DESIGN OF CLOSE-RANGE PHOTOGRAMMETRY FOR PRECISION, RELIABILITY AND SENSITIVITY

by

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A Thesis submitted for the Degree of Doctor of Philosophy in the Department of Civil Engineering at The City University

THE CITY UNIVERSITY
LONDON

NOVEMBER 1985

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ACKNOWLEDGEMENTS

The research described herein was undertaken in the Department of Civil Engineering of The City University. During the course of the research the author received valuable advice and helpful suggestions from many people.

In particular, I wish to express my gratitude to the following: Mr.Mike A.R.Cooper, who supervised this work, for his constant encouragement, his valuable advice and his guidance throughout the duration of the project.

Professor J.E.Alberda of Delft University of Technology, for sending some of his own publications and some of Prof.W.Baarda's. Dr.M.Molenaar of International Institute for Aerial Survey and Earth Sciences who kindly sent me some of his publications.

Mr.S.I. Granshaw of Oxford Polytechnic for his generous assistance and long useful discussions and for providing some of Prof.A.Gruen's publications.

[redacted], for their useful discussions.

[redacted] for providing the real data used in this study.

[redacted] for his help in gaining familiarity with photography by the UMK 10/1318 camera produced by Zeiss (Jena).

The staff of The City University Computer Centre, particularly [redacted] whose effort is appreciated.

The staff of the University of London Computer Centre (ULCC) especially [redacted] whose help is acknowledged.

[redacted] who typed the manuscript under a tight schedule.

The EGYPTIAN Government for their financial support.

Finally, but not least significantly, I wish to express my thanks and love to my wife and my daughters [redacted]

ABSTRACT

In the measurement of structural deformation by analytical close-range photogrammetry, design is a necessity. The design of precision, reliability and sensitivity for photogrammetry alone and photogrammetry with measured distances is studied. The methods which have been developed for design based on the three criteria above mentioned can be used to meet the specifications in many deformation surveys.

The problem of optimal design can be classified in terms of four interconnected problems: zero-, first-, second-, and third-order design. Zero-order design problems have been solved by adopting the free bundle adjustment method (inner constraints). As a "one-number" indicator of the precision, for sake of comparative studies, the mean variance of the object point co-ordinates has been adopted. For reliability assessment, the redundancy number lying between 0 and 1 has been discussed and shown to be advantageous over other measures especially in pre-analysis studies.

Of primary concern in the design of a network is the sensitivity of that network for the detection of point movements at some given level of statistical significance.

A new method has been developed dealing with the sensitivity of a series of formulated deformation models and successfully applied to the different simulated networks.

The main conclusions are that (1) the more cameras used,the more precise the network becomes; (2) not only the number of cameras but also the configuration of the cameras affects reliability in a significant way; (3) the higher the number of cameras used,the less sensitive the network is to hypothesised deformation; and (4) the addition of distances, measured by the conventional methods, to a large degree, does not significantly improve the results of the precision, reliability and sensitivity. This leads to the advocation of the argument that photogrammetry alone, in deformation measurement, can be used without any survey measurements.

NOTATION

Most of the symbols and abbreviations used in this thesis are defined at their first usage in the text. Where a symbol has only a special localised meaning it is defined in the text and not included in this list.

ℓ	vector of observations
x	vector of unknown parameters
x^{o}	vector of approximate values of x
Δx	vector of corrections to x^{o} to give x
A	design matrix
ℓ^{o}	vector of computed values of the observations, $\ell^{o}=F(x^{o})$
$C_{\ell\ell}$	covariance matrix of the observations, $C_{\ell\ell}=\sigma_o^2 W^{-1}$
$\sigma_o{}^2$	a priori variance factor
$\hat{\sigma}_o^2$	a posteriori variance factor
W	weight matrix of the observations
$C_{\hat{x}\hat{x}}$	covariance matrix of the unknown parameters, $C_{\hat{x}\hat{x}}=\sigma_o^2 Q_{\hat{x}\hat{x}}$
$Q_{\hat{x}\hat{x}}$	cofactor matrix of the unknown parameters, $Q_{\hat{x}\hat{x}}=C_{\hat{x}\hat{x}}$ if and only if $\sigma_o^2 = 1$
$\hat{x}$	least squares estimate of x
b	vector related to the observations, or vector of observational discrepancies, $b = \ell - \ell^{o}$
v	vector of corrections to the observations or, vector of residuals
$Q_{\hat{v}\hat{v}}$	cofactor matrix of residuals
n_s	number of cameras or photographs
n_o	number of object points

σ^2	variance
σ	standard error
σ_m^2	mean variance of object point co-ordinates
tr	trace
H_o	null hypothesis
H_A	alternative hypothesis
α	probability of type I error
β	probability of type II error
r	redundancy number
c	sensitivity parameter
χ^2	chi-square distribution
F	Fisher distribution
$E\{x\}$	expected value of x
x^T	the transpose of x
Q^-	a generalised inverse of Q
Q^+	Moore-Penrose inverse of Q
Q^{-1}	standard Cayley inverse of Q
I	identity matrix
deg	degree
rad	radian

CHAPTER 1

INTRODUCTION

1.1. Background

The photogrammetric network is essentially formed through the processes of three-dimensional spatial resection and intersection, the resection phase being the determination of the position of the camera perspective centres and the attitude of the camera axes, and the multi-ray intersections being utilised to determine object point positions. Under the most rigorous restitution procedures, these two phases are carried out simultaneously using a method termed the bundle adjustment.

A photogrammetric network shares many common features with three-dimensional geodetic networks in Euclidean space. Therefore techniques employed in the design optimisation of monitoring networks in geodesy are applicable to photogrammetry.

Due to the complexity of the general problem of optimisation of design, it is convenient to consider it as being made up of four distinctly different problems originated by Grafarend (1974) of which the first two are especially important for this study.

(i) Zero Order Design - the datum problem, i.e. the choice of the reference system.

(ii) First Order Design - the configuration problem, i.e. the selection of the network configuration.

(iii) Second Order Design - the generalised problem, i.e. the selection of the weights of observations.

(iv) Third Order Design - the densification problem, i.e. the improvement of an existing network.

The zero order design problem involves the choice of an optimal reference system for co-ordinates, given the object points, the photogrammetric network and the precision of the observations.

The first order design problem is concerned with the search for an optimal network geometry, given both the precision of the observations and criteria for the structure of the covariance matrix of the unknown parameters.

In the second order design problem the covariance matrix of the unknown parameters $C_{\hat{x}\hat{x}}$ ($C_{\hat{x}\hat{x}} = \sigma_0^2 Q_{\hat{x}\hat{x}}$, where σ_0^2 is the a priori variance factor, and $Q_{\hat{x}\hat{x}}$ is the cofactor matrix of the unknown parameters and $Q_{\hat{x}\hat{x}} = C_{\hat{x}\hat{x}}$ when $\sigma_0^2 = 1$) and the design matrix A are known. What is wanted is the weight matrix, W, required for the observations in order to achieve the ideal covariance matrix (often called criterion matrix) of the unknown parameters.

The third order design problem is associated with the improvement of an existing network by alteration of either or both of A and W. For example, this type of problem includes the densification of an existing network by the addition of new stations and/or new observations.

Before seeking any solution to the design problems, the required quality of the proposed network should be fixed. The most important criteria for a monitoring network are precision, reliability, and sensitivity.

1. Precision - This is a numerical assessment of the influence of the random observational errors on the estimated parameters (e.g. co-ordinates).

2. Reliability - For a network, reliability can be defined as its ability to detect gross errors in the observations and the determination of the effect of undetected gross errors on the co-ordinates.

3. Sensitivity - As the main task of monitoring networks is the determination of movements or the proof of the non-existence of movements, the ability to test specific deformation models, given by the sensitivity of the network (first introducted by Pelzer, 1972), is of further importance. For computation of sensitivity measures, some prediction of the movements of interest must be possible.

The last criterion, namely, sensitivity, may be more important than either precision or reliability because the sensitivity analysis produces results which can easily be explained to, and understood by people unfamiliar with the methods and principles of surveying and photogrammetry.

Clearly, if a design of a network is required, then there is need for investigating the costs. However, if the optimal design is sought, a search must be made for the observation scheme which will meet the precision, reliability and sensitivity requirements with the least cost. Obviously this cannot be undertaken unless the proposed scheme is costed. This aspect of detailed cost analysis is out of the scope of this thesis, although consideration is given to it.

There are two approaches to the solution of the aforementioned design problems. They can be solved for in an iterative (simulation) manner or by a direct (mathematical) solution. Using the simulation approach, the designer first solves for $C_{\hat{x}\hat{x}}$ using a first approximation to a good solution to the problem. Then the differences between the computed and desired $C_{\hat{x}\hat{x}}$ matrix are derived and the first approximation is updated. This process continues until the solution provides a $C_{\hat{x}\hat{x}}$ matrix close enough to the desired one. The simulation technique has been greatly improved by the use of computer interactive graphics which allow the designer to draw a proposed network on a graphics screen and to have a visual picture of how its precision relates to the design specifications (e.g. Krakiwsky et al, 1982). Moreover, Cross and Whiting (1982) have automated the technique for level networks (Fagir, 1984). However, the main disadvantage with this method is that the optimal solution may not be achieved in practice, and these systems do not employ all the criteria necessary for the design of truly optimum monitoring networks. It is important to notice that a network or part of it may be precise without being reliable at all (Ashkenazi and Crane, 1982).

The direct approach in which the required information is solved for mathematically is intuitively more pleasing. It does, however, have several problems. The W matrix (second order design problem) that results is generally fully populated and is frequently singular. Both of these properties are in direct conflict with reality (Krakiwsky et al, 1982). To the knowledge of the writer, no mathematical (analytical) optimisation has been carried out for geodetic or photogrammetric networks, which uses reliability or sensitivity as target functions.

1.2. Objectives and Methodology

Like any engineering project, a close-range photogrammetric project should be planned and designed to produce the best solution to a problem. The recent sudden collapse of Starra Dam in Italy (July 1985) indicates the practical importance of a properly implemented monitoring survey.

There is a number of methods and instruments to detect and monitor deformation in structures. Some of these instruments, inclinometers, extensometers, strain gauges, etc., are built into the structures and provide information about their internal changes and conditions. The main disadvantages of the use of such instrumentation for monitoring deformations are:

1. Costly maintenance.
2. Difficulty of interpreting the large amounts of data supplied,

and 3. One-dimensional information (e.g. case of extensometers) is often provided.

On the other hand, ground survey (e.g. triangulation and/or trilateration, levelling) are often used to accomplish the data gathering needed for monitoring surface structural deformations. A number of targets placed on the surface of the structure whose positions, carefully chosen, are determined by using one or more of these methods. Such a survey is repeated at predetermined intervals and in order to quantify the deformations, positions of the targets at each time are compared. At least two epochs are necessary. In general, no statement can be made about the moment of occurrence of the deformation. It should be noticed that one- or two- or three-dimensional information is the end product of such methods, however, they have their limitations especially in the following situations:

Firstly, when large number of targets on the structure have to be recorded.

Secondly, when hazardous or inaccessible positions are to be monitored.

Finally, if the rate of deformation is rapid,deformation inevitably would occur during the survey.

Consequently, close-range photogrammetry incorporated with a few spatial distances is thought to be the most economical alternative to the above mentioned methods. The choice of the distances in the object space is made due to the fact that it is more easy to measure distances than angles or height differences (e.g. in offshore structures or oil extraction platforms).

A photograph near-instantaneously records almost an infinite number of points, hence replacing hundreds of angular measurements.

In such a case, three-dimensional information about the structure as a whole can be provided and deformation is not likely to occur during the relatively short time of photography.

Close-range photogrammetry is based on the basic theoretical concept that the photograph, being a perfect plane, is a central projection of the object. Implicit in this concept is the condition of collinearity of the image point, the projection centre, and the object point. In reality, however, this collinearity condition is not exactly met because of the following reasons:

1. Lens distortion owing to the fact that when a light ray traverses from the object to its image it is deflected by the media through which it passes.
2. The image point is displaced from its theoretical position in a plane because of (a) film shrinkage or expansion, and (b) lack of film flatness.
3. Systematic and random errors in data reduction due to the instrumental errors of the photogrammetric instruments and the human factor.

To overcome such pitfalls, the camera supposed to be in use in this pre-analysis study, Zeiss (Jena) UMK 10/1318 available at The City University, is a metric camera. Most close-range cameras operate with glass plates which are known to be generally stable emulsion bases within narrow limits. Therefore, glass plates of format (130 x 180 mm) are considered. The best photogrammetric solution is attained when all systematic errors are corrected for and the random errors are minimised. Accordingly, in the analytical method of close-range photogrammetry, the mathematical procedure followed to correct the image co-ordinates for systematic errors in image positions on the photograph is ascribed as image refinement and the co-ordinates are referred to as the refined image co-ordinates which are then used in the collinearity equations discussed in Chapter 4.

The free bundle adjustment approach (inner constraints method) has been adopted as it provides a very useful way of making a priori estimates of precision so that a particular optimum configuration of camera positions can be arrived at (Cooper, 1981) and as a solution to the zero order design problem.

Such a method has the following characteristics:

(i) a least squares solution

(ii) a minimum norm solution

(iii) minimum trace of the covariance matrix of the unknown parameters.

In addition, from practical standpoint the following two points are made:

(a) the object point co-ordinates which are of primary importance can be constrained without bothering about the camera stations.

(b) the alternative, namely, the fixed (constrained) network adjustment is invalid due to the false assumption that the selected object points defining the co-ordinate system are stable. Especially in deformation analysis such stability is questionable.

At the present time, the detection of smaller movements has become critical for ensuring the safety of structures and in order to satisfy the increased demands placed on the monitoring of structures in a more reliable way an attempt is made to achieve optimal design of precision, reliability and sensitivity for deformation monitoring networks.

The data acquisition in close-range photogrammetry is defined by:

1. the camera in use (metric or non-metric).
2. the configuration of data acquisition, or the arrangement of the camera stations with respect to each other and with regard to the object space and it is this, coupled with the number of cameras to be used, which is of primary importance in this investigation.

The main objectives of this thesis are:

1. The development of efficient algorithms for the optimal design of precision, reliability and sensitivity of deformation monitoring networks.
2. Study of effect of incorporating survey measurements (slope distances) with photogrammetric observations in a simultaneous free bundle adjustment.
3. Investigating the relationship between the geometry of the network, i.e. its configuration and number of cameras and the three design criteria.

It is important to note that utilising a combination of both photogrammery and survey measurements (slope distances) proved to give flexibility to the designer so as to obtain an acceptable geometry of the monitoring network for the purpose in hand. The free bundle adjustment method (inner constraints) is used in an appropriate manner in order to give rigorous a priori cofactor matrices for both the unknowns and residuals. The redundancy number is examined and shown to be a very successful tool in assessing the reliability of the networks and has a number of advantages especially in pre-analysis studies.

A new method for the design of sensitivity has been developed through a series of simulated deformation models where displacements of 10 mm are to be detected, with 95% confidence, by the simulated networks. These models are:

1. Settlement, expansion and deflexion models in the cube case study.
2. Settlement and deflexion models in the bridge case study.
3. Settlement and drift models in the dam case study.

The method proved to be very successful and gave important findings concerning the sensitivity analysis of structural deformations. These findings can be summarised as follows:

(i) The more cameras used the less sensitive the network would be.

(ii) The networks are found to be least sensitive to a single point movement and most sensitive to multiple point displacements.

(iii) Effect of including slope distances on the settlement model is insignificant in the cube case study while they slightly degrade the sensitivity in the bridge case study.

(iv) Incorporating slope distances has not significantly increased the sensitivity when using the deflexion and expansion models (cube case study) whereas for the deflexion model in the bridge case study the sensitivity has been slightly decreased.

Finally, the developed algorithms are successfully tested using a total of seventeen case studies.

1.3. Scope and Organisation

Chapter 2 outlines a critical review of some of the recent applications of close-range photogrammetry to engineering and structural deformation studies. Such applications cover large, medium, and small, sized structures.

Chapter 3 reports on the design criteria specified as precision, reliability and sensitivity. Among the precision criteria, the mean variance of object space co-ordinates is chosen to represent a global indicator of precision of the different configurations. To represent the reliability, the redundancy number is chosen besides other indicators.

In Chapter 4 the development of the mathematical model for the simultaneous adjustment of the photogrammetric and survey measurements is outlined. Slope distances were selected to be incorporated with photogrammetric data.

To demonstrate the suitability of the mathematical procedures developed, their application to simulated deformation analysis of a cube are addressed in Chapter 5.

The three criteria were computed from data, some of which are real and others are fictitious, acquired through application to a medium-sized bridge in Chapter 6.

Chapter 7 encompasses the results of the application of the design criteria to a typical concrete dam. Three different configurations are discussed in the photogrammetric mode only.

Chapter 8 presents the conclusions drawn from the investigation and suggestions made for future research.

The appendices are concerned with the properties of generalised matrix inverses and the development of the inner constraints approach.

CHAPTER 2

REVIEW OF SOME RECENT CLOSE-RANGE PHOTOGRAMMETRY APPLICATIONS TO ENGINEERING

2.1. Introduction

Photogrammetry (as derived from three Greek words: photos = light; gramma = something drawn or written; metron = to measure) (Ghosh, 1979) has been defined by the American Society of Photogrammetry as "The art, science and technology of obtaining reliable information about physical objects and the environment through the process of recording, measuring and interpreting photogrammetric images and patterns of electromagnetic radiant energy and other phenomena". The photographs are commonly aerial photographs, before the advent of remote sensors, and are geometrically very close to an ideal central projection (Torlegard, 1980).

The classic and still the major application of photogrammetry is in aerial mapping wherein a sequence of stereoscopically viewed pairs of overlapping photographs taken from an aeroplane is used to generate topographic maps.

2.2. Close-range photogrammetry

Close-range photogrammetry is covered by the definition aforementioned but lies outside the specialised field of map production. Close-range means that the object to camera distance is limited. Some advocate 300 m as a maximum limit (Karara, 1985).

In close-range photogrammetry, not only central projections (bundles and pencils of rays) are used for mathematical models of image formation, but also parallel projections, scanner generated images and others. Not only is camera film used for the sensor system but also X-rays, scanning and transmission electron microscopes, photodiode arrays with analogue or digital output are also suitable for photogrammetric measurements.

Time is the fourth dimension in close-range photogrammetry when detection of deformation of structures is required.

In any photogrammetric process, there are two major phases:-

(a) Acquiring data from the object to be measured by taking the necessary photographs.

(b) Reducing the photos into plans, profiles, sections or spatial co-ordinates or combinations of these.

Thus the total photogrammetric system can be subdivided into two major divisions: data acquisition and data reduction.

The data acquisition system is concerned with procuring what may be termed as the raw data or raw information, i.e. necessary and suitable photography. Whereas the data reduction system is concerned with converting the raw data or photographs into a final form suitable for the intended use of the data. The final data form may be analogue such as map or digital such as printed spatial co-ordinates.

In this research, the data acquisition tool is the metric camera (UMK 10/1318) produced by Zeiss (Jena).

2.2.1. Analogue method

The introduction of stereoscopic measurements at the turn of this century marks a significant development in photogrammetry. The application of its principle led to the development of plotting instruments. Stereometric cameras are a pair of cameras rigidly attached to a fixed base with the optical axes of the cameras being parallel to each other and both perpendicular to the base. So, the stereometric camera - plotting instruments systems were manufactured. The arrangement, then, was that the photographs taken with the stereometric camera were used in the plotting instruments for a reconstruction (or restitution) of an optical model of the object space from which measurements were taken, or plotting done, directly on the instruments.

When the optical axes of the two cameras are both perpendicular to the base, the photography obtained is referred to as the "normal case". Due to their construction, stereometric cameras are restricted to taking only "normal case" photographs (Figure 2.1).

The normal case, however, has the disadvantage that the accuracy of the co-ordinate parallel to the direction of the camera axes is three to four times less than the co-ordinate perpendicular to this direction (Marzan and Karara, 1976). It suffices for some applications but as accuracy requirements are more and more stringent, means have to be sought to at least equalise accuracies in the three dimensions. Furthermore, the traditional stereoplotting instruments are severely limited in the principal distance and size of photo-format that can be used in them (Ibid).

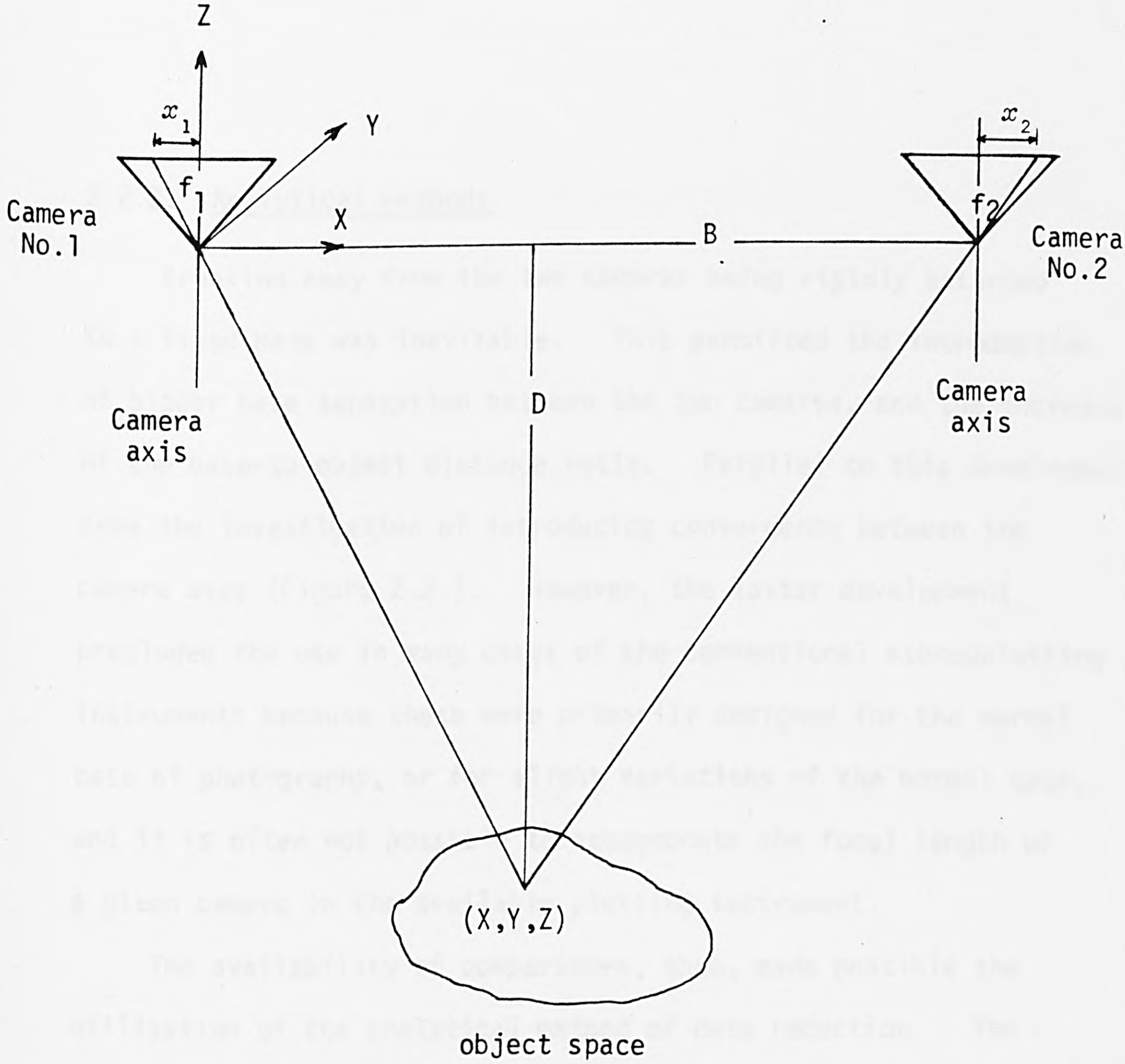


Figure 2.1. Data acquisition set-up.... the normal case of photogrammetry

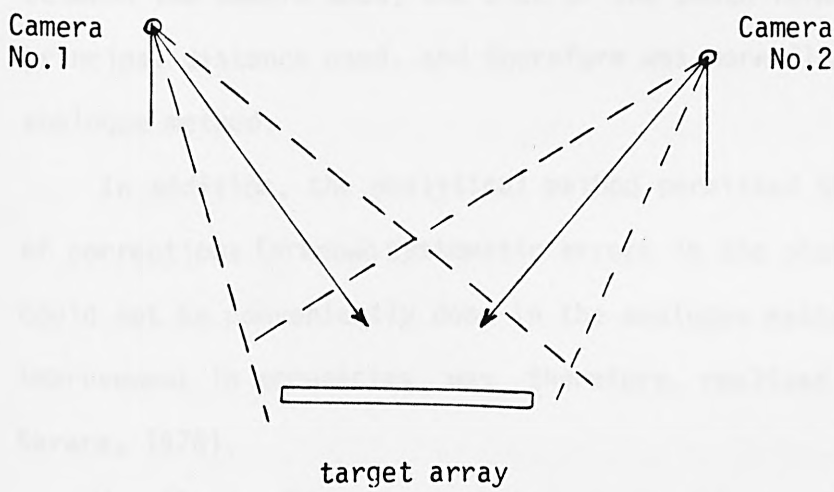


Figure 2.2. Convergent photography.

2.2.2. Analytical methods

Breaking away from the two cameras being rigidly attached to a fixed base was inevitable. This permitted the introduction of bigger base separation between the two cameras, and the increase of the base-to-object distance ratio. Parallel to this development came the investigation of introducing convergence between the camera axes (Figure 2.2.). However, the latter development precluded the use in many cases of the conventional stereoplotting instruments because these were primarily designed for the normal case of photography, or for slight variations of the normal case, and it is often not possible to accommodate the focal length of a given camera in the available plotting instrument.

The availability of comparators, then, made possible the utilisation of the analytical method of data reduction. The analytical method was not restricted by the amount of convergence between the camera axes, the size of the photo format, nor the principal distance used. and therefore was more flexible than the analogue method.

In addition, the analytical method permitted the application of corrections for known systematic errors in the photographs which could not be conveniently done in the analogue method. A great improvement in accuracies was therefore, realised (Marzan and Karara, 1976).

Usually in close-range photogrammetry the search is for great flexibility and homogeneous accuracy so the determination of dimensions of the object photographed should be an analytical solution applying the least squares method rather than by making use of optical and mechanical devices in analogue methods. The use of photogrammetric data for further calculations, data analysis, and data banks has also promoted analytical methods(Torlegard, 1980).

2.2.2.1. Modification of methods used for air photographs

Cooper (1981) shows that when the close-range photogrammetric task resembles in several ways the normal case for topographic mapping from air photographs, it may be satisfactory to use basic functional models that have been developed primarily from aerial triangulation and to modify them to take account of the particular differences brought by close-range photography. One of the major changes from the methods of analytical aerial triangulation was the use of full terms in the rotation matrix of orientations of one camera axis in relation to the next.

2.2.2.2. Direct Linear Transformation (DLT) method

Abdel-Aziz and Karara (1971) developed an approach for data reduction not requiring the classic elements of orientation (principal point and principal distance). This approach was placed on a more rigorous foundation by Bopp and Kraus (1978). Such a method is particularly suitable for non-metric cameras.

The approach involves a direct linear transformation from comparator co-ordinates into object-space co-ordinates. Since the image co-ordinate system is not involved (on the contrary with metric camera) in this approach, fiducial marks are not needed.

DLT is a direct solution and does not involve initial approximations for the unknown parameters of inner and outer orientation of the camera.

The basic equations used in thismethod are (Karara, 1972):

$$x + \Delta x + \frac{\ell_1 X + \ell_2 Y + \ell_3 Z + \ell_4}{\ell_9 X + \ell_{10} Y + \ell_{11} Z + 1} = 0 \qquad (2.1)$$

$$y + \Delta y + \frac{\ell_5 X + \ell_6 Y + \ell_7 Z + \ell_8}{\ell_9 X + \ell_{10} Y + \ell_{11} Z + 1} = 0 \qquad (2.2)$$

where x, y - comparator co-ordinates of an image point

X,Y,Z - object-space co-ordinates of the point

$\ell_1, \ell_2 \ldots \ell_{11}$ - tranformation coefficients

$\Delta x, \Delta y$ - errors due to lens distortion and film deformation; the mathematical modelling of which by power series involves at least 3 coefficients (a_1, a_2, a_3).

The number of unknowns in this case is 14$(\ell_1, \ell_2 \ldots\ldots \ell_{11}, a_1, a_2, a_3)$. From the knowledge of at least 7 control points and their comparator co-ordinates, the 14 unknowns can be determined.

This approach, DLT, which is originally tailored to reduce the data acquired via non-metric cameras,cannot be applied when higher precision, as in the case of deformation analysis, is required. Lenses of the non-metric cameras are designed for high resolution at the expense of high distortion which results in excluding the restitution with analogue plotters. Unlike metric cameras, most of the non-metric cameras operate with small format film which has questionable dimensional stability particularly because no flattening mechanism is used. The parameters of orientation in DLT technique are not physical characteristics of photogrammetry, so if their estimates are sought, they have to be found out through additional computations. Finally, the non-metric cameras, confirmed by Karara (1980) cannot completely replace metric cameras in close-range photogrammetry.

2.2.2.3. Sequential Adjustment method (Fixed number of unknowns)

The two methods described above include the assumption that the co-ordinates of the control points are much more accurate than the inherent accuracy of the photogrammetry.Therefore in the least squares estimation of the parameters of the transformations, the stochastic models include the assumption that the values of the co-ordinates of the control points are fixed, error free quantities (Cooper 1981). Such an assumption is seldom justified in close-range photogrammetry of objects that are essentially three-dimensional.

One important advantage of sequential adjustment is that the observer can, after a few measurements, examine the solution and then take more measurements to improve the results if necessary.

Following Cooper (1981), suppose that the first sequence is to estimate Δ from p observation equations and associated weight matrix:

$$A_p\Delta = V_p + b_p \qquad : W_p \qquad (2.3)$$

The least squares solution is:

$$\hat{\Delta}_1 = (A_p^T W_p A_p)^{-1} A_p^T W_p b_p \qquad (2.4)$$

and the covariance matrix of $\hat{\Delta}_1$ is $(A_p^T W_p A_p)^{-1}$.

The solution vector $\hat{\Delta}_1$ and its covariance matrix are regarded as measurements to be incorporated in the second sequence with q additional measurements:

$$\begin{bmatrix} I \\ A_q \end{bmatrix} \Delta = \begin{bmatrix} V_p \\ V_q \end{bmatrix} + \begin{bmatrix} \hat{\Delta}_1 \\ b_q \end{bmatrix} : \begin{bmatrix} A_p^T W_p A_p & 0 \\ 0 & W_q \end{bmatrix} \qquad (2.5)$$

The least squares solution to the second sequence is:

$$\hat{\Delta}_2 = \left(\begin{bmatrix} I \\ A_q \end{bmatrix}^T \begin{bmatrix} A_p^T W_p A_p & 0 \\ 0 & W_q \end{bmatrix} \begin{bmatrix} I \\ A_q \end{bmatrix} \right)^{-1} \begin{bmatrix} I \\ A_q \end{bmatrix}^T \begin{bmatrix} A_p^T W_p A_p & 0 \\ 0 & W_q \end{bmatrix} \begin{bmatrix} \hat{\Delta}_1 \\ b_q \end{bmatrix}$$

$$(2.6)$$

Equation (2.6) after reduction and substitution of $\hat{\Delta}_1$ from (2.4) becomes:

$$\hat{\Delta}_2 = (A_p^T W_p A_p + A_q^T W_q A_q)^{-1} (A_p^T W_p b_p + A_q^T W_q b_q) \qquad (2.7)$$

Now, if the solution for Δ is obtained from (p+q) observation equations in one sequence, the equations, then, are:

$$\begin{bmatrix} A_p \\ A_q \end{bmatrix} \Delta = \begin{bmatrix} V_p \\ V_q \end{bmatrix} + \begin{bmatrix} b_p \\ b_q \end{bmatrix} : \begin{bmatrix} W_p & 0 \\ 0 & W_q \end{bmatrix} \qquad (2.8)$$

The least squares solution to the second sequence is:

$$\hat{\Delta} = \left(\begin{bmatrix} A_p \\ A_q \end{bmatrix}^T \begin{bmatrix} W_p & 0 \\ 0 & W_q \end{bmatrix} \begin{bmatrix} A_p \\ A_q \end{bmatrix} \right)^{-1} \begin{bmatrix} A_p \\ A_q \end{bmatrix}^T \begin{bmatrix} W_p & 0 \\ 0 & W_q \end{bmatrix} \begin{bmatrix} b_p \\ b_q \end{bmatrix} \tag{2.9}$$

Equation (2.9) after reduction becomes:

$$\hat{\Delta} = (A_p{}^T W_p A_p + A_q{}^T W_q A_q)^{-1} \quad (A_p{}^T W_p K_p + A_q{}^T W_q b_q) \tag{2.10}$$

Equations (2.7) and (2.10) are identical. Hence, the result of making an adjustment in two sequences is the same as that which would be obtained from a simultaneous adjustment.

2.2.2.4. <u>Bundle Adjustment method</u>

The photogrammetric network is essentially formed through the processes of three-dimensional spatial resection and intersection. The resection phase being the determination of the position of the camera perspective centres and the attitude of the camera axes, and the multi-ray intersections being utilised to determine the object point positions. Under the most rigorous restitution procedures, these two phases are carried out simultaneously using a method termed the bundle adjustment. System self-calibration is also afforded using this technique when the model is extended to include inner orientation parameters.

By virtue of the bundle adjustment method, one can reconstruct what is, in effect, a three-dimensional model of an object from measurements made on two or more photographs of the object. The difference between the self-calibration technique and other methods is that self-calibration attempts this modelling without requiring any additional observations to be made specifically for the purpose of systematic error compensation.

The bundle method is based on the collinearity of the object point, the perspective centre and the imaged point.

Granshaw (1980) suggests that convergent multi-station photography (bundle method) may permit the recovery of the inner orientation elements, even if there are no control points, because of much lower correlation between the interior and exterior orientation elements. He concludes that the bundle adjustment is a powerful computational technique which provides the necessary flexibility which is essential in the various situations that may be encountered when co-ordinating engineering and industrial structures by photogrammetric methods.

2.2.2.5. Free adjustment method

The concept of inner accuracy introduced by Meissl (1962) allows the effect of an arbitrary set of parameters to be filtered out of a covariance matrix. If the filter parameters are selected to be rotational, translational and scaling parameters, then the inner accuracy refers to the precision of a free network adjustment (Granshaw, 1980).

Consider a bundle adjustment where the shape of an engineering structure is sought, the filter parameters are seven (in three-dimensional space). The shape is determined by purely photogrammetric measurements without control. In a free network adjustment an attempt is made to determine a solution vector and a covariance matrix when the system of normal equations is singular due to rank defect. Any solution vector will be biased, statistically, but as only the shape defined by the co-ordinates, and not the co-ordinates themselves, is important, this is not of significance.

In a free network adjustment a solution where the trace of the covariance matrix is a minimum is chosen. The geometrical interpretation of the minimum trace is that there should be no overall translational, rotational or scaling changes from the approximate values. Thus precision estimates are referred not to particular (arbitrary) points, but to the network of points as a whole.

Despite the fact that one of the earliest applications of a free network adjustment was to the relative orientation of a photogrammetric model (Meissl, 1965), the method has received most attention in connection with geodetic networks, although one notes the wider concept of inner accuracy has been used by Ebner (1974) and Gruen (1976) in a photogrammetric context (Granshaw, 1980). The results can be determined by transformations on arbitrary covariance matrices (Meissl, 1962,1964), generalised inverses (Mittermayer, 1972), and zero eigenvalue concepts (Mittermayer,1973) amongst others. Other useful summaries are given by Ashkenazi (1973) and Welsch (1979). Details of generalised inverses are given in Appendix A.

2.2.3. Semi-Analytical Method

This approach is similar to the semi-analytical approach in aerotriangulation. A stereoscopic pair of photographs is mounted in a stereoplotter, and relative orientation is performed to construct a three-dimensional optical model of the object being mapped. The spatial co-ordinates of any discrete point in the model can be measured with respect to the co-ordinate system of the stereoplotter.

Linking of the overlapping models and absolute orientation to an object space co-ordinate system can be performed analytically using the appropriate analytical technique.

Since a stereoplotter is used for the construction of any optical model, the type of cameras to be used and the geometric configuration of the photography must be restricted by the mechanical limitation of the stereoplotter (Wong, 1975).

2.4. Merits and drawbacks of close-range photogrammetry

In comparison with other measuring techniques in general and to surveying in particular, close-range photogrammetry has many advantages;

(i) the object is untouched during measurement (non-contact)
(ii) the data acquisition is rapid
(iii) the photographs are a permanent record
(iv) not only rigid and fixed objects but also deformation and movement can be measured, especially inaccessible or hazardous objects.

(v) time dependent parameters such as velocity, acceleration and frequency can be determined

(vi) photography and evaluation are flexible and can be optimised to the project requirements as, for example, precision and reliability

(vii) analytical methods provide a means of integration with succeeding calculations and data handling.

In order to make photogrammetry more effective and better suited to new fields of application, there are some drawbacks to be overcome.

(i) the results of the measurement are not immediately at hand, because time is needed for processing the photographs and for evaluation

(ii) it must be possible to photograph the object

(iii) errors during photography and development of the film/plate can ruin the whole measuring project, so expertise is required.

2.5. Representative examples of applications

An attempt has been made to categorise the applications according to the size of the structure. Cheffins and Chisholm (1980) chose an arbitrary subdivision which classifies structures of 20 m to over 200 m as large, 2 m to 20 m as medium-sized and 0.2 m to 2 m as small.

2.5.1. Large structures

Examples of photogrammetric measurements of structures in the range 20 m to 200 m or larger are generally to be found in the Civil Engineering branches of industry, although there are notable exceptions such as in the shipbuilding industry. Very often, the engineer chooses to use those methods and equipment with which he is familiar, unless the alternative offers distinct advantages. If photogrammetry is just as good as, but not better than, an established technique, it is unlikely to be adopted because it requires expertise and equipment which are relatively unusual.

(a) Rockfill Dams

1. The Building Research Station and Hunting Surveys Ltd. were jointly responsible for the photogrammetric study of the constructional displacements of the rockfill dam at Llyn Brianne in mid-Wales (Atkinson, 1976). The dam, when completed, was 300 m wide and 90 m high (Cheffins and Chisholm, 1980). A Wild P30 phototheodolite (principal distance 163.65 mm) was used to provide stereoscopic coverage at eight different stages of construction, of both upstream and downstream shoulders. Control points were fixed on the valley sides by theodolite observations. Eighty targetted points at various levels of the dam as the construction progressed were established. Three-dimensional displacements were determined to an average accuracy of 0.05 m according to Moore (1973).

2. Brandenberger (1974) used six camera stations to obtain 16 photos of the downstream face of a rockfilled dam (at Outardes, Quebec, Canada). The photographs were taken not to provide conventional stereomodels by normal case photography but to give adequate coverage of the area of interest, given the limitation of the topography. Geodetically surveyed control points (including the camera positions) were used as constraints in the estimation of co-ordinates of the photogrammetric points. The standard error of the positions of photogrammetrically determined points was found to be of the order of 28 mm (vector) over a dam length of about 240 m (or 1/8600), relative to the control.

(b) Buildings and Structural models

1. Cooper and Shortis (1980) used stereophotogrammetry with relative and absolute orientation for measuring two large structures; elevation of south transcept of St.Paul's Cathedral, London, and a tower crane under load. They concluded that the deformation was predominantly in the XZ plane, where the X-axis was roughly parallel to the facade of the building and the Z-axis was vertically upwards. Added to that, the accuracy can be improved by: first, the disposition of the cameras so as to get stronger geometrical configuration. Secondly, the case study of the crane, the crane has to be closer to a background where control can be better distributed.

2. Nooshin and Butterworth (1974) describe experimental investigation of a 1:50 scale model of a prestressed cable roof (Atkinson, 1976). The roof consists of a number of suspension cables which are connected

at one end to a reinforced concrete spatial beam and at the other end to a flexible cable which is in turn supported by two pin-ended steel pylons. A family of prestressing cables is arranged orthogonally to the suspension cables. The object of the investigation was to check the preliminary design and to provide information for the final design of the cable roof. Stereometric photography of the model and five premarked control points was taken with an Officine Galileo camera mounted on the laboratory ceiling above the model. The co-ordinates of all points of cable intersection were determined with a standard deviation of ±0.1 mm in plan and ±0.3 mm in elevation.

(c) Cooling Towers and Storage Tank Calibration

1. Chisholm (1977) used a Wild RC5 A aerial camera (nominal principal distance 152 mm) to check for any distortion in the shape of three cooling towers, owned by Imperial Chemical Industries Ltd. (ICI) in north-east England, from their original design shape. The use of a large format (230 mm x 230 mm) allowed fairly large scale photography of the complete extent of each tower, the tallest of which is approximately 115 m high. For each of the first two towers seven overlapping stereopairs were taken. However, for the third one 14 overlapping stereopairs had to be taken at distances varying from 25 m to 40 m. Classical triangulation methods, using a theodolite from a series of baselines around each tower, established the co-ordinates of at least six identifiable points on each stereoscopic overlap in a local origin Cartesian system. Contours defining the shape of the towers were plotted;

these were compared with the theoretical design contours. The maximum deviations were of the order of 0.3 m. The subsequent strength analysis using these data from comparisons showed that one tower would have to be demolished, one would survive with appropriate strengthening, while the third tower was relatively free from deformation.

2. Papo and Perelmuter (1980) used free network analysis of storage tank calibration. A control network of 12 points symmetrically located with respect to the tank bottom, together with the bottom centre was established. These points were used as camera stations as well. At height 14 m, 160 points were targetted to be photographed. They came to conclusion that the residuals of a free network adjustment can best disclose the existence of certain unmodelled systematic effects.

(d) Shipbuilding

Because of the need to make very large vessels, they are sometimes built in two separate parts owing to the limited size of existing berths. Cheffins and Chisholm (1980) have reported that Newton (1974) used a Galileo Santoni (nominal focal length 150 mm) with a workable format area of 110 mm x 160 mm to assess the quality control in shipbuilding using photogrammetry. A 50 m x 30 m cross section of the vessel was photographed with two overlapping photographs, each comprising 13 exposures, for each section of the ship. 27 premarked points were used as control established by traditional triangulation from a measured baseline.

Three-dimensional photogrammetric measurement was carried out in a Wild A7 stereoplotter. Some of the photogrammetric measurements were repeated using a stereocomparator and the accuracy in co-ordinate position within the plane of the mating surface was ± 3 mm.

2.5.2. Medium-sized structures

(a) Bridges

1. Christensen (1980) used stereophotogrammetry for observing displacements of a bridge loaded to failure. Vertical displacements were checked by comparison with similar results achieved by levelling to a series of rods hung up under the bridge. There was accordance between the two methods. The results of strain were inconsistent (Christensen, 1980) because the camera stations and control points were lying almost on the same circle.

2. Veress (1980) used photogrammetry for dimensional control of a curved segmental concrete bridge. He used control survey, both distances and angles, and adjusted a horizontal network by trilateration. He reached a conclusion that one of the most influential factors on achievable accuracy is the precision of a control network.

3. Cheffins and Chisholm (1980) used a Zeiss (Jena) phototheodolite (nominal focal length 162 mm) and format of 180 mm x 130 mm to photograph a low skew brick built arch carrying a commuter railway by an armoured lining which would be prepared so that it closely followed the distorted contour. 16 premarked reference points were established in the abutments at the springing of the arch and

signalised with white paint. Stereoscopic photogrammetry achieved by setting up 4 camera stations (2 at each end of the arch). Complete profiles, which include the arch, the abutments and the roadway, were plotted photogrammetrically at a scale of 1:12.

(b) Marine propeller

A study conducted by Cooper (1979) to see how far photogrammetry could be used to determine how the profiles of the finished propeller blade surfaces deviate from the design dimensions. The propeller had six blades and was about 7.5 m in diameter. The camera used was Zeiss (Jena) UMK (nominal focal length 100 mm) and usable format size of approximately 120 mm x 165 mm. Stereoscopic photography was taken of both sides of the object, which was contained within a single overlap on each view. Four external markers were placed on the floor surrounding the propeller, and these were co-ordinated by steel band and precise level to an estimated accuracy better than 1 mm. 16 premarked control points had been defined on the aft face of the propeller and were measured with a cylindrical polar co-ordinate measuring device to an estimated accuracy of 2 mm to 3 mm (r.m.s.) (Cooper, 1979).

(c) Measurement of Car body

Cooper (1979) studied the potential of applying photogrammetry to the measurement of a car body. He used 39 premarked control points on the car body, their three-dimensional co-ordinates were measured by the manufacturer using a conventional mechanical probe with readings taken from steel scales. A strip of 6 photographs was obtained by setting the camera (UMK 10/1318) successively at about 0.75 m intervals along a line roughly parallel to and about 1.5 m from the side of the car. Block adjustment was carried out using only 13 of the 39 measured points. The other 26 were regarded as check points. The r.m.s. in the longitudinal direction was 0.77mm, 0.34 mm in the height, and 1.88 mm in the transverse direction. He attributed the unexpected figures of accuracy to inherent random errors present in the measurements of the control which were greater than the internal random errors in the photogrammetry.

2.5.3. Small structures

The models of chemical plants are of crucial importance in both design and maintenance. These plants in general consist of pipelines of different dimensions and are mainly of complicated structure. A survey according to conventional methods often is impossible or possible only with extreme difficulties (Jaensch, 1979).

ICI used a 1000 mm x 750 mm x 500 mm model which represents the pipe system they need (Cheffins and Chisholm, 1980). The model is assembled in 24 sections which could be separated to gain access to the inside. Stereoscopic photography was taken, using a specially made Galileo Santoni camera of focal length 75 mm, from distances as close as 500 mm and complete cover of all sections of the model might need 50 to 100 stereopairs. The pair of photographs were set in the plotting instrument Galileo Santoni Stereosimplex IIB to obtain an oriented scaled model after a small adjustment. Design drawings of pipe layouts in both plan and elevation were plotted simultaneously at 1:16 scale.

All the foregoing applications are typical examples representing the use of close-range photogrammetry in engineering and monitoring of large structures' deformations. The methods used in these examples differ from each other in the way of formulation of the adjustment technique. On the other hand, such methods are similar in having no proper or rigorous a priori planning. For most of the applications mentioned before, if a prior design criteria mostly represented by precision, reliability, and sensitivity had been performed, better results could have been attained in more economical ways.

Therefore, such criteria, addressed in detail in the next Chapter (Chapter 3) are taken into consideration as grounds for this investigation.

Rather than using the stereophotogrammetry or nearly vertical photography techniques, the free bundle adjustment procedure is adopted as it leads to more homogeneous and most precise specifications which are to be met in monitoring deformation of structures.

CHAPTER 3

OPTIMAL DESIGN OF NETWORKS

3.1. Network Design

By network optimisation it is meant the design or reconnaissance of such networks subject to criteria derived from and determined by the purpose of these networks (Baarda, 1977).

In planning an optimal multi-station photogrammetric network for some special purpose, such as for monitoring structural deformation or for determining the precise shape characteristics of an object, due attention must be paid to the quality of the network design.

Quality is usually expressed in terms of the precision and reliability of the photogrammetric network (Fraser, 1984) but it also may include aspects of economy and testability (Dodson, 1983).

Alberda (1980) hints that in the design of a network, although powerful mathematical techniques are available, it does not seem possible so far to optimise all aspects in a unified model. Baarda's criterion theory for precision gave for the first time a consistent framework for decisions to be made by geodesists when planning a network (Molenaar; personal communication).

Following the widely accepted classification scheme of Grafarend (1974) which may be thought of in terms of the fixed and free parameters of the least squares adjustment to be carried out, the interconnected problems of network design can be identified as:

(i) Zero Order Design : the datum problem.

(ii) First Order Design : the configuration problem.

(iii) Second Order Design : the weight problem.

(iv) Third Order Design : the densification problem.

3.1.1. Zero Order Design

The datum problem involves the choice of an optimal reference system for the object space co-ordinates, given the photogrammetric network design and the precision of the observations. That is, for fixed A, the design matrix, and W, the weight matrix, one usually seeks, through the selection of an appropriate datum, an optimum form of the cofactor matrix of the unknown parameters, $Q_{\hat{x}\hat{x}}$.

It is usually required to express the spatial position of object points within a three-dimensional XYZ Cartesian co-ordinate system, however the observations (photo co-ordinates, and possibly object space distances, and angles) do not contain any information about the datum of this co-ordinate system, other than perhaps its scale if spatial distances are observed. Thus, a datum must be defined by the imposition of constraints which establish the origin, orientation and scale of the XYZ reference co-ordinate system.

Parameters of the shape of network, namely distance ratios and space angles, are determined solely as a function of the observations, and they are invariant with respect to changes in the datum, or zero-variance computational base. On the other hand, object space co-ordinates relate to the datum, and thus when the minimal constraints (Appendix B) are changed so one can expect the solution vector and the cofactor matrix of the unknown parameters to be altered.

In situations where the datum is arbitrarily assigned as in the case of relative deformation networks in which all points are assumed unstable, zero order design can be thought of as being the process of establishing a particular zero-variance computational base. Such a base for a given network geometry yields a cofactor matrix $Q_{\hat{x}\hat{x}}$ of the unknown parameters (exterior orientation, object space co-ordinates and additional parameters, if any) which is "best" in some sense (Fraser, 1984).

The solution for x which is optimal in the sense of minimising the mean variance σ_m^2 of the object point co-ordinates (which are of main concern) is provided by a free network adjustment. Such an approach, either using the Moore-Penrose inverse technique or the method of inner constraints (Blaha, 1971) yields a minimum Euclidean norm of the object point co-ordinate corrections (and hence for the co-ordinates) as well. The latter technique need not apply to all object points, and the imposition of this implicit minimal constraint may simply refer to a chosen subset of the target array.

According to Fraser (1984), the common, computationally simpler approach of "fixing" object point co-ordinates to remove the seven network defects of translation (three), rotation (three), and scale (one) will yield a mean variance σ_m^2 for the object points, which is larger in magnitude than that obtained from the inner constraints adjustment. Difference in the precision of functions of unknown parameters (e.g. distances) derived from $\hat{x}_2$ a subset of $\hat{x}$ corresponding to object point unknown parameters, may, however, be insignificant from a practical point of view.

An S-transformation (Baarda, 1973) could be used (Fraser, 1985) to transform both $\hat{x}_2$ and its corresponding cofactor matrix $Q_{\hat{x}\hat{x}}^{(2)}$ relating to one zero-variance computational base into their corresponding values for any other zero computational base including minimal constraints. For example, after the cofactor matrix of object point XYZ co-ordinates is computed for a datum of seven explicitly fixed co-ordinate values, the corresponding solution for $Q_{\hat{x}\hat{x}}^{(2)}$ is obtained simply by applying an S-transformation.

In close-range photogrammetric networks with dense target arrays, where a full cofactor matrix may not be sought, it is often computationally more practical to re-adjust the network with a different datum rather than applying an S-transformation as the latter (Strang Van Hees, 1982) does necessitate the computation of a full $Q_{\hat{x}\hat{x}}^{(2)}$ matrix.

3.1.2. First Order Design

The configuration problem is concerned with the search for an optimal network geometry, given both the precision of the observations and criteria for the structure of the covariance matrix of the unknown parameters. In other words, this procedure entails the finding of an optimal design matrix A given weight matrix W subject in certain cases to satisfying criteria imposed by an ideal covariance matrix. For example, a criterion may be that $Q_{\hat{x}\hat{x}}^{(2)}$ has a structure which is both homogeneous and isotropic, i.e. all point error ellipsoids are spheres of equal radius (Fraser, 1984).

The more flexible and seemingly the simplest part of network design, first order design is the most difficult problem. There are constraints on the choice of the positions of the points as these are largely dictated by topography, geology, accessibility, and aesthetics (Murnane,1984). The locations of the target array points are determined by engineering requirements.

Although its principal component is imaging geometry, first order design also embraces such aspects as camera and target point locations and camera selection.

The most common approach to first order design is through network simulation (Fraser, 1984) and such a process is adopted in this investigation as pre-analysis is undertaken.

(a) Imaging geometry

Simulations conducted by Granshaw (1980) have clearly indicated the significant overall accuracy enhancement that can be anticipated through the use of convergent rather than "normal" photography (Chapter 2).

In numerous practical applications, the site of a photogrammetric survey can impose restraints on the selection of an ideal imaging geometry, especially in industrial photogrammetry.

(b) Number of Camera Stations

Depending on the imaging geometry adopted, the use of additional camera stations can be expected not only to improve precision, but also significantly to enhance the network's reliability. Additional imaging rays increase the redundancy in a spatial intersection, and also alter the intersection geometry.

Fraser (1984) suggests that more attention should be paid to the imaging geometry, rather than concentrating too much on obtaining a coverage of a certain arbitrary number of photographs.

(c) Image scale and focal length

According to Fraser (1984) there is a linear relationship between object point precision and imaging scale. Concerning the focal length of the taking camera, as it increases, so the geometry of multi-ray intersections tends to become more homogeneous, thus leading to a reduction in the range of object point standard errors. Kenefick (1971) proposes that long focal length cameras are less subject to the critical influence of film unflatness, however, the choice of focal length is most often limited by both camera availability and the physical layout of the survey site.

3.1.3. Second Order Design

The second order design is defined as the problem of finding optimal weights for the observations which are projected in a geodetic or photogrammetric network with given point positions (Schmitt, 1978). This problem is characterised by an unknown W and fixed A and $Q_{\hat{x}\hat{x}}$, i.e. by a solution to the matrix equation:

$$(A^T W A)^- = Q_{\hat{x}\hat{x}} \qquad (3.1)$$

The generalised inverse $(\ )^-$ becomes either $(\ )^+$, i.e. Moore-Penrose inverse for a minimum norm solution of a free network or $(\ )^{-1}$, i.e. standard Cayley inverse for a constrained network (Schmitt, 1980). In the area of geodetic networks, considerable research attention has recently been directed to the analytical solution of equation (3.1) to mention but few; Cooper and Leahy (1978), Cross and Fagir (1982), Fagir (1984), Grafarend (1977), and Schaffrin (1977).

3.1.4. Third Order Design

The densification problem concerns the question how best to improve an existing network by additional stations and observations. Regarding the photogrammetric network optimisation, Fraser (1984) considers the densification problem is solved at the first order design stage due to the fact that object point precision is largely independent of target array density in networks with "strong" geometries. This problem can be considered as a special case of the first and second orders.

3.2. Quality of networks

The photogrammetric network is essentially formed through the process of three-dimensional spatial resection and intersection, the resection phase being the determination of the position of the camera perspective centres and the attitude of the camera axes, and the multi-ray intersections being utilised to determine object point positions. These two phases are carried out simultaneously in the case of bundle adjustment.

The quality of a geodetic or photogrammetric network is characterised by quantitative assessment of the influence of observational errors on the estimated co-ordinates and derived functions. If these observational errors are of known distribution (often normally distributed),then quality is appraised by the size of errors that can be anticipated with a certain probability.

The errors in quantitative measurements can be broadly classified into the following four types (Slama, 1980):

(i) blunders

(ii) constant errors

(iii) systematic errors, and

(iv) random errors.

Therefore quality could be assumed with regard to the following:

(i) <u>Precision</u>

This is a numerical assessment of the effect of the random observational errors on the final results, e.g. co-ordinates.

(ii) <u>Reliability</u>

A definition may be given to reliability as the ability of a network to detect gross errors in the observations and the determination of the effect of undetected gross errors on the co-ordinates.

(iii) <u>Systematic error compensation</u>

Systematic errors in observations can be of tremendous importance in monitoring networks. The reason for this is that, if undetected, they may be mistaken for a systematic deformation (Dodson, 1983). While there is no a priori knowledge about blunders, neither about their location nor about their size, we have a fairly well documented knowledge about possible systematic errors that are likely to occur in the data. This allows us to model those anticipated systematic errors as additional parameters in the estimation model. This procedure is called self calibration.

A widely accepted strategic approach in the treatment of additional parameters is to include a fairly large set in order to be sure to cover all possible errors. Because this creates the danger of overparameterization, a procedure for the deletion of non determinable additional parameters must be incorporated (Gruen, 1985). Alternatively prior calibration is required.

(iv) Sensitivity

If a three-dimensional network is subjected to deformation the sensitivity of that netowrk is expressed in terms of the minimum level of detectable object point movement (Fraser, 1982b). Prior information on quantity and direction of expected deformations gives important hints on network configuration and observation scheme (Welsch, 1982).

It should be pointed out that the problems of control of systematic errors is not considered here and what follows pertains only to precision, reliability and sensitivity as criteria for quality.

3.2.1. Precision

The precision of a network can be defined as its ability to propagate random errors. Information about precision is derived from the covariance matrix of the unknown parameters.

The Gauss-Markov model is the estimation model most widely used in photogrammetric linear or linearised estimation problems. An observation vector ℓ of dimension n x 1 is functionally related to a u x 1 unknown parameter vector x ($x = x^o + \Delta x$) through:

$$A \Delta x = b + v \tag{3.2}$$

where A is the design matrix, Δx is a vector of corrections to x^o (approximate values of x) to give x, b is a vector related to the observations, and v is a vector of corrections to the observations. If the observations have an associated weight matrix W, then the normal equations are given by:

$$A^T W A \Delta x = A^T W b \tag{3.3}$$

and the covariance matrix is given as (Cross, 1983; Bomford, 1980): (assuming the variance factor $\sigma_o^2 = 1$)

$$\sigma_o^2 Q_{\hat{x}\hat{x}} = (A^T W A)^{-1}$$

$$Q_{\hat{x}\hat{x}} = (A^T W A)^{-1} \tag{3.4}$$

where $Q_{\hat{x}\hat{x}}$ is the cofactor matrix of the unknown parameters.

If it happened that matrix $(A^T W A)$ is singular the standard Cayley inverse is to be replaced by a particular generalised inverse, the Moore-Penrose inverse. Mittermayer(1972) has shown that when using the latter inverse for solving equation (3.4) it leads to a covariance matrix of minimum trace.

An insight into the covariance matrix shows that precision depends on the geometry of the network characterised by the design matrix, the quality of the observations expressed by the weight matrix and the sort of inverse used.

It is notable that the covariance matrix, parallel to what was mentioned in the preceding paragraph, depends on the choice of the reference system or the so-called datum parameters. In other words how the measurements had been adjusted.

In practice, there are two alternatives: constrained (or fixed network) adjustment and a free (network) adjustment.

3.2.1.1. Constrained Network Adjustment

In the constrained adjustment two of the observation stations are assumed to be fixed (or their co-ordinates are given known variances), or fixing one point, an azimuth and a distance in the two-dimensional network. If we change the datum parameters, different solutions for equations (3.4) are produced. Such different solutions are related to each other by Baarda (1973) S-transformations.

The main disadvantage of such a network is the probably false postulation of an umoving (stable) station(s).

3.2.1.2. Free Network

Literally, free network means a network with which adjustment is made free from any kind of constraint such as given earlier. In this method of adjustment the corrections to the approximate co-ordinates are derived by selecting the "best" co-ordinate system (Blaha, 1971). In this sense "best" is interpreted as resulting in the smallest trace of the covariance matrix for the unknowns, namely, the aforementioned corrections. In other words, neither the length of the solution vector nor the sum of the corresponding variances can be decreased any further by a change in location of fixed point, observed azimuth or measured distance. It can be shown (Mittermayer,1972) that, in a two-dimensional network, the position is fixed by the centre of gravity (centroid) of the network and the scale and orientation are determined by the average distance and bearing from that centre to the points.

From the estimation viewpoint, it can be shown that the solution for free network adjustment characterised by singular normal equations is biased (Cooper, 1980). This means that the expected values of computed co-ordinates are different from their true values (Appendix B). In the meantime, whereas the co-ordinates and their variances depend on the arbitrarily chosen reference system, the variances of a distance or bearing, if any provided, are invariant. Equivalence of both unbiased estimable quantities and invariant quantities has been proved by Grafarend and Schaffrin (1976) (Fagir, 1984). Bearing in mind one of the objectives of this study of deformation monitoring it should be noticed that the co-ordinates themselves are of no concern but their difference is of utmost importance. Such a difference is datum independent if the same network was used in two or more epochs.

It is why the precision of estimable quantities which are invariant with respect to datum transformations should be considered as criteria to define the quality of networks.

3.2.1.3. Common Precision Criteria

(i) Standard error of co-ordinates

The standard errors of co-ordinates in the direction of the co-ordinate system are obtained by taking the square root of the corresponding diagonal elements of the covariance matrix of the unknown parameters. As mentioned before, the covariance matrix is dependent on the chosen reference system, these quantities are datum dependent. Therefore they are not valid representations of the precision of the network.

(ii) Absolute error ellipses and ellipsoid

Error ellipse and ellipsoid are used to evaluate the precision of geodetic or photogrammetric determination of position in two- and three- dimensional spaces respectively (Slama, 1980). The absolute ellipses are informative, to some extent, but for some monitoring purposes may be misleading due to the fact that their size is dependent on their distance from the points selected as datum points. The closer to the datum the lesser the size (higher precision) of the ellipse and vice versa.

Let the covariance matrix (Σ) of a point j which has the co-ordinates X_j, Y_j and Z_j be defined as follows (dropping the index j):

$$\Sigma = \sigma_0^2 Q_{\hat{X}\hat{X}} = \begin{bmatrix} \sigma_X^2 & \sigma_{XY} & \sigma_{XZ} \\ \sigma_{XY} & \sigma^2{}_Y & \sigma_{YZ} \\ \sigma_{XZ} & \sigma_{YZ} & \sigma_Z^2 \end{bmatrix} \qquad (3.5)$$

Following Mikhail (1976), the ellipsoid of constant probability is then given by the following quadratic form:

$$X^T \Sigma^{-1} X = [X \quad Y \quad Z] \, \Sigma^{-1} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = k^2 \tag{3.6}$$

when k = 1, it is called the standard ellipsoid. The semi-axes of the ellipsoid (a,b,c) are determined by diagonalising Σ by writing:

$$\begin{bmatrix} \sigma_u^2 & 0 & 0 \\ 0 & \sigma_v^2 & 0 \\ 0 & 0 & \sigma_w^2 \end{bmatrix} = \begin{bmatrix} a^2 & 0 & 0 \\ 0 & b^2 & 0 \\ 0 & 0 & c^2 \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} = T^T \Sigma T \tag{3.7}$$

where

T is an orthogonal matrix whose columns are the normalised eigenvectors of Σ;

$\lambda_1, \lambda_2, \lambda_3$ are the eigenvalues of Σ; and

u,v,w is a rotated co-ordinate system such that the random variables in the directions of its axes are uncorrelated.

In regard to the statistical significance of the error ellipsoid, the possibility of a point falling inside or on the ellipsoid defined by $a = k\sigma_u$, $b = k\sigma_v$, $c = k\sigma_w$ is expressed as:

$$P\left[\left(\frac{u^2}{\sigma_u^2} + \frac{v^2}{\sigma_v^2} + \frac{w^2}{\sigma_w^2}\right) < k^2\right] = P\left[\chi_3^2 < k^2\right] = 1 - \alpha \tag{3.8}$$

For the standard ellipsoid $(1 - \alpha) = 0.199$ which is obtained from χ^2 with three degrees of freedom. Confidence regions are established through selecting the significance level α and then the multiplier k is to be computed. For example, for $\alpha = 0.05$

$$P[\chi_3^2 < \chi^2_{0.05,3}] = P[\chi_3^2 < 7.81] = 0.95$$

and

$$k = \sqrt{7.81} = 2.79.$$

Thus for the ellipsoid whose semi-axes are

$$a = 2.79\sigma_u, \quad b = 2.79\sigma_v, \quad c = 2.79\sigma_w$$

there is 95 percent probability that the computed position of point j would fall within that ellipsoid or simply there is 95 percent confidence region.

The eigenvalues (the squares of the semi-axes of the ellipsoid) are the roots of the following characteristic equation:

$$\lambda^3 - (\sigma_X^2 + \sigma_Y^2 + \sigma_Z^2)\lambda^2 + (\sigma_X^2\sigma_Y^2 + \sigma_Y^2\sigma_Z^2 + \sigma_X^2\sigma_Z^2 - \sigma_{XY}^2 - \sigma_{YZ}^2 - \sigma_{XZ}^2)\lambda$$

$$-\sigma_X^2\sigma_Y^2\sigma_Z^2 - 2\sigma_{XY}\sigma_{YZ}\sigma_{XZ} + \sigma_X^2\sigma_{YZ}^2 + \sigma_Y^2\sigma_{XZ}^2 + \sigma_Z^2\sigma_{XY}^2 = 0 \qquad (3.9)$$

Having determined the eigenvalues, the eigenvectors are determined and normalised to obtain the T-matrix. Then, the error ellipsoid can be plotted by automatic plotting devices (Veress et al, 1979).

Because of the involved computation, it is neither practical nor necessary to perform analysis using error ellipsoids for all points in the photogrammetric solution (Slama, 1980). However, for many applications, the two-dimensional equivalent, error ellipse, is enough to determine the precision of the system. The formulae for the error ellipse are the same as for the error ellipsoid, except that terms which belong to two selected dimensions are retained and the third dimension terms are neglected.

For the X,Y selected directions, equation (3.6) will read:

$$[X \quad Y]\, \Sigma^{-1} \begin{bmatrix} X \\ Y \end{bmatrix} = k^2 \tag{3.10}$$

where

$$\Sigma = \begin{bmatrix} \sigma_X^2 & \sigma_{XY} \\ \sigma_{XY} & \sigma_Y^2 \end{bmatrix} \tag{3.11}$$

Then the semi major and semi minor axes a,b may be computed from (Mikhail, 1976; Bomford, 1980; Cross, 1983):

$$\begin{aligned} a^2 &= \tfrac{1}{2}(\sigma_X^2 + \sigma_Y^2) + \sqrt{\tfrac{1}{4}(\sigma_X^2 + \sigma_Y^2) - \sigma_{XY}^2} \\ b^2 &= \tfrac{1}{2}(\sigma_X^2 + \sigma_Y^2) - \sqrt{\tfrac{1}{4}(\sigma_X^2 + \sigma_Y^2) - \sigma_{XY}^2} \end{aligned} \tag{3.12}$$

and the angle θ between the semi major axis of the ellipse and the X axis is obtained from:

$$\tan 2\theta = \frac{2\sigma_{XY}}{\sigma_X^2 - \sigma_Y^2} \tag{3.13}$$

It is clear that a and b are the square roots of the eigenvalues of the characteristic equation :

$$\lambda^2 - (\sigma_X^2 + \sigma_Y^2)\lambda + \sigma_X^2\sigma_Y^2 - \sigma_{XY}^2 = 0 \tag{3.14}$$

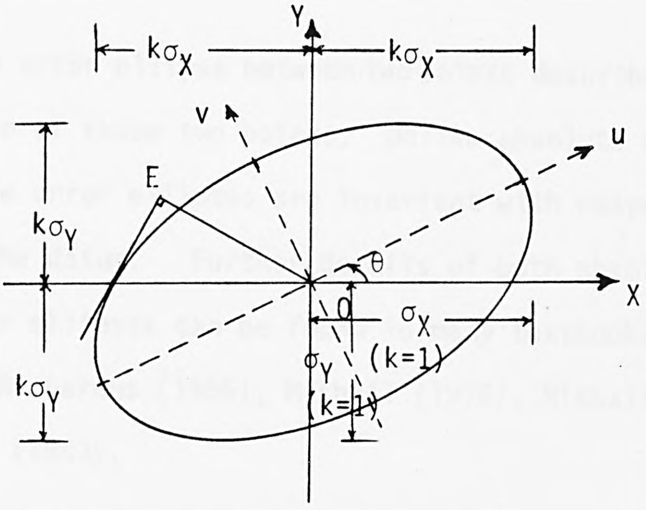


Figure 3.1. The standard Ellipse (after Mikhail, 1976)

Unlike the standard errors of co-ordinates which define the precision in the directions of the co-ordinates system chosen only, the standard error ellipse can be used to determine the standard error of any point (e.g. point E) at any direction (e.g. O E) by drawing the foot point curve or the pedal curve. However, it is the error ellipse rather than its pedal curve that is generally most useful in practice (Cross, 1983).

Statistically, if the observational errors are normally distributed, then there is 39.4 percent confidence region (Cross, 1983; Schofield, 1984).

(iii) Relative measures of Precision

The standard errors of quantities derived from the co-ordinates (e.g. co-ordinate differences, distances, angles) are often used as criteria of precision for networks due to the reason that they are not only invariant but they also related to the purpose of the network.

The relative error ellipse between two points describes the relative precision of those two points. Unlike absolute error ellipses, relative error ellipses are invariant with respect to translations of the datum. Further details of both absolute and relative error ellipses can be found in many textbooks to mention but few: Richardus (1966), Mikhail (1976), Mikhail et al (1981), and Cross (1983).

3.2.1.4. Global measures of Precision

To compare alternative network configurations, global measures which describe the precision of a network as a whole are desirable. The following are single numbers which could be used for this task.

(i) Mean Variance of Object Points

The most widely accepted indicator of statistical quality is simply the covariance matrix ($\sigma_o^2\ Q_{\hat{x}\hat{x}}$) of co-ordinates X,Y,Z. Near homogeneous precision is often desired for the X,Y,Z co-ordinates and a single estimator σ_m^2 can be employed to express the mean variance of the n_o object point co-ordinates (assuming $\sigma_o^2 = 1$):

$$\sigma_m^2 = \frac{1}{3n_o} \operatorname{tr} Q_{\hat{x}\hat{x}} \tag{3.15}$$

The magnitude of σ_m^2 can be expected to vary dramatically with different imaging geometries and also with changes in minimal and redundant control configurations.

It is this criterion which was chosen to be used in the assessment of global precision in this investigation.

(ii) Average Standard Error of Derived Quantities

Distances and directions are examples of such derived quantities. It is important to point out that the choice of the quantities would be a function of the purpose of the network. Ashkenazi and Cross (1972) have used the average standard of errors of selected distances and directions to analyse the scale and orientation of the U.K. network (Fagir, 1984)

(iii) Maximum Eigenvalue of the Covariance Matrix

The largest eigenvalue of the covariance matrix can be considered as a viable global scalar measure of precision. Its square root is the largest semi-axis of the hypersphere associated with the covariance matrix. Use of the largest eigenvalue as a criterion is therefore tantamount to saying that the best determination of a point is the one that produces the standard ellipse whose major axis is smallest. However, the main drawback of using the maximum eigenvalue as criterion is that the eigenvalues are dependent on the reference base.

3.2.2. Reliability of Networks

Reliability is concerned with the control of quality of conformance of an observed network to its design, i.e. to see if the assumptions made in the design are not invalidated by disturbances (Alberda, 1976). It should describe the qualities of the network with respect to the possibility of detecting gross erros in the observations and the influence of undetected gross erros on the co-ordinates.

Due to the increasing use of automatic data acquisition and the great amount of data captured, it is essential to have an objective measure to recognise the possible presence of gross errors or outliers. Hawkins (1980) defines an outlier as an observation which deviates so much from other observations as to arouse suspicions that it was generated by a different mechanism.

Baarda (1967, 68, 73, 76) developed an extensive reliability theory where the problem of gross error detection is treated on statistical bases (Molenaar; personal communication).

3.2.2.1. Internal Reliability

Internal reliability may be defined as the ability of a network to detect gross observational errors. Baarda (1968), Pelzer (1977), Niemier et al (1982), Ashkenazi and Crane (1982) and Cross (1983) have developed different criteria for internal reliability based on statistical tests examining the residuals or their cofactor matrix. The latter can be derived as follows:

Restating equation (3.3):

$$A^TWA\Delta\hat{x} = A^TWb \tag{3.16}$$

then

$$\Delta\hat{x} = (A^TWA)^{-1} A^TWb \tag{3.17}$$

and for the residuals v we obtain from equation (3.2):

$$\hat{v} = A\Delta\hat{x} - b \tag{3.18}$$

$$= A(A^TWA)^{-1}A^TWb - b \tag{3.19}$$

$$= [A(A^TWA)^{-1} A^TW - I]b \tag{3.20}$$

Applying the Gaussian propagation of error laws, the cofactor matrix of the residuals $Q_{\hat{v}\hat{v}}$ can be found from:

$$Q_{\hat{v}\hat{v}} = [A(A^TWA)^{-1} A^TW-I]\, C_{\ell\ell} [A(A^TWA)^{-1} A^TW-I]^T \qquad (3.21)$$

$$= \sigma_o^2[A(A^TWA)^{-1} A^TW-I]W^{-1}[WA(A^TWA)^{-1}A^T -I] \qquad (3.22)$$

(with $C_b = C_{\ell\ell} = \sigma_o^2 W^{-1}$; $C_{\ell\ell}$ is the covariance matrix of the observations, $\sigma_o^2 = 1$)

Hence;

$$Q_{\hat{v}\hat{v}} = W^{-1} - A(A^TWA)^{-1}A^T \qquad (3.23)$$

The product $Q_{\hat{v}\hat{v}}W$ is an idempotent matrix (Latin: idem = same, potent = power), and singular as the only idempotent matrices that are nonsingular are identity matrices (Searle, 1982).

From equation (3.23) we get:

$$\hat{v} = - Q_{\hat{v}\hat{v}}Wb \qquad (3.24)$$

A popular method of checking for gross errors after initial adjustment(post adjustment) is to compare the weighted residuals with the a posteriori variance factor $\hat{\sigma}_o^2$. For instance, if

$$|\hat{v}_i W_{ii}^{\frac{1}{2}}| > 3\hat{\sigma}_o \qquad (3.25)$$

we suspect a gross error in the ith observation. However, such a technique assumes that all the residuals can be represented by a common a posteriori variance factor which is not the case and there is, additionally, no account taken of the design matrix A.

Gruen (1978,1979) has suggested the adoption of the data snooping technique for gross error detection in photogrammetry. The method is based upon $Q_{\hat{v}\hat{v}}$ given in equation (3.23) and requires not all the elements of $Q_{\hat{v}\hat{v}}$ but the diagonal elements only. Such a technique requires a considerable amount of computation, although the problem is not so severe in engineering or large structures monitoring applications as it is with the large blocks of photography encountered in aerial triangulation.

If the jth observation, ℓ_j, is suspected of containing a gross error Δ_j and all other observations have only random, normally distributed errors ε_j, since the systematic errors are supposed to be eliminated or compensated for, we can set the following test:

$$H_o : \ell_j = \bar{\ell}_j + \varepsilon_j$$
$$H_A : \ell_j = \bar{\ell}_j + \varepsilon_j + \Delta_j \qquad (3.26)$$

where $\bar{\ell}_j$ is the observation vector without errors.

Baarda (1968) introduced a test statistic

$$\hat{\omega}_j = \frac{\hat{d}_j}{\sigma_{\hat{d}_j}} \qquad (3.27)$$

where

$$\hat{d}_j = \ell_j - \hat{\ell}_j^* \qquad (3.28)$$

in which $\hat{\ell}_j^*$ is the jth observed quantity computed from the parameters derived from least squares computation of all observations except ℓ_j (Cross, 1983).

Following the notions of Pelzer (1979), Fagir (1984) proved that equation (3.27) could be reduced to the so-called standardised residuals test given as:

$$\hat{\omega}_j = \hat{v}_j / \sigma_{\hat{v}_j} \tag{3.29}$$

As we are in pre-analysis stage, $\hat{v}$ is not at hand so we will apply the approach proposed by Cross (1983) for assigning the test statistics for reliability appraisal. Such an approach presents itself as an ideal candidate for such kind of study.

Now, if H_o is true, $\hat{\omega}_j \sim N(0,1)$ but under H_A the normal distribution will have a mean not zero but δ_j as given by:

$$\delta_j = \Delta_j / \sigma_{\hat{d}_j} \tag{3.30}$$

Due to the fact that Δ_j is not known a choice should be made so that the probability (α) of type I error (rejecting H_o although it is true) and the probability (β) of type II error (accepting H_o although it is false) are as small as possible and hence determine an upper bound δ_o (Figure 3.2) on δ_j. Therefore the largest gross error which will remain undetected is given by:

$$\Delta_o = \delta_o \cdot \sigma_{\hat{d}_j} \tag{3.31}$$

and

(Fagir, 1984)
$$\sigma_{\hat{d}_j} = 1/(c_j^T W Q_{\hat{v}\hat{v}} W c_j)^{\frac{1}{2}} \tag{3.32}$$

where c_j^T is a null vector but for the jth element which is unity, i.e.

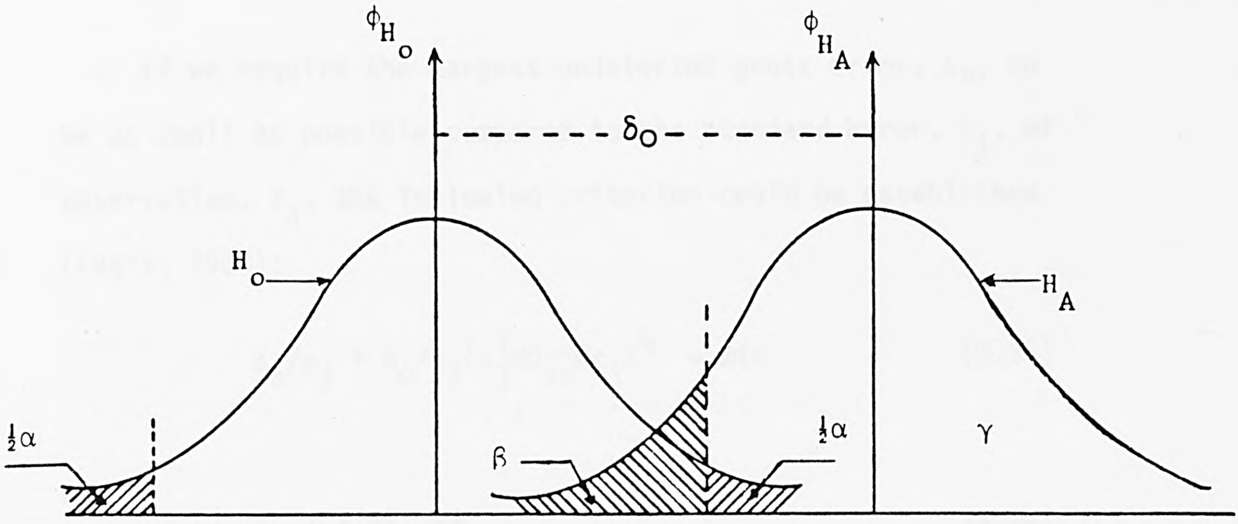


Figure 3.2. Probability density function of statistic ω_j.

$$c_j = [0 \;\; 0 \;\ldots\; 1 \;\;\; 0]^T \tag{3.33}$$

Substituting (3.32) into (3.31) we get:

$$\Delta_o = \delta_o/(c_j^T W Q_{\hat{v}\hat{v}} W c_j)^{\frac{1}{2}} \tag{3.34}$$

If we require the largest undetected gross error, Δ_0, to be as small as possible compared to the standard error, σ_j, of observation, ℓ_j, the following criterion could be established (Fagir, 1984):

$$\Delta_o/\sigma_j = \delta_o/\sigma_j(c_j^T W Q_{\hat{v}\hat{v}} W c_j)^{\frac{1}{2}} \rightarrow \min \tag{3.35}$$

where

$$\sigma_j = (c_j^T W^{-1} c_j)^{\frac{1}{2}} \tag{3.36}$$

Substituting (3.36) into((3.35) yields:

$$\Delta_o/\sigma_j = \delta_o/(c_j^T W^{-1} c_j c_j^T \, W Q_{\hat{v}\hat{v}} W c_j)^{\frac{1}{2}} \tag{3.37}$$

or

$$\Delta_o/\sigma_j = \delta_o \, \tau_j \tag{3.38}$$

Hence:

$$\Delta_o = \delta_o \sigma_j \tau_j \tag{3.39}$$

Since δ_o depends only on the chosen probabilities α and β, it could be regarded as constant and the criterion (3.35) reduced to:

$$\tau_j^2 \rightarrow \text{minimum} \tag{3.40}$$

It is extremely important that for uncorrelated observations τ_j^2 is given as:

$$\tau_j^2 = \frac{\sigma_j^2}{\sigma^2_{\hat{v}_j}} \qquad (3.41)$$

From equation (3.41) the limits of τ_j^2 are:

$$1 \leqslant \tau_j^2 \leqslant \infty \qquad (3.42)$$

It is worth mentioning that equation (3.41) was used in this thesis as criterion for the reliability but one important point against its use is that, as seen from equation (3.42) its upper bound is infinity so τ could take any value up to infinity. However, it is very useful tool for screening the maximum undetected gross error in any observation (equation 3.39) which could be used as a global measure for reliability, i.e.

$$\Delta_o \rightarrow \text{ a minimum} \qquad (3.43)$$

Further, τ is independent of both $\hat{v}$ and the reference system.

An alternative was sought to circumvent the loose upper bound of τ. The redundancy number technique proved to be very efficient criterion to assess reliability of different configurations of networks simulated in this study.

Redundancy Number Technique

Recall equation (3.24) which reads as follows:

$$\hat{v} = - Q_{\hat{v}\hat{v}}Wb$$

The effect of a gross error $\Delta \ell_j$ on the residual $\hat{v}_j$ of an observation ℓ_j is therefore:

$$\Delta \hat{v}_j = -r_j \Delta \ell_j \qquad (3.44)$$

where

r_j is the jth diagonal element of the matrix $Q_{\hat{v}\hat{v}}W$.

and is called redundancy number (Förstner, 1979; El Hakim, 1981). The redundancy number indicates the reliability of the adjustment of a particular observation. Zero redundancy means no reliability while increasing redundancy indicates increasing reliability.

So the limits for r_j are as follows:

$$0 \leq r_j \leq 1 \qquad (3.45)$$

Thus if r_j is large, the gross error $\Delta \ell_j$ is well revealed in the residual $\hat{v}$ ($\hat{v}$ is the visible part of the true error ε (Pope, 1975) and can easily be detected.

As has been demonstrated earlier $Q_{\hat{v}\hat{v}}W$ is an idempotent matrix with rank r (r = redundancy). Hence it yields:

$$t_r(Q_{\hat{v}\hat{v}}W) = \text{rank}(Q_{\hat{v}\hat{v}}W) = \text{rank}\,(Q_{\hat{v}\hat{v}}) = r \qquad (3.46)$$

That is, the trace of $Q_{\hat{v}\hat{v}}W$ equals the redundancy of the system and r_j can be interpreted as the contribution of the observation ℓ_j to the total redundancy r.

It is the author's point of view that for the evaluation of reliability, the local redundancy indicated by r_j is more important than the total redundancy r.

However for the sake of comparison between different systems the following criterion can serve as a global reliability measure (Gruen, 1978, 1979):

$$R = \frac{tr(Q_{\hat{v}\hat{v}}W)}{m} = \frac{r}{m} \qquad (3.47)$$

where m denotes number of observations.

The main restraint on using (3.47) is that such measure presumes that the redundancy is distributed homogeneously on the observations.

For the optimisation standpoint another global criterion has been introduced to indicate the overall reliability of different networks' configurations. This criterion is the maximum redundancy number of network. That is:

$$r(x) = r_{j_x} \rightarrow \text{a maximum} \qquad (3.48)$$

and

$$r(y) = r_{j_y} \rightarrow \text{a maximum}$$

where x, y are the observed photographic co-ordinates and

$$r\ (\text{measurement}) = r_j \rightarrow \text{a maximum} \qquad (3.49)$$

for survey measurements.

The local redundancies again are mainly influenced by parameters as:

(i) number of rays determining an object point.

(ii) type of an object point (control or non-control point).

(iii) number and distribution of image points.

(iv) number and distribution of control points.

(v) focal length of the taking camera.

The parameters mentioned in (iii), (iv), (v) may play a certain role, which however is not expected to be as significant as the influence of the first two parameters (Gruen, 1979).

In this thesis, the central point is the adoption of free network approach, i.e. there is no use of control points. So, the parameter No.(i) (above) will only be considered.

3.2.2.2. External Reliability

External reliability indicates the influence of undetected gross errors on an arbitrary function of the co-ordinates. Such a function can be: the co-ordinate itself, an angle, a distance, a difference of co-ordinates or an area computed with the adjusted co-ordinates.

Consider a quantity F, with least squares estimate $\hat{F}$, computed from the parameters and let $\Delta\hat{F}_j$ be the effect of a gross error of size Δ^u_j in the jth observation on $\hat{F}$. Then it can be shown that for uncorrelated observations:

(Cross, 1983) $$\Delta\hat{F}_j \leq \delta^u_j \gamma_j \sigma_{\hat{F}} \qquad (3.50)$$

where

$$\gamma_j = \hat{\sigma}_j / \sigma_{\hat{v}_j} \qquad (3.51)$$

Inspection of equation (3.50) leads to the fact that γ_j could be used as an indicator of the external reliability since by its multiplication by the standard error of the wanted function of the parameters it gives the maximum effect on that function. It is important to point out that for the evaluation of external reliability one must consider the precision of the co-ordinates and their functions respectively.

We could have an appriasal for external reliability which is independent of the precision through soliciting the relationship between τ_j and γ_j.

Recalling equation (3.41) and (3.51) we formulate:

$$\tau_j^2 - \gamma_j^2 = (\sigma_j^2 - \hat{\sigma}_j^2)/\sigma_{\hat{v}_j}^2$$

with

$$\sigma_{\hat{v}_j}^2 = \sigma_j^2 - \hat{\sigma}_j^2$$

Thence:

$$\tau_j^2 - \gamma_j^2 = 1$$

Finally,

$$\gamma_j^2 = \tau_j^2 - 1 \qquad (3.52)$$

Hence the computation of τ_j automatically leads to γ_j.

As it can be noticed from (3.52) that for an observation, the higher its internal reliability the higher is its external reliability.

γ_{max} could be proposed to serve as global external reliability indicator as follows:

$$\gamma_{max} \rightarrow \text{a minimum} \qquad (3.53)$$

Such a criterion will ensure that the influence on the parameters of undetected gross error of a designated size is minimum.

As has been mentioned before with regard to τ_j, γ_j is independent of the selection of reference system.

It is worth mentioning that γ_j computed as in equation (3.52) was used in this study but here, of crucial importance, the term reliability is chosen to be more strongly connected with the detection than with the effect of gross errors. Therefore a brief mention has been given on the topic of external reliability.

3.2.3. Sensitivity of Networks

3.2.3.1. Introduction

The concept of sensitivity analysis can be outlined in terms of a deformation analysis which employs tests for departures from congruency (Fraser, 1983) in other words, the proof of existence or non-existence of point movements. Since neither "true" co-ordinates nor "true" deformations can be obtained through a physical measuring process it is necessary to turn to statistical or "estimated" deformations in order to establish whether or not significant movements have occurred between two measuring epochs.

Essentially, the congruence of the two networks is examined within the tolerance implied by their respective covariance matrices. It is therefore not surprising that the covariance matrix of object points co-ordinates $\sigma_0^2 Q_{\hat{x}\hat{x}}^{(2)}$ plays a key role in assessing how sensitive a network is to the detection of systematic point displacements which are likely to occur under formulated deformation models. Such deformation models are formulated under the assumption that there is prior information regarding the magnitude, direction and extent of expected deformations provided by other disciplines (e.g. civil engineering, geology, etc.)

3.2.3.2. The Global Congruency Test

Fundamentally, this test examines the null hypothesis that the object target point array is stable over all measuring epochs. For the case of a two-epoch analysis, the null hypothesis H_0 can be written as:

$$H_0 : E\{\hat{x}_2^{(i+1)}\} - E\{\hat{x}_2^{(i)}\} = E\{d\} = 0 \qquad (3.54)$$

in which $\hat{x}_2^{(i)}$ is the vector of XYZ object point co-ordinates at epoch i, and d is the vector of co-ordinate differences.

Pelzer (1971) introduced the following test statistic which determines whether or not H_0 will be rejected.

$$\bar{F} = \frac{d^T Q_d^+ d}{h\sigma_0^2} \qquad (3.55)$$

where

$$Q_d = Q_{\hat{x}\hat{x}}^{(2)} + Q_{\hat{x}\hat{x}}^{(1)} \qquad (3.56)$$

The cofactor matrix Q_d of co-ordinate differences has a rank h; in photogrammetric context rank $(Q_d) = 3n_0 - 7$ where n_0 is the number of object points. The Moore-Penrose inverse in equation (3.55) is necessitated because of the rank defect of Q_d.

If the null hypothesis is accepted, i.e. no point displacement occur, $\bar{F}$ follows the central Fisher distribution. Should the null hypothesis is rejected, the point movements are implied and an alternative hypothesis is to be specified. It could be given as follows:

$$H_A = E\{d\} = \tilde{d} \neq 0 \qquad (3.57)$$

Now $\bar{F}$ is distributed according to the non-central F-distribution with non-centrality parameter.

$$\omega = \frac{\tilde{d}^T Q_d^+ \tilde{d}}{\sigma_o^2} \qquad (3.58)$$

The probability $\gamma = 1 - \beta$ that $\tilde{d}$ will lead to the rejection of the null hypothesis at significance level α is termed the power of the test with respect to the alternative hypothesis. For a specified probabilities α and β a critical value for ω (say ω^u) can be computed as a function of α, β and rank (Q_d)

Nomograms for ω^u, for three power values (γ = 0.70, 0.80, 0.90) are given in Baarda (1968). For this investigation values of α = 0.05 and γ = 0.80 were selected.

If $\omega > \omega^u$ for a given $\tilde{d}$ then it can be concluded that $\tilde{d}$ represents a detectable movement at the assigned probability levels α and β.

It should be noted that the global congruency test only tests the magnitude of the movement on the bearing designated by the vector $\tilde{d}$.

An alternative approach for sensitivity analysis is given by Niemier et al (1982) in which the use of external reliability measures in sensitivity analysis has been discussed (Fraser, 1983).

3.2.3.3. Sensitivity Measure

The cofactor matrices $Q_{\hat{x}\hat{x}}^{(1)}$, $Q_{\hat{x}\hat{x}}^{(2)}$ given in equation (3.56) can be determined from simulation procedures and for sensitivity analysis are generally assumed to be equal. Hence, the test statistic given in equation (3.58) can be calculated a priori from the knowledge of the proposed measuring system and the movements required to be detected.

The importance of the test of ω given in equation (3.58) in terms of sensitivity analysis is that any so-called form vector $\tilde{d}$ of modelled point displacements can be assumed in order to ascertain a just-detectable deformation.

Let c be a scalar value, called sensitivity parameter, then if:

$$\omega^{u} = c^{2}\omega = \frac{(c\tilde{d})^{T} Q_{d}^{+} (c\tilde{d})}{\sigma_{o}^{2}} \qquad (3.59)$$

then $c\tilde{d}$ represents a critical amount of movement, i.e. a just-detectable deformation.

For a sensitivity analysis of a single network which is remeasured at another epoch $c\tilde{d}$ can be considered as measure of the sensitivity of the network.

Finally, it is worth mentioning that the criterion given in equation (3.59) was used coupled with the use of the nomograms of Baarda (1968) in this research to assign the sensitivity of different simulated networks.

CHAPTER 4

MATHEMATICAL DEVELOPMENT

4.1. Introduction

In technical practice, as well as in all experimental sciences, one is faced with the following problem: evaluate quantitatively parameters describing properties, features, relations, or behaviour of various physical objects. The parameters can be evaluated usually only on the basis of the results of some measurements or observations.

The problem gets more complicated as the system whose parameters we are trying to determine gets more complex. The problem to be treated has to be first translated into the language of mathematics. That is the problem has to be first mathematically formulated.

The mathematical formulation of the problem would really be the mathematical formulation of the relation between the observed quantities and the wanted quantities. This relationship is called the mathematical model.

4.2. The Mathematical Model

The mathematical model links observable reality with the mechanism generating the observations (Morrison, 1976). In this investigation, the mathematical model denotes the relationships between observations (measurements), for instance, image co-ordinates and slope distances; and the three-dimensional co-ordinates of the object points.

Mathematical models may be divided into two general parts: (1) functional model (deterministic), and (2) stochastic. The functional model should represent, as far as possible, the physical relationship between measurements and three-dimensional co-ordinates. Morrison (1976) suggests that the functional model should be sufficiently tractable to permit the sort of mathematical manipulations required for the estimation of its parameters and other inferences about its nature. The stochastic model describes the properties of random errors inherent in the observations and how these errors will propagate through the used mathematical model and result in certain errors in the derived quantities of the three-dimensional co-ordinates of the object points.

4.2.1. The Functional Model

It is proposed that slope distances between object points are to be incorporated with the photogrammetric observations in an attempt to investigate their effect on the different adopted design criteria discussed in the preceding chapter. Therefore, in addition to the model of the photogrammetric measurements the model for slope distances is to be addressed in this section.

4.2.1.1. Functional model for a photograph

Assuming that light rays travel in straight lines, that all rays entering a camera lens system pass through a single point and that the lens system is distortionless, then a projective relationship exists between the photographic co-ordinates of the image points and the object space co-ordinates of the corresponding object points as illustrated in Figure 4.1.

This projective relationship can be represented as follows:

$$\begin{bmatrix} X_j - X_i^c \\ Y_j - Y_i^c \\ Z_j - Z_i^c \end{bmatrix} = \lambda \begin{bmatrix} r_{11} & r_{21} & r_{31} \\ r_{12} & r_{22} & r_{32} \\ r_{13} & r_{23} & r_{33} \end{bmatrix}_i \begin{bmatrix} x_{ij} - x_{p_i} \\ y_{ij} - y_{p_i} \\ - f_i \end{bmatrix} \tag{4.1}$$

By manipulation of matrices

$$\begin{bmatrix} x_{ij} - x_{p_i} \\ y_{ij} - y_{p_i} \\ - f_i \end{bmatrix} = \frac{1}{\lambda} \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}_i \begin{bmatrix} X_j - X_i^c \\ Y_j - Y_i^c \\ Z_j - Z_i^c \end{bmatrix} \tag{4.2}$$

where subscript i refers to the photo i (i = 1,2,....m) and subscript j refers to the object point A (j=1,2,....n). Here x,y are image co-ordinates. x_p and y_p are the photo co-ordinates of the principal point (p). f is the focal length of the camera. λ is a scale factor and r's are elements of R, a (3,3) orthogonal matrix, functions of three independent rotation parameters (ω,ϕ,κ) of the camera. The latter represent the rotations of the image space co-ordinate system (x,y,z) with respect of the object space co-ordinate system (X,Y,Z). X_j, Y_j and Z_j are the spatial co-ordinates of the object point A. Superscript c refers to the camera, so X^c,Y^c and Z^c are the spatial co-ordinates of the

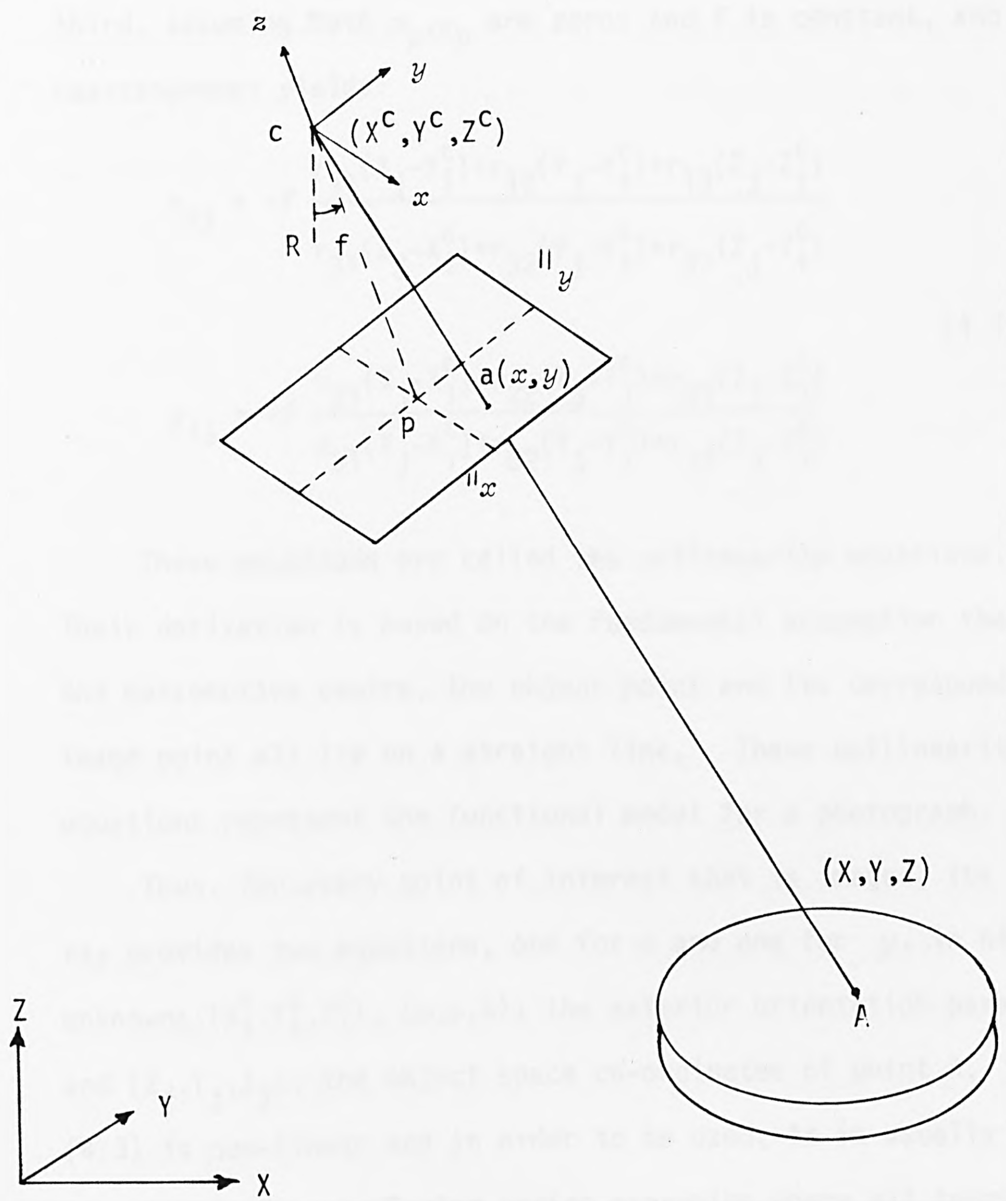


Figure 4.1. The geometrical basis of analytical photogrammetry

perspective centre in the object space co-ordinate system.

Dividing the first two equations in system (4.2) by the third, assuming both x_p, y_p are zeros and f is constant, and rearrangement yields:

$$x_{ij} = -f \frac{r_{11}(X_j - X_i^c) + r_{12}(Y_j - Y_i^c) + r_{13}(Z_j - Z_i^c)}{r_{31}(X_j - X_i^c) + r_{32}(Y_j - Y_i^c) + r_{33}(Z_j - Z_i^c)}$$

$$y_{ij} = -f \frac{r_{21}(X_j - X_i^c) + r_{22}(Y_j - Y_i^c) + r_{23}(Z_j - Z_i^c)}{r_{31}(X_j - X_i^c) + r_{32}(Y_j - Y_i^c) + r_{33}(Z_j - Z_i^c)} \quad (4.3)$$

These equations are called the collinearity equations. Their derivation is based on the fundamental assumption that the perspective centre, the object point and its corresponding image point all lie on a straight line. These collinearity equations represent the functional model for a photograph.

Thus, for every point of interest that is imaged, its light ray provides two equations, one for x and one for y, in nine unknowns: (X_i^c, Y_i^c, Z_i^c), (ω, ϕ, k); the exterior orientation parameters and (X_j, Y_j, Z_j), the object space co-ordinates of point A. System (4.3) is non-linear and in order to be used, it is usually reduced to a linear form by Taylor series expansion where all terms above and including the second order are dropped.

On the other hand, as is the case in practice, several photographs are taken from different locations of the same object so an over-determined model is available and hence, least squares is applied to get a unique solution vector and its covariance matrix.

4.2.1.2. Functional model for a Slope distance

The relationship between the measured slope distance (ℓ_{dh}) between points D,H and their three-dimensional co-ordinates (X_d, Y_d, Z_d) and (X_h, Y_h, Z_h) depicted in Figure 4.2 can be expressed as follows:

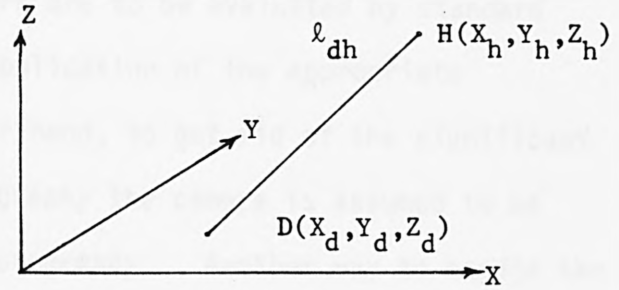


Figure 4.2. Typical slope distance

$$\ell_{dh} = [(X_h-X_d)^2 + (Y_h-Y_d)^2 + (Z_h-Z_d)^2]^{\frac{1}{2}} \qquad (4.4)$$

4.2.1.3. Notes on the functional models

The aforementioned functional model for a photograph as a perspective projection described by the collinearity equations (4.3) is rather theoretical. Due to some non-perspective projection parameters the photograph departs from such a projection. The following can be considered as non-perspective projection parameters:

i) the photographic co-ordinates of the principal point (p) are probably not (0,0).

ii) the axes of the photograph defined by the lines connecting the fiducial marks might be non-orthogonal.

iii) the radial and decentring lens distortion.

iv) deformation of the emulsion carrier (film or glass).

v) the atmospheric refraction.

So, these factors affecting the functional model must be eliminated or corrected for in the functional model for the photograph. For the functional model for distances no mention of the systematic errors which would occur has been given. It is assumed that these errors are to be evaluated by standard calibration methods, and application of the appropriate corrections. On the other hand, to get rid of the significant systematic errors of photography the camera is assumed to be calibrated prior to the photography. Another way to handle the problem of systematic errors of photography, as has been mentioned in section (3.2), is to use an additional parameter set to take account of these errors (Gruen, 1978).

It has been decided that no additional parameters be adopted, thus avoiding instability of the solution which might occur if additional parameters were introduced (Cooper, 1983). Moreover, inclusion of additional parameters might lead to the singularity of the augmented normal matrix given in section (4.2.3.2). Such instability and singularity are due to significant correlations between the additional parameters and the exterior orientation elements.

The general form of the functional model including both photogrammetric and slope distance observations can be written as:

$$\ell = F(x,c) \tag{4.5}$$

where

ℓ is the vector of observations;

x is the vector of unknown parameters; and

c is the vector of constants.

4.2.2. The Stochastic Model

In this investigation, as mentioned in section (4.2.1), the observations are photographic co-ordinates and slope distances. The stochastic model is the means by which the random errors are dealt with. Therefore equation (4.5) expresses the non-linear relationship between the vector of observations namely, the photographic co-ordinates and the slope distances, and the vector of unknown parameters namely, the exterior orientation elements and the object point co-ordinates.

The random or residual errors designated as v are introduced to give the stochastic model based on two assumptions. Firstly, $E\{v\} = 0$. This yields the linearised observation equations:

$$A\Delta x = b + v \tag{4.6}$$

where

A is the design matrix, $A = \left(\frac{\partial F}{\partial x}\right)_{x=x^o}$

Δx is the vector of corrections to x^o (the approximate co-ordinates of x) to give x via $x = x^o + \Delta x$.

b is the vector related to the observations or, vector of observational discrepancies, $b = \ell - \ell^o$, $\ell^o = F(x^o)$,

and v is the vector of residuals or, vector of corrections to the observations.

Secondly, the covariance matrix of the observations ℓ is symmetric positive-definite matrix $C_{\ell\ell} = \sigma_o^2 W^{-1}$, in which σ_o^2 is the a priori variance factor and W is the weight matrix of the observations.

The weight matrices are proportional to the inverse of the estimated covariance matrix of the observations. For example, the covariance matrix of the image co-ordinates x and y takes the form:

$$C_{xy} = \sigma_0^2 \begin{bmatrix} \sigma_x^2 & \sigma_{xy} \\ \sigma_{xy} & \sigma_y^2 \end{bmatrix} \tag{4.7}$$

in which σ_x^2 and σ_y^2 are the variances of x and y respectively and σ_{xy} is the covariance of the two measurements. From equation (4.7) the weight matrix of these measurements will be:

$$W_{xy} = \bar{\sigma}_0^2 \, C_{xy}^{-1} = \bar{\sigma}_0^2 \begin{bmatrix} \sigma_x^2 & \sigma_{xy} \\ \sigma_{xy} & \sigma_y^2 \end{bmatrix}^{-1} \tag{4.8}$$

Although the co-ordinates for a given point may be correlated it is assumed that co-ordinates for different points are uncorrelated. Further assumptions have been made on the weight matrix in this study. That is x and y co-ordinates of a given point are also uncorrelated. Although such an assumption may be invalid it is accepted because it is not possible to account for such a correlation in the covariance matrix of the observations. As a consequence, the covariance matrix for the image co-ordinates x and y and the corresponding weight matrix W have a diagonal form:

$$W_{xy} = \bar{\sigma}_0^2 C_{xy}^{-1} = \bar{\sigma}_0^2 \begin{bmatrix} \frac{1}{\sigma_x^2} & 0 \\ 0 & \frac{1}{\sigma_y^2} \end{bmatrix} \tag{4.9}$$

Applying the same assumptions stated above, the weight matrix for the other observed quantities can be formulated.

In other words both $C_{\ell\ell}$ and W are diagonal matrices. Consequently, the entire weight matrix can be written as:

$$W = \begin{bmatrix} W_p & 0 \\ 0 & W_g \end{bmatrix} \qquad (4.10)$$

where W_p is the diagonal weight matrix of the photogrammetric observations, and

W_g is the diagonal weight matrix of the slope distance observations.

4.2.3. The Least Squares estimation and Datum definition

It is relevant to mention at the outset that, briefly, the adjustment process determines the unknown parameters based on the information contained in the discrepancies between the measured values and those computed from the assumed model.

The operator that relates the corrections in the unknown parameters to these discrepancies is the design matrix of the problem. In the usual case where there are redundant observations available, the row space of the design matrix has a dimension larger than that of its column space. If its columns are linearly independent, then the rank is equal to their number, the dimension of the column space, and the problem will have a unique solution. In the event that two or more of its columns are linearly dependent, the design matrix is rank deficient, its deficiency determined by the number of interdependent columns.

We can now, in light of the above discussion, start as follows: Recalling equations (4.6) and (4.10) the problem is to estimate Δx from A and W using the principle of least squares which minimises the quadratic form:

$$\Omega = \hat{v}^T W \hat{v} \tag{4.11}$$

If A is of full column rank, the Best Linear Unbiased Estimate (BLUE) $\Delta\hat{X}$ can be found (e.g. Mikhail, 1976; Welsch, 1979) by solving the normal equations:

$$(A^T WA)\Delta\hat{x} = A^T Wb \tag{4.12}$$

then
$$\Delta\hat{x} = (A^T WA)^{-1} A^T Wb \tag{4.13}$$

and the covariance matrix of the least squares estimate is given as:

$$\hat{C}_{\hat{x}\hat{x}} = \hat{C}_{\Delta\hat{x}\,\Delta\hat{x}} = \hat{\sigma}_o^2 (A^T WA)^{-1} \tag{4.14}$$

in which $\hat{\sigma}_o^2 = \hat{v}^T W \hat{v}/\nu$: the a posteriori variance factor and ν is the number of degrees of freedom. The latter represents the number of observations in excess of the minimum required for a unique solution.

Because the functional model includes co-ordinates whose reference system has not been defined in the observation equations (4.6) nor in the normal equations (4.12), the matrix $(A^T WA)$ will be singular.

The datum definition is referred to as the Zero Order design problem which has been discussed in Chapter 3. It has been mentioned that rank deficiency of the design matrix as well as the normal equations matrix can be as high as seven: position of the origin of the co-ordinates (three elements); direction of co-ordinate axes (three elements); and the scale (one element). Therefore such elements have to be defined in order to define the datum. Conventionally these elements can be defined by assigning fixed values to all three co-ordinates of two selected points and to one co-ordinate of a third, non-collinear point. In deformation analysis, however, such a procedure has the main disadvantage that it is obligatory at the second epoch to relocate the same datum points which were chosen at the first epoch. These points, themselves, are likely to undergo deformation. In other words there is no guarantee they are stable.

To overcome the problem of dependency on stability of any points in the network, the free network technique (inner constraints method) was used to define the datum.

From the computational viewpoint, there are two approaches to get the solution and the covariance matrix of the unknown parameters in a free network adjustment.

4.2.3.1. The Moore-Penrose inverse approach

Appendix A demonstrates some of the details concerning the definition and characteristics of such an inverse.

Now, let $N = A^TWA$, $u = A^TWb$

then the normal equations(4.12) can be expressed as:

$$N\Delta\hat{x} = u \tag{4.15}$$

As matrix N is singular, the Moore-Penrose inverse N^+ can be applied to replace the standard Cayley inverse given in equation (4.14). However, the solution in such a case will be Best Linear Biased Estimate (BLBE) (Welsch, 1979).

Then

$$\Delta\hat{x} = N^+ u \tag{4.16}$$

The following properties are associated with the above solution:

(i) it is a least square solution (i.e. $\hat{v}^T W \hat{v}$ is a minimum)

(ii) it is a minimum norm solution (i.e. $\Delta x^T \Delta x$ is a minimum),

and (iii) its covariance matrix and hence that of $\hat{x}$ via $\hat{x} = \overset{o}{x} + \Delta\hat{x}$ has a minimum trace (i.e. trN^+ = minimum).

4.2.3.2. <u>The minimal set of constraints approach</u>

An alternative method to achieve the minimum trace covariance matrix (Granshaw, 1980) is to apply an appropriate set of constraints $G^T \Delta x = 0$ to the observation equations. Such set is a subset of minimal constraints and is called inner constraints (Appendix B).

$$A\,\Delta x = b + v$$

$$G^T \Delta x = 0 \tag{4.17}$$

where

G is of order $(6n_s+3n_o)x7$: $\begin{bmatrix} n_s = \text{no.of cameras used} \\ n_o = \text{no.of object points} \end{bmatrix}$

and satisfies the following two conditions:

(i) the columns of G are linearly independent, between themselves and the columns of A.

(ii) $G^T \Delta x = 0$

Denoting $\hat{k}$ as the estimator for the vector of Lagrangian multipliers of order (7x1) the least squares solution of the system (4.17) is given by:

$$\begin{bmatrix} A^T W A & G \\ G^T & 0 \end{bmatrix} \begin{bmatrix} \Delta\hat{x} \\ \hat{k} \end{bmatrix} = \begin{bmatrix} A^T W b \\ 0 \end{bmatrix} \qquad (4.18)$$

The matrix of coefficients is now regular and the solution $\Delta\hat{x}$ can be found in the conventional way.

It should be noted that what we are primarily concerned about, are the object point co-ordinates and not the exterior orientation parameters of the cameras. Therefore it is reasonable to apply inner constraints not to minimise the trace of the covariance matrix of the full solution but to minimise only the trace of the covariance matrix of the object point co-ordinates.

Let us partition Δx as $\begin{bmatrix} \Delta x_1 \\ \Delta x_2 \end{bmatrix}$ corresponding to $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ where

Δx_1 represents the vector of corrections to the approximate orientation elements to give x_1 and Δx_2 represents the vector of corrections to the approximate co-ordinates of the n_0 object points to give x_2 and G as $\begin{bmatrix} 0 \\ G \end{bmatrix}$. The linearised observation equations with no constraints will read:

$$\begin{bmatrix} A_1 & A_2 \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \end{bmatrix} = b + v \tag{4.19}$$

Then the photogrammetric normal equations are given by

$$\begin{bmatrix} A_1^T W A_1 & A_1^T W A_2 \\ A_2^T W A_1 & A_2^T W A_2 \end{bmatrix} \begin{bmatrix} \Delta \hat{x}_1 \\ \Delta \hat{x}_2 \end{bmatrix} = \begin{bmatrix} A_1^T W b \\ A_2^T W b \end{bmatrix} \tag{4.20}$$

Bordering (4.20) with $G = \begin{bmatrix} 0 \\ G \end{bmatrix}$ yields the augmented photogrammetric normal equations:

$$\begin{bmatrix} A_1^T W A_1 & A_1^T W A_2 & 0 \\ A_2^T W A_1 & A_2^T W A_2 & G \\ 0 & G^T & 0 \end{bmatrix} \begin{bmatrix} \Delta \hat{x}_1 \\ \Delta \hat{x}_2 \\ \hat{k} \end{bmatrix} = \begin{bmatrix} A_1^T W b \\ A_2^T W b \\ 0 \end{bmatrix} \tag{4.21}$$

The standard Cayley inverse of the coefficient matrix of the augmented photogrammetric normal equations gives the minimum trace solution for $\Delta\hat{x}_2$ (and hence for $\hat{x}_2$) if the following transformation matrix G is applied (see Appendix B).

$$G_j = \begin{bmatrix} 1 & 0 & 0 & 0 & Z_j^o & -Y_j^o & X_j^o \\ 0 & 1 & 0 & -Z_j^o & 0 & X_j^o & Y_j^o \\ 0 & 0 & 1 & Y_j^o & -X_j^o & 0 & Z_j^o \end{bmatrix} \tag{4.22}$$

in which the subscript j denotes the jth object point with approximate co-ordinates (X_j^o, Y_j^o, Z_j^o). The first three columns of G_j define the origin of the datum, the columns 4, 5 and 6 define the direction of the axes and the last column (7th column) defines the scale.

It should be noticed that if, in the network used, one or more slope distances are measured, then the 7th column is omitted from G. Moreover, if height differences are measured in the object space, then the 4th, 5th, and the 7th columns are to be dropped.

Investigation of the product $G^T\Delta x$ shows that the inner constraints are equivalent to the following equations:

$$\text{(i)} \quad \sum_{i=1}^{n_o} \delta X_i = \sum_{i=1}^{n_o} \delta Y_i = \sum_{i=1}^{n_o} \delta Z_i = 0$$

or

$$\sum_{i=1}^{n_o} X_i = \sum_{i=1}^{n_o} X_i^o$$

(i.e. the co-ordinate system's origin is the centroid of the approximate co-ordinates x_i^o).

$$\text{(ii)} \quad \sum_{i=1}^{n_o} (Y_i^o \delta Z_i - Z_i^o \delta Y_i) = \sum_{i=1}^{n_o} (Z_i^o \delta X_i - X_i^o \delta Z_i) = \sum_{i=1}^{n_o} (X_i^o \delta Y_i - Y_i^o \delta X_i) = 0$$

(i.e the mean orientation of the system of points will not change after adjustment).

$$\text{(iii)} \quad \sum_{i=1}^{n_o} (X_i^o \delta X_i + Y_i^o \delta Y_i + Z_i^o \delta Z_i) = 0$$ (i.e. the mean scale of the network will be held fixed).

Thus the zero variance datum is implicitly defined in terms of the initial values of the co-ordinates (approximate co-ordinates) of the object points. Hence, the derived covariance matrices of the object points are related to this datum. It is notable that the co-ordinates are, in this way, datum dependent whereas some of their derived functions (e.g. adjusted measurements) are not.

To conclude, in this research, equations (4.10), (4.21) and (4.22) are of crucial importance to the simulated cases discussed in the coming chapters when assessing the design criteria.

CHAPTER 5

SIMULATED NETWORKS FOR DEFORMATION ANALYSIS (CUBE)

5.1. Introduction

To design and plan a project before it is carried out is a common engineering practice. Pre-analysis is the simulation of the propagation of uncertainties in observations to uncertainties in results. In this way it is possible during the design of a photogrammetric network to predict the accuracies of the results and compare them with the desired accuracies. If the predicted results are too accurate or not accurate enough the design can be changed before any expensive work is carried out.

In this Chapter the simulation procedures used to demonstrate the concepts of precision, reliability and sensitivity analysis are described.

5.2. Network Configurations

Suppose a solid cube (Figure 5.1) of 4 m side comprises 26 regularly distributed points on its six faces. Such points represent the target array. Eight convergent photographs shown (each camera axis being directed towards one corner and the centre of the cube) will image the object with considerable redundancy. Fictitious provisional co-ordinates were assumed for the targets and the camera stations based on an origin at the centre of the cube. The taking camera is supposed to be the Zeiss (Jena) UMK 10/1318 with nominal principal distance 100 mm. Table (5.1) shows the

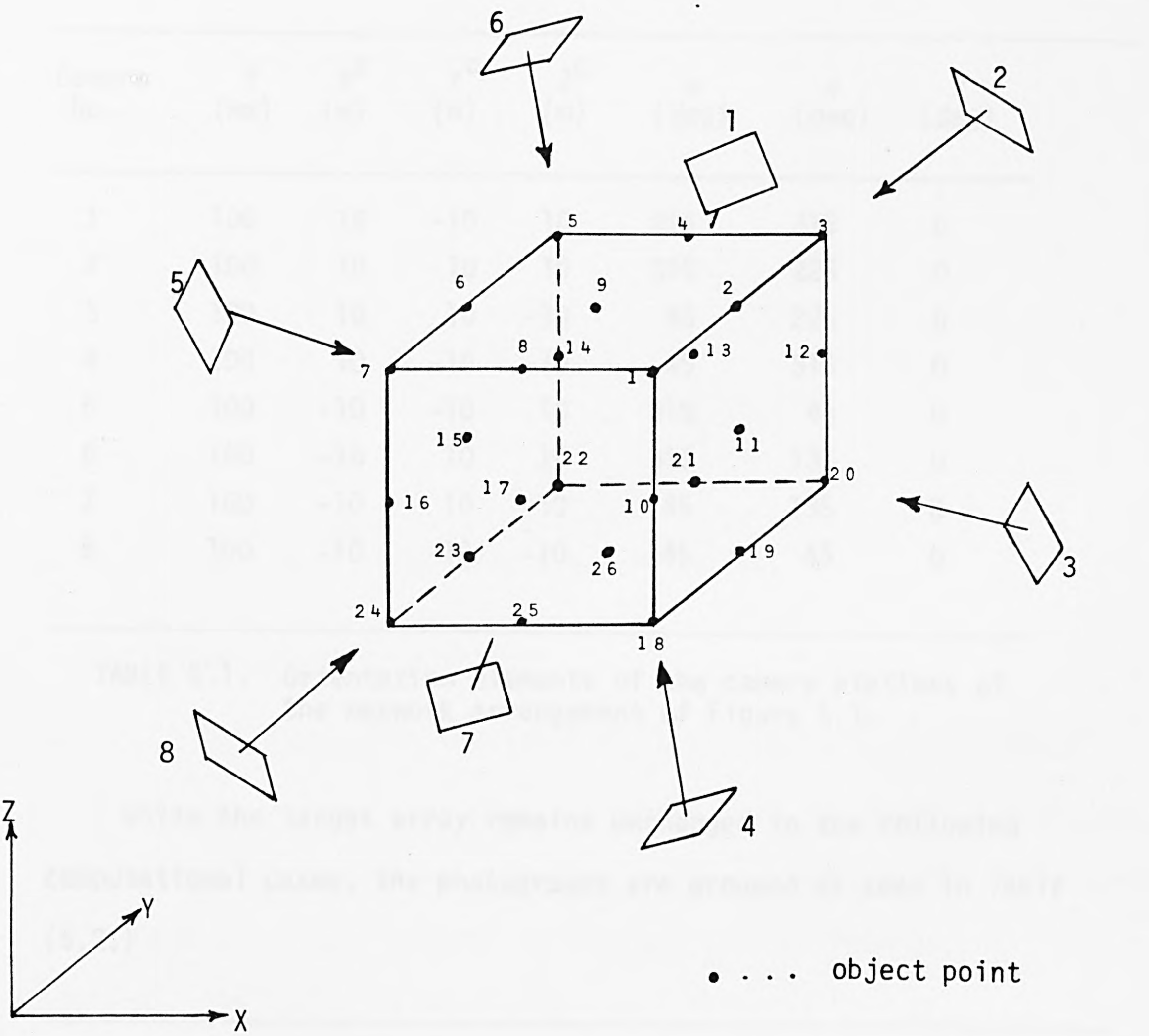


Figure 5.1. Object target array and Camera station configuration.

orientation elements of the individual photographs.

Camera No.	f (mm)	X^c (m)	Y^c (m)	Z^c (m)	ω (deg)	ϕ (deg)	κ (deg)
1	100	10	-10	10	315	315	0
2	100	10	10	10	315	225	0
3	100	10	10	-10	45	225	0
4	100	10	-10	-10	45	315	0
5	100	-10	-10	10	315	45	0
6	100	-10	10	10	315	135	0
7	100	-10	10	-10	45	135	0
8	100	-10	-10	-10	45	45	0

TABLE 5.1. Orientation elements of the camera stations of the network arrangement of Figure 5.1.

While the target array remains unchanged in the following computational cases, the photographs are grouped as seen in Table (5.2.)

Case	Photo-arrangement	Total number of object points	Number of slope distances
1	1-2-3-4	25	-
2	Case 1 + slope dist.	25	8
3	1-2-3-4-5	25	-
4	Case 3 + slope dist.	25	8
5	1-2-3-4-5-6	26	-
6	Case 5 + slope dist.	26	8
7	1-2-3-4-5-6-7	26	-
8	Case 7 + slope dist.	26	8
9	1-2-3-4-5-6-7-8	26	-
10	Case 9 + slope dist.	26	8

TABLE 5.2 Photo-arrangements, number of targets and slope distances.

Observation No.	Target Object Points From	To
1	1	20
2	3	18
3	7	18
4	1	24
5	5	24
6	7	22
7	3	22
8	5	20

TABLE 5.3. Simulated slope distances.

5.3. The Observation equations

5.3.1. The Observation Equations of Photogrammetric Measurements

The observation equations are confined to the image co-ordinates. As the camera can only be used to recreate directions, the determination of object point co-ordinates must be achieved by intersection from at least two spatially separated camera stations.

As has been mentioned in Chapter 4, the collinearity condition equations(4.3) constitute the functional model for a photograph. Therefore, for any object point imaged and measured on a photograph, two equations can be written. One equation relates the x-co-ordinate of the photogrammetric measurement to the six elements of exterior orientation (X^c_i, Y^c_i, Z^c_i, ω_i, ϕ_i, κ_i) of the ith camera and to the three object point co-ordinates of the point (X_j, Y_j, Z_j), and other equation relates the y-co-ordinate of the photogrammetric measurement to the same items.

Linearisation of equations (4.3) using Taylor series expansion and neglecting second and higher order terms yields the following observation equations for photogrammetric measurements based on approximate values x^o for x.

$$A_p \Delta x = V_p + b_p \qquad (5.1)$$

Adopting the partitioning forms: $A_p = [A_{p_1}, A_{p_2}]$, $\Delta x = \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \end{bmatrix}$, $b_p = \begin{bmatrix} b_{p_1} \\ b_{p_2} \end{bmatrix}$, equation (5.1) could be put in the form:

$$A_{p1}\Delta x_1 + A_{p_2}\Delta x_2 = V_p + b_p \qquad (5.1a)$$

where: Δx_1 is the vector of corrections to photo orientation elements

Δx_2 is the vector of corrections to object point co-ordinates

b_p is the vector of photo observational discrepancies

V_p is the vector of photo co-ordinate residuals,

and A_{p1}, A_{p_2} are the design matrices.

For one object point imaged on one photograph:

$$\underset{(2,6)}{A_{p1}} = \begin{bmatrix} \frac{\partial x}{\partial X^c} & \frac{\partial x}{\partial Y^c} & \frac{\partial x}{\partial Z^c} & \frac{\partial x}{\partial \omega} & \frac{\partial x}{\partial \phi} & \frac{\partial x}{\partial \kappa} \\ \frac{\partial y}{\partial X^c} & \frac{\partial y}{\partial Y^c} & \frac{\partial y}{\partial Z^c} & \frac{\partial y}{\partial \omega} & \frac{\partial y}{\partial \phi} & \frac{\partial y}{\partial \kappa} \end{bmatrix}; \qquad \underset{(2,3)}{A_{p2}} = \begin{bmatrix} \frac{\partial x}{\partial X} & \frac{\partial x}{\partial Y} & \frac{\partial x}{\partial Z} \\ \frac{\partial y}{\partial X} & \frac{\partial y}{\partial Y} & \frac{\partial u}{\partial Z} \end{bmatrix}$$

$$\underset{(6,1)}{\Delta x_1} = \begin{bmatrix} \delta X^c \\ \delta Y^c \\ \delta Z^c \\ \delta\omega \\ \delta\phi \\ \delta\kappa \end{bmatrix}; \qquad \underset{(3,1)}{\Delta x_2} = \begin{bmatrix} \delta X \\ \delta Y \\ \delta Z \end{bmatrix}; \qquad \underset{(2,1)}{V_p} = \begin{bmatrix} v_x \\ v_y \end{bmatrix}; \qquad \underset{(2,1)}{b_p} = \begin{bmatrix} b_x \\ b_y \end{bmatrix}$$

If there are n_s camera stations and n_o object points, the design matrix, A, is of order $2n_i \times (6n_s + 3n_o)$ in which n_i is the number of image points measured.

5.3.2. The Observation Equations of Survey measurements

Observation equations were developed for the slope distances in Chapter 4. It should be noted that in order to combine photogrammetric and survey measurements in a simultaneous adjustment, all the observation equations must be expressed in the same reference co-ordinate system. The utilisation of survey data will require the development of a solution which can combine both photogrammetric and survey measurements in a simultaneous solution.

In matrix form, the observation equations for slope distances based on approximate values x^o for x can be:

$$A_g \, \Delta x_2 = v_g + b_g \qquad (5.2)$$

where

b_g is the vector of survey observational discrepancies

v_g is the vector of survey observation residuals,

and A_g is the design matrix.

For slope distance, ℓ_{ij},

$$\underset{(1,6)}{A_g} = \left[\frac{\partial \ell_{ij}}{\partial X_j} \; \frac{\partial \ell_{ij}}{\partial Y_j} \; \frac{\partial \ell_{ij}}{\partial Z_j} \; \frac{\partial \ell_{ij}}{\partial X_i} \; \frac{\partial \ell_{ij}}{\partial Y_i} \; \frac{\partial \ell_{ij}}{\partial Z_i} \right]$$

and $\underset{(1,1)}{v_g} = [v_{\ell_{ij}}]$; $\underset{(1,1)}{b_g} = [b_{ij}]$

Equation (5.1a) coupled with equation (5.2) constitute the complete extended observation equations in the simultaneous bundle adjustment model.

$$\begin{bmatrix} \underset{(2mn,6m)}{A_{p_1}} & \underset{(2mn,3n)}{A_{p_2}} \\ \underset{(k,6m)}{0} & \underset{(k,3n)}{A_g} \end{bmatrix} \begin{bmatrix} \underset{(6m,1)}{\Delta X_1} \\ \underset{(3n,1)}{\Delta X_2} \end{bmatrix} = \begin{bmatrix} \underset{(2mn,1)}{v_p} \\ \underset{(k,1)}{v_g} \end{bmatrix} + \begin{bmatrix} \underset{(2mn,1)}{b_p} \\ \underset{(k,1)}{b_g} \end{bmatrix} \tag{5.3}$$

where

m,n,k represent orders of the above matrices as follows:-

m = number of photographs

n = number of object points imaged on m (or less) photographs,

k = number of survey measurements (slope distances) measured between targets.

5.4. Formation of the Normal Equations

5.4.1. Structure of the Normal equations matrix (N) with strictly photogrammetric measurements

From the linearised observations equations (5.1a) and their corresponding weights we can form the normal equations. The partitioned normal equations can be given by:

$$\begin{bmatrix} A_1^T W_p A_1 & A_1^T W_p A_2 \\ A_2^T W_p A_1 & A_2^T W_p A_2 \end{bmatrix} \begin{bmatrix} \Delta\hat{x}_1 \\ \Delta\hat{x}_2 \end{bmatrix} = \begin{bmatrix} A_1^T W_p b \\ A_2^T W_p b \end{bmatrix} \tag{5.4}$$

in which W_p is the image co-ordinate weight matrix. With $N_s = A_1^T W_p A_1$, $N_o = A_2^T W_p A_2$ and $N_{so} = A_1^T W_p A_2$, equation (5.4) could be written in the following simplified form:

$$\begin{bmatrix} N_s & N_{so} \\ N_{so}^T & N_o \end{bmatrix} \begin{bmatrix} \Delta\hat{x}_1 \\ \Delta\hat{x}_2 \end{bmatrix} = \begin{bmatrix} A_1^T W_p b \\ A_2^T W_p b \end{bmatrix} \qquad (5.4a)$$

where the suffix o indicates object point co-ordinates and the suffix s indicates the camera parameters.

If there are n_s camera stations; each photograph will produce a 6 x 6 entry in N_s. In the meantime, if we have n_o object points; N_o will be a series of 3 x 3 diagonal blocks, with a block for each object point. N_{so} represents the interaction between an object point and the photographs on which it is imaged. Consequently, the entire coefficient matrix of normal equations is of order $(6n_s + 3n_o) \times (6n_s + 3n_o)$. Typical structures of normal equations are represented in Figures (5.2) through (5.6). It is notable that such structures show the property of being sparse banded-bordered matrices as only a few of the total n_o are imaged on each photograph.

It should be noted that if an object point does not image on a particular photograph, the corresponding 6 x 3 submatrix of N_{so} is null and the structure of N_{so} is not regular.

Due to the lack of datum definition, the normal equations matrix (5.4a) is singular and the observation equations (5.1a) have column rank deficiency of 7 (3 translations, 3 rotations and one scale) as discussed in Chapter 4.

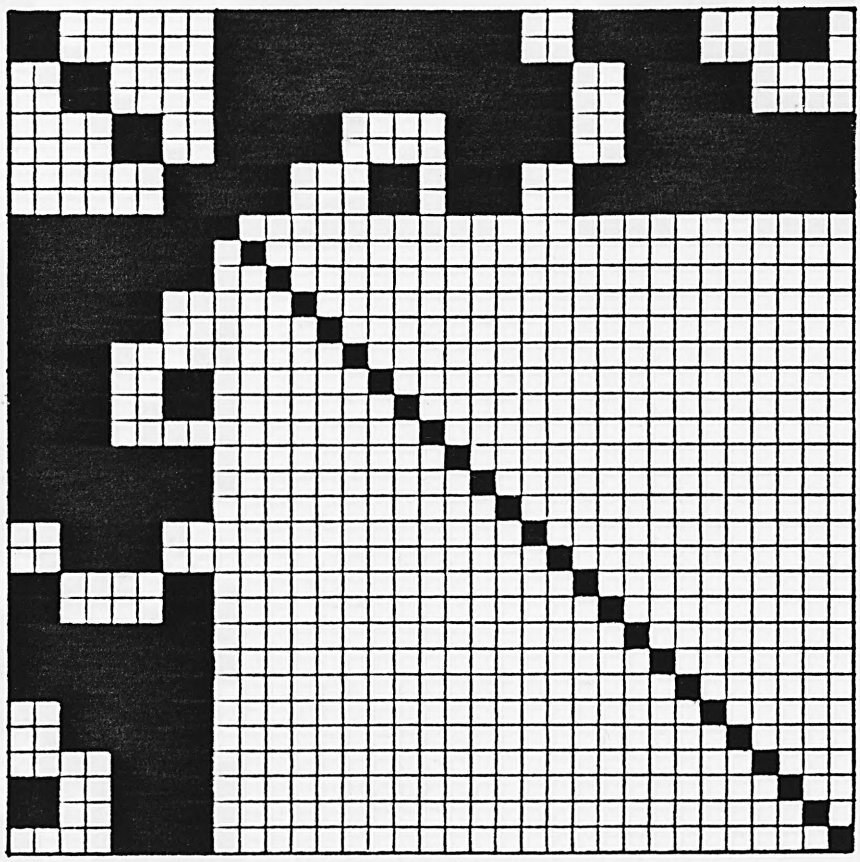


Figure 5.2. Structure of the coefficient matrix of the normal equations for 4 photos and 25 object points.

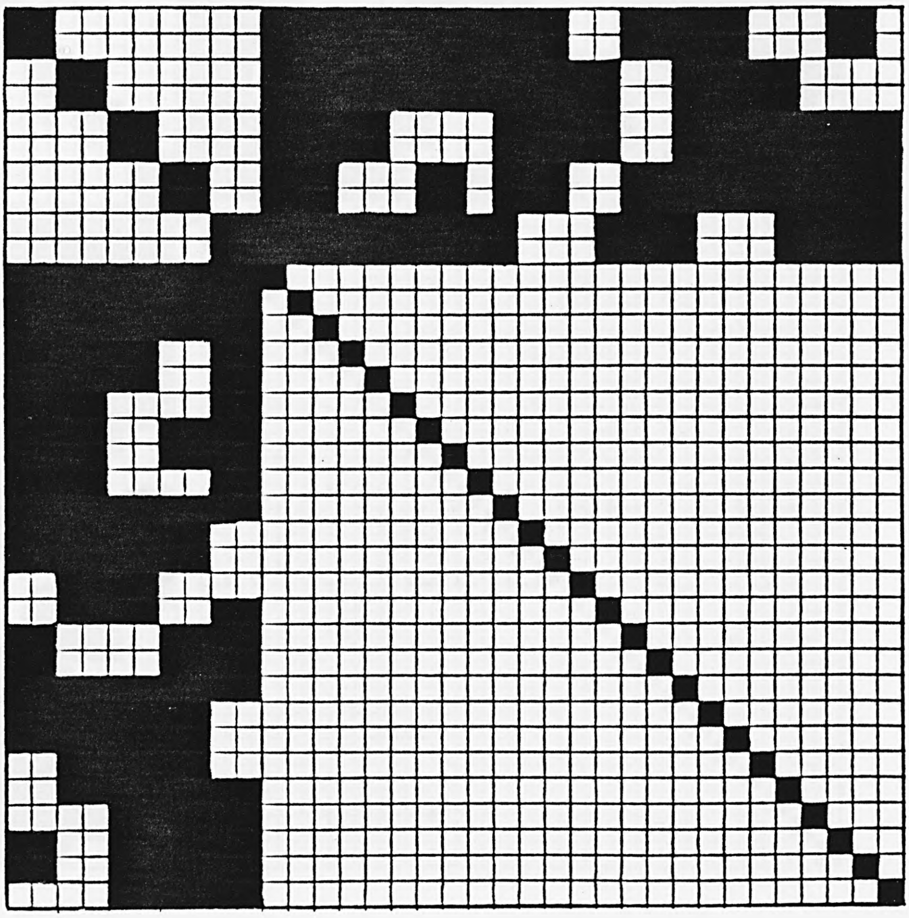


Figure 5.3. Structure of the coefficient matrix of the normal equations for 5 photos and 25 object points.

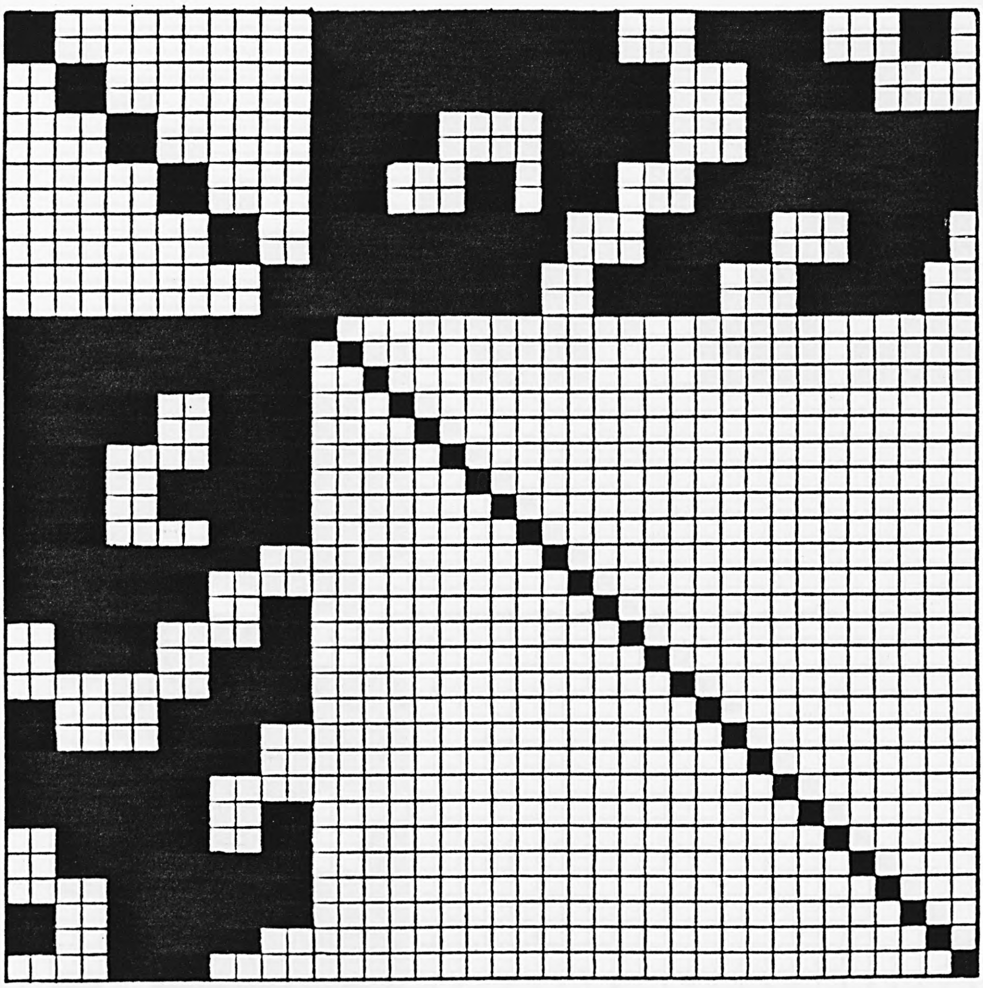


Figure 5.4. Structure of the coefficient matrix of the normal equations for 6 photos and 26 object points.

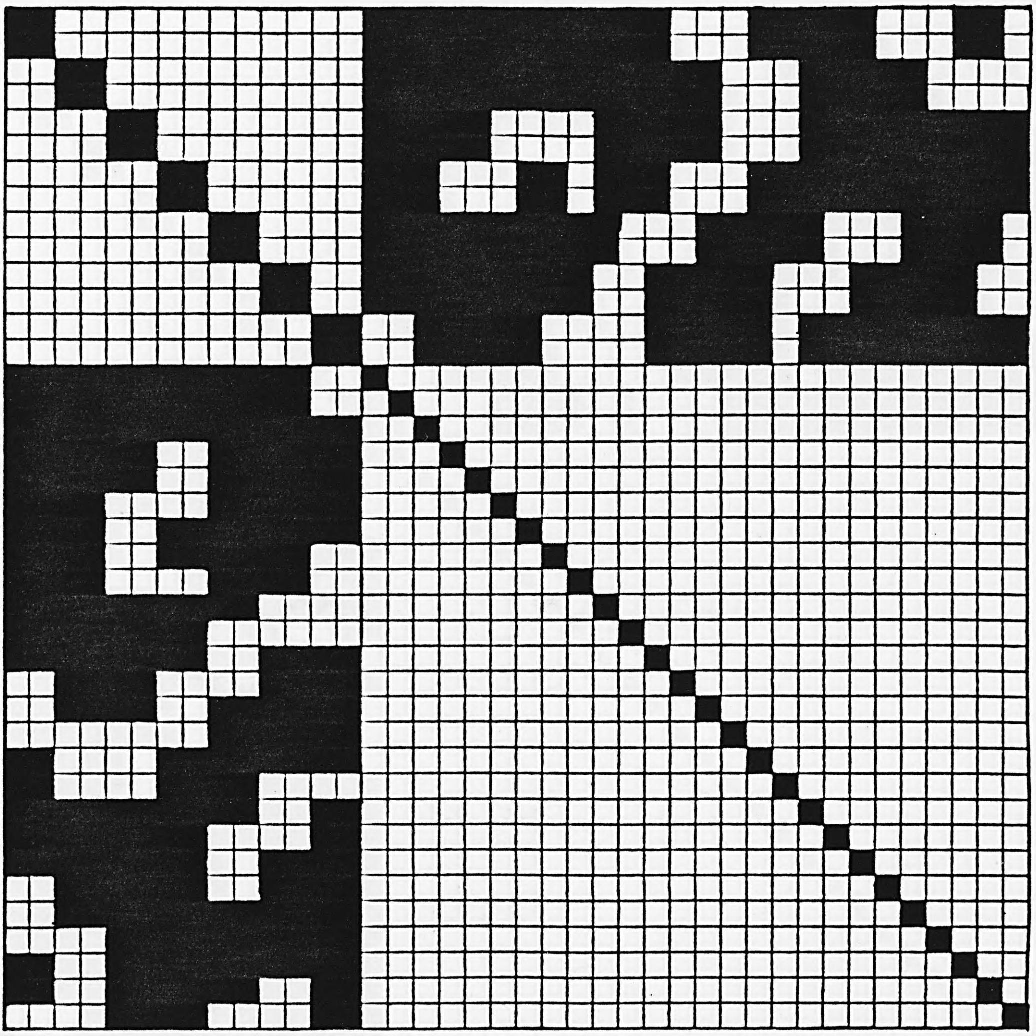


Figure 5.5. Structure of the coefficient matrix of the normal equations for 7 photos and 26 object points.

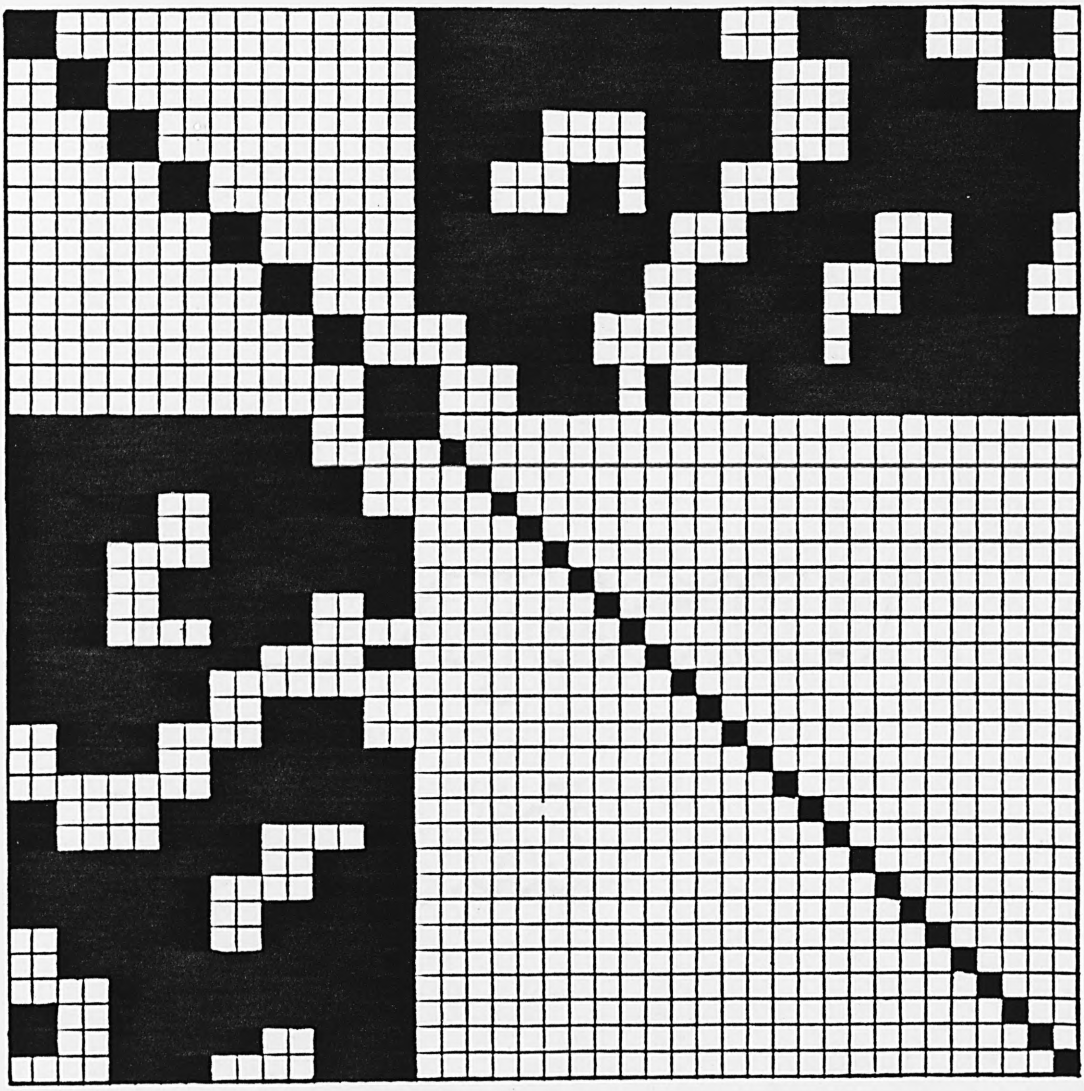


Figure 5.6. Structure of the coefficient matrix of the normal equations for 8 photos and 26 object points.

5.4.2. Effect of Survey measurements on the structure of N

The survey measurements are confined to the targets in this study. The slope distances between object points represent the survey measurements subject to investigation. Hence, their contribution to the normal equations will be in the submatrix N_o keeping the order of the normal equations constant for each two successive cases of the same number of cameras.

The structure of the normal equations with only photogrammetric observations as discussed in Section (5.4.1) can be exploited efficiently by reducing the size of the normal equations when seeking their solution. Such reduction can be performed through successive inversion of n_o (3 x 3) submatrices or n_s (6 x 6) ones depending on the size of n_o and n_s. However, the inclusion of n_d distances between object points, given in Table (5.3) necessitates the inversion of n_d (6 x 6) submatrices in addition to the usual inversion of a series of one of the two categories mentioned before.

Such an inclusion of survey observations in object space has a devastating effect upon the matrix N in terms of computer storage as the quality of sparsity is no longer retained. Further, the correlation between targets of object points increases the bandwidth of N_o significantly.

With the presence of survey measurements (slope distances) the normal equations (5.4) will be modified to count for the contribution of these survey measurements and could be written as:-

$$\begin{bmatrix} A_1^T W_p A_1 & A_1^T W_p A_2 \\ A_2^T W_p A_1 & A_2^T W_p A_2 + A_g^T W_g A_g \end{bmatrix} \begin{bmatrix} \Delta\hat{x}_1 \\ \Delta\hat{x}_2 \end{bmatrix} = \begin{bmatrix} A_1^T W_p b \\ A_2^T W_p b + A_g^T W_g b_g \end{bmatrix} \tag{5.5}$$

in which subscripts p and g designate photogrammetric and survey respectively, and W_g denotes the survey measurements weight matrix. Of prime computational importance is the fact that the normal equations matrix is created here as a point-wise accumulation without actually forming the observation equations matrix A. This means that the different observation equations need never be formed and considerable savings in computing and storage requirements is attained.

5.5. Inversion Algorithms and Covariance matrix of Co-ordinates

It has been mentioned, Section (4.2.2) that the common assumption that the a priori covariance matrix of the photogrammetric observations is a scalar matrix is in use; however it is almost certainly fallacious, yet it has the merit of being common to all the results which we wish to compare. As one of the primary goals of this investigation is to compare different network configurations to assess precision, reliability and sensitivity criteria for different designs, the most universal estimate of precision is the covariance matrix derived from the inverse of the coefficient matrix of the normal equations as discussed in Section (4.2.3).

Owing to the column rank deficiecy of (5.4a) and (5.5), there is no standard Cayley inverse as has been addressed in the above-mentioned sections, but a generalised inverse, namely the Moore-Penrose can be used. Such an inverse yields minimum trace covariance matrix of the unknowns. Through this investigation, F01BLF subroutine from the National Algorithm Group (NAG) Library was used to determine the Moore-Penrose inverse of the normal equations in cases 1, 2, 3 and 4. Unfortunately, its use was inefficient as it is rather expensive in terms of execution time and computer storage.

Alternatively, the likely most straight-forward approach is the method of inner constraints as mentioned in Section (4.2.3.2) and discussed, in detail, in Appendix B. It involves the bordering of the submatrix corresponding to the object point co-ordinates of the singular matrix, equation (5.5) with a transformation matrix, G, to minimise only the trace of their cofactor matrix.

G satisfies the condition $G^T \Delta x_2 = 0$. For an object point, j, in a network with a datum defect of seven, the appropriate 3 x 7 matrix G_j is given as follows (see Appendix B):

$$G_j = \begin{bmatrix} 1 & 0 & 0 & 0 & Z_j^o & -Y_j^o & X_j^o \\ 0 & 1 & 0 & -Z_j^o & 0 & X_j^o & Y_j^o \\ 0 & 0 & 1 & Y_j^o & -X_j^o & 0 & Z_j^o \end{bmatrix} \qquad (5.6)$$

Under the scheme of inner constraints, the augmented normal equations matrix (non-singular) can be written as:

$$\begin{bmatrix} A_1^T W_p A_1 & A_1^T W_p A_2 & 0 \\ A_2^T W_p A_1 & A_2^T W_p A_2 + A_g^T W_g A_g & G \\ 0 & G^T & 0 \end{bmatrix} \begin{bmatrix} \Delta\hat{X}_1 \\ \Delta\hat{X}_2 \\ \hat{K} \end{bmatrix} = \begin{bmatrix} A_1^T W_p b \\ A_2^T W_p b + A_g^T W_g b_g \\ 0 \end{bmatrix} \qquad (5.7)$$

in which $\hat{K}$ is an estimator for the vector of Langrangian multipliers.

With the relatively small networks encountered in deformation monitoring, a direct solution can be considered for equation (5.7) using the standard Cayley inverse. The Cholesky factorisation algorithm best suited symmetric matrices cannot be used for the solution of equation (5.7) due to the introduction of a null principal submatrix which causes the matrix to become indefinite. Thus, an alternative to the Cholesky factorisation must be found.

At first sight, Gaussian elimination with partial or complete pivoting may appear to represent a useful alternative solution technique. In applying complete pivoting, the first trailing submatrix produced will generally no longer be symmetric. However, to preserve the symmetry property, the choice of pivots is restricted to the diagonal elements, yet the augmented matrix has a null principal submatrix which will give rise to zero pivots.

Further, the recursive partitioning technique, which could have been used to reduce the size of the normal equations matrix, cannot be applied as it necessitates the presence of the inverse of one of the main block diagonal submatrices which is not the case.

An efficient inversion subroutine, available at the London University Computer Centre: Scientific Subroutine Package (SSP), was used. Such a subroutine was tested and found to give very accurate results when double precision arithmetic was used. The computer programs were coded in Fortran 77 Language and were run on the Honeywell, Amdahl and Cray 1S systems.

Now, if the standard Cayley inverse of equation(5.7) is expressed as:

$$M = \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ & M_{22} & M_{23} \\ \text{Symmetric} & & \\ & & M_{33} \end{bmatrix}$$

then M_{22} is identical to the minimum trace cofactor matrix $Q_{\hat{x}\hat{x}}^{(2)}$ obtained when using the Moore-Penrose inverse.

As has been mentioned in Section (4.2.3.2) the incorporation of slope distances, the seventh column of G in equation (5.6) is to be suppressed.

The inner constraints can be imposed through G related in some cases to all object points and in others to a sub-set of targets. However, the former situation can be advantageous especially in sensitivity analysis stage.

5.6. Simulated Networks

5.6.1. Description

In order to examine the numerical behaviour of the different criteria for design, namely, precision, reliability and sensitivity, an experiment was conducted in which data were obtained through simulation. Of primary concern was examination of the magnitude of the change of the mean variance σ_m^2 of the object point co-ordinates, the maximum internal reliability r_{max} of the observations and the indicative parameter of sensitivity c which accompany addition of slope distance measurements and/or changes in the number of cameras used.

Ten distinct cases(Table (5.2)) were considered, 5 μm standard deviation has been assumed for the photogrammetric co-ordinates. Photography on glass plates with format 130 x 180 mm was assumed. The odd-numbered cases comprise pure photogrammetric observations whereas the rest (even-numbered) include in addition to the photogrammetric observations, 8 diagonal distances, with presupposed 0.5 mm standard deviation on the four upright faces (Tables (5.2), (5.3)) of the cube. The object target array comprises 26 points as mentioned earlier in Section (5.2).

The least number of photographs to cover the cube was found to be 4, otherwise some of the object points would appear only on one photograph, in which case its co-ordinates and their variances would be indeterminate. Therefore the start arrangement was to have 4 photographs as shown in Figure (5.1).

Density of object points was such that, on average, 19 image points would appear on each photograph. Examination of Table (5.2) shows that the number of object points in the first four cases is 25. There is one missing point (point 15) which does not appear in cases 1, 2, however it appears in cases 3,4 on photograph 5. The computer programs are coded in a way to override any point which appears only on one photograph. This assumption helps comparison between two identical sets of object points. On the other hand, addition of photographs 6,7, 8 (Figure (5.1)) leads to imaging point 15 on 2,3, 4 photographs respectively. So the number of targetted object points will be 26, instead of 25, in cases 5-10.

5.6.2. Results and Analysis

It is relevant to mention at the outset that due to the bulk of the results, it is not possible to include all of them in the text but a sample is to be included. Results of cases 9 and 10 were chosen to be typical representations as they give comprehensive illustration for the concepts to be discussed herein.

5.6.2.1. Mean Variance of Object Point co-ordinates

An indication of the effect of number of cameras employed in cases 1-10 is perhaps best gained by equation (3.15) which reads

$$\sigma_m^2 = \frac{1}{3n_o} \text{ tr } Q_{\hat{x}\hat{x}}^{(2)} \qquad (5.8)$$

in which n_o is the number of object points and $Q_{\hat{x}\hat{x}}^{(2)}$ is their cofactor matrix. Because the geometry of the target array is the same for each case, the derived a priori precision of the object point co-ordinates depends almost solely on the number of cameras.

The variances of the estimated object point co-ordinates for each case have been computed using the free network bundle adjustment procedure. Figures (5.7) and (5.8) illustrate the variation in the mean variance σ_m^2 with changes in the configuration of camera stations in both pure and combined cases, which comprise slope distances, respectively. It is obvious from the two figures that precision increases with the increase of the number of cameras used. As expected, the incorporation of the 8 slope distances does improve the precision. However, such an improvement is marginal "within"

POINT	VARIANCES		
NO	X (mm²)	Y (mm²)	Z (mm²)
1	0.14190372D 00	0.14768672D 00	0.14768672D 00
2	0.15479080D 00	0.16350965D 00	0.16273012D 00
3	0.14190372D 00	0.14768672D 00	0.14768672D 00
4	0.16447366D 00	0.16453512D 00	0.16453512D 00
5	0.14190372D 00	0.14768672D 00	0.14768672D 00
6	0.15479080D 00	0.16350965D 00	0.16273012D 00
7	0.14190372D 00	0.14768672D 00	0.14768672D 00
8	0.16447366D 00	0.16453512D 00	0.16453512D 00
9	0.24412594D 00	0.24300722D 00	0.20952573D 00
10	0.15479080D 00	0.16273012D 00	0.16350965D 00
11	0.18626339D 00	0.23267389D 00	0.23267389D 00
12	0.15479080D 00	0.16273012D 00	0.16350965D 00
13	0.24412594D 00	0.20952573D 00	0.24300722D 00
14	0.15479080D 00	0.16273012D 00	0.16350965D 00
15	0.18626339D 00	0.23267389D 00	0.23267389D 00
16	0.15479080D 00	0.16273012D 00	0.16350965D 00
17	0.24412594D 00	0.20952573D 00	0.24300722D 00
18	0.14190372D 00	0.14768672D 00	0.14768672D 00
19	0.15479080D 00	0.16350965D 00	0.16273012D 00
20	0.14190372D 00	0.14768672D 00	0.14768672D 00
21	0.16447366D 00	0.16453512D 00	0.16453512D 00
22	0.14190372D 00	0.14768672D 00	0.14768672D 00
23	0.15479080D 00	0.16350965D 00	0.16273012D 00
24	0.14190372D 00	0.14768672D 00	0.14768672D 00
25	0.16447366D 00	0.16453512D 00	0.16453512D 00
26	0.24412594D 00	0.24300722D 00	0.20952573D 00

TABLE 5.4. Estimates of the variances of the 26 object point co-ordinates for case 9.

POINT	VARIANCES		
NO	X (mm^2)	Y (mm^2)	Z (mm^2)
1	0.12732023D 00	0.13122635D 00	0.11095951D 00
2	0.16808970D 00	0.16320010D 00	0.17301027D 00
3	0.12732023D 00	0.13122635D 00	0.11095951D 00
4	0.16421463D 00	0.17833809D 00	0.17498007D 00
5	0.12732023D 00	0.13122635D 00	0.11095951D 00
6	0.16808970D 00	0.16320010D 00	0.17301027D 00
7	0.12732023D 00	0.13122635D 00	0.11095951D 00
8	0.16421463D 00	0.17833809D 00	0.17498007D 00
9	0.24385061D 00	0.24276900D 00	0.22268668D 00
10	0.16791256D 00	0.17611423D 00	0.16312118D 00
11	0.20169366D 00	0.23265213D 00	0.23240389D 00
12	0.16791256D 00	0.17611423D 00	0.16312118D 00
13	0.24410036D 00	0.22590193D 00	0.24277629D 00
14	0.16791256D 00	0.17611423D 00	0.16312118D 00
15	0.20169366D 00	0.23265213D 00	0.23240389D 00
16	0.16791256D 00	0.17611423D 00	0.16312118D 00
17	0.24410036D 00	0.22590193D 00	0.24277629D 00
18	0.12732023D 00	0.13122635D 00	0.11095951D 00
19	0.16808970D 00	0.16320010D 00	0.17301027D 00
20	0.12732023D 00	0.13122635D 00	0.11095951D 00
21	0.16421463D 00	0.17833809D 00	0.17498007D 00
22	0.12732023D 00	0.13122635D 00	0.11095951D 00
23	0.16808970D 00	0.16320010D 00	0.17301027D 00
24	0.12732023D 00	0.13122635D 00	0.11095951D 00
25	0.16421463D 00	0.17833809D 00	0.17498007D 00
26	0.24385061D 00	0.24276900D 00	0.22268668D 00

TABLE 5.5. Estimates for the variances of the 26 object point co-ordinates for case 10.

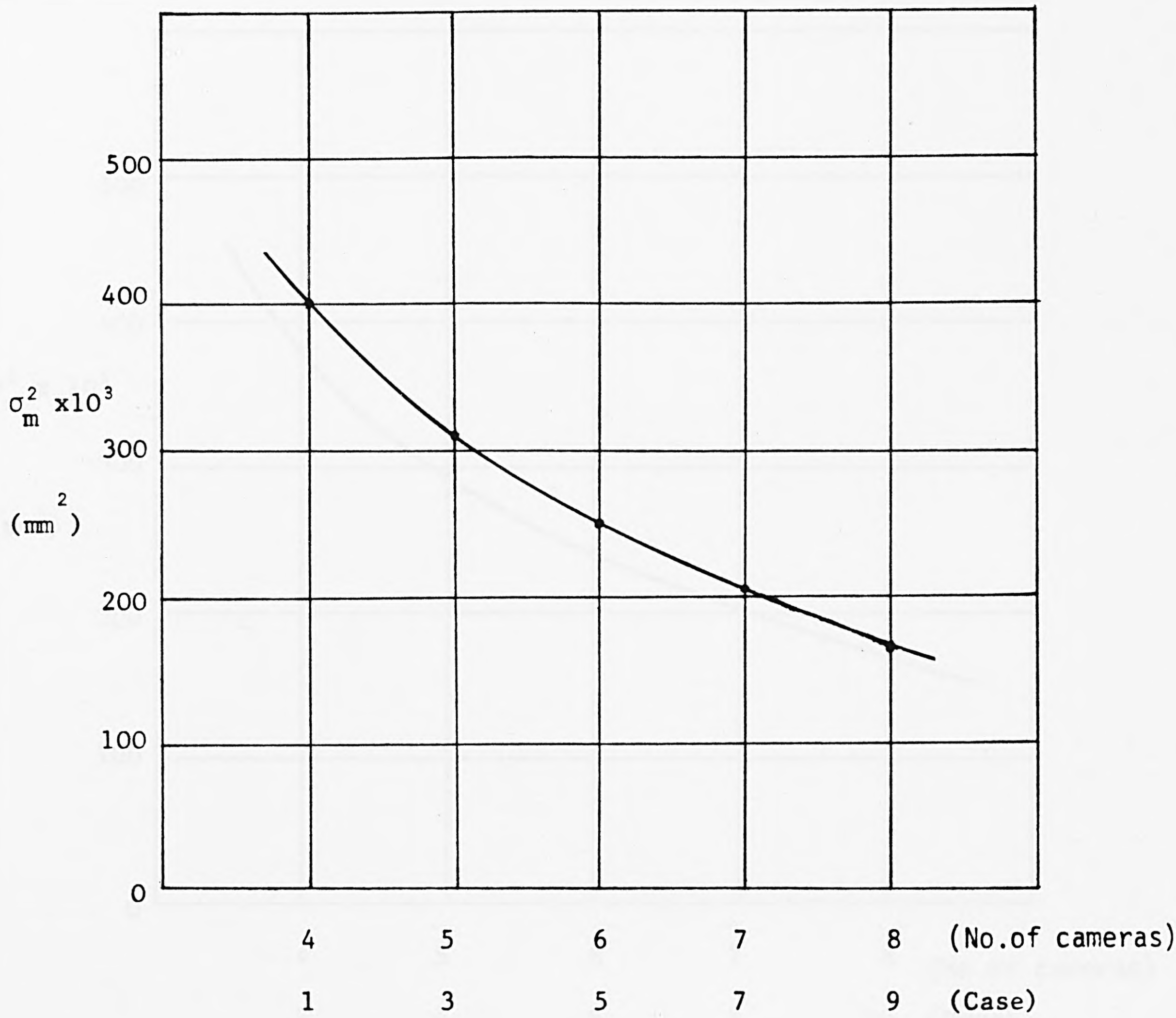


Figure 5.7. Relation between no.of cameras and precision (Photogrammetry)

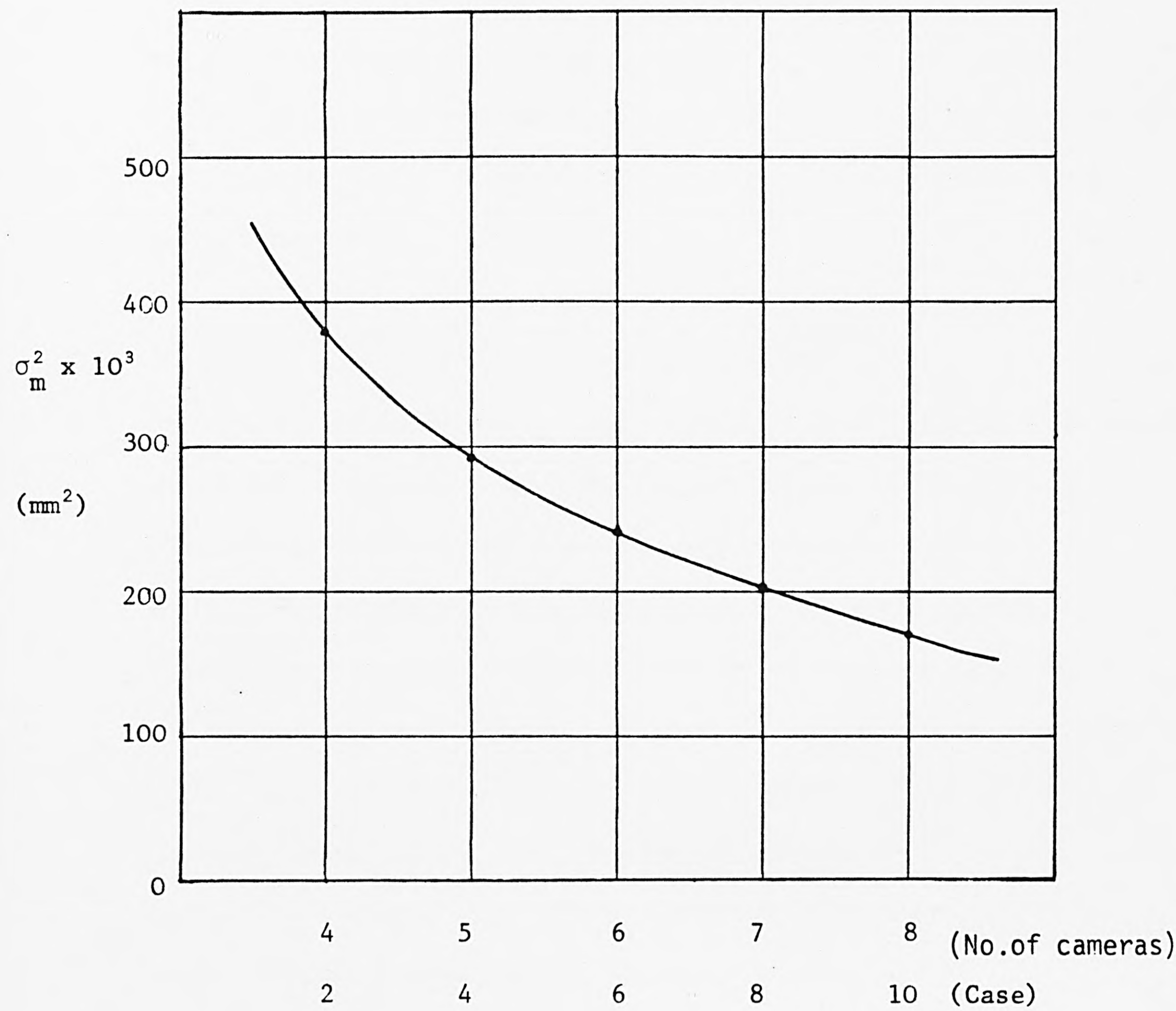


Figure 5.8. Relation between no.of cameras and precision (Photogrammetry + 8 distances)

the same number of camera cases. On the other hand, the improvement in precision "between" different number of camera cases is more significant. On the average there is a 24% and 23% improvement in precision should the number of cameras be increased to be 8 instead of 4 for the pure cases and combined cases respectively. The highest precision level is exhibited by cases 9 and 10.

5.6.2.2. r_{max} and other reliability indicators

In the design phase of a monitoring network, reliability values are computed to ensure that the network geometry and planned observation scheme exhibit a high degree of self-checking. Reliability values were attained by applying the concept of "redundancy numbers" (r_i), for each observation, computed from the diagonal elements ($q_{\hat{v}_i \hat{v}_i}$) of the cofactor matrix of the residuals $Q_{\hat{v}\hat{v}}$. Equation (5.9) represents such redundancy numbers. It can be argued that by computing the redundancy numbers in simulated tests, it is possible to discover weak situations in advance and avoid them while planning the photogrammetric project.

The global reliability of the x- and y- co-ordinate observations are represented by r_{max} (x) and r_{max} (y) respectively. Equation (5.9) can be given as:

$$r_i = q_{\hat{v}_i \hat{v}_i} \times w_{\ell_i} \qquad (5.9)$$

where w_{ℓ_i} is the a priori weight of observation ℓ_i.

PHOTO	POINT	r(x)	r(y)
1	1	0.44025168D 00	0.43848660D 00
	2	0.54790760D 00	0.57142772D 00
	3	0.60831004D 00	0.62991064D 00
	4	0.64829768D 00	0.67034212D 00
	5	0.63374572D 00	0.62120360D 00
	6	0.68769284D 00	0.67192076D 00
	7	0.67060720D 00	0.63990168D 00
	8	0.59726324D 00	0.55551532D 00
	9	0.55777472D 00	0.58055732D 00
	10	0.54943220D 00	0.56990312D 00
	11	0.58843348D 00	0.54713792D 00
	12	0.64854132D 00	0.64928228D 00
	16	0.68656064D 00	0.67305296D 00
	17	0.55435300D 00	0.58397904D 00
	18	0.60866396D 00	0.62955676D 00
	19	0.64922772D 00	0.64859588D 00
	20	0.58118024D 00	0.61239384D 00
	24	0.63434732D 00	0.62060200D 00
	25	0.64748640D 00	0.67115344D 00
2	1	0.60866396D 00	0.62955676D 00
	2	0.54943220D 00	0.56990312D 00
	3	0.44025168D 00	0.43848660D 00
	4	0.59726324D 00	0.55551532D 00
	5	0.67060720D 00	0.63990168D 00
	6	0.68656064D 00	0.67305296D 00
	7	0.63434732D 00	0.62060200D 00
	8	0.64748640D 00	0.67115344D 00
	9	0.55435300D 00	0.58397904D 00
	10	0.64922772D 00	0.64859588D 00
	11	0.58843348D 00	0.54713792D 00
	12	0.54790760D 00	0.57142772D 00
	13	0.55777472D 00	0.58055732D 00
	14	0.68769284D 00	0.67192076D 00
	18	0.58118024D 00	0.61239384D 00
	19	0.64854132D 00	0.64928228D 00
	20	0.60831004D 00	0.62991064D 00
	21	0.64829768D 00	0.67034212D 00
	22	0.63374572D 00	0.62120360D 00

TABLE 5.6. Computed values of redundancy numbers for case 9.

PHOTO	POINT	$r(x)$	$r(y)$
3	1	0.58118024D 00	0.61239384D 00
	2	0.64854132D 00	0.64928228D 00
	3	0.60831004D 00	0.62991064D 00
	4	0.64829768D 00	0.67034212D 00
	5	0.63374572D 00	0.62120360D 00
	10	0.64922772D 00	0.64859588D 00
	11	0.58843348D 00	0.54713792D 00
	12	0.54790760D 00	0.57142772D 00
	13	0.55777472D 00	0.58055732D 00
	14	0.68769284D 00	0.67192076D 00
	18	0.60866396D 00	0.62955676D 00
	19	0.54943220D 00	0.56990312D 00
	20	0.44025168D 00	0.43848660D 00
	21	0.59726324D 00	0.55551532D 00
	22	0.67060720D 00	0.63990168D 00
	23	0.68656064D 00	0.67305296D 00
	24	0.63434732D 00	0.62060200D 00
	25	0.64748640D 00	0.67115344D 00
	26	0.55435300D 00	0.58397904D 00
4	1	0.60866396D 00	0.62955676D 00
	2	0.64922772D 00	0.64859588D 00
	3	0.58118024D 00	0.61239384D 00
	7	0.63434732D 00	0.62060200D 00
	8	0.64748640D 00	0.67115344D 00
	10	0.54943220D 00	0.56990312D 00
	11	0.58843348D 00	0.54713792D 00
	12	0.64854132D 00	0.64928228D 00
	16	0.68656064D 00	0.67305296D 00
	17	0.55435300D 00	0.58397904D 00
	18	0.44025168D 00	0.43848660D 00
	19	0.54790760D 00	0.57142772D 00
	20	0.60831004D 00	0.62991064D 00
	21	0.64829768D 00	0.67034212D 00
	22	0.63374572D 00	0.62120360D 00
	23	0.68769284D 00	0.67192076D 00
	24	0.67060720D 00	0.63990168D 00
	25	0.59726324D 00	0.55551532D 00
	26	0.55777472D 00	0.58055732D 00

TABLE 5.6. (Continued)

PHOTO	POINT	r(x)	r(y)
5	1	0.67060720D 00	0.63990168D 00
	2	0.68656064D 00	0.67305296D 00
	3	0.63434732D 00	0.62060200D 00
	4	0.64748640D 00	0.67115344D 00
	5	0.60866396D 00	0.62955676D 00
	6	0.54943220D 00	0.56990312D 00
	7	0.44025168D 00	0.43848660D 00
	8	0.59726324D 00	0.55551532D 00
	9	0.55435300D 00	0.58397904D 00
	10	0.68769284D 00	0.67192076D 00
	14	0.64922772D 00	0.64859588D 00
	15	0.58843348D 00	0.54713792D 00
	16	0.54790760D 00	0.57142772D 00
	17	0.55777472D 00	0.53055732D 00
	18	0.63374572D 00	0.62120360D 00
	22	0.58118024D 00	0.61239384D 00
	23	0.64854132D 00	0.64928228D 00
	24	0.60831004D 00	0.62991064D 00
	25	0.64829768D 00	0.67034212D 00
6	1	0.63434732D 00	0.62060200D 00
	2	0.68656064D 00	0.67305296D 00
	3	0.67060720D 00	0.63990168D 00
	4	0.59726324D 00	0.55551532D 00
	5	0.44025168D 00	0.43848660D 00
	6	0.54943220D 00	0.56990312D 00
	7	0.60866396D 00	0.62955676D 00
	8	0.64748640D 00	0.67115344D 00
	9	0.55435300D 00	0.58397904D 00
	12	0.68769284D 00	0.67192076D 00
	13	0.55777472D 00	0.53055732D 00
	14	0.54790760D 00	0.57142772D 00
	15	0.58843348D 00	0.54713792D 00
	16	0.64922772D 00	0.64859588D 00
	20	0.63374572D 00	0.62120360D 00
	21	0.64829768D 00	0.67034212D 00
	22	0.60831004D 00	0.62991064D 00
	23	0.64854132D 00	0.64928228D 00
	24	0.58118024D 00	0.61239384D 00

TABLE 5.6. (Continued)

PHOTO	POINT	r(x)	r(y)
7	3	0.63434732D 00	0.62060200D 00
	4	0.64748640D 00	0.67115344D 00
	5	0.60866396D 00	0.62955676D 00
	6	0.64922772D 00	0.64859588D 00
	7	0.58118024D 00	0.61239384D 00
	12	0.68656064D 00	0.67305296D 00
	13	0.55435300D 00	0.58397904D 00
	14	0.54943220D 00	0.56990312D 00
	15	0.58843348D 00	0.54713792D 00
	16	0.64854132D 00	0.64928228D 00
	18	0.63374572D 00	0.62120360D 00
	19	0.68769284D 00	0.67192076D 00
	20	0.67060720D 00	0.63990168D 00
	21	0.59726324D 00	0.55551532D 00
	22	0.44025168D 00	0.43848660D 00
	23	0.54790760D 00	0.57142772D 00
	24	0.60831004D 00	0.62991064D 00
	25	0.64829768D 00	0.67034212D 00
	26	0.55777472D 00	0.58055732D 00
8	1	0.63434732D 00	0.62060200D 00
	5	0.58118024D 00	0.61239384D 00
	6	0.64922772D 00	0.64859588D 00
	7	0.60866396D 00	0.62955676D 00
	8	0.64748640D 00	0.67115344D 00
	10	0.68656064D 00	0.67305296D 00
	14	0.64854132D 00	0.64928228D 00
	15	0.58843348D 00	0.54713792D 00
	16	0.54943220D 00	0.56990312D 00
	17	0.55435300D 00	0.58397904D 00
	18	0.67060720D 00	0.63990168D 00
	19	0.68769284D 00	0.67192076D 00
	20	0.63374572D 00	0.62120360D 00
	21	0.64829768D 00	0.67034212D 00
	22	0.60831004D 00	0.62991064D 00
	23	0.54790760D 00	0.57142772D 00
	24	0.44025168D 00	0.43848660D 00
	25	0.59726324D 00	0.55551532D 00
	26	0.55777472D 00	0.58055732D 00

TABLE 5.6. (Continued)

PHOTO	POINT	r(x)	r(y)
1	1	0.45306624D 00	0.44588760D 00
	2	0.55059676D 00	0.57300392D 00
	3	0.61660472D 00	0.69182500D 00
	4	0.64899364D 00	0.67148416D 00
	5	0.67285792D 00	0.66269208D 00
	6	0.68923632D 00	0.67244096D 00
	7	0.71771256D 00	0.65747924D 00
	8	0.60020404D 00	0.55682400D 00
	9	0.55858808D 00	0.58092880D 00
	10	0.55180292D 00	0.57169932D 00
	11	0.58934152D 00	0.54741052D 00
	12	0.65030220D 00	0.65009592D 00
	16	0.68777572D 00	0.67470936D 00
	17	0.55474932D 00	0.58480120D 00
	18	0.65571284D 00	0.67888004D 00
	19	0.65050276D 00	0.64878764D 00
	20	0.61613624D 00	0.62712248D 00
	24	0.63705084D 00	0.66724204D 00
	25	0.64783432D 00	0.67232240D 00
2	1	0.61742428D 00	0.69100544D 00
	2	0.55232836D 00	0.57127232D 00
	3	0.45385160D 00	0.44510228D 00
	4	0.60037972D 00	0.55664836D 00
	5	0.71505732D 00	0.66013444D 00
	6	0.68809260D 00	0.67358472D 00
	7	0.67317528D 00	0.66237472D 00
	8	0.64823648D 00	0.67224128D 00
	9	0.55522972D 00	0.58428716D 00
	10	0.65101648D 00	0.64938164D 00
	11	0.58934096D 00	0.54741108D 00
	12	0.55034128D 00	0.57316096D 00
	13	0.55820176D 00	0.53134876D 00
	14	0.68896468D 00	0.67352036D 00
	18	0.61326738D 00	0.62999084D 00
	19	0.64982140D 00	0.64946900D 00
	20	0.65498720D 00	0.67960568D 00
	21	0.64869280D 00	0.67146396D 00
	22	0.63608020D 00	0.66821268D 00

TABLE 5.7. Computed values of redundancy numbers for case 10.

PHOTO	POINT	r(x)	r(y)
3	1	0.61326788D 00	0.62999084D 00
	2	0.64982140D 00	0.64946900D 00
	3	0.65498720D 00	0.67960568D 00
	4	0.64869280D 00	0.67146396D 00
	5	0.63608020D 00	0.66821268D 00
	10	0.65101648D 00	0.64938164D 00
	11	0.58934096D 00	0.54741108D 00
	12	0.55034128D 00	0.57316096D 00
	13	0.55820176D 00	0.58134876D 00
	14	0.68896468D 00	0.67352036D 00
	18	0.61742428D 00	0.69100544D 00
	19	0.55232836D 00	0.57127232D 00
	20	0.45385160D 00	0.44510228D 00
	21	0.60037972D 00	0.55664836D 00
	22	0.71505732D 00	0.66013444D 00
	23	0.68809260D 00	0.67358472D 00
	24	0.67317528D 00	0.66237472D 00
	25	0.64823648D 00	0.67224128D 00
	26	0.55522972D 00	0.58428716D 00
4	1	0.65571284D 00	0.67838004D 00
	2	0.65050276D 00	0.64878764D 00
	3	0.61613624D 00	0.62712248D 00
	7	0.63705084D 00	0.66724204D 00
	8	0.64783432D 00	0.67232240D 00
	10	0.55180292D 00	0.57169932D 00
	11	0.58934152D 00	0.54741052D 00
	12	0.65030220D 00	0.65009592D 00
	16	0.68777572D 00	0.67470936D 00
	17	0.55474932D 00	0.58480120D 00
	18	0.45306624D 00	0.44588760D 00
	19	0.55059676D 00	0.57300392D 00
	20	0.61660472D 00	0.69182500D 00
	21	0.64899364D 00	0.67148416D 00
	22	0.67285792D 00	0.66269208D 00
	23	0.68923632D 00	0.67244096D 00
	24	0.71771256D 00	0.65747924D 00
	25	0.60020404D 00	0.55682400D 00
	26	0.55858808D 00	0.58092880D 00

TABLE 5.7. (Continued)

PHOTO	POINT	r(x)	r(y)
5	1	0.71505732D 00	0.66013444D 00
	2	0.68809260D 00	0.67358472D 00
	3	0.67317528D 00	0.66237472D 00
	4	0.64823648D 00	0.67224128D 00
	5	0.61742428D 00	0.69100544D 00
	6	0.55232836D 00	0.57127232D 00
	7	0.45385160D 00	0.44510228D 00
	8	0.60037972D 00	0.55664836D 00
	9	0.55522972D 00	0.58428716D 00
	10	0.68896468D 00	0.67352036D 00
	14	0.65101648D 00	0.64938164D 00
	15	0.58934096D 00	0.54741108D 00
	16	0.55034128D 00	0.57316096D 00
	17	0.55820176D 00	0.58134876D 00
	18	0.63608020D 00	0.66821268D 00
	22	0.61326788D 00	0.62999084D 00
	23	0.64982140D 00	0.64946900D 00
	24	0.65498720D 00	0.67960568D 00
	25	0.64869280D 00	0.67146396D 00
6	1	0.67317528D 00	0.66237472D 00
	2	0.68809260D 00	0.67358472D 00
	3	0.71505732D 00	0.66013444D 00
	4	0.60037972D 00	0.55664836D 00
	5	0.45385160D 00	0.44510228D 00
	6	0.55232836D 00	0.57127232D 00
	7	0.61742428D 00	0.69100544D 00
	8	0.64823648D 00	0.67224128D 00
	9	0.55522972D 00	0.58428716D 00
	12	0.68896468D 00	0.67352036D 00
	13	0.55820176D 00	0.58134876D 00
	14	0.55034128D 00	0.57316096D 00
	15	0.58934096D 00	0.54741108D 00
	16	0.65101648D 00	0.64938164D 00
	20	0.63608020D 00	0.66821268D 00
	21	0.64869280D 00	0.67146396D 00
	22	0.65498720D 00	0.67960568D 00
	23	0.64982140D 00	0.64946900D 00
	24	0.61326788D 00	0.62999084D 00

TABLE 5.7.(Continued)

PHOTO	POINT	r(x)	r(y)
7	3	0.63705084D 00	0.66724204D 00
	4	0.64783432D 00	0.67232240D 00
	5	0.65571284D 00	0.67888004D 00
	6	0.65050276D 00	0.64878764D 00
	7	0.61613624D 00	0.62712248D 00
	12	0.63777572D 00	0.67470936D 00
	13	0.55474932D 00	0.58480120D 00
	14	0.55180292D 00	0.57169932D 00
	15	0.58934152D 00	0.54741052D 00
	16	0.65030220D 00	0.65009592D 00
	18	0.67285792D 00	0.66269208D 00
	19	0.68923632D 00	0.67244096D 00
	20	0.71771256D 00	0.65747924D 00
	21	0.60020404D 00	0.55682400D 00
	22	0.45306624D 00	0.44588760D 00
	23	0.55059676D 00	0.57300392D 00
	24	0.61660472D 00	0.69182500D 00
	25	0.64899364D 00	0.67148416D 00
	26	0.55858808D 00	0.58092880D 00
8	1	0.63705084D 00	0.66724204D 00
	5	0.61613624D 00	0.62712248D 00
	6	0.65050276D 00	0.64878764D 00
	7	0.65571284D 00	0.67888004D 00
	8	0.64783432D 00	0.67232240D 00
	10	0.68777572D 00	0.67470936D 00
	14	0.65030220D 00	0.65009592D 00
	15	0.58934152D 00	0.54741052D 00
	16	0.55180292D 00	0.57169932D 00
	17	0.55474932D 00	0.58480120D 00
	18	0.71771256D 00	0.65747924D 00
	19	0.68923632D 00	0.67244096D 00
	20	0.67285792D 00	0.66269208D 00
	21	0.64899364D 00	0.67148416D 00
	22	0.61660472D 00	0.69182500D 00
	23	0.55059676D 00	0.57300392D 00
	24	0.45306624D 00	0.44588760D 00
	25	0.60020404D 00	0.55682400D 00
	26	0.55858808D 00	0.58092880D 00

TABLE 5.7. (Continued)

Figure (5.9) illustrates the relation between number of cameras and the reliability of the x-co-ordinate photogrammetric observations whereas Figure (5.10) shows that with respect to the y-co-ordinate. Illustrated in Figures (5.11) and (5.12) are the reliability of the x- and y- co-ordinate combined observations respectively.

The reliability of the x- and y- co-ordinates of cases 1,2 is sufficient (Case 1: $r(x) = 0.583$, $r(y) = 0.540$; Case 2: $r(x) = 0.586$, $r(y) = 0.610$). On the other end of the scale, cases 9,10 display the highest reliability (Case 9: $r(x) = 0.688$, $r(y) = 0.673$; Case 10: $r(x) = 0.718$, $r(y) = 0.692$). One plausible explanation for such a situation is due to the symmetric network arrangement. The latter cases are the optimum configuration in reliability sense.

Again in reliability analysis of the x- and y- co-ordinate, as in precision, there is slight improvement "within" the same number of camera cases.

With regard to the "between" percentage of improvement for purely photogrammetric cases, it can be noticed that there is some similarity in the trend of the rate of improvement in x- and y- co-ordinate observations.

For the combined different number of camera cases x- and y- co-ordinate reliability improvements, it seems that the improvement rate is insignificant especially after the 6 camera case.

The reliability of the slope distances are plotted against the number of cameras in Figure (5.13). As can be seen from this figure, with the number of slope distances kept constant, the more

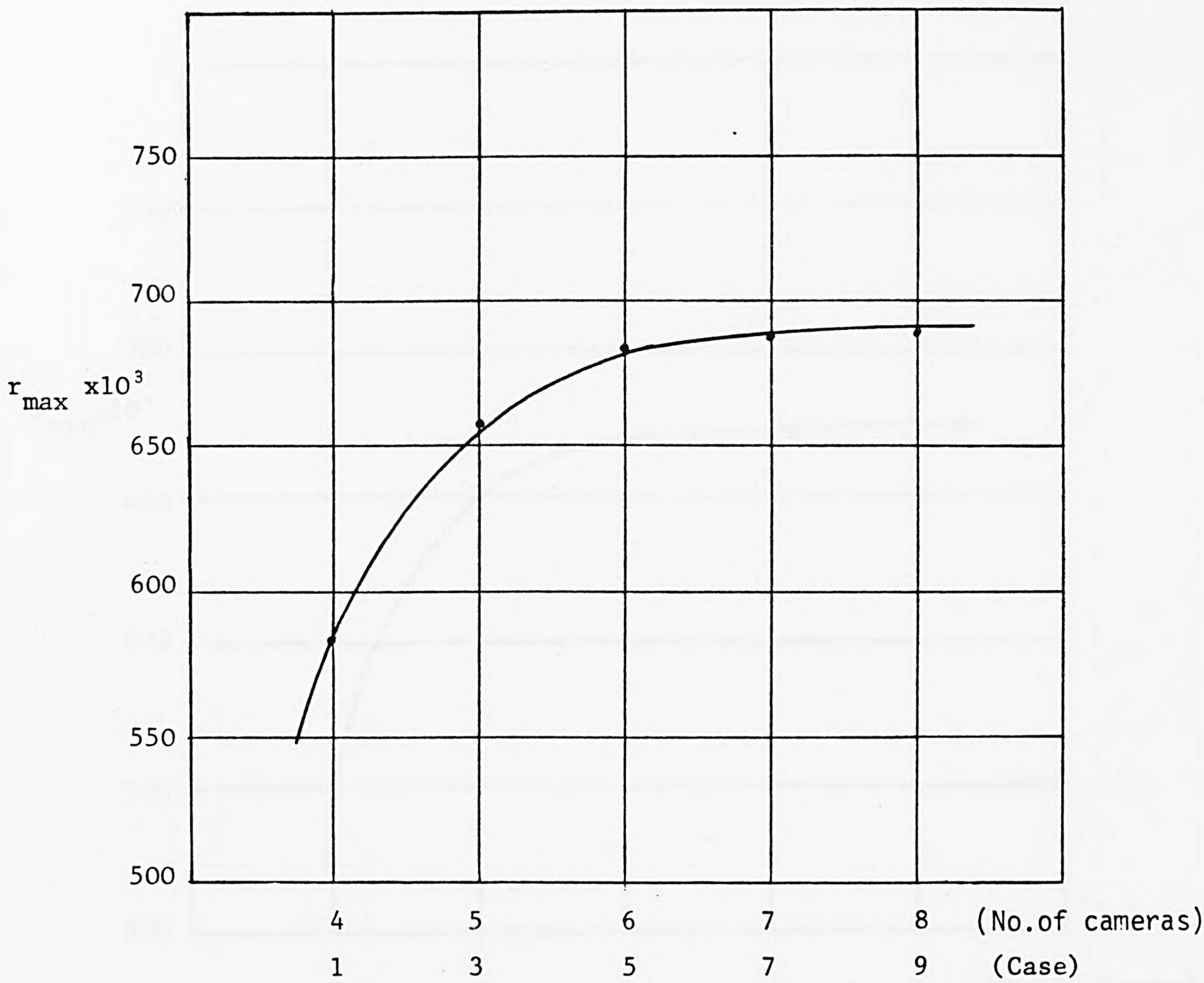


Figure 5.9. No.of cameras against Reliability (internal) of photo x-co-ordinates. (Photogrammetry)

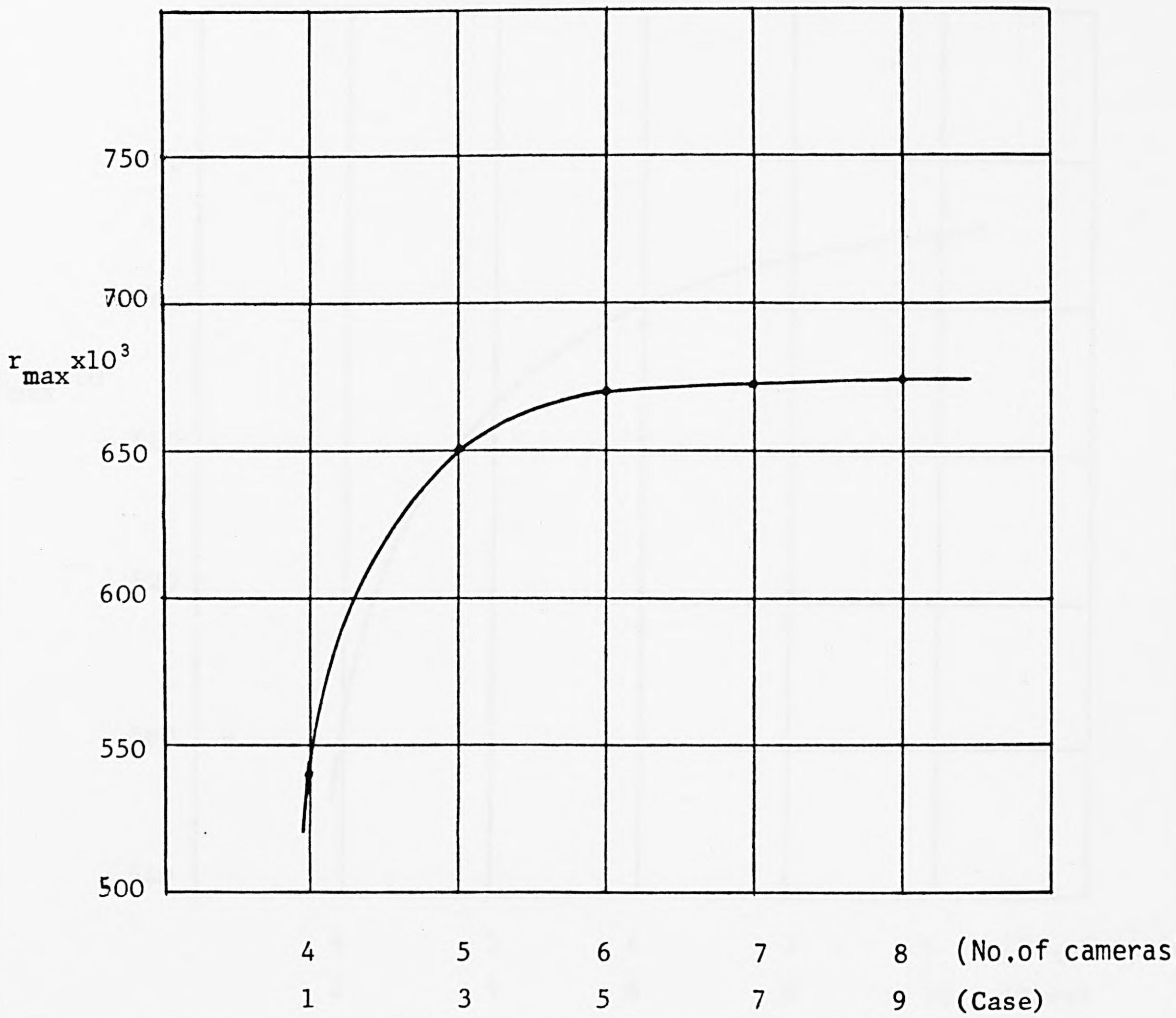


Figure 5.10. No.of cameras against Reliability (internal) of photo y-co-ordinates.

(Photogrammetry)

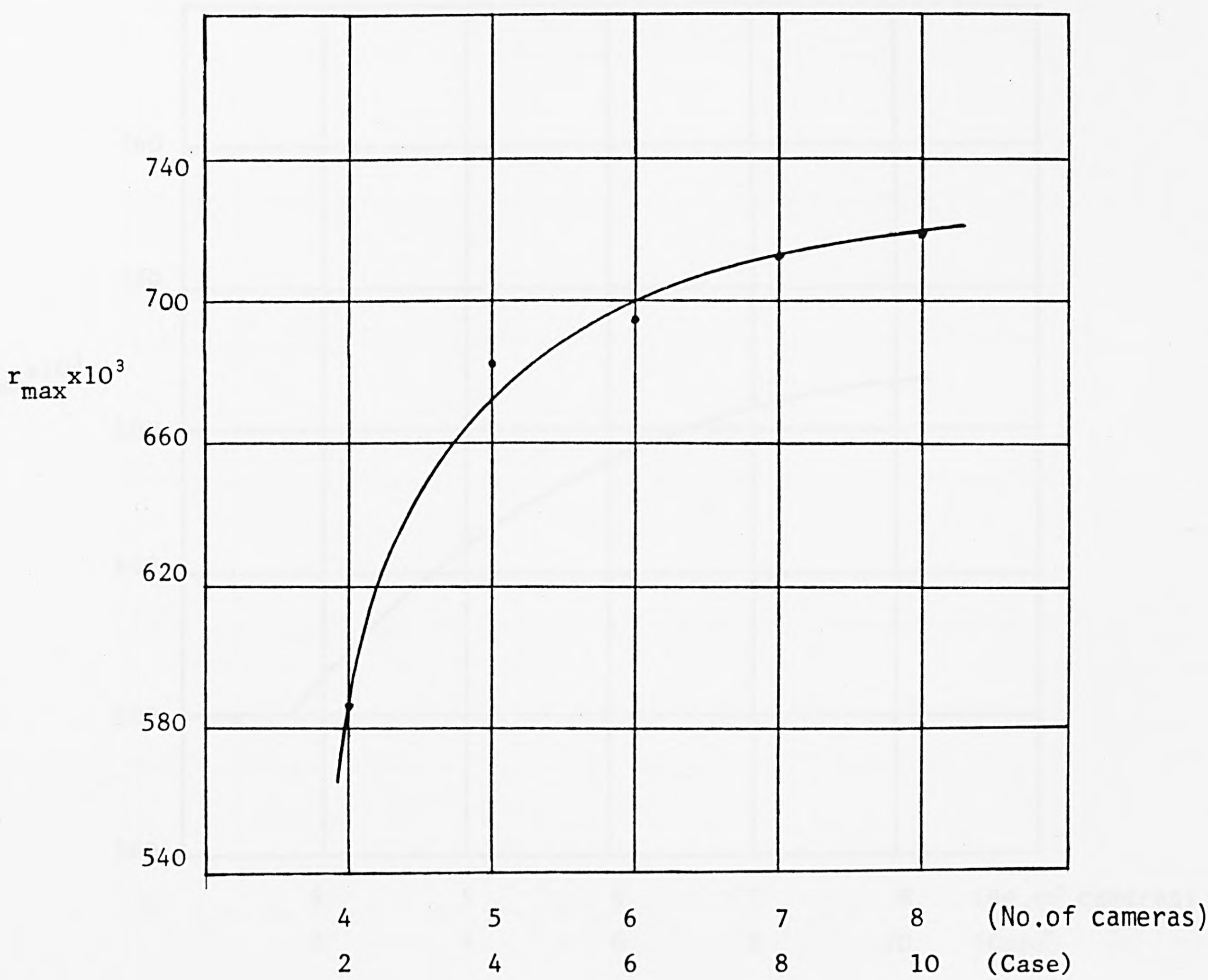


Figure 5.11. No.of cameras against Reliability (internal) of photo x-co-ordinates. (Photogrammetry + 8 distances)

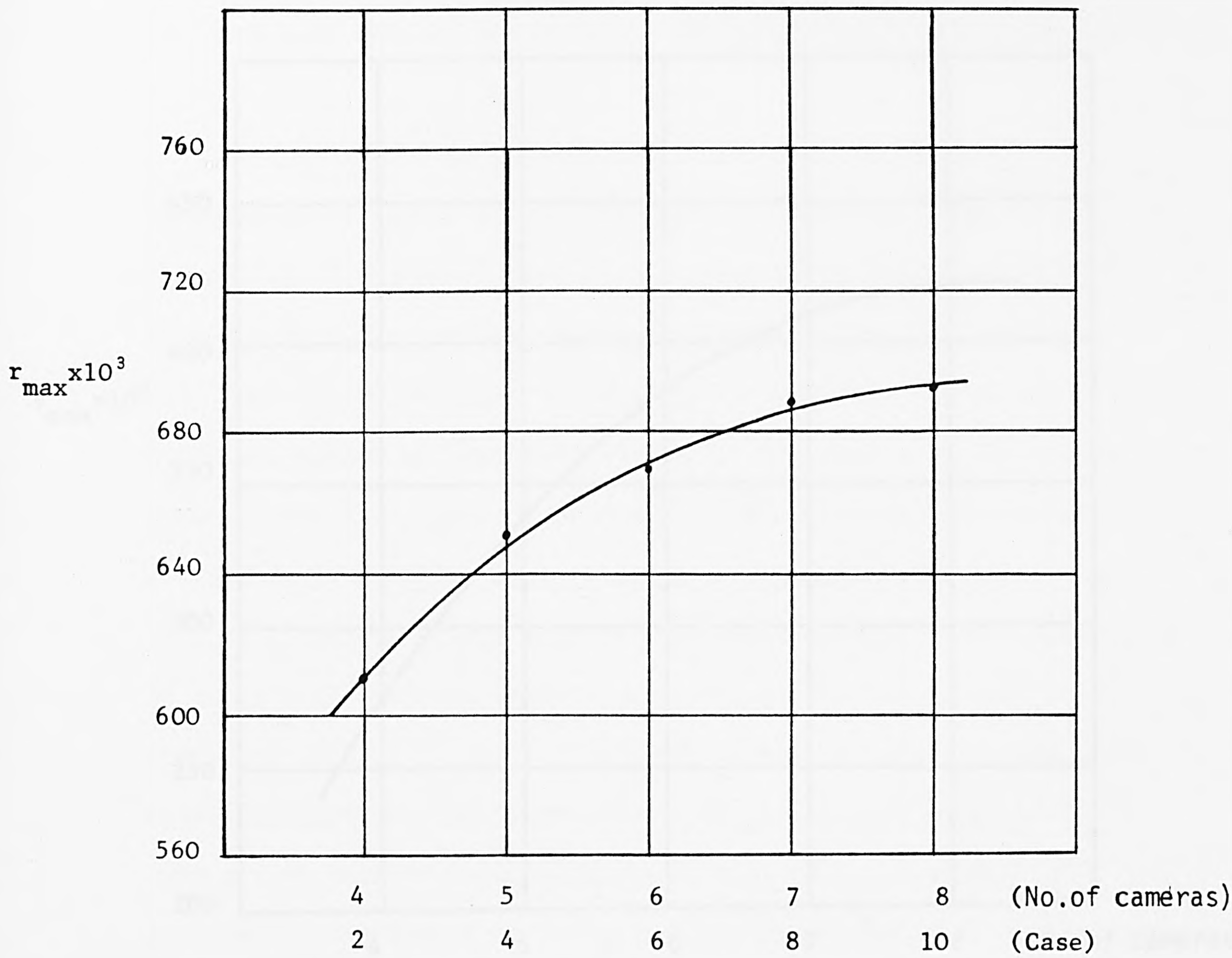


Figure 5.12. No.of cameras against Reliability (internal) of photo y-co-ordinates.
(Photogrammetry + 8 distances)

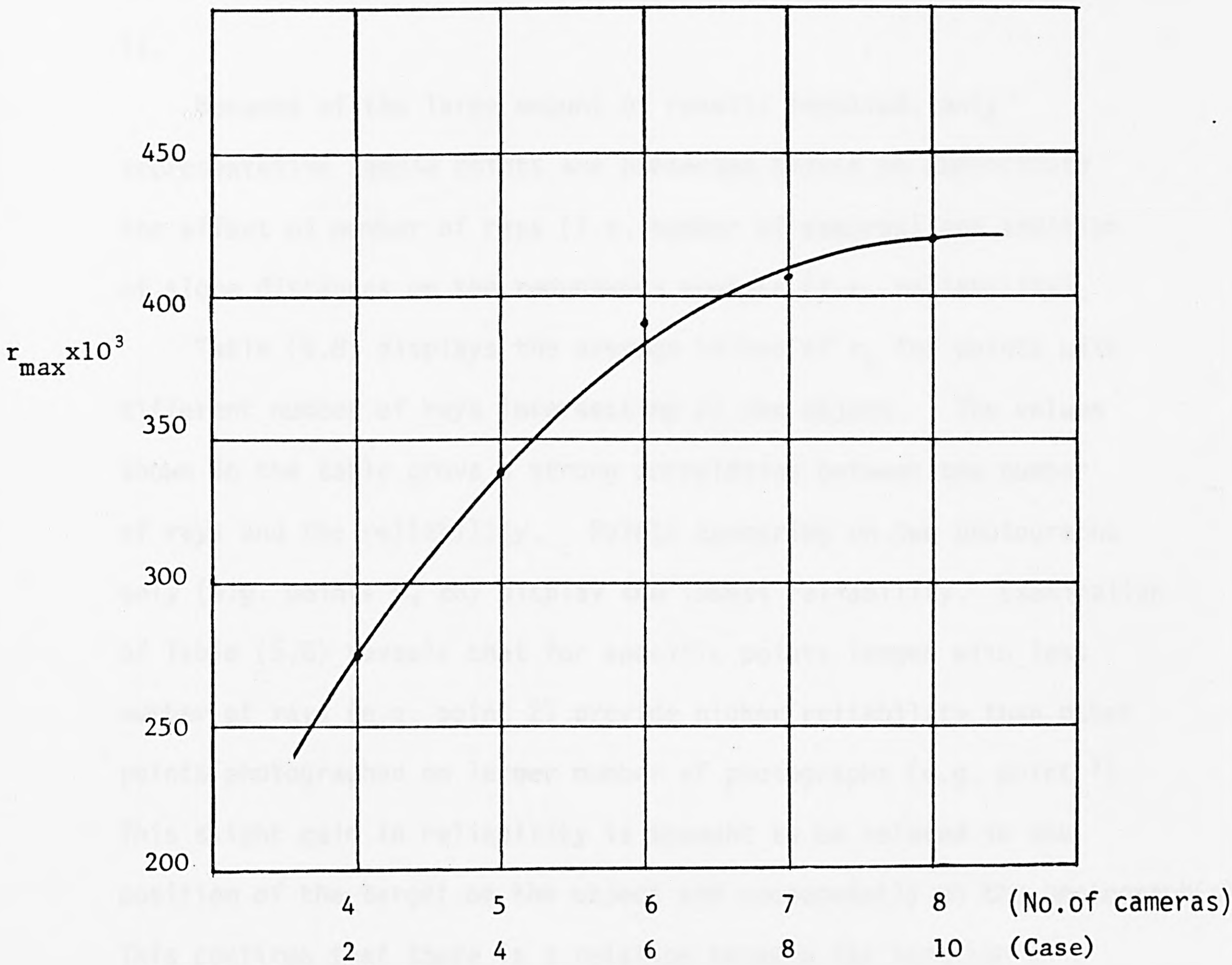


Figure 5.13. No.of cameras against Reliability (internal) of survey measurements (8 distances).

the number of cameras the more the reliability of slope distances is.

Because of the large amount of results involved, only representative sample points are presented herein to demonstrate the effect of number of rays (i.e. number of cameras) and addition of slope distances on the redundancy numbers (i.e. reliability).

Table (5.8) displays the average values of r_i for points with different number of rays intersecting at the object. The values shown in the table prove a strong correlation between the number of rays and the reliability. Points appearing on two photographs only (e.g. points 9, 26) display the lowest reliability. Examination of Table (5.8) reveals that for specific points imaged with less number of rays (e.g. point 2) provide higher reliability than other points photographed on larger number of photographs (e.g. point 1). This slight gain in reliability is thought to be related to the position of the target on the object and consequently on the photograph(s). This confirms that there is a relation between the location of a point on the photograph and the reliability.

When two measured distances originate from a point, the redundancy number increases substantially. Table (5.9) lists the values of redundancy numbers with and without adding distances and the percentage improvement. The latter varies between 14 to 4 for the x-co-ordinate whereas it ranges from 18 to 5 for the y-co-ordinate. It is also noted that the improvement in redundancy numbers slows down considerably after 6 rays. Therefore, it is not economically desirable to try to improve the redundancy if the object points already have 6 rays and display redundancy numbers of the order of 0.6 or more.

Object Point Number	Number of Rays	Redundancy numbers	
		$r(x)$	$r(y)$
1	4	0.450	0.431
	5	0.522	0.525
	6	0.564	0.563
	7	0.597	0.599
2	4	0.516	0.488
	5	0.579	0.573
	6	0.623	0.626
9	2	0.314	0.139
	3	0.440	0.464
	4	0.552	0.581
26	2	0.314	0.139
	3	0.436	0.474
	4	0.557	0.581

TABLE (5.8) Relation between number of rays and redundancy numbers.

Object Point Number	Number of Rays	r (without distances)		r (with (distances)		Improvement %	
		r(x)	r(y)	r(x)	r(y)	r(x)	r(y)
1	4	0.450	0.431	0.490	0.508	8.9	17.9
	5	0.522	0.525	0.561	0.576	7.5	9.7
	6	0.564	0.563	0.596	0.604	5.7	7.3
	7	0.597	0.599	0.624	0.634	4.5	5.8
3	4	0.451	0.430	0.493	0.506	9.3	17.7
	5	0.511	0.499	0.546	0.555	6.9	11.2
	6	0.564	0.563	0.597	0.603	5.9	7.1
	7	0.593	0.594	0.619	0.629	4.4	5.9
5	4	0.478	0.479	0.547	0.567	14.4	18.4
	5	0.516	0.518	0.563	0.574	9.1	10.8
	6	0.567	0.575	0.602	0.615	6.2	7.0
	7	0.597	0.599	0.624	0.633	4.5	5.7
7	4	0.421	0.453	0.502	0.515	19.2	13.7
	5	0.515	0.518	0.564	0.573	9.5	10.6
	6	0.556	0.559	0.593	0.604	6.7	8.1
	7	0.597	0.599	0.624	0.633	4.5	5.7

TABLE (5.9) Effect of distances between object points and their image co-ordinate reliability (Sample of points having 4, 5, 6, 7 rays).

On the other hand, for object points which have no distances, their image co-ordinates reliabilities are insignificantly improved. Table (5.10) represents the improvement gained by the inclusion of slope distances in the photogrammetric network as a whole. It should be noted that points with 2 or 3 rays can be seen to have had no significant improvement compared with that of "distance-connected" points.

Object Point Number	Number of Rays	r (without distances)		r (with distances)		Improvement %	
		r(x)	r(y)	r(x)	r(y)	r(x)	r(y)
	4	0.516	0.488	0.523	0.491	1.4	0.6
2	5	0.579	0.573	0.583	0.575	0.7	0.4
	6	0.623	0.626	0.626	0.628	0.5	0.3
	4	0.524	0.518	0.526	0.520	0.4	0.4
4	5	0.584	0.581	0.586	0.584	0.4	0.5
	6	0.629	0.630	0.631	0.631	0.4	0.2
	4	0.502	0.527	0.506	0.529	0.8	0.4
6	5	0.577	0.591	0.580	0.592	0.5	0.2
	6	0.629	0.630	0.631	0.631	0.4	0.2
	4	0.500	0.520	0.509	0.522	1.8	0.4
8	5	0.583	0.582	0.585	0.584	0.4	0.4
	6	0.631	0.633	0.632	0.634	0.2	0.2
	2	0.314	0.139	0.314	0.139	0.0	0.0
9	3	0.440	0.464	0.441	0.465	0.2	0.2
	4	0.552	0.581	0.554	0.581	0.4	0.0
	2	0.314	0.139	0.314	0.139	0.0	0.0
26	3	0.436	0.474	0.437	0.474	0.2	0.0
	4	0.557	0.581	0.558	0.582	0.2	0.2

TABLE (5.10) Effect of slope distances on redundancy numbers.

Table (5.11) summarises the precision and reliability values ($r_{i\ max}$) of the photogrammetric measurements.

Case	Precision σ_m^2 (mm^2)	Reliability	
		max $r(x)$	max $r(y)$
1	0.40027546	0.583039	0.540271
2	0.38408995	0.585583	0.610033
3	0.30379145	0.657194	0.650357
4	0.29312554	0.681672	0.651317
5	0.25444611	0.682046	0.669123
6	0.24798812	0.694132	0.670347
7	0.20352679	0.685850	0.671341
8	0.19979033	0.712184	0.688901
9	0.17192943	0.687693	0.673053
10	0.16986719	0.717713	0.691825

TABLE (5.11) Summary of precision and reliability (r_{max}) of photogrammetric measurements.

Naturally the global indicators don't provide in each case for sufficient precision/or reliability of all individual object points/and observations, e.g. if some observations have been cancelled or if points cannot be observed from certain camera stations. Such a situation is encountered in the computer programs, when dealing with object point (no.15) in cases 1-4, and observations from or to that point. So, in addition to the computation of the exact redundancy numbers other individual indicators, namely Tau (internal reliability), Gam (external reliability) and Delta (max. undetected gross error) were computed according to the equations presented in

PHO	PT.	TAU		GAM	
NO	NO	x	y	x	y
1	1	0.15071257D 01	0.15101561D 01	0.11275762D 01	0.11316233D 01
	2	0.13509720D 01	0.13228766D 01	0.90836406D 00	0.86602691D 00
	3	0.12321461D 01	0.12599709D 01	0.80243290D 00	0.76650294D 00
	4	0.12419747D 01	0.12213826D 01	0.73654686D 00	0.70126712D 00
	5	0.12561528D 01	0.12687703D 01	0.76021045D 00	0.78088295D 00
	6	0.12058763D 01	0.12199470D 01	0.67389730D 00	0.69876372D 00
	7	0.12211412D 01	0.12500960D 01	0.70084656D 00	0.75016003D 00
	8	0.12939488D 01	0.13416894D 01	0.82115992D 00	0.89450007D 00
	9	0.13389692D 01	0.13124339D 01	0.89041480D 00	0.84998988D 00
	10	0.13490963D 01	0.13246449D 01	0.90557207D 00	0.86872562D 00
	11	0.13036209D 01	0.13519219D 01	0.83631778D 00	0.90977620D 00
	12	0.12417414D 01	0.12410327D 01	0.73615338D 00	0.73495725D 00
	16	0.12068702D 01	0.12189205D 01	0.67567417D 00	0.69697000D 00
	17	0.13430952D 01	0.13085833D 01	0.89660735D 00	0.84403212D 00
	18	0.12817733D 01	0.12603250D 01	0.80183707D 00	0.76708482D 00
	19	0.12410348D 01	0.12416892D 01	0.73504530D 00	0.73606528D 00
	20	0.13117304D 01	0.12778639D 01	0.84890318D 00	0.79557280D 00
	24	0.12555570D 01	0.12693852D 01	0.75922557D 00	0.78188149D 00
	25	0.12427526D 01	0.12206442D 01	0.73785770D 00	0.69998018D 00
2	1	0.12817733D 01	0.12603250D 01	0.80183707D 00	0.76708482D 00
	2	0.13490963D 01	0.13246449D 01	0.90557207D 00	0.86872562D 00
	3	0.15071257D 01	0.15101561D 01	0.11275762D 01	0.11316233D 01
	4	0.12939488D 01	0.13416894D 01	0.82115992D 00	0.89450007D 00
	5	0.12211412D 01	0.12500960D 01	0.70084656D 00	0.75016003D 00
	6	0.12068702D 01	0.12189205D 01	0.67567417D 00	0.69697000D 00
	7	0.12555570D 01	0.12693852D 01	0.75922557D 00	0.78188149D 00
	8	0.12427526D 01	0.12206442D 01	0.73785770D 00	0.69998018D 00
	9	0.13430952D 01	0.13085833D 01	0.89660735D 00	0.84403212D 00
	10	0.12410843D 01	0.12416892D 01	0.73504530D 00	0.73606528D 00
	11	0.13036209D 01	0.13519219D 01	0.83631778D 00	0.90977620D 00
	12	0.13509720D 01	0.13228766D 01	0.90836406D 00	0.86602691D 00
	13	0.13389692D 01	0.13124339D 01	0.89041480D 00	0.84998988D 00
	14	0.12058763D 01	0.12199470D 01	0.67389730D 00	0.69876372D 00
	18	0.13117304D 01	0.12778639D 01	0.84890318D 00	0.79557280D 00
	19	0.12417414D 01	0.12410327D 01	0.73615338D 00	0.73495725D 00
	20	0.12321461D 01	0.12599709D 01	0.80243290D 00	0.76650294D 00
	21	0.12419747D 01	0.12213826D 01	0.73654686D 00	0.70126712D 00
	22	0.12561528D 01	0.12687703D 01	0.76021045D 00	0.78088295D 00
3	1	0.13117304D 01	0.12778639D 01	0.84890318D 00	0.79557280D 00
	2	0.12417414D 01	0.12410327D 01	0.73615338D 00	0.73495725D 00
	3	0.12321461D 01	0.12599709D 01	0.80243290D 00	0.76650294D 00
	4	0.12419747D 01	0.12213826D 01	0.73654686D 00	0.70126712D 00
	5	0.12561528D 01	0.12687703D 01	0.76021045D 00	0.78088295D 00
	10	0.12410848D 01	0.12416892D 01	0.73504530D 00	0.73606528D 00
	11	0.13036209D 01	0.13519219D 01	0.83631778D 00	0.90977620D 00
	12	0.13509720D 01	0.13228766D 01	0.90836406D 00	0.86602691D 00
	13	0.13389692D 01	0.13124339D 01	0.89041480D 00	0.84998988D 00
	14	0.12058763D 01	0.12199470D 01	0.67389730D 00	0.69876372D 00
	18	0.12817733D 01	0.12603250D 01	0.80183707D 00	0.76708482D 00
	19	0.13490963D 01	0.13246449D 01	0.90557207D 00	0.86872562D 00
	20	0.15071257D 01	0.15101561D 01	0.11275762D 01	0.11316233D 01
	21	0.12939488D 01	0.13416894D 01	0.82115992D 00	0.89450007D 00
	22	0.12211412D 01	0.12500960D 01	0.70084656D 00	0.75016003D 00
	23	0.12068702D 01	0.12189205D 01	0.67567417D 00	0.69697000D 00
	24	0.12555570D 01	0.12693852D 01	0.75922557D 00	0.78188149D 00
	25	0.12427526D 01	0.12206442D 01	0.73785770D 00	0.69998018D 00
	26	0.13430952D 01	0.13085833D 01	0.89660735D 00	0.84403212D 00

TABLE 5.12. Values of Tau and Gam for Case 9 (at $\alpha = 0.05$ and $\gamma = 0.8$)

PHO NO	PT. NO	TAU x	TAU y	GAM x	GAM y
4	1	0.12817733D 01	0.12603250D 01	0.80183707D 00	0.76708482D 00
	2	0.12410848D 01	0.12416892D 01	0.73504530D 00	0.73606528D 00
	3	0.13117304D 01	0.12778639D 01	0.84890318D 00	0.79557280D 00
	7	0.12555570D 01	0.12693852D 01	0.75922557D 00	0.78188149D 00
	8	0.12427526D 01	0.12206442D 01	0.73785770D 00	0.69998018D 00
	10	0.13499963D 01	0.13246449D 01	0.90557207D 00	0.86872562D 00
	11	0.13036209D 01	0.13519219D 01	0.83631778D 00	0.90977620D 00
	12	0.12417414D 01	0.12410327D 01	0.73615338D 00	0.73495725D 00
	16	0.12068702D 01	0.12189205D 01	0.67567417D 00	0.69697000D 00
	17	0.13430952D 01	0.13085833D 01	0.89660735D 00	0.84403212D 00
	18	0.15071257D 01	0.15101561D 01	0.11275762D 01	0.11316233D 01
	19	0.13509720D 01	0.13228766D 01	0.90836406D 00	0.86602691D 00
	20	0.12821461D 01	0.12599709D 01	0.80243290D 00	0.76650294D 00
	21	0.12419747D 01	0.12213826D 01	0.73654686D 00	0.70126712D 00
	22	0.12561528D 01	0.12637703D 01	0.76021045D 00	0.78088295D 00
	23	0.12058763D 01	0.12199470D 01	0.67389730D 00	0.69876372D 00
	24	0.12211412D 01	0.12500960D 01	0.70084656D 00	0.75016003D 00
	25	0.12939488D 01	0.13416894D 01	0.82115992D 00	0.89450007D 00
	26	0.13339692D 01	0.13124339D 01	0.89041480D 00	0.84998988D 00
5	1	0.12211412D 01	0.12500960D 01	0.70084656D 00	0.75016003D 00
	2	0.12063702D 01	0.12189205D 01	0.67567417D 00	0.69697000D 00
	3	0.12555570D 01	0.12693852D 01	0.75922557D 00	0.78188149D 00
	4	0.12427526D 01	0.12206442D 01	0.73785770D 00	0.69998018D 00
	5	0.12317733D 01	0.12603250D 01	0.80183707D 00	0.76708482D 00
	6	0.13499963D 01	0.13246449D 01	0.90557207D 00	0.86872562D 00
	7	0.15071257D 01	0.15101561D 01	0.11275762D 01	0.11316233D 01
	8	0.12939488D 01	0.13416894D 01	0.82115992D 00	0.89450007D 00
	9	0.13430952D 01	0.13085833D 01	0.89660735D 00	0.84403212D 00
	10	0.12058763D 01	0.12199470D 01	0.67389730D 00	0.69876372D 00
	14	0.12410348D 01	0.12416892D 01	0.73504530D 00	0.73606528D 00
	15	0.13036209D 01	0.13519219D 01	0.83631778D 00	0.90977620D 00
	16	0.13509720D 01	0.13228766D 01	0.90836406D 00	0.86602691D 00
	17	0.13389692D 01	0.13124339D 01	0.89041480D 00	0.84998988D 00
	18	0.12561528D 01	0.12637703D 01	0.76021045D 00	0.78088295D 00
	22	0.13117304D 01	0.12778639D 01	0.84890318D 00	0.79557280D 00
	23	0.12417414D 01	0.12410327D 01	0.73615338D 00	0.73495725D 00
	24	0.12821461D 01	0.12599709D 01	0.80243290D 00	0.76650294D 00
	25	0.12419747D 01	0.12213826D 01	0.73654686D 00	0.70126712D 00
6	1	0.12555570D 01	0.12693852D 01	0.75922557D 00	0.78188149D 00
	2	0.12068702D 01	0.12189205D 01	0.67567417D 00	0.69697000D 00
	3	0.12211412D 01	0.12500960D 01	0.70084656D 00	0.75016003D 00
	4	0.12939438D 01	0.13416894D 01	0.82115992D 00	0.89450007D 00
	5	0.15071257D 01	0.15101561D 01	0.11275762D 01	0.11316233D 01
	6	0.13499963D 01	0.13246449D 01	0.90557207D 00	0.86872562D 00
	7	0.12317733D 01	0.12603250D 01	0.80183707D 00	0.76708482D 00
	8	0.12427526D 01	0.12206442D 01	0.73785770D 00	0.69998018D 00
	9	0.13430952D 01	0.13085833D 01	0.89660735D 00	0.84403212D 00
	12	0.12058763D 01	0.12199470D 01	0.67389730D 00	0.69876372D 00
	13	0.13339692D 01	0.13124339D 01	0.89041480D 00	0.84998988D 00
	14	0.13509720D 01	0.13228766D 01	0.90836406D 00	0.86602691D 00
	15	0.13036209D 01	0.13519219D 01	0.83631778D 00	0.90977620D 00
	16	0.12410343D 01	0.12416892D 01	0.73504530D 00	0.73606528D 00
	20	0.12561528D 01	0.12637703D 01	0.76021045D 00	0.78088295D 00
	21	0.12419747D 01	0.12213826D 01	0.73654686D 00	0.70126712D 00
	22	0.12821461D 01	0.12599709D 01	0.80243290D 00	0.76650294D 00
	23	0.12417414D 01	0.12410327D 01	0.73615338D 00	0.73495725D 00
	24	0.13117304D 01	0.12778639D 01	0.84890318D 00	0.79557280D 00

TABLE 5.12 (Continued)

PHO NO	PT. NO	TAU x	TAU y	GAM x	GAM y
7	3	0.12555570D 01	0.12693852D 01	0.75922557D 00	0.78188149D 00
	4	0.12427526D 01	0.12206442D 01	0.73785770D 00	0.69998018D 00
	5	0.12817733D 01	0.12603250D 01	0.80183707D 00	0.76708482D 00
	6	0.12410348D 01	0.12416892D 01	0.73504530D 00	0.73606528D 00
	7	0.13117304D 01	0.12778639D 01	0.84890318D 00	0.79557280D 00
	12	0.12063702D 01	0.12189205D 01	0.67567417D 00	0.69697000D 00
	13	0.13430252D 01	0.13085833D 01	0.89660735D 00	0.84403212D 00
	14	0.13490263D 01	0.13246449D 01	0.90557207D 00	0.86872562D 00
	15	0.13036209D 01	0.13519219D 01	0.83631778D 00	0.90977620D 00
	16	0.12417414D 01	0.12410327D 01	0.73615338D 00	0.73495725D 00
	18	0.12561528D 01	0.12687703D 01	0.76021045D 00	0.78088295D 00
	19	0.12058763D 01	0.12199470D 01	0.67389730D 00	0.69876372D 00
	20	0.12211412D 01	0.12500960D 01	0.70084656D 00	0.75016003D 00
	21	0.12939488D 01	0.13416894D 01	0.82115992D 00	0.89450007D 00
	22	0.15071257D 01	0.15101561D 01	0.11275762D 01	0.11316233D 01
	23	0.13509720D 01	0.13228766D 01	0.90836406D 00	0.86602691D 00
	24	0.12821461D 01	0.12599709D 01	0.80243290D 00	0.76650294D 00
	25	0.12419747D 01	0.12213826D 01	0.73654686D 00	0.70126712D 00
	26	0.13389692D 01	0.13124339D 01	0.89041480D 00	0.84998988D 00
8	1	0.12555570D 01	0.12693852D 01	0.75922557D 00	0.78188149D 00
	5	0.13117304D 01	0.12778639D 01	0.84890318D 00	0.79557280D 00
	6	0.12410348D 01	0.12416892D 01	0.73504530D 00	0.73606528D 00
	7	0.12817733D 01	0.12603250D 01	0.80183707D 00	0.76708482D 00
	8	0.12427526D 01	0.12206442D 01	0.73785770D 00	0.69998018D 00
	10	0.12063702D 01	0.12189205D 01	0.67567417D 00	0.69697000D 00
	14	0.12417414D 01	0.12410327D 01	0.73615338D 00	0.73495725D 00
	15	0.13036209D 01	0.13519219D 01	0.83631778D 00	0.90977620D 00
	16	0.13490263D 01	0.13246449D 01	0.90557207D 00	0.86872562D 00
	17	0.13430252D 01	0.13085833D 01	0.89660735D 00	0.84403212D 00
	18	0.12211412D 01	0.12500960D 01	0.70084656D 00	0.75016003D 00
	19	0.12058763D 01	0.12199470D 01	0.67389730D 00	0.69876372D 00
	20	0.12561528D 01	0.12687703D 01	0.76021045D 00	0.78088295D 00
	21	0.12419747D 01	0.12213826D 01	0.73654686D 00	0.70126712D 00
	22	0.12821461D 01	0.12599709D 01	0.80243290D 00	0.76650294D 00
	23	0.13509720D 01	0.13228766D 01	0.90836406D 00	0.86602691D 00
	24	0.15071257D 01	0.15101561D 01	0.11275762D 01	0.11316233D 01
	25	0.12939488D 01	0.13416894D 01	0.82115992D 00	0.89450007D 00
	26	0.13389692D 01	0.13124339D 01	0.89041480D 00	0.84998988D 00

TABLE 5.12 (Continued)

PHO NO	PT. NO	TAU x	TAU y	GAM x	GAM y
1	1	0.14856590D 01	0.14975706D 01	0.10987187D 01	0.11147725D 01
	2	0.13476638D 01	0.13210559D 01	0.90344407D 00	0.86324316D 00
	3	0.12734930D 01	0.12022696D 01	0.78853315D 00	0.66742209D 00
	4	0.12413036D 01	0.12203436D 01	0.73542311D 00	0.69945579D 00
	5	0.12190971D 01	0.12284122D 01	0.69727890D 00	0.71343988D 00
	6	0.12045253D 01	0.12194751D 01	0.67147686D 00	0.69793940D 00
	7	0.11803878D 01	0.12332723D 01	0.62714867D 00	0.72177598D 00
	8	0.12907750D 01	0.13401118D 01	0.81614955D 00	0.89213205D 00
	9	0.13379940D 01	0.13120142D 01	0.88894767D 00	0.84934171D 00
	10	0.13461951D 01	0.13225624D 01	0.90124427D 00	0.86554678D 00
	11	0.13026162D 01	0.13515852D 01	0.83475087D 00	0.90927585D 00
	12	0.12400591D 01	0.12402558D 01	0.73331208D 00	0.73364470D 00
	16	0.12058036D 01	0.12174234D 01	0.67376728D 00	0.69434836D 00
	17	0.13426154D 01	0.13076631D 01	0.89588839D 00	0.84260478D 00
	18	0.12349323D 01	0.12136780D 01	0.72460874D 00	0.68776034D 00
	19	0.12398679D 01	0.12415057D 01	0.73298874D 00	0.73575566D 00
	20	0.12739771D 01	0.12627687D 01	0.78931468D 00	0.77109328D 00
	24	0.12528900D 01	0.12242167D 01	0.75480688D 00	0.70619156D 00
	25	0.12424188D 01	0.12195826D 01	0.73729543D 00	0.69812724D 00
2	1	0.12726476D 01	0.12029824D 01	0.78716695D 00	0.66870517D 00
	2	0.13455546D 01	0.13230566D 01	0.90028729D 00	0.86630171D 00
	3	0.14843731D 01	0.14988911D 01	0.10969792D 01	0.11165458D 01
	4	0.12905861D 01	0.13403232D 01	0.81585082D 00	0.89244959D 00
	5	0.11825774D 01	0.12307896D 01	0.63126008D 00	0.71752557D 00
	6	0.12055259D 01	0.12184393D 01	0.67327021D 00	0.69612804D 00
	7	0.12188098D 01	0.12287064D 01	0.69677630D 00	0.71394639D 00
	8	0.12420334D 01	0.12196562D 01	0.73664571D 00	0.69825578D 00
	9	0.13420344D 01	0.13082382D 01	0.89501750D 00	0.84349701D 00
	10	0.12393786D 01	0.12409378D 01	0.73216079D 00	0.73479591D 00
	11	0.13026168D 01	0.13515845D 01	0.83475183D 00	0.90927483D 00
	12	0.13479316D 01	0.13208749D 01	0.90391057D 00	0.86296616D 00
	13	0.13384569D 01	0.13115403D 01	0.88964428D 00	0.84860936D 00
	14	0.12047627D 01	0.12184975D 01	0.67190268D 00	0.69622993D 00
	18	0.12769529D 01	0.12598907D 01	0.79410881D 00	0.76637110D 00
	19	0.12405178D 01	0.12408543D 01	0.73408744D 00	0.73465595D 00
	20	0.12356162D 01	0.12130299D 01	0.72577365D 00	0.68661598D 00
	21	0.12415964D 01	0.12203619D 01	0.73590878D 00	0.69948782D 00
	22	0.12538456D 01	0.12233272D 01	0.75639196D 00	0.70464851D 00
3	1	0.12769529D 01	0.12598907D 01	0.79410881D 00	0.76637110D 00
	2	0.12405178D 01	0.12408543D 01	0.73408744D 00	0.73465595D 00
	3	0.12356162D 01	0.12130299D 01	0.72577365D 00	0.68661598D 00
	4	0.12415964D 01	0.12203619D 01	0.73590878D 00	0.69948782D 00
	5	0.12538456D 01	0.12233272D 01	0.75639196D 00	0.70464851D 00
	10	0.12393786D 01	0.12409378D 01	0.73216079D 00	0.73479591D 00
	11	0.13026168D 01	0.13515845D 01	0.83475183D 00	0.90927483D 00
	12	0.13479816D 01	0.13208749D 01	0.90391057D 00	0.86296616D 00
	13	0.13384569D 01	0.13115403D 01	0.88964428D 00	0.84860936D 00
	14	0.12047627D 01	0.12184975D 01	0.67190268D 00	0.69622993D 00
	18	0.12726476D 01	0.12029824D 01	0.78716695D 00	0.66870517D 00
	19	0.13455546D 01	0.13230566D 01	0.90028729D 00	0.86630171D 00
	20	0.14843731D 01	0.14988911D 01	0.10969792D 01	0.11165458D 01
	21	0.12905861D 01	0.13403232D 01	0.81585082D 00	0.89244959D 00
	22	0.11825774D 01	0.12307896D 01	0.63126008D 00	0.71752557D 00
	23	0.12055259D 01	0.12184393D 01	0.67327021D 00	0.69612804D 00
	24	0.12188098D 01	0.12287064D 01	0.69677630D 00	0.71394639D 00
	25	0.12420334D 01	0.12196562D 01	0.73664571D 00	0.69825578D 00
	26	0.13420344D 01	0.13082382D 01	0.89501750D 00	0.84349701D 00

TABLE 5.13. Values of Tau and Gam for Case 10
(at $\alpha = 0.05$ and $\gamma = 0.8$)

PHO NO	PT. NO	TAU x	TAU y	GAM x	GAM y
4	1	0.12349323D 01	0.12136780D 01	0.72460874D 00	0.68776034D 00
	2	0.12398679D 01	0.12415057D 01	0.73298874D 00	0.73575566D 00
	3	0.12739771D 01	0.12627687D 01	0.76931468D 00	0.77109328D 00
	7	0.12528900D 01	0.12242167D 01	0.75480688D 00	0.70619156D 00
	8	0.12424188D 01	0.12195826D 01	0.73729543D 00	0.69812724D 00
	10	0.13461951D 01	0.13225624D 01	0.90124427D 00	0.86554678D 00
	11	0.13026162D 01	0.13515852D 01	0.83475087D 00	0.90927585D 00
	12	0.12400591D 01	0.12402558D 01	0.73331208D 00	0.73364470D 00
	16	0.12058036D 01	0.12174234D 01	0.67376728D 00	0.69434836D 00
	17	0.13426154D 01	0.13076631D 01	0.89588839D 00	0.84260478D 00
	18	0.14356590D 01	0.14975706D 01	0.10987187D 01	0.11147725D 01
	19	0.13476683D 01	0.13210559D 01	0.90344407D 00	0.86324316D 00
	20	0.12734930D 01	0.12022696D 01	0.78853315D 00	0.66742209D 00
	21	0.12413086D 01	0.12203436D 01	0.73542311D 00	0.69945579D 00
	22	0.12190071D 01	0.12284122D 01	0.69727890D 00	0.71343988D 00
	23	0.12045253D 01	0.12194751D 01	0.67147686D 00	0.69793940D 00
	24	0.11803873D 01	0.12332723D 01	0.62714867D 00	0.72177598D 00
	25	0.12907750D 01	0.13401118D 01	0.81614955D 00	0.89213205D 00
	26	0.13379940D 01	0.13120142D 01	0.88894767D 00	0.84934171D 00
5	1	0.11825774D 01	0.12307896D 01	0.63126008D 00	0.71752557D 00
	2	0.12055259D 01	0.12184393D 01	0.67327021D 00	0.69612804D 00
	3	0.12188098D 01	0.12287064D 01	0.69677630D 00	0.71394639D 00
	4	0.12420334D 01	0.12196562D 01	0.73664571D 00	0.69825578D 00
	5	0.12726476D 01	0.12029824D 01	0.78716695D 00	0.66870517D 00
	6	0.13455546D 01	0.13230566D 01	0.90028729D 00	0.86630171D 00
	7	0.14343731D 01	0.14988911D 01	0.10969792D 01	0.11165458D 01
	8	0.12905361D 01	0.13403232D 01	0.81585082D 00	0.89244959D 00
	9	0.13420344D 01	0.13082382D 01	0.89501750D 00	0.84349701D 00
	10	0.12047627D 01	0.12184975D 01	0.67190268D 00	0.69622993D 00
	14	0.12393786D 01	0.12409378D 01	0.73216079D 00	0.73479691D 00
	15	0.13026168D 01	0.13515845D 01	0.83475183D 00	0.90927483D 00
	16	0.13479816D 01	0.13208749D 01	0.90391057D 00	0.86296616D 00
	17	0.13384569D 01	0.13115403D 01	0.88964428D 00	0.84860936D 00
	18	0.12538456D 01	0.12233272D 01	0.75639196D 00	0.70464851D 00
	22	0.12769529D 01	0.12598907D 01	0.79410881D 00	0.76637110D 00
	23	0.12405178D 01	0.12408543D 01	0.73408744D 00	0.73465595D 00
	24	0.12356162D 01	0.12130299D 01	0.72577365D 00	0.68661598D 00
	25	0.12415964D 01	0.12203619D 01	0.73590878D 00	0.69948782D 00
6	1	0.12188098D 01	0.12287064D 01	0.69677630D 00	0.71394639D 00
	2	0.12055259D 01	0.12184393D 01	0.67327021D 00	0.69612804D 00
	3	0.11825774D 01	0.12307896D 01	0.63126008D 00	0.71752557D 00
	4	0.12905361D 01	0.13403232D 01	0.81585082D 00	0.89244959D 00
	5	0.14343731D 01	0.14988911D 01	0.10969792D 01	0.11165458D 01
	6	0.13455546D 01	0.13230566D 01	0.90028729D 00	0.86630171D 00
	7	0.12726476D 01	0.12029824D 01	0.78716695D 00	0.66870517D 00
	8	0.12420334D 01	0.12196562D 01	0.73664571D 00	0.69825578D 00
	9	0.13420344D 01	0.13082382D 01	0.89501750D 00	0.84349701D 00
	12	0.12047627D 01	0.12184975D 01	0.67190268D 00	0.69622993D 00
	13	0.13384569D 01	0.13115403D 01	0.88964428D 00	0.84860936D 00
	14	0.13479816D 01	0.13208749D 01	0.90391057D 00	0.86296616D 00
	15	0.13026168D 01	0.13515845D 01	0.83475183D 00	0.90927483D 00
	16	0.12393786D 01	0.12409378D 01	0.73216079D 00	0.73479691D 00
	20	0.12538456D 01	0.12233272D 01	0.75639196D 00	0.70464851D 00
	21	0.12415964D 01	0.12203619D 01	0.73590878D 00	0.69948782D 00
	22	0.12356162D 01	0.12130299D 01	0.72577365D 00	0.68661598D 00
	23	0.12405178D 01	0.12408543D 01	0.73408744D 00	0.73465595D 00
	24	0.12769529D 01	0.12598907D 01	0.79410881D 00	0.76637110D 00

TABLE 5.13 (Continued)

PHO NO	PT. NO	TAU x	TAU y	GAM x	GAM y
7	3	0.12528900D 01	0.12242167D 01	0.75480688D 00	0.70619156D 00
	4	0.12424138D 01	0.12195826D 01	0.73729543D 00	0.69812724D 00
	5	0.12349323D 01	0.12136780D 01	0.72460874D 00	0.68776034D 00
	6	0.12398679D 01	0.12415057D 01	0.73298874D 00	0.73575566D 00
	7	0.12739771D 01	0.12627687D 01	0.78931468D 00	0.77109328D 00
	12	0.12058036D 01	0.12174234D 01	0.67376728D 00	0.69434836D 00
	13	0.13426154D 01	0.13076631D 01	0.89588839D 00	0.84260478D 00
	14	0.13461951D 01	0.13225624D 01	0.90124427D 00	0.86554678D 00
	15	0.13026162D 01	0.13515852D 01	0.83475087D 00	0.90927585D 00
	16	0.12400591D 01	0.12402558D 01	0.73331208D 00	0.73364470D 00
	18	0.12190971D 01	0.12284122D 01	0.69727890D 00	0.71343988D 00
	19	0.12045253D 01	0.12194751D 01	0.67147686D 00	0.69793940D 00
	20	0.11803878D 01	0.12332723D 01	0.62714867D 00	0.72177598D 00
	21	0.12907750D 01	0.13401118D 01	0.81614955D 00	0.89213205D 00
	22	0.14856590D 01	0.14975706D 01	0.10987187D 01	0.11147725D 01
	23	0.13476638D 01	0.13210559D 01	0.90344407D 00	0.86324316D 00
	24	0.12734930D 01	0.12022696D 01	0.78853315D 00	0.66742209D 00
	25	0.12413086D 01	0.12203436D 01	0.73542311D 00	0.69945579D 00
	26	0.13379940D 01	0.13120142D 01	0.88894767D 00	0.84934171D 00
8	1	0.12528900D 01	0.12242167D 01	0.75480688D 00	0.70619156D 00
	5	0.12739771D 01	0.12627687D 01	0.78931468D 00	0.77109328D 00
	6	0.12398679D 01	0.12415057D 01	0.73298874D 00	0.73575566D 00
	7	0.12349323D 01	0.12136780D 01	0.72460874D 00	0.68776034D 00
	8	0.12424188D 01	0.12195826D 01	0.73729543D 00	0.69812724D 00
	10	0.12058036D 01	0.12174234D 01	0.67376728D 00	0.69434836D 00
	14	0.12400591D 01	0.12402558D 01	0.73331208D 00	0.73364470D 00
	15	0.13026162D 01	0.13515852D 01	0.83475087D 00	0.90927585D 00
	16	0.13461951D 01	0.13225624D 01	0.90124427D 00	0.86554678D 00
	17	0.13426154D 01	0.13076631D 01	0.89588839D 00	0.84260478D 00
	18	0.11803878D 01	0.12332723D 01	0.62714867D 00	0.72177598D 00
	19	0.12045253D 01	0.12194751D 01	0.67147686D 00	0.69793940D 00
	20	0.12190971D 01	0.12284122D 01	0.69727890D 00	0.71343988D 00
	21	0.12413086D 01	0.12203436D 01	0.73542311D 00	0.69945579D 00
	22	0.12734930D 01	0.12022696D 01	0.78853315D 00	0.66742209D 00
	23	0.13476638D 01	0.13210559D 01	0.90344407D 00	0.86324316D 00
	24	0.14856590D 01	0.14975706D 01	0.10987187D 01	0.11147725D 01
	25	0.12907750D 01	0.13401118D 01	0.81614955D 00	0.89213205D 00
	26	0.13379940D 01	0.13120142D 01	0.88894767D 00	0.84934171D 00

TABLE 5.13 (Continued)

PHOTO NO	POINT NO	TAU x	TAU y	DELTA(um) x	DELTA(um) y
1	1	0.15071257D 01	0.15101561D 01	0.2110D 02	0.2114D 02
	2	0.13509720D 01	0.13228766D 01	0.1891D 02	0.1852D 02
	3	0.12821461D 01	0.12599709D 01	0.1795D 02	0.1764D 02
	4	0.12419747D 01	0.12213826D 01	0.1739D 02	0.1710D 02
	5	0.12561528D 01	0.12687703D 01	0.1759D 02	0.1776D 02
	6	0.12058763D 01	0.12199470D 01	0.1688D 02	0.1708D 02
	7	0.12211412D 01	0.12500960D 01	0.1710D 02	0.1750D 02
	8	0.12939488D 01	0.13416894D 01	0.1812D 02	0.1878D 02
	9	0.13389692D 01	0.13124339D 01	0.1875D 02	0.1837D 02
	10	0.13490963D 01	0.13246449D 01	0.1889D 02	0.1855D 02
	11	0.13036209D 01	0.13519219D 01	0.1825D 02	0.1893D 02
	12	0.12417414D 01	0.12410327D 01	0.1738D 02	0.1737D 02
	16	0.12068702D 01	0.12189205D 01	0.1690D 02	0.1706D 02
	17	0.13430952D 01	0.13085833D 01	0.1880D 02	0.1832D 02
	18	0.12817733D 01	0.12603250D 01	0.1794D 02	0.1764D 02
	19	0.12410848D 01	0.12416892D 01	0.1738D 02	0.1738D 02
	20	0.13117304D 01	0.12778639D 01	0.1836D 02	0.1789D 02
	24	0.12555570D 01	0.12693852D 01	0.1758D 02	0.1777D 02
	25	0.12427526D 01	0.12206442D 01	0.1740D 02	0.1709D 02
2	1	0.12817733D 01	0.12603250D 01	0.1794D 02	0.1764D 02
	2	0.13490963D 01	0.13246449D 01	0.1889D 02	0.1855D 02
	3	0.15071257D 01	0.15101561D 01	0.2110D 02	0.2114D 02
	4	0.12939488D 01	0.13416894D 01	0.1812D 02	0.1878D 02
	5	0.12211412D 01	0.12500960D 01	0.1710D 02	0.1750D 02
	6	0.12068702D 01	0.12189205D 01	0.1690D 02	0.1706D 02
	7	0.12555570D 01	0.12693852D 01	0.1758D 02	0.1777D 02
	8	0.12427526D 01	0.12206442D 01	0.1740D 02	0.1709D 02
	9	0.13430952D 01	0.13085833D 01	0.1880D 02	0.1832D 02
	10	0.12410848D 01	0.12416892D 01	0.1738D 02	0.1738D 02
	11	0.13036209D 01	0.13519219D 01	0.1825D 02	0.1893D 02
	12	0.13509720D 01	0.13228766D 01	0.1891D 02	0.1852D 02
	13	0.13389692D 01	0.13124339D 01	0.1875D 02	0.1837D 02
	14	0.12058763D 01	0.12199470D 01	0.1688D 02	0.1708D 02
	18	0.13117304D 01	0.12778639D 01	0.1836D 02	0.1789D 02
	19	0.12417414D 01	0.12410327D 01	0.1738D 02	0.1737D 02
	20	0.12821461D 01	0.12599709D 01	0.1795D 02	0.1764D 02
	21	0.12419747D 01	0.12213826D 01	0.1739D 02	0.1710D 02
	22	0.12561528D 01	0.12687703D 01	0.1759D 02	0.1776D 02
3	1	0.13117304D 01	0.12778639D 01	0.1836D 02	0.1789D 02
	2	0.12417414D 01	0.12410327D 01	0.1738D 02	0.1737D 02
	3	0.12821461D 01	0.12599709D 01	0.1795D 02	0.1764D 02
	4	0.12419747D 01	0.12213826D 01	0.1739D 02	0.1710D 02
	5	0.12561528D 01	0.12687703D 01	0.1759D 02	0.1776D 02
	10	0.12410848D 01	0.12416892D 01	0.1738D 02	0.1738D 02
	11	0.13036209D 01	0.13519219D 01	0.1825D 02	0.1893D 02
	12	0.13509720D 01	0.13228766D 01	0.1891D 02	0.1852D 02
	13	0.13389692D 01	0.13124339D 01	0.1875D 02	0.1837D 02
	14	0.12058763D 01	0.12199470D 01	0.1688D 02	0.1708D 02
	18	0.12817733D 01	0.12603250D 01	0.1794D 02	0.1764D 02
	19	0.13490963D 01	0.13246449D 01	0.1889D 02	0.1855D 02
	20	0.15071257D 01	0.15101561D 01	0.2110D 02	0.2114D 02
	21	0.12939488D 01	0.13416894D 01	0.1812D 02	0.1878D 02
	22	0.12211412D 01	0.12500960D 01	0.1710D 02	0.1750D 02
	23	0.12068702D 01	0.12189205D 01	0.1690D 02	0.1706D 02
	24	0.12555570D 01	0.12693852D 01	0.1758D 02	0.1777D 02
	25	0.12427526D 01	0.12206442D 01	0.1740D 02	0.1709D 02
	26	0.13430952D 01	0.13085833D 01	0.1880D 02	0.1832D 02

TABLE 5.14. Undetected Gross Errors for case 9.
(at $\alpha = 0.05$ and $\gamma = 0.8$)

PHOTO NO	POINT NO	TAU x	TAU y	DELTA(um) x	DELTA(um) y
4	1	0.12817733D 01	0.12603250D 01	0.1794D 02	0.1764D 02
	2	0.12410848D 01	0.12416892D 01	0.1738D 02	0.1738D 02
	3	0.13117304D 01	0.12778639D 01	0.1836D 02	0.1789D 02
	7	0.12555570D 01	0.12693852D 01	0.1758D 02	0.1777D 02
	8	0.12427526D 01	0.12206442D 01	0.1740D 02	0.1709D 02
	10	0.13490963D 01	0.13246449D 01	0.1889D 02	0.1855D 02
	11	0.13036209D 01	0.13519219D 01	0.1825D 02	0.1893D 02
	12	0.12417414D 01	0.12410327D 01	0.1738D 02	0.1737D 02
	16	0.12068702D 01	0.12189205D 01	0.1690D 02	0.1706D 02
	17	0.13430952D 01	0.13085833D 01	0.1880D 02	0.1832D 02
	18	0.15071257D 01	0.15101561D 01	0.2110D 02	0.2114D 02
	19	0.13509720D 01	0.13228766D 01	0.1891D 02	0.1852D 02
	20	0.12821461D 01	0.12599709D 01	0.1795D 02	0.1764D 02
	21	0.12419747D 01	0.12213826D 01	0.1739D 02	0.1710D 02
	22	0.12561528D 01	0.12687703D 01	0.1759D 02	0.1776D 02
	23	0.12058763D 01	0.12199470D 01	0.1688D 02	0.1708D 02
	24	0.12211412D 01	0.12500960D 01	0.1710D 02	0.1750D 02
	25	0.12939488D 01	0.13416894D 01	0.1812D 02	0.1878D 02
	26	0.13389692D 01	0.13124339D 01	0.1875D 02	0.1837D 02
5	1	0.12211412D 01	0.12500960D 01	0.1710D 02	0.1750D 02
	2	0.12068702D 01	0.12189205D 01	0.1690D 02	0.1706D 02
	3	0.12555570D 01	0.12693852D 01	0.1758D 02	0.1777D 02
	4	0.12427526D 01	0.12206442D 01	0.1740D 02	0.1709D 02
	5	0.12817733D 01	0.12603250D 01	0.1794D 02	0.1764D 02
	6	0.13490963D 01	0.13246449D 01	0.1889D 02	0.1855D 02
	7	0.15071257D 01	0.15101561D 01	0.2110D 02	0.2114D 02
	8	0.12939488D 01	0.13416894D 01	0.1812D 02	0.1878D 02
	9	0.13430952D 01	0.13085833D 01	0.1880D 02	0.1832D 02
	10	0.12058763D 01	0.12199470D 01	0.1688D 02	0.1708D 02
	14	0.12410848D 01	0.12416892D 01	0.1738D 02	0.1738D 02
	15	0.13036209D 01	0.13519219D 01	0.1825D 02	0.1893D 02
	16	0.13509720D 01	0.13228766D 01	0.1891D 02	0.1852D 02
	17	0.13389692D 01	0.13124339D 01	0.1875D 02	0.1837D 02
	18	0.12561523D 01	0.12687703D 01	0.1759D 02	0.1776D 02
	22	0.13117304D 01	0.12778639D 01	0.1836D 02	0.1789D 02
	23	0.12417414D 01	0.12410327D 01	0.1738D 02	0.1737D 02
	24	0.12821461D 01	0.12599709D 01	0.1795D 02	0.1764D 02
	25	0.12419747D 01	0.12213826D 01	0.1739D 02	0.1710D 02
6	1	0.12555570D 01	0.12693852D 01	0.1758D 02	0.1777D 02
	2	0.12068702D 01	0.12189205D 01	0.1690D 02	0.1706D 02
	3	0.12211412D 01	0.12500960D 01	0.1710D 02	0.1750D 02
	4	0.12939488D 01	0.13416894D 01	0.1812D 02	0.1878D 02
	5	0.15071257D 01	0.15101561D 01	0.2110D 02	0.2114D 02
	6	0.13490963D 01	0.13246449D 01	0.1889D 02	0.1855D 02
	7	0.12817733D 01	0.12603250D 01	0.1794D 02	0.1764D 02
	8	0.12427526D 01	0.12206442D 01	0.1740D 02	0.1709D 02
	9	0.13430952D 01	0.13085833D 01	0.1880D 02	0.1832D 02
	12	0.12058763D 01	0.12199470D 01	0.1688D 02	0.1708D 02
	13	0.13389692D 01	0.13124339D 01	0.1875D 02	0.1837D 02
	14	0.13509720D 01	0.13228766D 01	0.1891D 02	0.1852D 02
	15	0.13036209D 01	0.13519219D 01	0.1825D 02	0.1893D 02
	16	0.12410848D 01	0.12416892D 01	0.1738D 02	0.1738D 02
	20	0.12561528D 01	0.12687703D 01	0.1759D 02	0.1776D 02
	21	0.12419747D 01	0.12213826D 01	0.1739D 02	0.1710D 02
	22	0.12821461D 01	0.12599709D 01	0.1795D 02	0.1764D 02
	23	0.12417414D 01	0.12410327D 01	0.1738D 02	0.1737D 02
	24	0.13117304D 01	0.12778639D 01	0.1836D 02	0.1789D 02

TABLE 5.14 (Continued)

PHOTO NO	POINT NO	TAU x	TAU y	DELTA(um) x	DELTA(um) y
7	3	0.12555570D 01	0.12693852D 01	0.1758D 02	0.1777D 02
	4	0.12427526D 01	0.12206442D 01	0.1740D 02	0.1709D 02
	5	0.12817733D 01	0.12603250D 01	0.1794D 02	0.1764D 02
	6	0.12410848D 01	0.12416892D 01	0.1738D 02	0.1738D 02
	7	0.13117304D 01	0.12778639D 01	0.1836D 02	0.1789D 02
	12	0.12068702D 01	0.12189205D 01	0.1690D 02	0.1706D 02
	13	0.13430952D 01	0.13085833D 01	0.1880D 02	0.1832D 02
	14	0.13490963D 01	0.13246449D 01	0.1889D 02	0.1855D 02
	15	0.13036209D 01	0.13519219D 01	0.1825D 02	0.1893D 02
	16	0.12417414D 01	0.12410327D 01	0.1738D 02	0.1737D 02
	18	0.12561528D 01	0.12687703D 01	0.1759D 02	0.1776D 02
	19	0.12058763D 01	0.12199470D 01	0.1688D 02	0.1708D 02
	20	0.12211412D 01	0.12500960D 01	0.1710D 02	0.1750D 02
	21	0.12939488D 01	0.13416894D 01	0.1812D 02	0.1878D 02
	22	0.15071257D 01	0.15101561D 01	0.2110D 02	0.2114D 02
	23	0.13509720D 01	0.13228766D 01	0.1891D 02	0.1852D 02
	24	0.12821461D 01	0.12599709D 01	0.1795D 02	0.1764D 02
	25	0.12419747D 01	0.12213826D 01	0.1739D 02	0.1710D 02
	26	0.13389692D 01	0.13124339D 01	0.1875D 02	0.1837D 02
8	1	0.12555570D 01	0.12693852D 01	0.1758D 02	0.1777D 02
	5	0.13117304D 01	0.12778639D 01	0.1836D 02	0.1789D 02
	6	0.12410848D 01	0.12416892D 01	0.1738D 02	0.1738D 02
	7	0.12817733D 01	0.12603250D 01	0.1794D 02	0.1764D 02
	8	0.12427526D 01	0.12206442D 01	0.1740D 02	0.1709D 02
	10	0.12068702D 01	0.12189205D 01	0.1690D 02	0.1706D 02
	14	0.12417414D 01	0.12410327D 01	0.1738D 02	0.1737D 02
	15	0.13036209D 01	0.13519219D 01	0.1825D 02	0.1893D 02
	16	0.13490963D 01	0.13246449D 01	0.1889D 02	0.1855D 02
	17	0.13430952D 01	0.13085833D 01	0.1880D 02	0.1832D 02
	18	0.12211412D 01	0.12500960D 01	0.1710D 02	0.1750D 02
	19	0.12058763D 01	0.12199470D 01	0.1688D 02	0.1708D 02
	20	0.12561528D 01	0.12687703D 01	0.1759D 02	0.1776D 02
	21	0.12419747D 01	0.12213826D 01	0.1739D 02	0.1710D 02
	22	0.12821461D 01	0.12599709D 01	0.1795D 02	0.1764D 02
	23	0.13509720D 01	0.13228766D 01	0.1891D 02	0.1852D 02
	24	0.15071257D 01	0.15101561D 01	0.2110D 02	0.2114D 02
	25	0.12939488D 01	0.13416894D 01	0.1812D 02	0.1878D 02
	26	0.13389692D 01	0.13124339D 01	0.1875D 02	0.1837D 02

TABLE 5.14 (Continued)

PHOTO	POINT	TAU		DELTA(um)	
NO	NO	x	y	x	y
1	1	0.14856570D 01	0.14975706D 01	0.2080D 02	0.2097D 02
	2	0.13476688D 01	0.13210559D 01	0.1887D 02	0.1849D 02
	3	0.12734930D 01	0.12022696D 01	0.1783D 02	0.1683D 02
	4	0.12413086D 01	0.12203436D 01	0.1738D 02	0.1708D 02
	5	0.12190971D 01	0.12284122D 01	0.1707D 02	0.1720D 02
	6	0.12045253D 01	0.12194751D 01	0.1686D 02	0.1707D 02
	7	0.11803878D 01	0.12332723D 01	0.1653D 02	0.1727D 02
	8	0.12907750D 01	0.13401118D 01	0.1807D 02	0.1876D 02
	9	0.13379940D 01	0.13120142D 01	0.1873D 02	0.1837D 02
	10	0.13461951D 01	0.13225674D 01	0.1885D 02	0.1852D 02
	11	0.13026162D 01	0.13515852D 01	0.1824D 02	0.1892D 02
	12	0.12400591D 01	0.12402558D 01	0.1736D 02	0.1736D 02
	16	0.12058036D 01	0.12174234D 01	0.1688D 02	0.1704D 02
	17	0.13426154D 01	0.13076631D 01	0.1880D 02	0.1831D 02
	18	0.12349323D 01	0.12136780D 01	0.1729D 02	0.1699D 02
	19	0.12398679D 01	0.12415057D 01	0.1736D 02	0.1738D 02
	20	0.12739771D 01	0.12627687D 01	0.1784D 02	0.1768D 02
	24	0.12528900D 01	0.12242167D 01	0.1754D 02	0.1714D 02
	25	0.12424188D 01	0.12195826D 01	0.1739D 02	0.1707D 02
2	1	0.12726476D 01	0.12029824D 01	0.1782D 02	0.1684D 02
	2	0.13455546D 01	0.13230566D 01	0.1884D 02	0.1852D 02
	3	0.14843731D 01	0.14988911D 01	0.2078D 02	0.2098D 02
	4	0.12905861D 01	0.13403232D 01	0.1807D 02	0.1876D 02
	5	0.11825774D 01	0.12307896D 01	0.1656D 02	0.1723D 02
	6	0.12055259D 01	0.12184393D 01	0.1688D 02	0.1706D 02
	7	0.12188098D 01	0.12287064D 01	0.1706D 02	0.1720D 02
	8	0.12420334D 01	0.12196562D 01	0.1739D 02	0.1708D 02
	9	0.13420344D 01	0.13082382D 01	0.1879D 02	0.1832D 02
	10	0.12393786D 01	0.12409378D 01	0.1735D 02	0.1737D 02
	11	0.13026168D 01	0.13515845D 01	0.1824D 02	0.1892D 02
	12	0.13479816D 01	0.13208749D 01	0.1887D 02	0.1849D 02
	13	0.13384569D 01	0.13115403D 01	0.1874D 02	0.1836D 02
	14	0.12047627D 01	0.12184975D 01	0.1687D 02	0.1706D 02
	18	0.12769529D 01	0.12598907D 01	0.1788D 02	0.1764D 02
	19	0.12405178D 01	0.12408543D 01	0.1737D 02	0.1737D 02
	20	0.12356162D 01	0.12130299D 01	0.1730D 02	0.1698D 02
	21	0.12415964D 01	0.12203619D 01	0.1738D 02	0.1709D 02
	22	0.12538456D 01	0.12233272D 01	0.1755D 02	0.1713D 02
3	1	0.12769529D 01	0.12598907D 01	0.1788D 02	0.1764D 02
	2	0.12405178D 01	0.12408543D 01	0.1737D 02	0.1737D 02
	3	0.12356162D 01	0.12130299D 01	0.1730D 02	0.1698D 02
	4	0.12415964D 01	0.12203619D 01	0.1738D 02	0.1709D 02
	5	0.12538456D 01	0.12233272D 01	0.1755D 02	0.1713D 02
	10	0.12393786D 01	0.12409378D 01	0.1735D 02	0.1737D 02
	11	0.13026168D 01	0.13515845D 01	0.1824D 02	0.1892D 02
	12	0.13479816D 01	0.13208749D 01	0.1887D 02	0.1849D 02
	13	0.13384569D 01	0.13115403D 01	0.1874D 02	0.1836D 02
	14	0.12047627D 01	0.12184975D 01	0.1687D 02	0.1706D 02
	18	0.12726476D 01	0.12029824D 01	0.1782D 02	0.1684D 02
	19	0.13455546D 01	0.13230566D 01	0.1884D 02	0.1852D 02
	20	0.14843731D 01	0.14988911D 01	0.2078D 02	0.2098D 02
	21	0.12905861D 01	0.13403232D 01	0.1807D 02	0.1876D 02
	22	0.11825774D 01	0.12307896D 01	0.1656D 02	0.1723D 02
	23	0.12055259D 01	0.12184393D 01	0.1688D 02	0.1706D 02
	24	0.12188098D 01	0.12287064D 01	0.1706D 02	0.1720D 02
	25	0.12420334D 01	0.12196562D 01	0.1739D 02	0.1708D 02
	26	0.13420344D 01	0.13082382D 01	0.1879D 02	0.1832D 02

TABLE 5.15. Undetected Gross Errors for case 10.
(at α = 0.05 and γ = 0.8)

PHOTO NO	POINT NO	TAU x	TAU y	DELTA(um) x	DELTA(um) y
4	1	0.12349323D 01	0.12136780D 01	0.1729D 02	0.1699D 02
	2	0.12398679D 01	0.12415057D 01	0.1736D 02	0.1738D 02
	3	0.12739771D 01	0.12627687D 01	0.1784D 02	0.1768D 02
	7	0.12528900D 01	0.12242167D 01	0.1754D 02	0.1714D 02
	8	0.12424188D 01	0.12195826D 01	0.1739D 02	0.1707D 02
	10	0.13461951D 01	0.13225624D 01	0.1885D 02	0.1852D 02
	11	0.13026162D 01	0.13515852D 01	0.1824D 02	0.1892D 02
	12	0.12400591D 01	0.12402558D 01	0.1736D 02	0.1736D 02
	16	0.12058036D 01	0.12174234D 01	0.1688D 02	0.1704D 02
	17	0.13426154D 01	0.13076631D 01	0.1880D 02	0.1831D 02
	18	0.14856590D 01	0.14975706D 01	0.2080D 02	0.2097D 02
	19	0.13476688D 01	0.13210559D 01	0.1887D 02	0.1849D 02
	20	0.12734930D 01	0.12022696D 01	0.1783D 02	0.1683D 02
	21	0.12413086D 01	0.12203436D 01	0.1738D 02	0.1708D 02
	22	0.12190971D 01	0.12284122D 01	0.1707D 02	0.1720D 02
	23	0.12045253D 01	0.12194751D 01	0.1686D 02	0.1707D 02
	24	0.11803878D 01	0.12332723D 01	0.1653D 02	0.1727D 02
	25	0.12907750D 01	0.13401118D 01	0.1807D 02	0.1876D 02
	26	0.13379940D 01	0.13120142D 01	0.1873D 02	0.1837D 02
5	1	0.11325774D 01	0.12307896D 01	0.1656D 02	0.1723D 02
	2	0.12055259D 01	0.12184393D 01	0.1688D 02	0.1706D 02
	3	0.12188098D 01	0.12287064D 01	0.1706D 02	0.1720D 02
	4	0.12420334D 01	0.12196562D 01	0.1739D 02	0.1708D 02
	5	0.12726476D 01	0.12029824D 01	0.1782D 02	0.1684D 02
	6	0.13455546D 01	0.13230566D 01	0.1884D 02	0.1852D 02
	7	0.14843731D 01	0.14988911D 01	0.2078D 02	0.2098D 02
	8	0.12905861D 01	0.13403232D 01	0.1807D 02	0.1876D 02
	9	0.13420344D 01	0.13082382D 01	0.1879D 02	0.1832D 02
	10	0.12047627D 01	0.12184975D 01	0.1687D 02	0.1706D 02
	14	0.12393786D 01	0.12409378D 01	0.1735D 02	0.1737D 02
	15	0.13026168D 01	0.13515845D 01	0.1824D 02	0.1892D 02
	16	0.13479816D 01	0.13208749D 01	0.1887D 02	0.1849D 02
	17	0.13384569D 01	0.13115403D 01	0.1874D 02	0.1836D 02
	18	0.12538456D 01	0.12233272D 01	0.1755D 02	0.1713D 02
	22	0.12769529D 01	0.12598907D 01	0.1788D 02	0.1764D 02
	23	0.12405178D 01	0.12408543D 01	0.1737D 02	0.1737D 02
	24	0.12356162D 01	0.12130299D 01	0.1730D 02	0.1698D 02
	25	0.12415964D 01	0.12203619D 01	0.1738D 02	0.1709D 02
6	1	0.12188098D 01	0.12287064D 01	0.1706D 02	0.1720D 02
	2	0.12055259D 01	0.12184393D 01	0.1688D 02	0.1706D 02
	3	0.11325774D 01	0.12307896D 01	0.1656D 02	0.1723D 02
	4	0.12905861D 01	0.13403232D 01	0.1807D 02	0.1876D 02
	5	0.14843731D 01	0.14988911D 01	0.2078D 02	0.2098D 02
	6	0.13455546D 01	0.13230566D 01	0.1884D 02	0.1852D 02
	7	0.12726476D 01	0.12029824D 01	0.1782D 02	0.1684D 02
	8	0.12420334D 01	0.12196562D 01	0.1739D 02	0.1708D 02
	9	0.13420344D 01	0.13082382D 01	0.1879D 02	0.1832D 02
	12	0.12047627D 01	0.12184975D 01	0.1687D 02	0.1706D 02
	13	0.13384569D 01	0.13115403D 01	0.1874D 02	0.1836D 02
	14	0.13479816D 01	0.13208749D 01	0.1887D 02	0.1849D 02
	15	0.13026168D 01	0.13515845D 01	0.1824D 02	0.1892D 02
	16	0.12393786D 01	0.12409378D 01	0.1735D 02	0.1737D 02
	20	0.12538456D 01	0.12233272D 01	0.1755D 02	0.1713D 02
	21	0.12415964D 01	0.12203619D 01	0.1738D 02	0.1709D 02
	22	0.12356162D 01	0.12130299D 01	0.1730D 02	0.1698D 02
	23	0.12405178D 01	0.12408543D 01	0.1737D 02	0.1737D 02
	24	0.12769529D 01	0.12598907D 01	0.1788D 02	0.1764D 02

TABLE 5.15 (Continued)

PHOTO	POINT	TAU		DELTA(um)	
NO	NO	x	y	x	y
7	3	0.12528900D 01	0.12242167D 01	0.1754D 02	0.1714D 02
	4	0.12424188D 01	0.12195826D 01	0.1739D 02	0.1707D 02
	5	0.12349323D 01	0.12136780D 01	0.1729D 02	0.1699D 02
	6	0.12398679D 01	0.12415057D 01	0.1736D 02	0.1738D 02
	7	0.12739771D 01	0.12627687D 01	0.1784D 02	0.1766D 02
	12	0.12058036D 01	0.12174234D 01	0.1688D 02	0.1704D 02
	13	0.13426154D 01	0.13076631D 01	0.1880D 02	0.1831D 02
	14	0.13461951D 01	0.13225624D 01	0.1885D 02	0.1852D 02
	15	0.13026162D 01	0.13515852D 01	0.1824D 02	0.1892D 02
	16	0.12400591D 01	0.12402558D 01	0.1736D 02	0.1736D 02
	18	0.12190971D 01	0.12284122D 01	0.1707D 02	0.1720D 02
	19	0.12045253D 01	0.12194751D 01	0.1686D 02	0.1707D 02
	20	0.11803878D 01	0.12332723D 01	0.1653D 02	0.1727D 02
	21	0.12907750D 01	0.13401118D 01	0.1807D 02	0.1876D 02
	22	0.14856590D 01	0.14975706D 01	0.2080D 02	0.2097D 02
	23	0.13476688D 01	0.13210559D 01	0.1887D 02	0.1849D 02
	24	0.12734930D 01	0.12022696D 01	0.1783D 02	0.1683D 02
	25	0.12413086D 01	0.12203436D 01	0.1738D 02	0.1708D 02
	26	0.13379940D 01	0.13120142D 01	0.1873D 02	0.1837D 02
8	1	0.12528900D 01	0.12242167D 01	0.1754D 02	0.1714D 02
	5	0.12739771D 01	0.12627687D 01	0.1784D 02	0.1768D 02
	6	0.12398679D 01	0.12415057D 01	0.1736D 02	0.1738D 02
	7	0.12349323D 01	0.12136780D 01	0.1729D 02	0.1699D 02
	8	0.12424188D 01	0.12195826D 01	0.1739D 02	0.1707D 02
	10	0.12058036D 01	0.12174234D 01	0.1688D 02	0.1704D 02
	14	0.12400591D 01	0.12402558D 01	0.1736D 02	0.1736D 02
	15	0.13026162D 01	0.13515852D 01	0.1824D 02	0.1892D 02
	16	0.13461951D 01	0.13225624D 01	0.1885D 02	0.1852D 02
	17	0.13426154D 01	0.13076631D 01	0.1880D 02	0.1831D 02
	18	0.11803878D 01	0.12332723D 01	0.1653D 02	0.1727D 02
	19	0.12045253D 01	0.12194751D 01	0.1686D 02	0.1707D 02
	20	0.12190971D 01	0.12284122D 01	0.1707D 02	0.1720D 02
	21	0.12413086D 01	0.12203436D 01	0.1738D 02	0.1708D 02
	22	0.12734930D 01	0.12022696D 01	0.1783D 02	0.1683D 02
	23	0.13476688D 01	0.13210559D 01	0.1887D 02	0.1849D 02
	24	0.14356590D 01	0.14975706D 01	0.2080D 02	0.2097D 02
	25	0.12907750D 01	0.13401118D 01	0.1807D 02	0.1876D 02
	26	0.13379940D 01	0.13120142D 01	0.1873D 02	0.1837D 02

TABLE 5.15 (Continued)

Chapter 3 and are listed in Tables (5.12) through (5.15) for cases 9,10. The maximum values of Tau, Gam and undetected gross errors for the different cases are summarised in Table (5.16) for the photogrammetric measurements. Those for slope distances are tabulated in Table (5.17).

Case	Tau		Gam		Maximum undetected gross error (μm)	
	x	y	x	y	x	y
1	1.9922311	2.8393368	1.7230743	2.6574110	27.89	39.75
2	1.9846228	2.8387769	1.7142778	2.6568128	27.78	39.74
3	1.9257858	2.8345602	1.6457980	2.6523068	26.96	39.68
4	1.9174364	2.8341615	1.6360202	2.6518807	26.84	39.68
5	1.8218268	3.0545315	1.5228437	2.8862021	25.51	42.76
6	1.8216204	3.0508413	1.5225967	2.8822964	25.50	42.71
7	1.6778018	1.7862221	1.3472264	1.4800640	23.49	25.01
8	1.6774491	1.7860925	1.3467870	1.4799076	23.48	25.01
9	1.5071257	1.5101561	1.1275762	1.1316233	21.10	21.14
10	1.4856590	1.4988911	1.0987187	1.1165458	20.80	20.98

TABLE (5.16) Max Tau, Gam and undetected gross errors for photogrammetric measurements (at probability levels $\alpha = 0.05$, $\gamma = 0.8$)

As anticipated, the reliability increases with the increase in number of cameras and higher internal reliability reflects higher external reliability as can be seen from the values shown in that table. Moreover, the more number of cameras used, the larger the chance to detect gross errors with less magnitudes.

Case	Tau	Gam	Maximum undetected gross error (mm)
2	1.9964749	1.7279792	2.795
4	1.9215369	1.6408242	2.690
6	1.8127073	1.5119219	2.538
8	1.6707385	1.3384196	2.339
10	1.5575812	1.1941771	2.181

TABLE (5.17) Max Tau, Gam and undetected gross errors for the 8 slope distances (at $\alpha = 0.05$, $\gamma = 0.8$)

5.6.2.3. Models for Sensitivity analysis

Simulated models to investigate the sensitivity of the photogrammetric network and its relation to the number of cameras and to the incorporation of slope distances are undertaken. In the design phase we assume, for simplicity, that we have deformations between two epochs only to be detected. Presumably, the deformation monitoring networks have the same datum and identical cofactor matrices. To assess the just-detectable deformations, we apply equation (3.59) given in Section (3.2.3.3).

Restating equation(3.59):

$$\omega^u = \frac{(c\tilde{d})^T Q_d^+ (c\tilde{d})}{\sigma_0^2} \qquad (5.10)$$

in which $c\tilde{d}$ represents a just-detectable deformation and $\tilde{d}$ is a form vector which characterises the deformation model to be tested.

The models chosen are designated deflexion, expansion and settlement models. Before delving into the analysis of these models, a brief mention is to be given about the examination of the impact of single and multiple point movements on sensitivity. Table (5.18) reveals that the more the number of points in movement, the more sensitive the network is and vice versa.

Form vector description (units are in mm)	Sensitivity parameter c
$\tilde{d} = (\overset{dx_1}{10},0,\overset{dz_1}{5},0,0,0,\ldots.0,0,0)$	0.7807
$\tilde{d} = (0,0,0,\ldots\overset{dx_{24}}{-10},0,\overset{dz_{24}}{-5},\ldots.,0,0,0)$	0.5458
$\tilde{d} = (\overset{dx_1}{10},0,0,\overset{dx_2}{10},0,0,\ldots.,\overset{dx_9}{10},0,0,\ldots0,0,0)$	0.2104
$\tilde{d} = (\overset{dx_1}{10},0,0,\overset{dx_2}{10},0,0,\ldots\overset{dx_9}{10},\ldots,\overset{dx_{18}}{-10},0,0,\overset{dx_{19}}{-10},\ldots\overset{dx_{26}}{-10},0,0)$	0.1569

TABLE (5.18) Values of sensitivity parameter (c) for case 1 (4 photos).

It is meant by deflexion model that the targets on both the upper and lower surfaces of the cube are subjected to 10 mm downward displacements. The same magnitude of displacement was assumed but in outward radial direction for all the targets representing what we called the expansion model. Unlike deflexion model, in the settlement model all the targets are supposed to be moving 10 mm downwards. It is important to notice that both expansion and settlement models were applied to cases 5,6,7,8,9,10 only as cases 1-4 suffer from lack of determination of point 15. Deflexion model is applicable to all cases.

Table (5.19) lists the values of c for both expansion and settlement models while Table (5.20) displays these values for the deflexion model. The results summarised in such tables are graphically represented in Figures (5.14) and (5.15) respectively. These figures suggest that the sensitivity decreases with the increase of the number of cameras (larger values of (c) mean less sensitivity). This can be thought of as the effect of more correlation between the co-ordinates of different object points imaged on more photographs.

Number of cameras	Model designation	Photogrammetry c	Photo+ distances c
6	Expansion	0.2018	0.1003
6	Settlement	0.1531	0.1530
7	Expansion	0.2304	0.1076
7	Settlement	0.1736	0.1729
8	Expansion	0.2435	0.1130
8	Settlement	0.1899	0.1886

TABLE (5.19) Relation between sensitivity parameter (c) and number of cameras for Expansion and Settlement models (at $\alpha = 0.05$ and $\gamma = 0.8$)

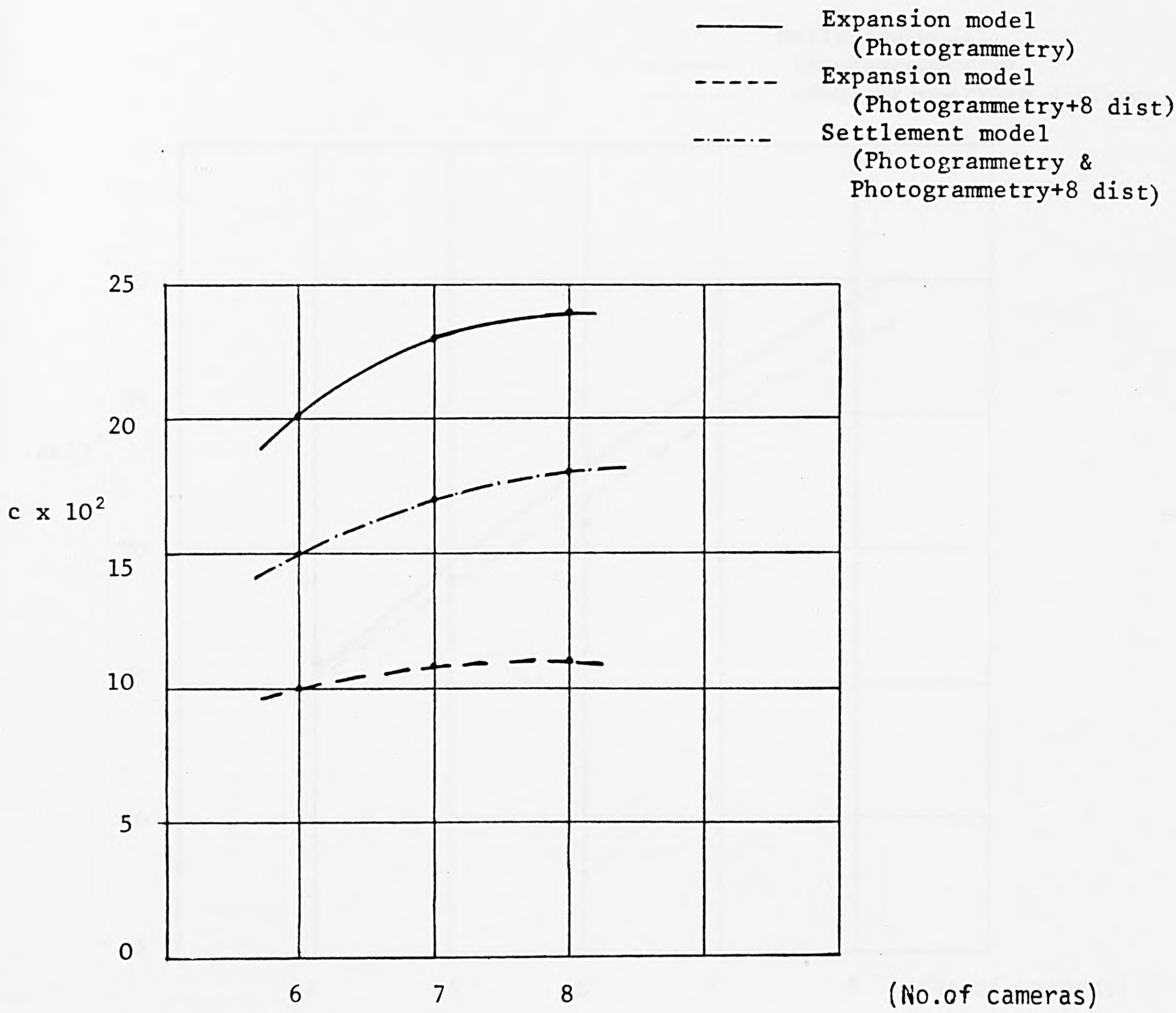


Figure 5.14. No.of cameras versus sensitivity parameter (c) for Expansion and Settlement models.

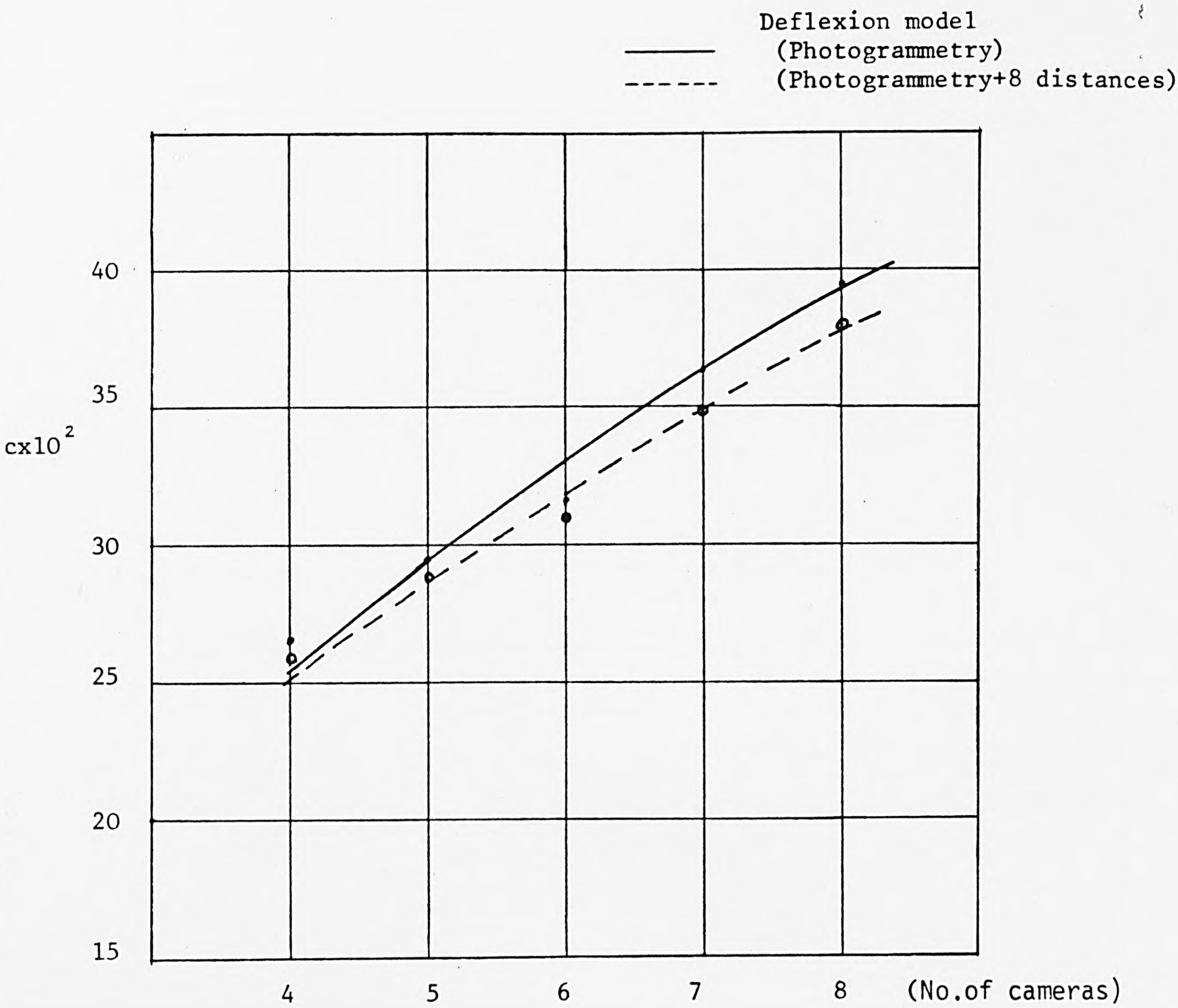


Figure 5.15. No.of cameras versus sensitivity parameter (c) for Deflexion model.

Number of cameras	Model designation	Photogrammetry c	Photos+distances c
4	Deflexion	0.2661	0.2599
5	Deflexion	0.2961	0.2877
6	Deflexion	0.3164	0.3064
7	Deflexion	0.3643	0.3494
8	Deflexion	0.3963	0.3781

TABLE (5.20) Relation between sensitivity parameter (c) and number of cameras for Deflexion model (at $\alpha = 0.05$ $\gamma = 0.8$)

Additionally, the inclusion of distances between object points, as anticipated, does not enhance the sensitivity for settlement model. On the other hand, it has slightly increased the sensitivity for both deflexion and expansion models.

5.6.3. Conclusions

On the basis of these simulations one can conclude that both the precision and reliability increase with the increase of the number of cameras used. However, sensitivity was found to decrease with the increase of that number, which can be thought of as being due to the increased correlation between the object point co-ordinates. This explanation is conformal with that of Fraser (1982a) where he states that the trace of the parameter covariance matrix is minimised at the expense of higher correlation between the parameters which is a well-known property of inner constraint adjustment. Comparing the global indicators r_{max} and σ_m^2 of the different cases it can be noticed that good precision does correspond to good reliability.

With regard to precision, the more cameras the better precision, but the benefit gained decreases as the number of cameras increases. Should the number of cameras be doubled, an average percentage improvement of the order of 24 and 23 in purely photogrammetric and combined, which comprise slope distances, cases respectively would be achieved.

Addition of slope distances provides a marginal improvement "within" the cases of the same number cameras, while it considerably enhances the improvement "between" the different number of camera cases. All the cases produce submillimetre level for σ_m^2 but case 9 (8 photographs) and case 10 (same as case 9 but with 8 slope distances) exhibit the highest precision ($\sigma_{m(9)}^2 = 0.172$ mm², $\sigma_{m(10)}^2 = 0.170$ mm²) which is attributed to the full coverage of the cube.

The main factor affecting the reliability is the number of images per object point. More intersecting rays from an object point give better reliability (larger redundancy numbers). However, it is noticed that improvement in redundancy numbers slows down considerably after 6 rays. Therefore, it can be said that it is not economically desirable to try to improve the redundancy if the object point already has 6 rays and its image co-ordinates display redundancy numbers of the order of 0.6 or more.

Distance observations between object points, in general, increase the reliability substantially when adjusted simultaneously with the photogrammetric data. Cases 9 and 10, again, show fully homogeneous and highest reliability ($r_{max(9)}(x) = 0.688$, $r_{max(9)}(y) = 0.673$;

$r_{max(10)}(x) = 0.718$, $r_{max(10)}(y) = 0.672$) which is justified by the symmetric arrangement of camera positions and distances measured on the cube's surface. Moreover, they can detect a gross error of the order of 4-fold the a priori standard error. It is notable that high internal reliability leads to higher external reliability, and the more the number of cameras the greater is the chance of detection of small gross errors.

Regarding the sensitivity it is worth mentioning that a network is least sensitive to a single point movement and most sensitive to multiple point displacements depending on the assumed pattern of movements of that cluster of points. Inclusion of distance observations between object points does not affect the sensitivity for the settlement model. On the other hand, such an addition has slightly increased the sensitivity for both the deflexion and expansion models.

Summarising the experiences gained by these investigations and indicated by the precision, reliability and sensitivity criteria, we have to prefer case 6 (6 cameras + slope distances) because in this case the x- and y- observations are of sufficient reliability ($r_{max}(x) = 0.682$, $r_{max}(y) = 0.670$) together with a satisfactory sensitivity (c (expansion, setllement models) = 0.10, 0.15 respectively; c (deflexion model) =0.31) and precision ($\sigma_m^2 = 0.248$ mm²).

One more important point is the considerable saving in computer storage and processing time due to the strategies implemented in this research: first, the direct editing of the normal equations without need to form the observation equations and, secondly, the computation of only the diagonal elements of the cofactor matrix of the residual.

CHAPTER 6

SIMULATED NETWORKS FOR DEFORMATION ANALYSIS (BRIDGE)

6.1. Introduction

The general need for monitoring structures is well known to the people of all walks of life due to the loss of human lives when large structures fail. Present methods of detecting and monitoring structural deformations can be divided into two basic groups: internal and external (Erlandson and Veress, 1975).

Internal methods generally utilize sophisticated instruments such as extensometers, inclinometers, and strain gauges. The main disadvantages of their use is their costly maintenance and the difficulty of interpretation of the large amount of data they supply. On the other hand, external methods utilize conventional field surveys (e.g. triangulation and/or trilateration, levelling) where a number of targets whose positions are determined by using some of these methods are placed on the surface of the structure. Such a survey is repeated at predetermined intervals, and positions of the targets each time are compared to quantify the deformations. If the number of targets is fairly large, these methods are considerably time consuming because an individual observation is required for each target on the structure. Erlandson and Veress (1975) claim that a complete survey of a large structure may take more than two weeks during which, deformation can occur but would be undetected.

Photogrammetry, with the inclusion of a few spatial distances, is thought to be the most economical alternative to survey methods as it can replace hundreds of angular measurements thus providing information about the structure as a whole and not to mention the common advantages such as near-instantaneous, complete and permanent recording of a situation. Bridge deformation presents itself as an ideal candidate for such kind of study to be presented in this Chapter.

6.2. Network Configurations

A plan of the bridge, under investigation, and the camera locations is shown in Figure (6.1). Both sides of the bridge (Figures 6.2a, 6.2b) and the columns (Figures 6.3a, b,c,d) with 41 targetted object points are based on real data which were made available to the author. Such data were considered as the approximate values required for the evaluation of the propagated covariance matrices. The wide angle metric camera Zeiss (Jena) UMK 10/1318 having a nominal focal length of 100 mm was assumed to be implemented with the use of glass plates with a negative format of 130 mm x 180 mm. Four camera stations were assumed to represent the start configuration. Subsequently, cameras 5 and 6 were added to provide the different network configurations as displayed in Table (6.2) keeping the number of targets unchanged. The orientation elements of the individual photographs are given in Table (6.1) assuming that the camera principal axis is directed towards the centre of the bridge. The photogrammetric co-ordinates

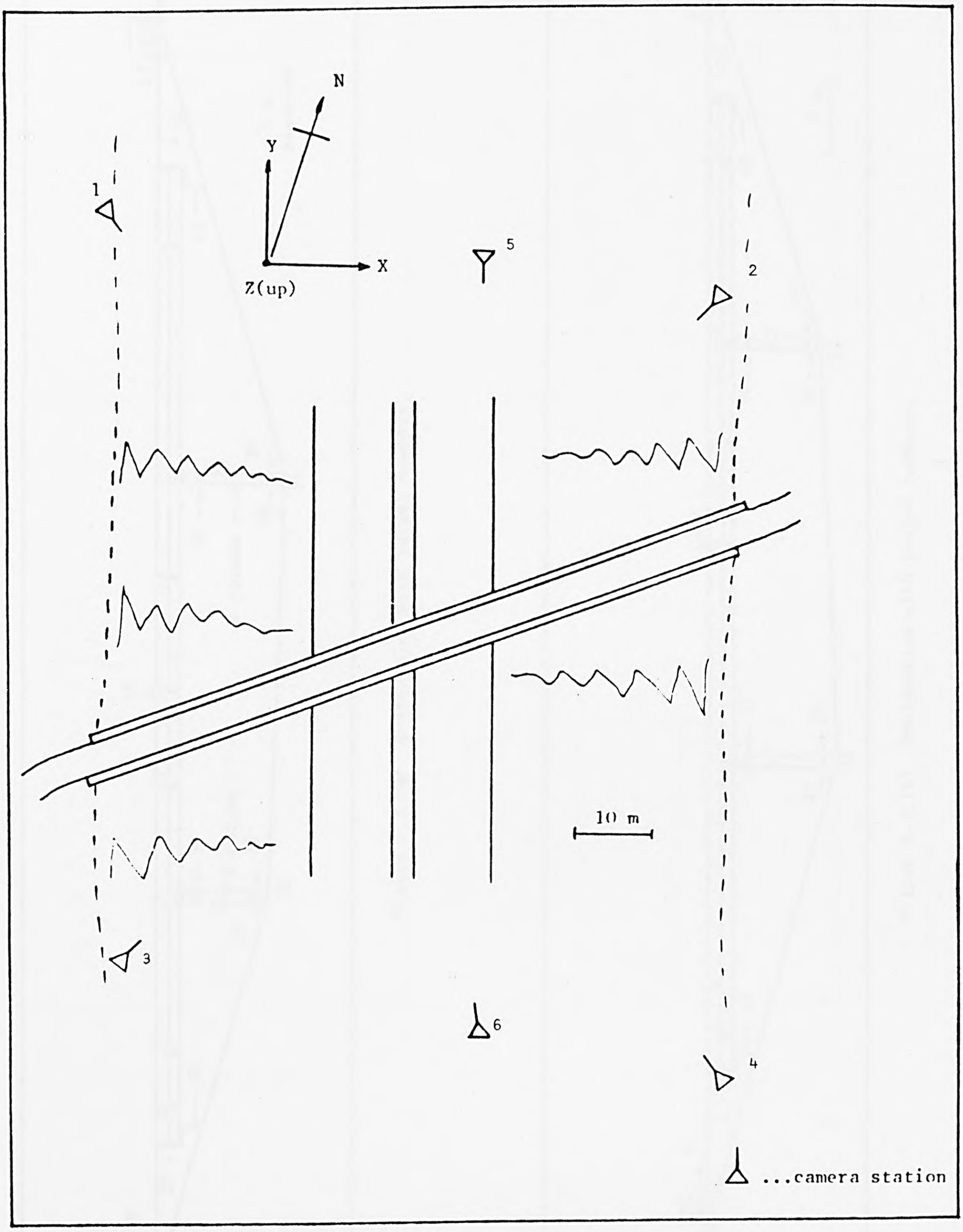


Figure 6.1. Camera station configurations (after Cooper, 1984)

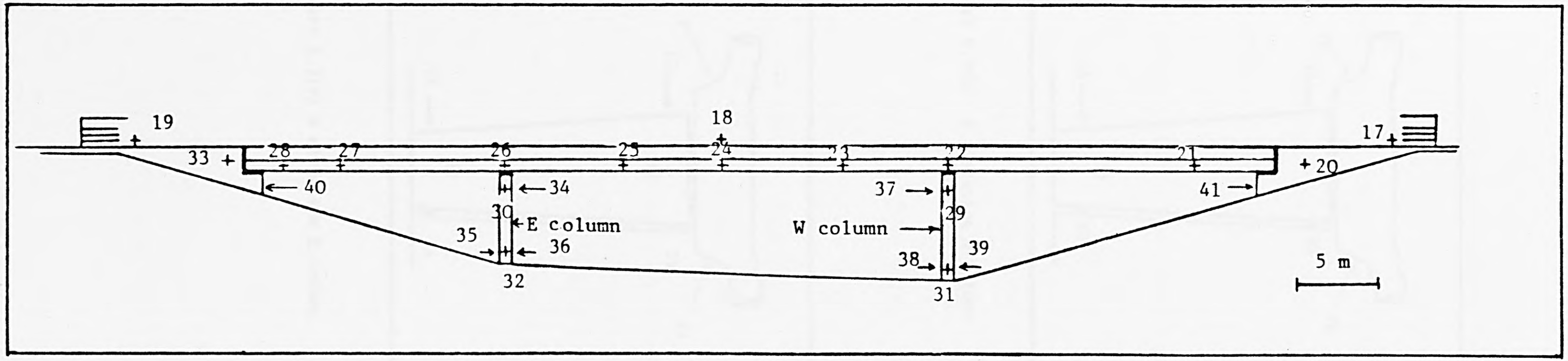


Figure 6.2.(a) S-elevation with target numbers.

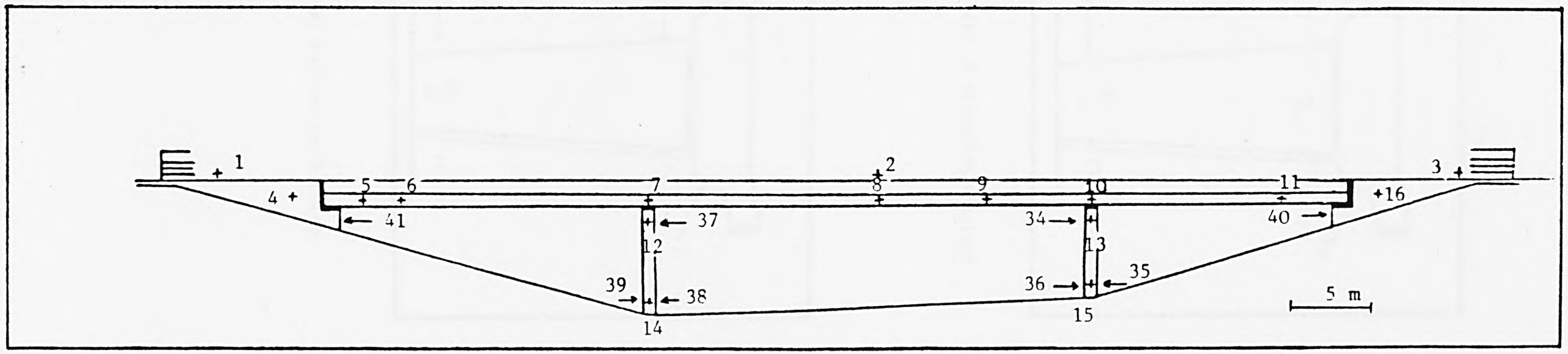


Figure 6.2.(b) N-elevation with target numbers.

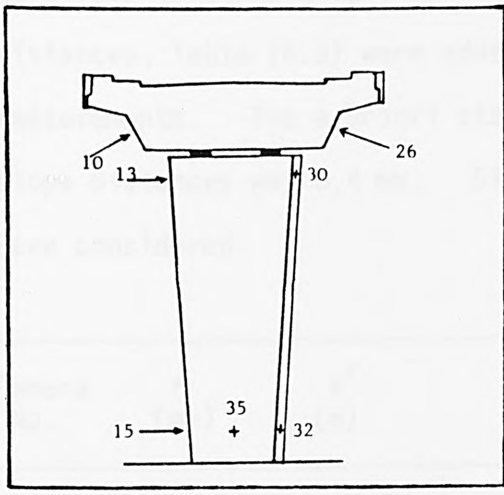


Figure 6.3(a) W elevation W column

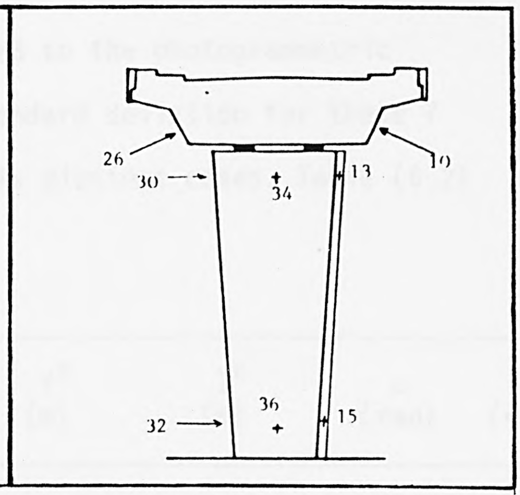


Figure 6.3(b) E elevation W column

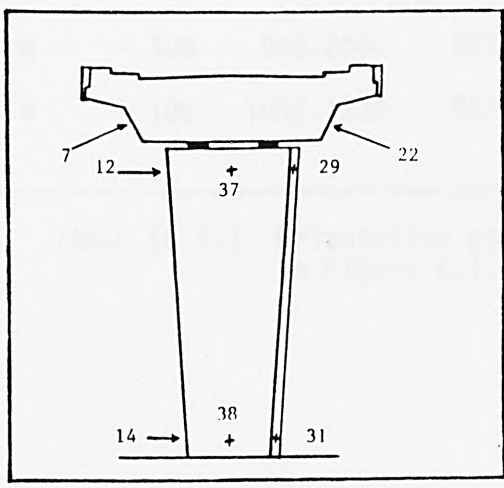


Figure 6.3(c) W elevation E column

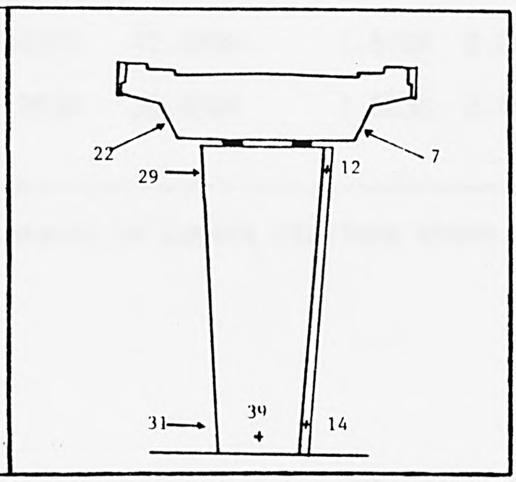


Figure 6.3(d) E elevation E column

were assigned a standard deviation of $\pm$ 3 μm. Seven spatial distances, Table (6.3) were added to the photogrammetric measurements. The a priori standard deviation for these 7 slope distances was 0.4 mm. Six distinct cases, Table (6.2) were considered.

Camera No.	f (mm)	X^c (m)	Y^c (m)	Z^c (m)	ω (rad)	φ (rad)	κ (rad)
1	100	968.8800	1054.2146	18.6406	1.5728	3.6220	0.0040
5	100	1008.0000	1049.0000	11.0000	1.5700	3.1000	0.0000
2	100	1038.9038	1044.0665	18.9812	1.5719	2.6949	0.0063
3	100	964.1530	966.4781	19.7393	1.5752	-0.4506	-0.0077
6	100	998.0000	957.0000	12.0000	1.5700	0.0000	0.0000
4	100	1032.1230	947.9636	20.4986	1.5838	0.4517	-0.0088

TABLE (6.1.) Orientation elements of camera stations shown in Figure 6.1.

Case	Photo arrangement	Total number of Targets	Number of slope distances
1	1-2-3-4	41	-
2	Case 1 + slope dist.	41	7
3	1-2-3-4-5	41	-
4	Case 3 + slope dist.	41	7
5	1-2-3-4-5-6	41	-
6	Case 5 + slope dist.	41	7

TABLE (6.2) Photo arrangement, No.of targets and slope distances.

Observation No.	Target Object Point	
	From	To
1	1	3
2	1	2
3	17	19
4	17	18
5	1	17
6	2	18
7	3	19

TABLE 6.3. Simulated slope distances.

6.3. Results and Analysis

Both the observation equations and the normal equations were formed in the same way as has been previously discussed in Sections (5.3) and (5.4) respectively. The same inversion routine used for the cube cases in Section (5.5) was adopted. Again, results of cases 5 and 6 were selected to represent the different criteria of design. Figure (6.4) depicts the structure of the coefficient matrix of the normal equations for 6 photographs and 41 target object points. The same levels of probability adopted throughout this study, i.e. $\alpha = 0.05$ and $\beta = 0.20$ were applied.

6.3.1. Mean Variance of Object point co-ordinates

As has been previously discussed in Section (5.6.2.1), the mean variance, σ_m^2, given in Equation (5.8) for each configuration is to be compared in order to show the influence of the number of cameras without and with the addition of spatial distances on the precision.

Restating Equation (5.8):

$$\sigma_m^2 = \frac{1}{3n_o} trQ_{\hat{x}\hat{x}}^{(2)} \qquad (6.1)$$

in which $Q_{\hat{x}\hat{x}}^{(2)}$ is the a priori cofactor matrix of the n_o object points. The free network bundle adjustment was performed to compute the variances of the object point co-ordinates.

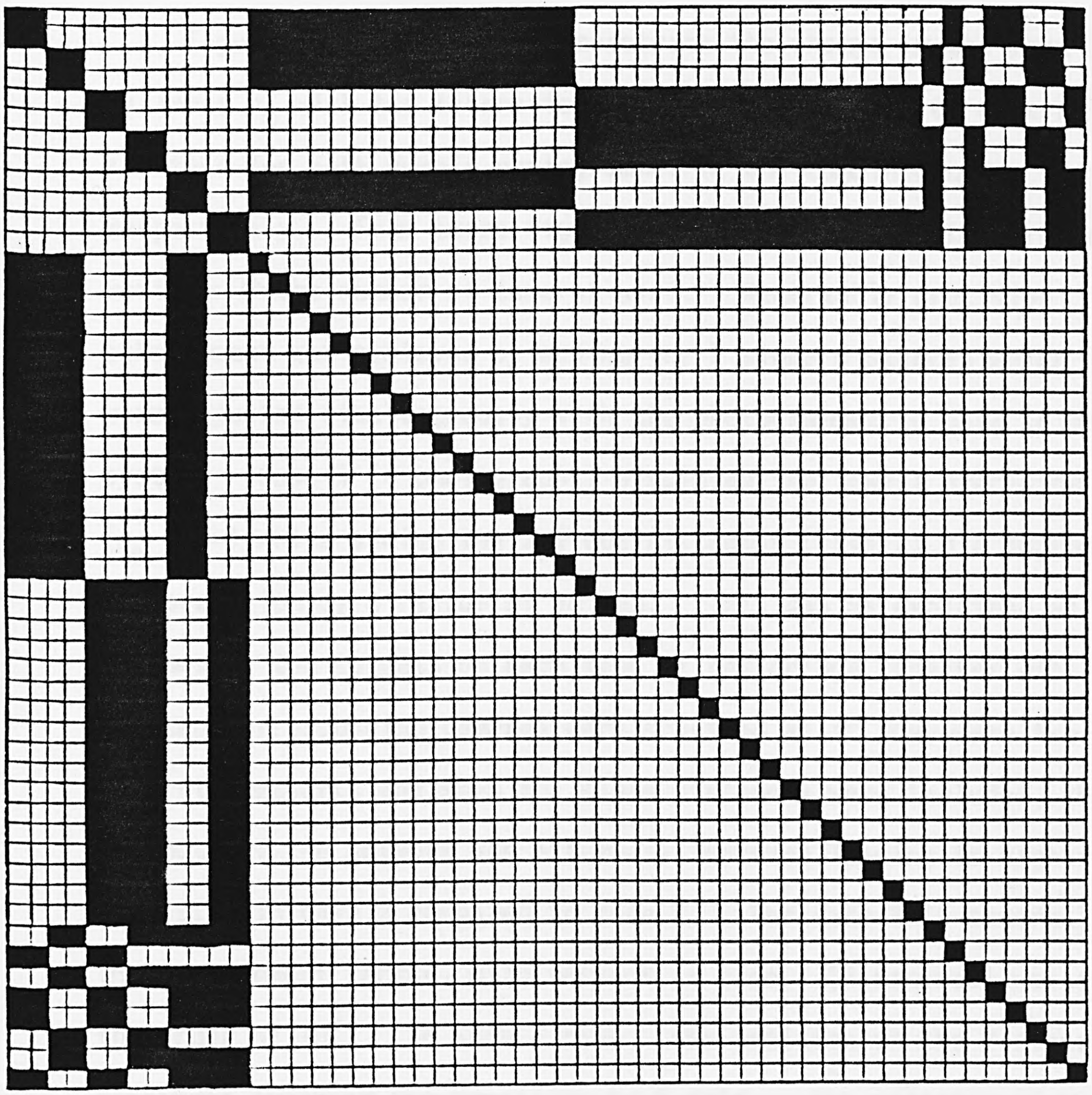


Figure 6.4. Structure of the coefficient matrix of the normal equations for 6 photos and 41 object points.

Figures (6.5) and (6.6) demonstrate the change in the mean variance σ_m^2 with the alteration of the configuration of camera stations for both purely photogrammetric and combined cases respectively. From these figures it is evident that the precision varies with the number of cameras used. Remarkable precision enhancement was achieved by adding the 7 slope distances which were adjusted simultaneously with the photogrammetric data in one system. Case 1 (4 photographs) displayed the lowest precision ($\sigma_m^2 \simeq \infty$) while adding the 7 slope distances (case 2) did improve the precision by a factor of 30 000. This can be explained as follows: the targets on each side of the bridge together with the two corresponding cameras were adjusted as if they were two separate networks and when the slope distances were included, the strength of the network was extremely improved leading to higher precision in case 2 compared to case 1. For case 3 (5 cameras) the improvement factor, with respect to case 1 (4 cameras) was of the order of 10^5. In such a case, the addition of the 7 slope distances improved the precision by 6-fold. The addition of the sixth camera improved the precision by only 3-fold and the same amount of improvement was attained when the 7 slope distances were added. Case 4 gave rise to 16-fold improvement in precision over case 2 whereas case 6 indicated the highest level of precision (σ_m^2 = 2.644 mm^2) and the improvement compared to case 4 was only by 2-fold. Such higher precision can be thought of being due to the symmetric arrangements of the cameras around the bridge, in addition to the inclusion of the 7 slope distances.

POINT	VARIANCES		
NO	X (mm^2)	Y (mm^2)	Z (mm^2)
1	0.24163152D 02	0.29953240D 02	0.19628969D 01
2	0.71986609D 01	0.14006010D 02	0.14757933D 01
3	0.44445133D 02	0.41674834D 02	0.20644292D 01
4	0.95549585D 01	0.12646087D 02	0.14528327D 01
5	0.53752395D 01	0.80427465D 01	0.11061516D 01
6	0.31215124D 01	0.84136279D 01	0.10467402D 01
7	0.36888358D 01	0.93660310D 01	0.10431978D 01
8	0.60308295D 01	0.11268172D 02	0.11039966D 01
9	0.58296754D 01	0.10665871D 02	0.11592637D 01
10	0.45936713D 01	0.99620294D 01	0.12413489D 01
11	0.42484368D 01	0.99485022D 01	0.12674155D 01
12	0.31060624D 01	0.79227854D 01	0.95685298D 00
13	0.40556911D 01	0.89118917D 01	0.12282922D 01
14	0.27855104D 01	0.75066368D 01	0.12290065D 01
15	0.42692011D 01	0.86405314D 01	0.13027218D 01
16	0.14741085D 02	0.21294315D 02	0.18270420D 01
17	0.33769444D 02	0.28717922D 02	0.21388992D 01
18	0.64727868D 01	0.10740730D 02	0.12653904D 01
19	0.26016797D 02	0.32751371D 02	0.16819860D 01
20	0.13454835D 02	0.16699169D 02	0.19638953D 01
21	0.43818406D 01	0.77600944D 01	0.11900035D 01
22	0.48241972D 01	0.78283719D 01	0.11444078D 01
23	0.56271914D 01	0.82311865D 01	0.10237010D 01
24	0.53107548D 01	0.84432326D 01	0.94398531D 00
25	0.43020786D 01	0.81288074D 01	0.89510581D 00
26	0.30671150D 01	0.71915454D 01	0.86812562D 00
27	0.28507014D 01	0.68342767D 01	0.86880858D 00
28	0.52398856D 01	0.80706095D 01	0.93466639D 00
29	0.41408709D 01	0.67452417D 01	0.11524851D 01
30	0.26670214D 01	0.62394393D 01	0.83413387D 00
31	0.41474005D 01	0.74365126D 01	0.12792271D 01
32	0.25164372D 01	0.58692302D 01	0.10714101D 01
33	0.10176257D 02	0.13596261D 02	0.13061435D 01
34	0.20651361D 01	0.48902693D 01	0.13653383D 01
35	0.31667963D 01	0.13539343D 02	0.19498410D 01
36	0.23489165D 01	0.61255505D 01	0.13884503D 01
37	0.22685415D 01	0.41660863D 01	0.13054571D 01
38	0.24742833D 01	0.56251690D 01	0.13142269D 01
39	0.36057441D 01	0.11731626D 02	0.25928749D 01
40	0.66260762D 01	0.88035861D 01	0.14260921D 01
41	0.60850567D 01	0.77548435D 01	0.13999430D 01

TABLE 6.4. Estimates of the variances of the 41 object point co-ordinates for case 5.

POINT	VARIANCES		
NO	X (mm^2)	Y (mm^2)	Z (mm^2)
1	0.22724083D 01	0.32666649D 01	0.11681276D 01
2	0.15058243D 01	0.21296390D 01	0.10907140D 01
3	0.21314310D 01	0.36448794D 01	0.17106786D 01
4	0.20701229D 01	0.29442528D 01	0.10638992D 01
5	0.23247712D 01	0.28145963D 01	0.89368538D 00
6	0.24847800D 01	0.40625645D 01	0.87501431D 00
7	0.23399733D 01	0.44060884D 01	0.88783011D 00
8	0.20760774D 01	0.51261660D 01	0.98554491D 00
9	0.22711561D 01	0.51432827D 01	0.10599255D 01
10	0.26472401D 01	0.51007248D 01	0.11507761D 01
11	0.27671774D 01	0.51128380D 01	0.11779480D 01
12	0.21834189D 01	0.43000195D 01	0.89790086D 00
13	0.26329768D 01	0.52040074D 01	0.11737439D 01
14	0.21685283D 01	0.63741590D 01	0.95621624D 00
15	0.28735149D 01	0.74688871D 01	0.12098195D 01
16	0.41113367D 01	0.68965323D 01	0.15790139D 01
17	0.17386422D 01	0.30714846D 01	0.18435850D 01
18	0.14718877D 01	0.21220209D 01	0.99938882D 00
19	0.18208657D 01	0.37425000D 01	0.10451263D 01
20	0.37357463D 01	0.62155300D 01	0.17009718D 01
21	0.24159331D 01	0.41032205D 01	0.11176232D 01
22	0.23139617D 01	0.40732438D 01	0.10744571D 01
23	0.20826972D 01	0.40256702D 01	0.95703593D 00
24	0.20375681D 01	0.39325149D 01	0.87035402D 00
25	0.21684951D 01	0.37092280D 01	0.80862819D 00
26	0.24664349D 01	0.32253536D 01	0.76147909D 00
27	0.25708645D 01	0.29916155D 01	0.75261272D 00
28	0.23887162D 01	0.22778428D 01	0.77789789D 00
29	0.22817957D 01	0.40951037D 01	0.10918912D 01
30	0.23417873D 01	0.31872514D 01	0.76764076D 00
31	0.25151512D 01	0.64569862D 01	0.11780530D 01
32	0.23305348D 01	0.46488414D 01	0.84374174D 00
33	0.18822087D 01	0.25942848D 01	0.98853228D 00
34	0.10886638D 01	0.44839039D 01	0.10753694D 01
35	0.24418412D 01	0.11107779D 02	0.14451397D 01
36	0.17994596D 01	0.56598707D 01	0.11705361D 01
37	0.11323138D 01	0.36325982D 01	0.10042579D 01
38	0.16470187D 01	0.49773893D 01	0.10992460D 01
39	0.28200357D 01	0.10227289D 02	0.19675617D 01
40	0.16174946D 01	0.53092456D 01	0.11120572D 01
41	0.17199145D 01	0.42508005D 01	0.10918484D 01

TABLE 6.5. Estimates of the variances of the 41 object point co-ordinates for case 6.

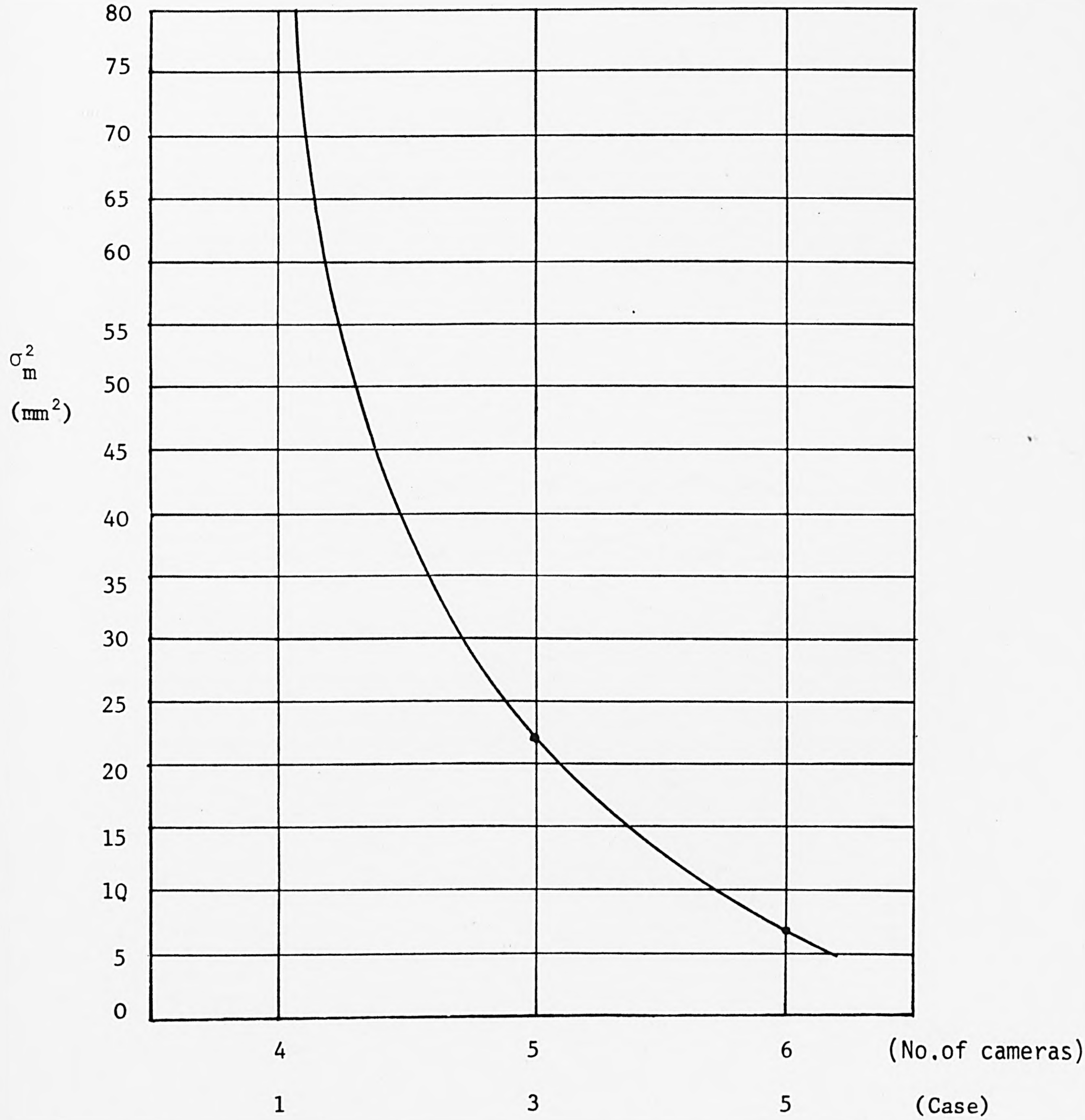


Figure 6.5. Relation between no.of cameras and precision (Photogrammetry)

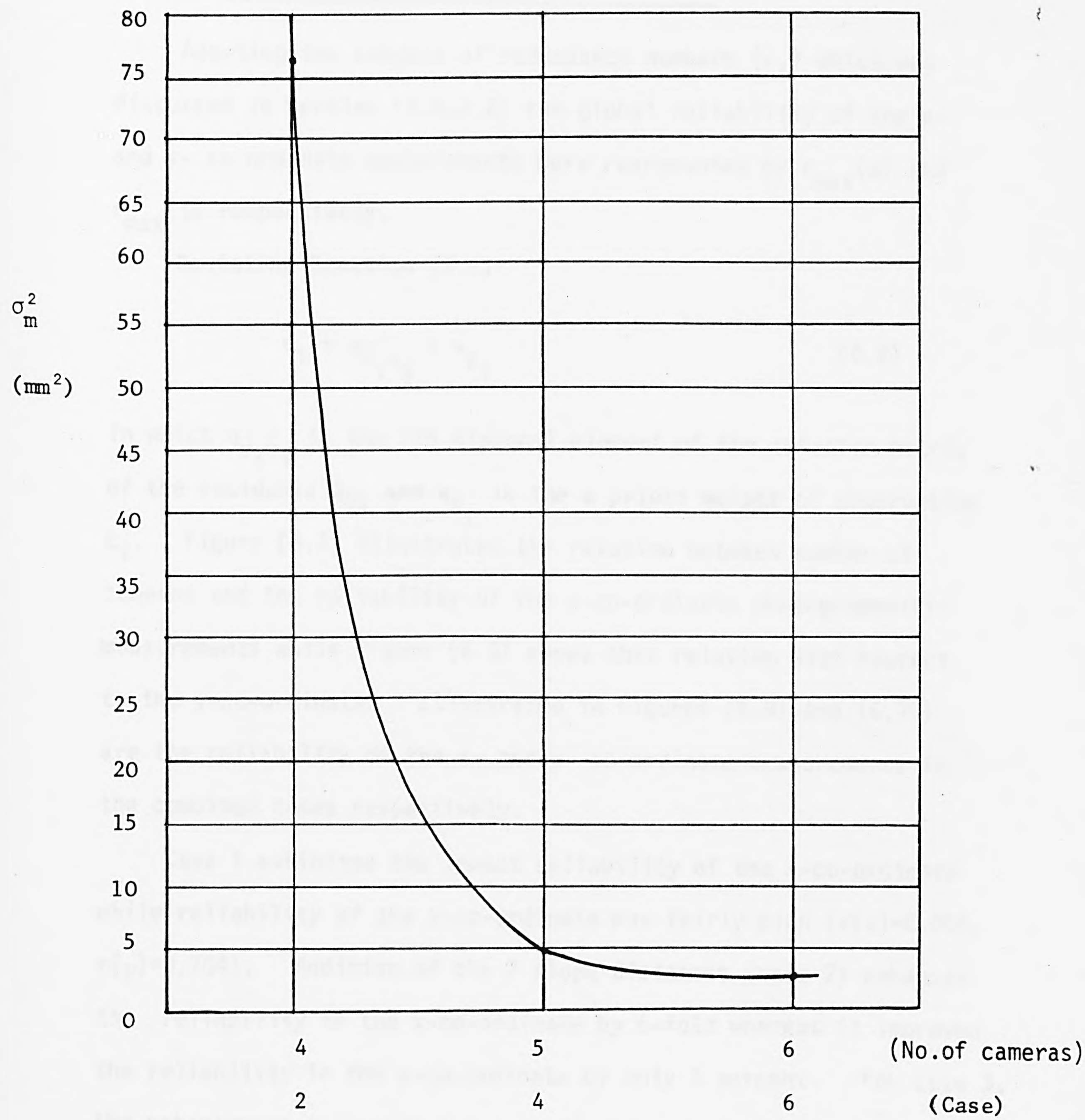


Figure 6.6. Relation between no.of cameras and precision (Photogrammetry + 7 distances)

6.3.2. r_{max} and other reliability indicators

Adopting the concept of redundancy numbers (r_i) which was discussed in Section (5.6.2.2) the global reliability of the x- and y- co-ordinate measurements were represented by $r_{max}(x)$ and $r_{max}(y)$ respectively.

Restating Equation (5.9):

$$r_i = q_{\hat{v}_i\hat{v}_i} \times w_{\ell_i} \qquad (6.2)$$

in which $q_{\hat{v}_i\hat{v}_i}$ is the ith diagonal element of the cofactor matrix of the residuals $Q_{\hat{v}\hat{v}}$ and w_{ℓ_i} is the a priori weight of observation ℓ_i. Figure (6.7) illustrates the relation between number of cameras and the reliability of the x-co-ordinate photogrammetric measurements while Figure (6.8) shows that relation with respect to the y-co-ordinate. Illustrated in Figures (6.9) and (6.10) are the reliability of the x- and y- co-ordinate measurements in the combined cases respectively.

Case 1 exhibited the lowest reliability of the x-co-ordinate while reliability of the y-co-ordinate was fairly high ($r(x)$=0.002, $r(y)$=0.704). Addition of the 7 slope distances (case 2) enhanced the reliability in the x-co-ordinate by 6-fold whereas it improved the reliability in the y-co-ordinate by only 3 percent. For case 3, the enhancement over case 1 x-co-ordinate was of the factor of 224 while adding the 7 slope distances (case 4) did improve the reliability in the same direction by 5.4 percent. As for the y-co-ordinate there was a slight improvement, 3.6 percent for the purely photogrammetric

PHOTO	POINT	$r(x)$	$r(y)$
1	1	0.28015677D 00	0.58967773D 00
	2	0.21004204D 00	0.62362729D 00
	3	0.42024963D-01	0.37855436D 00
	4	0.40218839D 00	0.65542409D 00
	5	0.41265136D 00	0.73007523D 00
	6	0.33402621D 00	0.72571040D 00
	7	0.30953574D 00	0.71831480D 00
	8	0.23010791D 00	0.67118528D 00
	9	0.19550256D 00	0.63758557D 00
	10	0.16242081D 00	0.59796567D 00
	11	0.15350490D 00	0.58622104D 00
	12	0.31319662D 00	0.71577990D 00
	13	0.16221498D 00	0.58989361D 00
	14	0.21582006D 00	0.58904646D 00
	15	0.11043007D 00	0.48724389D 00
	16	0.75160844D-01	0.43202322D 00
	35	0.22824388D-01	0.36534500D 00
	37	0.22553634D 00	0.65045783D 00
	38	0.12662371D 00	0.65064241D 00
	41	0.21817491D 00	0.60429701D 00
2	1	0.21569689D-01	0.21949242D 00
	2	0.21667916D 00	0.58210620D 00
	3	0.26508098D 00	0.61848141D 00
	4	0.51062866D-01	0.30700026D 00
	5	0.75116888D-01	0.38635122D 00
	6	0.13753816D 00	0.50918539D 00
	7	0.15519682D 00	0.53470054D 00
	8	0.23163484D 00	0.61097686D 00
	9	0.27593857D 00	0.63992082D 00
	10	0.31497930D 00	0.65938378D 00
	11	0.32362146D 00	0.66307969D 00
	12	0.16112689D 00	0.53924613D 00
	13	0.31493908D 00	0.66049373D 00
	14	0.11040055D 00	0.39370861D 00
	15	0.21441819D 00	0.60417933D 00
	16	0.33176513D 00	0.64649916D 00
	34	0.32008486D 00	0.67331043D 00
	36	0.26831517D 00	0.63545768D 00
	39	0.16141312D-01	0.21110894D 00
	40	0.24110592D 00	0.65310748D 00

TABLE 6.6. Computed values of redundancy numbers for case 5.

PHOTO	POINT	r(x)	r(y)
3	17	0.12694602D 00	0.55757614D 00
	18	0.95630826D-01	0.48093507D 00
	19	0.86027758D-02	0.13948892D 00
	20	0.16039902D 00	0.57550808D 00
	21	0.14950011D 00	0.53625096D 00
	22	0.14302578D 00	0.57888600D 00
	23	0.12135060D 00	0.54948182D 00
	24	0.10195535D 00	0.51247259D 00
	25	0.84383516D-01	0.47114627D 00
	26	0.65232409D-01	0.41528237D 00
	27	0.58534561D-01	0.39243320D 00
	28	0.35692492D-01	0.29375453D 00
	29	0.14412663D 00	0.58081474D 00
	30	0.67411350D-01	0.41588807D 00
	31	0.97791387D-01	0.49193267D 00
	32	0.53138453D-01	0.31755560D 00
	33	0.22705149D-01	0.20641230D 00
	35	0.93485238D-02	0.14999684D 00
	37	0.16052340D 00	0.60257052D 00
	38	0.11272736D 00	0.55088510D 00
	41	0.10682780D 00	0.58528942D 00
4	17	0.68046212D-01	0.38106392D 00
	18	0.31377858D 00	0.67293800D 00
	19	0.29191202D 00	0.64818152D 00
	20	0.11581334D 00	0.42829906D 00
	21	0.24300122D 00	0.61717850D 00
	22	0.25637543D 00	0.63397108D 00
	23	0.29692262D 00	0.68080481D 00
	24	0.33572789D 00	0.71675432D 00
	25	0.37435209D 00	0.74351460D 00
	26	0.42155338D 00	0.76583993D 00
	27	0.43805807D 00	0.77111091D 00
	28	0.47085617D 00	0.77286893D 00
	29	0.25676367D 00	0.62496688D 00
	30	0.42248753D 00	0.75817346D 00
	31	0.17436343D 00	0.48246812D 00
	32	0.33004836D 00	0.64776511D 00
	33	0.44346677D 00	0.69968042D 00
	34	0.29928969D 00	0.65533643D 00
	36	0.23762969D 00	0.63941747D 00
	39	0.19815631D-01	0.33996096D 00
	40	0.27252370D 00	0.61832836D 00

TABLE 6.6.(Continued)

PHOTO	POINT	$r(x)$	$r(y)$
5	1	0.17477961D 00	0.42376967D 00
	2	0.39268318D 00	0.49231847D 00
	3	0.22878283D 00	0.45260671D 00
	4	0.30545643D 00	0.49142319D 00
	5	0.36075477D 00	0.54996301D 00
	6	0.40498936D 00	0.53792861D 00
	7	0.41095731D 00	0.53600032D 00
	8	0.43314439D 00	0.53831110D 00
	9	0.44303333D 00	0.54426381D 00
	10	0.44191984D 00	0.54687531D 00
	11	0.43882466D 00	0.54659460D 00
	12	0.42912400D 00	0.55541673D 00
	13	0.44887356D 00	0.55168610D 00
	14	0.29502662D 00	0.53534492D 00
	15	0.30449006D 00	0.51893618D 00
	16	0.33352909D 00	0.47730827D 00
	34	0.36280867D 00	0.58819756D 00
	36	0.32664970D 00	0.59168454D 00
	37	0.25468462D 00	0.50803439D 00
	38	0.16975366D 00	0.48952971D 00
	40	0.17055701D 00	0.51233116D 00
	41	0.12776831D 00	0.34655372D 00
6	17	0.28960276D 00	0.48777587D 00
	18	0.42933118D 00	0.56315816D 00
	19	0.16377817D 00	0.48161702D 00
	20	0.41359022D 00	0.50718331D 00
	21	0.51133303D 00	0.60151348D 00
	22	0.50715249D 00	0.60449006D 00
	23	0.48758097D 00	0.61045342D 00
	24	0.46463504D 00	0.61251942D 00
	25	0.44263071D 00	0.61474710D 00
	26	0.41582593D 00	0.62234796D 00
	27	0.40497003D 00	0.62709759D 00
	28	0.35457701D 00	0.64257117D 00
	29	0.51591310D 00	0.60969132D 00
	30	0.42918544D 00	0.63458361D 00
	31	0.34773652D 00	0.57052762D 00
	32	0.34013873D 00	0.60112934D 00
	33	0.29727560D 00	0.56471612D 00
	34	0.31950857D 00	0.52072160D 00
	36	0.27346560D 00	0.51738536D 00
	37	0.38843398D 00	0.63144432D 00
	38	0.37250340D 00	0.62206044D 00
	40	0.15649817D 00	0.39268303D 00
	41	0.13943610D 00	0.54221696D 00

TABLE 6.6.(Continued)

PHOTO	POINT	r(x)	r(y)
1	1	0.33333120D 00	0.62479424D 00
	2	0.27276418D 00	0.64172388D 00
	3	0.54305994D-01	0.38089391D 00
	4	0.41173863D 00	0.66520301D 00
	5	0.41933811D 00	0.73113232D 00
	6	0.33641536D 00	0.73007066D 00
	7	0.31192694D 00	0.72353729D 00
	8	0.23193600D 00	0.67768799D 00
	9	0.19652081D 00	0.64271983D 00
	10	0.16315291D 00	0.60106111D 00
	11	0.15430738D 00	0.58881838D 00
	12	0.31440646D 00	0.72274987D 00
	13	0.16337301D 00	0.59647646D 00
	14	0.24936701D 00	0.65709177D 00
	15	0.11933776D 00	0.56281212D 00
	16	0.79688962D-01	0.43753183D 00
	35	0.26665249D-01	0.42682484D 00
	37	0.38937534D 00	0.67591648D 00
	38	0.15679564D 00	0.65342046D 00
	41	0.46761749D 00	0.60775776D 00
2	1	0.30225223D-01	0.22482112D 00
	2	0.26019042D 00	0.60237819D 00
	3	0.36876822D 00	0.65406587D 00
	4	0.52087010D-01	0.30975847D 00
	5	0.76089227D-01	0.38722546D 00
	6	0.13925404D 00	0.51986484D 00
	7	0.15700298D 00	0.54573766D 00
	8	0.23323071D 00	0.61904966D 00
	9	0.27717950D 00	0.64507852D 00
	10	0.31668591D 00	0.66287923D 00
	11	0.32572184D 00	0.66649162D 00
	12	0.16221422D 00	0.54404353D 00
	13	0.31717204D 00	0.66436774D 00
	14	0.13358424D 00	0.48045136D 00
	15	0.23214823D 00	0.65101677D 00
	16	0.34796338D 00	0.65418629D 00
	34	0.40781686D 00	0.68396557D 00
	36	0.28615232D 00	0.66713750D 00
	39	0.18920166D-01	0.24745300D 00
	40	0.48831763D 00	0.67120773D 00

TABLE 6.7. Computed values of redundancy numbers for case 6.

PHOTO	POINT	r(x)	r(y)
3	17	0.24005258D 00	0.58903822D 00
	18	0.13714389D 00	0.49861256D 00
	19	0.12531084D-01	0.14476249D 00
	20	0.16854221D 00	0.58578960D 00
	21	0.15050298D 00	0.58921793D 00
	22	0.14391476D 00	0.58214256D 00
	23	0.12250466D 00	0.55473961D 00
	24	0.10257140D 00	0.52056959D 00
	25	0.85100510D-01	0.48123808D 00
	26	0.66075249D-01	0.42488866D 00
	27	0.59366873D-01	0.40079556D 00
	28	0.36064970D-01	0.29445927D 00
	29	0.14495931D 00	0.53423861D 00
	30	0.67966358D-01	0.42094961D 00
	31	0.10666073D 00	0.55458790D 00
	32	0.61734139D-01	0.37149944D 00
	33	0.23192442D-01	0.21126934D 00
	35	0.10921683D-01	0.17523813D 00
	37	0.24578559D 00	0.62082241D 00
	38	0.13386169D 00	0.60494676D 00
	41	0.34961841D 00	0.60751478D 00
4	17	0.80693723D-01	0.33304664D 00
	18	0.35158071D 00	0.68164034D 00
	19	0.36399072D 00	0.66636263D 00
	20	0.12291260D 00	0.43183744D 00
	21	0.24478144D 00	0.61899510D 00
	22	0.25836938D 00	0.63590896D 00
	23	0.29954800D 00	0.68296868D 00
	24	0.33853526D 00	0.71907709D 00
	25	0.37683196D 00	0.74562652D 00
	26	0.42363629D 00	0.76739440D 00
	27	0.44025081D 00	0.77246768D 00
	28	0.47623059D 00	0.77386942D 00
	29	0.25836191D 00	0.63037363D 00
	30	0.42451131D 00	0.76324732D 00
	31	0.18681746D 00	0.56538360D 00
	32	0.36818690D 00	0.69025227D 00
	33	0.45187561D 00	0.70837843D 00
	34	0.39800486D 00	0.67129869D 00
	36	0.25296382D 00	0.64237739D 00
	39	0.23227046D-01	0.39848789D 00
	40	0.49161849D 00	0.62513928D 00

TABLE 6.7.(Continued)

PHOTO	POINT	r(x)	r(y)
5	1	0.18297387D 00	0.45669809D 00
	2	0.44004954D 00	0.53803099D 00
	3	0.25253703D 00	0.47808023D 00
	4	0.31231551D 00	0.49866997D 00
	5	0.36632547D 00	0.55052638D 00
	6	0.40826497D 00	0.55884716D 00
	7	0.41419048D 00	0.55847217D 00
	8	0.43590259D 00	0.55546876D 00
	9	0.44497687D 00	0.55388747D 00
	10	0.44409034D 00	0.55014971D 00
	11	0.44141500D 00	0.54866330D 00
	12	0.43109153D 00	0.56494450D 00
	13	0.45209997D 00	0.55283267D 00
	14	0.36010933D 00	0.56019491D 00
	15	0.33050120D 00	0.54211592D 00
	16	0.35227867D 00	0.49444052D 00
	34	0.40417021D 00	0.59668931D 00
	36	0.34785383D 00	0.60444961D 00
	37	0.36114688D 00	0.54686260D 00
	38	0.20719214D 00	0.54380997D 00
	40	0.30531772D 00	0.52006616D 00
	41	0.25826968D 00	0.36138849D 00
6	17	0.33486993D 00	0.50794486D 00
	18	0.48648132D 00	0.60550130D 00
	19	0.17905588D 00	0.52595521D 00
	20	0.43781203D 00	0.52194056D 00
	21	0.51511992D 00	0.60367073D 00
	22	0.51092374D 00	0.60789590D 00
	23	0.49117803D 00	0.61929214D 00
	24	0.46783778D 00	0.62793977D 00
	25	0.44555252D 00	0.63525861D 00
	26	0.41906244D 00	0.64343821D 00
	27	0.40849212D 00	0.64602191D 00
	28	0.35858262D 00	0.64351890D 00
	29	0.51894403D 00	0.61013399D 00
	30	0.43172243D 00	0.64200130D 00
	31	0.37997738D 00	0.59632301D 00
	32	0.39859231D 00	0.63242559D 00
	33	0.30379880D 00	0.57456528D 00
	34	0.37783071D 00	0.54467739D 00
	36	0.29074759D 00	0.54790237D 00
	37	0.45550140D 00	0.64137252D 00
	38	0.42213713D 00	0.63961482D 00
	40	0.26429494D 00	0.40369917D 00
	41	0.39134632D 00	0.54957816D 00

TABLE 6.7. (Continued)

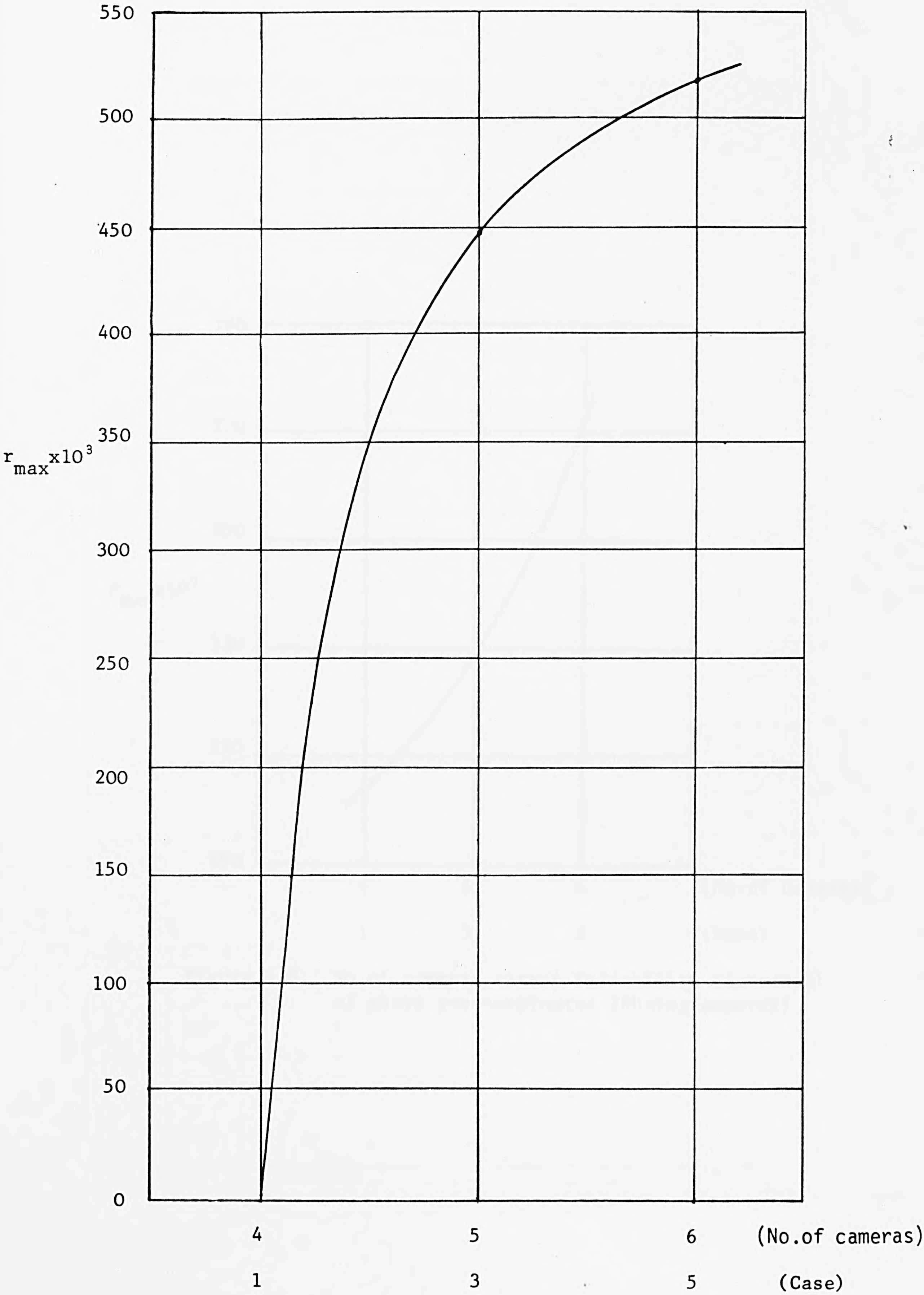


Figure 6.7. No.of cameras versus Reliability (internal) of photo x-co-ordinates (Photogrammetry)

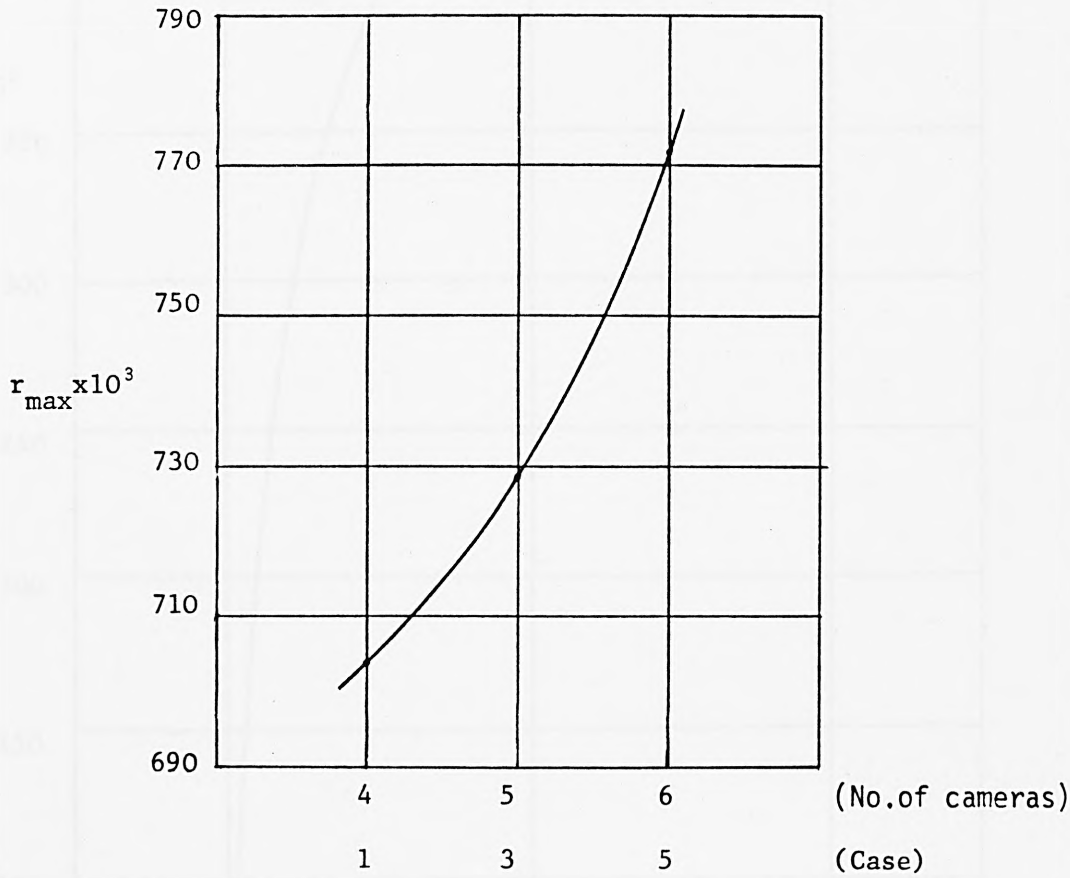


Figure 6.8. No.of cameras versus Reliability (internal) of photo y-co-ordinates (Photogrammetry)

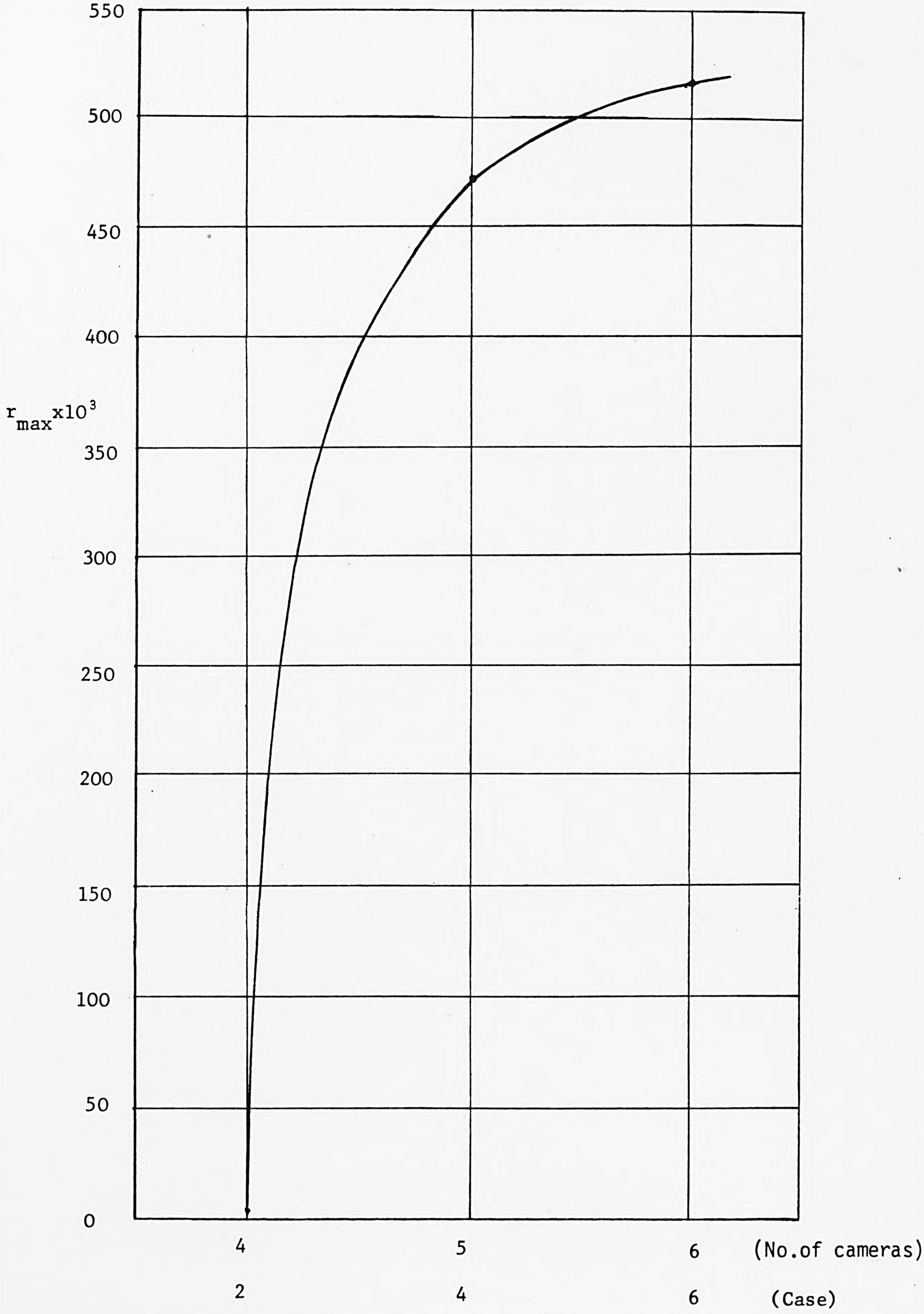


Figure 6.9. No.of cameras versus Reliability (internal) of photo x-co-ordinates (Photos + 7 distances)

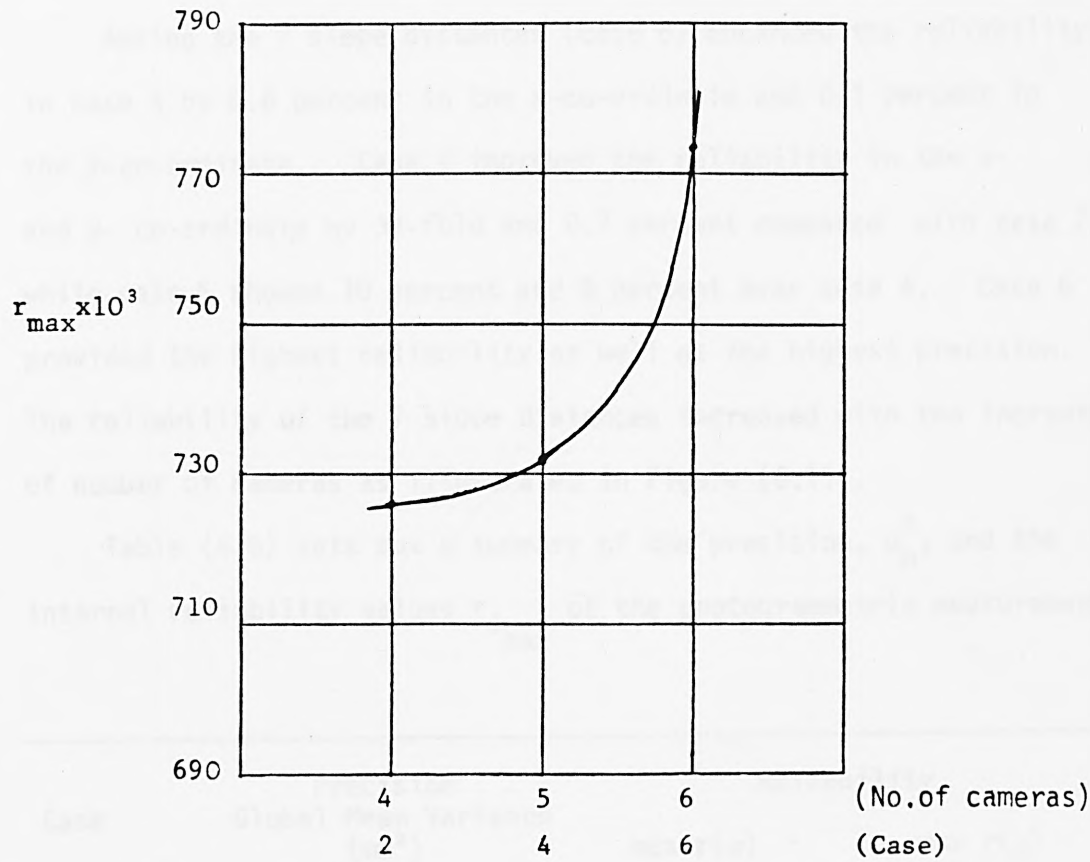


Figure 6.10. No.of cameras versus Reliability (internal) of photo y-co-ordinates (Photos + 7 distances)

measurements while only 0.3 percent for the combined case (case 4). 6 percent improvement was accomplished in both the x- and y-co-ordinate owing to the addition of the sixth camera (case 5) compared with case 3.

Adding the 7 slope distances (case 6) enhanced the reliability in case 5 by 0.6 percent in the x-co-ordinate and 0.1 percent in the y-co-ordinate. Case 4 improved the reliability in the x- and y- co-ordinate by 39-fold and 0.7 percent compared with case 2 while case 6 showed 10 percent and 6 percent over case 4. Case 6 provided the highest reliability as well as the highest precision. The reliability of the 7 slope distances increased with the increase of number of cameras as illustrated in Figure (6.11).

Table (6.8) sets out a summary of the precision, σ_m^2, and the internal reliability values $r_{i_{max}}$ of the photogrammetric measurements.

Case	Precision Global Mean Variance (mm^2)	Reliability max $r(x)$	max $r(y)$
1-4 cameras	2312988.800000	0.002028	0.704284
2-4 cameras+ 7 distances	76.306048	0.011553	0.726046
3-5 cameras	22.382190	0.447481	0.728521
4-5 cameras+ 7 distances	4.887012	0.472039	0.730573
5-6 cameras	6.859595	0.515913	0.772869
6-6 cameras+ 7 distances	2.644175	0.518944	0.773869

TABLE (6.8) Summary of precision and reliability of photogrammetric measurements.

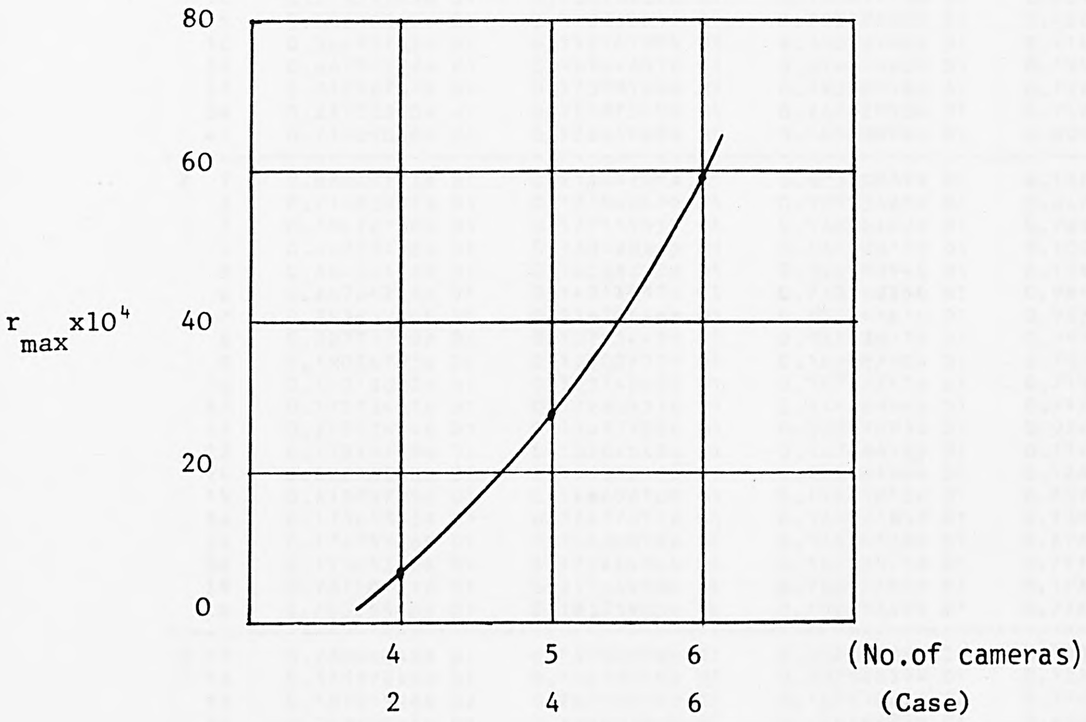


Figure 6.11. No.of cameras versus Reliability (internal) of 7 slope distances.

PHO NO	PT. NO	TAU x	TAU y	GAM x	GAM y
1	1	0.18892935D 01	0.13022448D 01	0.16029442D 01	0.83417117D 00
	2	0.21819605D 01	0.12663024D 01	0.19393173D 01	0.77686670D 00
	3	0.48780509D 01	0.16253088D 01	0.47744508D 01	0.12812605D 01
	4	0.15768313D 01	0.12352043D 01	0.12191788D 01	0.72507220D 00
	5	0.15567123D 01	0.11703512D 01	0.11930437D 01	0.60804757D 00
	6	0.17302535D 01	0.11738655D 01	0.14120117D 01	0.61478462D 00
	7	0.17973994D 01	0.11798929D 01	0.14935343D 01	0.62621665D 00
	8	0.20846552D 01	0.12206152D 01	0.18291493D 01	0.69992969D 00
	9	0.22616416D 01	0.12523645D 01	0.20285519D 01	0.75393430D 00
	10	0.24812994D 01	0.12931886D 01	0.22708691D 01	0.81996147D 00
	11	0.25523421D 01	0.13060786D 01	0.23482866D 01	0.84014355D 00
	12	0.17868639D 01	0.11819803D 01	0.14808384D 01	0.63014086D 00
	13	0.24328731D 01	0.13020065D 01	0.22725886D 01	0.83379910D 00
	14	0.21525542D 01	0.13029424D 01	0.19061715D 01	0.83525982D 00
	15	0.30092365D 01	0.14326061D 01	0.28382220D 01	0.10258461D 01
	16	0.36475745D 01	0.15214107D 01	0.35078198D 01	0.11465995D 01
	35	0.66191227D 01	0.16544301D 01	0.65431480D 01	0.13180056D 01
	37	0.21056769D 01	0.12399108D 01	0.18530718D 01	0.73306117D 00
	38	0.28102340D 01	0.12397349D 01	0.26262930D 01	0.73276363D 00
	41	0.21409060D 01	0.12863963D 01	0.18930078D 01	0.80920665D 00
2	1	0.68089173D 01	0.21344709D 01	0.67350839D 01	0.18857269D 01
	2	0.21482327D 01	0.13106867D 01	0.19013465D 01	0.84728955D 00
	3	0.19422750D 01	0.12715595D 01	0.16650622D 01	0.78540656D 00
	4	0.44253478D 01	0.18048064D 01	0.43108819D 01	0.15024401D 01
	5	0.36436416D 01	0.16088252D 01	0.35089294D 01	0.12602851D 01
	6	0.26964254D 01	0.14013997D 01	0.25041385D 01	0.98179491D 00
	7	0.25383914D 01	0.13675546D 01	0.23331161D 01	0.93284816D 00
	8	0.20777728D 01	0.12793448D 01	0.18213017D 01	0.79794938D 00
	9	0.19036793D 01	0.12500773D 01	0.16198750D 01	0.75012887D 00
	10	0.17313002D 01	0.12314899D 01	0.14747243D 01	0.71872630D 00
	11	0.17578481D 01	0.12280531D 01	0.14456936D 01	0.71282141D 00
	12	0.24912424D 01	0.13617785D 01	0.22817293D 01	0.92435964D 00
	13	0.17819139D 01	0.12304548D 01	0.14748618D 01	0.71695112D 00
	14	0.30096338D 01	0.15937219D 01	0.28386486D 01	0.12409470D 01
	15	0.21595795D 01	0.12865216D 01	0.19141012D 01	0.80940578D 00
	16	0.17361395D 01	0.12437011D 01	0.14192183D 01	0.73945414D 00
	34	0.17675326D 01	0.12186874D 01	0.14574538D 01	0.69656229D 00
	36	0.19305337D 01	0.12544596D 01	0.16513511D 01	0.75740935D 00
	39	0.78710121D 01	0.21764399D 01	0.78072295D 01	0.19331039D 01
	40	0.20365546D 01	0.12373930D 01	0.17741349D 01	0.72879459D 00
3	17	0.28066642D 01	0.13392076D 01	0.26224728D 01	0.89077327D 00
	18	0.32337098D 01	0.14419718D 01	0.30752039D 01	0.10388854D 01
	19	0.10781538D 02	0.26775041D 01	0.10735062D 02	0.24837528D 01
	20	0.24963885D 01	0.13181787D 01	0.22678925D 01	0.85883355D 00
	21	0.25863020D 01	0.13060452D 01	0.23851537D 01	0.84009175D 00
	22	0.26441911D 01	0.13143271D 01	0.24478045D 01	0.85291022D 00
	23	0.28647463D 01	0.13490354D 01	0.26845430D 01	0.90548132D 00
	24	0.31318070D 01	0.13968979D 01	0.29678637D 01	0.97535833D 00
	25	0.34424782D 01	0.14568744D 01	0.32940334D 01	0.10594730D 01
	26	0.39153293D 01	0.15517732D 01	0.37854727D 01	0.11865918D 01
	27	0.41332704D 01	0.15963096D 01	0.40104768D 01	0.12442686D 01
	28	0.52931178D 01	0.18450482D 01	0.51977973D 01	0.15505492D 01
	29	0.26340734D 01	0.13121430D 01	0.24368715D 01	0.84954069D 00
	30	0.38515313D 01	0.15506428D 01	0.37194485D 01	0.11851131D 01
	31	0.31977382D 01	0.14257624D 01	0.30374083D 01	0.10162670D 01
	32	0.43380599D 01	0.17745576D 01	0.42212278D 01	0.14659655D 01
	33	0.66364805D 01	0.22010617D 01	0.65607068D 01	0.19607837D 01
	35	0.10342570D 02	0.25820161D 01	0.10294113D 02	0.23805056D 01
	37	0.24959209D 01	0.12882379D 01	0.22868365D 01	0.81213102D 00
	38	0.29784159D 01	0.13473161D 01	0.28055233D 01	0.90291781D 00
	41	0.30595522D 01	0.13071176D 01	0.28915151D 01	0.84175793D 00

TABLE 6.9. Values of Tau and Gam for Case 5.
(at α = 0.05 and γ = 0.8)

PHO NO	PT. NO	TAU x	TAU y	GAM x	GAM y
4	17	0.38335226D 01	0.16199480D 01	0.37007966D 01	0.12744535D 01
	18	0.17852061D 01	0.12190246D 01	0.14788376D 01	0.69715206D 00
	19	0.18508619D 01	0.12420860D 01	0.15574626D 01	0.73673449D 00
	20	0.29384662D 01	0.15280109D 01	0.27630750D 01	0.11553429D 01
	21	0.20285970D 01	0.12729009D 01	0.17649945D 01	0.78757653D 00
	22	0.19749759D 01	0.12559295D 01	0.17030942D 01	0.75984145D 00
	23	0.18351787D 01	0.12119611D 01	0.15387920D 01	0.66472607D 00
	24	0.17258629D 01	0.11811766D 01	0.14066281D 01	0.62863200D 00
	25	0.16344057D 01	0.11597256D 01	0.12927807D 01	0.58733594D 00
	26	0.15401879D 01	0.11426968D 01	0.11714004D 01	0.55295197D 00
	27	0.15108946D 01	0.11387846D 01	0.11326087D 01	0.54482135D 00
	28	0.14573232D 01	0.11374887D 01	0.10600900D 01	0.54210741D 00
	29	0.19734822D 01	0.12649446D 01	0.17013618D 01	0.77465140D 00
	30	0.15384842D 01	0.11484596D 01	0.11691593D 01	0.56476490D 00
	31	0.23948168D 01	0.14396791D 01	0.21760394D 01	0.10357006D 01
	32	0.17406490D 01	0.12424852D 01	0.14247312D 01	0.73740726D 00
	33	0.15016526D 01	0.11955015D 01	0.11202502D 01	0.65515183D 00
	34	0.18279071D 01	0.12352869D 01	0.15301125D 01	0.72521292D 00
	36	0.20513967D 01	0.12505693D 01	0.17911528D 01	0.75094840D 00
	39	0.71038870D 01	0.17150843D 01	0.70331508D 01	0.13933823D 01
	40	0.19155693D 01	0.12717168D 01	0.16338316D 01	0.78566131D 00
5	1	0.23919639D 01	0.15361551D 01	0.21728993D 01	0.11660928D 01
	2	0.15958014D 01	0.14244805D 01	0.12436166D 01	0.10144677D 01
	3	0.20906835D 01	0.14864130D 01	0.18360167D 01	0.10997380D 01
	4	0.18093616D 01	0.14265013D 01	0.15079089D 01	0.10173033D 01
	5	0.16649223D 01	0.13484451D 01	0.13311522D 01	0.90460163D 00
	6	0.15713691D 01	0.13634452D 01	0.12121059D 01	0.92681321D 00
	7	0.15599176D 01	0.13658955D 01	0.11972229D 01	0.93041417D 00
	8	0.15194403D 01	0.13629607D 01	0.11439838D 01	0.92610035D 00
	9	0.15023869D 01	0.13554867D 01	0.11212343D 01	0.91506518D 00
	10	0.15042785D 01	0.13522464D 01	0.11237677D 01	0.91025842D 00
	11	0.15095743D 01	0.13525936D 01	0.11308468D 01	0.91077411D 00
	12	0.15265414D 01	0.13418084D 01	0.11533988D 01	0.89467866D 00
	13	0.14925813D 01	0.13463376D 01	0.11080609D 01	0.90145714D 00
	14	0.18410662D 01	0.13667313D 01	0.15458087D 01	0.93164080D 00
	15	0.18122305D 01	0.13881043D 01	0.15113502D 01	0.96272197D 00
	16	0.17315424D 01	0.14474398D 01	0.14135909D 01	0.10464617D 01
	34	0.16602029D 01	0.13038823D 01	0.13252448D 01	0.83672521D 00
	36	0.17496310D 01	0.13000345D 01	0.14357519D 01	0.83071642D 00
	37	0.19815203D 01	0.14029863D 01	0.17106796D 01	0.98405828D 00
	38	0.24271154D 01	0.14292575D 01	0.22115355D 01	0.10211645D 01
	40	0.24213926D 01	0.13970907D 01	0.22052533D 01	0.97563444D 00
	41	0.27976126D 01	0.16986923D 01	0.26127832D 01	0.13731553D 01
6	17	0.18582265D 01	0.14318247D 01	0.15662075D 01	0.10247546D 01
	18	0.15261731D 01	0.13325540D 01	0.11529112D 01	0.88073839D 00
	19	0.24709757D 01	0.14409506D 01	0.22596062D 01	0.10374674D 01
	20	0.15549444D 01	0.14041630D 01	0.11907360D 01	0.98573511D 00
	21	0.13984536D 01	0.12893693D 01	0.97758508D 00	0.81392453D 00
	22	0.14042057D 01	0.12861909D 01	0.98579589D 00	0.80888005D 00
	23	0.14321108D 01	0.12798932D 01	0.10251543D 01	0.79882828D 00
	24	0.14670470D 01	0.12777329D 01	0.10734183D 01	0.79536237D 00
	25	0.15030701D 01	0.12754157D 01	0.11221496D 01	0.79163450D 00
	26	0.15507537D 01	0.12676033D 01	0.11852647D 01	0.77898534D 00
	27	0.15714065D 01	0.12627938D 01	0.12121545D 01	0.77113432D 00
	28	0.16793635D 01	0.12474966D 01	0.13491708D 01	0.74582026D 00
	29	0.13722324D 01	0.12806929D 01	0.96866451D 00	0.80010889D 00
	30	0.15264322D 01	0.12553233D 01	0.11532542D 01	0.75883890D 00
	31	0.16958008D 01	0.13239198D 01	0.13695768D 01	0.86761945D 00
	32	0.17146361D 01	0.12897812D 01	0.13928305D 01	0.81457688D 00
	33	0.18340888D 01	0.13307146D 01	0.15374921D 01	0.87795286D 00
	34	0.17691259D 01	0.13857893D 01	0.14593857D 01	0.95938104D 00
	36	0.19122675D 01	0.13902501D 01	0.16299592D 01	0.96581330D 00
	37	0.16045062D 01	0.12584399D 01	0.12547669D 01	0.76398356D 00
	38	0.16384564D 01	0.12678962D 01	0.12978980D 01	0.77946188D 00
	40	0.25278155D 01	0.15958017D 01	0.23216053D 01	0.12436170D 01
	41	0.26780112D 01	0.13580428D 01	0.24842995D 01	0.91884722D 00

TABLE 6.9 (Continued)

PHO NO	PT. NO	TAU x	TAU y	GAM x	GAM y
1	1	0.17320564D 01	0.12651193D 01	0.14142204D 01	0.77493671D 00
	2	0.19147247D 01	0.12483199D 01	0.16328413D 01	0.74719651D 00
	3	0.42911739D 01	0.16203095D 01	0.41730293D 01	0.12749129D 01
	4	0.15584368D 01	0.12260915D 01	0.11952929D 01	0.70943673D 00
	5	0.15442503D 01	0.11695048D 01	0.11767372D 01	0.60641691D 00
	6	0.17240986D 01	0.11703548D 01	0.14044629D 01	0.60805464D 00
	7	0.17904968D 01	0.11756270D 01	0.14852201D 01	0.61814139D 00
	8	0.20764234D 01	0.12147450D 01	0.18197621D 01	0.68964144D 00
	9	0.22557747D 01	0.12473523D 01	0.20220088D 01	0.74557889D 00
	10	0.24757260D 01	0.12898544D 01	0.22647780D 01	0.81469279D 00
	11	0.25456967D 01	0.13031948D 01	0.23410621D 01	0.83565337D 00
	12	0.17334226D 01	0.11762672D 01	0.14766842D 01	0.61935817D 00
	13	0.24740573D 01	0.12948020D 01	0.22629543D 01	0.82250356D 00
	14	0.20025368D 01	0.12336359D 01	0.17349794D 01	0.72239702D 00
	15	0.28947500D 01	0.13329636D 01	0.27165378D 01	0.88135797D 00
	16	0.35424270D 01	0.15118029D 01	0.33983510D 01	0.11338201D 01
	35	0.61238872D 01	0.15306474D 01	0.60416880D 01	0.11583276D 01
	37	0.16025655D 01	0.12163358D 01	0.12522843D 01	0.69243970D 00
	38	0.25254164D 01	0.12370967D 01	0.23189929D 01	0.72829126D 00
	41	0.14623611D 01	0.12827285D 01	0.10670052D 01	0.80336319D 00
2	1	0.57519518D 01	0.21090236D 01	0.56643579D 01	0.18568739D 01
	2	0.19604436D 01	0.12884435D 01	0.16862203D 01	0.81245718D 00
	3	0.16467333D 01	0.12364862D 01	0.13083312D 01	0.72725374D 00
	4	0.43816258D 01	0.17967531D 01	0.42659870D 01	0.14927564D 01
	5	0.36252533D 01	0.16070080D 01	0.34846040D 01	0.12579646D 01
	6	0.26797612D 01	0.13869307D 01	0.24861858D 01	0.96102908D 00
	7	0.25237484D 01	0.13536551D 01	0.23171763D 01	0.91234986D 00
	8	0.20706521D 01	0.12709757D 01	0.18131740D 01	0.78446115D 00
	9	0.18994132D 01	0.12450698D 01	0.16148593D 01	0.74175392D 00
	10	0.17769926D 01	0.12282387D 01	0.14639121D 01	0.71314123D 00
	11	0.17521713D 01	0.12249057D 01	0.14387857D 01	0.70738529D 00
	12	0.24323789D 01	0.13557549D 01	0.22725949D 01	0.91546235D 00
	13	0.17756303D 01	0.12268620D 01	0.14672638D 01	0.71076752D 00
	14	0.27360396D 01	0.14426975D 01	0.25467455D 01	0.10398924D 01
	15	0.20754741D 01	0.12393784D 01	0.18186788D 01	0.73216033D 00
	16	0.16952467D 01	0.12363723D 01	0.13688906D 01	0.72706022D 00
	34	0.15659122D 01	0.12091575D 01	0.12050233D 01	0.67975136D 00
	36	0.18693962D 01	0.12243126D 01	0.15794437D 01	0.70635782D 00
	39	0.72700523D 01	0.20102665D 01	0.72009486D 01	0.17438955D 01
	40	0.14310302D 01	0.12205948D 01	0.10236442D 01	0.69989408D 00
3	17	0.20419179D 01	0.13029515D 01	0.17792566D 01	0.83527403D 00
	18	0.27002935D 01	0.14161798D 01	0.25083086D 01	0.10027787D 01
	19	0.89331715D 01	0.26282821D 01	0.88770239D 01	0.24306104D 01
	20	0.24558226D 01	0.13065594D 01	0.22210880D 01	0.84089094D 00
	21	0.25776703D 01	0.13027528D 01	0.23757918D 01	0.83496402D 00
	22	0.26360117D 01	0.13106458D 01	0.24389665D 01	0.84722624D 00
	23	0.28570336D 01	0.13426271D 01	0.26763698D 01	0.89590599D 00
	24	0.31223879D 01	0.13859916D 01	0.29579226D 01	0.95967326D 00
	25	0.34279456D 01	0.14415178D 01	0.32788429D 01	0.10382551D 01
	26	0.38902776D 01	0.15341310D 01	0.37595558D 01	0.11634250D 01
	27	0.41941943D 01	0.15795688D 01	0.39805038D 01	0.12227173D 01
	28	0.52657133D 01	0.18428390D 01	0.51698875D 01	0.15479197D 01
	29	0.26264972D 01	0.13082926D 01	0.24286802D 01	0.84358131D 00
	30	0.38357739D 01	0.15412921D 01	0.37031286D 01	0.11728518D 01
	31	0.30619473D 01	0.13426107D 01	0.28940493D 01	0.89618116D 00
	32	0.40247351D 01	0.16406688D 01	0.38985244D 01	0.13006898D 01
	33	0.65663913D 01	0.21756136D 01	0.64897993D 01	0.19321735D 01
	35	0.95637503D 01	0.23338325D 01	0.95163534D 01	0.21694517D 01
	37	0.20170738D 01	0.12691598D 01	0.17517382D 01	0.78151557D 00
	38	0.27332023D 01	0.12857053D 01	0.25436976D 01	0.80810769D 00
	41	0.16912307D 01	0.12829850D 01	0.13639140D 01	0.80377266D 00

TABLE 6.10. Values of Tau and Gam for Case 6.
(at α = 0.05 and γ =0.8)

PHO NO	PT. NO	TAU x	TAU y	GAM x	GAM y
4	17	0.35203036D 01	0.16157500D 01	0.33752833D 01	0.12691131D 01
	18	0.16865044D 01	0.12112181D 01	0.13580490D 01	0.68341006D 00
	19	0.16575050D 01	0.12250242D 01	0.13218634D 01	0.70759055D 00
	20	0.28523433D 01	0.15217379D 01	0.26713035D 01	0.11470336D 01
	21	0.20212068D 01	0.12710317D 01	0.17564957D 01	0.78455189D 00
	22	0.19673403D 01	0.12540144D 01	0.16942337D 01	0.75667176D 00
	23	0.18271188D 01	0.12100397D 01	0.15291707D 01	0.68131930D 00
	24	0.17186920D 01	0.11792673D 01	0.13978205D 01	0.62503718D 00
	25	0.16290190D 01	0.11580820D 01	0.12859637D 01	0.58408391D 00
	26	0.15363969D 01	0.11415388D 01	0.11664114D 01	0.55055507D 00
	27	0.15071272D 01	0.11377841D 01	0.11275782D 01	0.54272695D 00
	28	0.14490766D 01	0.11367531D 01	0.10487245D 01	0.54056236D 00
	29	0.19673637D 01	0.12595081D 01	0.16942667D 01	0.76574197D 00
	30	0.15343126D 01	0.11446359D 01	0.11643238D 01	0.55694820D 00
	31	0.23136160D 01	0.13299288D 01	0.20863410D 01	0.87676146D 00
	32	0.16480327D 01	0.12036385D 01	0.13099664D 01	0.66988484D 00
	33	0.14376150D 01	0.11881392D 01	0.11013621D 01	0.64161891D 00
	34	0.15850969D 01	0.12205121D 01	0.12298505D 01	0.69974986D 00
	36	0.19832491D 01	0.12476848D 01	0.17134686D 01	0.74613491D 00
	39	0.65614982D 01	0.15841359D 01	0.64848484D 01	0.12286116D 01
	40	0.14262179D 01	0.12647701D 01	0.10169059D 01	0.77436653D 00
5	1	0.23377898D 01	0.14797400D 01	0.21131165D 01	0.10907018D 01
	2	0.15074719D 01	0.13633154D 01	0.11280387D 01	0.92662236D 00
	3	0.19892285D 01	0.14462708D 01	0.17204114D 01	0.10448441D 01
	4	0.17393827D 01	0.14160983D 01	0.14838768D 01	0.10026636D 01
	5	0.16522146D 01	0.13477549D 01	0.13152235D 01	0.90357257D 00
	6	0.15659526D 01	0.13376838D 01	0.12039060D 01	0.88848074D 00
	7	0.15533173D 01	0.13381329D 01	0.11892637D 01	0.88915664D 00
	8	0.15146255D 01	0.13417456D 01	0.11375810D 01	0.89458442D 00
	9	0.14991023D 01	0.13436595D 01	0.11168294D 01	0.89745244D 00
	10	0.15005979D 01	0.13432162D 01	0.11188360D 01	0.90426049D 00
	11	0.15051385D 01	0.13500413D 01	0.11249186D 01	0.90697929D 00
	12	0.15230538D 01	0.13304456D 01	0.11487789D 01	0.87754509D 00
	13	0.14372458D 01	0.13449407D 01	0.11008634D 01	0.89936956D 00
	14	0.16664136D 01	0.13360737D 01	0.13330170D 01	0.88605472D 00
	15	0.17394561D 01	0.13581693D 01	0.14232736D 01	0.91903423D 00
	16	0.16843329D 01	0.14221420D 01	0.13559726D 01	0.10111815D 01
	34	0.15729606D 01	0.12945710D 01	0.12141685D 01	0.82213992D 00
	36	0.16955149D 01	0.12862339D 01	0.13692227D 01	0.80894847D 00
	37	0.16640182D 01	0.13522621D 01	0.13300212D 01	0.91028176D 00
	38	0.21969155D 01	0.13560522D 01	0.19561283D 01	0.91590265D 00
	40	0.18097725D 01	0.13367423D 01	0.15084020D 01	0.96075708D 00
	41	0.19677200D 01	0.16634618D 01	0.16946745D 01	0.13293251D 01
6	17	0.17280724D 01	0.14031100D 01	0.14093382D 01	0.98423455D 00
	18	0.14337235D 01	0.12851164D 01	0.10274130D 01	0.80717044D 00
	19	0.23632235D 01	0.13788773D 01	0.21412260D 01	0.94936957D 00
	20	0.15113190D 01	0.13841701D 01	0.11331748D 01	0.95704075D 00
	21	0.13933038D 01	0.12870634D 01	0.97020386D 00	0.81026674D 00
	22	0.13990137D 01	0.12825827D 01	0.97838604D 00	0.80313044D 00
	23	0.14263573D 01	0.12707269D 01	0.10178024D 01	0.78405790D 00
	24	0.14620168D 01	0.12619467D 01	0.10665332D 01	0.76974634D 00
	25	0.14981336D 01	0.12546561D 01	0.11155287D 01	0.75773482D 00
	26	0.15447586D 01	0.12466558D 01	0.11774036D 01	0.74441305D 00
	27	0.15646174D 01	0.12441604D 01	0.12033402D 01	0.74022639D 00
	28	0.16697574D 01	0.12465777D 01	0.13374444D 01	0.74428215D 00
	29	0.13831607D 01	0.12802282D 01	0.96280324D 00	0.79936491D 00
	30	0.15219406D 01	0.12480502D 01	0.11473025D 01	0.74674577D 00
	31	0.16222625D 01	0.12949685D 01	0.12773941D 01	0.82276577D 00
	32	0.15839284D 01	0.12574632D 01	0.12283441D 01	0.76237372D 00
	33	0.18142911D 01	0.13192597D 01	0.15138204D 01	0.86049188D 00
	34	0.16263645D 01	0.13549720D 01	0.12832334D 01	0.91430256D 00
	36	0.18545645D 01	0.13509784D 01	0.15618609D 01	0.90837365D 00
	37	0.14816825D 01	0.12486618D 01	0.10933357D 01	0.74776754D 00
	38	0.15391226D 01	0.12503763D 01	0.11699993D 01	0.75062703D 00
	40	0.19451611D 01	0.15738780D 01	0.16684279D 01	0.12153568D 01
	41	0.15985248D 01	0.13489171D 01	0.12471092D 01	0.90530515D 00

TABLE 6.10 (Continued)

PHOTO NO	POINT NO	TAU x	TAU y	DELTA(um) x	DELTA(um) y
1	1	0.13892935D 01	0.13022448D 01	0.1587D 02	0.1094D 02
	2	0.21819605D 01	0.12663024D 01	0.1833D 02	0.1064D 02
	3	0.43780509D 01	0.16253088D 01	0.4098D 02	0.1365D 02
	4	0.15768313D 01	0.12352043D 01	0.1325D 02	0.1038D 02
	5	0.15567123D 01	0.11703512D 01	0.1308D 02	0.9831D 01
	6	0.17302535D 01	0.11738655D 01	0.1453D 02	0.9860D 01
	7	0.17973994D 01	0.11798929D 01	0.1510D 02	0.9911D 01
	8	0.20846552D 01	0.12206152D 01	0.1751D 02	0.1025D 02
	9	0.22616416D 01	0.12523645D 01	0.1900D 02	0.1052D 02
	10	0.24812994D 01	0.12931886D 01	0.2084D 02	0.1086D 02
	11	0.25523421D 01	0.13060786D 01	0.2144D 02	0.1097D 02
	12	0.17868639D 01	0.11819803D 01	0.1501D 02	0.9929D 01
	13	0.24823731D 01	0.13020065D 01	0.2086D 02	0.1094D 02
	14	0.21525542D 01	0.13029424D 01	0.1808D 02	0.1094D 02
	15	0.30092365D 01	0.14326061D 01	0.2528D 02	0.1203D 02
	16	0.36475745D 01	0.15214107D 01	0.3064D 02	0.1278D 02
	35	0.66191227D 01	0.16544301D 01	0.5560D 02	0.1390D 02
	37	0.21056769D 01	0.12399108D 01	0.1769D 02	0.1042D 02
	38	0.23102340D 01	0.12397349D 01	0.2361D 02	0.1041D 02
	41	0.21409060D 01	0.12863963D 01	0.1798D 02	0.1081D 02
2	1	0.63089173D 01	0.21344709D 01	0.5719D 02	0.1793D 02
	2	0.21482827D 01	0.13106867D 01	0.1805D 02	0.1101D 02
	3	0.19422750D 01	0.12715595D 01	0.1632D 02	0.1068D 02
	4	0.44253478D 01	0.18048064D 01	0.3717D 02	0.1516D 02
	5	0.36486416D 01	0.16088252D 01	0.3065D 02	0.1351D 02
	6	0.26964254D 01	0.14013997D 01	0.2265D 02	0.1177D 02
	7	0.25383914D 01	0.13675546D 01	0.2132D 02	0.1149D 02
	8	0.20777728D 01	0.12793448D 01	0.1745D 02	0.1075D 02
	9	0.19036793D 01	0.12500773D 01	0.1599D 02	0.1050D 02
	10	0.17818002D 01	0.12314899D 01	0.1497D 02	0.1034D 02
	11	0.17573481D 01	0.12280531D 01	0.1477D 02	0.1032D 02
	12	0.24912424D 01	0.13617785D 01	0.2093D 02	0.1144D 02
	13	0.17819139D 01	0.12304548D 01	0.1497D 02	0.1034D 02
	14	0.30096388D 01	0.15937219D 01	0.2528D 02	0.1339D 02
	15	0.21595795D 01	0.12865216D 01	0.1814D 02	0.1081D 02
	16	0.17361395D 01	0.12437011D 01	0.1458D 02	0.1045D 02
	34	0.17675326D 01	0.12186874D 01	0.1485D 02	0.1024D 02
	36	0.19305337D 01	0.12544596D 01	0.1622D 02	0.1054D 02
	39	0.78710121D 01	0.21764399D 01	0.6612D 02	0.1828D 02
	40	0.20365546D 01	0.12373930D 01	0.1711D 02	0.1039D 02
3	17	0.28066642D 01	0.13392076D 01	0.2358D 02	0.1125D 02
	18	0.32337098D 01	0.14419718D 01	0.2716D 02	0.1211D 02
	19	0.10781538D 02	0.26775041D 01	0.9056D 02	0.2249D 02
	20	0.24968885D 01	0.13181787D 01	0.2097D 02	0.1107D 02
	21	0.25863020D 01	0.13060452D 01	0.2172D 02	0.1097D 02
	22	0.26441911D 01	0.13143271D 01	0.2221D 02	0.1104D 02
	23	0.28647463D 01	0.13490354D 01	0.2406D 02	0.1133D 02
	24	0.31318070D 01	0.13968979D 01	0.2631D 02	0.1173D 02
	25	0.34424782D 01	0.14568744D 01	0.2892D 02	0.1224D 02
	26	0.39153293D 01	0.15517732D 01	0.3289D 02	0.1303D 02
	27	0.41332704D 01	0.15963096D 01	0.3472D 02	0.1341D 02
	28	0.52931178D 01	0.18450482D 01	0.4446D 02	0.1550D 02
	29	0.26340734D 01	0.13121430D 01	0.2213D 02	0.1102D 02
	30	0.38515318D 01	0.15506428D 01	0.3235D 02	0.1303D 02
	31	0.31977882D 01	0.14257624D 01	0.2686D 02	0.1198D 02
	32	0.43380599D 01	0.17745576D 01	0.3644D 02	0.1491D 02
	33	0.66364805D 01	0.22010617D 01	0.5575D 02	0.1849D 02
	35	0.10342570D 02	0.25820161D 01	0.8688D 02	0.2169D 02
	37	0.24959209D 01	0.12882379D 01	0.2097D 02	0.1082D 02
	38	0.29784159D 01	0.13473161D 01	0.2502D 02	0.1132D 02
	41	0.30595522D 01	0.13071176D 01	0.2570D 02	0.1098D 02

TABLE 6.11. Undetected Gross Errors for case 5.
(at $\alpha = 0.05$ and $\gamma = 0.8$)

PHOTO NO	POINT NO	TAU x	TAU y	DELTA(um) x	DELTA(um) y
4	17	0.38335226D 01	0.16199480D 01	0.3220D 02	0.1361D 02
	18	0.17852061D 01	0.12190246D 01	0.1500D 02	0.1024D 02
	19	0.18508619D 01	0.12420860D 01	0.1555D 02	0.1043D 02
	20	0.29384662D 01	0.15280109D 01	0.2468D 02	0.1284D 02
	21	0.20285970D 01	0.12729009D 01	0.1704D 02	0.1069D 02
	22	0.19749759D 01	0.12559295D 01	0.1659D 02	0.1055D 02
	23	0.18351787D 01	0.12119611D 01	0.1542D 02	0.1018D 02
	24	0.17258629D 01	0.11811766D 01	0.1450D 02	0.9922D 01
	25	0.16344057D 01	0.11597256D 01	0.1373D 02	0.9742D 01
	26	0.15401879D 01	0.11426968D 01	0.1294D 02	0.9599D 01
	27	0.15108946D 01	0.11387846D 01	0.1269D 02	0.9566D 01
	28	0.14573232D 01	0.11374887D 01	0.1224D 02	0.9555D 01
	29	0.19734822D 01	0.12649446D 01	0.1658D 02	0.1063D 02
	30	0.15384842D 01	0.11484596D 01	0.1292D 02	0.9647D 01
	31	0.23948168D 01	0.14396791D 01	0.2012D 02	0.1209D 02
	32	0.17406490D 01	0.12424852D 01	0.1462D 02	0.1044D 02
	33	0.15016526D 01	0.11955015D 01	0.1261D 02	0.1004D 02
	34	0.18279071D 01	0.12352869D 01	0.1535D 02	0.1038D 02
	36	0.20513967D 01	0.12505693D 01	0.1723D 02	0.1050D 02
	39	0.71038870D 01	0.17150843D 01	0.5967D 02	0.1441D 02
	40	0.19155693D 01	0.12717168D 01	0.1609D 02	0.1068D 02
5	1	0.23919639D 01	0.15361551D 01	0.2009D 02	0.1290D 02
	2	0.15958014D 01	0.14244805D 01	0.1340D 02	0.1197D 02
	3	0.20906835D 01	0.14864130D 01	0.1756D 02	0.1249D 02
	4	0.18093616D 01	0.14265013D 01	0.1520D 02	0.1198D 02
	5	0.16649223D 01	0.13484451D 01	0.1399D 02	0.1133D 02
	6	0.15713691D 01	0.13634452D 01	0.1320D 02	0.1145D 02
	7	0.15599176D 01	0.13658955D 01	0.1310D 02	0.1147D 02
	8	0.15194403D 01	0.13629607D 01	0.1276D 02	0.1145D 02
	9	0.15023869D 01	0.13554867D 01	0.1262D 02	0.1139D 02
	10	0.15042785D 01	0.13522464D 01	0.1264D 02	0.1136D 02
	11	0.15095743D 01	0.13525936D 01	0.1268D 02	0.1136D 02
	12	0.15265414D 01	0.13418084D 01	0.1282D 02	0.1127D 02
	13	0.14925313D 01	0.13463376D 01	0.1254D 02	0.1131D 02
	14	0.18410662D 01	0.13667313D 01	0.1546D 02	0.1148D 02
	15	0.18122305D 01	0.13881043D 01	0.1522D 02	0.1166D 02
	16	0.17315424D 01	0.14474398D 01	0.1454D 02	0.1216D 02
	34	0.16602029D 01	0.13038823D 01	0.1395D 02	0.1095D 02
	36	0.17496810D 01	0.13000345D 01	0.1470D 02	0.1092D 02
	37	0.19815208D 01	0.14029863D 01	0.1664D 02	0.1179D 02
	38	0.24271154D 01	0.14292575D 01	0.2039D 02	0.1201D 02
	40	0.24213926D 01	0.13970907D 01	0.2034D 02	0.1174D 02
	41	0.27976126D 01	0.16986923D 01	0.2350D 02	0.1427D 02
6	17	0.18582265D 01	0.14318247D 01	0.1561D 02	0.1203D 02
	18	0.15261731D 01	0.13325540D 01	0.1282D 02	0.1119D 02
	19	0.24709957D 01	0.14409506D 01	0.2076D 02	0.1210D 02
	20	0.15549444D 01	0.14041630D 01	0.1306D 02	0.1179D 02
	21	0.13984536D 01	0.12893693D 01	0.1175D 02	0.1083D 02
	22	0.14042057D 01	0.12861909D 01	0.1180D 02	0.1080D 02
	23	0.14321103D 01	0.12798932D 01	0.1203D 02	0.1075D 02
	24	0.14670470D 01	0.12777329D 01	0.1232D 02	0.1073D 02
	25	0.15030701D 01	0.12754157D 01	0.1263D 02	0.1071D 02
	26	0.15507587D 01	0.12676033D 01	0.1303D 02	0.1065D 02
	27	0.15714065D 01	0.12627938D 01	0.1320D 02	0.1061D 02
	28	0.16793635D 01	0.12474966D 01	0.1411D 02	0.1048D 02
	29	0.13922324D 01	0.12806929D 01	0.1169D 02	0.1076D 02
	30	0.15264322D 01	0.12553233D 01	0.1282D 02	0.1054D 02
	31	0.16958008D 01	0.13239198D 01	0.1424D 02	0.1112D 02
	32	0.17146361D 01	0.12897812D 01	0.1440D 02	0.1083D 02
	33	0.18340388D 01	0.13307146D 01	0.1541D 02	0.1118D 02
	34	0.17691259D 01	0.13857893D 01	0.1486D 02	0.1164D 02
	36	0.19122675D 01	0.13902501D 01	0.1606D 02	0.1168D 02
	37	0.16045062D 01	0.12584399D 01	0.1348D 02	0.1057D 02
	38	0.16384564D 01	0.12678962D 01	0.1376D 02	0.1065D 02
	40	0.25278155D 01	0.15958017D 01	0.2123D 02	0.1340D 02
	41	0.26780112D 01	0.13580428D 01	0.2250D 02	0.1141D 02

TABLE 6.11 (Continued)

PHOTO	POINT	TAU		DELTA(um)	
NO	NO	x	y	x	y
1	1	0.17320564D 01	0.12651193D 01	0.1455D 02	0.1063D 02
	2	0.19147247D 01	0.12483199D 01	0.1608D 02	0.1049D 02
	3	0.42911739D 01	0.16203095D 01	0.3605D 02	0.1361D 02
	4	0.15584368D 01	0.12260915D 01	0.1309D 02	0.1030D 02
	5	0.15442503D 01	0.11695048D 01	0.1297D 02	0.9824D 01
	6	0.17240936D 01	0.11703543D 01	0.1448D 02	0.9831D 01
	7	0.17904963D 01	0.11756270D 01	0.1504D 02	0.9875D 01
	8	0.20764234D 01	0.12147450D 01	0.1744D 02	0.1020D 02
	9	0.22557747D 01	0.12473523D 01	0.1895D 02	0.1048D 02
	10	0.24757260D 01	0.12898544D 01	0.2080D 02	0.1083D 02
	11	0.25456967D 01	0.13031948D 01	0.2138D 02	0.1095D 02
	12	0.17334226D 01	0.11762672D 01	0.1498D 02	0.9881D 01
	13	0.24740573D 01	0.12948020D 01	0.2078D 02	0.1088D 02
	14	0.20025368D 01	0.12336359D 01	0.1682D 02	0.1036D 02
	15	0.23947500D 01	0.13329636D 01	0.2432D 02	0.1120D 02
	16	0.35424270D 01	0.15113029D 01	0.2976D 02	0.1270D 02
	35	0.61238872D 01	0.15306474D 01	0.5144D 02	0.1286D 02
	37	0.16025655D 01	0.12163358D 01	0.1346D 02	0.1022D 02
	38	0.25254164D 01	0.12370967D 01	0.2121D 02	0.1039D 02
	41	0.14623611D 01	0.12827285D 01	0.1228D 02	0.1077D 02
2	1	0.57519518D 01	0.21090236D 01	0.4832D 02	0.1772D 02
	2	0.19604436D 01	0.12884435D 01	0.1647D 02	0.1082D 02
	3	0.16467333D 01	0.12364362D 01	0.1383D 02	0.1039D 02
	4	0.43816258D 01	0.17967531D 01	0.3681D 02	0.1509D 02
	5	0.36252538D 01	0.16070080D 01	0.3045D 02	0.1350D 02
	6	0.26797612D 01	0.13869307D 01	0.2251D 02	0.1165D 02
	7	0.25237484D 01	0.13536551D 01	0.2120D 02	0.1137D 02
	8	0.20706521D 01	0.12709757D 01	0.1739D 02	0.1068D 02
	9	0.18994132D 01	0.12450698D 01	0.1596D 02	0.1046D 02
	10	0.17769926D 01	0.12282387D 01	0.1493D 02	0.1032D 02
	11	0.17521713D 01	0.12249057D 01	0.1472D 02	0.1029D 02
	12	0.24828789D 01	0.13557549D 01	0.2086D 02	0.1139D 02
	13	0.17756303D 01	0.12268620D 01	0.1492D 02	0.1031D 02
	14	0.27360396D 01	0.14426975D 01	0.2298D 02	0.1212D 02
	15	0.20754741D 01	0.12393784D 01	0.1743D 02	0.1041D 02
	16	0.16952467D 01	0.12363723D 01	0.1424D 02	0.1039D 02
	34	0.15659122D 01	0.12091575D 01	0.1315D 02	0.1016D 02
	36	0.18693962D 01	0.12243126D 01	0.1570D 02	0.1028D 02
	39	0.72700523D 01	0.20102665D 01	0.6107D 02	0.1689D 02
	40	0.14310302D 01	0.12205948D 01	0.1202D 02	0.1025D 02
3	17	0.20410179D 01	0.13029515D 01	0.1714D 02	0.1094D 02
	18	0.27002985D 01	0.14161798D 01	0.2268D 02	0.1190D 02
	19	0.89331715D 01	0.26282821D 01	0.7504D 02	0.2208D 02
	20	0.24358226D 01	0.13065594D 01	0.2046D 02	0.1098D 02
	21	0.25776703D 01	0.13027528D 01	0.2165D 02	0.1094D 02
	22	0.26360117D 01	0.13106458D 01	0.2214D 02	0.1101D 02
	23	0.28570886D 01	0.13426271D 01	0.2400D 02	0.1128D 02
	24	0.31223879D 01	0.13859916D 01	0.2623D 02	0.1164D 02
	25	0.34279456D 01	0.14415178D 01	0.2879D 02	0.1211D 02
	26	0.38902776D 01	0.15341310D 01	0.3268D 02	0.1289D 02
	27	0.41041943D 01	0.15795688D 01	0.3448D 02	0.1327D 02
	28	0.52657133D 01	0.18428390D 01	0.4423D 02	0.1548D 02
	29	0.26264972D 01	0.13082926D 01	0.2206D 02	0.1099D 02
	30	0.38357739D 01	0.15412921D 01	0.3222D 02	0.1295D 02
	31	0.30619473D 01	0.13428107D 01	0.2572D 02	0.1128D 02
	32	0.40247351D 01	0.16406688D 01	0.3381D 02	0.1378D 02
	33	0.65663913D 01	0.21756136D 01	0.5516D 02	0.1828D 02
	35	0.95687503D 01	0.23888325D 01	0.8038D 02	0.2007D 02
	37	0.20170738D 01	0.12691598D 01	0.1694D 02	0.1066D 02
	38	0.27332023D 01	0.12857053D 01	0.2296D 02	0.1080D 02
	41	0.16912307D 01	0.12829850D 01	0.1421D 02	0.1078D 02

TABLE 6.12. Undetected Gross Errors for case 6.
(at $\alpha = 0.05$ and $\gamma = 0.8$)

PHOTO NO	POINT NO	TAU x	TAU y	DELTA(um) x	DELTA(um) y
4	17	0.35203036D 01	0.16157500D 01	0.2957D 02	0.1357D 02
	18	0.16365044D 01	0.12112181D 01	0.1417D 02	0.1017D 02
	19	0.16575050D 01	0.12250242D 01	0.1392D 02	0.1029D 02
	20	0.28523433D 01	0.15217379D 01	0.2396D 02	0.1278D 02
	21	0.20212068D 01	0.12710317D 01	0.1698D 02	0.1068D 02
	22	0.19673403D 01	0.12540144D 01	0.1653D 02	0.1053D 02
	23	0.18271188D 01	0.12100397D 01	0.1535D 02	0.1016D 02
	24	0.17186920D 01	0.11792673D 01	0.1444D 02	0.9906D 01
	25	0.16290190D 01	0.11580820D 01	0.1368D 02	0.9728D 01
	26	0.15363969D 01	0.11415388D 01	0.1291D 02	0.9589D 01
	27	0.15071272D 01	0.11377841D 01	0.1266D 02	0.9557D 01
	28	0.14490766D 01	0.11367531D 01	0.1217D 02	0.9549D 01
	29	0.19673687D 01	0.12595081D 01	0.1653D 02	0.1058D 02
	30	0.15348126D 01	0.11446359D 01	0.1289D 02	0.9615D 01
	31	0.23136160D 01	0.13299288D 01	0.1943D 02	0.1117D 02
	32	0.16480327D 01	0.12036385D 01	0.1384D 02	0.1011D 02
	33	0.14876150D 01	0.11881372D 01	0.1250D 02	0.9980D 01
	34	0.15850969D 01	0.12205121D 01	0.1331D 02	0.1025D 02
	36	0.19882491D 01	0.12476848D 01	0.1670D 02	0.1048D 02
	39	0.65614982D 01	0.15841359D 01	0.5512D 02	0.1331D 02
	40	0.14262179D 01	0.12647701D 01	0.1198D 02	0.1062D 02
5	1	0.23377898D 01	0.14797400D 01	0.1964D 02	0.1243D 02
	2	0.15074719D 01	0.13633154D 01	0.1266D 02	0.1145D 02
	3	0.19899285D 01	0.14462708D 01	0.1672D 02	0.1215D 02
	4	0.17893827D 01	0.14160983D 01	0.1503D 02	0.1190D 02
	5	0.16522146D 01	0.13477549D 01	0.1388D 02	0.1132D 02
	6	0.15650526D 01	0.13376838D 01	0.1315D 02	0.1124D 02
	7	0.15538173D 01	0.13381329D 01	0.1305D 02	0.1124D 02
	8	0.15146255D 01	0.13417456D 01	0.1272D 02	0.1127D 02
	9	0.14991023D 01	0.13436595D 01	0.1259D 02	0.1129D 02
	10	0.15005979D 01	0.13482162D 01	0.1261D 02	0.1133D 02
	11	0.15051385D 01	0.13500413D 01	0.1264D 02	0.1134D 02
	12	0.15230538D 01	0.13304456D 01	0.1279D 02	0.1118D 02
	13	0.14872458D 01	0.13449407D 01	0.1249D 02	0.1130D 02
	14	0.16664136D 01	0.13360737D 01	0.1400D 02	0.1122D 02
	15	0.17394561D 01	0.13581693D 01	0.1461D 02	0.1141D 02
	16	0.16848329D 01	0.14221420D 01	0.1415D 02	0.1195D 02
	34	0.15729606D 01	0.12945710D 01	0.1321D 02	0.1087D 02
	36	0.16955149D 01	0.12862339D 01	0.1424D 02	0.1080D 02
	37	0.16640182D 01	0.13522621D 01	0.1398D 02	0.1136D 02
	38	0.21969155D 01	0.13560522D 01	0.1845D 02	0.1139D 02
	40	0.18097725D 01	0.13867423D 01	0.1520D 02	0.1165D 02
	41	0.19677200D 01	0.16634618D 01	0.1653D 02	0.1397D 02
6	17	0.17280724D 01	0.14031100D 01	0.1452D 02	0.1179D 02
	18	0.14337285D 01	0.12851164D 01	0.1204D 02	0.1079D 02
	19	0.23632285D 01	0.13788773D 01	0.1985D 02	0.1158D 02
	20	0.15113190D 01	0.13841701D 01	0.1270D 02	0.1163D 02
	21	0.13933038D 01	0.12870634D 01	0.1170D 02	0.1081D 02
	22	0.13990137D 01	0.12825827D 01	0.1175D 02	0.1077D 02
	23	0.14268573D 01	0.12707269D 01	0.1199D 02	0.1067D 02
	24	0.14620168D 01	0.12619467D 01	0.1228D 02	0.1060D 02
	25	0.14981336D 01	0.12546561D 01	0.1258D 02	0.1054D 02
	26	0.15447586D 01	0.12466558D 01	0.1298D 02	0.1047D 02
	27	0.15646174D 01	0.12441604D 01	0.1314D 02	0.1045D 02
	28	0.16699574D 01	0.12465777D 01	0.1403D 02	0.1047D 02
	29	0.13881607D 01	0.12802282D 01	0.1166D 02	0.1075D 02
	30	0.15219406D 01	0.12480502D 01	0.1278D 02	0.1048D 02
	31	0.16222625D 01	0.12949685D 01	0.1363D 02	0.1088D 02
	32	0.15839284D 01	0.12574632D 01	0.1330D 02	0.1056D 02
	33	0.18142911D 01	0.13192597D 01	0.1524D 02	0.1108D 02
	34	0.16268645D 01	0.13549720D 01	0.1367D 02	0.1138D 02
	36	0.18545645D 01	0.13509784D 01	0.1558D 02	0.1135D 02
	37	0.14316825D 01	0.12486618D 01	0.1245D 02	0.1049D 02
	38	0.15391226D 01	0.12503763D 01	0.1293D 02	0.1050D 02
	40	0.19451611D 01	0.15738760D 01	0.1634D 02	0.1322D 02
	41	0.15985248D 01	0.13489171D 01	0.1343D 02	0.1133D 02

TABLE 6.12 (Continued)

In addition to the global indicators of reliability, the individual indicators Tau, Gam and Delta were computed (see Chapter 3) and are listed in Tables (6.9) through (6.12) for cases 5 and 6. The maximum valuesof Tau, Gam and undetected gross errors for the six cases are summarised in Table (6.13) for the photogrammetric measurements. Those for the 7 slope distances are set out in Table (6.14).

Case	Tau		Gam		Max.undetected gross errors (μm)	
	x	y	x	y	x	y
1	8767.473900	36.059799	8767.473900	36.045931	73650.00	302.90
2	7975.718700	4.491667	7975.718700	4.378935	67000.00	37.73
3	6922.358400	4.661124	6922.358400	4.552590	58150.00	39.15
4	162.169730	4.384509	162.1666650	4.268948	1362.00	36.83
5	10.781538	2.677504	10.735062	2.483753	90.56	22.49
6	9.568751	2.628282	9.516353	2.430610	80.38	22.08

TABLE (6.13) Maximum Tau, Gam and undetected gross errors for photogrammetric measurements (at $\alpha = 0.05$ and $\gamma = 0.8$)

Case	Tau	Gam	Max.undetected gross errors (mm)
2	87.636129	87.630424	98.153
4	38.869742	38.856877	43.534
6	21.673544	21.650462	24.274

TABLE (6.14) Maximum Tau, Gam and undetected gross errors for the 7 slope distances (at $\alpha = 0.05$ and $\gamma = 0.8$)

Conformal with the findings of Chapter 5, an insight into the latter two tables reveals that reliability increases with the increase of the number of cameras and higher internal reliability leads to higher external reliability. Moreover, detection of gross errors is more likely, the greater the number of cameras used.

6.3.3. Simulated models for sensitivity analysis

Only were the two models, namely the settlement and deflexion models mentioned in Section (5.6.2.3) investigated in this section. The just-detectable deformations were computed according to Equation (3.59) which reads:

$$\omega^{u} = \frac{(c\tilde{d})^{T} \; Q_{d}^{+}((c\tilde{d})}{\sigma_{o}^{2}} \qquad (6.3)$$

in which $c\tilde{d}$ indicates a just-detectable deformation and $\tilde{d}$ is a form vector characterising the deformation model to be tested.

Listed in Table (6.15) are the values of c for both settlement and deflexion models. The results summarised in such a table are graphically represented in Figures (6.12) and (6.13). These figures suggest that the sensitivity decreases when the number of cameras increases.

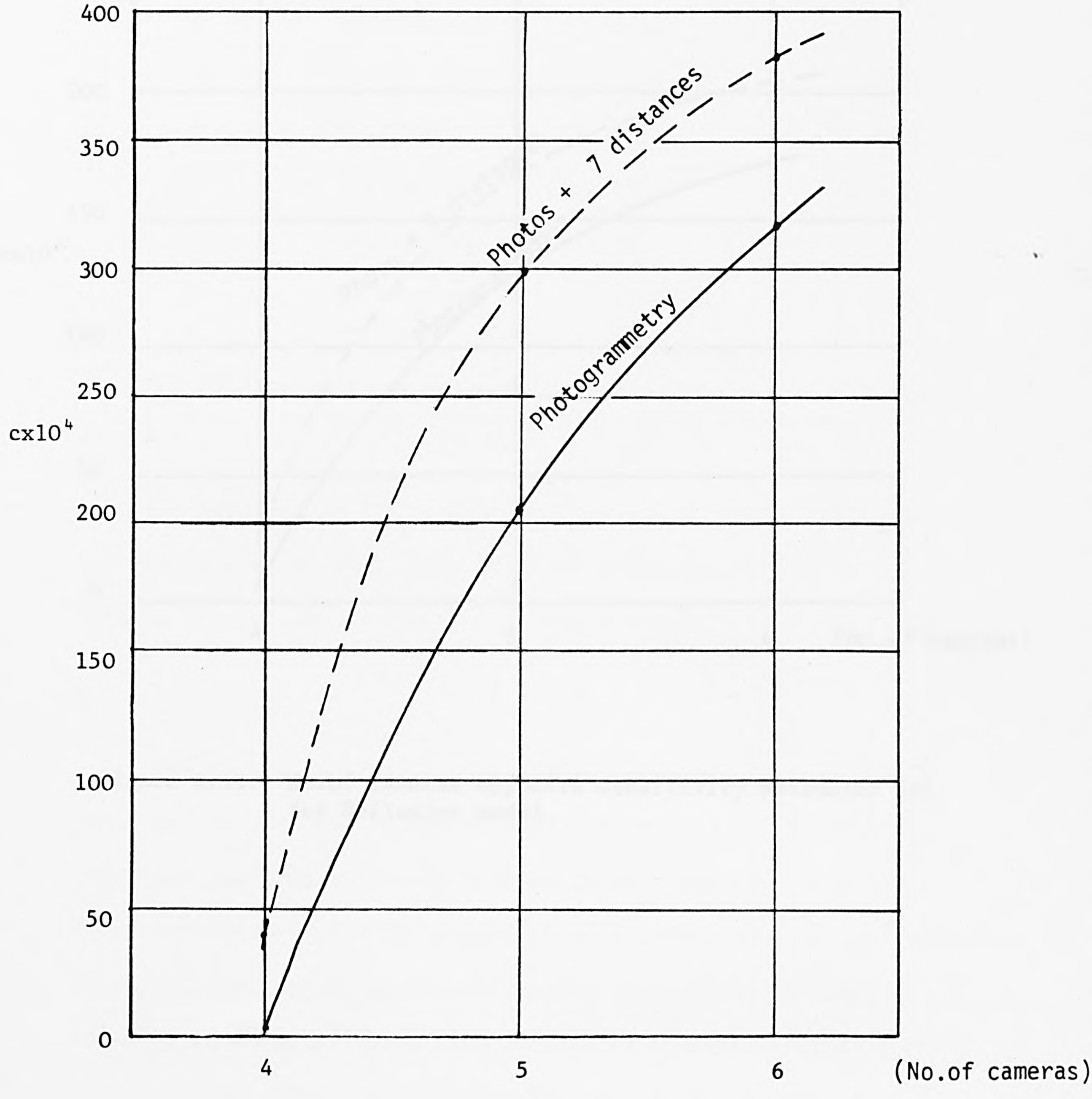


Figure 6.12. No.of cameras opposite sensitivity parameter (c) for Settlement model.

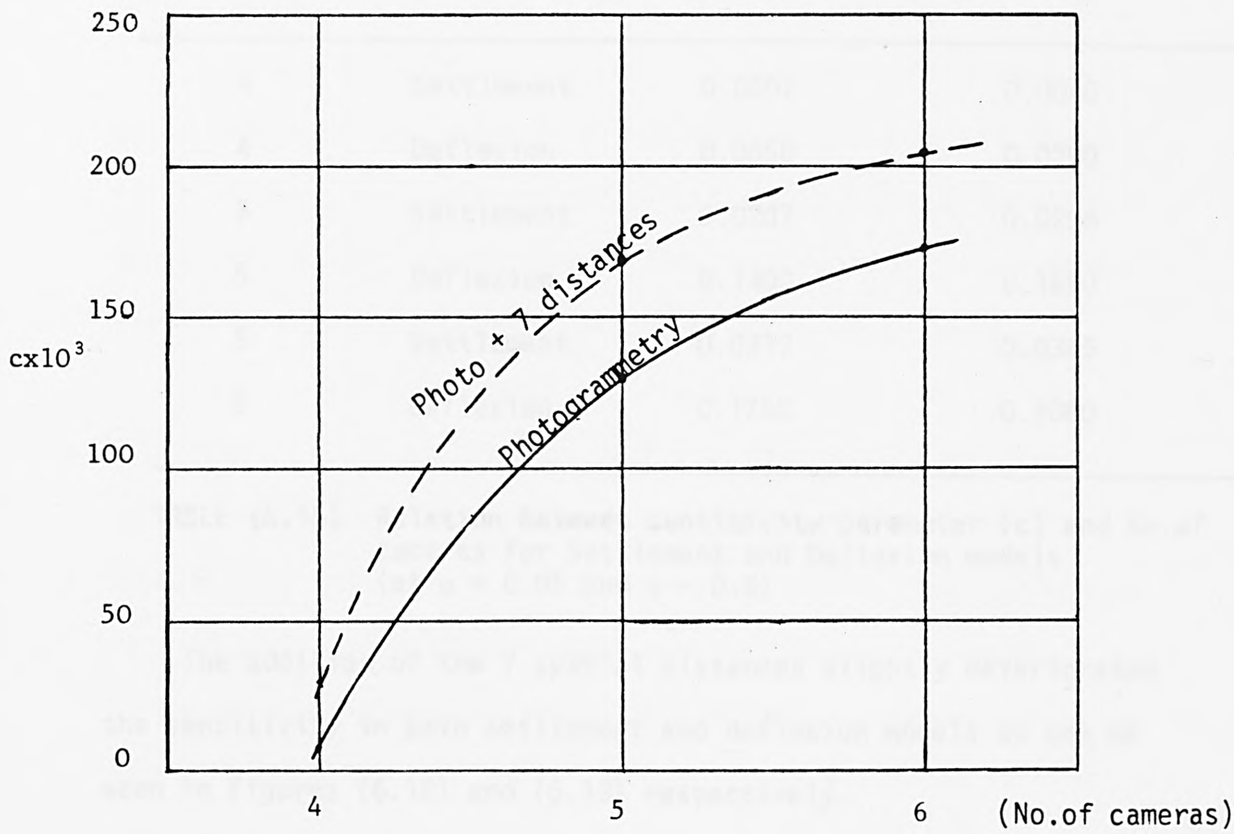


Figure 6.13. No.of cameras opposite Sensitivity parameter (c) for Deflexion model.

Number of cameras	Model	Photogrammetry	Photo+7 distances
		c	c
4	Settlement	0.0002	0.0040
4	Deflexion	0.0050	0.0290
5	Settlement	0.0207	0.0298
5	Deflexion	0.1300	0.1690
6	Settlement	0.0317	0.0385
6	Deflexion	0.1750	0.2060

TABLE (6.15) Relation between sensitivity parameter (**c**) and No.of cameras for Settlement and Deflexion models (at α = 0.05 and γ = 0.8)

The addition of the 7 spatial distances slightly deteriorated the sensitivity in both settlement and deflexion models as can be seen in Figures (6.12) and (6.13) respectively.

6.4. Conclusions

Some of the conclusions mentioned in Section (5.6.3) were confirmed here in the bridge deformation analysis. First, the precision and reliability were found to be increasing with the number of cameras with the exception that sensitivity was decreasing. Secondly, good precision did correspond to good reliability. Thirdly, addition of the 7 slope distances provided extremely high precision improvement especially with case 1 when augmented with those distances (case 2). At last, the higher internal reliability led to higher external reliability and the more the number of cameras the greater is the chance of detecting gross errors.

Case 6 (6 cameras + 7 slope distances) exhibited the highest precision (σ_m^2 = 2.644 mm) as well as the highest reliability ($r_{max}(x)$= 0.519, $r_{max}(y)$ = 0.774). Furthermore, in such a case gross errors of the order of 7- and 26-fold the a priori standard error in the x- and y-co-ordinates respectively can be detected. Consequently, the reliability in the y-co-ordinate direction was significantly much better than that of the x-co-ordinate.

With regard to sensitivity, the addition of the 7 slope distances, listed in Table (6.3), led to the results of slightly deteriorating the values of c in both settlement and deflexion models. On the contrary to the cube, where the slope distances were on each of its upright faces and no transverse distances, here in the bridge deformation investigation such transverse distances could be the reason for such a slight deterioration as it might have increased the correlation between the co-ordinates of the opposite linked targets.

CHAPTER 7

SIMULATED NETWORK DEFORMATION ANALYSIS (DAM)

7.1. Introduction

For safety reasons it is necessary to perform periodic control surveys of large engineering structures, such as dams, in order to determine whether the structure remains stable or whether it is subject to movements or local deformations as a function of time. Usually this is done by the classical methods mentioned in Section (6.1). Such methods are time consuming especially with large networks and therefore photogrammetric structural survey was chosen as an alternative. It offers an economical substitute and, rather, provides an instantaneous description due to the short time lapse between the data acquisition and data reduction.

During the preparation of this thesis (July 1985), the collapse of Starva dam, Italy, took place causing large scale losses and mortalities. It can be argued that the catastrophy happened, mainly, due to lack of surveillance and monitoring of that dam. Therefore, this investigation gets its utmost importance as a guard, if followed, against sudden failure of large structures among them are dams.

7.2. Network configurations

A typical concrete dam (Figure 7.1) was chosen to be under investigation. A total of 14 targets was placed in two rows on the face of the dam (Figure 7.3). A Zeiss (Jena) UMK 10/1318 camera, of nominal focal length of 100 mm, which accepts glass plates (130 mm x 180 mm) was assumed to be used for photography. An 3 μm standard error has been postulated for the photogrammetric measurements.

Three camera stations were selected (Figure 7.1) to represent the first configuration. To strengthen the network, an additional camera was chosen to constitute the second configuration which almost takes the "L-letter" shape as shown in Figure (7.1). The third configuration which, also comprises four camera stations is illustrated in Figure (7.2). Consequently, three distinct cases were obtained taking into account that the number of targets is constant through all these cases. Table (7.1) sets out the orientation elements of the individual cameras along with the photo arrangements.

Case	Photo arrangement	Camera No.	X^c (m)	Y^c (m)	Z^c (m)	ω (deg)	φ (deg)	κ (deg)
1	1-2-3	1	326.0	94.0	55.0	87	55	0
		2	310.0	208.0	55.0	87	125	0
		3	444.0	208.0	55.0	38	95	0
2	1-2-3-4L	4L	344.0	148.0	50.0	88	90	0
3	1-2-3-4-	1	326.0	94.0	55.0	87	55	0
		2	312.0	130.0	50.0	88	109	0
		3	306.0	168.0	50.0	88	88	0
		4	310.0	208.0	55.0	87	125	0

TABLE (7. 1) Orientation elements of camera stations shown in Figures 7.1 and 7.2.

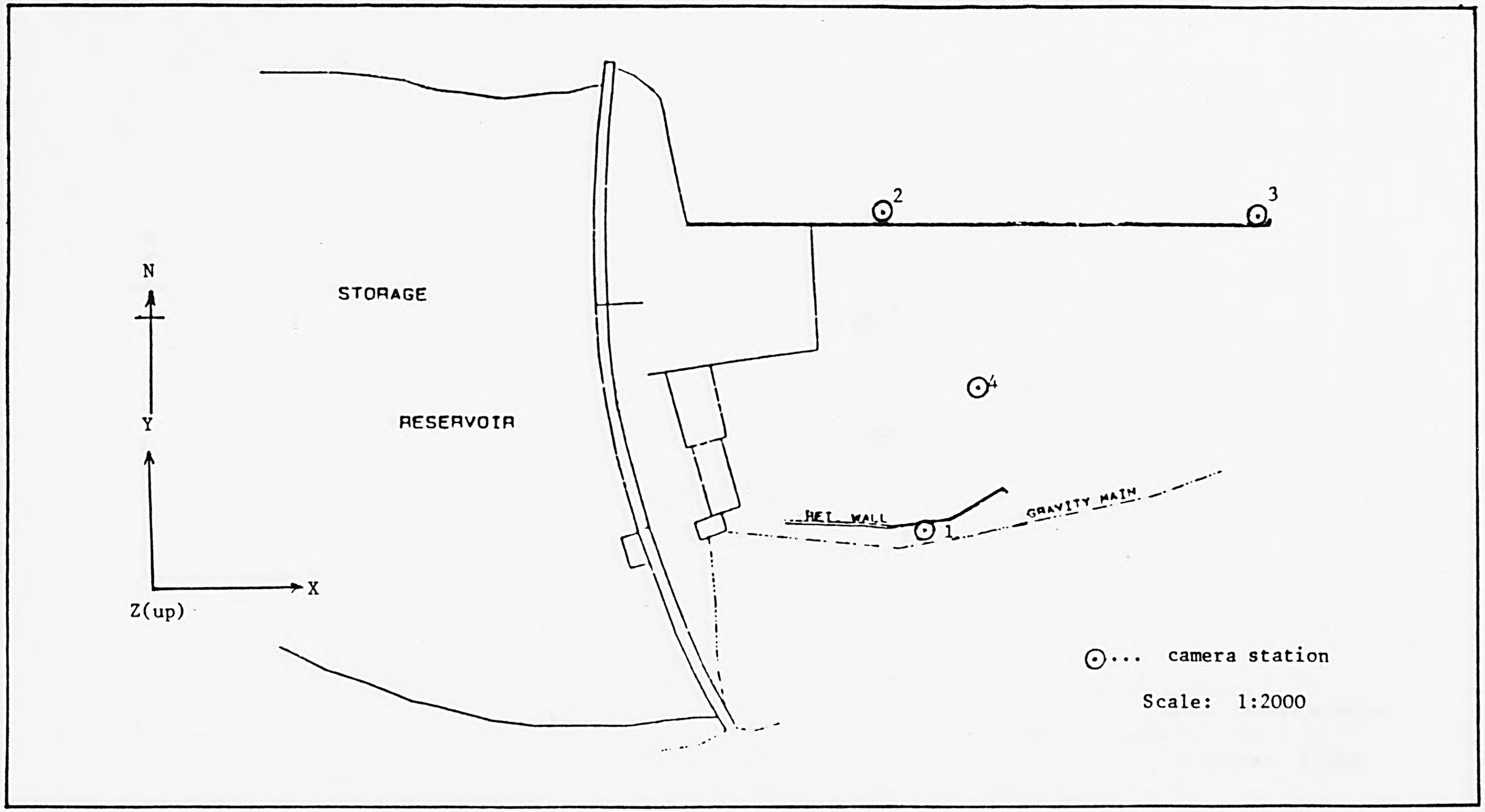


Figure 7.1. Camera Station Configurations for cases 1 and 2

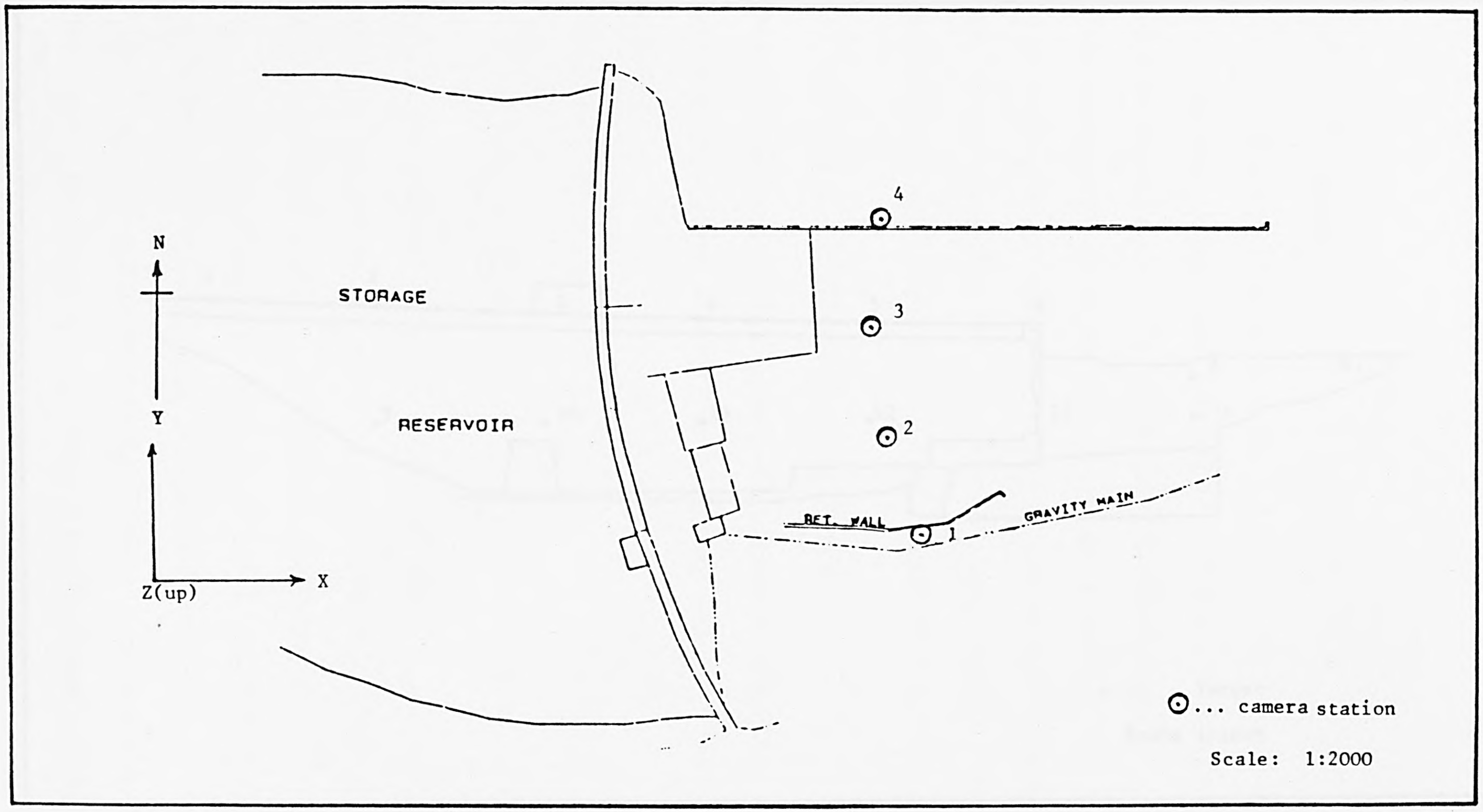


Figure 7.2. Camera station Configurations for case 3.

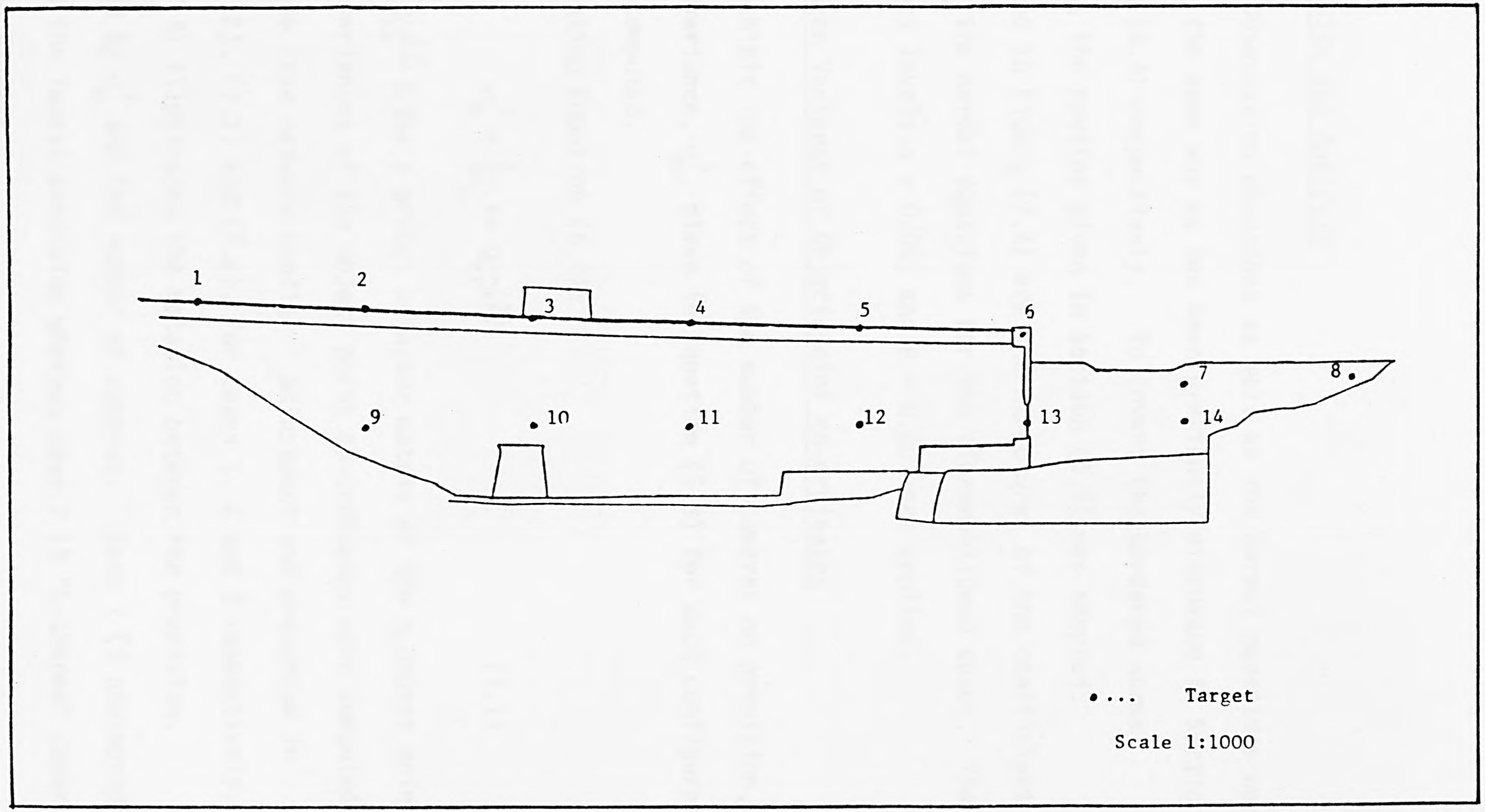


Figure 7.3. Dam Face Targets.

7.3. Results and Analysis

The observation equations as well as the normal equations were formed in the same way as has been previously discussed in Sections (5.3) and (5.4) respectively. To invert the bordered normal equations, the routine given in Section (5.5) was adopted. Illustrated in Figure (7.4) are the structures of the coefficient matrix of the normal equations for the aforementioned cases. The probability levels $\alpha = 0.05$, and $\beta = 0.20$ were applied.

7.3.1. Mean Variance of Object-point co-ordinates

To exhibit the effect of the number of cameras on precision, the mean variance, σ_m^2, given in Equation (5.8) for each configuration is to be computed.

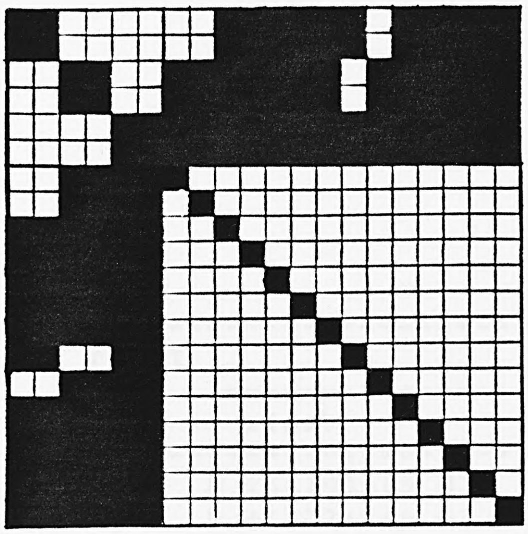
Restating Equation (5.8):

$$\sigma_m^2 = \frac{1}{3n_o} \operatorname{tr} Q_{\hat{x}\hat{x}}(2) \qquad (7.1)$$

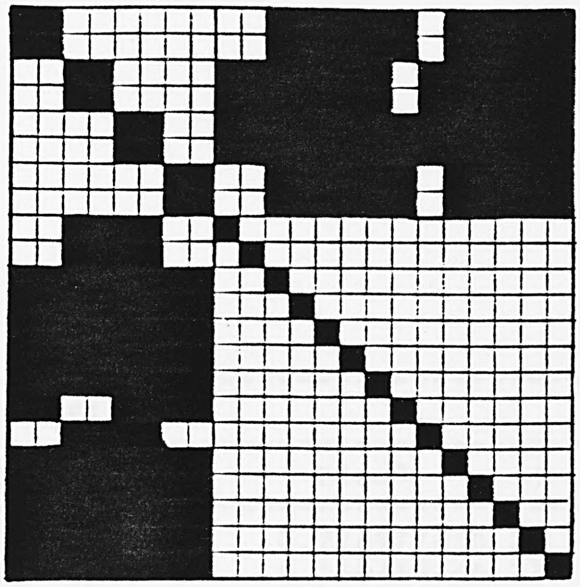
in which $Q_{\hat{x}\hat{x}}(2)$ is the a priori cofactor matrix of the n_o object points.

The variances of the object point co-ordinates were computed through the free network bundle adjustment and presented in Tables (7.2), (7.3) and (7.4), for cases 1, 2 and 3 respectively. Figure (7.5) illustrates the relation between the precision, designated by σ_m^2 and the number of cameras. Case 1 (3 photographs) exhibited the lowest precision whereas case 2 (4 "L-shaped" cameras) showed the highest.

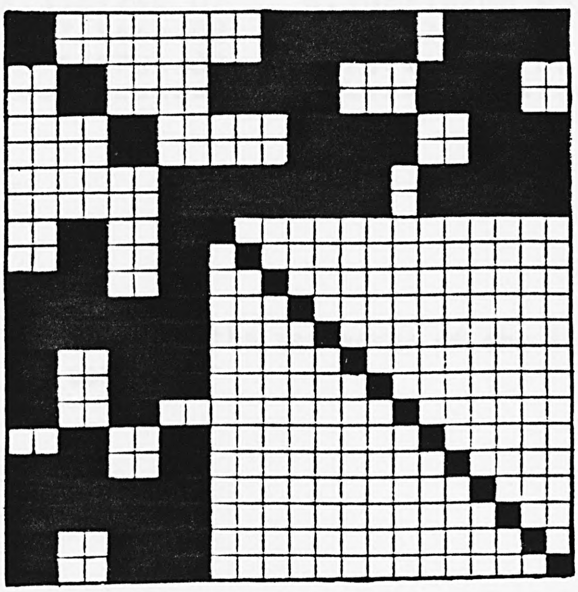
It is notable that the precision increases with the number of cameras, however it seems that the position of the cameras is of a



(a) Case 1-3 Photos and 14 object points.



(b) Case 2-4 Photos "L-Shaped" and 14 object points.



(c) Case 3-4 Photos and 14 object points.

Figure 7.4. Structure of coefficient matrix of the normal equations.

POINT	VARIANCES		
NO.	X (mm^2)	Y (mm^2)	Z (mm^2)
1	0.15579864D 03	0.75851794D 03	0.27697723D 02
2	0.64102636D 02	0.12910390D 03	0.16973876D 02
3	0.67034386D 02	0.12205808D 03	0.17316938D 02
4	0.93436383D 02	0.81382869D 02	0.87440373D 01
5	0.11267400D 03	0.50331720D 02	0.35239045D 01
6	0.77686039D 02	0.18364723D 02	0.11034078D 02
7	0.70229612D 02	0.28822716D 02	0.95578663D 01
8	0.24237027D 03	0.60454292D 03	0.20869646D 02
9	0.93998864D 02	0.12147482D 03	0.14933940D 02
10	0.42028943D 02	0.13959050D 03	0.88252461D 01
11	0.67633994D 02	0.13080968D 03	0.63270976D 01
12	0.81358774D 02	0.10671820D 03	0.70506774D 01
13	0.54076546D 02	0.49742875D 02	0.33214389D 01
14	0.93953078D 02	0.20794936D 02	0.99435672D 01

TABLE 7.2. Estimates of the variances of the 14 object point co-ordinates for case 1.

POINT	VARIANCES		
NO.	X (mm²)	Y (mm²)	Z (mm²)
1	0.11549312D 03	0.62744038D 03	0.25174333D 02
2	0.57822758D 02	0.12390681D 03	0.16689235D 02
3	0.61500726D 02	0.11029667D 03	0.14072725D 02
4	0.74595327D 02	0.71772139D 02	0.76675450D 01
5	0.84930228D 02	0.40981802D 02	0.72693806D 01
6	0.57894693D 02	0.13349450D 02	0.80222558D 01
7	0.59735377D 02	0.22942498D 02	0.62743356D 01
8	0.19793555D 03	0.45134698D 03	0.13776854D 02
9	0.81426096D 02	0.11954388D 03	0.13904356D 02
10	0.36555229D 02	0.12050678D 03	0.68435526D 01
11	0.53412632D 02	0.10039076D 03	0.48553650D 01
12	0.66409436D 02	0.74136143D 02	0.50800947D 01
13	0.45971737D 02	0.33384781D 02	0.58558595D 01
14	0.73414678D 02	0.17515877D 02	0.72147745D 01

TABLE 7.3. Estimates of the variances of the 14 object point co-ordinates for case 2.

POINT	VARIANCES		
NO.	X (mm^2)	Y (mm^2)	Z (mm^2)
1	0.25113954D 03	0.53588926D 03	0.40117740D 02
2	0.57160922D 02	0.79837316D 02	0.18167095D 02
3	0.71358760D 02	0.75757566D 02	0.13048678D 02
4	0.12942578D 03	0.39082091D 02	0.65716101D 01
5	0.15606869D 03	0.25894729D 02	0.57096680D 01
6	0.97924963D 02	0.32308468D 02	0.71719912D 01
7	0.70396229D 02	0.39625821D 02	0.69604839D 01
8	0.57157194D 03	0.58568238D 03	0.21918924D 02
9	0.99077473D 02	0.16746455D 03	0.15400927D 02
10	0.28087034D 02	0.77382462D 02	0.58629091D 01
11	0.44543375D 02	0.55861867D 02	0.42280104D 01
12	0.66144384D 02	0.57148683D 02	0.41884073D 01
13	0.45447573D 02	0.67745636D 02	0.49228660D 01
14	0.32012793D 02	0.48454779D 02	0.95714883D 01

TABLE 7.4. Estimate of the variances of the 14 object point co-ordinates for case 3.

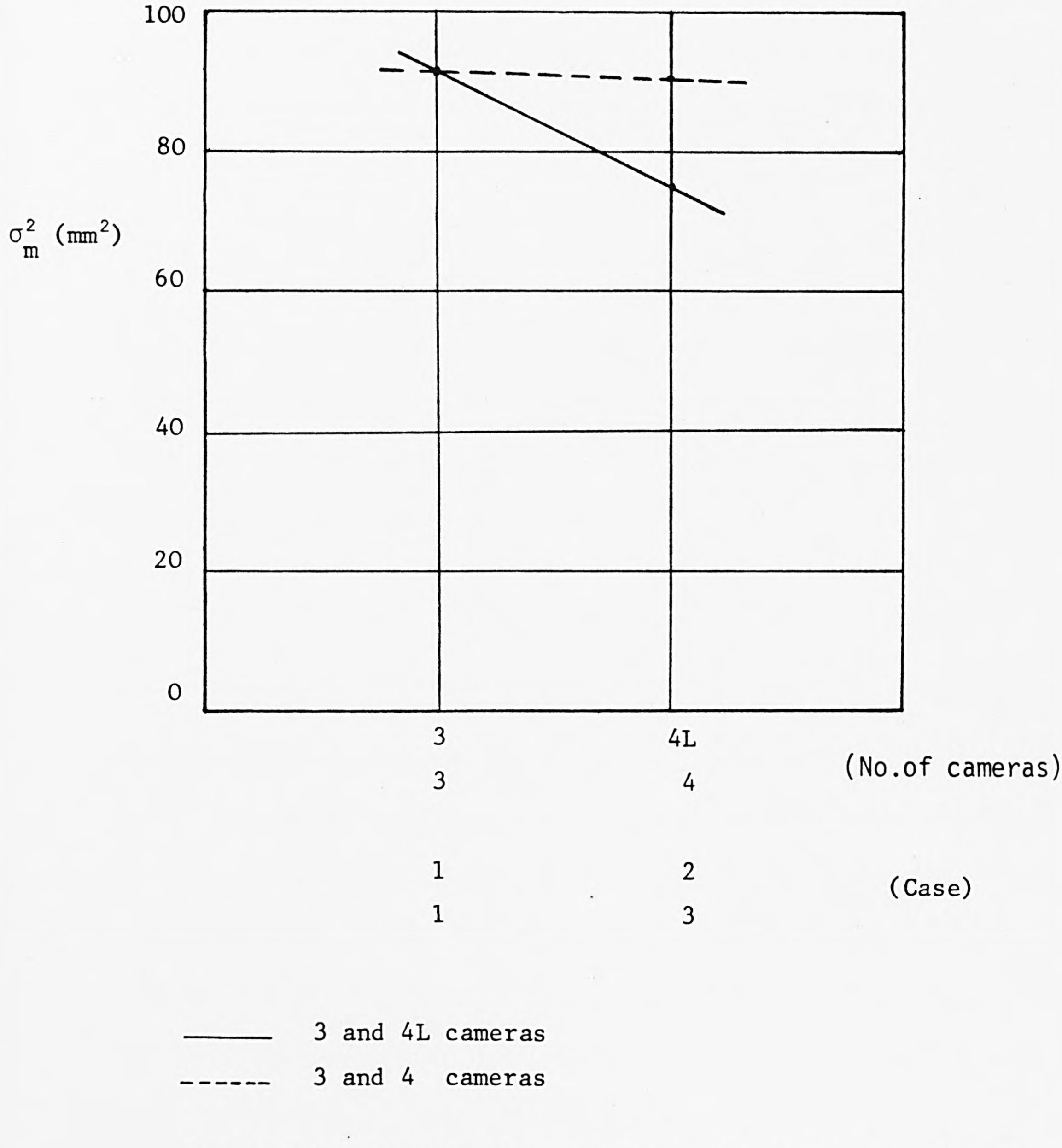


Figure 7.5. Relation between no.of cameras and precision

prime importance. This is evident from the comparison of the results of case 2 to those of case 3 where the presence of camera No.3 which contains images of all object points in the former case gave rise to considerable higher precision compared with the latter.

7.3.2. r_{max} and other reliability indicators

The global reliability of the x- and y- co-ordinate measurements represented by $r_{max}(x)$ and $r_{max}(y)$ respectively where r is the redundancy number mentioned in Section (5.6.2.2.) and computed as given by Equation (5.9) which reads:

$$r_i = q_{\hat{v}_i \hat{v}_i} \times w_{\ell_i} \tag{7.2}$$

in which $q_{\hat{v}_i \hat{v}_i}$ is the ith diagonal element of the cofactor matrix of the residuals $Q_{\hat{v}\hat{v}}$ and w_{ℓ_i} is the a priori weight of observation ℓ_i. The values of $r_{max}(x)$ and $r_{max}(y)$ for case 1 are displayed in Table (7.5) while Tables (7.6) and (7.7) set out these values for cases 2 and 3 respectively. Figure (7.6) shows the relation between number of cameras and the reliability of the x-co-ordinate photogrammetric measurements whereas Figure (7.7) demonstrates that relation with regard to the y-co-ordinate.

Agreeable with what has been mentioned in Section (7.3.1) the reliability does increase with number of cameras, yet such an increase depends on the location of the camera and number of imaged points caught by such a camera.

PHOTO	POINT	$r(x)$	$r(y)$
1	3	0.14812551D-01	0.65322413D-01
	4	0.32094878D-01	0.31512670D 00
	5	0.30857736D-01	0.37886178D 00
	6	0.22277267D-01	0.42015574D 00
	7	0.94958649D-02	0.52628449D 00
	8	0.72701631D-03	0.17137342D 00
	10	0.14689800D-01	0.68005474D-01
	11	0.24364516D-01	0.31395164D 00
	12	0.22353246D-01	0.37926780D 00
	13	0.12054237D-01	0.50978933D 00
	14	0.22048587D-02	0.46314536D 00
2	1	0.27270334D-02	0.17936933D 00
	2	0.54270334D-02	0.23632944D 00
	3	0.68986408D-01	0.33146943D 00
	4	0.11332489D 00	0.46941533D 00
	5	0.10501042D 00	0.35250050D 00
	6	0.82724446D-01	0.24144602D 00
	7	0.43059081D-01	0.15556134D 00
	9	0.78381384D-04	0.22383148D 00
	10	0.61894133D-01	0.36293057D 00
	11	0.91125800D-01	0.47878363D 00
	12	0.89223709D-01	0.36299330D 00
	13	0.78122333D-01	0.28219269D 00
	14	0.45987280D-01	0.13183309D 00
3	1	0.14166370D 00	0.19652531D 00
	2	0.21253589D 00	0.33801722D 00
	3	0.39657062D 00	0.62734946D 00
	4	0.69257962D 00	0.72464882D 00
	5	0.68558750D 00	0.72157501D 00
	6	0.72012084D 00	0.72050497D 00
	7	0.65308596D 00	0.53056592D 00
	8	0.21406696D 00	0.91247889D-01
	9	0.27434746D 00	0.22902998D 00
	10	0.57998047D 00	0.54405390D 00
	11	0.70298706D 00	0.65558293D 00
	12	0.71811326D 00	0.64735643D 00
	13	0.74460328D 00	0.67051712D 00
	14	0.59444232D 00	0.57877636D 00

TABLE 7.5. Computed values of redundancy numbers for case 1.

PHOTO	POINT	$r(x)$	$r(y)$
1	3	0.49946498D-01	0.11683104D 00
	4	0.12763513D 00	0.43420652D 00
	5	0.17621788D 00	0.49978817D 00
	6	0.22004413D 00	0.55506726D 00
	7	0.25369360D 00	0.64285686D 00
	8	0.12411210D 00	0.42839514D 00
	10	0.46838591D-01	0.13284726D 00
	11	0.12121478D 00	0.43738552D 00
	12	0.16770237D 00	0.53845107D 00
	13	0.22279466D 00	0.62208813D 00
	14	0.22427522D 00	0.58308570D 00
2	1	0.27705526D-02	0.18223229D 00
	2	0.54623176D-02	0.23786594D 00
	3	0.15676357D 00	0.45434454D 00
	4	0.21165098D 00	0.55842012D 00
	5	0.16860900D 00	0.46875322D 00
	6	0.12207003D 00	0.34055723D 00
	7	0.65139937D-01	0.24397371D 00
	9	0.79237732D-04	0.22627693D 00
	10	0.14111832D 00	0.47368966D 00
	11	0.18878256D 00	0.56109861D 00
	12	0.14910548D 00	0.43005196D 00
	13	0.12063828D 00	0.39751613D 00
	14	0.67192393D-01	0.21028133D 00
3	1	0.14392443D 00	0.19966154D 00
	2	0.21391770D 00	0.34021486D 00
	3	0.45784800D 00	0.65675639D 00
	4	0.72031912D 00	0.76377866D 00
	5	0.72013924D 00	0.75278410D 00
	6	0.75528759D 00	0.74537276D 00
	7	0.71209776D 00	0.61058668D 00
	8	0.33206080D 00	0.13535372D 00
	9	0.27734481D 00	0.23153222D 00
	10	0.62627771D 00	0.59198088D 00
	11	0.75249083D 00	0.69832270D 00
	12	0.76394610D 00	0.69328941D 00
	13	0.77729740D 00	0.70051762D 00
	14	0.66884448D 00	0.62923719D 00
4	3	0.29648033D 00	0.43054206D 00
	4	0.54431308D 00	0.55968950D 00
	5	0.54138720D 00	0.53818383D 00
	6	0.50781443D 00	0.56096523D 00
	7	0.44554467D 00	0.56717181D 00
	8	0.12214154D 00	0.29031391D 00
	10	0.29034911D 00	0.33936970D 00
	11	0.51222096D 00	0.59751051D 00
	12	0.49267503D 00	0.63392504D 00
	13	0.48430523D 00	0.63708113D 00
	14	0.40120271D 00	0.52399928D 00

TABLE 7.6. Computed values of redundancy numbers for case 2.

PHOTO	POINT	$r(x)$	$r(y)$
1	3	0.81348794D-01	0.13056028D 00
	4	0.25001080D 00	0.51016220D 00
	5	0.25934034D 00	0.57754200D 00
	6	0.21937220D 00	0.57391457D 00
	7	0.25105554D 00	0.65759607D 00
	8	0.46606946D-01	0.40481031D 00
	10	0.57790431D-01	0.14861567D 00
	11	0.18381612D 00	0.51840336D 00
	12	0.23370461D 00	0.61832912D 00
	13	0.18652634D 00	0.62170166D 00
	14	0.25837277D 00	0.55196444D 00
2	1	0.58850351D-02	0.56692304D-01
	2	0.14297114D-01	0.18840614D 00
	3	0.32671900D 00	0.30675681D 00
	4	0.47883033D 00	0.45434492D 00
	5	0.18473414D 00	0.33657492D 00
	9	0.35849134D-02	0.11120203D 00
	10	0.30088323D 00	0.31644744D 00
	11	0.31571898D 00	0.50507567D 00
	12	0.14699818D 00	0.30734634D 00
3	4	0.18085622D 00	0.35568579D 00
	5	0.40787343D 00	0.41658869D 00
	6	0.39510626D 00	0.36434789D 00
	7	0.25782504D 00	0.41607989D 00
	8	0.30124211D-02	0.12042178D 00
	11	0.13668288D 00	0.28301771D 00
	12	0.32040723D 00	0.52607893D 00
	13	0.31977449D 00	0.47994353D 00
	14	0.26094137D 00	0.35860757D 00
4	1	0.42192206D-01	0.15396679D 00
	2	0.84976707D-01	0.46087250D 00
	3	0.18010820D 00	0.55325300D 00
	4	0.29868226D 00	0.64085244D 00
	5	0.31787483D 00	0.54893352D 00
	6	0.16969219D 00	0.33708667D 00
	7	0.54004156D-01	0.28519157D 00
	9	0.17697634D-01	0.27599994D 00
	10	0.15316224D 00	0.54269304D 00
	11	0.27131442D 00	0.61887124D 00
	12	0.24841684D 00	0.56633768D 00
	13	0.13135193D 00	0.45341697D 00
	14	0.45472323D-01	0.23728546D 00

TABLE 7.7. Computed values of redundancy numbers for case 3.

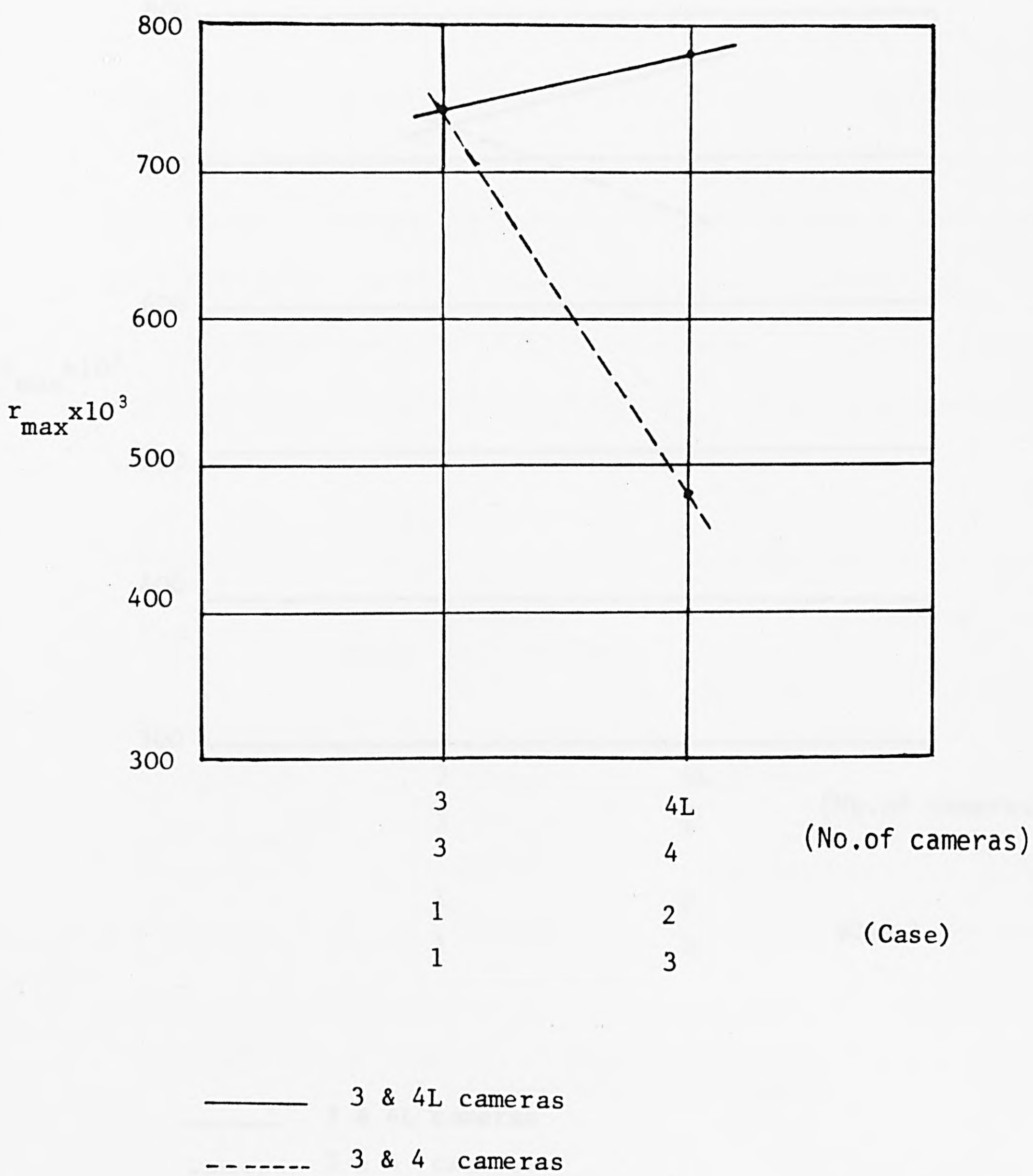


Figure 7.6. No.of cameras versus Reliability (internal) of photo x-co-ordinates.

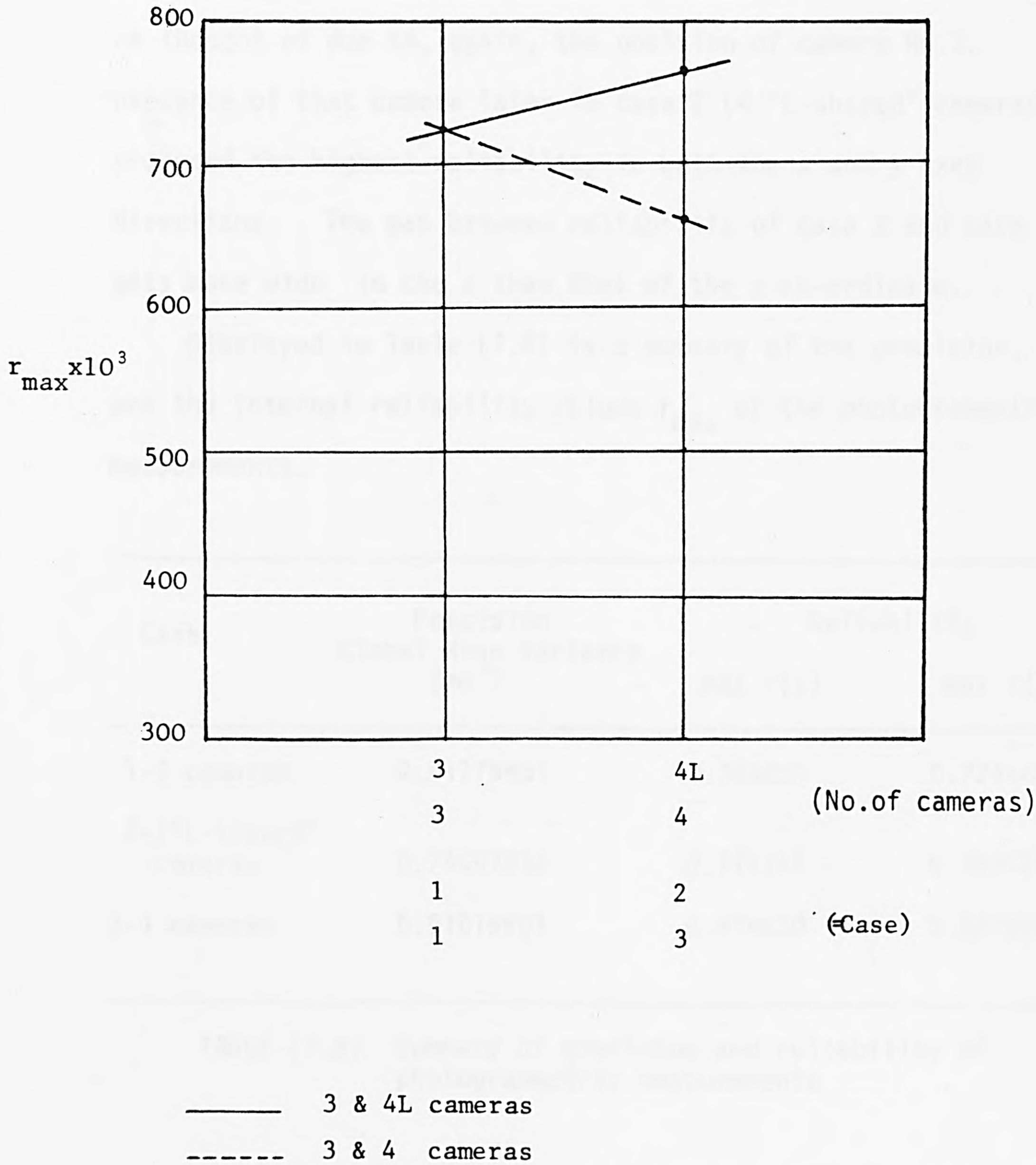


Figure 7.7. No.of cameras versus Reliability (internal) of the photo y-co-ordinates.

It is surprising and notable that case 3 (4 photographs) showed lower reliability than case 1 (3 photographs) which can be thought of due to, again, the position of camera No.3. The presence of that camera later in case 2 (4 "L-shaped" cameras) produced the highest reliability in both the x and y axes directions. The gap between reliability of case 2 and case 3 gets more wide in the x than that of the y co-ordinate.

Displayed in Table (7.8) is a summary of the precision, σ_m^2, and the internal reliability values r_{max} of the photogrammetric measurements.

Case	Precision Global Mean Variance (mm^2)	Reliability max r(x)	 max r(y)
1-3 cameras	0.91779957	0.744603	0.724649
2-4"L-shaped" cameras	0.74697933	0.777297	0.763779
3-4 cameras	0.91019901	0.478830	0.657596

TABLE (7.8) Summary of precision and reliability of photogrammetric measurements

Tau and Gam as individual indicators of reliability, were computed and are presented in Tables (7 9.) through (7. 14) for cases 1, 2 and 3.

The maximum values of Tau, Gam and the undetected gross errors for the three cases, at hand, are shown in Table (7.15).

PHO NO	PT. NO	TAU x	TAU y	GAM x	GAM y
1	3	0.82164661D 01	0.39126310D 01	0.31553857D 01	0.37626317D 01
	4	0.55819011D 01	0.17813834D 01	0.54915954D 01	0.14742207D 01
	5	0.56926958D 01	0.16246492D 01	0.56041757D 01	0.12304238D 01
	6	0.66799112D 01	0.15427475D 01	0.66248630D 01	0.11747637D 01
	7	0.10262017D 02	0.13784459D 01	0.10213178D 02	0.94874285D 00
	8	0.37087531D 02	0.24156180D 01	0.37074047D 02	0.21989112D 01
	10	0.82507240D 01	0.33346706D 01	0.31898990D 01	0.37019358D 01
	11	0.64065041D 01	0.17847140D 01	0.63279772D 01	0.14782435D 01
	12	0.66885150D 01	0.16237793D 01	0.66133375D 01	0.12793199D 01
	13	0.91081493D 01	0.14005694D 01	0.90530870D 01	0.98060930D 00
	14	0.21296568D 02	0.14694044D 01	0.21273077D 02	0.10766380D 01
2	1	0.19149382D 02	0.23611593D 01	0.19123254D 02	0.21389421D 01
	2	0.13574341D 02	0.20570322D 01	0.13537457D 02	0.17976043D 01
	3	0.38073100D 01	0.17369138D 01	0.36736370D 01	0.14201653D 01
	4	0.27705533D 01	0.14595580D 01	0.27971748D 01	0.10631602D 01
	5	0.30859138D 01	0.16843026D 01	0.29193945D 01	0.13553138D 01
	6	0.34768269D 01	0.20351198D 01	0.33299137D 01	0.17724876D 01
	7	0.43191137D 01	0.25354156D 01	0.47142237D 01	0.23298782D 01
	9	0.11295190D 03	0.21136809D 01	0.11294747D 03	0.18621619D 01
	10	0.40195278D 01	0.16599241D 01	0.38931504D 01	0.13248955D 01
	11	0.33126728D 01	0.14452030D 01	0.31581388D 01	0.10433725D 01
	12	0.33478028D 01	0.16597806D 01	0.31949622D 01	0.13247157D 01
	13	0.35777698D 01	0.18824659D 01	0.34351764D 01	0.15948912D 01
	14	0.46631638D 01	0.27541512D 01	0.45546837D 01	0.25661935D 01
3	1	0.26588725D 01	0.22557489D 01	0.24614978D 01	0.20219800D 01
	2	0.21691214D 01	0.17200085D 01	0.19248604D 01	0.15994339D 01
	3	0.15879606D 01	0.12625403D 01	0.12335392D 01	0.77071909D 00
	4	0.12016145D 01	0.11747250D 01	0.66624118D 00	0.61642419D 00
	5	0.12077264D 01	0.11772244D 01	0.57720235D 00	0.62117413D 00
	6	0.11784124D 01	0.11780982D 01	0.62342266D 00	0.62282352D 00
	7	0.12374134D 01	0.13728729D 01	0.72882921D 00	0.94062741D 00
	8	0.21613504D 01	0.33104613D 01	0.19160991D 01	0.31558133D 01
	9	0.19391917D 01	0.20895551D 01	0.16263495D 01	0.16347318D 01
	10	0.13139364D 01	0.13557482D 01	0.35099706D 00	0.91545244D 00
	11	0.11926366D 01	0.12350547D 01	0.65000100D 00	0.72481724D 00
	12	0.11300535D 01	0.12428773D 01	0.62652853D 00	0.73506778D 00
	13	0.11583775D 01	0.12212232D 01	0.58565949D 00	0.70093946D 00
	14	0.12970154D 01	0.13144516D 01	0.82598365D 00	0.85310204D 00

TABLE 7.9. Values of Tau and Gam for case 1. (at $\alpha = 0.05$ and $\gamma = 0.8$)

PHO NO	PT. NO	TAU x	TAU y	GAM x	GAM y
1	3	0.44745306D 01	0.29256399D 01	0.43613557D 01	0.27494306D 01
	4	0.27990772D 01	0.15175808D 01	0.26143514D 01	0.11415428D 01
	5	0.23821824D 01	0.14142303D 01	0.21621270D 01	0.10000237D 01
	6	0.21317933D 01	0.13422308D 01	0.18826956D 01	0.59531195D 00
	7	0.19353873D 01	0.12472175D 01	0.17151568D 01	0.74535321D 00
	8	0.23385264D 01	0.15278395D 01	0.26565451D 01	0.11551162D 01
	10	0.46205970D 01	0.27436184D 01	0.45110882D 01	0.25548859D 01
	11	0.23722493D 01	0.15120557D 01	0.26925438D 01	0.11341572D 01
	12	0.24419142D 01	0.13627835D 01	0.22277668D 01	0.92583961D 00
	13	0.21185934D 01	0.12678680D 01	0.18677361D 01	0.77941591D 00
	14	0.21115833D 01	0.13095853D 01	0.18597869D 01	0.84558486D 00
2	1	0.13998390D 02	0.23425417D 01	0.18972054D 02	0.21183724D 01
	2	0.13530423D 02	0.20503777D 01	0.13493423D 02	0.17899857D 01
	3	0.25256748D 01	0.14827520D 01	0.23192743D 01	0.10947846D 01
	4	0.21736512D 01	0.13381952D 01	0.19299636D 01	0.88925048D 00
	5	0.24353401D 01	0.14605885D 01	0.22205583D 01	0.10645744D 01
	6	0.28621703D 01	0.17135822D 01	0.26817939D 01	0.13915330D 01
	7	0.39181074D 01	0.20245499D 01	0.37883460D 01	0.17603415D 01
	9	0.11233939D 03	0.21022282D 01	0.11233544D 03	0.18491521D 01
	10	0.26620015D 01	0.14529560D 01	0.24670330D 01	0.10540810D 01
	11	0.23015429D 01	0.13349973D 01	0.20729447D 01	0.88443085D 00
	12	0.25897223D 01	0.14432976D 01	0.23888620D 01	0.10407247D 01
	13	0.23791045D 01	0.15860710D 01	0.26998593D 01	0.12311057D 01
	14	0.38578021D 01	0.21807187D 01	0.37259411D 01	0.19379200D 01
3	1	0.26359231D 01	0.22379624D 01	0.24388708D 01	0.20021178D 01
	2	0.21621043D 01	0.17144442D 01	0.19169494D 01	0.13925944D 01
	3	0.14778306D 01	0.12339508D 01	0.10881773D 01	0.72293471D 00
	4	0.11778415D 01	0.11442377D 01	0.62234282D 00	0.55612934D 00
	5	0.11783974D 01	0.11525633D 01	0.62339421D 00	0.57306381D 00
	6	0.11506515D 01	0.11578909D 01	0.56920909D 00	0.58370477D 00
	7	0.11850323D 01	0.12797535D 01	0.63584713D 00	0.79860448D 00
	8	0.17353664D 01	0.23227020D 01	0.14182724D 01	0.20964123D 01
	9	0.18988470D 01	0.20782332D 01	0.16141933D 01	0.18218269D 01
	10	0.12636201D 01	0.12997091D 01	0.77248672D 00	0.83020705D 00
	11	0.11527879D 01	0.11966632D 01	0.57351534D 00	0.65726913D 00
	12	0.11441123D 01	0.12009992D 01	0.55587127D 00	0.66513086D 00
	13	0.11342437D 01	0.11947869D 01	0.53526523D 00	0.65384595D 00
	14	0.12227493D 01	0.12606450D 01	0.70364471D 00	0.76761047D 00
4	3	0.18365470D 01	0.15240254D 01	0.15404236D 01	0.11500667D 01
	4	0.13554254D 01	0.13366768D 01	0.91497430D 00	0.66696389D 00
	5	0.13590831D 01	0.13631218D 01	0.92038407D 00	0.92633749D 00
	6	0.14032902D 01	0.13351560D 01	0.98449138D 00	0.88467037D 00
	7	0.14981468D 01	0.13278306D 01	0.11155464D 01	0.97357551D 00
	8	0.23613323D 01	0.18559492D 01	0.26808996D 01	0.15635048D 01
	10	0.18558367D 01	0.17165777D 01	0.15633713D 01	0.13952201D 01
	11	0.13972410D 01	0.12936811D 01	0.97584962D 00	0.82073792D 00
	12	0.14246879D 01	0.12559751D 01	0.10147538D 01	0.75991631D 00
	13	0.14369459D 01	0.12528602D 01	0.10318980D 01	0.75475744D 00
	14	0.15787671D 01	0.13814484D 01	0.12216815D 01	0.95309998D 00

TABLE 7.10. Values of Tau and Gam for case 2 (at $\alpha = 0.05$ and $\gamma = 0.8$)

PHO NO	PT. NO	TAU x	TAU y	GAM x	GAM y
1	3	0.35061011D 01	0.27675436D 01	0.33604680D 01	0.25805615D 01
	4	0.19999568D 01	0.14000575D 01	0.17320009D 01	0.97987301D 00
	5	0.19636540D 01	0.13158555D 01	0.16899517D 01	0.85526359D 00
	6	0.21350557D 01	0.13200074D 01	0.18863888D 01	0.56163775D 00
	7	0.19957911D 01	0.12331627D 01	0.17271891D 01	0.72158378D 00
	8	0.46320654D 01	0.15717165D 01	0.45228342D 01	0.12125563D 01
	10	0.41597960D 01	0.25939364D 01	0.40378092D 01	0.23934840D 01
	11	0.23324278D 01	0.13888844D 01	0.21071823D 01	0.96384639D 00
	12	0.20685516D 01	0.12717160D 01	0.18107749D 01	0.75660030D 00
	13	0.23154207D 01	0.12682620D 01	0.20883422D 01	0.78005677D 00
	14	0.19673274D 01	0.13459981D 01	0.16942187D 01	0.90095000D 00
2	1	0.13035433D 02	0.41993903D 01	0.12997020D 02	0.40791026D 01
	2	0.83632639D 01	0.23038408D 01	0.33032634D 01	0.20754957D 01
	3	0.17494254D 01	0.18055224D 01	0.14355257D 01	0.15033001D 01
	4	0.14451375D 01	0.14335670D 01	0.10432749D 01	0.10958832D 01
	5	0.23266251D 01	0.17236898D 01	0.21007581D 01	0.14039611D 01
	9	0.16701699D 02	0.29937733D 01	0.16671735D 02	0.28271260D 01
	10	0.13230602D 01	0.17776621D 01	0.15243190D 01	0.14697219D 01
	11	0.17797117D 01	0.14070897D 01	0.14722003D 01	0.98939967D 00
	12	0.26082188D 01	0.18037900D 01	0.24089013D 01	0.15012190D 01
3	4	0.23514366D 01	0.16767439D 01	0.21282044D 01	0.13459087D 01
	5	0.15658036D 01	0.15493383D 01	0.12048821D 01	0.11834058D 01
	6	0.15909006D 01	0.16566924D 01	0.12373216D 01	0.13208443D 01
	7	0.19694159D 01	0.15502853D 01	0.16966435D 01	0.11846454D 01
	8	0.18219739D 02	0.28816915D 01	0.18192276D 02	0.27026133D 01
	11	0.27048435D 01	0.18797201D 01	0.25132062D 01	0.15916494D 01
	12	0.17666432D 01	0.13787151D 01	0.14563750D 01	0.94913405D 00
	13	0.17683902D 01	0.14434606D 01	0.14584937D 01	0.10409507D 01
	14	0.19576206D 01	0.16698993D 01	0.16829375D 01	0.13373719D 01
4	1	0.48683735D 01	0.25081113D 01	0.47645630D 01	0.23001352D 01
	2	0.34304418D 01	0.14730233D 01	0.32814525D 01	0.10815718D 01
	3	0.23563145D 01	0.13444297D 01	0.21335928D 01	0.39860521D 00
	4	0.18297649D 01	0.12491684D 01	0.15323314D 01	0.74861312D 00
	5	0.17736664D 01	0.13497089D 01	0.14648865D 01	0.90648454D 00
	6	0.24275550D 01	0.17223809D 01	0.22120179D 01	0.14023538D 01
	7	0.43031493D 01	0.18725424D 01	0.41853427D 01	0.15831661D 01
	9	0.75169626D 01	0.19034677D 01	0.74501495D 01	0.16196262D 01
	10	0.25551955D 01	0.13574470D 01	0.23513877D 01	0.91796638D 00
	11	0.19198335D 01	0.12711589D 01	0.16388290D 01	0.78475792D 00
	12	0.20063629D 01	0.13283081D 01	0.17393941D 01	0.87506058D 00
	13	0.27591910D 01	0.14850843D 01	0.25716016D 01	0.10979415D 01
	14	0.46894988D 01	0.20528841D 01	0.45816371D 01	0.17928562D 01

TABLE 7.11. Values of Tau and Gam for case 3. (at α = 0.05 and γ = 0.8)

PHOTO NO	POINT NO	TAU x	TAU y	DELTA(um) x	DELTA(um) y
1	3	0.82164661D 01	0.39126310D 01	0.6902D 02	0.3287D 02
	4	0.55319011D 01	0.17813834D 01	0.4689D 02	0.1496D 02
	5	0.56926953D 01	0.16246492D 01	0.4782D 02	0.1365D 02
	6	0.66999112D 01	0.15427475D 01	0.5628D 02	0.1296D 02
	7	0.10262017D 02	0.13784459D 01	0.8620D 02	0.1158D 02
	8	0.37087531D 02	0.24156180D 01	0.3115D 03	0.2029D 02
	10	0.82507240D 01	0.38346706D 01	0.6931D 02	0.3221D 02
	11	0.64065041D 01	0.17847140D 01	0.5381D 02	0.1499D 02
	12	0.66885150D 01	0.16237793D 01	0.5618D 02	0.1364D 02
	13	0.91081493D 01	0.14005694D 01	0.7651D 02	0.1176D 02
	14	0.21296562D 02	0.14694044D 01	0.1789D 03	0.1234D 02
2	1	0.19149332D 02	0.23611593D 01	0.1609D 03	0.1983D 02
	2	0.13574341D 02	0.20570322D 01	0.1140D 03	0.1728D 02
	3	0.38073100D 01	0.17369138D 01	0.3198D 02	0.1459D 02
	4	0.29705533D 01	0.14595580D 01	0.2495D 02	0.1226D 02
	5	0.30859133D 01	0.16843026D 01	0.2592D 02	0.1415D 02
	6	0.34768269D 01	0.20351198D 01	0.2921D 02	0.1710D 02
	7	0.48191187D 01	0.25354156D 01	0.4048D 02	0.2130D 02
	9	0.11295190D 03	0.21136809D 01	0.9488D 03	0.1775D 02
	10	0.40195293D 01	0.16599241D 01	0.3376D 02	0.1394D 02
	11	0.33126738D 01	0.14452080D 01	0.2783D 02	0.1214D 02
	12	0.33473028D 01	0.16597806D 01	0.2812D 02	0.1394D 02
	13	0.35777698D 01	0.18824659D 01	0.3005D 02	0.1581D 02
	14	0.46631638D 01	0.27541512D 01	0.3917D 02	0.2313D 02
3	1	0.26568725D 01	0.22557489D 01	0.2232D 02	0.1895D 02
	2	0.21691214D 01	0.17200085D 01	0.1822D 02	0.1445D 02
	3	0.15879606D 01	0.12625403D 01	0.1334D 02	0.1061D 02
	4	0.12016145D 01	0.11747250D 01	0.1009D 02	0.9868D 01
	5	0.12077264D 01	0.11772244D 01	0.1014D 02	0.9889D 01
	6	0.11784124D 01	0.11780982D 01	0.9899D 01	0.9896D 01
	7	0.12374134D 01	0.13728729D 01	0.1039D 02	0.1153D 02
	8	0.21613504D 01	0.33104618D 01	0.1816D 02	0.2781D 02
	9	0.19091917D 01	0.20895551D 01	0.1604D 02	0.1755D 02
	10	0.13130864D 01	0.13557482D 01	0.1103D 02	0.1139D 02
	11	0.11926366D 01	0.12350547D 01	0.1002D 02	0.1037D 02
	12	0.11800585D 01	0.12428773D 01	0.9912D 01	0.1044D 02
	13	0.11588775D 01	0.12212232D 01	0.9735D 01	0.1026D 02
	14	0.12970154D 01	0.13144516D 01	0.1089D 02	0.1104D 02

TABLE 7.12. Undetected Gross Errors for case 1. (at $\alpha = 0.05$ and $\gamma = 0.8$)

PHOTO	POINT	TAU		DELTA(um)	
NO	NO	x	y	x	y
1	3	0.44745306D 01	0.29256399D 01	0.3759D 02	0.2458D 02
	4	0.27990772D 01	0.15175808D 01	0.2351D 02	0.1275D 02
	5	0.23821824D 01	0.14142303D 01	0.2001D 02	0.1188D 02
	6	0.21317933D 01	0.13422308D 01	0.1791D 02	0.1127D 02
	7	0.19853873D 01	0.12472175D 01	0.1668D 02	0.1048D 02
	8	0.28385264D 01	0.15278395D 01	0.2384D 02	0.1283D 02
	10	0.46205970D 01	0.27436184D 01	0.3881D 02	0.2305D 02
	11	0.28722498D 01	0.15120557D 01	0.2413D 02	0.1270D 02
	12	0.24419142D 01	0.13627835D 01	0.2051D 02	0.1145D 02
	13	0.21185934D 01	0.12678680D 01	0.1780D 02	0.1065D 02
	14	0.21115883D 01	0.13095853D 01	0.1774D 02	0.1100D 02
2	1	0.18998390D 02	0.23425417D 01	0.1596D 03	0.1968D 02
	2	0.13530428D 02	0.20503777D 01	0.1137D 03	0.1722D 02
	3	0.25256743D 01	0.14827520D 01	0.2122D 02	0.1246D 02
	4	0.21736512D 01	0.13381952D 01	0.1826D 02	0.1124D 02
	5	0.24353401D 01	0.14605885D 01	0.2046D 02	0.1227D 02
	6	0.28621703D 01	0.17135822D 01	0.2404D 02	0.1439D 02
	7	0.39181074D 01	0.20245499D 01	0.3291D 02	0.1701D 02
	9	0.11233989D 03	0.21022282D 01	0.9437D 03	0.1766D 02
	10	0.26620015D 01	0.14529580D 01	0.2236D 02	0.1220D 02
	11	0.23015429D 01	0.13349973D 01	0.1933D 02	0.1121D 02
	12	0.25897223D 01	0.14432976D 01	0.2175D 02	0.1212D 02
	13	0.28791045D 01	0.15860710D 01	0.2418D 02	0.1332D 02
	14	0.38578021D 01	0.21807187D 01	0.3241D 02	0.1832D 02
3	1	0.26359231D 01	0.22379624D 01	0.2214D 02	0.1880D 02
	2	0.21621043D 01	0.17144442D 01	0.1816D 02	0.1440D 02
	3	0.14778806D 01	0.12339508D 01	0.1241D 02	0.1037D 02
	4	0.11773415D 01	0.11442377D 01	0.9894D 01	0.9612D 01
	5	0.11783974D 01	0.11525633D 01	0.9899D 01	0.9682D 01
	6	0.11506515D 01	0.11578909D 01	0.9665D 01	0.9726D 01
	7	0.11850323D 01	0.12797535D 01	0.9954D 01	0.1075D 02
	8	0.17353664D 01	0.23227020D 01	0.1458D 02	0.1951D 02
	9	0.18988470D 01	0.20782332D 01	0.1595D 02	0.1746D 02
	10	0.12636201D 01	0.12997091D 01	0.1061D 02	0.1092D 02
	11	0.11527879D 01	0.11966632D 01	0.9683D 01	0.1005D 02
	12	0.11441123D 01	0.12009992D 01	0.9611D 01	0.1009D 02
	13	0.11342437D 01	0.11947869D 01	0.9528D 01	0.1004D 02
	14	0.12227493D 01	0.12606450D 01	0.1027D 02	0.1059D 02
4	3	0.13365470D 01	0.15240254D 01	0.1543D 02	0.1280D 02
	4	0.13554254D 01	0.13366768D 01	0.1139D 02	0.1123D 02
	5	0.13590831D 01	0.13631218D 01	0.1142D 02	0.1145D 02
	6	0.14032902D 01	0.13351560D 01	0.1179D 02	0.1122D 02
	7	0.14931468D 01	0.13278306D 01	0.1253D 02	0.1115D 02
	8	0.28613323D 01	0.13559492D 01	0.2404D 02	0.1559D 02
	10	0.18558367D 01	0.17165777D 01	0.1559D 02	0.1442D 02
	11	0.13972410D 01	0.12936811D 01	0.1174D 02	0.1087D 02
	12	0.14246879D 01	0.12559751D 01	0.1197D 02	0.1055D 02
	13	0.14369459D 01	0.12528602D 01	0.1207D 02	0.1052D 02
	14	0.15787671D 01	0.13814484D 01	0.1326D 02	0.1160D 02

TABLE 7.13. Undetected Gross Errors for case 2. (at $\alpha = 0.05$ and $\gamma = 0.8$)

PHOTO NO	POINT NO	TAU x	TAU y	DELTA(um) x	DELTA(um) y
1	3	0.35061011D 01	0.27675436D 01	0.2945D 02	0.2325D 02
	4	0.19999568D 01	0.14000575D 01	0.1680D 02	0.1176D 02
	5	0.19636540D 01	0.13158555D 01	0.1649D 02	0.1105D 02
	6	0.21350557D 01	0.13200074D 01	0.1793D 02	0.1109D 02
	7	0.19957911D 01	0.12331627D 01	0.1676D 02	0.1036D 02
	8	0.46320654D 01	0.15717165D 01	0.3891D 02	0.1320D 02
	10	0.41597960D 01	0.25939864D 01	0.3494D 02	0.2179D 02
	11	0.23324273D 01	0.13888844D 01	0.1959D 02	0.1167D 02
	12	0.20685516D 01	0.12717160D 01	0.1738D 02	0.1068D 02
	13	0.23154207D 01	0.12682620D 01	0.1945D 02	0.1065D 02
	14	0.19673274D 01	0.13459981D 01	0.1653D 02	0.1131D 02
2	1	0.13035433D 02	0.41998903D 01	0.1095D 03	0.3528D 02
	2	0.83632639D 01	0.23038408D 01	0.7025D 02	0.1935D 02
	3	0.17494954D 01	0.18055224D 01	0.1470D 02	0.1517D 02
	4	0.14451375D 01	0.14835670D 01	0.1214D 02	0.1246D 02
	5	0.23262251D 01	0.17236898D 01	0.1954D 02	0.1448D 02
	9	0.16701699D 02	0.29987733D 01	0.1403D 03	0.2519D 02
	10	0.18230602D 01	0.17776621D 01	0.1531D 02	0.1493D 02
	11	0.17797117D 01	0.14070897D 01	0.1495D 02	0.1182D 02
	12	0.26082133D 01	0.18037900D 01	0.2191D 02	0.1515D 02
3	4	0.23514366D 01	0.16767439D 01	0.1975D 02	0.1408D 02
	5	0.15658036D 01	0.15493383D 01	0.1315D 02	0.1301D 02
	6	0.15909006D 01	0.16568924D 01	0.1336D 02	0.1392D 02
	7	0.19694159D 01	0.15502853D 01	0.1654D 02	0.1302D 02
	8	0.18219739D 02	0.28816915D 01	0.1530D 03	0.2421D 02
	11	0.27048485D 01	0.18797201D 01	0.2272D 02	0.1579D 02
	12	0.17666432D 01	0.13787151D 01	0.1484D 02	0.1158D 02
	13	0.17683902D 01	0.14434606D 01	0.1485D 02	0.1213D 02
	14	0.19576206D 01	0.16698993D 01	0.1644D 02	0.1403D 02
4	1	0.48683735D 01	0.25081113D 01	0.4089D 02	0.2107D 02
	2	0.34304418D 01	0.14730233D 01	0.2882D 02	0.1237D 02
	3	0.23563145D 01	0.13444297D 01	0.1979D 02	0.1129D 02
	4	0.18297649D 01	0.12491684D 01	0.1537D 02	0.1049D 02
	5	0.17736664D 01	0.13497089D 01	0.1490D 02	0.1134D 02
	6	0.24275550D 01	0.17223809D 01	0.2039D 02	0.1447D 02
	7	0.43031493D 01	0.13725424D 01	0.3615D 02	0.1153D 02
	9	0.75169626D 01	0.19034677D 01	0.6314D 02	0.1599D 02
	10	0.25551955D 01	0.13574470D 01	0.2146D 02	0.1140D 02
	11	0.19198335D 01	0.12711589D 01	0.1613D 02	0.1068D 02
	12	0.20063629D 01	0.13288081D 01	0.1635D 02	0.1116D 02
	13	0.27591910D 01	0.14350343D 01	0.2318D 02	0.1247D 02
	14	0.46894980D 01	0.20528841D 01	0.3939D 02	0.1724D 02

TABLE 7.14. Undetected Gross Errors for case 3. (at α = 0.05 and γ = 0.8)

Case	Tau		Gam		Max.undetected gross errors (μm)	
	x	y	x	y	x	y
1	112.951900	3.912631	112.947470	3.782682	948.80	32.87
2	112.339890	2.925640	112.335440	2.749431	943.70	24.58
3	18.219739	4.199890	18.192276	4.079103	153.00	35.28

TABLE (7.15) Maximum Tau, Gam and Undetected Gross Errors for photogrammetric measurements (at α = 0.05, and γ =0.8)

An insight into Table (7.15) reveals that, agreeable with the findings of Chapters 5 and 6, higher internal reliability reflects higher external reliability. With one exception, all the results given in Table (7.15) are in line with computed values of $r_{max}(x)$ and $r_{max}(y)$ for the different cases. Case 3 provides the best chance of detecting gross errors in the x-axis direction, yet it manifests the lowest level of detection in the y-axis direction.

7.3.3. Simulated models for sensitivity analysis

In addition to the settlement model discussed in Section (5.6.2.3) the drift model was regarded. By drift model it is meant all the targets were to undergo displacement of the order of 10 mm in the positive direction of the X-axis, i.e. in a direction which is nearly perpendicular to the dam wall. Recalling Equation (3.59) to compute the just-detectable deformation ($c\tilde{d}$):

$$\omega^u = \frac{(c\tilde{d})^T Q_d^+ (c\tilde{d})}{\sigma_o^2} \qquad (7.3)$$

where $\tilde{d}$ is a form vector which characterises the deformation model under test.

The values of the sensitivity parameter, c, for both Settlement and Drift models are shown in Table (7.16) and graphically represented in Figures (7.8) and (7.9).

Number of cameras	Model designation	Sensitivity parameter c
3	Settlement	0.020460
3	Drift	0.005428
4L	Settlement	0.023120
4L	Drift	0.006224
4	Settlement	0.022920
4	Drift	0.004877

TABLE (7. 16) Number of cameras against the sensitivity parameter (c) for Settlement and Drift models. (at $\alpha = 0.05$ and $\gamma = 0.8$)

In general, the two figures pertain to the outcomes of Chapters 5 and 6 concerning the sensitivity analysis in such a way that the more the number of cameras, the less the sensitivity of the photogrammetric network.

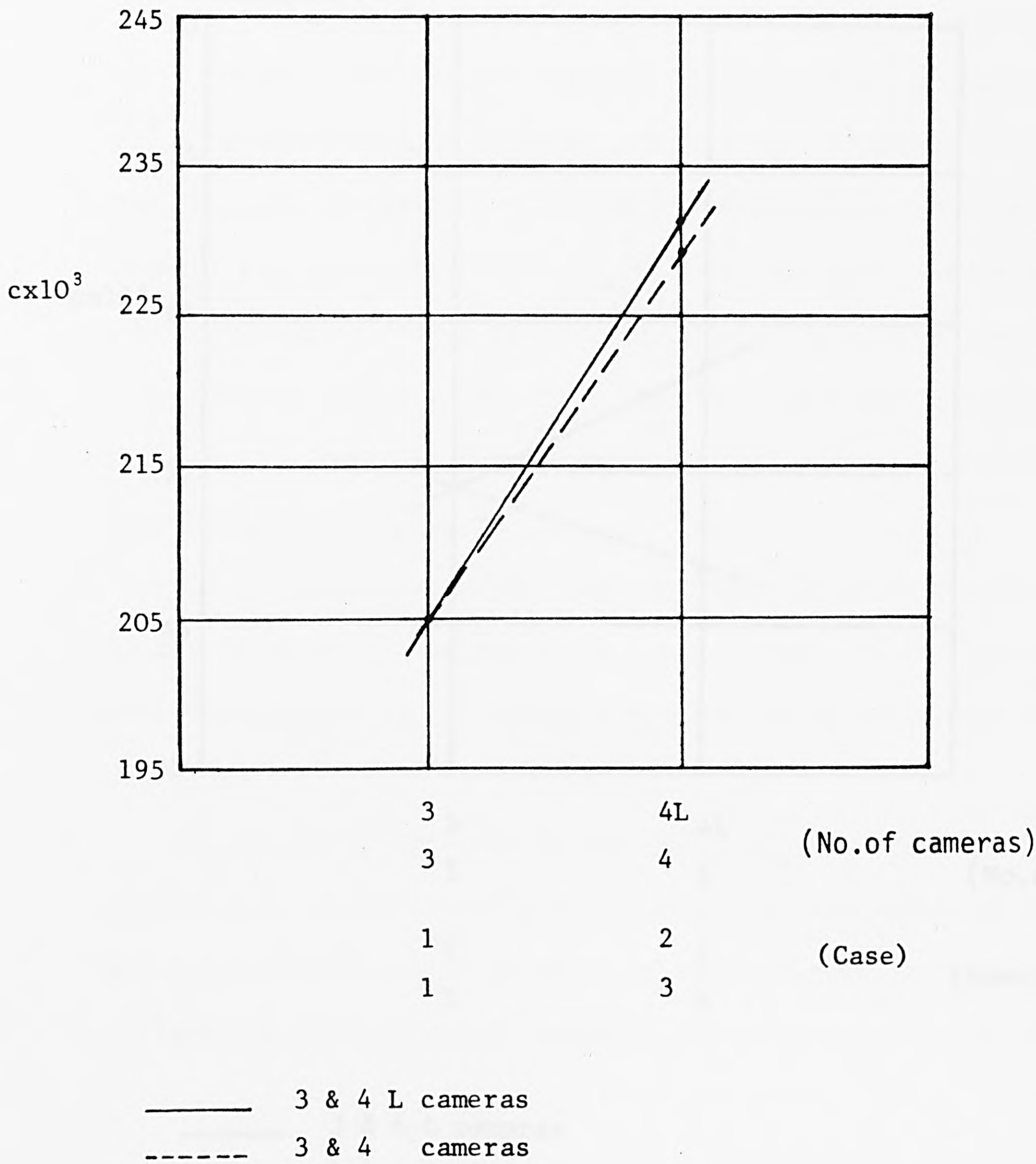


Figure 7.8. Relation between sensitivity parameter (c) and no.of cameras (Settlement model).

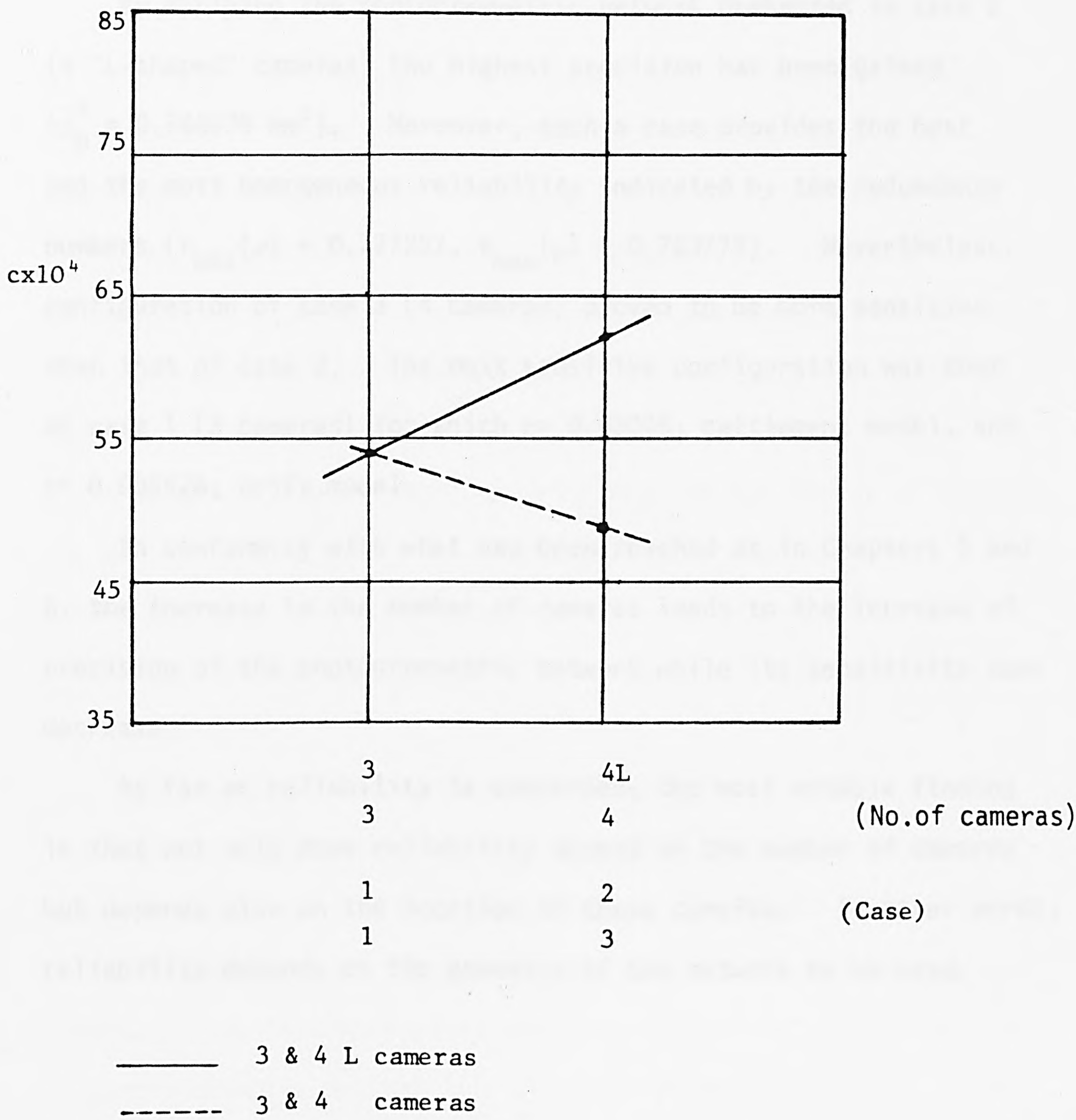


Figure 7.9. Relation between sensitivity parameter (c) and no.of cameras (Drift model).

7.4. Conclusions

By applying the photogrammetric network presented in case 2 (4 "L-shaped" cameras) the highest precision has been gained (σ_m^2 = 0.746979 mm^2). Moreover, such a case provides the best and the most homogeneous reliability indicated by the redundancy numbers ($r_{max}(x)$ = 0.777297, $r_{max}(y)$ = 0.763779). Nevertheless, configuration of case 3 (4 cameras) proved to be more sensitive than that of case 2. The most sensitive configuration was that of case 1 (3 cameras) for which c= 0.02046; settlement model, and c= 0.005428; drift model.

In conformity with what has been reached at in Chapters 5 and 6, the increase in the number of cameras leads to the increase of precision of the photogrammetric network while its sensitivity does decrease.

As far as reliability is concerned, the most notable finding is that not only does reliability depend on the number of cameras but depends also on the location of these cameras. In other words, reliability depends on the geometry of the network to be used.

CHAPTER 8

CONCLUSIONS AND RECOMMENDATIONS

Detailed discussion of results and conclusions are given in Chapters 5, 6 and 7. The following is a summary of the important findings pertaining to all aspects of the research.

1. The techniques which have been developed for design of a photogrammetric network for precision, reliability and sensitivity can be adapted to the required specifications in any deformation survey.

2. The precision is almost proportional to the number of cameras in use. However, there is an insignificant improvement in precision especially after implementing six cameras as in the case study of the cube. Such an increase cannot be justified economically.

3. The redundancy number, which lies between 0 and 1, as an indicator of reliability has been used successfully in testing the reliability of the simulated networks in Chapters 5, 6 and 7.

4. The inclusion of additional observations through increasing the number of cameras used in order to improve the reliability of a photogrammetric network is a precocupied misconception. Such an increase of number of cameras does lead to the degradation of the reliability unless the cameras are properly configured. Therefore the geometrical configuration or disposition of the cameras, is a very important factor in reliability which should be accounted for when designing networks.

5. A network, whether it be photogrammetric or geodetic, must be precise as well as reliable. In practice, one could encounter a very precise network which is not reliable at all.

6. Based on the simulation studies it was found that the main factor which affects the reliability is the number of images per object point. More intersecting rays from an object point give better reliability, i.e. larger redundancy numbers. Nevertheless it is noticed that improvement in reliability slows down considerably after 6 rays (cube case study). Therefore it is advisable not to try to improve the redundancy,in turn the reliability, if the object point already has 6 rays and its image co-ordinates display redundancy numbers of the order of 0.6 or more.

7. It is proved that at 95% confidence the networks are least sensitive to a single point movement and most sensitive to multiple point displacements depending on the hypothetical pattern of movement of that cluster of points.

8. The more cameras used the less sensitive the networks would be at 95% confidence. This can be thought of as being due to the increasing correlation between the object point co-ordinates when more cameras are used.

9. The addition of survey measurements namely, slope distances, provides a marginal improvement in precision "within" the cases of the same number cameras, while it considerably enhances the improvement in both precision and reliability "between" the different number of cameras cases. As for its effect on sensitivity it can be exhibited as follows:

(a) It does not affect the sensitivity for the settlement model of the cube case study whereas it has slightly increased the sensitivity for both the deflexion and expansion models postulated in that case.

(b) On the contrary, for the bridge case study the settlement and deflexion models lead to slightly worse values of the sensitivity parameter (c). This can be explained as follows: the slope distances are measured on the upright faces of the cube but in the bridge it is the presence of transverse distances which led to what might be an increase in the correlation between the co-ordinates of the opposite linked targets.

10. The photogrammetric monitoring system is both independent of the structure, which is a necessity when hazardous deformations occur, and economical in a sense that if the number of targets were increased it would not affect the costs too much. In addition, sometimes unexpected deformation occurs. In this case, the photographic record is a valuable source of information about the state of the object at the time of photography.

11. The set of equations used in the pre-analysis in this research is a subset of the equations used in the least squares adjustment.

12. Considerable savings in computer storage and processing time has been obtained by:

(a) The direct formation of the normal equations matrix without the need to form the observation equations.

(b) The computation and storage of only the diagonal elements of the cofactor matrix of the residuals.

Moreover, the software accepts a normal equations matrix of any size. The size would be limited by the hardware only. In other words, any number of cameras, object points, or slope distances can be accommodated.

With these thoughts in mind, the following recommendations are made.

1. It is highly recommended that after arriving at the suitable design for the three criteria, the resulting network configuration is to be executed, adjusted and checked for the goodness of the adjustment. This can be achieved by applying χ^2 and F tests. Above all, a posteriori analysis must be carried out to emphasise that the functional analysis complies with the assumptions made before.

2. The banded bordered structure of the normal equations matrix has not been fully exploited as the number of targets in this investigation is too small. If it happened that such a number is large (more than a few hundred) then further work should be oriented towards the full utilisation of that structure which would save a great deal of computer storage.

3. The effect of incorporating slope distances between the camera stations and the object points, rather than distances between the object points only, on the three design criteria is an important aspect for future research. However, it is difficult with metric cameras in current production to locate the perspective centre. It might be undertaken with the Centrax camera, produced by the National Physical Laboratory, London.

4. In-situ measurements (inclinometers, extensometers, strain gauges, etc.) can be usefully incorporated with the photogrammetric and survey measurements in a widened deformation study. This is a useful but undeveloped area of further research into design of monitoring deformation. In such a case, structural deformation models will play an important part.

APPENDIX A

SOME PROPERTIES OF GENERALISED MATRIX INVERSES

A.1. Significance of generalised inverses

Recent years have seen development for rectangular and singular matrices of an analogue of the inverse of non-singular matrices. The resulting matrices, known generally as generalised inverses, play an important role in understanding the solutions to linear equations AX=Y when A has no inverse (Searle, 1982).

Given here is a brief account of generalised inverses in addition to their properties, especially as they pertain to the least squares estimation.

It has been shown that the column rank of A, the design matrix, and hence that of N, the normal equations matrix, is deficient by 7, in a three-dimensional space, due to the orientation, translation and scale defects so that:

$$\text{rank}(A) = 6n_s + 3n_o - 7$$

in which n_s denotes the number of photographs and n_o is the number of object points. Consequently, the normal equations matrix cannot be inverted using the standard Cayley inverse. Then, generalised matrix algebra is required, particularly to search for an inverse so that the solution is unique, the primary pre-requisite property.

A.2. Definitions

Penrose (1955) on foundations laid by Moore (1920) shows that for any matrix A there is a unique matrix K which satisfies the following conditions:

$$\text{(i)} \quad AKA = A \qquad \text{(A.1)}$$

$$\text{(ii)} \quad KAK = K \qquad \text{(A.2)}$$

$$\text{(iii)} \quad (KA)^T = KA \qquad \text{(A.3)}$$

$$\text{(iv)} \quad (AK)^T = AK \qquad \text{(A.4)}$$

Such a matrix K exists and is unique no matter what the form of A is, be it square (singular or non-singular) or rectangular (Boullion and Odell, 1971; Searle, 1971). It was named after Penrose as Moore-Penrose inverse and Greville (1957) used instead the term "pseudoinverse". Table (A.1) presents some names for matrices satisfying some or all of the Penrose conditions (equations A.1 through A.4)

Conditions satisfied	Name of matrix	Symbol
i	Generalised inverse	A^-
i and ii	Reflexive generalised inverse	A^r
i,ii and iii	Left weak generalised inverse	A^w
i,ii and iv	Right weak generalised inverse	A^n
i,ii,iii and iv	Moore-Penrose inverse (pseudoinverse)	A^+

TABLE A.1. Names of matrices satisfying some or all of Penrose conditions.

A.3. Useful identities related to the Moore-Penrose inverses

The following well-known identities involving the Moore-Penrose inverses will be listed without proof (e.g. Rao and Mitra, 1971; Blaha, 1982a, 1982b, 1982c).

$$(A^+)^+ = A \quad (A.5)$$

$$(A^T)^+ = (A^+)^T \quad (A.6)$$

$$(AA^T)^+ = (A^+)^T A^+ \quad (A.7)$$

$$A^+A = AA^+ \text{ iff } A \text{ is normal, i.e.} A^TA=AA^T \quad (A.8)$$

$$(A^n)^+ = (A^+)^n \text{ iff } A \text{ is normal} \quad (A.9)$$

$$(AA^T)^+AA^T = AA^+ \quad (A.10)$$

$$A^+ = (A^TA)^+A^T = A^T(AA^T)^+ \quad (A.11)$$

$$A^+ = A^+(A^+)^TA^T = A^T(A^+)^TA^+ \quad (A.12)$$

$$A^T = A^TAA^+ = A^+AA^T \quad (A.13)$$

$$A^T = A^TAA^+ = A^+AA^T \quad (A.14)$$

$$A = (A^+)^TA^TA = AA^T(A^+)^T \quad (A.15)$$

$$(A^TA)^+ = A^+(A^+)^T \quad (A.16)$$

$$A^+A = (A^TA)^+A^TA = A^TA(A^TA)^+ \quad (A.17)$$

It is very important to notice that: $A^+A \neq I$ (A.18)

$$(A^-)^- \neq A \quad (A.19)$$

A.4. Some computational methods of Moore-Penrose inverses

A rapid review of some of the theorems for computing the Moore-Penrose inverses is provided. They are to be construed as neither the best nor the only possible. Strictly speaking, not all of which may be suitable for computer coding.

A.4.1. Computation of A^+ when independent rows or columns are identifiable

Let, if possible, A be partitioned in the form:

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$$

by a permutation of rows and columns, if necessary, such that A_{11} is an rxr matrix of rank r; A_{12}, A_{21} and A_{22} are matrices of suitable orders such that the order of A is mxn.

In such a case $A_{22} = A_{21} A_{11}^{-1} A_{12}$

Then one choice of a generalised inverse of A is:

$$A^- = \begin{bmatrix} A_{11}^{-1} & 0 \\ 0 & 0 \end{bmatrix} \qquad (A.20)$$

and the Moore-Penrose inverse A^+ (Graybille, 1969; Rao and Mitra, 1971) is as follows:

$$A^+ = \begin{bmatrix} A_{11}^T B A_{11} & A_{11}^T B A_{21}^T \\ A_{12}^T B A_{11}^T & A_{12}^T B A_{21}^T \end{bmatrix} \qquad (A.21)$$

where

$$B = (A_{11}A_{11}^T + A_{12}A_{12}^T)^{-1} A_{11}(A_{11}^T A_{11} + A_{21}^T A_{21})^{-1}$$

A.4.2. Computation of A^+ based on factorisation of matrices

(a) Rank factorisation

If the mxn matrix A is of rank r then it can be factorised in the form:

$$A = BC \qquad (A.22)$$

in which B is an mxr matrix and c an rxn matrix and both of rank r.

Although the factorisation is not unique since if Y is any non-singular rxr matrix, BY^{-1} are also factors of the specific type.

The Moore-Penrose inverse (Greville, 1960; Peters and Wilkinson, 1970; Rao and Mitra, 1971) is given by:

$$A^{+} = C^{T}(CC^{T})^{-1}(B^{T}B)^{-1}B \qquad (A.23)$$

(b) <u>The LU and related factorisations (Square matrices only)</u>

According to Peters and Wilkinson (1970) the simplest factorisation used in solving linear systems are those related to Gaussian elimination technique. These effectively give a decomposition of A of the form A = LU where L is lower triangular and U is upper triangular matrices. Either L or U can be chosen to have a unit diagonal or alternatively one can have A = LDU where D is a diagonal matrix and both L and U have unit diagonals, then:

$$A^{+} = U^{T}(L^{T}LUU^{T})^{-1}L^{T} \qquad (A.24)$$

The matrix $Y = (L^{T}LUU^{T})$ is of dimension rxr, $L^{T}L$ and UU^{T} are symmetric positive definite.

(c) <u>Singular value decomposition</u>

Any matrix A of order mxn can be written (Rao and Mitra,1971) in the following form:

$$A = \lambda_1 V_1 U_1^T + \ldots\ldots + \lambda_r V_r U_r^T \qquad (A.25)$$

in which $\lambda_1^2, \ldots\ldots \lambda_r^2$ are the non-zero eigenvalues of AA^T or A^TA and the vectors $V_1, \ldots\ldots V_r$ are orthonormal eigenvectors of AA^T and $U_1, \ldots\ldots U_r$ are orthonormal eigenvectors of A^TA corresponding to the eigenvalues $\lambda_1^2, \ldots\ldots, \lambda_r^2$. Then:

$$A^+ = \lambda_1^{-1} U_1 V_1^T + \ldots\ldots + \lambda_r^{-1} U_r V_r^T \qquad (A.26)$$

(d) <u>Diagonal reduction</u>

Searle (1971) defines A^- as a generalised inverse which satisfies the first condition of Penrose (equation A.1) such an inverse is not unique as will be shown below.

If a matrix A has order mxn then it can be reduced to a diagonal form as follows:

$$\underset{m\times m}{P} \; \underset{m\times n}{A} \; \underset{n\times n}{Q} = \underset{m\times n}{\Delta} = \begin{bmatrix} \underset{r\times r}{D} & \underset{r\times(n-r)}{0} \\ \underset{(m-r)\times r}{0} & \underset{(m-r)(n-r)}{0} \end{bmatrix} \qquad (A.27)$$

or simply, as:

$$PAQ = \Delta = \begin{bmatrix} D_r & 0 \\ 0 & 0 \end{bmatrix}$$

where P and Q are products of elementary matrices (non-singular), r is the rank of A and D_r is a diagonal matrix of order r.

Analogous to Δ, Δ^- can be defined (Searle, 1971) as:

$$\Delta^- = \begin{bmatrix} D_r^{-1} & 0 \\ 0 & 0 \end{bmatrix} \qquad (A.28)$$

then

$$A^- = Q\Delta^- P \qquad (A.29)$$

Clearly A^- as given by (A.29) is not unique because neither P nor Q by their definition is unique, neither Δ nor Δ^-. However, the Moore-Penrose inverse A^+ can be established (Searle, 1972) as:

$$A^+ = FA^T \qquad (A.30)$$

According to Searle (1971) the derivation of F can be done as follows.

Consider A^TA, for some integer t there will be a series of scalars $\lambda_1, \lambda_2, \ldots\ldots\lambda_t$ not all zero, such that:

$$\lambda_1 A^TA + \lambda_2(A^TA)^2 + \ldots\ldots + \lambda_t(A^TA)^t = 0$$

If λ_r is the first λ in this identity that is non-zero, then F is defined as (Searle, 1971):

$$F = (-1/\lambda_r)[\lambda_{r+1}I + \lambda_{r+2}(A^TA) + \ldots + \lambda_t(A^TA)^{t-r-1}]$$

APPENDIX B

INNER CONSTRAINTS APPROACH

B.1. Minimal adjustment constraints

In a free network adjustment because the columns of the design matrix A are linearly dependent the system of normal equations is singular. Any solution vector will be biased statistically as shown below.

Let the functional model be given by the linearised observation equations:

$$A\Delta x = v + b \qquad \text{(B.1)}$$

in which Δx, v and b are the u-vector of unknown parameter corrections, the n-dimensional vector of residuals and the n-vector related to the observations, respectively.

With the W matrix as the weight matrix of the observations, the classical least squares solution is:

$$\Delta\hat{x} = (A^TWA)^{-1} A^TWb \qquad \text{(B.2)}$$

$$E\{\Delta\hat{x}\} = E\{(A^TWA)^{-1} A^TWb\}$$

$$= (A^TWA)^{-1} A^TWE\{A\Delta x - v\}$$

with $E\{v\} = 0$, then

$$E\{\Delta\hat{x}\} = (A^TWA)^{-1} A^TWAE\{\Delta x\}$$

$$= E\{\Delta x\} \qquad \text{(B.3)}$$

On the other hand if A is not of full (column) rank, then the standard Cayley inverse will be replaced by the Moore-Penrose inverse (see Appendix A). Then:

$$\Delta\hat{x} = (A^TWA)^+A^TWb \qquad (B.4)$$

$$E\{\Delta\hat{x}\} = E\{(A^TWA)^+A^TWb\}$$

$$= (A^TWA)^+A^TWAE\{\Delta x\}$$

with $(A^TWA)^+ A^TWA \neq I$, then:

$$E\{\Delta\hat{x}\} \neq E\{\Delta x\} \qquad (B.5)$$

However, as only the shape defined by the co-ordinates, and not the co-ordinates themselves, is important, such a bias (equation B.5) is not of significance.

The singularity of the normal equations matrix will disappear if information about the definition of the co-ordinate system is introduced (Leick, 1982). This is accomplished by imposing as many constraints upon the adjustment as there is rank deficiency.

Adjustments which incorporate no more and no less conditions than are necessary to define the reference system lead to the so-called minimal constraint solutions. Each specific choice of constraints results in a different adjustment. Some quantities remain invariant with respect to each choice; others vary. The adjusted observations represent an invariant set (Cooper, 1980; Blaha, 1982b) while the vector of unknown parameters x, and their covariance matrix $\sigma_0^2\, Q_{\hat{x}\hat{x}}$ are identified as variants.

One widely used scheme for imposing minimal constraints to overcome the defect inherent in the normal equations matrix is to suppress d, rank deficiency, appropriate columns from the design matrix. Nevertheless, there is one pitfall in such an approach, the reference system may be ill-defined by some particular set of minimal constraints.

B.2. Inner adjustment constraints

It is advisable to define the reference system the best way bringing the inherent numerical difficulties ((ill- definition) pointed out in Section (B.1), to a minimum. In this sense "best" is interpreted as resulting the smallest trace of the covariance matrix of the unknown parameters (co-ordinates) (Blaha, 1971) and minimises $\Delta x^T \Delta x$. The set of minimal constraints which materialise this "best" co-ordinate system is called inner adjustment constraints; first developed by Meissl (1962) for geodetic networks (Ashkenazi, 1973) and reported in detail by Blaha (1971,1982a,1982b,1982c).

According to Meissl (1966) a free network can be obtained from a given arbitrary network by a Helmert transformation (Papo and Perelmuter, 1982) G^T matrix. This transformation is to be applied to the corrections to the approximate co-ordinates so that $G^T \Delta x = 0$ (inner constraints). Such a transformation can be arrived at by subjecting the object point network to three translations, three rotations and a scale change all differentially small (Fraser, 1982a) What follows is a detailed discussion concerning the construction of the G matrix.

Let $(X_i^o)^T = [X^o \; Y^o \; Z^o]_i$ denotes the co-ordinates of point i in an original co-ordinate system; and

$X_i^T = [X \; Y \; Z]_i$ its co-ordinates in a new system;

and

$$dt^T = [dt_1 \quad dt_2 \quad dt_3]$$

where

$dt_1^T = [\delta X \quad \delta Y \quad \delta Z]$ differential translations; and

$dt_2^T = [\delta\omega \quad \delta\phi \quad \delta\kappa]$ differential rotations; and

$dt_3 = \delta L$ change in scale

Let G be partitioned as $G = [G_t \quad G_r \quad G_s]$ where t, r and s refer to translation, rotation and scale respectively.

For the differential translations, one can write:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_i = \begin{bmatrix} X^o \\ Y^o \\ Z^o \end{bmatrix}_i + \begin{bmatrix} \delta X \\ \delta Y \\ \delta Z \end{bmatrix} \quad (B.6)$$

thus the differential changes due to the translations δX, δY, δZ are as follows:

$$\begin{bmatrix} dX \\ dY \\ dZ \end{bmatrix}_{\delta X, \delta Y, \delta Z} = I_{3x3} \begin{bmatrix} \delta X \\ \delta Y \\ \delta Z \end{bmatrix} \quad (B.7)$$

which gives:

$$G_{t_i} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \; ; \; dt_1 = \begin{bmatrix} \delta X \\ \delta Y \\ \delta Z \end{bmatrix} \quad (B.8)$$

The differential orientations can be found as follows:

As is known, it is possible to relate the co-ordinates of two Cartesian co-ordinate systems having the same origin by the equation:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = R^T \begin{bmatrix} X^o \\ Y^o \\ Z^o \end{bmatrix} \tag{B.9}$$

where the rotation matrix R can be written as follows (Ghosh, 1979):

$$R = R_\omega R_\phi R_\kappa$$

$$R = \begin{bmatrix} \cos\phi \cos\kappa & \cos\omega\sin\kappa + \sin\omega\sin\phi\cos\kappa & \sin\omega\sin\kappa - \cos\omega\sin\phi\cos\kappa \\ -\cos\phi\sin\kappa & \cos\omega\cos\kappa - \sin\omega\sin\phi\sin\kappa & \sin\omega\cos\kappa + \cos\omega\sin\phi\sin\kappa \\ \sin\phi & -\sin\omega\cos\phi & \cos\omega\cos\phi \end{bmatrix}$$

(B.10)

Now assuming differentially small rotations $\delta\omega$, $\delta\phi$, $\delta\kappa$ around X^o, Y^o, Z^o axes respectively it follows:

$$\begin{aligned} \sin\omega &= \delta\omega \\ \sin\phi &= \delta\phi \\ \sin\kappa &= \delta\kappa \\ \cos\omega &= \cos\phi = \cos\kappa \simeq 1 \end{aligned} \tag{B.11}$$

and neglecting second-order terms, the rotation matrix becomes:

$$R = \begin{bmatrix} 1 & \delta\kappa & -\delta\phi \\ -\delta\kappa & 1 & \delta\omega \\ \delta\phi & -\delta\omega & 1 \end{bmatrix} \tag{B.12}$$

It should be noted that R (equation B.12) is orthogonal only up to first-order terms. Therefore, the differential changes due to the rotations $\delta\omega$, $\delta\phi$, $\delta\kappa$ are given by:

$$\begin{bmatrix} X - X^o \\ Y - Y^o \\ Z - Z^o \end{bmatrix}_i = R_i^T \begin{bmatrix} X^o \\ Y^o \\ Z^o \end{bmatrix}_i - \begin{bmatrix} X^o \\ Y^o \\ Z^o \end{bmatrix} = (R_i^T - I) \begin{bmatrix} X^o \\ Y^o \\ Z^o \end{bmatrix} \qquad (B.13)$$

or in a compact form:

$$\begin{bmatrix} dX \\ dY \\ dZ \end{bmatrix}_{i \atop \delta\omega,\delta\phi,\delta\kappa} = \begin{bmatrix} 0 & -\delta\kappa & \delta\phi \\ \delta\kappa & 0 & -\delta\omega \\ -\delta\phi & \delta\omega & 0 \end{bmatrix} \begin{bmatrix} X^o \\ Y^o \\ Z^o \end{bmatrix} \qquad (B.14)$$

Thus,

$$G_{r_i} = \begin{bmatrix} 0 & Z^o & -Y^o \\ -Z^o & 0 & X^o \\ Y^o & -X^o & 0 \end{bmatrix} ; \quad dt_2 = \begin{bmatrix} \delta\omega \\ \delta\phi \\ \delta\kappa \end{bmatrix} \qquad (B.15)$$

For the change in scale, one has:

$$\begin{bmatrix} dX \\ dY \\ dZ \end{bmatrix}_{i \atop \delta L} = \begin{bmatrix} X^o\delta L \\ Y^o\delta L \\ Z^o\delta L \end{bmatrix} = \delta L \begin{bmatrix} X^o \\ Y^o \\ Z^o \end{bmatrix} \qquad (B.16)$$

Hence,

$$G_{s_i} = \begin{bmatrix} X^o \\ Y^o \\ Z^o \end{bmatrix} \quad \text{and } dt_3 = \delta L \qquad (B.17)$$

Finally, if we put $C = G^T = \begin{bmatrix} G_t^T \\ G_r^T \\ G_s^T \end{bmatrix}$ then C constitutes a set of minimal constraints, namely the set of inner constraints (Blaha, 1971). Then:

$$
\underset{(7\times3)}{G_i^T} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -Z^o & Y^o \\ Z^o & 0 & -X^o \\ -Y^o & X^o & 0 \\ X^o & Y^o & Z^o \end{bmatrix} \quad \text{(B.18)}
$$

This pattern of G-matrix repeats for each object point.

B.3. Geometrical interpretation of inner constraints

Let x_i^o denote the set of approximate co-ordinates of the ith unknown point, Δx_i denote the corrections to these co-ordinates, and x_i denote the adjusted co-ordinates, i.e.

$$x_i = x_i^o + \Delta x_i \quad \text{(B.19)}$$

For those inner adjustment constraints which correspond to G_t^T it holds that:

$$G_t^T \Delta x = 0 \quad \text{(B.20)}$$

In vector notation this can be expressed as:

$$\sum_{i=1}^{n_o} \delta x_i = 0 \quad \text{(B.21)}$$

in which n_o = number of object points in a network.

The geometrical interpretation of these conditions is that the centre of gravity of all points will not change after adjustment, i.e.

$$\sum_{i=1}^{n_o} x_i = \sum_{i=1}^{n} x_i^o \qquad (B.22)$$

The second set of constraint equations,

$$G_r^T \Delta x = 0 \qquad (B.23)$$

corresponds to the conditions:

$$\sum_{i=1}^{n_o} x_i^o \times \delta x_i = 0 \qquad (B.24)$$

If the centre of the system remains fixed, then the cross products $x_i^o \times \delta x_i$ reflect rotations of the points around the fixed centre. These constraint equations ensure that the sums of the rotations around all three co-ordinate axes are zero. Geometrically, this means that the mean orientation of the sytem of points involved in forming G_r will not change after adjustment either.

Finally, the part

$$G_s^T \Delta x = 0 \qquad (B.25)$$

with the previous notations and interpretations yields:

$$\sum_{i=1}^{n_o} (x_i^o . \delta x) = 0 \qquad (B.26)$$

which means that the average scale of the network will be held fixed.

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