.01%	11.06%	11.16%	11.30%	11.49%	11.72%	11.99%	12.31%
Difference (O-P)	1.86%	1.39%	0.97%	0.61%	0.30%	0.04%	-0.16%	-0.30%	-0.39%	-0.43%	-0.41%	-0.34%	-0.22%

Panel B: Daily Average

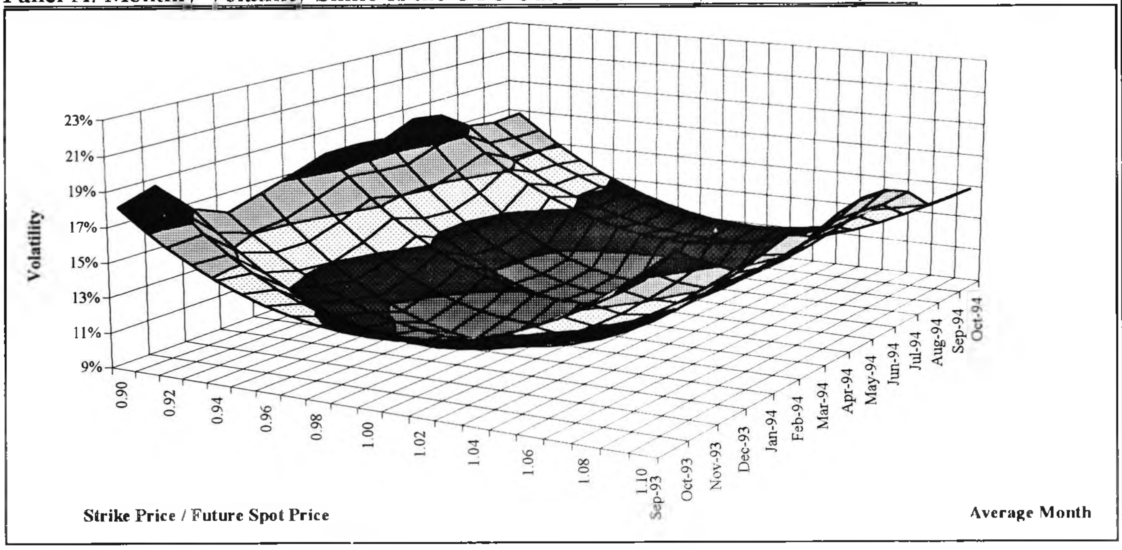
Volatility	Out-of-the-money Puts						ATM	Out-of-the-money Calls					
Strike Prices	-6%	-5%	-4%	-3%	-2%	-1%	Forward	+1%	+2%	+3%	+4%	+5%	+6%
Average (OTC)	13.28%	12.54%	11.93%	11.45%	11.10%	10.86%	10.76%	10.78%	10.93%	11.20%	11.61%	12.13%	12.78%
Average (PHLX)	11.85%	11.51%	11.25%	11.07%	10.97%	10.94%	11.00%	11.14%	11.35%	11.65%	12.02%	12.48%	13.01%
Difference (O-P)	1.42%	1.03%	0.68%	0.38%	0.13%	-0.08%	-0.24%	-0.35%	-0.42%	-0.44%	-0.42%	-0.35%	-0.23%

NB: *The Strike Prices are (X/F) Moneyness (Strike Price / Future Spot Rate).*
The Forward -% is out-of-the-money Puts and Forward +% is out-of-the-money Calls.
ATM is at-the-money options (X/F = 1)
O-P is [the Volatility of OTC - the Volatility of PHLX], the values show are the implied volatility, +ve indicates OTC Price > PHLX Price and -ve indicates OTC Price < PHLX Price across the strike prices.

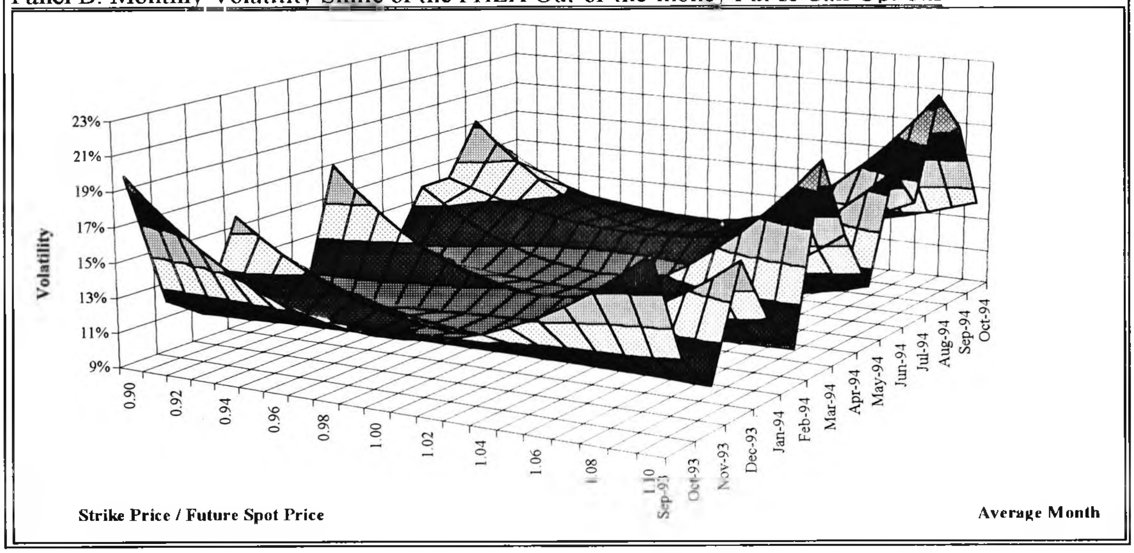
Panel A in Figure 7.3 shows that the OTC volatility has a valley-shaped surface while in Panel B the PHLX volatility has an irregular wave-shaped surface. The graphs show that the OTC smile is consistently left skewed while PHLX smile is consistently right skewed.

Figure 7.3: Monthly Volatility Smile of OTC and PHLX Options

Panel A: Monthly Volatility Smile of the OTC Out-of-the-money Put & Call Options



Panel B: Monthly Volatility Smile of the PHLX Out-of-the-money Put & Call Options



When comparing volatilities at ±2% on either side of the forward price, Table 7.2 shows that OTC at-the-money and out-of-the-money call volatilities are relatively lower (cheaper) than the PHLX, while the OTC out-of-the-money puts are relatively higher (more expensive) than the PHLX. The daily average differences between OTC and PHLX for out-of-the-money puts, at-the-money calls and out-of-the-money calls are 0.13%, -0.24% and -0.42% respectively.

Table 7.2: Monthly Volatility of the OTC and PHLX Options

	OTC Volatility			PHLX Volatility			Difference btw OTC & PHLX		
Date	OTM Put	ATM	OTM Call	OTM Put	ATM	OTM Call	OTM Put	ATM	OTM Call
Sep-93	13.20%	12.97%	13.15%	13.38%	13.06%	13.26%	-0.19%	-0.09%	-0.12
Oct-93	12.46%	12.19%	12.46%	12.31%	12.40%	12.50%	0.15%	-0.20%	-0.05
Nov-93	11.10%	10.94%	11.22%	11.09%	11.12%	11.13%	0.01%	-0.18%	0.09
Dec-93	10.73%	10.54%	10.78%	10.95%	10.84%	11.18%	-0.23%	-0.30%	-0.40
Jan-94	9.84%	9.56%	9.84%	10.02%	9.84%	10.02%	-0.18%	-0.29%	-0.18
Feb-94	10.34%	10.01%	10.30%	10.10%	10.28%	10.32%	0.24%	-0.26%	-0.02
Mar-94	11.13%	10.72%	10.80%	10.91%	10.94%	11.61%	0.22%	-0.22%	-0.81
Apr-94	9.98%	9.55%	9.79%	9.83%	9.81%	10.19%	0.16%	-0.26%	-0.40
May-94	9.90%	9.57%	9.81%	9.72%	9.78%	9.97%	0.18%	-0.21%	-0.17
Jun-94	10.18%	9.72%	9.91%	9.96%	10.14%	10.81%	0.22%	-0.42%	-0.90
Jul-94	12.18%	11.83%	11.82%	11.96%	12.02%	12.31%	0.22%	-0.19%	-0.49
Aug-94	11.77%	11.38%	11.37%	11.42%	11.54%	12.30%	0.35%	-0.15%	-0.93
Sep-94	12.00%	11.63%	11.63%	11.56%	11.77%	12.41%	0.44%	-0.13%	-0.79
Oct-94	11.30%	10.91%	10.91%	11.00%	11.06%	11.30%	0.30%	-0.16%	-0.39
Daily Averag	11.10%	10.76%	10.93%	10.97%	11.00%	11.35%	0.13%	-0.24%	-0.42

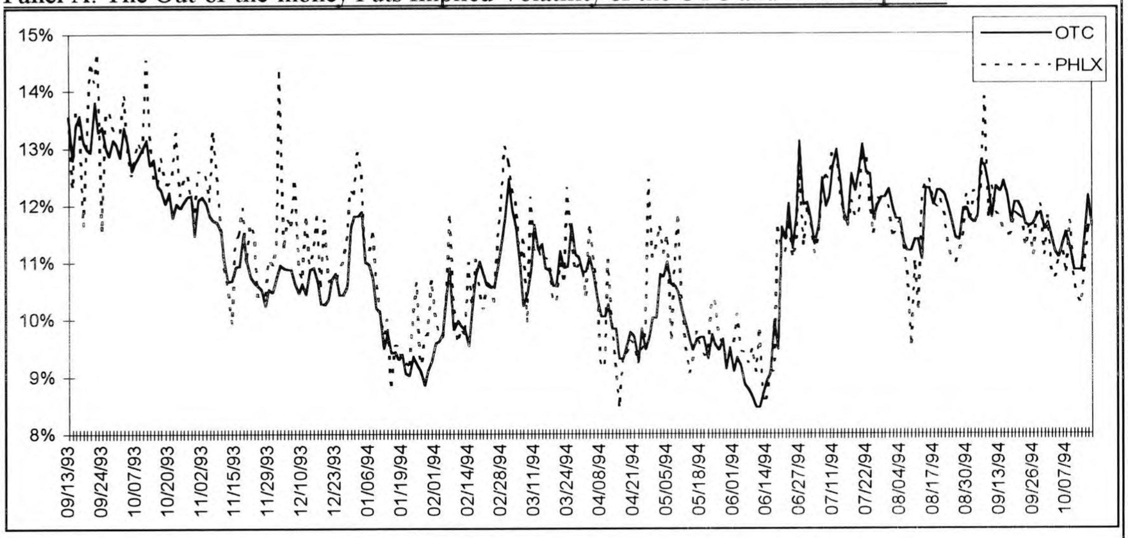
NB: OTM means Out-of-the-money Options
The Difference between OTC and PHLX is (OTC - PHLX)

Daily comparisons of volatility by moneyness are plotted in Figure 7.4. Panel A of Figure 7.4 plots the out-of-the-money puts for the OTC and PHLX. The PHLX volatility has more fluctuations over the period than the OTC, but the level of volatility on the PHLX is lower than for the OTC during the later period. Panel B of Figure 7.4 shows the out-of-the-money calls for the OTC and PHLX. The PHLX volatility is again more volatile than that of the OTC, but the level of volatility is higher than for the OTC during the later period. These plots confirm the results in Table 7.2. The bottom Panel C in Figure 7.4 shows the at-the-forward volatility for the OTC and PHLX. The at-the-forward PHLX volatility[55] is higher than the OTC volatility for almost all days. The averages is a 0.25% gap between the OTC and PHLX options, which is significantly different for out-of-the-money calls (-ve 2% moneyness).

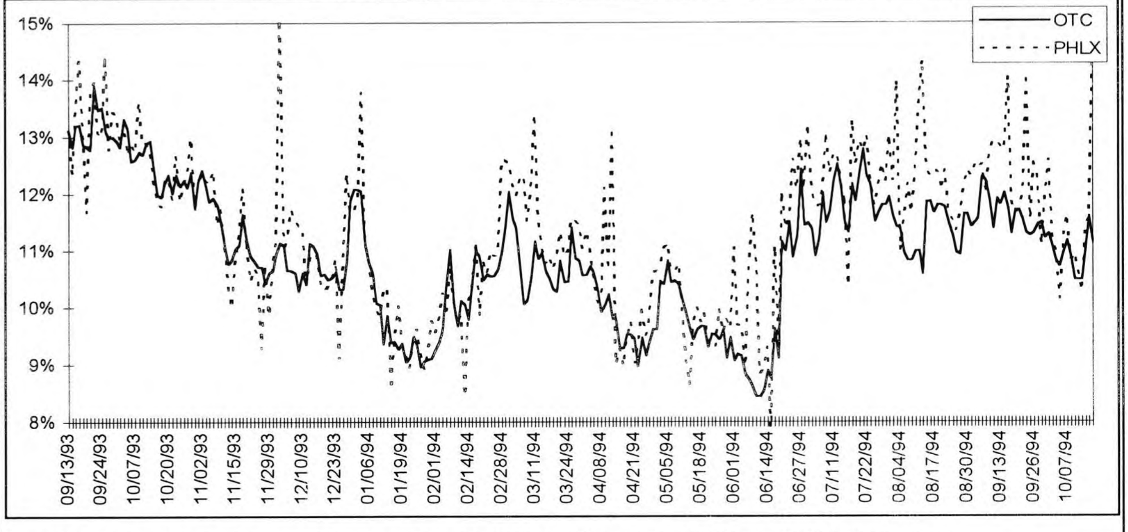
[55] The at-the-forward PHLX volatility is an average of at-the-forward call volatility and at-the-forward put volatility.

Figure 7.4: Daily Implied Volatility for Out-the-money Puts and Calls and the At-the-forward Options of the OTC & PHLX Options

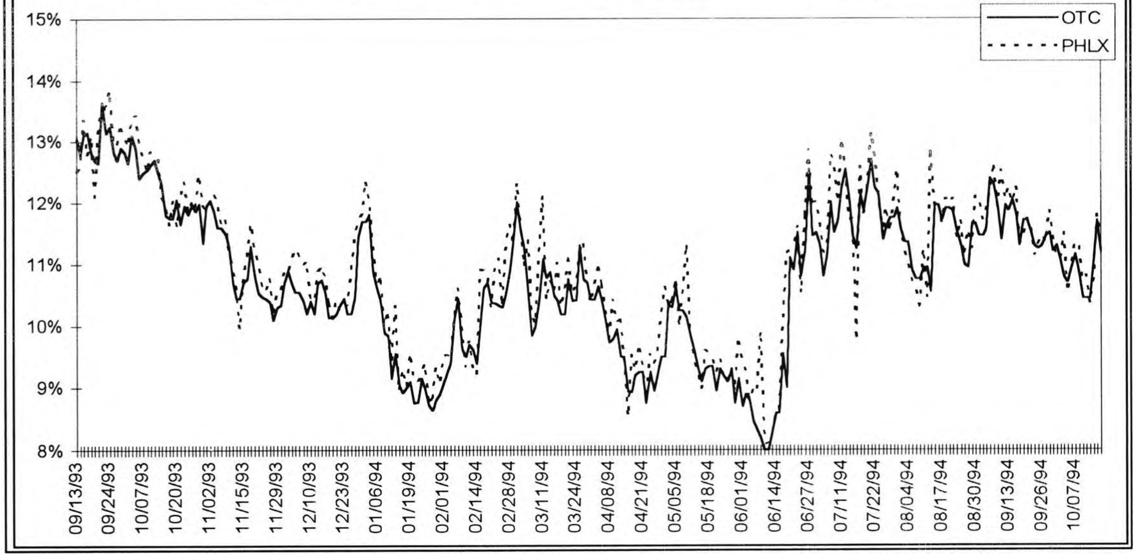
Panel A: The Out-of-the-money Puts Implied Volatility of the OTC and PHLX Options



Panel B: The Out-of-the-money Calls Implied Volatility of the OTC and PHLX Options



Panel C: The At-the-forward Options Implied Volatility of the OTC and PHLX Options



7.5.2 The Volatility Smile Skewness

Table 7.3 gives the monthly average volatility smile skewness, as measured by Equation (7.12). It shows that the PHLX has more positive skewness than the OTC market. The OTC volatility skewness shows a negative value while the PHLX volatility skewness shows a positive value. For the average of 278 trading days, it shows an OTC average of -1.47% smile skewness while the PHLX has +0.01% smile skewness.

Table 7.3: Monthly Average Volatility Smile Skewness of OTC and PHLX Options

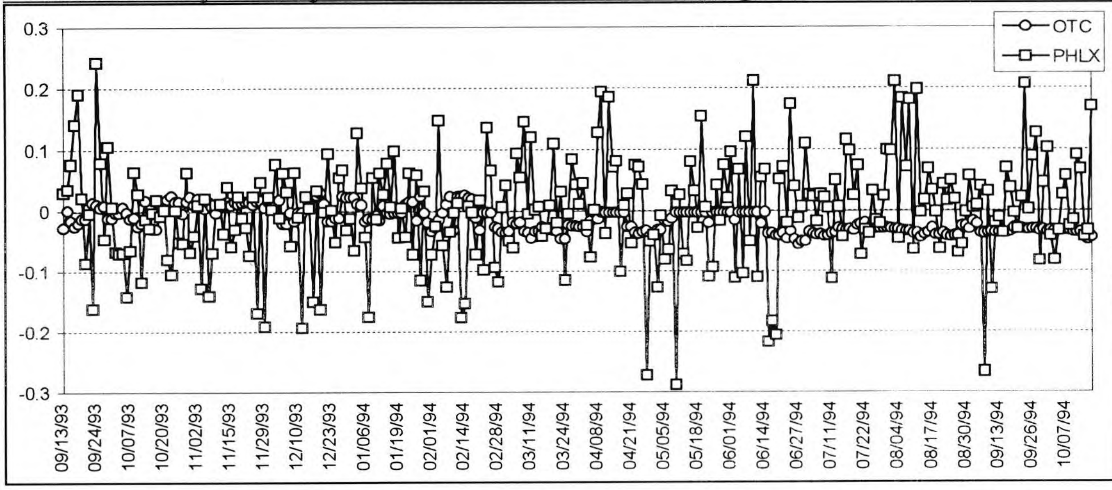
Date	Average Skewness		
	OTC	PHLX	Diff
Sep-93	-0.60%	4.50%	-5.10%
Oct-93	-0.03%	-3.62%	3.59%
Nov-93	1.18%	-3.65%	4.83%
Dec-93	0.58%	-0.78%	1.36%
Jan-94	0.02%	-0.34%	0.36%
Feb-94	-0.33%	-3.37%	3.04%
Mar-94	-3.09%	1.99%	-5.07%
Apr-94	-1.82%	1.66%	-3.48%
May-94	-1.00%	-2.59%	1.59%
Jun-94	-2.56%	0.35%	-2.91%
Jul-94	-3.12%	0.75%	-3.87%
Aug-94	-3.47%	4.52%	-7.99%
Sep-94	-3.16%	0.95%	-4.12%
Oct-94	-3.61%	1.14%	-4.75%
Daily Average	-1.47%	0.01%	-1.48%

NB: Diff means (OTC - PHLX)

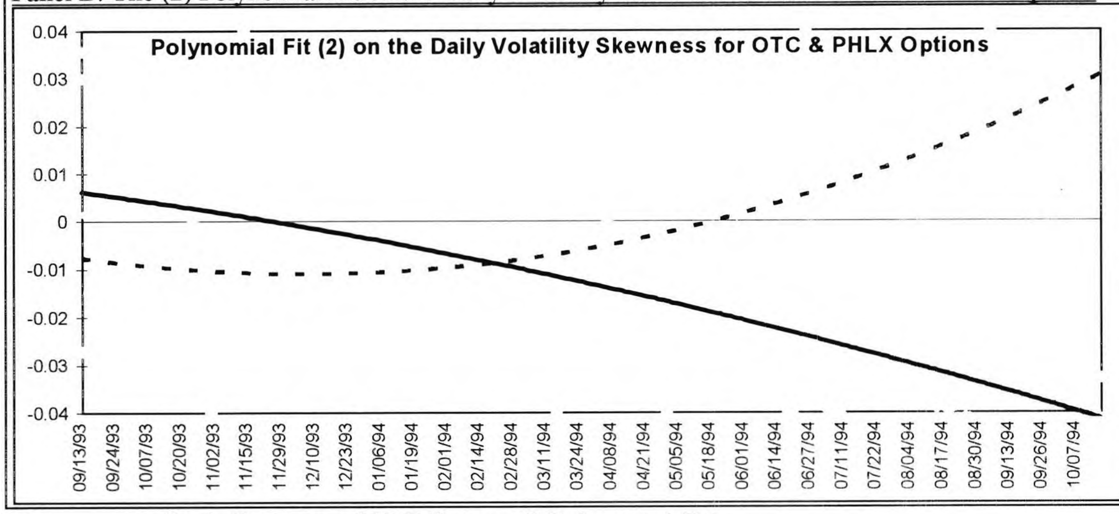
The values show the difference between the traded PHLX options and quoted OTC options. Panel A of Figure 7.5 shows the daily skewness of the PHLX over the 278 trading days. The PHLX volatility skewness lies within the wide range of ± 0.3 (±30%) and is mainly above zero for the later period, while the OTC volatility skewness lies within the small range of ±0.1 (±10%) and remains below zero after Jan-94. The trend for PHLX options is to move negative smile skewness while for OTC options, it is to move positive smile skewness (see Panel B of Figure 7.5).

Figure 7.5: Volatility Smile Skewness of OTC and PHLX Options

Panel A: The Daily Volatility Smile Skewness of OTC and PHLX Options



Panel B: The (2) Polynomial Fit on the Daily Volatility Smile Skewness of OTC and PHLX Options

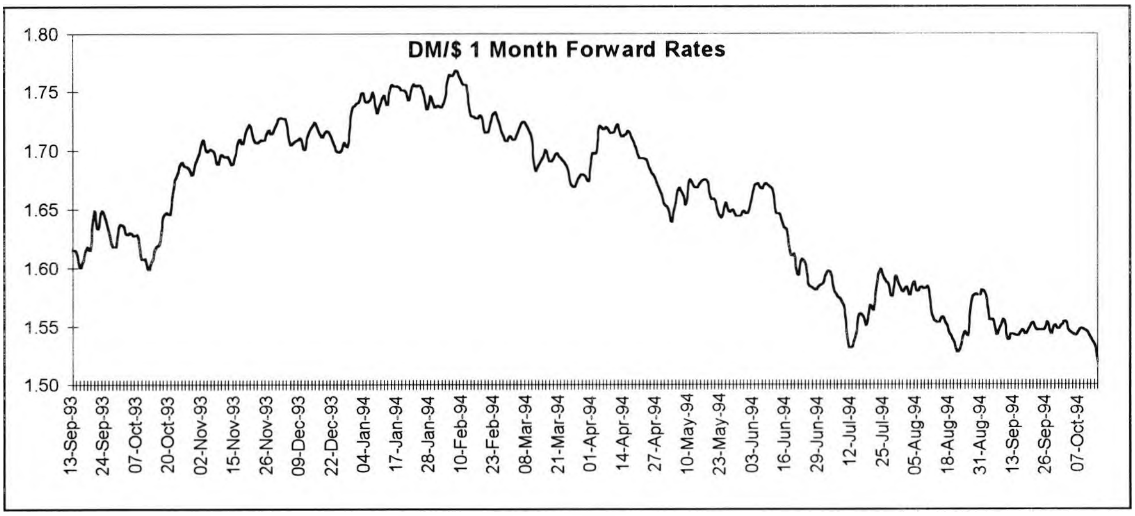


NB: —— *Smile Skewness of OTC Options (2 Polynomial Fit)*
----- *Smile Skewness of PHLX Options (2 Polynomial Fit)*

7.5.3 Volatility Level and the Exchange Rate

Figure 7.6 shows the daily movement of the one-month DM/$ forward rates. The inverted V-shape is the opposite of the simple V-shape for volatility in Figure 7.4. It suggests (weakly) that a high DM/$ rate (strong dollar) was associated with low volatility.

Figure 7.6: One-Month DM/$ Forward Exchange Rates



7.5.4 Skewness and Kurtosis of Implied Distributions

Table 7.4 shows the results for skewness and kurtosis of implied distributions. These are relatively small. The implied distributions for both the OTC and PHLX options are leptokurtic (see third column of Table 7.4). The kurtoses calculated from the implied distributions for both the OTC and PHLX are larger than 3 (except Mar-94 of PHLX) and larger than the kurtosis of the log-normal distribution (see right-hand column of Table 7.4). The observed statistical skewness (see second column of Table 7.4) for both OTC and PHLX options change over time.

The implied distribution graphs for the OTC and PHLX are presented in Panels A and B of Figure 7.7 respectively. The surface distributions are very similar except at the end of the tails: the PHLX has irregular distribution at the tails compared with the OTC. This is another way of viewing the same phenomenon as the earlier irregular wave-shaped volatility smiles. However, individual period plots provide better confirmation of the distributions. Figure 7.8 shows the plot of implied distributions for OTC and PHLX options in February 1994, when it is easier to identify the difference between the two distributions.

The results show the means of both distributions are less than 1 for all monthly average. This confirms the average volatility difference in Table 7.1, with the lowest difference of both volatilities at around -1% away-for-the-money, i.e., 1% out-of-the-money puts.

Table 7.4: Kurtosis & Skewness of Implied Distribution for the OTC & PHLX Options

Panel A: Kurtosis and Skewness from the Implied Distribution of the OTC Options

	Observed Distribution						Lognormal Distribution	
Month	$Skewness_D$	$Kurtosis_D$	$Variance_D$	$Mean_D$	Coeff of Var	Imp Vol	$Skewness_N$	$Kurtosis_N$
Sep-93	0.0829	3.7459	0.0014	0.9952	3.8180%	8.71%	0.1146	3.2127
Oct-93	-0.0252	3.5128	0.0012	0.9952	3.4919%	7.97%	0.1048	3.1775
Nov-93	-0.0231	3.6955	0.0010	0.9958	3.2063%	7.32%	0.0962	3.1494
Dec-93	0.1073	3.7220	0.0010	0.9943	3.1021%	7.08%	0.0931	3.1398
Jan-94	0.0266	3.6935	0.0008	0.9956	2.8071%	6.41%	0.0842	3.1143
Feb-94	0.0641	3.6096	0.0008	0.9951	2.9132%	6.65%	0.0874	3.1232
Mar-94	0.0468	3.6557	0.0010	0.9947	3.1305%	7.14%	0.0939	3.1424
Apr-94	0.2394	3.4806	0.0007	0.9931	2.7471%	6.27%	0.0824	3.1094
May-94	0.2460	3.4804	0.0007	0.9932	2.7529%	6.28%	0.0826	3.1099
Jun-94	0.2216	3.4489	0.0008	0.9931	2.7837%	6.35%	0.0835	3.1124
Jul-94	-0.0542	3.8142	0.0012	0.9958	3.4959%	7.98%	0.1049	3.1780
Aug-94	-0.0352	3.9460	0.0011	0.9952	3.4010%	7.76%	0.1021	3.1683
Sep-94	0.0390	3.9141	0.0012	0.9948	3.4728%	7.93%	0.1042	3.1756
Oct-94	-0.1028	3.9847	0.0011	0.9954	3.2632%	7.45%	0.0979	3.1548

Panel B: Kurtosis and Skewness from the Implied Distribution of the PHLX Options

							Lognormal Distribution	
Month	$Skewness_D$	$Kurtosis_D$	$Variance_D$	$Mean_D$	Coeff of Var	Imp Vol	$Skewness_N$	$Kurtosis_N$
Sep-93	-0.1170	3.6997	0.0015	0.9970	3.8258%	8.73%	0.1148	3.2136
Oct-93	-0.0036	3.5751	0.0013	0.9956	3.5775%	8.16%	0.1074	3.1865
Nov-93	-0.0011	3.9363	0.0011	0.9950	3.3241%	7.59%	0.0998	3.1607
Dec-93	0.2838	3.0181	0.0008	0.9947	2.8048%	6.40%	0.0842	3.1141
Jan-94	0.0952	3.6266	0.0008	0.9943	2.8716%	6.55%	0.0862	3.1196
Feb-94	0.0915	3.8379	0.0009	0.9947	3.0745%	7.02%	0.0923	3.1373
Mar-94	0.3659	2.8732	0.0007	0.9921	2.7383%	6.25%	0.0822	3.1087
Apr-94	-0.1216	3.6341	0.0008	0.9975	2.8822%	6.58%	0.0865	3.1205
May-94	-0.0452	3.7443	0.0008	0.9951	2.8795%	6.57%	0.0864	3.1203
Jun-94	-0.2388	3.1160	0.0008	0.9982	2.7816%	6.35%	0.0835	3.1122
Jul-94	0.1241	3.7676	0.0012	0.9948	3.5465%	8.09%	0.1064	3.1832
Aug-94	-0.3986	3.1592	0.0010	0.9996	3.2058%	7.32%	0.0962	3.1494
Sep-94	-0.3449	3.1801	0.0010	0.9986	3.2380%	7.39%	0.0972	3.1524
Oct-94	0.2917	3.6176	0.0011	0.9943	3.2802%	7.49%	0.0984	3.1565

NB:

$Skewness_D$	is the skewness of the Implied Distribution.
$Kurtosis_D$	is the kurtosis of the Implied Distribution.
$Variance_D$	is the variance of the Implied Distribution, translated to return form, gives a unique measure of the instantaneous implied volatility.
$Mean_D$	is the mean of the Implied Distribution.
Coeff of Var	is the coefficient of variation for the Implied Distribution.
Imp Vol	is the unambiguous implied return volatility for the Implied Distribution.
$Skewness_N$	is the skewness of the Log-normal Distribution.
$Kurtosis_N$	is the kurtosis of the Log-normal Distribution.

Figure 7.7: Implied Distributions of the OTC and PHLX Options

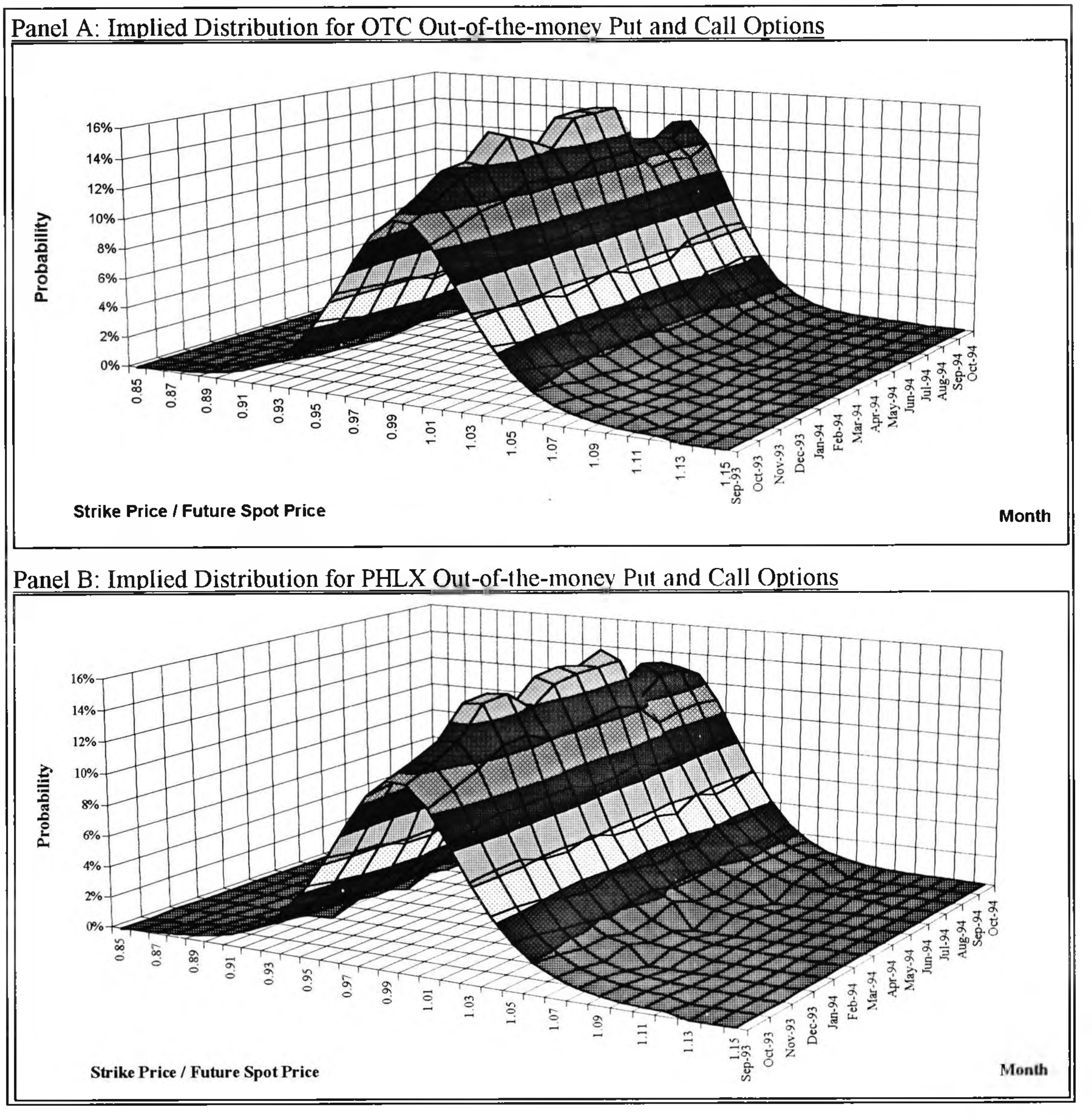
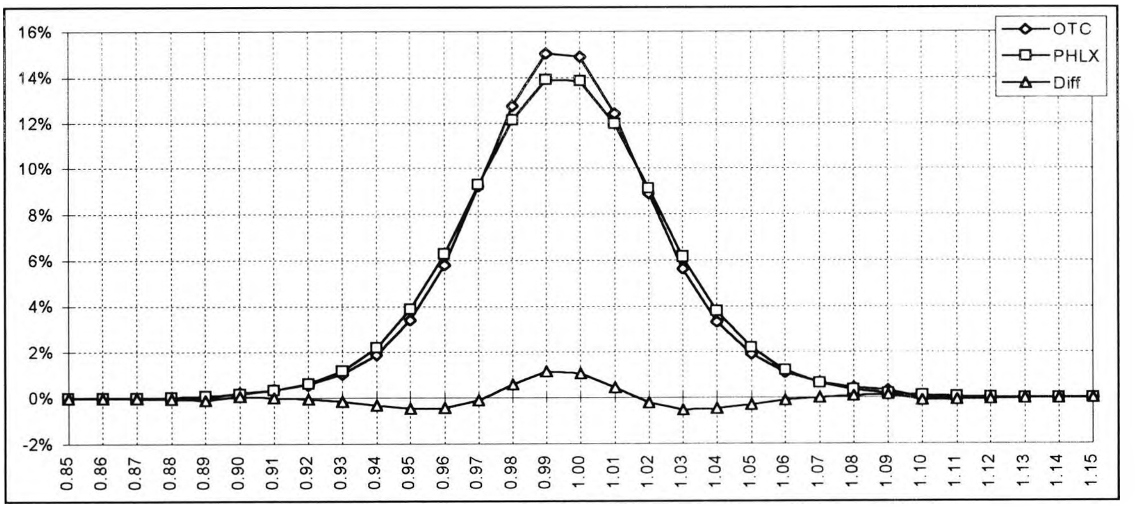


Figure 7.8: Implied Distributions of Feb '94 OTC and PHLX Options



7.6. Conclusions and Discussions

This study is the first to make a systematic comparison of OTC and PHLX options. The results show that the volatility smiles for the OTC and PHLX options are different, which is surprising as they depend on the same asset distribution. The OTC distribution is left skewed and the PHLX distribution is right skewed. The PHLX volatilities are more variable than the OTC quotes over the observed period. The volatility smile skewness indicates that puts are expensive on the OTC[56] and calls on the PHLX.

Some possible reasons for the differences are as follows. The OTC options' **quotes** are collected from the pricing system at noon London, and the PHLX options' prices are daily **traded** contracts. Moreover the OTC options have expiration fixed at one-month and the PHLX options have fixed calendar date, so it is necessary to approximate one-month option volatility. However, the PHLX daily volatility movement has a close resemblance to the OTC volatility (as in Figure 7.4).

The differences between the volatility smiles represent an arbitrage opportunity when the prices of the OTC quotes hold. However, after accounting for the transaction costs[57], we require a minimum of 4% difference in volatility levels. The average difference from our sample is about 1%, therefore only market-makers would be able to do the arbitrage profitably.

McCauley and Melick (1996) suggested that the OTC volatility smile (as observed from a risk-reversal) reflected market sentiment and that there was a strong relationship between price of a risk-reversal and the skewness of the implied distribution. However, the OTC information is sparse, i.e., only three options on any day. The smile skewness derived from the PHLX data is much more reliable, given both call and put volatilities across a range of strike prices. The PHLX smile and implied distribution show that the volatilities are not consistent over the observed period.

[56] The OTC options are quoted in Deutsche Mark per Dollar options. However, the call options allow holder to buy Deutsche Mark and the put options allow holder to sell Deutsche Mark. It is the same as call and put of the PHLX options. This has confirmed with the NatWest Markets.

[57] Transaction costs is assumed as $50, please refers to section 7.4.7 for more details.

The results confirm the variability of volatility on the PHLX options: forecasting of option prices required a more stable smile and reliable model (see Chapter 6). It is very different from the OTC results. In order to have a more complete analysis, further research in same area with more reliable OTC options is necessary to provide volatilities across a wide range of strike prices.

7.7 Appendix: An Example of OTC & PHLX Smiles vs Strikes and Arbitrage Opportunities for Different Volatility Level

Figure 7.9 shows the curves of an example "25-delta" calls' volatilities for OTC and the PHLX options. The differences between OTC and PHLX volatilities represent arbitrage opportunities. In Table 7.5, we show how the market prices the risk-reversals of "25-delta" calls from the given volatilities with spot exchange rate and both interest rates. The volatilities in first column of the Table indicate a range of possible volatility quotes for the options. The traders work-out the strike price and option price with the given spot foreign exchange rate, both interest rates and expiration. To achieve a shift of 10 basis-point on option price, the volatility quotes must have a spread of 4% as shown in example.

Figure 7.9: Example of OTC & PHLX Volatility Smiles versus Strike Prices

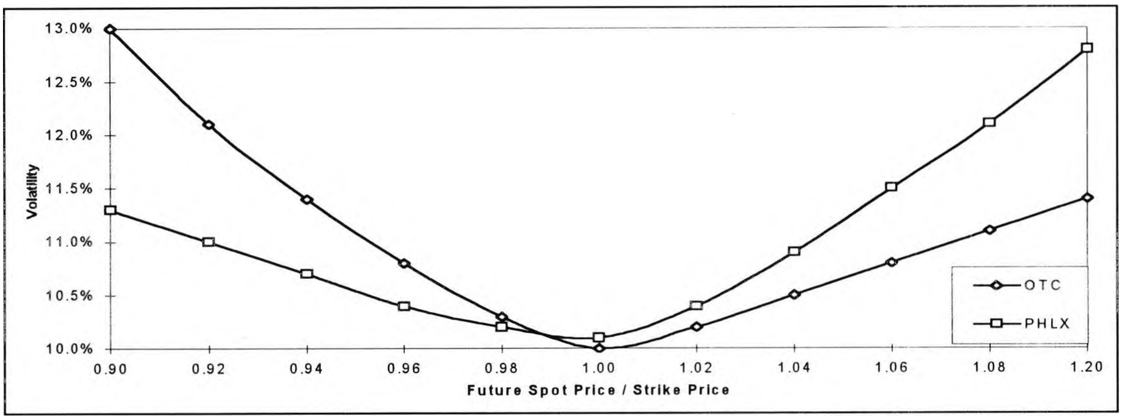


Table 7.5: Examples of Option Prices Using Different Volatility Levels

Volatility	C-Strike	C-Price	US-Ri	DM-Ri	Time	Spot
10.0%	0.5989	0.0027	5.00%	5.50%	0.0833	0.5582
10.5%	0.5995	0.0028	5.00%	5.50%	0.0833	0.5582
11.0%	0.6001	0.0030	5.00%	5.50%	0.0833	0.5582
11.5%	0.6006	0.0031	5.00%	5.50%	0.0833	0.5582
12.0%	0.6012	0.0032	5.00%	5.50%	0.0833	0.5582
12.5%	0.6018	0.0034	5.00%	5.50%	0.0833	0.5582
13.0%	0.6023	0.0035	5.00%	5.50%	0.0833	0.5582
13.5%	0.6029	0.0036	5.00%	5.50%	0.0833	0.5582
14.0%	0.6035	0.0038	5.00%	5.50%	0.0833	0.5582
14.5%	0.6041	0.0039	5.00%	5.50%	0.0833	0.5582

NB: Volatility is Volatility of the Option Price (change for comparing the option price).
C-Strike is the Strike Price of Call Option at 25% delta call.
C-Price is Call Option Price.
Spot is the given Spot Foreign Exchange Rate of Dollar/Deutsche Mark.
US-Ri is the given US Dollar Risk-free Interest Rate.
DM-Ri is the given Deutsche Mark Risk-free Interest Rate.
Time is Option's Time-to-expiration (one-month).

Chapter 8: Conclusion

This first part of final chapter summarises the findings of this study and presents the main conclusions. The second part makes some suggestions for further research.

8.1 Main Conclusion from Analysis

This thesis attempted to address the question of whether the currency option market is efficient or not. The availability of intra-day trade-by-trade data allowed us to analyse the option trades more accurately than other studies which have mostly used day-end prices or bid-ask quotes. The sample was from August 1987 to October 1994. We tested 869,303 American-style transactions and 62,579 European-style transactions (a total sample of 36,719,372 put options and 36,607,832 call options).

The study started with an analysis of put-call parity for the European options (in Chapter 2). This required no assumption about model. Beside the simple put-call approach, a box trading strategy was also tested. The tests involved pairing options in an arbitrage position which were traded within a limited period of time and within a limited range of spot rates. Both methods found that many (more than 90% of sample) option prices violated the arbitrage bound. Approximately 94% of European put-call pairs did not accord with put-call parity, of which half were on average significant at the 95% confidence level. After accounting for transactions costs of $50 per round turn, the results on the put-call pairs still showed more than 25% risk-free arbitrage opportunities. On the box trading strategy, the results showed more than 80% of the box-groups violated the box-condition. However, taking transactions costs of $150 into consideration, the risk-free arbitrage opportunities fell to 10% of the box-groups.

Analysis of two sub-periods showed that there was no systematic change in mispriced values over time. Deutsche Mark, Canadian Dollar, Swiss Franc and French Franc options had larger mispricing in the second period, while other options (the Australian Dollar, British Pound, and Japanese Yen) had more efficient pricing in the later period.

In the second study (Chapter 3), the put-call test was extended to the American calls and puts. The method involved using Barone-Adesi and Whaley's (1987) American model to compute the implied volatilities of the option prices and then pairing the options in an arbitrage position, (as before, traded within a limited period of time also within a limited range of spot rates). In this test, put option prices were used to estimate the call prices and then compared with the call prices in the market.

The results showed that almost all traded call prices in the sample were not equal to call prices estimated from puts. After accounting for transactions costs of $50, the numbers of risk-free arbitrage still remained at about 20% of call options in the put-call pairs. Tests of two sub-periods showed that only Australian Dollar and Swiss Franc calls systematically improved in pricing over time, while all other currencies worsened. Deutsche Mark calls shifted from being overpriced to being under-priced. The results for Deutsche Mark call options in the second period indicated an approximate risk-free profit[58] of $2,500 per trading day, but other currencies showed lesser amounts. The Japanese Yen calls had an approximate risk-free profit of $5,250 per fortnight. The mispriced European and American options again showed that the PHLX options market was not efficient.

In order to understand better the mispriced American options, an analysis of the early-exercise premium was then conducted (Chapter 4). The method involved pairing American and European options in an arbitrage position, (as before, traded within a limited period of time and within a limited range of spot rates). Both Garman and Kohlhagen (1983) and Barone-Adesi and Whaley (1987) models were used for this study.

The results showed that approximately 60% of American puts and 20% of American calls were priced below the equivalent European options. This violated rational pricing, i.e., American prices must always be more than or equal to European prices. When transactions costs of $50 were applied, the numbers of arbitrage opportunities on the under-priced American options fell, but was still 30% for puts (although only 2% for calls).

[58] After accounting for the transaction costs of $50.

American call options improved in pricing over time (except French Franc and Swiss Franc), however mispriced American puts increased over time. Deutsche Mark puts shifted from a "positive" early-exercise premium to a "negative" early-exercise premium[59]. The other put options (except the Japanese Yen) were also relatively under-priced in the second period. Deutsche Mark puts indicated risk-free arbitrage profits of more than $5,000[60] per trading day in the second period.

The analysis indicated that many American puts were priced below European equivalent. This result was so surprising that a data error was suspected, but the Philadelphia Stock Exchange[61] confirmed the data. In general, the Exchange had no explanation on the level of mispriced values. Once again the PHLX options market was found to be far from efficient.

In order to understand further the character of the mispricing, the study then turned (in Chapter 5) to the behaviour of the volatility smile, smile skewness and implied distributions. For these tests, the Deutsche Mark options were selected as they represented 40% of all trades on the PHLX market and also had the largest arbitrage opportunities.

The results showed that a volatility smile existed in the traded Deutsche Mark options on the PHLX, which was skewed. Implied volatilities had an upward skew for moneyness [F/X] greater than 1. The volatility skewness was more obvious for short-term options and during the second period of observation. The volatility skewness had a mean of 1.2%, i.e., the out-of-the-money calls had 1.2% more volatility that out-of-the-money puts (at ±2% of the forward price), a significant difference at the 95% confidence level.

In each observed quarter, the calls and puts had different distributions (except quarter 2 of 1992). On average, the implied distribution had small positive statistical skewness. However, the kurtosis averaged 3, so the distribution was not fat tailed.

[59] The "negative" early-exercise premium means the price of the American option is less than the European option.

[60] Please refer to section 4.5.2

[61] I have spoken to the Vice-President of PHLX's European Office in London. He has verified with his colleagues in Philadelphia that the source data are correct for the sample period. That is, data received after end 1995 are correct. I received the data tape in middle December 1995.

The differences between calls' and puts' implied distributions verified the mispricing of calls relative to puts which was found in the earlier chapters.

In Chapter 6 (also using with the Deutsche Mark options), we decided to test four alternative option-pricing models, taking account of the smile, in order to ascertain whether the smile was useful in forecasting. Tomorrow's option prices with volatility smile were forecast from today's market prices. The sample period covered more than 8 months in 1994. The four alternatively option pricing models used were Stochastic Volatility [Hull and White (1987a)], American [Barone-Adesi and Whaley (1987)], CEV Square-root [Beckers (1980)] and European [Garman and Kohlhagen (1983)] models.

The results showed pricing with volatility smile gave larger errors than assuming a constant at-the-money volatility smile. The adjusted volatility with smile effect (i.e., allowing for time-decay) performed better than a simple volatility smile. The results showed that daily options' volatilities are very unstable and prediction requires constant adjustment to the deterministic function of the volatility. Simple constant volatility (which is easy) provided the most accurate forecast for use in the models to estimate tomorrow's option prices. This result tends to reject the use of complicated implied binomial trees.

The results so far were focused on Philadelphia options. In Chapter 7, we see whether the Over-The-Counter (OTC) options were similar in their behaviour to PHLX options. The OTC data are quotes collected at noon in London. The comparison was based on Deutsche Mark options traded at different locations, OTC in London and PHLX in Philadelphia, however, they were in approximately the same trading time period.

The volatility smiles of OTC and PHLX options were found to be different. The PHLX volatilities were less stable, changing their shape over the observed months. They had wave-shaped surface across the strikes and the months. The OTC smiles were stable in their shape over the observed months, giving a valley-shaped surface across the strike prices and the months. The greater PHLX variation was confirmed by the results from the estimated implied distributions. The PHLX distribution had larger kurtosis and coefficient of variation. The smile skewness also indicated that OTC puts were expensive relative to PHLX puts, while OTC calls were cheap relative to PHLX calls.

The difference in the volatility smiles for the two markets represented a potential arbitrage opportunity. However, after accounting the transactions costs[62], we would need a 4% different in volatility levels to allow an investor to write high volatility options and buy low volatility options in order to earn risk-free arbitrage profits. The average difference in the sample was about 1%, therefore only low-cost market-makers would have been able to arbitrage the difference in volatility levels. It would appear that the OTC and PHLX markets are not well integrated.

[62] Transaction costs is assumed as $50, please refer to section 7.4.7 for more details.

8.2 Suggestions for Further Research

This thesis has provided important new empirical evidence on currency option pricing and it has also raised many new questions. The following areas for further research appear to be fruitful. While we analyse the put-call pricing relationship, the interesting question is why such a high level of mispricing continues over time?

The period of our study is from August 1987 to October 1994. It would be interesting to study more recent data to test the efficiency for the last three years. The early-exercise premium analysis provided stronger evidence of inefficiency in option pricing. Therefore, further analysis in this area is strongly recommended to see if the market has corrected the mispricing, i.e., more data on PHLX and tests in real time.

Volatility is the key element in pricing. However, taking account of the volatility smile did not help to improve the prediction of tomorrow's option prices. It created larger errors than assuming constant at-the-money volatility. This suggests the need for further study in the area of deterministic volatility function. Other areas to be considered may be the diffusion-jump, stochastic volatility process and local volatility surface [c.f. Bates (1996a and 1996b)], i.e., better models.

The implied distribution provides important explanation for the analyses of options mispricing. The earlier analysis of volatility smile has confirmed that volatility is skewed. The analysis of calls' and puts' distributions reflected the pricing errors. This implied distribution methodology can be applied to other derivatives and also incorporated into pricing models, i.e., enhanced option models.

In the area of comparative among markets, more analyses of OTC options with exchange options is necessary. OTC data are not easily available, therefore, it is important to have more traded OTC option prices in order to provide better explanation on the behaviour for both markets, i.e., more studies of OTC versus exchange markets.

Appendix-A: The Garman and Kohlhagen (1983) European currency option pricing model[63].

$$C_{GK} = Se^{-Rt}N(d_1) - Xe^{-rt}N(d_2)$$

$$where \quad d_1 = \frac{\ln\left(F/X\right) + \left(\sigma^2 t/2\right)}{\sigma\sqrt{t}}$$

$$d_2 = d_1 - \sigma\sqrt{t}$$

$$F = Se^{(r-R)t}$$

$$P_{GK} = Se^{-Rt}\left[N(d_1) - 1\right] - Xe^{-rt}\left[N(d_2) - 1\right]$$

[63] See Black and Scholes (1973), Black (1976) and Garman and Kohlhagen (1983) for full detail of the option model.

Appendix-B: The modified version of Beckers (1980) CEV Square-root European option pricing model[64]

$$C_{Sqr} = Se^{-Rt}N\left(q\{4\}\right) - Xe^{-rt}N\left(q\{0\}\right) + A_2\left(S/S_i\right)^{q_2}$$

Let w is 4 or 0, $\sigma = \sigma_1 S^{\theta-1}$, and $\theta = 1/2$

$$\textit{Where}\quad q(w) = \frac{1 + k(k-1)p - \frac{kp^2}{2}(k-1)(2-k)(1-3k) - \left(\frac{z}{w+y}\right)^k}{\sqrt{2k^2 p\left(1-(1-k)(1-3k)p\right)}}$$

$$k(w) = 1 - \frac{2(w+y)(w+3y)}{3(w+2y)^2}$$

$$p(w) = \frac{w+2y}{(w+y)^2}$$

$$y = \frac{4rSe^{-Rt}}{\sigma^2\left(\left|e^{-Rt} - e^{-rt}\right|\right)}$$

$$z = \frac{4rXe^{-rt}}{\sigma^2\left(\left|e^{-rt} - e^{-Rt}\right|\right)}$$

$$P_{Sqr} = Se^{-Rt}\left[N\left(q\{4\}\right) - 1\right] - Xe^{-rt}\left[N\left(q\{0\}\right) - 1\right]$$

[64] See Beckers (1980) for the full detail of the option model.

Appendix-C: The Hull and White (1987a) European stochastic volatility option pricing model[65].

$$C_{SV} = C(\sigma^2) + \frac{C''(\sigma^2)}{2} Var(\sigma^2) + \frac{C'''(\sigma^2)}{6} Skew(\sigma^2)$$

where $C(\sigma^2) = Se^{-Rt}N(d_1) - Xe^{-rt}N(d_2)$

$$C''(\sigma^2) = \frac{S\sqrt{t}\left[n(d_1)(d_1 d_2 - 1)\right]}{4\sigma^3}$$

$$C'''(\sigma^2) = \frac{S\sqrt{t}n(d_1)\left[(d_1 d_2 - 1)(d_1 d_2 - 3) - (d_1^2 + d_2^2)\right]}{8\sigma^5}$$

$$Var(\sigma^2) = \left[\frac{2\sigma^4(e^k - k - 1)}{k^2} - \sigma^4\right]$$

$$Skew(\sigma^2) = \sigma^6\left[\frac{e^{3k} - (9 + 18k)e^k + (8 + 24k + 18k^2 + 6k^3)}{3k^3}\right]$$

$$d_1 = \frac{\ln(S/X) + (r - R + \sigma^2/2)t}{\sigma\sqrt{t}}$$

$$d_2 = d_1 - \sigma\sqrt{t}$$

$$n(d_1) = \frac{e^{-\frac{(d_1)^2}{2}}}{\sqrt{2\pi}}$$

$$k = t$$

$$P_{SV} = C(\sigma^2) + \frac{C''(\sigma^2)}{2!} Var(\sigma^2) + \frac{C'''(\sigma^2)}{6} Skew(\sigma^2) - Se^{-Rt} + Xe^{-rt}$$

[65] See Hull and White (1987a) for the full detail of the option model.

Appendix-D1: The modified version of Barone-Adesi and Whaley (1987) American call option pricing model[66].

$$C_{BW} = C_{GK} + A_2\left(S/S_i\right)^{q_2} \qquad when\ S < S_i$$

$$C_{BW} = S - X \qquad when\ S \geq S_i$$

where $C_{GK} = Se^{-Rt}N(d_1) - Xe^{-rt}N(d_2)$

$$q_2 = \left[(1-N) + \sqrt{(N-1)^2 + \frac{4M}{K}}\right] / 2$$

$$M = 2r/\sigma^2,\ N = 2(r-R)/\sigma^2,\ K = 1 - e^{-rt}$$

$$a_2 = \left[1 - e^{-Rt}N\left(d_{1(S_i)}\right)\right] / Kq_2S_i^{(q_2-1)}$$

$$S_i = X + \left(S_i^c - X\right) * \left(1 - e^{H_2}\right)$$

$$S_i = S_i^c - \left(S_i^c - X\right)e^{H_2}$$

$$H_2 = -\left((r-R)t + 2\sigma\sqrt{t}\right)\left(X / \left(S_i^C - X\right)\right)$$

$$S_i^c = X / \left(1 - \left(1/q_2^i\right)\right)$$

$$q_2^i = \left[(1-N) + \sqrt{(N-1)^2 + 4M}\right] / 2$$

<u>Algorithm of critical spot S_i for Call (an iterative solution)</u>

$$S_i - X = C_{GK} + \left[1 - e^{-Rt}N\left(d_{1(S_i)}\right)\right]S_i / q_2$$

$$Let\ LHS_{S_i}^c = S_i - X, and\ RHS_{S_i}^C = C_{GK} + \left[1 - e^{-Rt}N\left(d_{1(S_i)}\right)\right]S_i / q_2$$

$$RHS_{S_i}^c + b_i^c(S - S_i) = S - X$$

$$b_i^C = \left[e^{-Rt}N\left(d_{1(S_i)}\right)\left(1 - (1/q_2)\right)\right] + \left[1 - \left(e^{-Rt}n\left(d_{1(S_i)}\right) / \sigma\sqrt{t}\right)\right] / q_2$$

$$n\left(d_{1(S_i)}\right)\ is\ normal\ density\ function,\ n\left(d_{1(S_i)}\right) = \frac{1}{\sqrt{2\pi}}e^{\frac{\left(d_{1(S_i)}\right)^2}{2}}$$

$$S_{(i+1)}^c = \left[X + RHS_{S_i}^c - b_i^cS_i\right] / \left(1 - b_i^c\right)$$

$$\frac{\left|LHS_{S_i} - RHS_{S_i}\right|}{X} < 0.00001,\ [Acceptable\ Tolerance\ Level\ for\ Iteration]$$

---

[66] See Barone-Adesi and Whaley (1987) for full detail of the option model.

Appendix-D2: The Modified version of Barone-Adesi and Whaley (1987) American put option pricing model.

$$P_{BW} = P_{GK} + A_1\left(S/S_i\right)^{q_1} \qquad when\ S > S_i$$

$$P_{BW} = X - S \qquad when\ S \le S_i$$

where $P_{GK} = Se^{-Rt}\left[N(d_1)-1\right] - Xe^{-rt}\left[N(d_2)-1\right]$

$$q_1 = \left[(1-N) - \sqrt{(N-1)^2 + \frac{4M}{K}}\right]/2$$

$$M = 2r/\sigma^2,\ N = 2(r-R)/\sigma^2, K = 1-e^{-rt}$$

$$a_1 = \left[e^{-Rt}N\left(-d_{1(S_i)}\right)-1\right]/Kq_1S_i^{(q_1-1)}$$

$$S_i = X + \left(S_i^p - X\right)*\left(1-e^{H_1}\right)$$

$$S_i = S_i^p - \left(S_i^p - X\right)e^{H_1}$$

$$H_1 = \left((r-R)t - 2\sigma\sqrt{t}\right)\left(X/\left(X-S_i^P\right)\right)$$

$$S_i^p = X/\left(1-\left(1/q_1^i\right)\right)$$

$$q_1^i = \left[(1-N) - \sqrt{(N-1)^2 + 4M}\right]/2$$

<u>Algorithm of critical spot S_i for Put (an iterative solution)</u>

$$X - S_i = P_{GK} - \left[1-e^{-Rt}N\left(-d_{1(S_i)}\right)\right]S_i/q_1$$

$$Let\ LHS_{S_i}^p = X - S_i, and\ RHS_{S_i}^P = P_{GK} - \left[1-e^{-Rt}N\left(-d_{1(S_i)}\right)\right]S_i/q_1$$

$$RHS_{S_i}^p - b_i^p(S-S_i) = X - S$$

$$b_i^P = -\left[e^{-Rt}N\left(-d_{1(S_i)}\right)\left(1-(1/q_1)\right)\right] - \left[1+\left(e^{-Rt}n\left(d_{1(S_i)}\right)/\sigma\sqrt{t}\right)\right]/q_1$$

$n\left(d_{1(S_i)}\right)$ *is normal density function,* $n\left(d_{1(S_i)}\right) = \frac{1}{\sqrt{2\pi}}e^{-\frac{\left(d_{1(S_i)}\right)^2}{2}}$

$$S_{(i+1)}^p = \left[X + b_i^pS_i - RHS_{S_i}^p\right]/\left(1+b_i^p\right)$$

$$\frac{\left|LHS_{S_i} - RHS_{S_i}\right|}{X} < 0.00001\text{, [Acceptable Tolerance Level for Iteration]}$$

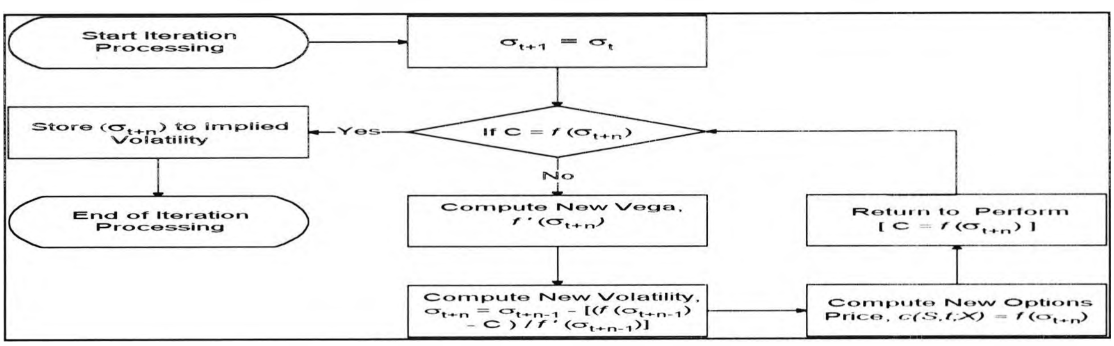
Appendix-E: The Methods to Estimate the Implied Volatility

The implied volatility is imputed from the traded option price. The option pricing model is not invertible, however, the implied volatility is solved-for by trial and error until a given level of accuracy is attained. In general, a quadratic approximation [see Equations (E.1) and (E.2)] is used to provide a close estimation. This quadratic procedure is a modified version of the Corrado and Miller, Jr. (1996a) method, and is based on the European-style option model and its available variables. When the close range is estimated, an iterative process[67] [see Figure E.1] is used. The procedure jumps to the closer implied volatility based on the rate of change of the model error for a volatility estimate. It consistently decreases the model error relative to the market price until the convergence criterion is met. Obviously, the appropriate model to use in the procedure is the model which is consistent with the terms of the option prices.

$$\sigma_{Call} \cong \frac{\left[\frac{\sqrt{2\pi}\left(C - \frac{Se^{-Rt} - Xe^{-rt}}{2}\right)}{Se^{-Rt} + Xe^{-rt}} + \sqrt{2\pi\left(\frac{C - \frac{Se^{-Rt} - Xe^{-rt}}{2}}{Se^{-Rt} + Xe^{-rt}}\right)^2 - \frac{2\left(Se^{-Rt} - Xe^{-rt}\right)\left(\ln\left(\frac{F}{X}\right)\right)}{Se^{-Rt} + Xe^{-rt}}}\right]}{\sqrt{t}} \qquad (E.1)$$

$$\sigma_{Put} \cong \frac{\left[\frac{\sqrt{2\pi}\left(P + \frac{Se^{-Rt} - Xe^{-rt}}{2}\right)}{Se^{-Rt} + Xe^{-rt}} + \sqrt{2\pi\left(\frac{P + \frac{Se^{-Rt} - Xe^{-rt}}{2}}{Se^{-Rt} + Xe^{-rt}}\right)^2 - \frac{2\left(Se^{-Rt} - Xe^{-rt}\right)\left(\ln\left(\frac{F}{X}\right)\right)}{Se^{-Rt} + Xe^{-rt}}}\right]}{\sqrt{t}} \qquad (E.2)$$

Figure E.1 - Iterative Process for Implied Volatility



---

[67]The iteration method is to find a root of the equation C - $f(\sigma)$. The process guarantees that only one such root exists for σ, where $\sigma_{t+1} = \sigma_t - [\,f(\sigma_t) - C\,] / f'(\sigma_t)$. The Vega is equal to $\delta f(\sigma) / \delta\sigma$, and $f'(\sigma) = \delta f(\sigma) / \delta\sigma$, therefore, it is applied to the formula. The equation of Vega is as follow.

$$\frac{\partial C}{\partial \sigma} = Xe^{-rt}\sqrt{t}\left[n(d_2)\right] > 0,\ n(d_2) = \frac{1}{\sqrt{2\pi}}e^{-\frac{d_2^2}{2}} \text{ and } d_2 = \frac{\ln\left(\frac{F}{X}\right) - \frac{\sigma^2 t}{2}}{\sigma\sqrt{t}}$$

Bibliography:

Abken, P. A., and Nandi, S., (1996), "Options and Volatility", Federal Reserve Bank of Atlanta, Economic Review, December 1996, Vol. 81, No 3-6, pp 21-35

Adams, P. D., and Wyatt, S. B., (1987a), "On the Pricing of European and American Foreign Currency Call Options", Journal of International Money and Finance, Vol. 6, pp 315-338

Adams, P. D., and Wyatt, S. B., (1987b), "Biases in Option Prices: Evidence from the Foreign Currency Option Market", Journal of Banking and Finance, Vol. 11, pp 549-562

Adams, P. D., and Wyatt, S. B., (1989), "On the Pricing of European and American Foreign Currency Call Options: A Clarification", Journal of International Money and Finance, Vol. 8, pp 305-311

Amin, K. I., and Ng, V., (1993), "Option Valuation with Systematic Stochastic Volatility", Journal of Finance, Vol. 48, pp 881-910

Bahra, B., (1996), "Probability Distributions of Future Asset Prices Implied by Option Prices", Bank of England Quarterly Bulletin, Vol. 36, No 3, pp 299-311

Barone-Adesi, G., and Whaley, R. E., (1987), "Efficient Analytic Approximations of American Option Values", Journal of Finance, Vol. 42, pp 301-320

Bates, D. S., (1991), "The Crash of '87: Was It Expected? The Evidence from Options Markets", Journal of Finance, Vol. 46, pp 1009-1044

Bates, D. S., (1996a), "Dollar Jump Fears, 1984-1992: Distributional Abnormalities Implicit in Currency Futures Options", Journal of International Money and Finance, Vol. 15, pp 65-93

Bates, D. S., (1996b), "Jumps and Stochastic Volatility: Exchange Rate Process Implicit in Deutsche Mark Options", The Review of Financial Studies, Vol. 9, pp 69-107

Beckers, S., (1980), "The Constant Elasticity of Variance Model and its Implications for Option Pricing", Journal of Finance, Vol. 35, pp 661-674

Bernnan, M., (1979), "The Pricing of Contingent Claims in Discrete-time Models", Journal of Finance, Vol. 34, pp 53-68

Bharadia, M. A. J., Christofides, N., and Salkin, G. R., (1996), "A Quadratic Method for the Calculation of Implied Volatility Using the Garman-Kohlhagen Model", Financial Analysts Journal, Vol. 52, No 2, pp 61-64

Biger, N., and Hull, J., (1983), "The Valuation of Currency Options", Journal of the Financial Management Association, Vol. 12, pp 24-28

Black, F., (1975), "Fact and Fantasy In the Use of Options", Financial Analysts Journal, Vol. 31, No 4, pp 36-41 & 61-72

Black, F., (1976), "The Pricing of Commodity Contracts", Journal of Financial Economics, Vol. 3, pp 167-179

Black, F., and Scholes, M., (1972), "The Valuation of Option Contracts and a Test of Market Efficiency", Journal of Finance, Vol. 27, pp 399-418

Black, F., and Scholes, M., (1973), "The Pricing of Options and Corporate Liabilities", Journal of Political Economy, Vol. 81, pp 637-654

Bodurtha, J. N. Jr., and Courtadon, G. R., (1986), "Efficiency Tests of the Foreign Currency Option Market", Journal of Finance, Vol. 41, pp 151-62

Bodurtha, J. N. Jr., and Courtadon, G. R., (1987a), "Tests of American Option Pricing Model on the Foreign Currency Options Market", Journal of Financial and Quantitative Analysis, Vol. 22, pp 153-167

Bodurtha, J. N. Jr., and Courtadon, G. R., (1987b), "The Pricing of Foreign Currency Options", Monograph Series in Finance and Economics, No 1987-4/5, New York University Press

Boness, A. J., (1964), "Some Evidence on the Profitability of Trading in Put and Call Options", in: "The Random Character of Stock Market Prices", (Ed) by **Cootner, P. H.**, (Cambridge, MIT Press, pp 475-496)

Borensztein, E. R., and Dooley, M. P., (1987), "Options on Foreign Exchange and Exchange Rate Expectations", IMF Staff Paper, Vol. 34, pp 643-680

Breeden, D. T., and Litzenberger, R. H., (1987), "Prices of State-contingent Claims Implicit in Options Prices", Journal of Business, Vol. 51, pp 621-651

Brenner, M. G., and Subrahmanyam, M. G., (1988), "A Simple Formula to Compute the Implied Standard Deviation", Financial Analysts Journal, Vol. 44, No 5, pp 80-83

Briys, E., and Coruhy, M., (1988), "Creating and Pricing Hybrid Foreign Currency Options", Financial Management, Vol. 17, No 4, pp 59-65

Butler, J. S., and Schachter, B., (1986), "Unbiased Estimation of the Black/Scholes Formula", Journal of Financial Economics, Vol. 15, pp 341-357

Canina, L., and Figlewski, S., (1993), "The Informational Content of Implied Volatility", The Review of Financial Studies, Vol. 6, pp 659-681

Chance, D. M., (1987), "Parity Tests of Index Options", Advances in Futures and Options Research, Vol. 2, pp 47-64

Chance, D. M., (1993), "Leap Into The Unknown", RISK, Vol. 6, No 5, pp 60-66

Choi, J. J., and Hauser, S., (1990), "The Effects of Domestic and Foreign Yield Curves on the Value of Currency American Call Options", Journal of Banking and Finance, Vol. 14, pp 41-53

Chiras, D. P., and Manaster, S., (1978), "The Information Content of Option Prices and a Test of Market Efficiency", Journal of Financial Economics, Vol. 6, pp 213-234

Chriss, N., (1996), "TransAtlantic Trees", RISK, Vol. 9, No 7, pp 45-48

Corrado, C. J., and Miller, Jr., T. W., (1996a), "A Note on a Simple, Accurate Formula to Compute Implied Standard Deviations", Journal of Banking & Finance, Vol. 20, pp 595-603

Corrado, C. J., and Miller, Jr. T. W., (1996b), "Efficient Option-Implied Volatility Estimators", Journal of Futures Markets, Vol. 16, pp 247-272

Corrado, C. J., and Miller, Jr. T. W., (1996c), "Volatility without Tears", RISK, Vol. 9, No 7, pp 49-52

Corrado, C. J., and Su, T., (1997), "Implied Volatility Skews and Stock Index Skewness and Kurtosis Implied by S&P 500 Index Option Prices", Journal of Derivatives, Vol. 4, No 4, pp 8-19

Cox, J. C., Ingersoll, Jr. F. E., and Ross, S. A., (1985), "A Theory of the Term Structure of Interest Rates", Econometrica, Vol. 53, pp 385-407

Cox, J. C., and Ross, S. A., (1976), "The Valuation of Options for Alternative Stochastic Processes", Journal of Financial Economics, Vol. 3, pp 145-166

Cox, J. C., Ross, S. A., and Rubinstein, M., (1979), "Option Pricing: A Simplified Approach", Journal of Financial Economics, Vol. 7, pp 229-263

Derman, E., and Kani, I., (1994), "Riding On A Smile", RISK, Vol. 7, No 2, pp 32-39

Derman, E., Kani, I., and Zou, J. Z., (1996), "The Local Volatility Surface: Unlocking the Information in Index Option Prices", Financial Analysts Journal, Vol. 52, No 4, pp 25-36

Dumas, B., Fleming, J., and Whaley, R. E., (1996), "Implied Volatility Functions: Empirical Tests", HEC School of Business Working Paper, February 1996

Dupire, B., (1993), "Model Art", RISK, Vol. 6, No 9, pp 118-124

Dupire, B., (1994), "Pricing With A Smile", RISK, Vol. 7, No 1, pp 18-20

Eisenberg, L., (1993), "One Step Beyond", RISK, Vol. 6, No 11, pp 60-68

Engle, R., and Mezrich, J., (1995), "Grappling With Garch", RISK, Vol. 8, No 9, pp 112-117

Fabozzi, F. J., Hauser, S., and Yaari, U., (1990), "Early Exercise of Foreign Currency Options: Determinants of American Premium and the Critical Exchange Rate", Advance in Futures and Options Research, Vol. 4, pp 219-236

Feiger, G., and Jacquillat, B., (1979), "Currency Option Bonds, Puts, and Calls on Spot Exchange and the Hedging of Contingent Foreign Earnings", Journal of Finance, Vol. 34, pp 1129-1139

Feinstein, S. P., (1988), "A Source of Unbiased Implied Volatility Forecasts", Federal Reserve Bank of Atlanta, Working Paper 88-9, December 1988

Figlewshi, S., (1989a), "Current Issues: Options - What Does an Option Pricing Model Tell us about Option Prices?", Financial Analysts Journal, Vol. 45, No 5, pp 12-15

Figlewshi, S., (1989b), "Options Arbitrage in Imperfect Markets", Journal of Finance, Vol. 44, pp 1289-1311

Finucane, T. J., (1989), "A Simple Weighting Scheme for Black-Scholes Implied Volatilities", Journal of Banking and Finance, Vol. 13, pp 321-326

Gadkari, V., (1986), "Relative Pricing of Currency Options: A Tutorial", Advance in Futures and Options Research, Vol. 1, Part A, pp 227-245

Garcia, P., Sherricj, B. J., and Tirupattur, V., (1996), "Recovering Probabilistic Information from Option Markets: Tests of Distributional Assumptions", Journal of Futures Markets, Vol. 16, pp 545-560

Garman, M. B., and Kohlhagen, S. W., (1983), "Foreign Currency Option Values", Journal of International Money and Finance, Vol. 2, pp 231-237

Gemmill, G., (1986), "A Primer on the Pricing of Options on Currencies and Short Term Interest Rates", The Investment Analyst, Vol. 81, July 1986, pp 16-22

Gemmill, G., (1996), "Did Option Traders Anticipate the Crash? Evidence from Volatility Smiles in the UK with US Comparisons", Journal of Futures Markets, Vol. 16, pp 881-897

Gemmill, G., and Thomas, D., (1996), "Warrants on the London's Stock Exchange: Pricing Biases and Investor Confusion", Seminar Paper in City University Business School, July 1996

Gendreau, B., (1984), "New Market in Foreign Currency Options", Federal Reserve Bank of Philadelphia, Business Review, July/August 1984, pp 3-12

Giddy, I. H., (1983a), "Foreign Exchange Options", Journal of Futures Markets, Vol. 3, pp 143-166

Giddy, I. H., (1983b), "The Foreign Exchange Option as a Hedging Tool", Midland Corporate Financial Journal, Vol. 1, No 3, Fall 1983, pp 32-42

Goodman, L. S., Ross, S., and Schmidt, F., (1985), "Are Foreign Currency Options Overvalued? The Early Experience of the Philadelphia Stock Exchange", Journal of Futures Markets, Vol. 5, pp 349-359

Gould, J. P., and Galai, D., (1974), "Transaction Costs and the Relationship between Put and Call Prices", Journal of Financial Economics, Vol. 1, pp 105-129

Grabbe, J. O., (1983), "The Pricing of Call and Put Options on Foreign Exchange", Journal of International Money and Finance, Vol. 2, pp 239-253

Guo, D., (1996a), "The Information Content of Implied Stochastic Volatility from Currency Options", Canadian Journal of Economics, Vol. 29, Special Issue 2, April 1996, pp S559-S561

Guo, D., (1996b), "The Predictive Power of Implied Stochastic Variance from Currency Options", Journal of Futures Markets, Vol. 16, pp 915-942

Hauser, S., and Lauterbach, B., (1997), "The Relative Performance of Five Alternative Warrant Pricing Models", Financial Analysts Journal, Vol. 53, No 1, pp 55-61

Heaton, H., (1986), "Volatilities Implied by Options Premia: A Test of Market Efficiency", The Financial Review, Vol. 21, pp 37-49

Heston, S. L., (1993), "A Closed-Form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options", The Review of Financial Studies, Vol. 6, pp 327-343

Heynen, R. C., (1994), "An Empirical Investigation of Observed Smile Patterns", The Review of Futures Markets, Vol. 13, pp 317-353

Hicks, A., (1990), "OTC Options Go Every Which Way", Foreign Exchange Special 1990, pp 11-13

Hicks, A., (1992), "Volatility Smiles, but Beware the Teeth!", Foreign Exchange Special 1992, pp 11-12

Hilliard, J. E., and Tucker, A. L., (1991), "Market-Determined Premia for American Currency Spot Options", Advances in Futures and Options Research, Vol. 5, pp 227-240

Hopper, G. P., (1995), "A Primer on Currency Derivatives", Federal Reserve Bank of Philadelphia, Business Review, May/June 1995, pp 3-14

Hull, J., and White, A., (1987a), "The Pricing of Options on Assets with Stochastic Volatilities", Journal of Finance, Vol. 42, pp 281-300

Hull, J., and White, A., (1987b), "Hedging the Risks from Writing Foreign Currency Options", Journal of International Money and Finance, Vol. 6, pp 131-152

Jackwerth, J. C., (1997), "Recovering Risk Aversion from Option Prices and Realised Returns", Seminar Paper, Institute of Finance and Accounting Seminar, LSE - February 14 1997 & University of California at Berkeley - August 29 1996

Jackwerth, J. C., and Rubinstein, M., (1996), "Recovering Probability Distributions from Option Prices", Journal of Finance, Vol. 51, pp 1611-1631

Jacquier, E., and Jarrow, R., (1995), "Vital Statistic", RISK, Vol. 8, No 4, pp 62-64

Jarrow, R. A., and Rudd, A., (1982), "Approximate Option Valuation for Arbitrary Stochastic Processes", Journal of Financial Economics, Vol. 10, pp 347-369

Johnson, L. J., (1986), "Foreign-Currency Options, Ex Ante Exchange-Rate Volatility, and Market Efficiency: An Empirical Test", The Financial Review, Vol. 21, pp 433-450

Jorion, P., (1995), "Predicting Volatility in the Foreign Exchange Market", Journal of Finance, Vol. 50, pp 507-528

Jorion, P., and Stoughton, N. M., (1989), "An Empirical Investigation of the Early Exercise Premium of Foreign Currency Options", The Journal of Futures Markets, Vol 9, pp 365-375

Kamiyama, N., (1997), "Stability of Implied Volatility Functions: A Test on the Nikkei Options", memo, City University Business School

Kritzman, M. P., (1991), "What Practitioners Need to Know ... About Estimating Volatility - Part 1", Financial Analysts Journal, Vol. 47, No 4, pp 22-25

Koehler, G., and Manaster, S., (1982), "The Calculation of Implied Variances from the Black-Scholes Model: A Note", Journal of Finance, Vol. 37, No 1, pp 227-230

Lai, T. S., Lee, C. F., and Tucker, A. L., (1992), "An Alternative Method for Obtaining the Implied Standard Deviation", Journal of Financial Engineering, Vol. 1, pp 369-375

Latané, H. A., and Rendleman, Jr. R. J., (1976), "Standard Deviations of Stock Price Ratios Implied in Option Prices", Journal of Finance, Vol. 31, pp 369-381

MacBeth, J. D., and Merville, L. J., (1979), "An Empirical Examination of the Black-Scholes Call Option Pricing Model", Journal of Finance, Vol. 34, pp 1173-1186

MacMillan, L. W., (1986), "Analytic Approximation for the American Put Option", Advances in Futures and Options Research, Vol. 1, Part A, pp 119-139

Malz, A. M., (1996), "Using Option Prices to Estimate Realignment Probabilities in the European Monetary System: the Case of Sterling-Mark", Journal of International Money and Finance, Vol. 15, pp 717-748

Mayhew, S., (1995), "Implied Volatility", Financial Analysts Journal, Vol. 51, No 4, pp 8-20

McCauley, R., and Melick, W., (1996), "Risk Reversal Risk", RISK, Vol. 9, No 11, pp 54-57

Melick, W. R., and Thomas, C. P., (1997), "Recovering an Asset's Implied PDF from Option Prices: An Application to Oil Prices During the Gulf Crisis", Journal of Financial and Quantitative Analysis, Vol. 32, pp 91-115

Melino, A., and Turnbull, S. M., (1990), "Pricing Foreign Currency Options with Stochastic Volatility", Journal of Econometrics, Vol. 45, pp 239-265

Melino, A., and Turnbull, S. M., (1995), "Misspecification and the Pricing and Hedging of Long-Term Foreign Currency Options", Journal of International Money and Finance, Vol. 14, pp 373-393

Merton, R. C., (1973a), "Theory of Rational Option Pricing", Bell Journal of Economics and Management Science, Vol. 41, pp 141-183

Merton, R. C., (1973b), "The Relationship Between Put and Call Option Prices: Comment", Journal of Finance, Vol. 38, pp 183-184

Mizrach, B., (1996), "Did Option Prices Predict the ERM Crises?", Department of Economics, Rutgers University, Working Paper No: 96-10, April 1996

Murphy, G., (1994), "When Options Price Theory Meets the Volatility Smile", Euromoney, March 1994, pp 66-74

Nusbaum, D., (1996), "Futures Shock", RISK, Vol. 9, No 11, pp 16-20

Ogden, J. P., and Tucker, A. L., (1987), "Empirical Tests of the Efficiency of the Currency Futures Options Market", Journal of Futures Markets, Vol. 7, pp 695-703

Parsley, M., (1995), "Playing it Safe with Currency Options", Euromoney, December, pp 104-106

Pasmantier, A. B., (1992), "Currency Options: From Inception to Present", Review of Business, Vol. 13, No 4, pp 43-45

Paxson, D. A., (1994), "Wipe the Smile Off - Implied Volatilities", Fourth Annual Intensive and Advanced Seminar on Volatility - London, October-10-1994

Phillips, S. M., and Smith, Jr. C. W., (1980), "Trading Cost for Listed Options: The Implications for Market Efficiency", Journal of Financial Economics, Vol. 8, pp 179-201

Pinkava, P., (1994), "A New Way to Look at Options - Implied Probability Distributions", LIFFE Money Market Review, 1st Quarter 1994, pp 1-4

Roger, C., (1997), "One for All", RISK, Vol. 10, No 3, pp 5X

Ronn, A. G., and Ronn, E. I., (1989), "The Box Spread Arbitrage Conditions: Theory, Tests, and Investment Strategies", The Review of Financial Studies, Vol. 2, pp 91-108

Rubinstein, M., (1976), "The Valuation of Uncertain Income Steams and the Pricing of Options", Bell Journal of Economics and Management Science, Vol. 7, pp 407-425

Rubinstein, M., (1994), "Implied Binomial Trees", Journal of Finance, Vol. 49, pp 771-818

Shastri, K., and Tandon, K., (1985), "Arbitrage Tests of the Efficiency of the Foreign Currency Options Market", Journal of International Money and Finance, Vol. 4, pp 455-468

Shastri, K., and Tandon, K., (1986a), "On the Use of European Models to Price American Options on Foreign Currency", Journal of Futures Markets, Vol. 6, pp 93-108

Shastri, K., and Tandon, K., (1986b), "The Valuation of Foreign Currency Options: Some Empirical Tests", Journal of Financial and Quantitative Analysis, Vol. 21, pp 145-160

Shastri, K., and Tandon, K., (1986c), "An Empirical test of a Valuation Model for American Options on Futures Contracts", Journal of Financial and Quantitative Analysis, Vol. 21, pp 377-392

Shastri, K., and Wethgavivorn, K., (1987), "Valuation of Currency Options for Alternate Stochastic Processes", Journal of Financial Research, Vol. 10, pp 283-293

Shimko, D. C., (1993), "Bounds of Probability", RISK, Vol. 6, No 4, pp 33-37

Shimko, D. C., (1994), "A Tail of Two Distributions", RISK, Vol. 7, No 9, pp 123-130

Stein, E. M., and Stein, J. C., (1991), "Stock Price Distributions with Stochastic Volatility: An Analytic Approach", The Review of Financial Studies, Vol. 4, pp 727-752

Stoll, H. R., (1969), "The Relationship Between Put and Call Option Prices", Journal of Finance, Vol. 24, pp 801-824

Stoll, H. R., (1973), "The Relationship Between Put and Call Option Prices: Reply", Journal of Finance, Vol. 29, pp 185-187

Stoll, H. R., and Whaley, R. E., (1986), "New Options Instruments: Arbitrageable Linkages and Valuation", Advances in Futures and Options Research, Vol. 1, Part A, pp 25-62

Stulz, R. M., (1982), "Options on the Minimum or Maximum of Two Risky Assets", Journal of Financial Economics, Vol. 10, pp 161-185

Taylor, S. J., and Xu, X., (1994a), "The Magnitude of Implied Volatility Smiles: Theory and Empirical Evidence for Exchange Rates", The Review of Futures Markets, Vol. 13, pp 355-380

Taylor, S. J., and Xu, X., (1994b), "The Term Structure of Volatility Implied by Foreign Exchange Options", Journal of Financial and Quantitative Analysis, Vol. 29, pp 57-74

Taylor, S. J., and Xu X., (1994c), "Implied Volatility Shapes when Price and Volatility Shocks are Correlated", University of Lancaster, Working Paper, August 1994

Taylor, S. J., and Xu, X., (1995), "Conditional Volatility and the Informational Efficiency of the PHLX Currency Options Market", Journal of Banking and Finance, Vol. 19, No 4, Coming Edition

Taylor, S. J., and Xu, X., (1996), "The Incremental Volatility Information in One Million Foreign Exchange Quotations", Financial Seminar Paper in City Business School February 28 1996, (Second Draft, June 1995).

Thanassoulas, C., (1992), "OTC: Derivatives for the Blooming Generation", Foreign Exchange Special 1992, pp 25-26

Whaley, R. E., (1982), "Valuation of American Call Options on Dividend-Paying Stocks: Empirical Test", Journal of Financial Economics, Vol. 10, pp 29-58

Whaley, R. E., (1986), "Valuation of American Futures Options: Theory and Empirical Test", Journal of Finance, Vol. 41, pp 127-150

Yang, H. C., (1985), "A Note on Currency Option Pricing Models", Journal of Business Finance & Accounting, Vol. 12, pp 429-438