Leasehold extensions: The Relativity conundrum and the time value of housing

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Abstract
We address a conundrum in short leasehold apartment valuations. Leaseholders have a statutory right to extend their leases by paying a premium. The unenfranchised Relativity, the leasehold value without extension rights relative to its long leasehold value, is an important component. Unenfranchised leasehold values are unobserved. Our insight is that transacted prices contain an embedded option value which can be used to obtain the unenfranchised Relativity by applying hedonic and option pricing models. Our unenfranchised Relativities appear plausible, match Tribunal decisions, suggest premium payments were not grossly unfair and highlight implications for deriving household's net discount rates from lease lengths.

Keywords: leasehold valuation, Relativity curves, option value

JEL Classification: G10, R20, R30

Introduction

The aim of this paper is to understand the valuation of short leaseholds in England and Wales from market transactions. There are four legal forms of ownership of a dwelling, the freehold, share of freehold, commonhold and leasehold. Most apartments are owned as a leasehold. A leasehold is a contract which grants a legal interest in the dwelling for a pre-defined length of time. The contract may also stipulate that the leaseholder must make regular ground rent payments to the freeholder. As ownership reverts to the freeholder upon expiration, a leasehold is a deteriorating asset.

Leaseholds are tradable in a secondary market and can be transferred to another person or entity before a lease expires. The buyer inherits the existing lease agreement in full, including the duration of the remaining use rights of the contract. Finance theory predicts that the length of a lease influences the dwelling's purchase price, as it reflects a decision involving the comparison of present costs and benefits against future benefits and costs. Its price should trade at a discount to an identical dwelling on a very long lease or freehold. In professional practice, the term Relativity describes the value of a leasehold relative to its value on a very long lease or freehold. The shorter the lease length, the larger the percentage price discount and the lower the Relativity.

The Leasehold Reform, Housing and Urban Development 1993 Act confers leaseholders of apartments the right to buy their freeholds or to extend the lease and extinguish any ground rent payable. The leaseholder has the right to extend the lease by 90 years in return for paying a premium to the freeholder. In this context, the freeholder is referred to as the landlord and the leaseholder as the tenant. The components of the premium are set by the 1993 Act.

For short leases, the value of the extended (or 'married') leasehold interest is greater than the value obtained by separately adding together the values of the existing leasehold and a 90-year leasehold. The Commonhold and Leasehold Reform Act 2002 clarified that this additional benefit should be equally shared between the tenant and landlord and that the marriage value payment to capture it is only applicable for short leases, defined as 80 and less years remaining.

There is however a conundrum. An important component in the marriage value calculation is the unenfranchised Relativity but unenfranchised leasehold values have not been observable in the market since the 1993 Act was passed. Valuers acting in their client's (leaseholder or freeholder) interests apply their own proprietary unenfranchised Relativity curve alongside heuristics in determining the rate to be applied. The Law Commission valuation report (2020) concluded that uncertainty about the unenfranchised Relativity contributes to higher transactions costs, disputes in lease extension and enfranchisement negotiations and claims being brought to Tribunals.

There is the related concern that the unenfranchised Relativity curves applied by agents have been "out of kilter" with the market and resulted in grossly unfair cash transfers between leaseholders and freeholders (Bracke et al., 2018). More generally, there have been attempts to recover the implied discount rates from leasehold sale transactions to analyse how UK households discount long-term cash-flows (Giglio et al., 2015). Many existing studies do not explicitly take into consideration the institutional context which could confound the interpretation of the implied net discount rates estimated. For example, the net discount rates reported in Bracke et al. (2018) imply that enfranchised short leaseholds are less valuable than identical unenfranchised short leaseholds. Lai and Milcheva (2021) treated the implied net

discount rates derived from their enfranchised Relativity curves as the rates relevant to UK households in discounting cash-flows.

Consequently, a better understanding of how the market values short leasehold apartments could shed light on these issues. Our insight is that as the 1993 Act confers the right but not the obligation to extend a lease, it implies that there is an embedded option value in the purchase price of a leasehold. Our objectives are to derive a Relativity curve under the Act World, the embedded option value and a Relativity curve under a No Act World[1]. There are practical applications in our investigation as both Relativity curves are currently required to calculate the marriage value and to extract the implied net discount rates to analyse the time value of housing. Our methodology comprises of a mix of empirical methods. It involves hedonic price modelling to extract the observed enfranchised Relativities from Prime Central London apartment transactions. An option pricing model is applied to derive the embedded option value, the unconditional unenfranchised Relativities and finally the implied net discount rates. As far as we are aware, this is the first time an attempt has been made to derive the unenfranchised Relativities from the enfranchised Relativities using an option pricing model.

We find that apartments on short leases trade at a significant price discount to an identical apartment on a very long lease. Since the sale price of a leasehold apartment contains an embedded option value, the price discounts reflect the enfranchised Relativity. We show how the embedded option value can be extracted to reveal the unenfranchised Relativity curve. Our enfranchised and unenfranchised Relativity curves are monotonically decreasing with lease expiration. The latter fall within the range of Relativity curves applied by different agents, implying that marriage value payments made between 2010-2016 broadly reflected market fundamentals. Furthermore, our unenfranchised Relativity curves appear to match past

Tribunal outcomes reasonably well. Finally, our results demonstrate that the institutional setting should be taken into consideration when attempting to derive and interpret the implied net discount rates from leasehold sale prices which existing empirical studies ignore.

The next section reviews the literature, highlighting the pertinent issues. Section three outlines the methodology comprising of the study context, our aims and objectives, the theoretical model, our data and the estimation strategies employed. The fourth section reports our hedonic model and simulation model results. Conclusions are then drawn.

Literature Review

The first relevant literature strand is motivated by the arguments that property is a durable and inheritable asset, containing information about intergenerational discount rates which can be used in cost-benefit analysis to address environmental issues and assess public infrastructure projects. Using data on freehold and leasehold properties in the UK and Singapore, Giglio et al. (2015) recovered the implied net discount rates over a long-time horizon using hedonic regression models. They used lease 'buckets' (bands) to represent the effect of different lease maturities. The authors reported that leaseholds with maturities of 100 years and 125–150 years are valued 10–15% and 5–8% respectively less than otherwise identical freeholds. The biggest price discounts to freeholds concern leases with 80–99 years remaining, approximately about a 16% discount. They reported that there were no price differences between leaseholds with maturities of more than 700 years and freeholds. Their estimated net discount rates for very long leases are around 1.9%. Giglio et al. (2015) excluded short leaseholds from their analysis. Lai and Milcheva (2021) reported similar net discount rates as Giglio et al. (2015) for long leaseholds in England and Wales. Their sample included short leaseholds, where implied net discount rates for the shorter leases rose from 3% to 12%. But they did not take into

consideration the implications of the 1993 Act. Fesselmeyer et al. (2016) used a different method on a sample of long leases in Singapore and reported similar results as Giglio et al. (2015) for that country. Wong et al. (2008) found support for declining intergenerational discount rates in Hong Kong and concluded that asset values are still sensitive to the length of tenure even when it is very long.

The second relevant literature strand examines price discounts within the context of a legal or institution influence, often over relatively shorter periods. For example, Gautier and van Vuuren (2011) exploited the fact that land lease contracts in Amsterdam are typically issued with a 50-year duration to reveal an exponential discount function with a corresponding discount rate of 8%. The recent UK literature in this strand investigates how leasehold valuations are affected by legislation. Grover (2014) discussed enfranchisement and the issues raised in determining the unenfranchised Relativity curve. He argued that the differences in existing graphs of Relativity are caused by deriving them from small samples containing dwellings with different characteristics. He recommended that attempts should be made to obtain a more definitive Relativity graph using larger samples.

Badarinza and Ramadorai (2015) and Dixon et al. (2000) inferred price discounts from Leasehold Valuation Tribunal (LVT) data. Both studies reported that shorter leaseholds had a higher price discount. They also found that the relative difference in the valuation of an extended lease and its existing term tended to be lower when its term to maturity was long, and that this differential disappeared for unexpired leases greater than 90 years. Bracke et al. (2018) argued that this dataset is inappropriate for obtaining the profile of discount rates due to the complexities of the case law and statute, and the relatively small number of cases available for analysis. They acknowledged that as LVT records reflect the opinions of experts, it could

provide useful information against which evidence obtained from the market could be compared.

Bracke et al. (2018), the Parthenia model in Sloane Stanley Estate v Mundy and Lagesse and Aaron v Welcome Trust Limited' (2016), and Savills (2016) explicitly attempted to derive Relativities for short leases (80 years and under). These studies used Prime Central London (PCL) data, applied hedonic models in estimation and reported significantly larger price discounts as the lease length shortens. The Upper Tribunal rejected the validity of the Relativity curve derived from the Parthenia model, which was later upheld by the Court of Appeal. Bracke et al. (2018) used PCL data from 1987 to 1991 (unenfranchised sample) and from 2004 to 2013 (enfranchised sample) to capture periods before and after the 1993 Act was passed to assess the robustness of their findings. For the unenfranchised sample, they reported net discount rates for leases around 100 years long to be 3% and for short leases to be 5% to 6%. The authors found that the net discount rates estimated from the enfranchised sample are significantly lower by an average of 1.6 percentage points for short leases but were similar for long leases, implying that short leaseholds with rights to extend are worth less than identical ones without rights. Bracke et al. (2018) speculated that this could have been due to the state of the market in different time periods and the change in institutional setting but were unable to distinguish between these explanations. The authors concluded that their unenfranchised Relativity curve obtained from pre-1993 Act market transactions is significantly different from the propriety curves used by professionals in valuing lease extensions and raised a concern that lease extension premiums led to grossly unfair cash transfers between leaseholders and freeholders. Savills (2016) derived enfranchised Relativity curves using PCL data from 2010 to 2015. They then used four reference points from Upper Tribunal data to derive the unenfranchised Relativity from an enfranchised Relativity curve that did not pass through the origin.

Apart from Badarinza and Ramadorai (2015) and Dixon et al. (2000), the empirical studies adopted similar methodologies. Giglio et al. (2015) and Lai and Micheva (2021) used lease buckets to capture the effect of different maturities in their hedonic price model. Wong et al (2008) included a single dummy variable in their hedonic price model for dwellings with 999-year leases to proxy the freehold premium in extracting the intergenerational discount rate. Fesselmeyer et al. (2016) applied non-linear least squares to their hedonic model to obtain estimates of the discount rate from actual lease maturities. They had to impose restrictions to achieve convergence in estimation. Bracke et al. (2018), the Parthenia model and Savills (2016) adopted a two-step estimation approach. They employed dummy variables to capture the unexpired lease effect on dwelling prices in the hedonic models but applied different techniques in the second step to obtain Relativities. The second step is necessary because the lack of sufficient observations at certain lease lengths leads to imprecise and implausible estimates. Savills (2016) imposed theoretical restrictions to justify an exponential function to fit a curve to reveal the Relativities from their lease dummy estimates. Bracke et al. (2018) and the Parthenia model fitted a second-degree local polynomial with an adjusted bandwidth to the lease dummy coefficients to obtain the Relativity curves. Our literature review has revealed that there have not been any studies which have applied an option pricing framework to analyse Relativity or adequately assessed the implications of deriving implied net discount rates from leaseholds with extension rights.

Methodology

Study Context

Under the terms of the 1993 Act, the tenant can exercise the option to extend at any lease length by serving a section 42 notice to the landlord. The landlord can opt to issue a counter-notice.

The tenant then has six months to put in an application to the First-Tier Tribunal to keep the claim alive to continue negotiations. Otherwise, it lapses leaving the tenant to either accept the landlord's offer or not proceed. For non-collective enfranchisements, there is a legal requirement to have owned the lease for a minimum of two years before a notice can be served. The premium payable is based on the relevant considerations at the date the notice is served.

There are three components that determine the premium: (i) the landlord's forgone ground rent; (ii) the reversion value to the landlord; and (iii) the marriage value. The components in the premium (S^T) for extending a lease with 80 years and less remaining are:

$$S^T = \frac{\left(1-(1+\tau)^{-T}\right)}{\tau}\kappa + \left(\frac{1}{(1+\lambda)^T}V^\infty - \frac{1}{(1+\lambda)^{T+90}}V^\infty\right)$$
$$+\frac{1}{2}\left[\left(VE^{T+90} + \frac{1}{(1+\lambda)^{T+90}}V^\infty\right) - \left(V^T + \frac{1}{(1+\lambda)^T}V^\infty + \frac{\left[1-(1+\tau)^{-T}\right]}{\tau}\kappa\right)\right] \qquad (1)$$

where:

κ =per period ground rent

τ =capitalisation rate for ground rent

λ = deferment (discount) rate

VE^{T+90} = the extended enfranchised lease value

V^T = the existing unenfranchised lease value

V^∞ = the infinite lease assumed to be equivalent to a freehold[2]

γ = per period ground rent

The first term in equation (1) represents foregone ground rent, the second term the reversionary value to the landlord (landlord taking possession of the dwelling); and the third term half the marriage value, $\frac{1}{2}[.]$. The latter's inclusion increases the premium. The equation reveals that a

lower value for an unenfranchised leasehold, V^T, in the marriage value increases the premium, benefiting the landlord at the expense of the tenant. The marriage value component of the premium is not applicable for extending leases with more than 80 years left.

The existing unenfranchised and the extended enfranchised lease values can be expressed as a percentage price discount to an infinite lease to reveal how Relativity affects the premium:

$$V^T = RC^T V^\infty \quad (2)$$

$$VE^{T+90} = RCE^{T+90} V^\infty \quad (3)$$

RC^T and RCE^{T+90} are respectively known as the unenfranchised (No Act World) and enfranchised (Act World) Relativity. The conundrum is that the unenfranchised Relativity is not observed in a market where tenants have the right to extend their leases. There is no consensus about the form of either Relativity curves in the industry. In practice the premium paid is a negotiated price between the two parties. Unresolved disputes can be referred to the First-Tier Tribunal, a very costly process which is normally avoided. Attempts to obtain an industry standard have met with little success. Dixon et al. (2000) were not able to identify a discernible pattern in Relativity from analysing market data from the Valuation Office Agency. A working group established by the Royal Institution of Chartered Surveyors (RICS) could not agree on a definitive Relativity graph that could be adopted as an industry standard.

Aim and Objectives

Our aim is to understand the valuation of short leases by deriving the unenfranchised and enfranchised Relativity curves and the option value embedded in a lease from recorded apartment market transactions. Our first objective is to derive the enfranchised Relativity curve

from the data by estimating hedonic price models. We adopt lease dummies, linear spline and a right-tailed restricted cubic spline functions to represent lease length. Our second objective is to apply option pricing to reveal the unenfranchised Relativity curve. We derive the *conditional* unenfranchised Relativities using a one-step binomial option pricing model, which is subsequently used as an input in the Longstaff and Schwartz (2001) Least Squares Monte-Carlo simulation procedure to obtain the embedded option value and *unconditional* unenfranchised Relativities. Our third objective is to assess whether premiums determined in leasehold extensions in recent years are likely to have resulted in a hugely unfair distribution of cash transfers between leaseholders and freeholders. Bracke et al. (2018) could not distinguish between possible explanations why the implied net discount rates obtained from unenfranchised sample were higher for short leases than the enfranchised sample. Our final objective is to consider the implications of deriving implied net discount rates inherent in short leases.

Theoretical Model

From the Gordon growth model, the value of a leasehold producing a finite cash-flow lasting for T periods at a point in time, t, can be expressed as (Giglio et al., 2015; Bracke et al., 2018):

$$V_t^T = RC_t^T V^\infty = \left[1 - \frac{(1+g)^T}{(1+r)^T}\right] \frac{rent}{r-g} \quad (4)$$

where:

$rent$ =periodic cash-flow

r =discount rate

g = constant growth rate for periodic cash-flows

The percentage discount in the purchase price compared to an identical apartment with an infinite lease is:

$$\frac{V_t^T}{V_t^\infty} - 1 = -\frac{(1+g)^T}{(1+r)^T} \quad (5)$$

where:

$\frac{(1+g)^T}{(1+r)^T}$ = implied net discount rate

Our insight is that the 1993 Act implies that there is an option embedded in value of a lease and consequently in any price discount observed in the market. The value of the enfranchised lease with T years unexpired ($\mathrm{VE_t^T}$) is the value of the unenfranchised lease (V_t^T) plus the option value (C_t^T):

$$\mathrm{VE_t^T} = V_t^T + C_t^T \quad (6)$$

Equation (6) states that the unenfranchised and enfranchised Relativities are distinct when the option value is non-zero. Lease expiration is an important issue for the valuation of short leaseholds. Having an option to extend it is valuable, implying $\frac{V_t^T}{V_t^\infty} < \frac{VE_t^T}{V_t^\infty}$ when $C_t^T \neq 0$. Expiration is not an issue for sufficiently long leaseholds. The embedded option value is zero, $C_t^T = 0$, and the enfranchised and unenfranchised Relativities are equal, $\frac{V_t^T}{V_t^\infty} = \frac{VE_t^T}{V_t^\infty}$. This informs us that the implied net discount rate households use to discount cash-flows as depicted in valuing an infinite lease $\left(V^\infty = \frac{rent}{r-g}\right)$ is different from the implied rates inherent in the enfranchised Relativity $\left(RCE_t^T \neq \left[1 - \frac{(1+g)^T}{(1+r)^T}\right]\right)$ in short leases. They only converge when the

option has no value, $\left(RC_t^T = \left[1 - \frac{(1+g)^T}{(1+r)^T}\right]\right)$. The existing literature often fails to make this distinction.

Rearranging equation (6) yields the option value:

$$C_t^T = VE_t^T - V_t^T \qquad (7)$$

Equation (7) reveals that the unenfranchised leasehold value can be obtained by subtracting the enfranchised leasehold value from the option value. Similar to an American option, the tenant can exercise the right to extend at any lease length but once exercised the transaction has to be completed within six months.

However, there is an implementation problem as the unenfranchised leasehold value is required to determine the premium which in turn is needed to obtain the option value. We adopt a two-step approach to overcome it. In the first-step, we derive the unenfranchised leasehold value *conditional* on a lease extension at the existing lease length and use this to calculate the implied premium. This ensures that tenants and landlord share the benefit from any additional uplift in leasehold values when the right to extend the lease is exercised immediately. This premium and the resulting expected payoff do not reflect the benefit from being able to delay an extension to a later date. In the second-step, we use the conditional expected payoff at each lease length as an input in the Longstaff and Schwartz (2001) Least Squares Monte-Carlo simulation method to obtain the *unconditional* expected payoff to capture this benefit, and from it the embedded option value. Using a rearranged equation (6), the unenfranchised Relativity is subsequently obtained.

More formally, the embedded option value in cash-flow terms is the expected payoff from exercising the option over the remaining length of the lease, $E_t[\eta^T]$. Using the law of iterated expectations, the expected payoff conditional on exercising the option at a particular lease length may be considered separately from the unconditional expected payoff over the entire unexpired term. The law implies:

$$C_t^T = E_t[\eta^T] = E_t[E(\eta^T|exercised)] \quad (8)$$

A binominal outcome is used to depict the anticipated payoff from going ahead with extending a lease at length T[3]. The anticipated conditional payoff involves consideration of the conditional payoff in an upstate, where apartment prices rise, and in a downstate, where apartment prices fall (Copeland and Antikarov 2003):

$$E(\eta^T|exercised) = \frac{C_u^T + C_d^T}{1+r_f} \quad (9)$$

where:

C_u^T = p $\eta^T u$ = anticipated payoff in the upstate

C_d^T = $(1 - p)\ \eta^T d$ = anticipated payoff in the downstate

r_f = risk-free rate

$u = e^{\sigma\sqrt{\delta}}$, upstate

$d = \frac{1}{u}$, downstate

σ = volatility of the asset price

δ = number of time-steps

$p = \frac{e^{r_f} - d}{u - d}$

We assume that the transaction is completed in six months as this is the time available to a tenant to submit a claim to the Tribunal to keep it `alive'. A risk-free rate is used to discount the expected payoff back to the present (when the section 42 notice is served). p and $(1-\mathrm{p})$ are the risk neutral probabilities for apartment market prices rising and falling respectively. u and d respectively capture the extent to which prices rise and fall, which depend on the apartment price volatility (σ) and how long it takes to complete the transaction (δ). η^T represent the difference between the net financial gain from the uplift in the leasehold value as a result of an extension and the premium payable. The net financial gains in each state comprise of:

$$\eta^T u = u[VE^{T+90-0.5} - VE^{T-0.5}] - S^T$$
$$\eta^T d = d[VE^{T+90-0.5} - VE^{T-0.5}] - S^T \qquad (10)$$

The premium (strike price), S^T, is revealed after the tenant has served the notice. The information used to determine its value is fixed at this date and therefore unaffected by the state of the market in the future. The uplift in the leasehold value from the 90-year extension, $[VE^{T+90-0.5} - VE^{T-0.5}]$, are only realised six months later. By that time, the lease has expired by half a year. The uplift also depends on whether apartment prices in the market have risen (u) or fallen (d).

Expanding (9) using (10):

$$E(\eta^T|exercised)\left(1+r_f\right)$$
$$= p(u[VE^{T+90-0.5} - VE^{T-0.5}]) + (1-\mathrm{p})d[VE^{T+90-0.5} - VE^{T-0.5}] - S^T \qquad (11)$$

as $S^T = pS^T + (1-p)S^T$.

The anticipated payoff $E(\eta^T|exercised)$ is equivalent to the difference between the enfranchised and conditional unenfranchised leasehold value. Expanding S^T by its constituent terms (equation (1)), collecting common terms together, simplifying and expressing leasehold values as a Relativity (relative to an infinite leasehold value) yields the equation for the conditional unenfranchised Relativity for short leases[4]:

$$RC^T = \left\{ \begin{array}{c} (1+r_f)RCE^T - pu[RCE^{T+90-0.5} - RCE^{T-0.5}] \\ -(1-p)d[RCE^{T+90-0.5} - RCE^{T-0.5}] \\ +\frac{1}{2}\left[\frac{1}{(1+\lambda)^T} - \frac{1}{(1+\lambda)^{T+90}} + RCE^{T+90} + \frac{\kappa}{V^\infty}\frac{[1-(1+\tau)^{-T}]}{\tau}\right] \end{array} \right\} \left(\frac{1}{1+r_f+\frac{1}{2}}\right) \quad (12)$$

Equation (12) represents the implied unenfranchised Relativity required to ensure that tenants and landlord share the benefit from an additional uplift in leasehold values conditional on extending the lease. It does not capture the benefit from being able to delay an extension to a later date.

The unconditional expected payoffs account for the decision to delay exercising the option at each lease length. The Longstaff and Schwartz (2001) Least Squares Monte-Carlo simulation method is employed to obtain the unconditional expected payoff and the embedded option value. The Longstaff and Schwartz (2001) procedure is a recursive algorithm which determines the optimal strategy (stopping rule) of when to exercise the option before its expiration. In our application it is used to determine at which point in the remaining term of the lease a tenant should exercise the right to extend by comparing appropriately discounted conditional expected payoffs. A required input to calculate the expected payoffs over the entire unexpired lease term

are the expected apartment prices in the future. We use the conventional geometric Brownian motion model to capture future changes in the asset price:

$$dVE^T = \mu VE^T dt + \sigma VE^T dZ \quad (13)$$

where:

$\mu =$ growth rate (the drift)

$\sigma =$ volatility

$dZ =$ Wiener process

$dt = \delta$, time step

The parameter values in our simulation model are based on statutory values applied in professional practice. These are outlined in the determination of the deferment rate by the Lands Tribunal in Earl Cadogan v Sportelli (2007) 1 EGLR 153. The Tribunal recommended the adoption of real rates in discounting real cash-flows when calculating the reversionary value to the landlord for leases with unexpired terms of 20 years and above, namely: $\lambda = 5.0\%$; $\tau = 6.0\%$; $r_f = 2.5\%$; $\sigma = 4.5\%$ and $\mu = 2.0\%$. In our simulation model, we use the equivalent semi-annual rates for r_f, σ (using the standard assumption that asset prices are normally distributed) and μ as we assume it takes six months complete a transaction. For very short leases with 11 to 20 years unexpired, the Tribunal indicated that the real rates would need to be adjusted to take into account the house price cycle at the date of the valuation. In Cadogan Square Properties Ltd v Earl Cadogan (2011) 1 EGLR 155, the Upper Tribunal adjusted the real growth rate μ to account for a period of above average growth and a sub-period of below average growth. For extremely short leases (10 years and under), net rental yields are used as proxies for the unenfranchised Relativities. We do not make any adjustments to our parameter values as this part of the unenfranchised Relativity curve is determined subjectively. Our results

may not be applicable for very and extremely short leases. We ran 100,000 simulations for each lease length up to 90 years. The unenfranchised Relativity is subsequently derived by subtracting the enfranchised Relativity from the embedded option value.

Empirical Implementation

Empirical implementation requires the enfranchised Relativity as an input. Expressing a finite leasehold value relative to an infinite leasehold value and taking logarithms:

$$\frac{VE_t^T}{V_t^\infty} = RCE_t^T$$

$$lnVE_t^T = lnV_t^\infty + lnRCE_t^T \quad (14)$$

Since transactions occur in an Act world, $ln\, RCE_t^T$ represents the enfranchised Relativity. However, price differences between identical apartments may not be solely due to lease expiration. For example, apartments with shorter leases may be under-maintained relative to those on long leases and consequently transact at lower prices in the market (Iwata and Yamaga 2009). We attempt to address heterogeneity among apartments and market conditions by controlling for their hedonic characteristics and time periods:

$$lnVE_t^T = X'\beta + lnRCE_t^T \quad (15)$$

where:

$X'\beta$ =apartment and location characteristics and time period

Giglio et al. (2015), Bracke et al. (2018) and Savills (2016) included variables representing dwelling and block characteristics. Giglio et al. (2015), Bracke et al. (2018) and Savills (2016) employed fixed effects to control for location and general market conditions at different time

periods (quarter and years). Giglio et al. (2015) included a three-digit postcode while Bracke et al. (2018) and Savills (2016) used street dummies.

Giglio et al. (2015) adopted lease buckets to obtain estimates of the percentage price discount. Savills (2016) used dummies for each lease length in estimation. Bracke et al. (2018) and Lai and Milcheva (2021) employed both approaches. Bracke et al. (2018) used a default dummy representing 999 years to represent the very long lease which differed from the definition of 900 years and above used by Savills (2016) and the 925 years and above by Lai and Milcheva (2021).

The specification to capture the enfranchised Relativity in the lease dummy model is:

$$lnRCE_t^T = \sum \gamma D^T \quad (16)$$

where:

$\gamma = RCE^T$

$D^T =$ dummy taking value of 1 if the lease length is T.

This approach relies on having sufficient number of transactions at each lease length. Using location and time controls compounds this data requirement problem as there have to be at least two transactions in each location in each time period in order to identify the fixed effects. Savills (2016) included singletons in estimation, which is likely to overstate the statistical significance of the regression coefficients and lead to incorrect inference. Insufficient observations at each lease length may also lead to implausible magnitudes such as apartments on shorter lease lengths having a lower price discount than those on longer leases. Consequently, studies adopting this approach apply a second estimation step to smooth the

estimates in order to reveal the Relativity curve. Bracke et al. (2018) fitted a second-degree local polynomial to the lease dummy estimates[5] while Savills (2016) imposed an exponential functional form to fit them[6]. However, curve fitting procedures can lead to undulations in the estimated Relativity at the longer lease lengths, which in our application would affect the uplift calculation in leasehold values from an extension.

We additionally employ a linear and a right-tailed restricted cubic spline function to avoid these problems. A linear spline is known to be a good approximation for capturing an unknown non-linear relationship, having the added advantages of being easy to interpret and amenable to testing propositions. The continuous variable measuring the length of the lease with T years remaining relative to an infinite lease in our model is:

$$L = L^{\infty} - L^{T} \tag{17}$$

where:

L = the difference between the existing lease and infinite lease

L^{∞} = the infinite lease (999 years)

L^{T} = the existing lease

The enfranchised Relativity is captured by:

$$lnRCE_t^T = \gamma_1 L + \sum_{k=1}^{K} \gamma_{k+1}(L - k_k)_+ \tag{18}$$

where:

k are the knot points.

The γ_k represent the marginal decline in the relative value of an apartment within the expiry interval defined by the knot (k). The enfranchised Relativity is the exponent of the sum of the products of the relevant spline segment estimates and expired lease length. A linear spline function comprising of 5-year intervals was initially estimated and then consolidated using a general to specific approach[7]. While the linear spline model reveals the general shape of the enfranchised Relativities, the resulting kinks may lead to over and under estimation at the knots. We address this concern by employing a penalty spline smoothing technique to eliminate these kinks.

We also estimate a right-tailed restricted cubic spline model. Restricted cubic spline functions are relatively robust as it is the number rather than the location of knots which primarily reveals the shape in the data. A restriction is imposed at the right tail as the decline due to expiration on longer leases is gradual as implied by the theory, professional practice and supported by the results from our lease bucket estimates. In most applications four knots is sufficient for an adequate fit (Harrell, 2001). The restricted cubic spline function is:

$$lnRCE_t^T = \gamma_1 L + \sum_{k=1}^{K} \gamma_{k+1} (L - k_k)^3{}_+ \qquad (19)$$

Being cubic, the estimates are not directly interpretable. The enfranchised Relativity is the exponent of the sum of the products of the relevant spline segment estimates and expired lease length.

Data

The existing statute in England and Wales has evolved from case law involving leasehold extension and enfranchisement claims in Prime Central London (PCL). Data on dwelling

attributes, location and lease length are extracted from apartment listings in Lonres database in PCL between 2010 to 2016[8] and matched to transacted prices recorded by the Land Registry. This is supplemented by additional lease length and property characteristics information from the Land Registry and Ordinance Survey Address Base respectively. The analysis is conducted only on leaseholds. As in Bracke et al. (2018) and Savills (2018), we eliminated leases between 250 to 899 years from estimation as there are relatively few observations within this range. Their exclusion is unlikely to make a significant difference to explaining Relativity. Lai and Milcheva (2021) also eliminated leases lying between 300 to 925 years due to this constraint. We retrieved 16,892 observations but 4,308 are dropped by the estimator for being singletons. The descriptive statistics of the 12,584 observations used in estimation are displayed in the appendix as table A1.

Results

Hedonic Model Results

Table 1 reports the results of the estimated hedonic models. The dependent variable apartment price is in natural logarithms. All models control for time and location influences using fixed effects. The full postcode is our preferred location control. As in Giglio et al. (2015) and Bracke et al. (2018) the standard errors are clustered by location[9]. Model A excludes controls for lease length. To account for lease expiration, model B includes lease buckets, model C lease dummies, models D and E contain linear and the right-tailed restricted cubic spline functions respectively. The default category in the lease bucket and lease dummy models are very long leases, defined to be 900 years and above, the same definition employed by Savills (2016). We also estimated linear and cubic spline models using 900 years as a long lease but as the results are almost the same as using 999 years, we only report the results for the latter.

Insert Table 1 Hedonic Model Results here

Broadly, the physical dwelling attribute estimates across all models using postcode as a location control are similar and plausible. Size (area), amenities (e.g., private garden, view) and physical condition are important. New builds and refurbished apartments are sold at a premium and those below average condition at a discount. The only hedonic model in table 2 that fails the link test for functional form is model A which does not control for lease length. The pattern of its residuals reported in figure 1 reveals that this specification over predicts apartment values for short leaseholds and that the over predictions worsen as the lease length shortens.

Insert Figure 1 Residuals of hedonic model excluding lease controls here

This residual pattern disappears in models incorporating lease length and empirically validates the existence of enfranchised Relativity.

Model B includes buckets constructed using dummies at 10-year intervals for shorter lease lengths. Longer lease lengths have buckets with larger intervals as Relativity changes gradually. The estimates are precise. e^{γ} reveals the enfranchised Relativity. The extent of the price discount relative to an apartment on a very long lease is obtained using the expression $1 - e^{\gamma}$. For example, the discounted price for lease lengths 60 to 69 years is $1 - e^{-0.133} = 12.5\%$ compared to 6.4% for leases with 81 to 89 years remaining. Giglio et al. (2015) reported that the price discount for leases between 80 to 99 years to be around 16-17%[10] while Bracke et al. (2018) reported it to be 10.5%. Lai and Micheva (2021) reported 4% discounts for leases above 80 years and 8% discounts for leases under 80 years. For long leases 150 to 250 years, our results indicate that there is a price discount of around 3%. A test of equality restrictions reveals that the price discount for leases from 90 to 250 years are the same,

F(2, 3379) = 0.06, supporting the findings in Dixon et. el. (2000) and the observation made by the Law Commission Valuation Report (2020) that steep discounts only occur when leases have less than 90 or 85 years left respectively. The bucket estimates also reveal that the non-linearities in price discounts are only significant for short leases, which supports our assumptions about gradual decreases in the price discount profiles at longer lease lengths inherent in our linear and right-tail restricted cubic spline models.

For brevity, the point estimates of the lease dummies in model C are not displayed. The main advantage of this approach is that it is data driven but in implementation it suffers from problem of insufficient data points at each lease length for a given location and time period, resulting in imprecise and implausible dummy point estimates as some of the estimates imply that short leases are more valuable than long leases. Following Bracke et al. (2018), we attempt to overcome this problem using a local polynomial model as a second estimation step.

In models D and E, the linear and cubic splines represent the difference between the very long lease (999 years) and the current lease length. The spline segments are labelled using the current lease length. For example, the spline segment labelled 'LS_85' captures the annual rate of decline (marginal effect) of the value of leasehold for leases 85 years or more years unexpired, the segment 'LS_70' the marginal rate of decline for leases with terms lying between 70 to 85 years, and so on. We explicitly tested for cut-off points at 250, 150, 125, 99 and 90 years, the lease length thresholds identified by the buckets specified in Giglio et al. (2015) and Bracke et al. (2018). However, the initial cut-off at 85 years dominates, in line with the observation made in the Law Commission Valuation report (2020). The segment estimates reveal that the rate of decline increases as the lease length shortens. The cubic spline model E has four knots, derived from the set revealed by the empirical results of the lease bucket and linear spline models. The

knot locations in this model, at 90, 85, 40 and 15 years remaining on the lease, dominate the alternatives in precision and robustness.

We undertook a simple robustness test of our linear and cubic spline functions using a simple dummy taking a value equal to 1 if the leasehold has 80 years or less remaining. Our spline functions completely explain the effects of lease expiration as this dummy is insignificant. Its inclusion also does not significantly affect the segment estimates. This also suggests that very steep price falls are not observed at the point when the marriage value become payable as the slope estimates dominate the intercept estimate. The reported results of the robustness tests can be found in the appendix.

Relativity Curve Results

The enfranchised Relativity curves are obtained from our linear and cubic spline and lease dummies estimates. As in Bracke et al. (2018) a local polynomial regression model is employed to derive the smoothed Relativity curve from our lease dummy estimates. We apply a penalized smooth regression to smooth out the kinks in the enfranchised curve derived from our linear spline model.

The unenfranchised Relativity curves are obtained by subtracting the value of the option obtained from the Longstaff and Schwartz (2001) simulation procedure from the enfranchised Relativities. Figure 2 displays the results.

Insert Figure 2 Enfranchised and Unenfranchised Relativity Curves

The vertical axis is the Relativity scale in decimals and the horizontal axis the unexpired lease length. Panel A compares the enfranchised Relativities obtained from the linear spline

(RCET_LS), smoothed linear spline (RCET_Smooth_LS), the cubic spline (RCET_CS) and the lease dummies (RCET_LD) models against the Savills (2016) curve (RCET_Savills). Overall, the enfranchised Relativities derived from our hedonic models are similar but deviate from the Relativity curve reported by Savills, probably due to different hedonic and second-step model specifications. Our curves lie above Savills from 10 to 38 years and then again above 57 years. Between those years, the differences are marginal.

Our enfranchised Relativities are monotonically decreasing with lease expiry except for the curves derived from the lease dummies model – the curve exhibits undulations at longer lease lengths (not shown in the diagram) caused by the smoothing procedure. This poses a problem as it affects the uplift calculations required by our simulation model. Our linear and restricted cubic spline models do not suffer from this problem.

Panel B displays the enfranchised and unenfranchised Relativity curves derived from our linear (RCT_LS), smoothed linear (RCT_Smooth_LS) and cubic spline (RCT_CS) and lease dummy (RCT_LD) models. The curves exhibit similar profiles. The Relativities obtained from the lease dummy hedonic model are slightly lower as the lease turns short. Theoretical consistency for short leases requires that (i) the enfranchised Relativities to be greater than the unenfranchised Relativities, as the gap between them reflects the total value of the embedded option and (ii) the unenfranchised Relativities should be monotonically decreasing as the benefit from receiving an income from owning the leasehold falls as the lease expires. All our curves meet these conditions.

We are now able to assess a conclusion reached by Bracke et al. (2018) that the unenfranchised Relativity curves applied by agents resulted in grossly unfair premiums in leasehold extensions because it did not reflect market conditions.

Insert Figure 3 Comparing Unenfranchised Relativities and Implied discount rates

Panel A in figure 3 compares our unenfranchised Relativity curves against those used by agents, namely, Savills, Gerald Eve and Knight Frank. To provide context a scatter plot of decisions reached by the Leasehold Valuation Tribunal (LVT) represented by the grey dots is also included. The Gerald Eve (1996) and Knight Frank curves are based on proprietary data and heuristics. The Savills (2016) unenfranchised Relativity curve is derived from market transactions but atheoretical, obtained by fitting an exponential function using eight Upper Tribunal observations and their enfranchised Relativity curve as reference points. Our unenfranchised Relativity curves are derived from market transactions and using the Gordon growth and option pricing models as a theoretical framework.

The Savills (2016) curve lies beneath all the other curves which could partly reflect the steeper price discounts of its enfranchised Relativity. For the most part, our unenfranchised curves lie in between the agents' curves. From 70 years remaining, our Relativities begin to converge with the Gerald Eve and Knight Frank curve until about 25 years when it starts to diverge. The gap is largest for unexpired leases with less than 20 years remaining. This could be partly due to the fact that the premium for leases under 20 years are not subject to the Sportelli ruling with which we calibrated values in our simulation model. Another potential explanation is that there are fewer transactions for leases below 20 years (the cumulative total is less than 2 percent of our sample) which makes it difficult to obtain the true enfranchised Relativity values from the

hedonic model. The third possible explanation concerns certain estates such as Grosvenor which only grant 20 year unenfranchisable leases confounding our hedonic model estimates at very short lease lengths.

However, our unenfranchised Relativity curves appear to fit the LVT data better for leases below 20 years than the agents' curves. In general, our unenfranchised curves pass through the LVT outcomes at the lower end of their Relativities. This could be attributable to the LVT Relativities being unadjusted for dwelling, block and location heterogeneity since it is better quality apartments that are brought to the LVT due to the high legal costs. Based on the above considerations, we do not find that the agents' unenfranchised Relativity curves are wildly out of line with the market in 2010-2016. It does not mean that unfair cash transfers did not take place, only that the marriage value payments based on agents' proprietary curves did not result in very large unfair premiums.

The implied net discount rates derived from our Relativity curves obtained from the smoothed linear and cubic spline models are displayed in Panel B in figure 3. The vertical axis depicts the rate in decimals. Our implied discount rates derived from the enfranchised Relativity curve (RCET) can be compared to those reported in the literature. As in Bracke et al. (2018) and Lai and Milcheva (2021), our implied net discount rates are declining but not constant across time. Our very long-term discount rates are just under 2%, the same figure reported by Giglio et al. (2015) and Lai and Milcheva (2021). Bracke et al. (2018) reported net discount rates of around 5-7% for very short leases and 2% for leases close to 100 years in their enfranchised sample, while Lai and Milcheva (2021) reported declines from 13% to below 3% for leases between 1 to 100 years[11]. The implied net discount rates from the enfranchised Relativity curves follow a similar hump shaped pattern and range from 3.5% to 6% for leases between 3 to 99 years.

Bracke et al. (2018) could not explain why the implied net discount rates obtained from the enfranchised sample were lower than the unenfranchised sample for short leases but were similar for long leases. Panel B shows that the implied net discount rates obtained from our unenfranchised Relativity curve (RCT) are lower than those derived from the enfranchised curve, which is consistent with an enfranchised leasehold apartment being less heavily price discounted than an identical unenfranchised one. In the case of short leaseholds, the implied net discount rates derived from each type of Relativity are distinct. The rates obtained from the enfranchised Relativity curve incorporate the value of the option to extend the lease before its expiration. However, it is the implied rates from the unenfranchised Relativity which are the discount rates households use to assess cash-flows (equation 4). For long leases, the unenfranchised and enfranchised Relativity curves converge because lease expiration is not a concern and the option value tends to zero. The implied net discount rates used by households can be derived from the enfranchised Relativity in long leases, as in Giglio et al. (2015).

Conclusion

Our research contributes to the understanding of the valuation of short leaseholds in England and Wales. We build on the research of Bracke et al. (2018) and Giglio et al. (2015) by explicitly acknowledging the embedded option value in distinguishing between the unenfranchised and enfranchised Relativities. The enfranchised Relativities capture the value of the legal right but not the obligation to extend a lease. We employed linear and right-tailed restricted cubic spline functions and lease dummies in our hedonic price models to capture the enfranchised Relativities. The linear and cubic spline models yield more plausible point estimates given the number of available observations at each lease length. Our derived enfranchised and unenfranchised Relativities are consistent with financial theory. Our

unenfranchised Relativity curve lies within the range of those applied by agents in the market and are consistent with judicial decisions made by the LVT, suggesting that premiums for short leasehold extensions were not hugely out of line with market fundamentals in 2010-2016. In contrast to the Bracke et al. (2018) approach of comparing confounding estimates from two samples to capture different institutional arrangements, our methodology is able to empirically validate short leaseholds with enfranchisement rights are more valuable than those without and yield the required implied net discount rates to examine how households value longer term cash-flows from leasehold prices with enfranchisement rights. Finally, our results have the potential to contribute to more equitable premiums for leasehold extensions and enfranchisement as it enables the unenfranchised Relativity curve to be derived from market evidence in an Act world.

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Figure 1: Residuals of hedonic model excluding lease controls

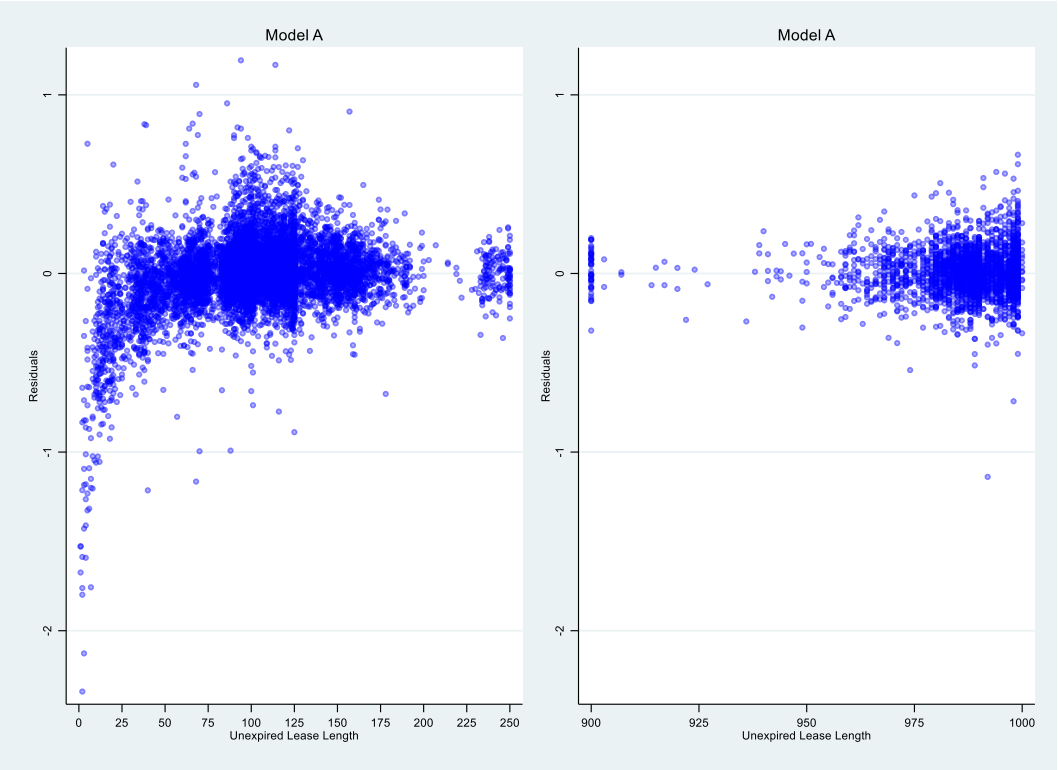

Figure 2: Enfranchised and Unenfranchised Relativity curves

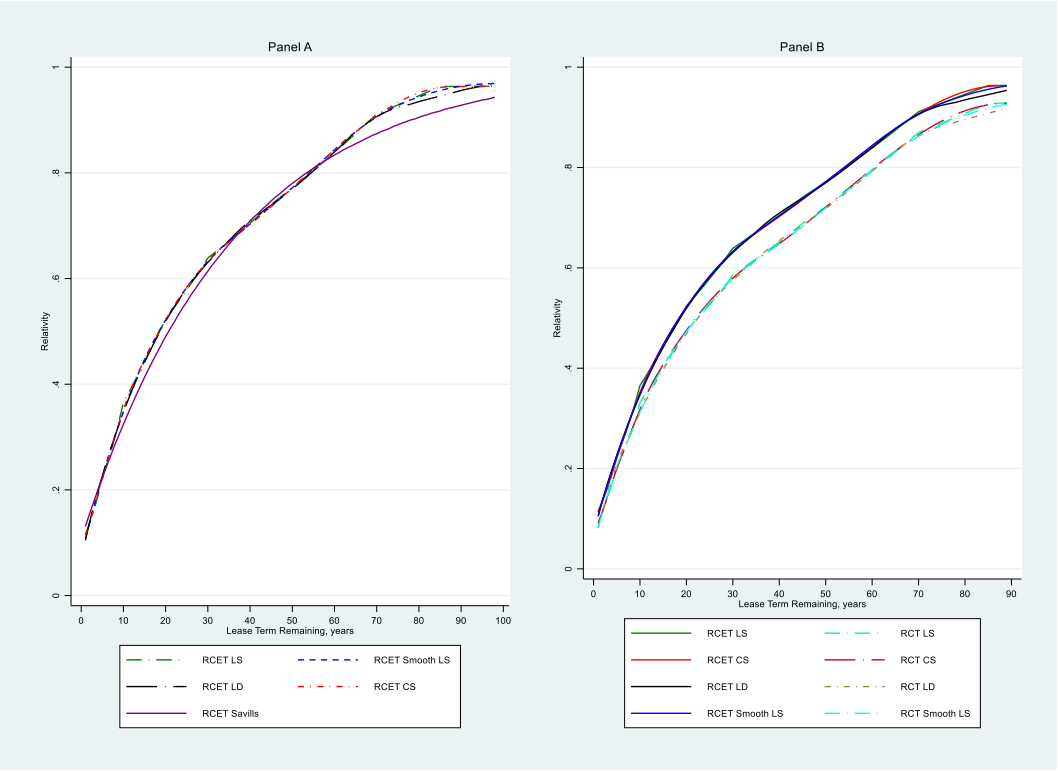

Figure 3: Comparing Unenfranchised Relativities and Implied discount rates

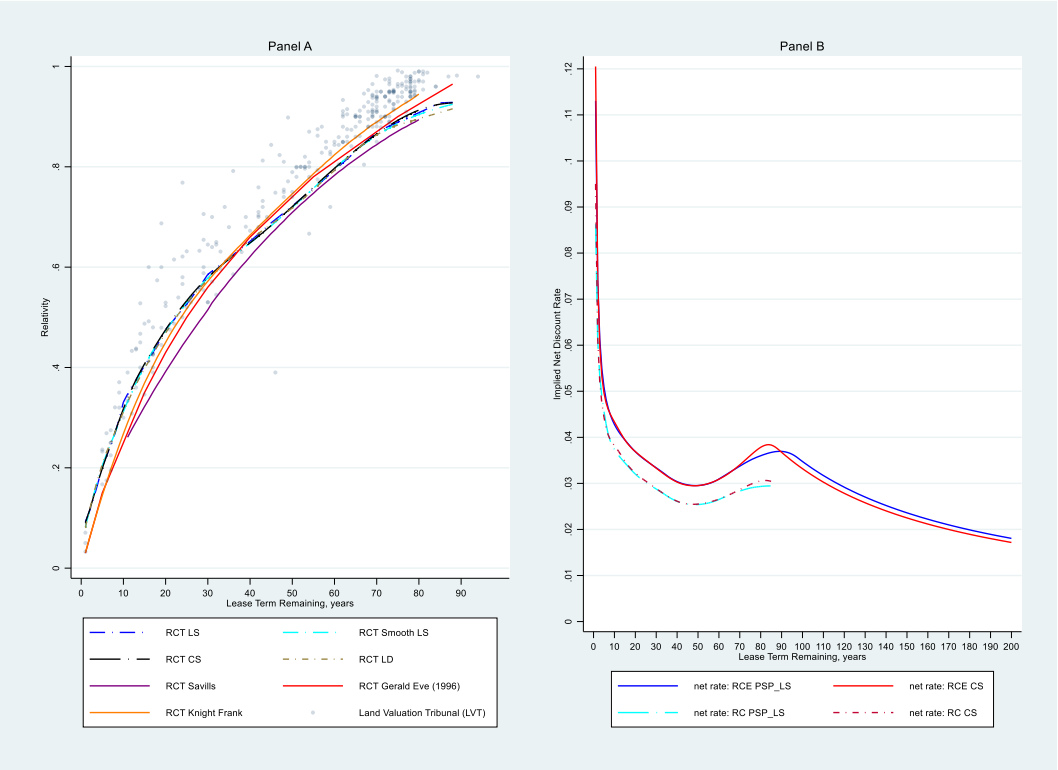

Table 1: Hedonic Model Results

Explanatory Variables	Model A	Model B	Model C	Model D	Model E
log area	0.979***	0.959***	0.962***	0.962***	0.963***
Bathrooms (Default 1 Bathroom)					
Two plus Bathrooms	0.0664***	0.0555***	0.0547***	0.0546***	0.0543***
Floor (Default Basement)					
Ground Floor	0.193***	0.194***	0.196***	0.195***	0.195***
First Floor	0.244***	0.246***	0.247***	0.247***	0.247***
Second Floor	0.213***	0.217***	0.219***	0.218***	0.218***
Third Floor	0.197***	0.205***	0.207***	0.206***	0.206***
Fourth Floor	0.182***	0.192***	0.195***	0.193***	0.193***
Fifth or more	0.236***	0.240***	0.244***	0.241***	0.241***
Multiple Floors (Default Single level)					
Multiple Floors	-0.0173*	-0.0169**	-0.0179**	-0.0190**	-0.0191**
Dwelling Condition (Default Average)					
New Build	0.0703***	0.0661***	0.0642***	0.0660***	0.0661***
Refurbished	0.106***	0.0710***	0.0698***	0.0706***	0.0705***
In Need of Refurbishment	-0.152***	-0.0805***	-0.0800***	-0.0824***	-0.0823***
In Need of Full Refurbishment	-0.184***	-0.0900***	-0.0833***	-0.0811***	-0.0805***
Garden					
(Default No Garden)					
Private Garden	0.0925***	0.0932***	0.0931***	0.0919***	0.0918***
Communal Garden	-0.019	-0.00573	-0.00461	-0.00696	-0.00713
Amenities (Default none)					
Balcony	0.0415***	0.0363***	0.0359***	0.0354***	0.0355***
Terrace	0.0477***	0.0373***	0.0382***	0.0378***	0.0377***
Patio	-0.00382	-0.00266	-0.0088	-0.00554	-0.00541
Roof Terrace	0.0206*	0.0193*	0.0210**	0.0187*	0.0187*
View	0.0395***	0.0431***	0.0414***	0.0412***	0.0413***
Car Parking					
(Default No Parking)					
Parking	0.00872	0.01	0.00961	0.0109*	0.0109*
Flat Type					
(Default Purpose Built)					
Converted Flat	-0.0190*	-0.0162**	-0.0142**	-0.0162**	-0.0163**
Lift (Default - No lift)					
Lift	0.0164**	0.0164***	0.0166***	0.0169***	0.0169***
Penthouse Unit (Default - No)					
Penthouse	0.126***	0.121***	0.120***	0.119***	0.118***
Size of Block (Default - 1 to 3 units)					
4-14	0.0162	0.0191	0.0187	0.0179	0.0178
15-24	0.0415*	0.0331*	0.0358**	0.0357**	0.0358**
25-49	0.00344	0.0305	0.0341	0.0322	0.032
50 plus	-0.00125	0.00895	0.00529	0.00651	0.00621
missing	0.0354*	0.0298	0.0299	0.0284	0.0284
	-0.00662	-0.00213	0.00533	0.003	0.00299
Listed Building					
Peppercorn rent	0.114***	0.000669	-0.00222	-0.000344	-0.000955
Land Registry Verified	-0.002	-0.0000674	-0.00027	-0.00103	-0.00107

	Lease Buckets		Lease dummies	Spline Segments		Spline Knots	
	1-9	-1.521***		LS_85	-0.0000407***	CS	-0.0000410***
	10-19	-0.831***		LS_70	-0.00369***	CS_knot90	-0.0000140***
	20-29	-0.554***		LS_45	-0.00831***	CS_knot85	0.0000158***
	30-39	-0.386***		LS_30	-0.00984***	CS_knot40	-0.0000169***
	40-49	-0.315***		LS_20	-0.0199***	CS_knot15	-0.000219***
	50-59	-0.233***		LS_15	-0.0326***		
	60-69	-0.133***		LS_10	-0.0376**		
	70-80	-0.0728***		LS_5	-0.1000***		
	81-89	-0.0666***		LS_Rest	-0.165***		
	90-99	-0.0309***					
	100-149	-0.0286***					
	150-250	-0.0308***					

Location FE	Postcode	Postcode	Postcode	Postcode	Postcode
Year and Quarter FE	Yes	Yes	Yes	Yes	Yes
N	12584	12584	12584	12584	12584
F	(32,3379) = 447.2	(44,3379) =542.6	(258,3375) = 262.4	(41,3379) =610.8	(37,3379) =661.7
Adjusted R2	0.9217	0.9569	0.9569	0.9581	0.9582
Link Test	Fail	Pass	Pass	Pass	Pass

*** significant at 1%; ** significant at 5%; * significant at 10%.

[1] The legislation sets out that the subject (or pariah) lease is the only one that has no Act rights.

[2] Freehold apartments do not exist in the UK. We prefer to express Relativity in terms of an infinite or very long lease rather than a freehold value based on a house due to their very different physical characteristics and differences in redevelopment potential.

[3] Since it is only six months when the payoff is realised, a binomial outcome is a reasonable approximation.

[4] A derivation is not necessary for long leases as there is little uplift and no marriage value payable.

[5] Bracke et al. (2014, 2018) fitted a second-degree local polynomial with a 15-year bandwidth weighted by the number of sales at that specific lease length.

[6] $RCE^T = 0.106 + (1 - 0.106)(1 - 0.972^T)$

[7] Equality restrictions were also imposed on redundant (insignificant) knots as a further check.

[8] We are able to matched 65% of the information in Lonres to transacted prices recorded by the Land Registry. For the remainder we used asking prices and included a dummy variable (KF_Verified) to control for the non-matches in estimation.

[9] We also obtained boot-strapped standard errors for the spline models as a further check on statistical significance.

[10] They report that their price discount is relative to a freehold in a flat only sample. But freehold flats do not exist in the UK, only the share of freehold.

[11] Giglio et al. (2015) excluded short leases in their investigation.