MATHEMATICAL MODELLING OF INDUSTRIAL THERMOMETERS

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CONTENTS

ABSTRACT

This thesis is concerned with the mathematical modelling and identification of industrial temperature sensors. Instruments of concern are the most common : resistance thermometers and thermocouples.

Both distributed and lumped parameter models are developed and validated for a common industrial thermometer assembly. The former are based on the finite-element method which is particularly suitable for industrial thermometers because of the different materials contained in them. These distributed models enabled (i) sensitivity of response to material dimensions and heat transfer coefficient and (ii) reduced (lumped-parameter) models to be determined. The models developed were fully validated with experimental results.

The models were then applied to investigate the most sensitive parameters which contribute to the dynamics of a typical industrial thermometer. The results indicated that air gap tolerances and variations in heat transfer coeifficients lead to substantial variations in time constants.

Steady-state models which assume both radial heat flow to the sensor and axial heat flow along the well and/or sheath wall allow estimates of stem correction errors to be made. Both the reduced (dynamic) and steady-state models were simulated on MEDIEM, an interactive modelling package developed at the City University.

Design can be based on models and a methodology is presented which could form a basis for a future program for computer-aided-design. Because the dynamic and steady-state models can be simulated interactively using MEDIEM, they can conveniently be used for design, which is an iterative process.

System identification is used here to predict the response of a fluid temperature transient when response data are obtained by applying a large step in the electrical current through the sensing element leading to deliberate self heating.

Problems associated with identification of the electrical step transient data and the subsequent use of this data for prediction of fluid transients is discussed. It is concluded that the technique maybe used in the future for practical in-situ testing of industrial thermometers in, for example, the nuclear industry.

ACKNOWLEDGEMENTS

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Thanks are due to [redacted] of this department and his technical staff. [redacted] assisted greatly in the design and operation of the experimental apparatus.

I would also like to thank [redacted] of Rosemount Engineering Co. for full cooperation. This enabled the project to be highly industrially oriented. Thanks are due to [redacted] of the National Physical Laboratory, Teddington, Middlesex for assisting in carrying out some experiments at NPL. I would also like to thank [redacted] (University of Tennessee) for interest in the project.

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CHAPTER 1: INTRODUCTION

1.1 TEMPERATURE AND ITS MEASUREMENT

Temperature can simply be defined as the degree of hotness or coldness measured with respect to an arbitrary zero and a 'hot' point. This zero can be, for example, the melting point of ice at standard pressure (and for the Celsius scale). Temperature is one of the basic variables in science. Extensive discussions on the thermodynamic viewpoint of temperature can be found in almost any textbook on thermodynamics.

Measurement can be defined as the assignment of numbers to qualities in such a way as to describe them (Finkelstein (1977)). Finkelstein has given a formal definition of measurement and also the relation between the formal theory of measurement and pattern recognition. A discussion can also be found in Roberts (1978).

The classical instrument for the measurement of temperature is, of course, the common thermometer which makes use of the fact that the expansion of a fluid (usually mercury) results when it is subjected to an increase in temperature. Industrial thermometers tend to be much more complex and also work on different physical principles.

The processes of measurement and control/monitoring are, of course, inseparable. Any control system can only function if proper and correct information about the controlled system is available. This information can, in most cases, only be acquired by measurement.

After the last war, a number of publications have appeared on control. However, as Mylroi (1979) has pointed out recently, there has been negligible work in the instrumentation area in this period. In particular, temperature measurement in industry is inadequate - frequently measurements have high errors and/or poor dynamic performance.

Temperature measurement is essential for the control of many chemical, nuclear, aeronautical engineering systems. All these industrial applications require thermal transducers to be much more complex than liquid glass thermometers used in laboratories. A common industrial thermometer is described in the next chapter.

1.2 PREVIOUS WORK

There have not been many studies in the dynamic behaviour of thermal transducers. Because of this lack of knowledge, many results could not be interpreted properly. This was found to be the case when Chohan (1978 (a), (b);) carried out a study of hot-film anemometer usage in unsteady single and two-phase flow. In this case, the situation is aggravated by the complexity of the fluid dynamics and heat transfer mechanisms involved - unfortunately the same remarks apply to invasive thermal transducers like resistance thermometers.

More than twenty years ago, Moffat (1958) carried out a study of the dynamic response of thermocouples. He was concerned with bare junctions so that heat transfer occurs directly between the fluid and the junction. An analysis of experimental data led him to propose an empirical equation for the time constant of the transducer in terms of the geometry and flow conditions. In a discussion on thispaper (and following it), Glawe has laid down a formula for the thermocouple time constant without neglecting radiation:

$$\tau = \frac{\left(\frac{d}{4}\right)\rho c}{h + 4\sigma\varepsilon T^3}$$

where d is the junction wire diameter, h is the heat transfer coefficient, ρ and c are the density and heat capacity of the junction, σ is Boltzman's constant, ε the emmissivity and T is the junction temperature.

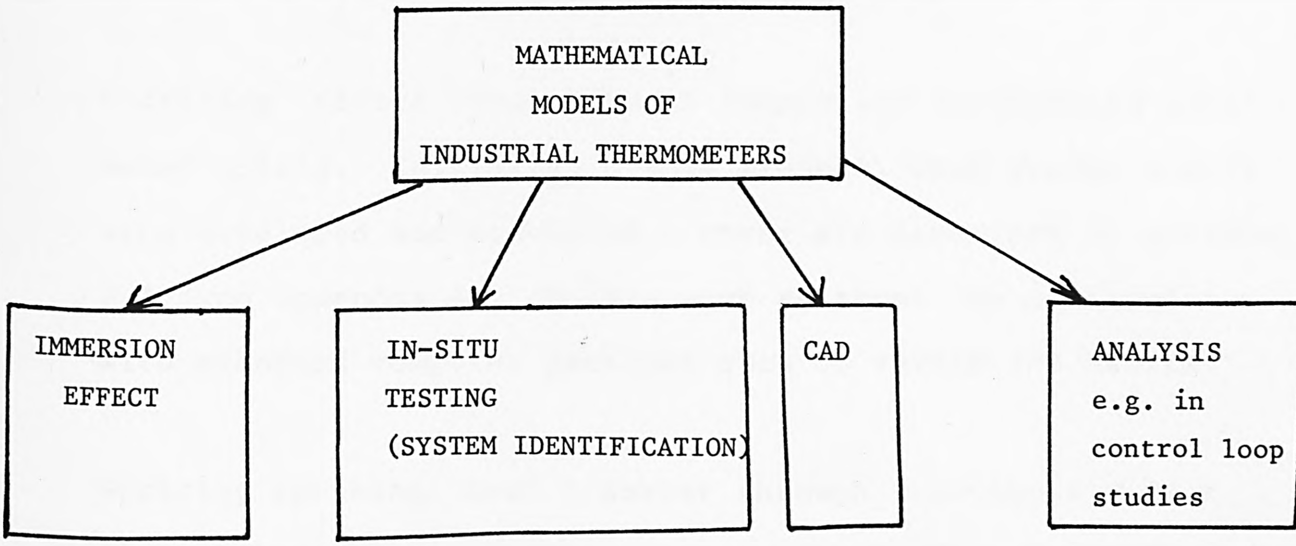
Almost all theoretical studies of thermometers assume a first or second order or (at most) third order system representation with parameters determined by arguments guessed or estimated (e.g. Lefkowitz (1955), Louis & Hartman (1964)). In fact a number of textbooks on control and instrumentation follow these lines in introducing instrument and general system dynamics (e.g. Doebelin (1975).

Perhaps the most extensive work to date has been carried out at the University of Tennessee by Kerlin and Co-workers (Kerlin et al (1978,1977)). They were concerned with in-situ testing of transducers - in particular, three prospective methods for testing were investigated: loop current step response, self-heating and noise analysis. In the loop current step response method, instead of applying a step transient from the fluid side, a step is introduced in the electrical port. An analytical transformation was developed to transform these results into the one obtained when a fluid transient is applied. This transformation is only strictly valid when the sensing element is centrally situated (or there is no heat transfer to the central core) and also when there is negligible heat transfer along the axis of the transducer assembly. Since this method was the only one capable of being used for in situ response testing, Kerlin et. al. put a lot of effort into the development of this method. This will be discussed in detail later in the thesis.

For purposes of this thesis, the time constant will refer to the time for 63.2% of the final response of any system. As Kerlin et al point out time constant is strictly only defined for first-order systems but this is relaxed for their and our work.

1.3 OUTLINE OF THESIS

The purpose of the present project was to develop mathematical models for industrial thermometers so that decisions, of industrial significance, could be arrived at with their assistance. These decisions could be related to thermometer design, system identification, control system performance analysis. This is depicted below:



Work described in this thesis provides a foundation for model-assisted decisions for (a) design, (b) in-situ testing (system identification), (c) immersion effects (errors due to different immersion lengths). Interest is restricted to immersion-type industrial thermometers (thermocouples and resistance thermometers); fortunately these are by far, the most common thermometers used for industrial temperature measurement.

The development and validation of mathematical models is described in Chapter 2, and their application in Chapter 3.

Chapter 2 contains a discussion of the physical processes connected with the use of immersion transducers. A common form of resistance thermometer is described (manufactured by Rosemount Engineering Co.). Method of testing procedure are presented.

Modelling effort involved both lumped and distributed parameter models. Initially in this project, bond graphs models were developed and simulated - these are described in section 2.4 (and appendix A). Models such as these can be simulated with standard computer packages such as THTSIM and MEDIEM.

Strictly speaking, heat transfer through thermometers is a distributed-parameter system governed by a (or a set of) parabolic type differential equations. Distributed-parameter (finite-element based) models are described next, which were simluated using another (finite-element) package, FINEL, developed at Imperial College.

These distributed-parameter models were, in turn , used to refine/redevelop lumped models and subsequent simulation using MEDIEM. These developments examplify the fact that modelling is an iterative process. (Chohan & Abdullah(1980)).

The above are dynamic models. Steady-state, two-dimensional models allow one to assess errors due to different immersion lengths of the thermometers.

Some model development work has been carried out and also presented in Chapter 2. Formulas for steady state errors could be useful at the design stage. An attempt toward this end is described here.

Chapter 3 discusses the use of models developed. A possible design procedure is described. A brief section is devoted to sensitivity of thermal response to variations in dimensions due to production tolerances (which can also occur in normal operation) and heat transfer coefficient.System identification is important in the use of thermometers because it can allow in-situ testing. Results are presented for cases when certain 'expediating' conditions (Kerlin et al (1978)) are fully satisfied and when they only hold approximately.

The final chapter deals with conclusions and suggestions for further work.

CHAPTER 2 DEVELOPMENT AND VALIDATION OF MATHEMATICAL MODELS

2.1 DESCRIPTION OF COMMERCIAL THERMOMETER

Thermometers can be classified as invasive or non-invasive. The former would include all those which require them to be inserted physically into the measured system, e.g. resistance thermometers and thermocouples. Non-invasive instruments include pyrometers (discussions can be found in Doebelin (1975) and Considine (1974)).

The two most widely used thermal transducers in industry are theremocouples and resistance thermometers. In the present thesis, attention is confined to these types, which are the major instruments used in the process industry for monitoring of temperature and for control of stationary and moving fluids. Extensive discussion of physical processes can be found in Doebelin (1975) and therefore only very brief mention will be made to this.

Resistance thermometers work on the fact that the electrical resistance of solid materials is temperature dependent. Thermocouples make use of the thermoelectric effect: if two wires of different materials are connected to form a circuit (so that there are two junctions) and if there exists a temperature difference between the two junctions, then there exists a electromotive force.

The sensing element of a thermocouple is, of course, a junction which occupies a very small volume of a transducer assembly.

This is also true, to a lesser degree, for resistance thermometers. The mass and the heat capacity of these sensing elements are much smaller than those of the other components of the thermometer and can be neglected for dynamic analysis. Therefore, the modelling philosophy developed for one type of transducer should be applicable, except for the transduction effect, to the other. Whilst in this thesis results of modelling and simulation of a particular resistance thermometer are presented, the general methodology and most of the results are applicable to thermocouples found in industry.

Finally, in this section we discuss, at some length, the resistance thermometer that has been modelled and results obtained. This is manufactured by the Rosemount Engineering company* and described in their manual and also by Johnston (1975). The sensing element (which is pure platinum) takes the form of coils (of wire diameter 0.025 mm.). Four of these coils are contained in holes drilled in a ceramic former (of 3.2 mm diameter). This former is covered with aluminium alloy of depth 0.1 mm (approximately). This subassembly is mounted in a stainless steel sheath (of external diameter 4.8 mm.(approximately) The sheath, in turn, is mounted into a well of external diameters 10 mm. The length of the ceramic former is 25 mm, but the lengths of the sheath and well are approximately ten times this. A schematic drawing of the assembly cross-section is presented as figure 1a. Notice that there are two air

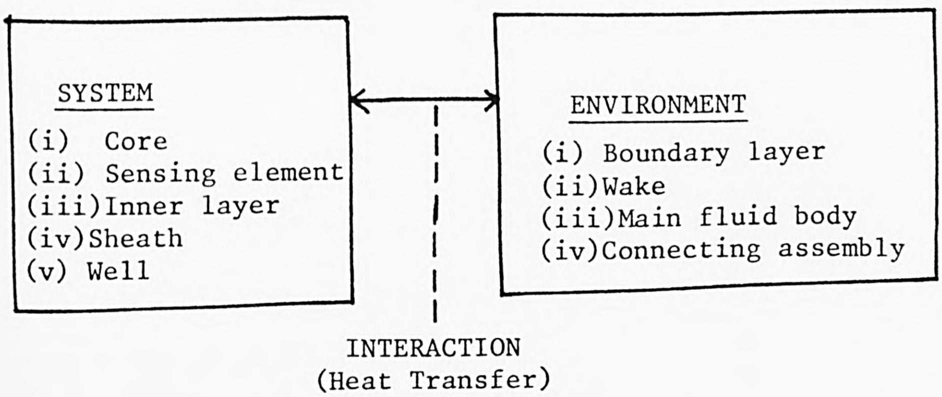
* Thermometer type E12382-392-A
  Well type E15043-225-11NAP1-316

gaps - one between the well and sheath, and the other between the sheath and the aluminium cover. As stated earlier, the sheath containing the transducer can be withdrawn from the well, which remain fixed to the pipeline or vessel containing the fluid whose temperature is to be monitored. Also note that there are air gaps where the platinum sensing coils are inserted.

2.2 PHYSICAL PROCESSES AND FUNDAMENTAL EQUATIONS

2.2.1 INTRODUCTION

In the present section, a brief discussion will be provided of the major physical processes connected with the use of thermometers. Since these transducers are used in a moving and stationary fluids, one will have to consider the interaction of the fluid dynamics/statics with heat transfer. This certainly complicates the problem. It appears wise to represent the thermometer as the system and the surrounding fluid (and other connections) as the environment. We can, then, isolate subsystems (or components) as follows:



The main interacting process between the system and environment is heat exchange between the two. This is the effect that carries information which we wish to retrieve.

Processes of interest in thermal transducer analysis and design can be broadly classified as follows:

(i) Short term processes - heat transfer, momentum transfer, thermoelectric effect, vibration

(ii) Long term processes - corrosion, fatigue, crack and crack propagation, scale formation, erosion, wear.

In the present work, we are concerned with the first of these two classes of processes. Of all the heat transfer mechanisms, three are dominant - conduction, free and forced convection. Radiation, which is neglected here, is only important at very high temperatures and rarefied gaseous conditions.

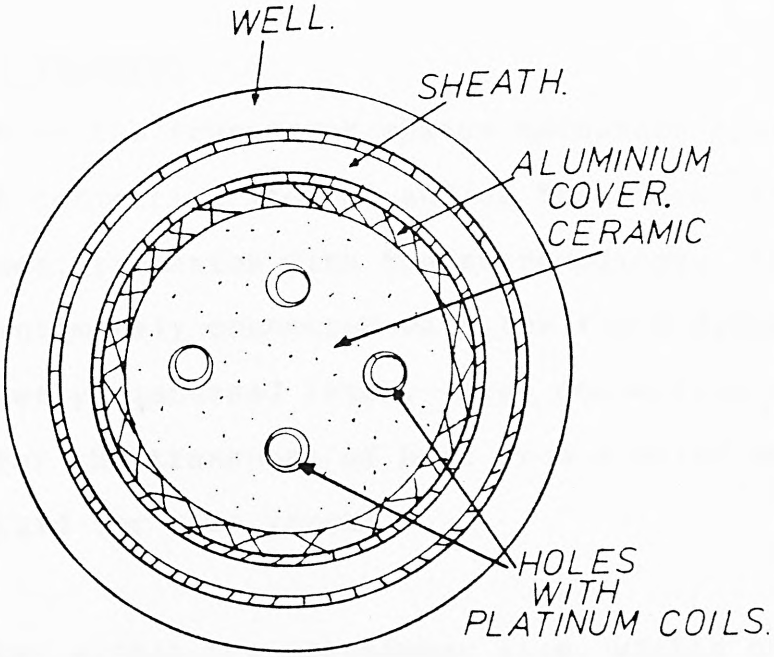


FIG. 1(a)

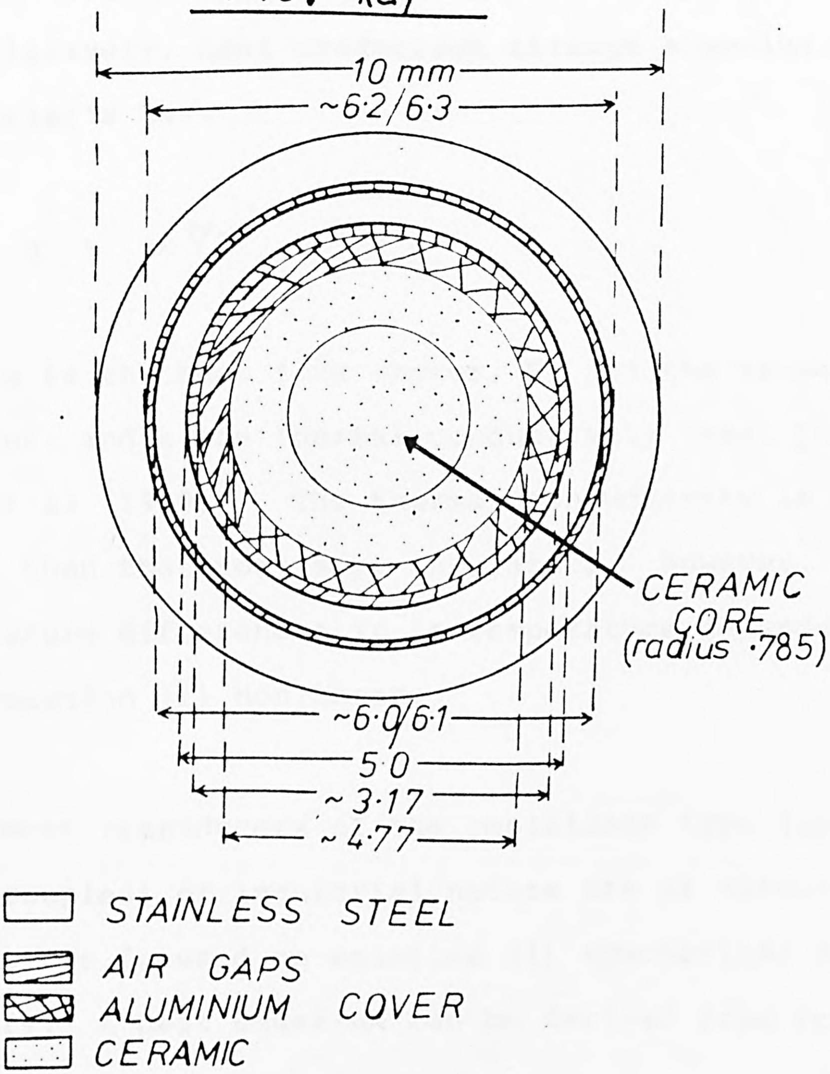


FIG 1(b)

CROSS SECTION OF 1(a) THERMOMETER PLUS WELL AND 1(b) MODEL

2.2.2 HEAT TRANSFER

The surface of the transducer system exchanges energy (heat) by means of convection (forced and/or free) and, in some circumstances, radiation with the surroundings. forced convection is intimately connected with the fluid dynamics, and this is briefly discussed later. Free convection is the major mechanism for the transport of heat from a solid surface to a static fluid (or vice versa).

Heat transfer within the transducer (i.e. within the "system", as described in section 3.1) is largely by heat conduction. Quantitatively, heat conduction through a medium is described by Fourier's Law:

$$q = -k \nabla T \qquad (1)$$

Where q is the heat flux vector, ∇T is the temperature gradient, and k the thermal conductivity (see, for example, Bird et al (1960)). The thermal conductivity is larger for metals than that for, say, insulators. However, for large temperature differences it is temperature dependent and this would make equation (1) nonlinear.

Since most transducers of the resistance type (and also most thermocouples) of industrial nature are of circular geometry, interest is focused on equation (1) specialised for such a geometry. A heat equation can be derived from Fourier's Law and the energy balance equation (Eckert & Drake (1972)), which, after assuming k to be constant is:

$$\rho c \frac{\partial T}{\partial t} = k\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2}\right) + Q \qquad (2)$$

Where the additional term Q accounts for heat addition into the system (sink or source). The other terms take the normal meaning: ρ - density, c - specific heat capacity, T - the temperature.

Almost all transducers are designed so that the length is much greater than the radius. This implies negligible heat transport in the axial direction. Assuming further that there is negligible circumferential heat transfer, equation (2) reduces to the linear, one-dimensional equation:

$$\frac{\partial T}{\partial t} = \frac{k}{\rho c}\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right) + \frac{Q}{\rho c} \qquad (3)$$

The simplest thermometer one could think of is a thin, sufficiently long wire (which can be considered to be a cylinder for purposes of analysis). A bond-graph model of this simple type of thermometer (resistance) has been constructed and simulated, and this is discussed elsewhere (Chohan and Abdullah (1980).

Most commercial transducers can be thought of as multi-layered cy linders with the sensing element - usually platinum or a junction - embedded in the interior. A multi-cylinder model for the transducer can be proposed such that for a transducer having n layers, one has the following equation for each layer:

$$\frac{\partial T_i}{\partial t} = \frac{k_i}{\rho_i c_i}\left(\frac{\partial^2 T_i}{\partial r^2} + \frac{1}{r}\frac{\partial T_i}{\partial r}\right) + \frac{Q_i}{\rho_i c_i} \qquad (4)$$

$$i = 1, \ldots, n.$$

where ρ_i , c_i , k_i are the physical properties of the i^{th} layer. The appropriate boundary conditions are:

$$\left.\begin{aligned} T_i\Big)_{r=r_i} &= T_{i+1}\Big)_{r=r_i} \\ -k_i\left(\frac{\partial T_i}{\partial r}\right)_{r=r_i} &= -k_{i+1}\left(\frac{\partial T_{i+1}}{\partial r}\right)_{r=r_i} \quad \forall i \\ \left(\frac{\partial T_1}{\partial r}\right)_{r=0} &= 0 \\ &\text{and at the boundary with the fluid:} \\ -k_n\left(\frac{\partial T_n}{\partial r}\right) &= h_1\left(T_f - T_n\Big|_{r_n}\right) \end{aligned}\right\} (5)$$

T_f is the bulk fluid temperature and h_1 is proportional to the heat transfer coefficient (see, for example, Orszik (1967)). However, solving these coupled sets of equations is time-consuming and it appears wise topursue approximate models, initially, which lead to quicker problem solution. Lumped parameter models are usually, in this respect, superior to distributed-parameter models.

In this thesis both distributed and lumped models are developed and described in this chapter.

2.2.3: FREE AND FORCED CONVECTION

Heat transfer by convection occurs when there is energy exchange between a solid surface and a fluid. The rate of heat transferred between the two media can be expressed by:

$$\frac{q}{A} = h \Delta T \tag{6}$$

where q/A is the heat flux, h is the (convective) heat transfer coefficient, and ΔT is the temperature difference. As most authors point out (e.g. Welty, Wicks and Wilson (1969), the determination of h is not at all a simple undertaking. It is dependent upon the mechanism of fluid flow, the properties of the fluid, the geometry of the specific system of interest, and perhaps some other factors.

For flow around a solid, e.g. a cylinder, the fluid particles immediately adjacent to the boundary are stationary, and a thin layer of fluid close to the surface will be in laminar flow (even if the free stream is turbulent). Because of this (momentum) boundary layer, molecular energy exchange or conductor effects will always be present and play a major role in the convection forces. If the external fluid flow is laminar, then all heat transfer between solid and fluid, or between adjacent fluid layers is by molecular means. If the flow is turbulent, then there is bulk mixing of fluid particles between regions at different temperatures and this increases the heat transfer rate. The distinction between static fluid, laminar, transitional and turbulent flow is a major consideration in convective heat transport.

There are two main classifications of convective heat transfer:

i) Free convection - this is the heat transfer process when the fluid motion results from the heat transfer and therefore is applicable when the fluid is static.

ii) Forced convection - these are those convection situations where the fluid motion is created by other forces like pumps and fans and not by the heat transfer.

There exist a few dimensionless parameters in the heat transfer literature which have assumed importance over the years and which will be relevant in this thesis. The Prandtl number is defined by:

$$Pr = \frac{\nu}{\alpha} = \frac{\mu C_p}{k} \tag{7}$$

Where ν is the momentum diffusivity, μ the viscosity, Cp the the specific heat at constant pressure, and k the thermal conductivity (all pertaining to the fluid). α is the thermal diffusivity.

The Nusselt number is given by:

$$Nu = \frac{hL}{k} \tag{8}$$

Where h is the heat transfer coefficient, L is a characteristic dimension (in the case of a cylinder - the diameter), and k is the fluid thermal conductivity. The Grasshoff number is defined by:

$$Gr = \frac{\beta g \rho^2 L^3 \Delta T}{\mu^2} \tag{9}$$

Where β is the coefficient of thermal expansion, g is the gravitational constant, and the other parameters take on the meaning assigned earlier.

It is possible to prove, by dimensional analysis, that the following general relationships hold (see, for example, Bird et al. (1960)):

(a) forced convection

$$Nu = f_1 (Re, Pr) \tag{10}$$

(b) free convection

$$Nu = f_2 (Gr, Pr) \tag{11}$$

Where Re is the Reynolds number equal to $\rho LV/\mu$ where V is the bulk fluid velocity.

If the Reynolds number and Grasshoff/Reynolds number - Prandtl number are available, then it may be possible to predict the Nusselt number from correlations developed by workers in heat transfer. The Nusselt number, in turn, gives the heat transfer coefficient which can be used to calculate resistances (on the fluid side) for heat transfer whcih are required for simulation of lumped models described later in this chapter.

For forced convection, a number of correlations can be expressed generally as:

$$NU = (A + B\,Re^{n}) \tag{12}$$

where A and B are functions of the Prandtl number (e.g. Comte-Bellot (1976). A typical expression is:

$$Nu = 0.42\,Pr^{0.26} + 0.57\,Pr^{0.33}\,Re^{0.5} \tag{13}$$

For free convection, a number of correlations can be expressed as:

$$Nu = C\,(PrGr)^{n} \tag{14}$$

where C and n are constants and $n = 0.25$ for the GrPr product in the range $10^3 - 10^8$ and C equals about 0.5 for cylinders (Schlichting (1968), Comte-Bellot (1976)). Hatton et al (1970) propose the following:

$$Nu = 0.36 + 0.52\,(GrPr)^{0.25} \tag{15}$$

This expression (except for the first term) compares well with that presented in Geb hart (1971).

$$Nu = 0.59\,(Gr\,Pr)^{0.25} \tag{16}$$

Schlichting (1968) presents two expressions for Nussett number (for free convection about vertical cylinders), each corresponding to a limiting value of the Prandtl number:

$$\begin{aligned} Nu &= 0.8\,(Gr\,Pr)^{0.25} \\ Nu &= 0.67\,(Gr\,Pr)^{0.25} \end{aligned} \tag{17}$$

Schclichting also presents a graph of experimental results and theoretical calculations (fig. 12.25 of Schlichting (1968)).

Analogous discussion for forced convection can be found in any textbook on heat transfer (for example Gebhart (1972)).

2.2.4 HEAT TRANSFER COEFFICIENT CALCULATIONS

It was pointed out above that the product of the Grashof and Prandtl number (the Rayleigh number (Ra) is required:

$$Gr\ Pr = Ra = aL^3\ (t_o - t_\infty) \tag{18}$$

where a is a factor denoting a combination of physical properties

$$a = \frac{g\ C_p{}^2 \rho^2 \beta}{\mu k} \tag{19}$$

This factor has been tabulated, for different temperatures, by Gebhart (1971) in British Units ($ft^{-3}\ {}^{O}F^{-1}$). The average temperature depends, of course, on the experiment.

The well (outer cyclinder) analysed in this project had a diameter of 10 mm and an effective length of 60 mm. Because of the large length, the cylinder can be considered to be a flat plate. Typical calculations are presented below, based on flat plate correlation:

$$Nu_L = \frac{4}{3}\ F(Pr)\ (Gr)^{0.25} \tag{20}$$

(see p. 336 of Gebhardt). The functions F(Pr) can be found from a graph and table presented by Gebhardt. For an average temperature of 70° (say plunging a thermometer from room temperature to boiling water).(Graph of Function F(Pr) and table for the factor 'a' presented in Appendix B).

$$Ra_L = 8.383 \times 10^7$$

for a 6 cm. length of cylinder, giving

$$Nu_L = 63.01$$

$$= 6\ 82.62 \quad w/m^2C$$

2.3 LUMPED PARAMETER MODELLING

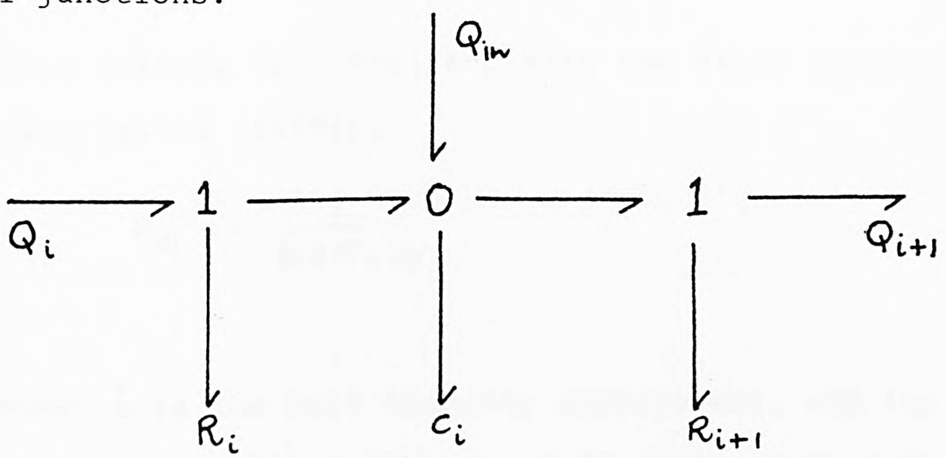
Since our interest is mainly in mechanistic (rather than 'black-box') modelling, one has to carry out as a precursor, an analysis of the physical processes connected with the use of thermal transducers -not just a catalogue of available results from a limited literature search. This has been presented above (section 2.2). Since experiments involved static water, only the case of free convection was examined in any detail.

It has been recognised for some time that a common concept for many systems is power i.e. these systems handle power. Power has connected to it two conjugate variables - the so called through and across variables. For heat flow, these correspond to entropy flow rate and temperature, respectively. For bond-graphs, each bond represents a power flow i.e. rate of energy flow (e.g. Karnopp and Rosenberg (1975). Conventionally thermal modelling is conducted with a pair of psuedo variables, these being heat flow rate (through variables) and temperature difference (across variable). The product of these variables is not power. Indeed heat flow itself is power. In this work we adopt these psuedo variables. Since the information desired from a thermometer is carried by heat diffusing from a surrounding medium to the sensing element, it would appear that bond-graph and network modelling would be beneficial here.

The thermometer was decomposed into the following subsystems:

1. Outer well, 2. Air gap, 3. Steel sheath, 4. Air Gap, 5. Aluminium cover, 6. Ceramic former containing the sensing element (here four sets of platimun coil). Subsystems 3, 4, 5, 6 form the internal sensor assembly. As is well known in system dynamic studies, there is no unique decomposition of a system. For exmaple, item 6 above has been further decomposed into two components here - an outer (annualar) ceramic and an inner cylindrical core (the outer radius of the annular region is twice that of the core). An important point that is made here is that the dimensions of the air gaps vary because of tolerances inherent to manufacturing processes. Dimensions were given in the first section (2.1).

In the present theses, effects of connecting assembly are neglected and only radial energy flow is considered for dynamic analysis. This was shown to be a very reasonable assumption by a number of simulations. The effects of the environment are represented by a fluid temperature and a resistance to heat flow to the well surface (which we shall hereafter refer to as boundary layer (BL) resistance). Each of the seven subsystems described in the last paragraph were considered as 'lumped' elements. Each 'lump' can, then, be represented (for the heat conduction process) by a bond-graph (see Karnopp and Rosenberg (1975), p.114) using 0 and 1 junctions:



where the R's and C represent the respective resistance and capacitance and Q_{in} is the heat generated (e.g. electrically) or added from the outside of the 'lump'. For each 'lump' the resistance was taken to be the steady-state value corresponding to an annular region of a cylinder (Myers (1971)):

$$R_i = \frac{\mathrm{In}\left(\dfrac{r_{i+1}}{r_i}\right)}{2\pi kL} \quad °C\ J^{-1}s \qquad (21)$$

where r_i , r_{i+1} are the inner and outer radii of the annulus, and L the length of the cylinder which contains the sensing element. The subsystem capicitance is just:

$$C = mc \qquad J°C^{-1} \tag{22}$$

where m is the mass and c the (specific) thermal capacity. The boundary-layer resistance was evaluated as:

$$R_{BL} = (Nu.\ k\pi L)^{-1} \qquad °CJ^{-1}s \tag{23}$$

This follows from the fact that the fluid film resistance is (see Kerlin (1978)):

$$R_{BL} = \frac{1}{h \cdot 2\pi r_o L} \tag{24}$$

where h is the heat transfer coefficient, and the above expression follows from the definition of Nussett number. (equation (8)).

Most of the experiments carried out are for still water in the range 50 - 100°C. The product of the Grashoff number and the Pradtl number was estimated to be about 10^5(Gr x Pr $10^{5)}$ and this gives from formula equation (16), a nusselt number of

$$Nu \approx 10.0 \tag{25}$$

Taking for water in the above temperature range, an average value of 0.65 for thermal conductivity (k), then the fluid side resistance is calculated to be:

$$R \approx 2.0 \tag{26}$$

Here the effective length of the cylinder is 0.025 m, corresponding to the approximate length of the platinum coils. The calculated resistances and capacitances for each of the annular regions are as follows:

well wall

$$R = 0.0598 \tag{26}$$

$$C = 4.36 \tag{27}$$

Air Gap

For this subsystem, results have shown that it is wise to consider the possible range of values the resistance can take:

$$R_{max} \approx 12.8 \tag{28}$$

$$R_{min} \approx 4.0 \tag{29}$$

$$C = (10^{-5}) \text{ - negligible} \tag{30}$$

Sheath

$$R = 0.0232 \tag{31}$$

$$C = 0.79 \tag{32}$$

Air Gap

As for the first air gap, we look at the possible range of resistance values. It is expected that at the manufacturing level, every attempt is made to reduce these air gaps even to below those quoted for some transducers.

$$R_{min} \approx 1.64 \tag{33}$$

$$R_{max} \approx 11.40 \tag{34}$$

$$C \quad O(10^{-5}) \text{ - negligible} \tag{35}$$

Aluminium Cover

$$R = 0.0123 \tag{36}$$

$$C = 0.537 \tag{37}$$

Ceramic annulus

$$R = 0.152 \tag{38}$$

$$C = 0.146 \tag{39}$$

Ceramic Core

It can be shown that the resistance of a cylinder is given by:

$$R = \frac{\frac{1}{2} r_o}{kA} = \frac{1}{4\pi kL} \quad \text{where} \quad A = 2\pi r_o L \text{ (surface area)} \tag{40}$$

Then,

$$R = 0.109 \tag{41}$$

$$C = 0.049 \tag{42}$$

A lumped model is presented in appendix A where typical simulation results are presented as time constant (i.e. time for response to a step change reach 63.2% of the final value) against a resistance. This early work was reported in a progress report (Chohan and Abdullah (1980)).

Before we leave these lumped models, it is wise to see if useful conclusions can be derived from the results.

The expression for thermal resistance (eqn (21) can be written as:

$$R_i = \frac{\ln\left\{\dfrac{r_i + (r_{i+1} - r_i)}{r_i}\right\}}{2\pi kL}$$

$$\text{or :} \quad R_i = \frac{\ln\{1 + d\}}{2\pi kL} \tag{43}$$

$$\text{where} \quad d = \frac{(r_{i+1} - r_i)}{r_i} \tag{44}$$

Equation (50) can be expanded as:

$$R_i = \frac{d - \tfrac{1}{2}d^2 + O(d^3)}{2\pi kL} \tag{45}$$

For small changes in <u>fractional</u> radial widths:

$$R_i \simeq \frac{d}{2\pi kL} \tag{46}$$

$$\text{or} \quad \delta R_i \simeq \frac{\delta d}{2\pi kL} \tag{46b}$$

Previous results (Appendix A; Chohan & Abdullah (1980)) suggest following relations for small changes.

$$\frac{\partial \tau}{\partial R_i} \simeq C_i \quad \text{Where } C_i \text{ is a constant} \tag{47}$$

with

$$C_{i+1} > C_i \qquad \text{if} \quad r_{i+1} > r_i \tag{48}$$

or:
$$\delta\tau \simeq c_i\,\delta R_i \qquad (49)$$

substituting $\overset{43}{(53\cancel{b})}$ into $\underset{49}{\overset{\cancel{46}}{(56)}}$:

$$\delta\tau \simeq \frac{c_i}{2\pi kL}\,\delta d \qquad (50)$$

This result implies that for the same material, changes in dimensions in the <u>outer</u> layers results in greater changes in the effective time constants.

2.4 EXPERIMENTAL RIG

Since this was the first project of its kind in the present department, it is not surprising to find that a great deal of time and effort was spent on the experimental apparatus. Only the final apparatus, which was used to obtain results presented later, is described. An earlier form of the apparatus , comprising a modified Wheatstone bridge, was used to obtain results presented in a conference (Chohan (1981)).

The objectives of the apparatus were:

i) to provide a stable, controllable current supply of between 1 mA and 50 mA to a 100 Ω nominal resistance thermometer.

ii) to measure the changes in output voltage resulting from either

a) a change of thermometer environment temperature (i.e. a fluid temperature transient denoted by FT).

b) a change in (self-heating) current (i.e. an electrically produced transient donoted by ET).

The objectives were realised as follows (see figure 2):

1) An emitter follower transistor provides a 100% feedback circuit. The base input e.m.f. is controlled by a potentiometer supplied by a zener stabilised e.m.f. The (variable) resistance thermometer is connected in the collector circuit so that the resistance changes have a minor effect on the current.
2) The e.m.f.'s from the resistance thermometer and a stable resistor adjusted to be equal to the initial value of the thermometer resistance are compared and the difference e.m.f. amplified to be in the range 0 - 1.0 V.

Operational Procedure

The constant current circuit requires that the base supply is stabilised by the zener and that the collector voltage is above the base voltage. It was estimated that Vcc (Vcc = common supply to transistor) is about 25V for a maximum of 50 mA and a 20V source is adequate for a maximum current of 20mA.

The two preset resistors (resistance decade boxes) have to be adjusted to be equivalent to the resistance between the e.m.f. tappings of the resistance thermometer at the initial condition. This can be done by adjusting them so that a small transient (i.e. impulse) change in the current does not cause the output voltage to change. The reason that this was done was to ensure preset resistors were exactly equal to the thermometer resistance, the large current being applied for such a small time that there could not be any appreciable change in the thermometer resistance due to self heating. The initial settings should be with the larger resistance set at 5,550 ohms and the lower resistance at 100 ohms. The latter resistor should be adjusted so that an increase of 1 ohm causes the voltage change to reverse as current is increased. The larger resistor can then be reduced until $\frac{\delta V}{\delta i} = 0$.

The R_2 should then be adjusted to make the output voltage zero. It may be necessary to re-adjust the resistance boxes. The amplification can be increased by adjusting resistor 4.70 K.

Having set the apparatus, the appropriate experiments were carried out, viz.

i) FT - where the thermometer (with or without a well) is plunged into a hot water bath (at a temperature of about 80 - 100°C).

ii) ET - where the sensing current is stepped up to a higher current with the thermometer (with or without a well) in a constant temperature water (at room temperature).

The vessel used for holding hot or cold water was lagged so as to minimise heat loss and thus to maintain the temperature effectively constant.

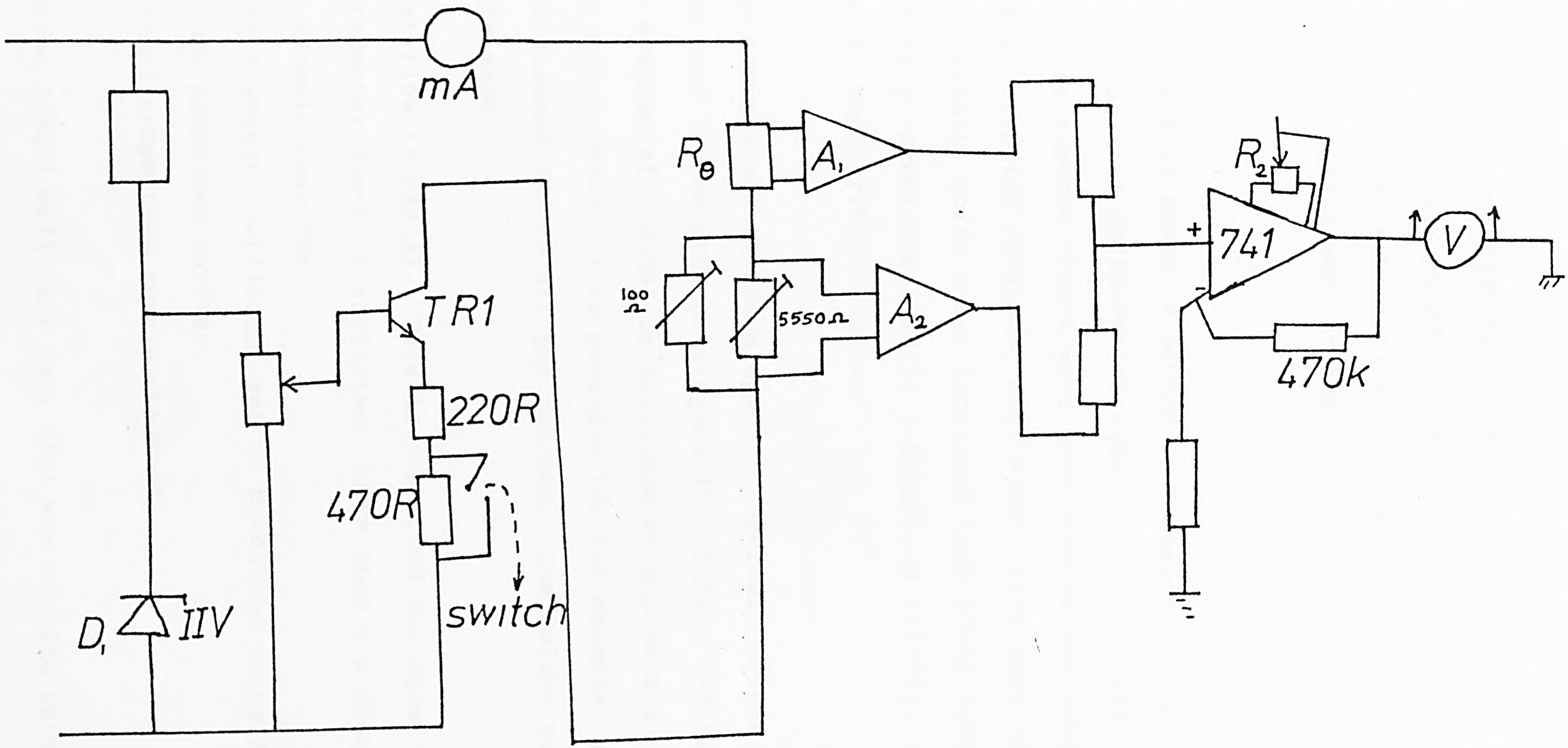


Figure 2. CIRCUIT DIAGRAM OF RESISTANCE THERMOMETER APPARATUS

2.5 DISTRIBUTED PARAMETER MODELLING

The advantage of distributed parameter models is, of course, they are capable of representing the real system very accurately . This is, as is well known, at the expense of computational load.

In the past twenty years, a method is freqeuntly used for solution of partial differential equations - the finite element method. Vast amounts ofwork have been done on the subject and a number of computer packages, e.g. FINEL, have been developed . Not surprisingly, quite a few excellent text books have appeared (e.g. Zienkewicz (1979) , Segerlind (1977)), to which we refer to for further details.

A finite element package developed at Imperial College, London, has been used in the present project (Hitchings, 1981). A typical program of a model is presented as Appendix B. By editing this program, it is possible to, for example, look at response changes due to air gap changes. The following assumptions were made:

(i) heat flow is radial. This is justified for cases where the thermometer depth is sufficient (more than 5 x diameter) and for dynamic response.
(ii) heat transfer coefficient can be predicted using flat plate correlation presented earlier.
iii) thermal properties remain constant.

Each region (steel well, air gap, etc) was divided into one four-noded axisymmetric line element with appropriate material properties k (thermal conductivity) and c (thermal capacity). At the well (or sheath if well is not used) surface, a surface heat transfer element was added with an appropriate value of h (the surface heat transfer coefficient). The resulting model (initial) had 8 elements and a total of 22 nodes: (1) heat transfer element, (2) well, (3) air gap, (4) sheath, (5) air gap, (6) aluminium cover, (7) first half (annular) ceramic, (8) ceramic core. Some results from simulation of this model are presented in Appendix A (Figures 3,4). Significant results are: a linear variation of the 'time constant' with the air gaps a_1 (the air gap nearest the fluid) and a_2 (the inner air gap) and a non-linear variation with the heat transfer coefficient h. All Finite-Element (i.e. distributed-parameter) models will also be referred to as FINEL. Earlier results had shown that air gaps were important in the dymnamic response of thermometers. The question next arises: does the air space around the platinum coils have an effect? To incorporate the effects of these air spaces in a one-dimensional model, the air spaces were replaced by an air gap of width such that the volume was the same as that for the air spaces. The average platinum sensing wire temperature was taken to be that of the third node of this (extra) element . The number of elements increases,of course, to nine (with the extra air gap between the annular ceramic and the core ceramic) and the total number of nodes is 25. Figure lb shows a schematic representation of the model.

It is convenient to enter the data in the FINEL model in mm units and present them as follows:

$$h : \xi_1 \; \xi_2 \; \xi_3 \; \xi_4 \; \xi_5 \; \xi_6 \; \xi_7 \; \xi_8 \; \xi_9 \; \xi_{10}$$

where $\xi_1 - \xi_2$ accounts for the heat transfer element, $\xi_2 - \xi_3$ the well element, etc. It has been assumed that thermal properties remain constant. The ξ_i are in mm units and the heat transfer coefficient h in J sec °K^{-1} mm^{-2}. The ordered set - $\{\xi_1 \ldots\ldots, \xi_N\}$ will be referred to as a parameter set. Refer to Appendix B for a listing of a FINEL model where a typical parameter set is used.

It is not possible to confidently determine the air gap widths and therefore this will have to be estimated. This can be done by trial and error and from a knowledge of general sensitivity analysis. A model validity can be obtained if both fluid transient and electrical transient results are found to satisfactorily agree with experiments. One can go further and carry out these procedures for both sheath-plus-well system and sheath alone. (i.e. thermometer alone).

Two thermometers were tested of the same type (described in the last chapter). These will be referred to as A and B, without the wells, and AA and BB with the wells.

Results for A and AA are shown in figures 3a,3b for thermometer alone (fig. 3a), the electrical transient results gave:

$$\tau_{ET}^{MODEL} = 11.5 \text{ sec.}$$

$$\tau_{ET}^{EXPERIMENT} = 11.6 \text{ sec.}$$

The corresponding fluid temperature step change results are:

$$\tau_{FT}^{MODEL} = 19.6 \text{ sec.}$$

$$\tau_{FT}^{EXPERIMENT} = 19.6 \text{ sec.}$$

As can be seen, agreement between model and experiment is excellent.

For the thermometer in well system (AA), the electrical transient results gave

$$\tau_{ET}^{MODEL} = 43.1 \text{ sec.}$$

$$\tau_{ET}^{EXPERIMENT} = 38.6 \text{ sec.}$$

The agreement (Fig.3b)is not as good as for the thermometer. However, the model reproduces the electrical transient data up to about 75% of the final value.

The corresponding results for fluid transient are:

$$\tau_{FT}^{MODEL} = 75.6 \text{ sec.}$$

$$\tau_{FT}^{EXPERIMENT} = 75.7 \text{ sec.}$$

Not only the 'time constants', but the entire time historics agree very well. There is some deviation during the late period, which is insignificant and can be attributed to axial and lead wise conduction.

For thermometer B the results for thermometer alone are (Figures 4a):

$$\tau_{ET}^{MODEL} = 9.95 \text{ sec.}$$

$$\tau_{ET}^{EXPERIMENT} = 9.4 \text{ sec.}$$

The agreement is very good throughout the time history except, to a lesser extent, during the late phase. For the fluid transients:

$$\tau_{FT}^{MODEL} = 17.3 \text{ sec.}$$

$$\tau_{FT}^{EXPERIMENT} = 17.2 \text{ sec.}$$

For the thermometer in well system, (figure 4b), the results are:

$$\tau_{ET}^{MODEL} = 28.5 \text{ sec.}$$

$$\tau_{ET}^{EXPERIMENT} = 27.5 \text{ sec.}$$

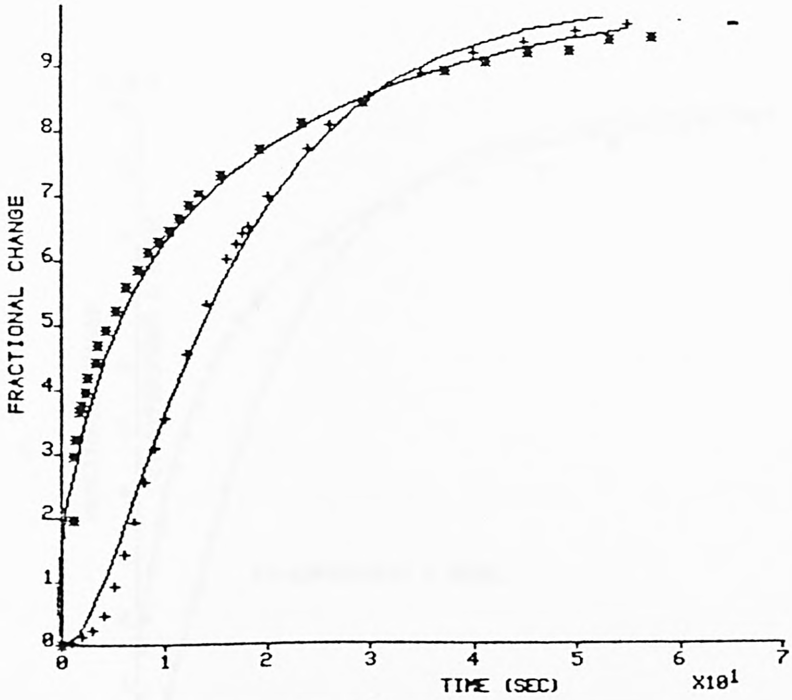


FIGURE 3(a)

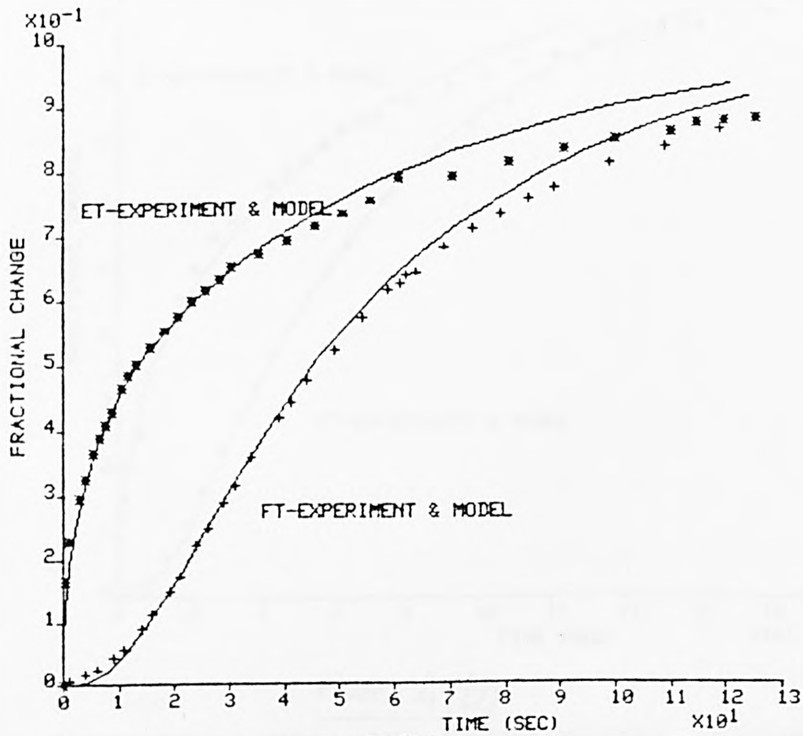


FIGURE 3(b)

MODEL (CONTINUOUS LINE) AND EXPERIMENTAL (CROSSES AND HASHES) RESULTS FOR THERMOMETER A (ABOVE) AND THERMOMETER PLUS WELL(BELOW)

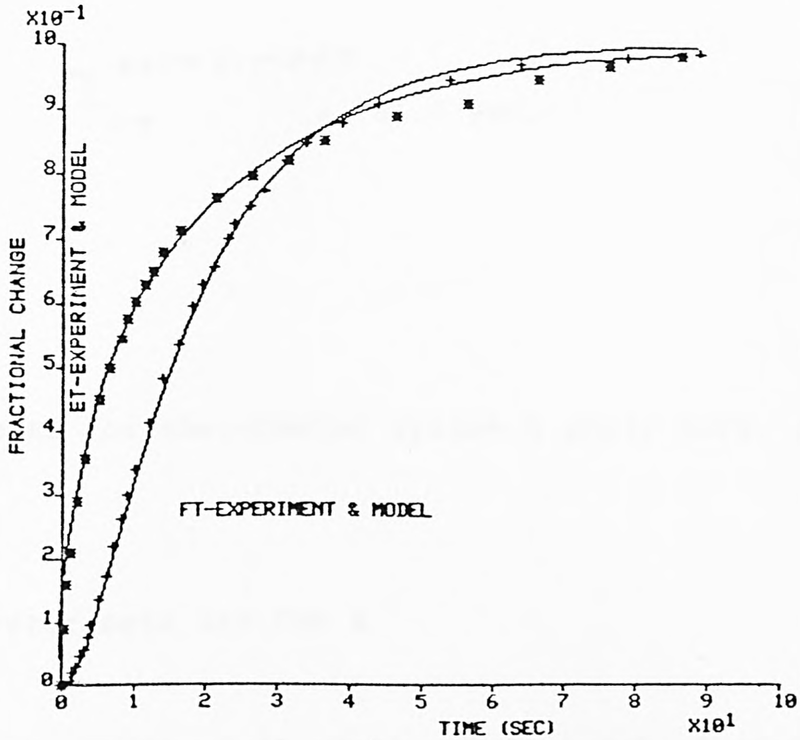


FIGURE 4(a)

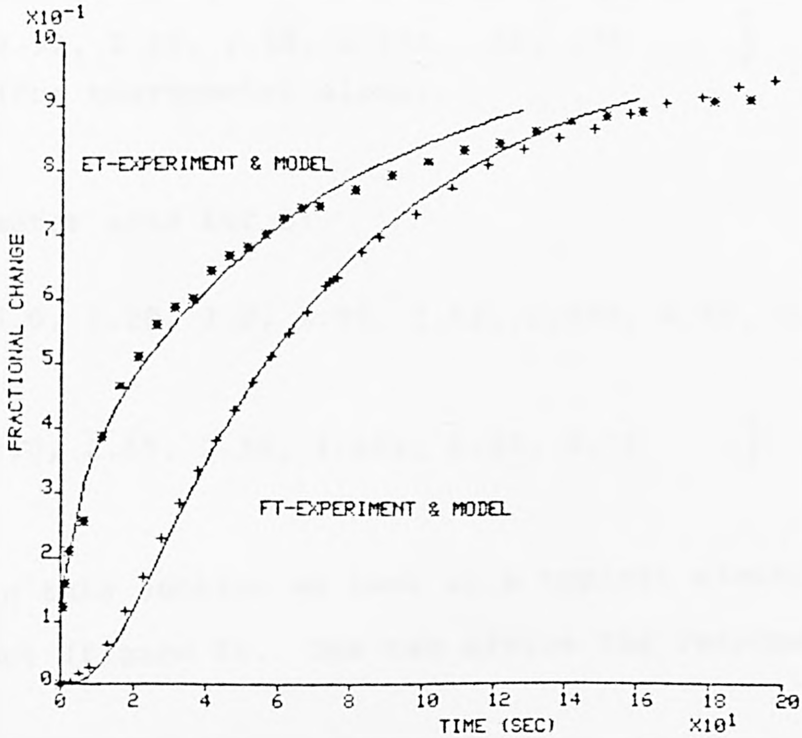


FIGURE 4(b)

MODEL (CONTINUOUS LINES) AND EXPERIMENTAL (HASHES AND CROSSES) RESULTS FOR THERMOMETER B (ABOVE) AND THERMOMETER - PLUS WELL (BELOW)

$$\tau_{FT}^{MODEL} = 59.1 \text{ sec.}$$

$$\tau_{FT}^{EXPERIMENT} = 61.6 \text{ sec.}$$

The comments for thermometer system A apply here, as can be seen.

The parameter sets are for A

$$\left\{ 5.0,\ 3.220,\ 2.92,\ 2.55,\ 2.48,\ 1.585,\ 0.85, 0.75 \right\}$$

and

$$\left\{ 2.92,\ 2.55,\ 2.48,\ 1.585,\ .85,\ .75 \right\}$$

(for thermometer alone).

The parameter sets for B:

$$\left\{ 5.0,\ 3.20,\ 3.0,\ 2.55,\ 2.50,\ 1.585,\ 0.85,\ 0.75 \right\}$$

$$\left\{ 3.0,\ 2.55,\ 2.50,\ 1.585,\ 0.85,\ 0.75 \right\}$$

Finally in this section we look at a typical electrical transient (ET) output (figure 5). One can divide the response into three zones:

(a) early time - to account for recovery from 0% to around 10 - 15%.

FIGURE 5. POSSIBLE DISSECTIONOF ELECTRICAL TRANSIENT (ET) DATA INTO THREE TIME PERIODS.

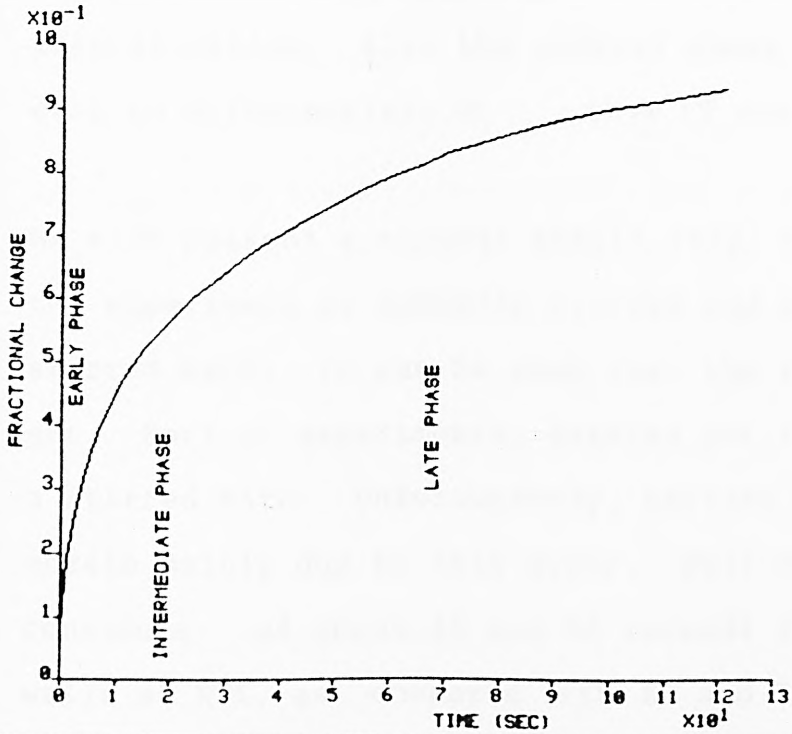
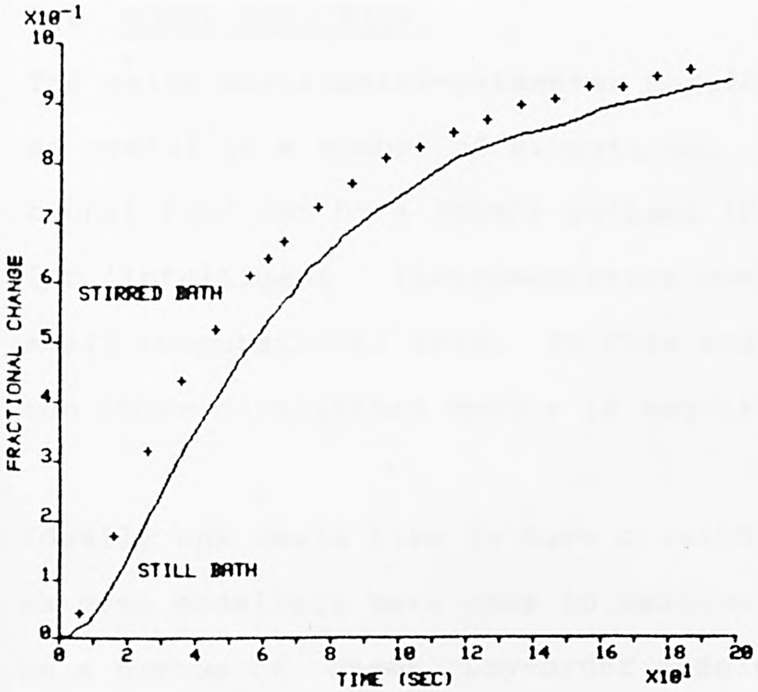


FIGURE 6. COMPARISON OF THERMOMETER PLUS WELL SYSTEM RESPONSE FOR A FLUID TEMPERATURE STEP (FT) FOR STIRRED AND STILL BATH.



(b) Intermediate time - the region approx 15% - approx 60%

(c) Late time - the region for recovery greater than 65%.

This dissection is convenient for later discussion concerning identification. Also the general shape of ET curve makes it easy to differentiate it from FT curve.

We also present a typical result (FIg. 6) when the bath in the experiment is manually stirred and compare it with non-stirred bath. It can be seen that the results are quite different. Earlier experiments, carried out for us by NPL, did have a stirred bath. Unfortunately, earlier work led to incorrect models mainly due to this error. Well-stirred bath gave FT time constants of about 45 and 50 seconds for thermometers with wells at NPL, as compared with 60 and 75 seconds for non-stirred. Figure 6 shows clearly that stirring quickens response.

2.6 MODEL REDUCTION

The valid distributed-parameter models developed can, of course, be useful in a number of situations. But the heavy computational load can be a severe setback in some. For example, for 'intelligent ' instrumentation one would like to have a small computational load. To this end, model reduction of the above distributed models is required.

Ideally one would like to have a valid first-order model . But as most modellers have come to realise, this is not possible in a number of cases. Low-order models require some form of 'lumping'. The following strategy was one that was implemented:- capacitances: well, sheath, aluminium cover, ceramic annulus with the resistances computed as described in the bond-graph section. However, this did not lead to a valid model when compared with the distributed (FINEL) models. Even further discretisation, for example dissection of well wall into two 'lumps' did not lead to any better models. In all these models, the underlying assumption was that there was no centre effect. In fact it was found that centre effect was not entirely negligible and its satisfactory inclusion leads to valid models. Below, we describe thermometer only and thermometer in-well models based on typical parameter sets.

(a) THERMOMETER SYSTEM

It had been suspected and found from earlier results that the thermometer system behaves at least as a second-order system.

A third-order model, including the centre effect, is represented schematically:

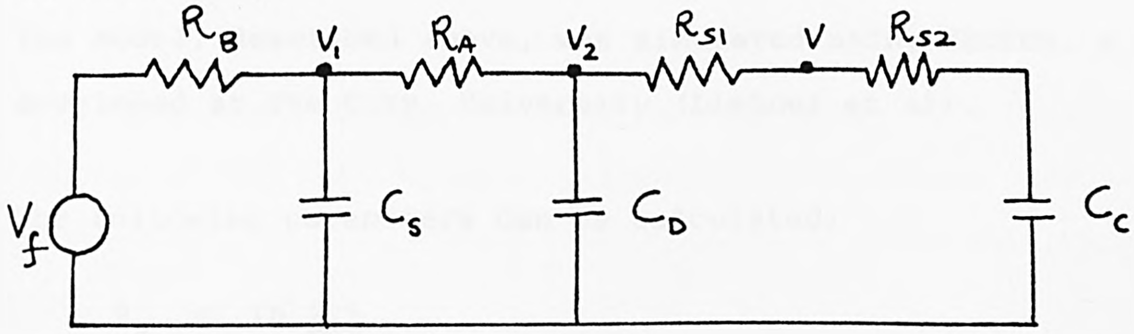


FIGURE 7

where: V_f = fluid temperature

R_B = boundarly - layer resistance

V_1 = Temperature of sheath

C_s = Sheath capacitance

V_2 = Temperature of aluminium cover

R_A = Resistance of air gap

V = Temperature of sensor

C_C = Capacitance of core

C_D = capacitance of aluminium cover and ceramic annulus

R_{S1} = 2/3 x resistance of air gap surrounding(term 1) platinum sensor plus that of aluminium cover and ceramic annular (both negligible)

R_{S2} = 1/3 x resistance of air gap around sensing wire plus that of ceramic core

Note that the resistances R_{S1} and R_{S2} contain the resistances due to air holes containing platinum wire coils.

Typical results will bepresented for the following thermometer:

13.4 : 3.0, 2.5, 2.375, 1.585, .85, .75, .07

The model, described above, was simulated using MEDIEM, a package developed at The City University (Liebner et al).

The following parameters can be calculated:

$$R_A = \frac{\ln \frac{2.5}{2.375}}{2\pi k L} = 13.61$$

$$R_B = 1.56$$

$$C_S = 0.79$$

C_D = 1.031 (Alluminium cover and ceramic annulus

$$R_S = \underbrace{22.13}_{1} + \underbrace{11.07}_{2} + \underbrace{0.522}_{\text{ceramic core resistance}}$$

$$= 33.722$$

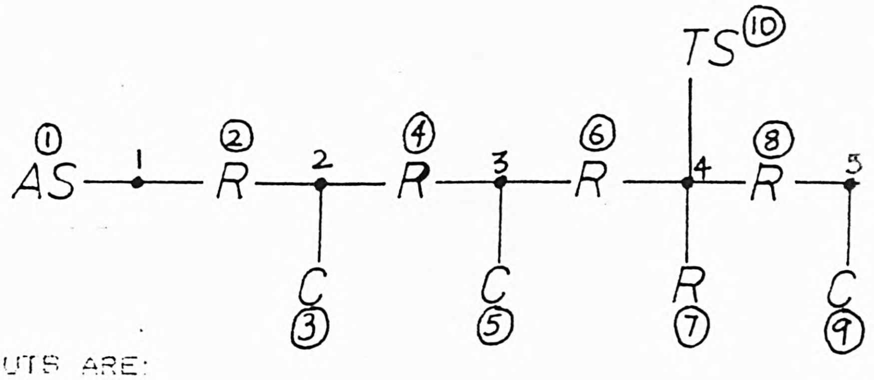
$$C_C = 0.1343$$

Figure 8 shows a typical Mediem model. It should be noted that the centre capacity is comparatively small. The resistance due to air gap surrounding the platinum sensing wire is not negligible - in fact it is quite large (the sum of terms 1 and 2. This shows that the air holes containing the platinum sensing coils contributes quite a large thermal resistance. But does this imply that their influence is negligible? Figure 9 shows the reduced model (Full curve) compared to the FE mdoel (CROSSES). The agreement can be seen to be excellent. The ET curves show that there is a difference of time constants in the ET curve

$$\tau_{ET}^{FE} = 9.95 \text{ secs}$$

$$\tau_{ET}^{REDUCED} = 9.05 \text{ sec.}$$

Even then the agreement overall is very good. The circles represent the reduced model output with the centre capacity neglected. It can be seen that there is a large error when this is neglected.




```
OUTPUTS ARE:
+    AC  7
+

TIME:  0.000E-01    6.000E 01    5.000E-02   PLOTTING RATE IS :     20

FREQ:  1.000E-07    1.000E 01    1.000E 00    LOG.  IS -1.0

PLOTTING IN TIME - RANGE IS :   0.000E-01    1.000E 02

PLOTTING IN FREQUENCY - RANGE IS :   0.000E-01    0.000E-01         0
```

NO	ELEMENT	ARROW	NODE	CONNECT	DOM.	VALUE1	VALUE2
1	AS	TO	1	0	T	1.000E 02	0.000E-01
2	R	FR	1	0	T	1.560E 00	0.000E-01
2	R	TO	2				
3	C	FR	2	0	T	7.900E-01	0.000E-01
4	R	FR	2	0	T	1.361E 01	0.000E-01
4	R	TO	3				
5	C	FR	3	0	T	1.031E 00	0.000E-01
7	R	FR	4	0	T	1.000E 06	0.000E-01
8	R	FR	4	0	T	1.155E 01	0.000E-01
8	R	TO	5				
9	C	FR	5	0	T	1.343E-01	0.000E-01
6	R	TO	4	0	T	2.215E 01	0.000E-01
6	R	FR	3				
10	TS	TO	4	0	T	2.000E-02	0.000E-01

```
PLEASE GIVE COMMAND - TYPE KEY FOR COMMANDS               OPTIONS
*KEY
```

FIGURE 8. MEDIEM MODEL AND PROGRAM FOR REDUCED MODEL OF FIG. 7. THE ELEMENTS ARE NUMBERED WITH CIRCLES, THE NODES WITHOUT CIRCLES.

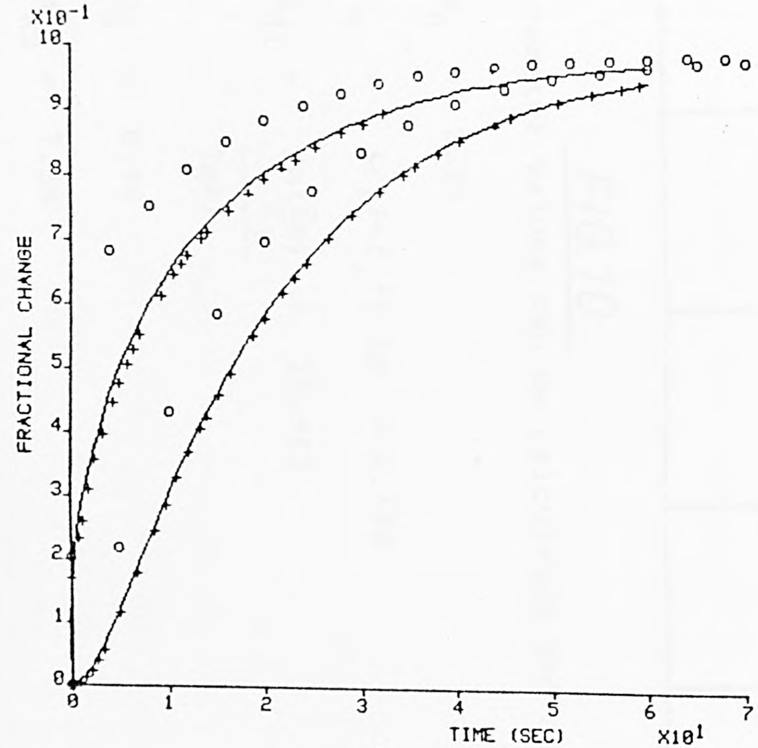


FIGURE 9

FIGURE 9. COMPARISON OF DISTRIBUTED-PARAMETER MODEL (CONTINUOUS LINES) WITH LUMPED MODEL (CROSSES).

CIRCLES ARE RESULTS FOR LUMPED MODEL WITH NEGLIGIBLE CENTRE CAPACITANCE.

A THERMOMETER-PLUS-WELL SYSTEM

Typical Model reduction results for a Thermometer-plus-well system are described here.

SYSTEM:

5.74: 5.0, 3.22, 2.92, 2.55, 2.48, 1.585, .85, .75, .07

the reduced model is represented below:

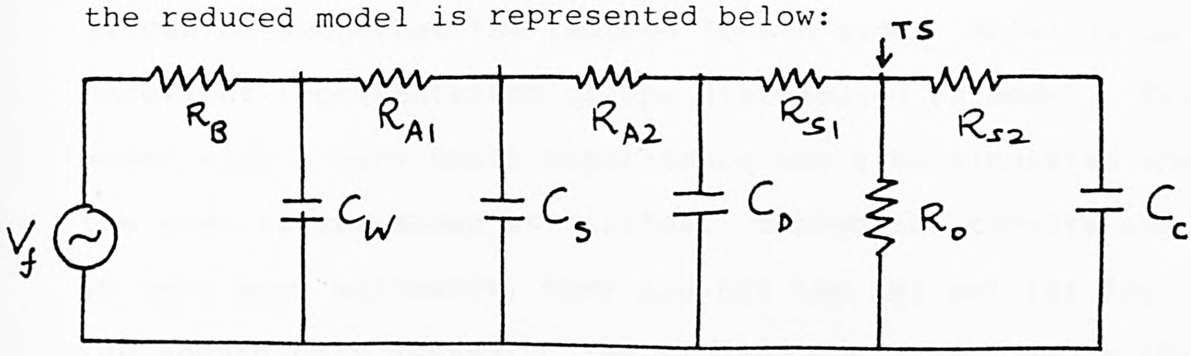


FIG.10

The parameter values can be calculated and obtained as follows:

$R_B = 2.22$

$C_W = (r_o^2 - r_i^2)\ Lc = 4.192$

$$R_{A1} = \frac{\ln\left(\frac{r_o}{r_i}\right)}{2\pi kL} = 25.942$$

$C_S = 0.58$

$R_{A2} = 7.38$

$C_D = 1.031$

$R_S = 33.722$

$C_C = 0.1343$

R_o = a large resistance

TS = Heat Input (i.e. current input) for ET

The model was simulated using MEDIEM - Figure 11 (as were the thermometer-alone systems). Results are shown in Figure 12 as continuous curves. Crosses are the data of the FE model. It can be seen that the reduced fourth order model is an excellent representation of the distributed FE model. The model with a very small capacitance was also simulated and the results are shown as circles. Though the results are not in very good agreement, they are not too far out (as for the sheath-only system). The FT data are within about 10% . MEDIEM took about 5 - 10minutes to compute if the PRIME computer used had few co-users at the time. Usually, it was faster than this.

Numerical values of the capacitances above show that the centre capacity is quite small. But its neglect leads to somewhat less satisfactory model. This would appear to be due to large resistances contributed by the air holes containing the platinum coils. That this is the case is shown, from typical simulation for a thermometer, as figure 13. it can be seen that low values of internal resistances make the centre capacitances effect neglible.

Some of the results to date are very briefly presented in Appendix C.

```
OK, SEG EMOND3.TEST


TIME: 0.000E-01   7.000E 01   1.000E-02  PLOTTING RATE IS :  500

FREQ: 0.000E-01   0.000E-01   0.000E-01   LOG. IS -1.0

PLOTTING IN TIME - RANGE IS :  0.000E-01   0.000E-01

PLOTTING IN FREQUENCY - RANGE IS :  0.000E-01   0.000E-01      0


NO   ELEMENT ARROW NODE  CONNECT DOM.    VALUE1       VALUE2
 1     AS     TO    1       0     T   1.000E 02    0.000E-01

 2     R      FR    1       0     T   2.220E 00    0.000E-01
 2     R      TO    2

 3     C      FR    2       0     T   4.192E 00    0.000E-01

 4     R      FR    2       0     T   2.594E 01    0.000E-01
 4     R      TO    3

 5     C      FR    3       0     T   5.800E-01    0.000E-01

 6     R      FR    3       0     T   7.280E 00    0.000E-01
 6     R      TO    4

 7     C      FR    4       0     T   1.131E 00    0.000E-01

 8     R      FR    4       0     T   2.213E 01    0.000E-01
 8     R      TO    5

 9     R      FR    5       0     T   1.000E 07    0.000E-01

10     R      FR    5       0     T   1.159E 01    0.000E-01
10     R      TO    6

11     C      FR    6       0     T   1.343E-01    0.000E-01

12     TS     TO    5       0     T   0.000E-01    0.000E-01
PLEASE GIVE COMMAND - TYPE KEY FOR COMMANDS          OPTIONS
*TIM
```

Fig. 11. MEDIEM model for thermometer plus well

FIGURE 12. DISTRIBUTED MODEL(CONTINUOUS LINES) AND LUMPED MODEL WITH (CROSSES) AND WITHOUT (CIRCLES) CENTRE CAPACITANCE FOR THERMOMETER PLUS WELL.

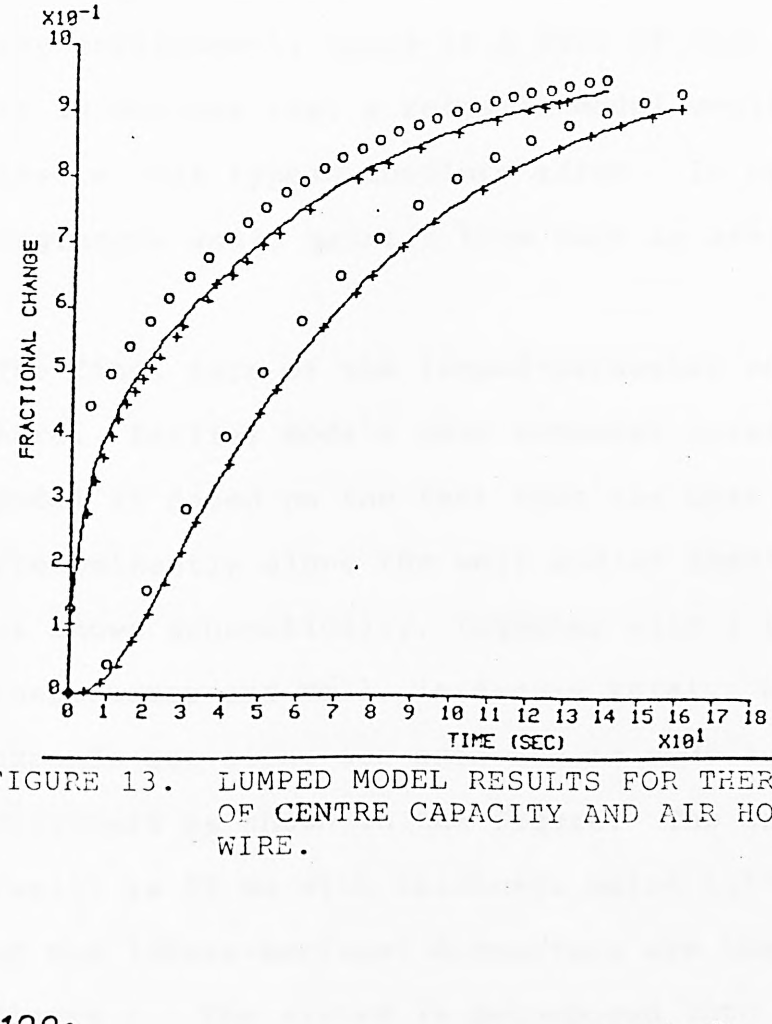
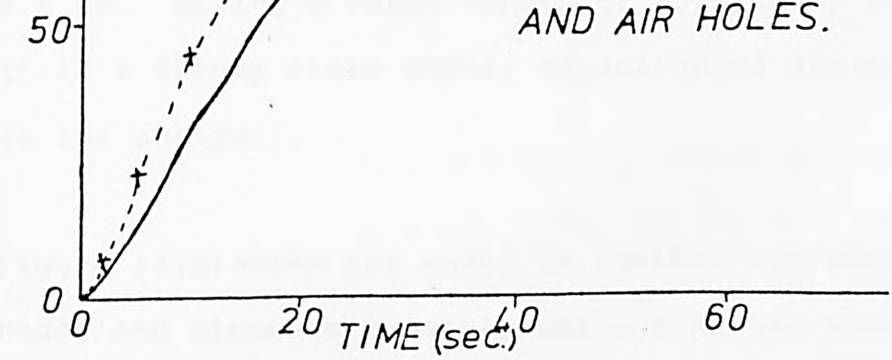


FIGURE 13. LUMPED MODEL RESULTS FOR THERMOMETER FOR EFFECT OF CENTRE CAPACITY AND AIR HOLES AROUND SENSING WIRE.



2.7 STEADY-STATE ERROR ANALYSIS

Because a thermometer assembly is usually connected from the environment, there is a loss of heat to this environment. It is obvious that a reliable model would assist to objectively assess this type of absolute error. In particular, design engineers would benefit from such an assessment.

The final form of the lumped-parameter model is presented here. Earlier models gave somewhat unreasonable errors. The model is based on the fact that the heat transfer is predominantly along the well and/or sheath wall. The model is shown schematically, together with a sketch of a Rosemount thermometer and Well, in figure 14(a). Note that the well expands out along its axis and is modelled here as two cylinders as shown in the figure. The expanded outer diameter (well) is 15 mm with thickness being 1.87 mm. The rest of the (cross-section) dimensions are the same as shown in figure 1. The system is decomposed into three sections A, B, and C corresponding those (in the model) shown in the figure. The length of the section A is 6.5 cm and, for full immersion, the length of sections B and C are both 6.5 cm. giving a total length of 23.5 cm of well. Because it is a steady state model, capacitances are not involved in the analysis.

Figure 14(b) shows the model in another representation with nodes and elements (resistances and Across Sources (AS)) numbered as required for simulation using MEDIEM (Liebner et al (1982)). Figure 14(c) shows the MEDIEM model which can directly be used, for example, for design.

The radial resistances (2,3,4,5,6,14,15,16,17) are as follows:

2,16,17 are boundary-layer resistances determined by equation (24). The other radial resistances are calculated using equation (21);3,14,15 are outer air gap resistances of the three sections of the thermometer plus well. 4 is the inner air gap resistance, and 5 and 6 are the central core resistances including air gaps surrounding the sensing wire . Note that for the sections B and C, the outer air gap radii (outer and inner) are (in the mean) 5.63 mm and 3.0 mm.

The axial resistances for the well wall (11,12 and 13) and sheath wall (8,9, 10) are given by:

$$R_{axial} = \frac{L}{2\pi r_m \Delta r k}$$

where L = axial length

r_m = Mean radius

Δr = Annular width

Resistance 7 is a large resistance for measuring the output.

For a typical thermometer-in-well assembly, it was found that the model gave an absolute error of 0.01%. Immersion lengths were varied and the following errors obtained:

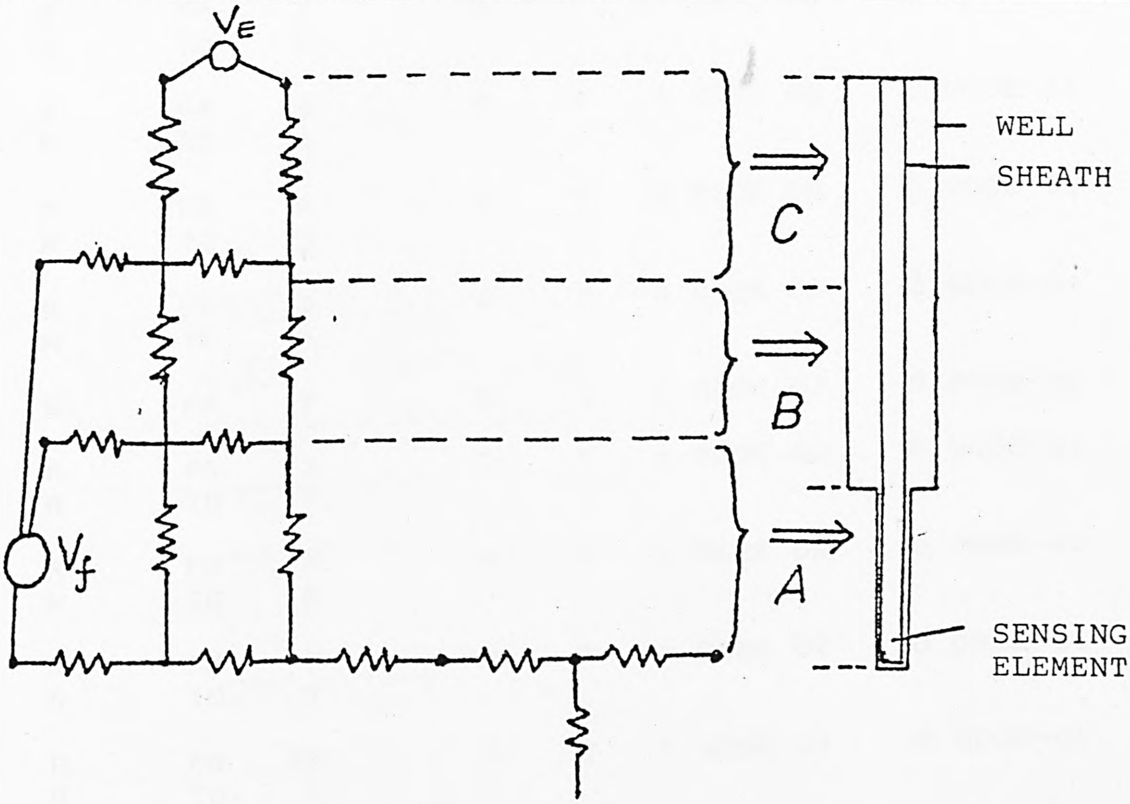
17.0 cm immersion - error 0.13°C (or 0.13%)

13.0 cm immersion - error 0.61°C (or 0.61%).

All simmulations were carried out with fluid temperature 100°C and environmental temperature 20°C. Unfortunately no experimental data could be found for comparison.

The above model can easily be applied to a thermometer on its own. This was done and simulated using MEDIEM. Here there is limited data made available by Carr(1972). He found that when the insertion length, L, is changed from 10 cm. to 25 cm., output does not change by more than 0.09°C in an oil bath. The above model gave a difference of 0.01°C in water. This appears to be in good agreement with Carr's result.

FIGURE 14(a) Steady state model (Left) and sketch of Rosemount Thermometer plus well (Right)



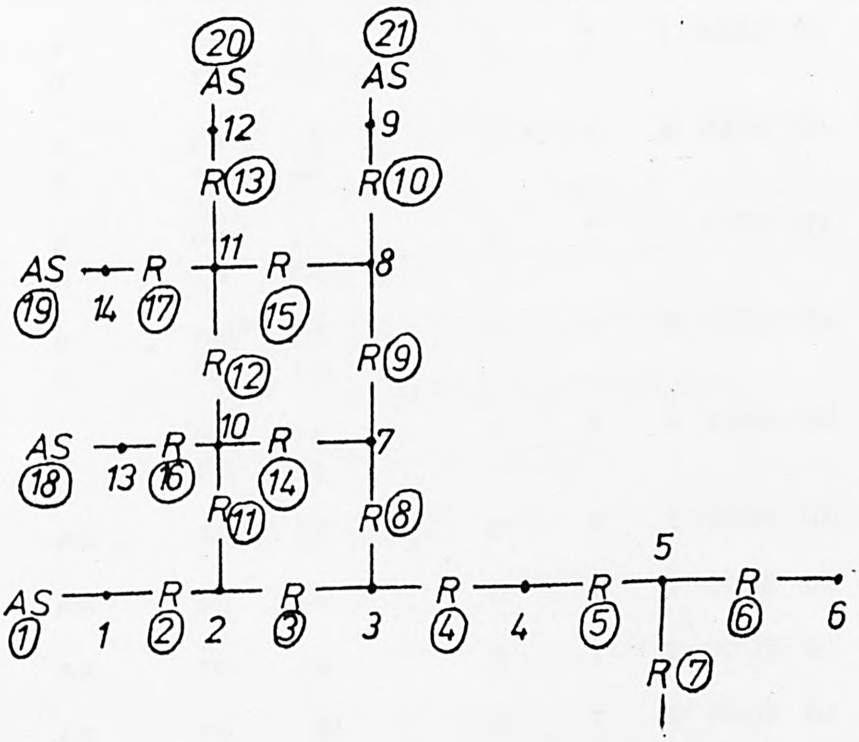


FIGURE 14(b) Network model of above suitable for MEDIEM simulation (see fig. 14(c)). Elements numbered with circles.

NO	ELEMENT	ARROW	NODE	CONNECT	DOM.	VALUE1	VALUE2
1	AS	TO	1	0	T	1.000E 02	0.000E-01
2	R	FR	1	0	T	7.200E-01	0.000
2	R	TO	2				
3	R	FR	2	0	T	4.330E 00	0.000E-01
3	R	TO	3				
4	R	FR	3	0	T	5.230E 00	0.000E-01
4	R	TO	4				
5	R	FR	4	0	T	8.510E 00	0.000E-01
5	R	TO	5				
6	R	FR	5	0	T	4.260E 00	0.000E-01
6	R	TO	6				
7	R	FR	5	0	T	1.000E 07	0.000E-01
8	R	FR	3	0	T	1.505E 02	0.000E-01
8	R	TO	7				
9	R	FR	7	0	T	1.968E 02	0.000E-01
9	R	TO	8				
10	R	FR	8	0	T	1.968E 02	0.000E-01
10	R	TO	9				
14	R	FR	10	0	T	1.606E 01	0.000E-01
14	R	TO	7				
15	R	FR	11	0	T	1.606E 01	0.000E-01
15	R	TO	8				
11	R	FR	2	0	T	2.723E 01	0.000E-01
11	R	TO	10				
13	R	FR	11	0	T	1.475E 01	0.000E-01
13	R	TO	12				
16	R	FR	13	0	T	9.150E-02	0.000E-01
16	R	TO	10				
17	R	FR	14	0	T	9.150E-02	0.000E-01
17	R	TO	11				
18	AS	TO	13	0	T	1.000E 02	0.000E-01
19	AS	TO	14	0	T	1.000E 02	0.000E-01
20	AS	TO	12	0	T	2.000E 01	0.000E-01
21	AS	TO	9	0	T	2.000E 01	0.000E-01
12	R	FR	10	0	T	1.475E 01	0.000E-01
12	R	TO	11				

FIGURE 14(c) Steady state analysis:MEDIEM MODEL

3. USE OF MODELS

3.1 INTRODUCTION

The previous chapter present results for model development, simulation and validation. The present chapter concerns the use of models.

The topics to be discussed here are as follows:

(a) design of thermometer assemblies:
This is a very brief procedure which could be refined and incorporated into a scientific design procedure.

(b) Sensitivity analysis : Presents results for response changes due to variations of air gap dimensions and heat transfer coefficient

(c) Model identification and in-situ response testing presents results for the whole time history for implementation of Kerlin's algorithm for testing when the expediating conditions are strictly and approximately satisfied.

As this is the first project at the Instrumentation Centre on thermal transducers, time did not allow a proper investigation into the design of transducers. But it is hoped that the framework provided here will form a base for thorough investigation in the future.

3.2 DESIGN OF THERMOMETER ASSEMBLIES

In the present section a brief framework is given for the design for thermal response. As Finkelstein (1977b) points out, instrument design is a special case of general engineering design methodology. He says further:

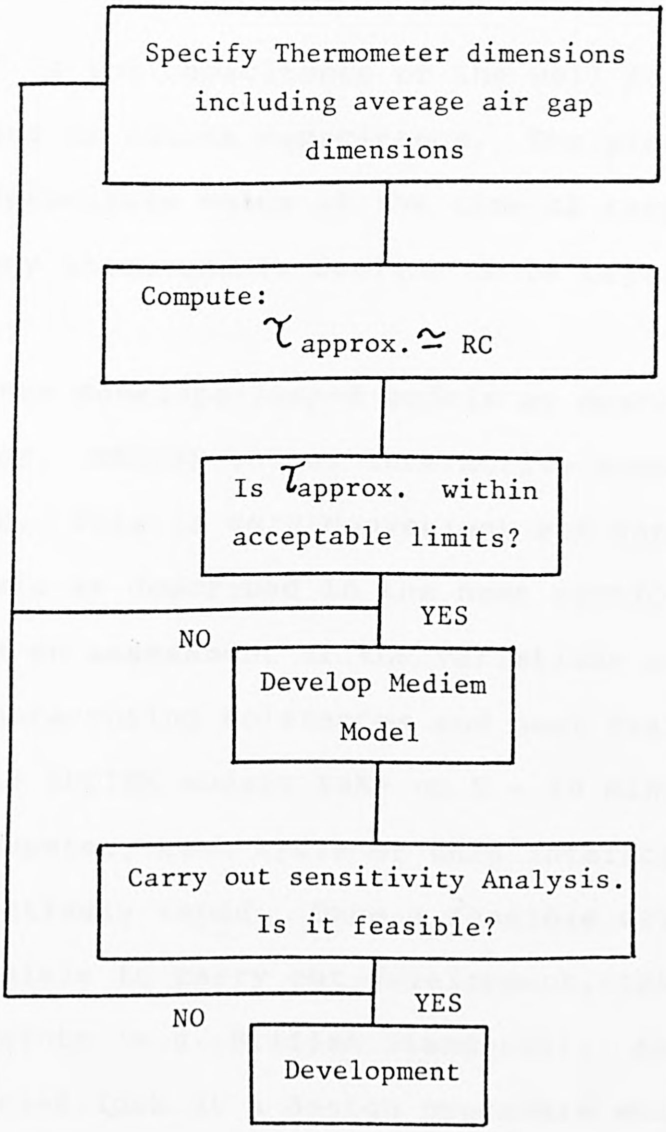
> "The basic stages of the engineering design process of a device such as an instrument consists of the establishment from an analysis of need of the design criteria and constraints , the generation of candidate designs, their analysis and evaluation in terms of design criteria or constraints, and a decision either to accept a particular candidate design or to generate a new candidate design typically by variation of parameters or structure of a candidate".

We assume that a candidate design is chosen: a cylindrical geometry with stainless stell well, stainless steel sheath, aluminium cover and ceramic core with platinum sensing wire embedded in the ceramic. Specification of dimensions of the system is then required.

The following procedure could be adopted for response design. See Flowchart 1.
As before, the dimensions will be presented as a parameter set.

$$\{\xi_1, \xi_2, \ldots\ldots, \xi_N\}$$

where ξ_1 is the outer most dimension.For the case of thermometer-in-well, the outer well diameter equals ξ_1 . Design is an interactive process and hence an initial parameter set is required.



FLOWCHART 1

DESIGN METHODOLOGY

R stands for the total resistance of the air gaps

$$R = R_{A1} + R_{A2}$$

and C is the capacitance of the well for thermometer-in-well systems or sheath capacitance. The proauct of R and C gives an approximate value of the time of response. This allows for any unreasonable designs to be rejected at a earlier stage.

One then develops lumped models as described in the last chapter. MEDIEM allows interactive simulations to be performed. This is very convenient for carrying out a sensitivity analysis as described in the next section. This in turn allows an assessment of the variations of response times due to manufacturing tolerances and heat transfer coefficient. Because MEDIEM models take up 5 - 10 minutes on a multi-user minicomputer, each cycle of this interactive design phase would be relatively rapid. Once a feasible design is arrived at, it is possible to carry out development, taking into account other constraints (e.g. British Standards). As stated earlier this is a brief look at a design procedure which should lead to further work in future. It has to be pointed out that sensitivity analysis, discussed below, should be incorporated into a design procedure.

3.3 SENSITIVITY ANALYSIS

In the early stages of the work leading to this doctoral thesis, sensitivity analysis of thermometer-in-well systems was carried out for lumped models using THT simulation. This was reported earlier (Chohan and Abdullah (1980)). Here, we present some results obtained using the distributed parameter (FINEL) models.

It is well known that manufacturing tolerances lead to variable air gap widths. Sensitivity to heat transfer coefficient is also important.Perhaps response differences due to these parameter variations are the most important. We present typical results of these from simulations of the distributed-parameter models. It is thought that these and similar sensitivity analysis would be useful in the design of thermometer systems.

The system simulated had the following parameter set:

6.8: 5.0, 3.110, 3.05, 2.5, 2.380, 1.585, .85, .75, 0.7

All the parameters are kept constant in a set of simulations except those corresponding to, for example, the first air gap (outer). Figure 15 shows the results of Time constant (both FT and ET) against the outer air gap width (a_1) in mm. Note the different scales of ET and FT time constant scales. The sensitivity coefficients can be found from the graphs:

$$\frac{\partial \tau_{FT}}{\partial a_1} \simeq 158.8 \qquad \text{sec. mm}^{-1}$$

$$\frac{\partial \tau_{ET}}{\partial a_1} \simeq 98.4$$

These show that unit changes of air gap widths produces higher change in fluid termperature time constant.

Figure 16 shows similar results but for the inner (a_2) air group. Again the results show the time constant are to a good approximation linear in a_2 The sensitivity coefficients are:

$$\frac{\partial \tau_{FT}}{\partial a_2} \simeq 127.1$$

$$\frac{\partial \tau_{ET}}{\partial a_2} \simeq 73.5$$

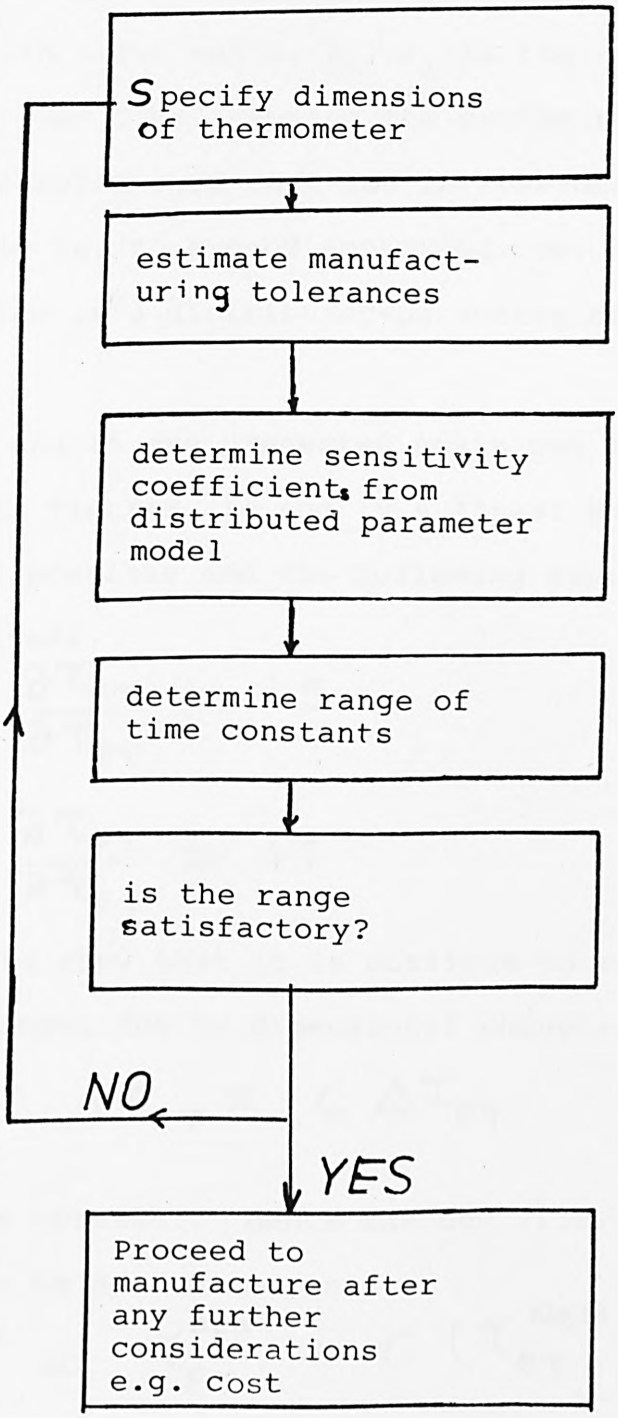
These sensitivities can be useful at the design stage of a thermometer system. For a given average dimension one can obtain from general manufacturing experience,the variation of widths due to manufacture tolerances. This would give, via the sensitivity coefficients, range of time constants one expects from a design. Hence one can have the following interactive cycle incorporated in a design process, see flow chart 2.

Figure 17 shows the variation of time constant with heat transfer coefficients. The results are for the system:

h: 5.0, 3.130, 3.0, 2.5, 2.375, 1.585, .85, .75, 07

The results show that the time constant is inversely related to the heat transfer coefficent. This agrees with the result of Kerlin et al (1982), who present the following expression for time constant:

$$\tau_{FT} = \frac{\rho c r_0^2}{2} \left\{ \frac{\ln (r_0/r_i)}{k} + \frac{1}{h r_0} \right\}$$



FLOWCHART 2

Use of sensitivity analysis in a design procedure where ρ , c are the density and capacity of the lumped cylinder with outer radius r_o, r_i is the radius of sensing element derived from assuming the system to be a first-order one. Our results show that the inverse heat-transfer coefficient relationship is, to a good approximation, valid for the real system (which is a distributed-parameter one).

Figures 15 and 16 are presented again but as τ_{FT} variation with τ_{ET} in figures 18 and 19. A linear approximation to the data is possible and the following approximate relationships can be derived:

$$\frac{\partial \tau_{FT}}{\partial \tau_{ET}} \approx 1 \cdot 5$$

$$\frac{\partial \tau_{FT}}{\partial \tau_{ET}} \approx 1 \cdot 7$$

These results show that it is possible to represent time constant changes due to dimensional changes of air gaps:

$$\Delta \tau_{FT} \simeq C \, \Delta \tau_{ET}$$

where C is a constant. Hence the new (i.e. current) value of τ_{FT} can be determined as:

$$\tau_{FT}^{NEW} \simeq \tau_{FT}^{OLD} + C \left(\tau_{ET}^{NEW} - \tau_{ET}^{OLD} \right)$$

where τ_{FT}^{OLD}, τ_{ET}^{OLD} are last recorded value and τ_{ET}^{NEW} is the value obtained from an ET carried out most recently. The utility of such a procedure is obvious - it is simple and quick though approximate. Some of 'coarser' decisions can be made using this approximation - e.g. whether degradation is severe enough to justify change of thermometer.

The results show that a sensitivity analysis using distributed parameter models developed in the present thesis would be beneficial both to the thermometer manufacturer and user.

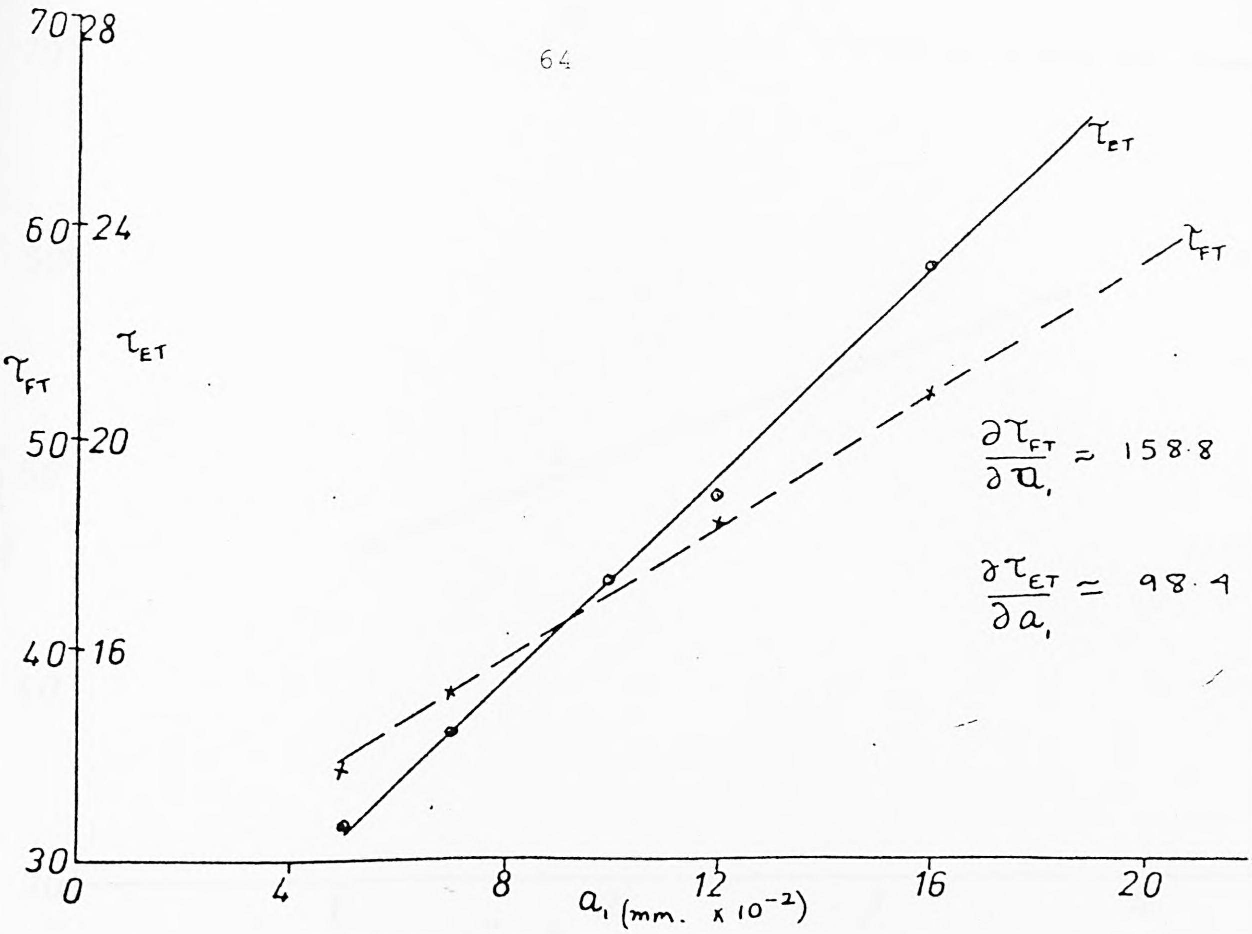
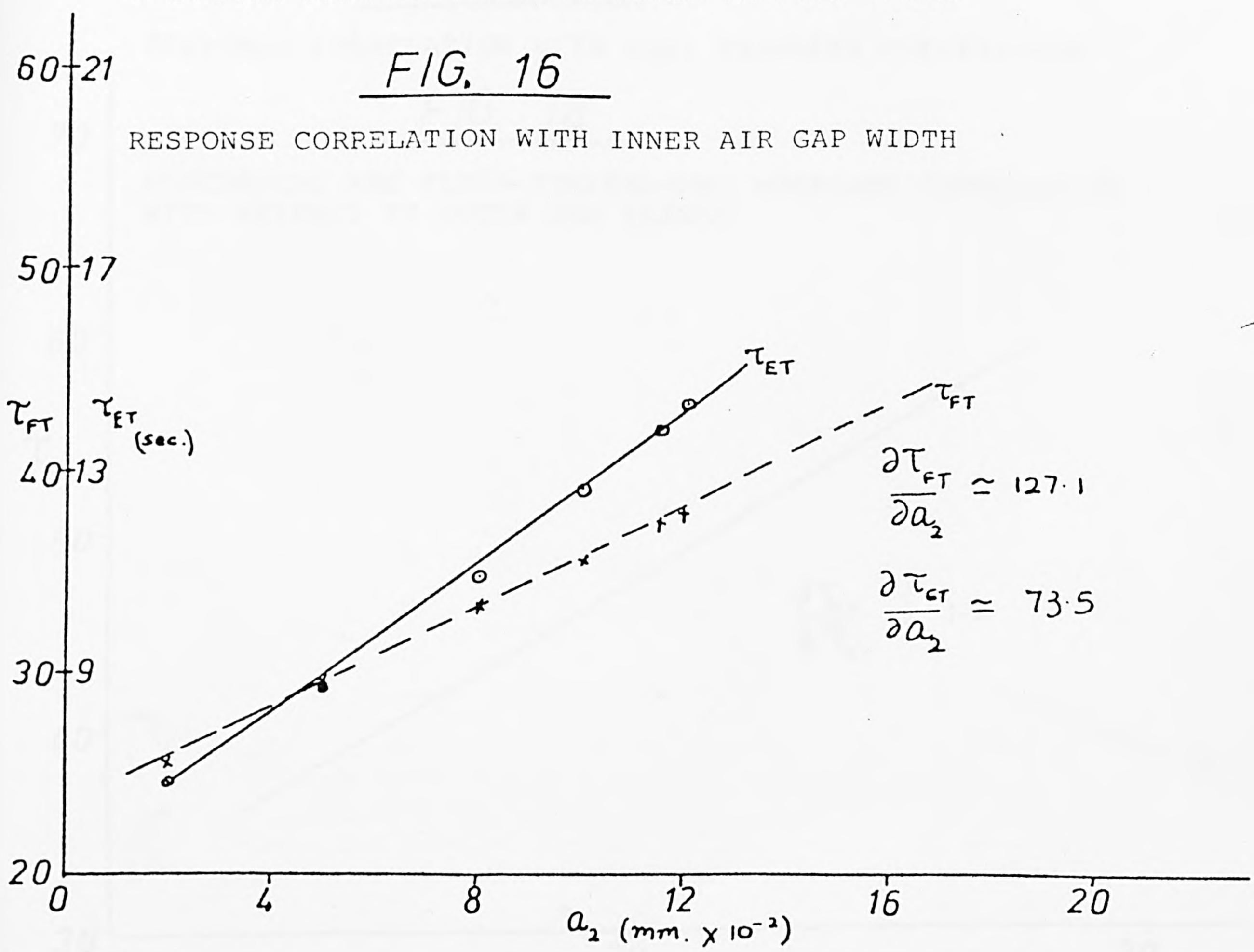


FIG. 15

RESPONSE CORRELATION WITH OUTER AIR GAP WIDTH

FIG. 16

RESPONSE CORRELATION WITH INNER AIR GAP WIDTH



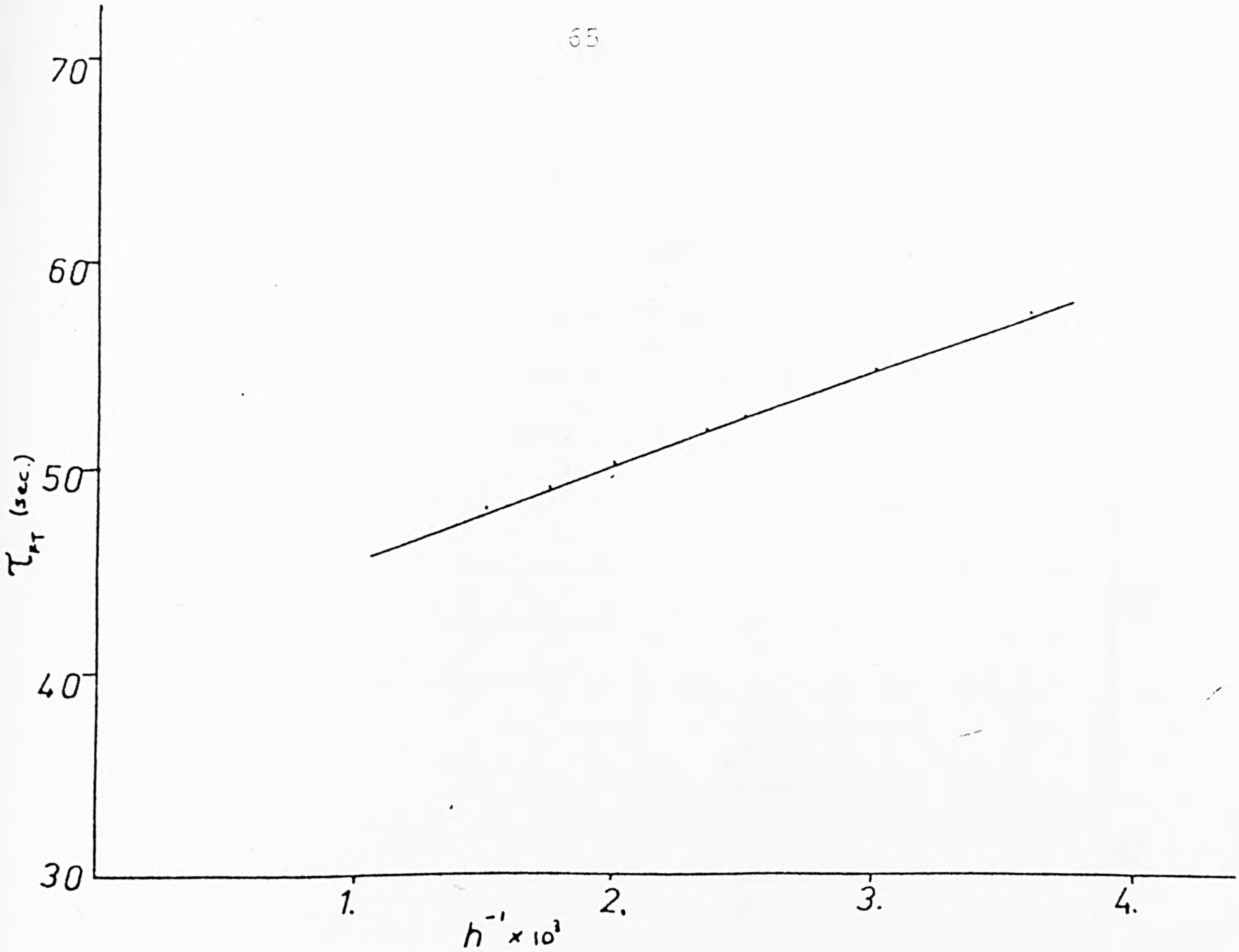
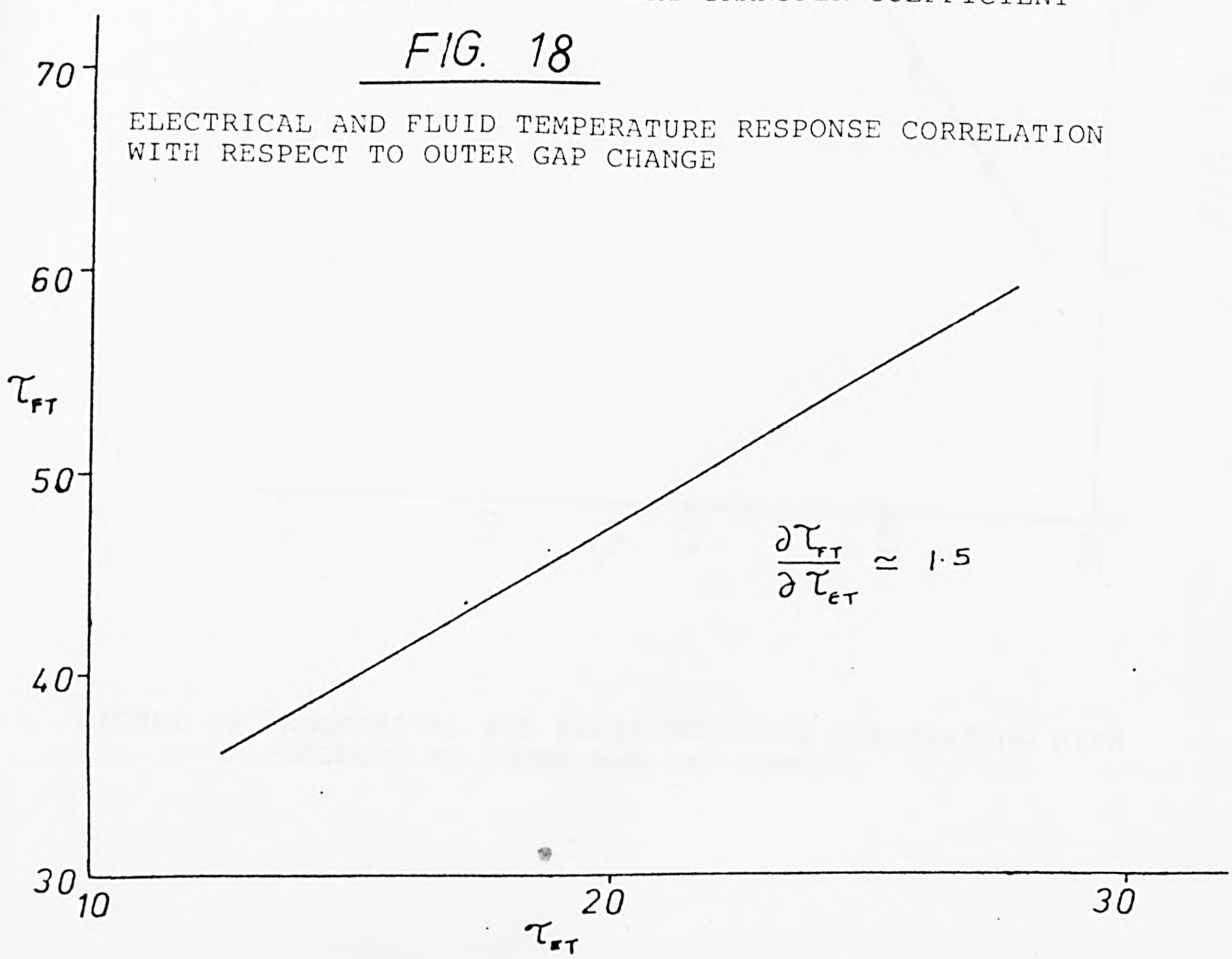


FIG. 17

RESPONSE CORRELATION WITH HEAT TRANSFER COEFFICIENT

FIG. 18

ELECTRICAL AND FLUID TEMPERATURE RESPONSE CORRELATION WITH RESPECT TO OUTER GAP CHANGE



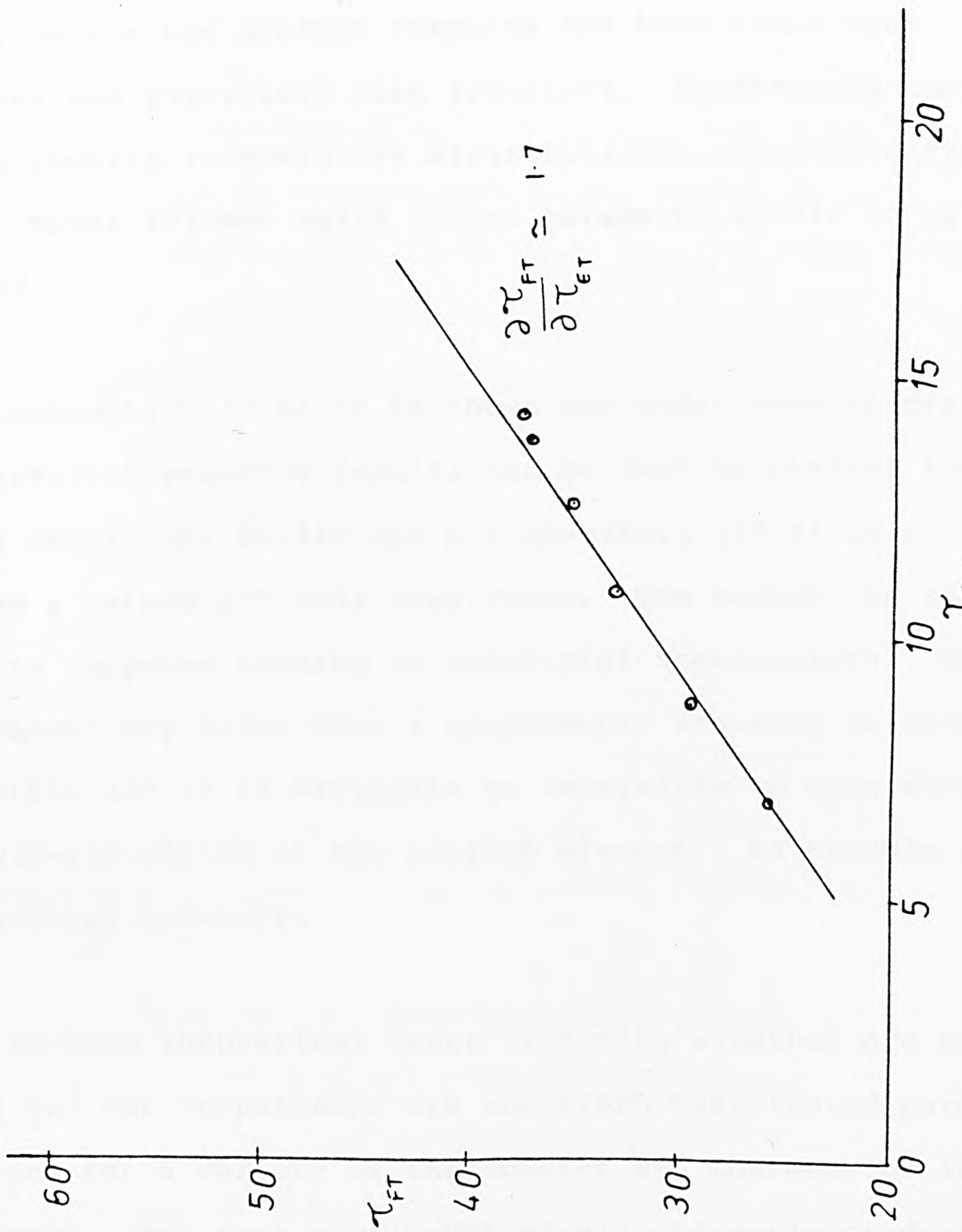


FIG. 19

FIGURE 19 ELECTRICAL AND FLUID RESPONSE CORRELATION WITH RESPECT TO INNER AIR GAP CHANGES

3.4 MODEL IDENTIFICATION

In Chapter 2, mathematical models for a resistance thermometer assembly were developed and tested against experimental results. In that chapter it was shown that distributed parameter models based on the finite element technique can provide accurate predictions for the dynamic response for both fluid step transients and electrical step transient. Furthermore the detailed results (temperature distributions) obtainable from the F.E. model allowed valid lumped parameter models to be developed.

In this subsection (3-4) it is shown how model identification from electrical response results can be used to predict fluid response behaviour. Kerlin and his coworkers (1978) have developed a method for this prediction. The method can lead to in situ response testing of industrial thermometers. Such a requirement may exist when a thermometer assembly is relatively inaccessible and it is desirable or imperative to know whether there is a malfunction in the sensing element. An example is in the nuclear industry.

In this section theoretical tests of Kerlin's method are made. The test bed for comparisons are the FINEL distributed parameter simulations for a variety of thermometer and thermometer in well systems. The fact that FINEL models correctly predict the experimental situation allows us to proceed with theoretically generated FINEL responses and use these as though they had arisen from experiments for subsequent testing of identification methods. By generating entire time histories for both ET and FT results a more detailed comparison than Kerlin's results can be made. In particular the likely errors that may arise in curve fitting of electrical transient data and their effect on fluid transient predictions can be assessed.

Identification can be simply defined as the determination of valid system models using input-output data from an experiment or normal operation or from a valid distributed / lumped parameter model (as done here). What Kerlin et al (1978) call testing is basically an identification excercise. In situ testing can be, as stated above, a requirement or desirable in an industrial situation.

In situ testing is the solution of the following problem - called the Electrical Transient (ET) or loop current step response (LCSR) problem: Determine the time response to fluid temperature step change by injecting a step electrical current into the thermometer and observing the output.

Successful solution of this problem requires, first, to the determination of a valid model (identification) and subsequent use of this information contained in the model to obtain the fluid temperature response. Kerlin and his colleagues (1978) put forward a neat procedure for this under the following general assumptions: (called the expediating conditions).

i) Heat transfer is one-dimensional (radial)

ii) the heat capacity of materials between the sensing element and the centre line of the sensor is insignificant .

These assumptions lead to the resulting responses to an electrical transient (ET) and a fluid transient (FT) to be given by:

$$\text{ET: } T_{ET}(t) = 1 + a_1 e^{-p_1 t} + a_2 e^{-p_2 t} + \ldots \qquad (1)$$

$$\text{FT: } T_{ET}(t) = 1 + b_1 e^{-p_1 t} + b_2 e^{-p_2 t} + \ldots$$

Details can be found in Kerlin et al. (1978). An important point to note is that the exponent parameters, p_i, are the same for ET and FT. Furthermore the b_i parameters are explicit functions of p_i and given as:

$$b_1 = \frac{p_2 p_3 \ldots p_N}{(-1)\ (p_2 - p_1)(p_3 - p_1) \ldots (p_N - p_1)}$$

$$b_2 = \frac{p_1 p_3 \ldots p_N}{(p_1 - p_2)(p_3 - p_2) \ldots (p_N - p_2)} \qquad (2)$$

where N is system order i.e. the highest value for i in p_i.

Two model types were considered for system identification - In both cases the data (ET and FT) were generated by the FINEL distributed parameter simulations.

(1) Identified models. These models are obtained by curve fitting second or higher order exponetnial functions:

$$T_{ET}(t) = 1 + a_1 e^{-p_1 t} + a_2 e^{-p_1 t} + \ldots$$

From this, according to (2), one obtains a fluid transient (FT) prediction, e.g.

$$T_{FT}(t) = 1 + \frac{p_2}{(p_1 - p_2)} e^{-p_1 t} + \frac{p_1}{(p_2 - p_1)} e^{-p_2 t}$$

for a second-order approximation.

(2) Empirical models. Some work was done to obtain models from input-output data. However results are not presented here because they were considered to be incomplete and time constraints did not allow further work.

Since heat transfer systems (such as a thermometer) are essentially distributed parameter, it would appear that a proper representation of electrical and fluid transient curves should take the form:

$$T(t) = 1 + \sum_{i=1}^{\infty} a_i e^{-p_i t}$$

Experience of Kerlin and coworkers (1978) and ours seem to suggest that two exponent terms approximate the electrical transient results adequately:

$$T(t) = 1 + a_1 e^{-p_1 t} + a_2 e^{-p_2 t}$$

The $a_i p_i$ can be estimated by exponential stripping if the exponents are well separated and the noise level is not significant. Then, for times $t >> \frac{1}{p_1}$ $(p_1 < p)$:

$$T(t) \approx 1 + a_1 e^{-p_1 t}$$

or $$T(t) - 1 \simeq a_1 e^{-p_1 t}$$

which allows estimation of a_1, p_1 from two(or more)data points (from the late time period - see figure 5). Having evaluated a_1, p_1, one can obtain two or more early time data points.

$$T_2(t) = T(t) - 1 - a_1 e^{-p_1 t} \simeq a_2 e^{-p_2 t}$$

One can then evaluate a_2, p_2, by taking more than two data points, one can check for consistency as well as obtain average values for the parameters.

There is a practical limitation with this method, however. Past experience has shown that it is not usually possible to estimate more than two exponents (Kerlin et al (1978)). The fluid temperature time response constant, τ_{FT}, can be predicted by the following (Kerlin et al (1978)):

$$\tau_{FT} = \frac{1}{p_1} [1 - \ln (1 - \frac{p_1}{p_2}) - \ln(1 - \frac{p_1}{p_3}) \; . \; . \; .] \tag{7}$$

$$p_1 < p_2 < p_3 \; . \; . \; .$$

Past experience, as far as we know, only that of Kerlin's group, has been confined to experimental, and therefore, noisy results. To see if the method works for noiseless data, one has to have access to such data or a distributed-parameter model. The present work has made such a model which can be used to generate noiseless data for our thermometer assembly. This modelling procedure can easily be applied to any industrial RTD or thermocouple systems. Noiseless data allow Kerlin's algorithm to be investigated without noise errors.

We looked at estimation errors of Kerlin et al (1978) for a number of thermometer and thermometer-in-well systems. It appears that, when represented as second-order approximations, thermometer-well system generally have higher errors associated with them. Our work, presented in the last chapter, shows that a valid representation of thermometer-in-well system is of order four and that of a thermometer of order three. Therefore the finding of Kerlin et al are qualitatively in agreement with this. Results presented later show the kind of errors one can expect.

PARAMETER ESTIMATION FROM ELECTRICAL TEST DATA

It would appear that the best approach to estimate parameters would be to fit a curve to the (ET) data. We are trying to fit a curve of the following form:

$$T(t) = 1 + \sum_{i=1}^{N} a_i e^{-p_i t}$$

where N is specified. To estimate the parameters a_i, p_i the idea is to minimise the following function F with respect to these same parameters:

$$\min_{p_i,\, a_i} F = \sum_{i=1}^{M} \left\{ y_i - (1 + \sum_{j=1}^{N} a_j \, e^{-p_j t_i}) \right\}$$

where y_i is the data at time $t = t_i$. A number of efficient and effective subroutines exist to carry out this minimisation. The theory of such optimisation procedures is well documented in any modern text on the subject. One such subroutine, which was used here is EO4CGF of the NAG library. It is for this reason that parameter estimated by curve fitting will be referred to as NAG. Those using exponential stripping will be referred to as PEEL (since the method is also known as peeling).

The results to be presented can be classified into two:

a) estimation of parameters of a system satisfying Kerlin's expediating conditions i.e. no central capacity and no axial heat transfer. (section 3.4.1)

b) estimation of parameters of real systems (one dimensional/lumped parameter systems with some central capacity i.e. cases where Kerlin's expediating conditions are only approximately valid. (section 3.4.2)

3.4.1 ESTIMATION OF PARAMETERS FOR SYSTEMS WITHOUT CENTRE CAPACITY

To simulate systems without centre capacity, it is only necessary to change the data of the capacity corresponding to that element in the model to a negligible value. It makes no difference whether the distributed-parameter (FINEL) or validated lumped models are used (and discussed in the last chapter) to present typical results.

The following system considered as typical thermometer-in-well systems:

5.74: 5.0, 3.22, 2.92, 2.55, 2.48, 1.585, .85, .75, .07

The fluid temperature time response constant (τ_{FT}) is 66.7 seconds. The reduced fourth-order model was simulated, with negligible centre capacity, using MEDIEM as described earlier. The data for an electrical transient (ET) were used to simulate the NAG procedure. Three NAG procedures were carried out as follows: (a) two exponent i.e. second-order model (b) three exponents, (c) four exponents. A typical set of results was the following:- 59 data points of ET; initial estimates of parameters:

$a_1 = -0.1$, $p_1 = 0.01$, $a_2 = -0.1$, $p_2 = 0.4$, $a_3 = -1.40$,

$p_3 = 0.8$

NAG gave the following estimates of the exponents for the three cases:

(a) $p_1 = 0.0187$, $p_2 = 54.8$

(b) $p_1 = 0.0185$, $p_2 = 0.3350$, $p_3 = 31.45$

(c) $p_1 = 0.0173$, $p_2 = 0.0190$, $p_3 = 1.57$, $p_4 = 5.54$

The number of data points were successively increased to 180 and 280 points (including detailed coverage of near and far time). Results for 280 points for second, third and fourth-order identification are:

(a) $p_1 = 0.0183$, $p_2 = 28.38$

(b) $p_1 = 0.0182$, $p_2 = 0.2822$, $p_3 = 32.033$

(c) $p_1 = 0.0182$, $p_2 = 0.2775$, $p_3 = 5.3372$, $p_4 = 31.2$

Figure 20 (59 points) and 21 (280 points) show ET fits (third-order) and FT predictions together with FINEL results. The crosses (++++) are fits and FT predictions; the continuous lines represent the model output without centre effect. The ET fits can be seen to be excellent and close inspection of data for fit near zero time showed very good fits. The FT predictions are satisfactory.

The results show that the dominant time constant is quite well-predicted even by low-order approximations and a relatively small number of data points.

In the NAG fitting procedure it is unwise to choose wild initial parameter estimates. How does one do this? One can choose the last predicted or otherwise available estimates or use the following procedure: as $p_1 \rightarrow \frac{1}{\tau_{ET}}$, $p_2 \rightarrow \quad \geqslant 5p_1$, $p_3 \rightarrow \quad \geqslant 5p_2$, etc. These two procedures will minimise the errors connected with initial estimates.

That it is possible to obtain very good FT predictions with a choice of initial parameter estimates:

$a_1 = -0.5,\ p_1 = 0.01,\ a_2 = 0.1,\ p_2 = 0.07,\ a_3 = -1.4,$

$p_3 = 0.8$, The exponents obtained by NAG were:

$p_1 = 0.0187,\ p_2 = 0.0988,\ p_3 = 0.8695$

180 data (ET) points were used at 1 second intervals. The ET data (continuous curve) and fit (++++) are presented together with FT true curve (continuous line) and FT prediction (++++) are presented in figure 22. It can be seen that the prediction is very good. Of course, in general the prediction will not be as good as here but experience with a number of initial estimates has shown that prediction overall is satisfactory and within 20%.

A fourth-order (or higher) model can be attempted:

$$T(t) = 1 + a_{,}e^{-p_1 t} + 1_2 e^{-p_i t} + a_3 e^{-p_3 t} + a_4 e^{-p_4 t}$$

An initial choice of parameter set:

$a_1 = -0.1,\ p_1 = 0.016,\ a_2 = -0.5,\ p_2 = 0.05,$
$a_3 = -1.2,\ p_3 = 2.0,\ a_4 = 1.4,\ p_4 = 4.0$

gave the estimates of exponents as:

$$p_1 = 0.0188,\ p_2 = 0.2618,\ p_3 = 4.45,\ p_4 = 4.45$$

The prediction of fluid temperature transient (FT) together with actual values (continuous line) is shown in figure 23.

The prediction is satisfactory. However, there is nothing to be gained by introducing extra parameter - a third-order model is just as good, if not better,for FT prediction as previous results have shown.

A typical thermometer (i.e. without a well) is the following:

13.4: 3.0, 2.5, 2.375, 1.585, .85, .75, .07.

The figure 24, shows ET and FT curves corresponding to the reduced model (full curve), a very good representation of the distributed model, without centre capacity and simulated using MEDIEM. The circles show the NAG fit and prediction . The exponents for the third-order prediction were:

-0.0635, -2.1645, - 24.2623

By introducing further data very near the starting point of the transient (i.e. zero), one gets the following exponents:

-0.0635, -3.28, -30.49

and there is very little significant change in the FT prediction.

It can be seen that Kerlin's theory is in agreement as far as the practical use of model data for FT prediction is concerned. Notice the very rapid initial heating.

The ET data were used to perform NAG identification using second order representation. The exponents were:

- 0.0634, -31.6

There was no noticeable difference in the predicted curves.

Initial parameter specifications did not appear to have a significant effect on the prediction. Indeed for thermometers, all predictions were within 10%. For thermometers, a second-order approximation is adequate. For thermometers in-well systems, most predictions were within 15% and a third-order approximation is adequate.

Above, we looked at systems with negligible effects of the centre capacity. However, when this is the case, the reduced models developed in chapter 2 shows that a thermometer becomes a second-order system and thermometer-in-well a third-order. Our experience with fitting of Et data is in agreement with this. This also happens to be a fortunate circumstance: Our experience, that of Kerlin et al (1978) and Boroujerdi (1983), shows that usually only up to two or three exponents can be reliably estimated.

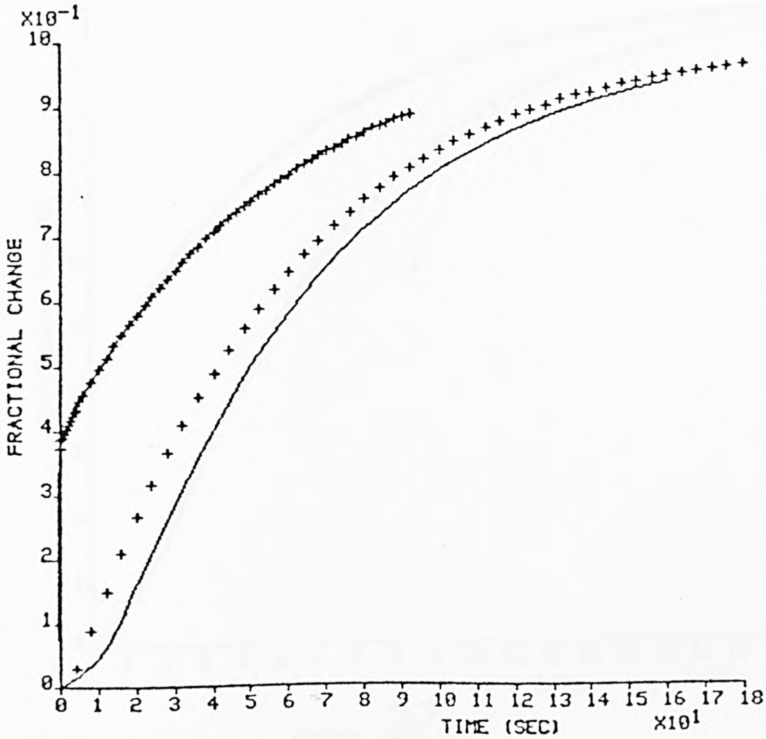


FIGURE 20

ET DATA/FIT AND FT DATA (——) AND PREDICTION(+++)

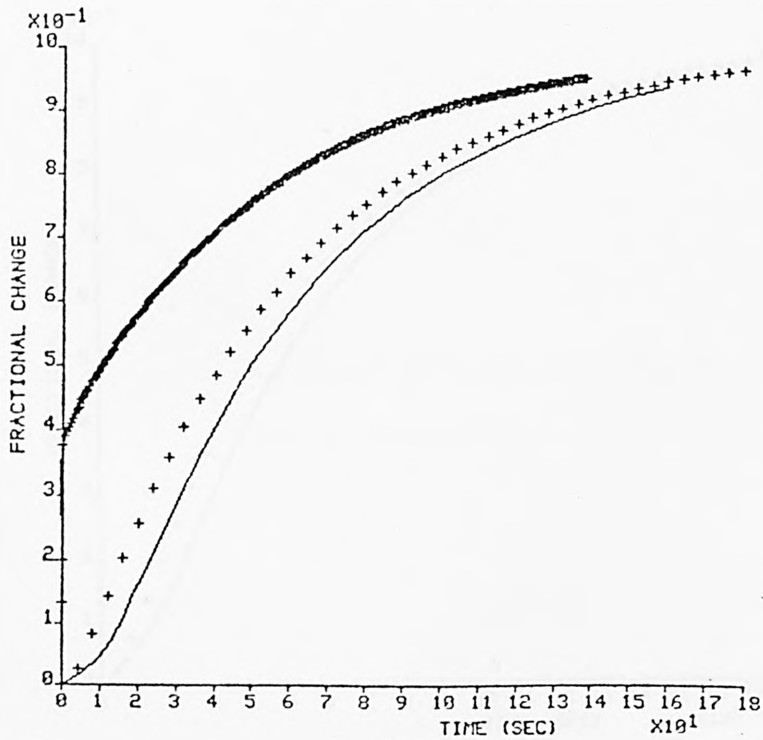


FIGURE 21

ET DATA/FIT AND FT DATA/PREDICTION

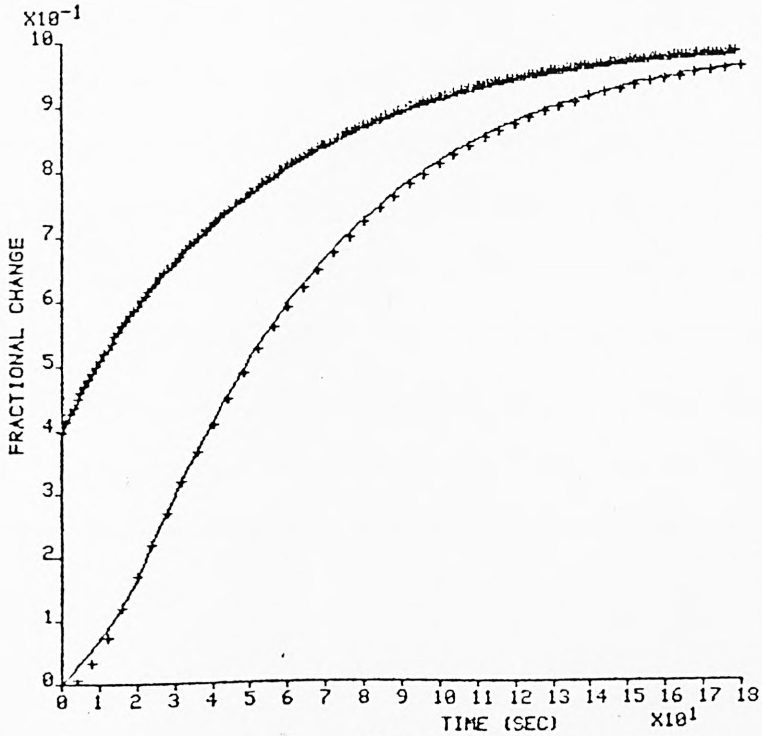


FIGURE 22

ET DATA/FIT AND FT DATA PREDICTION

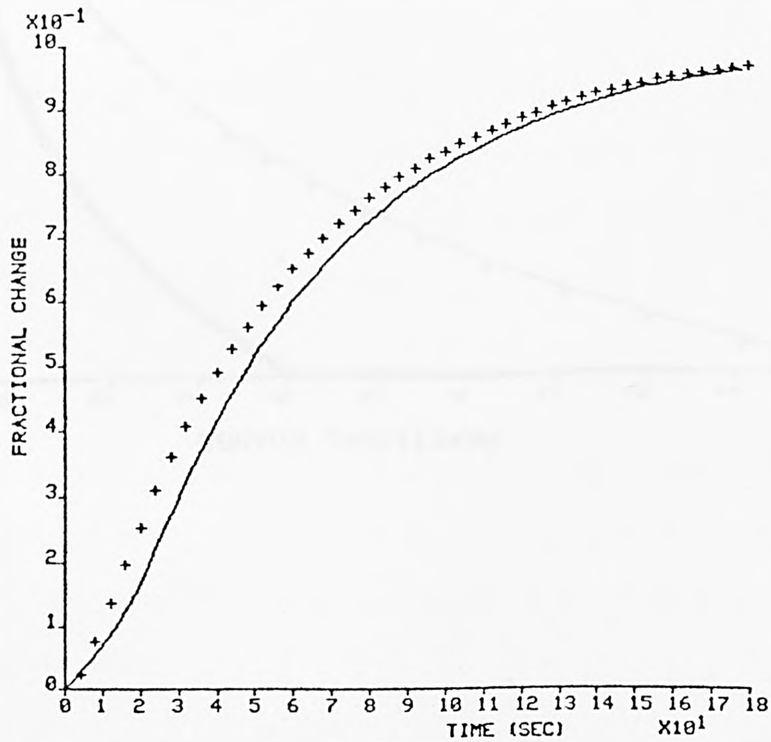


FIGURE 23

FT DATA (——) AND PREDICTION (+++)

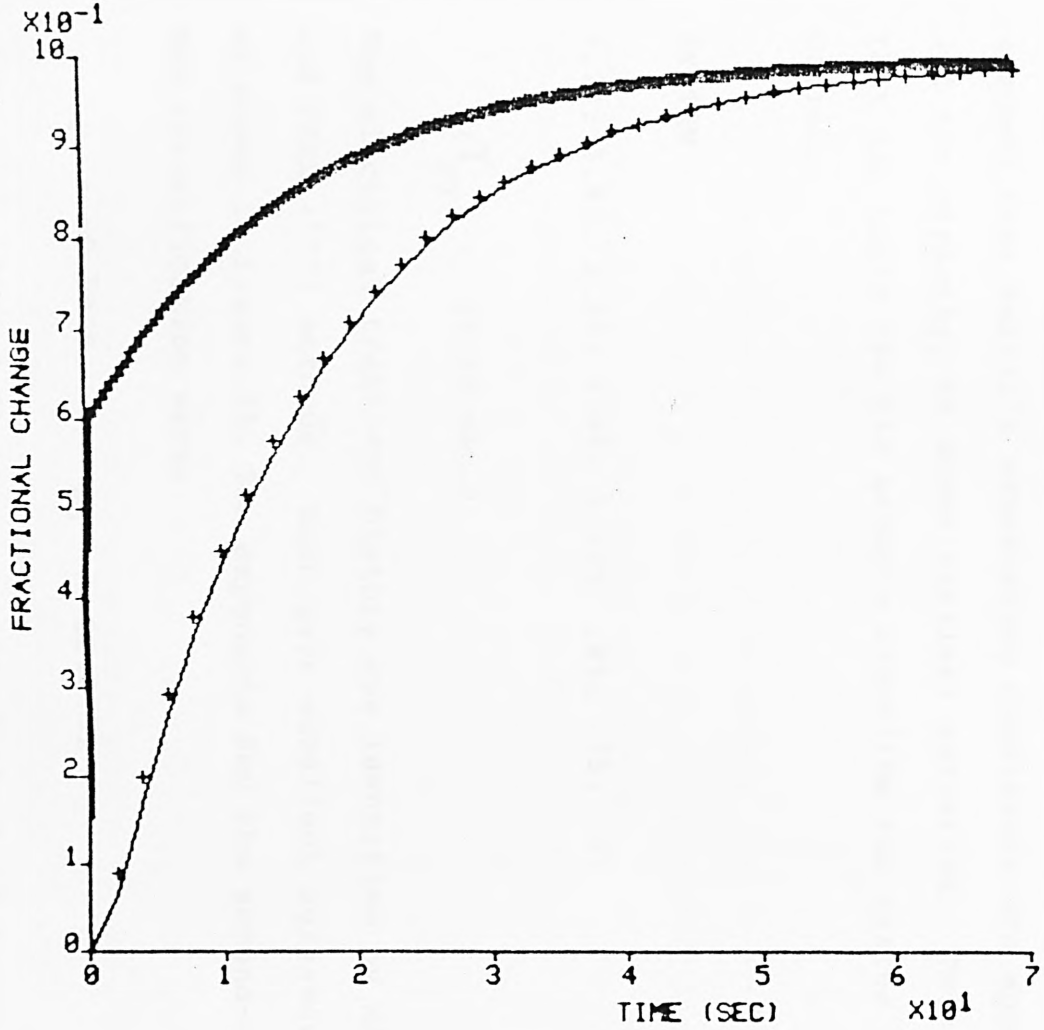


FIGURE 24. ET DATA FIT(TOP CURVES) AND FT DATA/PREDICTION

IDENTIFICATION OF SIMULATED REAL SYSTEMS

In the following, we shall present results for identifying of simulated real thermometer with and without wells i.e. without neglecting the centre capacity. As for the above results, typical results for thermometers will be presented first followed by those for thermometer-in-wells. It is assumed that Kerlin's expediating conditions are approximately (if not strictly, as shown earlier) satisfied. This implies that one could use his group's algorithm for system identification.

SYSTEM :

5.74: 2.92, 2.55, 2.48, 1.585, .85, .75, .07

(τ_{FT} = 19.55 sec.)

The electrical transient history was identified by NAG(***) and PEEL (°°°) methods. Both gave excellent agreement with ET as shown in figure 25. The exponents for the second-order NAG identification were:

$p_1 = -0.0805, \qquad p_2 = -10.6772$

giving a predicted fluid transient of about 12.5 seconds. For the PEEL identification, typical late time points were at 8.3 and 10.35 seconds (when recovery was 65 and 70 per cent). The exponents were:

$p_1 = -0.07654, \qquad p_2 = -0.4423$

giving a predicted τ_{FT} 15.55 seconds. From this and detailed predicted fluid transient, it can be seen that NAG (crosses) is faster and PEEL (squares) approximates the distributed-parameter model better (full curve). PEEL was also run with late times 28.8 and 16.5 seconds (91.5 and 80.9 per cent recovery) and earlier times being kept unchanged (2.15 and 4.2 sec., 37.1 and 49.4 per cent recovery). The exponents determined were:

$$p_1 = -0.0658, \qquad p_2 = -0.2851$$

giving a predicted $\tau_{FT} = 19.2$ seconds. The prediction of fluid transient (FT) was found to be excellent. A further identification with PEEL using as late times 41.1 and 20.6 seconds (recovery being 96.1 and 85.5 per cent) gave exponents:

$$p_1 = -0.06406 \quad \text{and} \quad p_2 = -0.2614$$

giving a predicted τ_{FT} 20.0 seconds.

The predicted FT (□□□) is in excellent agreement with FINEL (full curve) as seen in figure 26. Notice that the ET fitting by PEEL (000) is in very good agreement with the distributed parameter model. Therefore, PEEL gives excellent results if late times are 'late enough' - over 80%). NAG gives, generally, comparatively inferior results (+++).

SYSTEM:

13.6: 3.0, 2.5, 2.375, 1.585, .85, .75, .07

(τ_{FT} = 22.5 sec.)

PEEL (□□□□) see fig. 27 was run with late time points at 20.0 and 12.0 seconds (76.6% and 67.6% recovery) and earlier times 6.4 and 4.3 seconds (53.1 and 44.7%). The exponents were: $p_1 = -0.0575$ and $p_2 = -0.3682$, giving an approximate fluid transient time constant of 20.3 sec. This compares well with FINEL. NAG (++++) second-order attempt gave almost identical results to this PEEL run. A NAG third-order (▽▽▽▽) identification gave identical results to the distributed (FINEL) model - exponents : $p_1 = -0.055$, $p_2 = -0.2737$, $p_3 = 47.9$. It appears, therefore, that a third-order identification may lead to quite satisfactory, if not very good, prediction of fluid temperature transient. As with other other identification results presented in this thesis, the top curves (with non-zero slope at zero time) are fitted curves with FINEL electrical transient results. Agreement , as with most results for ET presented, is excellent, PEEL ET fit is shown as circles and not all results are presented in the figures for the sake of clarity.

It is quite clear that PEEL may offer an easy and quick basis for prediction of fluid temperature transients. For this purpose one needs to have some knowledge of 'late time' effects. Figure 28 shows the predicted FT results with details as follows (recovery factors in brackets):

A: 25.3 (84.7%), 20.5 (79.6%) sec;

$p_1 = -0.0548$, $p_2 = -0.2801$. (□□□□)

B: 32.3, 20.5 sec. : p_1 = -0.0582, p_2 = -0.4145
(90.0%),(79.6%) (ΔΔΔΔ)

C: 20.05 (79.6%) and 13.4 (70.2%) seconds;
p_1 = -0.057, p_2 = -0.3454. (++++)

D: 30.2, 13.4 sec., p_1 = -0.0556, p_2 = 0.3252
(88.3%)(70.2%) (0000)

Results show that very late times do not necessarily give better results than not-so-late times (like case A).

However, it can be seen that all the PEEL predictions are quite satisfactory. Therefore, if fairly noise-free or smoothed data of ET are available, predictions based on exponential peeling, would be useful for thermometers.

SYSTEM:

5.74: 5.0, 3.22, 2.92, 2.55, 2.48, 1.585, .85, .75, .07

$$\tau_{FT} = 74.8 \text{ sec.}$$

As this is a combination of thermometer and well, a third-order identification is more appropriate than a second-order. The NAG fit of FINEL ET data (***) is, as for most other ET fits, is good and the exponents are: 0.0167, -0.2244, -31.6 and the fluid temperature transient is shown as crosses (+++) in figure 29, giving a τ_{FT} of about 68 seconds.

It has been found by Caroll (reported in Kerlin et al (1978), p 52) that an approximate (and empirical) relation could be developed between poles p_1, p_2, etc.

$$p_2 = p_1 (1 + c)^2$$

where C is a constant. Such a formula would be most useful because, as we have shown that the dominant pole, p_1, can be determined to a satisfactory degree and it has been known (and also found by us) that two dominant poles are enough to predict fluid temperature response of thermometers in wells. A number of previously carried out results seemed to suggest, that, for the thermometer in well systems considered here, under staticfluid conditions, an approximate relation between dominant poles is:

$$p_2 = 0.355 \hat{p}_2$$

where $\hat{p}_2$ is determined by exponential peeling. The results for the system considered here are shown as triangles (▽▽▽▽▽). Agreement with the system is seen to be quite good. The 'hashed' curve (***) shows the fluid transient prediction using PEEL with late times 90.3 and 61.6 seconds (84.7% and 75.3% recovery); early times 26.75, 16.5 sec. (55.7%, 47.1%). The exponents were -0.0167 and -0.1378. Notice that the smallest (i.e. the dominant) exponent is the same as for NAG (above). The prediction is somewhat better than NAG.

It is clear that exponential peeling (PEEL) could be useful for fluid temperature response prediction. A program to study the effects of choice of late times on fluid temperature response prediction using PEEL. The late times together with other pertinent information is shown below (figure 30).

Late Times(Sec)	% Recovery	Exponents	Points
129.25	92.0%	-0.0167	****
61.6	75.3%	-0.122	
129.25	92.0%	-0.0167	++++
45.2	67.5%	-0.1566	
90.3	84.7%	-0.0163	0000
73.9	80.0%	-0.0495	
49.3	70.0%	-0.0181	ΔΔΔΔ
41.1	65.5%	-0.0534	

As can be seen all give acceptable results. Surprisingly , the 'late times' corresponding to recovery 65.5% and 70% give relatively better, and good, prediction of FT.

SYSTEM:
13.4: 5.0, 3.120, 3.05, 2.5, 2.385, 1.585, .85, .75, .07

$$\tau_{FT} = 38.5 \text{ sec.}$$

This is a thermometer-in-well system, and a third-order NAG identification gave excellent fit fo FINEL ET data and gave exponents: -0.0372, -0.2421, -6.3380 (τ_{FT} 31.5 sec.). A PEEL identification, using as late times 41.0 and 33.8 seconds (86.6%, 82.5% recovery) - exponents -0.03725, -0.2437. It can be seen that the dominant exponents are almost identical to those of NAG and, not surprisingly, the fluid temperature transient prediction are almost identical (***). The circles denote PEEL fit of ET data and it can be seen to be good (as for NAG) - fig. 31.

The importance of empiricism was noted earlier. As pointed out earlier a number of hard calculations had suggested the following empirical relation.

$$p_2 = c_1 \times \hat{p}_2$$

where p_2 is determined by Peeling or Nag methods and c_1 is a constant for a particular class of thermometer (with or without Wells). For the thermometer-in-well systems, an approximate relation was found to hold:

$$p_2 = 0.355 \times \hat{p}_2$$

The FT prediction is shown in figure 32 as hashes (***) and the normal PEEL as crosses. Note that the ET fit (000) does not agree closely to the real system for time less than about 1 second. The exponents of the 'improved' prediction: -0.03725, - 0.0865 .

The above results show that both curve fitting and exponential peeling give satisfactory predictions for fluid temperature transient response from electrical response data. Exponential peeling does notalways give the best result if 'late' time points are very late. However, it is easy to check for consistency of the dominant exponenent predictions by taking more than two data points in the late time period and obtaining estimates of the exponent. It may be necessary to filter ET data if significant noise is present when using exponential peeling.

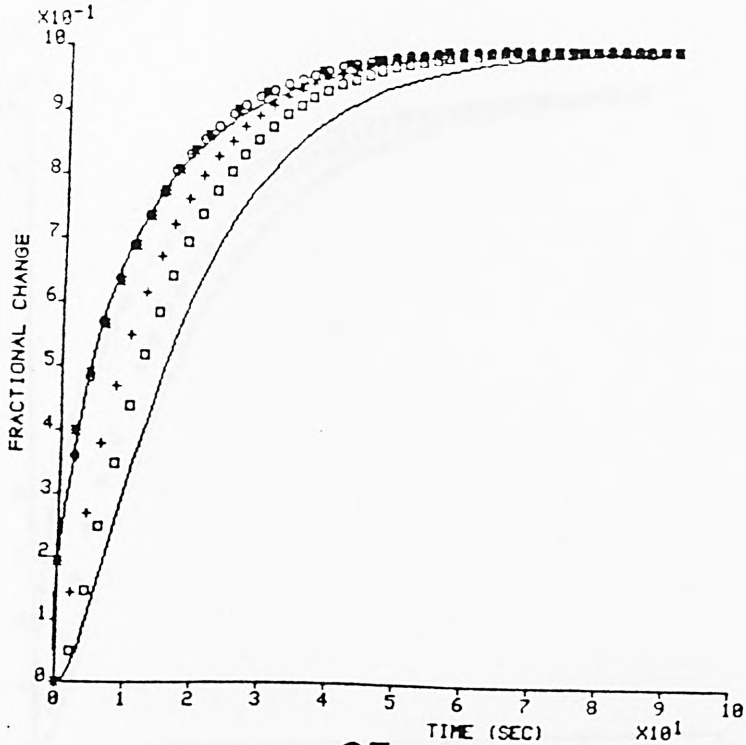


FIGURE 25

FIGURES 25 - 32: ET DATA/FIT AND FT DATA/PREDICTION

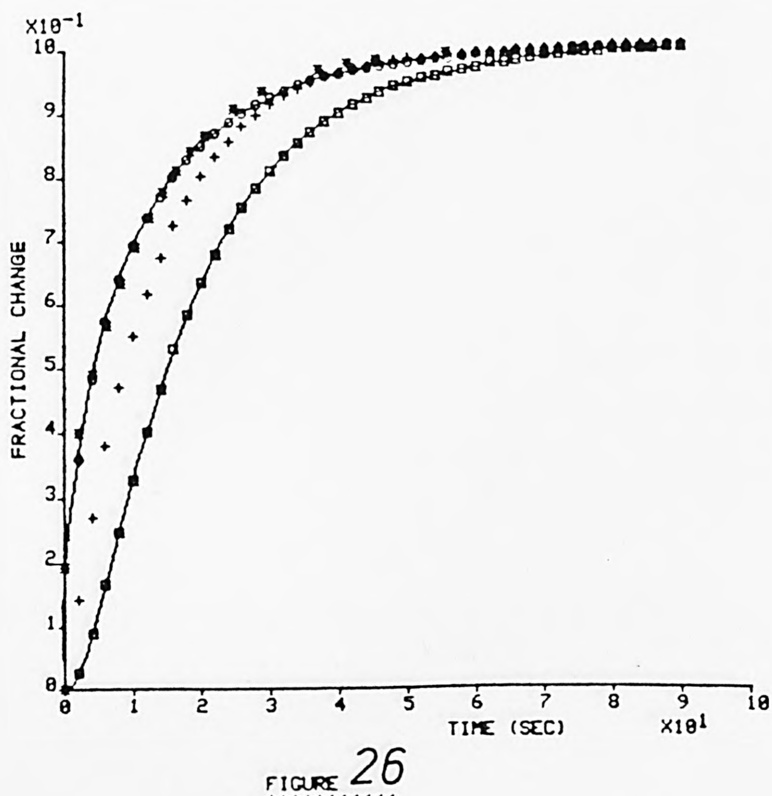


FIGURE 26

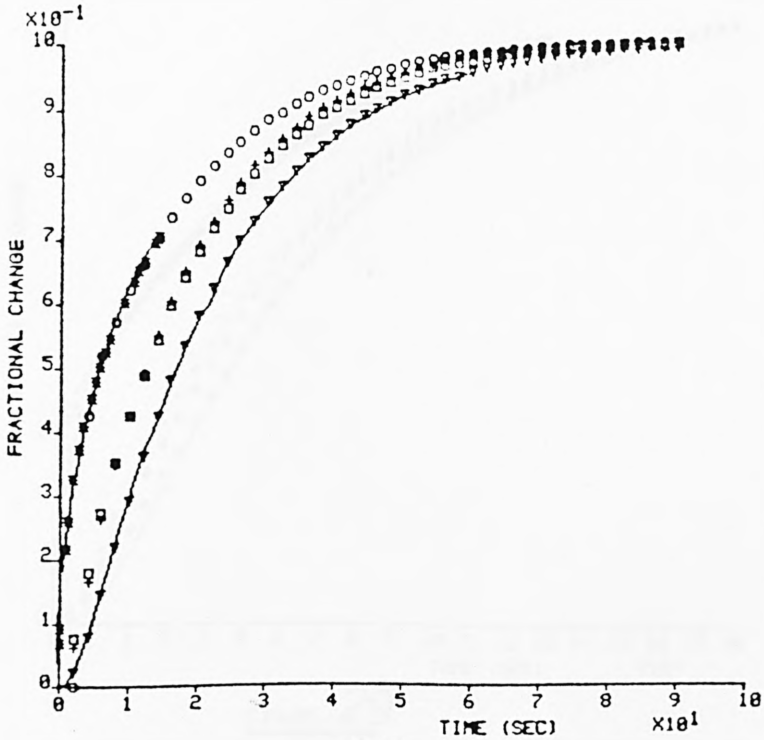


FIGURE 27

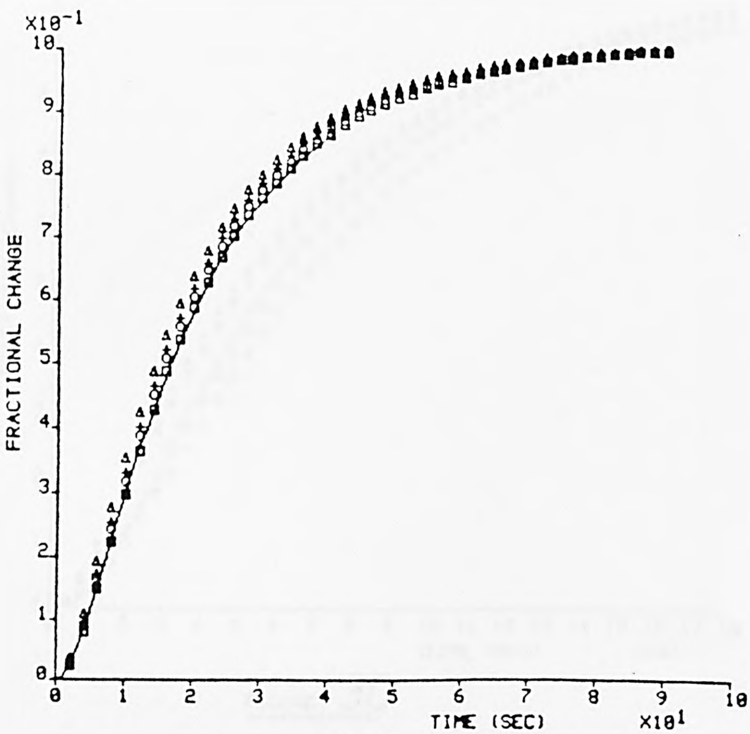


FIGURE 28

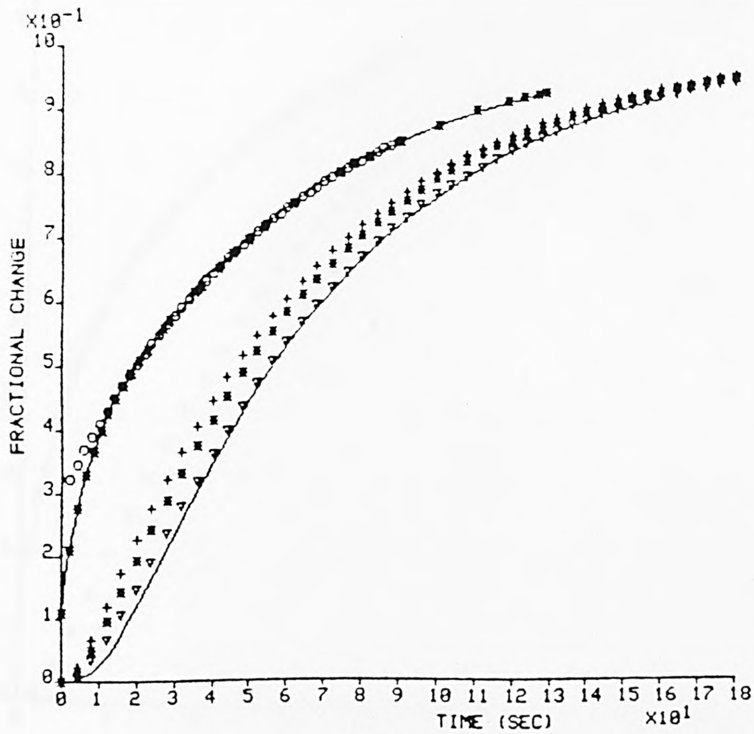


FIGURE 29

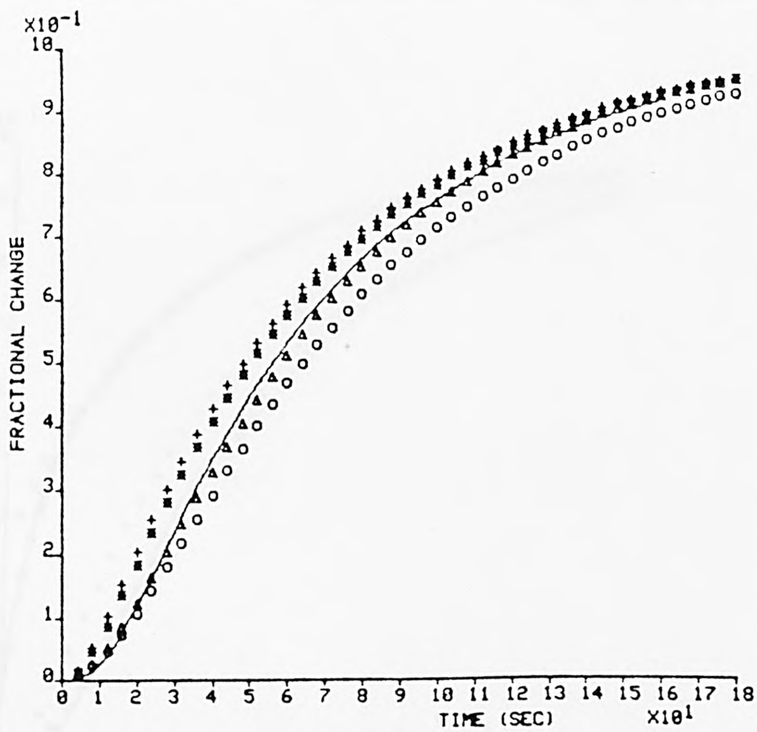


FIGURE 30

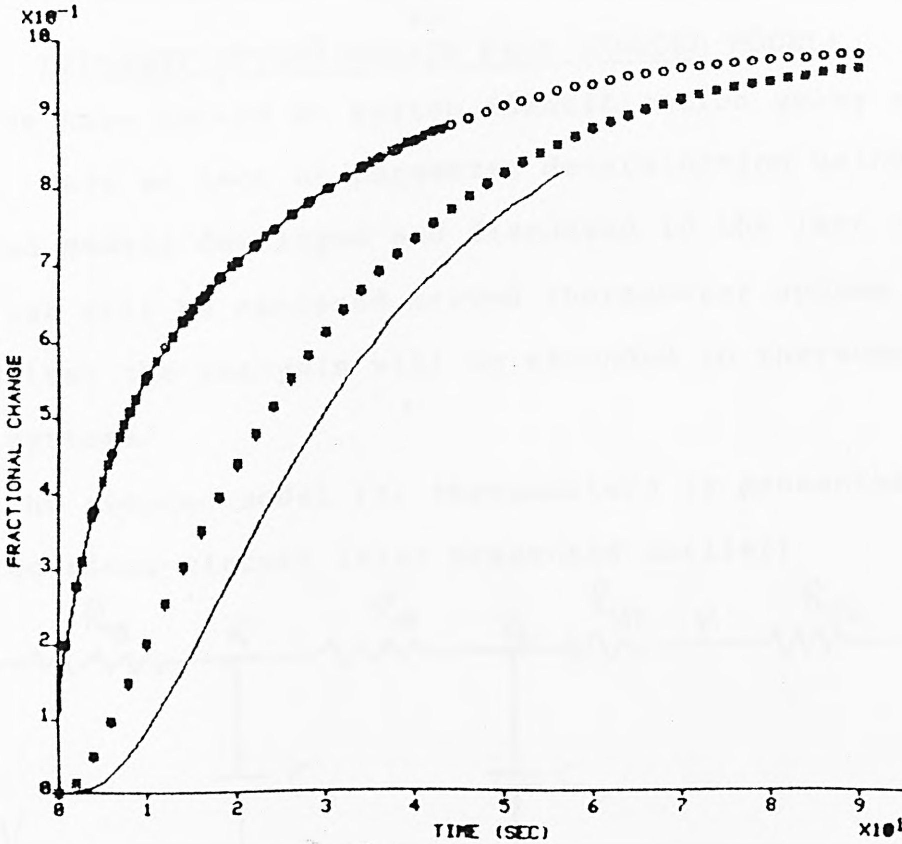


FIGURE 31

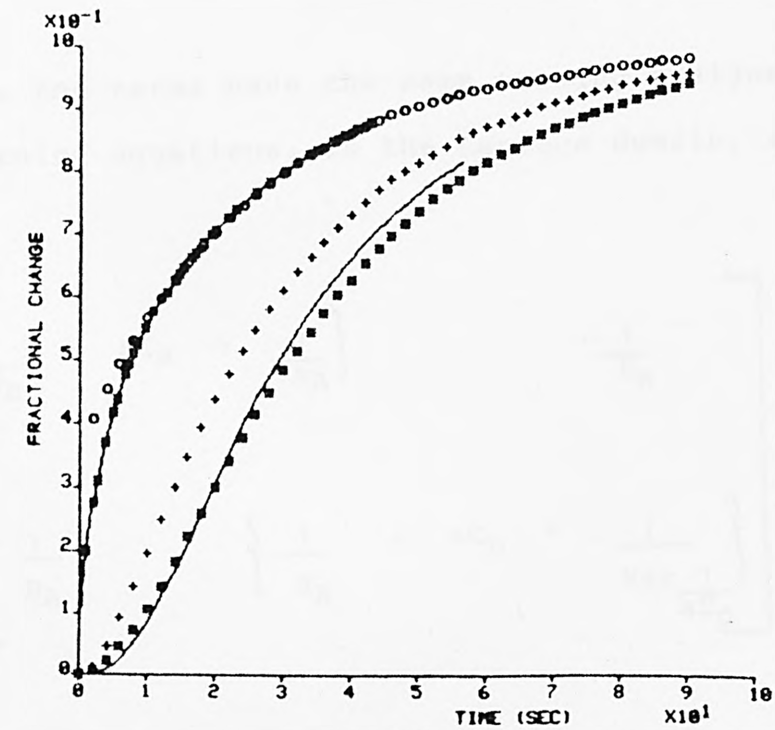


FIGURE 32

3.4.3 EXPONENT DETERMINATION FROM REDUCED MODELS

We have looked at system identification using system data. Here we look at parameter determination using valid reduced models developed and discussed in the last chapter. Interest will be centered around thermometer system and it is hoped that the analysis will be extended to thermometer-in-well systems.

The reduced model for thermometers is presented below as an electrical circuit (also presented earlier)

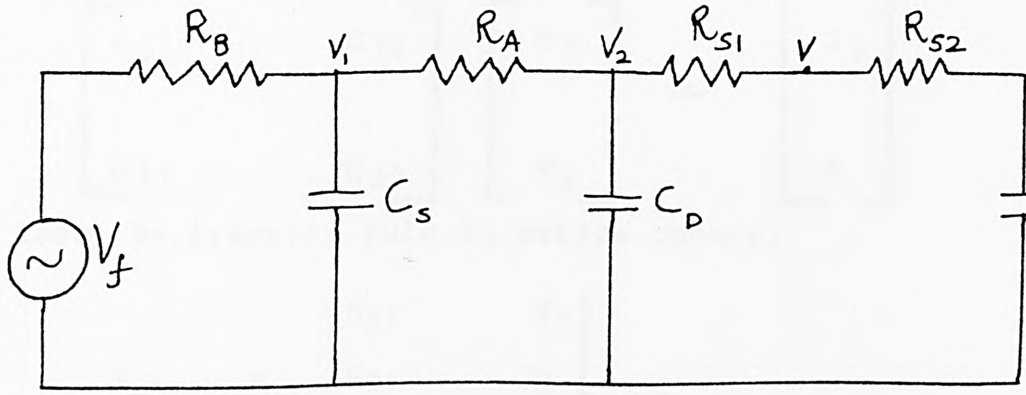


FIG. 33

where the terms have the same meaning assigned earlier. The governing equations, in the Laplace domain, are:

$$\begin{bmatrix} \left\{\frac{1}{R_B} + sC_S + \frac{1}{R_A}\right\} & -\frac{1}{R_A} \\ -\frac{1}{R_A} & \left\{\frac{1}{R_A} + sC_D + \frac{1}{R_S + \frac{1}{sC_C}}\right\} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} \frac{V_f}{R_B} \\ 0 \end{bmatrix}$$

or,
$$\begin{bmatrix} \left\{\frac{1}{R_B} + sC_s + \frac{1}{R_A}\right\} & -\frac{1}{R_A} \\ -\frac{1}{R_A} & \left\{\frac{1}{R_A} + sC_D + \frac{sC_C}{sR_sC_C+1}\right\} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} \frac{V}{R_B} \\ 0 \end{bmatrix}$$

Where $R_s = R_{s1} + R_{s2}$

Writing above as:
$$\begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} I_1 \\ 0 \end{bmatrix}$$

then, by Cramer's rule in matrix theory,

$$V_2 = \frac{\begin{vmatrix} G_{11} & I_1 \\ G_{21} & 0 \end{vmatrix}}{|G|}$$

$$= \frac{-G_{21} \; I_1}{G_{11}G_{22} - G_{12}G_{21}}$$

The voltage at the sensor is

$$V = V_2 - R_{s1} \; I$$

Where $I = \frac{V_2}{R_s + \frac{1}{sC_C}}$

Therefore,

$$V = V_2 \left(1 - \frac{R_{s1}}{R_s + \frac{1}{sC_C}}\right)$$

Substituting for V_2, one gets:

$$V = \frac{-G_{21} I_1}{G_{11}G_{22}-G_{12}G_{21}} \left(1 - \left\{\frac{R_{s1}}{R_s+\frac{1}{sC_c}}\right\}\right)$$

The expression on the right-hand side, divided by I_1, is the transfer function of the thermometer. The poles are given by the denominator when equated to zero. After substituting for the G's, multiplying bottom and top by $(sR_sC_c + 1)$, and collecting together the terms, one gets the following expression:

$$s^3C_SC_DC_CR_S + s^2(C_SC_C\frac{R_S}{R_A} + C_DC_S + C_cC_s + G_1C_cC_DR_s) + s(G_1C_C\frac{R_S}{R_A} + G_1C_D + G_1C_C + \frac{C_S}{R_A} - C_C\frac{R_S}{R_A^2}) \quad \text{(A)}$$

Where $G_1 = \frac{1}{R_B} + \frac{1}{R_A}$

Equating this expression to zero gives the poles (exponents) of the system.

The test thermometer system chosen was the following:
13.4 : 3.0, 2.5, 2.375, 1.585, 0.85, 0.75, .07
The variables in the above expression can be calculated to be

$$R_A = \frac{\ln \frac{2.5}{2.37}}{2\pi kL} = 13.61$$

R_B = 1.56

C_S = 0.79

C_D = 1.031

R_S = R_{S1} + R_{S2}

= 33.722

C_C = 0.1342

Roots of expression (A) can be determined by a suitable computer program (developed by NAG) :

-0.0541, -0.9127, -0.2585

Exponents determined by NAG, using ET data and a third-order model :

-0.055, -0.2737, -47.9

It can be seen that the first (i.e. dominant) exponents can be determined to quite a good accuracy.

A second-order model with ET data, gave exponents, determined by NAG, as:

-0.0597, -0.9603.

It appears, therefore, that a third-order model should be used for exponent prediction using ET data. This is not surprising since the physical model is third-order. The first (i.e. dominant) exponent is quite well predicted by the second order model. Note that when the centre capacity is absolutely negligible (results presented earlier) the exponents were

-0.0635, -2.156, -24.3.

Results show that the dominant exponent is satisfacorily predicted by curve-fitting but the other exponents not so well.

CHAPTER 4 CONCLUSIONS AND RECOMMENDATIONS FOR FURTHER WORK

Modelling effort, reported in the literature for industrial thermometers, has neglected to include detailed physical modelling which considered dimensions and physical properties . The present project, the first of its kind at Measurement and Instrumentation centre, had an objective to obtain detailed physical models for model-based decisions of industrial significance. It is to form a base for further work on thermal transducers at the centre.

The main conclusion of this thesis are as follows: Distributed and lumped-parameter models of a typical industrial thermometer have been developed and validated. The former were based on the finite-element method. This is particularly convenient since it easily allows physical property variations due to different materials, which are present in industrial resistance thermometers and thermocouples. The assumptions involved are quite modest:

a) radial heat flow

b) constant physical parameters.

These models have been fully validated with experimental results for a resistance thermometer-in-well assembly for entire time histories and for two cases:-

i) a fluid step change and negligible self-heating(abbreviated FT) and (ii) a deliberate self-heating step change produced by injecting electrical energy into the platinum sensing element (abbreviated ET).

It has been shown that distributed parameter (finite-element) models assist in the developments of reduced and valid lumped models. The latter can be easily simulated, in an interactive manner, by using MEDIEM, a package developed at the Measurement and Instrumentation centre (Liebner et al (1982)).

Sensitivity analysis using the dynamic models developed has indicated that time constant variations in a typical industrial thermometer arise from tolerances in air gaps and from variations in heat transfer coefficients . For air gaps it was found that the sensitivity coefficients $\partial \tau_{FT}/\partial a_i$ and $\partial \tau_{ET}/\partial a_i$ (a_i :- width of either air gap) are approximately constant. Similarily it was found that the variations of τ_{FT} with surface heat transfer is approximately inversely proportional to the heat transfer coefficient.

Regarding identification of electrically produced transients (ET) and their subsequent use for prediction of fluid step transients (FT) the following conclusions can be drawn:-

(a) Generally, curve-fitting does not produce unique parameter estimates for ET data. However, both curve fitting and exponential peeling gave satisfactory predictions of the fluid temperature transient (FT) response. The latter may require some filtering of electrical transient (ET) data if noise is significant. The dominant exponent can be reliably estimated even with relatively few data points and or low-order models.

(b) When Kerlin's expediating conditions are satisfied , errors were found to be within 10-15%. Further, when these assumptions hold, the thermometer and thermometer-plus-well systems can be represented as second-order and third-order systems respectively. Curve-fitting experience also shows that this is so. This also has an important overtone since only two or more exponents can be reliably predicted. Therefore,this work has shown that Kerlin's algorithm is useful from a practical point of view besides being theoretically sound. However, not all thermometers have negligible centre capacitances as the Rosemount sensor described earlier.

A study of the so called stem correction or immersion depth absolute error was made using lumped steady-state models developed and simulated using MEDIEM. These indicated errors of expected magnitudes although further experimental data is needed in this respect.

Future work should be concerned with the use of models to assist the designer and user of industrial thermometers. A first and brief approach at a design methodology has been made. No doubt this will need to be expanded and refined in the light of practical design requirements. The validity of the models developed in this thesis indicate that reliable design decisions can be made on the bases of mathematical models. These models being interactive, provide a powerful aid to an experienced industrial designer who should be part of an interactive computer-aided-design loop.

For the user of industrial thermometers, the prediction of expected fluid step behaviour entirely from the electrical port provides an attractive method of in-situ response testing.

Theoretically, the approach appears to be sound and if practically implemented, it can allow remote monitoring of faults in industrial sensors. For this in-situ testing, some work will need to be done on microprocessor based implementation of Kerlin's algorithm. This would involve looking into (a) appropriate curve-fitting routines implementable on a micro-processor and (b) appropriate filtering of ET data. The latter aspect would also allow the exponential peeling method to be used.

The future will also see the emergence of modelling software such as that used in this thesis implemented on smaller and cheaper microcomputers and single user computers.

In fact,this area is being actually persued at present in another research project at the centre. This will, it is believed, result in much wider use of the modelling techniques in industry.

APPENDIX A

Paper presented at IMEKO conference, Czechoslovakia, October 1981, and published in:"Temperature Measurement in Industry and Science", Dun Techtiniky CSTVTS, Prague, pp.238-45.

MATHEMATICAL MODELLING OF THERMAL TRANSDUCERS FOR TEMPERATURE MEASUREMENT

by

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Abstract

Lumped parameter and distributed parameter mathematical models are developed for an industrial platinum resistance thermometer assembly assuming one dimensional radially directed heat flow. A network type lumped parameter model is developed and solved using available computer packages including a bond graph package. Some uncertainties in the model 'lumping' led the authors to investigate the solution of the governing partial differential equations using the finite element technique. For this purpose an 8 element 22 noded distributed parameter model was developed using a finite element package. The results of a step response test on this model were found to be in close agreement with experiments conducted at the National Physical Laboratory. Additional sensitivity tests showed that air gap tolerances and uncertainties in the fluid/thermometer surface resistance can produce a ±20% variation in the thermometer time constant.

It is concluded that distributed parameter finite element models may provide a powerful aid towards the development of simpler lumped parameter models as well as being of interest in their own right (for calculating stem correction errors for example). The authors' broader aims are to develop a general mathematical modelling philosophy for industrial thermal sensors which they believe will help both the designer and user. Coupled with microprocessors this may lead to the development of "intelligent" instruments.

Key words: platinum resistance thermometers, mathematical modelling, finite element analysis, thermal transducers.

1. Introduction

A number of studies have been carried out in the last three decades on the dynamic response of industrial thermometers. As far as we know, all these assume a first or second-order system representation with parameters determined by argument, guessed or estimated (e.g. Lefkowitz (1955), Louis & Hartman (1964)). In fact a number of textbooks on control and instrumentation follow these lines in introducing instrument and general system dynamics (e.g. Doebelin (1975)).

Strictly speaking, heat transfer through thermometers is a distributed-parameter system governed by a parabolic-type partial differential equation. System representation by low-order models and estimated (or guessed/evaluated) parameters have some appeal as they are relatively easy to simulate. Unfortunately, such models may not be valid or easily be shown to be valid generally. What is required, for low-order models, is a systematic study of system order and the parameters involved. We have made an attempt in this direction and some work, which will hopefully lead to the development of valid low-order models, is reported here.

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Valid low-order models for thermometers can be useful in many industrial decisions. For example, computer-aided design of instruments could be implemented using such models. Instrument analysis based on models could lead to a better understanding of a control system performance. We also believe that low-order models would lead to development of algorithms/procedures for testing of thermometers especially in-situ electrical port testing and to the realization of 'intelligent' instruments based on microprocessor technology. These are practical problems and because of the importance of temperature measurement in industry, there is a need to solve them.

2. Commercial Thermometers

Commercial thermometers have been broadly classified as immersion and non-immersion types (Chohan and Abdullah (1980)). Only immersion type transducers will be considered here. The most important immersion thermometers are thermocouples and resistance thermometers. We are only concerned with these two types in this project. The working principle of these transducers is, of course, well documented (e.g. Dobelin (1975)).

Industrially used thermocouples and resistance thermometers can be 'decomposed' into two parts - the outer well or pocket and the internal transducer containing the sensing element (junction or metal resistor). The inside of the transducer can be quite complex and vary from manufacturer to manufacturer. Most of the immersion thermometers take a cylindrical geometry.

We have analysed one type of commercial resistance theremometer made by Rosemount Engineering but the analysis can be extended/modified to suit other makes of resistance thermometers and thermocouples. One of our aims is to build a general philosophy of thermal transducer modelling.

The thermometer can be decomposed into the following subsystems: 1. Outer well, 2. air gap, 3. steel sheath, 4. air gap, 5. aluminium cover, 6. ceramic former containing the sensing element (here a platinum coil). Subsystems 3, 4, 5, 6 form the internal sensor assembly. As is well known in system dynamic studies, there is no unique decomposition of a system. For example, item 6 above has been further decomposed into two components here - an outer (annular) ceramic and an inner cylindrical core. An important point that is made here is that the dimensions of the air gaps vary because of tolerances inherent to manufacturing processes. Dimensional and other details are contained in Chohan and Abdullah (1980)).

3. Lumped parameter modelling

Since our interest is in mechanistic (rather than 'black-box') modelling, one has to carry out as a precursor an analysis of the physical processes connected with the use of the thermal transducers - not just a catalogue of available results from a limited lierature search. This has been done to a certain extent(and is continuing). Since plunge tests, carried out at the National Physical Laboratory on the thermometers werein static water, only the case of free convection was examined in any detail.

It has been recognised for some time that a common concept for many systems is energy, i.e. these systems handle energy. Energy has connected to it two conjugate variables - the so-called through and across variable. For hear transfer, these correspond to heat flow rate and temperature, respectively. For bond-graphs, each bond represents a power flow, i.e. rate of energy flow (e.g. Karnopp and Rosenberg (1975)). (Strictly speaking the product of heat flow rate and temperature is not power but these variables (called pseudo variables) are used because of their convenience). Since the information desired from a thermometer is carried by heat diffusing from a surrounding medium to the sensing element, it would appear that bond-graph and network modelling would be beneficial here.

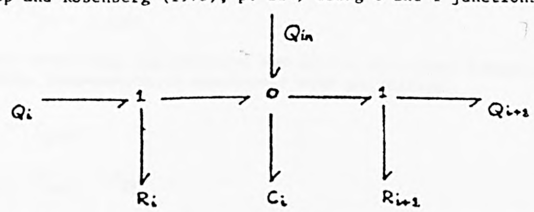
The thermometer together with its environment can be decomposed and represented, approximately, as follows:

SYSTEM	ENVIRONMENT
(i) core (ceramic)	(i) Boundary-layer
(ii) Annular ceramic (with Pt. wire)	(ii) main fluid body
(iii) Aluminium cover	(iii) connecting assembly
(iv) air gap	
(v) sheath	
(vi) air gap	
(vii) well	

INTERACTION

(heat transfer)

In this paper, effects of connecting assembly are neglected and only radial energy flow is considered. The effects of the environment are represented by a fluid temperature and a resistance to heat flow to the well surface (which we shall hereafter refer to as boundary layer (BL) resistance). Each of the seven sub-systems in the above diagram were considered as 'lumped' elements. Each 'lump' can, then, be represented (for the heat conduction process) by a bond-graph (see Karnopp and Rosenberg (1975), p. 114) using 0 and 1 junctions:



where the R's and C represent the respective resistance and capacitance and Q_{in} is the heat generated (e.g. electrically) or added from the outside of the 'lump'. For each 'lump', the resistance was taken to be the steady-state value corresponding to an annular region of a cylinder (Myers (1971)):

$$R_i = \frac{\ln\left\{\frac{r_{i+1}}{r_i}\right\}}{2\pi k L} \quad {}^{o}C\ J^{-1}\ s$$

where r_{i+1}, r_i are the inner and outer radii of the annulus, and L the length of the cylinder which contains the sensing element. The subsystem capacitance is just:

$$C = mc \quad J\ {}^{o}C^{-1}$$

where m is the mass and c the (specific) thermal capacity. The boundary-layer resistance was evaluated as:

$$R_{bL} = (Nu.\ k\ \pi\ L)^{-1}$$

All the details can be found in Chohan and Abdullah (1980).

The models developed were simulated using THTSIM, a package developed at the Twente Institute of Technology, the Netherlands (Van Dixhoorn (1977)). However, the seventh order model did not allow practically acceptable time step and therefore further model reduction was required. A fifth-order model was attempted and a bond-graph representation is presented as Figure 1. The air gap resistance was added to that of bL resistance (and well resistance) - this was considered as a composite first-order system. The second air gap was added to the sheath resistance. These composite resistances were denoted by R_{A1} and R_{A2} and their variation due to manufacturing tolerances were determined. The bL resistance was taken to be that of an equivalent flat plate. The results were represented as time constants (for step inputs). The time constant τ, being defined as the time at which the output reaches 63.2% of its final value. A typical set of results are presented in Figure 2 showing an almost linear variation of time constant with respect to the resistance. The results gave the following average values of the sensitivity coefficients (time constant change w.r.t. resistance):

$$\left.\frac{\partial \tau}{\partial R_{A1}}\right)_{R_{A2} = \text{const.}} \approx 5.6$$

$$\left.\frac{\partial \tau}{\partial R_{A2}}\right)_{R_{A1} = \text{const.}} \approx 1.5$$

Another model that was proposed had as the outermost subsystem (i.e. next to the fluid whose temperature is monitored) with parameters:

$$C = C_{well}$$

$$R_{A1} = R_{well} + R_{BL}$$

and the sub-system following this:

$$C = C_{sheath}$$

$$R_{A2} = R_{sheath} + R_{air\ gap}$$

with the second air gap resistance being added to the aluminium cover subsystem. This model was simulated on MEDIEM, a package developed at The City University Instrument Systems Centre (Liebner and Abdullah (1980)). The results showed that the time constants were lower than those from experiments with the flat plate boundary layer resistance. It was found that:

$$\frac{\partial \tau}{\partial R_{A2}} \approx 1.5$$

which corresponds with the first model but R_{A2} accounts for the outermost air gap.

In view of the discrepancies between the two lumped parameter model developed we thought it necessary to solve the heat flow equations exactly using the finite element techniques as described in the next section.

4. Distributed Parameter Modelling

As mentioned in the introduction the dynamic heat transfer process through thermometers is governed by a system of parabolic type partial differential equations. The finite element technique (Zienkiewicz (1975), Huebner (1975), Gallager (1975)), is a numerical technique which enables one to solve such equations for practical engineering systems. We used a finite element package FINEL developed at Imperial College of Science and Technology, London (Hitchings, 1980), to model one dimensional radially directed heat flow through the commercial thermometer assembly described already. Each region (steel well, air gap, etc.) was divided into one four noded axisymmetric line element with appropriate material properties k (thermal conductivity) and c (thermal capacity). At the well surface a surface heat transfer element was added with an appropriate value of h (the surface heat transfer coefficient). The resulting model had 8 elements and a total of 22 nodes. Results of the dynamic response simulation using FINEL are presented in Figure 3 where results of the NPL plunge test and a simple first order exponential low are also presented. As can be seen the finite element model results compare favourably with the experimental ones. The first order response law is a poor approximation, particularly during the initial time period. It should be emphasised that the finite element model results are an absolute prediction and not a mere fit. The input to the model is the geometry of the thermometer and the material properties of each region. In contrast the first order result is a mere fitting of the data to match the 63.2% output point. The finite element model predicts the time constant τ = 44 secs. correctly. Additional checks on the model validity were performed by increasing the number of elements. In fact each region was divided into two elements rather than one. The resulting model gave virtually identical results.

We then performed sensitivity tests on some of the more uncertain variables such as the air gaps and the boundary layer resistance. This resistance is related to the surface heat transfer coefficient h, by:

$$R_{bL} = \frac{1}{2\pi\, hrL}$$

where L is defined as before and r is the outer radius of the steel well. Results presented in Figure 4 show a linear variation of τ with the air gaps a_1 (the air gap nearest the fluid) and a_2 (the inner air gap) and a non-linear variation with h. These results indicate that the expected spread in τ could be in the range 40 - 60 secs., a result confirmed by the thermometer manufacturer, Rosemount Engineering.

5. Conclusions

The lumped parameter modelling described in this paper led to two models the first of which showed fair agreement with experiment whereas the second did not. A distributed parameter model was then developed based on a finite element solution of the governing parabolic differential equations assuming one dimensional radially directed heat flow. This was found to give good agreement with experimental results conducted at the National Physical Laboratory. Such finite element models can then be used as a "test bed" for the development of accurate and valid lumped parameter models.

Work is proceeding on these lines towards the development of a general modelling philosophy for industrial thermometers to include among other things:- (i) both fluid and electrical port loadings; (ii) axial as well as radial heat flow effects.

It is envisaged that such models will find increasing applications in industrial thermometer design and use particularly when coupled with microprocessors. We believe this trend may lead to "intelligent" selt checking instruments that will rely on some interrogating electrical signal.

6. Acknowledgements

The authors would like to thank Mr. J. Johnston of Rosemount Engineering for the supply of and information on their resistance thermometer assembly which was tested at N.P.L. Also we wish to thank Dr. A. Colclough for organising the tests conducted at N.P.L. Finally, Mr. R.K. Chohan wishes to acknowledge the support of an S.R.C. Research Studentship.

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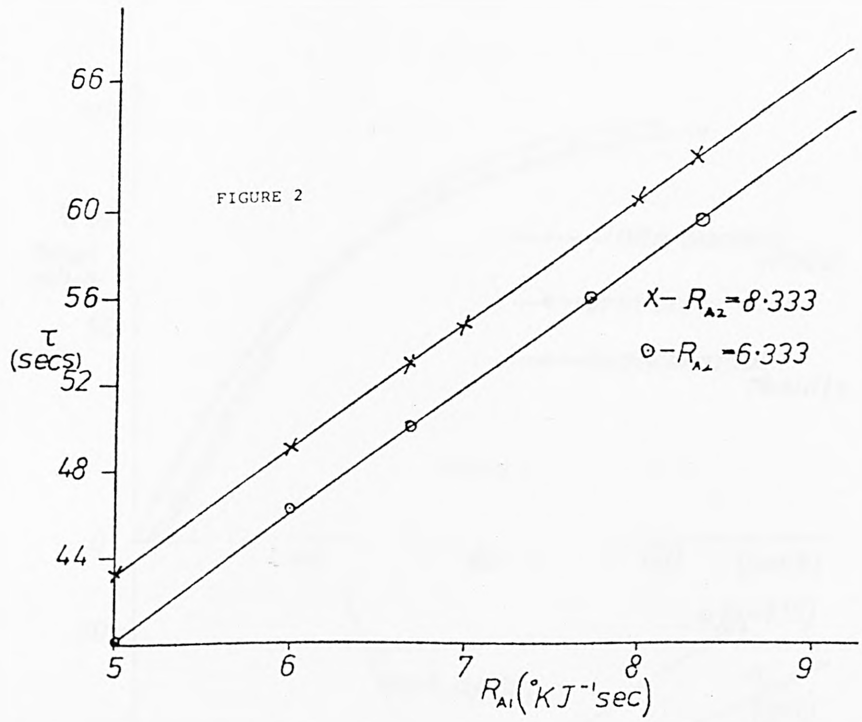

FIGURE 2

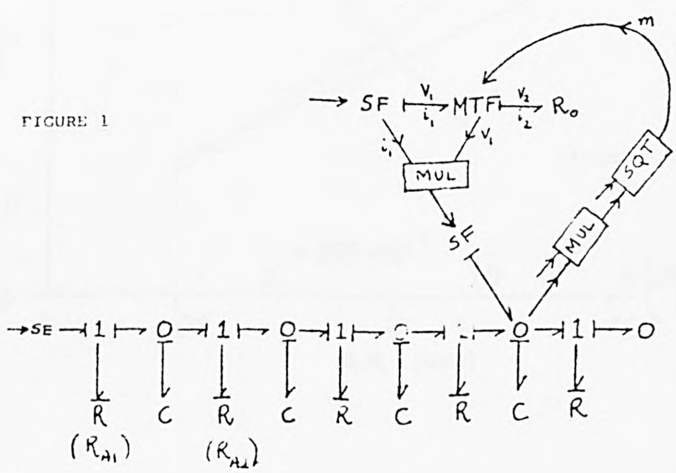

FIGURE 1

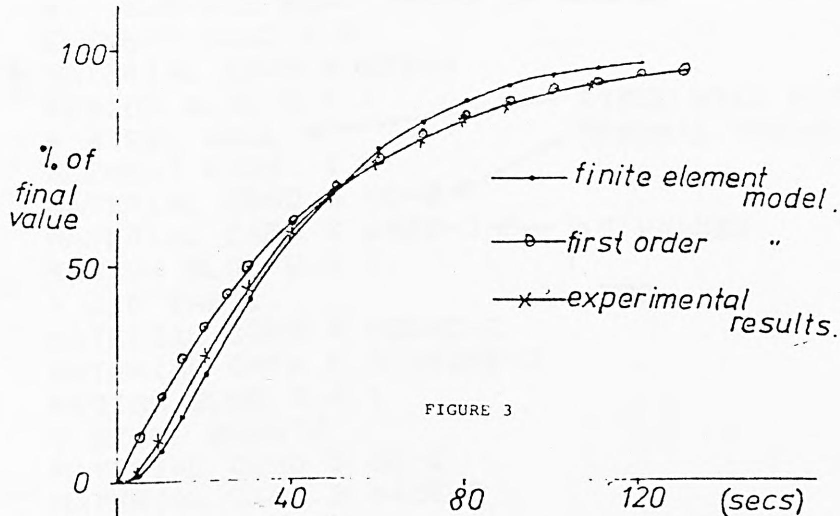

FIGURE 3

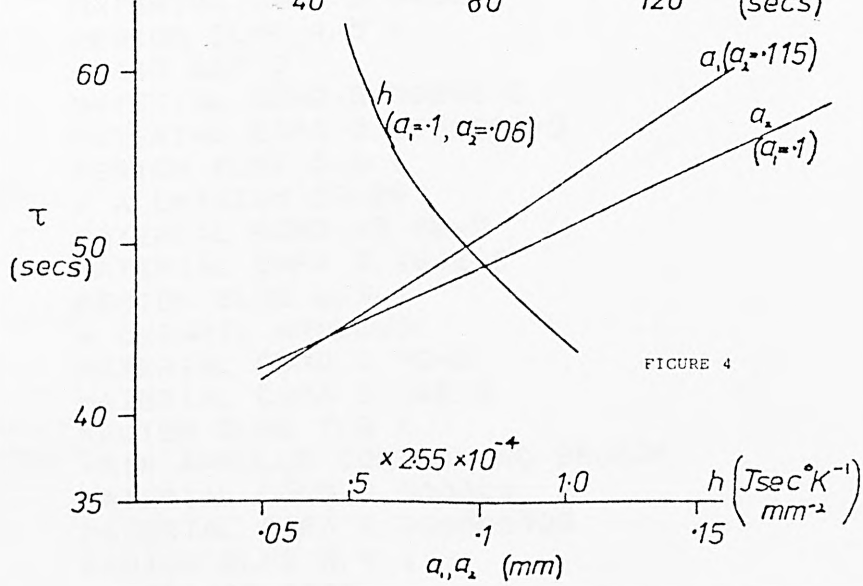

FIGURE 4

A FINEL MODEL

INPUT DATA

```
ANALYSIS TCOND
LET JO 20
COMMAND AAAA TIME CASE HISTORY
*{REGI COOR 5.1,0. 5.,0. 3.130,0. 3.00,0. 2.5,0. 2.375,0.
 {1.585,0. .85.0,0. .75,0. .0.07,0.
ELEMENT KX04 1.0
*   SURFACE HEAT TRANSFER ELEMENT
ELEMENT OWN3 1.0
†MATERIAL COND 4.25E-4
REGION SLNE 2,1 1                    STEEL WELL WITH
* STEEL WELL                         THERMAL CONDUCTIVITY (k)
ELEMENT KX04  1.0
MATERIAL COND 5.0E-2
MATERIAL CAPA 3.648E-3               ρC VALUES
REGION SLNE 2,3 1
* AIR GAP 1                          ETC.
MATERIAL COND 0.0024E-2
MATERIAL CAPA 0.000928E-3
REGION SLNE 3,4 1
* STEEL SHEATH
MATERIAL COND 5.0E-2
MATERIAL CAPA 3.648E-3
REGION SLNE 4,5 1
* AIR GAP 2
MATERIAL COND 0.0024E-2
MATERIAL CAPA 0.000928E-3
REGION SLNE 5,6 1
* ALUMINIUM COVER
MATERIAL COND 19.0E-2
MATERIAL CAPA 2.464E-3
REGION SLNE 6,7 1
* CERAMIC ANNULUS
MATERIAL COND 2.9E-2
MATERIAL CAPA 3.04E-3
REGION SLNE 7,8 1
*AIR ANNULUS CONTAINING SENSOR
MATERIAL COND 0.000024
MATERIAL CAPA 0.000000928
REGION SLNE 8,9 1
* CERAMIC CORE
MATERIAL COND 0.029
MATERIAL CAPA 0.00304
REGION SLNE 9,10 1
FIXED FREEDOM 2,1
LOAD SHTR 100.0 1,1,1
AAAA TIME 0.0 125.0 0.1
AAAA CASE 1
AAAA HISTORY 0.0 0.0 0.0001 1. 101.0 1.0
END JOB
```

* the set (5.1, 5.0, 3.130, 3.0, 2.5, 2.375, 1.585, .85, .75, .07,) is the parameter set ($\xi_1, \xi_2, \ldots \xi_{10}$). The first ξ_1, here 5.1, is only used for FINEL programs.

† The value entered here is h/2. For calculation of this heat transfer coefficient, h, see text and table and graph following.

Material Properties

	Density ρ (kgm^{-3})	C_v ($J\ kg^{-1}\ {}^oK^{-1}$)	k ($Wm^{-1}\ {}^oK^{-1}$)
Stainless Steel	7930	460	50
Aluminium Alloy	2800	880	180
Sintered Alumina Ceramic	3800	800	29
Platinum	21,450	136	69
Air	1.293	718.2	0.024

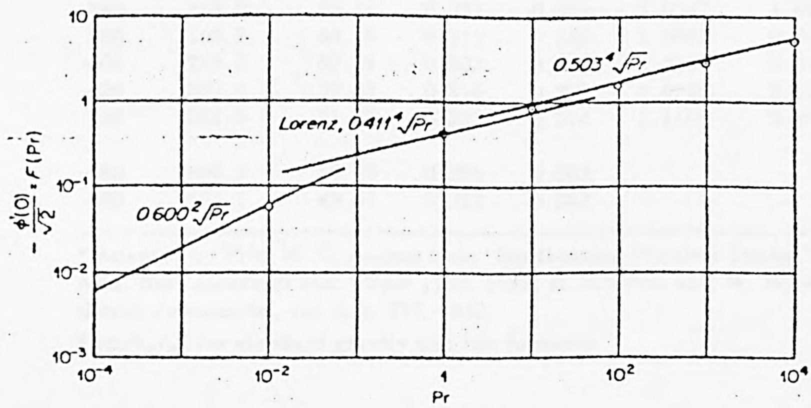

Fig. B-1 **Heat-transfer characteristics for laminar natural convection from an isothermal, vertical surface.**

Table B1 Properties of water*

t, °F	Saturation pressure, psia	ρ, lb/ft³	μ, lb/hr ft	k, Btu/hr ft °F	c_p, Btu/lb °F	Pr	a† × 10^{-6}, 1/ft³ °F
40	0.122	62.43	3.74	0.326	1.0041	11.5	
50	0.178	62.41	3.16	0.334	1.0013	9.49	
60	0.256	62.36	2.72	0.341	0.9996	7.98	154
70	0.363	62.30	2.37	0.347	0.9987	6.81	238
80	0.507	62.22	2.08	0.353	0.9982	5.89	330
90	0.698	62.12	1.85	0.358	0.9980	5.15	435
100	0.949	62.00	1.65	0.363	0.9980	4.55	547
110	1.28	61.86	1.49	0.367	0.9982	4.06	670
120	1.69	61.71	1.35	0.371	0.9985	3.64	804
130	2.22	61.55	1.24	0.375	0.9989	3.29	949
140	2.89	61.38	1.13	0.378	0.9994	3.00	1100
150	3.72	61.20	1.05	0.381	1.0000	2.74	1270
160	4.74	61.00	0.968	0.384	1.0008	2.52	1450
170	5.99	60.80	0.900	0.386	1.0017	2.33	1640
180	7.51	60.58	0.839	0.388	1.0027	2.17	1840
190	9.34	60.36	0.785	0.390	1.0039	2.02	2050
200	11.53	60.12	0.738	0.392	1.0052	1.89	2270
212	14.696	59.83	0.686	0.394	1.0070	1.76	2550
220	17.19	59.63	0.655	0.394	1.0084	1.67	2750
240	24.97	59.11	0.588	0.396	1.0124	1.51	3270
260	35.43	58.86	0.534	0.397	1.0173	1.37	3830
280	49.20	57.96	0.487	0.397	1.0231	1.26	4420
300	67.01	57.32	0.449	0.396	1.0297	1.17	5040
320	89.66	56.65	0.418	0.395	1.0368	1.10	5660
340	118.01	55.94	0.393	0.393	1.0451	1.04	6300
360	153.0	55.19	0.371	0.391	1.0547	1.00	6950
380	195.8	54.38	0.351	0.388	1.0662	0.97	7610
400	247.3	53.51	0.333	0.384	1.0800	0.94	8320
420	308.8	52.61	0.316	0.379	1.0968	0.92	9070
440	381.6	51.68	0.301	0.374	1.1168	0.90	9900
460	466.9	50.70	0.285	0.368			
480	566.1	49.67	0.272	0.362			

* Adapted by Prof. W. C. Andrae from "Smithsonian Physical Tables," 9th ed., 1954; *Natl. Bur. Standards Res. Paper* 1228, 1939; E. Schmidt and W. Sellschopp, *Forsch. Gebiete Ingenieurw.*, vol. 3, p. 277, 1932.

† $g\beta\rho^2 c_p/\mu k$ for standard gravity and low pressure.

Graph B1 and Table B1 are taken from Gerbhart (1971).

APPENDIX C *

Understanding the Dynamic Response of Industrial Temperature Sensors:- An Approach based on Mathematical Models.

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Abstract

Understanding the factors affecting the dynamic response of industrial temperature sensors is of importance to both the user and designer of such systems. This paper reports on some research being conducted at the authors' institution in the development and application of mathematical models to predict the dynamic behaviour of a complex thermometer in well assembly. It is shown that such models can be used to predict step fluid responses and deliberate self heating responses both to within a few percent. It is also shown that an identification of the electrical self heating response can be used to predict the fluid step response. The use of this technique for in situ response testing of industrial thermometers is a possibility. The other use of such mathematical models is for sensor design because the models are related to material properties and constructional features. This aspect is also discussed.

Introduction

This paper describes, briefly, some of the work carried out as a research project on the modelling of thermometers (Chohan 1983).

Models can predict the behaviour of complex, industrial, thermometers. Results presented in the paper show that models reproduce thermometer dynamic response to a few percent. Also one can reproduce two kinds of dynamic responses : (a) when fluid temperature external to the thermometer is changed and (b) when the electrical current through the sensing element is changed.

Models for thermometers are useful for design and analysis because industrial thermometers are complex. A typical industrial thermometer (which is analysed here) can be considered as a multilayer cylinder. For a thermometer-in-well system, the following annular regions can be identified: Well, air gap, sheath, air gap, aluminium cover, ceramic annulus (containing platinum sensing wire), and ceramic core.

This paper looks at our recent modelling work on distributed - parameter and lumped models. Earlier developmental work was presented earlier (Chohan and Abdullah (1980, 1981)).

* Paper presented at Transducer/Tempcon conference, Earls Court, London, June 14th-18th 1983 and published in proceedings.

Development of a Model for a Rosemount Thermometer Assembly.

Heat transfer through a thermometer carries information which is used to arrive at the temperature of the measured substance. This process, however, should strictly be looked at as distributed-parameter system governed by a parabolic-type differential equation. Such models are capable of quite accurate representations of the thermometer, with or without a well.

In the past two decades, a powerful method for solution of such partial differential equations has been made available - the Finite Element (FE) method. In fact, there now exist a number of very good computer packages for example, FINEL developed at Imperial College (Hitchings, D. (1981)), which was used in the present project.

For modelling purposes, it was assumed that one-dimensional radial heat flow through the thermometer (i.e. no significant heat flows along the stem) and that the thermalproperties remained constant. The thermometer-in-well system can be decomposed into the following regions for (i) well, (ii) air gap, (iii) sheath, (iv) air gap, (v) aluminium cover, (vi) ceramic former containing the sensing element (here a platinum coil). Item, (vi) was further decomposed into (a) an outer (annular) ceramic, (b) air gap with sensing wire (assumed to be mass less) imbedded in it, (c) inner cylindrical core.

Each of these regions (steel well, air gap, etc) was divided into one four noded axisymmetric line element with appropriate material properties κ (thermal conductivity) and c (thermal capacity). At the well surface (or at the sheath surface for a thermometer system) a surface heat transfer element was added with an appropriate value of h (the surface heat transfer coefficient). The resulting model had 9 elements with 25 nodes, with node 5 from the centre of the cylinder being the temperature of the sensing element.

Simulation of this model was carried out using FINEL package on a PRIME computer at the City University. Details can be found in a doctoral thesis to be released this year (Chohan, (1983)).

(i) Plunge test (or FT - our abbreviation) - where the thermometer is plunged into a hot water bath (unstirred) at about 70-90°C.

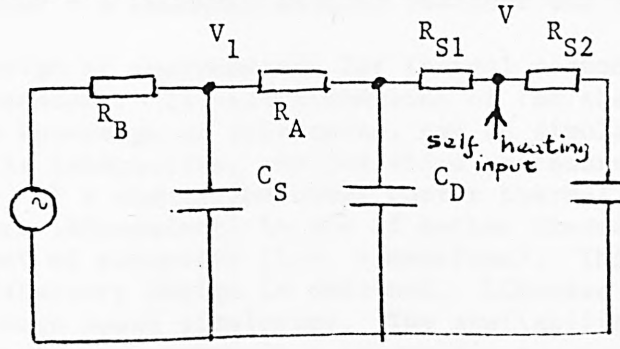
(ii) Electrical Transient test (ET) where the thermometer is held at a constant temperature water bath at room temperature and the current input into the thermometer undergoing a step change, from approximately 1-5 mA to about 30-45 mA, so that substantial self heating does occur.

The heat transfer coefficients was computed using correlations pertaining to a flat plate (Gebhardt, (1972)), and free convection conditions and average temperature of the experiments. Figure 1 and 2 show the results of model output and experiment for both case (i) and (ii) (which shall be referred to as FT and ET) for a thermometer and thermometer-in-well system. Agreement is seen to quite good showing the validity of the distributed parameter model. It should be noted that there are two sets of experiments which are distinct and validity is tested with respect to both.

The major disadvantage of distributed-parameter models is the relatively large computer time required for simulation. Valid low-order models have some appeal in this respect. They may even allow simulation on present generation of microprocessors. Indeed, the present project is expected to form a base for such work at the City University (Gardners and Abdullah (1983)).

Previous papers discussed earlier efforts in lumped modelling (Chohan and Abdullah (1980), (1981)), there the validated models are briefly presented. Validity of lumped model can be tested against distributed models, which are, for our purposes, very good representation of the real system.

The reduced (i.e. lumped) model implemented can be represented as an electrical circuit as follows for a thermometer.



where:

V_f = Fluid temperature.

R_B = Boundary layer resistance.

V_1 = Temperature of sheath

R_A = Resistance of air gap between sheath and Aluminium cover.

R_{S1} = Resistance of aluminium cover and ceramic annulus

R_{S2} = Resistance of core of ceramic

C_s = Sheath capacitance.

C_D = Capacitance of aluminium cover plus annular ceramic.

C_c = Capacitance of central core.

To simulate this model, a computer package MEDIEM was used. MEDIEM was developed at the City University (Liebner and Abdullah, (1982)), for simulation of models of Instrument systems and is interactive allowing the user to change model structure and/or parameters very easily.

The results for a typical thermometer are shown in figure 3. The continuous curve shows the ET (upper curve) and FT (lower) curves with the corresponding distributed-parameters results (+ +++). The agreement is seen to be excellent. As a comparison results are also shown for the lumped model when the central core capacitance is neglected. It can be seen that the neglect of centre capacitance, leads to a somewhat poorer model. This is inspite of the fact that the value of this capacitance is quite small.

Application of Such a Model for Design and In-Situ Response Testing,

System design is carried out at the design state where more care is required in time-history prediction then, say, when the thermometer is in use. Therefore, exact (or very good) representations of the system to be designed is most useful. As the discussion above has shown, our work has led to very satisfactory lumped models which can be used for computer-aided design of thermometers.

The lumped model can be simulated using MEDIEM which has been transferred to a microcomputer - a CROMEMCO machine (Gardner and Abdullah (1983)).

Briefly, the design of thermometers for thermal response can be looked at as an iterative procedure. Initial dimensions of the thermometer are provided together with a knowledge of tolerances, can be simulated using MEDIEM. Because MEDIEM is interactive, any iterative procedure is well-suited for its use. Hence, if a simulation shows poorer thermal response, one can change parameters (dimensions) to see if better thermal response is obtained from this new set of parameter (i.e. dimensions). This procedure is carried on until a satisfactory design is obtained. Likewise different materials can be explored through model simulation. The availability of valid lumped models and microprocessors can assist in a scientific approach to design. Models can also assist, together with present day computers, the user of thermometers. Testing of thermometers is important in that it allows the user to decide whether a thermometer, after some service, is still performing satisfactorily.

A theory for testing was developed by Kerlin and his associates at the University of Tennessese (Kerline et. al. (1982)). The general assumptions for their method of testing are:

(i) radial heat flow.
(ii) Negligible centre capacity.

For thermocouples with normal length stems, these conditions are satisfied. For resistance thermometers, the second assuption and, as shown in the last section, leads to only approximate models. However, it is of interest to the user to know if approximate analysis do lead to satisfactory prediction of fluid temperature response from an in-situ test.

Our work differs from Kerlins in that we have validated distributed and lumped models which can be used as 'test beds' for any testing analysis. Kerlin's theory shows that an electrical transient response can be represented approximately as:

$$T_{ET}(t) = 1 + a_1, e^{-p_1 t} + a_2 e^{-p_2 t} \qquad (1)$$

and the predicted Fluid Temperature transient is given by

$$T_{FT}(t) = 1 + \frac{p_2}{(p_1 - p_2)} e^{-p_1, t} + \frac{p_1}{(p_1 - p_2)} e^{-p_2 t} \qquad (2)$$

Important point to note is that only exponents are required for predictions.

As explained in the previous section, our models can produce ET and FT data. These can be used to produce 'best' curve fits to ET data i.e. obtain a_1, p_1, a_2, p_2, (equn. (1)), and hence determine equation (2). This can be compared with FT data. We have found that when Kerlin's expediating conditions hold, predictions - can obtained to a satisfactory error levels (within 10%).

In a practical industrial environment a microprocessor could be programmed to inject an electrical test signal and process the sensor response to obtain the parameters of equation (1). These could be used to predict the expected fluid step response. Any significant deviations of this from some previously stored "ideal" response could be monitored by the microprocessor and an error message sent to a control room. In this way in situ response testing in industry (eg. the nuclear industry) could take place to periodically check the integrity of sensors. This would be a step towards intelligent instrumentation based on mathematical models.

Conclusions.

The work presented here indicates that mathematical models can provide considerable insight into the design and performance of industrial temperature sensors. The potential of these modelling techniques has been demonstrated. In the future it is expected that distributed parameter (finite element) modelling and lumped parameter modelling (using for example MEDIEM) will be used to aid in the evaluation and design of industrial sensors. The decreasing cost and increasing power of digital computers will make the CAD approach extremely attractive. Furthermore, the use of microprocessor based test systems developed for in-situ response testing in hazardous or inaccessible environments would appear to be an attractive possibility. Development of such systems will rely on sound mathematical models to represent sensor dynamics.

Acknowledgements

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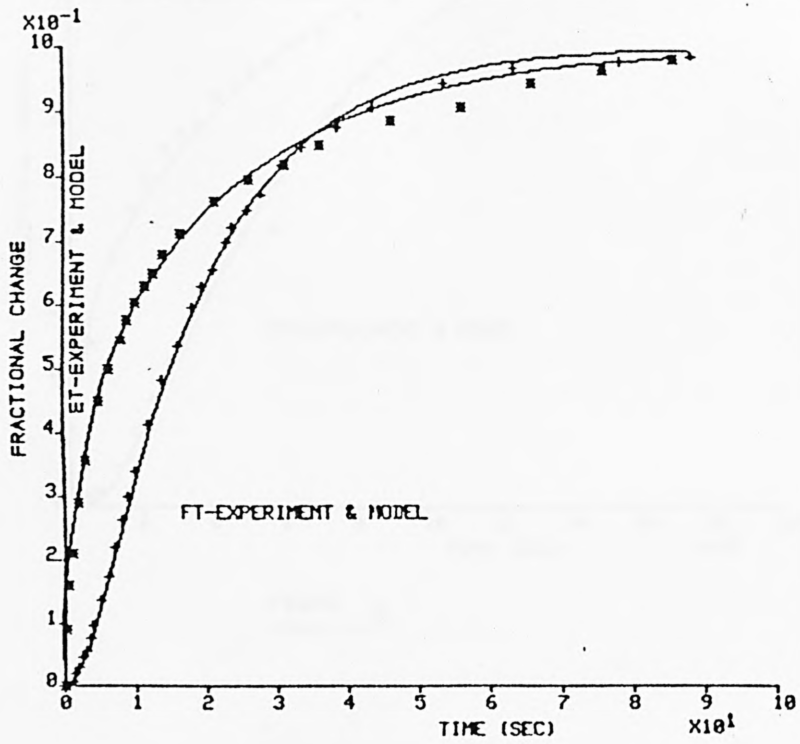
X10^-1
FRACTIONAL CHANGE
ET-EXPERIMENT & MODEL
FT-EXPERIMENT & MODEL
TIME (SEC)
X10^1
0 1 2 3 4 5 6 7 8 9 10
*+ experiment
— model results.

FIGURE 1

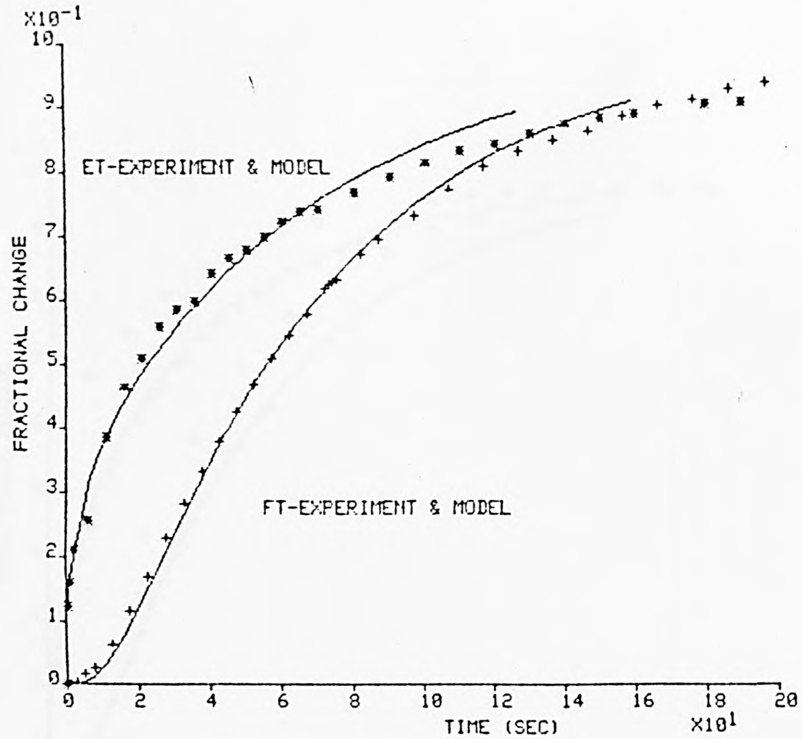

FIGURE 2

*+ experiment
— model results

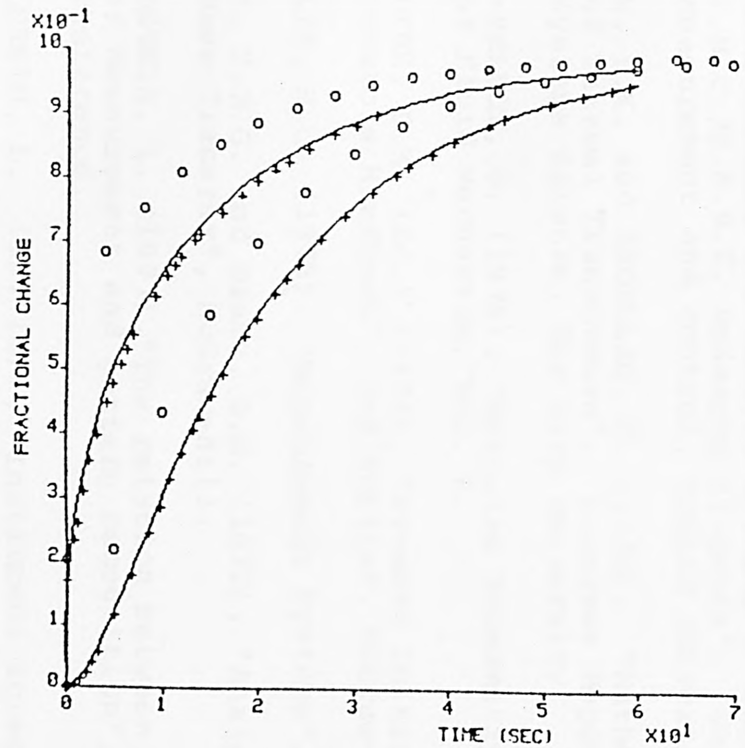

FIGURE 3

— distributed (finite element) model
+ reduced model with centre capacity.
0 reduced model without centre capacity.

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