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Interfaces with other disciplines

Bi-level optimisation of subsidy and capacity investment under competition and uncertainty

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ABSTRACT

In this paper, we develop a bi-level real options framework for deriving the equilibrium Government subsidisation and firm-level capacity investment policy in a duopoly market structure. We find that strategic interactions with the Government may impact a firm's capacity investment decision significantly and that the equilibrium subsidisation policy depends on both the market structure and the type of duopolistic competition. Interestingly, the provision of greater subsidy to the leader raises the follower's incentive to invest earlier and in a bigger project. The loss in value of the leader, due to the follower's entry, relative to the monopolist increases with economic uncertainty and, although a subsidy can mitigate this loss, its effect becomes less pronounced as economic uncertainty increases. We also find that a profit (welfare)-maximising Government does not offer (offers) a subsidy in a highly uncertain environment or upon low tax rate, while higher tax rate does not always decelerate investment. Finally, we find that while competition is always desirable for a social planner, a profit-maximising Government may benefit more under pre-emptive competition.

1. Introduction

Firms devising strategies for capacity investment in deregulated industries face the formidable challenge of managing not only the uncertainty in future revenue streams, but also the likely presence of a rival. It further complicates capacity investment decisions the fact that they are often made in light of support schemes designed to incentivise investment in infrastructure projects, promote research and development (R&D) or accelerate the structural transformation of many industries due to pressing climate change concerns.[1] The literature on methods for identifying *ex-ante* the level of Government support that aligns firm and Government-level optimisation objectives has grown considerably. However, existing bi-level models for optimal subsidy design are developed under the assumption of monopoly or perfect competition (Lukas & Thiergart, 2019; Sarkar, 2012) and, consequently, the implications of strategic interactions at the firm level for optimal subsidy design remain an important open research question. Additionally, it remains unclear how the optimal subsidisation strategy would differ if a Government pursued a social welfare rather than a profit-maximisation objective.

Analysing the joint implications of firm-level strategic interactions and the non-cooperating game between a private firm and the Government for optimal subsidy design is a challenging task, whereby the following trade-offs must be balanced. First, capacity investment decisions are particularly risky since a large capacity raises the downside risk during recession, whereas a low capacity may result in forgone revenues upon a sudden upturn in the economy. Second, the level of subsidy should be designed so that investment intensity targets are met in a timely manner. However, a high (low) subsidy may induce a firm to invest earlier (later) in a smaller (bigger) project. Third, upon a firm's investment, the Government receives a tax from the cash flows of the operating project and, therefore, it must balance the subsidy level so as to maximise its net tax income (i.e., profit) or social welfare. Finally, the Government needs to account for firm-level strategic interactions, since competition is likely to reduce the value of the subsidy, and, in turn, alter a firm's investment policy substantially.

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[1] For example, to encourage sustainable innovation, part of the American Recovery and Reinvestment Act of 2009 allocated $400 million to Advanced Research Projects Agency-Energy and $2.5 billion to renewable energy and energy efficiency R&D (CRS, 2009). Besides, Innovate UK, which supports business-led innovation in all sectors around UK, declares an increase in R&D funding from £700 million in 2021–22 to £1.1 billion in 2024–25 (Cookson, 2021). The design of support schemes may be subject to balancing conflicting objectives, as private firms pursue profit-maximisation objectives while a Government may maximise either profits associated with corporate tax (Lukas & Thiergart, 2019) or social welfare (Azevedo et al., 2021).

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In this paper, we embed these trade-offs in a real options framework to address open research questions such as: How is a Government's subsidisation policy affected by firm-level strategic interactions? Does the subsidy offset a firm's loss in value due to competition? How does the equilibrium subsidisation and capacity investment strategy vary when a Government pursues a social welfare instead of a profit maximisation objective? To address these questions, we consider a duopoly consisting of two identical firms that have the option to invest in a project. To incentivise investment, the Government will grant a subsidy to the first firm that enters the market (leader). Indeed, since the firms are identical, subsidising the follower is not a plausible option as this will reduce the incentive to invest first. This is consistent with Nie et al. (2016), who show that the efficiency of subsidies depends on the position of the subsidised firm. In particular, the Government's subsidisation incentive reaches the lowest (highest) level if the subsidised firm is a follower (leader). The subsidy takes the form of a lump-sum cash grant.

By addressing these questions, our work bridges two strands of literature: bi-level real options and duopolistic competition. Regarding the latter, we consider the case of pre-emptive competition, where both firms have the incentive to invest first to gain a leader advantage, and non-pre-emptive competition with the role of the leader being assigned exogenously. While only the leader benefits from the subsidy, the follower's entry reduces the leader's expected revenues, thus implicitly affecting the subsidisation policy, as the alignment of the Government's and leader's objectives should account for the latter's loss in market share. Thus, the contribution of our work is threefold. First, we develop a bi-level real options framework to analyse the non-cooperative game between a Government and two competing firms. Second, we obtain the equilibrium investment threshold, project scale and subsidisation policy, and demonstrate how each depends on strategic interactions. Finally, we derive and compare the optimal investment and subsidisation strategies for the case of a profit and social welfare-maximising Government,[2] and provide policy and managerial insights based on analytical and numerical results.

We proceed by discussing some related work in Section 2 and present assumptions and notation in Section 3.1. In Section 3.2, we present the analytical framework under monopoly and derive the equilibrium capacity investment policy of the firm as well as the equilibrium subsidisation policy of an income-maximising Government. We then expand Section 3.2 by allowing for non-pre-emptive and pre-emptive duopolistic competition in Sections 3.3 and 3.4, respectively. Next, in Section 3.5 we explore how the optimal subsidisation and capacity investment policy changes when the Government optimises social welfare. Section 4 proceeds with various numerical examples, results and policy implications, whereas Section 5 concludes the paper offering suggestions for further research.

2. Related research and our contribution

Although traditional real options models address the problem of optimal investment under uncertainty ignoring the implications of competition (McDonald & Siegel, 1985, 1986), the game-theoretic real options literature has grown considerably in recent years. Nevertheless, models that allow for strategic interactions often analyse their implications for investment timing without considering managerial discretion over project scale (Bar-Ilan & Strange, 1999; Bøckman et al., 2008; Dangl, 1999, Hagspiel et al., 2016a), or take no notice of the implications of the wide range of support schemes deployed to incentivise investment in many industries. Examples of real options models for strategic capacity investment include Huisman and Kort (2015), who analyse the problem of optimal capacity investment under duopolistic competition and demonstrate how discretion over project scale may be used strategically to deter or accommodate the entry of a rival. Other related examples include Huberts et al. (2015) and Jeon (2021).

Examples of policy-oriented real options models include Boomsma et al. (2012), Boomsma and Linnerud (2015) and Ritzenhofen et al. (2016). More recently, Bigerna et al. (2019) consider a firm that has the option to invest in renewable energy under economic uncertainty and empirically analyse how a subsidy, in the form of a feed-in premium, affects its capacity investment policy. For a given environmental target, they derive the required investment scale and determine the corresponding optimal subsidy level and investment threshold. The contribution of this line of work includes the provision of policy insights not only on how various support schemes, such as feed-in tariffs, renewable portfolio standards and green certificate trading, may differ in incentivising green investments, but also on how random revisions of support schemes may impact investment incentives. However, the optimal investment and subsidisation policies are determined *ex-post*, and, thus, they do not reflect the equilibrium from the strategic interaction between a firm's and a Government's optimisation objectives.

Such strategic interactions are analysed in bi-level real options models, with the objective to understand private firms' investment behaviours and Governments' optimal subsidisation strategies under uncertainty (Pennings, 2000, 2005; Yu et al., 2007). For example, Pennings (2000) studies how the Government's choice on the level of subsidy and taxation may impact a private firm's optimal investment strategy. Results indicate that, when the tax income is exactly offset by the subsidy, a firm can invest earlier as the tax rate increases, which renders subsidisation a more effective fiscal incentive. Other related examples include Danielova and Sarkar (2011), Barbosa et al. (2016), Tian (2018), Jin et al. (2021) and Silaghi and Sarkar (2021). Allowing for discretion over project scale, Lukas and Thiergart (2019) analyse the effect of uncertainty and investment stimulus (in the form of cash grants) on optimal investment timing, scaling and debt financing strategies. Their results indicate that, when the Government aims to maximise its profit, the relationship between the equilibrium subsidy and price uncertainty is ambiguous. Also, social welfare optimisation objectives are analysed in Pawlina and Kort (2006), Yang et al. (2018) and Azevedo et al. (2021). The latter demonstrate the effect of a Government's subsidisation and taxation policy on a monopolist's capacity investment strategy and show that, by choosing the appropriate tax-subsidy package, the Government is able to implement a welfare-maximising policy.

Although existing bi-level real options models do not extend beyond the strategic interactions between a Government and a private firm, recent game-theoretic, albeit static, models demonstrate that the market structure can influence both the design of subsidies and the private firms' investment incentives (Wang & Zhou, 2020; Yang et al., 2021). For example, Yang and Nie (2015) analyse the effectiveness of different subsidy strategies under asymmetric duopoly. They find that, while subsidising the smaller firm benefits the social welfare, subsidising the larger firm can improve the total R&D investment output, especially when the cost gap between the firms is significant. Also, Nie et al. (2016) consider a unilateral and a bilateral subsidy and show that the firms' positioning is critical to a Government's subsidisation policy since the output of the subsidised firm is the highest (lowest) if the firm is the leader (follower). More recently, Yang et al. (2021) developed a game-theoretic model between a Government and two symmetric firms and derived the equilibrium the two firms can reach regarding their technology improvement decisions. Their results confirm that a subsidy is critical for expanding the green product market and improving social

[2] Some Governments, like Germany, have phased out feed-in tariffs to mitigate excessive financial commitments and moved to lowest-bid auctions as a more sustainable approach (Appunn & Wehrmann, 2019). In this case, fiscal prudence becomes a priority, focusing on optimising the Government's budgetary standing. In contrast, the 2030 EU Climate and Energy Framework sets ambitious targets, including a 40% reduction in greenhouse gas emissions, compared to 1990 levels, and a 32.5% enhancement in energy efficiency (European Commission, 2021). In this case, Governmental financial interests may not take precedence, as greater emphasis is put on factors such as social welfare.

welfare. Allowing for economic uncertainty is an important extension of this line of work in analysing the implications of firm-level strategic interactions for optimal subsidy design in a deregulated environment.

Therefore, in this paper, we develop a stylised, game-theoretic real options model for analysing the interaction between a Government and two competing firms (Grenadier, 1996; Thijssen et al., 2012). More specifically, we assume that the Government offers a unilateral subsidy to the first investor (leader) in the form of a cash grant, while imposing a tax rate on the firms' revenues. Our work does not consider bilateral subsidisation schemes (Lahiri & Ono, 1999; Nie et al., 2020; Toshimitsu, 2003; Yang & Nie, 2015), but focuses on how the entry of a follower, in the context of duopolistic competition, impacts the alignment of the Government's and the leader's objectives regarding optimal subsidisation and capacity investment policies. We begin with the benchmark case of monopoly, which we subsequently extend to pre-emptive and non-pre-emptive duopolistic competition. In all cases, we derive the equilibrium subsidisation policy together with the equilibrium capacity investment policy for the monopolist, leader and follower. The subsidisation and capacity investment policies are derived under a profit and welfare-maximising Government, thus allowing comparison of results under optimisation objectives that have so far been considered separately in the existing literature.

Thus, our work contributes to the existing literature on strategic capacity investment that ignores either economic uncertainty (Nie et al., 2016) or strategic interactions between private firms and the Government (Huisman & Kort, 2015). By integrating these features in a bi-level real options framework, we are able to derive new insights on the Government's subsidisation policy and a firm's capacity investment policy. Our results complement prior contributions on duopolistic competition that ignores the interaction between private firms and a Government, as they indicate that both the market structure and the type of duopolistic competition can have a significant impact on the equilibrium subsidisation and capacity investment policy. Contrary to conventional intuition, we find that, even though the follower receives no support from the Government, they can actually benefit from the leader's subsidy and invest not only earlier but also in greater capacity. Moreover, it is shown that the loss in the value of the leader, due to the presence of a rival, relative to the monopolist increases with price uncertainty and that, although a subsidy can mitigate this loss, its effect becomes less pronounced as uncertainty increases. Our results also suggest that a profit (welfare)-maximising Government does not offer (offers) a subsidy in a highly uncertain environment or when the tax rate is low, while a higher tax rate does not always decelerate investment. Finally, we find that, while competition is always desirable for a social planner, a profit-maximising Government may benefit more under pre-emptive competition.

3. Problem formulation

3.1. Model setting

We consider two firms, each with a perpetual option to invest in a project of infinite lifetime. The firms have discretion over both the timing of investment and project scale and face demand uncertainty. The exogenous demand shock process is given by

$$dX_t = \mu X_t dt + \sigma X_t dW_t, \quad X_0 \equiv X, \tag{1}$$

where $t \geq 0$ denotes time, $\mu > 0$ is the annual growth rate, $\sigma > 0$ is the annual volatility and $W = \{W_t : t \geq 0\}$ is the standard Brownian motion. Also, we assume that both the private firms and the Government are risk-neutral and denote by $r > \mu$ the risk-free rate (Silaghi & Sarkar, 2021). Thus, as in Hagspiel et al. (2016b) and Jeon (2021), the output price P_t is given by

$$P_t = X_t(1 - \eta Q_t), \tag{2}$$

where Q_t is the total market output at time t and $\eta > 0$ is the price elasticity of the inverse demand function. Since the firms face no variable operating cost, we assume that, after investing, they both produce at full capacity (Dobbs, 2004).

To incentivise investment under market structure $i \in \{m, p, n\}$, that is, **m**onopoly, **p** re-emptive duopoly, or **n**on-pre-emptive duopoly, the Government provides a unilateral subsidy to the first investor in the form of a lump-sum cash grant, denoted by S_i. This is consistent with Nie et al. (2016) and Jung and Feng (2020), who show that the Government has less incentive to subsidise the follower over the leader. Indeed, granting a subsidy to the follower may reduce a firm's incentive to invest first and, therefore, we do not consider this case.[3]

Since the Government receives tax income from the projects' operating cash flow in the form of a corporate tax $\tau \in [0, 1]$, it may pursue a profit (π) maximisation objective. Alternatively, it may maximise social welfare (w). To distinguish between the two, we denote the equilibrium subsidy by $\widetilde{S}_i^k$, where $k \in \{\pi, w\}$; for example, $\widetilde{S}_m^\pi$ corresponds to the equilibrium subsidy under monopoly for a profit-maximising Government.

We denote by T_{ij} the random time at which firm $j \in \{l, f\}$, i.e., a **l**eader or a **f**ollower, respectively, enters the industry. Also, we denote the investment threshold of each firm by X_{ij} and its capacity by Q_{ij}, with optimal thresholds $\left(X_{ij}^*, Q_{ij}^*\right)$ and equilibrium thresholds $\left(\widetilde{X}_{ij}^k, \widetilde{Q}_{ij}^k\right)$. For example, if the subsidy is exogenously defined, then the optimal investment threshold and project scale of the non-pre-emptive leader is denoted by X_{nl}^* and Q_{nl}^*, respectively. If the subsidy is endogenously defined via alignment of the firm's and the Government's optimisation objectives, then the equilibrium investment threshold and project scale are denoted by $\widetilde{X}_{nl}^\pi = X_{nl}^*\left(\widetilde{S}_n^\pi\right)$ and $\widetilde{Q}_{nl}^\pi = Q_{nl}^*\left(\widetilde{S}_n^\pi\right)$, respectively. The investment cost $I(\cdot)$ is assumed to be a linear function of the installed capacity, i.e., $I(Q_{ij}) = \delta Q_{ij}$, $\delta > 0$ (Bigerna et al., 2019; Nagy et al., 2021). Finally, we denote by $F_{ij}(\cdot)$ the value of the firm's investment opportunity, by $V_{ij}(\cdot)$ the expected value of the active project, and by $G_i^k(\cdot)$ the Government's value function. As in, for example, Dangl (1999), each firm's optimisation objective at time t is summarised as

$$F_{ij}\left(X_t, t\right) = \max \left\{ e^{-r\Delta t}\mathbb{E}_{X_t}\left[F_{ij}(X_{t+\Delta t}, t + \Delta t)\right], \max_{Q_{ij}} V_{ij}\left(X_t, Q_{ij}\right) \right\}, \tag{3}$$

where $\mathbb{E}_{X_t}[\cdot]$ denotes the expectation conditional on the demand shock level X_t. The outer maximisation represents the firm's decision to either postpone investment or invest immediately at time t. As suggested by the first argument, if the firm defers investment for a time interval Δt, then its return is the discounted expected value (conditional on the current demand level X_t) of waiting to invest after Δt, reflecting the expected capital appreciation of the option to invest. This value is compared with the second argument that reflects the firm's value function under immediate investment, where the firm must choose the size of the project so as to maximise its expected net present value at investment.

3.2. Monopoly

This problem has already been examined by Lukas and Thiergart (2019), albeit in the absence of an inverse demand function. By contrast, we assume here that the firm has market power, and present the results under the assumptions of Section 3.1 for ease of reference and comparison with the case of duopoly. As shown at the top of Fig. 1, the

[3] The Government may threaten to subsidise the follower to force the leader accept a smaller subsidy and, to maintain the first-mover advantage, a firm will always accept a smaller subsidy. However, the Government does not necessarily benefit from such a bargain, as offering a smaller subsidy to the leader delays both firms' investment, and the present value of their revenues upon investment is reduced due to the discounting effect.

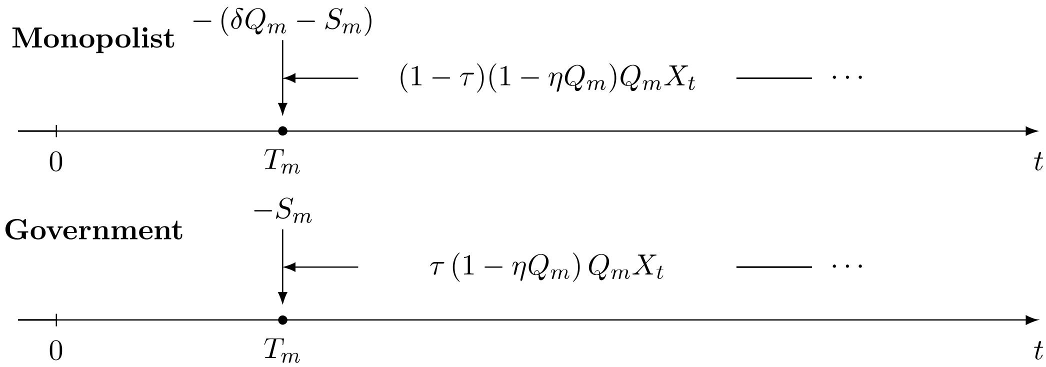

Fig. 1. Irreversible capacity investment and subsidy design under monopoly.

monopolist can choose the investment time, T_m, at which they install the capacity Q_m and incur the investment cost, δQ_m, less the cash grant, S_m. Meanwhile, upon the monopolist's investment, the Government receives a perceptual stream of tax income from the operating project.

We first assume that the monopolist does not have the option to postpone investment and, therefore, must invest immediately. By exercising a now-or-never investment opportunity, the monopolist knows the price of the output and must determine the corresponding size of the project by maximising, with respect to Q_m, the discounted to time zero expected project value given by

$$V_m(X,Q_m,S_m) = \mathbb{E}_X\left[\int_0^\infty (1-\tau)(1-\eta Q_m)Q_m X_t e^{-rt}dt - (\delta Q_m - S_m)\right]. \tag{4}$$

The optimal capacity satisfying

$$\Phi_m(X,S_m) = \max_{Q_m} V_m(X,Q_m,S_m)$$

is obtained by applying the first-order necessary condition (FONC) to (4) and is given by

$$Q_m^* = \frac{1}{2\eta}\left(1 - \frac{\delta(r-\mu)}{(1-\tau)X}\right). \tag{5}$$

Next, we assume that the demand is too low to justify immediate investment. Subject to the optimal capacity choice at investment, i.e., the inner maximisation in (3), the monopolist's objective upon deferred investment is to determine the random first-passage time of X_t through the investment threshold from below, i.e., $T_m = \inf\{t \geq 0 : X_t \geq X_m\}$. The monopolist's optimisation objective is

$$F_m(X,S_m) = \sup_{T_m\in\mathcal{S}} \mathbb{E}_X\left[\int_{T_m}^\infty (1-\tau)(1-\eta Q_m^*)Q_m^* X_t e^{-rt}dt - (\delta Q_m^* - S_m)e^{-rT_m}\right], \tag{6}$$

where $\mathcal{S}$ is the set of stopping times of the filtration generated by X_t. We can then rewrite (6) using the law of iterated expectations and the strong Markov property of the geometric Brownian motion[4]:

$$F_m(X,S_m) = \sup_{T_m\in\mathcal{S}} \mathbb{E}_X\left[e^{-rT_m}\right]\Phi_m(X_m,S_m) = \max_{X_m>X}\left(\frac{X}{X_m}\right)^\beta \Phi_m(X_m,S_m), \tag{7}$$

where the second equality follows from the stochastic discount factor $\mathbb{E}_X\left[e^{-rT_m}\right] = (X/X_m)^\beta$ (Dixit & Pindyck, 1994, p. 315), with $\beta > 1$ the positive root of $\sigma^2 x(x-1)/2 + \mu x - r = 0$.

By applying the FONC to the unconstrained optimisation problem (7) and integrating condition (5) for optimal capacity choice at investment, where we set $X = X_m^*$, we obtain the optimal investment policy. All proofs can be found in the online supplementary Appendix A.

[4] For each $s \geq 0$, X_{T_m+s} is independent of the past given X_{T_m}.

Proposition 1. *The following results hold:*

1. *The optimal investment threshold under monopoly is given by*

$$X_m^*(S_m) = \begin{cases} \max\{X, c(S_m)\}, & \text{if } S_m \leq \frac{\left(\beta-\sqrt{\beta^2-1}\right)\delta}{2\beta\eta} \\ X, & \text{if otherwise} \end{cases} \tag{8}$$

and the optimal capacity by

$$Q_m^*(S_m) = \begin{cases} \frac{\delta+2\beta\eta S_m+\sqrt{\delta^2-4\beta^2\eta S_m(\delta-\eta S_m)}}{2(\beta+1)\eta\delta}, & \text{if } X < X_m^*(S_m) \\ \frac{1}{2\eta}\left(1-\frac{\delta(r-\mu)}{(1-\tau)X}\right), & \text{if otherwise}, \end{cases} \tag{9}$$

where

$$c(S_m) = \frac{r-\mu}{1-\tau}\,\frac{(\beta+1)\delta^2}{\beta\delta - 2\beta\eta S_m - \sqrt{\delta^2 - 4\beta^2\eta S_m(\delta-\eta S_m)}}.$$

2. *Both the optimal investment threshold and optimal capacity decrease with increasing subsidy.*

As suggested in the bottom part of (8), if the subsidy is high enough, it outweighs the value of waiting and the monopolist is better off investing immediately and installing the capacity indicated in the bottom part of (9). Otherwise, the optimal investment policy is given in the top parts of (8) and (9).

Next, we analyse the Government's decision and derive the optimal (equilibrium) subsidy. Upon the monopolist's investment at T_m, the Government receives a perceptual stream of tax income from the operating project. Hence, the Government's discounted net income at time 0 is given by

$$\begin{aligned} G_m^\pi(X,S_m) &= \mathbb{E}_X\left[\int_{T_m^*}^\infty \tau(1-\eta Q_m^*(S_m))Q_m^*(S_m)X_t e^{-rt}dt - S_m e^{-rT_m^*}\right] \\ &= \left(\frac{X}{X_m^*(S_m)}\right)^\beta \left[\tau(1-\eta Q_m^*(S_m))Q_m^*(S_m)\frac{X_m^*(S_m)}{r-\mu} - S_m\right]. \end{aligned} \tag{10}$$

We assume that the Government chooses the level of subsidy so as to maximise its net income, i.e., $\widetilde{S}_m^\pi = \operatorname{argmax}_{S_m\geq 0} G_m^\pi(X,S_m)$. The equilibrium subsidisation strategy for the Government is given in the following proposition.

Proposition 2. *The equilibrium subsidy of a profit-maximising Government under monopoly is*

$$\widetilde{S}_m^\pi = \begin{cases} 0, & \text{if } \frac{r-\mu}{1-\tau}\frac{\beta+1}{\beta-1}\delta \leq X \\ \min\{S_1,S_2\}, & \text{if } \frac{r-\mu}{1-\tau}\frac{\beta+1}{\sqrt{\beta^2-1}}\delta \leq X < \frac{r-\mu}{1-\tau}\frac{\beta+1}{\beta-1}\delta \\ S_1, & \text{if otherwise} \end{cases} \tag{11}$$

with

$$S_2 = \frac{1}{4\beta\eta}\,\frac{\delta^2 - A^2}{\beta\delta - A} \tag{12}$$

and

$$S_1 = \begin{cases} \frac{\delta\theta}{2\beta\eta\psi}, & \text{if } \tau > \frac{1}{\beta+1} \\ 0, & \text{if otherwise}, \end{cases} \tag{13}$$

where

$$A = \beta\delta - \frac{r-\mu}{1-\tau}\frac{(\beta+1)\delta^2}{X}, \quad \psi = \left(\frac{\tau}{1-\tau} - \frac{1}{\beta}\right)^{-2} - 1 \quad \text{and} \quad \theta = \sqrt{\beta^2+\psi} - \beta.$$

As indicated in the top part of (11), the Government will not grant a subsidy if the output price is high enough to allow the firm to invest immediately. Conversely, if the output price is too low to justify immediate investment (bottom part), the Government will grant a subsidy given in the top part of (13) if the corporate tax rate is greater than the critical value $1/(\beta+1)$. Finally, according to the middle part of (11), the firm will postpone investment (invest immediately) in the absence (presence) of a sufficiently high subsidy, where S_2 is the minimum subsidy required by the firm to undertake immediate investment. Notice that S_1 is independent of X and that the equilibrium subsidy depends on X only when $\widetilde{S}_m^\pi = S_2$, where S_2 decreases with increasing X. Having derived the equilibrium subsidy, we can now introduce it into (8) and (9) to obtain the equilibrium investment threshold $\widetilde{X}_m^\pi = X_m^*\left(\widetilde{S}_m^\pi\right)$ and equilibrium capacity $\widetilde{Q}_m^\pi = Q_m^*\left(\widetilde{S}_m^\pi\right)$.

Corollary 1. *The equilibrium investment threshold under monopoly is given by*

$$\widetilde{X}_m^\pi = \begin{cases} \frac{r-\mu}{1-\tau}\frac{(\beta+1)\delta\psi}{\beta\psi - \theta\left(\sqrt{\psi+1}+1\right)}, & \text{if } \tau > \frac{1}{\beta+1} \\ \frac{r-\mu}{1-\tau}\frac{\beta+1}{\beta-1}\delta, & \text{if otherwise}, \end{cases}$$

and the equilibrium capacity is given by

$$\widetilde{Q}_m^\pi = \begin{cases} \frac{\psi+\theta\left(\sqrt{\psi+1}+1\right)}{2(\beta+1)\eta\psi}, & \text{if } \tau > \frac{1}{\beta+1} \text{ and } X < \widetilde{X}_m^\pi \\ \frac{1}{(\beta+1)\eta}, & \text{if } \tau \le \frac{1}{\beta+1} \text{ and } X < \widetilde{X}_m^\pi \\ \frac{1}{2\eta}\left(1 - \frac{\delta(r-\mu)}{(1-\tau)X}\right), & \text{if } X \ge \widetilde{X}_m^\pi. \end{cases}$$

Consistent with extant contributions, we find that a larger subsidy accelerates investment, thereby resulting in installing less capacity (see Proposition 1). We also confirm that a higher tax rate reduces the firm's incentive to invest and must be offset by a larger subsidy in order to stimulate investment, as shown in Proposition 3. Note that this subsidy can never be infinite and is bounded from above by $(\beta - \sqrt{\beta^2-1})\delta/(2\beta\eta)$. This is because as τ approaches 1, the firm's incentive to invest is extremely low and a large amount of subsidy is required to reduce the investment threshold by a small amount. However, from the Government's perspective, it is not worth it because the extra tax income is not sufficient to cover the cost of providing a greater subsidy. Consistent with conventional real options literature, the results suggest that uncertainty postpones investment and increases the amount of installed capacity, however, as we will show in Section 4, its effect on equilibrium subsidy is ambiguous and the Government is not willing to provide a subsidy when uncertainty is high.

Proposition 3. *The equilibrium subsidy increases with the corporate tax rate while the monopolist's equilibrium capacity decreases.*

Furthermore, by allowing for an inverse demand function, we observe that the optimal (equilibrium) investment threshold depends on (is independent of) the price elasticity of demand, η, as shown in Proposition 4. Intuitively, the investment scale shrinks as η increases, so the Government has a greater incentive to grant smaller subsidy. In turn, the reduction in subsidy postpones investment, thereby offsetting the effect of increased η.

Proposition 4. *An increase in the elasticity of demand decreases both the subsidy and firm's equilibrium capacity but has no effect on the investment threshold.*

3.3. Non-pre-emptive duopoly

In the light of Goto et al. (2008), Mason and Weeds (2010), Paxson and Pinto (2005), Siddiqui and Takashima (2012), we begin with non-pre-emptive duopoly, where the leader's role is assigned exogenously. As shown in Fig. 2, the leader enjoys monopoly profits from time T_{nl} until the follower's entry at time T_{nf}. Upon that, the total market output increases from Q_{nl} to $Q_{nl} + Q_{nf}$, whereas the market price per unit output drops from $\left(1-\eta Q_{nl}\right)X_t$ to $\left(1-\eta Q_{nl}-\eta Q_{nf}\right)X_t$. This trade-off directly affects the tax income of the Government along with the subsidy level, and, in what follows, we conduct a step-by-step analysis in the order of follower, leader and Government to obtain their optimal strategy.

3.3.1. Follower

We assume that the leader has already entered the market, and begin by analysing the follower's capacity investment policy. Given the leader's optimal capacity, the follower's value of the active project is

$$V_{nf}\left(X, Q_{nf}; Q_{nl}\right) = \mathbb{E}_X\left[\int_0^\infty (1-\tau)\left(1-\eta Q_{nl}-\eta Q_{nf}\right)Q_{nf}X_t e^{-rt}dt - \delta Q_{nf}\right]. \tag{14}$$

In solving $\Phi_{nf}\left(X;Q_{nl}\right) = \max_{Q_{nf}} V_{nf}\left(X, Q_{nf}; Q_{nl}\right)$, we get, by applying the FONC, the follower's condition for optimal capacity at investment

$$Q_{nf}^*\left(X;Q_{nl}\right) = \frac{1}{2\eta}\left(1-\eta Q_{nl} - \frac{\delta(r-\mu)}{(1-\tau)X}\right). \tag{15}$$

Next, we assume that immediate investment is not possible and, similarly to the case of monopoly, we derive the follower's expected option value

$$\begin{aligned} F_{nf}\left(X;Q_{nl}\right) &= \sup_{T_{nf}\in S}\mathbb{E}_X\left[\int_{T_{nf}}^\infty (1-\tau)\left(1-\eta Q_{nl}-\eta Q_{nf}^*\right)Q_{nf}^*X_t e^{-rt}dt \right. \\ &\quad \left. - \delta Q_{nf}^* e^{-rT_{nf}}\right] \\ &= \max_{X_{nf}>X}\left(\frac{X}{X_{nf}}\right)^\beta \Phi_{nf}\left(X_{nf};Q_{nf}\right). \end{aligned} \tag{16}$$

The FONC applied to (16) together with the condition for optimal capacity choice at investment yield the expression for the optimal investment threshold, $X_{nf}^*\left(Q_{nl}\right)$, and the optimal capacity, $Q_{nf}^*\left(Q_{nl}\right)$, as shown in Proposition 5. Note that, unlike the case of monopoly, the optimal investment policy of the follower depends on that of the leader.

Proposition 5. *The optimal investment threshold of the follower is given by*

$$X_{nf}^*\left(Q_{nl}\right) = \frac{\beta+1}{\beta-1}\frac{r-\mu}{1-\tau}\frac{\delta}{1-\eta Q_{nl}} \tag{17}$$

and the optimal capacity by

$$Q_{nf}^*\left(Q_{nl}\right) = \frac{1-\eta Q_{nl}}{(\beta+1)\eta}. \tag{18}$$

Proposition 5 indicates that the follower will invest later and in a smaller capacity as the leader's installed capacity increases. This also agrees with empirical evidence that incumbents tend to discourage potential competitors from entering the market by investing in more capacity (Crozet & Chassagne, 2013; Snider, 2009; Williams, 2013).

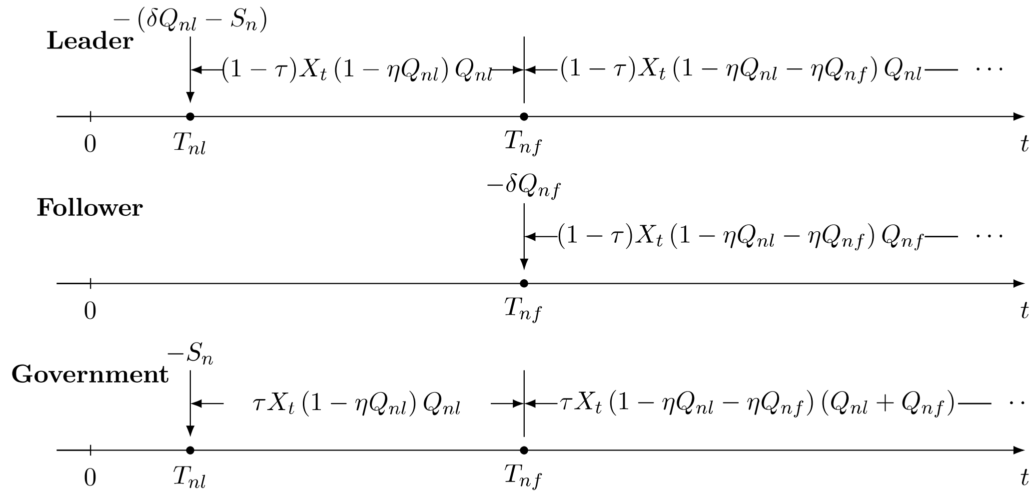

Fig. 2. Irreversible capacity investment and subsidy design under duopolistic competition.

3.3.2. Leader

As shown in Fig. 2, the leader enjoys monopoly profits from T_{nl} until T_{nf}. The active project value of the non-pre-emptive leader is given by

$$\begin{aligned} V_{nl}\left(X, Q_{nl}, S_n\right) &= \mathbb{E}_X\left[\int_0^{T^*_{nf}} (1-\tau)\left(1-\eta Q_{nl}\right) Q_{nl} X_t e^{-rt} dt - \left(\delta Q_{nl} - S_n\right)\right] \\ &\quad + \mathbb{E}_X\left[\int_{T^*_{nf}}^{\infty} (1-\tau)\left(1-\eta Q_{nl} - \eta Q^*_{nf}\right) Q_{nl} X_t e^{-rt} dt\right] \\ &= \frac{1-\tau}{r-\mu}\left(1-\eta Q_{nl}\right) Q_{nl} X - \frac{1-\tau}{r-\mu}\eta Q^*_{nf} Q_{nl} X^*_{nf}\left(\frac{X}{X^*_{nf}}\right)^{\beta} \\ &\quad - \left(\delta Q_{nl} - S_n\right), \end{aligned} \tag{19}$$

where the first term reflects the monopoly profits of the leader in the absence of the follower, the second term is the expected loss in value due to the follower's entry, and the third term is the investment cost reduced by the subsidy. We maximise the leader's active project value, that is, $\Phi_{nl}\left(X, S_n\right) = \max_{Q_{nl}} V_{nl}\left(X, Q_{nl}, S_n\right)$, and the option value of the non-pre-emptive leader is given by

$$F_{nl}(X, S_n) = \max_{X_{nl} > X}\left(\frac{X}{X_{nl}}\right)^{\beta} \Phi_{nl}\left(X_{nl}, S_n\right). \tag{20}$$

Solving (20) gives the optimal investment threshold, $X^*_{nl}(S_n)$, and optimal capacity, $Q^*_{nl}(S_n)$, of the leader, as we show in Proposition 6.

Proposition 6. *For $X < X^*_{nl}$, the optimal capacity of the leader under non-pre-emptive duopoly is obtained as the solution to*

$$\delta\left(1-\left(\frac{\beta}{\beta+1}\frac{\delta Q_{nl} - S_n}{\delta Q_{nl}}\right)^{\beta}\right)\left(1-(\beta+1)\eta Q_{nl}\right)Q_{nl} - \left(1-2\eta Q_{nl}\right)\beta S_n = 0, \tag{21}$$

and the optimal investment threshold is given by

$$X^*_{nl}(S_n) = \max\left\{X, \frac{\beta}{\beta-1}\frac{r-\mu}{1-\tau}\frac{\delta Q^*_{nl} - S_n}{\left(1-\eta Q^*_{nl}\right)Q^*_{nl}}\right\}. \tag{22}$$

For $X^*_{nf} > X \geq X^*_{nl}$, the optimal investment capacity of the leader is the solution to

$$\frac{1-\tau}{r-\mu}\left(1-2\eta Q_{nl}\right)X - \delta - \frac{\delta}{\beta-1}\left(\frac{\beta-1}{\beta+1}\frac{1-\tau}{r-\mu}\frac{1-\eta Q_{nl}}{\delta}X\right)^{\beta}\left(\frac{1-(\beta+1)\eta Q_{nl}}{1-\eta Q_{nl}}\right) = 0$$

and, for $X \geq X^*_{nf} \geq X^*_{nl}$,

$$Q^*_{nl}(S_n) = \frac{1}{2\eta}\left(1-\frac{r-\mu}{1-\tau}\frac{\delta}{X}\right). \tag{23}$$

Note that by setting $S_n = 0$ and $\tau = 0$, we retrieve the optimal investment policy presented in Huisman and Kort (2015), where the non-pre-emptive leader's investment decision aligns with that of the monopolist (see Proposition 1). More specifically, the optimal investment decisions, as indicated in (21)–(22), correspond to what Huisman and Kort (2015) refer to as the entry *deterrence* strategy. That is, when the initial price, X, is low, the leader can deter the follower from entering the market and enjoy a period of monopoly by investing in a capacity above a certain level.[5] Indeed, from (17) and (22), we show that $X^*_{nl}(S) < X^*_{nf}$ for any $S \geq 0$ if X is low. On the other hand, when X is high, the leader's ability to deter the follower's entry may diminish. Under these circumstances, the leader can implement an entry *accommodation* strategy by opting for a smaller capacity, which will trigger the follower to make its investment immediately after[6] (see the last scenario in Proposition 6). We restrict our analysis to the scenario where the initial price level is sufficiently low, to the extent that it always favours the leader's adoption of an entry deterrence strategy, leading to a situation where neither firm pursues immediate investment.

Next, we analyse the optimal subsidisation policy of the Government. Following from the bottom part of Fig. 2, the Government's value function is formulated as

$$\begin{aligned} G^{\pi}_n(X, S_n) &= \mathbb{E}_X\left[\int_{T^*_{nl}}^{T^*_{nf}} \tau\left(1-\eta Q^*_{nl}(S_n)\right) Q^*_{nl}(S_n) X_t e^{-rt} dt - S_n e^{-rT^*_{nl}}\right] \\ &\quad + \mathbb{E}_X\left[\int_{T^*_{nf}}^{\infty} \tau\left(1-\eta Q^*_{nl}(S_n) - \eta Q^*_{nf}(S_n)\right)\right. \\ &\quad \left.\left(Q^*_{nl}(S_n) + Q^*_{nf}(S_n)\right) X_t e^{-rt} dt\right] \\ &= \left(\frac{X}{X^*_{nl}(S_n)}\right)^{\beta}\left(\tau\left(1-\eta Q^*_{nl}(S_n)\right) Q^*_{nl}(S_n)\frac{X^*_{nl}(S_n)}{r-\mu} - S_n\right) \\ &\quad + \left(\frac{X}{X^*_{nf}(S_n)}\right)^{\beta} \tau\left(1-2\eta Q^*_{nl}(S_n) - \eta Q^*_{nf}(S_n)\right) Q^*_{nf}(S_n)\frac{X^*_{nf}(S_n)}{r-\mu} \end{aligned} \tag{24}$$

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[5] As Proposition 3 in Huisman and Kort (2015) highlights, there exists a specific range for the investment threshold wherein the leader will contemplate the deterrence strategy. In our context, this deterrence interval is given by (X_1^{det}, X_2^{det}), where X_1^{det} solves equation $(1-\tau)X_1^{det}/(r-\mu) - \delta - [(1-\tau)(\beta-1)X_1^{det}/((r-\mu)(\beta+1)\delta)]^{\beta}\delta/(\beta-1) = 0$ and $X_2^{det} = 2(\beta-1)(r-\mu)\delta/((\beta+1)(1-\tau))$.

[6] In particular, the leader will only employ the entry accommodation strategy if the optimal capacity, as indicated in (23), results in immediate investment of the follower, which is triggered when the initial price surpasses the threshold $X^{acc} = (\beta+3)(r-\mu)\delta/((\beta-1)(1-\tau))$. Interested readers may refer to Huisman and Kort (2015, Proposition 4).

where the first term is the discounted present value of the Government's net tax income with only one firm in the market, reduced by the subsidy; the second term reflects the trade-off, whereby the total market output (instantaneous revenue) increases (decreases) upon the follower's entry. Again, the Government will set the subsidy level so as to maximise its profit, i.e., $\widetilde{S}^{\pi}_{n} = \text{argmax}_{S_n \geq 0} G^{\pi}_{n}(X, S_n)$, which in this case is solved numerically. By inserting $\widetilde{S}^{\pi}_{n}$ in (17), (18), (21) and (22), we obtain the equilibrium capacity, $\widetilde{Q}^{\pi}_{nj}$, and investment threshold, $\widetilde{X}^{\pi}_{nj}$, for both firms.

3.4. Pre-emptive duopoly

Here, we consider a non-co-operative game in which both leader and follower roles are not assigned exogenously, therefore both firms have the incentive to pre-empt each other to receive financial support. Note that, since the follower enters the market after the leader has invested, the follower's optimal investment policy is the same as in Section 3.3.1. However, in contrast to Section 3.3.2, the optimal investment threshold of the pre-emptive leader must be determined by considering the strategic interactions with the follower. Therefore, we begin with the investment decision of the leader. For a given X_{pl}, the pre-emptive leader's capacity should maximise its active project value:

$$\Phi_{pl}\left(X_{pl}, S_p\right) = \max_{Q_{pl}\geq 0} \left\{ \frac{1-\tau}{r-\mu}\left(1-\eta Q_{pl}\right) Q_{pl} X_{pl} - \left(\delta Q_{pl} - S_p\right) - \frac{1-\tau}{r-\mu}\eta Q^{*}_{pf} Q_{pl} X^{*}_{pf} \left(\frac{X_{pl}}{X^{*}_{pf}}\right)^{\beta} \right\}. \quad (25)$$

The optimal capacity of the pre-emptive leader solves

$$\frac{1-\tau}{r-\mu}\left(1-2\eta Q_{pl}\right) X_{pl} - \delta - \frac{\delta}{\beta-1}\left(\frac{\beta-1}{\beta+1}\frac{1-\tau}{r-\mu}\frac{1-\eta Q_{pl}}{\delta} X_{pl}\right)^{\beta} \left(\frac{1-(\beta+1)\eta Q_{pl}}{1-\eta Q_{pl}}\right) = 0. \quad (26)$$

Next, to determine the pre-emptive leader's optimal investment threshold, we consider the strategic interactions between the leader and the follower. As in Huisman and Kort (2015), the pre-emption trigger is defined as the intersection point of the option value of the follower, $F_{pf}\left(X_{pl}; Q_{pl}(S_p)\right)$, and the active project value of the leader, $\Phi_{pl}\left(X_{pl}, S_p\right)$, as formulated in (16) and (25), respectively. Therefore, given a subsidy level S_p, the pre-emption trigger $\widehat{X}$ is the solution to equation

$$F_{pf}\left(\widehat{X}; Q^{*}_{pl}\left(\widehat{X}, S_p\right)\right) = \Phi_{pl}\left(\widehat{X}, S_p\right). \quad (27)$$

Intuitively, as illustrated in Fig. 3, for $X_{pl} < \widehat{X}$ ($X_{pl} > \widehat{X}$) the demand is low (high), and the expected option value of the follower is greater (smaller) than the active project value of the leader so that each firm is better off being the follower (leader). For a given investment threshold X_{pl}, each firm can pre-empt the other by investing at a lower threshold, i.e., $X_{pl} - \epsilon$ for $\epsilon > 0$. This continues until a firm is indifferent between being the leader or the follower, which happens at the intersection of the follower's option value and the leader's project value. Note also that $\Phi_{pl}\left(X_{pl}, S_p\right)$ is a linear function of S_p and that an increase in the subsidy shifts the leader's project value curve upwards (solid arrow).[7] This leads to a leftward movement of the intersection of the two curves along the follower's option value curve (grey), which lowers the investment trigger.

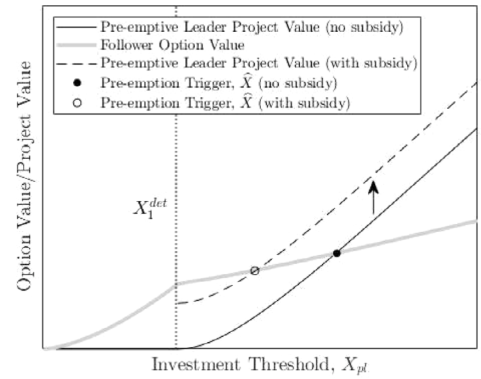

Fig. 3. Active project value of leader with $S_p = 0$ (solid line) and $S_p = 0.12$ (dashed line), and option value of follower (grey).

3.5. Social welfare

From a social planner's perspective, the goal of the Government is to maximise the social welfare or total surplus, i.e., the sum of producer surplus (PS), consumer surplus (CS) and Government's revenues. Given that the latter are expenses for the firm, a social planner maximises the sum of producer surplus and consumer surplus without taxes and subsidies (Azevedo et al., 2021). Hence, the discounted consumer surplus is given by

$$CS_m(X, S_m) = \frac{1}{2(r-\mu)}\eta X^{*}_{m}(S_m) Q^{*\,2}_{m}(S_m)\left(\frac{X}{X^{*}_{m}(S_m)}\right)^{\beta}, \quad (28)$$

and the expected present value of the producer surplus is

$$PS_m(X, S_m) = \left(\frac{1}{r-\mu} X^{*}_{m}(S_m)(1-\eta Q^{*}_{m}(S_m)) - \delta\right) Q^{*}_{m}(S_m)\left(\frac{X}{X^{*}_{m}(S_m)}\right)^{\beta}, \quad (29)$$

where $X^{*}_{m}(S_m)$ and $Q^{*}_{m}(S_m)$ are given in (8) and (9), respectively. Thus, the total social welfare is $G^{w}_{m}(X, S_m) = PS_m(X, S_m) + CS_m(X, S_m)$, and, using (28) and (29), we obtain

$$G^{w}_{m}(X, S_m) = \left(\frac{X}{X^{*}_{m}(S_m)}\right)^{\beta}\left[\left(2-\eta Q^{*}_{m}(S_m)\right)\frac{X^{*}_{m}(S_m)}{2(r-\mu)} - \delta\right] Q^{*}_{m}(S_m). \quad (30)$$

Note that given a fixed subsidy S_m, the firms' capacity investment policy is the same as in the previous sections. However, the conflict of interest between the firm and the Government is weakening. This is because the firm's value, reflected in the producer surplus, is now part of the Government's optimisation objective and the cost of providing the subsidy is no longer a concern for the Government. We derive the equilibrium subsidisation strategy for the social planner in the following proposition, where $\widetilde{S}^{w}_{m} = \text{argmax}_{S_m \geq 0} G^{w}_{m}(X, S_m)$.

Proposition 7. *The equilibrium subsidy of a social planner under monopoly is*

$$\widetilde{S}^{w}_{m} = \begin{cases} 0, & \text{if } \frac{r-\mu}{1-\tau}\frac{\beta+1}{\beta-1}\delta \leq X \\ \min\{S^{w}_{1}, S_2\}, & \text{if } \frac{r-\mu}{1-\tau}\frac{\beta+1}{\sqrt{\beta^2-1}}\delta \leq X < \frac{r-\mu}{1-\tau}\frac{\beta+1}{\beta-1}\delta \\ S^{w}_{1}, & \text{if otherwise} \end{cases} \quad (31)$$

[7] Noting that when $X_{pl} \leq X_1^{det}$, the leader chooses not to invest in any capacity, resulting in zero project value. In this scenario, the Government will not grant a subsidy to the leader, and, therefore, only the segment of the leader's project value curve that corresponds to $X_{pl} > X_1^{det}$ undergoes an upward shift.

with S_2 given in (12) and

$$S_1^w = \frac{\tau\delta\left(\sqrt{4\tau^2\beta^2 + 3(3-4\tau)} - 2\tau\beta\right)}{3\beta\eta(3-4\tau)}. \tag{32}$$

Unlike Section 3.2, here we find that the Government will offer a subsidy even when the tax rate is below $1/(\beta+1)$. This can be attributed to the Government no longer accounting for the tax income covering the cost of the subsidy. Next, we obtain the equilibrium investment threshold and capacity as follow.

Corollary 2. *The equilibrium investment threshold under monopoly is given by*

$$\widetilde{X}_m^w = \frac{r-\mu}{1-\tau}\frac{(\beta+1)\delta(3-4\tau)}{\beta(3-2\tau) - \sqrt{4\tau^2\beta^2 + 3(3-4\tau)}} \tag{33}$$

and the equilibrium capacity by

$$\widetilde{Q}_m^w = \begin{cases} \frac{(3-4\tau)-2\tau\beta+\sqrt{4\tau^2\beta^2+3(3-4\tau)}}{2(\beta+1)\eta(3-4\tau)}, & \text{if } X < \widetilde{X}_m^w \\ \frac{1}{2\eta}\left(1 - \frac{\delta(r-\mu)}{(1-\tau)X}\right), & \text{if } X \geq \widetilde{X}_m^w. \end{cases} \tag{34}$$

While Proposition 3 still holds in the case of a welfare-maximising Government, we find that the impact of economic uncertainty on the equilibrium subsidy is no longer the same. Indeed, contrary to Section 3.2, where a profit-maximising Government is better off providing a subsidy only when the uncertainty is low, Proposition 8 indicates that a social planner is willing to provide more subsidy to the firm when uncertainty is high.

Proposition 8. *The equilibrium subsidy, investment threshold and capacity increase with economic uncertainty.*

Under duopoly, the discounted social welfare is given in (35). More specifically, the first term is the total surplus of a monopoly market and the second term represents the increment in social welfare due to the entry of the second investor:

$$\begin{aligned} G_i^w(X, S_i) = &\left(\frac{X}{X_{il}^*(S_i)}\right)^\beta \left[\left(2 - \eta Q_{il}^*(S_i)\right)\frac{X_{il}^*(S_i)}{2(r-\mu)} - \delta\right] Q_{il}^*(S_i) \\ &+ \left(\frac{X}{X_{if}^*(S_i)}\right)^\beta \left[\left(2 - 2\eta Q_{il}^*(S_i) - \eta Q_{if}^*(S_i)\right)\frac{X_{if}^*(S_i)}{2(r-\mu)} - \delta\right] Q_{if}^*(S_i). \end{aligned} \tag{35}$$

Again, the equilibrium subsidy can be solved numerically by integrating the optimal investment strategies of the leader and follower into (35) and maximising it with respect to S_i, i.e., $\widetilde{S}_i^w = \text{argmax}_{S_i \geq 0} G_i^w(X, S_i)$. By inserting $\widetilde{S}_i^w$ back into the optimal investment strategies of the firms, we can obtain the equilibrium capacity, $\widetilde{Q}_{ij}^w$, and investment threshold, $\widetilde{X}_{ij}^w$, for both firms.

4. Numerical examples and analysis

In this section, we illustrate our model and key findings through a set of numerical examples. Specifically, we demonstrate how strategic interactions with the Government may impact a firm's capacity investment decision and how the equilibrium subsidisation policy depends on the market structure, type of duopolistic competition and the Government's optimisation objective. We adopt baseline parameter values from the real options, corporate finance and operational research literature (see Dixit & Pindyck, 1994; Hagspiel et al., 2016a; Huisman & Kort, 2015), in particular, r = 10% per year, μ = 6% per year, σ = 10% per year, τ = 0.4 per year, δ = £0.1 and η = 0.05 per unit output. We set X = £0.005 to ensure that firms do not undertake immediate investment while we analyse the impact of the different parameters on the firms' equilibrium investment threshold. In doing so, we additionally take into account the empirical analysis of renewable subsidy policy of Bigerna et al. (2019). We choose to perturb our base parameter values around their estimates according to $r \in (0.06, 0.15)$, $\mu \in (0.03, 0.08)$ and $\sigma \in (0, 0.5)$. We also consider $\tau \in (0, 0.7)$ and $\eta \in (0.04, 0.1)$; finally, we note that the ratios δ/X in our and their study are comparable. Overall, this way we first ensure that our analysis is not limited to a single set of parameter values; second, we show the robustness and range of our results under stressed parameter values.

Fig. 4 confirms the impact of an exogenous subsidy on the optimal investment threshold and optimal capacity of the monopolist, and extends the results to the duopoly case. Although conventional intuition suggests that the follower will be worse off if not subsidised, we interestingly find that the follower not only enters the market earlier (left side), but also installs more capacity (right side) if the Government provides more subsidy to the leader. This seemingly implausible result occurs because the follower's investment threshold (capacity) is positively (negatively) correlated with the leader's capacity (see also expressions (17)–(18)), despite the fact that the follower's investment strategy is not directly affected by the subsidy. Indeed, the total market output is bounded due to the inverse demand function, which means that a greater capacity of the leader will squeeze the follower's market share. However, by receiving a subsidy, the leader will invest earlier in a smaller project, stimulating the follower's motivation to gain a larger market share. This result is consistent with empirical evidence that a subsidy does not necessarily enhance market power, as demonstrated in Dai and Li (2020). On the other hand, the market output price after the leader's investment is higher if the leader decides to invest less, increasing the follower's incentive to invest earlier. This reveals an indirect effect of the subsidy that is not captured when firm-level strategic interactions are ignored and firms are price takers.

The implications of economic uncertainty for each firm's investment policy under an endogenously defined subsidy when the Government maximises the profit are illustrated in Fig. 5. As indicated on the left side, the equilibrium investment thresholds increase with uncertainty under all market structures. This is attributed to greater uncertainty increasing the opportunity cost of investment and, in turn, the value of waiting, as demonstrated empirically in Bulan (2005), Xie (2009) and Bigerna et al. (2019). However, interestingly, the right plots show that, while the leader's equilibrium capacity strictly increases with uncertainty, this is not true for the follower. This happens because greater uncertainty raises follower's incentive to invest later in more capacity, yet the follower's capacity is negatively correlated with the leader's, as suggested by (18). There are therefore two opposing effects with the former dominating (being dominated by) the latter when σ is small (large). Qualitative similar results hold for a Government that maximises social welfare; we do not report these here in the interest of space.

The left plots (black lines) in Fig. 6 confirm the ambiguous impact of price uncertainty on the equilibrium subsidy when the Government maximises the profit, and this is extended to the duopoly case, whereby an increase in uncertainty can lead to an increase (decrease) of the equilibrium subsidy if σ is small (large). The Government stops eventually subsidising the leader in a highly uncertain environment. This can be attributed to the rapid increase of the investment threshold when uncertainty is high (see left side of Fig. 5), which has a negative impact on the discounting of the Government's payoff. In this case, the effectiveness of the subsidy is reduced, so that the additional tax income cannot cover the cost of subsidisation. Also, although the Government's incentive to offer a subsidy is low under a pre-emptive duopoly, $\widetilde{S}_p^\pi > 0$ is still possible when the tax rate is sufficiently high.

As illustrated on the right side of Fig. 6, a larger subsidy is required to mitigate the leader's loss in project value due to an increase in the tax rate, yet the equilibrium subsidy of the non-pre-emptive leader is initially (eventually) higher (lower) than that of the monopolist. The former is driven by the need for financial support due to competition,

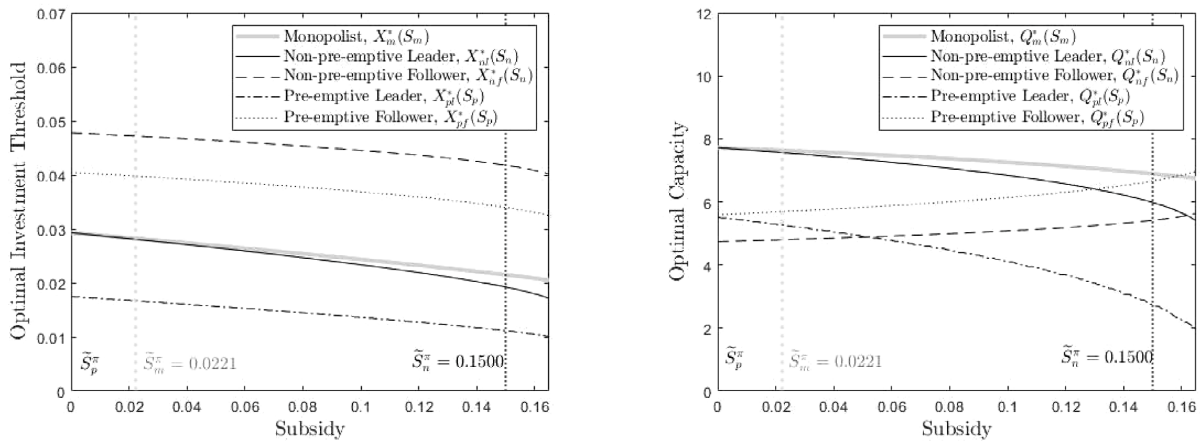

Fig. 4. Impact of exogenous subsidy on firms' optimal investment threshold (left) and capacity (right) under monopoly, non-pre-emptive duopoly and pre-emptive duopoly, with corresponding equilibrium subsidies $S_i^\pi = 0.0221$, 0.1500 and 0 (vertical lines) when the Government maximises the profit.

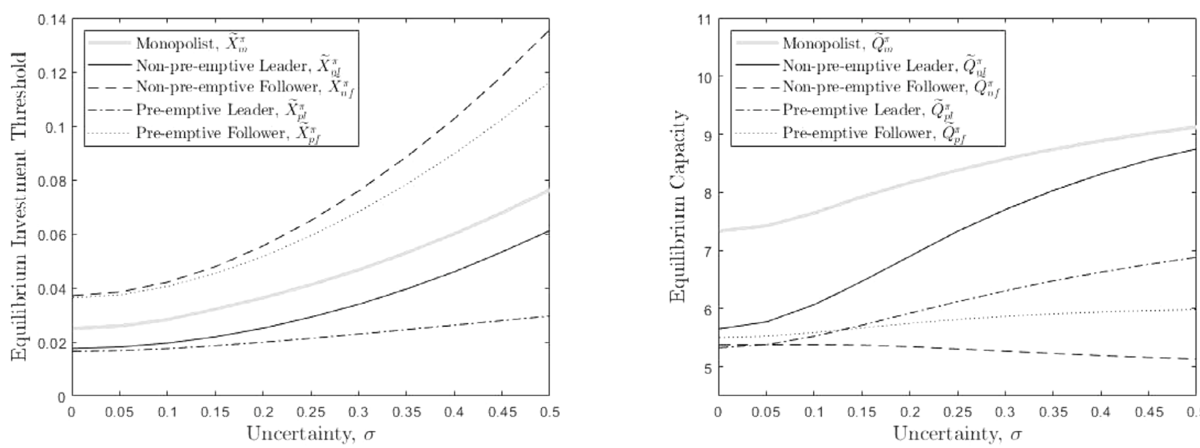

Fig. 5. Impact of uncertainty on the equilibrium investment threshold (left) and equilibrium capacity (right) under a profit-maximising Government.

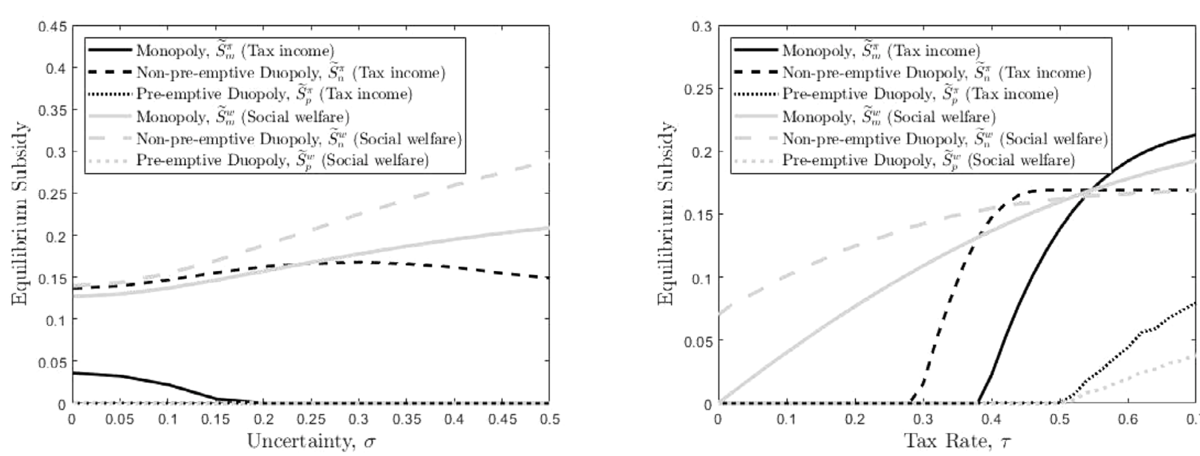

Fig. 6. Impact of price uncertainty on the equilibrium subsidy (left) and impact of tax rate on equilibrium subsidy (right) under a profit-maximising (black lines) and welfare-maximising (grey lines) Government.

i.e., the Government is willing to grant a subsidy to the leader even if the tax rate is relatively low (i.e., $0.29 < \tau < 0.38$), when no subsidy is offered to the monopolist if $\tau < 0.38$. The latter is due to the existence of a 'ceiling', so that a subsidy above this level (e.g., 0.17 for the non-pre-emptive leader) always induces immediate investment. Thus, we get the maximum subsidy of 0.23 under monopoly and 0.17 under non-pre-emptive duopoly, which explains why the equilibrium subsidy of the non-pre-emptive leader no longer increases when $\tau > 0.47$.

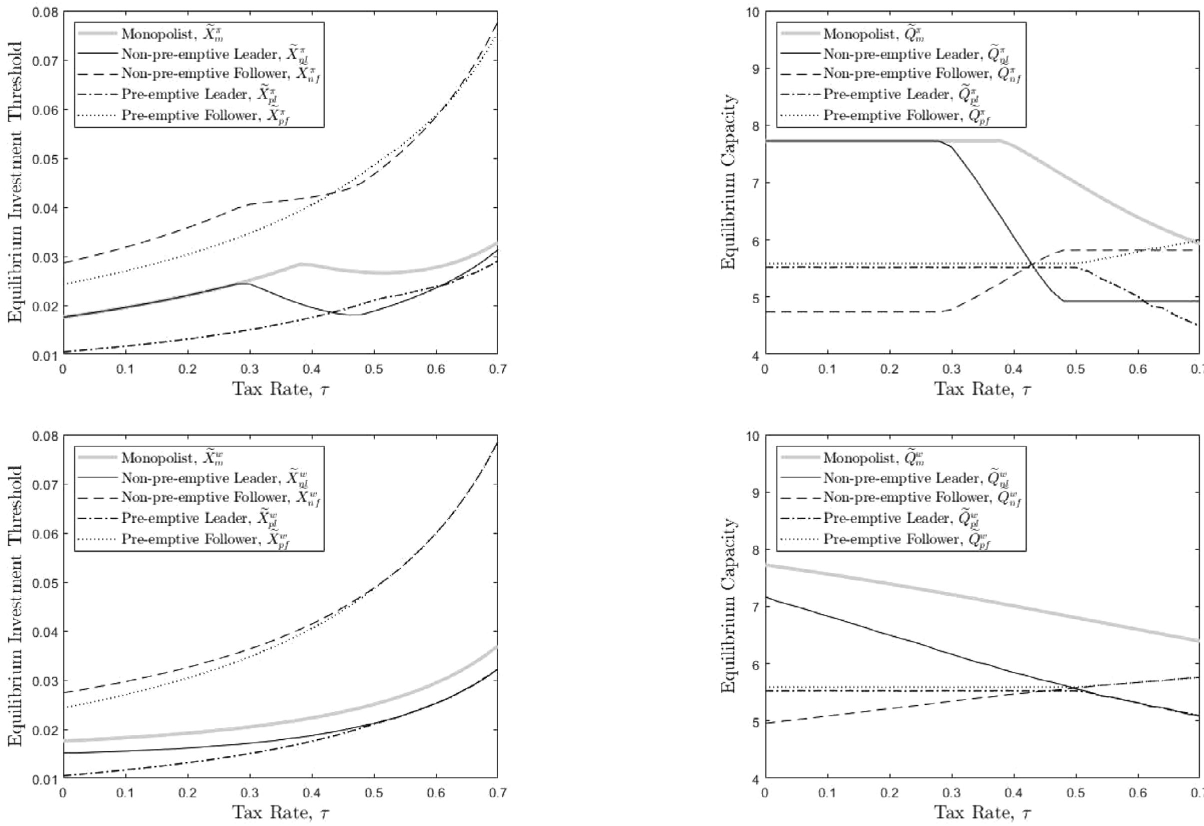

Fig. 7. Impact of tax rate on firms' equilibrium investment threshold (left) and equilibrium capacity (right) when the Government maximises the profit (top) or social welfare (bottom).

Additionally, the critical tax level leading to a positive subsidy is found to be the highest under a pre-emptive duopoly market ($\tau = 0.5$). This is because the pre-emptive leader invests earlier than the monopolist or non-pre-emptive leader, and, therefore, does not need as much fiscal stimulus as the latter.

Contrary to the profit-maximising Government, we find that a social planner (grey lines) is willing to provide a subsidy even when the tax rate is low (see right side) and, as shown in Proposition 8, this subsidy increases with uncertainty (see left side). This happens because the conflict of interest between the firm and the Government weakens when the Government maximises the social welfare, as a large proportion of the Government's value function (i.e., producer surplus) coincides with the firms' value functions (Cui et al., 2019). In addition, the cost of subsidy is no longer a concern for a social planner. Therefore, the social planner is more willing to provide larger subsidies whenever required by the firm and, as τ increases, the equilibrium subsidy also gradually increases. Finally, we observe that the equilibrium subsidy is still the lowest under pre-emptive duopoly, confirming that it is not optimal for the Government to grant too much subsidy, as it may induce more intense competition at an undesirable level.

Fig. 7 illustrates the impact of the tax rate on the firms' capacity investment policy when the Government maximises the profit (top part) and social welfare (bottom part). Interestingly, the top-left plots indicate that the equilibrium investment threshold does not necessarily increase with τ in all cases. This is because the subsidy accelerates the leader's investment and cancels out (all or a part of) the impact of a higher tax rate. Indeed, an increase in τ in the region $0.29 < \tau < 0.48$ leads to an abrupt growth of the equilibrium subsidy, especially when the Government maximises its profit (see black solid and dotted plots on right side of Fig. 6). This can outweigh the effect of larger τ and accelerate investment. However, increasing τ has a slow-down effect on the increase of the equilibrium subsidy, such that the increasing subsidy gradually becomes dominated by the rising tax rate, and the investment is deferred. Also, as shown in Propositions 1 and 6, the optimal (equilibrium) capacity of each firm is not (is) affected by the tax rate. Specifically, as shown by the top-right plots in Fig. 7, the equilibrium capacity of the monopolist, non-pre-emptive and pre-emptive leader decreases with increasing τ when this is greater than 0.38, 0.3 and 0.5, respectively, but it is constant when τ is too low to justify provision of a subsidy. In contrast, the capacity of the non-pre-emptive and pre-emptive follower exhibits a reverse pattern. Again, this can be attributed to the multiplicative demand function, which induces a bounded market output that has to be shared between the two firms. As shown in (18), if the leader decides to invest more (less), there will be a smaller (bigger) market left for the follower. In addition, as the leader's capacity decreases with rising τ (see also Proposition 3), the market price before the entry of the follower, i.e., $P_t = X_t(1 - \eta Q_{il})$ for $i \in \{n, p\}$, will be higher, raising the incentive of the follower to invest in larger capacity. On the other hand, the equilibrium investment threshold is strictly increasing with τ when the Government maximises the social welfare, as shown in the bottom-left plots. This is because, although the Government is willing to grant a subsidy when τ is low, this subsidy grows relatively slowly with τ (see grey lines on the right side of Fig. 6), so that the effect of extra subsidy is dominated by that of rising tax rate. Hence, our findings corroborate the empirical evidence supporting the effectiveness of both tax reductions and subsidies in spurring investments (Becker, 2015; David et al., 2000; Djankov et al., 2010; Klemm & Van Parys, 2012).

Next, Fig. 8 illustrates how the leader's loss in value (relative to the monopolist) due to the presence of the rival is affected by uncertainty.

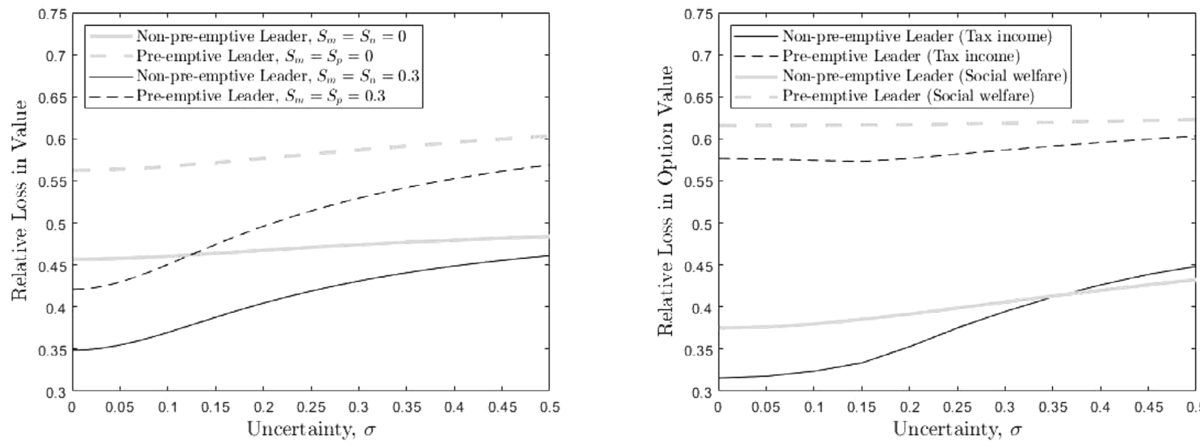

Fig. 8. Left: effect of price uncertainty on relative loss in value of the non-pre-emptive leader (solid lines) and pre-emptive leader (dashed lines) for fixed subsidy level 0 (grey) and 0.1 (black). Right: exogenous subsidy replaced with equilibrium subsidy.

On the left side, we assume an exogenous subsidy level, where $S_i = 0$ (dashed line) or 0.3 (solid line) for $i \in \{m, n, p\}$, and observe that the leader's relative loss in the value increases with uncertainty. Intuitively, this is because greater uncertainty delays the follower's entry but increases its impact. We also observe that the pre-emptive leader incurs a larger loss than the non-pre-emptive leader under a fixed subsidy. This is because the former invests earlier and scarifies substantial revenue due to the threat of pre-emption. Our results show that a subsidy can offset the leader's relative loss, however, as uncertainty increases, the impact of the subsidy becomes less pronounced due to the discounting effect, as the investment threshold of both the monopolist and leader increases rapidly with σ (left part of Fig. 5). Therefore, the relative loss in value of the leader with Government support (black lines) converges to that without Government support (grey lines) as uncertainty grows. The right side presents a similar trend, except that we use the equilibrium subsidy, investment threshold and capacity to obtain the leader's relative loss in value for the case of a profit-maximising and a welfare-maximising Government (black and grey lines, respectively).

Fig. 9 presents the effect of price elasticity of demand, η, on each firm's investment policy. The top-right plots indicate that a higher (lower) η allows for a larger (smaller) installed capacity. However, while the optimal investment thresholds are affected by η, as shown in Propositions 1 and 6, the top-left plots suggest that the equilibrium investment thresholds are actually independent of η. This is due to the fact that the equilibrium subsidy is endogenously chosen and thus varies with η. Intuitively, as the investment scale shrinks with increasing η, the total investment cost drops and so does the equilibrium subsidy (see bottom plots). In turn, a decrease in the subsidy delays investment, thereby offsetting the impact of an increase in η. Some more results related to the effect of varying growth rate μ and interest rate r on the equilibrium investment threshold are moved to the online supplementary Appendix B for space reasons.

Finally, Fig. 10 illustrates how the Government's value function depends on price uncertainty and market structure. We find that both Government's profit and social welfare increase when uncertainty is higher as this motivates investment at a higher price threshold and the installation of a larger project (see Fig. 5). The left side of Fig. 10 indicates that the Government's profit is greater under pre-emptive competition as both firms invest earlier and, thus, the effect of discounting on its profit is not significant. Note that earlier investment does not necessarily lead to a large loss in total market output, since even though the pre-emptive leader invests in less capacity, the investment intensity of the pre-emptive follower is greater than that of the non-pre-emptive follower. Also, the cost of the subsidy is minimum under pre-emptive competition. Interestingly, we observe a huge improvement in Government's value under non-pre-emptive duopoly under social welfare (right side of Fig. 10). This is because the Government is no longer concerned about the cost of the subsidy and, thus, the non-pre-emptive leader will receive more subsidy from a social planner that rapidly grows with σ (left side of Fig. 6). Therefore, competition is desirable for a social planner, while a profit-maximising Government may benefit more under pre-emptive competition.

5. Concluding discussion

Despite their increasing prominence, models for analysing the interaction between firm and Government-level policy-making do not account for critical features of a deregulated environment, such as competition. In this communication, we address this disconnect by developing a bi-level real options framework for deriving the equilibrium Government subsidisation and firm-level capacity investment policy under a pre-emptive and non-pre-emptive duopolistic competition.

Our results show that the insights of traditional bi-level real options models under monopoly cannot be naturally transposed to a deregulated environment. In particular, we find that strategic interactions with the Government can significantly affect a firm's capacity investment decision and that the equilibrium subsidisation policy crucially depends on market structure and the type of duopolistic competition. Contrary to conventional intuition, we find that providing a larger subsidy to the leader can actually increase the follower's incentive to invest earlier and in a bigger project. Furthermore, the results suggest that the loss in the value of the leader, due to the presence of a rival, relative to the monopolist increases with economic uncertainty and, although a subsidy can mitigate this loss, its effect becomes less pronounced. We confirm that a profit-maximising Government is less willing to offer a subsidy when uncertainty is high or the tax rate is low, and extend to demonstrating how the critical tax rate that leads to switching from a subsidy to a non-subsidy regime changes under different market structures and types of duopolistic competition. Furthermore, we demonstrate how results are different when the Government aims to maximise social welfare and show that competition can be desirable for a social planner, while a profit-maximising Government may benefit the most from pre-emptive competition.

Therefore, the policy-making and managerial relevance of our results is reflected in the new insights gained when firm-level strategic interactions are integrated into the evaluation of real options. In particular, not only is competition a key aspect of deregulated industries

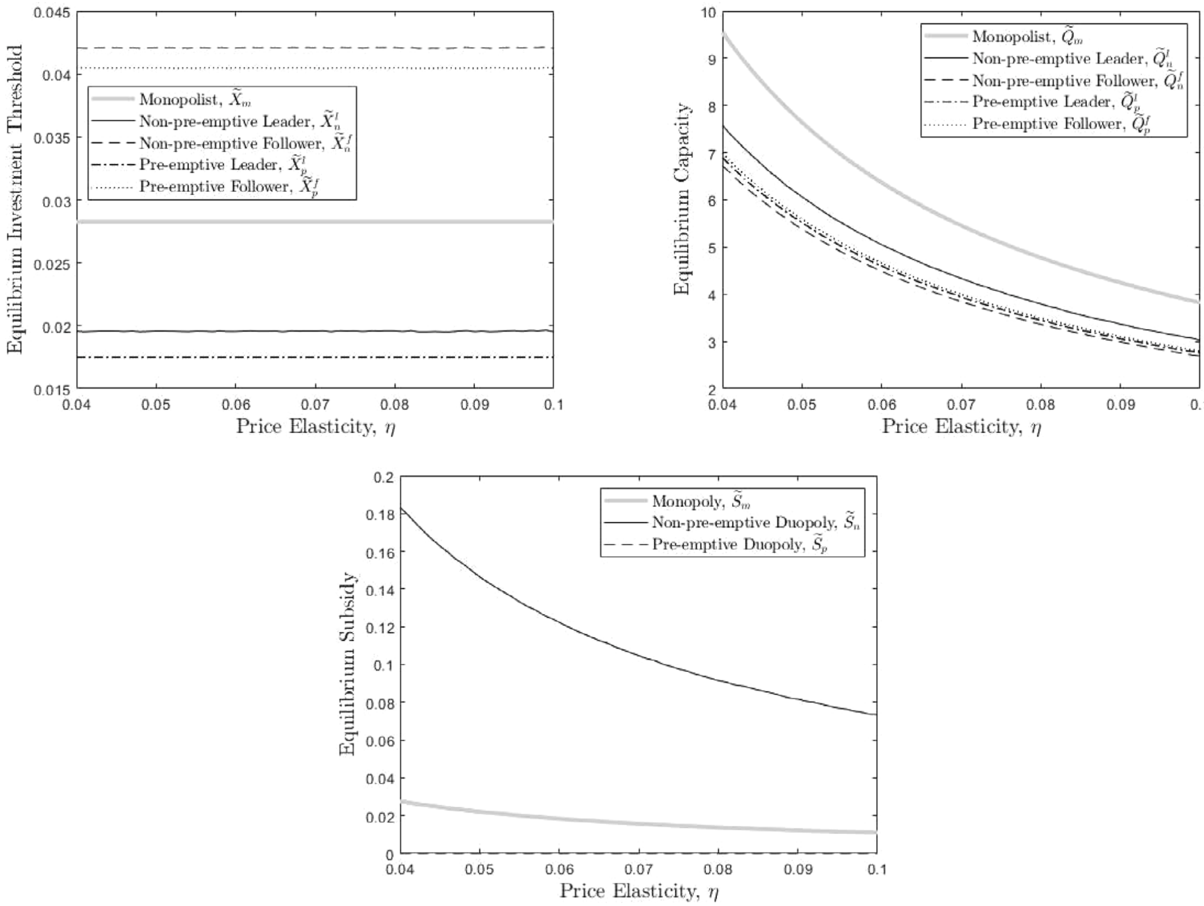

Fig. 9. Effect of price elasticity on firms' equilibrium investment threshold (top-left), equilibrium capacity (top-right) and equilibrium subsidy (bottom).

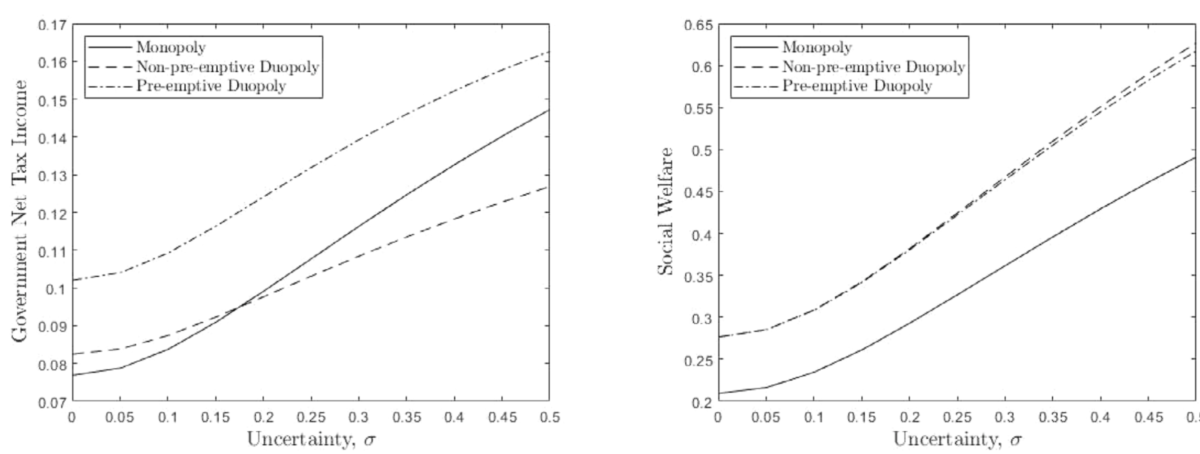

Fig. 10. The impact of price uncertainty on Government's profit (left side) and social welfare (right side) under different market structures.

and entails a loss in value relative to monopoly that must be taken into account when designing subsidisation policies, but also the type of competition can affect significantly a Government's subsidisation policy. Similarly, at the firm level, the interaction with Government policy-making produces dynamics under which the investment policy deviates from that of traditional duopolistic competition, which ignores such interactions. Such strategic interactions tend to be overlooked in the literature that values bi-level real options, yet if their implications are not properly understood, subsidisation policies will not be properly designed, thus potentially inducing under or over-investment cycles and increased regulatory risk following corrective policy actions. Indeed, the history of green investment in Europe since 2000 includes several examples of under or over-incentivised policies needing drastic adjustments. Also, in the context of public–private partnerships (PPPs), the Governments of Mexico and Spain had to pay, respectively, \$2.5 billion and \$8.9 billion to their private partners due to inefficient design (Silaghi & Sarkar, 2021).

We also investigate the robustness of the results by replacing (2) with an iso-elastic demand function, $P_t = X_t Q_t^{-\gamma}$, $\gamma \in (0, 1)$. We confirm that the subsidy can still accelerate the firms' investment and a higher price uncertainty induces later investment and greater capacity for both firms. While the follower can still benefit from the subsidy (by investing earlier), we find that its capacity decreases with higher subsidy amount. This can be attributed to the unbounded market output under an iso-elastic demand function, such that the follower can always choose to invest more if the leader's capacity increases (see also Boonman & Hagspiel, 2014). Thus, in this case, the follower becomes the larger capacity in the market and earns a greater profit. As a result, both firms want to be the follower and have no incentive to invest first under pre-emptive duopoly.

Directions for future work may include relaxation of the assumption of unilateral subsidy. More specifically, it would be interesting to analyse how the positioning and cost asymmetry of firms can affect the equilibrium subsidy, and whether the Government should offer either bilateral subsidies to both firms or a unilateral subsidy to the larger or smaller firm. Also, our model does not consider production flexibility (Hagspiel et al., 2016a) or sequential capacity expansion options, so the project size is fixed at investment and cannot be adjusted afterwards; both options would be meaningful additions to this work. Finally, the assumption of duopolistic competition could be relaxed to explore optimal investment and subsidisation policies under oligopoly when the *accordion effect* occurs (Bouis et al., 2009).

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Appendix. Supplementary data

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.ejor.2024.03.028.

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