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Highlights

The recent explosion in theories of consciousness, which aim to link subjectivity and physical

- substrates, require a better characterization of mathematical structure of quality of consciousness, or qualia.
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- In traditional and intuitive models of qualia, a particular quale is assumed to be a point in a high dimensional space.
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- Such models assume that qualia exist independent of measurements, but they are incompatible with the findings that qualia are generally affected by measurements.
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 To account for how the measurement can affect qualia, a Quantum-like Qualia (QQ) hypothesis proposes a mathematical structure employed in quantum theory.

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- We will outline how QQ can be tested with various experimental paradigms, building on the
- successful quantum cognition framework.
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Abstract

 To arbitrate theories of consciousness, scientists need to understand mathematical structures of quality of consciousness, or qualia. The dominant view regards qualia as points in a dimensional space. This view implicitly assumes that qualia can be measured without any effects on it. This contrasts with intuitions and empirical findings to show that by means of internal attention qualia can change when they are measured. What is a proper mathematical structure for entities that are affected by the act of measurement? Here we propose the mathematical structure used in quantum theory, in which we consider qualia as "observables" (i.e., entities that can, in principle, be observed), sensory inputs and internal attention as "states" that specify the context that a measurement takes place, and "measurement outcomes" with probabilities that qualia observables take particular values. Based on this mathematical structure, the Quantum-like Qualia (QQ) hypothesis proposes that qualia observables interact with the world, as if through an interface of sensory inputs and internal attention. We argue that this qualia-interface-world scheme has the same mathematical structure as observables-states-environment in quantum theory. Moreover, within this structure, the concept of an "measurement instrument" in quantum theory can precisely model how measurements affect qualia observables and states. We argue that QQ naturally explains known properties of qualia and predicts that qualia are sometimes indeterminate. Such predictions can be empirically determined by the presence of order effects or violations of Bell inequalities. Confirmation of such predictions substantiates our overarching claim that the mathematical structure of QQ will offer novel insights into the nature of consciousness.

1. Introduction

 Research on consciousness has recently entered a new phase. A burst of neuroimaging studies on consciousness since 1990 has produced a huge amount of empirical data, requiring a principled 80 explanation for consciousness and its neuronal substrate (Koch et al., 2016; Mashour et al., 2020; 81 Seth & Bayne, 2022). Over the last twenty years, many of the initial ideas about consciousness 82 and brains were abandoned in the face of empirical data. The remaining theories have retained their core principles in the form of variations that have branched out from these theories. Some 84 theories aspire to make quantitative predictions, a few of which are currently pitted against each other in an adversarial way (Melloni et al., 2021). Through empirical tests of rival theoretical predictions, substantial scientific progress is to be expected, as has happened in other fields, such 87 as physics and experimental psychology (Aspect et al., 1982; Bell, 1964; Einstein et al., 1935; Freedman & Clauser, 1972; Kahneman, 2003).

 As the science of consciousness matures, it has become increasingly clear that we lack an understanding of the target phenomenon, namely consciousness. While "consciousness" can mean the level or presence of consciousness, as in the clinical science of coma, general 93 anesthesia, or deep sleep (Casarotto et al., 2016), this article focuses on the issue of quality of consciousness, feelings of what-it-is-like-to-be, or, in short, qualia (Balduzzi & Tononi, 2009; Kanai & Tsuchiya, 2012; Lyre, 2022; Tsuchiya & Saigo, 2021; Tye, 2021). Qualia in consciousness research comes in two senses, broad and narrow. In the broad sense, we use a quale to mean a 97 moment of entire conscious experience across all sensory modalities and thoughts, that is, everything being experienced. Qualia in the narrow sense refers to one aspect of the experience, such as the "redness" of the sunset, the particular flavor and taste of tuna sashimi, and so on (Balduzzi & Tononi, 2009; Kanai & Tsuchiya, 2012). This article embraces both senses of qualia. What is not qualia concerns everything that is not part of our conscious experience.

 In this article, Section 2 reviews the popular models of qualia and their deficiencies. To address these deficiencies, Section 3 proposes the Quantum-like Qualia (QQ) hypothesis. Our hypothesis is inspired by the mathematical structure of quantum theory. None of our claims rests on whether or not microscopic quantum phenomena play a significant role in the brain and/or consciousness. Section 4 focuses on empirical research projects that can test the validity of the QQ hypothesis, followed by the conclusion in Section 5.

2. Traditional qualia models and their deficiencies

 Traditional models of qualia are founded on the notion of points in a putative metric space, 114 sometimes called a psychological space, quality space, qualia space, phenomenal space (Clark, 2000; Lee, 2021; Rosenthal, 2015) (Figure 1A). These models have been proposed for various modalities, such as color, time, pain, sound and smell (Churchland, 2005; Klincewicz, 2011; Kostic, 2012; Renero, n.d.; Shepard & Cooper, 1992; Young et al., 2014). In the cognitive domain, there 118 are strong arguments that concepts reside in such a space (Gärdenfors, 2000). Thus it seems natural to start with the idea to represent qualia as single points in a high dimensional space. Here, a definite point corresponds to a particular quale (either in the narrow or broad sense). To specify a

- combination of narrow qualia or a quale in the broad sense, multiple points are often considered as 122 well^1 .
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 In the case of narrow sense qualia, the distance between the two points relates to the "similarity" between the respective qualia (e.g., a red quale and an orange quale are close in similarity, but red and green are dissimilar). Inspired by early work by Shepard, many variants of such similarity models have been proposed (Ashby & Perrin, 1988; Krumhansl, 1978; Nosofsky, 1991), where visualization techniques such as multidimensional scaling (Borg & Groenen, 2005) have played a central role (Figure 1A). Under this framework, various types of qualia, e.g., color (Bujack et al., 2022; Churchland, 2005; Indow, 1988; Shepard & Cooper, 1992; Zeleznikow-Johnston et al., 2023), sound (Cowen et al., 2020; Renero, 2014; Shepard, 1982), object (Hebart et al., 2020), emotion (Cowen & Keltner, 2017; Nummenmaa et al., 2018), olfaction (Young et al., 2014), art (Graham et al., 2010) etc., have been investigated and visualized based on similarity ratings of pairwise comparisons between the set of qualia under investigation.

 Despite widespread use, the psychological space approach to modeling qualia encounters three challenges: the inability to adequately capture indeterminate and dynamic facets of qualia, as well as their intricate interactions with internal mental processes. The following summary briefly covers these three points.

 Firstly, as this approach assumes a quale is a definite entity (e.g., a point or points in a space), it is unable to capture the intuition that some qualia appear to be indeterminate entities. The indeterminacy of qualia becomes apparent when one introspects on the border of experience in space or time or the nature of unattended or barely attended experience. To determine the spatial border of experience, one can stretch their arms to estimate the limit of the visual field at the periphery, and experientially confirm that this limit is tenuous. Under complete darkness, it is not clear that any such boundary exists. Time also seems to have an indeterminate character. The 148 start and end times of an event often feel unsure and a moment rarely feels point-like, but is 149 typically experienced as having some duration (Filk, 2013). Even when one is focally attending to qualia, one can sense an uncertainty regarding the phenomenal appearance. Changes in certain aspects of qualia have been psychophysically confirmed. The very act of attending can alter the quality of the experience (Carrasco & Barbot, 2019).

 Qualia can be uncertain in two ways. Firstly, the "epistemic" uncertainty of qualia implies that qualia themselves are always determinate, i.e., in a definite state, but measurement processes inject noise so that there is uncertainty about the value of this definite state. Epistemic uncertainty can be captured by modifying the classical model by replacing a point with a cloud of points. However, we suspect that some qualia are "ontologically" indeterminate. Such qualia can be characterized as being in an indefinite "state" whereby properties can only be attributed by means of measuring an ensemble of like qualia. Consequently, indeterminate qualia cannot be modeled or represented as a cloud of dots.

 Secondly, the psychological space approach is by default static and does not account for the temporal dynamics of qualia, because it maps sensory inputs into qualia "at a given time" (See also Footnote 1). The temporal dynamics of qualia, however, are one of the most studied aspects of 166 gualia, from very fine time scales using masking and priming (Bachmann, 2000; Breitmeyer &

 Temporally extended and varying qualia can be represented as either a dynamically moving single point in high-dimensional space or a single point of a very high-dimensional space, where different time points are represented as different dimensions.

- Ogmen, 2007), to larger time scales involving adaptation, expectation (Melloni et al., 2011), and multistability (Brascamp et al., 2018; Maier et al., 2012). If the space itself changes dynamically, the traditional psychological space approach may require substantial updates to account for the spatio-temporal dynamics of qualia.
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 Thirdly, the psychological space approach is not well developed regarding how qualia interact with internal mental processes, such as attention. As alluded to above, how we attend to sensory inputs appears to significantly alter what we experience (Carrasco & Barbot, 2019), as implied from change blindness and inattentional blindness demonstrations (Pitts et al., 2018; Simons & Rensink, 176 2005). However, before we pay attention, we already experience something at the to-be-attended locations, and that is the reason why we can consciously direct attention there. The psychological space model is similarly unclear about how qualia relate to other internal processes, such as memory and expectation.

 Of course, any general framework can be in principle extended. Yet, since the pioneering work by Shepard (Shepard, 1970, 1962b, 1962a, 1980, 1987), subsequent extensions (e.g., concerning dynamics) have not been proposed. It is noteworthy that masking effects have been documented for over a century (Breitmeyer & Ogmen, 2007; Exner, 1868), and despite more than six decades of exploration within high dimensional point models, scant insights into these effects have emerged. We contend that the outlined QQ hypothesis presented here holds promise for

- 187 explicating such masking phenomena, even without properly fleshed out computational models².
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 Thus, the psychological space approach to modeling qualia as points in a dimensional space appears deficient in regard to psychophysically-informed intuitions that qualia are indeterminate, dynamic, and interact with other mental processes. But why do researchers continue to adhere to the psychological-space models? We surmise that this is due to the combination of the intuitive 193 appeal of such models and the lack of compelling alternatives³.

 Interestingly, a similar situation arose in the field of cognitive science, in particular decision making. In decision making, models based on standard probability theory and logic have been persistently

- challenged by many (apparently) paradoxical findings in human decision making. Some of these
- paradoxes in decision making have had fairly natural explanations by means of quantum
- probability theory, which was introduced in psychology with the quantum cognition framework
- (Busemeyer & Bruza, 2012; Haven & Khrennikov, 2013; A. Y. Khrennikov, 2010; Pothos &
- 201 Busemeyer, $2022)^4$. Notably, analogous qualia-related concerns have been raised in the context of

² One promising venue is dynamical models of consciousness and qualia (**Esteban et al., 2018**; Fekete & Edelman, 2011; Moyal et al., 2020). However, so far, such models do not address the issue of how measurements and observations affect qualia, one of the central points of our paper. ³ For more recent mathematically elaborated models, see (Hoffman et al., 2023; Kleiner, 2024; Kleiner & Ludwig, 2024) and references therein.

 Some studies in quantum cognition are highly relevant to our proposal (Atmanspacher & Müller-Herold, 2016; Filk, 2009; A. Khrennikov, 2015, 2021). Our Quantum-like Qualia (QQ) hypothesis is quite orthogonal to the Quantum Brain hypothesis, which considers quantum mechanical processes in the brain (Hameroff & Penrose, 2014) and the role of consciousness in quantum collapse (Chalmers & McQueen, 2021) (See also (Smolin, 2022)). QQ is completely consistent with the possibility that all physical events happening in the brain are purely classical. Our core idea is to utilize the mathematical formalism of quantum theory, as outlined below. For these and other related issues see (Atmanspacher, 2017).

- human decision-making. By incorporating the indeterminacy inherent in quantum theory and acknowledging the role of measurement in determining the state within cognitive processes, it has become possible to more effectively model these phenomena, propelling the growth of the quantum cognition field. Consequently, we posit that quantum cognition establishes the conceptual and theoretical foundation of the Quantum-like Qualia hypothesis.
- Decision making and other cognitive processes are inextricably linked to perception and sensation 209 (Barsalou, 2010) and also appear to share basic neural processing architectures. Thus, it seems natural to consider the application of quantum probability theory as an alternate mathematical framework for qualia, in order to address the challenges for the psychological space approach.
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Figure 1. Traditional psychological space models.

 Traditional psychological space models (Lee, 2021; Rosenthal, 2015; Shepard & Cooper, 1992) assume each quale occupies a point in space (or a combination of points). "Distances" between two points are assumed to be related to perceived experiential similarity (Ashby & Perrin, 1988; Krumhansl, 1978; Nosofsky, 1991). A) A classic color hue ring model for the representation of similarity relationships among 9 colors for color-typical and red-green color blind individuals (Shepard & Cooper, 1992). B) Similar representations (points-in-high dimensional spaces) have 224 been used in other domains of experience, such as emotional experience (Cowen & Keltner, 225 2017). Here, 2185 brief videos are represented as points using the tSNE algorithm (van der Maaten & Hinton, 2008).

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3. The Quantum-like Qualia hypothesis

 Three essential challenges for existing models for qualia (i.e., indeterminacy, dynamics, and interactions) are inherently related with the limitations in "classical" approaches. Classical approaches assume that qualia can be probed, observed, reported or "measured," without affecting them. To consider a more general mathematical structure, it is useful to start with the

- assumption that such "measurements" necessarily affect qualia. How much these measurements affect qualia can vary depending on various factors.
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 Quantum theory offers a mathematical structure that deals with entities whose properties can change upon measurement. As we argue below, such a mathematical structure, proposed as a

242 Quantum-like Qualia (QQ^5) hypothesis, attains the three desired features for qualia. QQ states that

243 gualia are like quantum entities, which are inextricably affected by measurement. We first give a

broad sketch of QQ (Figure 2), then explain technical concepts with familiar examples from

consciousness research. More detailed mathematical formulations will be pursued in future work.

3.1 Separating qualia observables from states (of sensory input and attention)

 To account for the indeterminacy of qualia, QQ distinguishes each instance of measured value of qualia (say, color qualia Q="red") from all possible measurable qualia. Inspired by quantum theory, we call all possible measured outcomes "observables". Observables are intrinsic properties of a system that can, in principle, be measured. For example, a color qualia observable at the fixation can be a coarse set of color labels, such as Q={"red","blue", "green", …}. QQ does not presuppose 254 that all aspects of qualia can be simultaneously measured and reported.

 Now consider a situation where you momentarily see many color patches (Figure 2A). Suppose you are attending to the right most red patch. This kind of "sensory input" and "attention" constitute a "state", separate from "observables". While each color quale can be indeterminate, under a particular "state", the expected value of a particular quale (modeled as an observable) is a given. Formally, states are like functions that return the expected value for a given quale, when a 261 particular observable is measured.

Figure 2. Conceptual framework of the QQ hypothesis.

 A) An exemplar sensory input of many colorful patches with the size of each patch proportional to 266 cortical magnification (Tyler, 2015). While you are fixating on the cross at the bottom center, you see the color of each patch without moving your eyes. However, you may feel your experience

changes depending on where you direct your attention. B) The conceptual diagram of QQ. QQ

 This is different from the quantum question (QQ) equality by Wang and Busemeyer (Z. Wang et al., 2014).
⁶ Note this statement is about measurement and reports on qualia. We assume that qualia exist before

measurements in the same way quantum particles exist before measurement.

 considers Qualia as observables that are properties of a system that can be in principle "measured", probed and reported. Sensory inputs and Attention act as an interface or a "state" between Qualia and the World. For example, here the state can be "the sensory input as in (A) AND attending to a red patch on the right". Then, we can define and measure a probability that a 273 particular value is assigned to the observable, for example, Prob("color Q for the leftmost circle" = "blue" | the state) =0.7. How Qualia (Q), Attention (A) and Sensory input (S) evolve over time with or without measurement is formalized by the theory of Instruments (Davies & Lewis, 1970; Ozawa 276 & Khrennikov, 2021). Informally, the putative interaction between the world and qualia, qualia and 277 subjective reports, and how reports alter attention and qualia through instruments are depicted by arrows in the panel.

3.2 Dynamics of qualia observables and states: updates through instruments

 In quantum theory, there are three mathematically equivalent ways to consider the dynamics of 285 observables and states (Sakurai & Napolitano, 2014) (See Table 1 for a summary). QQ considers both observables and states to change over time. This interpretation is called an "interaction" picture.

 In most quantum cognition studies, observables are possible response options, which are fixed, while (mental) states change dynamically. This idea of fixed-observables and dynamic-states is called the "Schrödinger" picture. In QQ, we consider sensory inputs and attention as "states". It is not difficult to imagine how these "states" can change measurement outcomes.

 In some fields of physics (e.g., particle physics), states are considered to be fixed, while observables change. This dynamic is called the "Heisenberg" picture. In QQ, it is natural to consider changes of qualia observables as a consequence of changes in the brain through 297 perceptual learning, sensory adaptation, and so on (C. Song et al., 2017). In this case, even if sensory inputs and attention are fixed, qualia can change.

 In this paper, we predominantly consider sensory inputs and internal attention as major foundational elements of states, but other mental elements, such as memories and expectations, can also constitute states. Thus, in this interaction picture, QQ explicitly considers how qualia (observables) interact with states (sensory inputs and attention). Without a state, we cannot consider a particular measurement outcome of any qualia observable.

 Finally, to formalize how qualia observables interact with other mental processes, we introduce the concept of an "instrument" (cf. the arrows in Figure 2; Davies & Lewis, 1970). In modern measurement theory, any measurement of the system is described by a mathematical structure called a (measurement) instrument, which offers a generalization of a conditional probability. In standard quantum physics, measurements are considered all-or-nothing. As the theory of quantum measurement matured, researchers arrived at the concept of instruments as the most general form of measurement. The formalism of instruments offers a bridge from nonlinear wave collapse (which is the result of a measurement in standard quantum theory) to the unitary dynamics of an isolated system and `unsharp' or weak measurements. We propose that this generalized formalism to characterize the effects of measurements would be particularly useful when considering the interaction between qualia and attention. Attention may not determine qualia in an all-or-nothing way, but rather in an unsharp or weak way.

- Instruments are utilized in modern quantum measurement theory and have started being applied in the field of quantum cognition (A. Khrennikov, 2015; Ozawa & Khrennikov, 2021). Instruments can describe how qualia observables and states of sensory inputs and attention dynamically develop upon measurements.
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- While the above descriptions are sufficient to understand the foundations of the QQ hypothesis, we now expand the conceptual framework and provide associated technical details.
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3.3 What counts as a system?

 We define qualia observables as all possible intrinsic properties of a system. But what is meant by the term "system"? We consider a system minimally as "that which is experiencing the qualia in 331 guestion". It would correspond to "the complex" in Integrated Information Theory (Albantakis et al., 2022). Over time, a system itself can change (then observables would change accordingly). Yet the system should still need to be identified as a coherent entity or phenomenon. A system has an associated set of qualia observables, which can be measured from the outside environment.

3.4 A state as an interface between qualia and the world

 The interrelationship between the system and the environment external to it is represented by the state of the system. In a sense, a state can be considered an interface. This idea may sound strange at first, but actually it is equally applicable across classical and quantum theory (Ojima, 2004; Saigo, 2021; Saigo et al., 2019). For example, the temperature of water in a cup as an observable needs to be determined in the context (= "state") of where and how the measurement 343 instruments are placed⁷.

 In QQ, such a context would involve at least sensory inputs and attention. In a particular state, call it "φ", the expected value of reporting a particular quale, P(Q=q|φ) can be established. For 347 example, in a state φ = "one is sitting at the sunset with the mind wandering", P(Q="seeing the 348 color of red" $\vert \phi \rangle$ can be established. Or, in a state ϕ = "sensory input to a participant is a weak 349 grating stimulus with masking under a particular attentional instruction", we may obtain P(Q="faint" | ϕ) = 0.7, when we assume Q as observables with outcomes of {highly visible, less visible, faint, not visible}. Note that in this framework, there is no point in talking about considering a single-trial 352 quale as in [Q="faint"] without considering the state. We can consider only an ensemble of measurement outcomes given a particular state.

 The notion of an interface between system and environment is an important idea, as discussed in many theories of consciousness. Just to name a few, "interface" in interface theory of consciousness (Hoffman et al., 2015, 2023; Prakash et al., 2020; Prentner, 2021), "background conditions" in the Integrated Information Theory of consciousness, Albantakis et al., 2022, "Markov blanket" in the free energy principle, Kirchhoff et al., 2018, and "mediation" in philosophy, (Taguchi, 2019).

 Consider all possible temperatures of water as observables. The temperature of water is a complex physical concept, which depends not only on the average kinetic energy of water molecules but also on the measuring probe device's temperature, surface areas, and many other factors. We treat all of these factors that relate to measurement as "states". In the case of measuring water temperature, depending on how invasive the measurement probe is (with a probe from either a very cold or very hot environment), the measured outcome of the temperature of water can change.

 Inspired by the mathematical structure of quantum theory, QQ aspires to establish principled associations among observables, states, and their interactions, not at the level of an individual event (or the qualia property at each moment) but at the level of collections of similar events. In fact, for every individual event, the set of all qualia properties would be unique and never identical to the other sets, especially when space and time are considered. Thus, QQ proposes that qualia should not be considered at the level that assumes definiteness of qualia properties for each event. Rather, QQ proposes to consider qualia at the level of ensembles where some "similar" qualia properties are grouped together (as in the above categorical set of observables). How to construe "similar" is an important question, which the authors have discussed elsewhere, using concepts 371 from category theory (Tsuchiya et al., 2016, 2022, 2023). In category theory, it is quite explicit what one considers as similar is a choice of mathematicians or scientists, not automatically or uniquely 373 'aiven' by the world (Cheng, 2022). In most theoretical and experimental contexts, qualia are similar as long as they are considered similar in some way by the observing individual, as in the everyday usage of "similar".

 In summary, "state" is an interface that assigns an "average" value to each observable, noting that measurement of a single event may not be possible.

3.5 Instrument formalism for dynamics of qualia and states

 Let us now consider the dynamics of qualia. For simplicity, in relation to a discrete time step, denote qualia, sensory input, and attention at time t as Q(t), S(t), and A(t). Their interdependency is illustrated by the arrows in Figure 2. The dynamical update rules are expressed as

386 $Q(t+1) = f(Q(t), S(t), A(t))$ and 387 $A(t+1) = g(Q(t), S(t), A(t))$

 This simple formulation is a primitive form of an instrument. Currently, we do not have enough data to constrain the form of the functions f and g. However the equations generally formalize how

391 changes of sensory inputs⁸ affect both what we experience and how we attend. They also capture how attending to uncertain aspects of qualia (e.g., a spatial boundary) can change qualia. For 393 specific and empirical applications of instruments in quantum cognition, see (Ozawa & Khrennikov, 2021).

3.6 A common mathematical and philosophical structure between quantum phenomena and qualia

 QQ proposes an application of some aspects of the mathematical structure from quantum theory (e.g., separation of observables, states and averaged measured outcomes, and instruments). In parallel with the mathematical structure, we surmise that there is a common philosophical stance covering both quantum phenomena and qualia. Through such a philosophical connection, QQ naturally situates some of the perplexing psychological findings in qualia and attention as detailed below.

⁸ While some theories consider a possible role of conscious agents on the control of $S(t+1)$ through motor control and intention, we consider that they are better left out from the formalism of this update rule of instrument for qualia. Consider the sensory input while you are looking at an everchanging shape and colors of a burning fireplace. Also, in an experimental situation, experimenters can change sensory input S(t+1) to a participant in any way they want.

3.6.1 Noncommutativity, complementarity, uncertainty relations in quantum theory, quantum cognition and QQ

 One of the foundational ideas behind quantum theory is "complementarity". In the context of qualia, two qualia are complementary when they cannot be experienced simultaneously, as we consider in 410 more detail below (Bruza et al., $2023)$ ⁹. Complementarity is a philosophical concept that one of the founders of quantum theory, Niels Bohr, introduced in physics, indirectly inspired by one of the founders of modern experimental psychology, William James, through Edgar Rubin (Holton, 1988).

The idea of complementarity can be mathematically expressed via the concept of

noncommutativity (Atmanspacher & Filk, 2018; Streater, 2007). Noncommutativity implies

sensitivity to the order of an operation. In general, the effect of processing A then B may not be the

 same as B then A. Noncommutativity is the default for many processes, from cooking to chemical 418 reactions¹⁰. In the brain, this could correspond to the effect of processing A leaving some trace, in

- 419 terms of synaptic plasticity or neuronal activity, which impacts on processing B. If this is the case, processes A and B are expected to be noncommutative and likewise for the corresponding qualia.
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If observables A and B are noncommutative, measuring A after B typically yields a different

 outcome to B after A. It is generally accepted that many aspects of human cognition are noncommutative. Even in arithmetic, subtraction and division are noncommutative. Whilst multiplication is commutative for numbers, it is not for matrices. Note that matrix operations are 426 fundamental to quantum theory (Busemeyer & Bruza, 2012). Noncommutative observables can be used to formalize important features of qualia, such as the aforementioned indeterminacy. Starting with the well established noncommutative formalization of quantum theory as a guiding framework, it should be possible to appropriately extend this formalism for QQ and, as we explain later, it should be possible to empirically demonstrate its necessity.

 Regarding qualia, in general, when we consider "processes", whereby the order of the processes 433 matters. In an example drawn from masking, presenting target T briefly before mask M at a particular interval can make T completely invisible. But swapping the order into M then T, both of them can become highly visible. This is an example of noncommutativity. Quantitative and coherent explanations of order effects, fallacies in decision making, conceptual combination, evidence accumulation, over/under distribution effects in memory and other cognitive phenomena is one of the hallmarks of the quantum cognition framework (Busemeyer & Bruza, 2012; 439 Busemeyer & Wang, 2017; Pothos & Busemeyer, 2022). Complementarity as noncommutativity is experimentally demonstrated as uncertainty relations (Atmanspacher & Filk, 2018).

Complementarity, noncommutativity and uncertainty relations are the basis of quantum theory,

from which the field of quantum cognition arose. Quantum cognition started from explaining

enigmatic phenomena in decision making (Aerts et al., 2018; Basieva et al., 2019; Broekaert et al.,

2020; Busemeyer et al., 2019; Mistry et al., 2018), concept combination (Aerts & Arguëlles, 2022;

Bruza et al., 2015; D. Wang et al., 2021), and judgment (Ozawa & Khrennikov, 2021; Z. Wang &

Busemeyer, 2013; White et al., 2020). It has recently expanded into modeling for language (Surov

 Note that we are not saying that all qualia are complementary to each other. Some combinations of qualia are likely to be complementary and cannot be experienced at the same time. Indeed, at each moment, we are experiencing multiple qualia at the same time. This is consistent with our introduction of a concept of "broad-sense" qualia. A broad-sense quale is composed of qualia in narrow sense in a unified way.

 Note that non-commutativity includes commutativity as a special case. This is similar to the statement that quantum probability theory includes classical probability theory as a special case.

- et al., 2021), emotion (Huang et al., 2022; Khrennikov, 2021), music (beim Graben & Blutner, 2019), and social judgments (Tesař, 2020). It is beginning to be applied to solve real-world problems (Arguëlles, 2018; Q. Song et al., 2022; Wojciechowski et al., 2022) and it has been influencing the design of artificial intelligence and robots that aim to interact with the world (Ho & Hoorn, 2022).
- To the extent that cognition is continuous with perception (Barsalou, 2010), quantum cognition is a relevant framework to consider quality of perceptual consciousness, or qualia. Indeed, certain applications of quantum cognition to perceptual judgements are already emerging (Asano et al., 2014; Atmanspacher & Filk, 2010; Bruza et al., 2023; Conte et al., 2009; Epping et al., 2023;
- Yearsley et al., 2022) as we will discuss below.
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3.6.2 A common philosophical structure between quantum phenomena and qualia

 On the philosophical side, both quantum phenomena and qualia arise from "interactions". In the above, we introduced "a state as an interface", which is an idea almost equivalent to the philosophical concept of "mediation" (Taguchi, 2019). Quantum phenomena arise from interactions between quantum objects, such as photons, and measurement devices (Plotnitsky, 2021).

- Notably, Niels Bohr stated that the "reality" responsible for quantum phenomena is indeterminate and beyond representation (Plotnitsky, 2021). By "reality", we mean a definite single event before any measurement. Such a concept is not problematic in the classical view, which assumes that anything can exist before measurement and it is in principle not affected by measurement. In quantum theory, a property of an observable is not defined without a state and there is no meaning to a single measurement outcome. In this sense, we adopt a view analogous to Bohr's that "reality" is "indeterminate" and "beyond representation" before any measurement.
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 Likewise, QQ proposes that the reality of qualia defies concrete representation in a similar way, such as points in a high dimensional space in classical models. Note that classical models can consider a distribution of points rather than a single point. However, this still assumes the existence of "reality" of qualia before measurement. Moreover, measurement is assumed to introduce noise so that a probability distribution is needed to model it. In this view, the underlying uncertainty is epistemic due to the limitation of our measurement technique or lack of knowledge. However, QQ proposes that measurement outcomes statistically arise from interrelationship between qualia observables and states of sensory inputs and attention. In other words, the underlying uncertainty of qualia is ontic due to the nature of the very "being-ness" of qualia phenomena. If qualia are ontologically uncertain, we would be unable to establish what property each qualia observable corresponds to, for at least some states at a single event, even if we had 486 all relevant information available¹¹. For such qualia, the act of measurement does not reveal pre-

 As "ontologically" indeterminate qualia, we consider several cases where measurements of qualia have non-ignorable impacts (periphery, similarity judgements, attention related experiments). In Section 4, we provided empirical experiments to address this issue. In classical physics objects exist independent of measurement. Similarly, classical qualia models tend to assume existence of qualia independent of measurement. For example, in encountering an unfamiliar painting, classical models tend to assume that you have some preference even if you do not articulate it or even if it is uncertain. Our QQ is more explicit about this. Some qualia are affected by measurement and measurement instrument theory (in the future) should specify how a particular type of measurement should affect qualia in what way. This also means that QQ also anticipates some qualia are not affected by measurements as well (say, the color of apple in front of you).

- existing properties of qualia observables. Rather the measured property emerges as part of the interrelationship between qualia observables and a state where a measurement takes place.
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 In classical philosophy literature, representationalism states that the phenomenal character of 491 experience is reducible to representational content (Block, 1998). These views typically conceive of

- a definitive single event, regardless of a state, which is reduced to a cognitive representation. By
- contrast, anti-representational views of consciousness propose that such a definitive
- representation does not exist (Gibson, 2014; Koenderink, 2010; Schlicht & Starzak, 2021; Varela,
- Francisco et al., 2017). While the precise reasoning behind the latter views is not the same, the QQ hypothesis shares the same conclusion.

 The point of quantum theory, as argued by Bohr, is to abandon the assumption that "reality" must be definitive and to argue that, due to indeterminacy, the underlying "reality" cannot be represented in a classical way. Instead, quantum theory offers a suitable predictive and explanatory framework.

 The analogy with qualia is that, due to their indeterminacy, some qualia cannot be "represented" as points in the dimensional space, as is usually assumed. Specifically, QQ points out that at least some qualia are indeterminate when they are in an unattended state. In many cases, when attention is directed to a particular qualia observable, measurement outcomes about the attended property would become more determinate. This corresponds to an intentional, content-bearing phenomenal object with an associated cognitive representation as proposed by the orthodox cognitive science. However, in an unattended state, these qualia observables have properties, which do not have well established values or qualities. Classical representationalism does not consider such a possibility. Further, as we elaborate later, QQ predicts that the measurement outcomes are not only statistical but they additionally violate some statistical laws that must be satisfied if qualia properties are always determinate.

3.7 Interim summary: What is the Quantum-like Qualia hypothesis?

 In summary, QQ hypothesizes the following. First, observables correspond to all possible aspects of experience that a system can have, including experiences from all sensory modalities, as well as thoughts, concepts, memories and feelings, that is, anything, as long as it is part of an experience (i.e., qualia in the broad sense). States are a particular arrangement of the system. When the system is in a given state, averaged measurement outcomes from qualia observables can be lawfully specified. States represent sensory inputs and any internal condition of the system, including how the system attends to or accesses observables. Second, averaged measurement outcomes are results of interactions between observables and states and they can be reported outside the system. Third, observables and states change dynamically and interact with each other, as formalized by the instrument theory. From mathematical and philosophical perspectives, qualia have an analogical correspondence with quantum phenomena. Table 1 summarizes these basic concepts and how they are used in quantum theory, quantum cognition, and QQ.

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 Table 1. Conceptual summary of quantum terminologies (columns: observables, states, averaged measurement outcomes) and how they are used in (rows) quantum theory, quantum cognition, and QQ (the Quantum-like Qualia hypothesis). Each cell entry explains a representative usage of each concept.

4. What are the benefits of QQ and how can we test QQ predictions?

 As explained above, QQ accords with fundamental intuitions about qualia, such as their indeterminacy, dynamics, and interaction with internal processes. Furthermore, QQ offers some important insights concerning our empirical knowledge about qualia and provides novel perspectives about the nature of qualia. Here we provide some details of three lines of investigation comprising order effects, violation of the Bell inequality, and relationships between qualia and attention, thereby showcasing how to empirically test various predictions from QQ.

4.1 Order effects in similarity judgments among color qualia.

 The QQ hypothesis is empirically testable in surprisingly simple ways. One way is to ask if the 550 order of questions or stimuli matters for the resulting reports. Epping and colleagues (Epping et al., 2023) presented a pair of color patches to participants, then asked if the reported similarities are symmetric with respect to the order of color patch presentation.

 Since seminal work by Rosch (Rosch, 1975) and Tversky (Tversky, 1977), perceptual similarity 555 judgments about colors, faces and objects have been repeatedly shown to be asymmetric (Best & Goldstone, 2019; Hodgetts & Hahn, 2012; Polk et al., 2002; Roberson et al., 2007). These studies challenge standard points-in-space type models, requiring arguably ad hoc modifications (Ashby & Perrin, 1988; Krumhansl, 1978; Nosofsky, 1991).

 The extremely high citation rate of Tversky's paper attests to the fact that researchers are aware of this asymmetry. Yet, it is not common to empirically take asymmetries into account in similarity studies, as this doubles the numbers of trials. Even when different orders are included, researchers often remove them by symmetrising the originally asymmetric similarity matrix, so that they can use popular, existing analytic algorithms, such as multidimensional scaling.

 While an isolated instance of asymmetry (e.g., "Is China similar to North Korea'' vs. ``Is North Korea similar to China", (Tversky, 1977)) can be explained in many possible models, a collection of perceptual reports for many stimuli, such as color patches, and a particular pattern of asymmetries across many stimuli represent a more substantial challenge (Figure 3A). Epping et al.'s quantum models, which consider a state as a density matrix (this is a generalization of the idea that a state

 can be a vector), and similarity as arising from sequential projections (Figure 3B), offered a better fit to the empirical data (Figure 3C), compared to points-in-space models of qualia (Figure 3D, E), with flexibility to accommodate asymmetry when mapping distance between points to similarity.

 As noted previously, most similarity experiments tend to ignore the effect of order of presentation, using a simultaneous presentation paradigm, or paradigms that allow longer and uncontrolled inspection of the items. This is understandable due to the increased cost of experiments that manipulate order, because the number of the trials increases quadratically with the number of items to examine. Distributing pairs of items across many participants in online samples may solve this issue (Kawakita et al., 2023).

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Figure 3. Quantum model of color similarity.

 A) Empirical asymmetry matrix. The raw similarity matrix is subtracted from its transpose to reveal 587 the degree of asymmetry in similarity judgements. Taken from (Epping et al., 2023). B) How quantum operations (projections) give rise to perceived similarity (Epping et al., 2023; Pothos et al., 2013; Yearsley et al., 2022). Assume an initial (mental) state as a unit vector Ψ (the black line). Color qualia observables {red and orange} are represented as two "subspaces" in a space (the red and orange axes). The vector is projected onto a subspace representing the color that is first experienced. From there, it is further projected onto the subspace corresponding to the second color. The resulting length of the final projection can be related to the perceived similarity between the two colors. Importantly, the resulting length can depend on the order with which the colors are experienced. C) The best fit quantum similarity model for the data in (A) (Epping et al., 2023). In the quantum model, each of 9 color qualia observables is modeled as a subspace in 3D space.

 Experienced similarity between the two subspaces is related to the square value of the cosine angle between them (e.g., the red and the pink subspaces have a narrow angle, but the red and the green subspaces have a near 90 deg angle). D) Traditional 3D MDS representation of 9 colors based on their pairwise similarity. E) Bayesian information criteria (BIC) for best fit 2D and 3D MDS and quantum models. Note that MDS models needed additional free parameters to account for 602 asymmetries in similarity judgements (Nosofsky, 1991), resulting in more complex models. The 3D quantum model offered the best fit to the empirical data.

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4.2 Violation of the Bell inequality in the domain of qualia.

 Quantum theory was developed in the 1920s by Bohr, Heisenberg, Shroedinger, Born and others. This theory challenged the predominant realist view of nature. In 1935, Einstein, Podolsky and Rosen (Einstein et al., 1935) (EPR) challenged this view, claiming that quantum theory is incomplete. In 1962, Bell discovered one fundamental inequality (Bell, 1964) must be satisfied assuming EPR's view is correct. Subsequently, the violation of the Bell inequality was empirically demonstrated (Aspect et al., 1982; Freedman & Clauser, 1972). The Nobel Prize for Physics in 2022 was awarded for the demonstration of violations of the Bell inequality.

 Since the initial EPR experiments, there has been debate about loopholes in the experiments that were being conducted. Over the years these loopholes have been successively closed. Nowadays, it is generally accepted that the EPR experiments do empirically verify that microscopic particles can violate the Bell inequalities and are therefore entangled. What this implies about the underlying nature of these particles has been debated (Zeilinger, 2010). In parallel, a classical realist view has 625 been questioned in relation to cognitive phenomena when these violate the Bell inequalities (Bruza et al., 2023).

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- Bell's inequality can be represented as follows:
- 629 $S = E(a,b) E(a,b') + E(a',b) + E(a',b'),$

 where a and a' are two measurement settings for system A, b and b' for B, and E(:) is the expected value of the corresponding measurements. These expected values have to be measured in separate experimental conditions. In classical systems, |S|<=2, unless there are direct influences or signaling, between measurements of system A and system B. Contextuality-by-Default (CbD) is a generalization of the Bell inequalities. CbD allows a determination of contextuality in the presence of direct influences. (For its application, see (Basieva et al., 2019; Cervantes & Dzhafarov, 2019)). The Bell inequality can be violated by quantum phenomena. A generally accepted explanation for the violation is that the properties of the phenomena do not have definite values at all times, that is, they are indeterminate.

For the QQ hypothesis, demonstrating that qualia violate the Bell inequality will play a similarly

fundamental role. If these types of inequalities are violated, qualia can be assumed to be quantum-

like (which implies additional properties, such as noncommutativity). There are many ways to

psychophysically test the Bell inequalities (Basieva et al., 2019; Bruza et al., 2023; Cervantes &

- Dzhafarov, 2019).
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- *4.2.1 Establishing violations of the temporal Bell inequality in multistable perception*
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- Multistable perception (Brascamp et al., 2018; Maier et al., 2012) can be used to demonstrate
- violations of a type of Bell inequality. Atmanspacher and Filk (Atmanspacher & Filk, 2010) focused
- on the number of reversals between three time points of an ambiguous figure. They proposed
- empirical tests involving the temporal version of the Bell inequality (Yearsley & Pothos, 2014).
- Specifically, Atmanspacher & Filk's proposal was to measure perceptual switches between times
- t1, t2, and t3, where t1<t2<t3, selecting two time points per condition and for all three possible combinations. The probability of the perceptual state being different at time i vs time j is denoted by
- pij. If qualia are determinate at all time (as hypothesized Figure 5 and Table 1 of Atmanspacher &
- 656 Filk 2010), then it has to be the case that $p12 + p23 \ge p13$. If violations of this inequality are found
- under some conditions, it gives reason to believe that the qualia are generally indeterminate, which is fundamental to the QQ hypothesis. (Note that qualia can be in a determinate state under some conditions under the QQ. Indeterminacy includes determinacy as a special case).
- On the other hand, if qualia are generally determinate and can never be indeterminate, p12+p23 ≥ p13 have to always apply. Without doubt, there will be many instances of qualia which indeed behave in such a classical way (as we noted above, the classical probability theory is a special case of the quantum probability theory). What is of interest is whether we can identify cases of qualia for which p12+p23≥ p13 is violated. When this happens, then we can conclude that the qualia should be considered quantum-like in general (even if they might be classical-like, in many 667 cases)¹². The research effort for identifying such violations is still in its infancy, but there are 668 already some promising results (Waddup et al., 2023) that showed violations of the temporal Bell inequality within a decision paradigm.
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A closely related phenomenon concerns quantum Zeno effects (Atmanspacher et al., 2004;

 Yearsley & Pothos, 2016). Quantum Zeno effects are the surprising prediction that, everything else being equal, an increased frequency of measurements can slow down change in the relevant state. Yearsley and Pothos (2016) demonstrated the Zeno effect at the cognitive level (i.e., the switch of opinion about someone to be judged from guilty to not guilty over the accumulated evidence. If "measurements" do not affect qualia, any kind of gradual changes in qualia should not be affected by measurements. While multistable percepts change spontaneously, other types of qualia changes, such as morph-induced categorical perception and gradual change blindness, can be used to test if the effects of measurement can be precisely predicted from the quantum formulation of the Zeno effects (Atmanspacher et al., 2004; Yearsley & Pothos, 2016).

4.2.2 Establishing violations of Bell inequality in multiple qualia about an object

 Another way to test the Bell inequality is to set up a task with at least three qualia observables, measuring two observables at a time, but against three different states. If qualia can be modeled classically and if measurements do not change qualia, then we expect the logical constraints, as exemplified by a Venn diagram (Figure 4A) to be satisfied by the set of probabilities. A simple

 It is worth repeating here that even if we were to find violations of temporal Bell inequality, it does not mean that brains that support qualia are operating in non-classical mechanisms. Instead, it would exclude mathematical structures for qualia that are purely based on classical notions (e.g., determinacy). Rather more broader mathematical structures, such as quantum-like, need to be considered.

- diagrammatic analysis reveals various inequalities, described by George Boole as "conditions of possible experience" (Pitowsky, 1994). Pitowsky convincingly argues that quantum phenomena violate Boole's "conditions of possible experience" as these are predicated on an assumption of realism. As quantum phenomena do not always have definite properties at all times, like marbles being pulled from an urn, they can violate probabilistic relationships expressed in these inequalities.
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 Figure 4A demonstrates probability relationships amongst the three averaged measurement outcomes about three qualia observables, Color={red, purple, orange, … }, Position={up, down, center, left, right}, and Shape={circle, octagon, hexagon,...}. Let's say, you are briefly presented with an object and you experience it with associated (narrow-sense) qualia. In classical theory, these qualia should stay the same regardless of which of two observables you report. Let Prob(C='red')=p(R), Prob(S='circle')=p(C), and Prob(P='left')=p(L) represent the probability that the averaged measurement outcomes of your qualia observables of the object is red, circular, and on 702 the left, respectively. Then, we obtain that $p(R)-p(R,C)-p(R,L)+p(C,L)$ has to be always non-negative. This is easily confirmed from a Venn diagram (Figure 4A).

 Now, imagine the object was "masked" to reduce its visibility or two such objects are simultaneously tested. The three properties can be randomly changed from trial to trial. In such a 707 situation, your answers are likely to become probabilistic, that is Prob(C='red'), Prob(S='circle'), 708 Prob(P='left') are all smaller than 1. But, answers will still have to satisfy various probabilistic 709 constraints. For example, $p(R)$ - $p(R, C)$ - $p(R, L)$ + $p(C, L)$ has to be greater than or equal to 0, if these qualia properties follow the common sense assumptions regarding the objects being observed. Boole termed such probabilistic constraints "conditions of possible experience". It is worth noting that classical intuitions regarding the averaged measurement outcomes are so entrenched, it is hard to imagine how things could be otherwise. Violations of such Venn diagram constraints can physically arise and are even easy to demonstrate in a classroom using just 3 polarizers (Figure 715 4B and C, https://www.youtube.com/watch?v=zcqZHYo7ONs). This is an excellent demonstration to become familiar with the interesting reality of quantum phenomena, directly observable at the macro level.

719 Bruza and colleagues (Bruza et al., 2023) examined this constraint for qualia of a face. They considered three qualia observables. Whether faces appear trustworthy={yes, no}, dominant={yes, no}, and intelligent={yes, no} (Figure 4D). It turned out that in this case, the Boole's "possibility of experience" was violated (i.e.,g p(A)-p(A,B)-p(B,C)+p(C,A)<0), implying that the simple classic 723 probabilistic picture in Figure 4A is inappropriate¹³.

 Several extensions to the above task are possible. For example, it is plausible that the degree of violation of the Bell inequality may depend on the characteristics of the qualia. If this were the case, performing the same face experiment but with reduced visibility might induce greater violations of the Bell inequality. Visual psychophysics offer a multitude of techniques to reduce visibility of an object (Kim & Blake, 2005; Stein & Peelen, 2021). As mentioned in the opening section, one of the fundamental visibility manipulations is masking. It is interesting to note that 731 masking among three objects (Breitmeyer et al., 1981; Dember & Purcell, 1967) has been reported to be quite complex and might reveal a promising alternative demonstration of Bell inequality violations.

 Note that this does not mean that the quantum-like explanation is unique and the only way to explain this result. Rather, quantum theory is able to bring together a body of insights and mechanisms, in a coherent, axiomatic framework.

 One might argue that properties of faces, such as trustworthiness, dominance, and intelligence are not directly experienced qualia, but rather they are cognitively inferred constructs or concepts (Kemmerer, 2015; McClelland & Bayne, 2016). It would be a fruitful future experiment to examine if similar conclusions can be obtained when using more perceptual aspects of qualia of an object, such as color, orientation, size, location, and so on.

 To sum up, one explanation for a violation of a Bell inequality is that the underlying phenomena do not have well-defined properties that exist prior to observation and are distributed in a certain manner (Pitowsky, 1994). Consequently, when the inequality is violated, there is reason to believe that the phenomena are indeterminate prior to measurement. While superficially simple, definitive 745 tests of such inequalities are subject to several checks and assumptions (Blasiak et al., 2021), and 746 this makes it hard to definitely establish the inference from violations to indeterminacy.

While the fundamental ideas are fairly simple, almost no research on qualia has adopted a task

design, where three qualia observables are measured under three states. This is understandable

- given that it would be difficult to motivate such a task or interpret the results, in the absence of a
- quantum-like theoretical framework. We believe there is a huge opportunity to test novel ideas
- about consciousness with the QQ formulation involving three or more observables.
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Figure 4. Classical probability predictions and their violations in perceptual and quantum phenomena.

 A) Venn diagram of Boole's idea of possible experience. B) Intuitive physical demonstration of the violation of the Venn diagram constraints using polarizers. See

 https://www.youtube.com/watch?v=zcqZHYo7ONs. The main idea is this: prepare 3 polarizers. By arranging two of them, you can completely block any light through them. That is, the probability of

passing photons across two polarizers can be set to 0. Then, insert a third polarizer between the

two. Depending on the angle of the third, the three filters can pass more photons, and thus the

- output beam would be brighter at the intersection of the three polarizers. C) An explanation of (B)
- with a quantum projection scheme. Assume the state can be influenced by measurement. After we
- 766 project the initial state Ψ to the ↑ axis, further projection to the \rightarrow gives 0 length, which corresponds
- 767 to a perfect block of photons. However, if we project to the ∧ axis, after the ↑ one, then third
- 768 projection to the \rightarrow gives a non-zero length, explaining why more photons pass through three filters
- 769 than just the two original ones. D) A face used in (Bruza et al., 2023), where the relationship in A)
- does not hold for three aspects of the face (dominance, trustworthiness, and intelligence).
- Consequently, there is reason to believe that some of these facial traits were indeterminate prior to judgment.
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4.3 Dual-task interference and non-interference between qualia in terms of incompatible and compatible observables.

 The relationship between consciousness and attention is one of the most debated topics in psychology, neuroscience and philosophy (Block, 2007; Bor & Seth, 2012; Bronfman et al., 2019; Cohen et al., 2012; Dehaene et al., 2006; Hardcastle, 1997; Iwasaki, 1993; Koch & Tsuchiya, 2007; Lamme, 2003; Maier & Tsuchiya, 2021; Mole, 2008; Pitts et al., 2018; Tallon-Baudry, 2011; van Boxtel et al., 2010a). QQ is quite consistent with the known empirical findings. Moreover, QQ makes further testable predictions which are critical to empirical research in this area.

 Traditionally, sensory inputs are considered to be filtered by attention first (Figure 5A), implying that attention is necessary for consciousness. Information selected with attention is experienced as qualia and subsequently reported in a feedforward manner. Only some aspects of sensory input are attended, which ostensibly give rise to particular qualia. Behavioral reports reflect the experienced qualia. In this model, typically, attention is considered as a single limited resource and any task consumes some amount of attention.

 This view goes against empirical findings concerning reports of sensory inputs outside of attention. Among many empirical findings, a particularly intriguing one is a pattern of the tasks that consume almost all attention and those that do not consume any attention, as shown in Figure 5B. These properties of task combinations have been documented over the years within the "dual task" research program (Braun & Julesz, 1998; Braun & Sagi, 1990; Bronfman et al., 2019; Fei-Fei et al., 797 2005; Matthews et al., 2018; Pastukhov et al., 2009; Reddy et al., 2004). For example, conscious experience of genders presented at the periphery do not differ with or without performing a difficult central letter task. Meanwhile, the experience of red/green bisected disks becomes totally unclear under a dual-task with the same central task (Reddy et al., 2004, 2006). Notably, this is even the 801 case when the disk and the face are superposed transparently at the same location (Matthews et 802 al., 2018). One possible explanation of this pattern is the existence of attention-free specialized 803 modules in the cortex, possibly due to biological significance or extended training (VanRullen et al., 2004).

- There are many alternatives to the traditional view of attention and consciousness. One view considers consciousness and attention to operate independently (Figure 5C) (Koch & Tsuchiya, 2007; Lamme, 2004). In this scheme, unattended conscious and attended unconscious processes are both possible. Attention and consciousness do not proceed in a feedforward manner. While 810 this view is consistent with empirical findings, it does not explain how consciousness and attention interact dynamically.
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- The QQ hypothesis (Figure 2, Figure 5D) explicitly considers how qualia can be affected by attention through the formalism of instruments. This does not mean that all qualia are equally affected by attention, as demonstrated by the dual task. In fact, QQ provides two novel explanations about why a given pair of tasks may not interfere with one another.
- One explanation has to do with the existence of "commutative" qualia. While any process is 819 generally noncommutative (See 3.6.1), in quantum theory, some observables, called "centers", are 820 always commutative with any other observables. Centers do not show any order effects. Such 821 observables include mass. It is plausible that some types of qualia (e.g., extreme pain, bright light, 822 loud sound) may also behave like centers and be commutative with other types of qualia. These would also be predicted to be less affected by states of measurement including attention. This is an empirical question for future research, which can be addressed by testing the presence of order effects in similarity experiments, for example.
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 Another explanation relates to the idea of "incompatibility". In quantum theory, when the properties 828 of two or more observables cannot not be generally established together, these observables are called "incompatible". According to QQ, pairs of qualia observables that cannot be simultaneously established are deemed "incompatible".

832 From the QQ perspective, it is important to point out that, in many dual tasks, a letter discrimination 833 task is used as the primary difficult fixation task (Matthews et al., 2018; Tsuchiya & Koch, 2015). Thus, the conclusions from these studies may be revealing "incompatibility" between qualia observables of letters and others. In other words, some qualia observables, such as face gender (Matthews et al., 2018) and the presence of animals in a natural scene (Li et al., 2002) (Figure 5B top row), may just be "compatible" with a letter qualia observable. These qualia observables may be "incompatible" with others. If the attentional interference happens only at the task level, we should not expect systematic patterns in interference *and* order effects. However, if interference is 840 a result of the incompatibility between specific qualia combinations, then interference would result 841 in specific order effects with a quantitative explanation based on a quantum-like model (Epping et al., 2023).

 Reconsidering the patterns of attentional limits in terms of incompatibilities between observables 845 might allow novel insights into the qualia-attention research. With traditional psychological theories, 846 we consider attention as a fixed resource (Joseph et al., 1997), which can amplify aspects of 847 qualia, it is hard to explain why in some visual illusions stronger attention leads to poorer visibility 848 of the target (Schölvinck & Rees, 2009; van Boxtel et al., 2010b). Further, it is also hard to understand why distracting participants sometimes leads to better psychological performance in various paradigms (Koch & Tsuchiya, 2007; Tsuchiya & Koch, 2015). Attention can change the 851 neuronal circuitry momentarily (Gilbert & Li, 2013; Harris & Thiele, 2011), thus it might be possible to understand such effects as a change, for a pair of observables, from incompatible into compatible. This change can be formalized as an instrument where attention as a state affects qualia observables. This explanation offers a coherent explanation of these seemingly odd relationships between qualia and attention.

 Unlike the limited resource model, QQ predicts an existence of pairs of "compatible" qualia observables, even though each one consumes a significant amount of a presumed attentional "resource". QQ also predicts pairs of "incompatible" qualia observables, which cannot be

- simultaneously established, even if each does not consume much attentional resource.
- Discoveries of such pairs of qualia observables would further support QQ.
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Figure 5. QQ is compatible with the empirical findings about the relationship between attention and qualia.

867 A) Traditional feedforward models of sensory input, attention, qualia, and reports (taken from Lamme, 2004). B) Top row: a list of peripheral perceptual discriminations that can be conducted simultaneously with difficult letter discrimination tasks at the fixation. For example, conscious experience of genders presented at the periphery does not differ with or without performing a 871 difficult central letter task (Matthews et al., 2018). Bottom row: a list of tasks that cannot be performed concurrently with the letter task. One novel interpretation of such results is using the 873 notion of incompatibility. Incompatibility is the inability to jointly establish the values of two or more 874 observables. Modified from (Tsuchiya & Koch, 2015). C) A static view of consciousness and 875 attention that is consistent with dissociations between qualia and attention (Maier & Tsuchiya, 2021). D) Quantum qualia hypothesis (reproduced from Figure 2).

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5. Conclusion

 We proposed a Quantum-like Qualia (QQ) hypothesis based on a quantum theoretical framework (e.g., noncommutative observables, states, and instruments; Figure 2, Table 1). QQ proposes qualia as observables, not the "things" or results of "cognitive processes" as traditionally assumed. QQ explains intuitive and known properties of qualia, such as their inherent indeterminacy, dynamics, and interaction with attention. Predictions from QQ can be empirically tested with demonstrations of asymmetry in perceptual similarity judgements, violations of the Bell inequality, 889 and apparent incompatibilities between particular qualia. Amongst these, particularly powerful are demonstrations of Bell inequality violations. In order to test them, we minimally need to measure 891 three observables, two at a time across three different states (Figure 4). Such experiments have been rarely conducted systematically, due to the lack of theoretical background and motivation.

- Additionally, there are subtle loopholes that need to be considered, before compelling empirical
- 894 evidence is provided that substantiates our claim that qualia are indeterminate (Atmanspacher & Filk, 2019; Basieva et al., 2019; Emary, 2017). In physics, it took more than twenty years from the
- 896 theoretical proposal by Bell through to the initial experiment by Clauser and then to the compelling
- demonstration by Aspect (Section 4.2.1). Will a similar pathway await the Quantum-like Qualia
- hypothesis in the future? Only time will tell. With increasing evidence that QQ provides a coherent
- explanation on the mathematical structure of qualia, QQ may well emerge as a promising
- mathematical and philosophical framework to link qualia and the brain.
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