Journal of Physics: Conference Series

PAPER • OPEN ACCESS

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To cite this article: K Rajana and A Giaralis 2024 *J. Phys.: Conf. Ser.* **2647** 032012

Optimal tuning and assessment of non-grounded regenerative tuned mass damper inerter (RE-TMDI) configurations for concurrent motion control and energy harvesting

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Abstract. This paper addresses the optimal tuning and numerical performance assessment of regenerative tuned mass damper inerters (RE-TMDIs) in three different configurations with non-grounded inerters attached to cantilevered primary structures under Gaussian white noise base excitation. The studied RE-TMDI configurations behave linearly and differ in the placement of the electromagnetic motor (EM), modelled as viscous damping element used for transforming kinetic energy to electricity, with respect to the inerter element. The primary structure is modelled as a linear damped generalized single-degree-of-freedom system, while a connectivity index is used to account for the location of the two RE-TMDI attachment points to the primary structure. A bi-objective optimization problem formulation is adopted and numerically solved for determining optimal RE-TMDI stiffness and EM damping coefficients that minimize primary structure displacement variance and maximize the available energy for harvesting by the EM. Parametric numerical results are reported for different RE-TMDI configurations, connectivity, inertance, secondary mass ratio and relative weighting between the two optimal design objectives. These results demonstrate that improved energy generation and vibration suppression is concurrently achieved with increasing inertance and/or increasing the distance of the host structure locations where the RE-TMDI is attached to. Recommendations are provided establishing the most advantageous RE-TMDI configuration.

1. Introduction

The efficacy and applicability of passive tuned mass dampers (TMDs) for motion control of dynamically excited engineering structures is well-established in the scientific literature and has been demonstrated in several real-life structures (e.g. [1]). Their effectiveness rely on the inertia of an oscillating (secondary) mass tuned, through stiffeners, to the dominant vibration mode of the host (primary) vibrating structure. Further, TMDs employ an energy dissipation device with judicially chosen damping coefficient which is inserted between the secondary mass and the primary structure to absorb efficiently the kinetic energy from the host structure and the secondary mass [2]. In this context, TMDs can further serve as kinetic energy harvesters by using dissipative devices capable of transforming kinetic energy to electricity [3]. In fact, for large-scale structures with low-frequency dynamics, electromagnetic motors (EMs) coupled with appropriate energy harvesting/storage circuitry have been found to be quite effective to endow energy generation capabilities to TMDs, oftentimes termed regenerative TMDs [4]. Nevertheless, the design/tuning of the stiffness and damping properties of dual functioning TMDs given secondary mass, primary structure, and excitation, becomes challenging as the objectives of motion control and energy harvesting may be conflicting [5,6].

Over the past decade, inerter-based dynamic vibration absorbers, such as the tuned mass damper inerter (TMDI) [7], have emerged in the literature, achieving enhanced motion control by leveraging the inertia property of inerters [8]. Inerters are mechanical devices resisting the relative acceleration developing at their two terminals (ends) by the inertance constant (inertia property) which is independent of the device physical mass [9]. In this respect, TMDI configurations with two different attachment locations to the primary structure were shown to provide enhanced motion control capability as inertance increases and/or as the difference of the modal coordinate of the two attachment locations increases [10-12] (see Fig.1a). Importantly, both these effects can be readily leveraged to improve the vibration suppression efficacy of TMDIs. First, actual inerter devices with scalable inertance of several orders of magnitude larger than the device physical mass (several thousands of tons of inertance) have been prototyped and tested [13-15]. Second, the modal coordinate difference of the TMDI attachment locations (see Fig.1b) increases by increasing the distance between the two locations (e.g. spanning several floors in multi-storey buildings) [16,17], or by implementing either local structural modifications to the primary structure (e.g. creating soft floors in multi-storey buildings) [18,19] or global primary structure shaping aiming for increased modal curvature [20].

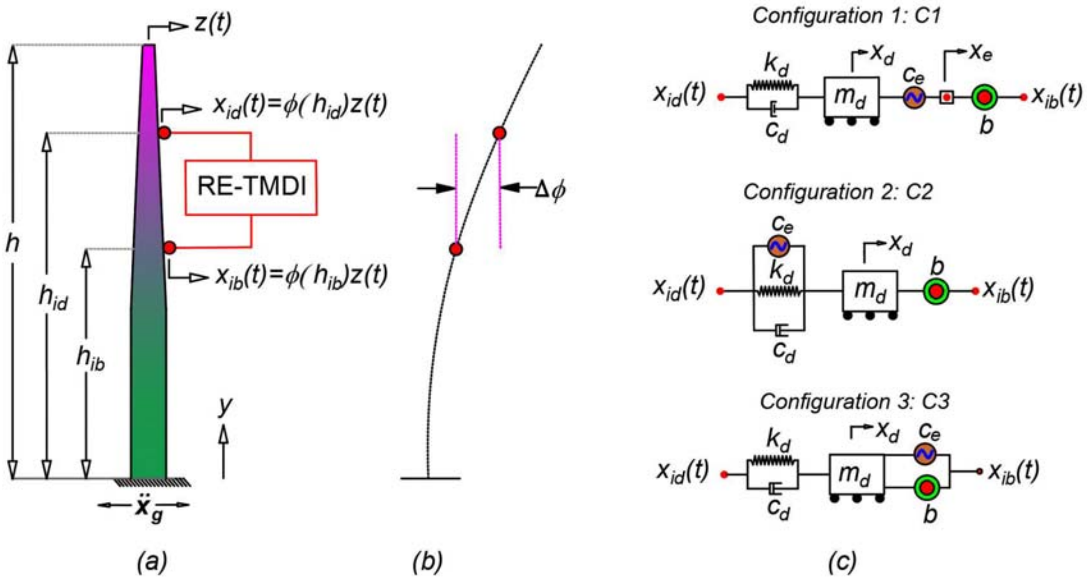

Fig. 1. (a) *Continuous generalized SDOF primary structural system model under base-excitation equipped with RE-TMDI (b) dominant mode shape of the uncontrolled primary structure (c) Considered regenerative tuned mass damper-inerter (RE-TMDI) configurations.*

Moreover, the potential of various regenerative tuned mass damper-inerter (RE-TMDI) configurations featuring EMs has been considered in the literature for concurrent motion/vibration control and energy harvesting, the most widely studied of which are shown in Fig.1c. Specifically, the potential of RE-TMDI configuration (C1) has been parametrically studied by Salvi and Giaralis [21] and Giaralis [22] without considering any rigorous optimal tuning criterion. Similarly, the concurrent efficacy for motion control and energy harvesting of the RE-TMDI configuration (C2) has been assessed by Marian and Giaralis [23] (with grounded inerter connection) and by Petrini et al [16] (non-grounded inerter connection) without optimizing for energy harvesting maximization. Lastly, RE-TMDI configuration (C3) has only been studied for grounded inerter connection [24,25]. Configuration 3 (C3) involves placing the EM damper in parallel with the inerter (Asai *et al*., 2017, Joubaneh and Barry, 2019).

In this regard, the RE-TMDI configurations in Fig.1c have not been systematically and comparatively studied under optimal tuning conditions for minimizing structural vibration and/or maximizing energy harvesting potential. To this end, this pilot study makes such a comparison by applying a bi-objective optimal tuning approach to all three different configurations attached to a generic cantilevered structure with a single dominant mode shape under white noise ground excitation. The aim is to reveal potential trade-offs between motion control and energy generation as inertance and/or RE-TMDI connectivity varies. The presentation begins by describing the dynamical modelling and the mechanical properties of cantilevered structure equipped with RE-TMDI.

2. Modelling of RE-TMDI equipped structures

Consider the base-excited cantilevered primary structure with height h, modelled as a generalized damped single-degree-of-freedom (SDOF) dynamic system with continuously distributed mass $m(y)$ and flexural rigidity $EI(y)$ properties, $0 \le y \le h$, shown in Fig. 1(a). The system response is governed by its first mode of vibration $\varphi(y)$, as shown in Fig.1(b) and the free-end displacement $z(t)$ is taken as the generalized coordinate, where t denotes time. In this regime, the generalized mass, m_s, stiffness, k_s, and damping, c_s, coefficients are determined as

$$m_s = \int_0^h m(y)\left(\varphi(y)\right)^2 dy \quad ; \quad k_s = \int_0^h EI(y)\left(\frac{d^2\varphi(y)}{dy^2}\right)^2 dy \quad \text{and} \quad c_s = a_c k_s \tag{1}$$

where a_c is the stiffness proportionality coefficient to define the inherent structural damping.

The considered primary structure is equipped with a RE-TMDI unit attached to the structure at two different arbitrary locations with heights h_{ib} and h_{id}. Three different RE-TMDI configurations are studied in this work, as detailed in the introduction, represented by the linear mechanical models shown in Fig.1(c). The configurations comprise a secondary mass m_d, a viscoelastic connection of the secondary mass to the structure at location h_{id} with a tuning stiffness k_d and damping coefficient c_d, modelling energy losses at the connection due to friction and other local effects, an ideal inerter device/element with inertance b which connects the secondary mass to location h_{ib} and an EM acting as energy harvester with equivalent damping coefficient c_e. It is noted that the modelling of the EM as a linear mechanical dashpot assumes a purely resistive energy harvesting circuit. Whilst this modelling assumption does not account for potential nonlinear behavior of the circuitry, it is deemed sufficient for the comparative quantification of the available energy for harvesting as the properties of EH-TMDI are let to vary [5,16].

Based on the above modelling assumptions, the equations of motion of RE-TMDI equipped cantilevered structure can be readily derived using standard structural dynamics techniques in terms of the generalized coordinate $z(t)$, the secondary mass displacement $x_d(t)$, and, in the case of configuration (i), the displacement $x_e(t)$. Derivations of equations of motion for each of the three RE-TMDI configurations are herein omitted due to lack of space. The interested reader is directed to the work of Wang and Giaralis [20] offering detailed derivation for the case of TMDI (configuration (C2) with c_e=0). The three different resulting dynamical systems are characterized by 8 quantities, defined as

$$\begin{aligned}
&\omega_s = \sqrt{\frac{k_s}{m_s}} \quad ; \quad \omega_d = \sqrt{\frac{k_d}{m_d + b}} \quad ; \\
&\beta = \frac{b}{m_s} \quad ; \quad \mu = \frac{m_d}{m_s} \quad ; \quad \Delta\varphi = \varphi(h_{id}) - \varphi(h_{ib}) \\
&\xi_s = \frac{c_s}{2m_s\omega_s} \quad ; \quad \xi_d = \frac{c_d}{2(m_d + b)\omega_d} \quad ; \quad \xi_p + \xi_e = \frac{c_e}{2(m_d + b)\omega_d}
\end{aligned} \tag{2}$$

These are the structural natural frequency, ω_s, the RE-TMDI frequency, ω_d, the inertance ratio, β, the mass ratio, μ, the difference of the modal coordinates of the RE-TMDI connecting locations, $\Delta\varphi$, hereafter termed connectivity index (Fig.1b), the inherent structural damping ratio, ξ_s, the parasitic damping ratio of the connection to the structure, ξ_d, and the EM damping ratio which is split into the energy harvesting part, ξ_e, and the energy loss part ξ_p.

3. Bi-objective optimal tuning of the RE-TMDIs under white noise excitation

To draw meaningful comparisons between motion control and energy harvesting performance, a bi-objective optimal design formulation is herein devised for tuning the different RE-TMDI configurations of Fig.1. Specifically, the formulation pursues minimization of the primary structure dynamic response in terms of the displacement variance of the h_{id} location under white noise excitation, $\sigma^2_{x,id}$, and maximization of the energy available for harvesting by RE-TMDI under white noise excitaiton which is proportional to the relative velocity variance across the EM terminals, $\sigma^2_{\dot{x}_{EM}}$. Mathematically, the optimal design problem is expressed as

$$\min_{\mathbf{p}}\left\{\gamma J_1(\mathbf{p})+(1-\gamma)J_2(\mathbf{p})\right\} \quad , \quad \mathbf{I_{lb}} \le \mathbf{p} \le \mathbf{I_{ub}} \tag{3}$$

where J_1 and J_2 are performance indices associated with the structural dynamic response and the harvested energy, respectively, γ is a weighting factor taking values within [0,1] to regulate the relative importance of J_1 and J_2 performances in the design problem, and **p** is the vector of design variables with lower and upper bounds $\mathbf{I_{lb}}$ and $\mathbf{I_{ub}}$, respectively. The performance indices in Eq.(3) are defined as

$$J_1 = \frac{\sigma^2_{x,id}}{\min\left\{\sigma^2_{x,id}\right\}} \quad \text{and} \quad J_2 = -\frac{\sigma^2_{\dot{x}_{EM}}}{\max\left\{\sigma^2_{\dot{x}_{EM}}\right\}} \tag{4}$$

where $\min\left\{\sigma^2_{x,id}\right\}$ is obtained by optimally designing the RE-TMDI for minimizing the structural response only (single objective optimization), while $\max\left\{\sigma^2_{\dot{x}_{EM}}\right\}$ is obtained by optimally designing the RE-TMDI for maximizing energy harvested with no provision for structural response mitigation (single objective optimization). Notably, the normalization in the performance indices ensure a numerically balance in the weighted sum definition of the objective function in Eq.(3), including the limiting cases of γ=0 (EH-TMDI tuning for energy generation maximization only) and γ=1 (EH-TMDI tuning for structural motion mitigation only).

In the ensuing numerical investigation, RE-TMDI optimal tuning is pursued using two design variables, that is, energy harvesting damping ratio and RE-TMDI frequency ratio

$$\mathbf{p} = \left[\xi_e, \lambda = \frac{\omega_d}{\omega_s}\right], \tag{5}$$

while the remaining properties in Eq.(2) are taking fixed values. The optimization problem in Eq.(3) is numerically solved using the standard pattern search algorithm implemented in the built-in MATLAB® function 'fminsearch' within sufficiently wide search range such that the design variables do not hit the boundaries of the search domain.

4. Parametric Numerical Investigation

In this section, optimal RE-TMDI tuning properties and performance of optimally tuned RE-TMDI configurations for motion control and energy harvesting are parametrically investigated. The performance is quantified in terms of motion control and energy harvesting under white noise base excitation using the following dimensionless indices

$$\frac{\sigma^2_{x,id}}{\sigma^2_{x,o}} \quad \text{and} \quad \frac{\mathrm{E_{em}}}{\mathrm{E_{e,o}}} = \frac{c_e\sigma^2_{\dot{x},id}}{c_s\sigma^2_{\dot{x},o}}, \tag{6}$$

respectively, where $\sigma^2_{x,o}$ is the displacement variance of the uncontrolled generalized SDOF primary structure, $\mathrm{E_{em}}$ is the available energy for harvesting by the RE-TMDI, and $\mathrm{E_{e,o}}$ = $c_s\sigma^2_{\dot{x},o}$ is the energy dissipated by the inherent damping of the uncontrolled primary structure. In all the considered cases, the same primary structure properties are taken ξ_s= 2% and ω_s= 2π, while the parasitic damping ratio is fixed at ξ_d= 1% and the damping ratio corresponding to EM energy loss is fixed at ξ_p= 0.5%.

The presentation begins by examining in Fig.2 the optimal design parameters for structural response minimization only (γ=*1.0*) and for energy harvesting maximization only (γ=0) for varying inertance ratios within the range β=5% to β=100% for Δφ= 0.25 and for two different values of secondary mass ratios (μ = *0.5%* and *5%)*.

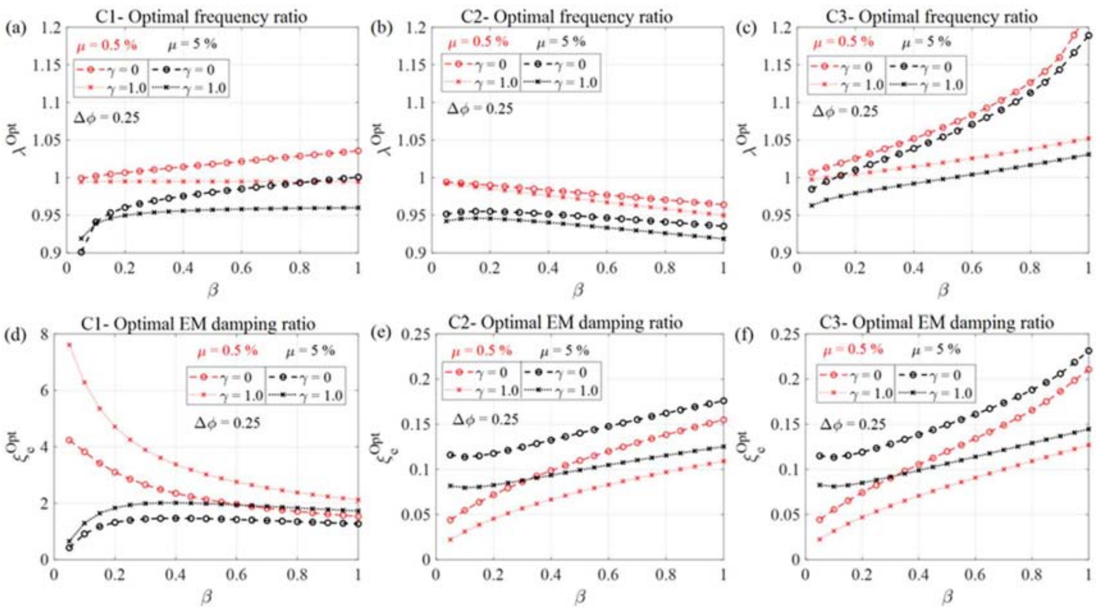

Fig 2. *Bi-objective optimal design variables for two secondary different mass ratios with respect to different inertance ratio for two different mass ratio and weighting factor values and for Δφ*=0.25.

It can be observed that in configuration C1, as the inertance ratio increases, there is an increase of the optimal frequency ratio for both secondary mass ratios. Further, configuration C3, appears to be sensitive to changes in the performance index definition with respect to the inertance ratio. Configuration C2, on the other hand, is not sensitive to the optimal design criterion. Furthermore, in Fig.2b, it can be seen that the optimal EM damping coefficient for C1 is significantly higher than that of C2 and C3. However, as the inertance ratio increases, the optimal EM damping coefficient for C1 drops, whereas for C2 and C3 the optimal EM damping ratio increases with higher inertance ratio. Additionally, when considering EM damper energy maximization as the performance function (e.g., for $\gamma=0$), a lower EM damping ratio is required for C1 as compared to $\gamma=1.0$. This means that maximising the EM damper energy necessitates the development of higher relative velocity across the EM terminals (higher kinematics) which enables the higher EM damping energy for C1. However, this is not the case for C2 and C3. Therefore, for EM damping energy maximisation, a lower relative velocity is required for energy maximisation. As the inertance ratio β increases, the difference between the optimal response of the ξ_e increases significantly for C2 and C3 when comparing $\gamma=0$ against $\gamma=1.0$.

Next, Fig. 3 illustrates that all three configurations exhibit the same trend with respect to the inertance ratio β. This trend is favourable as increasing β leads to a reduction in the displacement response of the primary structure and an increase in EM damping energy. Since the inertance level is scalable [9,15], target EM damping energy harvest requirements can be set without compromising on structural motion control. Additionally, the trade-off between displacement and EM damping energy responses is minimal as the weighting factor γ varies from 0 to 1. The performance trade-off between the performance index is slightly higher for Configurations 2 and 3. The secondary attached mass μ is critical for both motion control and energy harvesting for lower β which may be the case if the inertance control force is too high. However, as the inertance ratio increases, the need for a larger attached mass becomes unnecessary as the response converges with the use of larger inertance ratios.

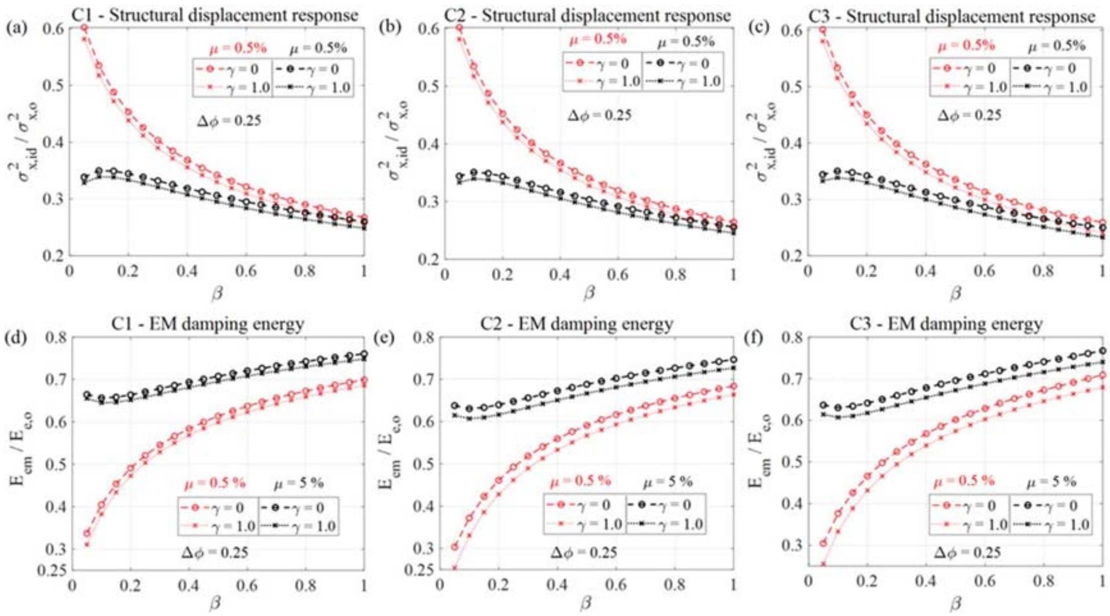

Fig 3. *Performance assessment for two secondary different mass ratios with respect to different inertance ratio for two different mass ratio and weighting factor values and for Δφ=0.25.*

Lastly, Fig.4 offers comparisons of the three optimal designed configurations (on the same plot) to assess motion control and EM damping energy harvesting with respect to the inertance ratio and modal connectivity index. On this occasion, minimization of structural displacement is set as the optimal tuning criterion. In the top row of panels, perofrmance indices against the inertance ratio β, with $\Delta\phi=0.25$ are plotted, while in the bottom row of panels, β is fixed to *0.25* and performance indices are plotted against $\Delta\phi$. The secondary attached mass μ is fixed at *0.5%*.

In Figs. 4(a) and 4(c), an increase in the inertance ratio β and inerter modal connectivity index $\Delta\phi$ results in a reduction in the normalised structural displacement variance for all three configurations. Configation 3 outperforms the other two configurations for higher β and $\Delta\phi$ values in terms of motion control performance. In Figs. 4(b) and 4(d), it can be observed that C1 performs best for energy harvesting potential at lower β and $\Delta\phi$ values, while C3 offers comparable energy harvesting capability as Configuration 1 as these variables increase. Therefore, it can be concluded that C3 is the preferred option since β can be scaled under the provision that accommodating inertance forces by the primary structural system does not pose significant design challenges.

In terms of performance, C1 is preferable as it leads to increased energy harvesting for any given value of β and Δφ, with negligible compromise in vibration suppression for $\beta>0.4$ and/or $\Delta\varphi>0.3$ However, this improved energy harvesting comes at the cost of significantly higher damping ratio for C1 which may not be practicable. At the same time, the series device connectivity in C1 is technologically more challenging to implement in practice than the parallel device connectivity in C3 which can be readily achieved through rotational electromagnetic motors driven by ball-screw inerter mechanisms. In this regard, C3 may be more beneficial in practical terms.

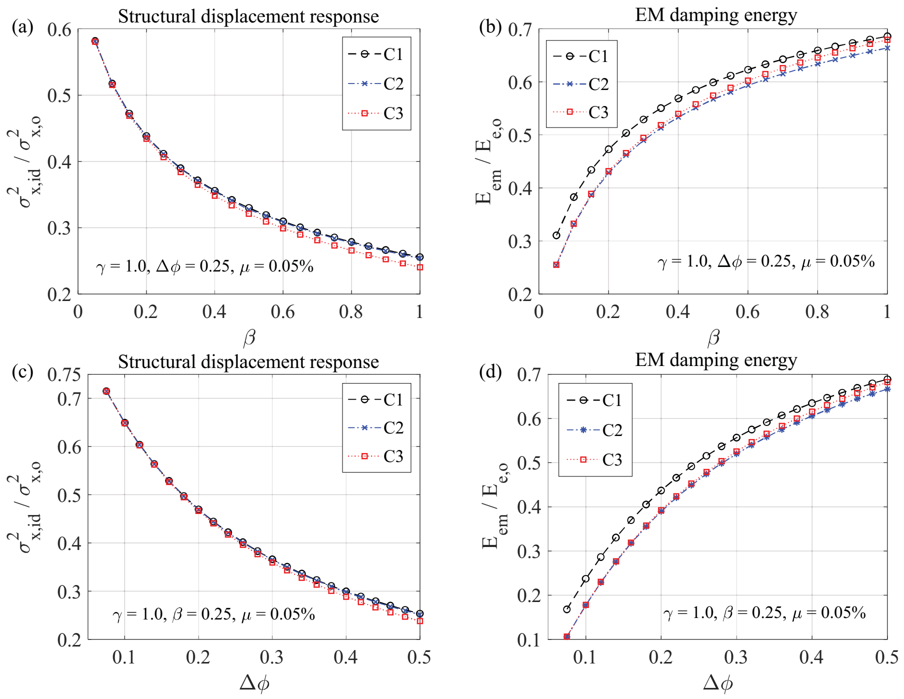

Fig 4. *Comparison of the optimal motion control performance and EM damping energy responses for γ =1.0 with respect to inertance ratio and modal connection of the inerter $\Delta\phi$. (a,b) variation of β for fixed $\Delta\phi$ = 0.25 and (c,d) variation of $\Delta\phi$ for fixed β = 0.25*

5. Concluding remarks

This study adopted a bi-objective optimization approach for the optimal design of three different non-grounded RE-TMDIs attached to cantilevered primary structures for either motion control or EM energy harvesting. The key findings are summarized as:

- The trade-off between motion control and energy harvesting is negligible in all the considered configurations under white noise excitation. This is a notable improvement compared to previous studies on the optimal design of RE-TMDI which did not consider EM damping ratio as a design variable.
- Increasing the inertance ratio β and modal connectivity index $\Delta\phi$ leads to an increase in EM damping energy and improved motion control for all three configurations. Therefore, it is recommended to use higher $\Delta\varphi$, when possible, and β with a lower mass ratio μ for the attached secondary mass.
- There is no significant variation in the optimal displacement response among the three optimally designed configurations. However, for higher values of β and $\Delta\phi$, Configuration 3 offers better motion control performance.
- In terms of energy harvesting, Configuration 1 performs best for lower inertance ratios (e.g., $\beta < 0.9$). However, as the β approaches 1.0, the energy that can be harvested is similar for both Configurations 1 and 3.

Acknowledgements

The first author gratefully acknowledges the financial support of City, University of London through a funded PhD studentship. The second author acknowledges the partial support of Royal Academy of Engineering and of ABL Group through the industrial Fellowship award IF2223-209 and the partial support of Khalifa University, Abu Dhabi through a Faculty Start-Up grant award.

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