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Flexural behaviour of rectangular double-opening concrete filled sandwich steel tube (DCFSST) beams

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Abstract: Experimental and numerical studies on flexural behaviour of rectangular double-opening concrete filled sandwich steel tube (DCFSST) beams were performed in this study. Six specimens under different cross-section of inner tube and nominal cross-sectional steel ratio (α_n) were tested under bending to assess the failure modes, moment-displacement and moment-strain relationships, flexural capacity and flexural stiffness of rectangular DCFSST beams. The experimental results demonstrate that the two design variables investigated in this paper have limited impact on the failure modes of typical specimens. The failure mode of outer tube includes multiple local bulges on top flange and one case of fracture on bottom flange that is located at the same plane as the primary local bulge on top flange, whilst the failure mode of inner tubes is mainly due to excessive global deflection. Simultaneously, sandwich concrete is crushed at the locations with evident local bulge on outer tube and cracks nearly uniformly within the lower 2/3 section depth. In general, the flexural capacity and stiffness of the specimens enhance as α_n increases, and are moderately affected by the cross-section of inner tube. In addition, finite element (FE) models were built to replicate the flexural behaviour of rectangular DCFSST beams, and the FE models were validated by comparison with test results. At the end of the paper, simplified formulae that can better predict the flexural capacity of rectangular DCFSST beams were developed.

Keywords: Double-opening concrete filled sandwich steel tube (DCFSST); Rectangular section; Flexural behaviour; Tests; Finite element (FE) model; Simplified formulae.

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31 1. Introduction

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Replacing concrete core near the centroid of conventional regular concrete filled steel tube (CFST) by a hollow steel tube, i.e. the so-called concrete filled double-skin steel tube (CFDST), can reduce the amount of concrete and self-weight while maintaining good structural performance [1]. Compared with the CFST, the CFDST members have the characteristics of higher flexural stiffness, lighter self-weight and better dynamic load resistance due to their unique section form [2]. In the past two decades, the performance of CFDST structures under various loadings have received extensive attention, such as static behaviour and design method of members and joints [3-5], hysteretic performance and seismic design method of members, connections and frames [6-8], response and design calculation of members and joints under impact loading [9,10], fire performance of columns [11-13], among others. Large numbers of in-depth studies have significantly promoted the practical application of CFDST [1,2].

Nevertheless, for structural members to be used as hollow piers or towers, pipes on land and underwater, main structure of tunnels, etc. they are designed to withstand heavy loads requiring high stiffness and stability, the CFDST members cannot be directly employed or need to be retrofitted or reinforced before being adopted. To meet the above challenges, on the basis of previous studies, a new type of composite member was developed by the authors [14], i.e. rectangular double-opening concrete filled sandwich steel tube (DCFSST), which evolves from the rectangular CFDST [15] and is formed from an outer steel rectangular hollow section (RHS), two inner steel tubes placed symmetrically within the infill concrete. Taking rectangular CFDST as a counterpart, the layout of inner tubes is more flexible for the rectangular DCFSST, the inner tubes, for example, can be placed further away from the centroid after a rational arrangement, resulting in greater flexural capacity and flexural stiffness. Given the unique cross-sectional configuration, the rectangular DCFSST is more suitable for members or structures that functionally need double holes, such as piers, bridge towers, principal body of two-way tunnels, etc.

56 Until now, there has been a certain amount of research on the flexural properties of CFDST with
157 outer rectangular and square steel section. The first experimental and theoretical study on the flexural
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58 behaviour of CFDST beams with outer and inner steel square hollow section (SHS) was conducted
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659 by Zhao and Grzebieta [1]. After that, the flexural behaviour of CFDST members with the similar
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60 and different cross-sections of the tube received further attention, such as the CFDST beams with
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61 outer steel SHS and inner steel circular hollow section (CHS) [16], the CFDST beams with outer and
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62 inner steel RHS [15], and the CFDST beams with outer and inner steel SHS [17]. Additionally, the
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63 available studies on the flexural behaviour of CFDST also included members with pairs of concentric
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64 steel SHS and sandwich grout under blast loading [18], and members with outer SHS and inner CHS
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65 after exposure to high temperatures [19]. Currently, the only research on the newly proposed
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66 composite member concerning the experimental and numerical study of rectangular DCFSSST short
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67 columns under concentric compression has been carried out by the research group of reference [14].
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68 The previous literature review shows that the investigation into the structural performance of
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69 rectangular DCFSSST members is very limited and there is almost no research on the flexural
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70 properties of rectangular DCFSSST beams. To provide theoretical basis for the flexural resistance
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71 design and thus facilitate engineering application of this new type of composite members, it is very
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72 crucial to understand their flexural behaviour. Hence, the goals of the present study are (1) to present
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73 the experimental results of 6 rectangular beams loaded in flexure, (2) to analyze the effect of the
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74 cross-section of the inner tube and nominal cross-sectional steel ratio on the flexural behaviour of the
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75 specimens, (3) to introduce the FE models that simulate the flexural behaviour of the rectangular
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76 DCFSSST beams and (4) to propose an accurate simplified model for the flexural capacity of the new
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77 composite beams.
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52 78 2. Experimental study

53 54 55 79 2.1 Preparation of specimens

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80 A total of six rectangular DCFSSST beam specimens, 3 with inner SHSs and 3 with inner CHSs, were
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81 designed and manufactured, and the cross-sections of the two inner tubes in each specimen were
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82 identical. Fig. 1 shows the configuration of the rectangular DCFSSST beams, where D_o , B_o and t_o
 183 are the depth, width and wall thickness of the outer steel RHS, respectively, d_i and t_i are the width
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 3 or diameter and wall thickness of the inner steel sections, respectively, and d_e is the centroidal
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 5 distance between the two inner tubes.
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 8 The main test parameter of the specimens was the cross-section of the inner tube and the nominal
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 10 cross-sectional steel ratio, α_n . The details of the specimens are presented in Table 1, where, ϕ is
 1187
 12 the void ratio, e_0 is the offset ratio of inner steel tube, f_{y_o} and f_{y_i} are the yield strengths of the
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 14 outer and inner steel tubes, f_{cu} is the compressive cube strength of the sandwich concrete, $M_{u,e}$ and
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 25 of the specimens (L) was set to be 1200 mm.

2693 The definition of α_n , ϕ and e_0 in the rectangular DCFSSST is as follows [14,15]:
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$$2994 \quad \alpha_n = \frac{A_{so}}{A_{ceq}} \quad (1)$$

$$30 \quad \phi = \sqrt{\frac{\sum A_{ssi}}{A_{ceq}}} \quad (2)$$

$$31 \quad e_0 = \frac{d_e}{D_o} \quad (3)$$

3295
 33 where, A_{so} is the cross-sectional area of outer steel RHS; A_{ceq} is the equivalent cross-sectional
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 35 area of concrete, that is, the cross-sectional area surrounded by the inner wall of outer steel RHS; and
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 43 $\sum A_{ssi}$ is the sum of cross-sectional area surrounded by the outer wall of inner steel tubes.

4400 The outer RHSs were produced by welding together two identical cold-formed C-profiles with
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 4601
 47 straight welds, while the inner SHSs and CHSs were fabricated from the finished cold-formed steel
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 50 tubes. For all the specimens, the dimensions of two rectangular endplates were: depth×width×
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 52 thickness=220 mm×160 mm×15 mm. The detailed fabrication process of rectangular DCFSSST beam
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 55 specimens is the same as that of composite short columns in [14].

5605 The properties measured for the steel tubes in the tensile tests of standard test coupons are listed in
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 5806
 59 Table 2. The characteristic compressive cube strength of the sandwich concrete, measured in cubes
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 6107
 62 with a side length of 150 mm, was designed as 40 MPa and its mix proportion includes: ordinary
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108 Portland cement (P.O 42.5) 398 kg/m³, fly ash (grade I) 170 kg/m³, fine aggregate (river sand) 770
1109 kg/m³, coarse aggregate (limestone gravel) 795 kg/m³, tap water 219 kg/m³, and water reducer 6.0
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1410 kg/m³. The measured properties of concrete include: $f_{cu,28}=40.5$ MPa, $f_{cu}=49.8$ MPa, and $E_c=32.1$
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1611 GPa, in which $f_{cu,28}$ is the compressive cube strength at the age of 28 days, and E_c is the elastic
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1912 modulus.

1113 2.2. Testing set-up and measurement

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1414 Four-point bending tests were conducted to investigate the flexural behaviour along the major axis of
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1615 rectangular DCFSSST beam specimens, and the vertical concentrated force monitored by a load cell
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1816 was applied by a 5000 kN capacity testing machine and distributed to the two quarter points on the
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2117 top surface of the specimens by a spreader beam. Fig. 2 presents the testing set-up and it is seen that
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2318 the effective span length L_e of the specimens is 1000 mm.

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2619 A hybrid force & displacement control loading scheme was used in the tests. Prior to reaching the
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2820 load corresponding to 90 % of the estimated ultimate value P_u , the force control rate of 0.5 kN/s was
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3121 employed, keeping the load constant at the levels of $P_u/10$ about 2 to 3 minutes. After that the
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3322 displacement control of the mid-span with the rate of 0.5 mm/min was adopted and each displacement
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3623 level of 5 mm in the mid-span was also held for 2 to 3 minutes. The tests were terminated, when the
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3824 tensile area of the outer steel section fractured or the displacement in the mid-span reached approx.
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4125 $L_e/12$.

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4326 To trace the development of deformations in the key positions of the specimens, five displacement
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4527 transducers, DTs, were arranged as presented in Fig. 2(a), two at the supports, two at the quarter
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4828 points and one at the mid-span. Several longitudinal and transverse strain gauges, SGs, were attached
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5029 on the outer walls of the outer and inner steel tubes in the mid-span, as shown in Fig. 2(b).

5130 2.3. Test results and discussion

54 55 5631 2.3.1. Failure modes

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5832 The test results indicated that, good deformability was achieved for all the specimens, considering
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6033 that the displacement in the mid-span eventually reached 6% to 9% of L_e . The overall failure modes

134 of rectangular DCFSSST beam specimens are similar, that is, there are one primary local bulge on top
1135 flange and adjacent part of sidewalls as well as two secondary local bulges just on top flange of outer
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136 steel RHS within two quarter points, and one case of fracture on bottom flange and adjacent part of
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137 sidewalls of outer steel RHS that locates at the same plane as the primary local bulge; however, the
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138 final deflection shape, positions of local bulges and fracture site of outer steel RHS show a certain
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1139 discrepancy for each specimen. Fig. 3(a1) shows the overall appearance of the specimens after the
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1140 end of the tests, where the arrows point to the positions of the local bulges on the outer tube, and the
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141 dashed coils denote the fracture site of outer tube. The distance between the fracture site of outer tube
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1142 and the mid-span section is approximately equal to B_0 .

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143 After removal of the outer steel tube, the failure mode of the sandwich concrete is displayed in Fig.
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2144 3(a2) and fine cracks of the length of approx. $2/3$ of the section depth are uniformly distributed in the
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145 tension zone of all the specimens. Moreover, in the places where the evident localized bulge of outer
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2146 tube occurs, the sandwich concrete is crushed, and the more obvious the localized bulge of outer tube,
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147 the greater the range and the higher the degree of concrete crushing.

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3148 Fig. 3(a3) demonstrates the failure mode of inner tubes and as seen in the figure, the failure in both
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3149 of the inner tubes is mainly due to the global deflection and at the position of the outer tube with the
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150 local bulge, the inward local buckling appears in the upper inner tube of all the specimens with inner
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41051 SHSs and one specimen with inner CHSs, since the CHS tubes have a better local stability as
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152 compressed than the SHSs that have a similar value of d_i/t_i , whilst the mid-span displacement of
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41553 specimen RC-c is the largest among the three specimens with inner CHSs.

46 47 154 2.3.2. Moment versus deformation curves

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51055 Fig. 4 presents the experimental curves for the bending moment M vs. mid-span displacement u_m
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5156 behaviour, where the hollow circles denote the flexural capacity and the relevant mid-span
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5157 displacement, and the inverted triangles indicate the initiation of the fracture, ITF, at the bottom
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5158 flange of the outer steel section. All the $M - u_m$ diagrams consist of three successive phases that
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159 are categorized as approximate elastic, elasto-plastic and plastic hardening ones and the third phase
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160 ends soon after the ITF. Moreover, while the cross-sections of the inner tubes are identical, a larger
1161 value of α_n causes a higher slope at each phase of the $M - u_m$ curve and a later appearing ITF.
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1462 Simultaneously, when the section sizes of inner tubes and e_0 are similar, the $M - u_m$ curve of the
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1663 specimens with inner SHSs is slightly higher than that of the specimens with inner CHSs, i.e. the
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1864 former type has a better flexural performance.
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1165 The distribution of displacements u along the effective span can be determined on account of the
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1366 data from all of the DTs in Fig. 2(a) as exhibited in Fig. 5 by the solid lines, and the relevant half-sine
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1667 curves having the same u_m as the measured results are indicated by the dashed lines, where x is
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1868 the distance from the fixed support to the measuring points, and m is the ratio of M to flexural
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2169 capacity as defined later on. When $m \leq 1.0$, the measured displacements and the respective half-sine
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2370 distribution are generally in good agreement, and there is a certain deviation between them while
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2671 $m > 1.0$. In addition, for the same specimen, the deflection at the half span at the moment of the fracture
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2872 in the outer tube is greater than that in the case without the fracture of the outer tube due to the
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3173 concentration of the damage in the fracture.
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3374 Fig. 6 shows the recorded relationship between strains ε_L and ε_T in the mid-span and bending
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3675 moment M , where the subscripts 'L' and 'T' stand for 'longitudinal' and 'transverse', respectively,
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3876 and ε_{yi} is the yield strain of inner steel tube. The measured results indicate that, for the flange of
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4177 outer steel RHS, the strains in the corner points 1-1 and 3-1 and in the middle points 1-2 and 3-2 are
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4378 very close to each other and therefore only the values from the latter are presented. Figs. 6(a) and 6(c)
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4679 indicate that in all of the specimens, the $M - \varepsilon_L$ curve of the top flange of point 1-2 in the top flange
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4880 has a similar development pattern, but there is a certain difference in the respective $M - \varepsilon_L$ curve of
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5181 point 3-2 of the bottom flange of the outer tube. This may be due to the difference in the width and
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5382 length of concrete cracks near the mid-span section. According to the method of reference [16], the
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5683 flexural capacity $M_{u,e}$ of the rectangular DCFSSST beams is defined as the bending moment that
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5884 causes strain ε_L equal to 0.01 at the bottom flange of the outer tube and the respective results are
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6185 presented in Table 1. In general, strains ε_L in points f and g of the lower inner tube increase
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186 continuously, whereas the values of ε_L in points d and e of the upper inner tube increase to a certain
187 extent but then decrease while the neutral axis of the composite beam shifts gradually upwards during
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1488 loading and thereafter downwards [20]. When bending moment $M_{u,e}$ indicated by the hollow circle
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1689 is reached, strains ε_L at points d, f and g of the inner tube are generally higher than ε_{yi} , and ε_L at
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1990 point e of the inner tube is slightly less than ε_{yi} , indicating that the whole section of the lower inner
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1191 tube and part of the upper inner tube have yielded and this is different from the yielding behaviour of
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1492 the inner tubes in the rectangular CFDST beams [15-17]. It is shown in Figs. 6(b) and 6(d) that the
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1693 $M - \varepsilon_T$ diagrams have generally similar development patterns with the $M - \varepsilon_L$ curves. Moreover,
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1994 due to the Poisson effect, the absolute value of ε_T is smaller than that of the corresponding ε_L under
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2195 the same moment.

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2496 The distribution of longitudinal strains ε_L is drafted on account of all the measurement results
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2697 presented in Fig. 2(b) and the typical output until reaching moment $M_{u,e}$ is presented in Fig. 7, where
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2998 y indicates the vertical location of the point considered from the centroidal axis. It can be seen that,
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3199 while $m \leq 0.8$, a linear distribution along the section depth is reached for the ε_L of both outer and
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33
3400 inner tubes, and the location of neutral axis determined by the distribution of ε_L of outer steel RHS
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3601 is generally the same as that determined by the distribution of ε_L of inner tubes. However, when
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3902 $m > 0.8$, the distribution of ε_L is no longer linear in both outer and inner tubes, due to the occurrence
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4203 of cracking of sandwich concrete and local bulge of the walls in outer tube. Additionally, the cross-
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4404 sectional form of inner tubes has no obvious effect on the distribution of ε_L , whilst the larger the α_n ,
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4605 the closer the neutral axis is to the centroid axis, meaning that increasing α_n can enhance the
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4906 confinement effect of outer tube on the sandwich concrete, and inhibit the upward shifting of neutral
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5207 axis to a certain extent such that the damage process of the specimens slows down.

208 2.3.3. Mechanical index

5209 The values of flexural capacity $M_{u,e}$ are compared in Fig. 8(a) and it is seen that when α_n is the
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5810 same and the inner tubes have similar sizes and e_0 , the specimens of RS series have similar values
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6211 of $M_{u,e}$ with the specimens of RC series and when α_n has values 0.08, 0.113 and 0.147, the ratio
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212 of capacities $M_{u,e}$ between RS series and RC series is equal to 1.004, 1.071 and 0.974, respectively.
 213 Simultaneously, with the same features of the inner tubes, $M_{u,e}$ improves with the increasing α_n .
 214 For the RS series specimens, when α_n is equal to 0.110 and 0.142, $M_{u,e}$ is 20.6% and 51.0% higher
 215 than the respective values when $\alpha_n = 0.079$, whereas for the RC series specimens the similar
 216 percentage values are 13.0% and 55.6%, respectively. These effects follow mainly from the improved
 217 confinement of the outer tube to the sandwich concrete when α_n increases.

218 The curvature of mid-span section can be acquired according to the assumption that the overall
 219 deflection of the specimens conforms to the half-sine curve form, which has been basically confirmed
 220 by the aforementioned discussion. The flexural stiffness K_e of the specimen at the elastic phase is
 221 equal to the ratio between the bending moment and curvature [15] and the values are listed in Table
 222 1. The values of K_e are compared in Fig. 8(b) and it is seen that the values of K_e in the RS series
 223 are higher than those of the RC series, excluding the case with $\alpha_n = 0.110$. When α_n varies from
 224 0.079 to 0.142, the values of K_e in the RS series of specimens are 1.104, 0.930 and 1.042 times those
 225 of the specimens in the RC series. When the sizes and e_0 of the inner tubes are kept unchanged, a
 226 larger value of α_n results in a greater value of K_e . In the specimens of the RS series, when α_n is
 227 equal to 0.110 and 0.142, K_e is respectively 11.2 % and 40.9 % higher than that when $\alpha_n = 0.079$,
 228 whereas in the specimens of the RC series the respective values are 32.0 % and 49.4 %. Clearly the
 229 increase of the stiffness of the inner tubes is the main reason for the improved values of K_e in the
 230 specimens of RS series.

231 According to references [20, 21], the flexural stiffness of CFST and CFDST members with
 232 rectangular cross-sections can be evaluated applying the superposition principle and the contribution
 233 of the concrete as cracked is close to 20 %. This principle is also applied when evaluating K_c for the
 234 rectangular DCFSSST beam specimens in Eq. (4):

$$235 \quad K_c = E_{so} \cdot I_{so} + 0.2E_c \cdot I_c + \sum(E_{si,i} \cdot I_{si,i}) \quad (4)$$

236 where E_{so} and E_c are the elastic moduli and I_{so} and I_c are the moments of inertia for the steel
 237 tube and sandwich concrete, respectively and $E_{si,i}$ and $I_{si,i}$ are the elastic modulus and moment of

238 inertia for the inner steel tube i , respectively. The results show that, the minimum and maximum
 2139 values of the ratio of K_c to K_e are 0.998 and 1.101, respectively, while its mean value and Cov are
 2 3
 2440 1.062 and 0.040, respectively, which means that the flexural stiffness of rectangular DCFSSST beams
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 2441 can be well predicted in accordance with the superposition principle while considering 20%
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 2442 contribution of sandwich concrete, and overall the prediction is slightly higher than the test result.
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12143 3. Simulations by FE model

12444 3.1. FE models

12645 The simulations for the flexural behaviour of the rectangular DCFSSST beams were carried out using
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 246 the FE package ABAQUS [23] and the reliability of the evaluations was validated against the
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 22147 experimental results. The numerical analysis of typical rectangular DCFSSST beams was further
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 248 performed by the FE model.
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22649 The behaviour of outer and inner steel tubes in ABAQUS was modelled as elasto-plastic material
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 2850 and the measured values for elastic modulus and Poisson's ratio were applied and the data pairs of
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 32151 real stress and plastic strain were transformed from the engineering values σ_s and ε_s to depict the
 32
 3352 plastic properties of steel materials. The $\sigma_s - \varepsilon_s$ relationship for the cold-formed steel RHS and SHS
 34
 35
 32653 proposed in [24] was adopted for the steel tubes, which contain 4 flat portions and 4 corner portions
 37
 32854 considering the local strengthening of corner area. The relationship between σ_s and ε_s of flat
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 40
 42155 portions in the cold-formed rectangular and square steel tube is as follows:
 42

$$\begin{aligned}
 \sigma_s = & \begin{cases} E_s \cdot \varepsilon_s & (\varepsilon_s \leq \varepsilon_{s0}) \\
 0.75f_y + 0.5E_s \cdot (\varepsilon_s - \varepsilon_{s0}) & (\varepsilon_{s0} < \varepsilon_s \leq \varepsilon_{s1}) \\
 0.875f_y + 0.1E_s \cdot (\varepsilon_s - \varepsilon_{s1}) & (\varepsilon_{s1} < \varepsilon_s \leq \varepsilon_{s2}) \\
 f_y + 0.005E_s \cdot (\varepsilon_s - \varepsilon_{s2}) & (\varepsilon_s > \varepsilon_{s2}) \end{cases} \quad (5)
 \end{aligned}$$

42957 where, E_s is the elastic modulus, f_y is the yield strength, $\varepsilon_{s0} = 0.75f_y/E_s$, $\varepsilon_{s1} = \frac{4}{3}\varepsilon_{s0}$, and $\varepsilon_{s2} =$
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 5258 $3\varepsilon_{s0}$.
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52459 The corner portions in the cold-formed rectangular and square steel tube are assumed to have the
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 5260 same expression for the $\sigma_s - \varepsilon_s$ relationship as the flat portions; but yet the improved yield strength
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 52961 of the former in comparison with that of the latter was related to the yield-to-tensile strength ratio and
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262 the corner radius to thickness ratio [24], and the corner radius in the cold-formed rectangular and
 2163 square steel tubes was determined as suggested in reference [25].

2 3
 2 64 The following five-phase constitutive model, which has been widely used in the previous FE
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 5
 2 65 simulations of composite members [29], was chosen to replicate the $\sigma_s - \varepsilon_s$ relationship of inner
 7
 2 66 steel CHSs:

$$\sigma_s = \begin{cases} \frac{E_s \cdot \varepsilon_s}{-A \cdot \varepsilon_s^2 + B \cdot \varepsilon_s + C} & (\varepsilon_{se} < \varepsilon_s \leq \varepsilon_{sa}) \\ f_y & (\varepsilon_{sa} < \varepsilon_s \leq \varepsilon_{sb}) \\ f_y \cdot \left(1 + 0.6 \frac{\varepsilon_s - \varepsilon_{sb}}{\varepsilon_{sc} - \varepsilon_{sb}}\right) & (\varepsilon_{sb} < \varepsilon_s \leq \varepsilon_{sc}) \\ 1.6f_y & (\varepsilon_s > \varepsilon_{sc}) \end{cases} \quad (6)$$

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 1 8 2 68 where, $\varepsilon_{se}=0.8 f_y/E_s$, $\varepsilon_{sa}=1.5\varepsilon_{se}$, $A = 0.2 f_y/(\varepsilon_{sa} - \varepsilon_{se})^2$, $B = 2A \cdot \varepsilon_{sa}$, $C = 0.8f_y + A \cdot \varepsilon_{se}^2 -$
 19
 20
 2 1 69 $B \cdot \varepsilon_{se}$, $\varepsilon_{sb}=15\varepsilon_{se}$, and $\varepsilon_{sc}=150\varepsilon_{se}$.

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 2 4 70 The ductile damage model for metal in ABAQUS was selected to replicate the fracture process of
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 2 6 71 steel tube, and there were two parameters of fracture strain and plastic damage factor that need to be
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 2 8 72 defined. When the equivalent plastic strain of the element accumulated to the fracture strain, which
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 3 2 73 was determined by the formulae provided in reference [26], the damage evolution began, and the
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 3 3 74 linear damage criterion in reference [27] was subsequently adopted while defining the displacement
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 3 6 75 at failure as the product of the average elongation and the standard distance of the coupons.
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 3 8 76 Additionally, the residual stresses in the finished rectangular and square steel tubes were not included
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 4 1 77 in the FE model, considering that the impact of them was significantly reduced after filling concrete
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 4 2 78 into the space between the outer and inner steel tubes [14, 28].

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 4 5 79 The concrete damaged plasticity (CDP) model in ABAQUS [23], which defines the failure of
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 4 8 80 concrete as compressive crushing and tensile cracking, shows good convergence. Therefore, it was
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 5 0 81 chosen to model the complicated behaviour of sandwich concrete. The elastic properties, including
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 5 3 82 the elastic modulus E_c and Poisson's ratio μ_c were determined according to the provisions of ACI
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 2 8 83 318-19 [30] and CEB [31], respectively, i.e. $E_c=4730\sqrt{f'_c}$ and $\mu_c=0.2$, where f'_c is the compressive
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 5 2 84 cylinder strength. With reference to documents [14] and [15] as well as the test results in this study,
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 6 0 85 it is assumed that the inner tubes can always reliably restrain the sandwich concrete before fracturing

286 of the bottom flange of the outer tube, which means that the behaviour of the sandwich concrete is
 2187 similar to that of the concrete core in the rectangular CFST. Therefore, the compressive $\sigma_c - \varepsilon_c$
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 2488 relationship, which is suitable for the FE simulation of rectangular DCFSSST short columns [14], was
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 2689 selected for the sandwich concrete in the rectangular DCFSSST beams, and the detailed formulae are
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 2890 expressed as:

$$\sigma_c/f'_c = \begin{cases} 2(\varepsilon_c/\varepsilon_{cp}) - (\varepsilon_c/\varepsilon_{cp})^2 & (\varepsilon_c/\varepsilon_{cp} \leq 1) \\ \frac{\varepsilon_c/\varepsilon_{cp}}{a_1 \cdot (\varepsilon_c/\varepsilon_{cp}) - 1} + \varepsilon_c/\varepsilon_{cp} & (\varepsilon_c/\varepsilon_{cp} > 1) \end{cases} \quad (7)$$

14
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 12692 where, $\varepsilon_{cp} = (12.5f'_c + 800\xi^{0.2} + 1300) \times 10^{-6}$, $\xi (= \alpha_n \cdot f_{yo}/f_{ck})$ is the nominal confinement factor,
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 18
 199 f_{ck} is the characteristic compressive strength of concrete [15], $a_1 = (f'_c)^{0.1}/(1.2\sqrt{1 + \xi})$, and $k =$
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 2194 $1.5(\varepsilon_{cp}/\varepsilon_c) + 1.6$.

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 2495 Moreover, the formula presented in reference [32] was employed to calculate the compression
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 2696 damage factor of sandwich concrete. Simultaneously, the stress-fracture energy relationship in
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 297 ABAQUS [23] was used to specify tension-stiffening behaviour of sandwich concrete, and the failure
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 32198 stress and the fracture energy were respectively taken as $0.1f'_c$ and 100 N/m [5, 31]. The plasticity
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 3499 parameters for the CDP model were determined by referring to the recommended values of the
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 3600 software package [23] and further corrected by the test results, as listed in Table 3.

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 3801 The 4-node general-purpose shell elements with full integration (S4) suitable for large-strain
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 43102 analysis were chosen to simulate all the steel tubes, and the Simpson's rule with 9 integral points in
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 43308 direction of wall thickness was specified to meet the calculation accuracy. The 8-node three-
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 4604 dimensional solid elements C3D8R with reduced integration were selected to model the sandwich
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 4805 concrete, as they are appropriate for treating large strains with geometric nonlinearity and possible
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 5106 serious mesh distortion. The structured meshing strategy included in the software package [23] was
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 53307 employed, and the corner portions of all steel tubes were subdivided to ensure the convergence and
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 5608 accuracy. In order to accurately capture the fracture moment and post-fracture damage evolution of
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 5809 the steel tube, the element encryption was set within two quarter points of the rectangular DCFSSST
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 6010 beam specimens, and the element was deleted once it met the fracture criteria. Fig. 9 exhibits the

311 meshing for the FE model of the rectangular DCFSSST beams.

3112 The interaction between various components of the rectangular DCFSSST beams, including outer
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313 and inner steel tubes as well as sandwich concrete, was simulated by defining the contact pairs. For
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314 the interface between the outer and inner steel tubes and sandwich concrete, the normal direction was
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315 simulated by the ‘hard contact’, and the tangent directions were modelled by the ‘Coulomb friction’
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316 with friction coefficient equal to 0.6 [14], while setting both the inner wall of outer RHS and the outer
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317 wall of inner tubes as the master surface and the surface of sandwich concrete in contact with the
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318 above walls as the slave surface. Moreover, the interface between sandwich concrete and two
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319 endplates was also simulated by the ‘hard contact’, and the interface between the outer and inner steel
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320 tubes and the two endplates was defined as the ‘shell-to-solid coupling’.
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23321 The boundary conditions for the FE model of the rectangular DCFSSST beams, shown in Fig. 9,
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322 have good agreement with those of the specimens tested within this study and literature references
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323 [15-17]. Four reference points, RPs, were defined in ABAQUS such that they are located at the
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3324 midpoint of the section width and they were set at the two loading positions and support positions
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3825 and each RP was coupled with the outer surface of outer tube at the corresponding area. The length
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4326 of coupling area was set to be $0.8B_0$ based on the comprehensive consideration of convergence,
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5327 accuracy and computational efficiency. For the RP corresponding to the fixed support, three
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53233 In addition, the impact of initial geometric defects (IGDs) on the flexural performance of DCFSSST
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5534 beams was analyzed based on the FE model. The buckling eigenmode of outer and inner tubes in the
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53735 composite beam was treated as the IGDs in the first analysis step, and the amplitude was equal to 0.1
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337 on the flexural behaviour of the rectangular DCFSSST beam specimens and as seen in the figure, the
3138 FE simulation results with and without IGDs are close to each other and generally agree well with
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339 the measured results.
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3640 3.2. Validation of the FE model

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3841 Fig. 3(b) illustrates the simulated failure modes of rectangular DCFSSST beam specimens in this study.
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13142 Comparison in Fig. 3 indicates that the results predicted by the FE model are located mainly within
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13343 two quarter points of the span and they include multiple local bulges on the top flange and part of
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13644 side walls of the outer tube and fracture of the bottom flange of the outer steel section, shown in Fig.
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13845 3(b1). Sandwich concrete is crushed at the site of local bulges of the outer tube and cracked within
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346 its lower part, as shown in Fig. 3(b2) and the global deflections of the inner tubes in Fig. 3(b3)
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23347 generally agree well with the observations from the tests. The difference between the simulated and
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348 measured failure area in the specimens is mainly due to the fact that, the FE models cannot reasonably
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23849 mimic the influence of the following factors: 1) the sizes and flatness of the specimens differ from
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3050 the design considerations, 2) there are manufacture defects in the materials of the specimens, 3) the
31
32
3351 loading devices in the testing set-up have the unavoidable minor position deviation.
34

3352 Fig. 11 presents the curves of $M - u_m$ behaviour such that those labelled with letter ‘M’ indicate
36
37
3353 the measured ones and those labelled with letter ‘P’ the predicted ones. The available test results from
39
43054 this study and the ones from references [15-17] are generally in good agreement with the FE-
41
42
355 predictions, except that the plastic hardening phase of the former is slightly higher, and the fracture
43
44
43556 moment and process of the former show a certain difference. This may be due to the deviation
46
47
357 between the material properties from the standard tests and the real properties of the tubes and
48
49
53058 sandwich concrete in the specimens, and in the plastic phase the rolling support between the spreader
51
53259 beam and the specimen gradually move inward due to the increase of displacement, resulting in
53
54
3560 additional bending moments in the specimen, whilst in the FE model, the initial position of the rolling
56
5761 support is simplified to a fixed loading point. The predicted $M - \varepsilon$ diagrams are compared with the
58
59
362 measured data in Fig. 12. As can be seen from these figures, the development trend of the simulated
61
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63
64
65

363 $M - \varepsilon$ curves generally fits well with that of the measured results, and the agreement between them
 3164 is good in elastic and elastic-plastic phase, but the plastic phases of the curves are deviated. Moreover,
 2
 3
 365 the simulated and measured $M - \varepsilon_T$ curves of the inner steel CHSs include obvious differences,
 5
 366 possibly due to the slight deviation in the positions of the attached transverse strain gauges. Fig. 13
 7
 367 demonstrates the typical distributions of ε_L in the mid-span section on account of predicted and
 9
 10 measured data. It is shown that, with the increase of m , the predicted distribution of ε_L and variation
 12
 13 amplitude of cross-sectional neutral axis generally accord with the measured results when $m \leq 0.9$.
 14
 15 However, for the results with $m = 1.0$, the discrepancy between the predicted and measured
 17
 18 distribution becomes more evident. This may be caused by the subtle difference between the
 19
 20 simulated and observed cracking characteristics of the concrete together with local bulge pattern of
 21
 22 outer tube walls.
 23
 24

25
 26 Fig. 14 presents a comparison between the simulated flexural capacity $M_{u,fe}$ and the measured
 27
 28 flexural capacity $M_{u,e}$ within this study and references [15-17] and overall $M_{u,fe}$ is close to $M_{u,e}$
 29
 30 with the discrepancy limited to 10%. The mean value and Cov of $M_{u,fe}/M_{u,e}$ are 0.976 and 0.039,
 31
 32 verifying that the FE model is capable of predicting well the flexural capacity of the rectangular
 33
 34 DCFSSST and CFDST beams.
 35
 36

37 38 39 **3.3. Further numerical study**

40
 41 The FE model is further used to undertake numerical analysis of rectangular DCFSSST beams, and the
 42
 43 simulation results indicate that the strength properties (f_{yo} , f_{yi} and f') and d_i/t_i have very limited
 44
 45 influence on the stress/strain distribution characteristics, while merely affecting the specific values.
 46
 47 Therefore, the effect of key parameters, including α_n , ϕ , e_0 and depth-to-breadth ratio of outer
 48
 49 tube (η), on the response of rectangular DCFSSST beams is numerically studied. The parameters for
 50
 51 the benchmark are as follows: $D_o=600$ mm, $L/D_o=8.0$, $\eta=2.0$, $\alpha_n=0.10$, $\phi=0.5$, $e_0=0.5$, $f'=50$
 52
 53 MPa, $f_{yo}=f_{yi}=355$ MPa and $d_i/t_i=40$ (SHS) or 60 (CHS). It should be noted that, while changing
 54
 55 η , the section circumference of outer steel RHS is kept fixed.
 56
 57
 58
 59
 60

61 Fig. 15 displays the influence of the key parameters on the stresses of the sandwich concrete in the
 62
 63
 64
 65

389 mid-span section at the moment of attaining the flexural capacity M_u and ‘S33’ represents the
3190 normal stress along the direction of the span indicated in Fig. 9, and the dashed line indicates the
2
3
3191 location of the neutral axis. Most of the sandwich concrete is stressed in tension with $S_{33} > 0$ and the
4
5
3192 tensile stresses increase with the decrease of α_n , ϕ and η and increase of e_0 . The depth of the
7
8
3193 compressed section decreases with the increase of α_n and ϕ and remains almost unchanged with
9
10
3194 the variation of e_0 , whereas η has no evident effect on the location of the neutral axis. Moreover,
12
13
3195 the stress distribution characteristics and maximum compressive stress of sandwich concrete change
14
15
3196 with the change of key parameters. In general, the maximum compressive stress appears at two top
17
18
3197 corners where sandwich concrete contacts with the outer tube, and under certain parameter conditions
19
20
3198 (e.g. $\alpha_n=0.16$, $\phi = 0.75$ or $e_0=0.6$) the maximum compressive stress simultaneously arises at two
22
23
3199 top corners where sandwich concrete contacts with the upper inner tube. However, as the change of
24
25
4000 key parameters leads to the variation in the area and cross-section characteristics of the sandwich
27
24001 concrete in compression, its maximum stress does not indicate any consistent varying characteristics
29
30
4002 while the values of the parameters are increased or decreased.

3403 The effects of ϕ and e_0 on the von Mises stress in the inner SHSs are demonstrated in Fig. 16 at
34
35
4004 the moment of reaching M_u , and the impact of other parameters have similar variation features. It is
36
37
3405 seen that within the centre-most quarters of the span the von Mises stress in the whole section of the
39
40
4006 lower tube exceeds f_{yi} in the tensile region of the sandwich concrete, but in most of the cases only
41
42
4407 the von Mises stress of the top flange and part of the sidewalls in the upper tube is greater than f_{yi} .
44
45
4008 For certain cases, e.g. with $\phi=0.25$, the von Mises stress of the upper tube remains less than f_{yi} ,
47
48
4009 indicating that whole section of the lower tube has yielded while only partial or no yielding has
49
50
4110 occurred in the upper tube. The variation of parameters mainly determines the distribution
52
53
4111 characteristics of the von Mises stress of the upper tube. In general, when there is a yield area in the
54
55
4112 upper tube, the gradient of the von Mises stress between the top and bottom flange increases with the
56
57
4113 increase of ϕ and the decrease of e_0 .

4114 Fig. 17 shows the influence of the key parameters on the interaction stress p in typical points of

415 the mid-span section. For a rectangular DCFSSST beam, p in the corner point 1 is higher than those
416 in the other three points where the top flange of the outer tube is in contact with the sandwich concrete,
2
3
417 and it decreases gradually after reaching the maximum value as the local bulge in the top flange
4
5
418 extends to its corner. In the later phase of loading, stress p of the corner point 4 exceeds that of point
6
7
419 1. Stress p of the corner point 2 is close to that of point 4 in the early phase of loading but the
8
9
10 difference in them increases gradually as u_m improves. Stress p of the corner point 3 is generally
11
12
13
14 very small and the four parameters have no regular influence on the $p - u_m$ relationship of points
15
16
17 1, 2 and 4, since the location of the neutral axis of the cross-section varies continuously with the
18
19
20 development of cracking in the sandwich concrete and local bulge in the walls of the tube.

4. Simplified calculation of flexural capacity

21
22
23 Fig. 18 presents the results for the flexural capacity M_u in the rectangular DCFSSST beams calculated
24
25
26 using the FE model under various parametric conditions and as compared with the composite beams
27
28
29 having inner CHSs, higher values of M_u exist for the beams with inner SHSs. With the improvement
30
31
32 of parameters α_n , ϕ , e_0 , f_{y0} , f_{yi} , and f'_c and reduction of d_i/t_i the flexural capacity increases
33
34
35 and the variation of α_n and f_{y0} has a greater impact on M_u .

36
37 By referring to the method for the rectangular CFDST beams in [15] and taking the above
38
39
40 numerical results into consideration, the flexural capacity of rectangular DCFSSST beams, M_u , can
41
42
43 be calculated as the compound sum of the capacity values for the outer steel RHS together with the
44
45
46 sandwich concrete, $M_{osc,u}$ and for the inner steel tubes, $M_{i,u}$:

$$M_u = M_{osc,u} + M_{i,u} \quad (8)$$

47
48 For the compound section of outer steel RHS together with sandwich concrete, coefficient $\gamma_{m,o}$
49
50
51 for the cross-sectional moment capacity is defined as:

$$\gamma_{m,o} = \frac{M_{osc,u}}{W_{scm} \cdot f_{oscy}} \quad (9-1)$$

$$W_{scm} = \begin{cases} \frac{1}{3} B_o \cdot D_o^2 - \frac{d^4}{3} + d^2 \cdot \frac{d_e^2}{3} / D_o & \text{(with inner SHSs)} \\ \frac{1}{6} B_o \cdot D_o^2 - \pi \left(\frac{d_i^4}{16} + \frac{d_i^2 \cdot d_e^2}{4} \right) / D_o & \text{(with inner CHSs)} \end{cases} \quad (9-2)$$

$$f_{oscy} = C_1 \cdot \phi^2 \cdot f_{y0} + C_2 \cdot (1.18 + 0.85\xi) \cdot f_{ck} \quad (9-3)$$

440 where, W_{scm} and f_{oscy} are the compound elastic section modulus and the characteristic value of
 441 compound axial compressive strength [15]; and the parameters C_1 and C_2 equal to $\alpha_1/(1 + \alpha_1)$
 442 and $(1 + \alpha_n)/(1 + \alpha_1)$ respectively, and $\alpha_1 = A_{so}/A_c$, A_c is the cross-sectional area of
 443 sandwich concrete.

444 The results in Fig. 16 indicate that when reaching M_u , yielding in the whole section and its parts
 445 generally appear in the lower and upper inner tubes of the DCFSSST beams, respectively, and
 446 accordingly the use of plastic section modulus, W_{psi} , is more suitable for calculating $M_{i,u}$.
 447 Coefficient $\gamma_{m,i}$ for the cross-sectional moment capacity of the inner steel tubes is defined as:

$$\gamma_{m,i} = \frac{M_{i,u}}{W_{psi} \cdot f_{yi}} \quad (10-1)$$

$$W_{psi} = \begin{cases} \frac{\pi d_e}{4} \cdot [d_i^2 - d_i^2 - 2t_i] & \text{(with inner SHSs)} \\ d_e \cdot [d_i^2 - (d_i - 2t_i)^2] & \text{(with inner CHSs)} \end{cases} \quad (10-2)$$

448 The simulation results indicate that, the impact of α_n , f_{yo} and f'_c on $\gamma_{m,o}$ and $\gamma_{m,i}$ can be
 449 unified into the parameter ξ , which is determined by the above three parameters. The influence of
 450 the key parameters on the relationship between the coefficients $\gamma_{m,o}$ and $\gamma_{m,i}$ for the cross-sectional
 451 moment capacity and the longitudinal strain $\varepsilon_{L,bf}$ of the bottom flange in the outer steel RHS is
 452 presented in Fig. 19 and the other parameters have only a moderate effect. In general, only the type
 453 of the inner tube has some influence on the form of the later phase in the $\gamma_{m,o} - \varepsilon_{L,bf}$ diagram, but
 454 in terms of M_u corresponding to $\varepsilon_{L,bf} = 0.01$, the difference in $\gamma_{m,o}$ and $\gamma_{m,i}$ of rectangular
 455 DCFSSST beams with different types of inner tubes is generally less than 5% and therefore this
 456 influence is ignored. In addition, $\gamma_{m,o}$ and $\gamma_{m,i}$ improve with the increase of ξ , ϕ and e_0 . Using
 457 the regression analysis for a suitable amount of the simulation data, the expressions for $\gamma_{m,o}$ and
 458 $\gamma_{m,i}$ are presented as:

$$\gamma_{m,o} = [1.27 + 0.35\ln(\xi)] \cdot (0.3\phi + 0.85) \cdot (0.62e_0 + 0.69) \quad (11)$$

$$\gamma_{m,i} = [0.85 + 0.12\ln(\xi)] \cdot (0.5\phi + 0.5) \cdot (e_0 + 0.85) \quad (12)$$

459 Fig. 20 presents the comparison between the simplified and simulated values of the coefficients,
 460 and it is shown that both $\gamma_{m,o}$ and $\gamma_{m,i}$ can be predicted well using the simplified formulae. The

465 final formula for the flexural capacity of rectangular DCFSSST beams is produced by inserting all the
 4166 relevant parameters into Eq. (8). In Fig. 21 the values of simplified formulae for $M_{u,s}$ are compared
 2
 3
 467 with a considerable number of results from the FE simulations, $M_{u,fe}$, and the actual models $M_{u,e}$
 5
 468 based on the measured data. The difference between the simplified models and the simulated and
 7
 8
 469 measured results is generally less than 10%. The statistical outcome is that the mean and Cov of
 10
 1170 $M_{u,s}/M_{u,fe}$ are 1.045 and 0.040, respectively, and for $M_{u,s}/M_{u,e}$ they are 0.911 and 0.050,
 12
 13
 1471 respectively, showing that the simplified formulae are capable of predicting accurately the flexural
 15
 1472 capacity of rectangular DCFSSST beams. The scope of application for the simplified equations is:
 17
 18
 1473 $\eta=1.2$ to 2.0 , $\alpha_n=0.04$ to 0.16 , $\phi=0.25$ to 0.75 , $e_0=0.4$ to 0.6 , $f_{y0}=f_{yi}=235$ MPa to 460 MPa,
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 20
 1474 $f'_c=31.9$ MPa to 65 MPa, and $d_i/t_i=20$ to 60 for SHS or 30 to 90 for CHS.

2475 5. Conclusions

2476 The main conclusions from the experimental and numerical investigation on the flexural behaviour
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 28
 2477 of rectangular DCFSSST beams reported in this paper are summarized as:

30
 3178 (1) The investigated beam specimens exhibit good deformability and their ultimate mid-span
 32
 33
 3479 displacement is between 6% and 9% of the effective span length. The failure patterns include mainly
 35
 3480 multiple local bulge regions on the top flange of the outer tube, global deformation of the inner tubes
 37
 38
 3481 and crushing of concrete at the evident sites of local bulge and nearly uniform cracks that extend to
 39
 40
 4182 approx. 2/3 of the section depth in the sandwich concrete.

42
 43
 4483 (2) Three successive phases in the $M - u_m$ diagrams are classified as approximately elastic,
 44
 45
 4484 elastic-plastic and plastic strengthening one before the fracture of the bottom flange in the outer tube.
 47
 48
 4485 The displacement distribution of the beam specimens fits well with the half-sine wave form up to
 49
 50
 4186 reaching the flexural capacity.

52
 53
 5487 (3) The cross-section form of the inner tubes has a limited effect on the behaviour, but α_n
 54
 55
 5488 influences significantly on the values of M_{ue} and K_{ie} in the rectangular DCFSSST specimens. In the
 57
 58
 5489 ones with inner SHSs and α_n equal to 0.110 and 0.142, M_{ue} is 1.206 and 1.510 times that with α_n
 59
 60
 6190 equal to 0.079, respectively, and K_{ie} is 1.112 and 1.409 times that with α_n equal to 0.079, and for

491 the specimens with inner CHSs, the values equal to 1.130 and 1.556 are associated with M_{ue} and
4192 respectively the values equal to 1.132 and 1.494 are associated with K_{ie} . The flexural stiffness of the
2
3
4493 specimens calculated using the superposition principle accords generally well with the ones based on
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5
4494 the measured results.

7
4495 (4) The established FE model predicts well the flexural performance of rectangular DCFSSST
9
10
14196 beams, and the FE model can be further used to reveal the overall stress and strain states of each
12
14397 component in the rectangular DCFSSST beams. The simplified formulae for the flexural capacity of
14
15
14698 rectangular DCFSSST beams suggested based on the data from the parametric analyses agree well with
17
14899 the numerical and experimental results.

2100 **Declaration of Competing Interest**

22
23
501 The authors declare that they have no known competing financial interests or personal relationships
24
25
25602 that could have appeared to influence the work reported in this paper.

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33
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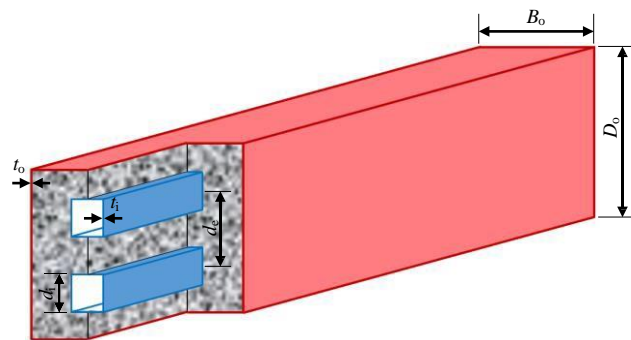
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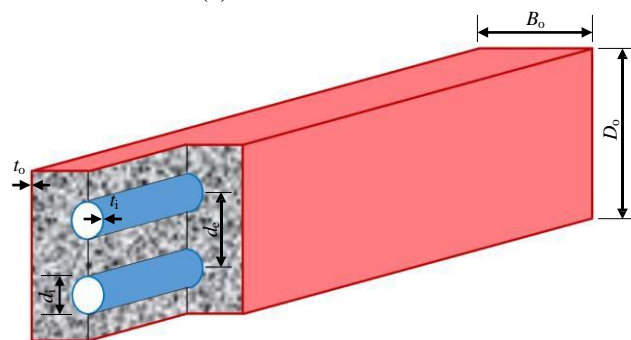
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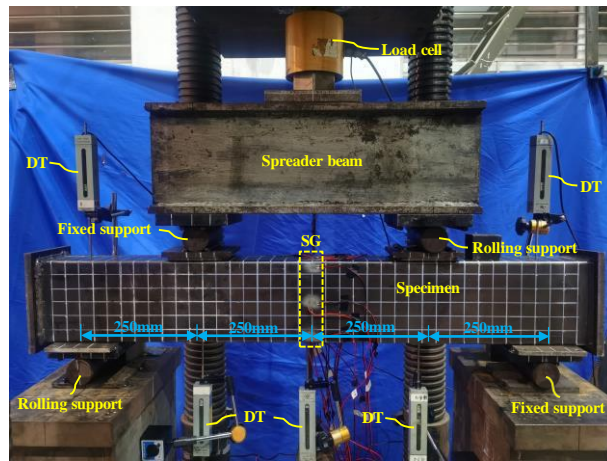
Figures:

(a) With inner SHSs

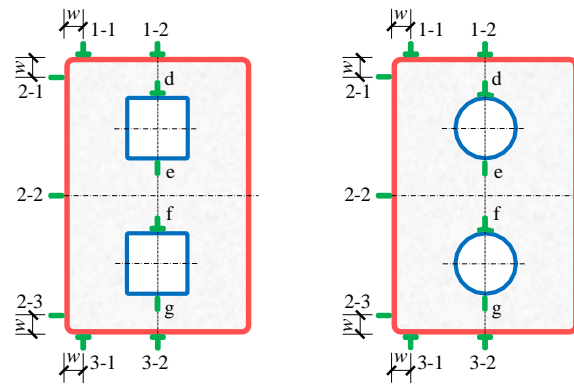


(b) With inner CHSs

Fig. 1. Configuration of the rectangular DCFSSST beams.

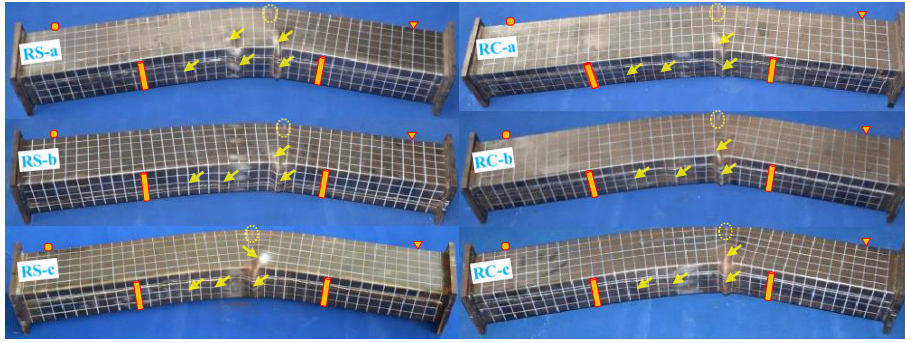


(a) General layout



(b) Layout of strain gauges ($w=20$ mm)

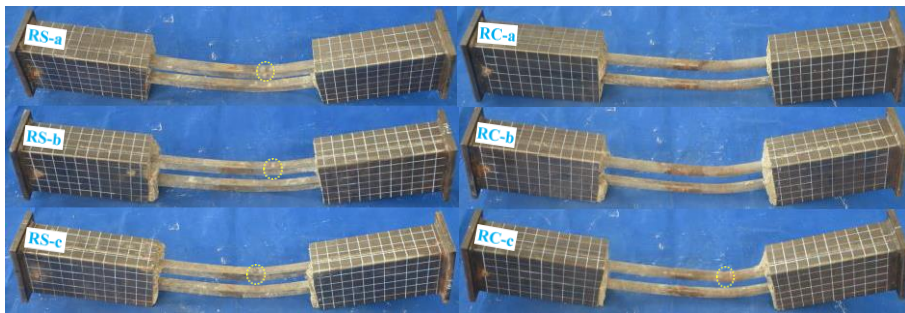
Fig. 2. Testing set-up.



(1) Overall appearance



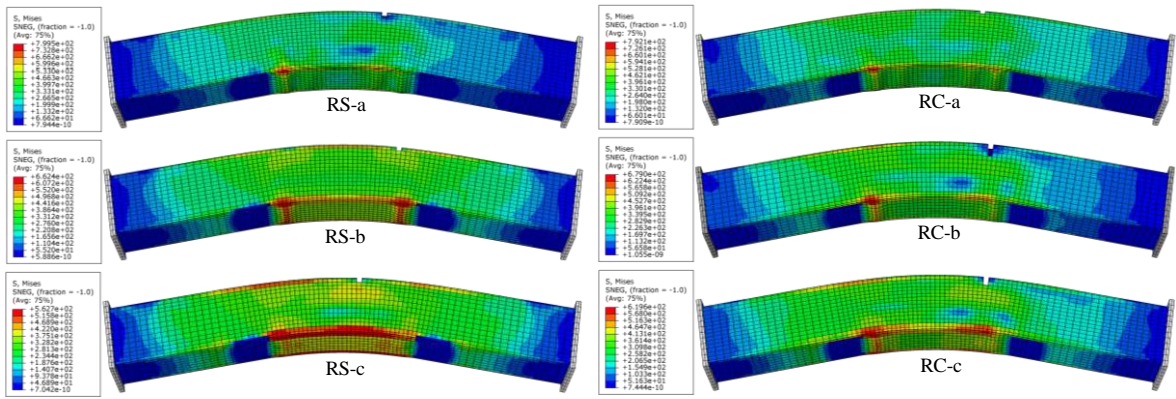
(2) Sandwich concrete



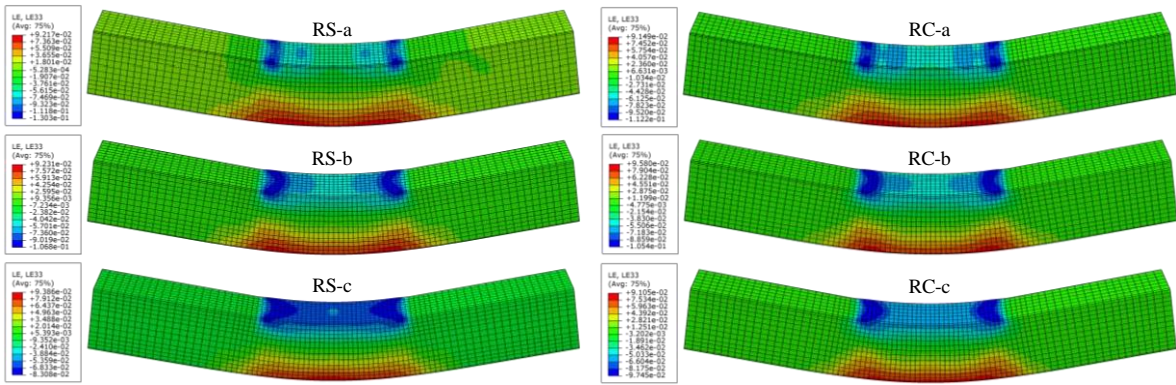
(3) Inner tubes

(a) Measured result

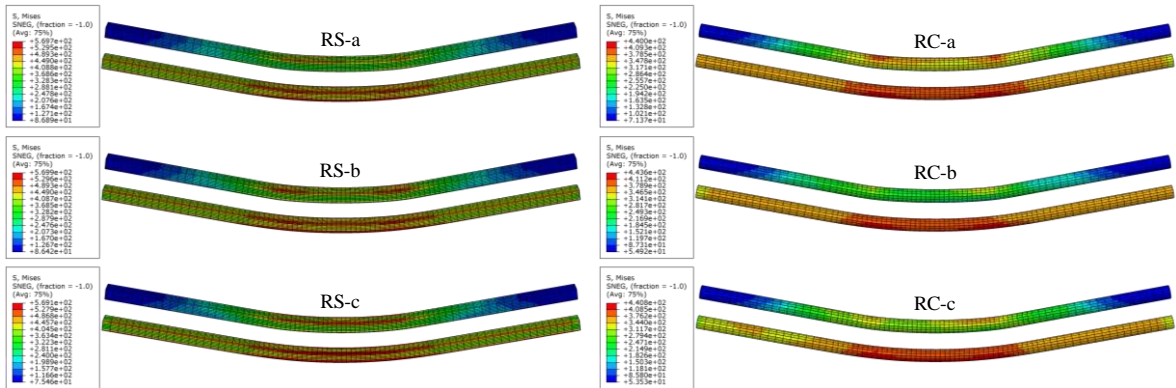
Fig. 3. (continued).



(1) Overall appearance



(2) Sandwich concrete



(3) Inner tubes

(b) Simulated result

Fig. 3. Failure modes of rectangular DCFSSST beam specimens.

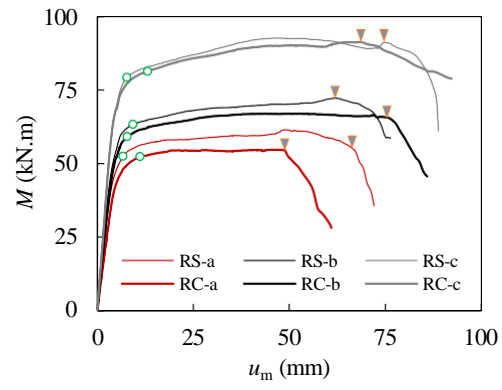
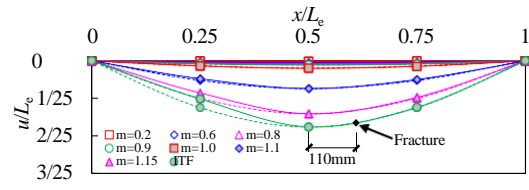
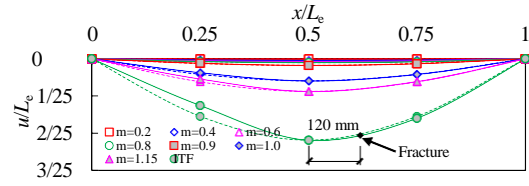


Fig. 4. $M - u_m$ curves for the specimens according to the measurements.



(a) RS-b



(b) RC-b

Fig. 5. Distribution of deflections along effective span of typical specimens.

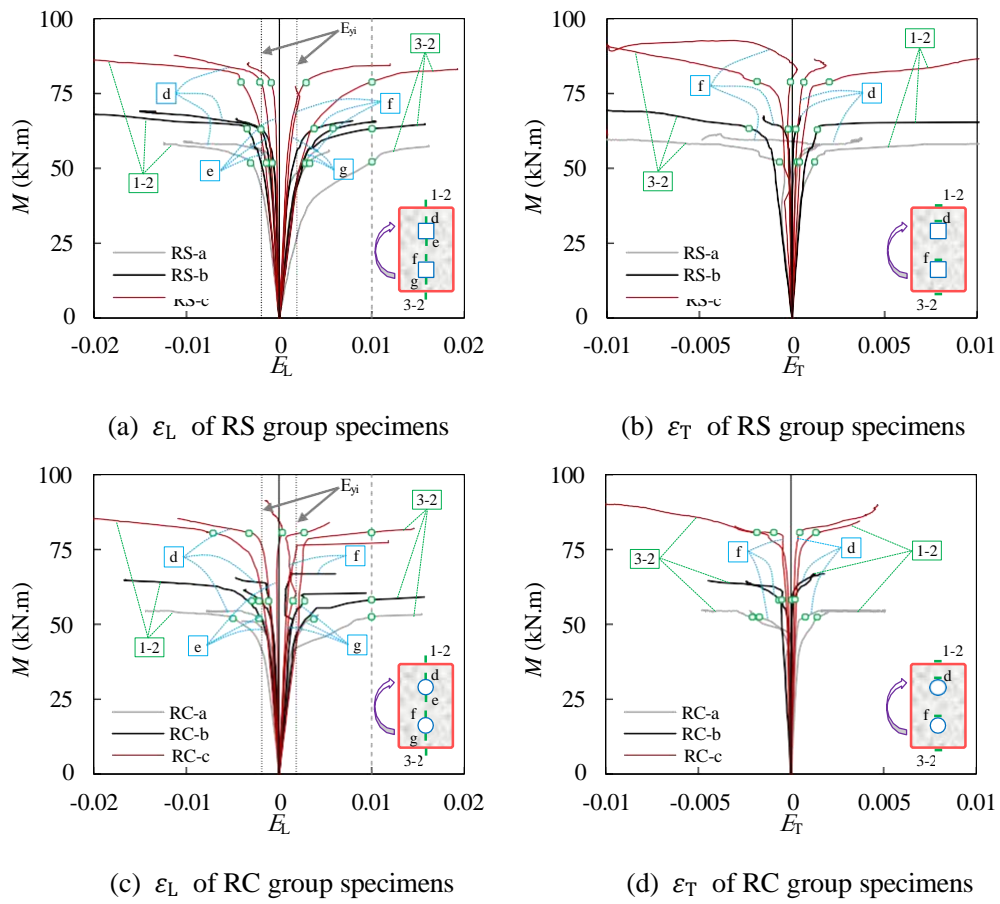
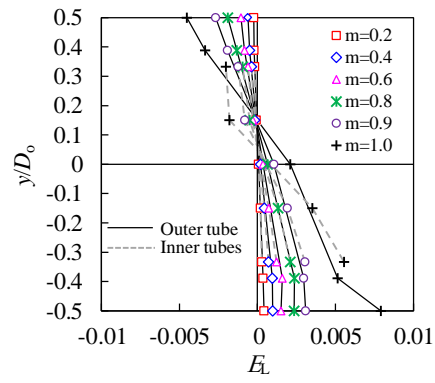
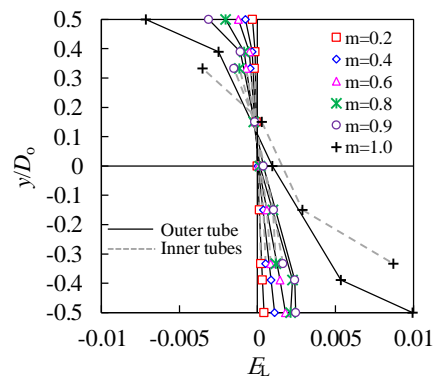


Fig. 6. Relationships between ε_L , ε_T and M according to the recorded data.

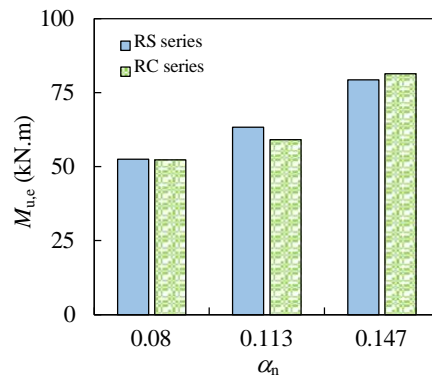


(a) RS-b

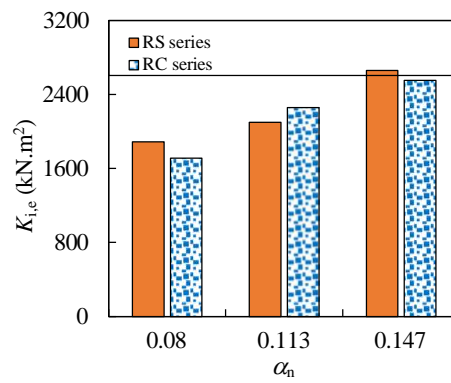


(b) RC-c

Fig. 7. Distribution of ϵ_L in the mid-span section of typical specimens.



(a) Flexural capacity



(b) Flexural stiffness

Fig. 8. Effects of varying α_n on the mechanical index in the specimens.

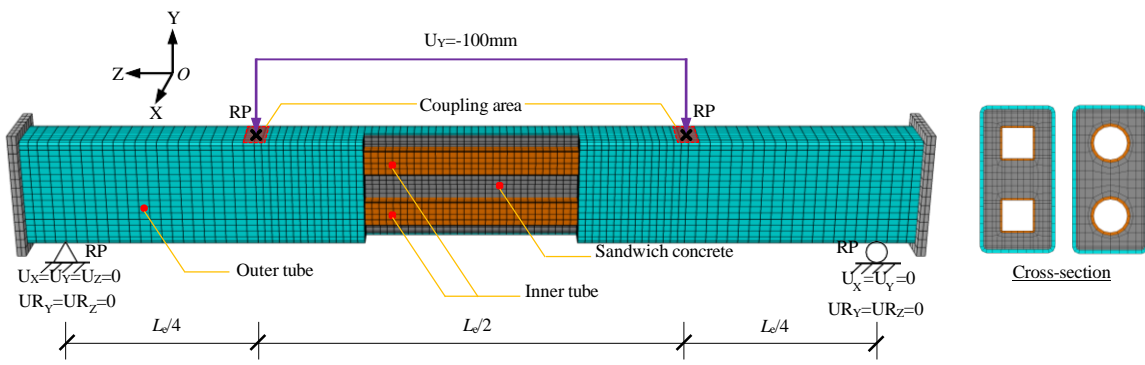
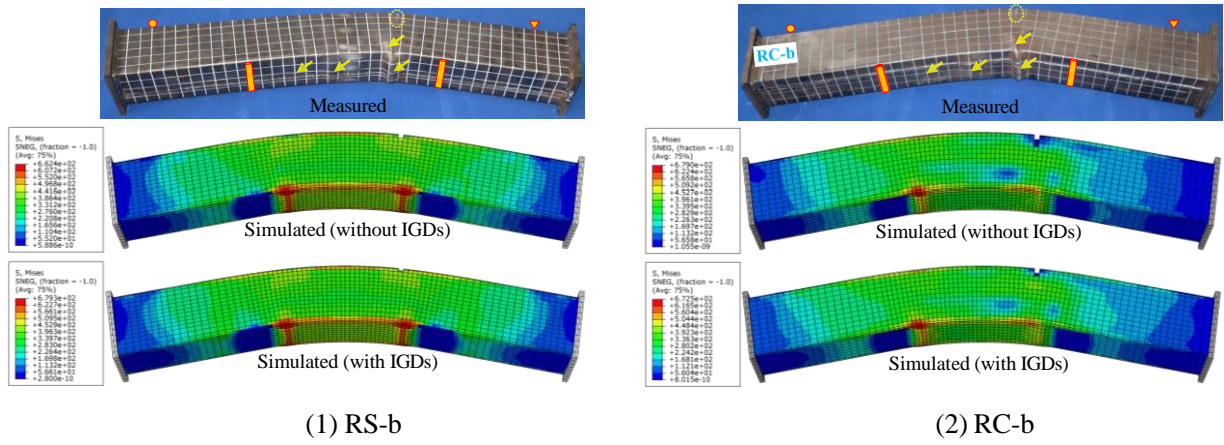
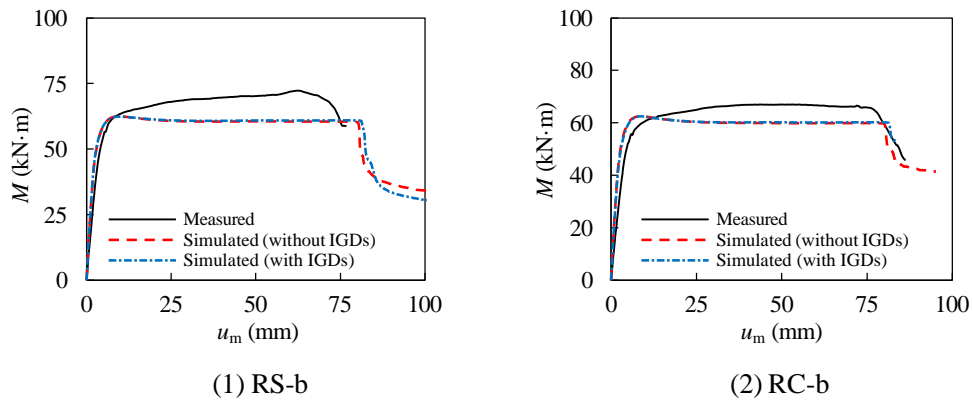


Fig. 9. Meshing and boundary conditions for the FE model of the rectangular DCFSSST beams.



(a) Failure modes



(b) $M-u_m$ curves

Fig. 10. Impact of the initial geometric defects (IGDs) on the flexural behaviour of the rectangular DCFSSST beam specimens.

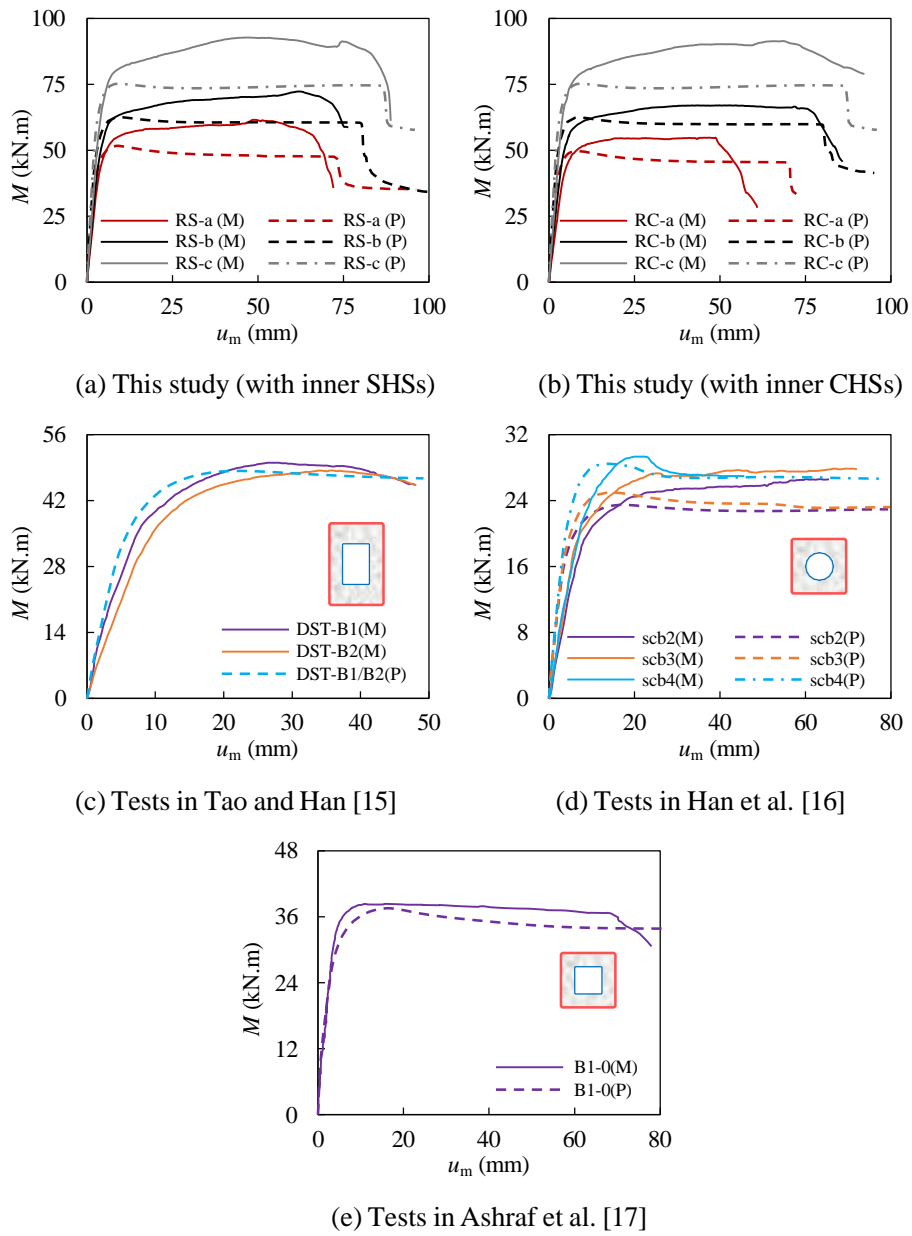


Fig. 11. Comparison between the predicted and measured $M - u_m$ curves.

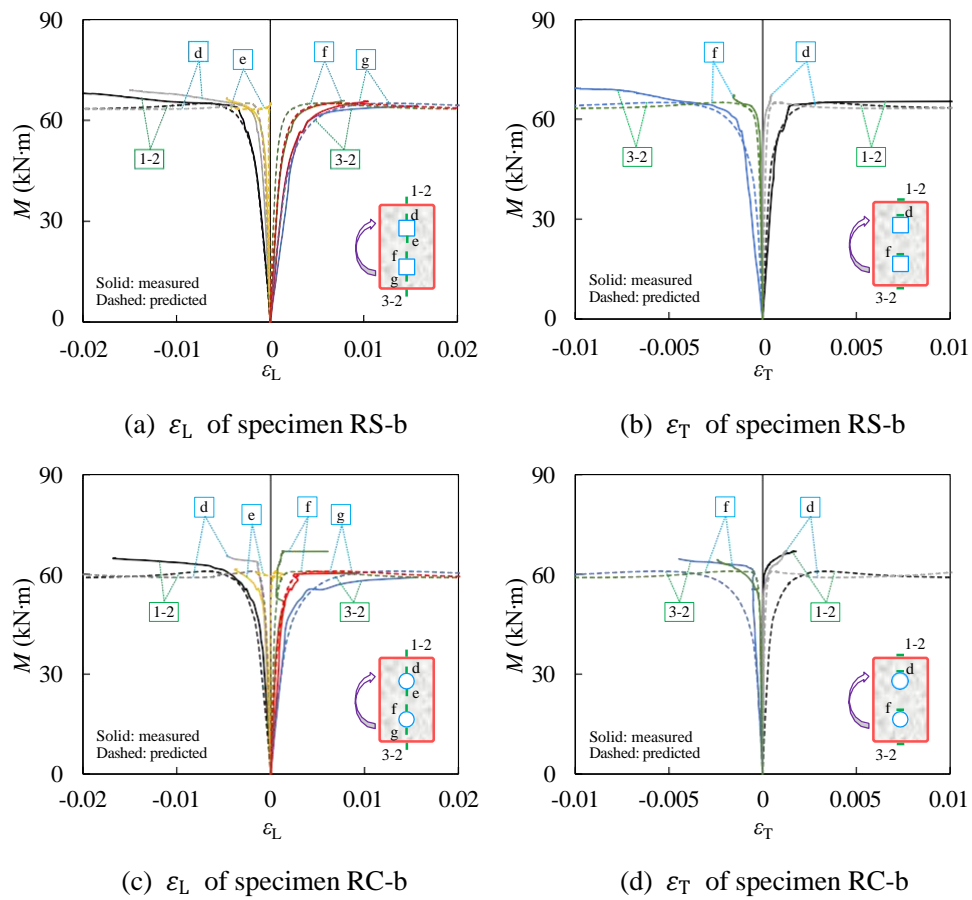
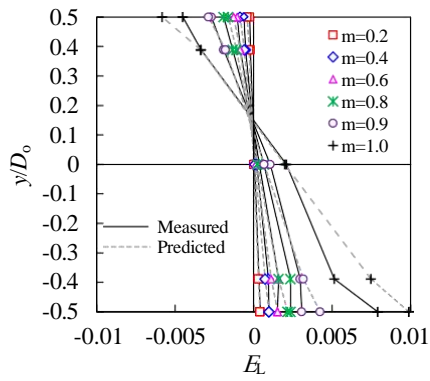
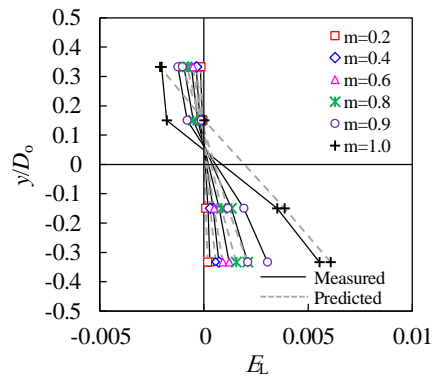


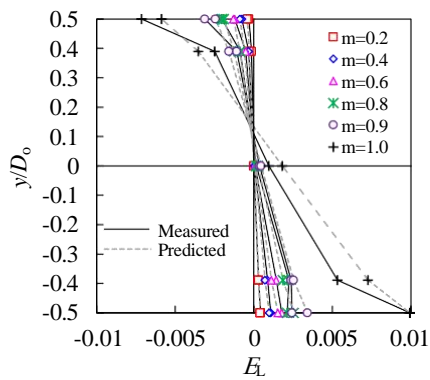
Fig. 12. Typical $M - \varepsilon_L$ and $M - \varepsilon_T$ diagrams according to predictions and measured data.



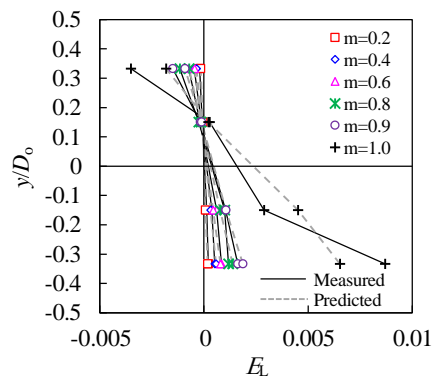
(a) Outer tube of specimen RS-b



(b) Inner tubes of specimen RS-b



(c) Outer tube of specimen RC-c



(b) Inner tubes of specimen RC-c

Fig. 13. Typical distributions of ϵ_L in the mid-span section on account of predicted and measured data.

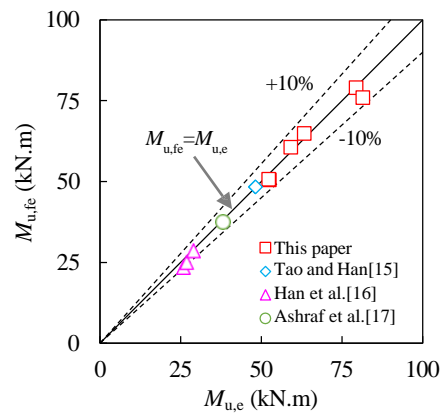


Fig. 14. Comparison of flexural capacities evaluated on account of predicted and measured data.

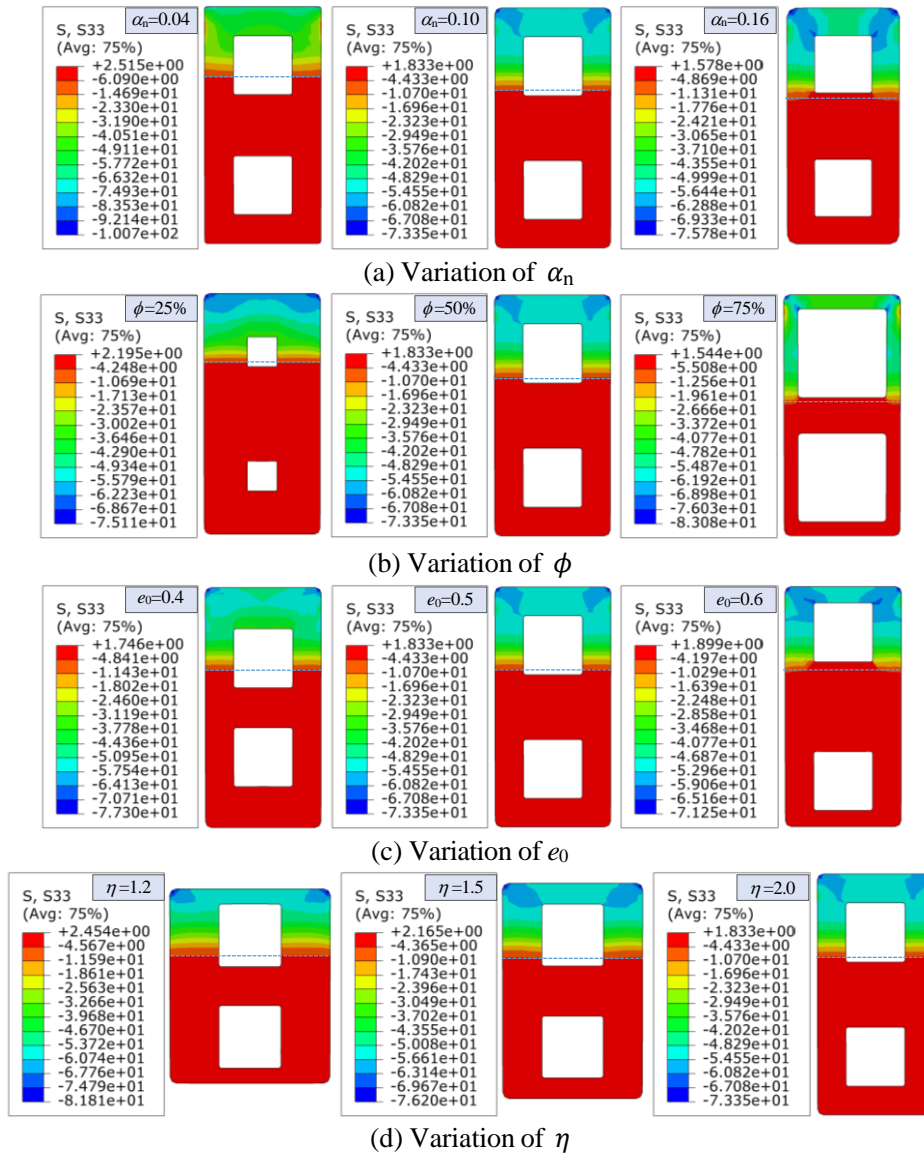
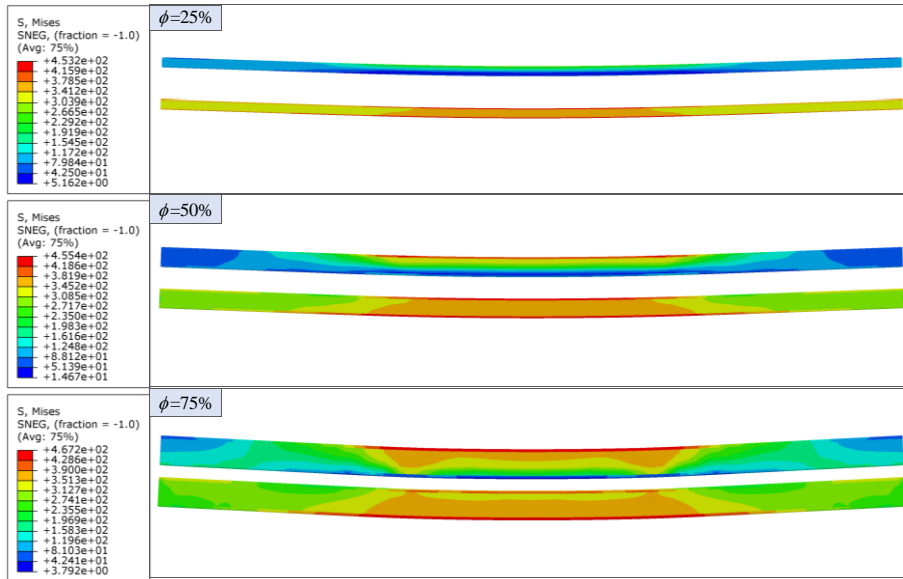
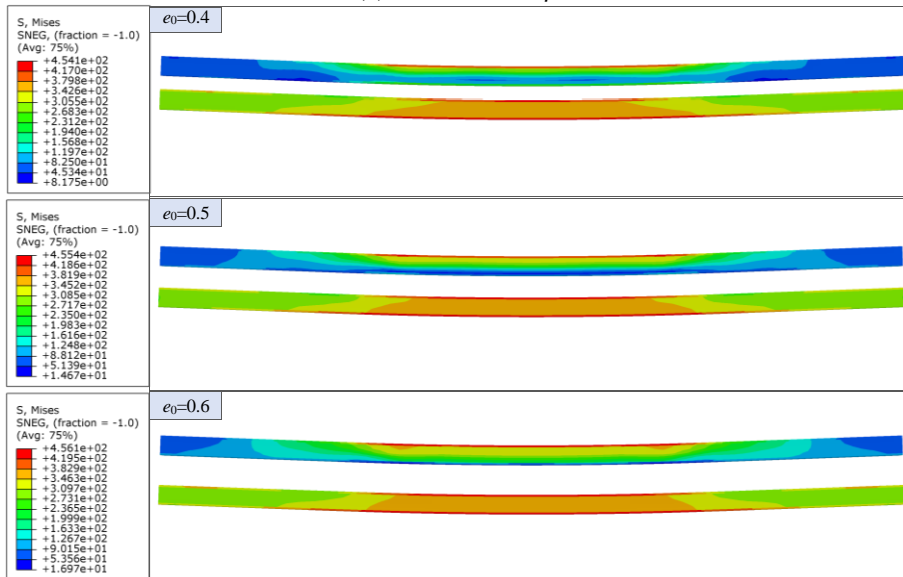


Fig. 15. Influence of the key parameters on the stresses of sandwich concrete in the mid-span section.



(a) Variation of ϕ



(b) Variation of e_0

Fig. 16. Effects of ϕ and e_0 on the von Mises stresses in the inner SHSs.

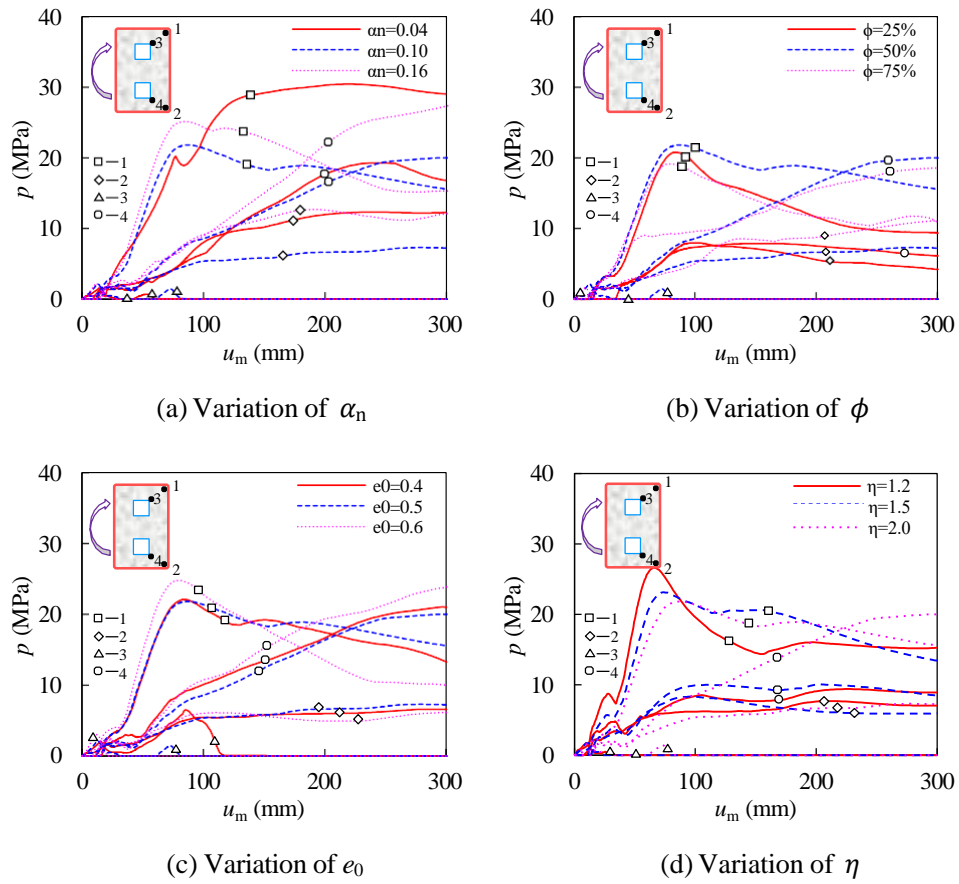


Fig. 17. Influence of the key parameters on the interaction stress p in typical points of the mid-span section.

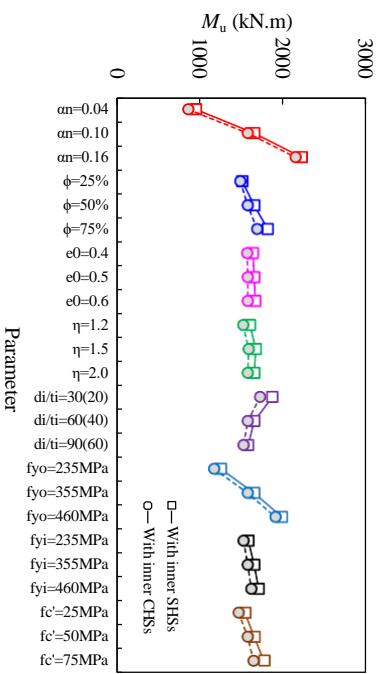
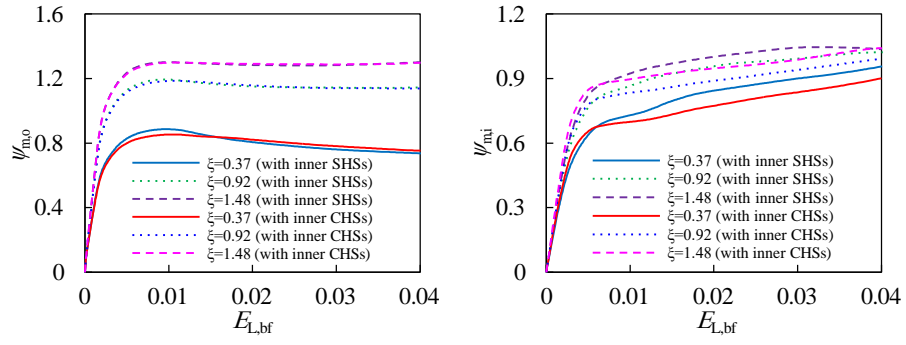
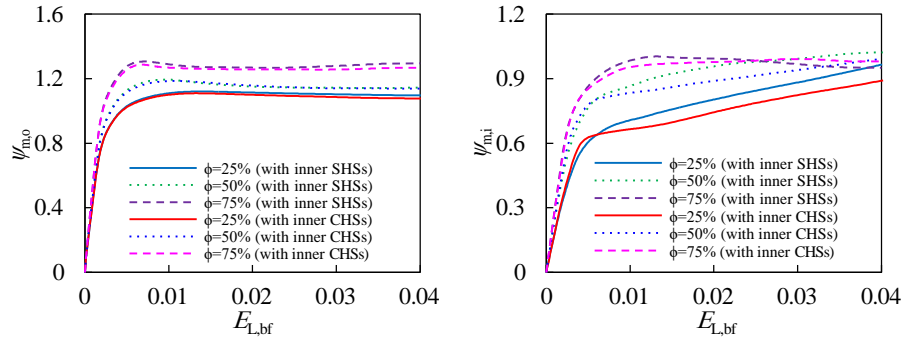


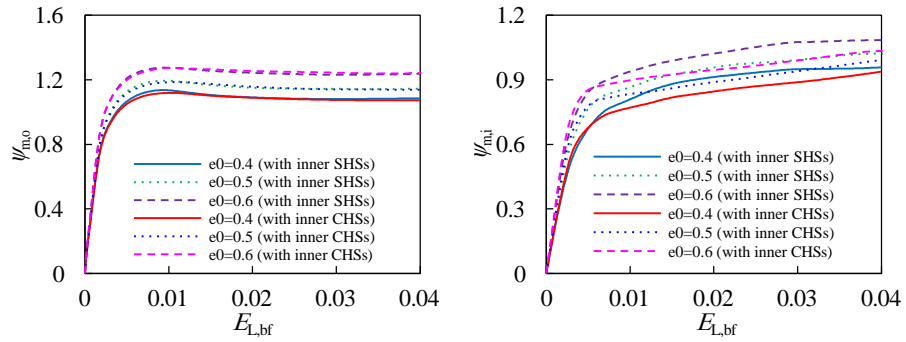
Fig. 18. Effect of key parameters on the values of M_u in rectangular DCFSSST beams.



(a) Confinement factor, ξ



(b) Void ratio, ϕ



(c) Offset ratio of inner steel tube, e_0

Fig. 19. Effect of key parameters on $\gamma_{m,o}$ and $\gamma_{m,i}$.

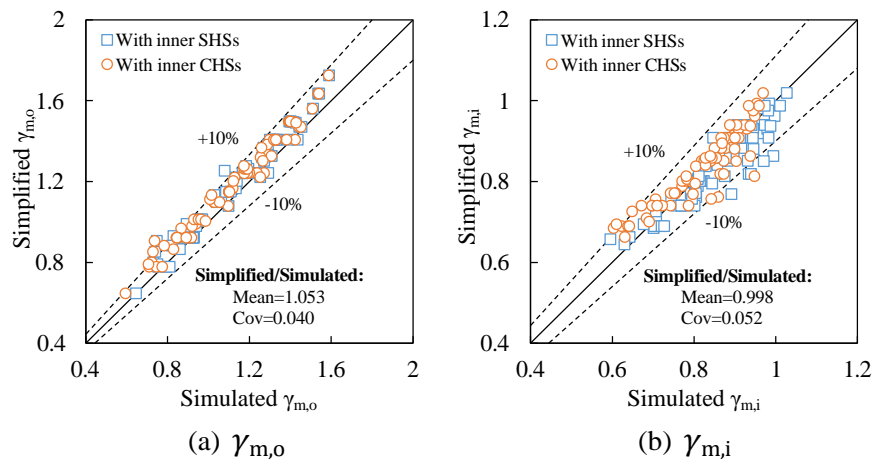


Fig. 20. Coefficients $\gamma_{m,o}$ and $\gamma_{m,i}$ for the cross-sectional moment capacity.

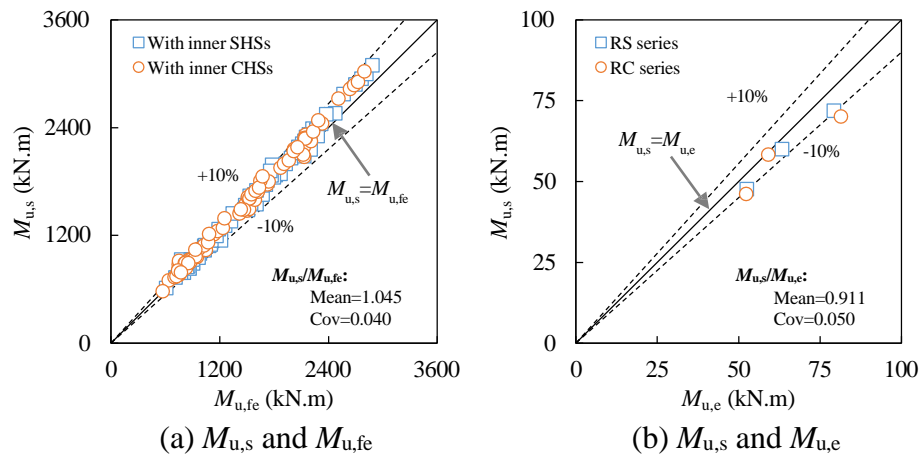


Fig. 21. Flexural capacities based on simplified equations, simulation data and measured data.

Tables:

Table 1. Details of the specimens.

No.	Label	$D_o \times B_o \times t_o$ (mm)	$d_i \times t_i$ (mm)	α_n	ϕ	d_e (mm)	e_0	f_{yo} (MPa)	f_{yi} (MPa)	f_{cu} (MPa)	$M_{u,e}$ (kN·m)	K_e (kN·m ²)
1	RS-a	180×120×2.68	30×1.94	0.079	30.0%	90	0.50	324.9	371.8	49.8	52.5	1888.4
2	RS-b	180×120×3.66	30×1.94	0.110	30.4%	90	0.50	333.7	371.8	49.8	63.3	2099.1
3	RS-c	180×120×4.63	30×1.94	0.142	30.8%	90	0.50	334.8	371.8	49.8	79.3	2261.8
4	RC-a	180×120×2.68	33×1.97	0.079	29.2%	87	0.48	324.9	352.9	49.8	52.3	1453.7
5	RC-b	180×120×3.66	33×1.97	0.110	29.6%	87	0.48	333.7	352.9	49.8	59.1	1919.1
6	RC-c	180×120×4.63	33×1.97	0.142	30.0%	87	0.48	334.8	352.9	49.8	81.4	2171.1

Table 2. Properties of steel tubes.

Type	Cross-section	$t_o(t_i)$ (mm)	Yield strength (MPa)	Tensile strength (MPa)	Elastic modulus (N/mm ²)	Poisson's ratio	Elongation (%)
Outer tube	Rectangular	2.68	324.9	462.6	2.03×10^5	0.273	27.2
		3.66	333.7	452.0	2.06×10^5	0.292	28.7
		4.63	334.8	468.2	2.03×10^5	0.277	32.3
Inner tube	Square	1.94	371.8	460.9	1.92×10^5	0.278	14.6
	Circular	1.97	352.9	451.2	1.88×10^5	0.270	15.4

Table 3. Plasticity parameters for CDP model

Plasticity parameter	Dilation angle	Flow potential eccentricity	Ratio of σ_{b0} to σ_{c0}	Ratio of $q_{(TM)}$ to $q_{(CM)}$	Viscosity parameter
Value	30°	0.1	1.16	2/3	5×10^{-6}

Note: σ_{b0} is initial equibiaxial compressive yield stress, σ_{c0} is initial uniaxial compressive yield stress, and $q_{(TM)}$ and $q_{(CM)}$ are the second stress invariant on the tensile and compressive meridian, respectively.