Optimising Investment Decisions under Uncertainty: A Study of Risk, Subsidies, Competition, and Technological Learning

by

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Declaration

I, Zixuan Zhang, hereby declare that the work presented in this thesis is entirely my own. This thesis is submitted as a partial fulfillment of the requirements for the award of the Doctor of Philosophy degree in Actuarial Science and Insurance at the Bayes Business School, and has not been previously submitted for any other degree or qualification at this or any other university.

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Abstract

The primary objective of this thesis is to offer effective methods for enhancing investment decision-making under escalating market uncertainty and the deregulation of numerous industries. Specifically, the first part of the thesis delves into the valuation and optimal planning of a multi-stage project, while the subsequent part addresses the strategic interaction between private firms and the Government under uncertainty. Various aspects including risk management, Government support, duopolistic competition, technological learning and subsidy retraction are thoroughly considered.

In this thesis, we begin by taking the perspective of a private firm interested in the sequential capacity expansion of a project and develop a framework for assessing the downside risk of the serial project and optimising the sequence of the stages. Under general distributional assumptions for the duration of each stage, we consider the trade-off between maximising the expected NPV and minimising the risk exposure, and obtain the optimal schedule for risk-averse decision-makers. Results show that both the duration variability of each stage and the decision-maker's risk preferences can significantly affect the optimal sequence of the stages and that high duration variability is not always undesirable, even for risk-averse decision-makers.

Subsequently, we bridge the gap between optimal subsidisation policy-making and duopolistic competition by constructing a bi-level real options framework for analysing the non-cooperative game between a Government and two symmetric firms under uncertainty and subsidy. We derive and compare the optimal investment and subsidisation strategies for the case of a profit and social welfare-maximising Government, and provide policy and managerial insights based on analytical and numerical results. Our results indicate that both the market structure and the type of duopolistic competition can have a significant impact on the equilibrium subsidisation and capacity investment policy. In addition, we show that a profit (welfare)-maximising Government does not offer (offers) a subsidy in a highly uncertain environment or when the tax rate is low, while a higher tax rate does not always decelerate investment.

Meanwhile, although traditional literature indicates that Governments tend to withdraw subsidies as the cost of alternative energy technologies approaches commercial maturity due to the learning effect, models for analysing the impacts of technological learning on capacity investment and optimal subsidy retraction remain underdeveloped. Therefore, we extend our model to account for the trade-off that although a higher learning rate enhances cost reduction and incentivises greater investment, it also triggers earlier subsidy retraction. Indeed, our results confirm that the appearance of technological learning and subsidy retraction may result in an ambiguous effect on a firm's investment capacity.

Chapter 1

Introduction

1.1 Topic of the thesis

Effective decision-making is a fundamental cornerstone of every business operation and investment strategy, with the overarching goal of maximising returns while managing risks and uncertainties. The need to incorporate uncertainty into decision-making processes prior to the 1980s was not particularly pronounced, as most industries were under state regulation. Nevertheless, the deregulation of various industries during that period exposed businesses to different forms of uncertainty, thus highlighting the significance of expanding conventional capital budgeting methods, such as the net present value (NPV) rule. More specifically, the NPV rule is a fundamental tool for measuring project performance that involves discounting future cash flows to their present value, and plays an important role in facilitating not only investment decisions and risk assessment but also decisions about scheduling of complex projects under uncertainty.

Examples of earlier literature on the deterministic project scheduling problem (with the NPV maximisation objective) include Russell (1970), Elmaghraby and Herroelen (1990) and Demeulemeester et al. (1996), where all relevant problem data is assumed known from the outset. More recently, Herroelen and Leus (2005) and Wiesemann and Kuhn (2015) highlight that the uncertainty inherent in the duration and cash flows of a project should be accounted for, explicitly. The former describe the stochastic project scheduling problem as a multi-stage decision process, in which the uncertainty in project activities is considered in order to prevent schedule disruptions. By introducing uncertainty into the duration of project completion time, costs, and revenues, Sobel et al. (2009) put forth a comprehensive reformulation of the stochastic NPV maximisation problem, where they also introduce an algorithm designed to identify an optimal adaptive strategy for stochastic project scheduling. More recent contributions in this stream of research have expanded upon this work, developing and refining various scheduling approaches and algorithms within a stochastic environment (Wiesemann et al., 2010; Leyman and Vanhoucke, 2017; Zheng et al., 2017).

While NPV-oriented models for stochastic project scheduling primarily emphasise the financial aspects of optimisation problems, they often make the assumption of a risk-neutral decision-maker and neglect risk considerations. However, in situations with extreme values of duration and cash flow distributions, relying solely on the expected NPV and disregarding attitudes towards risk may lead to less accurate decision-making (Blau et al., 2000; Browning, 2014; Chao et al., 2014; Rezaei et al., 2020). Examples of literature on the stochastic project scheduling problem that account for risk preferences include Ke and Liu (2005), Beraldi et al. (2012) and Zhao et al. (2016), who control the probability of the occurrence of undesirable investment outcomes using the chance-constrained method. De Reyck and Leus (2008) and Creemers et al. (2015) study a stochastic NPV maximisation problem when project activities carry a risk of failure, such that an activity's failure leads to overall project termination. Furthermore, the concept of Value at Risk (VaR) is devised to provide a more pragmatic assessment of the maximum potential loss tied to an investment at a specified probability level. Additionally, a coherent risk measure known as the Conditional Value at Risk (CVaR) is put forth to quantify the losses that may occur beyond the VaR threshold, as discussed in Rockafellar et al. (2000). Based on these risk measures, the trade-off between risk minimisation and profit maximisation is commonly involved in mean-risk approaches. Examples include Colvin and Maravelias (2011), Alonso-Ayuso et al. (2014), Huang et al. (2016) and Rezaei et al. (2020).

While the NPV rule and risk measures provide a valuable framework for assessing investments and planning multi-stage projects, they often fall short when faced with the price uncertainty and operational flexibility inherent in real-world investment projects. In contrast, real options theory introduces a solution that grants decision-makers the necessary flexibility to adapt and modify their strategies based on future events and new information. Real options pertain to strategic opportunities embedded within investment projects, enabling decision-makers to adjust operations, switch technologies, delay or abandon projects, or capitalise on favorable market conditions. This adaptable approach empowers firms to seize value-enhancing opportunities while mitigating risks, ultimately leading to improved investment outcomes.

The seminal work of McDonald and Siegel (1986), Dixit and Pindyck (1994) and Trigeorgis (1996) has spawned a substantial literature in the area of investment under uncertainty. A strand of this literature places special attention on exploring the implications of various support schemes designed to encourage investment across multiple industries. Examples of policy-oriented real options models include Boomsma et al. (2012), Boomsma and Linnerud (2015) and Ritzenhofen et al. (2016), where the impact of subsidies and tax cuts on a firm's investment incentive are examined. In particular, Bigerna et al. (2019) show that greater subsidy induces earlier investment with a smaller capacity size, implying that the Government can not achieve an environmental goal by simply providing more (less) subsidy to the firm since this could result in insufficient (delayed) investment. In the same line of work, Azevedo et al. (2021) confirm that a higher subsidy or a lower tax rate accelerates investments. Nonetheless, they indicate that the effect of subsidies on the investment capacity relies on whether the subsidy is fixed or variable: the former leads to smaller investments, whereas the latter promotes larger investments.

Although this stream of literature offers valuable insights on how Government support affects private investment, the optimal investment and subsidisation decisions are often determined *ex-post*. Consequently, these decisions may not accurately capture the equilibrium resulting from the strategic interaction between a firm's and a Government's optimisation goals. Such strategic interactions are often explored through the utilisation of bi-level real options models, which serve the purpose of comprehending the investment decisions of private firms and the optimal subsidisation strategies employed by Governments under uncertainty (Pennings, 2000; Yu et al., 2007). For instance, Pennings (2000) considers a zero-expected cost policy, where the Government can lower the firm's investment threshold by providing a lump-sum subsidy, while simultaneously imposing taxes on the firm's revenue equal to the amount of the subsidy. More recently, Lukas and Thiergart (2019) investigate the impact of uncertainty and Government support on the firm's optimal capacity investment and indicate a non-monotonic relationship between the equilibrium subsidy and price uncertainty when the Government seeks to maximise its own profit. Furthermore, maximising the social welfare is also recognised as a common objective for Governments, as discussed in Pawlina and Kort (2006) and Yang et al. (2018).

Nevertheless, the aforementioned bi-level real options models examining the optimal capacity investment and subsidisation policies under uncertainty often overlook the strategic interactions at the firm level, leaving important research questions unaddressed. While these models primarily focus on the strategic interactions between a Government and a private firm, recent (static) game-theoretic models, argue that market structure can play a significant role in shaping subsidy design and private firms' investment incentives (Nie et al., 2016; Wang and Zhou, 2020; Yang et al., 2021). Indeed, it is crucial for the Government to take firm-level strategic interactions into account, since competition tends to reduce the value of a subsidy, and, therefore, alter a firm's investment policy. In addition, the Government's decision regarding which firm to subsidise and the positioning of the subsidised firms can also yield different implications on firms' investment strategies (Yang and Nie, 2015; Nie et al., 2016).

Additional gaps in the optimal investment and subsidisation decision-making encompass considerations related to the learning effect and the possibility of gradually phasing out or retracting the subsidy. This is particularly relevant in the energy sector, where technological learning reduces the costs of alternative energy technology (AET) towards commercial maturity, at which point market forces take over and no further subsidy is required. Therefore, technological learning is among the key determinants of the intensity of Government support. However, existing real options models for analysing the correlation between technological learning and subsidy retraction, as well as their joint impacts on the decision-making of the firm and Government remain underdeveloped.

Hence, within this thesis, we aim to enhance the decision-making not only for investors but also for the Government. Firstly, we introduce a continuous-time framework that enables the derivation of probability distribution and risk measures of the NPV of a serial project under economic and technological uncertainty. The optimal sequence of stages for investors with different risk appetites is obtained. Subsequently, we develop a bi-level real options framework for analysing the non-cooperative game between a Government and two symmetric firms under uncertainty. We obtain the equilibrium investment threshold, capacity and subsidy level by taking into consideration the objectives of both firms and the Government. Lastly, our study extends to the joint impact of subsidy retraction and technological learning on the firm's capacity investment, from which we derive the optimal subsidy retraction decision of the Government.

1.2 Outline and contributions of the thesis

Having covered the core concepts of this thesis and the pertinent literature in the previous section, the following is an outline of the thesis chapters, along with a summary of the main contributions.

Chapter 2 presents a model for risk assessment and optimal scheduling of serial projects for risk-averse decision-makers. Due to rising market uncertainty and the deregulation of many industries, the valuation and planning of complex projects has become increasingly challenging, and has also raised the necessity for efficient risk management. While the traditional literature on project scheduling focuses on maximising the NPV or minimising the makespan of a project under risk-neutrality, the implications of attitudes towards risk remain an important open research direction.

Therefore, we take the perspective of a private firm interested in the sequential capacity expansion of a project and develop a framework for measuring the downside risk of the serial project and optimising the sequence of the stages. Under general distributional assumptions for the duration of each stage, we present an accurate representation of the project's NPV based on a Pearson curve fit, leading to closed-form expressions for the associated risk measures. We then assess the impact of duration variability on the VaR and demonstrate its role in stochastic project scheduling. We also account for the trade-off between maximising the expected NPV and minimising the risk exposure, and obtain the optimal schedule for risk-averse decision-makers. We demonstrate that both the duration variability of each stage and the decision-makers' risk preferences can significantly affect the optimal sequence of the stages and that high duration variability is not always undesirable, even for risk-averse decision-makers.

Chapter 3 constructs a strategic game between two firms and a Government, and investigates the optimal subsidy design and capacity investment under competition and uncertainty within a real options framework. Specifically, we develop a bi-level real options framework for deriving the equilibrium Government subsidisation and firm-level capacity investment policy in a duopoly market structure. This is motivated by pressing sustainability concerns that emphasise the need to meet timely ambitious targets that require green investment at unprecedented levels. In this context, Governments must support private firms to achieve the necessary investment intensity, while relying on them to tackle financial limitations and technology transfer. The interaction of firm and Government policy-making is often analysed in the real options literature, yet existing models do not extend beyond the case of monopoly or perfect competition to allow for strategic interactions at the firm level.

We find that strategic interactions with the Government may impact a firm's capacity investment decision significantly and that the equilibrium subsidisation policy depends crucially on both the market structure and the type of duopolistic competition. Interestingly, we also find that provision of a greater subsidy to the leader raises the follower's incentive to invest earlier and in a bigger project. The loss in value of the leader, due to the follower's entry, relative to the monopolist increases with economic uncertainty and, although a subsidy can mitigate this loss, its effect becomes less pronounced as economic uncertainty increases. The results also suggest that a profit(welfare)-maximising Government does not offer (offers) a subsidy in a highly uncertain environment or when the tax rate is low, while a higher tax rate does not always decelerate investment. Finally, we find that while competition is always desirable for a social planner, a profit-maximising Government may benefit more under pre-emptive competition.

In the light of Chapter 3, Chapter 4 investigates the impacts of technological learning and subsidy retraction on the equilibrium investment strategy of a firm. Indeed, while Governments worldwide have deployed a wide range of support schemes to incentivise investment in alternative energy technologies, the increasing cumulative energy production accelerates technological learning and drives down their costs towards commercial maturity. The maturity threshold, also referred to as *grid parity*, for a AET is reached when the technology achieves cost competitiveness, at which point market forces naturally take over, and there is no longer a necessity for ongoing Government support. Despite the significance of grid parity as a determinant of the duration of Government support, models for analysing its relationship with the technology learning rate remain underdeveloped. To bridge this gap, we develop a bi-level real options framework in order to derive a private firm's optimal investment strategy as well as a Government's optimal subsidy retraction policy.

We find that a greater subsidy may accelerate investment but its impact on project scale is ambiguous. More specifically, a bigger project accelerates the cost-reduction process, thereby incentivising a firm to install a greater capacity, however, it also speeds up the retraction of subsidy, as the operational cost will reach grid parity sooner. Results also indicate that although the duration of the subsidy is shorter when the learning rate is high, the firm is still willing to invest earlier. Interestingly, we also show that a higher tax rate does not necessarily delay the investment and induce a smaller project size under learning effect.

Chapter 5 provides a review of the overall contribution of this thesis and directions for future work.

Chapter 2

Risk assessment and optimal scheduling of serial projects

The valuation and planning of complex projects become increasingly challenging with rising market uncertainty and the deregulation of many industries, which have also raised the necessity for efficient risk management. In this chapter, we take the perspective of a private firm interested in the sequential capacity expansion of a project and develop a framework for measuring the downside risk of the serial project and optimising the sequence of the stages. Under general distributional assumptions for the duration of each stage, we present an accurate representation of the project's net present value (NPV) based on a Pearson curve fit, leading to closed-form expressions for the associated risk measures. We then assess the impact of duration variability on the value at risk and demonstrate its role in stochastic project scheduling. We also account for the trade-off between maximising the expected NPV and minimising the risk exposure, and obtain the optimal schedule for risk-averse decision-makers. It becomes obvious that both the duration variability of each stage and the decision-makers' risk preferences can significantly affect the optimal sequence of the stages and that high duration variability is not always undesirable, even for risk-averse decision-makers.

2.1 Introduction

The deregulation of many industries poses a formidable challenge to private firms that manage multi-stage projects, since the associated uncertainties over both future revenue streams and completion time of different stages (technological uncertainty) complicate the assessment of risk, and, in turn, critical managerial decisions, such as project scheduling. Examples of such projects include the Elizabeth line, London's new railway, which was originally designed to deliver a series of stages between 2017 and 2019, yet was not fully operational until May 2023 with an additional cost of £3 billion over the original budget (Tucker, 2017; Keay, 2022). In addition, the High Speed 2 and the Heathrow expansion can be treated as serial projects, where each stage has an uncertain duration, cost and benefit (Edgington, 2020; Thijssen, 2021). More specifically, the former has a full network of 330 miles and will be executed in two phases. While its first phase (140 miles) is under construction and due for completion between 2029 and 2033, the second phase is split into three sub-phases with a target completion date between 2040 and 2045. Similarly, the Heathrow expansion, which aims to increase capacity from about 80 to 142 million passengers per annum, will be delivered in four stages with Stage 1 to be completed by 2026 and all expansions by 2050 (Heathrow, 2019). Other examples include the development and capacity expansion of renewable energy projects, such as the Walney Extension and the Hornsea offshore wind farm (Vaughan, 2019) which is planned to have a total capacity of up to 6 gigawatts and whose construction has been split into four phases executed consecutively due to limited budget and workforce (Orsted, 2023).

While the traditional literature on project scheduling assumes discrete cash flows (Brucker et al., 1999; Herroelen and Leus, 2005; Demeulemeester and Herroelen, 2006), this is not suitable in the case of large infrastructure projects where revenues accrue continuously (Pogue, 2004; Almond and Remer, 1979; Tanchoco et al., 1981; Remer and Nieto, 1995). Additionally, as this literature focuses on maximising the net present value (NPV) or minimising the makespan of a project under risk neutrality, the implications of attitudes towards risk remain an important open research direction. To address these disconnects, we develop a continuous-time framework in which we derive the probability distribution and risk measures of the NPV of a serial project as well as the optimal sequence of stages under economic and technological uncertainty. The former is modelled via a continuous-time stochastic process, while the latter by a generic probability distribution. We contribute in three ways. First, we derive an accurate approximation for the probability distribution of the NPV of a multi-stage capacity expansion using a Pearson curve fit[1], from which we can obtain the closed-form expression for the value at risk (VaR) and the conditional VaR (CVaR) of the project. Second, we investigate the trade-off between expected NPV maximisation and risk minimisation, thereby deriving the solution to the optimal scheduling problem for risk-averse decision-makers. Third, we present the implications of economic and technological uncertainty on project scheduling and present managerial insights.

Our findings suggest that both the duration variability and the decision-makers' risk preferences can affect the optimal sequence of stages of a serial project significantly, and that their effect depends also on the expansion cost. More specifically, using a benchmark example (i.e., each stage with equal capacity size, cost and expected duration), we demonstrate that duration variability is undesirable if capacity expansions are costly, in which case stages with lower duration variability must be executed first. However, contrary to the intuition that an increase in uncertainty entails greater downside risk, we find that a project with higher duration variability is not always associated with higher risk exposure, especially when the cost of each stage is relatively low.

We proceed by first discussing some related work in Section 2.2. In Section 2.3, we introduce our model, the benchmark case of a single-stage capacity expansion, and the extension to a multi-stage project. In Section 2.4.1, we analyse the impact of duration variability on the project's expected NPV and risk exposure using a benchmark example, while Section 2.4.2 provides a general model for obtaining the optimal sequence of stages under risk aversion. Section 2.5 presents numerical results

[1] The Pearson distribution is a versatile family of probability distributions known for its ability to fit to different probability distributions, including the normal (Gaussian), exponential, gamma, and chi-squared distributions, making it a valuable tool in statistical analysis and modeling.

and managerial insights into the stochastic project scheduling problem and Section 2.6 concludes this chapter offering directions for further research.

2.2 Related work

Until the 1980s, the need to allow for uncertainty in decision-making was not particularly pronounced, as many industries were subject to state regulation. However, the deregulation of many industries in the 1980s exposed firms to various types of uncertainty, which, in turn, raised the importance of extending traditional capital budgeting techniques, such as the NPV rule, to account for uncertainty and risk assessment (Wiesemann and Kuhn, 2015). In the real options literature, this application potential has been exploited in decision-making under uncertainty by analysing the interaction between uncertainty in cash flows and managerial flexibility (McDonald and Siegel, 1986; Dixit and Pindyck, 1994; Trigeorgis, 1996). A strand of this literature focuses on the sequential nature of investment decisions and the value creation of modularity (Gollier et al., 2005; Gamba and Fusari, 2009; Baldwin et al., 2000; Kort et al., 2010; Chronopoulos et al., 2017). However, the underlying methodology, which is based on dynamic programming, is not particularly suitable to address critical aspects of serial projects, e.g., scheduling, that require robust optimisation techniques.

The stochastic project scheduling problem is addressed in (Herroelen and Leus, 2005), where it is described as a multi-stage decision process, in which the uncertainty in project activities is considered in order to prevent schedule disruptions. By allowing for uncertainty in projects' makespans, costs and revenues, a generic reformulation of the stochastic NPV maximisation problem is proposed by (Sobel et al., 2009) and an algorithm is presented for identifying an optimal adaptive policy for project scheduling. More recent examples in the same line of work, where various scheduling policies and algorithms are developed and tested in a stochastic environment, include (Wiesemann et al., 2010; Liang et al., 2019; Leyman and Vanhoucke, 2017; Zheng et al., 2017; Ding and Zhu, 2015). Although NPV-oriented models for stochastic project scheduling focus on the financial aspect of optimisation problems and provide significant flexibility in sequential decision processes, they tend to assume a risk-neutral decision-maker (Wiesemann and Kuhn, 2015; Gutjahr, 2015). However, for extreme values of duration and cash flow distributions, decision-making that is based only on the expected NPV and ignores attitudes towards risk may not be particularly accurate (Blau et al., 2000; Browning, 2014; Chao et al., 2014; Rezaei et al., 2020).

The variance of a project's revenues was often used to evaluate the risk of it (Markowitz, 1968; Van Horne, 1966), until the VaR was introduced as a more practical risk measure of the worst-case loss of an investment associated with a given probability. Nevertheless, despite its popularity, the VaR does not capture the shape of the tail of a loss distribution. To overcome the drawbacks of VaR while maintaining its advantages, (Rockafellar et al., 2000) introduced a coherent risk measure, known as the CVaR, aiming at quantifying the expected losses occurring beyond the VaR. Examples of literature on the stochastic project selection and scheduling problem that account for decision-makers' risk preferences include (Ke and Liu, 2005; Beraldi et al., 2012; Huang and Zhao, 2014; Wang and Ning, 2018), who control the probability of occurrence of undesirable investment outcomes (e.g., a negative expected NPV or positive VaR) using a chance-constrained method. Moreover, the trade-off between risk minimisation and profit maximisation is commonly considered in mean-risk models (Colvin and Maravelias, 2011; Chen et al., 2012; Bozorgi-Amiri et al., 2013; Alonso-Ayuso et al., 2014; Dupačová and Kozmík, 2015; Huang et al., 2016; Zhao et al., 2018, 2019). For example, (Alonso-Ayuso et al., 2014) consider a stochastic copper extraction planning problem under both risk neutrality and risk aversion, and their results clearly indicate the advantage of involving risk measures, such as the VaR and CVaR, of a project in the decision-making process.

The unknown distribution and variability (variance) of a project's makespan further emphasise the need for risk measures that facilitate efficient risk management. While different probability distributions for modelling project duration are examined within the stochastic resource-constrained project scheduling problem (RCPSP), the high- and low-variability settings of the duration distribution are often distinguished

(Ashtiani et al., 2011; Ballestin and Leus, 2009; Fang et al., 2015). The reason is that the optimal scheduling rule that minimises the expected makespan of a project often changes with respect to the variance of the duration variables. For example, (Chen et al., 2018) evaluate the efficiency of 17 priority rules and show that the optimal one for the deterministic RCPSP does not perform best for the stochastic RCPSP. Their results confirm that the performance of the priority rules depends on project characteristics, e.g., the resource demand and duration variability of each activity. Therefore, different scheduling rules could be chosen according to the amount of information on duration distributions that a decision-maker has. Similarly, we investigate the impact of duration variability on the NPV distribution and, more importantly, show how the optimal schedule of a serial project can be obtained for decision-makers with different risk preferences.

More pertinent to our work is Creemers (2018), who analyses the NPV of a multi-stage project assuming a discrete and deterministic cash flow stream as well as a generic probability distribution for the duration of each stage. Closed-form expressions for the moments of the NPV are derived and it is demonstrated how the optimal sequence of stages that maximises the project's NPV can be obtained by solving a least-cost fault detection problem. However, a discrete and deterministic cash flow incurred at the start of each stage is not particularly relevant in the case of large infrastructure projects where revenues usually accrue continuously[2], which are also influenced by future price uncertainty (Pogue, 2004; Almond and Remer, 1979; Tanchoco et al., 1981; Remer and Nieto, 1995). For example, the annual average electricity price rise between 2004 and 2021 in the UK is approximately 8% per year, from 4.16 pence per kilowatt hour (p/kWh) in 2004 to 15.08 p/kWh in 2019 (BEIS, 2023). This was followed by a dramatic increase to 20.86 p/kWh in 2022 due to high market volatility, which clearly demonstrates that the revenue stream of a project fluctuates with time and that price uncertainty should also be taken into account. In addition, (Cui et al., 2020) study the unbiased estimation by Monte

[2]The numerous discrete cash flows associated with infrastructure projects, such as electricity usage and train ticket purchases occurring every minute, can be effectively modelled using a continuous model as it is mathematically more tractable.

Carlo simulation of the expected present value of a cumulative cash flow over an infinite horizon, dependent on an underlying stochastic process such as a geometric Brownian motion or a Cox–Ingersoll–Ross process.

Therefore, in this chapter, we expand on Creemers (2018) by develop a continuous-time framework for sequential capacity expansion under economic and technological uncertainty and derive the project's VaR and CVaR. Furthermore, we consider the trade-off between NPV maximisation and downside risk minimisation of a project due to alternative scheduling options[3]. Our results indicate that both the duration variability and the risk preferences can have a significant effect on the optimal sequencing of a multi-stage project, and that this depends on the expansion cost of each stage. Interestingly, we also find that higher duration variability does not necessarily imply higher risk exposure; differently from conventional intuition, it can be beneficial even for risk-averse decision-makers.

2.3 Risk assessment of serial project

2.3.1 The model

We take the perspective of a private firm that considers the capacity expansion of a project sequentially in discrete stages. While the construction process takes a random but finite amount of time, the project has an infinite lifetime[4], accrues stochastic revenues and is subject to technological uncertainty, reflected in the random duration of each stage. Given a probability space $(\Omega, \mathcal{F}, \mathbb{P})$, the σ-algebra $\mathcal{F}_t \subset \mathcal{F}$ reflects the information available at time $t \geq 0$. Without being a real restriction, we assume, in light of (Dixit and Pindyck, 1994; Cui et al., 2020), that the price (per unit flow

[3]Note that while the value of waiting due to economic uncertainty is not the focus of this paper, we provide a real options framework in Appendix A.2 to investigate its impact on the risk assessment and optimal scheduling of a two-stage project. A thorough study of the implications of discretion over timing is left for future work.

[4]The assumptions of infinite lifetime and perpetual revenue stream are commonly made in the real options literature because they not only support analytical tractability, but are also key features of different projects (Dixit and Pindyck, 1994). For example, in the electricity sector, power generation facilities have an effective operation life of 30–50 years, while transmission facilities remain in service even longer. Hence, although the construction of a gas power plant or the installation of the wind farm has a finite duration, its lifetime is significantly longer.

of output) process for the project follows a geometric Brownian motion

$$dP_t = \alpha P_t dt + \beta P_t dW_t, \quad P_0 \equiv P, \tag{2.1}$$

where P is the initial price, $\alpha > 0$ the growth rate, $\beta > 0$ the volatility, and W is a standard Brownian motion. Here, we assume that the firm operates as a price taker in a perfectly competitive market, and, therefore, lacks the ability to influence the market price. Also, we denote by $r > \alpha$ the subjective discount rate defined exogenously[5]. Model (2.1) can be adapted to the users' preferences and the requirements of their respective application, as our build-up is general in terms of underlying model assumptions. Also, the particular choice, as said in Cui et al. (2020), is a viable candidate in project management.

The project comprises $n \in \mathbb{N}$ stages that are executed sequentially. For $j \in \{1, 2, \ldots, n\}$, we denote by $D_j > 0$ the deterministic scale of each capacity expansion. Thus, $P_t D_j$ is the instantaneous revenue of the project in stage j. Following (Huisman and Kort, 2015), we assume a linear investment cost function, where a deterministic cost, $C_j = cD_j$, is incurred at the beginning of each capacity expansion and $c \geq 0$ represents the expansion cost per unit output. After the completion of stage j, the accumulated capacity is $D'_j = \sum_{k=0}^{j} D_k$, where $D_0 \geq 0$ is the initial capacity.

The duration of each stage is denoted by τ_j and has a general, continuous probability distribution with cumulative distribution function (cdf) $F_{\tau_j}(t)$ and probability density function (pdf) $f_{\tau_j}(t)$. Assuming that $\{\tau_j\}_{j=1}^{n}$ are independent (Steyn, 2001; Chen et al., 2015), the completion time of stage j is given by $T_j = \sum_{k=1}^{j} \tau_k$. Finally, we denote by $V(\cdot)$ the resulting NPV of a project with associated cdf $G_V(v)$ and pdf $g_V(v)$.

[5]Risk neutrality is commonly assumed in corporate finance, however it also relies on market completeness. Hence, it may not be particularly relevant in our context of construction or capacity expansion in the absence of hedging instruments. For this, we use, instead, an exogenously defined (subjective) discount rate.

2.3.2 Single-stage capacity expansion

We begin with the basic case of a project that is subject to a single capacity expansion. As shown in Figure 2.1, the expansion begins at time $t = 0$, where a deterministic cost of cD_1 is incurred. Subsequently, the firm receives an instantaneous revenue of $P_t D_0$ from time $t = 0$ until T_1, at which point the capacity of the project is expanded to $D_1' = D_0 + D_1$ and the firm earns a perpetual stream of stochastic revenues $P_t D_1'$.

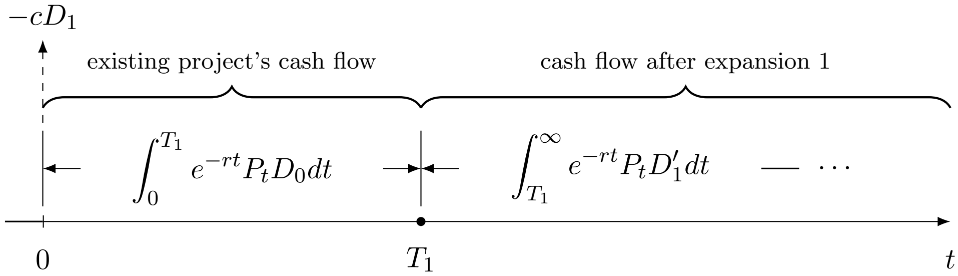

Fig. 2.1. Cash flow of single-stage capacity expansion taking place between time 0 and T_1.

We derive the NPV of the project by discounting the continuous cash flow over its lifetime. The discounted to time $t = 0$ expected NPV, $V(P, T_1)$, of this single-stage expansion conditional on the makespan of the project can be formulated as[6]

$$\begin{aligned} V(P, T_1) &= \mathbb{E}\left[\int_0^{T_1} e^{-rt} P_t D_0 dt + \int_{T_1}^{\infty} e^{-rt} P_t D_1' dt - C_1 \middle| P, T_1 \right] \\ &= \frac{PD_0}{r-\alpha} + \frac{PD_1}{r-\alpha} e^{-(r-\alpha)T_1} - cD_1. \end{aligned} \tag{2.2}$$

The mean, variance, skewness and kurtosis of $V(P, T_1)$ are given, respectively, by

$$\mu = \mathbb{E}[V(P, T_1)] = \frac{PD_0}{r-\alpha} + \frac{PD_1}{r-\alpha} M_{T_1}(\alpha - r) - cD_1, \tag{2.3}$$

$$\sigma^2 = m_2, \quad \psi = \frac{m_3}{m_2^{3/2}}, \quad \epsilon = \frac{m_4}{m_2^2}, \tag{2.4}$$

[6] In what follows, we abbreviate the random variable "conditional expected NPV" in (2.2) to simply "NPV", whereas the "expected NPV" is constant and is given by (2.3).

where

$$m_k = \mathbb{E}\left[\left(V\left(P, T_1\right) - \mu\right)^k\right] = \left(\frac{PD_1}{r-\alpha}\right)^k \mathbb{E}\left[\left(e^{-(r-\alpha)T_1} - M_{T_1}(\alpha - r)\right)^k\right]$$

and $M_{T_1}(\delta) = \mathbb{E}\left[e^{\delta T_1}\right]$, $\delta \in \mathbb{R}$.

Proposition 2.3.1. *The cdf and pdf of the NPV of a single-stage project are given by*

$$G_V(v) = 1 - F_{T_1}\left(-\frac{1}{r-\alpha}\ln\frac{(r-\alpha)(v+cD_1) - PD_0}{PD_1}\right), \tag{2.5}$$

$$g_V(v) = \frac{1}{(r-\alpha)(v+cD_1) - PD_0} f_{T_1}\left(-\frac{1}{r-\alpha}\ln\frac{(r-\alpha)(v+cD_1) - PD_0}{PD_1}\right), \tag{2.6}$$

for $v \geq PD_0/(r-\alpha) - cD_1$*.*

From Proposition 2.3.1, for a given probability distribution for $\tau_1 = T_1$, we derive the distribution of $V\left(P, T_1\right)$. In turn, this facilitates the evaluation of the risk associated with the project by looking at the left tail of its NPV distribution. For example, consider $\mathrm{VaR}_p(X) = -q_p^+(X)$, where $q_p^+(X) = \inf\{v \in \mathbb{R} : \mathbb{P}(X \leq v) > p\}$ is the p-quantile of a random variable X, for $p \in (0,1)$, while $\mathrm{CVaR}_p(X)$ denotes the expectation of X given that it is larger than $\mathrm{VaR}_p(X)$. Given a closed-form expression for G_V, we can also obtain the VaR and CVaR of the project NPV.

Proposition 2.3.2. *For the NPV of the project at level p, we have that*

$$\mathrm{VaR}_p(V) = -\frac{PD_0}{r-\alpha} - \frac{PD_1}{r-\alpha} e^{-(r-\alpha)F_{T_1}^{-1}(1-p)} + cD_1 \quad \text{and} \quad \mathrm{CVaR}_p(V) = \frac{1}{p}\int_0^p \mathrm{VaR}_q(V)dq. \tag{2.7}$$

2.3.3 Multi-stage project

In this section, we generalise to a multi-stage project and formulate its NPV, before moving on to scheduling these stages in Section 2.4. The stochastic cash flow stream of the multi-stage project is shown in Figure 2.2.

The NPV of a serial project with $n \geq 1$ stages is given by the sum of the NPVs of

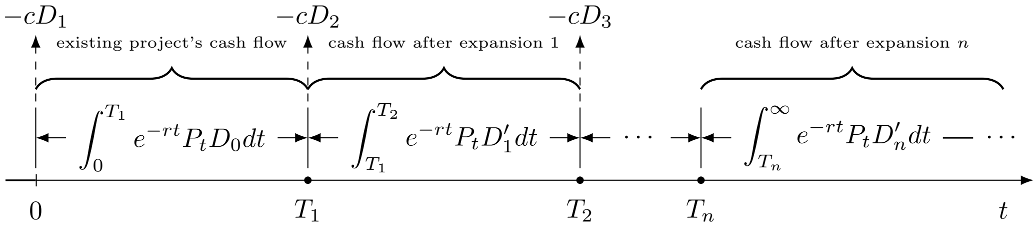

Fig. 2.2. Cash flow of multi-stage project.

the various capacity expansions, i.e.

$$\begin{aligned} V(P, T_1, \ldots, T_n) &= \sum_{j=1}^{n} \mathbb{E}\left[\int_{T_{j-1}}^{T_j} e^{-rt} P_t D'_{j-1} dt - C_j e^{-rT_{j-1}} \middle| P, T_{j-1}, T_j \right] \\ &\quad + \mathbb{E}\left[\int_{T_n}^{\infty} e^{-rt} P_t D'_n dt \middle| P, T_n \right] \\ &= \sum_{j=0}^{n} \mathbb{E}\left[\int_{T_j}^{\infty} e^{-rt} P_t D_j dt \middle| P, T_j \right] - \sum_{j=1}^{n} cD_j e^{-rT_{j-1}} \\ &= \frac{PD_0}{r-\alpha} + \sum_{j=1}^{n} V_j(P, T_{j-1}, T_j), \end{aligned} \tag{2.8}$$

where

$$V_j(P, T_{j-1}, T_j) \equiv V_j = \frac{PD_j}{r-\alpha} e^{-(r-\alpha)T_j} - cD_j e^{-rT_{j-1}} \tag{2.9}$$

corresponds to the increment of the NPV of the project's cash flows due to the jth capacity expansion, for $j \in \{1, 2, \ldots, n\}$; also, $T_0 \equiv 0$.

Expressions for the true NPV density and distribution functions are not available in closed form in the multi-stage case, however we can obtain very accurate analytical approximations that can be used subsequently for computing the risk measures of the sequential capacity expansion. To this end, we fit a Pearson curve type based on the first four moments of the true, but otherwise unknown, distribution of $V(P, T_1, \ldots, T_n)$. More specifically, the Pearson family of solutions $g_V(x)$ satisfies the differential equation

$$\frac{d \ln g_V(x)}{dx} = -\frac{a + x}{c_0 + c_1 x + c_2 x^2} \tag{2.10}$$

resulting in well-defined density functions. Solving equation (2.10) yields the general form of Pearson's density function

$$g_V(x) = \mathcal{C}\left(c_0 + c_1 x + c_2 x^2\right)^{-\frac{1}{2c_2}} \exp\left\{\frac{(c_1 - 2ac_2)\arctan\left(\frac{c_1+2c_2x}{\sqrt{4c_0c_2-c_1^2}}\right)}{c_2\sqrt{4c_0c_2 - c_1^2}}\right\}, \tag{2.11}$$

where $\mathcal{C}$ is the normalising constant and the parameters $\{a, c_0, c_1, c_2\}$ control the shape of the distribution. We estimate these based on the first four integer moments $\{\mu_1, \mu_2, \mu_3, \mu_4\}$ as

$$a = c_1 = \frac{\sqrt{\upsilon\gamma}\,(\theta+3)}{10\theta - 12\gamma - 18}, \quad c_0 = \frac{(4\theta - 3\gamma)\,\upsilon}{10\theta - 12\gamma - 18}, \quad c_2 = \frac{2\theta - 3\gamma - 6}{10\theta - 12\gamma - 18}, \tag{2.12}$$

where

$$\upsilon = \mu_2 - \mu_1^2, \quad \gamma = \frac{\left(\mu_3 - 3\mu_1\mu_2 + 2\mu_1^3\right)^2}{\upsilon^3}, \quad \theta = \frac{\mu_4 - 4\mu_1\mu_3 + 6\mu_1^2\mu_2 - 3\mu_1^4}{\upsilon^2} \tag{2.13}$$

are the variance, squared skewness and kurtosis, respectively. We can classify the Pearson distribution family types as in (Johnson et al., 1994) which is standard in the literature. First, we select a family according to the η-criterion proposed by (Elderton and Johnson, 1969): given $\sqrt{\gamma}$ and θ, we compute

$$\eta = \frac{\gamma\,(\theta+3)^2}{4\,(4\theta - 3\gamma)\,(2\theta - 3\gamma - 6)}. \tag{2.14}$$

We distinguish between the *main* types corresponding to $\eta < 0$ (I), $0 < \eta < 1$ (IV) and $\eta > 1$ (VI); and the *transition* types $\eta = 0, \theta = 3$ (normal), $\eta = 0, \theta < 3$ (II), $\eta = \pm\infty$ (III), $\eta = 1$ (V) and $\eta = 0, \theta > 3$ (VII). Then, we can approximate the NPV distribution accordingly. Important advantages of the Pearson fitting approach, as we will demonstrate next, are its excellent results for different skewness-kurtosis $(\gamma^{1/2}, \theta)$ levels (see also (Brignone et al., 2021)), for varying number of stages and general assumptions for the distribution of τ_j. More details about the proximity of distributions with shared moments is beyond the scope of this research; interested reader may refer, instead, to (Akhiezer, 1965, Corollary 2.5.4), (Lindsay and Basak, 2000, Theorems 1, 2) and Kyriakou et al. (2023). Given the Pearson fitted cdf of the project's NPV, $G_V(v)$, the VaR and CVaR follow:

$$\mathrm{VaR}_p(V) = -G_V^{-1}(p) \quad \text{and} \quad \mathrm{CVaR}_p(V) = \frac{1}{p}\int_0^p \mathrm{VaR}_q(V)dq. \tag{2.15}$$

To illustrate the Pearson curve fit, we set $P = 1$, $r = 0.1$, $\alpha = 0.08$, $\beta = 0.1$, $c = 30$, $D_0 \equiv 0$ and $D_j = 10$ for all $j \in \{1, 2, \ldots, n\}$, and study the Pearson curve approximation for $\tau_j \sim \mathrm{LogN}\,(m, s)$ and $\tau_j \sim \mathrm{Weibull}\,(\lambda, \kappa)$[7]. For the sake of comparison, we assume that they share the same mean and variance, e.g., $e^{m+\frac{1}{2}s^2} = \lambda\Gamma(1 + 1/\kappa) = 10$ and $e^{2m+s^2}\left(e^{s^2} - 1\right) = \lambda^2\left[\Gamma(1 + 2/\kappa) - (\Gamma(1 + 1/\kappa))^2\right] = 28$, from which we obtain parameters $m = 2.18$, $s = 0.50$, $\lambda = 11.28$ and $\kappa = 1.96$. We compare the density approximations with the true simulation estimates in Figure 2.3. The top and bottom panels of Table 2.1 also report the true mean, variance, skewness, kurtosis, $\mathrm{VaR}_{0.05}$ and $\mathrm{CVaR}_{0.05}$ of $V(P, T_1, \ldots, T_n)$, along with the values corresponding to the Pearson curve fit and the associated absolute percentage errors.

Two comments are in order. It is obvious from Figure 2.3 and Table 2.1 that the approximation of the NPV distribution by a Pearson curve fit is very accurate, regardless of the distribution of τ_j and the number of stages. While one can rely on Monte Carlo simulation estimates, using a Pearson approximation leads to an analytical expression that considerably reduces the computational effort and avoids unwanted simulation error. In particular, for the accuracies reported in Table 2.1 based on 10^7 simulation trials for the Monte Carlo estimates, we achieve a reduction in the computing time by a factor of 100 the least.

In addition, we recall that we have chosen the lognormal and Weibull parameter values for τ_j so that their mean and variance are matched. Nevertheless, the resulting variance, skewness and kurtosis of $V(P, T_1, \ldots, T_n)$ vary significantly between the two distributions. This implies that the assumptions about the distribution and the higher moments of the duration variables can affect substantially the risk characteristics of the NPV and, therefore, the valuation and planning of a project.

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[7] Both the lognormal (Chen et al., 2018; Trietsch et al., 2012) and exponential distributions (Sobel et al., 2009) are commonly used to model activity duration in the project management literature. Here, we use a Weibull distribution which generalises the exponential distribution with an extra parameter that offers added flexibility. Gamma, normal and uniform distributions have also been examined.

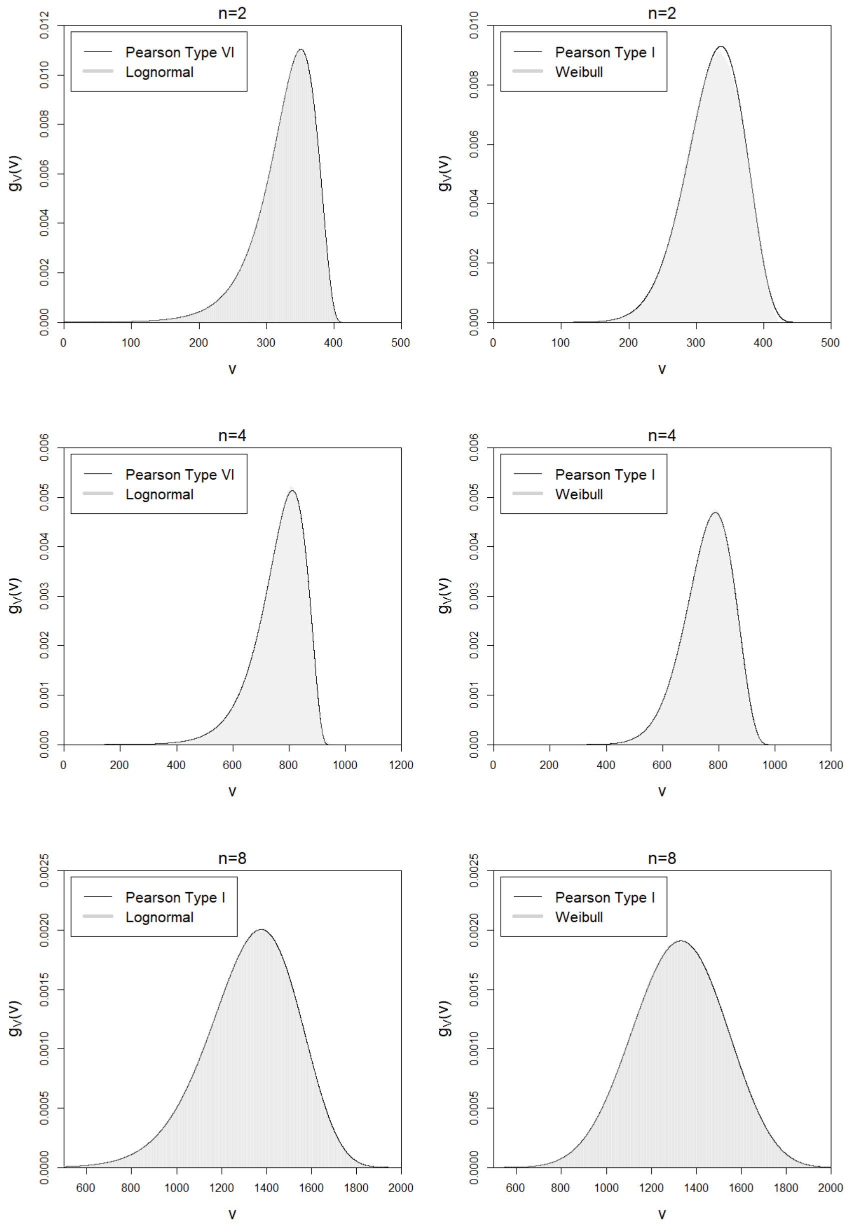

Fig. 2.3. Simulated and fitted Pearson pdf of NPV of n-stage capacity expansion for $\tau_j \sim \text{LogN}(m, s)$ (left panel) and $\tau_j \sim \text{Weibull}(\lambda, \kappa)$ (right panel) sharing same mean, 10, and variance, 28, with $m = 2.18$, $s = 0.50$, $\lambda = 11.28$ and $\kappa = 1.96$.

2.4 Optimal scheduling of serial project

2.4.1 Impact of duration variability on expected NPV and VaR

In this section, we investigate the optimal order in which the stages of a serial project should be executed, as well as the factors affecting the NPV distribution and the downside risk of the project. Specifically, we study the impact of duration variability

Table 2.1: Simulation estimates and fitted-Pearson mean, variance, skewness, kurtosis, $\mathrm{VaR}_{0.05}$ and $\mathrm{CVaR}_{0.05}$ of NPV of n-stage capacity expansion for $\tau_j \sim \mathrm{LogN}(m, s)$ (upper panel) and $\tau_j \sim \mathrm{Weibull}(\lambda, \kappa)$ (lower panel) sharing same mean, 10, and variance, 28, with $m = 2.18$, $s = 0.50$, $\lambda = 11.28$ and $\kappa = 1.96$.

Lognormal									
Stages	$n = 2$			$n = 4$			$n = 8$		
	Estimate	Pearson	Abs. error	Estimate	Pearson	Abs. error	Estimate	Pearson	Abs. error
Mean	327.03	327.04	0.0%	763.92	763.92	0.0%	1327.57	1327.57	0.0%
Var	1985.85	1981.25	0.2%	8275.57	8274.28	0.0%	41551.24	41551.02	0.0%
Skew	-1.3734	-1.3720	0.1%	-1.1607	-1.1588	0.1%	-0.4751	-0.4750	0.0%
Kurt	6.5072	6.5007	0.1%	5.2344	5.2290	0.1%	3.3347	3.3342	0.0%
$\mathrm{VaR}_{0.05}$	-243.52	-243.12	0.2%	-592.25	-592.25	0.0%	-967.28	-967.04	0.0%
$\mathrm{CVaR}_{0.05}$	-205.33	-205.32	0.0%	-521.53	-522.27	0.1%	-854.34	-854.16	0.0%
Weibull									
Stages	$n = 2$			$n = 4$			$n = 8$		
	Estimate	Pearson	Abs. error	Estimate	Pearson	Abs. error	Estimate	Pearson	Abs. error
Mean	325.31	325.31	0.0%	759.93	759.93	0.0%	1323.30	1323.30	0.0%
Var	1846.15	1846.15	0.0%	7444.58	7444.58	0.0%	41237.06	41237.06	0.0%
Skew	-0.4563	-0.4563	0.0%	-0.5591	-0.5590	0.0%	-0.0694	-0.0694	0.0%
Kurt	3.1260	3.1260	0.0%	3.2644	3.2642	0.0%	2.7969	2.7969	0.0%
$\mathrm{VaR}_{0.05}$	-249.98	-249.87	0.0%	-603.79	-604.31	0.1%	-984.84	-984.21	0.1%
$\mathrm{CVaR}_{0.05}$	-227.13	-226.61	0.2%	-556.77	-556.95	0.0%	-903.11	-903.10	0.0%

on the optimal sequence of a serial project and investigate the importance of risk considerations in stochastic project scheduling.

We start by showing how the expected NPV, μ, and VaR of a project with single capacity expansion depend on the variance of its makespan, τ_1, with the real-world example of the Hornsea offshore wind farm. More specifically, the construction (i.e., Stage 1) of the 1.2-gigawatt (GW) wind farm takes about three years with a cost around £4.2 billion (Orsted, 2023). Hence, we set $D_0 \equiv 0$ GW, $D_1 = 1.2$ GW, $P =$ £0.2 billion per GW, $r = 10\%$ per year, $\alpha = 8\%$ per year and $\beta = 10\%$ per year. Assuming that $\tau_1 \sim \mathrm{LogN}(m, s)$, then for $\mathbb{E}[\tau_1] = e^{m+\frac{1}{2}s^2} = \tau$ we get that $m = \ln \tau - s^2/2$. Thus, $\mathrm{Var}[\tau_1] = \left(e^{s^2} - 1\right) e^{2m+s^2} = \tau^2 \left(e^{s^2} - 1\right)$, which is increasing with s for given τ. For $\tau = 3$ years, we present in Figure 2.4 μ as an increasing function of s for $c =$ £2 billion and £4 billion per GW capacity installed[8]. This implies that, for a fixed makespan expectation, a capacity expansion with higher duration variability is expected to be more profitable.

[8]In this chapter, we focus on situations where each stage generates a positive profit, and do not consider the case when the investment cost exceeds the revenue, leading to a negative profit flow. Such situations become more pertinent within a real options framework, where the firm has the option to either abandon or forego the execution of subsequent stages.

Of particular interest is the U-shaped 5%-quantile curve, from which we observe that the risk exposure of the expansion, $\mathrm{VaR}_{0.05}(V) = -q^{+}_{0.05}(V)$, is surprisingly low when the duration variance is large. This counter-intuitive result can be attributed to the VaR_p of the project's NPV, which is determined by the quantile of τ_1 (see Proposition 2.3.2); for lognormal τ_1, the quantile as a function of s increases initially and then starts to decrease. For $\mathbb{E}[\tau_1] = \tau$ held fixed, the skewness of τ_1 given by $\left(e^{s^2}+2\right)\sqrt{e^{s^2}-1}$ increases in s, so that a higher duration variability implies a shorter makespan with increasing concentration around small values. This implies that high duration variability can benefit both risk-neutral and risk-averse decision-makers and, therefore, variance reduction is not always necessary, especially when the duration variability is moderate to high.

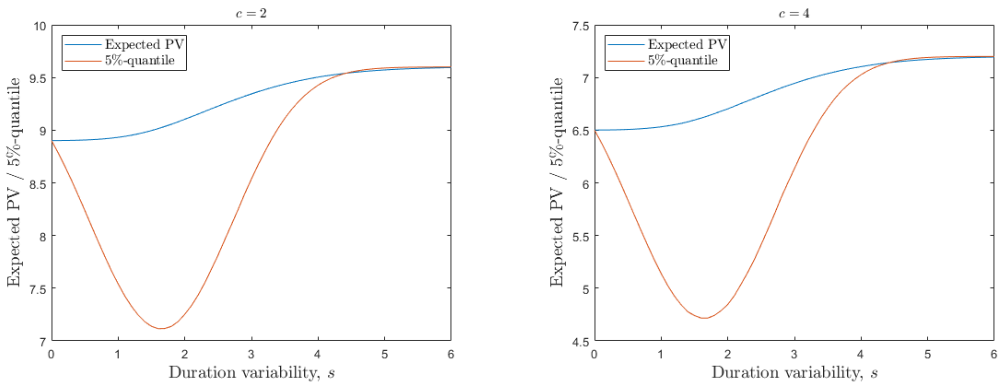

Fig. 2.4. Expected NPV and 5%-quantile of single capacity expansion as function of duration variability s when $\tau_1 \sim \mathrm{LogN}\left(\ln 3 - s^2/2, s\right)$, for $c = 2$ and 4.

Next, we add one more stage and consider two capacity expansions of equal size and cost to demonstrate the implications of risk aversion for optimal scheduling. We assume that each stage of the project has the same expected duration τ but different duration variability s. Therefore, the question now arises: how should the firm determine the order of execution for each stage to achieve a higher (lower) project value (risk exposure)?

Assuming that $\tau_1 \sim \mathrm{LogN}(\ln\tau - 1/2, 1)$, $\tau_2 \sim \mathrm{LogN}(\ln\tau - s^2/2, s)$ and $D_1 = D_2 = 1.2$ GW, Figure 2.5 illustrates how μ and the 5%-quantile of the project depend on s, for each of the two possible ways of scheduling. Consistent with Figure 2.4, we find that executing the stage with high duration variability first, that is, τ_2

for $s > 1$ (blue solid line) can result in larger expected NPV but only when c is small (e.g., $c \leq 2$). As c increases, this strategy is no longer appropriate. Indeed, μ eventually decreases with s as shown in the bottom-right panel of Figure 2.5. This happens because the discount factor of the project's cost, $\mathbb{E}[e^{-r\tau_2}]$, increases faster than that of the revenue, $\mathbb{E}[e^{-(r-\alpha)\tau_2}]$, as s rises. Consequently, duration variability is desirable for risk-neutral decision-makers when the cost is relatively low, as their preferences are independent of the risk associated with the schedule; but, it can be harmful to the project's value if capacity expansions are costly.

On the other hand, executing first the stage with lower duration variability, that is, $\tau_1 \sim \mathrm{LogN}\,(\ln 3 - 1/2, 1)$, when the other stage has duration $\tau_2 \sim \mathrm{LogN}\left(\ln 3 - s^2/2, s\right)$ with $s > 1$ (purple dashed line), does not guarantee lower risk exposure, particularly when the cost of each expansion is low compared to its revenues. Whereas this scheduling strategy can be quite safer if s is low-to-moderate, it performs poorly in terms of both the project's value and risk exposure if s is large (magenta line appears above the purple line, despite the smaller variance of τ_1 and stage 1 being implemented first). Again, this follows from the positively skewed distribution of τ_2 with increasing concentration around smaller values as s increases. In particular, the 95%-quantile linked to τ_2 becomes smaller if $s > 1.67$, thus indicating that the firm should execute stage 2 first despite its increasing duration variability. However, if the capacity expansions are costly (e.g., $c \geq 4$), then high duration variability becomes undesirable as it always leads to lower expected NPV and higher risk. Therefore, the decision-maker should always execute stages with lower duration variability first (see the bottom-right panel of Figure 2.5).

Consequently, our results indicate that duration variability can significantly affect the optimal sequence of stages for both risk-neutral and risk-averse decision-makers; moreover, its impact depends on the level of expansion cost. Indeed, for all stages with the same expected duration and capacity, risk-neutral decision-makers should always execute the stages with higher duration variability earlier if the cost is relatively low. However, under risk aversion, the optimal sequence of stages is less obvious as it depends on the trade-off between maximising the expected NPV

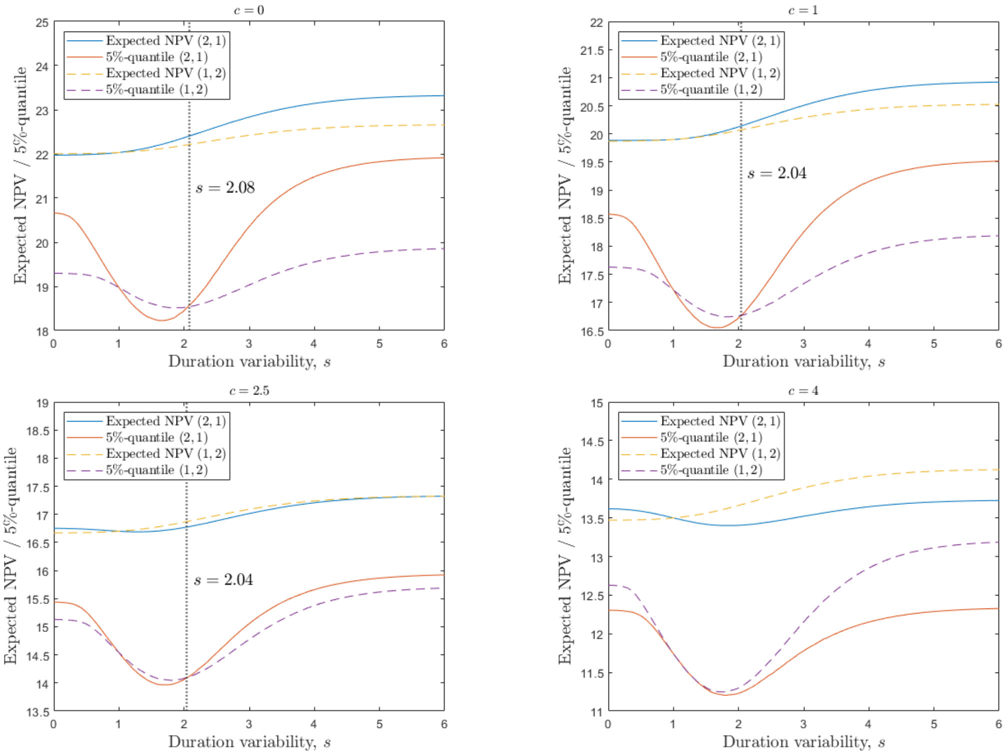

Fig. 2.5. Expected NPV and 5%-quantile of two-stage project as function of duration variability s when $\tau_1 \sim \text{LogN}(\ln 3 - 1/2, 1)$ and $\tau_2 \sim \text{LogN}\left(\ln 3 - s^2/2, s\right)$, and stage 2 is executed before stage 1 (solid lines) or vice versa (dashed lines).

and minimising the downside risk of a project. For example, a risk-averse decision-maker may be willing to bear a slightly higher risk in exchange for a larger expected NPV, and vice versa. Moreover, this trade-off can be even more complicated due to the fact that high duration variability is not always harmful. Due to the technological uncertainty reflected in the makespan of a project, it is implied that risk considerations have to be incorporated in stochastic project scheduling.

2.4.2 Risk management and optimal scheduling

To address the trade-off between the expected NPV and risk exposure of a serial project due to the different ways of scheduling, we incorporate risk measures, such as the VaR and CVaR, into the stochastic project scheduling problem. To this end, we define the symmetric group S_n on the set $N = \{1, 2, \ldots, n\}$ and $\boldsymbol{\pi} \in S_n$ a permutation of N (i.e., a bijection from N to N itself). In our context, a permutation $\boldsymbol{\pi} = (\pi(1), ..., \pi(n))$ encompasses the sequence of stages of a serial project; for any $j, k \in N$, $\pi(j) = k$ means that stage k is the jth term of the sequence. It is worth noting that our model is also capable of managing the optimal sequence of stages when there are precedence constraints between these stages. As the optimal schedule of a serial project is obtained through an enumeration of all possible permutations, any precedence constraints simply serve to eliminate certain permutations that are no longer feasible. Also, we introduce $\omega \in [0, 1]$ which reflects the risk appetite of a decision-maker, with large (small) ω corresponding to high (low) risk aversion. Thus, the optimal sequence of stages $\boldsymbol{\pi}^*_\omega$, which maximises a combination of the expected NPV and the risk measure of a serial project, can be formulated as

$$\boldsymbol{\pi}^*_\omega = \underset{\boldsymbol{\pi} \in S_n}{\operatorname{argmax}} V(\boldsymbol{\pi}), \tag{2.16}$$

where

$$\begin{aligned} V(\boldsymbol{\pi}) = (1-\omega)\mathbb{E}&\left[\sum_{j=1}^{n} \frac{PD_{\pi(j)}}{r-\alpha} e^{-(r-\alpha)\sum_{k=1}^{j}\tau_{\pi(k)}} - \sum_{j=1}^{n} cD_{\pi(j)} e^{-r\sum_{k=0}^{j-1}\tau_{\pi(k)}}\right] \\ &- \omega\mathcal{R}\left(\sum_{j=1}^{n} \frac{PD_{\pi(j)}}{r-\alpha} e^{-(r-\alpha)\sum_{k=1}^{j}\tau_{\pi(k)}} - \sum_{j=1}^{n} cD_{\pi(j)} e^{-r\sum_{k=0}^{j-1}\tau_{\pi(k)}}\right), \end{aligned} \tag{2.17}$$

$\pi(0) = \tau_0 = 0$ and $\mathcal{R}(\cdot)$ is a risk measure such that a larger value of it implies higher risk. In particular, we consider $\mathcal{R} \in \{\mathrm{VaR}_p, \mathrm{CVaR}_p\}$ to account for the left tail of the NPV distribution of a project, which we evaluate based on (2.15). Given a p level of confidence, ω controls the weights of the expected NPV and the risk exposure in this mean-risk model. A decision-maker is assumed to be risk-neutral if $\omega = 0$, in which case the second part of (2.17) vanishes and (2.16) reduces to the expected NPV maximisation model. In the next section, we obtain the optimal schedule of a serial project under various combinations of duration variability and decision-maker's risk appetite, and show how the results based on either the mean-VaR or mean-CVaR model differ.

2.5 Project scheduling: numerical illustration

We revisit, first, the example in Figure 2.5, where a firm considers scheduling the two phases of an offshore wind farm, each with capacity of 1.2 GW. The upper panel of Table 2.2 reports the expected NPV and $\mathrm{VaR}_{0.05}$ of the two-stage project for $c = \{2, 3\}$ and $s \in \{0.5, 1.5, 2.2, 3\}$. The optimal sequences of stages for decision-makers with different attitudes towards risk are also obtained based on the mean-risk model (2.16)–(2.17) with $\mathcal{R} \equiv \mathrm{VaR}_{0.05}$. Consistent with the bottom-right panel of Figure 2.5, we have $\pi^*_\omega = (2, 1)$ if $s < 1$ and $\pi^*_\omega = (1, 2)$ if $s > 1$, for any $\omega \in [0, 1]$. Therefore, our results indicate that decision-makers should always execute the stage with lower duration variability first when the cost of each stage is high.

Next, we examine whether the aforementioned scheduling strategy is still optimal if the expansion cost is lower, e.g., $c = 2$. The lower panel of Table 2.2 confirms that both the duration variability of each capacity expansion and the decision-makers' risk preferences can affect significantly the optimal schedule of a project in this case. Taking $\omega = 0.5$ as an example, we obtain $\boldsymbol{\pi}^*_{0.5} = (2, 1)$ if $s < 1$ or $s > 2.14$, whereas $\boldsymbol{\pi}^*_{0.5} = (1, 2)$ if $1 < s < 2.14$. This implies that a risk-averse decision-maker with $\omega = 0.5$ may choose to reduce the risk exposure of the project by a significant amount without foregoing too much revenue when $s > 2.14$.

Table 2.2: Optimal schedule of two-stage project with duration variability s for decision-makers with risk appetite ω obtained from mean-risk model (2.16), for $\mathcal{R} \equiv \mathrm{VaR}_{0.05}$ and $c = 3$ (upper panel) or 2 (lower panel). $\pi = (1, 2)$: execute stage 1 followed by stage 2; $\pi = (2, 1)$: execute stage 2 followed by stage 1.

$c = 3$									
	Expected NPV μ		$\mathrm{VaR}_{0.05}$		Optimal Sequence $\boldsymbol{\pi}^*_\omega$				
	$\pi = (1, 2)$	$\pi = (2, 1)$	$\pi = (1, 2)$	$\pi = (2, 1)$	$\omega = 0$	$\omega = 0.25$	$\omega = 0.5$	$\omega = 0.75$	$\omega = 1$
$s = 0.5$	15.6090	15.6858	-14.1864	-14.2613	$(2, 1)$	$(2, 1)$	$(2, 1)$	$(2, 1)$	$(2, 1)$
$s = 1.5$	15.6938	15.6031	-13.1931	-13.1262	$(1, 2)$	$(1, 2)$	$(1, 2)$	$(1, 2)$	$(1, 2)$
$s = 2.2$	15.8407	15.6771	-13.2537	-13.2625	$(1, 2)$	$(1, 2)$	$(1, 2)$	$(1, 2)$	$(1, 2)$
$s = 3.0$	16.0228	15.8501	-13.9066	-13.9731	$(1, 2)$	$(1, 2)$	$(1, 2)$	$(1, 2)$	$(1, 2)$
$c = 2$									
	Expected NPV μ		$\mathrm{VaR}_{0.05}$		Optimal Sequence $\boldsymbol{\pi}^*_\omega$				
	$\pi = (1, 2)$	$\pi = (2, 1)$	$\pi = (1, 2)$	$\pi = (2, 1)$	$\omega = 0$	$\omega = 0.25$	$\omega = 0.5$	$\omega = 0.75$	$\omega = 1$
$s = 0.5$	17.7423	17.7852	-15.8959	-16.2493	$(2, 1)$	$(2, 1)$	$(2, 1)$	$(2, 1)$	$(2, 1)$
$s = 1.5$	17.8268	17.7876	-15.0290	-14.8855	$(1, 2)$	$(1, 2)$	$(1, 2)$	$(1, 2)$	$(1, 2)$
$s = 2.2$	17.9748	17.9397	-15.0560	-15.1548	$(1, 2)$	$(1, 2)$	$(2, 1)$	$(2, 1)$	$(2, 1)$
$s = 3.0$	18.1547	18.1763	-15.6240	-16.1191	$(2, 1)$	$(2, 1)$	$(2, 1)$	$(2, 1)$	$(2, 1)$

Aiming to shed more light on the results, we present in Figure 2.6 the scenarios in which a schedule $\boldsymbol{\pi}$ optimises a mean-risk model. The left panel illustrates the difference between the mean-VaR model (2.17) with $\boldsymbol{\pi} = (2, 1)$ and $\boldsymbol{\pi} = (1, 2)$, i.e., $\Delta V = V((2,1)) - V((1,2))$, for different combinations of duration variability s and risk preference ω: positive differences mean $\boldsymbol{\pi}^*_\omega = (2,1)$; negative differences correspond to $\boldsymbol{\pi}^*_\omega = (1,2)$; intersections between the surface and the xy-plane refer to transitioning optimal results. Indeed, for $c = 2$, it can be observed that decision-makers prefer to execute stage 2 first if $s > 2.76$.

By analogy, the right panel shows the results corresponding to $\mathcal{R} \equiv$ $\text{CVaR}_{0.05}$ in (2.17). Here, we observe that a risk-averse decision-maker is more likely to execute stage 1 first, when the duration variability of stage 2 is of a moderate level, i.e., $1 < s < 3.72.$, still larger than that of stage 1. This can be attributed to CVaR being a more conservative risk measure than VaR, rendering a same expected NPV less attractive to decision-makers. Similar (unreported) results are obtained for $p < 0.05$.

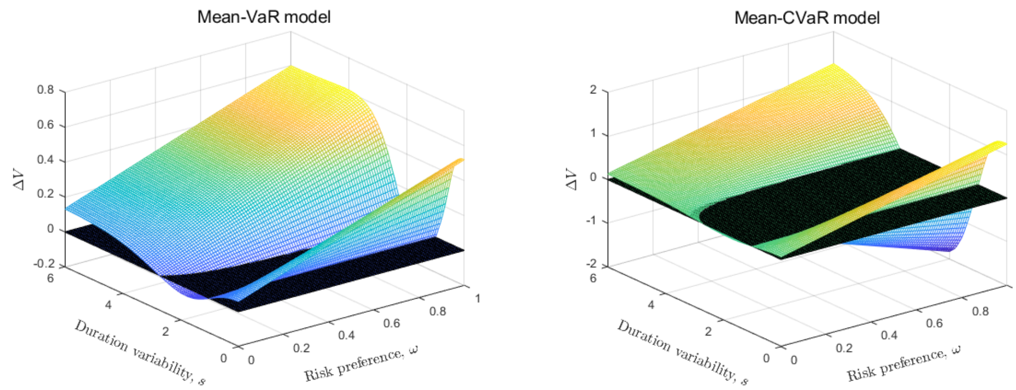

Fig. 2.6. Left panel: difference between mean-VaR model with schedule $\pi = (2,1)$ and $\pi = (1,2)$ for varying duration variability s and risk preference ω when $c = 2$. Right panel: same as left based, instead, on mean-CVaR model.

Next, the upper and lower panels of Table 2.3 show examples of the optimal schedule of three-stage and four-stage capacity expansions, respectively. As the number of stages increases, the optimal sequence of stages becomes more ambiguous. However, we can still observe that risk-averse decision-makers prefer to execute stages with lower duration variability first if the capacity expansions are very costly (e.g. $c = 6$ or 8). On the other hand, if the cost of each stage is relatively low (e.g. $c = 2$), it can be optimal for decision-makers to execute the stage with the highest duration variability first (shaded grid), which is also consistent with our previous results. Furthermore, we take a closer look at the two schedules that appear most frequently in Table 2.3 for each panel (i.e., $\pi = (3, 1, 2)$ and $\pi = (1, 2, 3)$ for $n = 3$, and $\pi = (4, 1, 2, 3)$ and $\pi = (1, 2, 3, 4)$ for $n = 4$), and present in the upper and lower panels of Figure 2.7 the expected NPV and 5%-quantile of the three-stage and four-stage project, respectively. Results suggest that, for projects with more stages, duration variability is still undesirable when the cost is high, while it can be beneficial if the cost is low due to higher expected NPV and lower risk exposure.

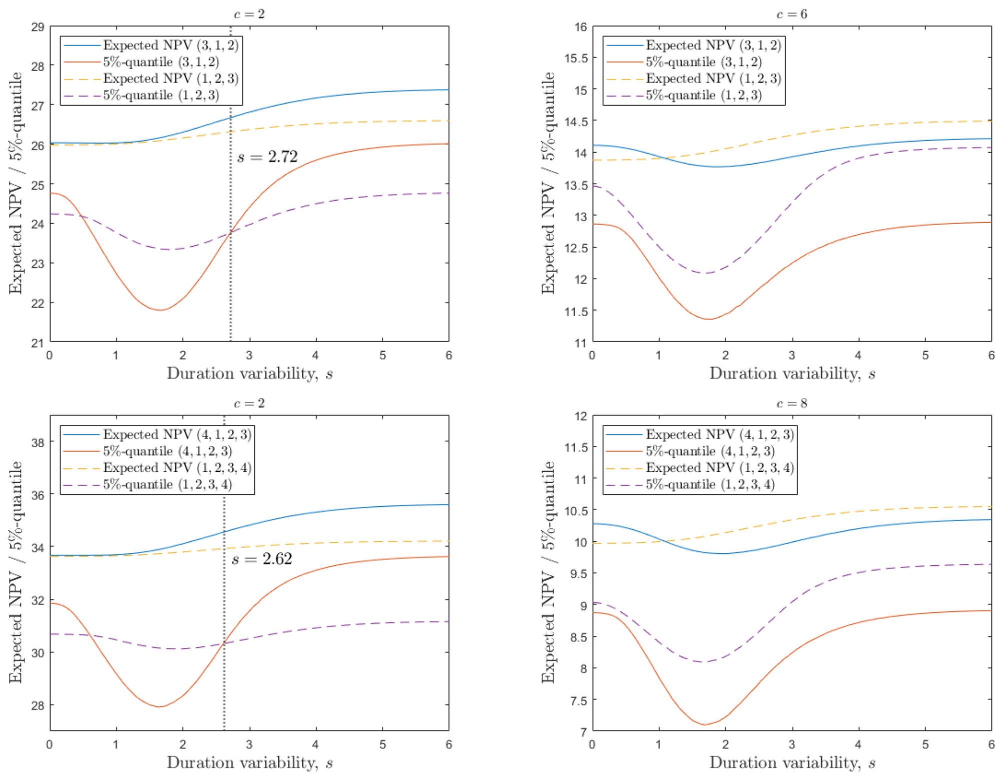

Fig. 2.7. Expected NPV and 5%-quantile of three-stage project (upper panel) as function of duration variability s for schedule $\pi = (3, 1, 2)$ (solid lines) and $\pi = (1, 2, 3)$ (dashed lines); and of four-stage project (lower panel) as function of duration variability s for schedule $\pi = (4, 1, 2, 3)$ (solid lines) and $\pi = (1, 2, 3, 4)$ (dashed lines).

In summary, the managerial insights of our results are threefold. First, for stages with equal size and expected duration, duration variability is unwanted when capacity expansions are costly, as this leads to lower expected NPV and higher risk of the project. Therefore, in this case, it is optimal for decision-makers (with any risk preference) to execute the stages with lower duration variability first. Second, we find that a project with higher duration variability can have larger expected

Table 2.3: Optimal schedule of three-stage project (upper panel) and four-stage project (lower panel) for decision-makers with risk appetite ω, unit cost c, and $D_j = 1.2$. For $n = 3$, $\tau_1 \sim \text{LogN}(\ln 3 - 0.09/2, 0.3)$, $\tau_2 \sim \text{LogN}(\ln 3 - 1/2, 1)$ and $\tau_3 \sim \text{LogN}(\ln 3 - s^2/2, s)$; for $n = 4$, $\tau_1 \sim \text{LogN}(\ln 3 - 0.09/2, 0.3)$, $\tau_2 \sim \text{LogN}(\ln 3 - 0.64/2, 0.8)$, $\tau_3 \sim \text{LogN}(\ln 3 - 1/2, 1)$ and $\tau_4 \sim \text{LogN}(\ln 3 - s^2/2, s)$.

$n = 3$										
	$c = 6$					$c = 2$				
	$\omega = 0$	$\omega = 0.25$	$\omega = 0.5$	$\omega = 0.75$	$\omega = 1$	$\omega = 0$	$\omega = 0.25$	$\omega = 0.5$	$\omega = 0.75$	$\omega = 1$
$s = 1.5$	(2, 1, 3)	(1, 2, 3)	(1, 2, 3)	(1, 2, 3)	(1, 2, 3)	(3, 1, 2)	(1, 2, 3)	(1, 2, 3)	(1, 2, 3)	(1, 2, 3)
$s = 2.5$	(2, 1, 3)	(1, 2, 3)	(1, 2, 3)	(1, 2, 3)	(1, 2, 3)	(3, 1, 2)	(3, 1, 2)	(1, 3, 2)	(1, 3, 2)	(1, 3, 2)
$s = 3.0$	(2, 1, 3)	(1, 2, 3)	(1, 2, 3)	(1, 2, 3)	(1, 2, 3)	(3, 1, 2)	(3, 1, 2)	(3, 1, 2)	(3, 1, 2)	(3, 1, 2)
$s = 4.0$	(2, 1, 3)	(1, 2, 3)	(1, 2, 3)	(1, 2, 3)	(1, 2, 3)	(3, 1, 2)	(3, 1, 2)	(3, 1, 2)	(3, 1, 2)	(3, 1, 2)
$n = 4$										
	$c = 8$					$c = 2$				
	$\omega = 0$	$\omega = 0.25$	$\omega = 0.5$	$\omega = 0.75$	$\omega = 1$	$\omega = 0$	$\omega = 0.25$	$\omega = 0.5$	$\omega = 0.75$	$\omega = 1$
$s = 1.5$	(3, 2, 1, 4)	(1, 3, 2, 4)	(1, 2, 3, 4)	(1, 2, 3, 4)	(1, 2, 3, 4)	(4, 2, 1, 3)	(1, 2, 3, 4)	(1, 2, 3, 4)	(1, 2, 3, 4)	(1, 2, 3, 4)
$s = 2.5$	(3, 2, 1, 4)	(1, 2, 3, 4)	(1, 2, 3, 4)	(1, 2, 3, 4)	(1, 2, 3, 4)	(4, 2, 1, 3)	(4, 1, 2, 3)	(1, 4, 2, 3)	(1, 4, 2, 3)	(1, 4, 2, 3)
$s = 3.0$	(3, 2, 1, 4)	(1, 2, 3, 4)	(1, 2, 3, 4)	(1, 2, 3, 4)	(1, 2, 3, 4)	(4, 2, 1, 3)	(4, 1, 2, 3)	(4, 1, 2, 3)	(4, 1, 2, 3)	(4, 1, 2, 3)
$s = 4.0$	(3, 2, 1, 4)	(1, 3, 2, 4)	(1, 2, 3, 4)	(1, 2, 3, 4)	(1, 2, 3, 4)	(4, 2, 1, 3)	(4, 1, 2, 3)	(4, 1, 2, 3)	(4, 1, 2, 3)	(4, 1, 2, 3)

NPV and is not always associated with worse downside risk if the expansion cost is relatively low. This suggests that, although variance reduction may be beneficial for scheduling a (resource-constrained) project with the aim of minimising its makespan, it can be harmful to the project financially. Finally, we demonstrate that the optimal schedule under risk aversion depends not only on decision-makers' risk preferences, but also on the level of duration variability and the cost of each stage.

2.6 Concluding discussion

In this chapter, we develop a risk assessment and optimal scheduling framework for sequential capacity expansion under output price and technological uncertainty. We derive the distribution, VaR and CVaR of the project's NPV. The novelty of our work lies in demonstrating the potentially positive impact of duration variability on the stochastic project scheduling problem, whereby we highlight the importance of risk considerations. We study the trade-off between maximising the expected NPV and minimising the downside risk of a serial project using a mean-risk model, and obtain the optimal investment strategies for risk-averse decision-makers.

We show that both the duration variability and the decision-makers' risk preferences can significantly affect the optimal sequence of stages of a serial project and that this also depends on the capacity expansion cost. More specifically, if the expansion cost of each stage is high, the duration variability is harmful to a project's NPV and risk exposure and, therefore, decision-makers should always execute stages with lower duration variability first. However, if the cost is relatively low, it can be optimal for risk-neutral decision-makers to execute the stages with higher duration variability first due to larger expected NPV. This coheres with Creemers et al. (2015), who show that operational variability may not lead to lower project values and, therefore, variance reduction strategies may not serve risk-neutral decision-makers. Taking also into account the decision-makers' attitudes towards risk, we find that executing stages with lower duration variability earlier does not guarantee lower risk exposure. Contrary to the intuition that increasing uncertainty entails greater risk exposure, our results indicate that higher duration variability may not lead to higher downside risk; instead, it may be beneficial not only for risk-neutral but also for risk-averse decision-makers.

This counter-intuitive result of non-monotonic relationship between the VaR of a project's NPV and its duration variability arises when the skewness of the duration increases with respect to its variance. Indeed, in such a case, high duration variability implies shorter makespan with increasing concentration around small values and that the project is expected to have larger profit and lower risk exposure (if expansion cost is low). Consequently, investing in variance reduction is only recommended when the duration variability of each stage is low. However, if the skewness of the duration distribution decreases with increasing variance, or the project is rather costly, then high duration variability can cause an opposite effect, which is unfavourable to both risk-neutral and risk-averse decision-makers.

Hence, this chapter conveys crucial implications for investment under technological uncertainty when the true distribution of a project's makespan is unknown, as ignoring or underestimating the uncertainty associated with the project may lead to inappropriate project scheduling and, therefore, lower NPV or greater downside risk. Directions for future research may include studying the potential effects of the price dynamics on the risk measures of the project, or the development of a real options framework to allow for discretion over investment timing (Heydari and Siddiqui, 2009; Jeon, 2021)[9]. The objective would be to investigate how managerial flexibility influences the distribution of the NPV and the risk measures of a serial project. Moreover, within a real options framework, the firm has the flexibility to choose not to proceed to the next stage, especially when investment costs are high or when operating costs are involved that may result in a negative profit flow.

In Chapter 3, we deviate from the stochastic scheduling problem and develop a real options framework in order to analyse how price uncertainty impacts investment and subsidisation policies in the light of strategic interactions between two firms and a Government. Unlike Chapter 2, the objective now is to facilitate the integration

[9] We do provide a real options framework in Appendix A.2 for risk assessment and optimal scheduling of a two-stage project, granting the firm the flexibility to postpone the investment before each stage.

of the necessary flexibility to adapt and modify strategies based on future events and new information.

Chapter 3

Optimal subsidy design and capacity investment under duopolistic competition and uncertainty

In this chapter, we develop a bi-level real options framework for deriving the equilibrium Government subsidisation and firm-level capacity investment policy in a duopoly market structure. We find that strategic interactions with the Government may impact a firm's capacity investment decision significantly and that the equilibrium subsidisation policy depends on both the market structure and the type of duopolistic competition. Interestingly, the provision of greater subsidy to the leader raises the follower's incentive to invest earlier and in a bigger project. The loss in value of the leader, due to the follower's entry, relative to the monopolist increases with economic uncertainty and, although a subsidy can mitigate this loss, its effect becomes less pronounced as economic uncertainty increases. We also find that a profit (welfare)-maximising Government does not offer (offers) a subsidy in a highly uncertain environment or upon low tax rate, while higher tax rate does not always decelerate investment. Finally, we find that while competition is always desirable for a social planner, a profit-maximising Government may benefit more under pre-emptive competition.

3.1 Introduction

Firms devising strategies for capacity investment in deregulated industries face the formidable challenge of managing not only the uncertainty in future revenue streams, but also the likely presence of a rival. It further complicates capacity investment decisions is the fact that they are often made in light of support schemes designed to incentivise investment in infrastructure projects, promote research and development (R&D) or accelerate the structural transformation of many industries due to pressing climate change concerns[1]. The design of support schemes may be subject to balancing conflicting objectives, as private firms pursue profit-maximisation objectives while a Government may maximise either profits associated with corporate tax (Lukas and Thiergart, 2019) or social welfare (Azevedo et al., 2021). The literature on methods for identifying *ex-ante* the level of Government support that aligns firm and Government-level optimisation objectives has grown considerably. However, existing bi-level models for optimal subsidy design are developed under the assumption of monopoly or perfect competition (Sarkar, 2012; Lukas and Thiergart, 2019) and, consequently, the implications of strategic interactions at the firm level for optimal subsidy design remain an important open research question. Additionally, it remains unclear how the optimal subsidisation strategy would differ if a Government pursued a social welfare rather than a profit-maximisation objective.

Analysing the joint implications of firm-level strategic interactions and the non-cooperating game between a private firm and the Government for optimal subsidy design is a challenging task, whereby the following trade-offs must be balanced. First, capacity investment decisions are particularly risky since a large capacity raises the downside risk during recession, whereas a low capacity may result in forgone revenues upon a sudden upturn in the economy. Second, the level of subsidy should be designed so that investment intensity targets are met in a timely manner. However, a high (low) subsidy may induce a firm to invest earlier (later) in a smaller

[1] For example, to encourage sustainable innovation, part of the American Recovery and Reinvestment Act of 2009 allocated $400 million to Advanced Research Projects Agency-Energy and $2.5 billion to renewable energy and energy efficiency R&D (CRS, 2009). Besides, Innovate UK, which supports business-led innovation in all sectors around UK, declares an increase in R&D funding from £700 million in 2021-22 to £1.1 billion in 2024-25 (Cookson, 2021).

(bigger) project. Third, upon a firm's investment, the Government receives a tax from the cash flows of the operating project and, therefore, it must balance the subsidy level so as to maximise its net tax income (i.e., profit) or social welfare. Finally, the Government needs to account for firm-level strategic interactions, since competition is likely to reduce the value of the subsidy, and, in turn, alter a firm's investment policy substantially.

In this chapter, we embed these trade-offs in a real options framework to address open research questions such as: How is a Government's subsidisation policy affected by firm-level strategic interactions? Does the subsidy offset a firm's loss in value due to competition? How does the equilibrium subsidisation and capacity investment strategy vary when a Government pursues social welfare instead of a profit maximisation objective? To address these questions, we consider a duopoly consisting of two identical (in terms of cost) firms that have the option to invest in a project. To incentivise investment, the Government will grant a subsidy to the first firm that enters the market (leader). Indeed, since the firms have the same level of investment cost per unit, subsidising the follower is not a plausible option as this will reduce the incentive to invest first. This is consistent with Nie et al. (2016), who show that the efficiency of subsidies depends on the position of the subsidised firm. In particular, the Government's subsidisation incentive reaches the lowest (highest) level if the subsidised firm is a follower (leader). The subsidy takes the form of a lump-sum cash grant.

By addressing these questions, our work bridges two strands of literature: bi-level real options and duopolistic competition. Regarding the latter, we consider the case of pre-emptive competition, where both firms have the incentive to invest first to gain a leader advantage, and non-pre-emptive competition with the role of the leader being assigned exogenously. While only the leader benefits from the subsidy, the follower's entry reduces the leader's expected revenues, thus implicitly affecting the subsidisation policy, as the alignment of the Government's and leader's objectives should account for the latter's loss in market share. Thus, the contribution of our work is threefold. First, we develop a bi-level real options framework to analyse the non-cooperative game between a Government and two symmetric firms. Second, we obtain the equilibrium investment threshold, project scale and subsidisation policy, and demonstrate how each depends on strategic interactions. Finally, we derive and compare the optimal investment and subsidisation strategies for the case of a profit and social welfare-maximising Government, and provide policy and managerial insights based on analytical and numerical results.

We proceed by discussing some related work in Section 3.2 and present assumptions and notation in Section 3.3.1. In Section 3.3.2, we present the analytical framework under monopoly and derive the equilibrium capacity investment policy of the firm as well as the equilibrium subsidisation policy of an income-maximising Government. We then expand Section 3.3.2 by allowing for non-pre-emptive and pre-emptive duopolistic competition in Section 3.3.3 and 3.3.4, respectively. Next, in Section 3.3.5 we explore how the optimal subsidisation and capacity investment policy changes when the Government optimises social welfare. Section 3.4 proceeds with various numerical examples, results and policy implications, whereas Section 3.5 concludes the chapter offering suggestions for further research.

3.2 Related research and our contribution

Although traditional real options models address the problem of optimal investment under uncertainty ignoring the implications of competition (McDonald and Siegel, 1985, 1986), the game-theoretic real options literature has grown considerably in recent years. Nevertheless, models that allow for strategic interactions often analyse their implications for investment timing without considering managerial discretion over project scale (Bar-Ilan and Strange, 1999; Dangl, 1999; Bøckman et al., 2008, Hagspiel et al., 2016a), or take no notice of the implications of the wide range of support schemes deployed to incentivise investment in many industries. Examples of real options models for strategic capacity investment include Huisman and Kort (2015), who analyse the problem of optimal capacity investment under duopolistic competition and demonstrate how discretion over project scale may be used strategically to deter or accommodate the entry of a rival. Other related examples include

Huberts et al. (2015) and Jeon (2021).

Examples of policy-oriented real options models include Boomsma et al. (2012), Boomsma and Linnerud (2015) and Ritzenhofen et al. (2016). More recently, Bigerna et al. (2019) consider a firm that has the option to invest in renewable energy under economic uncertainty and empirically analyse how a subsidy, in the form of a feed-in premium, affects its capacity investment policy. For a given environmental target, they derive the required investment scale and determine the corresponding optimal subsidy level and investment threshold. The contribution of this line of work includes the provision of policy insights not only on how various support schemes, such as feed-in tariffs, renewable portfolio standards and green certificate trading, may differ in incentivising green investments, but also on how random revisions of support schemes may impact investment incentives. However, the optimal investment and subsidisation policies are determined *ex-post*, and, thus, they do not reflect the equilibrium from the strategic interaction between a firm's and a Government's optimisation objectives.

Such strategic interactions are analysed in bi-level real options models, with the objective to understand private firms' investment behaviours and Governments' optimal subsidisation strategies under uncertainty (Pennings, 2000; Pennings, 2005; Yu et al., 2007). For example, Pennings (2000) studies how the Government's choice on the level of subsidy and taxation may impact a private firm's optimal investment strategy. Results indicate that, when the tax income is exactly offset by the subsidy, a firm can invest earlier as the tax rate increases, which renders subsidisation a more effective fiscal incentive. Other related examples include Danielova and Sarkar (2011), Barbosa et al. (2016), Tian (2018), Jin et al. (2021) and Silaghi and Sarkar (2021). Allowing for discretion over project scale, Lukas and Thiergart (2019) analyse the effect of uncertainty and investment stimulus (in the form of cash grants) on optimal investment timing, scaling and debt financing strategies. Their results indicate that, when the Government aims to maximise its profit, the relationship between the equilibrium subsidy and price uncertainty is ambiguous. Also, social welfare optimisation objectives are analysed in Pawlina and Kort (2006), Yang et al. (2018)

and Azevedo et al. (2021). The latter demonstrate the effect of a Government's subsidisation and taxation policy on a monopolist's capacity investment strategy and show that, by choosing the appropriate tax-subsidy package, the Government is able to implement a welfare-maximising policy.

Although existing bi-level real options models do not extend beyond the strategic interactions between a Government and a private firm, recent game-theoretic, albeit static, models demonstrate that the market structure can influence both the design of subsidies and the private firms' investment incentives (Wang and Zhou, 2020; Yang et al., 2021). For example, Yang and Nie (2015) analyse the effectiveness of different subsidy strategies under asymmetric duopoly. They find that, while subsidising the smaller firm benefits the social welfare, subsidising the larger firm can improve the total R&D investment output, especially when the cost gap between the firms is significant. Also, Nie et al. (2016) consider a unilateral and a bilateral subsidy and show that the firms' positioning is critical to a Government's subsidisation policy since the output of the subsidised firm is the highest (lowest) if the firm is the leader (follower). More recently, Yang et al. (2021) developed a game-theoretic model between a Government and two symmetric firms and derived the equilibrium the two firms can reach regarding their technology improvement decisions. Their results confirm that a subsidy is critical for expanding the green product market and improving social welfare. Allowing for economic uncertainty is an important extension of this line of work in analysing the implications of firm-level strategic interactions for optimal subsidy design in a deregulated environment.

Therefore, in this chapter, we develop a stylised, game-theoretic real options model for analysing the interaction between a Government and two symmetric firms (Grenadier, 1996; Thijssen et al., 2012). More specifically, we assume that the Government offers a unilateral subsidy to the first investor (leader) in the form of a cash grant, while imposing a tax rate on the firms' revenues. Our work does not consider bilateral subsidisation schemes, as these often arise in asymmetric competition (Lahiri and Ono, 1999; Toshimitsu, 2003; Yang and Nie, 2015; Nie et al., 2020), but focuses on how the entry of a follower, in the context of symmetric duopoly, impacts the alignment of the Government's and the leader's objectives regarding optimal subsidisation and capacity investment policies. We begin with the benchmark case of monopoly, which we subsequently extend to pre-emptive and non-pre-emptive duopolistic competition. In all cases, we derive the equilibrium subsidisation policy together with the equilibrium capacity investment policy for the monopolist, leader and follower. The subsidisation and capacity investment policies are derived under a profit and welfare-maximising Government, thus allowing comparison of results under optimisation objectives that have so far been considered separately in the existing literature.

Thus, our work contributes to the existing literature on strategic capacity investment that ignores either economic uncertainty (Nie et al., 2016) or strategic interactions between private firms and the Government (Huisman and Kort, 2015). By integrating these features in a bi-level real options framework, we are able to derive new insights on the Government's subsidisation policy and a firm's capacity investment policy. Our results complement prior contributions on duopolistic competition that ignores the interaction between private firms and a Government, as they indicate that both the market structure and the type of duopolistic competition can have a significant impact on the equilibrium subsidisation and capacity investment policy. Contrary to conventional intuition, we find that, even though the follower receives no support from the Government, they can actually benefit from the leader's subsidy and invest not only earlier but also in greater capacity. Moreover, it is shown that the loss in the value of the leader, due to the presence of a rival, relative to the monopolist increases with price uncertainty and that, although a subsidy can mitigate this loss, its effect becomes less pronounced as uncertainty increases. Our results also suggest that a profit (welfare)-maximising Government does not offer (offers) a subsidy in a highly uncertain environment or when the tax rate is low, while a higher tax rate does not always decelerate investment. Finally, we find that, while competition is always desirable for a social planner, a profit-maximising Government may benefit more under pre-emptive competition.

3.3 Problem formulation

3.3.1 Model setting

We consider two symmetric firms, each with a perpetual option to invest in a project of infinite lifetime. The firms have discretion over both the timing of investment and project scale and face demand uncertainty. The exogenous demand shock process is given by

$$dX_t = \mu X_t dt + \sigma X_t dW_t, \quad X_0 \equiv X, \tag{3.1}$$

where $t \geq 0$ denotes time, $\mu > 0$ is the annual growth rate, $\sigma > 0$ is the annual volatility and $W = \{W_t : t \geq 0\}$ is the standard Brownian motion. Also, we assume that both the private firms and the Government are risk-neutral and denote by $r > \mu$ the risk-free rate (Silaghi and Sarkar, 2021). Thus, as in Hagspiel et al. (2016b) and Jeon (2021), the output price P_t is given by

$$P_t = X_t(1 - \eta Q_t), \tag{3.2}$$

where Q_t is the total market output at time t and $\eta > 0$ is the price elasticity of the inverse demand function. Since the firms face no variable operating cost, we assume that, after investing, they both produce at full capacity[2] (Dobbs, 2004).

To incentivise investment under market structure $i \in \{m, p, n\}$, that is, **m**onopoly, **p**re-emptive duopoly, or **n**on-pre-emptive duopoly, the Government provides a unilateral subsidy to the first investor in the form of a lump-sum cash grant, denoted by S_i. This is consistent with Nie et al. (2016) and Jung and Feng (2020), who show that the Government has less incentive to subsidise the follower over the leader. Indeed, granting a subsidy to the follower may reduce a firm's incentive to invest first and, therefore, we do not consider this case. Subsidising the follower could be intended to offset a strategic disadvantage and would be pertinent within asymmetric competition, but less plausible in a symmetric duopoly where the firms

[2] In Chapter 3 and 4, we assume that the firms have to produce up to capacity. This assumption aligns with Dixit (1980) that entry deterrence strategies under competition are most effective when firms commit to operating at full capacity. Furthermore, when technology learning is considered, firms have a stronger motivation to maximise production capacity as it leads to cumulative experience, ultimately expediting cost reduction (Della Seta et al., 2012).

are assumed to be identical[3].

Since the Government receives tax income from the projects' operating cash flow in the form of a corporate tax $\tau \in [0,1]$, it may pursue a profit (π) maximisation objective. Alternatively, it may maximise social welfare (w). To distinguish between the two, we denote the equilibrium subsidy by $\widetilde{S}_i^k$, where $k \in \{\pi, w\}$; for example, $\widetilde{S}_m^\pi$ corresponds to the equilibrium subsidy under monopoly for a profit-maximising Government.

We denote by T_{ij} the random time at which firm $j \in \{l, f\}$, i.e., a **l**eader or a **f**ollower, respectively, enters the industry. Also, we denote the investment threshold of each firm by X_{ij} and its capacity by Q_{ij}, with optimal thresholds $\left(X_{ij}^*, Q_{ij}^*\right)$ and equilibrium thresholds $\left(\widetilde{X}_{ij}^k, \widetilde{Q}_{ij}^k\right)$. For example, if the subsidy is exogenously defined, then the optimal investment threshold and project scale of the non-pre-emptive leader is denoted by X_{nl}^* and Q_{nl}^*, respectively. If the subsidy is endogenously defined via alignment of the firm's and the Government's optimisation objectives, then the equilibrium investment threshold and project scale are denoted by $\widetilde{X}_{nl}^\pi = X_{nl}^*\left(\widetilde{S}_n^\pi\right)$ and $\widetilde{Q}_{nl}^\pi = Q_{nl}^*\left(\widetilde{S}_n^\pi\right)$, respectively. The investment cost $I(\cdot)$ is assumed to be a linear function of the installed capacity, i.e., $I(Q_{ij}) = \delta Q_{ij}$, $\delta > 0$ (Bigerna et al., 2019; Nagy et al., 2021). Finally, we denote by $F_{ij}(\cdot)$ the value of the firm's investment opportunity, by $V_{ij}(\cdot)$ the expected value of the active project, and by $G_i^k(\cdot)$ the Government's value function. As in, for example, Dangl (1999), each firm's optimisation objective at time t is summarised as

$$F_{ij}\left(X_t, t\right) = \max\left\{e^{-r\Delta t}\mathbb{E}_{X_t}\left[F_{ij}(X_{t+\Delta t}, t+\Delta t)\right], \max_{Q_{ij}} V_{ij}\left(X_t, Q_{ij}\right)\right\}, \tag{3.3}$$

where $\mathbb{E}_{X_t}[\cdot]$ denotes the expectation conditional on the demand shock level X_t. The outer maximisation represents the firm's decision to either postpone investment or invest immediately at time t. As suggested by the first argument, if the firm defers investment for a time interval Δt, then its return is the discounted expected value

[3] The Government may threaten to subsidise the follower to force the leader accept a smaller subsidy and, to maintain the first-mover advantage, a firm will always accept a smaller subsidy. However, the Government does not necessarily benefit from such a bargain, as offering a smaller subsidy to the leader delays both firms' investment, and the present value of their revenues upon investment is reduced due to the discounting effect.

(conditional on the current demand level X_t) of waiting to invest after Δt, reflecting the expected capital appreciation of the option to invest. This value is compared with the second argument that reflects the firm's value function under immediate investment, where the firm must choose the size of the project so as to maximise its expected net present value at investment.

3.3.2 Monopoly

This problem has already been examined by Lukas and Thiergart (2019), albeit in the absence of an inverse demand function. By contrast, we assume here that the firm has market power, and present the results under the assumptions of Section 3.3.1 for ease of reference and comparison with the case of duopoly. As shown at the top of Figure 3.1, the monopolist can choose the investment time, T_m, at which they install the capacity Q_m and incur the investment cost, δQ_m, less the cash grant, S_m. Meanwhile, upon the monopolist's investment, the Government receives a perceptual stream of tax income from the operating project.

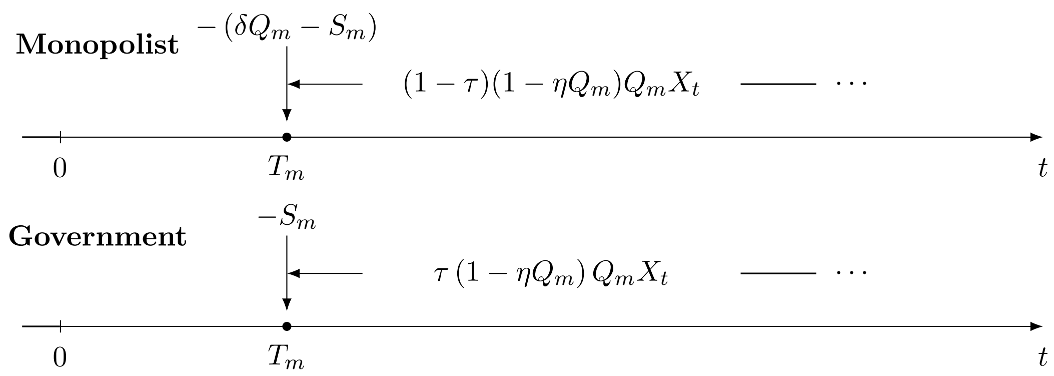

Figure 3.1: Irreversible capacity investment and subsidy design under monopoly.

We first assume that the monopolist does not have the option to postpone investment and, therefore, must invest immediately. By exercising a now-or-never investment opportunity, the monopolist knows the price of the output and must determine the corresponding size of the project by maximising, with respect to Q_m, the discounted to time zero expected project value given by

$$V_m(X, Q_m, S_m) = \mathbb{E}_X \left[\int_0^\infty (1-\tau)(1-\eta Q_m) Q_m X_t e^{-rt} dt - (\delta Q_m - S_m) \right]. \quad (3.4)$$

The optimal capacity satisfying

$$\Phi_m(X, S_m) = \max_{Q_m} V_m(X, Q_m, S_m)$$

is obtained by applying the first-order necessary condition (FONC) to (3.4) and is given by

$$Q_m^* = \frac{1}{2\eta}\left(1 - \frac{\delta(r-\mu)}{(1-\tau)X}\right). \tag{3.5}$$

Next, we assume that the demand is too low to justify immediate investment. Subject to the optimal capacity choice at investment, i.e., the inner maximisation in (3.3), the monopolist's objective upon deferred investment is to determine the random first-passage time of X_t through the investment threshold from below, i.e., $T_m = \inf\{t \geq 0 : X_t \geq X_m\}$. The monopolist's optimisation objective is

$$F_m(X, S_m) = \sup_{T_m \in \mathcal{S}} \mathbb{E}_X \left[\int_{T_m}^{\infty} (1-\tau)(1-\eta Q_m^*) Q_m^* X_t e^{-rt} dt - (\delta Q_m^* - S_m) e^{-rT_m}\right], \tag{3.6}$$

where $\mathcal{S}$ is the set of stopping times of the filtration generated by X_t. We can then rewrite (3.6) using the law of iterated expectations and the strong Markov property of the geometric Brownian motion[4]:

$$F_m(X, S_m) = \sup_{T_m \in \mathcal{S}} \mathbb{E}_X \left[e^{-rT_m}\right] \Phi_m(X_m, S_m) = \max_{X_m > X} \left(\frac{X}{X_m}\right)^{\beta} \Phi_m(X_m, S_m), \tag{3.7}$$

where the second equality follows from the stochastic discount factor $\mathbb{E}_X\left[e^{-rT_m}\right] = (X/X_m)^{\beta}$ (Dixit and Pindyck, 1994, p. 315), with $\beta > 1$ the positive root of $\sigma^2 x(x-1)/2 + \mu x - r = 0$.

By applying the FONC to the unconstrained optimisation problem (3.7) and integrating condition (3.5) for optimal capacity choice at investment, where we set $X = X_m^*$, we obtain the optimal investment policy. All proofs can be found in Appendix B.

Proposition 3.3.1. *The following results hold:*

[4] For each $s \geq 0$, X_{T_m+s} is independent of the past given X_{T_m}.

1. *The optimal investment threshold under monopoly is given by*

$$X_m^*(S_m) = \begin{cases} \max\{X, c(S_m)\}, & \text{if } S_m \leq \frac{\left(\beta - \sqrt{\beta^2-1}\right)\delta}{2\beta\eta} \\ X, & \text{if otherwise} \end{cases} \tag{3.8}$$

and the optimal capacity by

$$Q_m^*(S_m) = \begin{cases} \frac{\delta + 2\beta\eta S_m + \sqrt{\delta^2 - 4\beta^2\eta S_m(\delta - \eta S_m)}}{2(\beta+1)\eta\delta}, & \text{if } X < X_m^*(S_m) \\ \frac{1}{2\eta}\left(1 - \frac{\delta(r-\mu)}{(1-\tau)X}\right), & \text{if otherwise} \end{cases}, \tag{3.9}$$

where

$$c(S_m) = \frac{r-\mu}{1-\tau} \frac{(\beta+1)\delta^2}{\beta\delta - 2\beta\eta S_m - \sqrt{\delta^2 - 4\beta^2\eta S_m(\delta - \eta S_m)}}.$$

2. *Both the optimal investment threshold and optimal capacity decrease with increasing subsidy*[5].

As suggested in the bottom part of (3.8), if the subsidy is high enough, it outweighs the value of waiting and the monopolist is better off investing immediately and installing the capacity indicated in the bottom part of (3.9). Otherwise, the optimal investment policy is given in the top parts of (3.8) and (3.9).

Next, we analyse the Government's decision and derive the optimal (equilibrium) subsidy. Upon the monopolist's investment at T_m, the Government receives a perceptual stream of tax income from the operating project. Hence, the Government's discounted net income at time 0 is given by

$$\begin{aligned} G_m^\pi(X, S_m) &= \mathbb{E}_X\left[\int_{T_m^*}^{\infty} \tau(1 - \eta Q_m^*(S_m))Q_m^*(S_m)X_t e^{-rt}dt - S_m e^{-rT_m^*}\right] \\ &= \left(\frac{X}{X_m^*(S_m)}\right)^\beta \left[\tau(1 - \eta Q_m^*(S_m))Q_m^*(S_m)\frac{X_m^*(S_m)}{r-\mu} - S_m\right]. \end{aligned} \tag{3.10}$$

We assume that the Government chooses the level of subsidy so as to maximise

[5]The latter is due to the assumption of a lump-sum subsidy that remains constant regardless of the firm's investment size. However, if the Government offers a subsidy commensurate with the investment size, e.g., sQ for $s \geq 0$, the subsidy per unit of output, s, will have no impact on the firm's optimal capacity.

its net income, i.e., $\widetilde{S}_m^\pi = \text{argmax}_{S_m \geq 0} G_m^\pi(X, S_m)$. The equilibrium subsidisation strategy for the Government is given in the following proposition.

Proposition 3.3.2. *The equilibrium subsidy of a profit-maximising Government under monopoly is*

$$\widetilde{S}_m^\pi = \begin{cases} 0, & \text{if } \frac{r-\mu}{1-\tau}\frac{\beta+1}{\beta-1}\delta \leq X \\ \min\{S_1, S_2\}, & \text{if } \frac{r-\mu}{1-\tau}\frac{\beta+1}{\sqrt{\beta^2-1}}\delta \leq X < \frac{r-\mu}{1-\tau}\frac{\beta+1}{\beta-1}\delta \\ S_1, & \text{if otherwise} \end{cases} \tag{3.11}$$

with

$$S_2 = \frac{1}{4\beta\eta}\frac{\delta^2 - A^2}{\beta\delta - A} \tag{3.12}$$

and

$$S_1 = \begin{cases} \frac{\delta\theta}{2\beta\eta\psi}, & \text{if } \tau > \frac{1}{\beta+1} \\ 0, & \text{if otherwise} \end{cases}, \tag{3.13}$$

where

$$A = \beta\delta - \frac{r-\mu}{1-\tau}\frac{(\beta+1)\delta^2}{X}, \quad \psi = \left(\frac{\tau}{1-\tau} - \frac{1}{\beta}\right)^{-2} - 1 \quad \text{and} \quad \theta = \sqrt{\beta^2 + \psi} - \beta.$$

As indicated in the top part of (3.11), the Government will not grant a subsidy if the output price is high enough to allow the firm to invest immediately. Conversely, if the output price is too low to justify immediate investment (bottom part), the Government will grant a subsidy given in the top part of (3.13) if the corporate tax rate is greater than the critical value $1/(\beta+1)$[6]. Finally, according to the middle part of (3.11), the firm will postpone investment (invest immediately) in the absence (presence) of a sufficiently high subsidy, where S_2 is the minimum subsidy required by the firm to undertake immediate investment. Notice that S_1 is independent of X and that the equilibrium subsidy depends on X only when $\widetilde{S}_m^\pi = S_2$, where S_2 decreases with increasing X. Having derived the equilibrium subsidy, we can

[6] It is worth noting that this critical value, $1/(\beta+1)$, increases (decreases) with greater uncertainty and growth rate (interest rate). This implies that the Government tends not to provide a subsidy when uncertainty is high.

now introduce it into (3.8) and (3.9) to obtain the equilibrium investment threshold $\widetilde{X}_m^\pi = X_m^*\left(\widetilde{S}_m^\pi\right)$ and equilibrium capacity $\widetilde{Q}_m^\pi = Q_m^*\left(\widetilde{S}_m^\pi\right)$.

Corollary 3.3.1. *The equilibrium investment threshold under monopoly is given by*

$$\widetilde{X}_m^\pi = \begin{cases} \frac{r-\mu}{1-\tau}\frac{(\beta+1)\delta\psi}{\beta\psi-\theta\left(\sqrt{\psi+1}+1\right)}, & \text{if } \tau > \frac{1}{\beta+1} \\ \frac{r-\mu}{1-\tau}\frac{\beta+1}{\beta-1}\delta, & \text{if otherwise} \end{cases},$$

and the equilibrium capacity is given by

$$\widetilde{Q}_m^\pi = \begin{cases} \frac{\psi+\theta\left(\sqrt{\psi+1}+1\right)}{2(\beta+1)\eta\psi}, & \text{if } \tau > \frac{1}{\beta+1} \text{ and } X < \widetilde{X}_m^\pi \\ \frac{1}{(\beta+1)\eta}, & \text{if } \tau \leq \frac{1}{\beta+1} \text{ and } X < \widetilde{X}_m^\pi \\ \frac{1}{2\eta}\left(1-\frac{\delta(r-\mu)}{(1-\tau)X}\right), & \text{if } X \geq \widetilde{X}_m^\pi \end{cases}.$$

Consistent with extant contributions, we find that a larger subsidy accelerates investment, thereby resulting in installing less capacity (see Proposition 3.3.1). We also confirm that a higher tax rate reduces the firm's incentive to invest and must be offset by a larger subsidy in order to stimulate investment, as shown in Proposition 3.3.3. Note that this subsidy can never be infinite and is bounded from above by $(\beta - \sqrt{\beta^2-1})\delta/(2\beta\eta)$. This is because as τ approaches 1, the firm's incentive to invest is extremely low and a large amount of subsidy is required to reduce the investment threshold by a small amount. However, from the Government's perspective, it is not worth it because the extra tax income is not sufficient to cover the cost of providing a greater subsidy. Consistent with conventional real options literature, the results suggest that uncertainty increases the investment threshold and the amount of installed capacity, however, as we will show in Section 3.4, its effect on equilibrium subsidy is ambiguous and the Government is not willing to provide a subsidy when uncertainty is high.

Proposition 3.3.3. *The equilibrium subsidy increases with the corporate tax rate while the monopolist's equilibrium capacity decreases.*

Furthermore, by allowing for an inverse demand function, we observe that the optimal (equilibrium) investment threshold depends on (is independent of) the price elasticity of demand, η, as shown in Proposition 3.3.4. Intuitively, the investment scale shrinks as η increases, so the Government has a greater incentive to grant smaller subsidy. In turn, the reduction in subsidy postpones investment, thereby offsetting the effect of increased η.

Proposition 3.3.4. *An increase in the elasticity of demand decreases both the subsidy and firm's equilibrium capacity but has no effect on the investment threshold.*

3.3.3 Non-pre-emptive duopoly

We begin with the symmetric, non-pre-emptive duopoly (Goto et al., 2008; Mason and Weeds, 2010; Siddiqui and Takashima, 2012), where the leader's role is assigned exogenously. As shown in Figure 3.2, the leader enjoys monopoly profits from time T_{nl} until the follower's entry at time T_{nf}[7]. Upon that, the total market output increases from Q_{nl} to $Q_{nl} + Q_{nf}$, whereas the market price per unit output drops from $(1 - \eta Q_{nl}) X_t$ to $(1 - \eta Q_{nl} - \eta Q_{nf}) X_t$. This trade-off directly affects the tax income of the Government along with the subsidy level, and, in what follows, we conduct a step-by-step analysis in the order of follower, leader and Government to obtain their optimal strategy.

3.3.3.1 Follower

We assume that the leader has already entered the market, and begin by analysing the follower's capacity investment policy. Given the leader's optimal capacity, the follower's value of the active project is

$$V_{nf}\left(X, Q_{nf}; Q_{nl}\right) = \mathbb{E}_X \left[\int_0^{\infty} (1-\tau)\left(1 - \eta Q_{nl} - \eta Q_{nf}\right) Q_{nf} X_t e^{-rt} dt - \delta Q_{nf} \right]. \quad (3.14)$$

In solving $\Phi_{nf}\left(X; Q_{nl}\right) = \max_{Q_{nf}} V_{nf}\left(X, Q_{nf}; Q_{nl}\right)$, we get, by applying the FONC, the follower's condition for optimal capacity at investment

$$Q^*_{nf}\left(X; Q_{nl}\right) = \frac{1}{2\eta}\left(1 - \eta Q_{nl} - \frac{\delta(r-\mu)}{(1-\tau)X}\right). \quad (3.15)$$

[7]This plot corresponds to the leader's entry *deterrence* strategy as introduced in Huisman and Kort (2015). Considerations related to other strategies are discussed later on, e.g., when the follower undertakes immediate investment following the leader's actions.

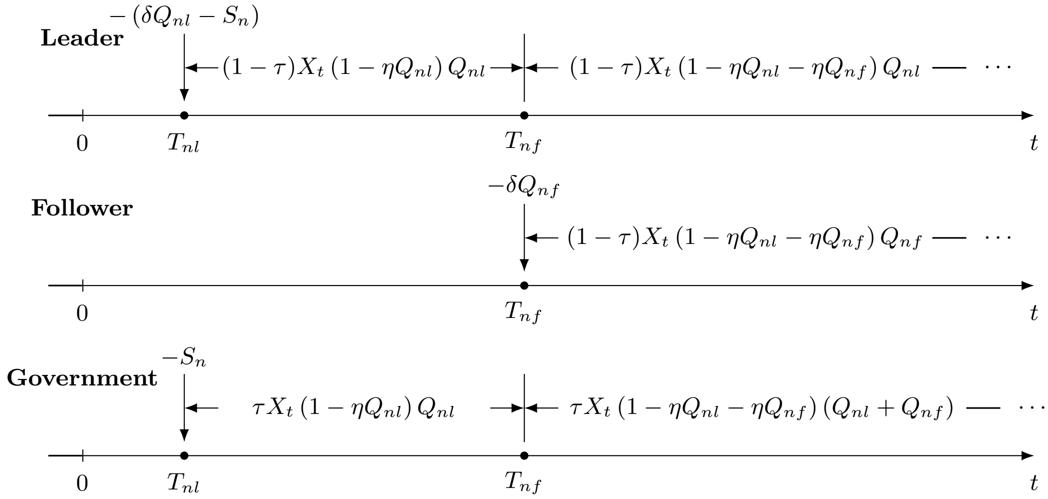

Figure 3.2: Irreversible capacity investment and subsidy design under duopolistic competition.

Next, we assume that immediate investment is not possible and, similarly to the case of monopoly, we derive the follower's expected option value

$$\begin{aligned} F_{nf}\left(X; Q_{nl}\right) &= \sup_{T_{nf} \in \mathcal{S}} \mathbb{E}_X \left[\int_{T_{nf}}^{\infty} (1-\tau)\left(1 - \eta Q_{nl} - \eta Q_{nf}^*\right) Q_{nf}^* X_t e^{-rt} dt - \delta Q_{nf}^* e^{-rT_{nf}} \right] \\ &= \max_{X_{nf} > X} \left(\frac{X}{X_{nf}} \right)^{\beta} \Phi_{nf}\left(X_{nf}; Q_{nf}\right). \end{aligned} \tag{3.16}$$

The FONC applied to (3.16) together with the condition for optimal capacity choice at investment yield the expression for the optimal investment threshold, $X_{nf}^*\left(Q_{nl}\right)$, and the optimal capacity, $Q_{nf}^*\left(Q_{nl}\right)$, as shown in Proposition 3.3.5. Note that, unlike the case of monopoly, the optimal investment policy of the follower depends on that of the leader.

Proposition 3.3.5. *For $Q_{nl} < 1/\eta$, the optimal investment threshold of the follower is given by*

$$X_{nf}^*\left(Q_{nl}\right) = \frac{\beta+1}{\beta-1} \frac{r-\mu}{1-\tau} \frac{\delta}{1-\eta Q_{nl}} \tag{3.17}$$

and the optimal capacity by

$$Q_{nf}^*\left(Q_{nl}\right) = \frac{1-\eta Q_{nl}}{(\beta+1)\eta}. \tag{3.18}$$

3.3.3.2 Leader

As shown in Figure 3.2, the leader enjoys monopoly profits from T_{nl} until T_{nf}. The active project value of the non-pre-emptive leader is given by

$$\begin{aligned} V_{nl}(X, Q_{nl}, S_n) &= \mathbb{E}_X\left[\int_0^{T^*_{nf}} (1-\tau)(1-\eta Q_{nl}) Q_{nl} X_t e^{-rt} dt - (\delta Q_{nl} - S_n)\right] \\ &\quad + \mathbb{E}_X\left[\int_{T^*_{nf}}^{\infty} (1-\tau)\left(1-\eta Q_{nl} - \eta Q^*_{nf}\right) Q_{nl} X_t e^{-rt} dt\right] \\ &= \frac{1-\tau}{r-\mu}(1-\eta Q_{nl}) Q_{nl} X - \frac{1-\tau}{r-\mu}\eta Q^*_{nf} Q_{nl} X^*_{nf}\left(\frac{X}{X^*_{nf}}\right)^{\beta} \\ &\quad - (\delta Q_{nl} - S_n), \end{aligned} \tag{3.19}$$

where the first term reflects the monopoly profits of the leader in the absence of the follower, the second term is the expected loss in value due to the follower's entry, and the third term is the investment cost reduced by the subsidy. We maximise the leader's active project value, that is, $\Phi_{nl}(X, S_n) = \max_{Q_{nl}} V_{nl}(X, Q_{nl}, S_n)$, and the option value of the non-pre-emptive leader is given by

$$F_{nl}(X, S_n) = \max_{X_{nl} > X}\left(\frac{X}{X_{nl}}\right)^{\beta} \Phi_{nl}(X_{nl}, S_n). \tag{3.20}$$

Solving (3.20) gives the optimal investment threshold, $X^*_{nl}(S_n)$, and optimal capacity, $Q^*_{nl}(S_n)$, of the leader, as we show in Proposition 3.3.6.

Proposition 3.3.6. *For $X < X^*_{nl}$, the optimal capacity of the leader under non-pre-emptive duopoly is obtained as the solution to*

$$\delta\left(1-\left(\frac{\beta}{\beta+1}\frac{\delta Q_{nl} - S_n}{\delta Q_{nl}}\right)^{\beta}\right)(1-(\beta+1)\eta Q_{nl}) Q_{nl} - (1-2\eta Q_{nl})\beta S_n = 0, \tag{3.21}$$

and the optimal investment threshold is given by

$$X^*_{nl}(S_n) = \max\left\{X, \frac{\beta}{\beta-1}\frac{r-\mu}{1-\tau}\frac{\delta Q^*_{nl} - S_n}{(1-\eta Q^*_{nl}) Q^*_{nl}}\right\}. \tag{3.22}$$

For $X^*_{nf} > X \geq X^*_{nl}$, the optimal investment capacity of the leader is the solution to

$$\frac{1-\tau}{r-\mu}(1-2\eta Q_{nl}) X - \delta - \frac{\delta}{\beta-1}\left(\frac{\beta-1}{\beta+1}\frac{1-\tau}{r-\mu}\frac{1-\eta Q_{nl}}{\delta}X\right)^{\beta}\left(\frac{1-(\beta+1)\eta Q_{nl}}{1-\eta Q_{nl}}\right) = 0$$

and, for $X \geq X^*_{nf} \geq X^*_{nl}$,

$$Q^*_{nl}(S_n) = \frac{1}{2\eta}\left(1 - \frac{r-\mu}{1-\tau}\frac{\delta}{X}\right), \text{ for } X \geq \frac{r-\mu}{1-\tau}\delta. \tag{3.23}$$

Note that by setting $S_n = 0$ and $\tau = 0$, we can retrieve the optimal investment policy presented in Huisman and Kort (2015), where the non-pre-emptive leader's investment decision aligns with that of the monopolist (see Proposition 3.3.1). More specifically, the optimal investment decisions as indicated in (3.21) and (3.22) correspond to what Huisman and Kort (2015) introduced as the entry *deterrence* strategy. That is, when the initial price, X, is low, the leader can deter the follower from entering the market and enjoy a period of monopoly by investing in a capacity greater than a certain level[8]. Indeed, from (3.17) and (3.22), we can show that $X^*_{nl}(S) < X^*_{nf}$ for any $S \geq 0$ if X is low. On the other hand, when the initial price, X, is high, the leader's ability to deter the follower's entry may diminish. Under such circumstances, the leader can apply an entry *accommodation* strategy by opting for a smaller capacity, which will trigger the follower to make its investment immediately afterward[9] (see the last scenario in Proposition 3.3.6). This study places its primary emphasis on comprehending the disparities between the investment thresholds of the leader and the follower. Accordingly, we restrict our analysis to the scenario where the initial price level is low, to the extent that it always favors the leader's adoption of an entry deterrence strategy, leading to a situation where neither firm pursues immediate investment.

Next, we analyse the optimal subsidisation policy of the Government. Following from the bottom part of Figure 3.2, the Government's value function is formulated

[8] As Proposition 3 in Huisman and Kort (2015) highlights, there exists a specific range for the investment threshold within which the leader will contemplate the entry deterrence strategy. In our particular context, this deterrence interval is defined as (X_1, X_2), with X_1 being the solution to the equation $\frac{1-\tau}{r-\mu}X_1 - \delta - \left(\frac{1-\tau}{r-\mu}\frac{\beta-1}{\beta+1}\frac{X_1}{\delta}\right)^{\beta}\frac{\delta}{\beta-1} = 0$ and $X_2 = 2\frac{\beta-1}{\beta+1}\frac{r-\mu}{1-\tau}\delta$.

[9] Particularly, the leader will only employ this entry accommodation strategy if the optimal capacity, as indicated in (3.23), results in the immediate investment of the follower. Interested readers may refer to Proposition 4 in Huisman and Kort (2015).

as

$$\begin{aligned}
G_n^\pi(X, S_n) &= \mathbb{E}_X\left[\int_{T_{nl}^*}^{T_{nf}^*} \tau\left(1-\eta Q_{nl}^*(S_n)\right) Q_{nl}^*(S_n) X_t e^{-rt} dt - S_n e^{-rT_{nl}^*}\right] \\
&+ \mathbb{E}_X\left[\int_{T_{nf}^*}^{\infty} \tau\left(1-\eta Q_{nl}^*(S_n)-\eta Q_{nf}^*(S_n)\right)\left(Q_{nl}^*(S_n)+Q_{nf}^*(S_n)\right) X_t e^{-rt} dt\right] \\
&= \left(\frac{X}{X_{nl}^*(S_n)}\right)^\beta \left(\tau\left(1-\eta Q_{nl}^*(S_n)\right) Q_{nl}^*(S_n) \frac{X_{nl}^*(S_n)}{r-\mu} - S_n\right) \\
&+ \left(\frac{X}{X_{nf}^*(S_n)}\right)^\beta \tau\left(1-2\eta Q_{nl}^*(S_n)-\eta Q_{nf}^*(S_n)\right) Q_{nf}^*(S_n) \frac{X_{nf}^*(S_n)}{r-\mu},
\end{aligned} \tag{3.24}$$

where the first term is the discounted present value of the Government's net tax income with only one firm in the market, reduced by the subsidy; the second term reflects the trade-off, whereby the total market output (instantaneous revenue) increases (decreases) upon the follower's entry. Again, the Government will set the subsidy level so as to maximise its profit, i.e., $\widetilde{S}_n^\pi = \text{argmax}_{S_n \geq 0} G_n^\pi(X, S_n)$, which in this case is solved numerically. By inserting $\widetilde{S}_n^\pi$ in (3.17), (3.18), (3.21) and (3.22), we obtain the equilibrium capacity, $\widetilde{Q}_{nj}^\pi$, and investment threshold, $\widetilde{X}_{nj}^\pi$, for both firms.

3.3.4 Pre-emptive duopoly

Here, we consider a non-cooperative game in which both leader and follower roles are not assigned exogenously, therefore both firms have the incentive to pre-empt each other to receive financial support. Note that, since the follower enters the market after the leader has invested, the follower's optimal investment policy is the same as in Section 3.3.3.1. However, in contrast to Section 3.3.3.2, the optimal investment threshold of the pre-emptive leader must be determined by considering the strategic interactions with the follower. Therefore, we begin with the investment decision of the leader. For a given X_{pl}, the pre-emptive leader's capacity should maximise its active project value:

$$\Phi_{pl}\left(X_{pl}, S_p\right) = \max_{Q_{pl} \geq 0} \left\{\frac{1-\tau}{r-\mu}\left(1-\eta Q_{pl}\right) Q_{pl} X_{pl} - \left(\delta Q_{pl} - S_p\right) - \frac{1-\tau}{r-\mu} \eta Q_{pf}^* Q_{pl} X_{pf}^* \left(\frac{X_{pl}}{X_{pf}^*}\right)^\beta\right\}. \tag{3.25}$$

The optimal capacity of the pre-emptive leader solves

$$\frac{1-\tau}{r-\mu}\left(1-2\eta Q_{pl}\right)X_{pl}-\delta-\frac{\delta}{\beta-1}\left(\frac{\beta-1}{\beta+1}\frac{1-\tau}{r-\mu}\frac{1-\eta Q_{pl}}{\delta}X_{pl}\right)^{\beta}\left(\frac{1-(\beta+1)\eta Q_{pl}}{1-\eta Q_{pl}}\right)=0. \tag{3.26}$$

Next, to determine the pre-emptive leader's optimal investment threshold, we consider the strategic interactions between the leader and the follower. As in Huisman and Kort (2015), the pre-emption trigger is defined as the intersection point of the option value of the follower, $F_{pf}\left(X_{pl};Q_{pl}(S_p)\right)$, and the active project value of the leader, $\Phi_{pl}\left(X_{pl},S_p\right)$, as formulated in (3.16) and (3.25), respectively. Therefore, given a subsidy level S_p, the pre-emption trigger $\widehat{X}$ is the solution to equation

$$F_{pf}\left(\widehat{X};Q_{pl}^{*}\left(\widehat{X},S_p\right)\right)=\Phi_{pl}\left(\widehat{X},S_p\right). \tag{3.27}$$

Intuitively, as illustrated in Figure 3.3, for $X_{pl}<\widehat{X}$ ($X_{pl}>\widehat{X}$) the demand is low (high), and the expected option value of the follower is greater (smaller) than the active project value of the leader so that each firm is better off being the follower (leader). For a given investment threshold X_{pl}, each firm can pre-empt the other by investing at a lower threshold, i.e., $X_{pl}-\epsilon$ for $\epsilon>0$. This continues until a firm is indifferent between being the leader or the follower, which happens at the intersection of the follower's option value and the leader's project value[10]. Note also that $\Phi_{pl}\left(X_{pl},S_p\right)$ is a linear function of S_p and that an increase in the subsidy shifts the leader's project value curve upwards (solid arrow). This leads to a leftward movement of the intersection of the two curves along the follower's option value curve (grey), which lowers the investment trigger (dashed arrow).

3.3.5 Social welfare

From a social planner's perspective, the goal of the Government is to maximise the social welfare or total surplus, i.e., the sum of producer surplus (PS), consumer surplus (CS) and Government's revenues. Given that the latter are expenses for the

[10] It is worth noting that the pre-emption game operates in a Stackelberg setup, wherein the pre-emptive follower optimally reacts to the decisions of the pre-emptive leader. Cournot competition, more relevant in an accommodation scenario (i.e., only when the initial price is high), is not applicable here due to the significantly lower investment threshold of the leader under pre-emptive duopoly.

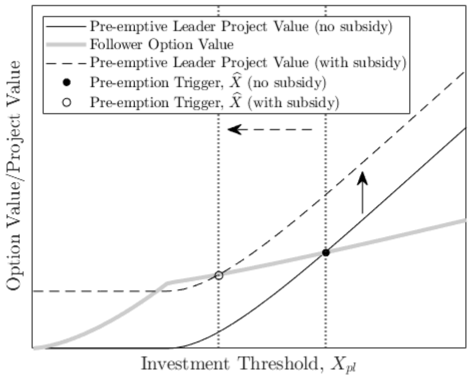

Figure 3.3: Active project value of leader with $S_p = 0$ (solid line) and $S_p = 0.15$ (dashed line), and option value of follower (grey).

firm, a social planner maximises the sum of producer surplus and consumer surplus without taxes and subsidies (Azevedo et al., 2021). Hence, following Huisman and Kort (2015), the discounted consumer surplus is given by

$$CS_m(X, S_m) = \frac{1}{2(r-\mu)} \eta X_m^*(S_m) {Q_m^*}^2(S_m) \left(\frac{X}{X_m^*(S_m)} \right)^{\beta}, \tag{3.28}$$

and the expected present value of the producer surplus is

$$PS_m(X, S_m) = \left(\frac{1}{r-\mu} X_m^*(S_m)(1 - \eta Q_m^*(S_m)) - \delta \right) Q_m^*(S_m) \left(\frac{X}{X_m^*(S_m)} \right)^{\beta}, \tag{3.29}$$

where $X_m^*(S_m)$ and $Q_m^*(S_m)$ are given in (3.8) and (3.9), respectively. Thus, the total social welfare is $G_m^w(X, S_m) = PS_m(X, S_m) + CS_m(X, S_m)$, and, using (3.28) and (3.29), we obtain

$$G_m^w(X, S_m) = \left(\frac{X}{X_m^*(S_m)} \right)^{\beta} \left[(2 - \eta Q_m^*(S_m)) \frac{X_m^*(S_m)}{2\,(r-\mu)} - \delta \right] Q_m^*(S_m). \tag{3.30}$$

Note that given a fixed subsidy S_m, the firms' capacity investment policy is the same as in the previous sections. However, the conflict of interest between the firm and the Government is weakening. This is because the firm's value, reflected in the producer surplus, is now part of the Government's optimisation objective and the cost of providing the subsidy is no longer a concern for the Government. We derive the equilibrium subsidisation strategy for the social planner in the following proposition, where $\widetilde{S}_m^w = \operatorname{argmax}_{S_m \geq 0} G_m^w(X, S_m)$.

Proposition 3.3.7. *The equilibrium subsidy of a social planner under monopoly is*

$$\widetilde{S}_m^w = \begin{cases} 0, & \text{if } \frac{r-\mu}{1-\tau}\frac{\beta+1}{\beta-1}\delta \le X \\ \min\{S_1^w, S_2\}, & \text{if } \frac{r-\mu}{1-\tau}\frac{\beta+1}{\sqrt{\beta^2-1}}\delta \le X < \frac{r-\mu}{1-\tau}\frac{\beta+1}{\beta-1}\delta \\ S_1^w, & \text{if otherwise} \end{cases} \tag{3.31}$$

with S_2 given in (3.12) *and*

$$S_1^w = \frac{\tau\delta\left(\sqrt{4\tau^2\beta^2 + 3(3-4\tau)} - 2\tau\beta\right)}{3\beta\eta(3-4\tau)}, \text{ for } \tau \neq 0.75. \tag{3.32}$$

Unlike Section 3.3.2, here we find that the Government will offer a subsidy even when the tax rate is below $1/(\beta+1)$. This can be attributed to the Government no longer accounting for the tax income covering the cost of the subsidy. Next, we obtain the equilibrium investment threshold and capacity as follow.

Corollary 3.3.2. *For $\tau \neq 0.75$, the equilibrium investment threshold under monopoly is given by*

$$\widetilde{X}_m^w = \frac{r-\mu}{1-\tau}\frac{(\beta+1)\delta(3-4\tau)}{\beta(3-2\tau) - \sqrt{4\tau^2\beta^2 + 3(3-4\tau)}} \tag{3.33}$$

and the equilibrium capacity by

$$\widetilde{Q}_m^w = \begin{cases} \frac{(3-4\tau)-2\tau\beta+\sqrt{4\tau^2\beta^2+3(3-4\tau)}}{2(\beta+1)\eta(3-4\tau)}, & \text{if } X < \widetilde{X}_m^w \\ \frac{1}{2\eta}\left(1 - \frac{\delta(r-\mu)}{(1-\tau)X}\right), & \text{if } X \ge \widetilde{X}_m^w \end{cases}. \tag{3.34}$$

While Proposition 3.3.3 still holds in the case of a welfare-maximising Government, we find that the impact of economic uncertainty on the equilibrium subsidy is no longer the same. Indeed, contrary to Section 3.3.2, where a profit-maximising Government is better off providing a subsidy only when the uncertainty is low, Proposition 3.3.8 indicates that a social planner is willing to provide more subsidy to the firm when uncertainty is high.

Proposition 3.3.8. *The equilibrium subsidy, investment threshold and capacity in-*

crease with economic uncertainty.

Under duopoly, the discounted social welfare is given in (3.35). More specifically, the first term is the total surplus of a monopoly market and the second term represents the increment in social welfare due to the entry of the second investor.

$$\begin{aligned} G_i^w(X, S_i) = & \left(\frac{X}{X_{il}^*(S_i)}\right)^\beta \left[(2 - \eta Q_{il}^*(S_i)) \frac{X_{il}^*(S_i)}{2(r-\mu)} - \delta\right] Q_{il}^*(S_i) \\ & + \left(\frac{X}{X_{if}^*(S_i)}\right)^\beta \left[(2 - 2\eta Q_{il}^*(S_i) - \eta Q_{if}^*(S_i)) \frac{X_{if}^*(S_i)}{2(r-\mu)} - \delta\right] Q_{if}^*(S_i) \end{aligned} \tag{3.35}$$

Again, the equilibrium subsidy can be solved numerically by integrating the optimal investment strategies of the leader and follower into (3.35) and maximising it with respect to S_i, i.e., $\widetilde{S}_i^w = \text{argmax}_{S_i \geq 0}\, G_i^w(X, S_i)$. By inserting $\widetilde{S}_i^w$ back into the optimal investment strategies of the firms, we can obtain the equilibrium capacity, $\widetilde{Q}_{ij}^w$, and investment threshold, $\widetilde{X}_{ij}^w$, for both firms.

3.4 Numerical examples and analysis

In this section, we illustrate our model and key findings through a set of numerical examples. Specifically, we demonstrate how strategic interactions with the Government may impact a firm's capacity investment decision and how the equilibrium subsidisation policy depends on the market structure, type of duopolistic competition and the Government's optimisation objective. We adopt baseline parameter values from the real options, corporate finance and operational research literature (see Dixit and Pindyck, 1994; Huisman and Kort, 2015; Hagspiel et al., 2016a), in particular, $r = 10\%$ per year, $\mu = 6\%$ per year, $\sigma = 10\%$ per year, $\tau = 0.4$ per year, $\delta =$ £0.1 and $\eta = 0.05$ per unit output. We set $X =$ £0.005 to ensure that firms do not undertake immediate investment while we analyse the impact of the different parameters on the firms' equilibrium investment threshold. In doing so, we additionally take into account the empirical analysis of renewable subsidy policy of Bigerna et al. (2019). We choose to perturb our base parameter values around their estimates according to $r \in (0.06, 0.15)$, $\mu \in (0.03, 0.08)$ and $\sigma \in (0, 0.5)$. We also consider $\tau \in (0, 0.7)$ and $\eta \in (0.04, 0.1)$; finally, we note that the ratios δ/X in our and their study are comparable. Overall, this way we first ensure that our analysis is not limited to a single set of parameter values; second, we show the robustness and range of our results under stressed parameter values.

Figure 3.4 confirms the impact of an exogenous subsidy on the optimal investment threshold and optimal capacity of the monopolist, and extends the results to the duopoly case. Although conventional intuition suggests that the follower will be worse off if not subsidised, we interestingly find that the follower not only enters the market earlier (left panel), but also installs more capacity (right panel) if the Government provides more subsidy to the leader. This seemingly implausible result occurs because the follower's investment threshold (capacity) is positively (negatively) correlated with the leader's capacity (see also expressions 3.17–3.18), despite the fact that the follower's investment strategy is not directly affected by the subsidy. Indeed, the total market output is bounded due to the inverse demand function, which means that a greater capacity of the leader will squeeze the follower's market share. However, by receiving a subsidy, the leader will invest earlier in a smaller project, stimulating the follower's motivation to gain a larger market share. On the other hand, the market output price after the leader's investment is higher if the leader decides to invest less, increasing the follower's incentive to invest earlier. This reveals an indirect effect of the subsidy that is not captured when firm-level strategic interactions are ignored and firms are price takers.

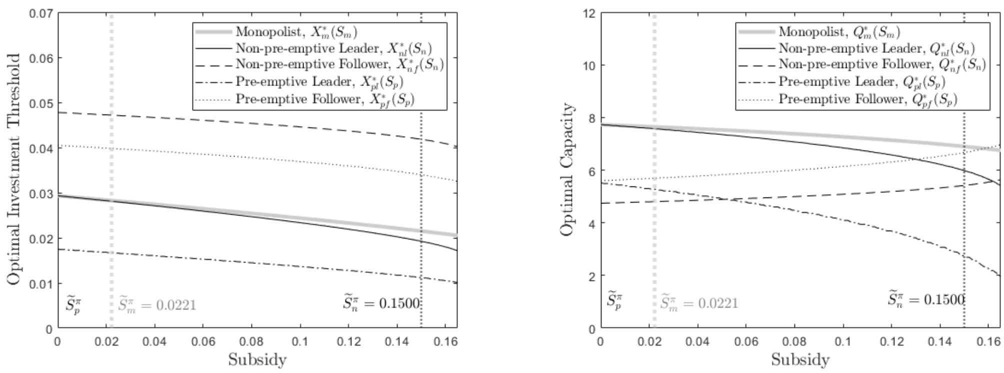

Figure 3.4: Impact of exogenous subsidy on firms' optimal investment threshold (left panel) and capacity (right panel) under monopoly, non-pre-emptive duopoly and pre-emptive duopoly, with corresponding equilibrium subsidies S_i^π = 0.0221, 0.1500 and 0 (vertical lines) when the Government maximises the profit.

The implications of economic uncertainty for each firm's investment policy under an endogenously defined subsidy when the Government maximises the profit are illustrated in Figure 3.5. As indicated on the left panel, the equilibrium investment thresholds increase with uncertainty under all market structures. This is attributed to greater uncertainty increasing the opportunity cost of investment and, in turn, the value of waiting. However, interestingly, the right panel shows that, while the leader's equilibrium capacity strictly increases with uncertainty, this is not true for the follower. This happens because greater uncertainty raises follower's incentive to invest later in more capacity, yet the follower's capacity is negatively correlated with the leader's, as suggested by (3.18). There are therefore two opposing effects with the former dominating (being dominated by) the latter when σ is small (large). Qualitative similar results hold for a Government that maximises social welfare.

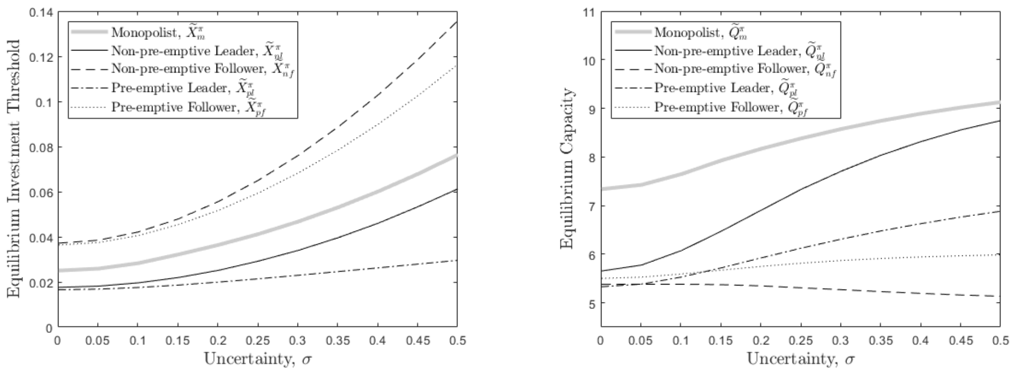

Figure 3.5: Impact of uncertainty on the equilibrium investment threshold (left panel) and equilibrium capacity (right panel) under a profit-maximising Government.

The left plots (black lines) in Figure 3.6 confirm the ambiguous impact of price uncertainty on the equilibrium subsidy when the Government maximises the profit, and this is extended to the duopoly case, whereby an increase in uncertainty can lead to an increase (decrease) of the equilibrium subsidy if σ is small (large). The Government stops eventually subsidising the leader in a highly uncertain environment. This can be attributed to the rapid increase of the investment threshold when uncertainty is high (see left panel of Figure 3.5), which has a negative impact on the discounting of the Government's payoff. In this case, the effectiveness of the subsidy is reduced, so that the additional tax income cannot cover the cost of subsidisation. Also, although the Government's incentive to offer a subsidy is low under a pre-emptive duopoly, $\widetilde{S}_p^{\pi} > 0$ is still possible when the tax rate is sufficiently high.

As illustrated on the right panel of Figure 3.6, a larger subsidy is required to mitigate the leader's loss in project value due to an increase in the tax rate, yet the equilibrium subsidy of the non-pre-emptive leader is initially (eventually) higher (lower) than that of the monopolist. The former is driven by the need for financial support due to competition, i.e., the Government is willing to grant a subsidy to the leader even if the tax rate is relatively low (i.e., $0.29 < \tau < 0.38$), when no subsidy is offered to the monopolist if $\tau < 0.38$. The latter is due to the existence of a 'ceiling', so that a subsidy above this level (e.g., 0.17 for the non-pre-emptive leader) always induces immediate investment. Thus, we get the maximum subsidy of 0.23 under monopoly and 0.17 under non-pre-emptive duopoly, which explains why the equilibrium subsidy of the non-pre-emptive leader no longer increases when $\tau > 0.47$. Additionally, the critical tax level leading to a positive subsidy is found to be the highest under a pre-emptive duopoly market ($\tau = 0.5$). This is because the pre-emptive leader invests earlier than the monopolist or non-pre-emptive leader, and, therefore, does not need as much fiscal stimulus as the latter.

Contrary to the profit-maximising Government, we find that a social planner (grey lines) is willing to provide a subsidy even when the tax rate is low (see right panel) and, as shown in Proposition 3.3.8, this subsidy increases with uncertainty (see left panel). This happens because the conflict of interest between the firm and the Government weakens when the Government maximises the social welfare, as a large proportion of the Government's value function (i.e., producer surplus) coincides with the firms' value functions (Cui et al., 2019). In addition, the cost of subsidy is no longer a concern for a social planner. Therefore, the social planner is more willing to provide larger subsidies whenever required by the firm and, as τ increases, the equilibrium subsidy also gradually increases. Finally, we observe that the equilibrium subsidy is still the lowest under pre-emptive duopoly, confirming that it is not optimal for the Government to grant too much subsidy, as it may induce more intense competition at an undesirable level.

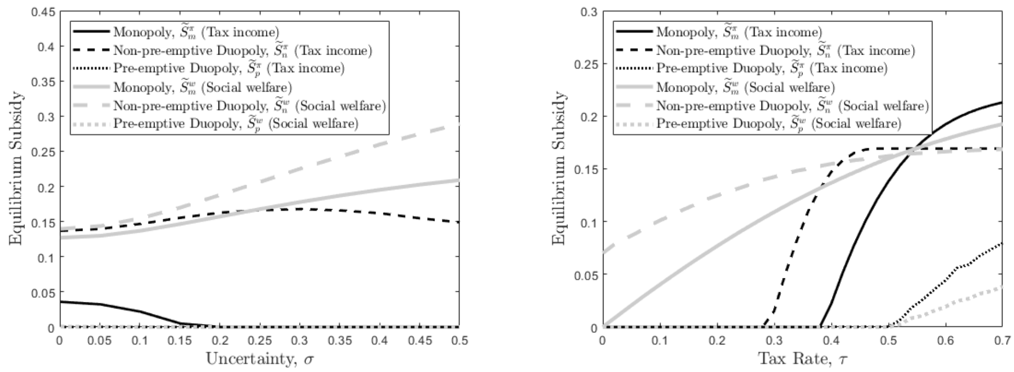

Figure 3.6: Impact of price uncertainty on the equilibrium subsidy (left panel) and impact of tax rate on equilibrium subsidy (right panel) under a profit-maximising (black lines) and welfare-maximising (grey lines) Government.

Figure 3.7 illustrates the impact of the tax rate on the firms' capacity investment policy when the Government maximises the profit (top panels) and social welfare (bottom panels). Interestingly, the top-left panel indicates that the equilibrium investment threshold does not necessarily increase with τ in all cases. This is because the subsidy accelerates the leader's investment and cancels out (all or a part of) the impact of a higher tax rate. Indeed, an increase in τ in the region $0.29 < \tau < 0.48$ leads to an abrupt growth of the equilibrium subsidy, especially when the Government maximises its profit (see black solid and dotted plots on the right panel of Figure 3.6). This can outweigh the effect of larger τ and accelerate investment. However, increasing τ has a slow-down effect on the increase of the equilibrium subsidy, such that the increasing subsidy gradually becomes dominated by the rising tax rate, and the investment is deferred. Also, as shown in Propositions 3.3.1 and 3.3.6, the optimal (equilibrium) capacity of each firm is not (is) affected by the tax rate. Specifically, as shown by the top-right panel in Figure 3.7, the equilibrium capacity of the monopolist, non-pre-emptive and pre-emptive leader decreases with increasing τ when this is greater than 0.38, 0.3 and 0.5, respectively, but it is constant when τ is too low to justify provision of a subsidy. In contrast, the capacity of the non-pre-emptive and pre-emptive follower exhibits a reverse pattern. Again, this can be attributed to the multiplicative demand function, which induces a bounded market output that has to be shared between the two firms. As shown in (3.18), if the leader decides to invest more (less), there will be a smaller (bigger) market left for the follower. In addition, as the leader's capacity decreases with rising τ (see also Proposition 3.3.3), the market price before the entry of the follower, i.e., $P_t = X_t(1-\eta Q_{il})$ for $i \in \{n, p\}$, will be higher, raising the incentive of the follower to invest in larger capacity. On the other hand, the equilibrium investment threshold is strictly increasing with τ when the Government maximises the social welfare, as shown in the bottom-left panel. This is because, although the Government is willing to grant a subsidy when τ is low, this subsidy grows relatively slowly with τ (see grey lines on the right panel of Figure 3.6), so that the effect of extra subsidy is dominated by that of rising tax rate.

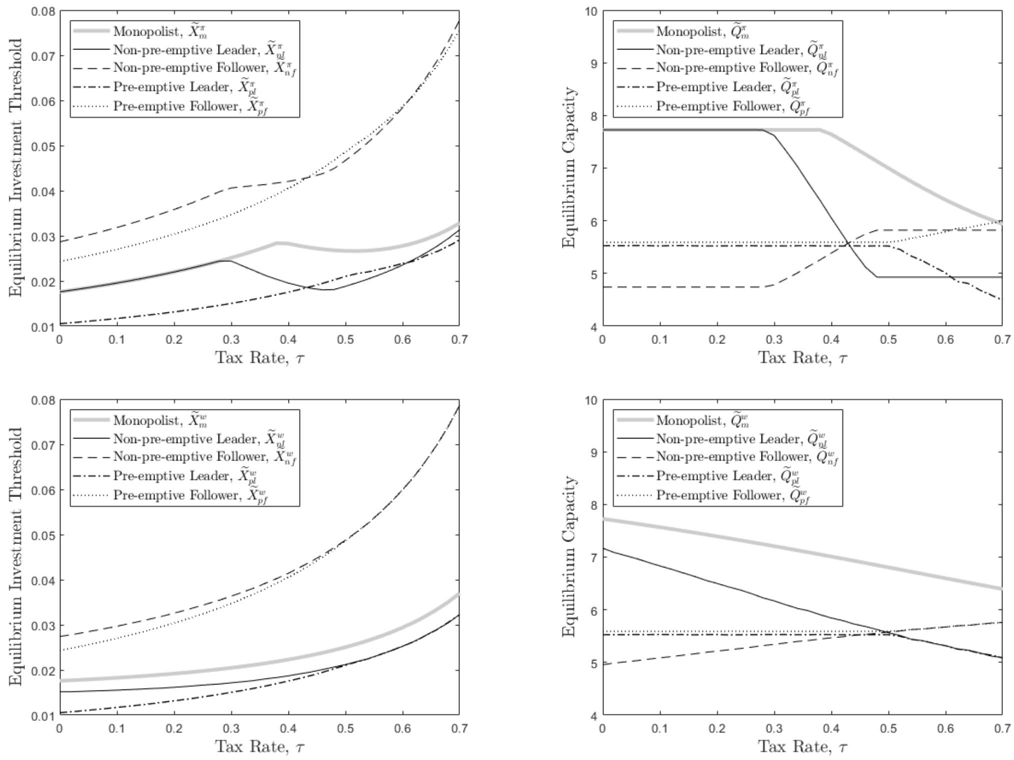

Figure 3.7: Impact of tax rate on firms' equilibrium investment threshold (left panels) and equilibrium capacity (right panels) when the Government maximises the profit (top panels) or social welfare (bottom panels).

Next, Figure 3.8 illustrates how the leader's loss in value (relative to the monopolist) due to the presence of the rival is affected by uncertainty. On the left panel, we assume an exogenous subsidy level, where $S_i = 0$ (dashed line) or 0.3 (solid line) for $i \in \{m, n, p\}$, and observe that the leader's relative loss in the value increases with uncertainty. Intuitively, this is because greater uncertainty delays the follower's entry but increases its impact. We also observe that the pre-emptive leader incurs a larger loss than the non-pre-emptive leader under a fixed subsidy. This is because the former invests earlier and scarifies substantial revenue due to the threat of pre-emption. Our results show that a subsidy can offset the leader's relative loss, however, as uncertainty increases, the impact of the subsidy becomes less pronounced due to the discounting effect, as the investment threshold of both the monopolist and leader increases rapidly with σ (left panel of Figure 3.5). Therefore, the relative loss in value of the leader with Government support (black lines) converges to that without Government support (grey lines) as uncertainty grows. The right panel presents a similar trend, except that we use the equilibrium subsidy, investment threshold and capacity to obtain the leader's relative loss in value for the case of a profit-maximising and a welfare-maximising Government (black and grey lines, respectively).

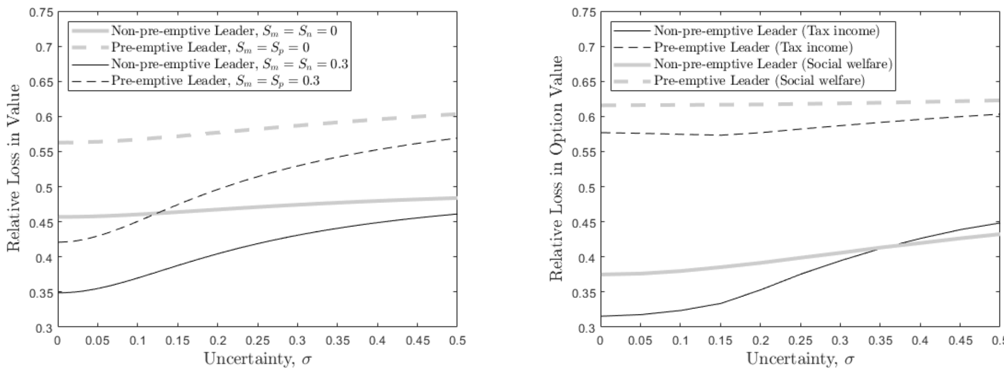

Figure 3.8: Left: effect of price uncertainty on relative loss in value of the non-pre-emptive leader (solid lines) and pre-emptive leader (dashed lines) for fixed subsidy level 0 (grey) and 0.1 (black). Right: exogenous subsidy replaced with equilibrium subsidy.

Figure 3.9 presents the effect of price elasticity of demand, η, on each firm's investment policy. The top-right panel indicates that a higher (lower) η allows for a larger (smaller) installed capacity. However, while the optimal investment thresholds are affected by η, as shown in Propositions 3.3.1 and 3.3.6, the top-left panel suggests that the equilibrium investment thresholds are actually independent of η. This is due to the fact that the equilibrium subsidy is endogenously chosen and thus varies with η. Intuitively, as the investment scale shrinks with increasing η, the total investment cost drops and so does the equilibrium subsidy (see bottom panel). In turn, a decrease in the subsidy delays investment, thereby offsetting the impact of an increase in η.

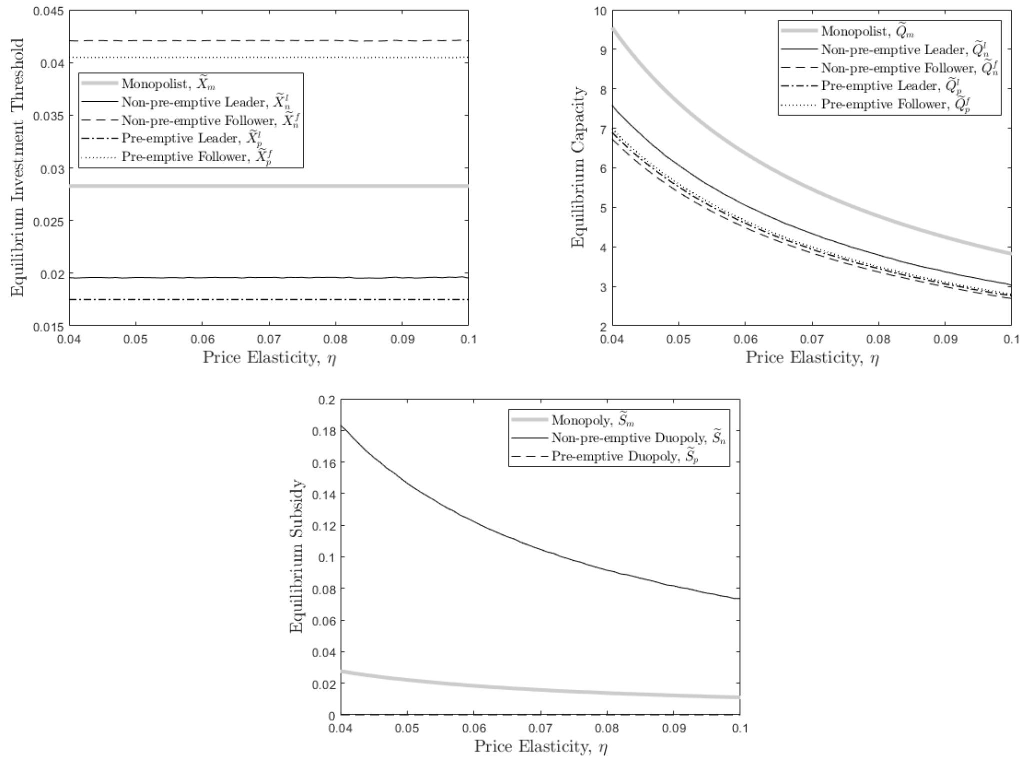

Figure 3.9: Effect of price elasticity on firms' equilibrium investment threshold (top-left panel), equilibrium capacity (top-right panel) and equilibrium subsidy (bottom panels).

Finally, Figure 3.10 illustrates how the Government's value function depends on price uncertainty and market structure. We find that both Government's profit and social welfare increase when uncertainty is higher as this motivates investment at a higher price threshold and the installation of a larger project (see Figure 3.5). The left panel of Figure 3.10 indicates that the Government's profit is greater under pre-emptive competition as both firms invest earlier and, thus, the effect of discounting on its profit is not significant. Note that earlier investment does not necessarily lead to a large loss in total market output, since even though the pre-emptive leader invests in less capacity, the investment intensity of the pre-emptive follower is greater than that of the non-pre-emptive follower. Also, the cost of the subsidy is minimum

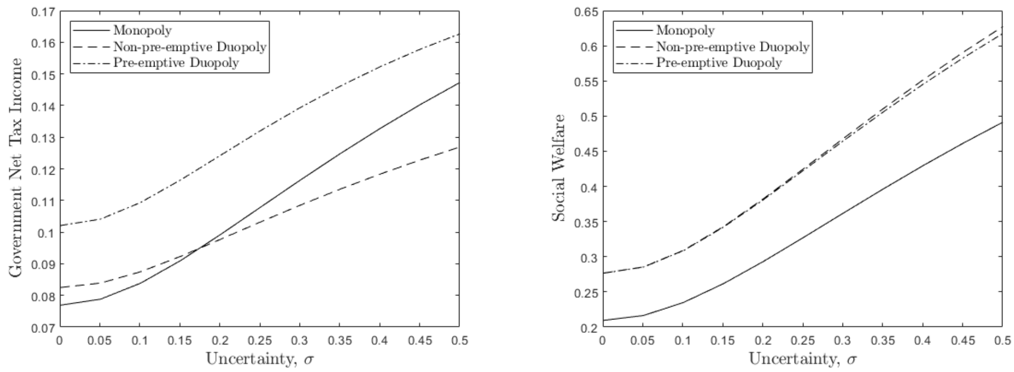

Figure 3.10: The impact of price uncertainty on Government's profit (left panel) and social welfare (right panel) under different market structures.

under pre-emptive competition. Interestingly, we observe a huge improvement in Government's value under non-pre-emptive duopoly under social welfare (right panel of Figure 3.10). This is because the Government is no longer concerned about the cost of the subsidy and, thus, the non-pre-emptive leader will receive more subsidy from a social planner that rapidly grows with σ (left panel of Figure 3.6). Therefore, competition is desirable for a social planner, while a profit-maximising Government may benefit more under pre-emptive competition.

3.5 Concluding discussion

Despite their increasing prominence, models for analysing the interaction between firm and Government-level policy-making do not account for critical features of a deregulated environment, such as competition. In this chapter, we address this disconnect by developing a bi-level real options framework for deriving the equilibrium Government subsidisation and firm-level capacity investment policy under a pre-emptive and non-pre-emptive duopolistic competition.

Our results show that the insights of traditional bi-level real options models under monopoly cannot be naturally transposed to a deregulated environment. In particular, we find that strategic interactions with the Government can significantly affect a firm's capacity investment decision and that the equilibrium subsidisation policy crucially depends on market structure and the type of duopolistic competition. Contrary to conventional intuition, we find that providing a larger subsidy to the leader can actually increase the follower's incentive to invest earlier and in a bigger project. Furthermore, the results suggest that the loss in the value of the leader, due to the presence of a rival, relative to the monopolist increases with economic uncertainty and, although a subsidy can mitigate this loss, its effect becomes less pronounced. We confirm that a profit-maximising Government is less willing to offer a subsidy when uncertainty is high or the tax rate is low, and extend to demonstrating how the critical tax rate that leads to switching from a subsidy to a non-subsidy regime changes under different market structures and types of duopolistic competition. Furthermore, we demonstrate how results are different when the Government aims to maximise social welfare and show that competition can be desirable for a social planner, while a profit-maximising Government may benefit the most from pre-emptive competition.

Therefore, the policy-making and managerial relevance of our results is reflected in the new insights gained when firm-level strategic interactions are integrated into the evaluation of real options. In particular, not only is competition a key aspect of deregulated industries and entails a loss in value relative to monopoly that must be taken into account when designing subsidisation policies, but also the type of competition can affect significantly a Government's subsidisation policy. Similarly, at the firm level, the interaction with Government policy-making produces dynamics under which the investment policy deviates from that of traditional duopolistic competition, which ignores such interactions. Such strategic interactions tend to be overlooked in the literature that values bi-level real options, yet if their implications are not properly understood, subsidisation policies will not be properly designed, thus potentially inducing under or over-investment cycles and increased regulatory risk following corrective policy actions. Indeed, the history of green investment in Europe since 2000 includes several examples of under or over-incentivised policies needing drastic adjustments. Also, in the context of public-private partnerships (PPPs), the Governments of Mexico and Spain had to pay, respectively, \$2.5 billion and \$8.9 billion to their private partners due to inefficient design (Silaghi and Sarkar, 2021).

We also investigate the robustness of the results by replacing (3.2) with an iso-elastic demand function, $P_t = X_t Q_t^{-\gamma}$, $\gamma \in (0, 1)$. We confirm that the subsidy can still accelerate the firms' investment and a higher price uncertainty induces higher investment threshold and greater capacity for both firms. While the follower can still benefit from the subsidy (by investing earlier), we find that its capacity decreases with higher subsidy amount. This can be attributed to the unbounded market output under an iso-elastic demand function, such that the follower can always choose to invest more if the leader's capacity increases (see also Boonman and Hagspiel, 2014). Thus, in this case, the follower becomes the larger capacity in the market and earns a greater profit. As a result, both firms want to be the follower and have no incentive to invest first under pre-emptive duopoly.

In Chapter 4, we ignore firm-level strategic interactions and transition from a lump-sum subsidy to a continuous price-based support, akin to a feed-in tariff. This allows us to incorporate policy uncertainty (i.e., subsidy retraction) considerations in our model, a common occurrence when new energy technologies reach cost competitiveness driven by the learning effect. Nevertheless, the joint effect of technological learning and subsidy retraction on a firm's capacity investment and the Government's subsidy design has not received enough attention, making it an important and unaddressed question.

Chapter 4

Optimal capacity investment and subsidy design under technological learning and uncertainty

Rising concerns about climate change have incentivised the deployment of alternative energy technologies at unprecedented levels, which, in turn, require Government support until they become cost-competitive. Although the duration of support often correlates with the cost reduction progression as a result of technological learning, this relationship is yet to be analysed to provide insights on optimal investment and subsidy retraction. To this end, we develop a bi-level real options framework to derive a private firm's optimal investment as well as the Government's optimal subsidy retraction policy. We find that a larger subsidy can accelerate investment, but its impact on project scale is ambiguous. A bigger project speeds up the cost-reduction process, thereby incentivising a firm to install more capacity, but also the retraction of subsidy as the operating and maintenance cost reaches grid parity sooner. The results further suggest that, although the duration of the subsidy is shorter when the learning rate is high, the firm is still willing to invest earlier. Interestingly, a higher tax rate does not necessarily delay the investment or induce a smaller project size.

4.1 Introduction

Investment in alternative energy technologies (AETs) is critical to addressing pressing climate change concerns. However, the substantial capital requirements of these technologies combined with increasing economic uncertainty raises private firms' incentive to act more cautiously, thus reducing the likelihood of meeting timely the required sustainability targets. Therefore, Governments worldwide have deployed a wide range of support schemes to incentivise investment in AETs, which in turn will facilitate faster technological learning and lower their investment and operating costs towards commercial maturity (Yao et al., 2015; Pan et al., 2021; Zahoor et al., 2022)[1]. Meanwhile, the maturity threshold, also referred to as *grid parity*, is attained when an AET becomes cost-competitive, at which point market forces take over and no further Government support is required. Although grid parity is among the key determinants of the duration of Government support (Sendstad et al., 2022), models for analysing the correlation between grid parity and subsidy duration remain underdeveloped and, therefore, the implications of technological learning for capacity investment and optimal subsidy retraction remain an important open research question.

Efficient subsidy design is critical, as under-subsidised projects may fail to incentivise the necessary capacity investments, while over-subsidised projects can strain the Government's financial position. Additionally, empirical evidence on renewable energy (RE) incentives and investment in Europe since 2000 emphasise how unexpected revisions of subsidy schemes can decelerate RE deployment, significantly. For example, the effects of subsidy revisions in the UK energy sector saw investment in wind and solar power fall by 56% after the Government banned subsidies for onshore wind and made massive cuts in funding for solar power in 2017 (Independent, 2018; Guardian, 2018). Empirical results also indicate that a retroactive subsidy change decreases the investment rate by approximately 45% for solar photovoltaic

[1] Here, technological learning refers to learning-by-doing, as originally introduced by Wright (1936). That is, the investment and operating costs of AETs decrease with cumulative energy produced or capacity installed, which can be attributed to the growing experience of technicians, technological breakthroughs, better use of preventive maintenance interventions and increasing digital capabilities of firms (Kobos et al., 2006; Steffen et al., 2020).

and 16% for onshore wind (Sendstad et al., 2022). Therefore, a stable policy environment with credible policy commitments is crucial for incentivising private firms' investments.

Analysing the implications of technological learning for capacity investment and optimal subsidy design is a challenging task, in which the following trade-offs must be balanced. First, from a private firm's perspective, capacity investment decisions are particularly risky, as a large capacity increases the downside risk in the event of an unexpected recession, while a small capacity can lead to forgone revenues if market conditions suddenly become favourable. Second, a large capacity accelerates the learning process, which may cause an earlier subsidy retraction, as the operating and maintenance (O&M) cost will reach grid parity sooner. Third, the duration of a subsidy should be designed so that investment intensity targets are met in a timely manner. However, a large (small) duration may induce a firm to invest earlier (later) in a smaller (bigger) project. Finally, the subsidy level, digression rate or grid parity should be determined endogenously aligning the profit-maximisation objective of a firm and the Government.

Analysing these trade-offs is amenable to real options theory, which addresses the problem of investment under uncertainty while reflecting the value from embedded managerial discretion. In this chapter, we develop a real options framework to analyse the interaction between a private firm's capacity investment and the Government's subsidy retraction policy. Specifically, the private firm holds the option to invest in an AET, factoring in considerations related to technological learning and price uncertainty. The firm receives price-based support, akin to a feed-in tariff (FIT), wherein it receives a fixed support payment on top of the remuneration obtained from selling the produced electricity in the market (Klein et al., 2008; Couture and Gagnon, 2010). However, the support is subject to retraction once the technology's O&M cost drops to a certain threshold optimally chosen by the Government so as to maximise its own net tax income. Thus, the contribution of our work is threefold. First, we develop a suitable framework to analyse the trade-off between technological learning and subsidy retraction in a firm's optimal investment policy.

Second, we obtain the equilibrium investment timing, project scale and subsidy retraction threshold by analysing the non-cooperative game between the Government and a firm. Finally, we provide managerial insights that contribute to informed decision-making for both parties.

We proceed by discussing some related work in Section 4.2 and present our assumptions and notation in Section 4.3.1. In Section 4.3.2, we derive the equilibrium investment policy of the firm as well as the equilibrium subsidy retraction policy of the Government under a fixed project scale. We then allow in Section 4.3.3 for a continuously scalable capacity. Section 4.4 proceeds with various numerical examples, results and policy implications, whereas Section 4.5 concludes this chapter. Proofs of results are given in Appendix C.

4.2 Related work

The implications and effectiveness of different support schemes, e.g., renewable portfolio standards, FITs, green-certificate trading and tax exemptions, for RE investment under uncertainty are addressed in the real options literature, with Danielova and Sarkar (2011), Boomsma et al. (2012), Zhang et al. (2014) and Kitzing et al. (2017) reflecting some relevant examples in the field. More recently, Bigerna et al. (2019) consider a firm that has the option to invest in RE under economic uncertainty and analyse how a feed-in premium affects the investment timing and project scale. For a given environmental target capacity, they determine the optimal subsidy level and investment threshold. Other examples in the same line of work include Kim and Lee (2012), Ritzenhofen et al. (2016) and Li et al. (2020). However, despite the important contribution, there are two key limitations. First, the effectiveness of a subsidy is typically assessed *ex post* by analysing the impact of an exogenous subsidy on the timing and intensity of RE investments, thus disregarding the interaction between firm and Government policy-making (Alizamir et al., 2016). Second, it overlooks that the subsidy level is often designed to gradually decline as RE becomes more cost-competitive. Consequently, the potential adverse impact of subsidy digression or retraction on RE investment has not been sufficiently explored (Ma et al., 2021).

The endogenous derivation of investment and subsidisation policies requires a bi-level framework that aligns the firm- and Government-level optimisation objectives. Examples of models where the investment and subsidisation policy are derived *ex ante* by aligning the profit-maximising objective of a firm with the Government's objective of providing a subsidy at zero-expected cost, include Pennings (2000), Yu et al. (2007) and Azevedo et al. (2021). Also, Lukas and Thiergart (2019) analyse a firm's optimal capacity investment and the Government's subsidisation policy when debt financing is possible. They derive the subsidy level that the Government should offer *ex ante* so as to maximise their net tax income at the time of the firm's investment. Although this work passes over the policy uncertainty and the implications of subsidy retraction, a key result is that the capacity investment and subsidisation policies can differ substantially when the firm's and Government's optimisation objectives are misaligned.

Allowing for policy uncertainty, Dalby et al. (2018) show that a decrease in the expected time of a policy change postpones investment and find that investors prefer a lower subsidy that is available for a longer time period to a larger subsidy with a high risk of retraction. Also, Nagy et al. (2021) find that increasing the subsidy size or the probability of subsidy retraction incentivises a firm to accelerate investment at the expense of a smaller project. Similarly, Barbosa et al. (2020) analyse two FITs under market and regulatory uncertainty, where the reduction of the tariff level is modelled via a Poisson process, and find that an increase in the likelihood or size of tariff reduction accelerates investment. Furthermore, Hagspiel et al. (2021) develop a real options model for investment in a RE project, where a subsidy is available for an exponentially distributed time period. They find that the effects of subsidy retraction risk on the firm's investment strategy differ significantly depending on whether it is assumed to be time-dependent or constant over time.

The aforementioned approaches deal with the effects of policy uncertainty on RE investment by making some probabilistic assumption about the time of subsidy withdrawal, yet they overlook how this depends on technological learning. However, Governments' policy-making often relies on technological learning, as the retraction of a subsidy occurs when a RE technology approaches its grid parity (IRENA, 2022). Hence, learning curves have been recognised as a critical factor behind a firm's production strategy and the Government's energy planning and policy-making (Smit and Junginger, 2007; Neij et al., 2017; Upstill and Hall, 2018), and various empirical models are developed to obtain the learning curves of RE technologies (McDonald and Schrattenholzer, 2001; Nemet, 2006; Söderholm and Sundqvist, 2007).

In particular, Steffen et al. (2020) estimate the O&M learning curves for onshore wind and solar photovoltaic in Germany by analysing the empirical relationship between cost reductions and the cumulative energy produced. Their results reveal a substantial decrease in O&M costs as cumulative experience increases. In addition, they demonstrate that learning-by-doing is one of the most pivotal mechanisms contributing to this cost reduction. As indicated in Steffen et al. (2020), O&M costs are non-negligible, typically accounting for 20%–25% of lifecycle costs for wind and solar plants in Europe. More specifically, the cost reductions associated with each doubling of cumulative experience range from 9.2% to 12.8% for onshore wind and from 15.7% to 18.2% for solar photovoltaic technologies. Similar studies on the impact of learning on O&M cost reduction of RE include Ederer (2015), Wiser et al. (2019) and Victoria et al. (2021).

Within the context of investment under uncertainty, the implications of technological learning are analysed in Majd and Pindyck (1987), who assume a fixed investment size and a declining marginal cost as a function of cumulative capacity. In the same line, Della Seta et al. (2012) allow for flexibility in both investment timing and project scale. Their results indicate that the presence of technological learning affects a firm's investment, significantly. Specifically, a firm will invest later (earlier) and in a larger (smaller) project if the learning process is slow (fast). Siddiqui and Fleten (2010) consider a firm that may choose either an existing RE technology or an unconventional energy technology that requires cost-reducing enhancement measures prior to deployment. More recently, Sarkar and Zhang (2020) examine the optimal investment and financing choices of a levered firm with a learning-curve technology and find that the optimal leverage ratio is an increasing function of the learning speed.

In light of the previous discussion, with this chapter we contribute to the existing literature by developing a stylised game-theoretic real options model for analysing the strategic interaction between the Government and a private firm, and the joint effect of technological learning and subsidy retraction on a firm's optimal AET investment policy. More specifically, the learning effect is reflected in the decreasing O&M cost of the AET, where an increase in the installed capacity accelerates the accumulation of experience, thus resulting in a faster cost-reduction process. Meanwhile, the Government provides a fixed subsidy (per unit output) to the firm that will be retracted when the AET become cost-competitive. We kick off with the benchmark case where the firm has discretion only over the investment timing, and derive the optimal subsidy retraction threshold for the Government along with the firm's equilibrium investment threshold. We then consider the joint determination of investment timing and capacity size and analyse the impact of subsidy retraction and technological learning on the firm's investment strategy.

Consistent with Lukas and Thiergart (2019), Azevedo et al. (2021) as well our results in Chapter 3, we find that a larger subsidy (i.e., a later subsidy retraction) accelerates the firm's investment; however, the optimal capacity size does not necessarily decrease with larger subsidy. Results also suggest that technological learning hastens investment, yet its impact on the equilibrium capacity is non-monotonic. This can be attributed to the trade-off between cost reduction and subsidy retraction. We show that the equilibrium duration of the subsidy is shorter when the learning rate is high, but, interestingly, the firm is still willing to invest earlier. In addition, greater economic uncertainty hastens subsidy retraction and delays the firm's investment. Notably, our findings suggest that a higher tax rate does not always discourage the firm's investment by causing delays or inducing smaller project sizes. Finally, we demonstrate the value of production flexibility.

4.3 Model

4.3.1 Problem formulation

We consider a private firm with a perpetual option to invest in an AET of infinite lifetime. The firm has discretion over both the time of the investment and the size of the project and faces demand uncertainty. The exogenous demand shock parameter is denoted by X_t, where $t \geq 0$ is continuous and denotes time, and is assumed to follow a geometric Brownian motion:

$$dX_t = \mu X_t dt + \sigma X_t dW_t, \quad X_0 \equiv x,$$

where $\mu > 0$ is the annual growth rate, $\sigma > 0$ the annual volatility and dW_t the increment of the standard Brownian motion. Thus, the output price P_t is given by

$$P_t = X_t \left(1 - \eta Q_t\right),$$

where Q_t is the total market output at time t and $\eta > 0$ the price elasticity parameter of the inverse demand function. Our analysis can accommodate a wide range of demand functions (e.g., iso-elastic demand function), in this chapter we focus on a linear demand function and leave the analysis related to other demand functions for further research.

As illustrated in Figure 4.1, the firm invests at time Λ with capacity size Q incurring a linear sunk cost δQ, $\delta > 0$ (Bigerna et al., 2019; Nagy et al., 2021). After investment has taken place, the project will operate at full capacity (Huisman and Kort, 2015). The variable O&M cost is

$$C_t(Q) = C_0 e^{-\gamma Q(t-\Lambda)}, \quad t \geq \Lambda, \quad C_0 \geq 0, \tag{4.1}$$

where $\gamma \in [0, \infty)$ is an exogenous parameter that determines the speed of the learning process (Della Seta et al., 2012; Sarkar and Zhang, 2020); a high (low) γ means that the unit O&M cost declines rapidly (slowly) with cumulative production.

To stimulate the firm's investment, the Government offers a subsidy S (per unit output per unit of time) to the firm that takes the form of a fixed premium on top of the firm's marginal revenues (Nagy et al., 2021; Barbosa et al., 2020; Nagy et al., 2023). The subsidy is available when the firm invests, but its retraction is triggered at time

$$T = \inf\{t : C_t(Q) < \omega C_0, \quad 0 \leq \omega \leq 1\}, \tag{4.2}$$

that is, once the O&M cost drops below a fraction ω of its starting level C_0, where ωC_0 represents the grid parity (or maturity threshold) of this AET. More in detail, ω signifies the ratio between the O&M cost when the subsidy is withdrawn at $t = T$ and the initial O&M cost at $t = \Lambda$. Thus, a small ω indicates that the initial cost of the AET is much higher than its grid parity, and, therefore, the Government is willing to provide the subsidy for a longer period; on the other hand, a large ω corresponds to the case where the AET is close to its commercial maturity, thereby the subsidy is expected to be withdrawn soon. According to (4.1) and (4.2), the duration of the subsidy is

$$C_0 e^{-\gamma Q(T-\Lambda)} = \omega C_0 \implies T - \Lambda = -\frac{\ln \omega}{\gamma Q}, \tag{4.3}$$

which implies that the firm will receive more subsidy if the Government retracts later (by setting a smaller ω), and less subsidy if the learning process is too fast, i.e., high learning speed or capacity size. Additionally, the Government receives corporate tax, subject to rate τ, on the firm's revenues. All cash flows are discounted at a constant rate r.

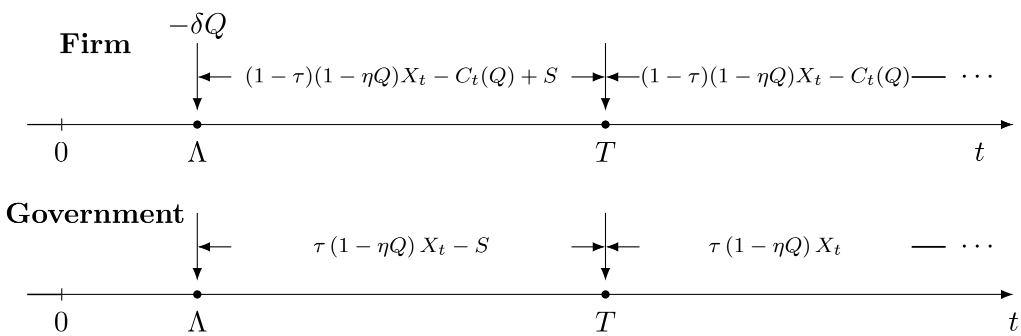

Figure 4.1: Instantaneous cash flows per unit output of the firm and Government.

The firm's investment threshold and installed capacity are denoted by X and Q, while the optimal investment threshold and capacity by X^* and Q^*. The equilibrium subsidy retraction threshold is denoted by $\widetilde{\omega}$, with $\widetilde{X} = X^*(\widetilde{\omega})$ and $\widetilde{Q} = Q^*(\widetilde{\omega})$ the firm's equilibrium investment threshold and capacity, respectively. In addition, $V(\cdot)$ denotes the expected value of the active project, $F(\cdot)$ the expected value of the firm's investment opportunity, and $G(\cdot)$ the Government's value function. To distinguish between the case of fixed and flexible capacity size, we denote the optimal (equilibrium) decision variable(s) in the former case as X_f^* $\left(\widetilde{X}_f \text{ and } \widetilde{\omega}_f\right)$ and the objective functions of the firm and the Government as $V_f(\cdot)$, $F_f(\cdot)$ and $G_f(\cdot)$.

To facilitate the exposition of our results, we state our research questions in the form of the following testable hypotheses:

H1: *Technological learning and subsidy retraction are opposing forces with an ambiguous combined impact on a firm's capacity investment policy.*

While a higher learning rate enhances cost reduction and incentivises greater investment, it drives the cost closer to grid parity, triggering earlier subsidy retraction. Consequently, the combined influence of technological learning and subsidy retraction may result in a non-monotonic effect on the firm's investment size.

H2: *Higher learning rate shortens the duration of the subsidy, yet may still accelerate the firm's investment decision.*

The duration of the subsidy given in (4.3) decreases with increasing γ, indicating that the firm may receive a smaller subsidy if the learning curve is steep. However, this does not necessarily discourage the firm's investment, as a higher learning rate speeds up the cost-reduction process and offsets the effect of a smaller subsidy.

H3: *An increase in the tax rate does not necessarily decelerate the firm's investment or decrease the size of the project.*

Within a bi-level framework, the Government may consider setting a lower subsidy retraction threshold, ωC_0, to offset the adverse effect of a higher tax rate that tends to delay the firm's investment. In addition, the presence of technological learning can raise the firm's incentive to opt for a bigger project that facilitates faster O&M cost reductions, even when the tax rate is high.

4.3.2 Benchmark: fixed scale

In this section, we analyse the firm's optimal investment and the Government's subsidy retraction policy, assuming that the firm has no discretion over project scale. The firm's value function under a now-or-never investment opportunity is given by

$$
\begin{aligned}
V_f(x,\omega) &= \mathbb{E}_x\left[\int_0^\infty \left((1-\tau)(1-\eta Q)X_t - C_0 e^{-\gamma Q t} + S\cdot \mathbb{1}_{\{e^{-\gamma Q t}>\omega\}}\right) Q e^{-rt}dt - \delta Q\right] \\
&= \frac{1-\tau}{r-\mu}x(1-\eta Q)Q - \frac{C_0 Q}{r+\gamma Q} + \frac{SQ}{r}\left(1-\omega^{\frac{r}{\gamma Q}}\right) - \delta Q, \qquad (4.4)
\end{aligned}
$$

where $\mathbb{1}_{\{\cdot\}}$ is the indicator function, which is equal to one if the condition is satisfied and zero otherwise; each of the four terms on the right-hand side corresponds to the gross revenue, accumulated O&M cost, total subsidy received (until revoked) and lump-sum investment cost, respectively.

Next, we assume that the demand is too low to justify immediate investment. The firm's objective when investment is deferred is to maximise the discounted expected value of all future cash flows, as shown in (4.5)

$$
F_f(x,\omega) = \sup_{\Lambda\in\mathcal{S}} \mathbb{E}_x\left[\int_\Lambda^\infty \left((1-\tau)(1-\eta Q)X_t - C_0 e^{-\gamma Q(t-\Lambda)} + S\cdot \mathbb{1}_{\{e^{-\gamma Q(t-\Lambda)}>\omega\}}\right) Q e^{-rt}dt - \delta Q e^{-r\Lambda}\right], \qquad (4.5)
$$

where $\Lambda = \inf\{t \geq 0 : X_t \geq X_f\}$ and $\mathcal{S}$ denotes the set of stopping times of the filtration generated by X_t. Based on the law of iterated expectations and the strong Markov property of process X_t, we can rewrite (4.5) as

$$
F_f(x,\omega) = \sup_{\Lambda\in\mathcal{S}} \mathbb{E}_x\left[e^{-r\Lambda}\right] V_f\left(X_f,\omega\right) = \max_{X_f>x}\left(\frac{x}{X_f}\right)^\beta V_f\left(X_f,\omega\right), \qquad (4.6)
$$

where the second equality follows based on the stochastic discount factor $\mathbb{E}_x\left[e^{-r\Lambda}\right] = (x/X_f)^\beta$ with $\beta > 1$ the positive root of the quadratic equation $\sigma^2\beta(\beta-1)/2+\mu\beta-r=0$. By applying the first-order necessary condition (FONC) to the unconstrained optimisation problem (4.6), we obtain the expression for $X_f^*(\omega)$.

Proposition 4.3.1. *For a given subsidy retraction threshold* ω*, the optimal investment threshold is given by*

$$X_f^*(\omega) = \frac{\beta}{\beta-1}\frac{r-\mu}{1-\tau}\frac{1}{1-\eta Q}\left[\frac{C_0}{r+\gamma Q} - \frac{S}{r}\left(1-\omega^{\frac{r}{\gamma Q}}\right)+\delta\right]. \tag{4.7}$$

We proceed with the optimal subsidy retraction policy of the Government. From the bottom part of Figure 4.1, the Government's value function, i.e., the present value of the tax income after subtracting the subsidy, is given by

$$\begin{aligned} G_f(x,\omega) &= \mathbb{E}_x\left[\int_\Lambda^\infty \left(\tau\left(1-\eta Q\right)X_t - S\cdot \mathbb{1}_{\{e^{-\gamma Q(t-\Lambda^*)}>\omega\}}\right)Qe^{-rt}dt\right] \\ &= \left(\frac{x}{X_f^*(\omega)}\right)^\beta \left[\frac{\tau}{r-\mu}\left(1-\eta Q\right)X_f^*(\omega) - \frac{S}{r}\left(1-\omega^{\frac{r}{\gamma Q}}\right)\right]Q. \end{aligned} \tag{4.8}$$

We assume that the Government will choose the subsidy retraction threshold so as to maximise its own net income, i.e., $\widetilde{\omega}_f = \mathrm{argmax}_{\omega\geq 0}\, G_f(x,\omega)$ (see Proposition 4.3.2). Plugging $\widetilde{\omega}_f$ into (4.7) yields the equilibrium investment threshold $\widetilde{X}_f = X_f^*(\widetilde{\omega}_f)$ given in (4.10). The upper part of (4.9) suggests that the Government will not provide a subsidy if the tax rate is below the critical level, $1/(\beta+1)$, since the extra tax income will not cover the cost of the subsidy. However, when $\tau > 1/(\beta+1)$ the Government is willing to grant a subsidy, and the subsidy retraction threshold is given in the lower part of (4.9). Note that without technological learning, i.e., $\gamma = 0$, $\widetilde{\omega}_f = 1$ implies a zero subsidy, whereas if the learning curve is steep, the Government will set a lower subsidy retraction threshold and $\widetilde{\omega}_f \to 0$ as $\gamma \to \infty$.

Proposition 4.3.2. *The equilibrium subsidy retraction threshold and the investment threshold are respectively given by*

$$\widetilde{\omega}_f = \begin{cases} 1 & \text{if } \tau \leq \frac{1}{\beta+1} \\ \left(1 - \frac{\left(\frac{\tau}{1-\tau}\beta-1\right)\left(\frac{C_0}{r+\gamma Q}+\delta\right)r}{\left(\frac{1}{1-\tau}\beta-1\right)S}\right)^{\frac{\gamma Q}{r}} & \text{if } \tau > \frac{1}{\beta+1} \end{cases} \tag{4.9}$$

and

$$\widetilde{X}_f = \begin{cases} \frac{\beta}{\beta-1}\frac{r-\mu}{1-\tau}\frac{1}{1-\eta Q}\left(\frac{C_0}{r+\gamma Q}+\delta\right) & \text{if } \tau \leq \frac{1}{\beta+1} \\ \frac{\beta}{\beta-1}\frac{\beta}{\beta-(1-\tau)}\frac{r-\mu}{1-\eta Q}\left(\frac{C_0}{r+\gamma Q}+\delta\right) & \text{if } \tau > \frac{1}{\beta+1} \end{cases}. \tag{4.10}$$

Consistent with existing literature, we show that greater price uncertainty raises the investment threshold and that the Government is less willing to provide a subsidy (or retracts the subsidy immediately) in a highly uncertain environment (Lukas and Thiergart, 2019), see also the left panel of Figure 3.6 in the previous chapter.

Proposition 4.3.3. *Greater uncertainty raises both the equilibrium subsidy retraction threshold $\widetilde{\omega}_f$ and the equilibrium investment threshold $\widetilde{X}_f$.*

In our next Proposition 4.3.4, we show that the Government will delay revoking the subsidy to compensate for the higher tax rate when $\tau > 1/(\beta+1)$. Interestingly, we find that the firm will invest earlier even though it pays more taxes. This can be attributed to the opposing effects of a higher tax rate and a later withdrawal of the subsidy, with the latter's effect dominating in accelerating investment.

Proposition 4.3.4. *An increase in tax rate decreases the equilibrium subsidy retraction threshold $\widetilde{\omega}_f$ and investment threshold $\widetilde{X}_f$.*

A higher subsidy level leads to an earlier withdrawal, but has no impact on the firm's investment timing, as we show in Proposition 4.3.5. Intuitively, while a larger subsidy expedites investment, its effect is offset by the shorter time horizon of the subsidy. Finally, an increase in the price elasticity parameter, η, leads to lower project prices, which means that the firm has to wait longer to enter the market until prices recover.

Proposition 4.3.5. *A greater subsidy (price elasticity) raises the equilibrium subsidy retraction threshold $\widetilde{\omega}_f$ (investment threshold $\widetilde{X}_f$), but has no effect on the equilibrium investment threshold $\widetilde{X}_f$ (subsidy retraction threshold $\widetilde{\omega}_f$).*

4.3.3 Joint determination of timing and scale

In what follows, we expand our paradigm to allow for a firm's flexibility to scale the size of the project. The firm's value function under a now-or-never investment opportunity is the same as (4.4), except that the size of the project is no longer exogenous. Let $\Phi(x,\omega) = \max_Q \{V_f(x,\omega,Q)\}$ be the maximised expected value of the active project. By applying the FONC, we get that the optimal capacity satisfies

$$\frac{1-\tau}{r-\mu}(1-2\eta Q)x - \frac{C_0}{r+\gamma Q} + \frac{\gamma C_0 Q}{(r+\gamma Q)^2} - \delta + \frac{S}{r}\left(1-\omega^{\frac{r}{\gamma Q}}\right) + \frac{S\ln\omega}{\gamma Q}\omega^{\frac{r}{\gamma Q}} = 0. \quad (4.11)$$

Subject to the optimal capacity choice at investment, the firm's optimisation objective when investment is deferred is given by

$$F(x,\omega) = \sup_{\Lambda\in\mathcal{S}} \mathbb{E}_x\left[e^{-r\Lambda}\right]\Phi(X,\omega) = \max_{X>x}\left(\frac{x}{X}\right)^{\beta}\Phi(X,\omega). \quad (4.12)$$

By applying the FONC to the unconstrained optimisation problem (4.12), we obtain the optimal investment threshold and project scale.

Proposition 4.3.6. *The optimal investment threshold is given by*

$$X^*(\omega) = \frac{\beta}{\beta-1}\frac{r-\mu}{1-\tau}\frac{1}{1-\eta Q^*(\omega)}\left[\frac{C_0}{r+\gamma Q^*(\omega)} - \frac{S}{r}\left(1-\omega^{\frac{r}{\gamma Q^*(\omega)}}\right)+\delta\right]. \quad (4.13)$$

while the optimal capacity Q^* satisfies the equation

$$\left(\frac{\beta}{\beta-1}\frac{1-2\eta Q^*}{1-\eta Q^*}-1\right)\left(\frac{C_0}{r+\gamma Q^*} - \frac{S}{r}\left(1-\omega^{\frac{r}{\gamma Q^*}}\right)+\delta\right) + \frac{\gamma C_0 Q^*}{(r+\gamma Q^*)^2} + \frac{S\ln\omega}{\gamma Q^*}\omega^{\frac{r}{\gamma Q^*}} = 0. \quad (4.14)$$

Proposition 4.3.7 confirms hypothesis H1 by showing that the effect of the learning rate on the optimal capacity is non-monotonic. This can be attributed to the opposing effects of technological learning on cost reduction and subsidy retraction. More specifically, a higher learning rate expedites the cost-reduction process, raising the firm's incentive to invest more. Meanwhile, the duration of the subsidy, $-\ln(\omega)/(\gamma Q)$, decreases with increasing learning rate or project scale as the O&M cost reaches the subsidy retraction threshold earlier. Intuitively, when technology learning is slow, the subsidy effect dominates that of cost reduction, thus the firm can invest in a smaller project. However, as the learning rate increases, the cost-reduction effect becomes more pronounced and the firm may be willing to invest more at the cost of a smaller subsidy (see Section 4.4).

Proposition 4.3.7. *There is a non-monotonic relationship between the optimal capacity Q^* and the learning rate γ.*

Finally, we study the optimal subsidy retraction threshold of the Government that maximises its net tax income. Based on the bottom illustration in Figure 4.1, the Government's value function is formulated as

$$\begin{aligned} G(x,\omega) &= \mathbb{E}_x\left[\int_{\Lambda^*}^{\infty}\left(\tau\left(1-\eta Q^*(\omega)\right)X_t - S\mathbb{1}_{\left\{e^{-\gamma Q^*(\omega)t}>\omega\right\}}\right)Q^*(\omega)e^{-rt}dt\right] \\ &= \left(\frac{x}{X^*(\omega)}\right)^{\beta}\left[\frac{\tau}{r-\mu}\left(1-\eta Q^*(\omega)\right)X^*(\omega) - \frac{S}{r}\left(1-\omega^{\frac{r}{\gamma Q^*(\omega)}}\right)\right]Q^*(\omega). \end{aligned} \tag{4.15}$$

The Government chooses the subsidy retraction level so as to maximise its own net tax income, i.e., $\widetilde{\omega} = \text{argmax}_{\omega\geq 0}\, G(x,\omega)$, which in this case is solved numerically. By inserting $\widetilde{\omega}$ in (4.14) and (4.13), we obtain the equilibrium capacity, $\widetilde{Q} = Q^*(\widetilde{\omega})$, and the investment threshold, $\widetilde{X} = X^*(\widetilde{\omega})$, for the firm.

4.4 Numerical results

In this section, we illustrate our model and key findings through a set of numerical examples. We adopt baseline parameter values from the real options literature (see Huisman and Kort, 2015), in particular, $r = 0.1$, $\mu = 0.06$, $\sigma = 0.1$, $\tau = 0.4$, $x = 0.005$, $\delta = 0.1$, $\eta = 0.05$, $\gamma = 0.01$, $C_0 = 0.01$ and $S = 0.005$.

We start our analysis with the impact of an exogenous subsidy retraction threshold on the optimal investment threshold and capacity. Consistent with Lukas and Thiergart (2019) and our results in Chapter 3, we find that a later subsidy retraction, corresponding to smaller ω, accelerates the firm's investment as shown in the left panel of Figure 4.2, the impact, though, on the firm's optimal capacity level can vary (see right panel). Under technological learning, the firm has a greater (smaller) incentive to invest more, as a larger (smaller) project advances (postpones) the cost-reduction process (subsidy withdrawal). More specifically, when ω is high the subsidy is likely to be withdrawn sooner and investing in cost reduction to become more attractive. Decreasing, however, ω increases the subsidy duration making it more significant, in which case the firm can choose to invest in a smaller project in exchange for a longer subsidy.

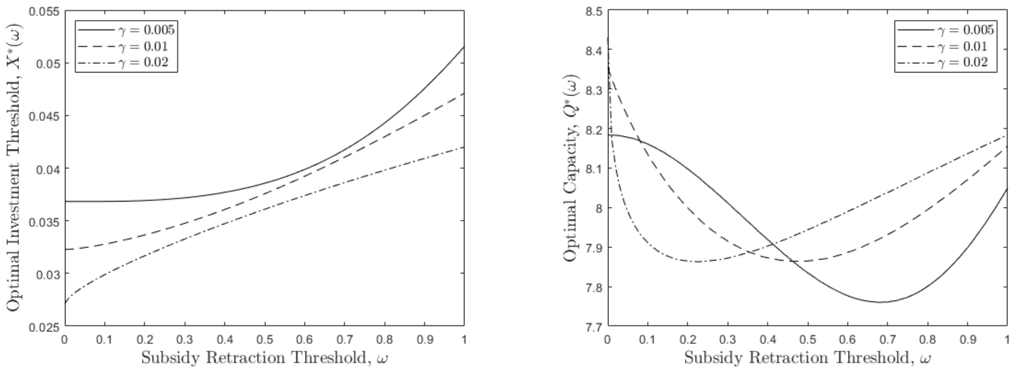

Figure 4.2: Effect of subsidy retraction threshold on firm's optimal investment threshold (left panel) and optimal capacity (right panel).

Consistent with conventional real options intuition, increasing price uncertainty raises the opportunity cost of investment and, in turn, the value of waiting and investment threshold, as shown in Figure 4.3. In addition, Figure 4.3 also demonstrates the impact of technological learning on the equilibrium investment threshold and project scale; while the former strictly decreases with increasing learning rate (left panel), the latter exhibits an inverted U-shape (right panel) that confirms hypothesis H1 and Proposition 4.3.7. The former result can be attributed to a higher learning rate reducing the O&M costs, thus expediting investment. As for the latter, when the learning rate is low (here, $\gamma < 0.02$), the firm invests more as the learning rate increases; however, if the learning rate is high enough ($\gamma > 0.02$), it can lead to smaller investment capacity (see Proposition 4.3.7).

Naturally, under a flat learning curve, the learning effect is so weak that investing in a larger project has little effect on cost reduction, therefore the capacity is low when γ is close to 0. As γ increases, the firm has a stronger incentive to invest more to reduce the O&M cost. However, if γ is large enough, the cost-reduction process is fast even with a smaller investment size. Thus, further investing in learning (by increasing capacity) becomes less attractive, as it is dominated by the cost of delaying the investment (higher discount rate). As a result, and in line with Della Seta et al. (2012), it becomes optimal for the firm to invest earlier in a smaller project when the learning curve is steep.

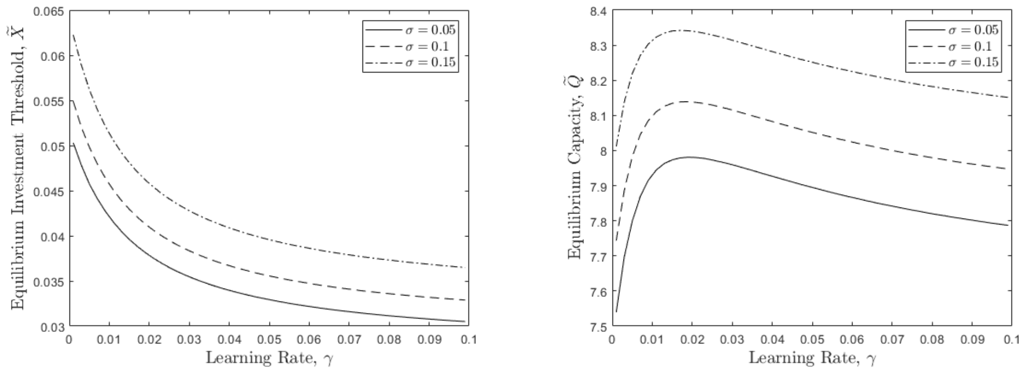

Figure 4.3: Effect of learning rate and price uncertainty on firm's equilibrium investment threshold (left panel) and equilibrium capacity (right panel).

The implications of technological learning for the Government's subsidy withdrawal decision are exhibited in Figure 4.4. We find that, although the Government is willing to retract the subsidy at a lower threshold if the learning rate increases (left panel), the duration of the subsidy is actually shorter (right panel), i.e., the total amount of subsidy the firm receives decreases with higher γ. Indeed, with a steep learning curve, the firm's O&M cost drops rapidly below grid parity and the Government sets a lower subsidy retraction threshold to ensure sufficient subsidy for the firm. However, the duration of the subsidy, $-\ln(\widetilde{\omega})/(\gamma\widetilde{Q})$, decreases much faster with increasing γ than with decreasing $\widetilde{\omega}$. This result confirms hypothesis H2 and contrasts with the conventional real options intuition that a firm tends to invest later when the subsidy is low (see left panel of Figure 4.3). The reason is that the learning rate and subsidy play a similar role in cost reduction and in stimulating investment, therefore in the presence of technological learning, the firm may rely less on Government support. Meanwhile, the Government tends to withdraw the subsidy earlier when price uncertainty is high, as the rapid push of the investment threshold (see left panel of Figure 4.3) has a negative impact on the Government's discounted payoff. In this case, the effect of the subsidy on accelerating investments is weak and the additional tax income cannot cover the cost of subsidisation, and, as a result, the Government eventually stops subsidising the firm.

Moving on to the tax rate effects, the left panel of Figure 4.5 exhibits the non-

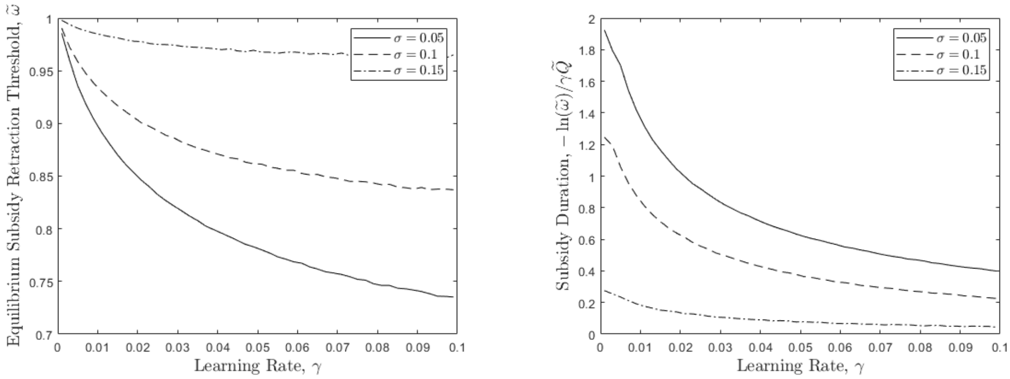

Figure 4.4: Effect of learning rate and price uncertainty on equilibrium subsidy retraction threshold (left panel) and equilibrium subsidy duration (right panel).

monotonic behaviour of the equilibrium investment threshold. While the the tax rate increases, we observe a particular region (here, $0.39 < \tau < 0.55$) in which the firm does not delay the investment. This counter-intuitive finding can be attributed to the fact that the rising tax rate effect is moderated by the deferral of subsidy withdrawal. Indeed, as examined by Pennings (2000) and Yu et al. (2007), subsidisation is more efficient in accelerating private investments compared to tax cuts.

More interesting, and in contrast to the existing literature (Lukas and Thiergart, 2019) (see also Proposition 3.3.3 and the top-right panel of Figure 3.7 in the previous chapter), is the V-shaped relationship between the equilibrium capacity and tax rate in the right panel that confirms hypothesis H3[2]. When the tax rate is low ($0.39 < \tau \leq 0.47$), the O&M cost is relatively low compared to the net income, and thus the firm's incentive for cost reduction is also low. As a result, the firm will choose to expedite investment with less capacity to offset the effect of rising tax rate. Conversely, when the tax rate is high ($\tau > 0.47$), the cost-cutting process becomes more attractive to the firm due to lower net income. Therefore, a further increase in the tax rate leads to larger investment scale, thereby to deferred investment. The lower panel shows that it is not optimal for the Government to grant a subsidy when the tax rate is below $1/(\beta + 1)$, since the additional tax income cannot cover the cost of the subsidy. As the tax rate increases, $\widetilde{\omega}$ decreases, indicating that the

[2]Noting that, under a fixed subsidy level (e.g., for $\tau < 0.39$ or $\tau > 0.55$), the investment threshold strictly decreases with an increasing tax rate, while the capacity remains constant. These findings align with Azevedo et al. (2021), who compared the distinct impacts of tax cuts and subsidies, demonstrating that a tax cut encourages earlier investments without affecting the capacity size.

Government is willing to provide the subsidy for a longer period to compensate for the higher tax rate. Furthermore, if the tax rate is high (here, $\tau > 0.55$), the Government should never revoke the subsidy.

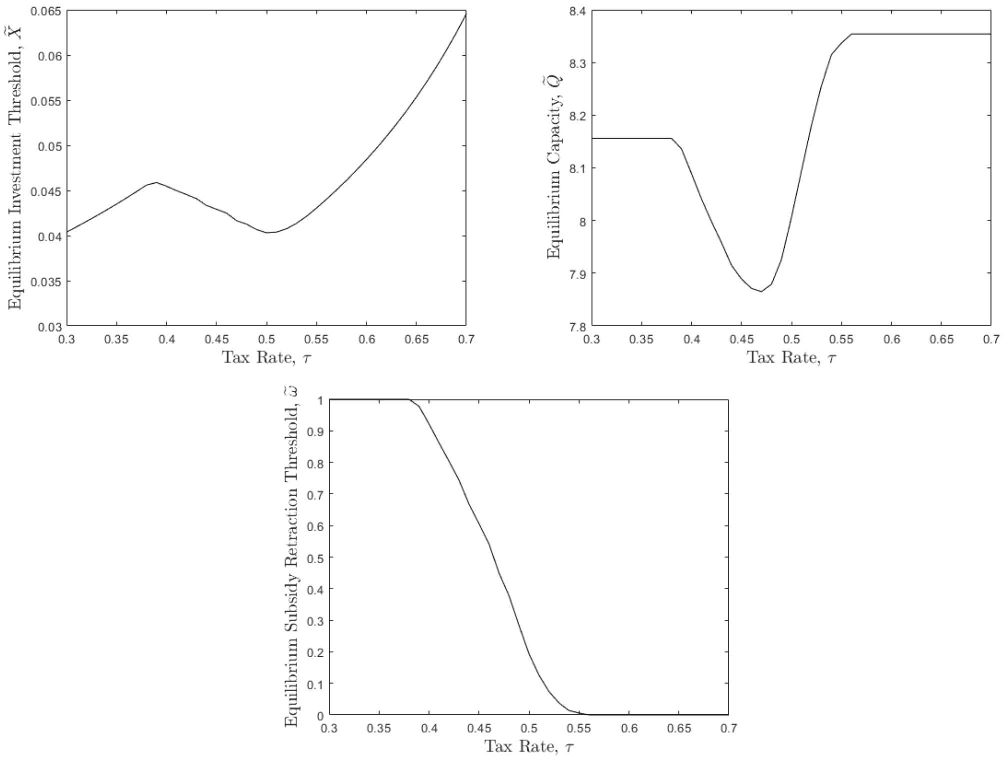

Figure 4.5: Tax rate effect on firm's equilibrium investment threshold (top-left panel), equilibrium capacity (top-right panel), and equilibrium subsidy retraction threshold (bottom panel).

We conclude our numerical analysis with Figure 4.6 illustrating how the firm's relative loss in value in the absence of discretion over project scale, as given by the percentage difference between the option value (4.5) when the firm invests at fixed capacity Q as opposed to equilibrium capacity $\widetilde{Q}$, is affected by economic uncertainty and technological learning. The left panel exhibits zero relative loss at the equilibrium capacity, but this increases with under- or over-investment with greater economic uncertainty leading to higher (lower) relative loss if the firm under-invests (over-invests). Naturally, this is because the equilibrium capacity increases with uncertainty (see right panel of Figure 4.3), and the relative loss due to under-investment (over-investment) with a fixed capacity is magnified (attenuated). The right panel indicates a rather ambiguous effect of learning rate on relative loss.

When γ is small, a larger learning rate hoists (reduces) the relative loss in value in the case of under-investment (over-investment); however, this effect reverses for large γ. This is due to the non-monotonic relationship of the equilibrium capacity with the learning rate, as shown in the bottom-right panel of Figure 4.3, where it first increases with γ before starting to decrease.

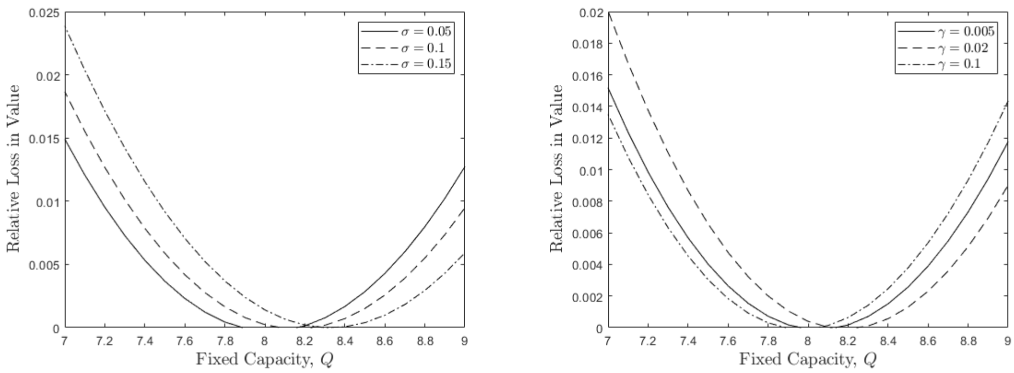

Figure 4.6: Relative loss in value due to fixed capacity, subject to varying σ (left panel) and γ (right panel).

4.5 Concluding discussion

In recent years, the literature on optimal subsidy design for AET has experienced rapid growth, with various models being developed to explore the implications of policy uncertainty on AET investments. While these models have provided valuable insights, they often rely on probabilistic assumptions about the timing of subsidy retraction and overlook the impact of technological learning. In practice, learning curves have emerged as a critical factor affecting a firm's production strategy and the Government's subsidisation policy-making. Indeed, subsidies are typically phased out as technological learning drives down the cost of AETs towards commercial maturity.

In this chapter, we examine the joint impact of technological learning and subsidy retraction on the equilibrium subsidisation policy of the Government and the capacity investment policy of a firm by developing a bi-level real options framework. Our results show that, while technology learning propels investment and encourages the installation of a larger project, it also speeds up subsidy retraction. Consequently, the firm faces the trade-off between expediting (or slowing down) cost reduction and receiving less (more) subsidy when deciding the investment capacity. Interestingly, even though the duration of the subsidy dwindles with increasing learning rate, the firm remains willing to invest earlier. This is because the learning effect moderates the impact of reduced subsidy, making early investment still attractive despite the shorter subsidy period. In addition, we show that, under highly uncertain environments, it is optimal for the Government to withdraw the subsidy earlier, while the firm tends to delay its investment and opt for larger capacity.

Our analysis offers valuable insights not only for investors but also for policymakers aiming to design efficient subsidisation schemes. Contrary to conventional intuition, which suggests that a higher tax rate would discourage the firm's investment, causing delays and leading to smaller capacity, our results challenge this ideology, particularly when subsidies and technological learning are taken into account. This occurs because, as the Government increases the tax rate, it also extends the duration of the subsidy provided to the firm offsetting part of the effect caused by the rising tax rate.

Chapter 5

Concluding remarks and future directions

The primary objective of this thesis is to develop models that support private firms and the Government in make more informed investment decisions. This is achieved through an in-depth investigation of two critical domains within the field of operational research: project scheduling and investment under uncertainty and strategic interactions. For the former, we depart from the common assumption that decision-makers are risk-neutral, thereby demonstrating the significance of risk consideration in the valuation and planning of complex projects. This is especially pertinent in today's landscape characterised by rising market uncertainty and the deregulation of many industries. As for the latter, we develop bi-level real options models in order to derive the equilibrium investment and subsidisation policies taking into account the strategic interaction between private firms and the Government. Furthermore, we expand our model to explore the relationship between technological learning and subsidy retraction. We reveal their joint impact on a firm's investment strategy and extract valuable insights for the Government in formulating well-informed subsidy retraction policies. Hence, we believe that this work contributes significantly to an important area of decision-making.

Below, we summarise the key findings of this thesis, discuss the limitations of each chapter and offer directions for further research.

In Chapter 2, we develop a risk assessment and optimal scheduling framework for sequential capacity expansion under output price and technological uncertainty. We derive the distribution, VaR and CVaR of the project's NPV. The novelty of our work lies in demonstrating the potentially positive impact of duration variability on the stochastic project scheduling problem, whereby we highlight the importance of risk considerations. We show that both the duration variability and the decision-makers' risk preferences can significantly affect the optimal sequence of stages of a serial project and that this also depends on the capacity expansion cost. Specifically, in cases where the expansion cost of each stage is high, the variability in duration negatively impacts a project's NPV and increases its risk exposure. Consequently, decision-makers should prioritise the execution of stages with lower duration variability. However, when the cost is relatively low, it might be optimal for risk-neutral decision-makers to initially execute stages with higher duration variability, as this can result in a larger expected NPV. Taking also into account the decision-makers' attitudes towards risk, we find that executing stages with lower duration variability earlier does not guarantee lower risk exposure. Contrary to the intuition that increasing uncertainty entails greater risk exposure, our results indicate that higher duration variability may not lead to higher downside risk; instead, it may be beneficial not only for risk-neutral but also for risk-averse decision-makers.

Hence, Chapter 2 conveys crucial implications for investment under uncertainty when the true distribution of a project's makespan is unknown, as ignoring or underestimating the uncertainty associated with the project may lead to inappropriate project scheduling and, therefore, lower NPV or greater downside risk. Future research directions for Chapter 2 may include studying the potential effects of the volatility of the price dynamics on the risk measures of the project, or the development of a real options framework to allow for discretion over investment timing (Heydari and Siddiqui, 2009; Jeon, 2021). The objective would be to investigate how managerial flexibility influences the distribution of the NPV and the risk measures of a serial project. Also, a computational comparison of different approximation methods and an algorithmic study on more elaborated project scheduling models taking risk aversion into account can also be meaningful extensions of this work.

Furthermore, we investigate the optimal capacity investment and subsidy design problem through a non-cooperative game involving both the firm and the Government. Despite their increasing prominence, models for analysing the interaction between firm and Government-level policy-making do not account for critical features of a deregulated environment, such as competition and policy uncertainty. Therefore, we address these disconnects in Chapter 3 and 4 by developing a bi-level real options framework for deriving the equilibrium Government subsidisation and firm-level capacity investment policy under duopolistic competition and under technological learning and subsidy retraction, respectively.

Our results in Chapter 3 clearly indicate that the insights of traditional bi-level real options models under monopoly cannot be naturally transposed to a deregulated environment. In particular, we find that strategic interactions with the Government can significantly affect a firm's capacity investment decision and that the equilibrium subsidisation policy crucially depends on market structure and the type of duopolistic competition. Contrary to conventional intuition, we find that providing a larger subsidy to the leader can actually increase the follower's incentive to invest earlier and in a bigger project. Furthermore, we demonstrate how results are different when the Government aims to maximise social welfare and show that competition can be desirable for a social planner, while a profit-maximising Government may benefit the most from pre-emptive competition. Therefore, the policy-making and managerial relevance of our results is reflected in the new insights gained when firm-level strategic interactions are integrated into the evaluation of real options. In particular, not only is competition a key aspect of deregulated industries and entails a loss in value relative to monopoly that must be taken into account when designing subsidisation policies, but also the type of competition can affect significantly a Government's subsidisation policy. Similarly, at the firm level, the interaction with Government policy-making produces dynamics under which the investment policy deviates from that of traditional duopolistic competition, which ignores such interactions. Such strategic interactions tend to be overlooked in the literature that values bi-level real options, yet if their implications are not properly understood, subsidisation policies will not be properly designed, thus potentially inducing under or over-investment cycles and increased regulatory risk following corrective policy actions.

Directions for future work of Chapter 3 may include relaxation of the assumption of unilateral subsidy and allowance for asymmetric competition. More specifically, it would be interesting to analyse how the positioning and cost asymmetry of firms can affect the equilibrium subsidy, and whether the Government should offer either bilateral subsidies to both firms or a unilateral subsidy to the larger or smaller firm. Also, our model does not consider production flexibility (Hagspiel et al., 2016a) or sequential capacity expansion options, so the project size is fixed at investment and cannot be adjusted afterwards; both options would be meaningful additions to this work. Finally, the assumption of duopolistic competition could be relaxed to explore optimal investment and subsidisation policies under oligopoly when the *accordion effect* occurs (Bouis et al., 2009).

While the literature on optimal subsidy design for AET has experienced rapid growth, with various models being developed to explore the implications of policy uncertainty on AET or RE investments, they often rely on probabilistic assumptions about the timing of subsidy retraction and overlook the effects of technological learning. Yet, learning curves have emerged as a critical factor affecting a firm's production strategy and the Government's subsidisation policy-making, where subsidies are typically phased out as the learning effect drives the cost of AET gradually towards its commercial maturity. Therefore, we examine in Chapter 4 the joint impacts of technological learning and subsidy withdrawal on the firm's capacity investment. Our results show that, while technology learning propels investment and encourages larger capacity size, it also speeds up the retraction of subsidies. Consequently, the firm faces the trade-off between expediting (or slowing down) cost reduction and receiving less (more) subsidy when deciding the investment capacity. Interestingly, our findings indicate that, even though the duration of the subsidy dwindles with increasing learning rates, the firm remains willing to invest earlier. This is because the learning effect moderates the impact of reduced subsidy, making early investment still attractive despite the shorter subsidy period. In addition, we show that, under highly uncertain environments, it is optimal for the Government to withdraw the subsidy earlier, while the firm tends to delay its investment and opt for larger capacity. Contrary to conventional intuition, our research reveals that a higher tax rate does not necessarily result in investment delays or reduced capacity size, particularly in the presence of subsidies and technological learning. This occurs because, as the Government increases the tax rate, it may simultaneously extend the duration of the subsidies provided to the firm to compensate the firm for paying more tax; meanwhile the learning effect diminishes the firm' incentive to under-invest. Hence, Chapter 4 offers valuable insights not only for investors but also for policy-makers aiming to design efficient subsidisation schemes.

A limitation of our model is that while the subsidy retraction threshold is determined endogenously by the Government, we assume an exogenous and fixed subsidy level S. Therefore, an important direction for future work would be to allow for both an endogenous subsidy level and subsidy retraction. Also, in addition to the Government's profit-oriented strategy, other objectives may be integrated within the same framework, such as social welfare optimisation (Yang et al., 2021) or revenue-neutral, i.e., zero-cost, tax-subsidy packages (Pennings, 2000; Azevedo et al., 2021).

Appendix A

Proofs and Supplementary Results of Chapter2

A.1 Proofs of the Propositions

Proof of Proposition 2.3.1. From (2.2), we have that

$$\begin{aligned} G_V(v) &= \mathbb{P}\left(\frac{PD_0}{r-\alpha} + \frac{PD_1}{r-\alpha} e^{-(r-\alpha)T_1} - cD_1 \le v\right) \\ &= \mathbb{P}\left(T_1 \ge -\frac{1}{r-\alpha} \ln \frac{(r-\alpha)(v+cD_1) - PD_0}{PD_1}\right), \end{aligned}$$

from which (2.5) follows for $v \ge PD_0/(r-\alpha) - cD_1$. The density function (2.6) follows from differentiating (2.5) with respect to v. ■

Proof of Proposition 2.3.2. By definition of VaR, we have that

$$\begin{aligned} \mathrm{VaR}_p(V) &= -\inf\{v \in \mathbb{R} : \mathbb{P}(V \le v) > p\} \\ &= -\inf\left\{v \in \mathbb{R} : 1 - F_{T_1}\left(-\frac{1}{r-\alpha} \ln \frac{(r-\alpha)(v+cD_1) - PD_0}{PD_1}\right) > p\right\} \\ &= -\inf\left\{v \in \mathbb{R} : v > \frac{PD_0}{r-\alpha} + \frac{PD_1}{r-\alpha} e^{-(r-\alpha)F_{T_1}^{-1}(1-p)} - cD_1\right\}, \end{aligned}$$

from which the final result follows. The CVaR follows by definition. ■

A.2 The case of managerial investment flexibility

In what follows, we develop a real options framework which incorporates the firm's discretion over investment timing. In this case, the firm is not obligated to invest immediately in the next capacity expansion after each stage is completed. Instead, it has the option to delay the next investment while waiting for more favourable price conditions (refer also to Heydari and Siddiqui (2009) for a study of the optimal interruption policy of multiple-exercise interruptible load contracts).

We denote by $T_i = \sum_{j=1}^{i}(w_j + \tau_j)$ the completion time of stage i, which now includes both waiting times $\{w_j\}_{j=1}^{i}$ and construction times $\{\tau_j\}_{j=1}^{i}$ of all stages up to i. Let $P^{(i)}$ be the investment threshold of stage i and $P^{(i)^*}$ the optimal investment threshold. Hence, w_i is the random first-passage time of the price process through the investment threshold from below, i.e., $w_i = \inf\left\{t \geq 0 : P_{T_{i-1}+t} \geq P^{(i)}\right\}$.

Figure A.1 illustrates the cash flows of a single-stage capacity expansion when the firm has discretion over investment timing. We assume that the initial output price of the project is too low to justify immediate investment, therefore the firm must defer it.

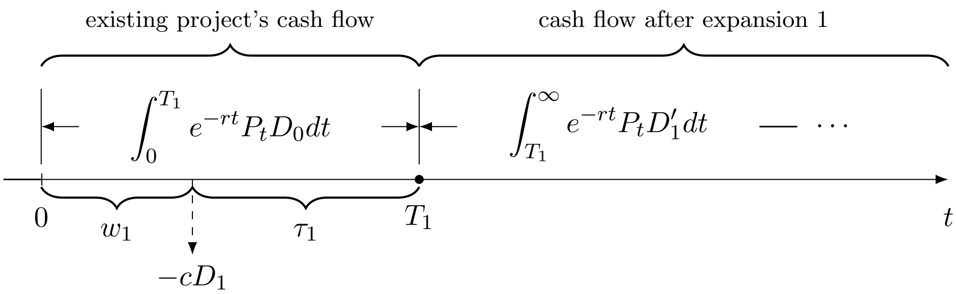

Fig. A.1. Single-stage capacity expansion.

The firm's expected option value is determined via backward induction. Therefore, we first assume that the project is already active and accrues stochastic revenues. The conditional expected NPV of the project is given by

$$V\left(P, \tau_1\right) = \frac{PD_0}{r-\alpha} + \frac{PD_1}{r-\alpha} e^{-(r-\alpha)\tau_1} - cD_1. \tag{A.1}$$

Moving backwards, we assume that the initial output price is too low to justify immediate investment, so the firm must wait for a period of time, w_1. The firm's optimisation objective is

$$
\begin{aligned}
F\left(P, \tau_1\right) &= \sup_{w_1 \in \mathcal{S}} \mathbb{E}\left[\int_0^{w_1+\tau_1} e^{-rt} P_t D_0 dt + \int_{w_1+\tau_1}^{\infty} e^{-rt} P_t D_1' dt - C_1 e^{-r w_1} \middle| P, \tau_1\right] \\
&= \sup_{w_1 \in \mathcal{S}} \mathbb{E}\left[\int_0^{\infty} e^{-rt} P_t D_0 dt \middle| P\right] \\
&\quad + \mathbb{E}\left[\left(\int_{\tau_1}^{\infty} e^{-rt} P_{w_1+t} D_1 dt - c D_1\right) e^{-r w_1} \middle| P, \tau_1\right] \\
&= \max_{P^{(1)} > P} \frac{P D_0}{r-\alpha} + \left(\frac{P}{P^{(1)}}\right)^{\rho} \left(\frac{P^{(1)} D_1}{r-\alpha} e^{-(r-\alpha)\tau_1} - c D_1\right), \qquad \text{(A.2)}
\end{aligned}
$$

where $\mathcal{S}$ is the set of stopping times of the filtration generated by the price process. Note that the last equality follows from the stochastic discount factor $\mathbb{E}\left[e^{-r w_1} \middle| P\right] = \left(P/P^{(1)}\right)^{\rho}$ ((Dixit and Pindyck, 1994, p. 315)), with $\rho > 1$ the positive root of $\beta^2 x(x-1)/2 + \alpha x - r = 0$. By applying the first-order necessary condition (FONC) to the unconstrained optimisation problem (A.2), we obtain the optimal investment threshold of the first capacity expansion:

$$
P^{(1)^*} = \frac{\rho}{\rho-1} c(r-\alpha) e^{(r-\alpha)\tau_1}. \qquad \text{(A.3)}
$$

We now consider the two-stage capacity expansion as illustrated in Figure A.2.

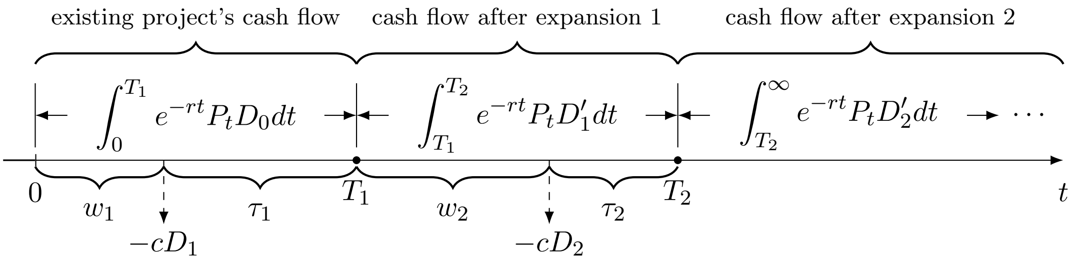

Fig. A.2. Two-stage capacity expansion.

Starting with the second capacity expansion, we first consider the case that once the construction of the first stage is completed at T_1, the price process is high enough for immediate investment, i.e., $P^{(2)} = P_{T_1+w_2} \leq P_{T_1}$. The conditional expected NPV of the second expansion (discounted to time $T_1 = w_1 + \tau_1$) is given by

$$V\left(P_{T_1}, \tau_2\right) = \frac{P_{T_1} D_2}{r-\alpha} e^{-(r-\alpha)\tau_2} - cD_2. \tag{A.4}$$

Next, if $P^{(2)} > P_{T_1}$, that is, the firm cannot invest directly in the second stage and must wait, the maximised value of the option to invest in the second stage is given by

$$\begin{aligned} F^{(2)}\left(P_{T_1}, \tau_2\right) &= \sup_{w_2 \in \mathcal{S}} \mathbb{E}\left[\int_{w_2+\tau_2}^{\infty} e^{-rt} P_{T_1+t} D_2 dt - C_2 e^{-rw_2} \middle| P_{T_1}, \tau_2\right] \\ &= \max_{P^{(2)} > P_{T_1}} \left(\frac{P_{T_1}}{P^{(2)}}\right)^{\rho} \left(\frac{P^{(2)} D_2}{r-\alpha} e^{-(r-\alpha)\tau_2} - cD_2\right). \end{aligned} \tag{A.5}$$

The optimal investment threshold of the second expansion is then

$$P^{(2)^*} = \frac{\rho}{\rho - 1} c(r-\alpha) e^{(r-\alpha)\tau_2}. \tag{A.6}$$

Working backwards to the first stage, if it is still optimal to wait, i.e., $P^{(1)^*} \geq P$, the maximised option value of the first capacity expansion is

$$\begin{aligned} F^{(1)}\left(P, \tau_1, \tau_2\right) &= \sup_{w_1 \in \mathcal{S}} \mathbb{E}\left[\int_{w_1+\tau_1}^{\infty} e^{-rt} P_t D_1 dt - C_1 e^{-rw_1} + e^{-rT_1} \mathbb{E}\left[F^{(2)}\left(P_{T_1}, \tau_2\right)\right] \middle| P, \tau_1, \tau_2\right] \\ &= \max_{P^{(1)} > P} \left(\frac{P^{(1)} D_1}{r-\alpha} e^{-(r-\alpha)\tau_1} - cD_1 + e^{-r\tau_1} \mathbb{E}\left[F^{(2)}\left(P_{T_1}, \tau_2\right)\right]\right) \left(\frac{P}{P^{(1)}}\right)^{\rho}, \end{aligned} \tag{A.7}$$

where the conditional expectation of the option value of the second expansion, given the information at time T_1, depends on whether or not the second stage is executed immediately, i.e., whether or not $P^{(2)^*} \leq P_{T_1}$:

$$\begin{aligned} \mathbb{E}\left[F^{(2)}\left(P_{T_1}, \tau_2\right) \middle| P_{T_1}, \tau_2\right] &= \left(\frac{P^{(2)^*} D_2}{r-\alpha} e^{-(r-\alpha)\tau_2} - cD_2\right) \left(\frac{P_{T_1}}{P^{(2)^*}}\right)^{\rho} \mathbb{P}\left(P^{(2)^*} > P_{T_1}\right) \\ &\quad + \left(\frac{P_{T_1} D_2}{r-\alpha} e^{-(r-\alpha)\tau_2} - cD_2\right) \mathbb{P}\left(P^{(2)^*} \leq P_{T_1}\right). \end{aligned} \tag{A.8}$$

In Table A.1, we present the optimal scheduling of a two-stage project when the firm has the option to delay the investment for each stage. Our results confirm that executing the stage with higher duration variability is not always harmful.

Table A.1: Optimal scheduling of a two-stage project, when the firm has the option to delay the next investment, for decision makers with risk appetite ω and duration variability s, where $\tau_1 \sim \text{LogN}(\ln 3 - 1/2, 1)$ and $\tau_2 \sim \text{LogN}\left(\ln 3 - s^2/2, s\right)$. $\pi = (1,2)$: execute stage 1 followed by stage 2; $\pi = (2,1)$: execute stage 2 followed by stage 1.

$c = 2$									
	Option Value		$\text{VaR}_{0.05}$		Optimal Sequence $\boldsymbol{\pi}^*_\omega$				
	$\pi = (1,2)$	$\pi = (2,1)$	$\pi = (1,2)$	$\pi = (2,1)$	$\omega = 0$	$\omega = 0.25$	$\omega = 0.5$	$\omega = 0.75$	$\omega = 1$
$s = 0.5$	17.7865	17.8093	-14.8091	-14.6098	$(2,1)$	$(1,2)$	$(1,2)$	$(1,2)$	$(1,2)$
$s = 1.5$	17.8716	17.8171	-14.4123	-13.7516	$(1,2)$	$(1,2)$	$(1,2)$	$(1,2)$	$(1,2)$
$s = 2.0$	17.9705	17.9143	-14.4424	-14.0803	$(1,2)$	$(1,2)$	$(1,2)$	$(1,2)$	$(1,2)$
$s = 2.5$	18.0860	18.0576	-14.5931	-14.8854	$(1,2)$	$(2,1)$	$(2,1)$	$(2,1)$	$(2,1)$
$s = 3.0$	18.1942	18.2050	-14.8093	-15.7409	$(2,1)$	$(2,1)$	$(2,1)$	$(2,1)$	$(2,1)$

We note that, while the generalised multi-stage problem does not admit an analytical solution, it is, nevertheless, possible to solve it numerically to obtain the optimal scheduling of the serial project.

Appendix B

Proofs of the Propositions of Chapter 3

Proof of Proposition 3.3.1.

1. If the demand is low, the firm must defer investment, i.e., $X < X_m^*(S_m)$, $\forall S_m$. From (3.7), the value of the monopolist's investment opportunity is given by

$$F_m(X, S_m) = \max_{X_m > X} \left(\frac{X}{X_m}\right)^{\beta} \left[\frac{1-\tau}{r-\mu}\left(1 - \eta Q_m^*\right) Q_m^* X_m - \delta Q_m^* + S_m\right]. \quad \text{(B.1)}$$

Applying the FONC to (B.1), the optimal investment threshold can be expressed as

$$X_m^*(Q_m^*, S_m) = \frac{\beta}{\beta-1}\frac{r-\mu}{1-\tau}\frac{\delta Q_m^* - S_m}{(1-\eta Q_m^*)Q_m^*}. \quad \text{(B.2)}$$

Substituting X_m^* into (3.5) and solving with respect to Q_m^* yields

$$Q_m^*(S_m) = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{for} \quad b^2 - 4ac \geq 0,$$

where $a = (\beta+1)\eta\delta/(\beta-1)$, $b = -(\delta + 2\beta\eta S_m)/(\beta-1)$ and $c = \beta S_m/(\beta - 1)$. Substituting $Q_m^*(S_m)$ into (B.2) gives the optimal investment threshold, $X_m^*(S_m) = c(S_m)$ if $c(S_m) \geq X$; otherwise $X_m^*(S_m) = X$. If $b^2 - 4ac < 0$, i.e., $S_m > (\beta - \sqrt{\beta^2 - 1})\delta/(2\beta\eta)$, then $Q_m^*(S_m)$ is not real and a local maximum of (B.1) does not exist. In this case, the firm invests either immediately or never. As $\lim_{X_m \to \infty} F_m(X, S_m) = 0$, we conclude that $X_m^* = X$ and the corresponding optimal investment capacity Q_m^* is given by (3.5).

2. Having derived analytical expressions for $X_m^*(S_m)$ and $Q_m^*(S_m)$, we can also determine how S_m affects the optimal investment policy. For $X < X_m^*(S_m)$, the optimal investment capacity is given in the top part of (3.9), and differentiating with respect to S_m yields

$$\frac{\partial Q_m^*(S_m)}{\partial S_m} = \frac{\beta}{(\beta+1)\delta}\left(1 - \frac{\beta(\delta - 2\eta S_m)}{\sqrt{\delta^2 - 4\beta^2\eta S_m(\delta - \eta S_m)}}\right). \tag{B.3}$$

Since

$$\beta > 1 \Longleftrightarrow 1 < \frac{\beta(\delta - 2\eta S_m)}{\sqrt{\delta^2 - 4\beta^2\eta S_m(\delta - \eta S_m)}},$$

we have that $\partial Q_m^*(S_m)/\partial S_m < 0$. From (3.5), we observe that the optimal capacity level is increasing in X_m, i.e., $\partial Q_m^*(S_m)/\partial X_m^* > 0$. Consequently,

$$\frac{\partial X_m^*(S_m)}{\partial S_m} = \frac{\partial X_m^*(S_m)}{\partial Q_m^*(S_m)}\frac{\partial Q_m^*(S_m)}{\partial S_m} < 0.$$

■

Proof of Proposition 3.3.2. From Proposition 3.3.1, we have that $S_m \in [0, (\beta - \sqrt{\beta^2 - 1})\delta/2\beta\eta]$. From the top part of (3.8), $X_m^*(0) = \max[X, (r-\mu)(\beta+1)\delta/((1-\tau)(\beta-1))]$ and $X_m^*\left((\beta - \sqrt{\beta^2-1})\delta/2\beta\eta\right) = \max[X, (r-\mu)(\beta+1)\delta/((1-\tau)(\beta^2-1)^{1/2})]$. We conduct a case-by-case analysis.

First, when the X is too low, the firm will always postpone the investment, i.e., $X < (r-\mu)(\beta+1)\delta/((1-\tau)(\beta^2-1)^{1/2})$, even if the Government grants the maximum subsidy level. Rearranging (3.5), we obtain $X_m^*(Q_m^*) = (r-\mu)\delta/((1-\tau)(1-2\eta Q_m^*))$, where $Q_m^*(S_m)$ is given by (3.9). Then, the discounted value of the Government's tax income at time 0 in (3.10) can be written as

$$G_m(X, S_m) = \left(\frac{X(1-\tau)\left(1 - 2\eta Q_m^*(S_m)\right)}{(r-\mu)\delta}\right)^\beta \left[\frac{\tau\left(1 - \eta Q_m^*(S_m)\right)Q_m^*(S_m)\delta}{(1-\tau)(1 - 2\eta Q_m^*(S_m))} - S_m\right]. \tag{B.4}$$

Differentiating $G_m(X, S_m)$ with respect to S_m, we get

$$\begin{aligned}\frac{\partial G_m(X, S_m)}{\partial S_m} = &\left(\frac{X(1-\tau)(1-2\eta Q_m^*(S_m))}{(r-\mu)\delta}\right)^\beta \left[\frac{\partial Q_m^*(S_m)}{\partial S_m}\left(\frac{\tau\delta}{1-\tau}\right.\right.\\ &\left.\left. - \frac{2(\beta-1)\eta\tau(1-\eta Q_m^*(S_m))Q_m^*(S_m)\delta}{(1-\tau)(1-2\eta Q_m^*(S_m))^2} + \frac{2\eta\beta S_m}{1-2\eta Q_m^*(S_m)}\right) - 1\right],\end{aligned}$$

where $\partial Q_m^*(S_m)/\partial S_m$ is given by (B.3). Solving $\partial G_m(X, \widetilde{S}_m^\pi)/\partial S_m = 0$ gives the equilibrium subsidy $\widetilde{S}_m^\pi = \theta/(2\beta\eta\psi)$ for $\tau > 1/(1+\beta)$. If $\tau \le 1/(1+\beta)$, the local maximum of (B.4) does not exist and $\widetilde{S}_m^\pi = 0$. Hence, in this case, $\widetilde{S}_m^\pi = S_1$.

Second, we consider the case when the initial price of the project is high enough that the firm is willing to invest immediately even without a subsidy, i.e., when $X \ge (r-\mu)(\beta+1)\delta/((1-\tau)(\beta-1))$. In this case, $X_m^*(S_m) = X$, $\forall S_m$ making no sense for the Government to provide any subsidy.

Finally, if $\exists S_m > 0$ such that $X \ge X_m^*(S_m)$, i.e., $(\beta+1)\delta/\sqrt{\beta^2-1} \le (1-\tau)X/(r-\mu) < (\beta+1)\delta/(\beta-1)$, it is still possible for the firm to invest immediately if it receives sufficient subsidy. We denote by S_2 the subsidy level that leads to immediate investment, i.e., $S_2 = \inf\{s \in [0, (\beta - \sqrt{\beta^2-1})\delta/(2\beta\eta)] : X_m^*(s) = X\} = (\delta^2 - A^2)/(4\beta\eta(\beta\delta - A))$. The Government's action set is as follows: if $S_1 \ge S_2$, it will not offer more subsidy than the firm needs and the optimal subsidy will be S_2; if $S_1 < S_2$, it will not be optimal for the Government to induce immediate investment resulting in $\widetilde{S}_m^\pi = S_1$. Hence, $\widetilde{S}_m^\pi = \min\{S_1, S_2\}$.

Notice that, as $\tau \to 1$, the firm will not invest immediately as $X < \frac{r-\mu}{1-\tau}\frac{\beta+1}{\sqrt{\beta^2-1}}\delta$ and, therefore, $\widetilde{S}_m^\pi = S_1$. Meanwhile, from (3.13), we can obtain $\lim_{\tau\to1}\widetilde{S}_m^\pi = \left(\beta - \sqrt{\beta^2-1}\right)\delta/2\beta\eta$, indicating the maximum subsidy that the Government is willing to provide to a firm (see also equation 3.8). From Corollary 3.3.1, we arrive at $\lim_{\tau\to1}\widetilde{X}_m^\pi = \infty$, that is, the firm will never invest as all the revenue goes to the Government. ■

Proof of Corollary 3.3.1. This follows directly from Propositions 3.3.1 and 3.3.2. ■

Proof of Proposition 3.3.3. For $\tau > 1/(\beta+1)$, differentiating $\widetilde{S}_m^\pi$ with respect to ψ, we get

$$\frac{\partial \widetilde{S}_m^\pi}{\partial \psi} = \frac{\delta(\beta^2+\psi)^{-\frac{1}{2}}}{4\beta\eta\psi^2}\left(\psi - 2(\beta^2+\psi) + 2\beta\sqrt{\beta^2+\psi}\right) = -\frac{\delta\theta^2(\beta^2+\psi)^{-\frac{1}{2}}}{4\beta\eta\psi^2} < 0.$$

Since $\partial\psi/\partial\tau < 0$, we have that $\partial\widetilde{S}_m^\pi/\partial\tau > 0$. Given that $\partial\theta/\partial\psi = (\beta^2+\psi)^{-1/2}/2$, we differentiate $\widetilde{Q}_m^\pi$ with respect to ψ and obtain

$$\frac{\partial \widetilde{Q}_m^\pi}{\partial \psi} = f(\psi)\left(\sqrt{1+\psi} - \sqrt{1+\frac{\psi}{\beta^2}}\right) > 0,$$

as $f(\psi) = \psi/(\beta(\sqrt{\beta^2+\psi}+\beta)(\sqrt{1+\psi}+1))$ is a strictly positive function. Therefore,

$$\frac{\partial \widetilde{Q}_m^\pi}{\partial \tau} = \frac{\partial \widetilde{Q}_m^\pi}{\partial \psi}\frac{\partial \psi}{\partial \tau} < 0.$$

■

Proof of Proposition 3.3.4. Given $\tau > 1/(\beta+1)$, $\partial \widetilde{S}_m^\pi/\partial \eta < 0$ follows from (3.13). In addition, $\partial \widetilde{Q}_m^\pi/\partial \eta < 0$ and $\partial \widetilde{X}_m^\pi/\partial \eta = 0$ follow directly from Corollary 3.3.1. ■

Proof of Proposition 3.3.5. This follows the lines of proof of Proposition 3.3.1. The only difference is that the subsidy term in (B.1) vanishes as the follower receives no support, while the inverse demand function becomes $1 - \eta Q_{nl} - \eta Q_{nf}$. ■

Proof of Proposition 3.3.6. Using the follower's investment strategy as shown in Proposition 3.3.5 in (3.20), the option value of the leader, given investment threshold, X_{nl}, and capacity, Q_{nl}, can be written as

$$\begin{aligned} F_{nl}(X,S_n) \quad = \quad & \left(\frac{X}{X_{nl}}\right)^\beta \left[\frac{(1-\tau)(1-\eta Q_{nl})Q_{nl}X_{nl}}{r-\mu} - \delta Q_{nl} + S_n \right. \\ & \left. - \frac{\delta Q_{nl}}{\beta - 1}\left(\frac{(\beta-1)(1-\tau)(1-\eta Q_{nl})X_{nl}}{(\beta+1)(r-\mu)\delta}\right)^\beta\right]. \end{aligned} \tag{B.5}$$

Then, by maximising (B.5) with respect to X_{nl}, we obtain the leader's optimal investment threshold as a function of Q_{nl}:

$$X_{nl}^*(S_n) = \frac{\beta}{\beta-1}\frac{r-\mu}{1-\tau}\frac{\delta Q_{nl} - S_n}{(1-\eta Q_{nl})Q_{nl}}. \tag{B.6}$$

Substituting (B.6) back into (B.5) and maximising it with respect to Q_{nl}, we obtain Q_{nl}^* as the solution to (3.21). If, however, $X > X_{nl}^*(S_n)$, the leader will invest immediately at time 0, with the corresponding Q_{nl}^* maximising (3.19). Additionally, if X is so large that $X > X_{nf}^*$, both firms will invest immediately, with Q_{nl}^* obtained in the same way. ∎

Proof of Proposition 3.3.7. It can be derived by following the same procedure as shown in the proof of Proposition 3.3.2 with $G_m(X, S_m)$ replaced by $SW_m(X, S_m)$ as formulated in (3.30). ∎

Proof of Corollary 3.3.2. This follows directly from Propositions 3.3.1 and 3.3.7. ∎

Proof of Proposition 3.3.8. Rewriting the upper part of (3.31) and (3.33) as

$$S_1^w = \frac{\tau\delta}{3\eta\,(3-4\tau)}\left(\sqrt{4\tau^2+\frac{3\,(3-4\tau)}{\beta^2}}-2\tau\right) \quad \text{and} \quad \widetilde{X}_m^w = \frac{r-\mu}{1-\tau}\frac{(1+\frac{1}{\beta})\delta(3-4\tau)}{(3-2\tau)-\sqrt{4\tau^2+\frac{3(3-4\tau)}{\beta^2}}},$$

respectively, we can obtain $\partial S_1^w/\partial\beta < 0$ and $\partial\widetilde{X}_m^w/\partial\beta < 0$, and, therefore, $\partial S_1^w/\partial\sigma > 0$ and $\partial\widetilde{X}_m^w/\partial\sigma > 0$. Next, we show that

$$\frac{\partial}{\partial\beta}\left(\sqrt{4\tau^2\beta^2+3\,(3-4\tau)}-2\tau\beta\right) = \frac{4\tau^2\beta}{\sqrt{4\tau^2\beta^2+3\,(3-4\tau)}}-2\tau$$

is negative, if $3 > 4\tau$, and positive, if $3 < 4\tau$. Substituting this to the upper part of (3.34), we obtain $\partial\widetilde{Q}_m^w/\partial\beta < 0$. ∎

Appendix C

Proofs of the Propositions of Chapter 4

Proof of Proposition 4.3.1. By applying the FONC to (4.6) with respect to X_f, the optimal investment threshold X_f^* is the solution to

$$-\beta V_f\left(X_f^*,\omega\right) + X_f^* \frac{\partial V_f}{\partial X_f}\left(X_f^*,\omega\right) = 0, \tag{C.1}$$

with $V_f\left(X_f^*,\omega\right)$ given in (4.4). Substituting (4.4) into (C.1), X_f^* satisfies

$$(\beta-1)\frac{1-\tau}{r-\mu}X_f^*(1-\eta Q)Q = \beta\left(\frac{C_0 Q}{r+\gamma Q} + \delta Q - \frac{SQ}{r}\left(1-\omega^{\frac{r}{\gamma Q}}\right)\right). \tag{C.2}$$

By rearranging (C.2), we obtain (4.7). ∎

Proof of Proposition 4.3.2. Differentiating $G_f(x,\omega)$ in (4.8) with respect to ω, we get

$$\begin{aligned}\frac{\partial G_f(x,\omega)}{\partial\omega} =& \left(\frac{\tau}{r-\mu}(1-\eta Q)\frac{\partial X_f^*(\omega)}{\partial\omega} + \frac{S}{\gamma Q}\omega^{\frac{r}{\gamma Q}-1}\right)Q\left(\frac{x}{X_f^*(\omega)}\right)^{\beta} \\ &-\beta\left(\frac{\tau}{r-\mu}(1-\eta Q)X_f^*(\omega) - \frac{S}{r}\left(1-\omega^{\frac{r}{\gamma Q}}\right)\right)Q\left(\frac{x}{X_f^*(\omega)}\right)^{\beta}\frac{\frac{\partial X_f^*(\omega)}{\partial\omega}}{X_f^*(\omega)},\end{aligned} \tag{C.3}$$

where from (4.7)

$$\frac{\partial X_f^*(\omega)}{\partial \omega} = \frac{\beta}{\beta-1}\frac{r-\mu}{1-\tau}\frac{1}{1-\eta Q}\frac{S}{\gamma Q}\omega^{\frac{r}{\gamma Q}-1}.$$

Setting

$$\frac{\partial G_f}{\partial \omega}(x, \widetilde{\omega}_f) = 0$$

yields

$$\begin{aligned} \frac{S}{\gamma Q}\widetilde{\omega}_f^{\frac{r}{\gamma Q}-1} + \beta\frac{S}{r}\left(1-\widetilde{\omega}_f^{\frac{r}{\gamma Q}}\right)\frac{1}{X_f^*(\omega)}\frac{\partial X_f^*(\omega)}{\partial \omega} &= (\beta-1)\frac{\tau}{r-\mu}(1-\eta Q)\frac{\partial X_f^*(\omega)}{\partial \omega} \\ \Longrightarrow \left(\frac{\beta}{1-\tau}-1\right)\frac{S}{r}\left(1-\widetilde{\omega}_f^{\frac{r}{\gamma Q}}\right) &= \left(\frac{\beta\tau}{1-\tau}-1\right)\left(\frac{C_0}{r+\gamma Q}+\delta\right), \end{aligned} \tag{C.4}$$

where both sides of (C.4) are positive if $\tau > 1/(\beta+1)$ and $\widetilde{\omega}_f$ is given in the lower part of (4.9). If $\tau \leq 1/(\beta+1)$, the local maximum of (4.8) does not exist and $\widetilde{\omega}_f = 1$, i.e., the Government will not provide a subsidy. Substituting $\widetilde{\omega}_f$ into (4.6), we obtain the equilibrium investment threshold, $\widetilde{X}_f = X_f^*(\widetilde{\omega}_f)$. ∎

Proof of Proposition 4.3.3. To understand how $\widetilde{\omega}_f$ varies with σ, we first analyse the impact of β. In relation to the lower part of (4.9), we have for

$$f(\beta) = \frac{\frac{\tau}{1-\tau}\beta - 1}{\frac{1}{1-\tau}\beta - 1}$$

that

$$\frac{\partial f(\beta)}{\partial \beta} = \frac{\tau}{\beta-1+\tau} - \frac{\tau\beta-1+\tau}{(\beta-1+\tau)^2} = \left(\frac{\beta}{1-\tau}-1\right)^{-2} > 0,$$

for $\tau > 1/(\beta+1)$. Substituting this to the lower part of (4.9) implies that $\partial\widetilde{\omega}_f/\partial\beta < 0$. Since $\partial\beta/\partial\sigma < 0$, we get that

$$\frac{\partial \widetilde{\omega}_f}{\partial \sigma} = \frac{\partial \widetilde{\omega}_f}{\partial \beta}\frac{\partial \beta}{\partial \sigma} > 0. \tag{C.5}$$

Next, we analyse how the equilibrium investment threshold $\widetilde{X}_f$ varies with σ.

Rearranging the first two terms of the lower part of (4.10), we obtain

$$\frac{\partial}{\partial\beta}\left(\left(1+\frac{1}{\beta-1}\right)\left(1+\frac{1-\tau}{\beta-(1-\tau)}\right)\right)<0, \tag{C.6}$$

which implies $\partial\widetilde{X}/\partial\beta<0$. Thus, we obtain

$$\frac{\partial\widetilde{X}_f}{\partial\sigma}=\frac{\partial\widetilde{X}_f}{\partial\beta}\frac{\partial\beta}{\partial\sigma}>0. \tag{C.7}$$

■

Proof of Proposition 4.3.4. To investigate the impact of tax rate on $\widetilde{\omega}_f$, we first derive how $\frac{\frac{\tau}{1-\tau}\beta-1}{\frac{1}{1-\tau}\beta-1}$, in relation to the lower part of (4.9), varies with τ and we obtain

$$\begin{aligned}\frac{\partial}{\partial\tau}\left(\frac{\frac{\tau}{1-\tau}\beta-1}{\frac{1}{1-\tau}\beta-1}\right) &= \frac{\beta+1}{\beta-1+\tau}-\frac{\tau\beta-1+\tau}{(\beta-1+\tau)^2}\\ &= \left(\frac{\beta}{\tau+\beta-1}\right)^2>0. \end{aligned}\tag{C.8}$$

Substituting this to the bottom part of (4.9), we can obtain $\partial\widetilde{\omega}_f/\partial\tau<0$ as required. Next, $\partial\widetilde{X}_f/\partial\tau<0$ can be obtained directly from the lower part of (4.10). ■

Proof of Proposition 4.3.5. The effects of the subsidy size can be directly obtained from the lower parts of (4.9) and (4.10). From (4.9) we observe that $\widetilde{\omega}_f$ is independent of η, and $\partial\widetilde{X}_f/\partial\eta>0$ can be obtained from the lower part of (4.10).

■

Proof of Proposition 4.3.6. Following the same steps as in the proof of Proposition 4.3.1, we can obtain the equilibrium investment threshold as a function of capacity Q, where

$$X^*(Q^*) = \frac{\beta}{\beta-1}\frac{r-\mu}{1-\tau}\frac{1}{1-\eta Q^*}\left(\frac{C_0}{r+\gamma Q^*}-\frac{S}{r}\left(1-\omega^{\frac{r}{\gamma Q^*}}\right)+\delta\right). \tag{C.9}$$

Substituting (C.9) into (4.11) with x replaced by $X^*(Q^*)$, we obtain Q^* as the solution to the equation

$$\left(\frac{\beta}{\beta-1}\frac{1-2\eta Q^*}{1-\eta Q^*}-1\right)\left(\frac{C_0}{r+\gamma Q^*}-\frac{S}{r}\left(1-\omega^{\frac{r}{\gamma Q^*}}\right)+\delta\right)+\frac{\gamma C_0 Q^*}{(r+\gamma Q^*)^2}+\frac{S\ln\omega}{\gamma Q^*}\omega^{\frac{r}{\gamma Q^*}}=0, \tag{C.10}$$

as shown in (4.14). ■

Proof of Proposition 4.3.7. From (4.14), we obtain the optimal capacity Q^* at the lower and upper boundaries of the learning rate: for $\gamma = 0$, we have that

$$Q^* = \frac{1}{(\beta+1)\eta} \quad \text{and} \quad X^* = \frac{\beta+1}{\beta-1}\frac{r-\mu}{1-\tau}\left(\frac{C_0}{r}+\delta\right),$$

and for $\gamma = \infty$,

$$Q^* = \frac{1}{(\beta+1)\eta} \quad \text{and} \quad X^* = \frac{\beta+1}{\beta-1}\frac{r-\mu}{1-\tau}\delta.$$

The identical equilibrium capacity at $\gamma = 0$ and $\gamma = \infty$ suggests that Q^* is either constant or has a non-monotonic relationship with γ. To show that Q^* is not constant, we use proof by contradiction. More specifically, we assume that $Q^* = 1/(\beta+1)\eta$ for all $\gamma \geq 0$. Next, by substituting $Q^* = 1/(\beta+1)\eta$ into (4.14), we get that

$$0 = \frac{\gamma C_0(\beta+1)\eta}{(r(\beta+1)\eta+\gamma)^2}+\frac{S\ln\omega(\beta+1)\eta}{\gamma}\omega^{\frac{(\beta+1)\eta r}{\gamma}} \tag{C.11}$$

for all $\gamma \geq 0$. Without loss of generality, we find that, for $\gamma = (\beta+1)\eta r$, (C.11) holds if and only if $C_0 = -4S\omega\ln\omega$, which contradicts our underlying assumption. Therefore, we conclude that $Q^* \neq 1/(\beta+1)\eta$ for all $\gamma \geq 0$, and there is a non-monotonic relationship between the optimal capacity Q^* and the learning rate γ. ■

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