(Un)Equal Tunings:
Exploring multiple levels of resolution between equal tunings and intonational practices in composition

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Declaration

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Abstract

This practice-based research explores the employment of equal tunings (beyond 12 Equal Divisions of the Octave and its subdivisions) in compositions for Western concert music performers. Drawing on current and historical discussions of microtonality, intonation and notation, and employing (novel) frameworks of 'rounding' and 'approximation', through a multi-modal approach I explore various ways in which pitch classes may be organised in my compositional practice. A portfolio of compositions for a variety of ensemble configurations was produced alongside a written thesis which problematises these discussions and offers a technical commentary for each composition. Several annexes, including a computer application, *Equal Tuning Lab,* a collection of spreadsheets, *Global Properties of EDOs*, were also produced.

Research Questions

1. What defines 'idiomatic writing' in the area of musical pitch, when composing for performers of Western concert music instruments, and how can an understanding of instrumental technique inform a compositional approach?
2. How may intervallic approximations facilitate an 'idiomatic writing' for such instruments when composing music based on equal tunings?
3. What are the most pragmatic notational strategies, given this context?
4. How do these questions influence my practice as a composer and what creative practices can be derived from the process of answering the previous questions?

Part I

Chapter 1. Personal background, Motivations, Influences and Statement of Positionality

Before defining the field of this research, I will position myself in terms of my background and musical influences, as well as other extra-musical interests which have shaped my practice and how I think about music.

My impetus for composing derives from my creative excitement in building scales and chords based on numerical combinations and then instinctively composing with those structures. 12 Equal Divisions of the Octave (hereafter 12EDO), the tuning system of the main instruments I play (the guitar and piano) and of MIDI technology, is one framework for realising these structures, but is not the only available option. Since 24 and 48EDO are subdivisions of 12EDO, it is easy for me to conceptualize them theoretically. Nonetheless, I do not find them particularly acoustically attractive by themselves. I therefore became interested in exploring other alternative equal tunings beyond the subdivisions of 12EDO. This presented an infinitude of possibilities, both acoustically and mathematically. Given current digital technology, it has never been easier to experiment with these tunings. However, as a composer of Western concert music, the task of writing for instruments introduces additional challenges, namely issues of instrumental technique and notation, as neither instrument design nor the inherited intonation techniques of performers are built with such tunings in mind. This means that compromises will have to be made somewhere between these alternative equal tunings and what is practically achievable and idiomatic for players of modern concert instruments, which also leads to questions regarding precision of performance. Even though multiple authors have addressed many of the issues relevant to this goal (as will be discussed in Chapter 2) there is still a lot to discover. Given the precedent that composers associated with 'spectralism' set in the employment of intervallic approximations to write music based on acoustic phenomena and manipulations of it, similar approximation strategies (which assume variable levels of precision) could be tested for their efficacy in achieving this goal. Due to the different structural characteristics of Equal Tunings and the materials which serve as the starting point for spectralist composition practice, it can be expected that a composition research project based on these premises will lead to new insights. They, in turn, might influence the development of this research and my own compositional practice into unforeseen avenues. Considering this contextualization, the research questions if this project have thus been developed:

1. What defines ‘idiomatic writing’ in the area of musical pitch, when composing for performers of Western concert music instruments, and how can an understanding of instrumental technique inform a compositional approach?
2. How may intervallic approximations facilitate an ‘idiomatic writing’ for such instruments when composing music based on equal tunings?
3. What are the most pragmatic notational strategies, given this context?
4. How do these questions influence my practice as a composer and what creative practices can be derived from the process of answering the previous questions?

Throughout this research, I understood that to be able to address an issue related to either of the first three research questions, that issue would often also relate to the other questions. Finding answers would generate new knowledge relevant to these questions and lead my own compositional practice in new avenues (i.e. leading to the fourth question). Therefore, notation, intonation technique, tuning systems, approximation strategies and idiomatic writing are most of the time not discussed separately but in relation to each other.

An early influence for this project has been composer Easley Blackwood’s writings about 5 and 7EDO in relation to the Meantone major scale.[1] Alongside 12EDO, these three tuning systems are also numerically and visually logical to me as a starting point for this research since I started working on this project with an electric piano – 5 black keys, 7 white ones and 12 in total - and software which allows its retuning (bitKlavier and Pianoteq). Additionally, the alternating and simultaneous employment of these tuning systems is enticing to me, suggesting a ‘microtonally[2] expanded modality’ around pentatonic, dorian and chromatic resembling structures.[3]

Another relevant early influence is the remark which composer and conductor Robert Lopez-Hanshaw makes relating to his exposure to ‘two completely different philosophies of

[1] See Chapter 2.5, last paragraph starting on page 37.

[2] For the definitions of ‘microtonality’, as used in this research, see Chapter 2.3, page 28.

[3] See Appendix 7 for a systematization of how these tuning systems are conceptualized in relation to each other in my music.

microtonality: Either escape The System, or help it to become somehow more itself'.[4] In my creative practice, Lopez-Hanshaw's remark simultaneously resonates with composer James Tenney's hypotheses from his article 'The Several Dimensions of Pitch'[5] and to composer Ivor Darreg's definition of 'xenharmonic',[6] which maps different tunings, temperaments and intonation practices into the categories of 'non-xenharmonic' and 'xenharmonic'. However, I imagine this variety of systems to be a spectrum that goes from the former to the latter. Such spectrum provides a rich framework for musical exploration.

During this research, I became fascinated with the music and research of Just Intonation innovator Marc Sabat and other composers from Plainsound Music (even though my research sits on the opposite side of the microtonal spectrum, *per se*), and some ideas from jazz bassist Adam Neely's YouTube channel about polyrhythms and irrational time signatures.

In mid-2021 I acquired a Lumatone[7] and have been improvising and building mappings of a variety of tuning systems on it. This greatly influenced the gradual evolution of how I think about pitch at the later stages of my project.

Upon embarking on this research, it was unclear which kind of musical sound I was searching. This meant that whenever I started to write a new piece, I had no clear idea of how it would

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[4] Robert Lopez-Hanshaw, 'The Journey In (Performing Microtonal Choral Music, Part 1)', *NEWMUSICBOX*, 2020 <https://nmbx.newmusicusa.org/the-journey-in-performing-microtonal-choral-music-part-1/> [accessed 25 September 2020] (para. 34 of 46).

[5] James Tenney, 'The Several Dimensions of Pitch', in *From Scratch: Writings in Music Theory*, ed. Lauren Pratt and others (Urbana: University of Illinois Press, 2015), pp. 368–82. See Chapter 2.2.

[6] '[…] a tuning is xenharmonic if most musically-trained or musically-inclined listeners to a permanence hear it as different from a performance in 12-tone equal temperament. Nearly all just-intonation performances are xenharmonic. […] Certain of the 12-tones-per-octave-only-and-no-more UNequal temperaments would not be called xenharmonic because the deviation from 12-equal is too trivial to be heard in a normal performance. A wild style of performing with wide vibrato and/or deviations from 12-tone-equal or the use of pitch-fringes and double-tracking, would not be xenharmonic even though small intervals occurred, because there would still be only 12 pitch-classes. […] Most non-twelve-tone equal and unequal temperaments are xenharmonic.' Ivor Darreg, 'Defining One's Terms', in *Tonalsoft*, 1998 <http://www.tonalsoft.com/sonic-arts/darreg/dar27.htm> [accessed 15 October 2020] (para. 32-37 of 96).

[7] I recommend the reader to watch the following playlist to familiarise themselves with some concepts related to this instrument: Lumatone Keyboard, *Learning Lumatone,* online video recording playlist, YouTube, <https://www.youtube.com/playlist?list=PL2fLlhfYL4JTcGYbPXnkrqaOUoLstk0WX> [accessed 9 August 2023]. Also, refer to the glossary in Appendix 1.

sound (even if some ideas regarding which tunings, rhythmical structures or extra musical influences may have already been delineated). To me, microtonal systems and aesthetic outcomes are neither intrinsically linked nor mutually exclusive. Nonetheless, different microtonal models lead to different materials being used.[8] Only halfway through my research I started to become more consciously aware of my own compositional tendencies, and certain musical gestures, microtonal and rhythmical structures became more prevalent than others. As shall be seen throughout this account of my artistic practice, the overarching narrative of this doctoral research is how my attitudes towards 'rounding' (as contextualized in Chapter 2.3) have shifted.

The focus of the composition commentaries (in Part II) will be primarily on pitch, its notational and performance contexts, and its practical systematization. Discussion of other parameters (such as rhythm, form, etc.) will be somewhat limited. The contribution to knowledge being made is on the implications of bridging alternative equal tunings and the pitch context of performers of modern concert music instruments (two fields with little overlap).

Chapter 2. Literature Review

During the pursuit of addressing the research questions, I understood that to be able to give insight into one, I first needed to unpack issues not directly related to it, but also address issues that were pertinent to the other research questions and to the compositional explorations of Part II. This pursuit would not necessarily follow the order in which these questions were presented. Therefore, this chapter is divided into multiple subchapters which address and intertwine these issues.

Chapter 2.1 gives an overview of the characteristics of equal tunings and of music which implicitly or explicitly converges with their structure.

[8] "Most models are designed not merely to provide a description of a pitch 'space' but to suggest or embody an explanation of it. All such models are attempts to circumscribe and make manifest the processes by which we form cognitive representations of musical materials. Clearly, the model and the observations that arise from it are linked: observation is done in the ambience of the model; the model is created in the context of an observation strategy. This interaction helps evolve the adequacy of the model and the sensitivity of the observation." Bob Gilmore, 'Changing the Metaphor: Ratio Models of Musical Pitch in the Work of Harry Partch, Ben Johnston, and James Tenney', *Perspectives of New Music*, 33.1/2 (1995), 458–503 (p. 458).

Chapter 2.2 addresses how intonation, notation and intervallic perception might intertwine (and therefore sheds insight into the first and third research questions).

Chapter 3.3 relates to the second and third questions by contextualizing microtonal approximations in 'spectral' music and microtonal notation systems.

Chapter 2.4 gives further insight into the second, third and fourth questions by addressing tuneable intervals and how they might be used as another approximation strategy, which also has its own notational and compositional implications.

Chapter 2.5 problematizes, classifies and correlates a variety of Equal Tunings (according to their internal structures) with historical temperaments, whilst Chapter 2.6 links them to historical string intonation techniques (therefore giving insight into the first question).

2.1. Equal tunings and alternative divisions of the (semi)tone

Equal tunings are tuning systems whose steps are logarithmically equally distanced in frequency.[9] They may be derived by dividing an arbitrary interval into an arbitrary number of equal parts. Their intervals cannot be represented by just frequency ratios since they correspond to irrational numbers. Equal tunings have also been referenced as 'equal temperaments' and 'rank 1 temperaments' in Regular Temperament theory[10] due to their historical employment for the tempering of Just Intonation ratios ('natural intervals').[11] Historically, Equal Divisions of the Octave (EDOs) have been the most addressed sub-category of equal tunings in theoretical works and in compositional and performative practice, from which 12EDO has been the most well-known example in Western music. However, equal tunings or structures close to them may be found in non-Western music and theoretical writings, such as 7EDO in the music

[9] The term 'equal tuning' is being used differently to how composer Easley Blackwood uses it in Easley Blackwood, *The Structure of Recognizable Diatonic Tunings* (Princeton, N.J: Princeton University Press, 1985). p. 226. Blackwood's definition only refers to EDOs, whereas in this research this term also encompasses tunings that repeat at other intervals.

[10] See Kyle Gann, *The Arithmetic of Listening: Tuning Theory and History for the Impractical Musician* (Urbana: University of Illinois Press, 2019), pp. 186-98. Also see 'Tour of Regular Temperaments', *Xenharmonic Wiki* <https://en.xen.wiki/w/Tour_of_Regular_Temperaments> [accessed 20 March 2021].

[11] Joe Monzo, 'EDO / edo' <http://www.tonalsoft.com/enc/e/edo.aspx> [accessed 9 August 2023].

from Solomon Islands[12] and Thai music, 17EDO in Arabian music, and 53EDO in Ancient Chinese theoretical writings.[13]

Equal tunings may be a convenient reference framework to map other tunings, notational systems, intonational practices and microtonal structures which imply the same number of steps per interval cycle, but whose adjacent steps may not fall exactly within equal distances. Therefore, for pragmatism, these practices and structures will be treated as equivalent to the equal tunings to which they are closest. The format 'xNPi' or 'x Notes Per *interval*' (e.g. '72NPO' or '72 Notes per Octave') shall be used to name these practices, structures and notation systems within an instrumental context in which precise equal divisions are not viable in all or some instruments (e.g. in pieces with non-fixed pitch concert instruments, such as bowed strings, woodwinds and brass, and/or with fixed-pitch instruments tuned to non-equal historical temperaments, such as harpsichords, pipe organs, etc.). However, when referring to a specific equal tuning, it shall be named in the format 'xEDi' or 'x Equal Divisions of *interval*' (e.g. '5ED3/2' or '5 Equal Divisions of 3/2', in which '3/2' means the ratio between the 3rd and 2nd partials of the harmonic series). Also, when referring to a particular interval comprised of a certain number of steps of an equal tuning, the format 'x\yEDi' or 'x steps of y Equal Divisions of *interval*' will be used (e.g. '4\7EDO' or '4 steps of 7 Equal Divisions of the Octave', and '1\2ED11/7' or '1 step of 2 Equal Divisions of 11/7').

Composers such as Alois Hába, Ivan Wyschnegradsky, Julián Carrillo, Ezra Simms and Charles Ives have pioneered the employment of 24, 36, 48, 72 and 96NPO (reached by subdividing the semitone into 'microtones': quarter, sixth, eighth, twelfth and sixteenth tones, respectively) in concert instruments.[14] They have also used special or retuned instruments (such as pianos) to precisely perform the EDO counterparts of these systems. Alternative divisions of the whole tone, such as third tones, implying 18EDO/NPO (such as in Maurice Ohana's *Tombeau de Claude Debussy*),[15] and fifth tones, implying 30EDO/NPO (such as in Alois

[12] *Shaping Bamboo*, dir. by Hugo Zemp (SERDDAV, 1979).

[13] J. Murray Barbour, *Tuning and Temperament: A Historical Survey* (Mineola, N.Y: Dover Publications, 2004). pp. 114, 116, 124. Also see

[14] Gann, pp. 205-214, 218-230.

[15] This piece employs simultaneously, according to my classification system, 18EDO (on the zyther), 18NPO (on the voice and flute), 12EDO (on the piano and pitched percussions), and 12NPO (on all pitched instruments except the zither). Also, see Gann, pp. 205-06.

Hába's *String Quartet No.16*)[16] have also been explored. Historical usages of other equal tunings not related to 12EDO have however been mostly confined to either unusual instruments (such as 31EDO or close to it in, respectively, the Fokker Organ and Nicola Vicentino's Archicembalo,[17] clarinets tuned to 13ED3/1 - the Bohlen-Pierce Scale -, special electric guitars, etc.),[18] retuned concert instruments (e.g. Robert Hasegawa's *Due Corde* in 19EDO for two retuned pianos)[19] and electronic media (e.g. Easley Blackwood's *Twelve Microtonal Etudes for Electronic Music Media*, employing all tunings from 13EDO to 24EDO, Stephen Weigel's *Six Macrotonal Etudes for Electronic Music Media*, similarly employing 5, 7, 8, 9, 10 and 11EDO and Karheinz Stockausen's *Studie II* in 25ED5/1).[20]

Recent examples of pieces for mixed ensembles, such as Fabio Costa's *Aphoristic Madrigal* for voice quartet and Fokker Organ and Christian Klinkenberg's opera *The Glacier* (which contains a variety of simultaneous tuning systems such as 19EDO, 41EDO, the Bohlen-Pierce Scale, Just Intonation and others),[21] may suggest a variety of insights for the adaptation of equal tunings unrelated to 12EDO when composing for special and modified concert instruments.

2.2. Intonation, Western Chromatic Notation and Tenney's hypotheses

According to composer and violinist Marc Sabat, 'intonation' is 'the art of selecting pitches, or (more accurately) pitch-"regions" along the glissando-continuum of pitch-height […]. The

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[16] Regarding Hába's explorations with fifth tones, see Gann, pp. 206, 211-12.

[17] See Gann, pp. 199-204.

[18] For a recent application of many of these instruments, see Christian Klinkenberg, *Microtonal Systems Combined: A Composer's Approach*, online video recording, media.AMU, 2020 <https://media.amu.cz/cs/media/7f7021d167584ef1a4e543693d951625?token=cfc37046d31f4c6c95a49c104463a4c6> [accessed 30 July 2024].

[19] See the excerpt between 16'56'' and 24'45'' in Robert Hasegawa, *Composing with Hybrid Microtonalities*, online video recording, media.AMU, 2020 <https://media.amu.cz/cs/media/43961354d2be41a88f293d82d326c7c5?token=61b239853c26464f8f65bd1c1aad3170> [accessed 30 July 2024].

[20] '81 steps each approximately in ratio to the next by a factor of $\sqrt[25]{5}$', meaning an 81-note scale in 25ED5/1. John Kelsall, 'Compositional Techniques in the Music of Stockhausen (1951-1970)' (unpublished PhD Thesis, University of Glasgow, 1975). p. 78.

[21] See Klinkenberg, *Microtonal Systems Combined*.

"tolerance" or exactitude of such regions varies based on the instrument and musical style'.[22] Tuning systems and temperaments, for fixed-pitch instruments (such as the harpsichord), are thus included in this definition, alongside the individual intonational choices of performers of non-fixed pitch instruments (such as the violin and the trombone).

Regarding intonation in modern orchestral performances, composer and musicologist James Murray Barbour states:

> The result is 'a very great lack of precision', with heterogeneous sounds that are a mixture of 'just, Pythagorean, tempered, or simply false'. Of course the ears of the audience, trained for years to endure such cacophony, actually are pleased by what seems to be a good performance.[23]

Additionally, James Tenney proposes two hypotheses:

> the auditory system would tend to interpret any given interval as thus 'representing'—or being a variant of—the simplest interval within the tolerance range around the interval actually heard (where 'simplest interval' means the interval defined by a frequency ratio requiring the smallest integers). The simpler just ratios thus become "referential" for the auditory system—not in any conscious or cognitive way but rather on a very primitive, precognitive, neurological level.
>
> [...] within the tolerance range, a mistuned interval will still carry the same harmonic sense as the accurately tuned interval does, although its timbral quality will be different—less 'clear' or 'transparent', for example, or more 'harsh', 'tense', or 'unstable', etc.[24]

Considering Sabat's definition of intonation and Barbour's remarks about the multiplicity of intonational approaches in orchestral performances, Tenney's proposals could justify the success of what Barbour would consider to be 'good' orchestral performances. The Western (five-line staff) Chromatic Notation would then be compatible with these remarks for its non-imposition of a specific intonational approach within a convention of 12 pitch classes (12NPO).

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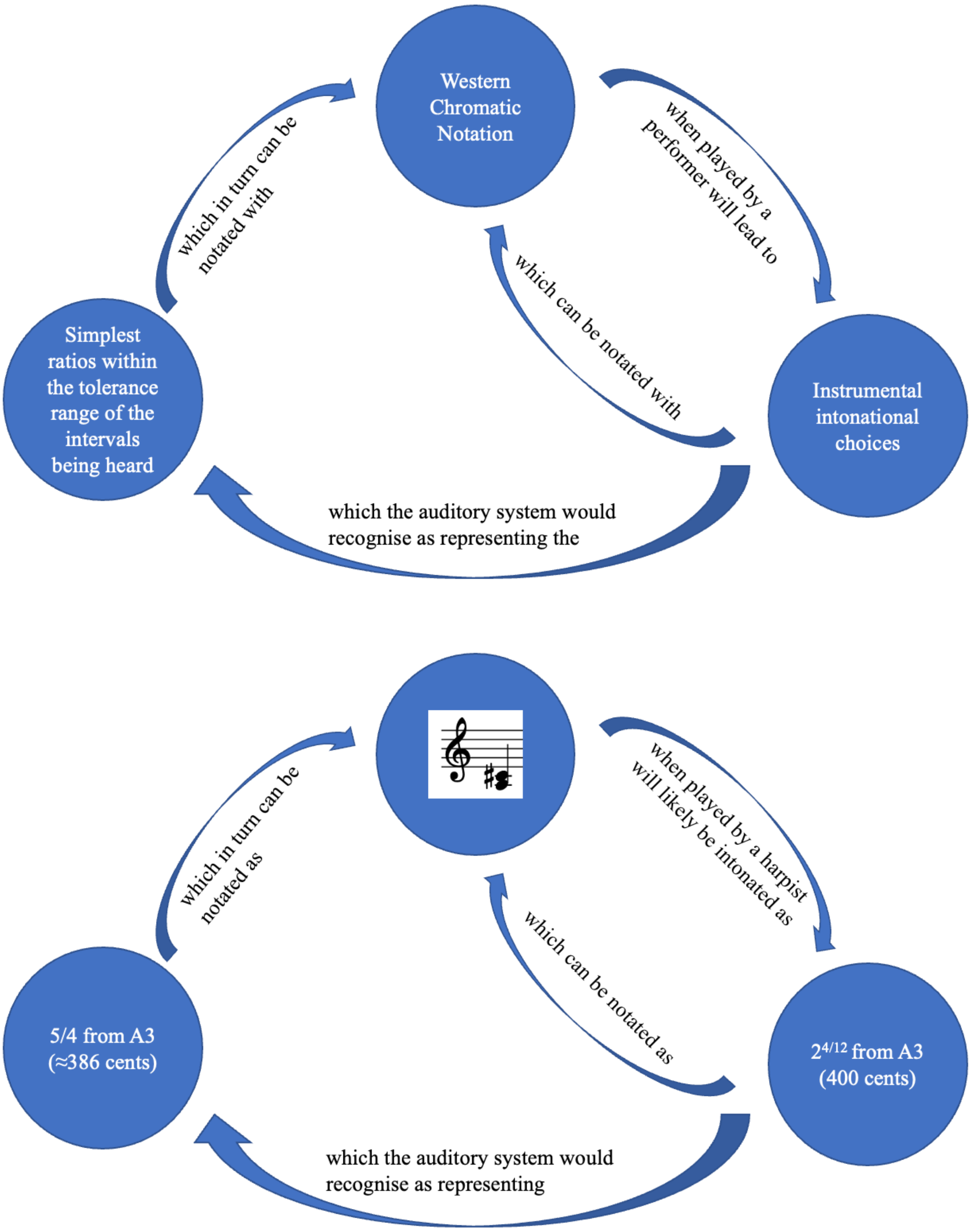
[22] Marc Sabat, 'Intonation and Microtonality', in *NEWMUSICBOX*, 2005 <https://nmbx.newmusicusa.org/intonation-and-microtonality/> [accessed 25 September 2020] (para. 2 of 19).

[23] Barbour, p. 200. Even though this remark can be read as a mockery of orchestral intonation, it captures the intonational variety that may happen during a performance.

[24] Tenney, pp. 368-82 (pp. 378-79). Related to Tenney's hypotheses are Marc Sabat and Robin Hayward's 'tuneable intervals'. See Chapters 2.4, 3.2 (two paragraphs before Figure 27 on pages 63 and 64) and 3.3.2. Also related to Tenney's hypothesis is Paul Erlich's writings on 'Harmonic Entropy'. See Paul Erlich, 'on Harmonic Entropy' <http://tonalsoft.com/enc/e/erlich/harmonic-entropy_original.aspx> [accessed 14 December 2020].

This system gives the performers the liberty to make their own intonational choices, unless otherwise specified by the composers or by the nature of their instruments. According to Tenney, these choices would then lead to intervals which would be recognized by the ear to represent the simplest ratios within their tolerance range. Figure 1 proposes a model to illustrate the relationship between notation, intonation and Tenney's hypotheses.



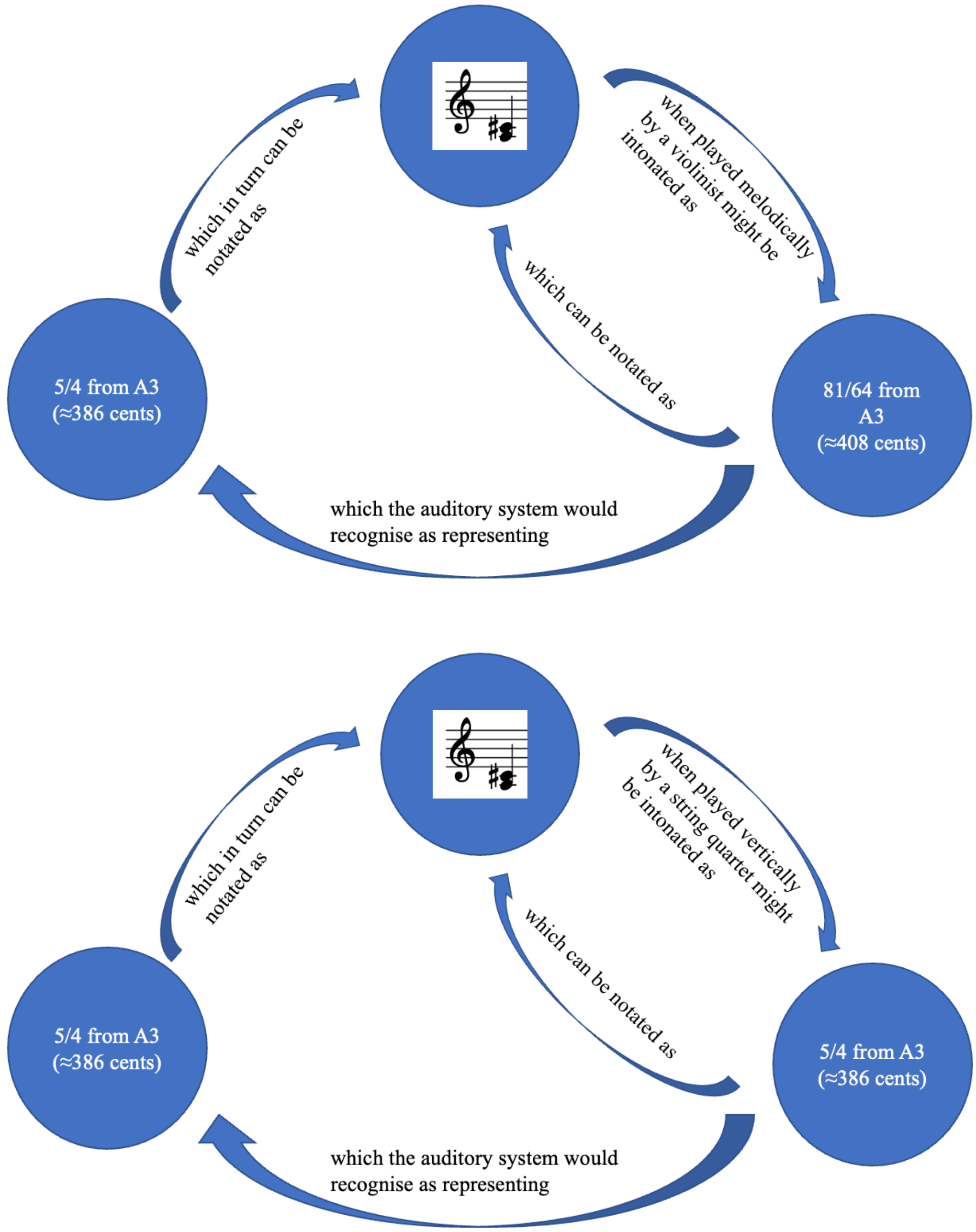


Figure 1. My model for the representation of the relationship between Western Chromatic Notation, Instrumental intonation and Tenney's hypotheses, with three possible examples.

Whilst this model proposes a framework for how the Western concert music repertoire relates to a variety of intonational approaches,[25] it assumes not only that intonational choices are made by performers, but that these choices are compatible (but not intrinsic) to the Western Chromatic Notation system. If intonational choices are then to be pre-determined within a score, then the intonation-agnostic characteristics of this notational approach should be superseded.

[25] See Chapter 2.6 for some insight on three different historical string intonation techniques.

2.3. EDOs, NPOs, Spectralism, Rounding, Microtonality and Notation

Composers associated with 'spectralism' such as Tristan Murail, Gérard Grisey, Horațiu Rădulescu, Claude Vivier and Georg Friedrich Haas have used a variety of microtones related to subdivisions of 12NPO to approximate the harmonic series, other acoustic phenomena, and distortions of these when writing for concert instruments,[26] often freely switching between variable levels of 'resolution'[27] according to circumstance, such as the speed of performance.[28] This practice of microtonal approximation could be adapted to similarly approximate materials built on any equal tuning into subdivisions of 12NPO or to any equal tuning unrelated to 12EDO (when composing for mediums which can be tuned into these systems). This proposed practice shall be named *rounding* (as if converting a size in a unit of measurement into another unit and rounding out the decimals). Figures 2 and 3 offer two practical examples of rounding.

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[26] Gann, p. 214.

[27] 'Semitones, quarter tones, eighth tones, twelfth tones (...). This is all a question of scaling, just as one decides on a certain screen resolution on the computer.' Original text: 'Halbtöne, Vierteltöne, Achteltöne, Zwölftetöne (…). Das alles ist eine Frage der Skalierung, so wie man sich etwa für eine bestimmte Bildschirmauflösung am Computer entscheidet.' (My translation). Georg Friedrich Haas, 'Mikrotonalität Und Spektrale Musik Seit 1980', in *Orientierungen: Wege Im Pluralismus Der Gegenwartsmusik*, ed. by Jörn Peter Hiekel, Veröffentlichungen Des Instituts Für Neue Musik Und Musikerziehung Darmstadt (Mainz: Schott Musik International, 2007), pp. 123–38. (p.126).

[28] Rozalie Hirs and Tristan Murail, 'Interview with Tristan Murail', in *Contemporary Compositional Techniques and OpenMusic*, ed. by Rozalie Hirs and Bob Gilmore, Musique-Sciences (Sampzon: Ed. Delatour France, 2009), pp. 7-14 (p. 14).

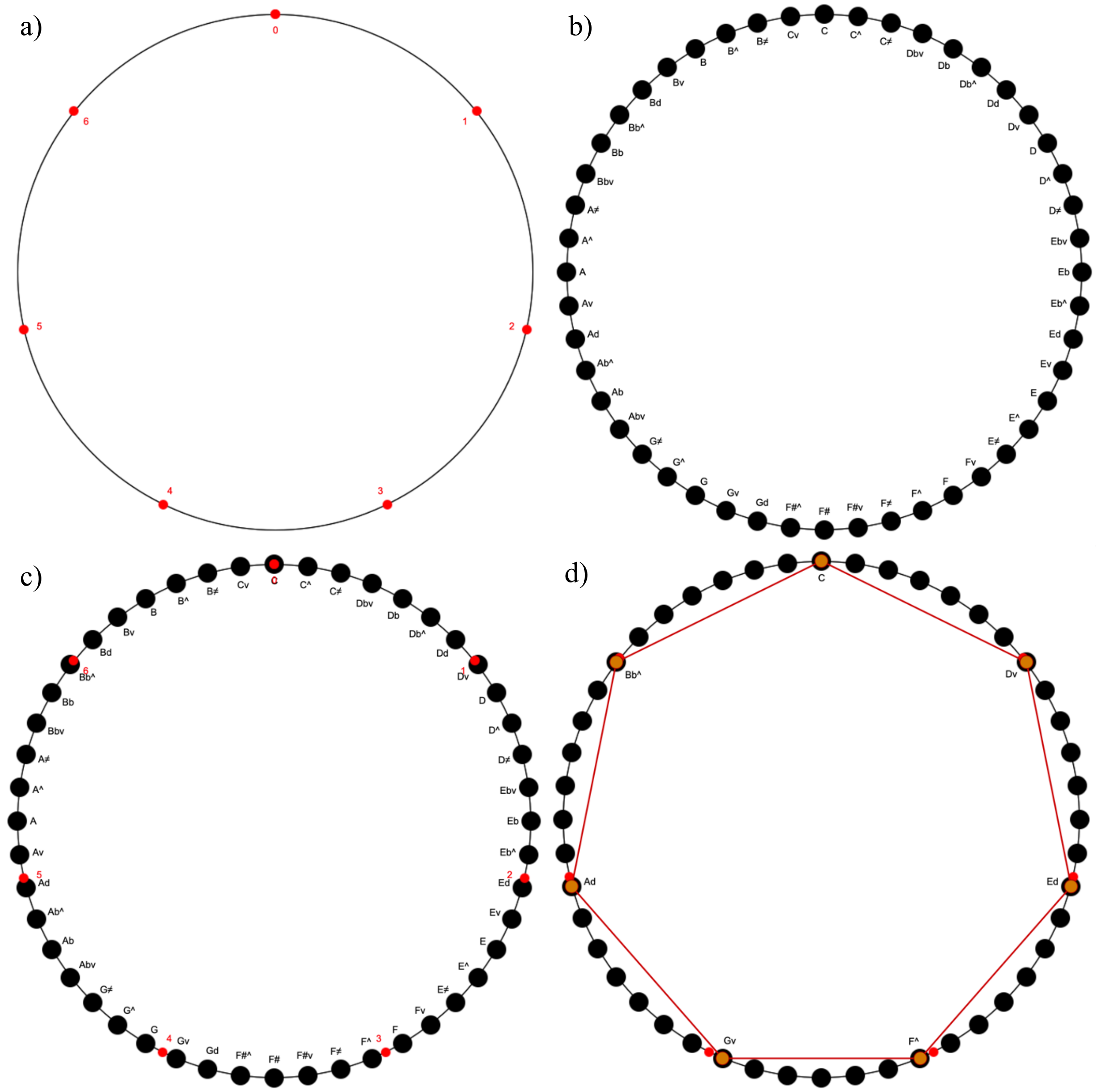

Figure 2. An example of 7EDO being rounded to 48NPO.[29]

[29] These and all similar looking figures have been built on Equal Tuning Lab, a tool which I have been programming for the visual representation of equal tunings, alongside other features.

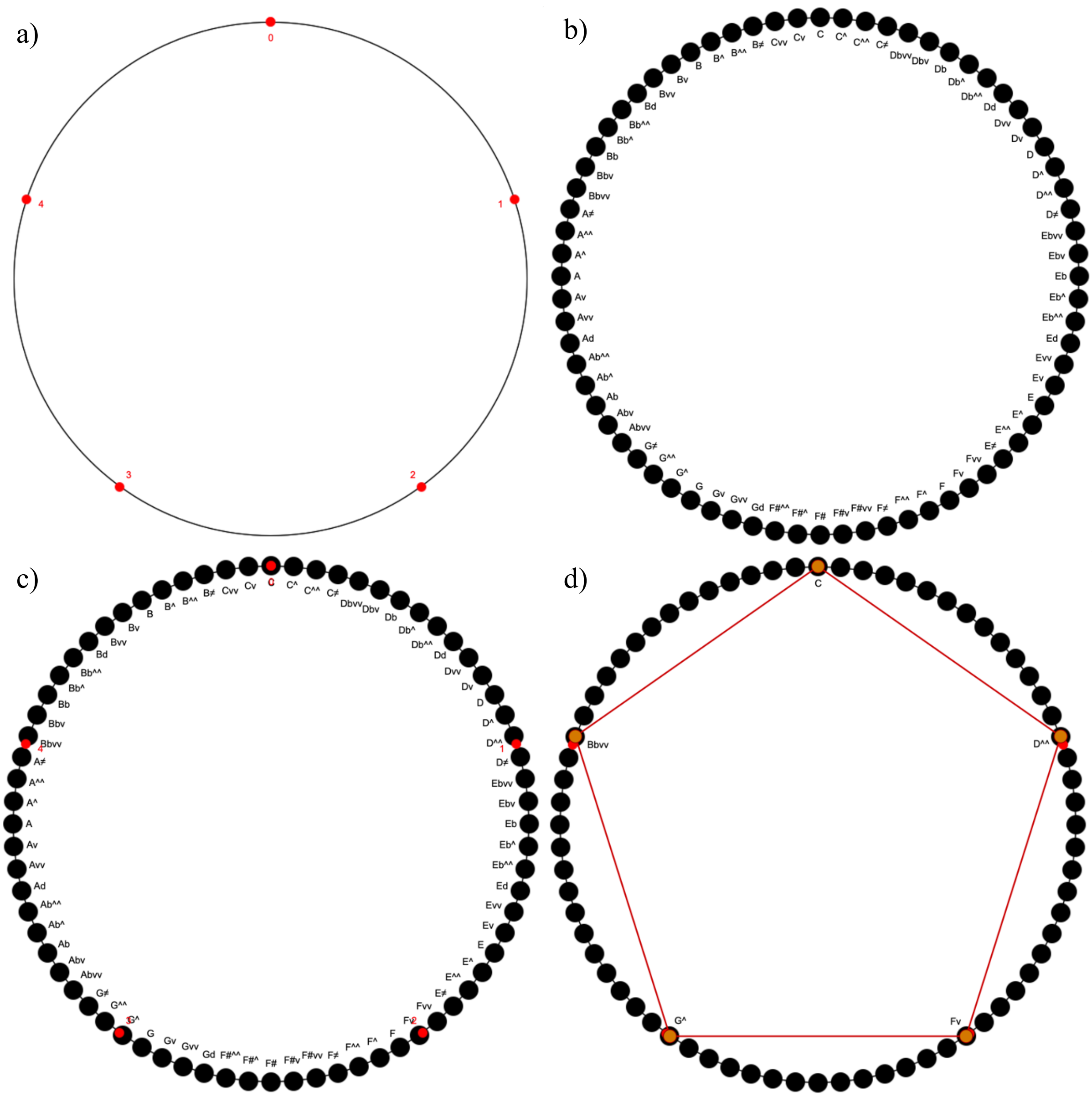

Figure 3. An example of 5EDO being rounded to 72NPO.

In the a) pictures of Figures 2 and 3, 5EDO and 7EDO are represented in red dots and each pitch is named with a number from 0 onwards. In the b) pictures, 48 and 72NPO are represented in black dots and each pitch is referred with note names. The c) pictures juxtapose 7EDO with 48NPO and 5EDO with 72NPO. In the d) pictures, the polygons' vertices ending in orange dots correspond to the notes of 48 and 72NPO which respectively 'round' the pitches of 7 and 5EDO. The accidentals are represented as follows: '#' and 'b' for a semitone higher or lower, '≠'' and 'd' for a quarter tone higher or lower. In 48NPO '^' and 'v' notate pitches an eighth tone higher and lower, respectively. In 72NPO, '^' and 'v' correspond to twelfth tone deviations, and '^^' and 'vv' to sixth tone deviations.[30]

The terms 'Microtonal' and 'Microtonality' have been described in a variety of ways. Composer and sound designer Terumi Narushima introduces three possible different definitions:

1. 'Microtonal suggests music with very small intervals'
2. 'it can also refer to music that uses any intervals not found in the standard Western system of 12-tone equal temperament' encompassing 'experimental tuning systems', 'scales found in different musical cultures around the world, as well as historic intonation systems, from ancient Greek scales to temperaments that predate the gradual adoption of 12 equal divisions of the octave in the West'
3. 'Microtonality is 'a musical continuum that embraces "all intervals and tuning systems", of which 12-tone equal temperament is "only one of the myriads of possibilities"'.[31]

Despite these various definitions, Sabat's is the most useful for this research, because it is premised on his own definition of intonation (as seen in Chapter 2.3). He states that microtonality is 'an approach to pitch which acknowledges the musical possibility of this entire glissando-continuum and is not limited to the conventional twelve equal tempered pitch-classes'.[32] According to this definition, both the spectral practice and rounding are, thus, microtonal, since this entire 'glissando-continuum' is at their disposal.[33] However, depending on each piece's instrumentation, these practices will be subject to instrumental intonation. This may imply that there is a tendency for 'microtonality' to be a model within the composers' agency, and 'intonation' a technique within the performers' agency. Notation would then be the tool from which microtonality would serve as the framework for the performers'

[30] See Appendix 2 to see how these typeface accidentals are notated in a score and to identify enharmonic equivalencies between different combinations of these accidentals.

[31] Terumi Narushima, *Microtonality and the Tuning Systems of Erv Wilson*, Routledge Studies in Music Theory (Abingdon, Oxfordshire), p. 1.

[32] Sabat, 'Intonation and Microtonality', (para. 3 of 19).

[33] This would mean that a piece for non-retuned piano based on approximations of the harmonic series (such as Radulescu's 2nd Piano Sonata), would be considered as being microtonal.

intonational choices. Violinist Mira Benjamin then proposes seven approaches to notation[34] (which I complement by adding categories 3.c. and 3.d.):

1. Ratio notation
2. Cent notation
3. Symbolic incremental alterations (alternative 'microtonal' accidentals)
    a. Subdivisions of 12EDO (as an extension of the Western Chromatic Notation)[35]
    b. Comma deviations (e.g. The Helmholtz-Ellis Just Intonation Notation) [36]
    c. Circle of Fifths Notation (as an adaptation of the Western Chromatic Notation)[37]
    d. A mixture of the previous (e.g. Sagittal Notation)[38]
4. Trajectories and continua (glissandi)
5. Tablatures and scordaturas
6. Qualitative descriptions (verbal text notation)
7. Implicit pitch behaviour (Western Chromatic Notation)

Throughout this research, most of these categories have been used, particularly 3.a. (while expecting performative intonation to simulate characteristics of 7). The other categories tend to serve as complements for specific aims.

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[34] Mira Benjamin, 'Thick Relationality: Microtonality and the Technique of Intonation in 21st Century String Performance' (unpublished doctoral thesis, University of Huddersfield, 2019) <http://eprints.hud.ac.uk/id/eprint/35116/>. pp. 121-31.

[35] For a broad compilation of these notational systems, see Gardner Read, *20th-Century Microtonal Notation*, Contributions to the Study of Music and Dance, no. 18 (New York: Greenwood Press, 1990).

[36] See Marc Sabat, 'The Extended Helmholtz-Ellis JI Pitch Notation', trans. by Natalie Pfeiffer, 2004 <https://marsbat.space/pdfs/notation.pdf> [accessed 11 August 2023].

[37] In this system, the A B C D E F G note names are placed in a circle of fifths, and sharps and flats are added as fifths are stacked in the circle of fifths until the necessary amount of note names is reached to represent all the pitches of a desired tuning. Even though this notational system looks visually identical to the Western Chromatic Notation system, it implies fixed tuning systems/temperaments. When this system is applied to EDOs other than 12EDO, enharmonic equivalencies between pitches change. For further information on this notational approach and other approaches related to it, see 'Circle-of-Fifths Notation', *Xenharmonic Wiki* <https://en.xen.wiki/w/Circle-of-fifths_notation> [accessed 9 August 2023].

[38] See George D. Secor and David C. Keenan, 'Sagittal: A Microtonal Notation System', 2023 <https://sagittal.org/sagittal.pdf> [accessed 9 August 2023].

2.4. Tuneable Intervals, Quasi-Tuneable Intervals, and Pitch Classes VS Tuneability

Following an empirical study, Marc Sabat and tubist Robin Hayward state that, within a three-octave range, there is a finite number of just ratios which may be tuned precisely by ear. They name them 'tuneable intervals' and classify them into three classes based on their difficulty of reproduction: 'easy', 'more difficult', and 'hard'. All other intervals would be thus 'untuneable intervals.'[39]

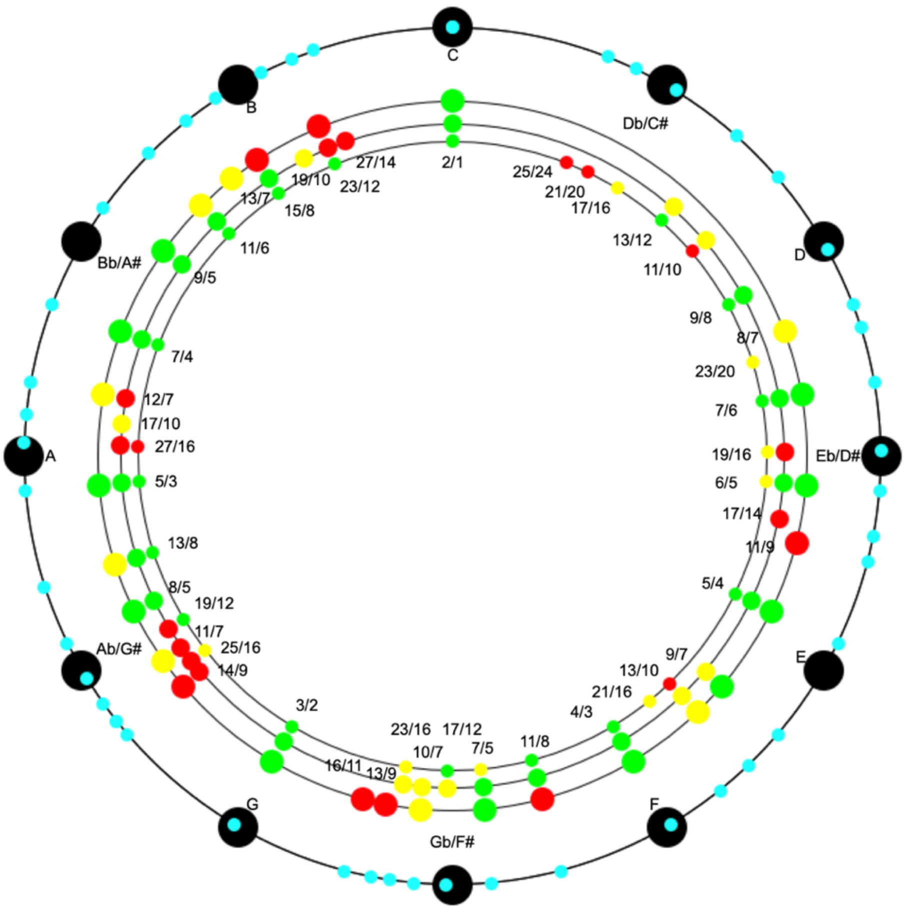


Figure 4. Representation of Sabat and Hayward's tuneable intervals against 12EDO.

In Figure 4, all tuneable intervals are represented with coloured dots inside the larger outer circle on top of three other inner circles (green for 'easy' intervals, yellow for 'more difficult' and red for 'hard'). The biggest dots on the larger inner circle correspond to tuneable intervals equal or larger than a unison and smaller than an octave. The medium-sized dots on the middle

[39] Marc Sabat and Robin Hayward, 'Towards an Expanded Definition of Consonance: Tuneable Intervals on Horn, Tuba and Trombone', 2006, pp. 1-5 <https://marsbat.space/pdfs/tuneable-brass.pdf> [accessed 21 March 2021].

inner circle correspond to tuneable intervals equal or larger than an octave and smaller than two octaves. The smallest dots on the smallest inner circle correspond to tuneable intervals equal or larger than two octaves and smaller than three octaves. All tuneable intervals are projected as one-octave range 'interval classes'[40] (cyan dots and ratios) to help compare their size with the 12EDO pitches (black dots and note names). All displayed ratios correspond to these one-octave interval classes. For example, between the C and Db/C# regions, the ratio 25/24 near a small red dot does not correspond to the tuneable interval represented by the dot but, instead, to its interval class. The actual ratio represented by that red dot is 25/6 since that red dot, due to its size and location, can only correspond to an interval larger than two octaves, but smaller than three octaves. Then, by stretching the 25/26 ratio by two octaves, one gets the 25/6 ratio.

By analysing the list of tuneable intervals,[41] one may observe that the intervals of 12EDO are not part of it. They are, however, close to some tuneable ones. Nonetheless, the intervals of 12EDO can be learned and reproduced by performers, although playing them would not follow the methodology required to tune tuneable intervals.[42]

The level of reproduction difficulty of tuneable intervals does not remain constant when octaves are added or subtracted from tuneable intervals. Tuneable intervals then do not share with their respective 'interval classes' the same properties regarding tuneability.[43] This is relevant for the instrumental adaptation of EDOs. While tuneable intervals, as defined by Sabat, and Hayward) do not inhere in EDOs (except for the octave), according to Tenney's hypotheses, some of their untuneable intervals might be within the tolerance range of tuneable ones. This category of untuneable intervals, which shall be referred as *quasi-tuneable intervals*, would thus represent tuneable intervals in line with Tenney's principle of 'regions of tolerance'. A

[40] I am using a more limited definition of 'interval classes' than that of set theory and applying it for Just Intonation intervals (instead of equally tempered ones). Interval classes, for the purposes of this research, are generalizations that treat intervals smaller than an octave and their associated compound intervals (such as the major second, the major ninth and the major seventeenth) as being part of the same category. In this case the ratios 9/8, 9/4 and 9/2 all share the same interval class: 9/8.

[41] For the full list of tested intervals and their difficulty level see Sabat and Hayward, pp. 6-10. For the full list of tuneable intervals written in HEJI notation, see Marc Sabat, 'The Extended Helmholtz-Ellis JI Pitch Notation'.

[42] Sabat and Hayward, p. 3.

[43] See Appendix 5 for a proposed model to grasp such properties of interval classes.

notational system which treats both categories as equivalent (i.e. assuming some level of implicit pitch behaviour), similarly to the model of Figure 1, would then seem to be appropriate to notate quasi-tuneable intervals.

Alongside rounding, approximations into tuneable intervals could be another viable microtonal tool to adapt equal tunings for non-fixed pitch instrumental composition.

2.5. EDOs, Circles of fifths, Diatonicism, Meantone temperaments and EDO categories

While EDOs are generated by equally dividing an octave (and, according to Italian theorist Patrizio Barbieri, are therefore 'closed-chain systems'),[44] they may also be conceptualized as being generated by a mechanism close to that of Meantone temperaments: taking Pythagorean Tuning, which is generated by staking multiple perfect fifths at the size of the 3/2 ratio (which serves as the generator interval) up to a desired number of pitches and reorganizing these pitches into a one-octave ascending scale, and reducing the size of all fifths so that the size of all major thirds become closer to the 5/4 ratio, instead of the 81/64 ratio of Pythagorean Tuning.[45] In a closed-chain system (i.e. EDOs), the size of the fifth allows for the initial pitch to be reached at some point. In an open-chain system (e.g. historical Meantone temperaments and Pythagorean Tuning), this does not happen.

There are some EDOs which contain intervals close to that of open-chain historical systems,[46] such as:

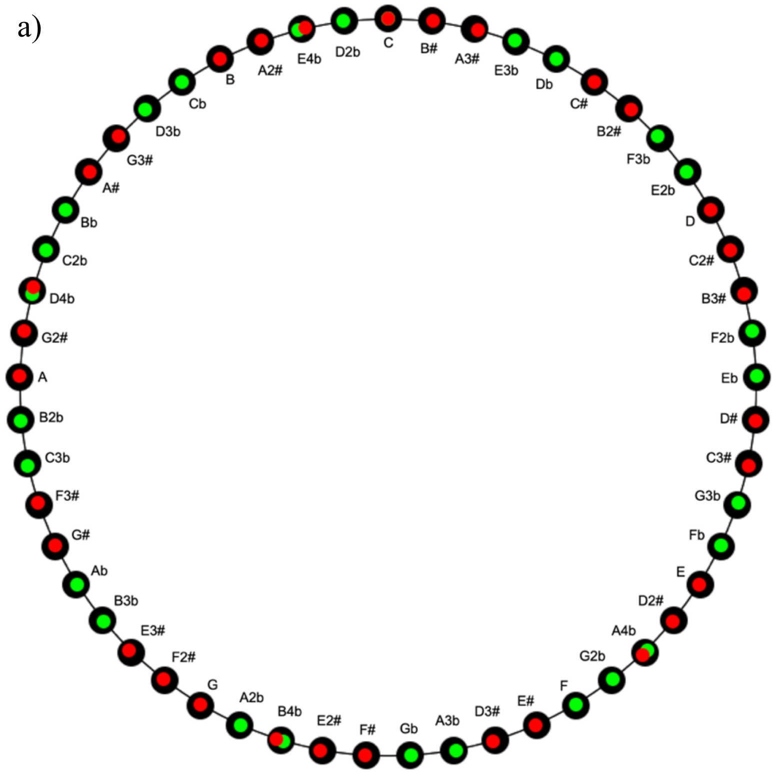
[44]Patrizio Barbieri, *Enharmonic Instruments and Music 1470-1900: Revised and Translated Studies*, Tastata, 2 (Latina: Il Levante Libreria, 2008). pp. 1, 277-545.

[45] See Barbour, pp. 25-44 for a more detailed explanation of this mechanism and the varieties of temperaments that can be generated through it.

[46] Barbieri, pp. 1, 279-280.

- 53EDO (Pythagorean Tuning)

a)



b)

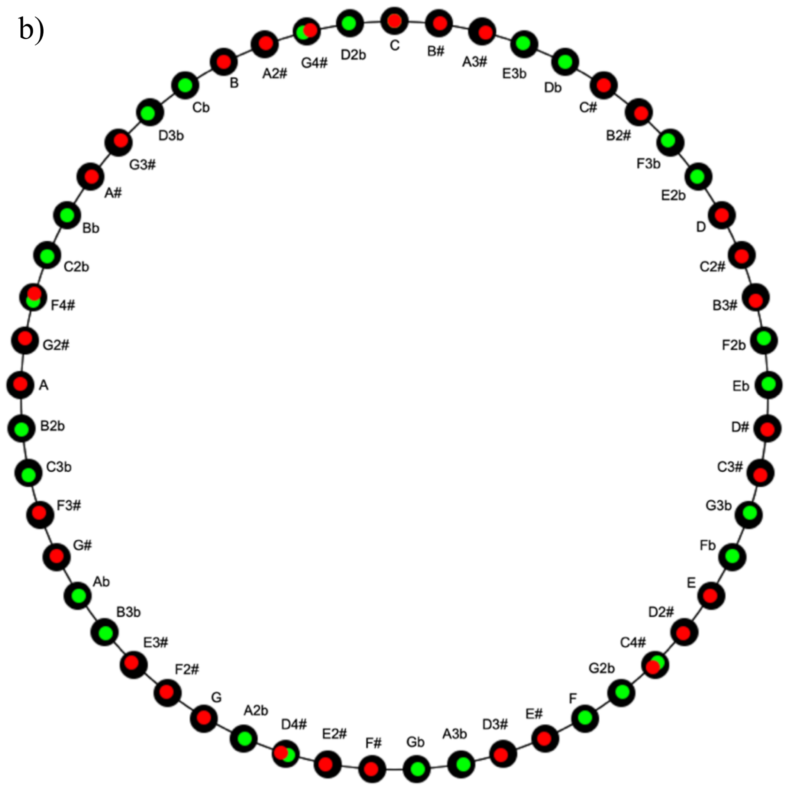
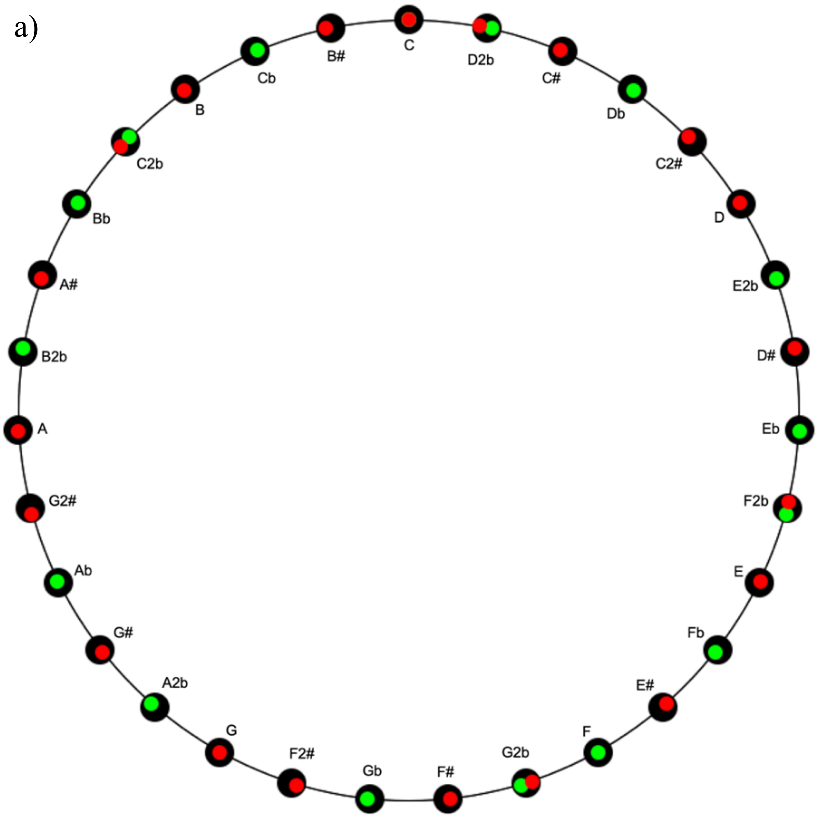


Figure 5. Comparison between Pythagorean Tuning represented through both an ascending (red dots) and descending (green dots) cycle of fifths from C, and 53EDO (black dots and Circle of Fifths Notation). Black dots which contain both red and green dots have been labelled with different enharmonic note names in each picture: a) with quadruple flats and b) with quadruple sharps. This means that those particular pitches of 53EDO treat different pitches of Pythagorean Tuning (derived from cycles of fifths in opposing directions) as being equivalent by tempering them into a single pitch.

- 31EDO (1/4-Comma Meantone Temperament)

a)



b)

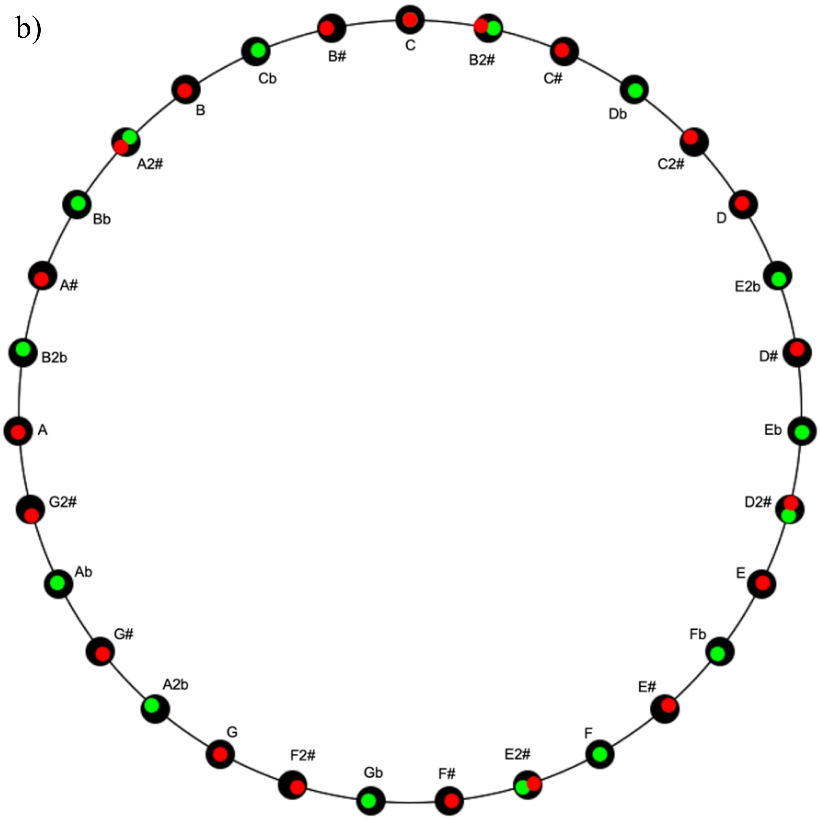
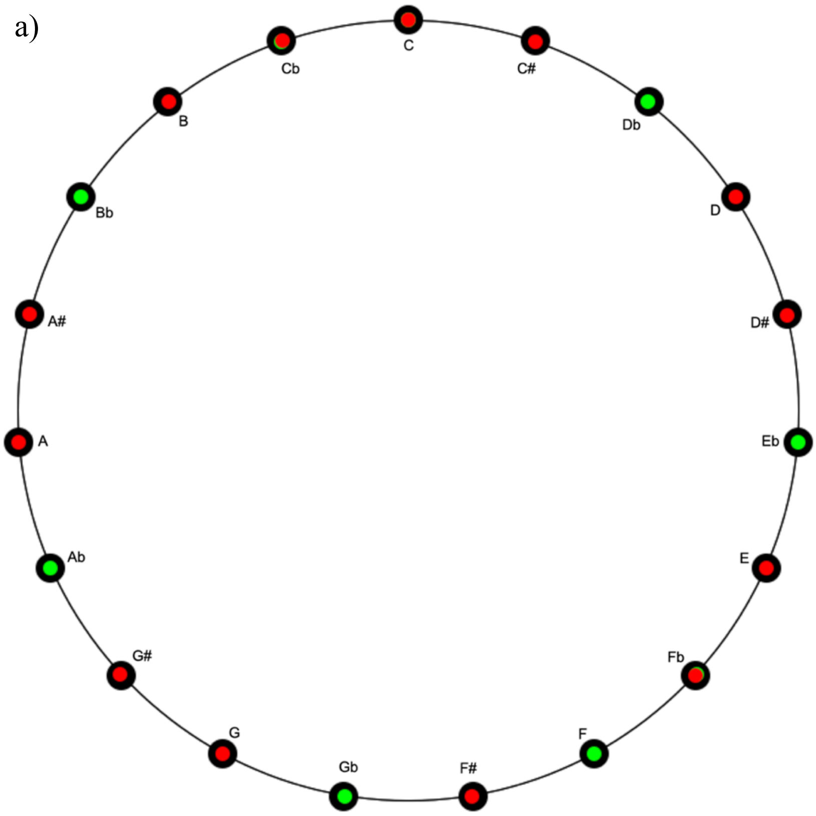


Figure 6. Comparison between 1/4-Comma Meantone Temperament represented through both an ascending and descending cycle of fifths from C, and 31EDO. a) Black dots with both red and green dots are labelled with double flat note names. b) Black dots with both red and green dots are labelled with double sharp note names.

- 19EDO (1/3-Comma Meantone Temperament)

a)



b)

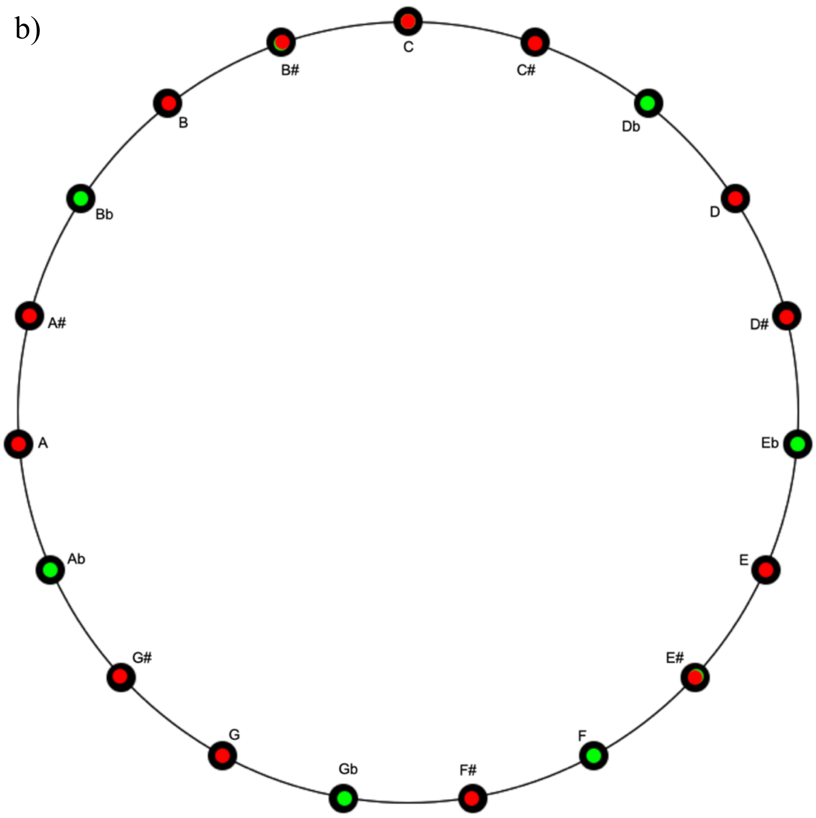


Figure 7. Comparison between 1/3-Comma Meantone Temperament represented through both an ascending and descending cycle of fifths from C, and 19EDO. a) Black dots with both red and green dots are labelled with single flat note names. b) Black dots with both red and green dots are labelled with single sharp note names.

- and 12EDO (1/11-Comma Meantone Temperament).[47]

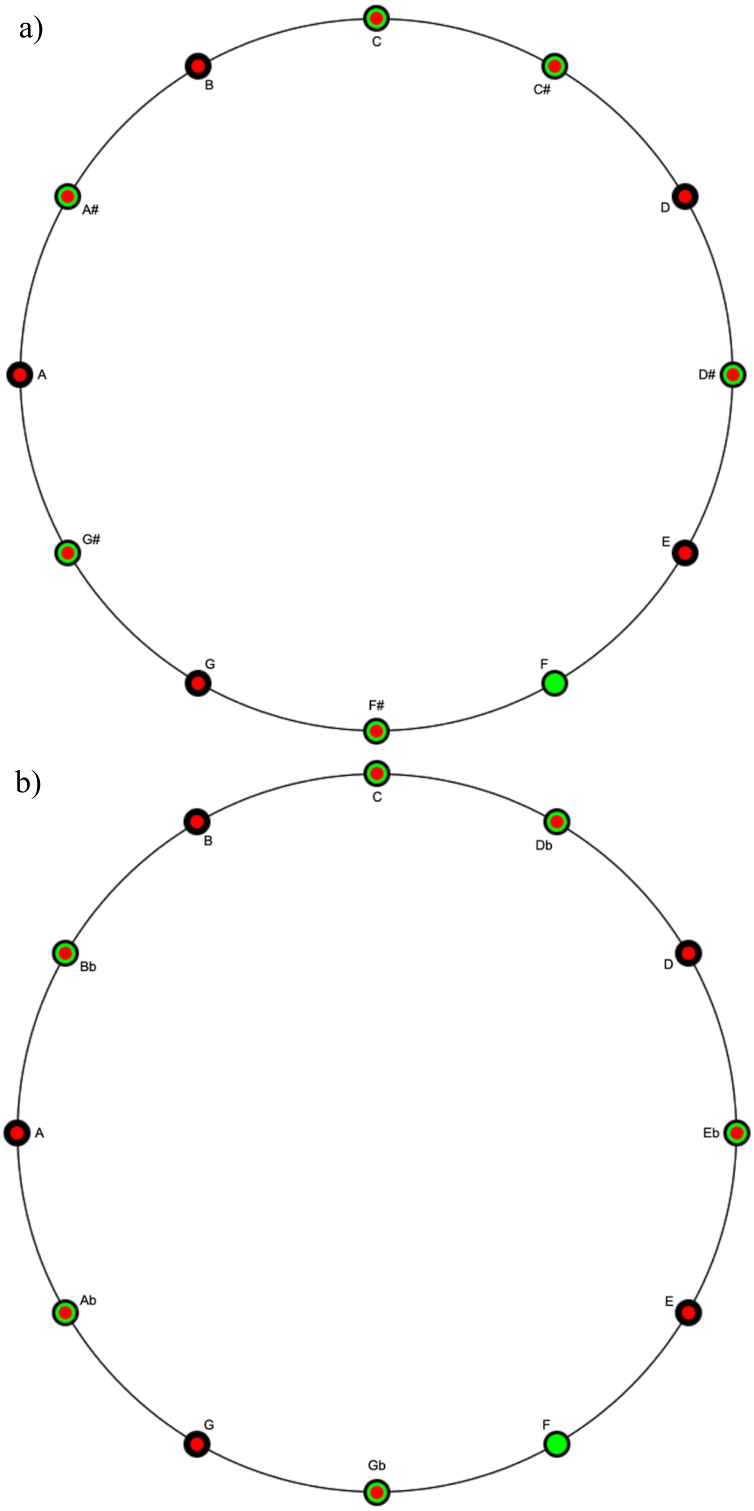


Figure 8. Comparison between 1/11-Comma Meantone Temperament represented through both an ascending and descending cycle of fifths from C, and 12EDO. a) Black dots with both red and green dots are labelled with single flat note names. b) Black dots with both red and green dots are labelled with single sharp note names.

[47] 1/11-Comma Meantone Temperament is usually referred as a theoretical curiosity, given how surprisingly close it is to 12EDO. One example is in Barbour, p. 35.

This mechanism of stacking multiple iterations of a generator interval, when used only up to seven pitches (and after rearranging them into a one-octave range), creates seven-note scales, with seven possible modes. Depending on the actual size of the generator interval, the resulting scales may be classified as 'Diatonic'. For the purposes of this research, this category only includes scales with an intervallic structure that resembles that of the major and natural minor scales, and of the seven Gregorian modes. EDOs which include diatonic scales (derived through this methodology) are then also considered to be Diatonic.

However, there are other EDOs which would not be classified as diatonic, since they do not include intervals that can be used as generators to create diatonic scales.[48] Microtonal practitioner and theorist Kite Giedraitis then classifies EDOs as falling within the following categories, according to the cent (hereafter '¢') sizes of their possible generator intervals (which, for convenience, will be referred as 'fifths'):

- Superflat – EDOs with a fifth narrower than four-sevenths of an octave (4\7EDO ≈ 686¢)
- Perfect – EDOs with a fifth of 4\7EDO
- Diatonic – EDOs with a fifth between 4\7EDO and 3\5EDO
- Pentatonic – EDOs with a fifth of three-fifths of an octave (3\5EDO = 720¢)
- Supersharp – EDOs with a fifth wider than 3\5EDO
- Trivial – EDOs with a fifth about 100¢ from just, and are notated as subsets of 12EDO[49]

Similarly to Giedraitis, composer Easley Blackwood also considers tuning systems (including EDOs) to be Diatonic if their fifth sizes are simultaneously larger than 4\7EDO and narrower than 3\5EDO.[50] He notes that 5EDO (in which the major seconds are so stretched that its minor

[48] For further exploration of the scalar possibilities based on what this mechanism allows, refer to the theory of Moments of Symmetry scales. Narushima, pp. 59-108. Also see 'MOS Scale', *Xenharmonic Wiki* <https://en.xen.wiki/w/MOS_scale> [accessed 30 July 2024].

[49] Kite Giedraitis, 'Alternative Tunings: Theory, Notation and Practice', 2019 <https://www.tallkite.com/misc_files/alt-tuner_manual_and_primer.pdf> [accessed 7 August 2023]. p. 206. Although, 2 and 4EDO could also be classified as Superflat, 3EDO as Supersharp, and 6EDO as both Superflat and Supersharp (depending on whether one chooses its fifth to measure 600¢ or 800¢). Another source with a possible different classification system would be Lumi - Music Theory, *Making Sense of Microtones by Stacking Fifths*, online video recording, YouTube, 16 September 2021 <https://www.youtube.com/watch?v=z486ScNJBOo> [accessed 9 August 2023].

[50] Blackwood, p. 199.

seconds become unisons) and 7EDO (in which its minor seconds and so stretched and its major seconds so compressed that both have the same size) are placed at the borders of these limits of Meantone temperaments.[51] Blackwood's remarks about 5 and 7EDO then also coincide with Giedraitis's Pentatonic and Perfect EDO classifications. Blackwood also adds that, within Diatonic systems (both open-chain or closed-chain, in Barbieri's model), for music which relies more extensively on chromatic intervals harmonically and melodically (instead of only the diatonic scale) there is a 'range of acceptability' for the size of the fifth within approximately 695¢ and 701.5¢.[52] Another method to conceptualize this range of acceptability would be for the proportion of the sizes of the major and minor seconds of a system (which Blackwood refers as 'R') being between 1.5 and 2.2, or between the proportions 3/2 (in 19EDO), and 11/5 (in 65EDO).[53]

James Murray Barbour also makes a distinction between what he calls 'positive' and 'negative' tuning systems: positive systems having a fifth larger than the 3/2 perfect fifth (approximately 702¢) and negative systems having narrower fifths than 3/2.[54] He then uses these two categories to classify what would be the Diatonic subcategory of EDOs.[55]

Theorist and generalized keyboard advocate Robert Holford Macdowall Bosanquet, however, defines 'negative' systems as those with a fifth narrower than 700¢ and 'positive' systems as those with a fifth larger than 700¢.[56]

The two ways that Barbour and Bosanquet differentiate negative and positive systems deserves further problematization.

Barbour, by using the 3/2 ratio as the border between negative and positive systems, places Pythagorean Tuning as his reference point. Historical Meantone temperaments are thus negative systems in this model. Blackwood's range of acceptability not only falls within Barbour's (Diatonic) negative category but is restricted into a fifth range size that includes a variety of Meantone temperaments. Moreover, amongst the online microtonal and xenharmonic communities (such as in the Xenharmonic Alliance online forums), alongside the tendency of referring to EDOs close to historical Meantone temperaments (such as 31EDO and

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51 Blackwood, pp. 195-99.

52 Blackwood, p. 204.

53 Blackwood, pp. 211-212.

54 Barbour, p. XI.

55 Barbour, pp. XI, 114-127.

56 Gann, p. 179. and Narushima, p. 22.

19EDO) as being ‘Meantone’ systems themselves, EDOs with fifths wider than 3/2 also tend to be referred as ‘Superpythagorean’.[57]

Bosanquet, by using 700¢ as the border between negative and positive systems, instead places 12EDO as his reference point to distinguish between negative and positive systems.

By mapping the already mentioned Diatonic EDOs (19, 31 and 53EDO) on the Lumatone with a Bosanquet-Wilson layout,[58] alongside two other positive (and Superpythagorean) ones (17 and 22EDO), and 12EDO (see Figures 9 to 14), one may notice fundamental features that differentiate Bosanquet’s negative and positive systems: whether sharp notes are lower or higher than what would be their flat enharmonic equivalents in 12EDO, and whether diatonic semitones are larger or narrower than chromatic semitones.[59]

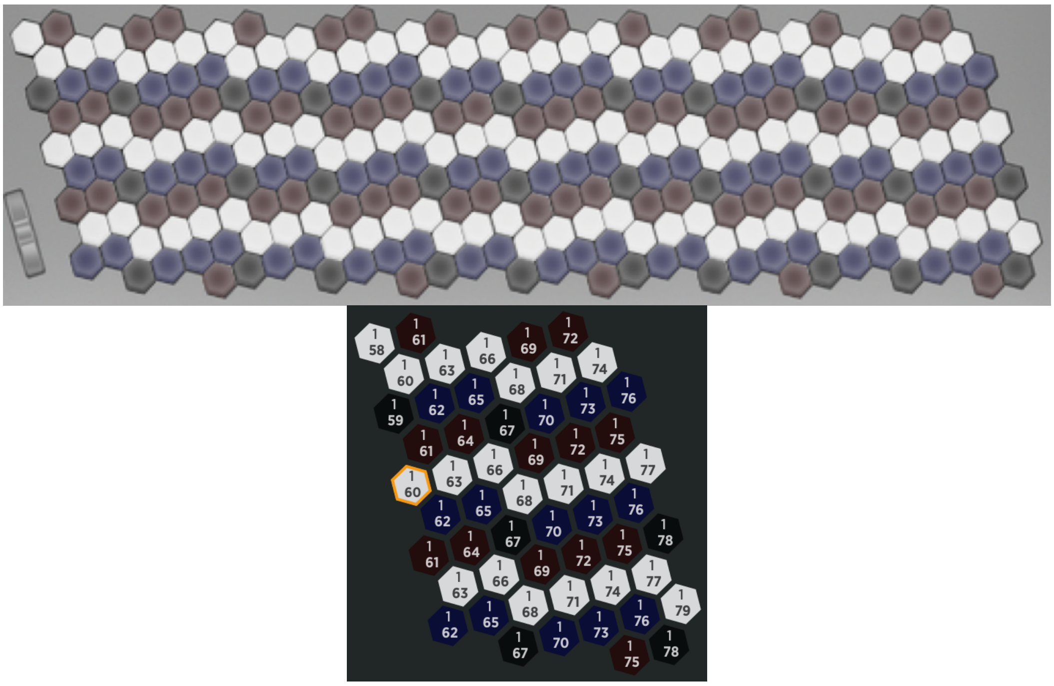

Figure 9. Bosanquet-Wilson mapping of 19EDO, a Diatonic EDO according to Giedraitis, a negative EDO according both to Barbour and Bosanquet, and close to 1/3-Comma Meantone Temperament. This mapping is a literal representation of Figure 7. Sharp notes (in dark red, diagonally right/up from a white ‘natural key’) are lower than what would otherwise be their flat enharmonic equivalents (in dark blue, diagonally left/down from a white ‘natural’ key), with the exception of the E#/Fb and B#/Cb pairs, which are enharmonic (by being represented by the same black key, similarly to the black keys of the 12EDO piano). Consequently, a diatonic semitone (2 steps) is wider than a chromatic semitone (1 step). Middle C is mapped as MIDI note 60.

[57] At around 2’30’’ see benyamind, *An Introduction to 17-Tone Equal Temperament*, online video recording, YouTube, 24 March 2023 <https://www.youtube.com/watch?v=LiMrUBo3qeQ> [accessed 9 August 2023].

[58] To become familiarized with the Bosanquet-Wilson layout, see Lumatone Keyboard, *LUMATONE | Quick Answers | “How Do You Know Which Note Is Which?”*, online video recording, YouTube, 15 June 2023 <https://www.youtube.com/watch?v=SPlmn5rbhZU> [accessed 21 June 2023].

[59] See Appendix 2 for Bosanquet-Wilson mappings of non-Diatonic EDOs.

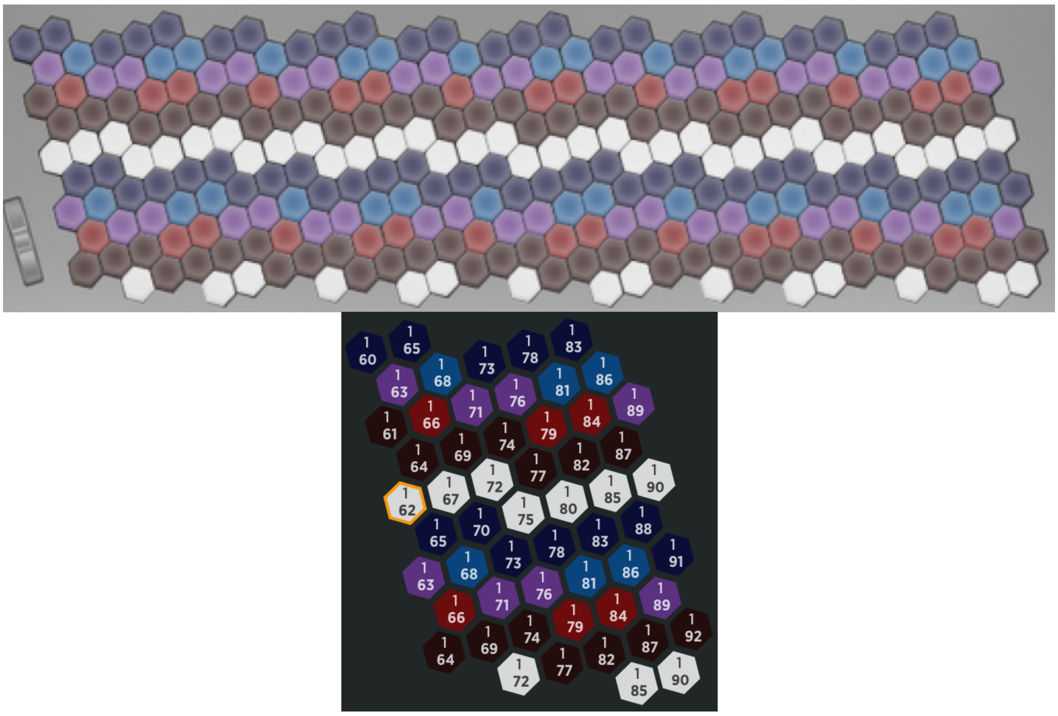

Figure 10. Bosanquet-Wilson mapping of 31EDO, a Diatonic EDO according to Giedraitis, a negative EDO according both to Barbour and Bosanquet, and close to 1/4-Comma Meantone Temperament. This mapping is a literal representation of Figure 6. The diatonic semitone corresponds to 3 steps and the chromatic semitone is 2 steps wide. Extra colours are introduced to represent double sharps (red), double flats (blue) and notes that are symmetrically notated with a double sharp and a double flat (purple). Enharmonic equivalencies are different from 19EDO (and will be different for any other EDOs).

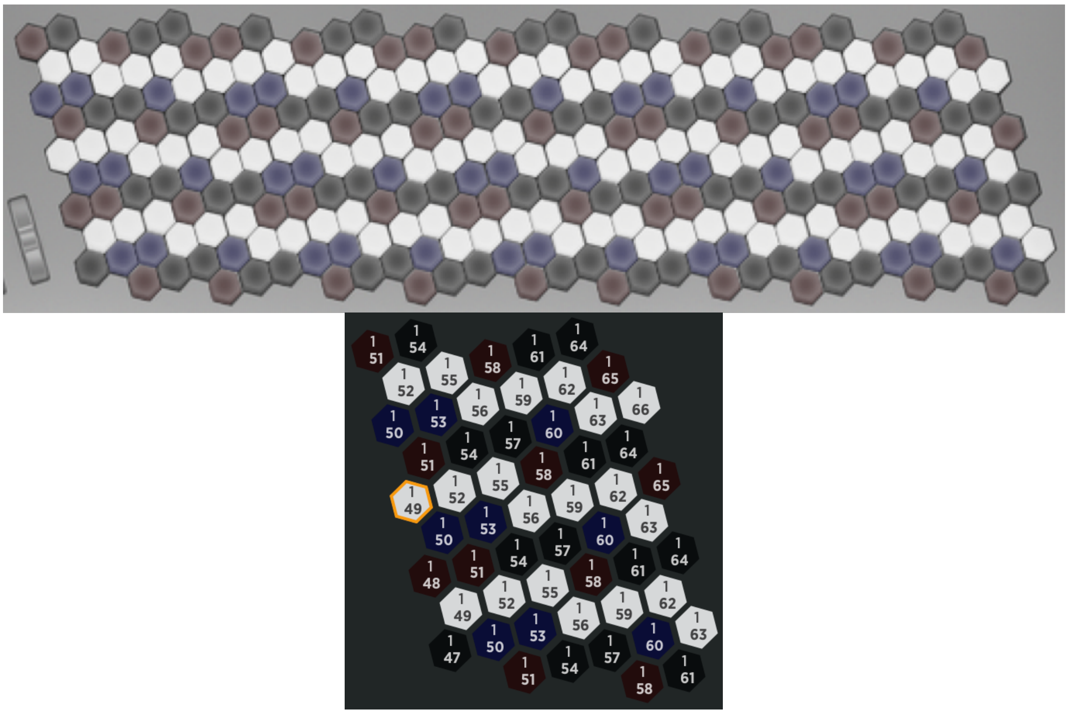

Figure 11. Bosanquet-Wilson mapping of 17EDO, a Diatonic EDO according to Giedraitis, a Superpythagorean EDO, and a positive EDO according to both Barbour and Bosanquet. Sharp notes are now higher than what would otherwise be their flat enharmonic equivalents. The diatonic semitone corresponds to 1 step and the chromatic semitone is now wider: 2 steps.

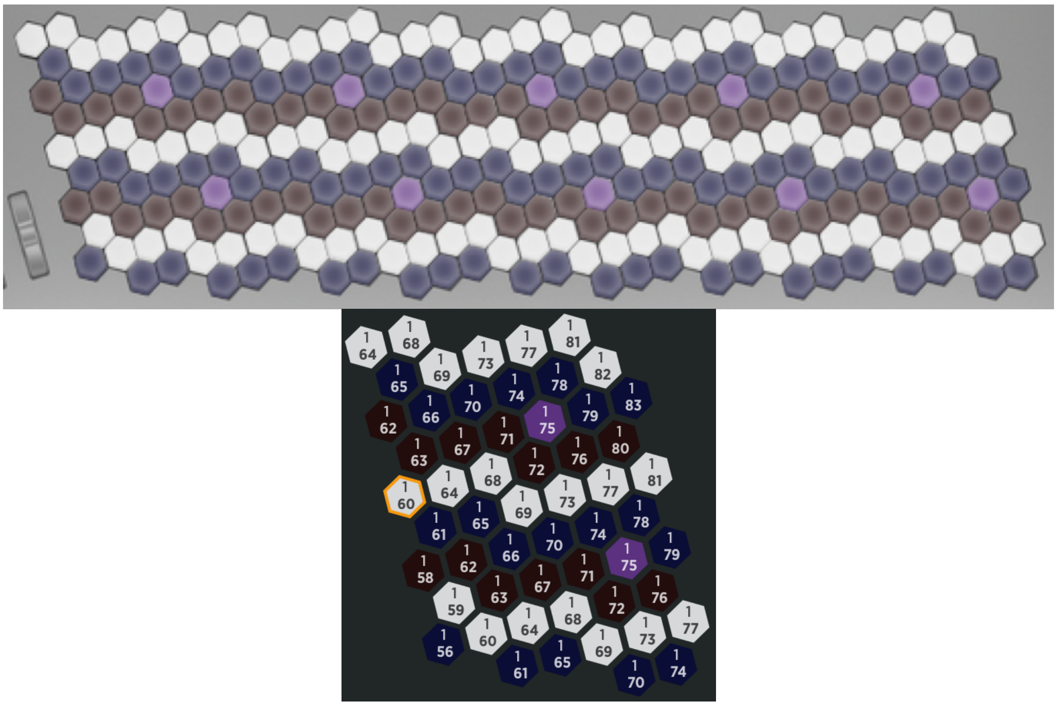

Figure 12. Bosanquet-Wilson mapping of 22EDO, a Diatonic EDO according to Giedraitis, a Superpythagorean EDO, and a positive EDO according to both Barbour and Bosanquet. The diatonic semitone corresponds to 1 step and the chromatic semitone to 3 steps.

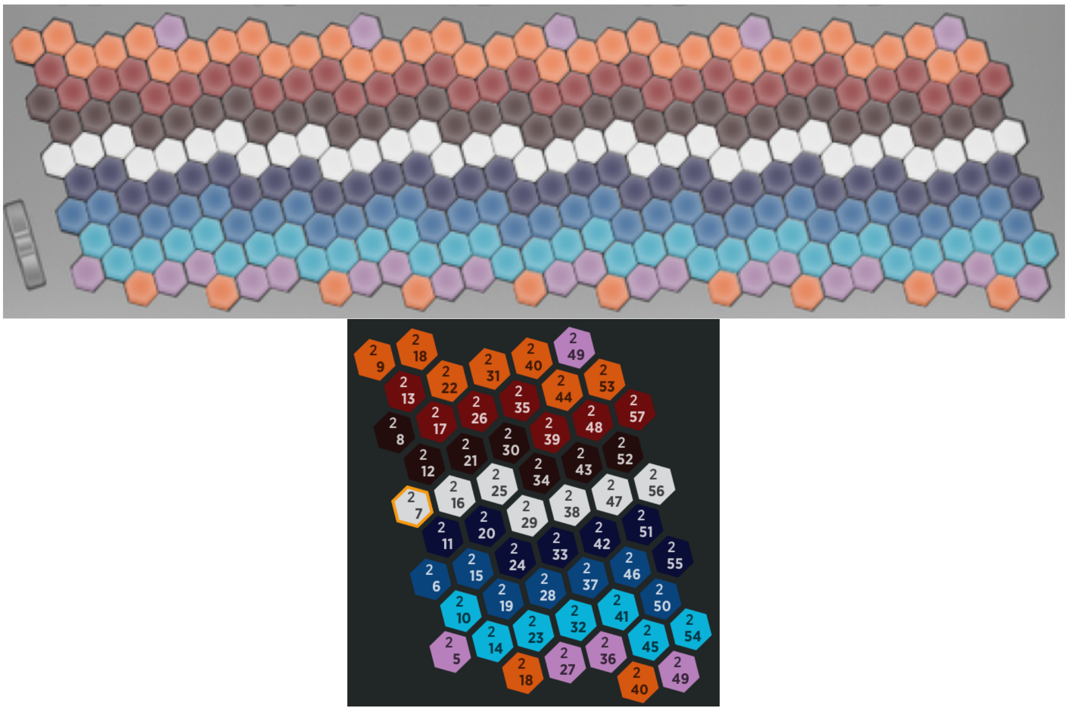

Figure 13. Bosanquet-Wilson mapping of 53EDO, a Diatonic EDO according to Giedraitis, a negative EDO according to Barbour (because its fifth size, approximately 701.9¢, is lower than the 3/2 ration, approximately 702¢) but a positive system according to Bosanquet, and close to Pythagorean Tuning. This mapping is a literal representation of Figure 5. Extra colours are added for triple sharps (orange), triple flats (lighter blue) and notes that are symmetrically quadruple sharps and quadruple flats (lighter purple). The diatonic semitone corresponds to 4 steps and the chromatic semitone corresponds to 5 steps.

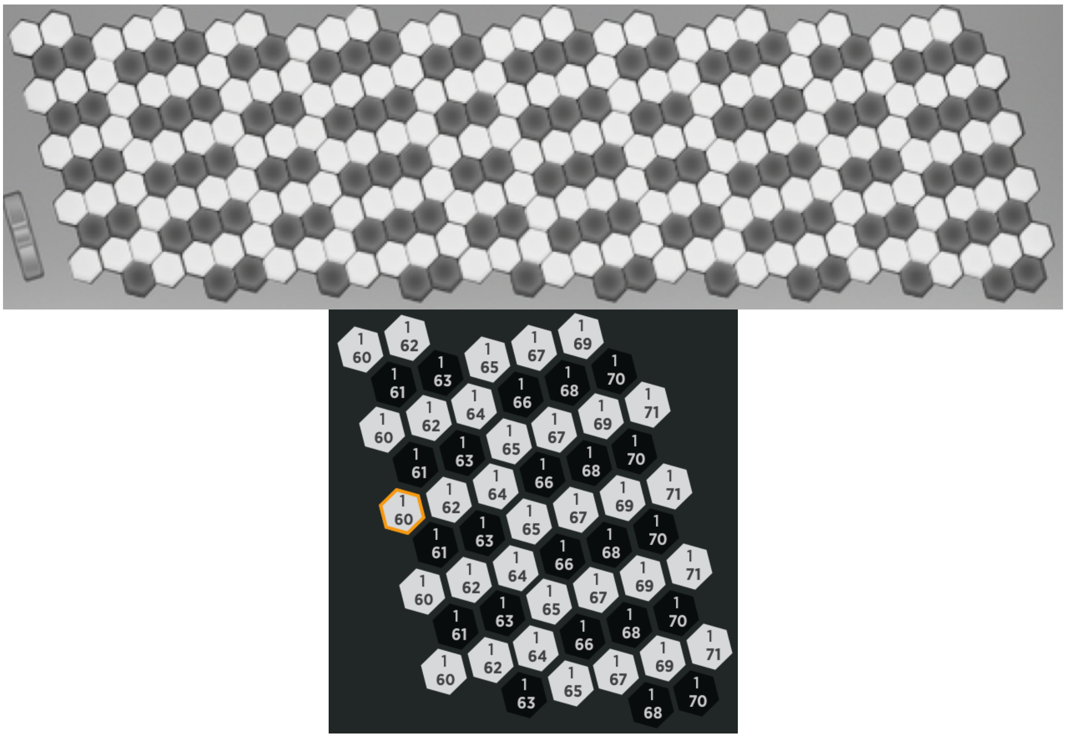

Figure 14. Bosanquet-Wilson mapping of 12EDO, a Diatonic EDO according to Giedraitis, a negative EDO only according to Barbour, and close to 1/11-Comma Meantone Temperament. This mapping is a literal representation of Figure 8. In Bosanquet's nomenclature, 12EDO would neither be negative nor positive. The diatonic and chromatic semitones have the same size of 1 step.

By analysing the differences between these systems, Bosanquet's choice of 700¢ as the reference point between negative and positive systems becomes clear: this is the fifth size which allows for the note pairs C#/Db, D#/Eb, F#/Gb, G#/Ab and A#/Bb to be enharmonic and makes the diatonic and chromatic semitones[60] sharing the same size. Negative EDOs have diatonic semitones wider than chromatic semitones and sharps notes lower than their flat equivalents. Positive EDOs have reversed features.

Despite Barbour and Bosanquet's models largely overlapping, their reference points, 700¢ and the 3/2 ratio (approximately 702¢), lead to Barbour considering 12EDO as negative (whereas Bosanquet would consider neither to be negative nor positive).

Blackwood's range of acceptability conforms to Barbour's negative category, and both include historical Meantone temperaments and 12EDO.

Pythagorean Tuning (positive for Bosanquet, but neither positive nor negative for Barbour) is the system from which Meantone temperaments are derived. 53EDO closely

[60] Respectively, semitones with pitches with different note names but with the same accidentals attached to them, and semitones with same note names but with different accidentals attached to them.

approximates it, but its fifth is around 0.1¢ narrower than the 3/2 Pythagorean fifth and, according to Barbour, that would make it a negative system (although, negligibly so).

Simultaneously, Barbour describes positive systems as having fifths wider than 3/2, and these systems are already referred as Superpythagorean.

Giedraitis, Bosanquet and Barbour offer rich insights with their models of pitch space. However, each model has a set of priorities that is not addressed by the other ones. Given that in this research I am looking at pitch through a multimodal approach, and all these models have useful insights which influence my practice (especially at the later stages of this research), I propose to fuse and adapt their models into the following new hybrid model:

- Superflat – EDO's with a fifth size narrower than 4\7 (686¢)
    - Trivial (Superflat) – Superflat EDOs contained in 12EDO (2, 4, & 6EDO)
- Equal Heptatonic[61] – EDOs with a fifth size of 4\7 (686¢ - 7EDO and its multiples)
- Diatonic – EDOs with a fifth size between 4\7 and 3\5, which can be subdivided into:
    - Meantone – EDOs with a fifth size between 7EDO's 4\7 (686¢) and the 3/2 ratio (702¢), from which an acceptable range between 696¢ and 701.5¢ may be highlighted
        - Negative – EDOs with a fifth size between 7EDO's 4\7 (686¢) and 700¢
        - Neutral – EDOs with a fifth size equal to 700¢
    - Positive – EDOs with a fifth size between 700¢ and 5EDO's 3\5 (720¢)
        - Pythagorean: EDOs with a fifth size close to the 3/2 ratio (702¢ with a 0.1¢ margin of error) which approximate Pythagorean Tuning, from which only 53EDO (or any multiple of it) would be included (although, technically and negligibly, such tunings would also be Meantone)
        - Superpythagorean – EDOs with a fifth size between the 3/2 ratio (702¢) and 5EDO's 3\5 (720¢)
- Equal Pentatonic[62] - EDOs with a fifth size of 3\5 (720¢ - 5EDO and its multiples)
- Supersharp – EDO's with a fifth size wider than 3\5 (720¢)
    - Trivial (Supersharp) – Supersharp EDOs contained in 12EDO (3 & 6EDO) [63]

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[61] This nomenclature was suggested by Stephen Weigel, instead of Giedraitis's 'Perfect' nomenclature.

[62] This nomenclature was also suggested by Stephen Weigel, instead of Giedraitis's 'Pentatonic' nomenclature.

[63] This model, as well as the contents of the next chapter, serve as the base structure for the spreadsheet 'EDO Diatonicism' in the *Global Properties of EDOs* spreadsheet collection.

2.6. Convergence between Diatonic EDOs and string intonation techniques[64]

Similarly with open-chain systems, Diatonic EDOs may also serve as models to represent properties of other historical intonation standards, including string intonation techniques, particularly of renowned baroque and early romantic period violinists Giuseppe Tartini and Bartolomeo Campagnoli, respectively, and 19th century violin technique theorists Louis Spohr and François-Antoine Habeneck.[65]

Tartini's approach is aimed towards the just intonation of major and minor thirds and sixths in performance (corresponding to the ratios 5/4, 6/5, 5/3 and 8/5), making major intervals narrower than their 12EDO counterparts, and minor intervals wider.[66] Within the context of the Western classical repertoire, diatonic semitones then also become wider. Sharp notes are therefore lower than the flat notes that would be enharmonic to them in 12EDO. This intonational behaviour displays intervallic characteristics that converge with Meantone and Negative EDOs - such 31 and 19EDO (see Figures 15 and 16).

[64] For a recent overview on string intonation technique, see Benjamin, 'Thick Relationality', pp. 73-117.

[65] Ross W. Duffin, *How Equal Temperament Ruined Harmony: And Why You Should Care* (New York, NY; London: W. W. Norton, 2008). pp. 46-63, 76-93, 134-136.

[66] In this chapter, a variety of tuning systems, intonation practices and intervals are compared against 12EDO for the reader's convenience.

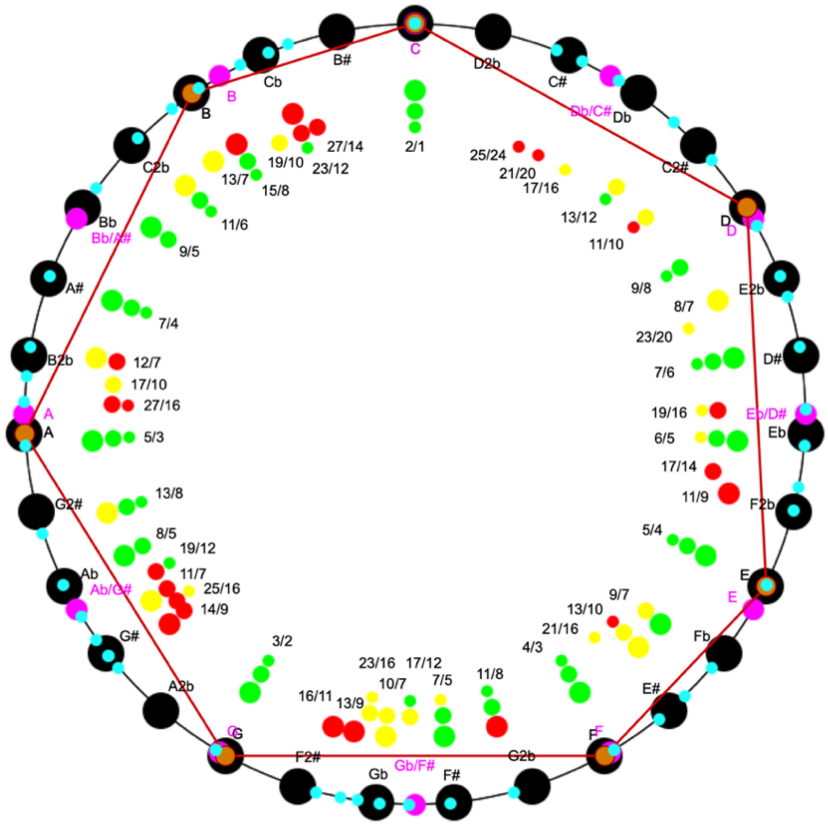

Figure 15. Circular representation of the diatonic scale (polygon and orange dots) of 31EDO (black dots and Circle of Fifths Notation), against 12EDO (magenta dots and notation) and Sabat and Hayward's tuneable intervals (green, yellow and red dots) and their projections into one octave interval classes (cyan dots and ratios).

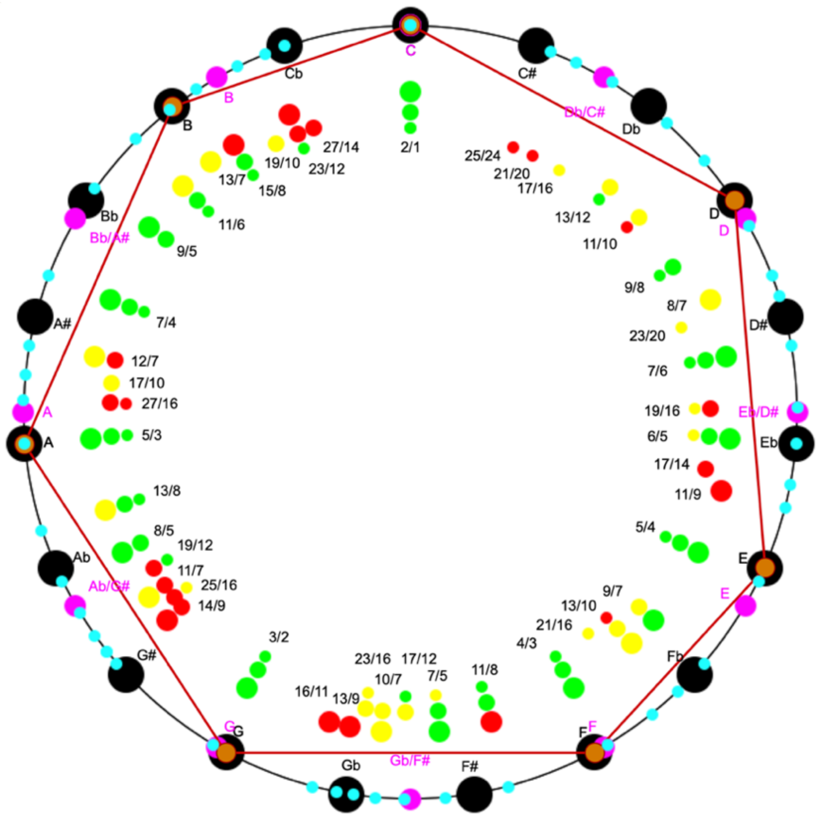

Figure 16. Circular representation of the diatonic scale of 19EDO, against 12EDO and Sabat and Hayward's tuneable intervals and their projections into one octave interval classes.

The intonational technique of famous late 19th century Austro-Hungarian violinist Joseph Joaquim, coined as the "Joaquim mode", also displays these characteristics.[67]

Campagnoli's approach can be understood as aiming for the intonation of sharp notes to be higher than the flat notes that would be enharmonic to them in 12EDO. Diatonic semitones then become narrower than the ones of 12EDO. Major thirds and sixths would then be wider than their 12EDO counterparts and minor ones would be narrower. This intonational behaviour displays intervallic characteristics closer to Positive Pythagorean and Superpythagorean EDOs (such as 53 and 17EDO – see Figures 17 and 18).

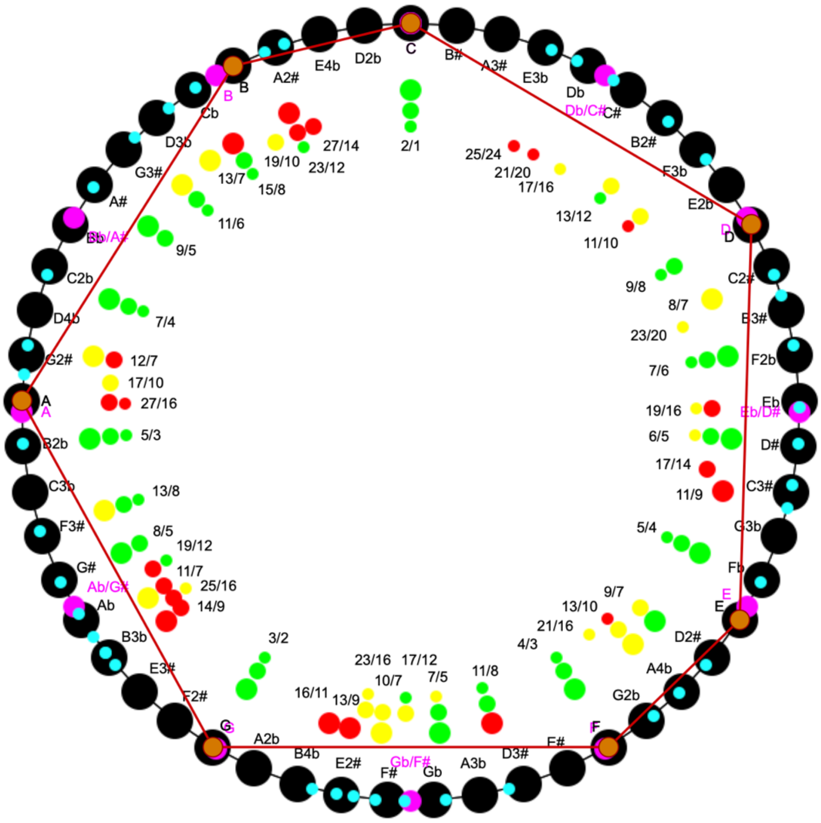

Figure 17. Circular representation of the diatonic scale of 53EDO, against 12EDO and Sabat and Hayward's tuneable intervals and their projections into one octave interval classes.

[67] Duffin, pp. 119-137.

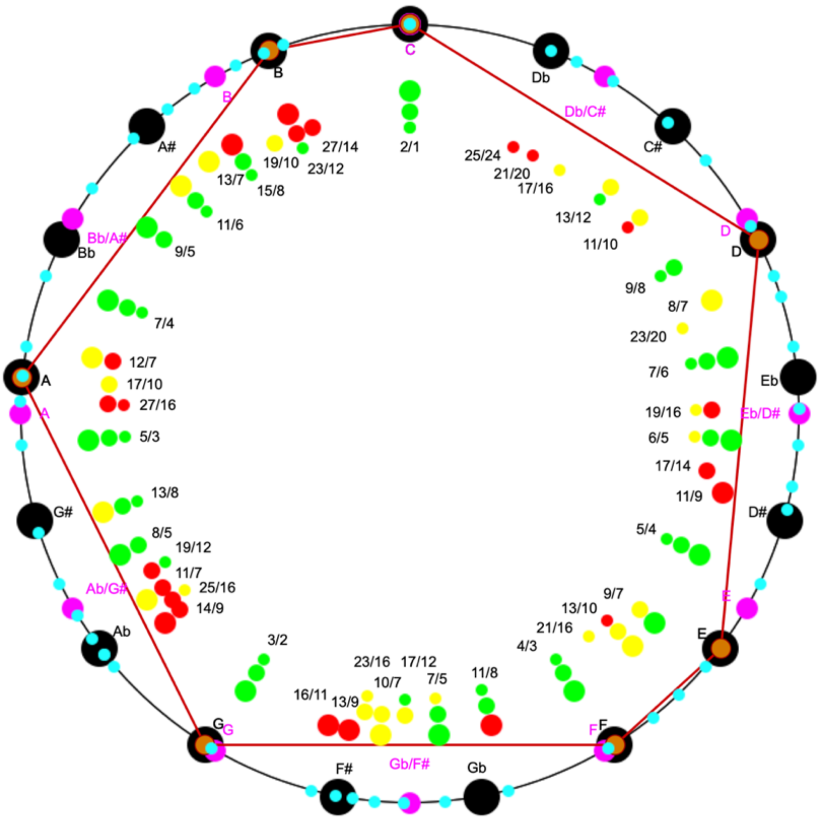

Figure 18. Circular representation of the diatonic scale of 17EDO, against 12EDO and Sabat and Hayward's tuneable intervals and their projections into one octave interval classes.

Famous Spanish 20th century cellist Pablo Casals also applied a similar approach, which he coined as 'expressive intonation'.[68] Violinist Theodor Podnos and composer Charles Schackford also report that, when analysing audio recordings of several string performers (including Casals), the cent sizes of the intervals played by these performers tend to be consistent with this approach.[69]

When comparing these cent sizes with a variety of Positive EDOs, they converge towards the cent sizes of 29 and 46EDO. The exception is the major second, which tends to be substantially wider, approximating the major seconds of 49, 27 and even 42EDO. This could suggest an 'expressive range' complementary to Blackwood's acceptable range, which instead converges with a limited range of Positive systems up to 46EDO. This list would include 53, 41, 29 and 46EDO.

[68] Theodor H. Podnos, *Intonation for Strings, Winds, and Singers: A Six-Month Course* (Metuchen, NJ: Scarecrow Press, 1981). p. 49, and Duffin, pp. 19-20, 154-156.

[69] Podnos, pp. 49-51.

Spohr and Habeneck's approach is based on simulating 12EDO in performance (a Neutral system in my revised model). With this approach there is enharmonic correlation between sharps and flats and the diatonic and chromatic semitones have now the same size.

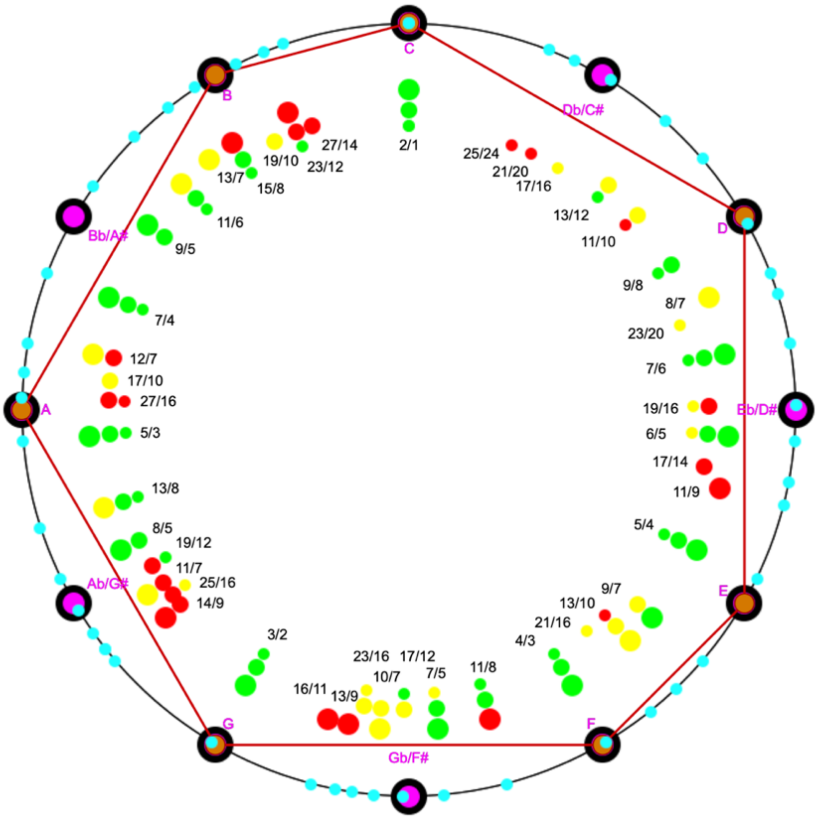

Figure 19. Circular representation of the diatonic scale of 12EDO, against Sabat and Hayward's tuneable intervals and their projections into one octave interval classes.

12EDO and most 'expressive range' EDOs are also compatible with the Schismatic family of temperaments (which includes 12, 53, 41 and 29EDO).[70] Given the results from Podnos and Schackford's analyses and the current prevalence of 12EDO in concert music, I hypothesize that modern intonation standards (at least for melody) would converge with this 'schismatic range' of EDOs.

The reflexions from this chapter and the previous one served as the starting point for the development of the 'EDO Diatonicism' spreadsheet from Global Properties of EDOs. It serves as a database to compare microtonal intervallic deviations between the diatonic scales (and

[70] See 'Schismatic Family', *Xenharmonic Wiki* <https://en.xen.wiki/w/Schismatic_family> [accessed 9 August 2023].

beyond) of a variety of EDOs with several reference intervals (including Sabat and Hayward's tuneable intervals), as well as associating these scales with historical tuning systems, temperaments, and string intonation practices. Other more speculative models beyond the scope of these research are also featured as further potential research avenues.

Part II. Research narrative on my compositional practice

Drawing creative stimulation from my exploratory models discussed in Chapter 2, I developed a portfolio of musical compositions. Commentaries on these compositions are given in each chapter, which in turn are grouped into different sections.

Section 1 corresponds to a set of two instrumental pieces reliant on electronic media:

- *Adam,* a chamber opera for four voices and mixed ensemble (Chapter 3)
- *Dreams from an Old Memory*, for electric piano and electronics (Chapter 4)

Section 2 includes two pieces for different types of string instruments:

- *Odd [s]Paces*, for string quartet (Chapter 6)
- *The Fable of the Pilgrim*, guitar quartet (Chapter 7)

I originally planned that a piece for wind quintet and electric guitar (which will be referred as 'Piece Z') would be written next, followed by a piece for brass quintet and a final piece for sinfonietta. However, this plan had to be adapted due to a mix of time constraints and having deemed 'Piece Z' to be unsuccessful as a concert piece (but appropriate to be adapted for a larger ensemble). Section 3 is then comprised of the following:

- 'Piece Z', an unfinished piece for wind quintet with electric guitar (Chapter 8)
- *Seeking Gnosis,* for mixed ensemble (Chapter 9)
- *Convergence*, for sinfonietta (Chapter 11). This piece includes an adapted/revised version of 'Piece Z' (see subchapter 11.2). It also features a heavier brass section than the standard sinfonietta formation and many 'brass only' passages (see subchapter 11.3.2).

In between commentaries of submitted compositions there are also two reflective chapters discussing which tuning systems I have been more inclined to use halfway through the project (Chapter 5) and the necessity to explore more extreme levels of approximation compromises in the final piece of this project (Chapter 10).

Aside from the main compositional process, there are some important extra-compositional elements which carried an important influence on the research agenda. These included the development of:

- *Equal Tuning Lab* – a computer application built on Max/MSP which, amongst a variety of other features, visually represents the intervals of equal tunings and Sabat and Hayward's tuneable intervals as dots on a circle and lets the user play these tunings. Many of the illustrations used on this submission were built on Equal Tuning Lab. Equal Tuning Lab played a key role on my pitch explorations and aided me cross-checking pitches derived from multiple systems and find 'happy coincidences', in which pitches derived from different systems were so close that I might as well consider them to be equivalent. Some speculative considerations derived from this cross-checking, which might have potential to go beyond this research, have also been annotated in Appendices 5 and 9.
- *Global Properties of EDOs* – a set of spreadsheets designed to give a global view of the properties, differences and resemblances between EDOs and other systems by focusing mostly on cent deviations. This document was developed in parallel to the composition process of *Convergence* and led both to the variety of tuning systems explored in it, but also to the development of Chapters 2.5 and 2.6 of the literature review in the later stages of this research.
- Several Bosanquet-Wilson mappings on the Lumatone, many of which are used as illustrations throughout this submission.

Section 1: Compositions with electric piano

Intrigued by Blackwood's remarks about 5 and 7EDO being at the extremes of diatonicism (in tuning systems generated by cycles of fifths),[71] I focused the two pieces included in this section (*Adam* and *Dreams from an Old Memory*) on these two EDOs either by employing them on a retuned electrical piano, mixing them with other materials, approximating them, or deriving from them other EDOs through numerical formulas. Ideally, this would lead to sonorities which could be perceived as 'somewhat diatonic' through microtonality.[72] Derived from this and the layout of the piano, the other premise shared by both pieces is the simultaneous mixing of EDOs with less than 12 pitches (also referred as 'macrotonal' EDOs)[73] for melodic and harmonic exploration.

The following chapters will focus on the composition process of each piece (Chapter 3 for *Adam*, and Chapter 4 for *Dreams from an Old Memory*) and a reflection on my tendencies towards certain tuning systems around this stage of this research project (Chapter 5).

Chapter 3. *Adam*

This chapter will address the composition process of *Adam*,[74] a chamber opera for four voices and mixed ensemble, written in 2020 and commissioned for the OperaFest Lisbon 2020. Its subchapters will focus predominantly on the choice of the diverse tuning systems for each section of the opera (3.1), approximation techniques and notational systems that were applied in each instrument (3.2) and reflections on their efficacy (3.3).

My main goal in the conception of *Adam* was to explore rounding and approximations into tuneable intervals of a variety of EDOs (as theorized in Chapters 2.3 and 2.4) and finding appropriate notational strategies to test their efficacy in performance. Even though

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[71] Blackwood, p. 199.

[72] The search for these sonorities proved to be a recurring tendency throughout this research.

[73] This nomenclature derives from the fact that the smallest intervals of these tunings are larger than the 12EDO semitone, hence the word 'macrotonal', in contrast with 'microtonal'. Stephen Weigel employs such tunings in *Six Macrotonal Etudes for Electronic Music Media* (2019).

[74] In this submission, the title of opera will be referred in the English translation of its Portuguese title, *Adão*.

compositional choices involved other parameters, my preoccupations were focused on pitch and its notation. After this piece, gradually, I became more consciously considering other parameters (particularly rhythm and form) and how they could conceptually relate to my pitch explorations.

3.1. Choosing EDOs for each scene in the libretto

Given the constraints of the maximum of 12 keys per octave of the piano, and the convenience of a limited number of pitches, I chose to work with macrotonal EDOs, particularly 5, 6, 7, 9, 10 and 11EDO. They would be the starting point from which I would derive intervallic approximations. While improvising with them on bitKlavier and Equal Tuning Lab, it became apparent to me that a varied microtonal quasi-modal/tonal sonority could be achieved with these materials, as their limited number of pitches suggests me to conceptualize them as scales/modes. Not only did I not mind this, but I also embraced it: I am always excited in exploring scales (both old and new) which can be conceptualized and related to each other through their numerical and acoustical properties.

While reading the libretto, each of its sections suggested to me a series of 'moods'. These moods could then serve as an opportunity for the employment of these macrotonal EDOs to serve as a conduit for their musical translation. In my improvisations, it was becoming clear that these EDOs (and combinations of them with other EDOs), to my sensibility, reflected those moods well, for each passage of the libretto. So, associations between the two began being made.

The strongest associations are with the LsLsLsL scale in 11EDO (in which 'L' stands for 'large step', comprising two steps of 11EDO, and 's' for 'small step', comprising of one step). This 11EDO scale in is known as 'orgone[7]',[75] but I informally refer to it as 'tense dorian' because, like the 12EDO LsLLLsL dorian mode, this scale's structure is also symmetrical, and their 'L' and 's' steps, respectively, also correspond to 2 and 1 steps. Most of this scale's intervals, to my perception, retain the qualitative identities of the 12EDO dorian mode (i.e. a minor third remains a minor third) except the fourth (which is almost a 11/8 with

[75] Orgone[7] is a name used within the context of Regular Temperament Theory. See the following pages for more information on this scale: 'Orgone', *Xenharmonic Wiki* <https://en.xen.wiki/w/Orgone> [accessed 9 August 2023] and 'Orgonia', *Xenharmonic Wiki* <https://en.xen.wiki/w/Orgonia> [accessed 9 August 2023]. See Appendix 7 for more contextualization of this scale in my compositional practice.

545¢) and the fifth (approximately 655¢). I associate this scale with the feelings of longing and sadness, which are particularly relevant for the first scene of the opera (see Table 1).

While improvising, when switching from this 'tense dorian' scale to its 12EDO rounding, which also happens to be the LsLLLsL dorian mode,[76] I feel a sense of hope. When mixing the 12EDO dorian mode with 5EDO, I associate the sonority of this compound scale with innocence, beauty, and wonder, which I find appropriate for the scene when Eve is born. In the end, the following structure was reached:

Section	Emotion/Dramatical Context	Initial Microtonal Resources
Overture	Eeriness	Rounding between 11 & 12EDO
1st Act Scene 1	Adam's hurt and longing for Lilith after she left him	Hybrid chromatic sonority derived from 11 and 12EDO
1st Act Scene 2	The confrontation between Lilith's resolve and the Angel's judgment	Whole tone/quasi-diatonic sonority derived from mixing 6 and 7EDO
Interlude	Divine Mystery	Harmonic series and 7EDO
2nd Act Scene 3 1st Half	A sense of wonder, naïve curiosity and emergent passion that grows within Eve and Adam	Pentatonic/diatonic/harmonic sonority derived by mixing 5EDO, approximations into tuneable intervals, the 12NPO dorian mode,[77] and sets of the harmonic series
2nd Act Scene 3 2nd Half	The progressive dismantlement of Adam and Eve's relationship	Chromatic sonority which retains intervallic properties from the previous section, derived from the mixture of 10EDO[78] with two notes from 9EDO[79]
Ending	Adam's decisions to leave Eve and Eve's mental breakdown	A mixture of materials from the previous sections

Table 1. Structuring of Adam.

[76] See Figure 105 in Appendix 7 for a visual representation of it.

[77] This scale can be conceptualized as a rounding of 7EDO into 12NPO. See Figure 22 in Chapter 3.2.

[78] 10EDO is a subdivision of 5EDO and therefore includes it, similarly to 24EDO in relation to 12EDO.

[79] See Appendix 4 and Figures 23 and 24 in Chapter 3.2.

3.2. Technical Insights

The instrumentation of this opera may lead to several key considerations regarding intonation to be taken.

The accordion is built to play in a fixed intonation converging towards 12EDO, although variation in the tuning of accordions may skew the tuning of particular performances. Nonetheless, the accordion part has been conceived through a 12EDO framework and henceforth will be referred assuming that it is in fact tuned to this tuning. The electric piano, however, may be tuned to any tuning system through software, when connected to a computer. The intonation of both these instruments is fixed prior to performance (even though the electric piano may be retuned, customized mappings of tunings systems need to be programmed beforehand).

By contrast, the intonation of voices and fretless strings is largely not pre-determined, but flexible. Woodwinds and their standard fingerings are designed to conform towards 12EDO, but this intonation might be altered through other factors, such as breath control and alternative fingerings. Also, intonational practices may vary from instrument to instrument and from performer to performer, according to their received intonation practices and the individual characteristics of their instruments. For example, the same woodwind instruments from different models may react differently to the same microtonal fingerings, which means that fingerings need to be tested for each instrument.

The chosen macrotonal EDOs are very likely to fall outside of current practices of performers of non-fixed pitch Western concert instruments. Therefore, asking professional performers of these instruments (not necessarily experienced with microtonality) to play in these tunings (similarly to doing the same for 12EDO) in a 17-minute opera might be too much of a demanding task for a performance with limited rehearsal time. Thus, subdivisions of 12NPO could instead be a more pragmatic approach to round intervals of these tuning systems (see Figures 20 to 25), while also considering live intonation.

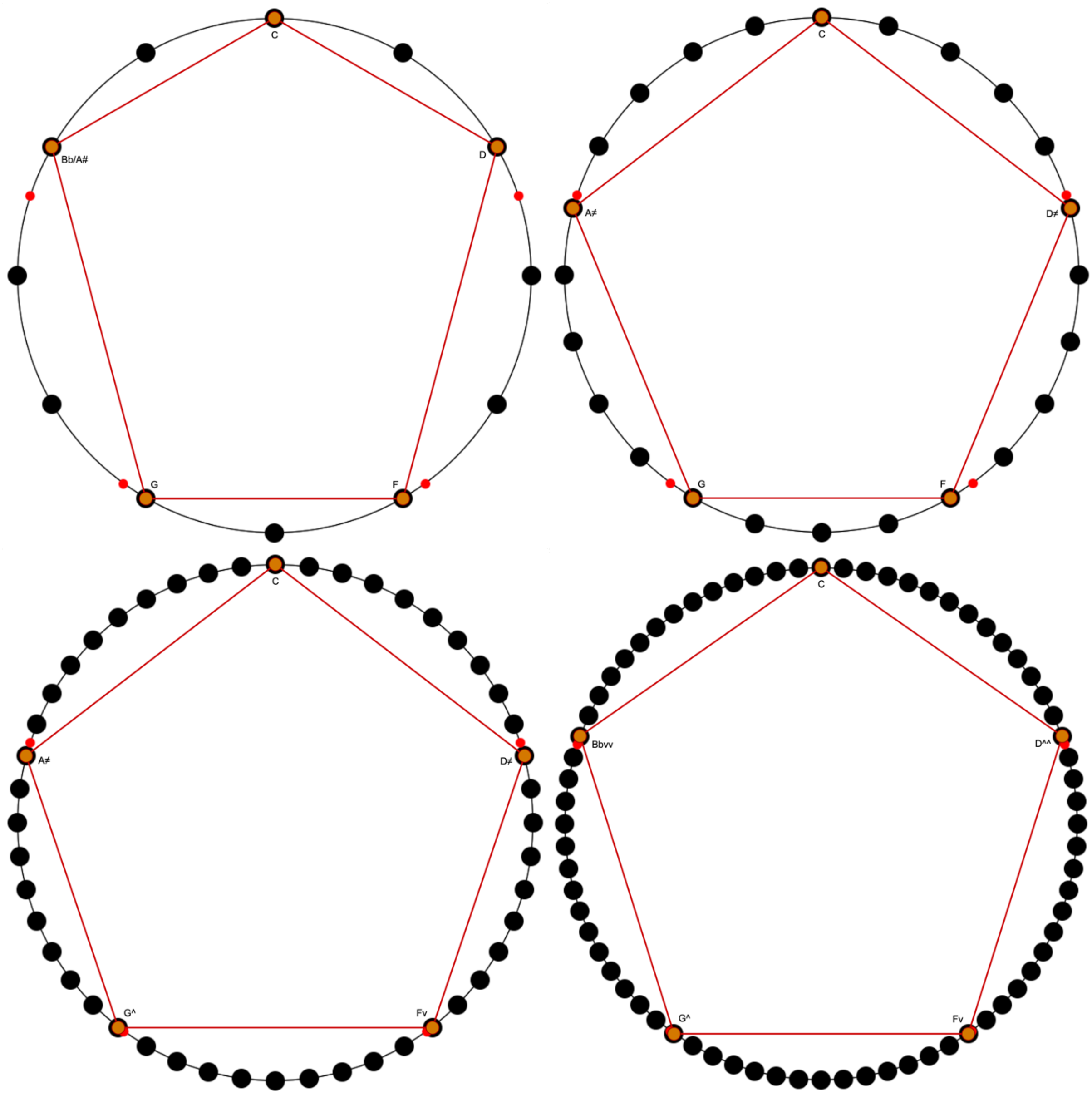

Figure 20. 5EDO rounded to 12, 24, 48 and 72EDO.

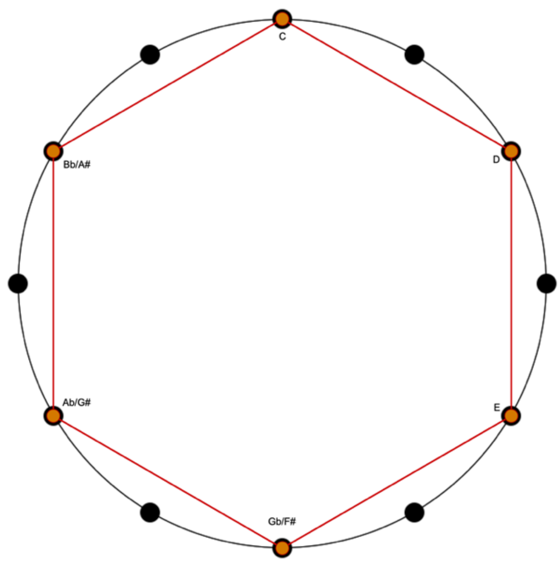

Figure 21. 6EDO rounded to 12EDO. As can be observed, 6EDO is contained within 12EDO.

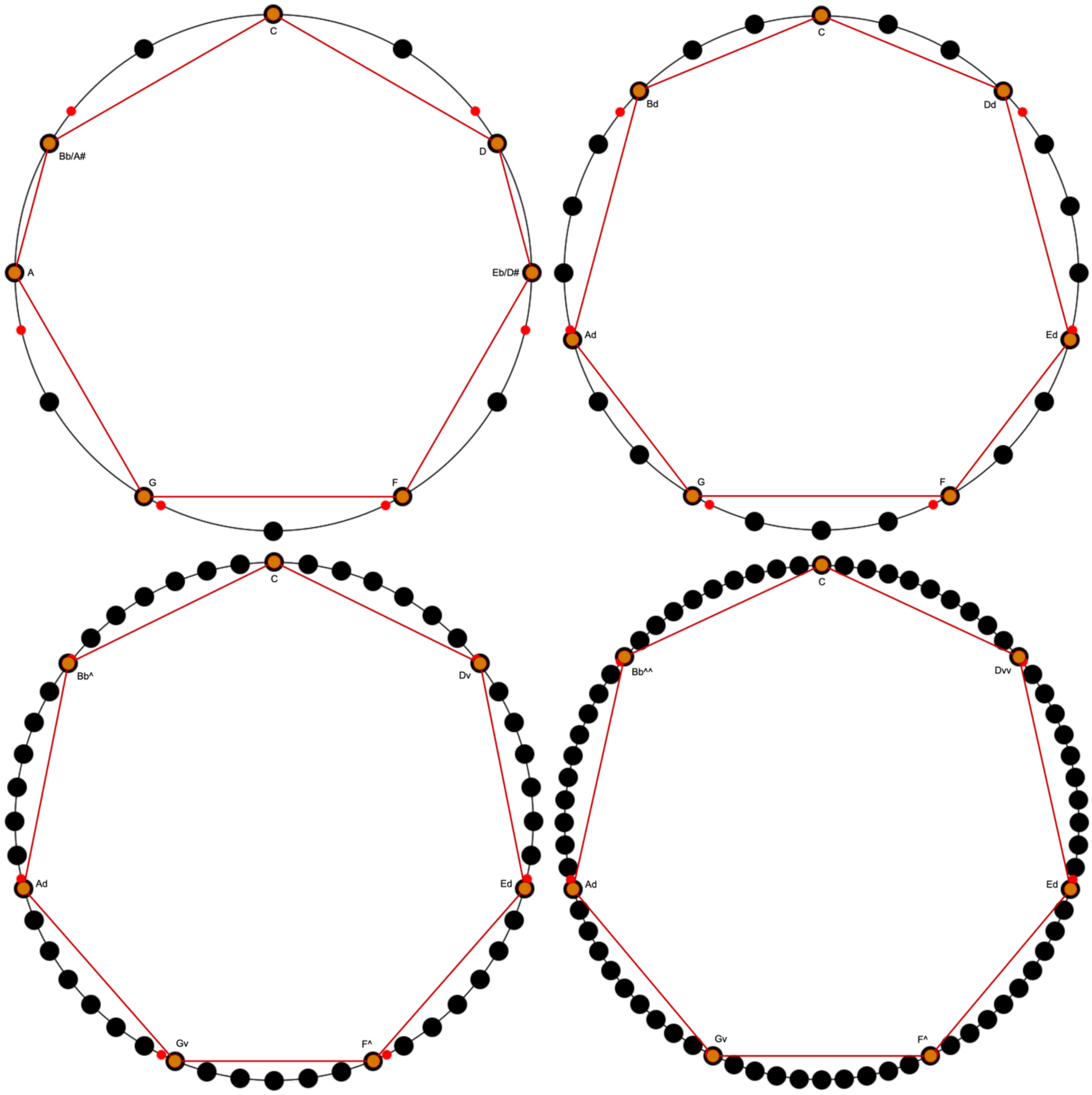

Figure 22. 7EDO rounded to 12, 24, 48 and 72EDO.

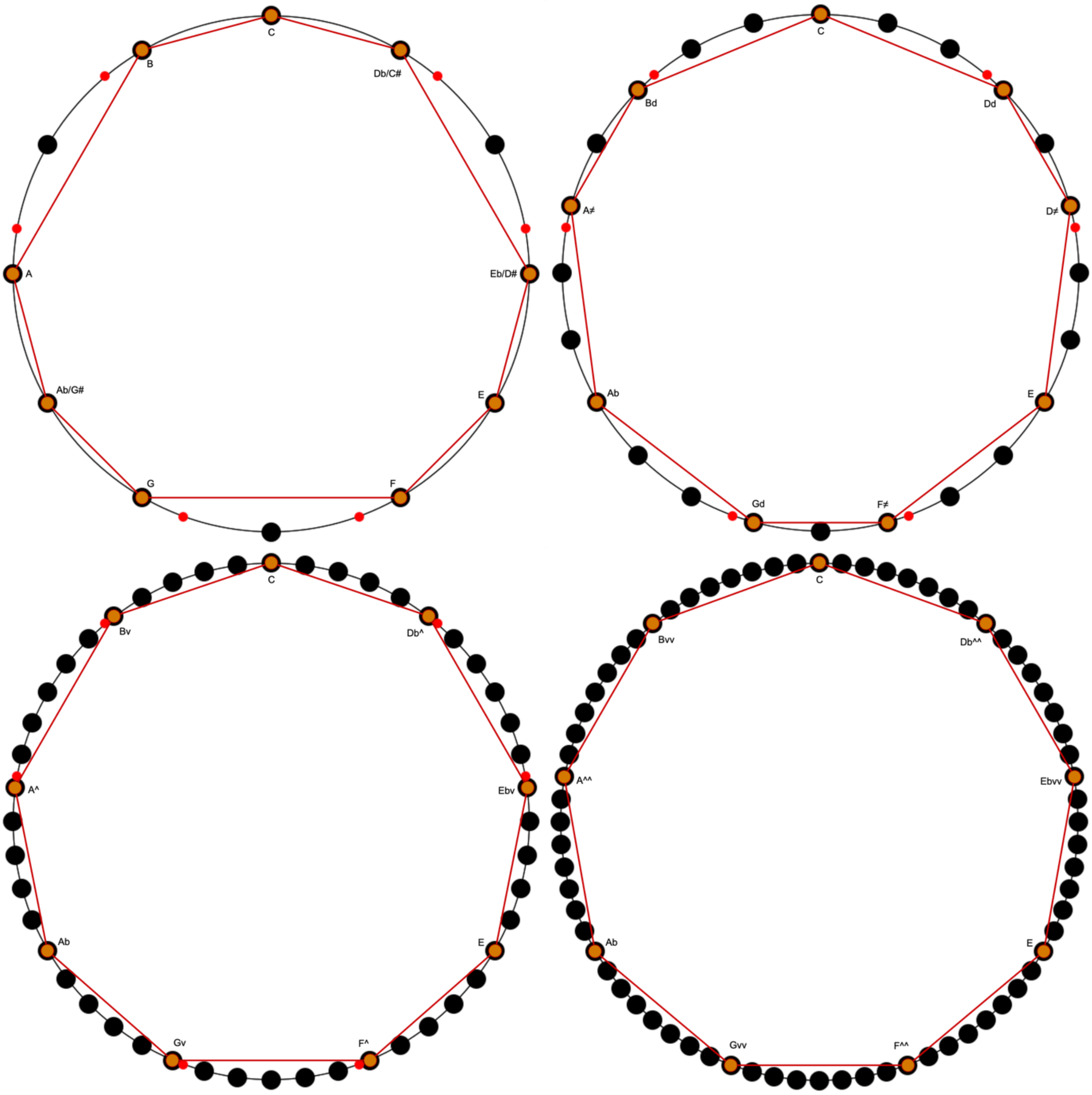

Figure 23. 9EDO rounded to 12, 24, 48 and 72EDO. As can be observed, 9EDO is contained within 72EDO.

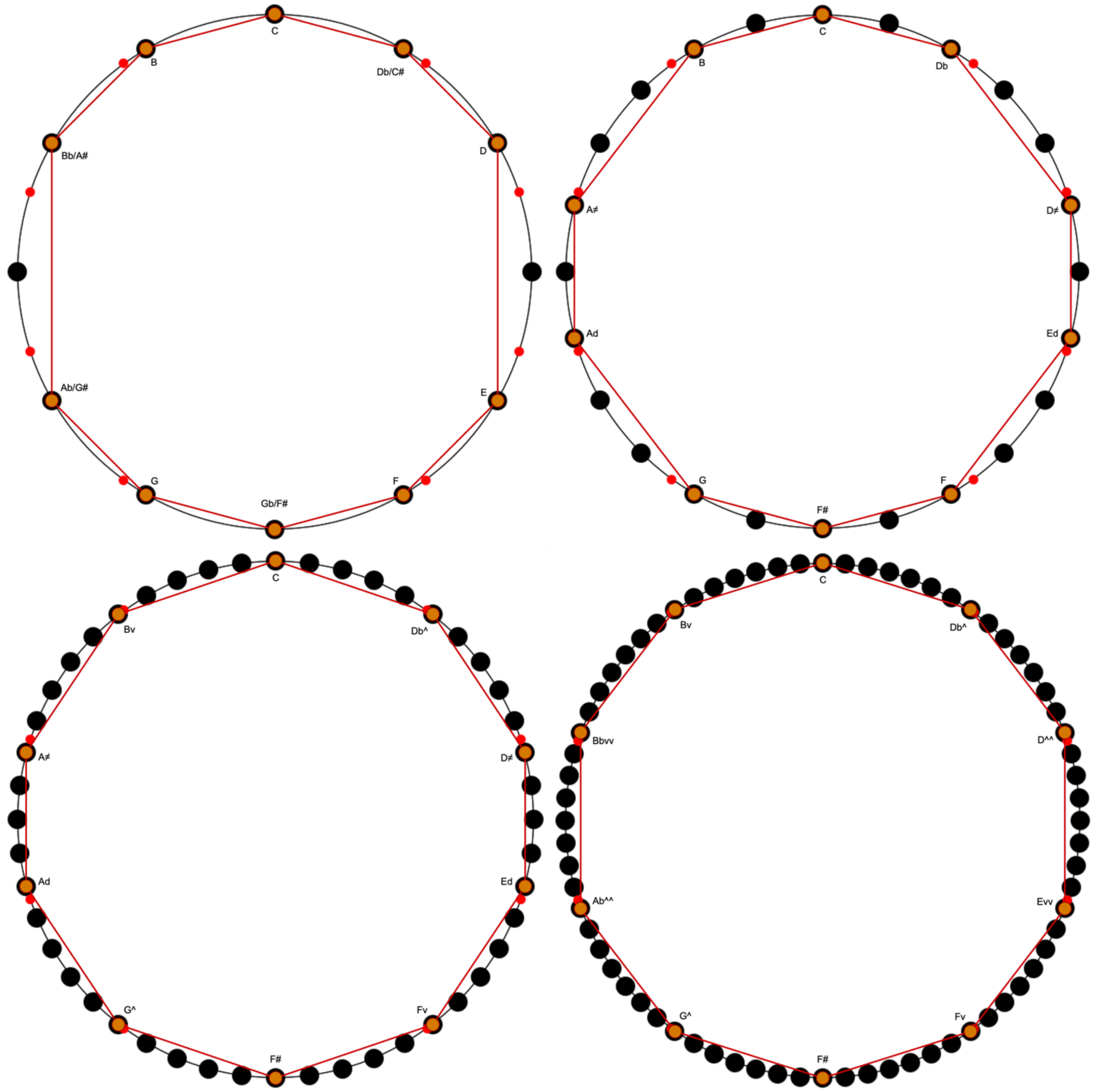

Figure 24. 10EDO rounded to 12, 24, 48 and 72EDO.

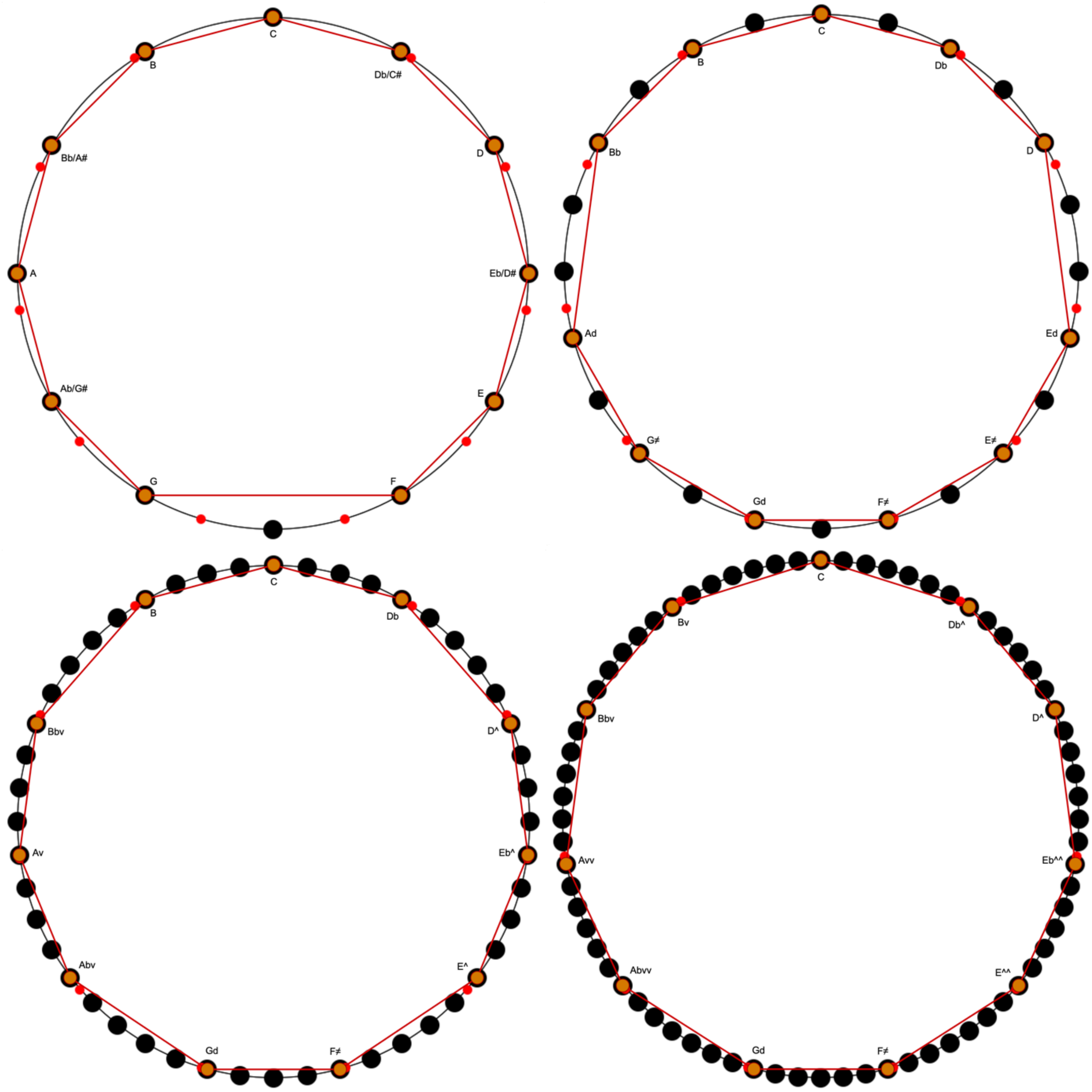

Figure 25. 11EDO rounded to 12, 24, 48 and 72EDO.

When I rounded these macrotonal EDOs into 48EDO and played them myself, the results tended to 'fool my ear' much better than 24 and 12EDO: 12EDO being the more distorted, but sounding familiar; 24EDO somewhat less distorted, but sounding more 'xenharmonic' to me.[80] 72EDO had a similar result to 48EDO, but roundings sounded more 'harmonic' to me in comparison, given that 72EDO approximates well many Just Intonation ratios (which is also a distortion of the quality of those EDOs), whereas I found that 48EDO better preserved the

[80] By this is meant that the intervallic qualities of EDOs rounded into 24EDO sound harsher to me than those into 12EDO and, at times, of the unrounded EDOs themselves. I do not consider this harshness to be undesirable by principle, but at this stage I did not find it particularly compelling. However, my views of 24EDO/NPO as a recipient for microtonal approximations changed through the process of writing 'Piece Z' and *Convergence*.

sensation of inharmonicity generally present in alternative EDOs. In varying degrees, all these approximations retain some kind of resemblance to the EDOs from which they are derived.

By being aware of these points, I started to wonder if allocating different subdivisions of 12EDO (each with its set of accidentals, trying to have as much similarity as possible between these systems)[81] to each instrument could be a viable strategy to approximate EDOs more idiomatically for performers (being aware of intonational tendencies, and their advantages and limitations). This would lead, in theory, to different intonations of the same materials between instruments. My hypothesis, however, was that, when employing them simultaneously, performers (according to what their instruments allow) would intonate in relation to each other. The result would not equate to 'MIDI intonation', perhaps not even to what I would imagine when writing the piece, but the live intonation would sound 'effective' (rather than 'defective') to me. It was at this time that I started using the Dorico score editor, due to its ease of building notational systems for different tuning systems, employing them simultaneously on different instruments, and having a reliable playback.

These considerations offer a rich framework, which may be conceptualized into an intonational spectrum from 12EDO (accordion) to any potential tuning system (electric piano), while in between (strings, woodwinds and voices) lies a grey area of intonational possibilities (see Figure 26).

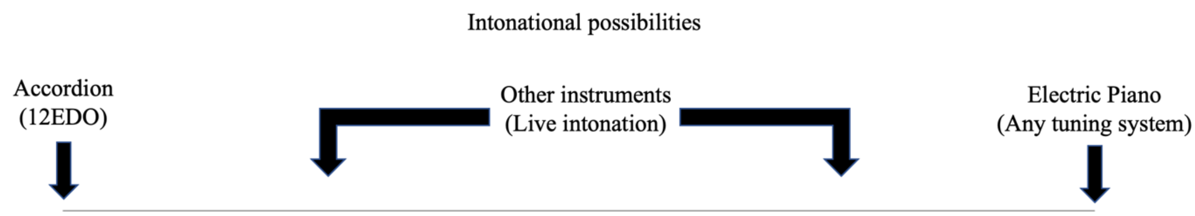

Figure 26. My model for the representation of the intonational variety of Adam's instrumentation in a spectrum.

This spectrum and the variety of materials that may be derived just from these EDOs and their intervallic approximations suggested to me that they would provide a rich basis for compositional explorations related to Darreg's xenharmonicity and Lopez-Hanshaw's

[81] See Appendix 3 to see how these systems are notated in this research.

dichotomy,[82] two concepts which could be useful guidelines for the musical/dramatical differentiation between each section of this opera, alongside the mood associations.

The employment and approximation of macrotonal EDOs throughout the score could then conform to the following guidelines:

1. The electric piano performs the macrotonal EDOs and scales derived from them as they are conceptualized and serves as an intonational reference for non-fixed pitch instruments.
2. The accordion performs rounded versions into 12EDO of these EDOs and scales and serves as an intonational reference for non-fixed pitch instruments.
3. All the other instruments and voices employ approximations of these EDOs and scales. Live intonation also needs to be considered.

It was at this time that I started sketching the Overture (bar 1) to illustrate, purposedly, the two extremes of Figure 26 by employing melodically 11EDO on the electric piano and its rounding to 12EDO on the accordion (or vice-versa). I used dual notational system in the electric piano, in which one system would notate which keys the pianist would play (serving as a tablature), and the other would notate the pitches which were actually being played. Given the variety of tuning systems which would be used throughout this opera in the electric piano, two viable approaches could be taken for the notation of this second system:

1. A system which precisely notates each pitch. This would likely require either a considerable amount of extra unfamiliar accidentals (e.g. The Sagittal Notation), add cent deviations to every single note, or employ Circle of Fifths Notation and making the score potentially not look microtonal.
2. A system which approximates each pitch to its closest one within a chosen grid (12, 24, 48, 72EDO, etc.). This would only require a few extra accidentals, at the expense of notational precision. However, this notation could be complemented with written descriptions of which tuning systems would be employed at each time.

For pragmatism, the latter approach was chosen, and 48NPO, from prior experience, seemed an appropriate system towards this aim. With this strategy, the model in Figure 1 (Chapter 2.2)

[82] See Chapter 1, last paragraph starting on page 18.

would be expanded by adding quarter and eighth tone (arrowed) accidentals to the Western Chromatic Notation system, similarly to spectral practice,[83] despite 50¢ quarter tones and 25¢ eighth tones (from 48EDO) never actually being used on the electric piano, but intervals close to them.[84] In this instrument, rounding is then only used for notational purposes, and not for intervallic approximations. In the accordion, rounding affects both notation and interval sizes.

The 48NPO notational system used in the electric piano suggested a possible path for the compositional approach for the remaining instruments (including voices). However, some considerations need to be considered.

In past compositions, I have used 48NPO to approximate and/or notate sets from the harmonic series and equal tunings (particularly in slow passages) and noticed that it has been somewhat effective in woodwinds (flutes and clarinets), whereas in the strings the results have been mixed. Regarding strings, when approximating the harmonic series (which is taught in many current classical music curriculums), 48NPO has worked satisfactorily, but when approximating equal tunings unrelated to 12EDO (not part of classical music curriculums to my knowledge), it was more problematic. Retrospectively, these past experiences of writing for strings seem to be in line with models proposing that string intonation tends to conform towards Just Intonation harmonically and Pythagorean Tuning melodically through embodied practice[85] whereas, in woodwind fingering driven practices, a different paradigm is in place. By using 48NPO to notate the harmonic series, Just Intonation is implied, but not when it is used to round alternative equal tunings. By personal experience, whereas 48NPO seemed to be a more idiomatic approach for woodwind writing, an alternative solution should be found for strings.

After becoming aware of Sabat and Hayward's tuneable intervals,[86] programming them into Equal Tuning Lab*,* speculating about possible Interval Class Ranks[87] and even creating a series of mathematical formulas to represent Just Intonation ratios as 'reverse-temperaments' of equal

[83] See Gann, pp. 214-17.

[84] See Figures 20 to 25 to compare the macrotonal EDOs used in this opera against 48EDO/NPO.

[85] Benjamin, 'Thick Relationality', pp. 73-117 (pp. 94-95). Also, see the last two pictures of Figure 1 from Chapter 2.2.

[86] See Chapter 2.4.

[87] See Appendix 5.

tunings,[88] I started visually comparing this list of intervals against a variety of equal tunings that I had been using and noticed that 'happy coincidences' were relatively common (intervals of equal tunings being within the proximity of tuneable intervals, to variable distances, making them 'quasi tuneable-intervals'). Some of these coincidences involved 'easy' tuneable intervals (which, for the multimodal approach that I was devising, were particularly convenient). However, at other times, they involved 'more difficult' and 'hard' tuneable intervals which could be classified as quarter tones. Even though these 'quarter tone quasi-tuneable intervals' would not always be easy to tune by ear using Sabat and Hayward's methodology, quarter tones have been vastly employed since the 20th century and have as conceptual reference the chromatic scale semitones. Therefore, they could be intoned by string players by relying on those references and mechanically dividing them in two.[89]

Alongside this, having come across articles by composers Thomas Nicholson and Marc Sabat[90] and Robert Lopez-Hanshaw,[91] I noticed that 72NPO (as I classify it) tends to be referenced as a satisfactory system to approximate Just Intonation ratios. If this correlation between Just Intonation and 72NPO was to be as satisfactorily as it seemed, it could then be a powerful tool for string writing (and potentially addressing my previous problems with string instruments and 48NPO). On Equal Tuning Lab, I compared Sabat and Hayward's tuneable intervals against 48EDO and 72EDO.

[88] See Appendix 9.

[89] However, this consideration was later problematized in consequence of the rehearsal process of a later piece for string quartet, *Odd [s]Paces.* See Chapter 6.3.1.

[90] Thomas Nicholson and Marc Sabat, 'Fundamental Principles of Just Intonation and Microtonal Composition', 2018. pp. 29-30. <https://marsbat.space/pdfs/JI.pdf> [accessed 11 March 2021].

[91] Robert Lopez-Hanshaw, 'Getting Your Hands Dirty (Performing Microtonal Choral Music, Part 2)', *NEWMUSICBOX*, 2020. (para. 48 of 66). <https://nmbx.newmusicusa.org/getting-your-hands-dirty-performing-microtonal-choral-music-part-2/> [accessed 17 March 2021].

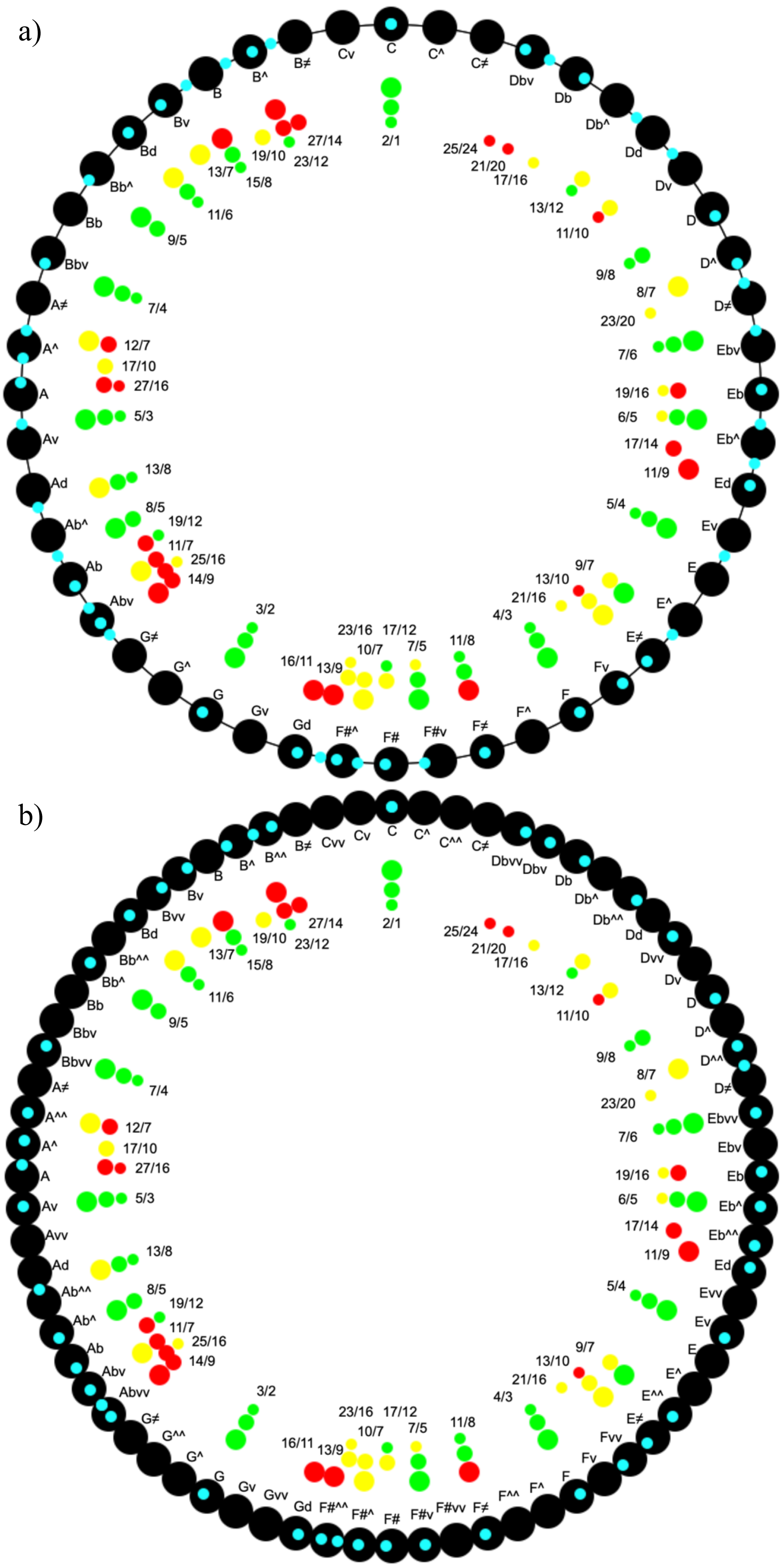

Figure 27. Sabat and Hayward's tuneable intervals against 48EDO (a) and 72EDO (b).[92]

[92] See Figure 4 on Chapter 2.4 and the paragraph after it (starting on page 30) for a description of how tuneable intervals are represented on Equal Tuning Lab.

My suspicions were confirmed: most tuneable intervals, visually, do align quite closely to 72EDO and not nearly as much with 48EDO. Therefore, a 72NPO notational system would indeed seem appropriate to approximate such intervals. These results would then present one possible reason for why 48NPO, as a notational approach and conceptual framework, may not have been effective to approximate equal tunings on strings in past pieces, as 48EDO intervals tend to deviate from tuneable intervals. Instead, 72NPO could restructure and reconceptualize these equal tunings so that, in performance, and especially in more harmonic based textures, some of their intervals converge with tuneable intervals. Also, by following a methodology to tune these intervals like Sabat and Hayward's, this approach could make them easier to perform (i.e. being more idiomatic for string performers).

These considerations suggested a compositional approach in which 48NPO roundings would be applied in the woodwinds and, in the strings, 72NPO for approximations into tuneable intervals (whenever appropriate). However, as precaution, mostly 'easy' and 'more difficult' tuneable intervals were employed in the strings. Even though I was excited about this hypothesis, it had not yet been tested, and I wanted to make sure that this piece would be performable. Relying too much on these speculations for a piece of this size would be a substantial risk. With rounding, however, I was more experienced.

Given that the accordion can only intonate in 12EDO, and inspired by Haas's 'renderings' of microtonal precision,[93] alongside 48 and 72NPO, approximations into 12NPO were also used in both the woodwinds and strings to introduce more intonational variety in these instruments, creating an 'on/off switch button' between 12NPO (non-xenharmonic) and 48/72NPO (more xenharmonic) with which I could play. The clarinet and cello parts in bars 162 to 166 illustrate this.[94] Roundings into 24NPO, however, were not used for personal taste.[95]

Another advantage of this approach would be to incorporate the 12NPO received intonational practices of each performer and establishing them as their intonational baseline, from which they would regularly deviate (into 48/72NPO), but (hopefully) firmly rely. I found this to be a very important feature to include in this piece and in future ones.

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[93] Haas, 'Mikrotonalität Und Spektrale Musik Seit 1980', p. 126.

[94] See Appendix 4 to compare how each pitch of every equal tuning and microtonal scale is rounded to 12, 48 and 72NPO in *Adam*.

[95] Although in *Convergence*, a piece with no electronic resources, 24NPO was the main strategy. This shows how my taste and approaches have since changed.

This partly justifies my choice to approximate all tunings and scales solely into 12NPO for the singers, who would still have to get accustomed to an entirely microtonal musical background while also having the added variable of acting their characters. However, I predicted that their intonation would be influenced by this same microtonal instrumental background.

Regarding notation, for consistency, given that a notational system was already in place for 48NPO for the electric piano and the woodwinds (even if for different purposes), an expansion of its accidental system seemed appropriate for 72NPO. Therefore, while maintaining semitone and quarter tone accidentals, the single arrowed eighth tone accidentals of 48NPO were reused for twelfth tones and double arrowed accidentals were introduced for sixth tones.[96] The individual string parts, when appropriate, also include written ratios, names of the target 'natural intervals' (such as '5/4 Natural Major Third'), information indicating when the harmonic series is being employed, and excerpts of other parts (for intonational references).[97] Treating the twelfth and sixth tone pitches of 72NPO as equivalent to the eighth tone ones of 48NPO was also a convenient choice, since the former accidentals tend to approximate the same pitches of the latter with a slightly higher level of resolution.[98]

3.3. Conclusions and further questions

3.3.1. Transpositions

When writing within a modal/tonal idiom for concert instruments, by employing multiple macrotonal EDOs and NPOs anchored to the same 12NPO pitch, if the composer wishes to remain consistent to these structures, transpositions seem to be problematic. There might be a great variety of internal microtonal and intonational options, but the anchor notes (the harmonic references) are fixed. Transpositions may lead to chords, scales and melodies whose fundamentals do not correspond to 12NPO (i.e. received intonational practices). This might lead to performance issues. However, in a non-tonal idiom, this may not be as problematic. This reasoning may justify the very limited number of transpositions used in this opera, and,

[96] See the performance notes of *Adam* for a more detailed outlook of this notational system.

[97] See Figures 107 and 108 in Appendix 10.

[98] See Figures 20 to 25 and Appendix 4 to compare roundings of the macrotonal EDOs and scales into 48 and 72NPO.

when they do happen, I retune an entire tuning system so that the new fundamental is anchored to 12EDO (e.g. from bar 22 onwards, 11EDO anchored to C is re-anchored to D).[99]

3.3.2. Tuneable intervals in the strings and 48NPO in the Woodwinds

The overall employment of approximations of intervals of equal tunings (particularly 5EDO) into some of the easiest tuneable intervals (according to Sabat and Hayward) in the strings through a 72NPO notation, whilst considering a degree of acceptable intonational variation, was, for me, satisfactory. Some successful examples happened in bars 205-220 (10’54’’-11’30’’ in the live recording, 10’33’’-11’08’’ in the computer mock-up) and 228-239 (11’53’’-12-23’’ and 11’30’’-11’57’’, similarly), particularly when the pitch Bbvv is used (which, above a G and a C respectively corresponds to the interval classes 7/6 and 7/4, both Rank A interval classes).[100] This suggests that further explorations including approximations into other ratios could be viable and thus relevant to explore in my string quartet. By contrast, melodically, variations between 12 and 72NPO roundings led to more mixed results, such as in the cello part in bars 162-166 (9’35’’-9’44’ in the recording and 9’15’’-9’25’’ in the mock-up), where the sixth tonal differentiation between D and D^^ (approximations of the interval 1\5EDO over C into 12 and 72NPO, respectively) was intonated closer to a twelfth tone (D and D^) or a unison (D and D). Perhaps, if the 1\5EDO interval was instead approximated into 12 and 24NPO, leading to D and D≠, then, in terms of notation, the desired melodic contour could be clearer to read and be conceptualized by the performer.

The employment of roundings into 48NPO on the clarinet lead also to mixed results, in contrast with the saxophone part, which was, for me, successful. Relevant examples would be the motive which reappears multiple times throughout the opera in the clarinet part in bars 113-123 (7’16’’-7’55’’ in the recording and 6’55’’-7’38’’ in the mock-up – arguably the most successful in this performance), 256 (13’03’’-13’17’’ and 12’35’’-12’47’’, similarly – which, apart from the pitch played instead of the F^, seemed to be successful particularly in the longer notes) and 310-313 (14’55’’-15’05’’ and 14’19’’-14’27’’ – which is arguably intonated in 12NPO). Another example on the clarinet, similar to the previous 12/72NPO rounding

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[99] This was a struggle that I constantly faced in this doctoral project and may be one of the greatest weaknesses of my compositional practice.

[100] See Figure 89 in Appendix 5 for a visual representation and Appendix 9 for mathematical considerations related to 7/4 being used to approximate 4\5EDO.

variation on the cello, is in bars 139-140 (8’30’’-8’35’’ in the recording, 8’13’’-8’17’’ in the mock-up) in which the 12 and 48NPO rounding variation of the intervals 2\5EDO (F and Fv, respectively) and 3\5EDO (G and G^, similarly), resulted more on timbral variation than in pitch variation (with the Fv being instead played as a G). This theme is reiterated and developed in the clarinet multiple times, such as in bars 205-206 (10’56’’-11’00’’ in the recording and 10’36’’-10’40’’ in the mock-up) which, in this case, the only microtonal variation was performed in the opposite direction. By contrast, in the saxophone part, in bars 162-163 (9’35’’-9’38’’ and 9’15’’-9’19’’, similarly), the aimed eighth tone pitch variation for this motive was achieved.

These performing inconsistencies have a variety of causes. The woodwind parts did not include any alternative fingerings. Instead, I had planned to work directly with the performers to find such fingerings. Also, the scheduled rehearsals were few and rehearsal time limited. However, it needs to be acknowledged that the ensemble which premiered this opera is one of the professional Portuguese ensembles which most regularly performs contemporary music. These less successful examples may then point to a compositional error of judgement on my part. Therefore, further explorations on the future wind ensemble piece should be made by including, perhaps, alternative approximation strategies.[101]

3.3.3. Using 12NPO in the voices against a microtonal instrumental section

The choice to round equal tunings into 12NPO to write the singer’s parts, while expecting that their intonation would be adjusted in performance to the microtonal instrumental section, was successful since the first rehearsal. Aided by the quasi-modal language of the opera, the singers intuitively adjusted their intonation to the ensemble. However, in the passage in bars 241-247 (12’24’’-12’39’’ in the recording and 11’57’’-12’12’’ in the mock-up), the 12EDO mock-up rendition of the voices may sound ‘out of tune’ to the instrumental part (or vice versa), which is polarized towards a pitch region below the 12EDO Bb and above the A (an A≠ and Bbv in the piano and Bbvv in the strings) present in the hybrid 9/10EDO scale being used and approximated in the instrumental section.[102] However, in the live performance, the singers’ intonation is seemingly adjusted to this ‘low Bb pitch region’. The successfulness of this

[101] See Chapter 8 to see how I explored alternative approximation techniques in ‘Piece Z’.

[102] See Appendix 4 for the notation of this scale.

section, through various layers of notational rendition, may imply that my model from Figure 1 may be more elastic than how I originally conceptualized it.

3.3.4. 'The Benjamin Test': A notational framework[103]

Violinist Mira Benjamin suggests that the notational model used in the strings' individual parts is the most appropriate approach overall for string writing.[104] This consists of:

- a quantitative reference of measurement (i.e. a notation system with a set of microtonal accidentals),[105]
- descriptions of the sonority of microtonal intervals[106] and
- acoustical references to tune intervals relationally (i.e. telling which pitches to tune to).[107]

This could be a useful model to test the efficacy of microtonal notational systems based on its three premises.

(Having finished composing and workshopping all compositions, I found this model to be somewhat irrelevant for writing for instruments with less relational intonational practices, such as woodwinds. Nonetheless, I consider this model to be a valuable basis to then derive microtonal notational approaches with which to compose according to the relevant necessities of each piece.)

3.3.5. The dual notation system in the electric piano part

While the electric piano part was musically successful, the amount of information included in it (the dual notation system) may have been excessive but did not seem to be disruptive for performance.

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[103] This model was coined as such by James Albany Hoyle, who conducted *Seeking Gnosis* and *Convergence*.

[104] Mira Benjamin, conversation with Pedro Finisterra, 13 September 2020. Also see Figures 107 and 108 in Appendix 10.

[105] This corresponds to notational model 3.a at the end of Chapter 2.3.

[106] This corresponds to notational model 6 in Chapter 2.3.

[107] This is one component behind notational model 1 in Chapter 2.3.

Chapter 4. *Dreams from an Old Memory*

This chapter will address the composition process of *Dreams from an Old Memory* for electric piano and electronics. Subsequent subchapters will mostly comment on the compositional process (4.1), and notational issues (4.2).

4.1. Contextualization within the research and early sketching process

I started writing *Dreams from an Old Memory* before I was commissioned to write *Adam*. After completing *Adam* I then returned to work on it. Because of this, many of its compositional choices ended up being a reaction to the ideas and considerations made while composing *Adam*.

Originally, this piece was planned to be named *O Sentido das Palavras*[108] and its form, musical gestures, rhythms, density and register variation would be based on a spectral and numerical analysis of this title. Another idea, based on Barbour's tendency to relate some EDOs to each other within a Fibonacci sequence[109] (which I found intriguing and relates to my interest in exploring numeric relations in my music), was to take the number of pitches of 5, 7 and 12EDO, and through them derive a Fibonacci sequence whose integers would serve as suggestions for a variety of parameters, including tuning (which would then lead also to 19, 31, 50EDO, etc.).

After sketching what would become the middle section of this piece (bars 35 to 114), by mixing 7EDO with its 12EDO rounding on the electric piano (see Figure 28) and mostly 5EDO on the electronics, I then understood that my pre-determined compositional strategies were too restrictive for the somewhat lyrical musical excerpt that I had written. However, the mixture of 5, 7 and 12EDO was melodically and harmonically pleasing to me.

[108] This title may be translated as 'the way of words', or 'the meaning of words'.

[109] 'Observe that 31 logically follows 19 in the Fibonacci series: 5, 7, 12, 19, 31, 50, 81, . . . ' Barbour, p. 117.

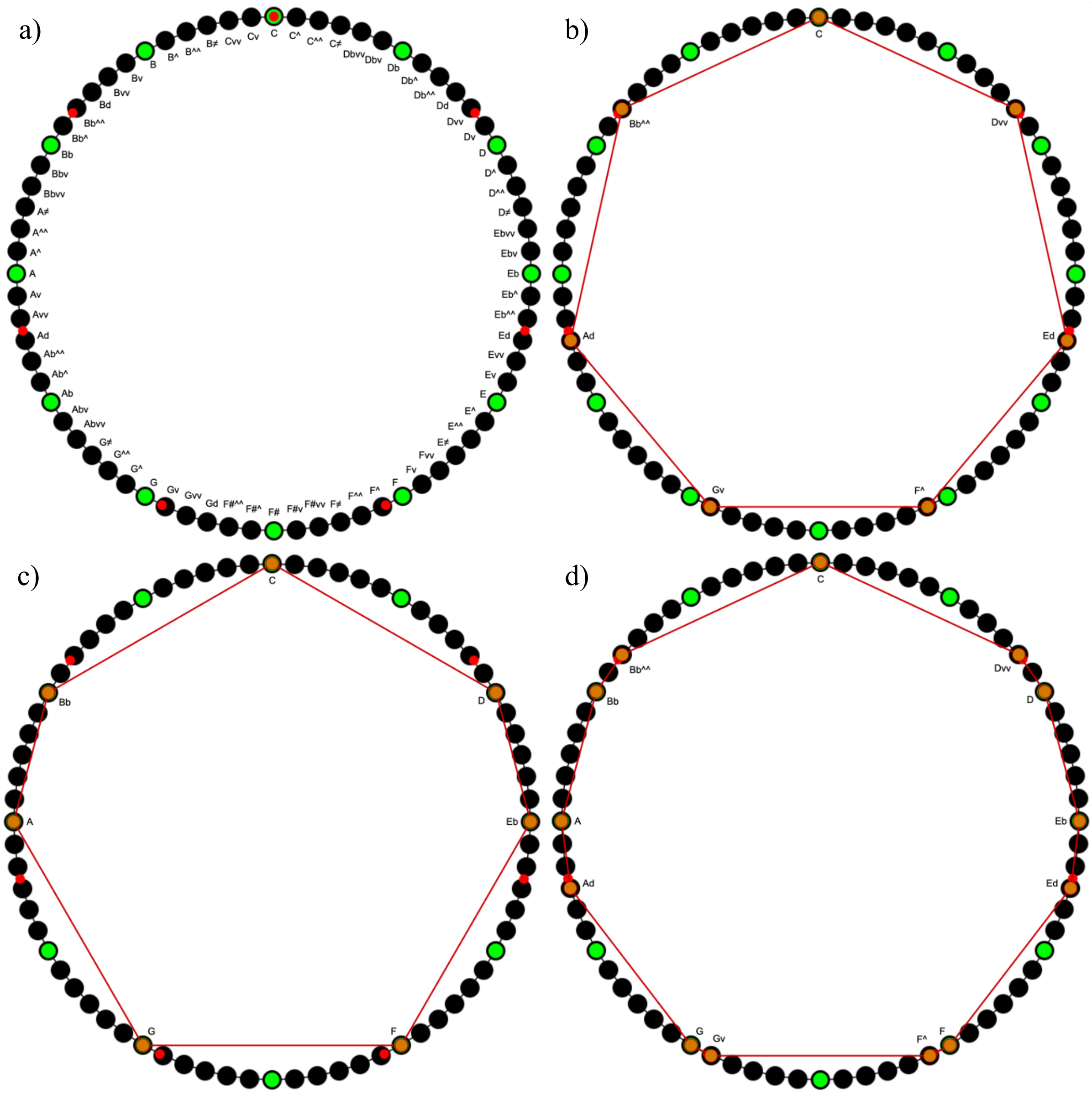


Figure 28. The creation process of the '7EDO+7EDO-rounded-to-12EDO scale', notated in 72NPO. a) 7EDO (red dots), 12EDO (green dots) and 72NPO (black dots and notation) are represented. b) The 72NPO notes which most closely notate the pitches of 7EDO are selected. c) The same process is repeated for the pitches of 12EDO which round the pitches of 7EDO. d) The created scale is notated by mixing the scales produced in b) and c).

After having paused the writing of this piece to work on *Adam*, I returned to the initial idea concerning the Fibonacci sequence. I was now also influenced by musicologist Joseph Yasser's proposal of 7EDO being a desirable EDO to write pentatonic music, 12EDO for diatonic music, and 19EDO for 12-tone chromatic music.[110] These EDOs happen to relate numerically to the numbers 5, 7, 12 and 19 of the Fibonacci sequence that I was already planning to use. I then experimented with building and rounding a scale comprising 5 and 7EDO (5EDO mapped on

[110] Barbour, pp. 115-16.

the black keys of the piano and 7EDO on the white keys) into 12, 19, 31 and 50EDO (see Figures 29 to 33).[111] This approach was also influenced by Darreg's considerations about how, when a piece of music which was written in one EDO is played in another, its musical qualities (sometimes drastically) change.[112] This change of quality was something that I wanted to explore, as a consequence of the intonational variety that would result from this approach.

[111] Also see the performance notes of *Dreams from an Old Memory* for the notation of the different roundings of this '5-7EDO scale' (which technically is a subset of 35EDO) in 72NPO.

[112] Gann, p. 186.

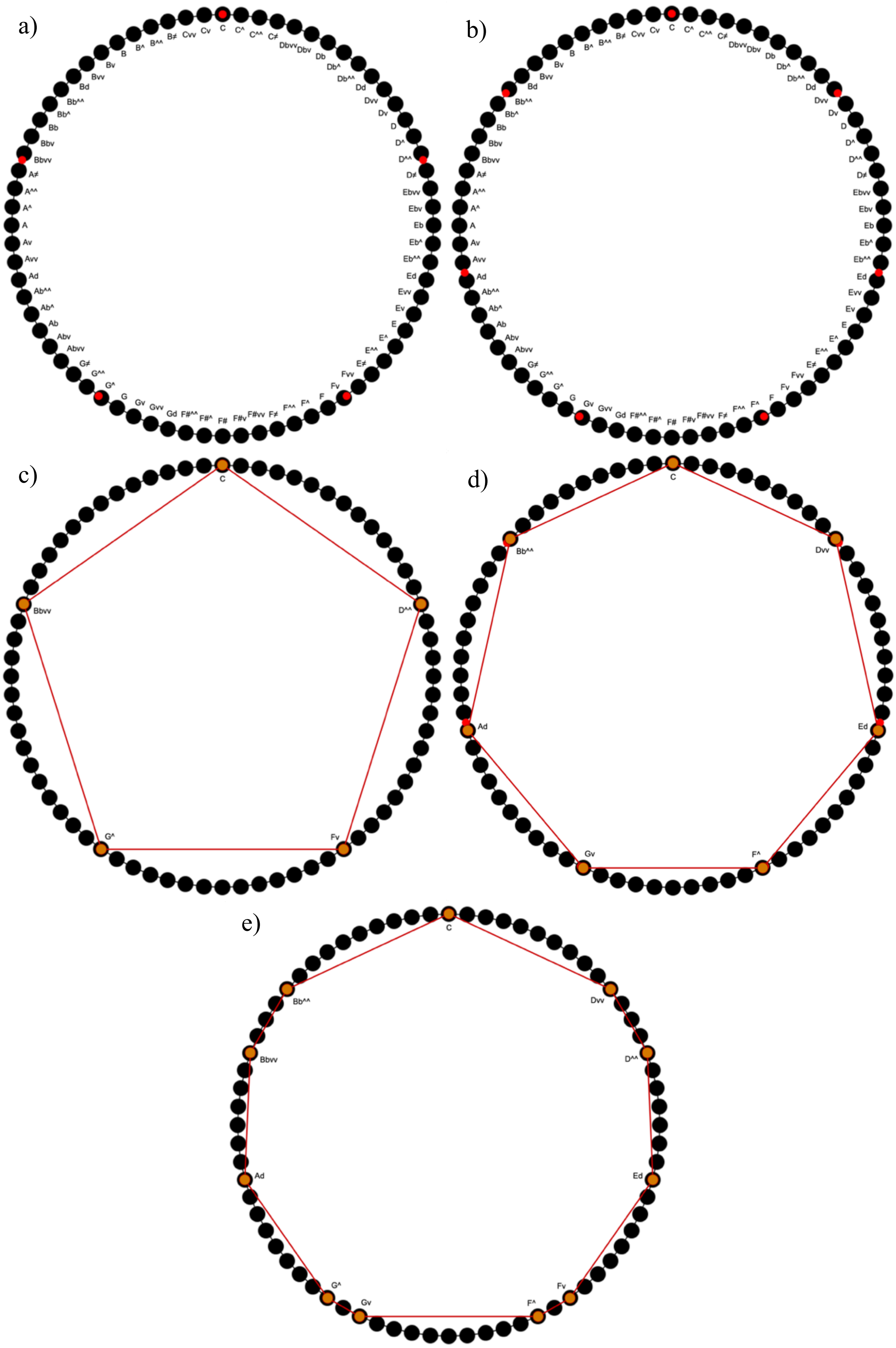


Figure 29. The creation process of the 5-7EDO scale and its notation in 72NPO: a) and b) illustrate in red dots 5EDO (a) and 7EDO (b) against 72NPO (black dots and notation); c) and d) illustrate the selection of the notes of 72NPO which more closely notate 5EDO (c) and 7EDO (d); e) corresponds to the notated 5-7EDO scale in 72NPO.

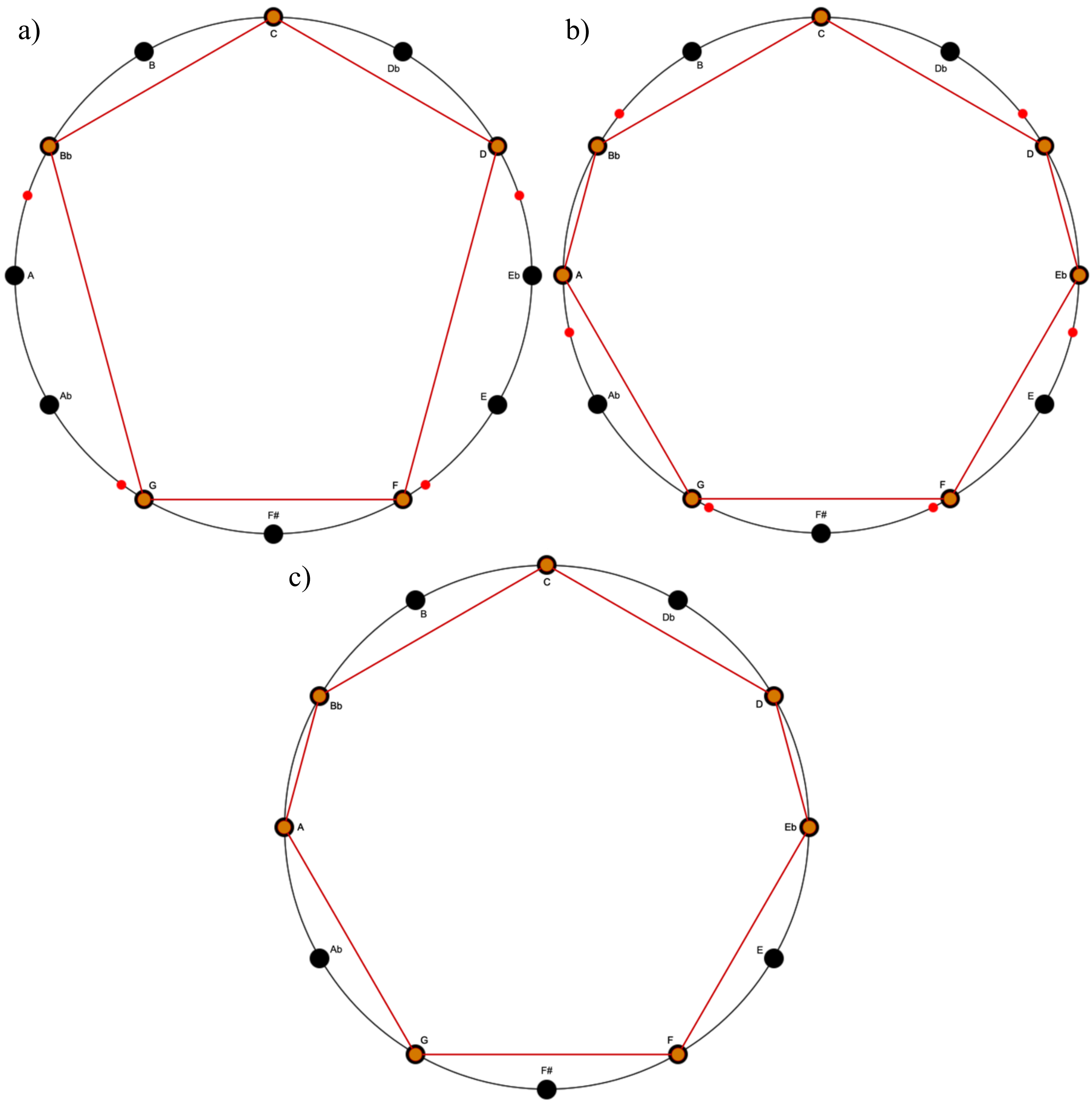


Figure 30. The creation process of the rounded 5-7EDO scale into 12EDO: a) and b) illustrate the rounding of 5EDO (a) and 7EDO (b) into 12EDO and c) corresponds to the rounded 5-7EDO scale into 12EDO. This results in the 12EDO dorian mode.

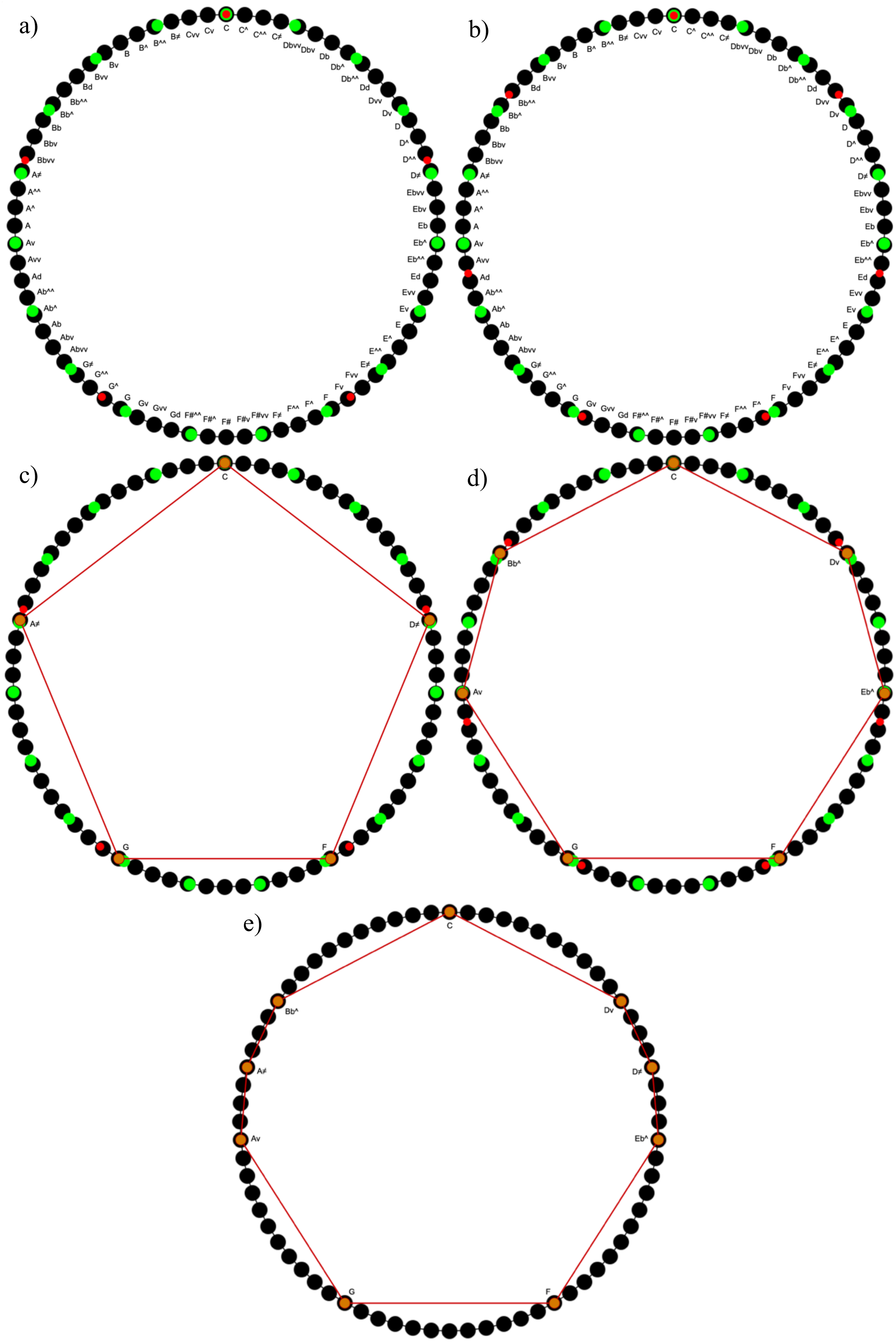


Figure 31. The creation process of the rounded 5-7EDO scale into 19EDO and its notation in 72NPO: a) and b) illustrate 5EDO (a) and 7EDO (b) in red dots, 19EDO in green dots and 72NPO in black dots; c) and d) illustrate the rounding of 5EDO (c) and 7EDO (d) into 19EDO and notated into 72NPO (polygons); e) shows the final sum of c) and d).

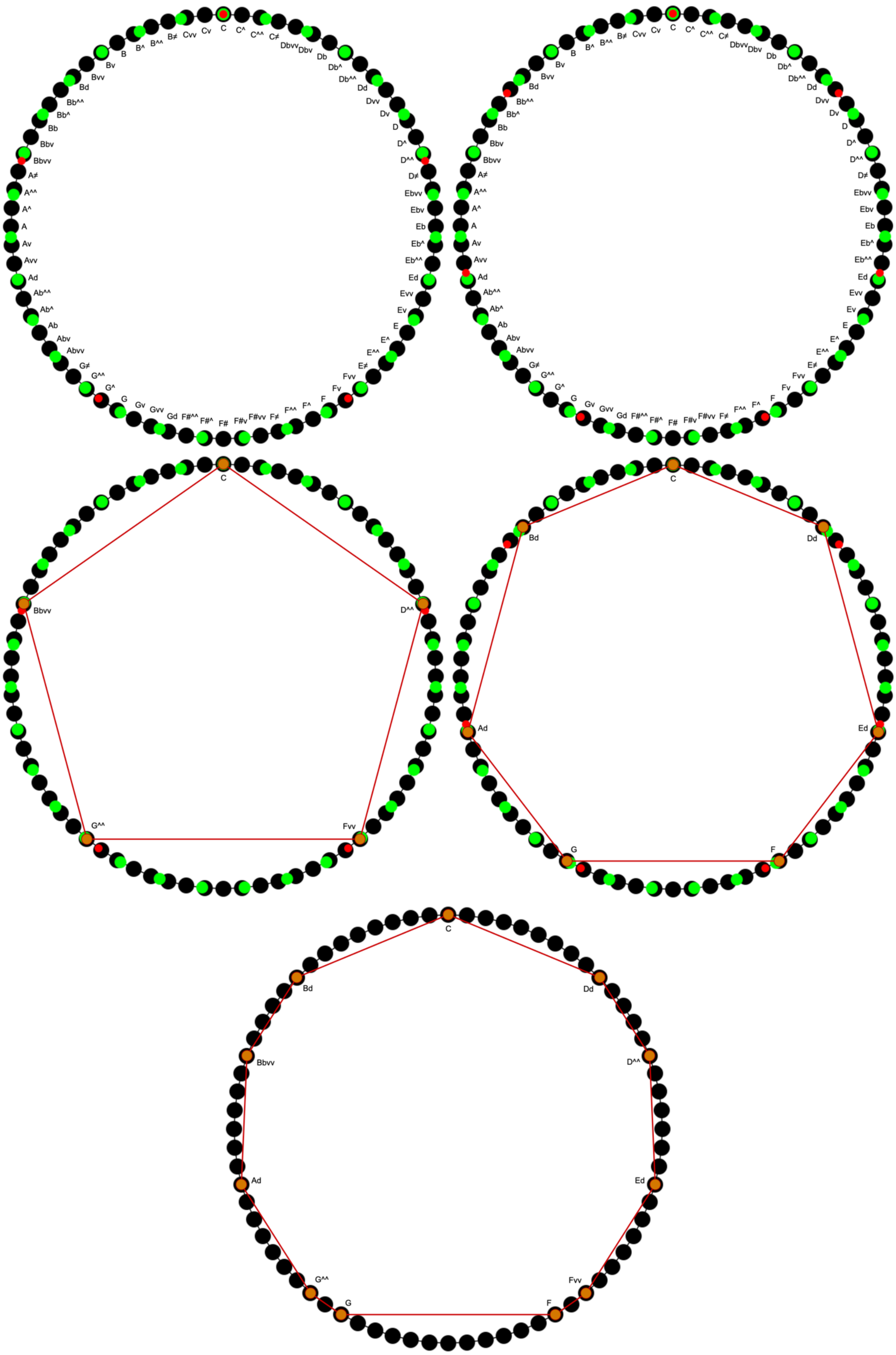


Figure 32. The creation process of the rounded 5-7EDO scale into 31EDO and its notation in 72NPO.

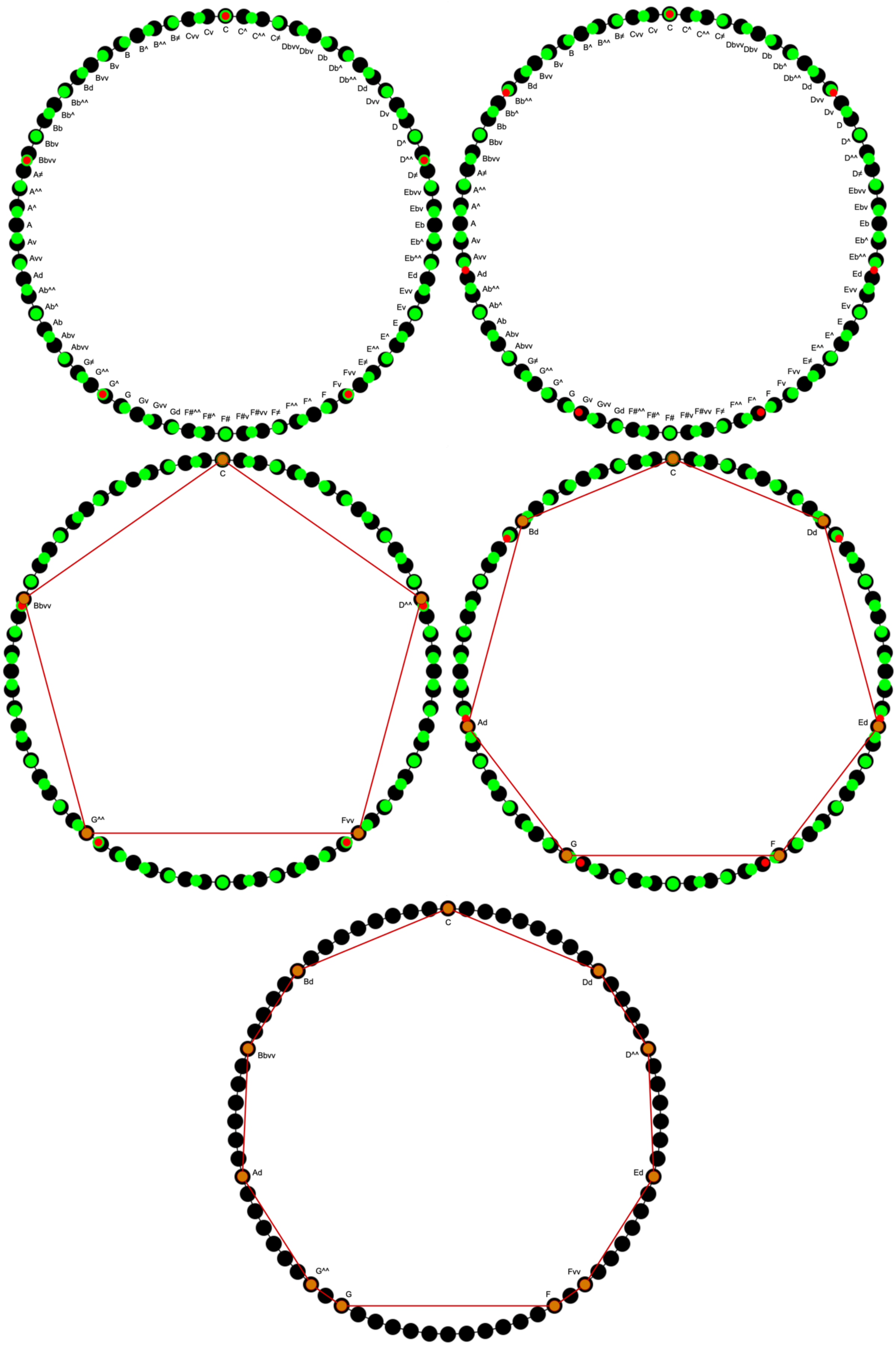


Figure 33. The creation process of the rounded 5-7EDO scale into 50EDO and its notation in 72NPO.

While improvising with these materials, I noticed that I could fairly accurately (to my ear) perform the beginning of Euripedes's Stasimon Chorus from *Orestes*[113] in its version in the enharmonic genus[114] with the 19EDO rounding of the 5-7EDO scale. Given that I have been fascinated by Ancient Greek music theory for years, such coincidence could not be left unnoticed, and so I decided to use this theme and develop it with the other roundings of this scale (see bars 134 to 161), simulating the constant alternation between hypothetical *genera* (each hypothetical *genus* being associated to a specific rounding into an EDO). Soon after, it became clear to me which would be the current title and that the form of this piece would vaguely resemble a theme and variation form, and rounding would be main source of variation.

4.2. Notation

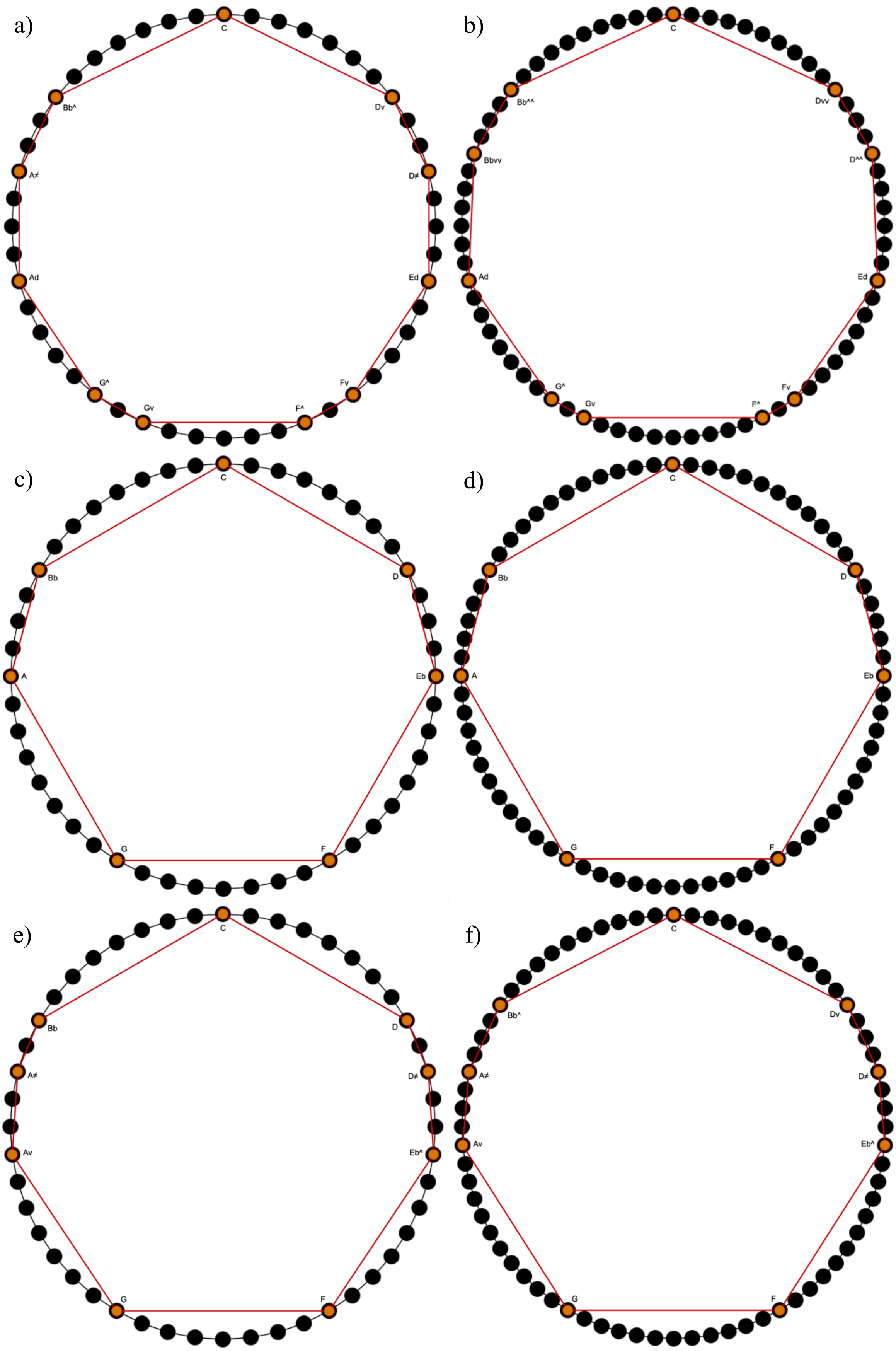
Similarly to *Adam*'s electric piano part, microtonal notation would not aid the performer intonating each pitch, given how the instrument works. Therefore, it would seem reasonable only to notate which keys would be performed, instead of using *Adam*'s dual notation. This would be the approach taken by Enno Poppe in *Rad* (2003) for two microtonally retunable electric keyboards. The score would also contain much less information graphically, if I followed this choice. However, given all the complexity derived from the technology involved in this piece and the choices that were being made to map pitches in the keyboard,[115] I still decided to include another system with microtonal notation. This way, the performer could reference which pitches they are supposed to hear when learning this piece.

Given the number of pitches that would be derived from the roundings of the 5-7EDO scale, 72NPO, in comparison to 48NPO, allows for a greater notational differentiation between different roundings because it has more pitches (see Figure 34).

[113] I knew as a teenager its performance in the 'enharmonic genus' by De Organigraphia, alongside a performance in the 'chromatic genus' by Atrium Musicae de Madrid. Refer to tracks 2 and 3 of the first CD of *Norton Recorded Anthology of Western Music*, 3 vols (New York; London: W.W. Norton, 2010), I. For a transcription of what has survived from the notation of this piece, see *Norton Anthology of Western Music*, ed. by J. Peter Burkholder and Claude V. Palisca, 6th ed, 3 vols (New York: W.W. Norton, 2010), I. p. 4.

[114] *Genera* (sing. *genus*) are the classes of tetrachords used to form 7 note scales in ancient Greek music. There are three classes of *genera*: diatonic, chromatic and enharmonic. Donald Jay Grout, Claude V. Palisca, and J. Peter Burkholder, *A History of Western Music*, 8th ed (New York: W. W. Norton & Company, 2010), pp. 15-16. From the three, the enharmonic genus is the one that I perceive to be the most xenharmonic.

[115] See the performance notes of *Dreams from an Old Memory.*


a)
C
Dv
D≠
Ed
Fv
F^
Gv
G^
Ad
A≠
Bb^
b)
C
Dvv
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Ed
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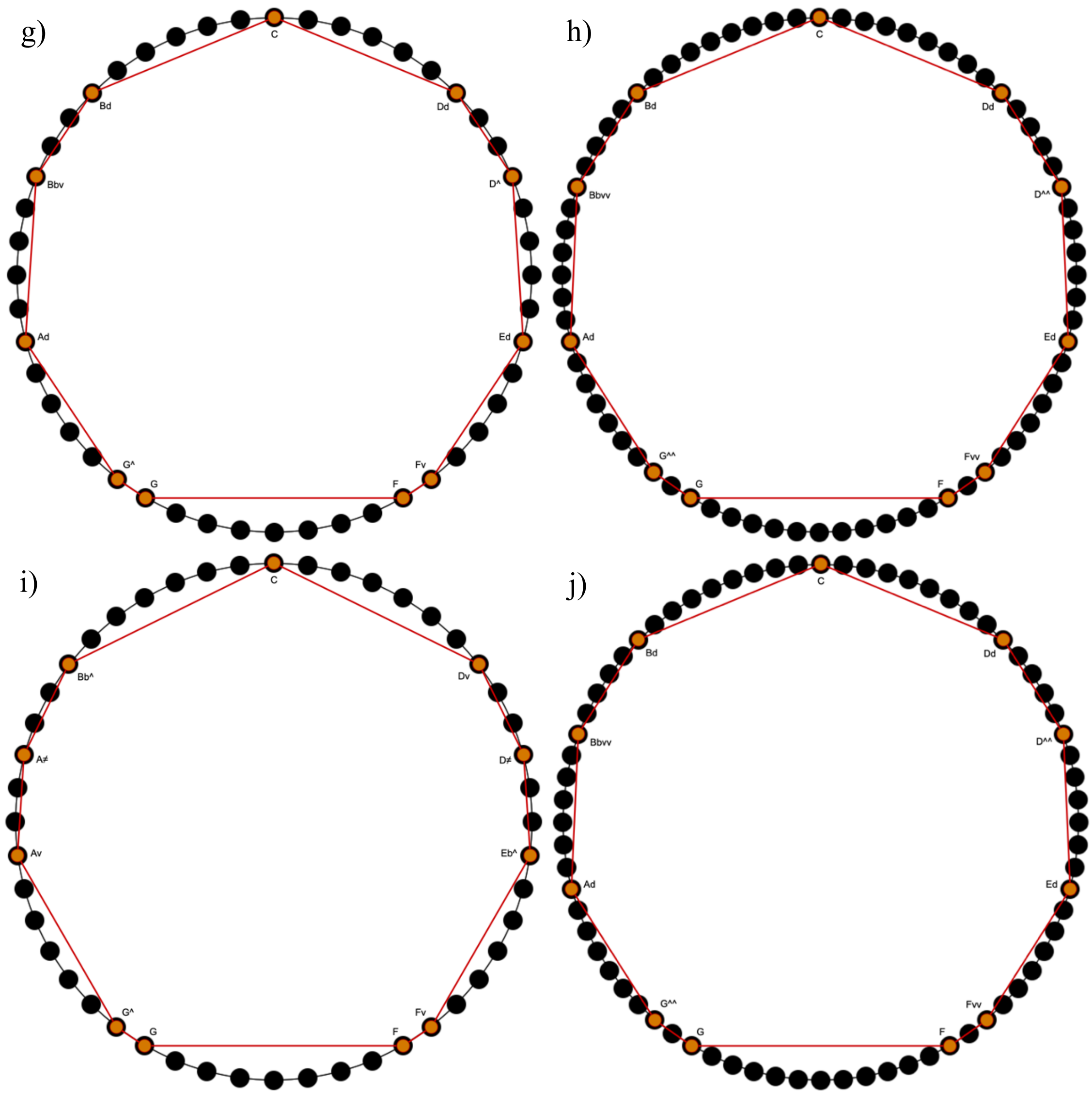


Figure 34. Comparison between the notation into 48NPO – a), c), e), g) and i) – and 72NPO – b), d), f), h) and j) – of the 5-7EDO scale – a) and b) – and its roundings into 12EDO – c) and d) –, 19EDO – e) and f) –, 31EDO – g) and h) – and 50EDO – i) and j).

While writing *Adam,* I developed an appreciation for the harmonic qualities of tuneable intervals. If, in the scales used in this piece and their rounded versions, were there any quasi-tuneable intervals to be found and musically exploited (see Figures 35 to 37), then 72NPO would more accurately notate them than 48NPO, even though the issues regarding tuneability would not apply in this piece. This would be an aesthetical choice, rather than a technical necessity.

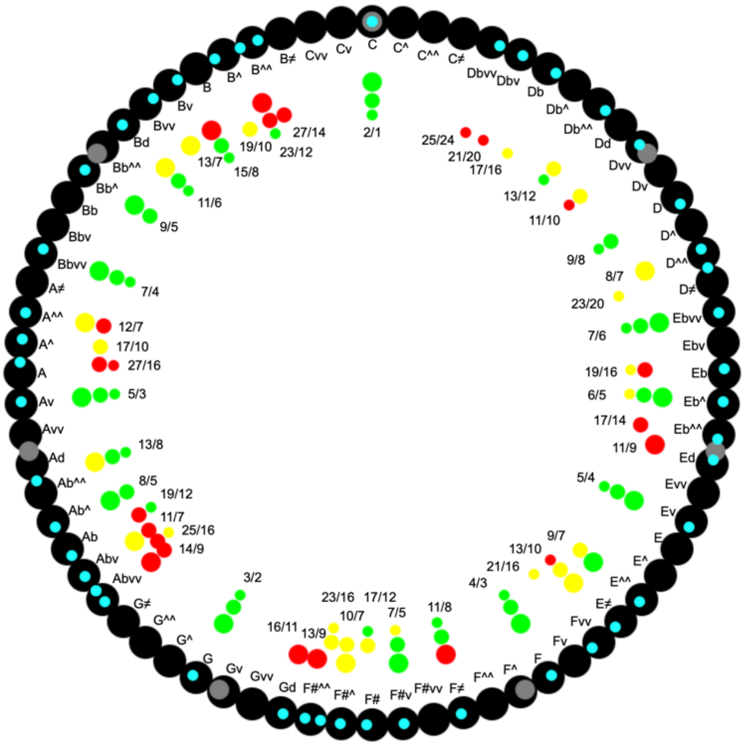


Figure 35. A representation of 7EDO (grey dots) and 72NPO (black dots and notation) against Sabat and Haywards's tuneable intervals.

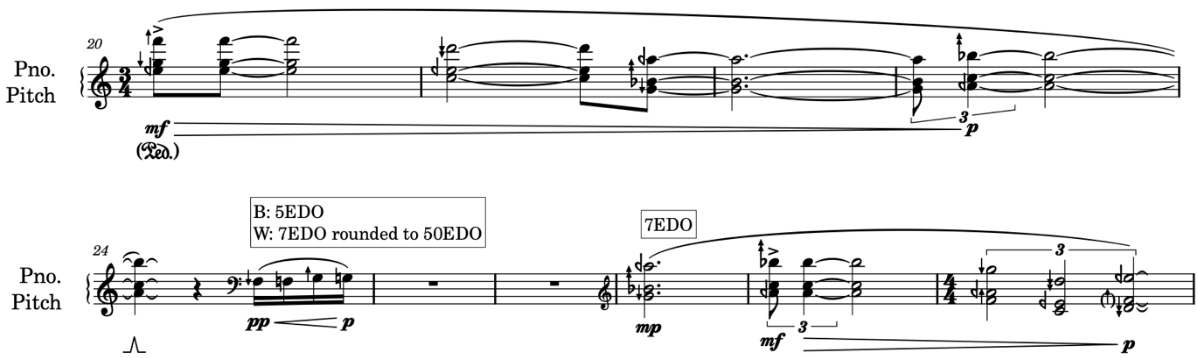


Figure 36. Excerpt from bars 20 to 29 of Dreams from an Old Memory*, which employs a faux bourdon set of parallel chords built from the 7EDO quasi-tuneable intervals close to 11/9 and 11/5 (which are respectively, Rank I and G intervals).*[116]

[116] See Appendix 5.

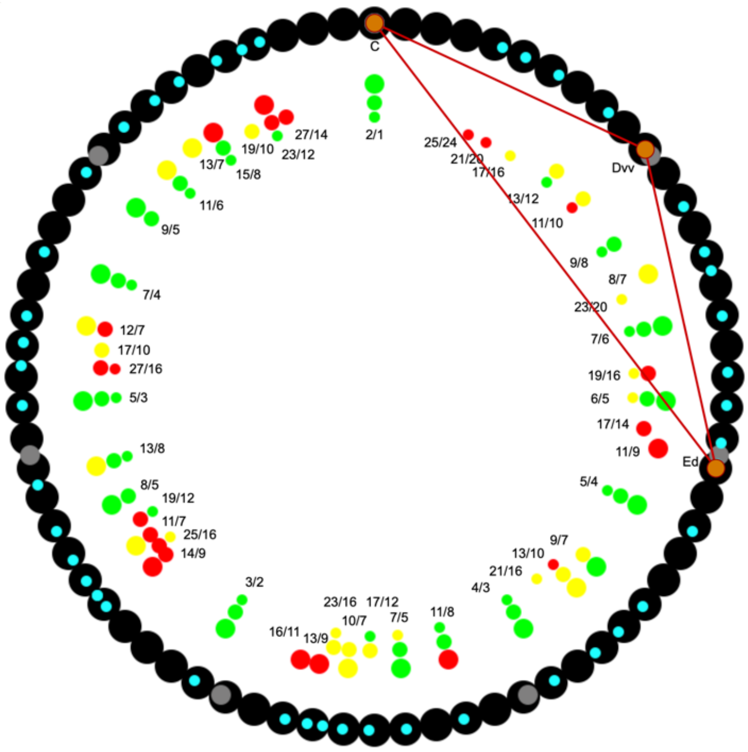


Figure 37. Selection of the intervals which form the chord of Figure 36 and all its transpositions. Do note: the ratio 11/10 when extended one octave becomes 11/5, since $\frac{11}{10}\times 2=\frac{22}{10}=\frac{11}{5}$.

As was established, for a piece almost entirely based on small microtonal variations derived from the intermixing of multiple roundings of the scales that are used in this piece, a notation system with a relatively high resolution, such as 72NPO, would be appropriate. However, if a higher subdivision of 12NPO was used, such as 96NPO (48×2) or 144NPO (72×2), then the number of extra accidentals that would be needed (see Figures 38 and 39) could start to become somewhat unsustainable, especially because this research project is not aimed towards specialist performers, but non-specialists. Also, neither 96NPO nor 144NPO seem much more effective than 72NPO to approximate and notate the pitches of these systems that can form quasi-tuneable intervals (which is a relevant insight for future pieces with strings).

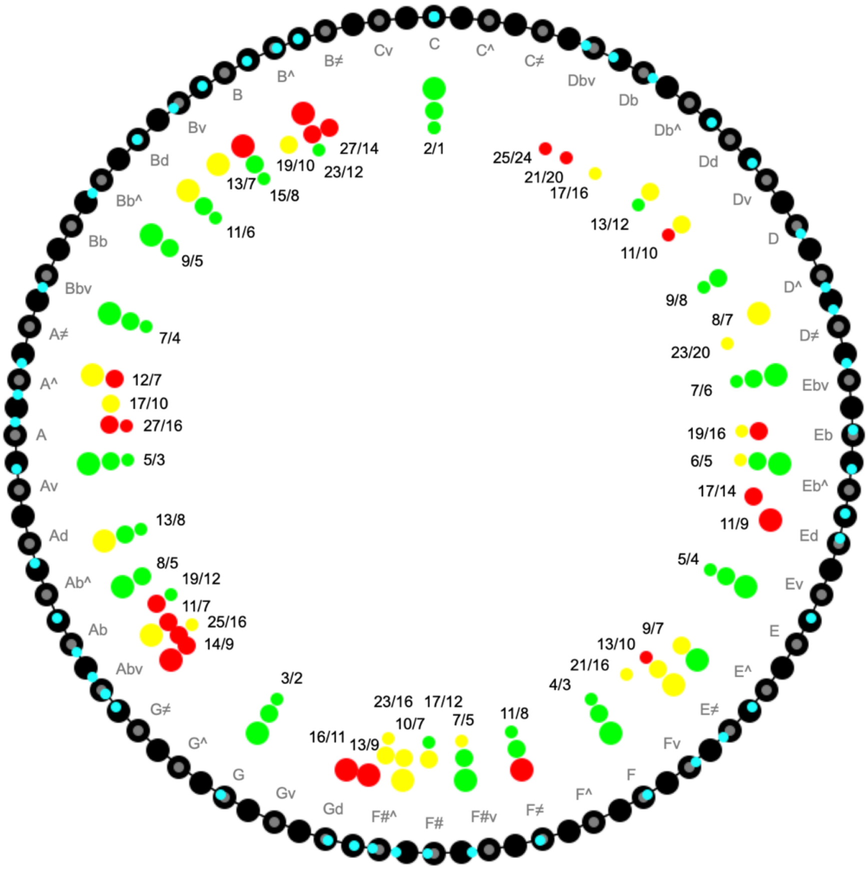


Figure 38. Sabat and Hayward's tuneable intervals against 96NPO (black dots) and 48NPO (grey dots and notation).

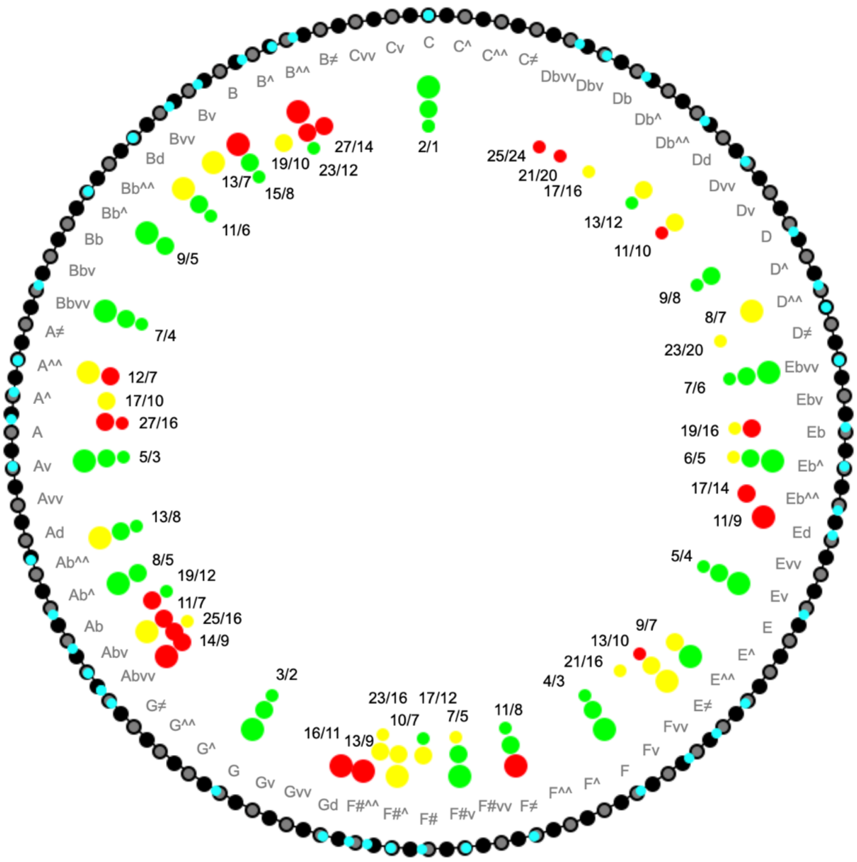


Figure 39. Sabat and Hayward's tuneable intervals against 144NPO (black dots) and 72NPO (grey dots and notation).

Chapter 5. Reflections on my tendencies towards certain EDOs

Having written the previous two pieces, it became clear that the EDOs (besides 12EDO and subdivisions) to which I have been mostly attracted are 5, 7, 11, 17[117] and 19EDO (all prime numbered). Some can be conceptualized as 'tuning/scales' (5, 7 and 11EDO) and all relate to scales in 12EDO to which I am already attracted.

5 and 7EDO, through rounding into 12EDO, correspond to the second pentatonic and dorian modes.[118]

11EDO melodically sounds to me as a variation of the chromatic scale (while harmonically sounding quite xenharmonic).[119]

17 and 19EDO[120] have recognizable diatonic scales and can approximate some Just Intonation intervals beyond the possibilities of 12EDO. When notated using the Circle of Fifths Notation system, their enharmonic equivalencies differ to 12EDO, but the harmonic/melodic characteristics of their major, minor and perfect intervals (plus the augmented fourth and diminished fifth), when notated with it, generally sound to me as acceptable renderings of those intervals, only differing in cent size. For example, a major sixth in 12EDO (9 steps), 17EDO (13 steps) and 19EDO (14 steps), all sound to me as acceptable renderings of a major sixth.

This research was designed to leave open-ended which equal tunings would be used. Being aware of these personal tendencies later led me to explore other equal tunings which are prime numbered and/or could be conceptualized as tuning/scales with analogous 12EDO scales to which I am already attracted, or others which are neither, but relate to them in some other form.[121]

[117] Even though 17EDO is not present in neither *Adam* nor *Dreams from an Old Memory*, it was used in some of my previous compositions. 17EDO is later explored in 'Piece Z' and *Convergence*.

[118] See Appendix 7.

[119] I recommend the reader to explore 11EDO in Equal Tuning Lab, but also to see the available information on it in Global Properties of EDOs.

[120] Similarly to 11EDO, I also recommend the reader to explore 17 and 19EDO with the same tools.

[121] In *Convergence* I employed 8 and 9EDO, which are not prime numbered, but can be conceptualized as tuning/scales which relate to the subdivisions of 12EDO (8EDO is included in 24EDO, and 9EDO is included in 36EDO – both are then included in 72EDO).

Section 2: String pieces

The following chapters will discuss the composition process of *Odd [s]Paces* for string quartet (Chapter 6) and *The Fable of the Pilgrim* for guitar quartet (Chapter 7).

One possible resource for microtonal exploration in both fretted and unfretted string instruments is the usage of scordatura. Having studied microtonal pieces which employ scordatura in bowed strings, such as Dan Trueman's *Songs that are Hard to Sing* and Ligeti's Violin Concerto, and in guitars, such as Georg Friedrich Haas's *Quartett für vier Gitarren*, it became clear to me that scordatura is more commonly used in guitar composition and is therefore also more easily absorbed by players in general. Therefore, the employment of scordatura to achieve microtonal results seems to be more idiomatic for guitar players than for bowed strings players. Therefore, even before choosing which specific tuning systems and microtonal approximations to explore, I decided not to use scordatura in *Odd [s]Paces* but use it in *The Fable of the Pilgrim*.

Another goal for these pieces was to develop other parameters, notably rhythm and form, with stronger conceptual relationships between them and my pitch choices. (In retrospect, I consider having made significant progress in terms of rhythm. Although, to this day I am still not satisfied with my use of form.)

Chapter 6. *Odd [s]Paces*

This chapter will address the composition process of *Odd [s]Paces* for string quartet. Subsequent subchapters will focus on the rationale behind the choice of exploring ED3/2s instead of EDOs (6.1), the sketching process of the beginning of the piece and notes on the workshopping process (6.2) and some insights that were discovered throughout the composition and workshopping process (6.3).

6.1. Choosing Equal Divisions of 3/2 instead of EDOs

In *Adam*, the string instruments were not the only instruments that approximated equal tunings, but they would be in this piece. Therefore, I had to approach this piece differently. Since, up to this point, the only equal tunings I had explored were EDOs, I thought this instrumentation would be ideal to explore, instead, equal divisions of the 3/2 perfect fifth. The fact that I had at my disposal the fifths of the open strings only made this decision even more obvious as a potential idiomatic approach to microtonal string writing. The open strings would also serve as a reference which the performers could regularly access throughout the piece and would also lead to sonorities encompassing the instruments' full resonance. However, similarly to *Adam*, I gravitated towards what would be considered macrotonal systems (albeit equal divisions of the fifth instead of the octave), as I used bitKlavier for sketching purposes, which allowed me to map up to 7 pitches between a selection of piano keys a perfect fifth apart. I then had access to:

- 2ED3/2 – which I perceive to be an 'extended neutral chord', a structure I associate with 24NPO

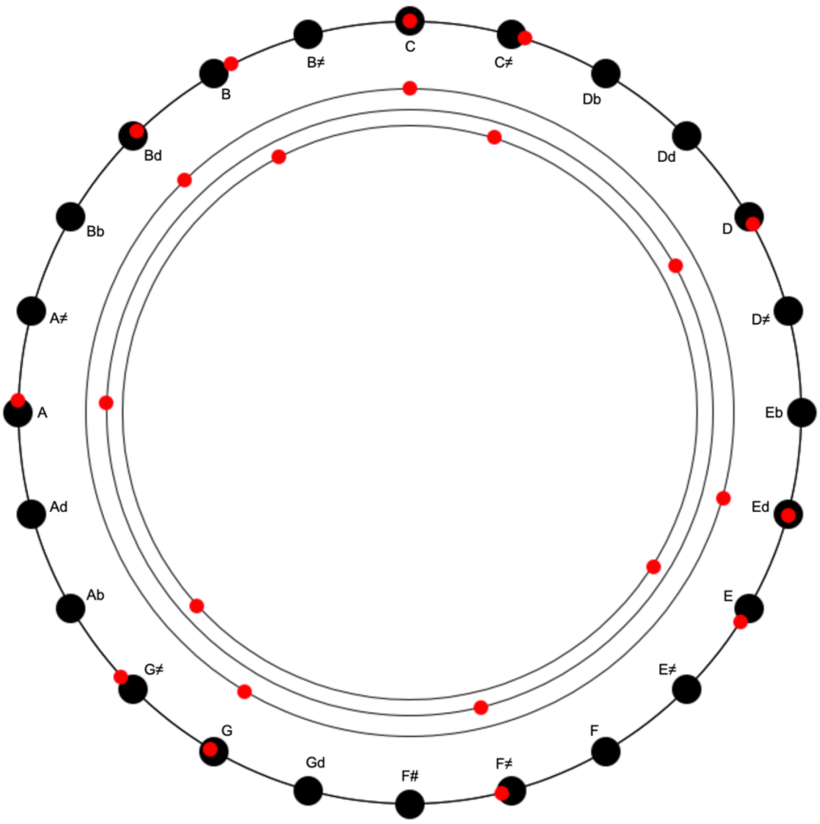


Figure 40. 2ED3/2 represented within a three-octave range (red dots on the 3 inner circles), and a projected collection of its pitch classes (red dots on the outer circle), against 24EDO (black dots and notation). This dual representation of non-octave-repeating equal tunings follows the same mechanism used to represent tuneable intervals and their projected interval classes in Figure 4 on Chapter 2.4.

- 3ED3/2 – which, within an octave span, resembles 5EDO but with ‘in tune’ fifths (and major ninths) and ‘out of tune narrow octaves’

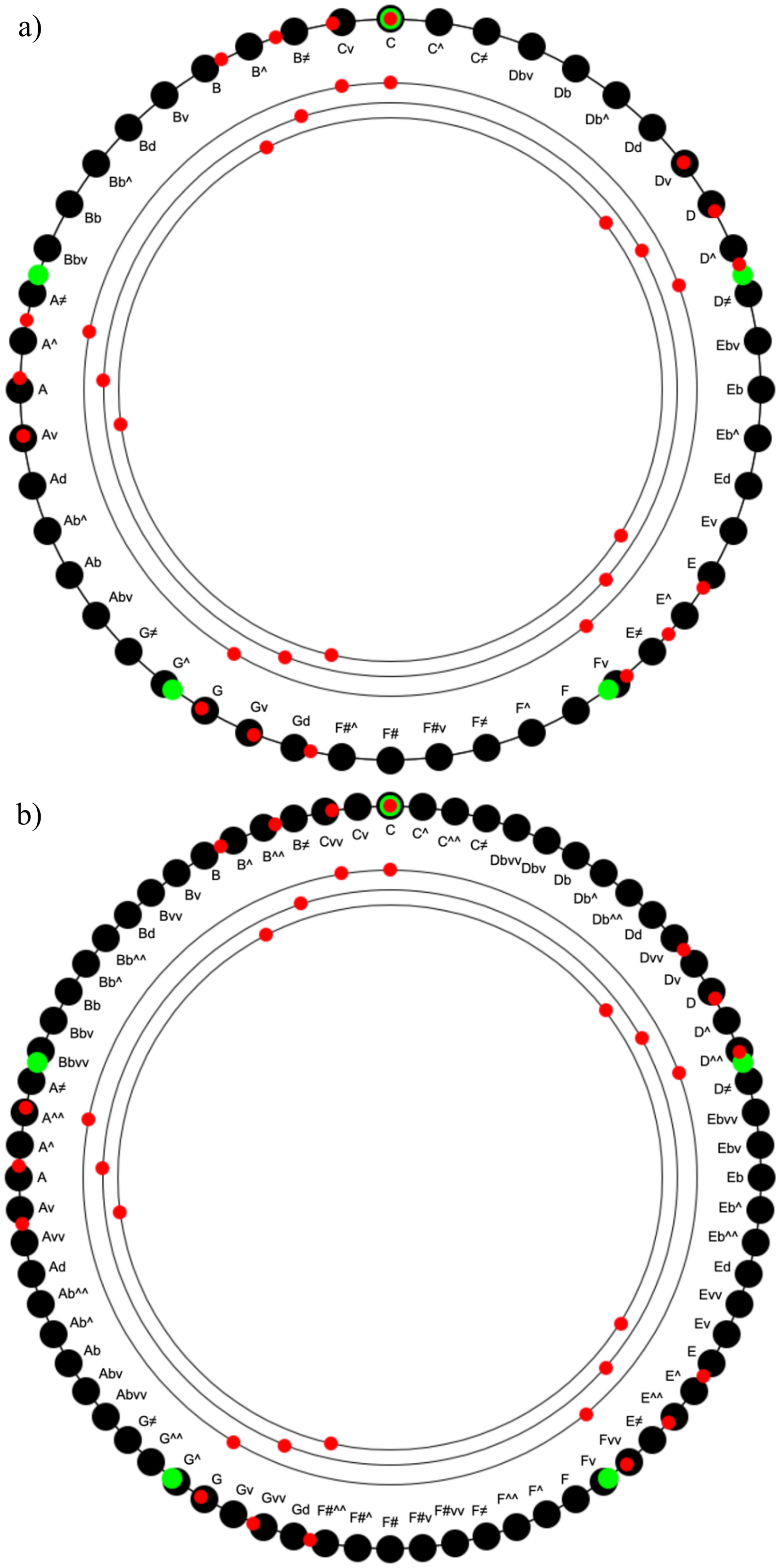


Figure 41. 3ED3/2 represented within a three-octave range (red dots on the 3 inner circles), and a projected collection of its pitch classes (red dots on the outer circle), against 5EDO (green dots), 48EDO and 72EDO – black dots and notation in a) and b), respectively.

- 4ED3/2 – which, within an octave span, resembles 7EDO, but can also approximate some partials of the harmonic series, alongside other inharmonic intervals

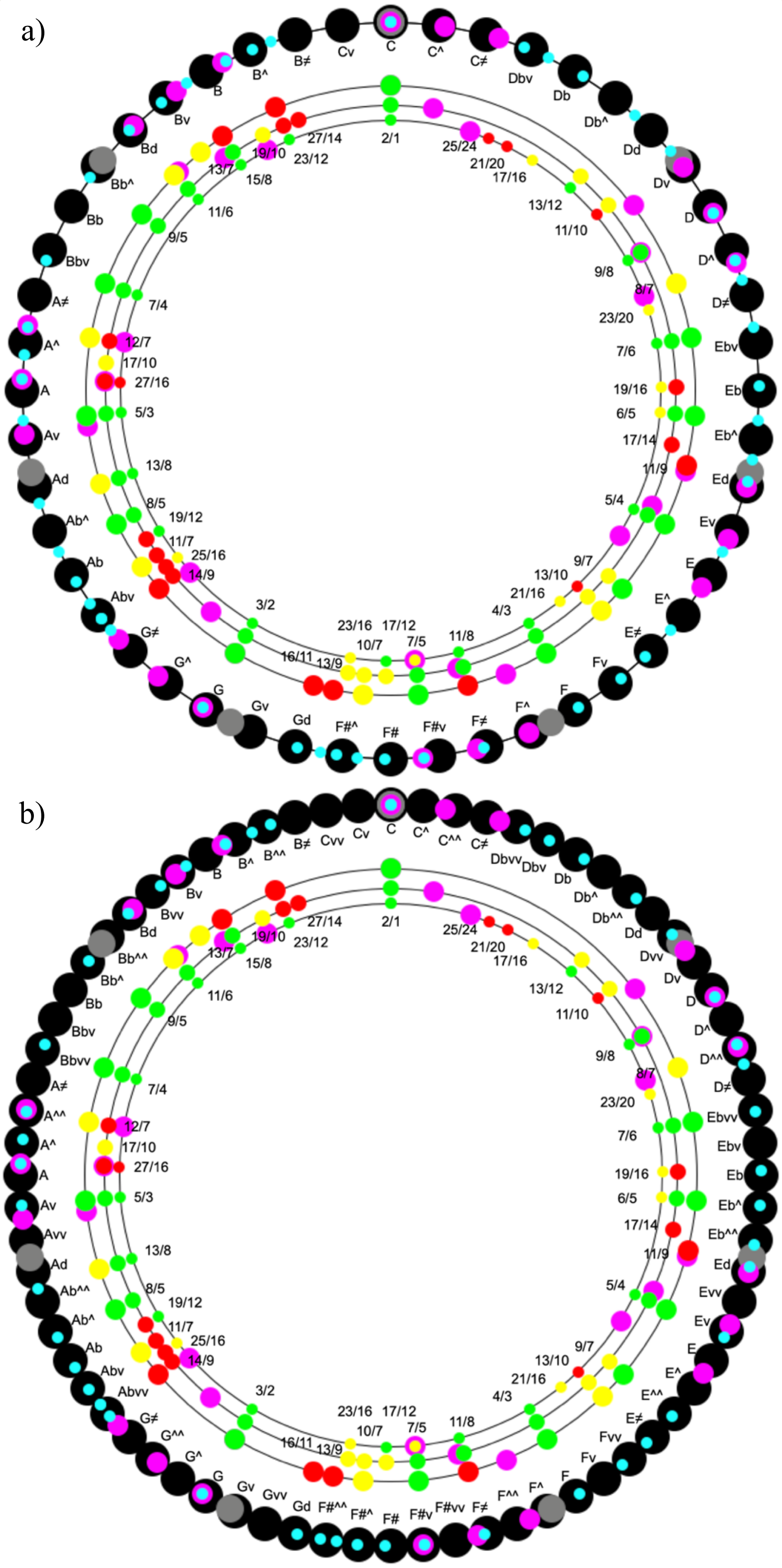


Figure 42. 4ED3/2 represented within a three-octave range (magenta dots on the 3 inner circles), and a projected collection of its pitch classes (magenta dots on the outer circle), against 7EDO (grey dots), 48EDO and 72EDO – black dots and notation in a) and b), respectively –, tuneable intervals (green, yellow, and red dots in the 3 inner circles) and their projected collection of interval classes (cyan dots in the outer circle).

- 5ED3/2 – which sounded enticingly xenharmonic to me and has many pitches close to 24EDO and 48EDO

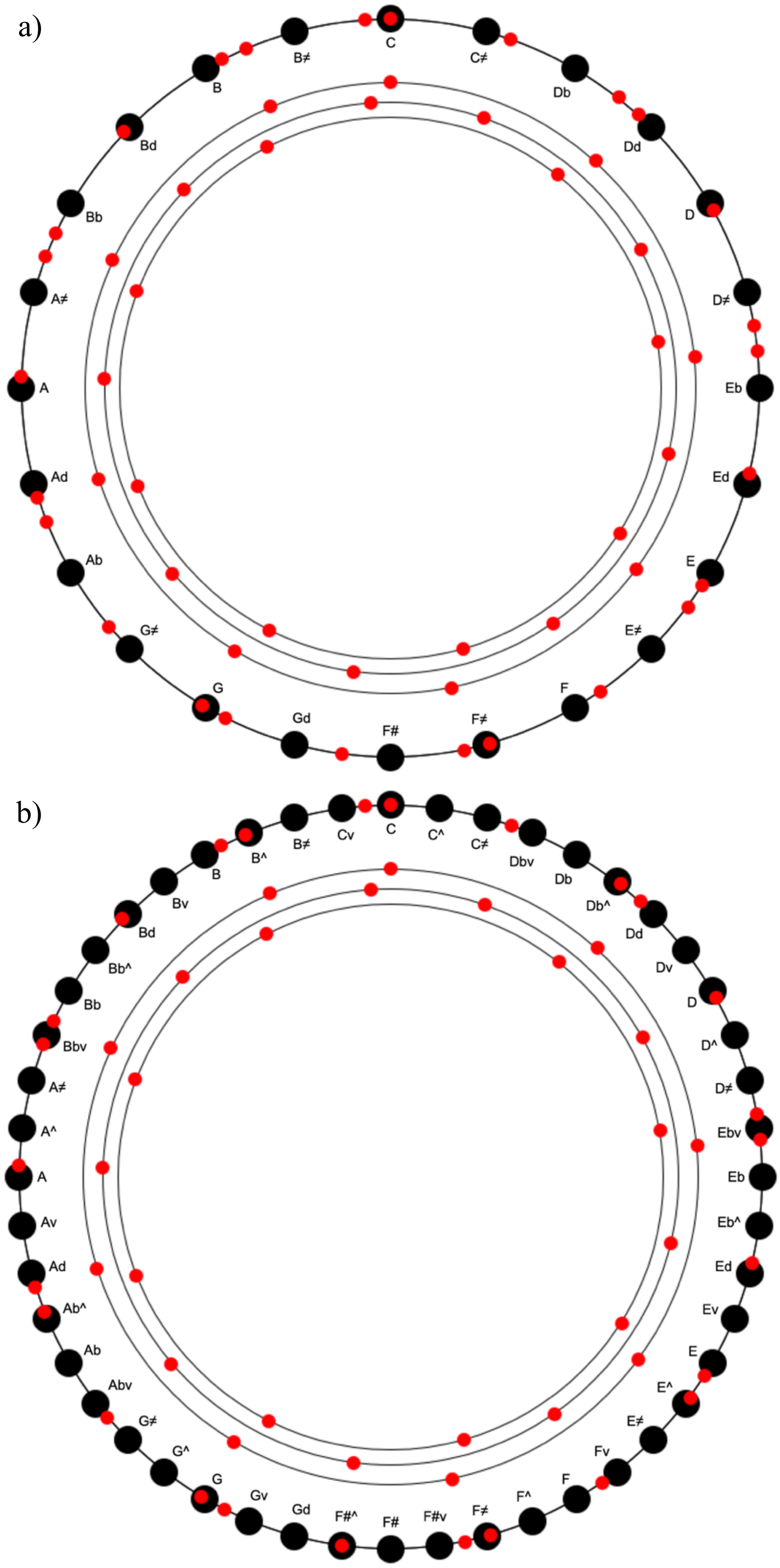
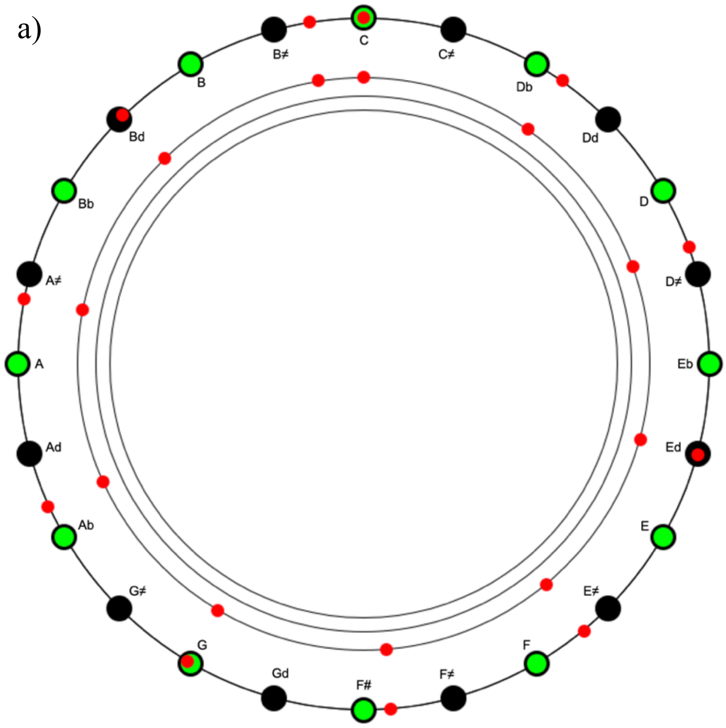


Figure 43. 5ED3/2 represented within a three-octave range (red dots on the 3 inner circles), and a projected collection of its pitch classes (red dots on the outer circle), against 24EDO and 48EDO – black dots and notation in a) and b), respectively.

- 6ED3/2 – which sounded xenharmonic to me, but less compelling than 5ED3/2, while also resembling an incomplete (non-octave repeating) chromatic scale which, to me, could be acceptably approximated with 24NPO,[122] despite 48NPO being more precise

a)



a)

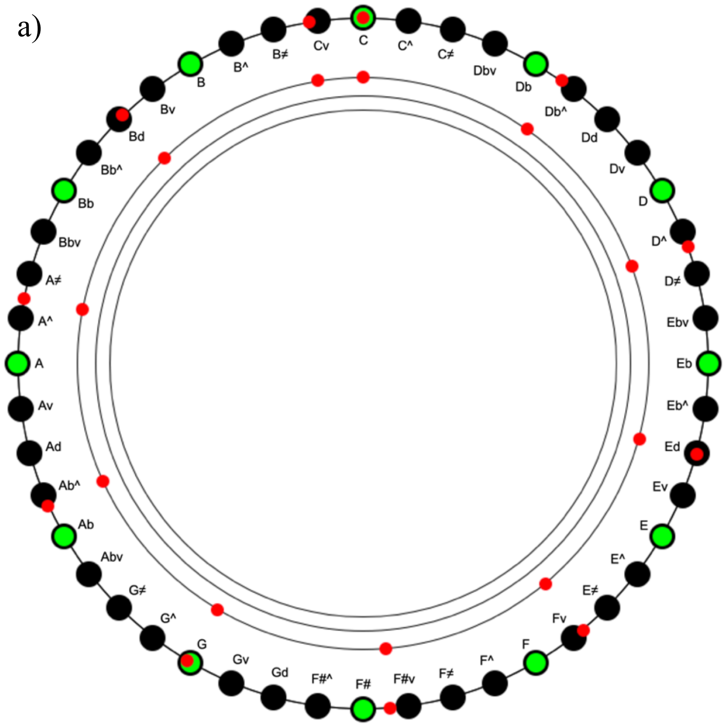


Figure 44. 6ED3/2 represented within a 1 octave range (red dots), against 12EDO (green dots), 24EDO and 48EDO – black dots and notation in a) and b), respectively.

[122] See Appendix 6 for an exercise I did around this time in which I tested my perception of a variety of intervals in 72EDO and the degree of notational precision I intuitively found reasonable to notate them.

- 7ED3/2 – which I perceive as a very slightly stretched (and 'brighter') version of 12EDO on the electric piano, but functionally trivial outside of a keyboard context.

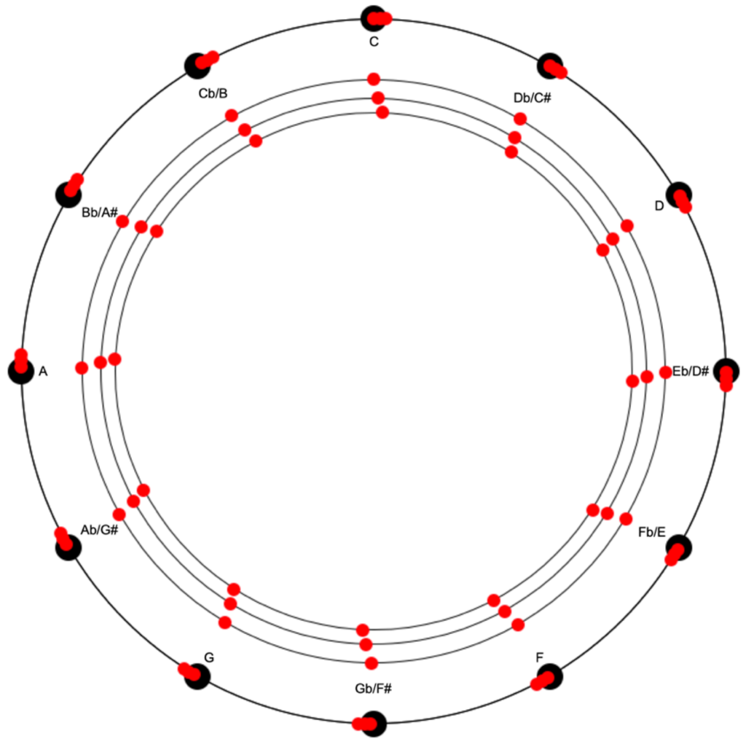


Figure 45. 7ED3/2 represented within a three-octave range (red dots on the 3 inner circles), and a projected collection of its pitch classes (red dots on the outer circle), against 12EDO (black dots and notation).

6.2. The sketching process

I was particularly intrigued by 4ED3/2, and therefore started sketching the slow opening of the piece (see Figure 46). With this structure, one has at their disposal a variety of harmonic/tuneable/consonant and inharmonic/untuneable/dissonant intervals. Having learned about Sabat's method for performing 'tuneable dissonances' – by playing a series of tuneable intervals starting from one pitch which would lead to another pitch which, together with the initial pitch, would form the desired 'tuneable dissonance'[123] – I understood that its notational requirements were not flexible enough to notate all the variety of intervals that 4ED3/2 offered, even though it presented a clear methodology to reach, in performance, intervals with similar characteristics to 4ED3/2's more dissonant intervals. I then followed a different approach by using a notation system in 72NPO (later substituted with 48NPO), which allows for the tuneable and untuneable intervals of 4ED3/2 to be notated with some precision and suggests intervallic referential measurements (the semitone and its subdivisions) to the performers.

[123] Sabat, 'The Extended Helmholtz-Ellis JI Pitch Notation', p. 6.

With this, I deduced that untuneable intervals could instead be produced by first playing a tuneable interval (ideally 'easy') close to it and then rely on the performers' calculations (informed by the inbuilt measurement standard of the notation system) and muscle memory to melodically approximately change from one pitch (forming a tuneable interval from a reference note) to another (forming an untuneable interval). Such notational approach, whilst approximating 4ED3/2 to an acceptable degree of precision to me, would be imbedded with my knowledge of string intonation practices. In theory, this would facilitate the composition of music written idiomatically for this instrumental setting.

Only later in the composition process it became clear to me that, overall, the intervals of 48EDO approximated more closely than 72EDO the pitches of 4ED3/2. Therefore, despite having chosen to use 72NPO notation for the string section in *Adam*, 48NPO notation seemed an appropriate choice for approximating and notating 4ED3/2.

Figure 46. Early sketch of the first section of the beginning of Odd [s]Paces*. A '*' is used for intervals which are not present in the 72NPO rounding of 4ED3/2, but usually serve as references to play untuneable intervals from them. Three different notehead shapes are being used to visually illustrate the different intervals involved: circles for open strings and intervals not notated with microtonal accidentals (perfect fifths, fourths, etc.), triangles for untuneable intervals and coloured squares for tuneable intervals (green for 'easy', orange for 'more difficult' and red for 'hard' intervals).*

Despite the limited number of rehearsals to produce a complete run-through performance of the piece, with repetitive trial-and-error readings of separate groups of bars (with me conducting), we managed to record this section successfully. For example, the second violin part on bars 12 to 17 (on Figure 47) was rehearsed by:

1. Having the cello play its C and G drones 'ad infinitum'
2. Inviting the second violinist to find by ear the Ev by sliding the pitch up and down and finding the most harmonically stable version of the pitch
3. Repeating step 2 for the F≠
4. Trying to play both pitches interchangeably (with no specified durations), and getting comfortable with that task
5. Trying to find the G^^ (a G^ in the final 48NPO version of the score) right after playing the F≠, by aiming to make it sound purposedly dissonant with the cello part
6. Repeating step 4 while also including the G^^ (G^)
7. Repeating step 6 while having the remaining instruments playing the pitches that do not serve as references for the performance of the second violin part
8. Playing all the parts with note durations.

This rehearsal methodology (and variations of it) was used throughout most of this piece. In the recording session, before recording each take of each cut section, a variation of this formula was used. Having gone through this rehearsal process, it is my belief that, were we to have more rehearsal sessions, the performers would be able to play the whole piece from beginning to end. Therefore, I consider this approach used to approximate 4ED3/2 to be idiomatically written for string quartet.

6.3. Insights

6.3.1. Quarter tones as variations of chromatic accidentals

The other passages of *Odd [s]Paces* are mostly based on approximating other equal divisions of 3/2 into 12 and 24NPO (semitones and quarter tones). While rehearsing these passages both in group and individually with each performer I noticed a tendency for the performers to conceptualize quarter tones not as being subdivisions of 12NPO (although they were acquainted with the concept). Instead, they conceptualized them as being either exaggerations or contractions of the pitch modifications produced by chromatic accidentals. For example, an F≠ could be conceptualized in two ways:

1. as a lowered F# (converging with Tartini's technique). This behaviour shares resemblances with 34EDO, which is a subdivision of 17EDO, a Superpythagorean tuning, and the one with the narrowest fifth size beyond the 'expressive range'.[124] For major thirds, such as D-F# (which are closer to the wider 9/7 than to 5/4), the lowering of the F# to F≠, approximates them to the 5/4 ratio. This behaviour between chromatic intonation and quarter tone accidentals appears to imply an exaggerated form of expressive intonation for chromatic and diatonic pitches, and quarter tones would encode a convergence towards Tartini's intonation technique.[125]
2. or as an exaggeratedly flattened Gb (converging with Campagnoli's technique). If the D-F# major third was substituted with the D-Gb diminished fourth, the closest intonation standard to achieve a 5/4 ratio would be Pythagorean Tuning/53EDO, which is within the 'expressive range' of fifth sizes.[126]

This phenomenon could be due to the fact none of the performers (who were students) were substantially exposed to microtonal music in their practice (some had never played microtonal music). However, I also observed that amongst the performers there didn't seem to exist a universal preference for conceptualizing quarter tones as either contractions or exaggerations of chromatic accidentals: each performer had their own tendencies. If there was an overall tendency, it seemed to be not to conceptualize quarter tones as subdivisions of semitones.

In retrospect, this should not be surprising. However, since I am a keyboard player and a guitarist, I tendentially take semitones for granted as fixed references, whereas it is understandable that string players would not, as their instruments do not have frets. Even during rehearsals, some performers would tell me that they would think that a sharp note (or a natural note a diatonic semitone lower than another natural note) should be positioned to 'lead' to the semitone above it (i.e. Campagnoli's approach). Curiously, when they were supposed to intone 5/4 major thirds on top of other players, the notation seemed initially confusing to them (given

[124] See the 'EDO Diatonicism' spreadsheet of Global Properties of EDOs.

[125] I recommend the reader to see the 34EDO section of the 'EDO Step Proximities' spreadsheet on Global Properties of EDOs.

[126] I also recommend the reader looking at the 53EDO section of the 'EDO Step Proximities' spreadsheet on Global Properties of EDOs.

that it employed down arrowed accidentals, with which they were not familiar). Nonetheless, minutes later, they would be able to tune those intervals by ear, with less effort each time they tried (probably not because of the intervals themselves, but because of the musical context, which does not conform with classical common practice music). This led me to consider if these particular performers have been taught intonation tendentially towards Campagnoli's model (alongside adjusting their intonation when playing with other instruments such as the piano), but not Tartini's. For example, the first violinist told me that they conceptualize the note C melodically in relation to a B (which would serve as a leading tone towards that C). In turn, they conceptualize that reference B as a 4/3 perfect fourth below the E open string. Because of this, for them, C would not necessarily be vertically in tune with the G open string.

Considering these considerations, despite these passages having been performed mostly to my satisfaction, an idiomatic microtonal approach for string writing should not take quarter tones for granted in the same way as for instruments of fixed pitch.

6.3.2. Following formulas VS following methodologies

Given that the recording of this piece was successful, I consider the beginning of this piece to be the first time my employment of 48NPO in string writing to have been effective in performance. This contradicts both my previous experience, and my remarks about 72NPO being a better system for microtonal string writing. Overall, this experience helped me reach the conclusion that I simply cannot rely on pre-determined formulas: which was the starting point of this research project, by relying so much on roundings into 12NPO and its subdivisions. Each context matters: what did not work in one context, may work in another, and vice versa.

In the following pieces, I then began to focus towards developing a set of methodologies to help me make appropriate choices for each situation (in other words, finding more idiomatic strategies to write for each instrument depending on each context). This led me to gradually move away from using rounding as an approximation strategy and basis for a notational framework, for every situation. Developing new tools beyond rounding on Equal Tuning Lab was central for this, as was later creating Global Properties of EDOs as a more data driven and visually global complement to Equal Tuning Lab (as it allows to compare a much bigger sample of tuning systems simultaneously, although with a completely different visual layout). These new tools were particularly useful during the sketching process of the last piece of this project, *Convergence*, and the unfinished 'Piece Z' (which was adapted and incorporated into *Convergence*). However, ironically, following this logic, I concluded that rounding was precisely the appropriate strategy to employ on the following piece, *The Fable of the Pilgrim*.

Chapter 7. *The Fable of the Pilgrim*

This chapter will address the composition process of *The Fable of the Pilgrim* for guitar quartet. Subsequent subchapters will focus on the rationale behind the choice of scordatura (7.1), commentary on the musical contents of some movements (7.2), and notational issues relating to the full score and parts (7.3).

7.1. Choosing which scordatura to use

Before writing this piece, I looked at several microtonal pieces for unmodified guitars in solo and chamber contexts with a variety of scordatura settings. Each setting, when compared to the kind of music I have been writing (based on roundings of a variety of EDOs into subdivisions of 12NPO), presented different advantages and disadvantages.

Some settings, by having multiple guitars tuned a microtone apart (usually a quarter tone), but having each guitar tuned to itself in 12EDO, and having them playing simultaneously, permitted to use almost all the pitches of the overarching system (for the example given, 24EDO) throughout the entirety of the registers of all combined instruments. However, this would not allow for a certain degree of melodic integrity if I wanted to write melodic lines based on rounding.

Other settings, by having within a single guitar some strings microtonally retuned, allowed for more melodic integrity, but at expense of access to some pitches throughout the entirety of the register of that guitar.

Having considered these and other variables, and decided not to use different microtonal scordatura for each guitar, because of the extra layer of complexity that it would introduce,[127] I designed the following scordatura which would be used in all guitars:

[127] Georg Friedrich Haas's *Quartett für vier Gitarren*, which is based on having different microtonal scordaturas for each guitar (although each based on different Just Intonation chords), is an example of such a piece.

String	Amount of retuning	Final scordatura
E	-200¢	D
B	-50¢	Bd
G	+25¢	G^
D	(no retuning)	D
A	-50¢	Ad
E	-25¢	Ev

Table 2. The scordatura used in The Fable of the Pilgrim.

This scordatura has ‘eighth tone down’ pitches mostly in the lower register (in the retuned low E string) and ‘eighth tone up’ pitches tendentially in the middle register (in the retuned G string). Throughout most of the register, it also has semitones (in the D and retuned high E strings) and quarter tones (in the retuned B and A strings). With this I would have a vast variety of microtonal intonational possibilities at my disposal:

- a versatile register for 12EDO for whenever I do not want to have microtones (or play around with a ‘on/off microtonal switch button’, such as in the beginning of the third movement),
- a versatile register for 24EDO roundings of alternative EDOs (for what I perceive to be more xenharmonic distortions of approximated scales), and
- a less versatile register for 48EDO (for closer approximations of whatever EDO I want to use).

7.2. Commentary on some movements

This piece was planned to span multiple movements and, except for the last two, each movement was inspired by Tony Bayfield’s book *Being Jewish Today*.

It was suggested to me that this quartet sounds as a ‘magical realist’ piece. However, I chose a title which still makes a religious reference, while also alluding to the idea of fairy tales (a type of story I have always enjoyed).

While experimenting with this scordatura, I found several left-hand positions which correspond to roundings into 24 and 48NPO of sets of 5EDO and 7EDO. These positions sometimes sounded like chords (such as in Figure 47), other times like sets of a scale (like in Figures 48 and 49). I found their reverberance so powerful that I essentially sketched the *Incipit* and the first movement so that all of them would be used and repeated multiple times. This compulsion to play and hear these harmonies repeatedly ultimately led this piece to be, if not minimalistic, then repetitive. Given my interest in additive and subtractive rhythms, I considered them to be an appropriate strategy to explore variation with these repetitive materials.

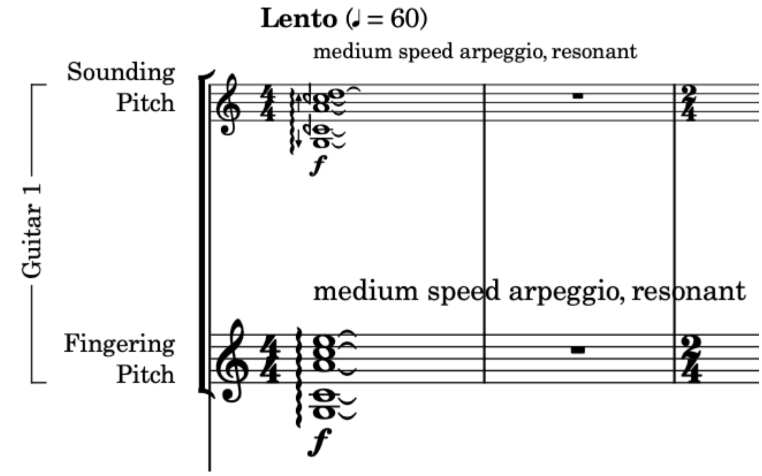


Figure 47. The initial chord of the Incipit *of* The Fable of the Pilgrim *on the first guitar, with pitches from 5EDO rounded into 48EDO, on D.*

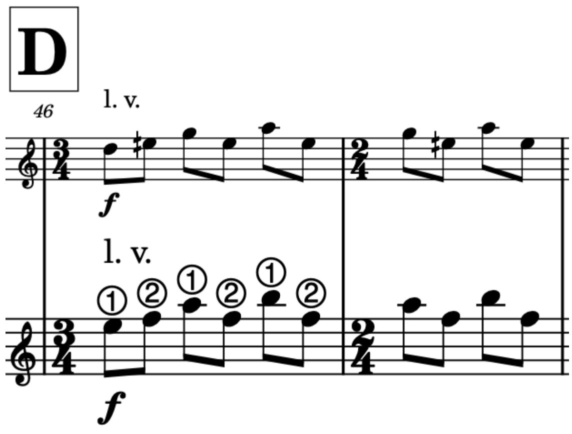


Figure 48. First guitar part on rehearsal mark D of the first movement of The Fable of the Pilgrim*, with pitches from 5EDO rounded into 24EDO, on D.*

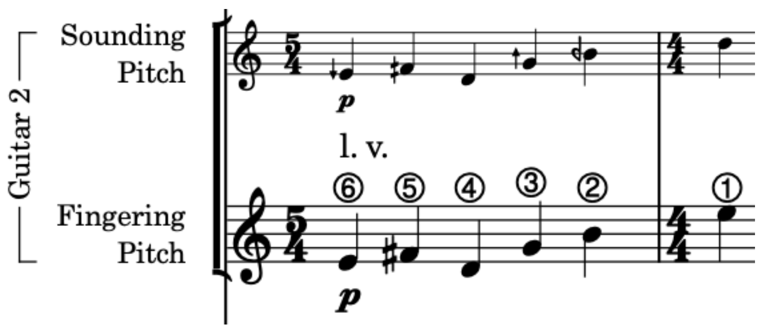


Figure 49. Second guitar part on the beginning of the first movement of The Fable of the Pilgrim*, with pitches from 7EDO rounded into 48EDO, on D.*

Regarding the first movement's form, given how these materials were generated, it also felt appropriate to apply a 'Stravinskian' approach by interlocking blocks of materials, regardless of EDO approximation. Curiously, to my perception, this did not lead to the sensation that tuning systems were being exchanged regularly, but instead harmonic fields within a unifying system. It is as if my ear recognized a common denominator between them, similarly to how switching from a major to a minor scale in 12EDO, to me, does not lead to the sensation of perceiving the new scale to be 'out of tune' from the previous one. This may be because 48EDO is the common denominator behind these structures (7EDO rounded to 48EDO and 5EDO rounded to 24EDO) which forces them to conform to certain common interval sizes. To my perception, my employment of rounding in this movement behaves similarly to how temperaments have historically been used to temper different scales in Just Intonation, making them conform into an overall structure.

In the second (entirely) and third movements (only partially), I wanted to explore a concept that had only been explored in my pieces with electric keyboard: rounding an EDO unrelated to 12EDO to another one also unrelated to 12EDO. Only after would the resulting scale be rounded into a subdivision of 12EDO, for performance. At the time, I was particularly curious of doing this with 5, 7EDO and 11EDO.[128]

The resulting 'tense dorian/orgone[7]' mode found when rounding 7EDO into 11EDO (Figure 50) seemed to me particularly appropriate for a slow movement (ideally lyrical) because its harmonic and melodic qualities evoke a sense of longing and sadness to me and I perceive it to be somewhat singable, given its melodic structure being close to that of the dorian mode. In the end, I don't think the second movement came out as lyrical, but it is still more melodically driven and transmits to me a sense of pain and longing (see Figure 51). This feeling is transmitted in the first guitar's melody from bars 7 and 8, which sounds mostly dorian, but whose quarter-tonally distorted fourth and fifth degrees add a particular tension. This is aided by the shape of the melody, which leaps to the tonic after an ascending scale from the first to its idiosyncratic fifth degree, and then resolves by descending to the dorian mode in D's idiomatic sixth degree: B. However, this is a somewhat masked resolution, as vertically this B lands on top of a Cv, making a (bright) stretched major seventh.

I enjoyed the resultant approximated sonorities in Figures 52 to 54 (which were used throughout this movement), despite not considering them to have grasped the 'tense

[128] See Appendix 7 for a systematization of this approach.

dorian/orgone[7]' sonority as I have it when playing it unapproximated on electronic media. They just sound too different to me, particularly the 12 and 24EDO approximations, which I used almost as often as the 48EDO approximation (which I perceive to be much more satisfactory), and therefore changing too much the overall sonority of the movement away from this scale. Another reason unrelated to pitch might relate to the fact that I play this scale usually with piano timbres constantly with a sustain pedal. Classical guitars, however, besides having different timbres, cannot sustain pitches for has long as a piano. Because of this, arguably this instrumentation may not have been the most appropriate to recreate the sonority of this scale with which I had been familiarised. Nevertheless, this may have been the first time in this research in which I consciously was not satisfied with the results of rounded material.[129]

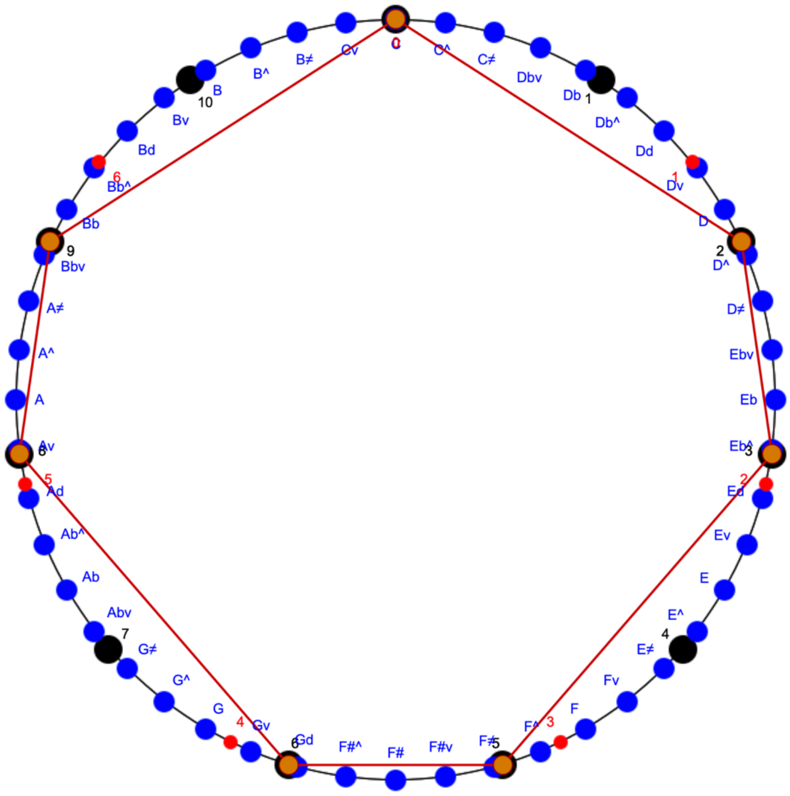


Figure 50. Rounding (polygon and orange dots) of 7EDO (red dots and numbers) into 11EDO (black dots and numbers) compared against 48EDO (blue dots and notation).

[129] See Chapter 9.4 for a further problematization of this.

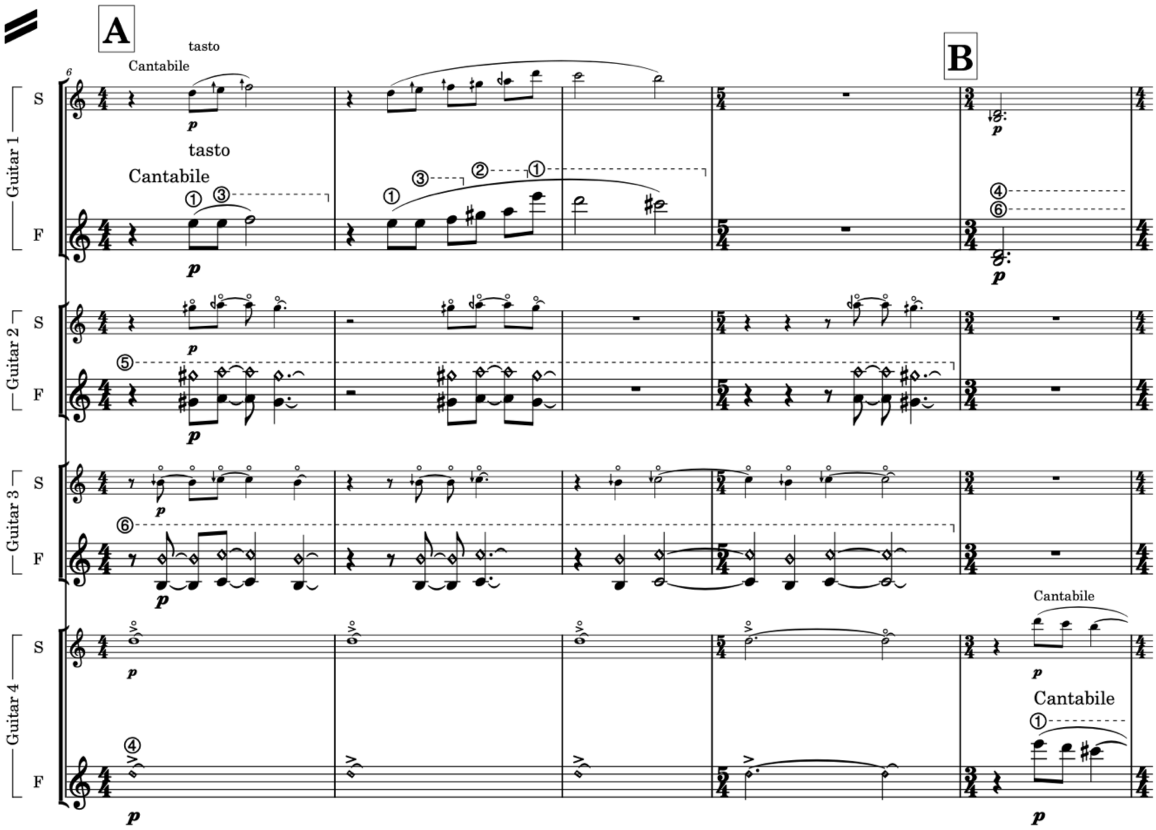


Figure 51. Rehearsal mark A and first bar of rehearsal mark B of the second movement of The Fable of the Pilgrim*. Except for the first guitar in bar 8 and the fourth guitar in bar 10 (both in 7EDO rounded to 11EDO rounded to 12EDO on D), this passage is written in 7EDO rounded to 11EDO rounded to 48EDO on D.*

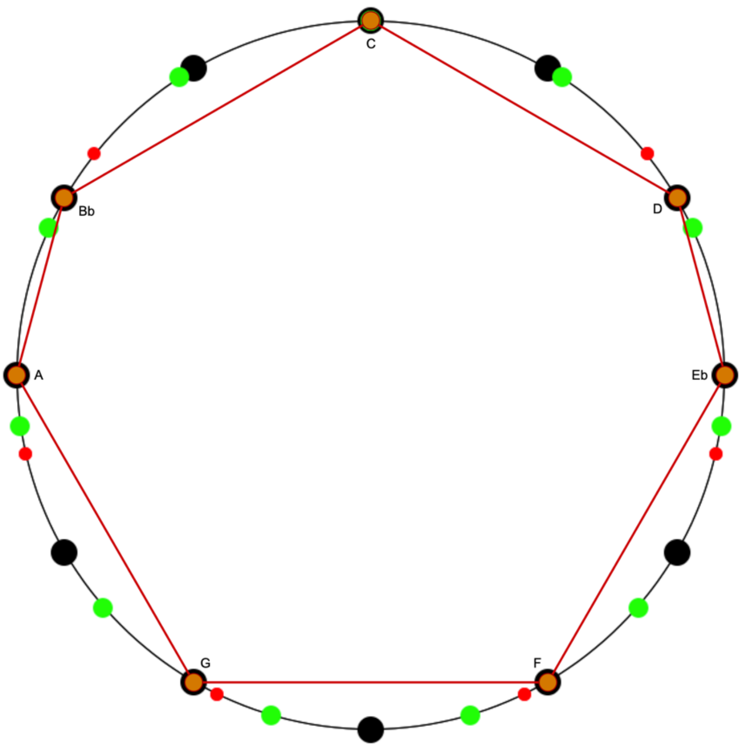


Figure 52. Double rounding (polygon, orange dots and notation) of 7EDO (red dots) into 11EDO (green dots) into 12EDO (black dots).

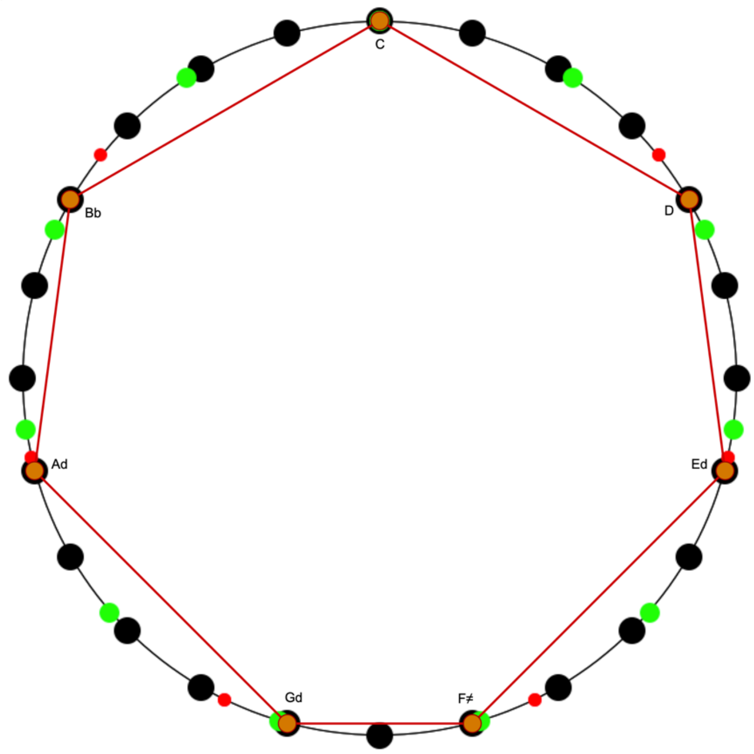


Figure 53. Double rounding of 7EDO into 11EDO into 24EDO.

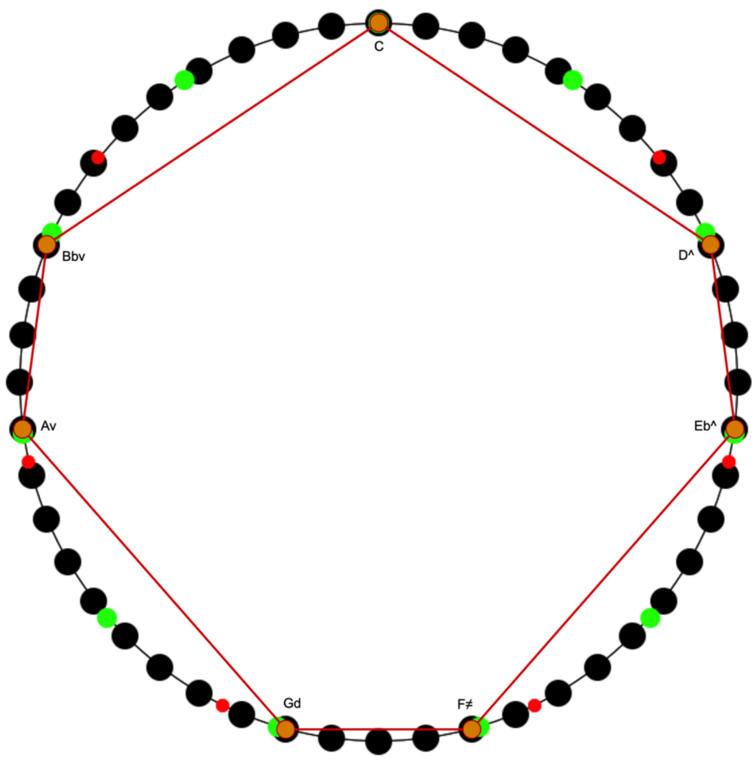


Figure 54. Double rounding of 7EDO into 11EDO into 48EDO.

While striving to write a fast and energetic third movement, I found that combining this 'double rounding' approach with 5EDO rounded into 11EDO rounded into subdivisions of 12EDO

(Figures 56 to 58) alongside the more explored ‘single rounding’ approach, with 5EDO rounded directly into 12EDO subdivisions, was particularly successful. I perceive the resulting ‘whole tone pentatonic scale’ (as I name it, given that it sounds to me as a hybrid between the pentatonic and whole tone scales) of 5EDO rounded into 11EDO (Figure 55) to have a certain energy, which is aided by the polyrhythmic and repetitive ascending and descending scales and trills with approximations of it (see Figure 59).

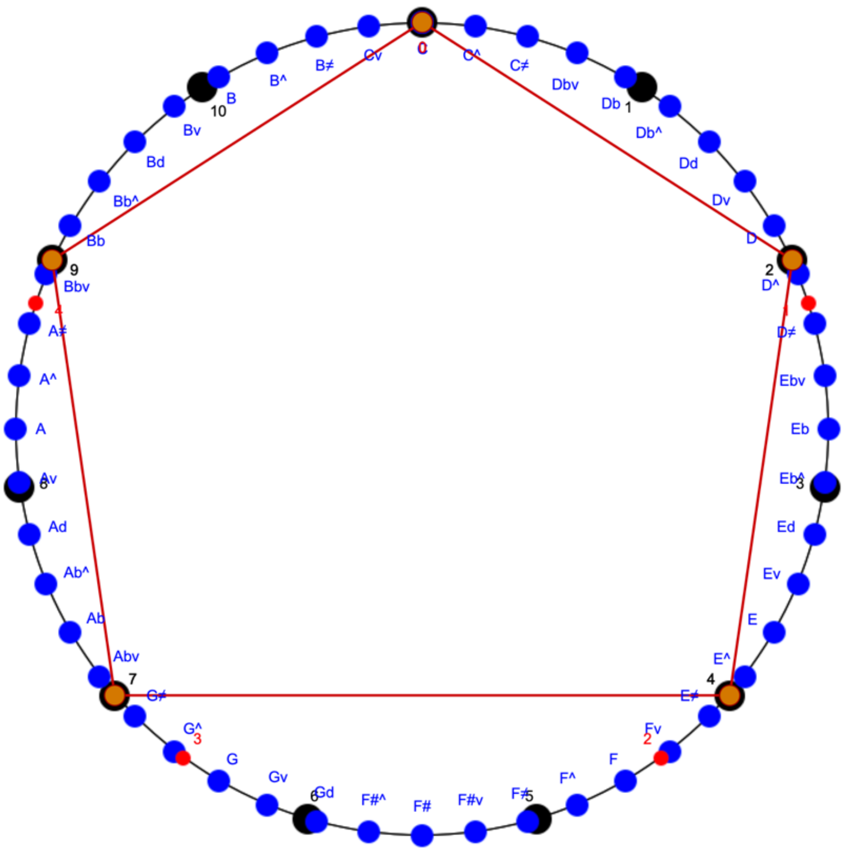


Figure 55. Rounding (polygon and orange dots) of 5EDO (red dots and numbers) into 11EDO (black dots and numbers) compared against 48EDO (blue dots and notation).

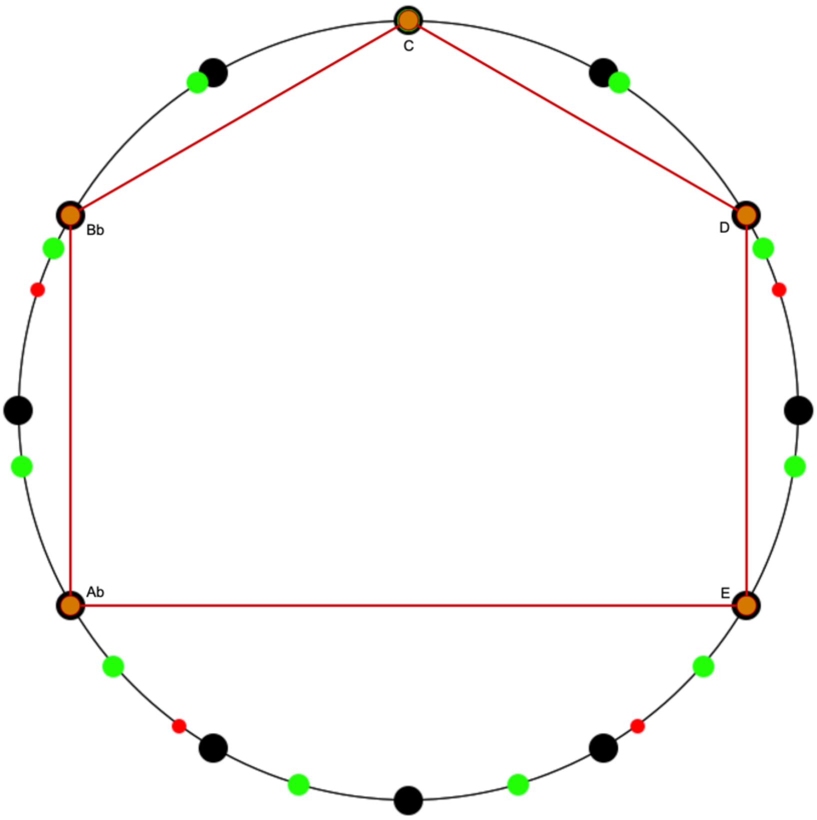


Figure 56. Double rounding (polygon, orange dots and notation) of 5EDO (red dots) into 11EDO (green dots) into 12EDO (black dots).

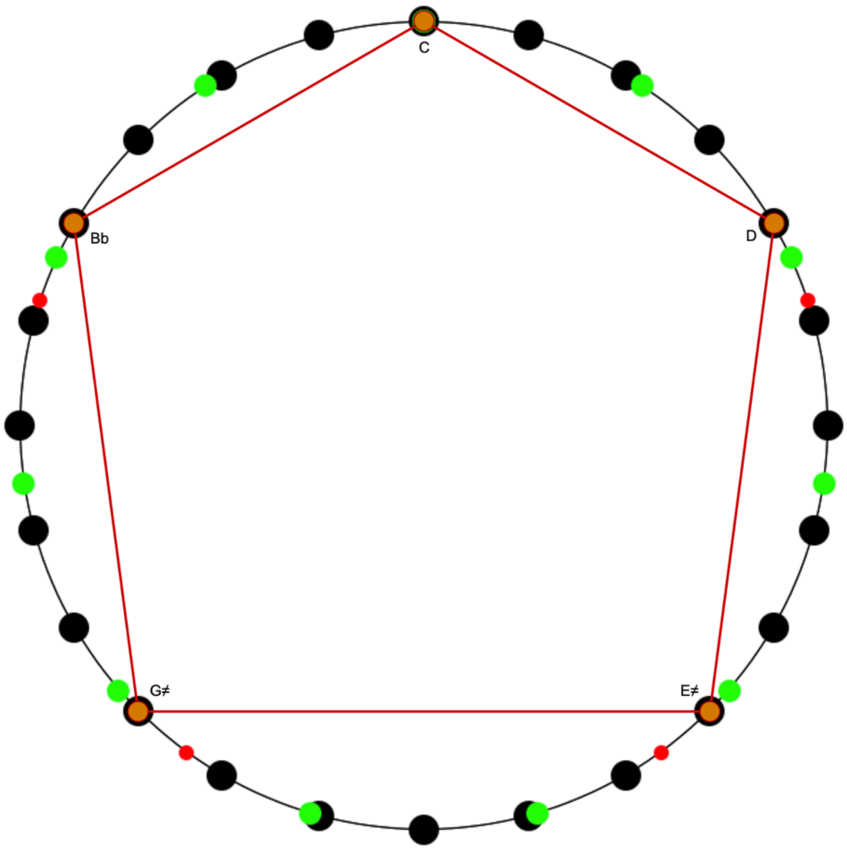


Figure 57. Double rounding of 5EDO into 11EDO into 24EDO.

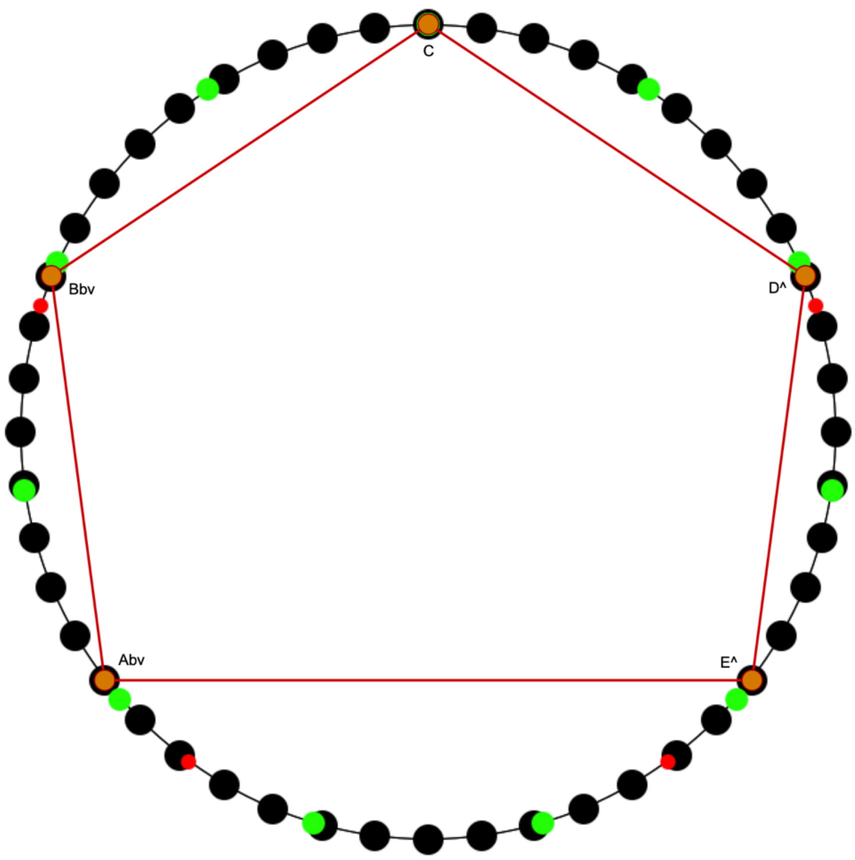


Figure 58. Double rounding of 5EDO into 11EDO into 48EDO.

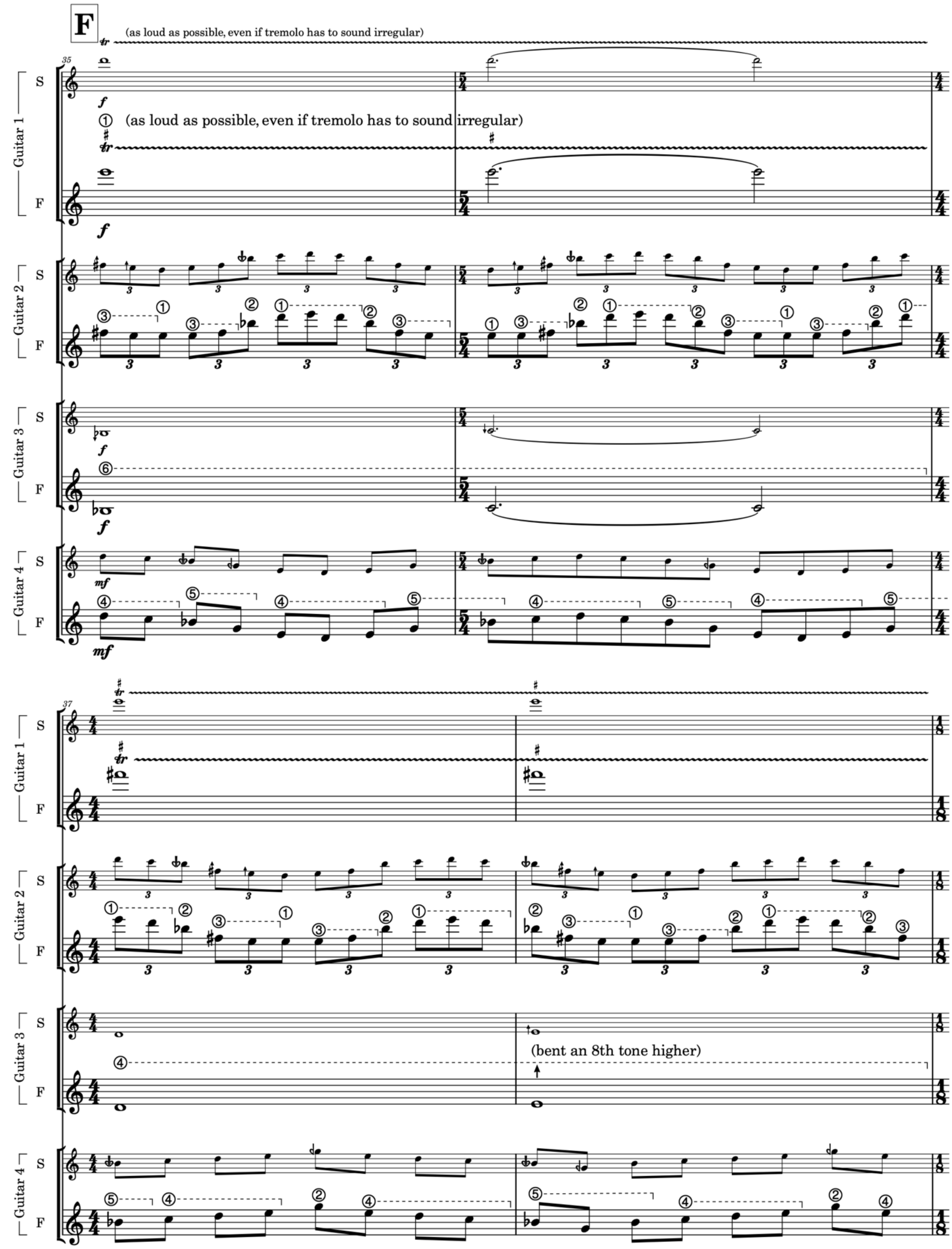


Figure 59. The first four bars of rehearsal mark F of the third movement of The Fable of the Pilgrim. *In this passage, each guitar is mixing, according to convenience, different double roundings of 5EDO rounded to 11EDO rounded to 12, 24 and 48EDO, on D.*

My perception of the different simultaneous roundings of this scale into 12, 24 and 48EDO, spread into all of the different guitars, is that they are much more successful than when this same strategy was applied for the 11EDO orgone[7] scale in the second movement, possibly because of the much faster speed of performance, which might reduce the ear’s capacity to distinguish that different roundings of the same materials are being employed. From my experience of having written the second and third movements of this piece, I propose that more precise intonational precision is desirable for slower music, and a more blurred hybrid approximation approach of a microtonal scale can be effective at faster speeds, since intonational deviations between different simultaneous roundings of the same scale would be less noticeable.

However, there are passages in this third movement in which repetitive single roundings into 12 and 24EDO of the same musical material (based on 5EDO), followed by additive and subtractive rhythmical variations of the initial 48EDO rounding of a 5EDO chord (from the *Incipit*), are used not simultaneously, but in blocks in a quasi-minimalistic context (see Figure 60).

Figure 60. Rehearsal marks A and B of the third movement of The Fable of the Pilgrim. *Rehearsal mark A intermixes 5EDO rounded to 12 and 24EDO. Rehearsal mark B uses 5EDO rounded to 48EDO.*

This intonational variation, while happening at a fast speed, is quite noticeable because it does not happen simultaneously, but in consecutive blocks.

I think that in this movement I finally felt that my creativity was completely unbounded by the schemes I had set up (even though I used them). However, I believe this was due mostly to my rhythmical explorations.

(The last movement is not addressed in this commentary because it was not written based on microtonal approximations of any particular equal tuning, even though, sometimes, it approximates some Just Intonation harmonies. Instead, it was freely written based on the 48EDO scordatura that was designed for this piece. Nonetheless, the last comment I made for the third movement also applies to this one.)

7.3. Notation and parts

From my experience on reading microtonal music for retuned (12EDO) guitars, I find it confusing to read a score when it is only written in 'concert pitch', because it is not clear in which strings each pitch should be played. Therefore, a notation approach which shows where each note should be played would still be needed for this piece. This could be achieved either with tablature, or by adapting my notational approach for microtonal keyboards from *Adam* and *Dreams from an Old Memory*, by adding an extra 'scordatura' system[130] with fingering and/or fret position markings. The latter was chosen for convenience. Similarly to the keyboard parts of previous pieces, with this dual notational approach, performers could anytime see how a note is to be played and how it is supposed to sound.

Initially all parts were made with this dual notation. However, attending to the feedback of the performers that would eventually play the piece, a second version of the parts with only the 'scordatura notation' was also produced. With this new version, fewer page turnings would be needed, and the parts would look less cluttered. During the earlier rehearsals, all performers used the dual notation parts, and in the latter rehearsals and concert, one performer migrated to a doubled sized printed 'scordatura only' part. Two of the performers chose to use the dual notation parts because they were using tablets and pedals to turn pages. Only one performer chose to use a double sided printed 'dual notation' part. All this variety of personal preferences

[130] See the performance notes of *The Fable of the Pilgrim* for a detailed description of its notation.

regarding parts gives even more reason that, for microtonal music, it is extremely important to be as conscientious as possible when making parts.[131]

Typos were easily found by the performers in the full score and parts during the rehearsal process exactly because of this notational approach. The performers were aware, through the notational system, how each note would sound and should be played and, therefore, any incongruences could be easily identified. I consider that, if this can happen, then the notational approach is effective.

[131] See Figures 110 and 111 in Appendix 10 for excerpts of both the 'dual' and 'scordatura' part versions of this piece.

Section 3: Mixed ensemble pieces

The following chapters will discuss the composition process of an unfinished piece for wind quintet and electric guitar, 'Piece Z' (Chapter 8), *Seeking Gnosis* for mixed ensemble (Chapter 9), a reflection on the emergence of a dual microtonal composition practice (Chapter 10) and *Convergence* for sinfonietta (Chapter 11) which includes, as its first movement, an adapted and finished version of 'Piece Z' (subchapter 11.2).

Chapter 8. 'Piece Z'

This chapter will address the composition process of an unfinished piece for wind quintet and electric guitar (hereafter referred as 'Piece Z'). Subsequent subchapters will focus on early technical considerations in relation to its instrumentation (8.1), and a commentary of selected music excerpts (8.2). A sketch score of this piece can be found in Appendix 8.

8.1. Context within the research project and early technical considerations

I worked on 'Piece Z' for three months. Further development of my rhythmical language and use of form was one of the goals behind this piece. I was also aware (also by experience) that woodwind intonation tends to be more unpredictable than that of strings. Therefore, an instrument with a more predictable intonation (retuned electric guitar) was included to serve as a pitch reference for the other performers.

Having already employed retuned guitar strings and studied compositions with retuned wind instruments, particularly Julian Anderson's *Eden*, I recognized that these strategies share similarities. Each guitar string can only play in 12EDO with itself melodically, but if tuned microtonally against other strings, this would be like having some wind instruments retuned microtonally with others who are not retuned.

Instead of using alternative fingerings, this concept, when applied to approximate EDOs in woodwinds, in theory would reduce the difficulty level of performance. The strategies for each instrument were then planned as follows:

- Flute: 12NPO
- Oboe: 12NPO

- Clarinet: retuned a quarter tone lower
- Bassoon: either retuned an eighth or quarter tone lower, or relying on a small amount of microtonal fingerings
- Horn: a mixture of 12NPO and quarter and eighth tones through alternative fingerings and natural harmonics
- Electric guitar: uses the same 48EDO scordatura from *The Fable of the Pilgrim*

This seemed to be a balanced foundation to explore microtonality. However, the pitch instability expected from the woodwinds' microtones was proving to be incompatible with my repetitive rhythmical gestures. I found myself constantly writing music with which I was not satisfied to make it performable. The instrumentation was proving to be too restrictive for such music. In contrast, *The Fable of the Pilgrim* allowed for a much more adventurous exploration of microtonality without having to constantly worry about performability. In the end, I decided to stop writing this piece and work on the next piece, *Seeking Gnosis*. However, the process of writing Piece Z led to the exploration of approximation strategies that had not been used in previous pieces which deserve careful consideration. Also, after finishing *Seeking Gnosis*, part of 'Piece Z' was adapted into what would become the first movement of *Convergence* for sinfonietta.[132] Therefore, two excerpts from passages that were incorporated into *Convergence* are analysed bellow.

8.2. Commentary on two excerpts

8.2.1. From an improvised guitar chord to 17EDO

In previous pieces, I always started their composition process by first choosing which tuning systems to use and then writing music based on approximations of materials that can be found in them. This time, I chose to 'reverse-engineer' this process and instead start by finding material which I enjoyed, and only later find a tuning system (or tuning systems) which could include approximated versions of it. By doing so, my goal was to generate material which otherwise I might not have reached and then use the structure of whichever compatible tuning system(s) I would choose as a basis for possible ways to develop it. Through improvisation on

[132] See Appendix 8 for the last sketch of this piece and Chapter 11.2 for the commentary of the adapted version of it in *Convergence*.

a retuned guitar, I developed what would become the guitar riff from rehearsal mark E from the final sketch of the piece (see Figure 61), as well as the rhythmically increasingly complex repetitive rearticulations of its harmony in the wind parts (see Figures 62 and 63).

Figure 61. The electric guitat riff from rehearsal mark E of 'Piece Z'.

Figure 62. Bars 98-110 of 'Piece Z'.

Figure 63. Bars 125-130 of ‘Piece Z’.

By analysing this riff in 48EDO, I could notice that its harmony approximates some partials of the harmonic series in Ed (8-Ed, 10-G^, 11-A, 12-Bd, 18-F≠ and 22-A), but also includes a ‘fake octave’ (Ev/Ed) and a neutral seventh (Ed/D). I went to Equal Tuning Lab to find an equal tuning which could approximate this harmony to my satisfaction and noticed that I was struggling to find a system which included all its intervallic characteristics. This suggests that, indeed, if I had started the composition process of this piece by having chosen a tuning system beforehand, I would likely not have written this guitar riff. Ultimately, I chose 17EDO for the convenience of it not having too many pitches, repeating at the octave, how many intervals from this riff it can approximate (except for the ‘fake octave’), and because it is one of my favourite EDOs (which I had not yet used in this research).

8.2.2. Expanding the piece with 17EDO and Lumatone experiments

Even though I had already acquired a Lumatone around a year before working on this piece, it was only around this time that I became proficient working with Bosanquet-Wilson mappings (by generating them on the *Hexagonal Keyboard Layout* webapp[133] and recolouring them) to a point where it started influencing the way I conceptualize pitch. One example relevant to this piece relates to how I had started recolouring a Bosanquet-Wilson mapping of 17EDO to help me understand its intervallic qualities when rounding it to subdivisions of 12NPO.

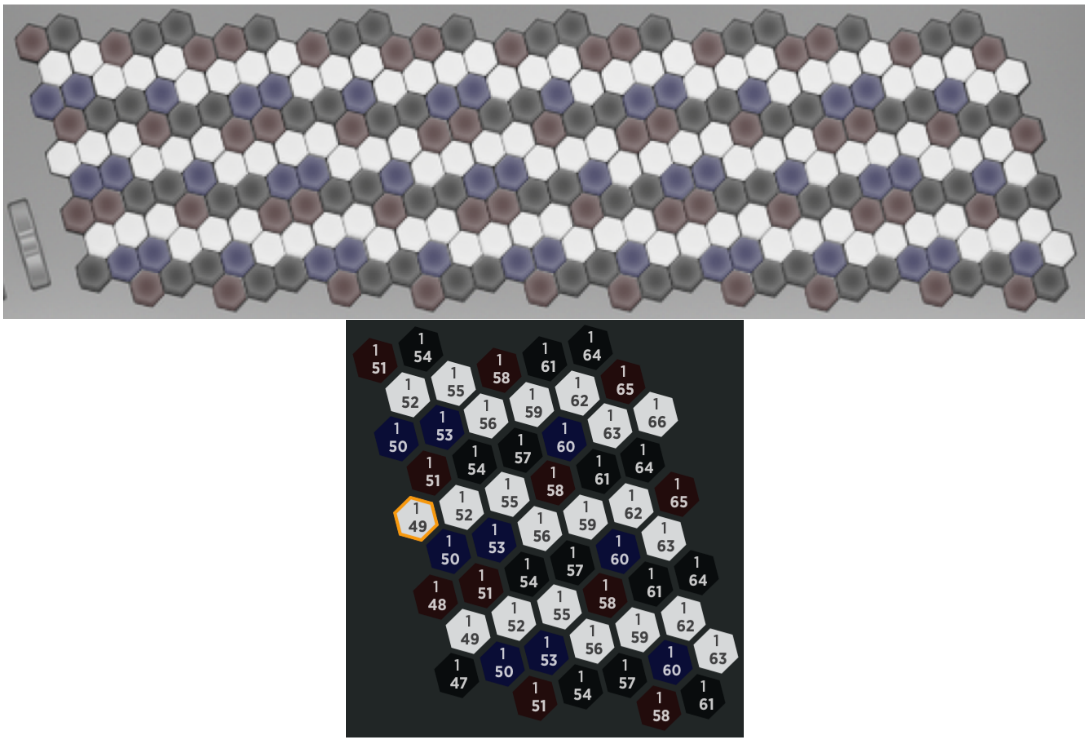


Figure 64. A Bosanquet-Wilson mapping of 17EDO with each key colour corresponding to a specific accidental in Circle of Fifths Notation. White keys correspond to natural notes, dark red to sharp notes, dark blue to flat notes, and black to pitches that are both sharp and flat notes enharmonically. Middle C is mapped into MIDI note 56. Note that Bosanquet-Wilson mappings which imply Circle of Fifths Notation do not suggest how each pitch sounds. Instead, they identify and locate pitches within a circle of fifths of variable cent sizes.

133 Sjoerd Visscher, 'Hexagonal Keyboard Layout', online computer application <https://sjoerdvisscher.glitch.me/keyboard/> [accessed 9 August 2023].

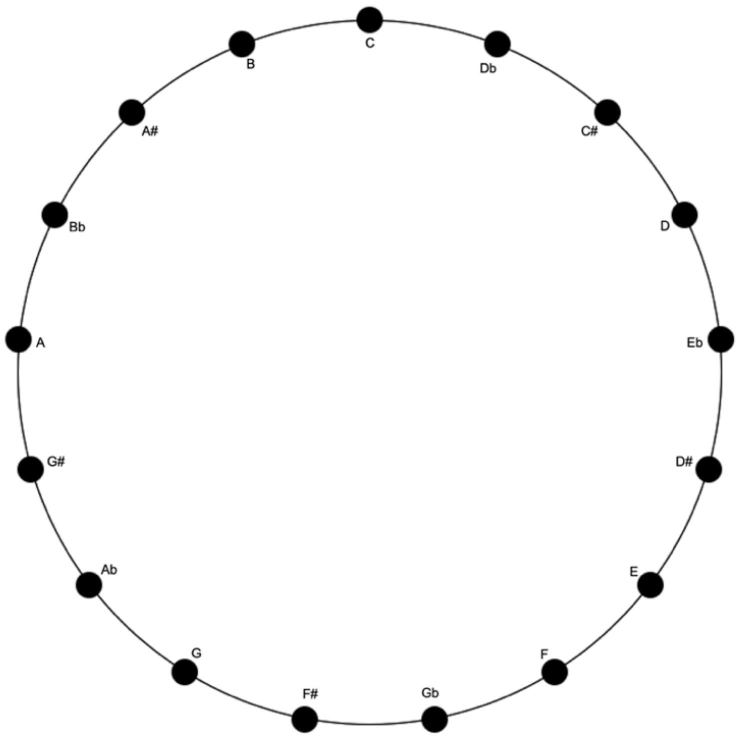


Figure 65. 17EDO notated with Circle of Fifths Notation.

Recolouring the keys of this mapping to represent the 24 and 48EDO roundings of each pitch of 17EDO helps finding which pitches are closer to semitones and which are to quarter and eighth tones (see Figures 66 and 68). This, I found, could be useful in distributing pitches amongst retuned and non-retuned instruments, with and without alternative fingerings.

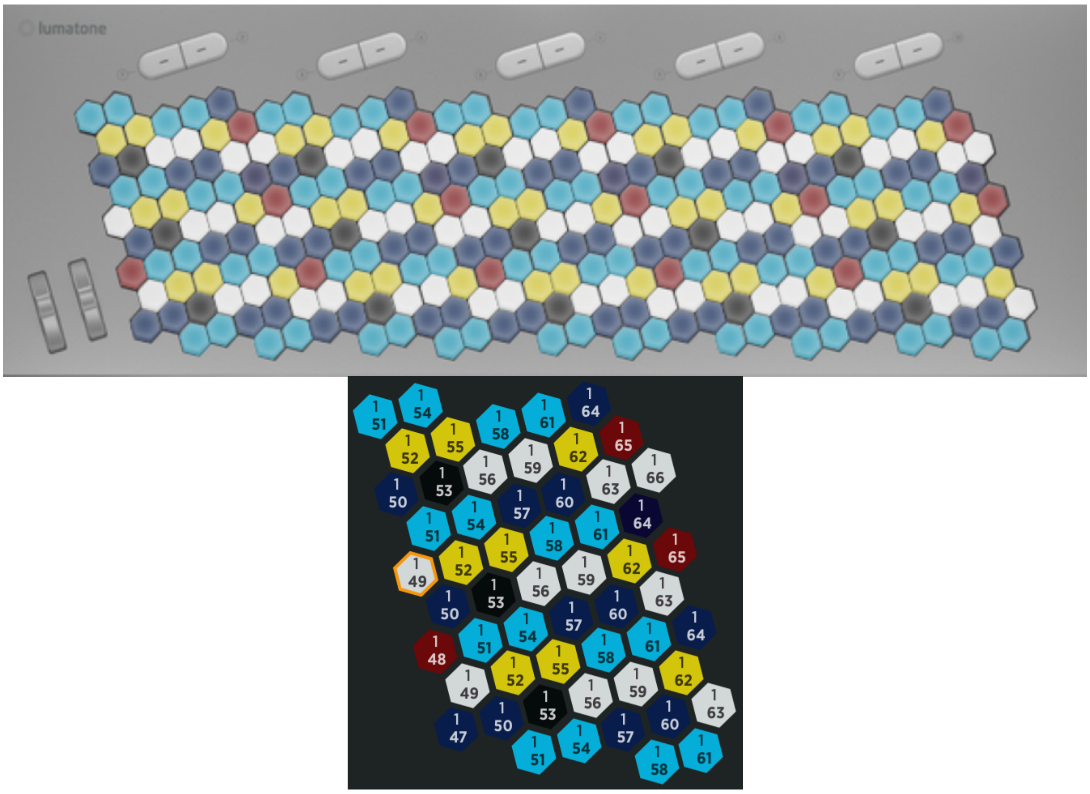


Figure 66. The same Bosanquet-Wilson mapping of 17EDO from Figure 64 but recoloured to represent the 48EDO rounding of each pitch. White keys correspond to pitches closer natural notes, black to sharp/flat notes, light blue to quarter flat notes, yellow to natural notes raised by an eighth tone, dark red to sharp/flat notes raised by an eighth tone, and dark blue to sharp/flat notes lowered an eighth tone. There are no pitches which would be rounded as quarter sharp notes.

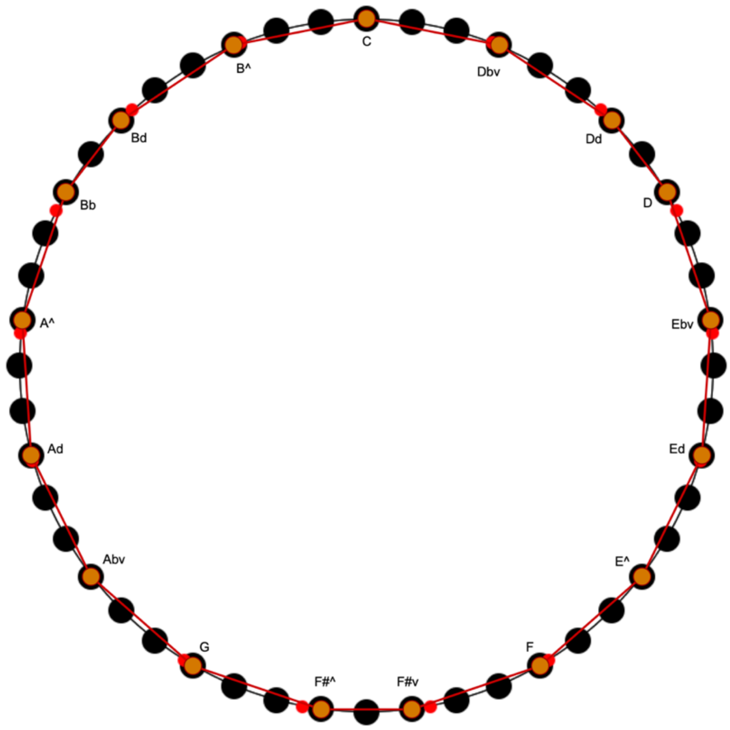


Figure 67. Rounding (polygon, orange dots and notation) of 17EDO (red dots) into 48EDO (black dots).

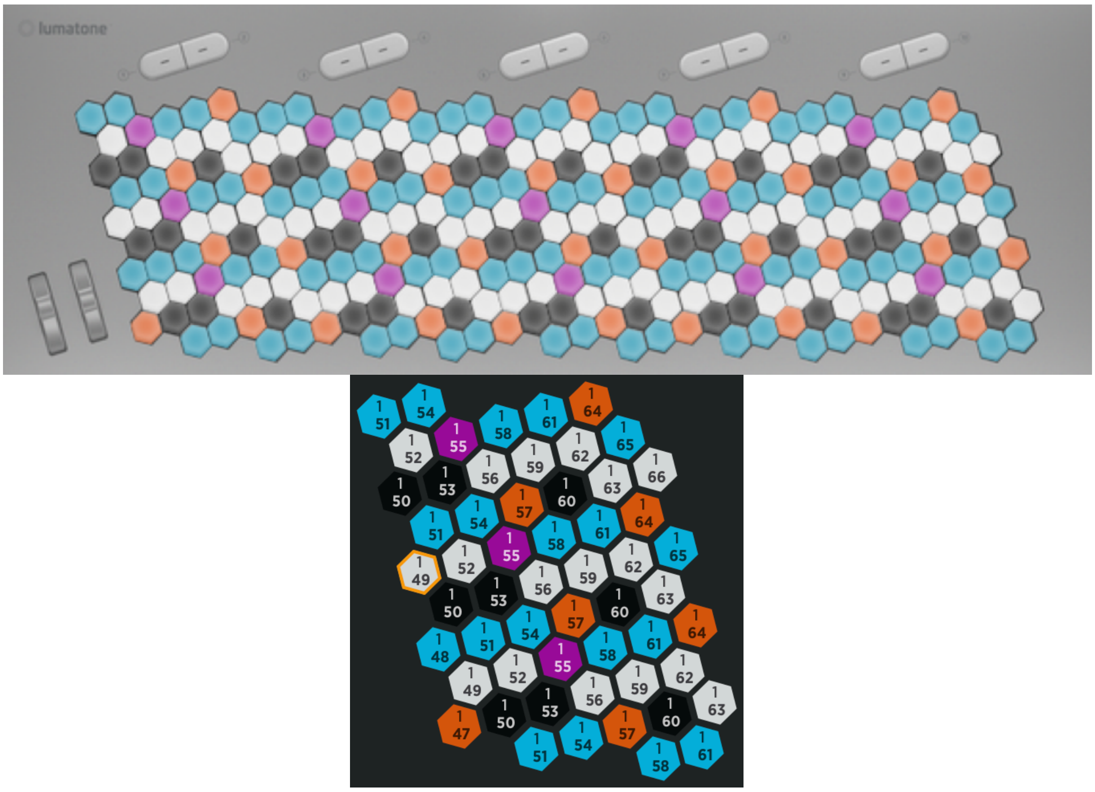


Figure 68. The same Bosanquet-Wilson mapping of 17EDO from Figure 64 but recoloured as to represent the 24EDO rounding of each pitch. White keys correspond to pitches closer natural notes, black to sharp/flat notes, orange to quarter sharp notes, blue to quarter flat notes, and purple to notes that are enharmonically both quarter sharp and quarter flat.

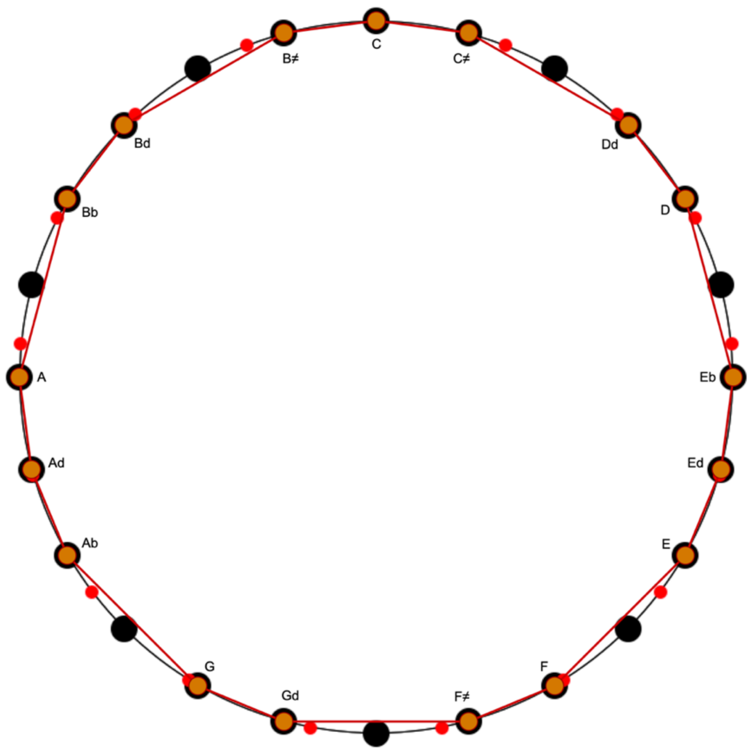


Figure 69. Rounding of 17EDO into 24EDO.

Each of these mappings highlights different intervallic aspects of 17EDO's diatonicism. The mapping from Figure 68 helps in identifying diatonic and pentatonic scales composed either of only quarter tone notes or semitones. The mapping from Figure 64 helps in identifying diatonic-sounding scales irrespective of how their pitches would be rounded in 24 or 48EDO. The mapping from Figure 66 suggests that the major thirds, sixths and sevenths of the 17EDO diatonic scales are somewhat stretched by an eighth tone, in comparison to their 12EDO counterparts, while the other intervals its diatonic scales do not deviate as much from 12EDO. I found this multimodal approach to diatonicism (as highlighted by these mappings) intriguing and used it to sketch the beginning of the piece.

Having already sketched a predominantly harmonic passage; it would now be reasonable to develop more melodically driven material. Partly inspired by aspects of Christopher Bochmann's 'isobematic' music,[134] particularly his linking of pitch with durations through numerical equivalencies, this initial section was designed to resemble this to some extent. For each instrument, the duration of each pitch would be determined by identifying the number of steps in 17EDO forming the melodic interval between that pitch and the following one. One 17EDO step would then be associated with one crotchet. After this, all pitches would be rounded to 24NPO. Different instruments would then repetitively play a fixed set of pitches from two different pentatonic scales:

- The Flute plays C, F and G (see Figure 70). Following this logic, in 17EDO the interval C-F corresponds to 7 crotchets, F-G to 3, and G-C to 10. In 12EDO, the durations would be 5, 2 and 7.

[134] '(…) a subsystem of atonal music which I call Isobematic (…). In this language, all elements of each musical aspect have an equal interval (they make a continuum) and equal importance.' Original text: "(...) um subsistema da música atonal que chamo de Isobemática (…). Nesta linguagem, todos os elementos em cada aspecto da música têm um intervalo igual (criam um continuum) e uma importância igual. (My translation). Christopher Bochmann, 'O Ritmo Como Factor Determinante Na Definição de Linguagens Musicais Do Século XX', *Modus*, 6, 2006, pp. 185–96. p. 191.

- The Oboe then adds the D and a Bb (bar 6 onwards), forming a pentatonic scale with the Flute.

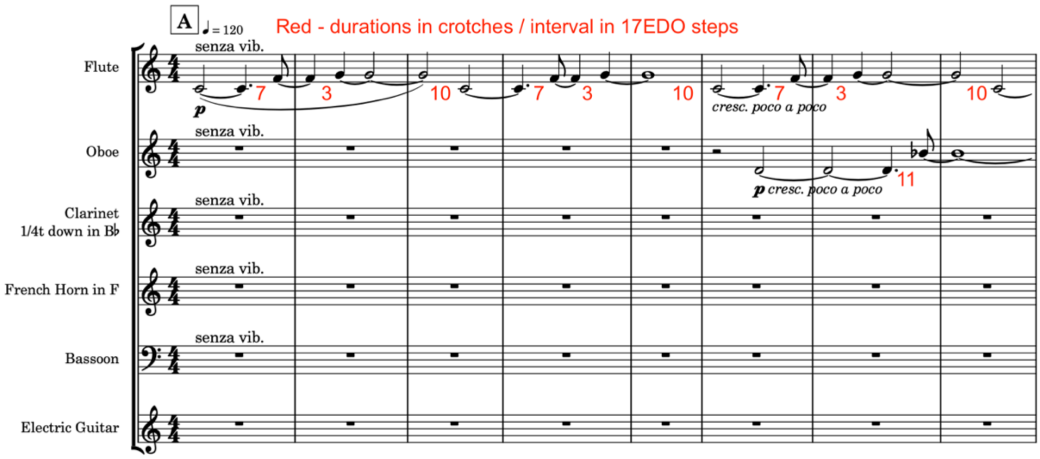


Figure 70. Bars 1-8 of 'Piece Z'.

- The Clarinet then plays Ad and Ed (bar 14 onwards).

Figure 71. Bars 9-22 of 'Piece Z'.

- The Horn then forms an incomplete pentatonic scale with the Clarinet by playing Dd and Bd (bar 24 onwards)

Figure 72. Bars 23-37 of 'Piece Z'.

By following this process, I reached a repetitive but fluid 'wave like' poly-pentatonic texture simulating 17EDO. This strategy has the advantage of presenting only a limited number of pitches per part and, given the simplicity of the horizontal intervallic relationships in each part, this could be a viable and idiomatic approximation technique for performance.[135]

In the following passage (such as in bars 49 to 65, in Figure 73), I used a hybrid approach instead: using rounding into 48NPO and 24NPO to approximate 17EDO harmonies (by relying on the modified Bosanquet-Wilson mappings of Figures 66 and 68), while writing diatonic melodies in the Oboe in 12NPO, based on the suggested Circle of Fifths Notation in the 17EDO Bosanquet-Wilson mapping of Figure 64 (which would look indistinguishable from 12NPO notation). The intention was that the oboe's intonation would gradually converge towards 12EDO. Being aware of how harmonic and melodic intonation tends to differ in string

[135] See Chapter 11.2 for comments on the adaptation of this passage in *Convergence*.

technique, I deduced that following such a hybrid approach could help trick my ear into believing that I was actually listening to music in 17EDO. Even though, in practice, this piece was written as a subset of 24NPO, my ear finds this rendition to be acceptably close to 17EDO. However, I need to recognize that other listeners trained to hear music in 17EDO might disagree, since this piece was, in fact, not written in 17EDO, but in a simulation of it in 24NPO.

Figure 73. Bars 49-65 of 'Piece Z'.

(The reader may now choose to progress to Chapter 9 for the commentary of *Seeking Gnosis* or jump to Chapter 11.2 to read about the adaptation of 'Piece Z' into the first movement of *Convergence.*)

Chapter 9. *Seeking Gnosis*

This chapter will address the composition process of *Seeking Gnosis* for mixed ensemble. Subsequent subchapters will focus on early considerations taken between choosing to leave 'Piece Z' unfinished and starting to write this piece (9.1), its early sketching process (9.2), the rehearsal process and recording session (9.3) and the gradual rejection of rounding as the main tool to explore approximations of equal tunings (9.4).

9.1. Early technical and conceptual considerations

Given what I considered to be the shortcomings of 'Piece Z', I wanted to retry some of the strategies from that piece, but recontextualizing them and adjusting them. I then chose a new instrumentation including woodwinds, alongside other instruments. The electric guitar scordatura used in 'Piece Z' and *The Fable of the Pilgrim* still seemed worth exploring; it was thus incorporated into this new work. I also decided that alternative fingerings would be employed instead of retuned woodwinds. Also, my rhythmical language was becoming more complex and gradually relying on metric modulations, additive and subtractive repetitive rhythms and irrational time signatures. For this reason, I included percussion so that it would serve a role like that of a drum set in the new work.

Another goal for this piece was to improve how I work with form. To this point, my tendency had been to write either short single-movement pieces or longer pieces with multiple movements. *Adam* was an exception, because it has a libretto. I ended up following a variation of a 'Stravinskian' block approach (like in the *Symphonies of Wind Instruments*) by interlocking different textures, which develop through time.

9.2. Early sketching process of two passages[136]

Being inspired by aspects of Gnostic theology, I started to wonder what kind of music could represent these ideas and started sketching the beginning of this piece. I decided that resonance would be a central part of (it to represent infinity), and that a fluid rhythmical structure would be an adequate way of developing musical material. As previously stated, I find it compelling to relate different musical parameters into a single principle. I then decided that the ratio 11/7 would represent this principle[137] and initially applied it in the cello part with material derived from the 4ED11/7 tuning/scale approximated into 72NPO, but not always through roundings (see Figure 74).[138]

This cello solo was then developed to move from a consonant 5/4 major third (which serves as a ‘intervallic tonic’) and, through ‘zig-zag-like’ melodic and dynamical movements, widen its vertical intervals over G. This process gradually achieves more dissonant intervals (while establishing the 7/4 minor seventh as the new ‘intervallic tonic’) and culminates in a climax around the ambit of the octave – an interval which, in its 2/1 ‘perfect’ form, does not exist in this scale – in many respects the most dissonant intervallic region of this passage.

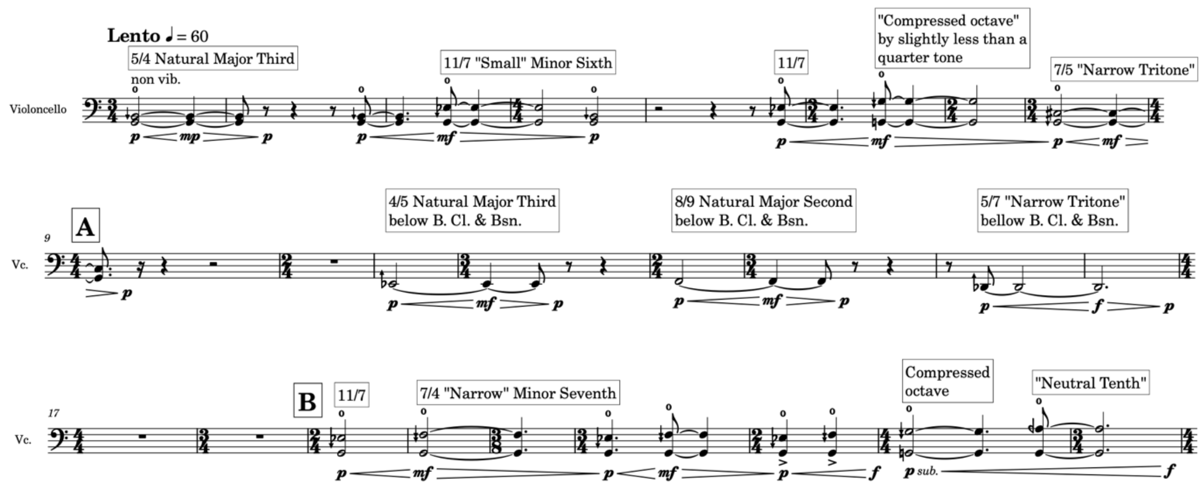


Figure 74. Cello part in bars 1-25 of Seeking Gnosis*.*

[136] See the performance notes of *Seeking Gnosis* for brief descriptions of musical structures used in other sections of this piece.

[137] This 11/7 ratio was inspired by Adam Neely, *7:11 Polyrhythms*, online video recording, YouTube, 25 February 2019 <https://www.youtube.com/watch?v=U9CgR2Y6XO4> [accessed 3 August 2023].

[138] See Chapter 9.4 for further reflections on my gradual abandonment of rounding as the main approximation technique.

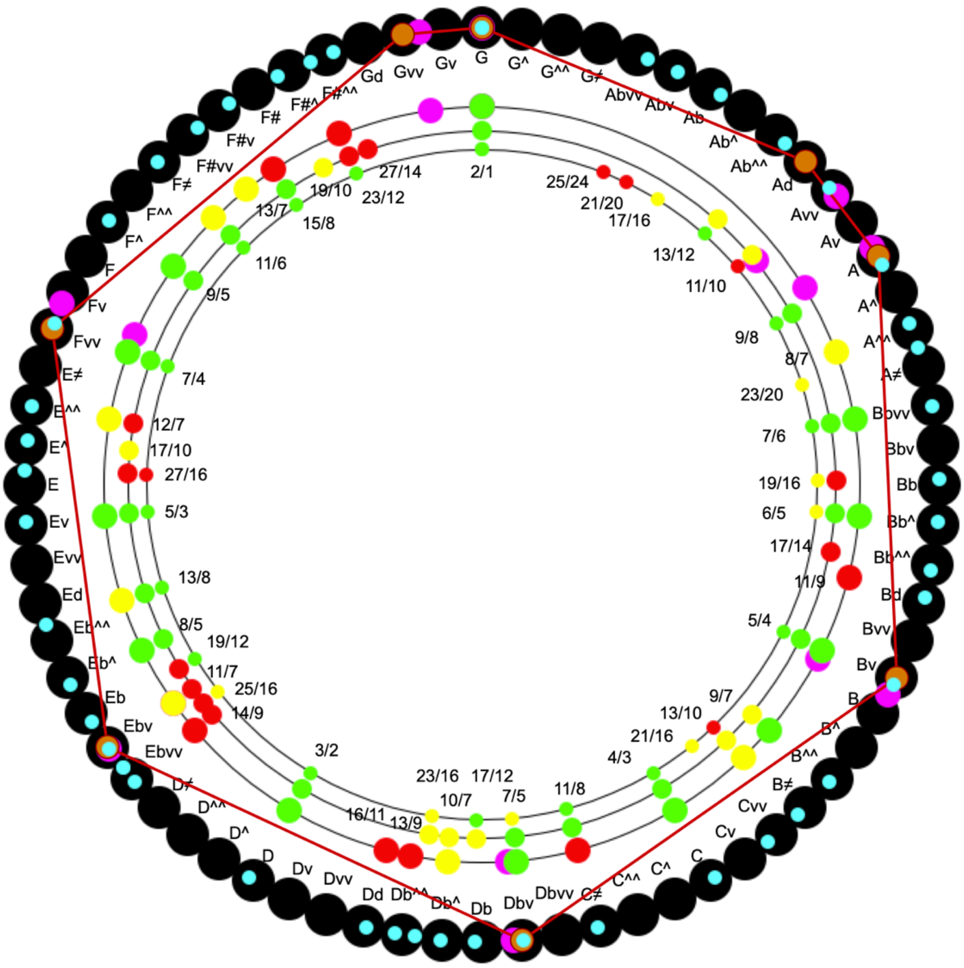


Figure 75. The first pitches of the 4ED11/7 tuning/scale (magenta dots in the three inner circles, and their respective pitch class projections in the outer circle), against 72NPO, Sabat ad Hayward's tuneable intervals (green, yellow and red dots) and their interval class projections (cyan dots), and the pitches of 72NPO chosen to approximate this scale in the cello part (orange dots and red polygon). Not all pitches of this tuning/scale were literally rounded into 72NPO.

From this unifying principle, several other musical structures were derived and employed beyond this initial section in different ways.[139] The sections from the beginning up to rehearsal marks F, H, O, and T-V are inspired by the aforementioned religious influences. From these influences I imagine a tendentially static and resonant music heavily based on Just Intonation intervals. Having employed 4ED11/7 (a 'compressed whole tone scale with near pure major thirds') on the cello part in the beginning of the piece, I wanted to write an even more static variation of this material in the viola (see Figure 76).

[139] See the performance notes of this piece for a description of these structures.

Figure 76. Viola and cello parts on rehearsal marks T, U and V of Seeking Gnosis.

Therefore 2ED11/7 ('a compressed augmented chord with near pure major thirds') was chosen as an ideal structure to explore since it is included inside 4ED11/7, and because it presented an appealing notational challenge.

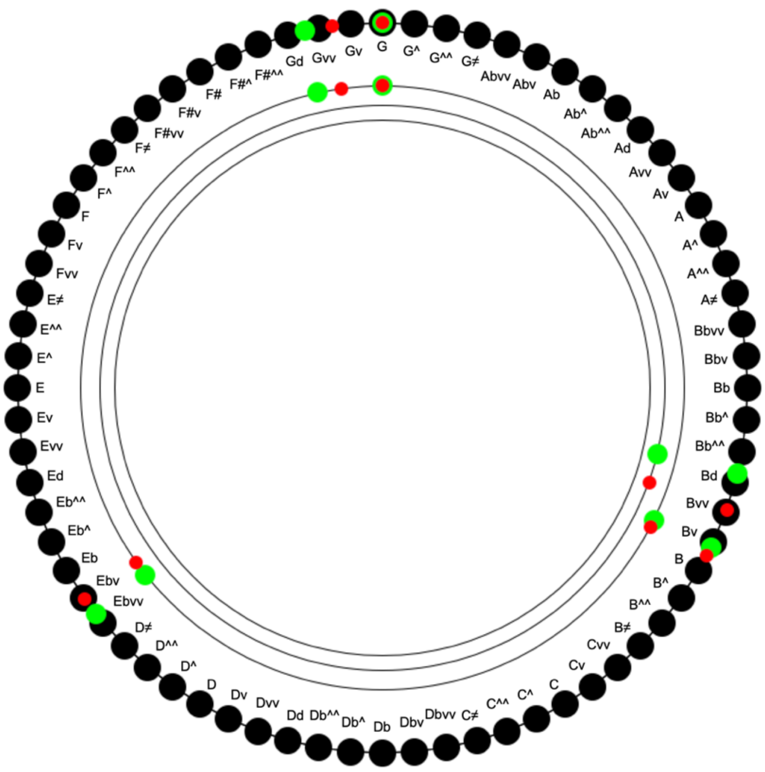


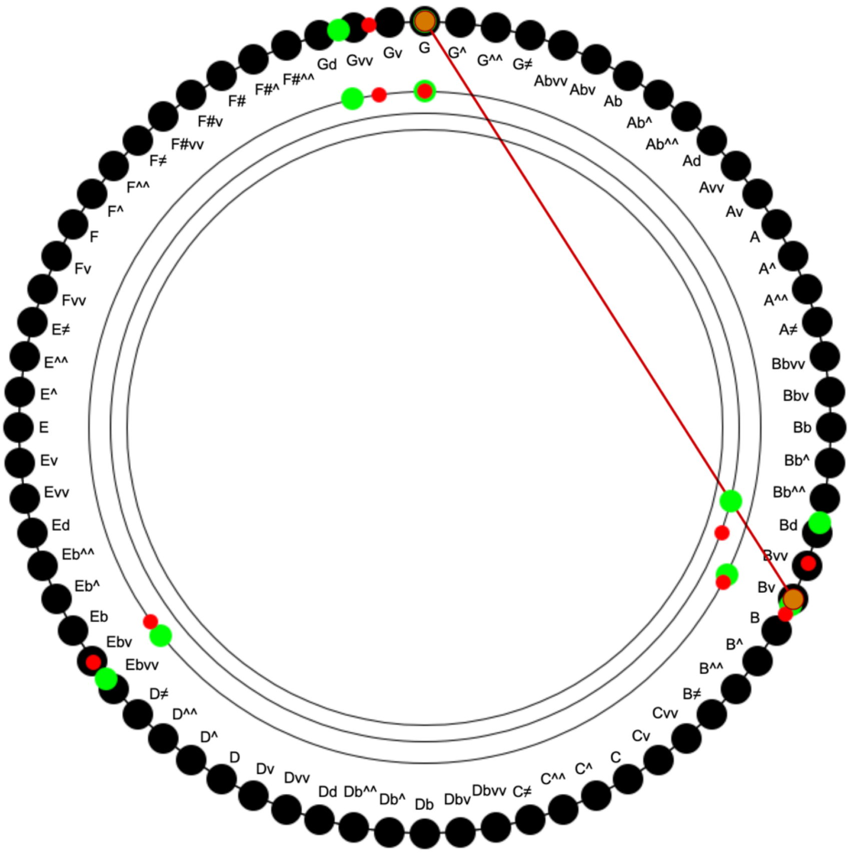
Figure 77. First five pitches of 2ED11/7 (red dots), five pitches a 5/4 major third apart (green dots) and 72EDO (black dots and notation). All three structures start on G.

For the passages which would be based mostly on 2ED11/7 (rehearsal marks O and T), the viola was to be the main part. It would simulate this tuning system by vertically always playing its smallest interval (henceforth referred as '1\2ED11/7', which sounds near to a pure 5/4 major third) in double stops and horizontally move up and down only by this step as well. This would sound as if parts of an expanded augmented chord are being played, but the diapason reference is constantly being readjusted – the music could sound vertically 'in tune' but horizontally 'out of tune'. This 'pitch drifting' within a piece of music is precisely one of the phenomena that temperaments are designed to avoid, but, in this passage, my goal is to purposedly recreate it.

As may be observed in Figure 77, a pitch a 1\2ED11/7 above G is close to another one a 5/4 major third above the same G. If one wants to notate both pitches in 72NPO by selecting the pitches of 72NPO closest to them, then both would be notated as Bv. However, when multiple iterations of these intervals are stacked, they would gradually become farther away from each other and be notated with different pitches of 72NPO. This dilemma explores the limits of my model from Chapter 2.2. The intonation model (2ED11/7), the auditory references (the 5/4 ratio) and the notation system (72NPO) all diverge (a phenomenon which historically would have been eliminated through temperament).

I decided that this passage would be then notated to look consistent with 72NPO: 5/4 major thirds would always be represented as 23 steps of 72NPO ('23\72NPO') – a 12NPO major third minus a twelfth tone (see Figure 78). This would be complemented with written notes giving qualitative descriptions of the intervals to play and how to tune them relationally. Even though multiple iterations of 23\72NPO would deviate in pitch from as many iterations of 5/4 ratios and of 1\2ED11/7, this solution would illustrate to the performer that a 5/4 is slightly narrower than a major third of 12EDO. This way, such notational approach could pass the 'Benjamin test',[140] since the performer would have access to a quantitative reference of measurement (72NPO), a description of the sonority of all intervals ('5/4 Natural Major Third'), and an acoustical reference to tune relationally all intervals (the common pitches between each double stop). This notational solution arguably also has some of the characteristics of a temperament, although it does not try to avoid what can be perceived as pitch drifting, but instead bends it to conform to the conceptual internal structure of 72NPO.

[140] See Chapter 3.3.4.



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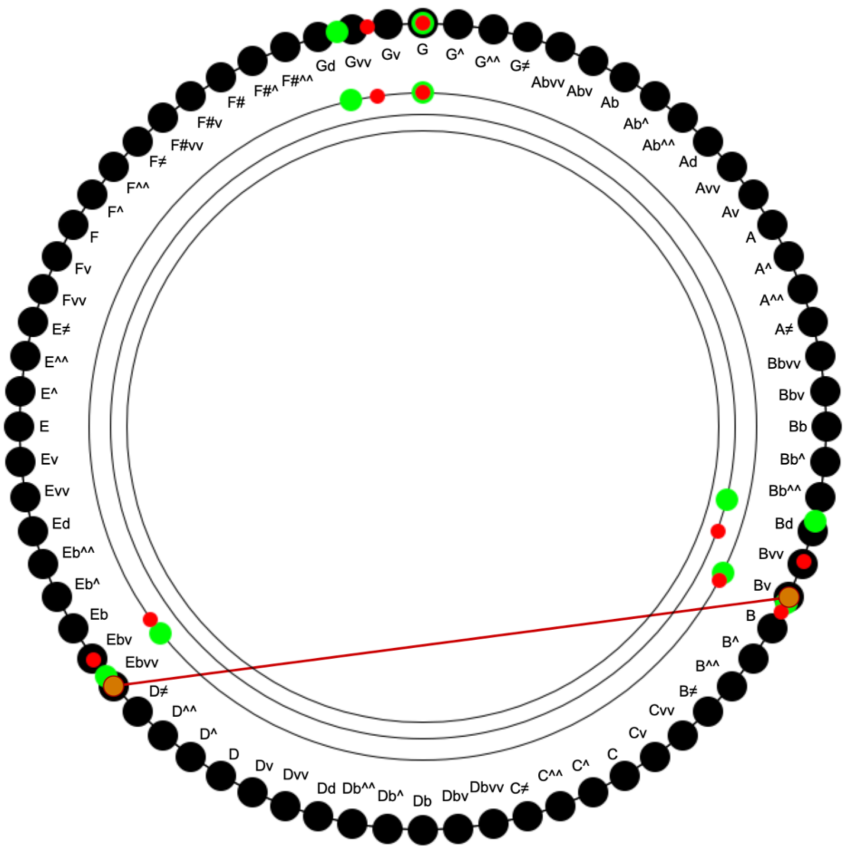

G
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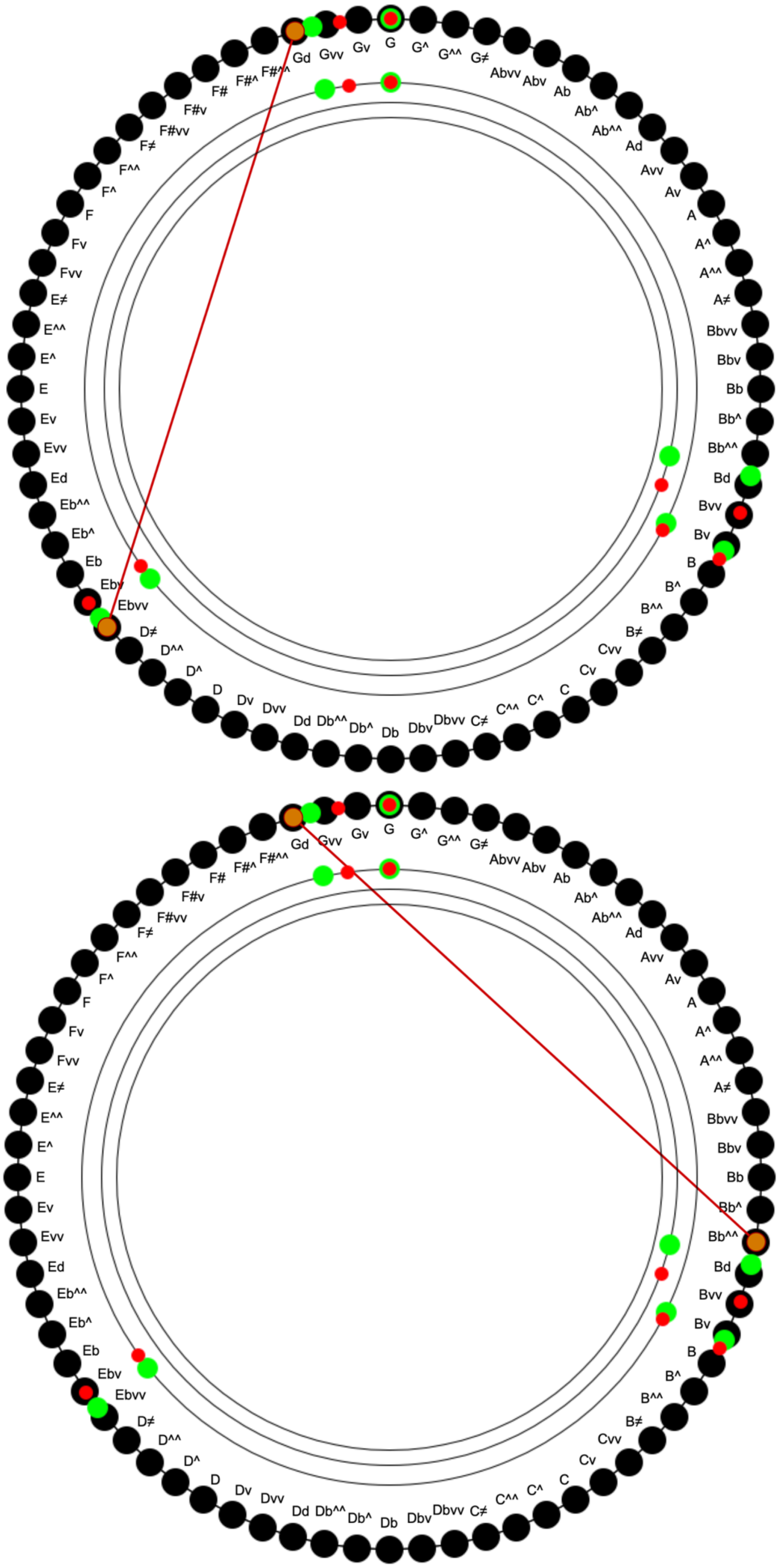

Figure 78. Representation of how the first five pitches of 2ED11/7 starting on G (as shown in the score in Figure 76) were notated in 72NPO in the viola part as double stops (orange dots and red lines), against the first five pitches of 2ED11/7 (red dots) and five iterations of 5/4 major thirds (green dots). Each new interval is built on top of the highest pitch from the previous interval in its corresponding system.

9.3. Commentary on the rehearsal process and recording session

The rehearsal and recording schedule of *Seeking Gnosis* included one three-hour rehearsal and a three-hour recording session three days later. This was preceded by individual sessions with most of the performers to test the performability of their parts and make some minor revisions. This piece proved to be too rhythmically complex for such a tight schedule. However, I consider the performance of its microtonality to have been successful overall.

9.3.1. The Woodwind parts

Due to health circumstances and schedule incompatibilities, no clarinet player was present on the rehearsal and a performer from the jazz department joined the project two days before the recording session. Their first instrument is the saxophone, whilst the clarinet is their second. This unusual circumstance led to a close collaboration between the performer and I on the day before the recording session, in which we went through the clarinet part, chose microtonal fingerings compatible with their instrument and revised the part whenever certain passages were not idiomatic for the instrument. This led to some of the microtonality of the clarinet part being adapted, but with no significant compromises.

Similarly, individual sessions with the flutist, saxophonist and bassoonist were held in which we workshopped alternative fingerings and made necessary revisions. Compatible fingerings were found for most (if not all) microtones in the flute and saxophone parts (and this is reflected in the recording). However, for the piccolo passages as well as the two microtones in the bassoon part, both performers chose to use embouchure to control pitch deviation instead of alternative fingerings. Particularly, the bassoonist and I tested various microtonal fingerings, but they found the fingerings to be unstable for their instrument.

While I consider that the microtones written for the piccolo passages were accurately performed, I do not consider the microtonal deviations in both microtones in the bassoon part (F≠ and Gd) in rehearsal mark DD to have been accurate, particularly the Gd (see Figure 79).

Figure 79. Bars 334-349 of Seeking Gnosis.

However, the moderately-sharper-than-written F≠ in the Bassoon part in the recording sounds musically appropriate to me, because, since this passage is emulating the 'tense dorian/orgone[7]' scale,[141] and when this scale is played melodically descending starting from C, I perceive its 545.4¢ 5\11EDO (less than 5¢ lower than a 24EDO F≠) to be a F#.

9.3.2. The string parts

While no individual session could be held with the cellist due to schedule incompatibilities, one session was held with the violist. Even though I was aware that the viola part was difficult to perform, I left that session feeling confident. The violist understood right away how the microtonality of their part was conceptualised and could play very accurately the passages based on Just Intonation (such as the one in Figure 76). They could also tune by ear with double stops a variety of Just Intonation tuneable intervals very fast, including 5/4, 7/5, 10/7, 5/3, 7/4 and 9/5. This leads me to wonder whether if this violist was trained with a technique close to that of Giuseppe Tartini.

The cellist, however, approached microtonality (as shown in the excerpt in Figure 74) by attentively following the microtonal accidentals and calculating melodically microtonal deviations by subdividing the semitone (is how I had initially deduced to be the approach that the performers of *Odd [s]Paces* would have taken). The cellist's approach was the opposite of the violist, which seemed to calculate microtonal intervals harmonically by tuning consonances. However, both methods proved effective, and I consider the string parts to have been performed overwhelmingly successfully.

Having used the same scordatura from *The Fable of the Pilgrim* (and 'Piece Z'), I was confident that the electric guitar part would be easily performable and that only some minor details would have to be revised. These expectations corresponded to reality. During the rehearsal and recording sessions, the conductor even relied on the retuning of the guitar to give accurate pitch references to the other instruments. This shows how easily standard guitars can be turned into microtonal instruments.

9.4. Abandoning rounding as the main approximation technique

As stated in previous chapters, there have been instances in which I was not completely satisfied with the results generated with rounding and others in which I've made individual

[141] The 'tense dorian/orgone[7]' scale in 11EDO is the rounding of 7EDO into 11EDO, which numerically relates to the 11/7 ratio, the unifying principle behind this piece.

modifications to rounded material. To a certain extent, the inclusion of approximations into tuneable intervals early in this research already points to a potential rethinking of how relevant rounding is to this research.

Having improvised on the Lumatone with the 'tense dorian/orgone[7]' scale in 11EDO and familiarized myself with its characteristics I found that its roundings into 12, 24 and 48EDO in *The Fable of the Pilgrim* still did not sound as close to this scale as I would have liked them to be. While sketching *Seeking Gnosis*'s final sections (centred on the bassoon line with electric guitar chords in 48EDO, as shown in Figure 79), I wanted to retry conveying this scale's sonority. Through improvisation on the guitar, I found the chord progression from Figure 79, which rendered this scale's harmonic qualities (when I play it on the Lumatone with a sustain pedal) more acceptably to my ear than the roundings of it in the second movement of *The Fable of the Pilgrim*. One factor for this perceptual difference lies in its sustained arpeggiated usage in the electric guitar part in this piece, whereas in *The Fable of the Pilgrim* this scale is mostly used melodically.

At the end of Chapter 9.3.1, I stated that the bassoon part, which is approximating this scale mostly in 12NPO, sounded effective because of how I perceive the 11EDO orgone[7] scale in its descending form. My perception might also be influenced by what I perceive to be the success of the much more xenharmonic sounding approximation of this scale in the electric guitar part, with its sustained harmonies. The electric guitar harmonies help trick my ear into perceiving what ended up being performed as a mostly 12NPO melodic approximation of this scale as converging to the sustained harmonies. It could be argued that a variation of the model from Figure 1 (in Chapter 2.2) might be at play, but my harmonic references (in the context of this passage) are not necessarily the simple ratios from James Tenney's hypotheses, but instead more complex intervals.

Rounding allows a tuning system to approximate another material which does not share its underlying structure. However, it consists of a pre-established mathematical formula which affects both harmony and melody but does not consider how they are perceived by the listener. I am still creatively drawn to it, but these experiences have led me to consider that rounding is only useful up to a certain point, having to be complemented by its user's ingenuity. One might say that this is exactly what I have been doing all along. Just the fact that I freely switch between different roundings of a material in the same musical passages, or simultaneously mix them already shows that, at least unconsciously, I already considered rounding not to be equally appropriate for every situation. My unsuccessful attempts at using 48NPO roundings in strings prior to this research project already pointed towards this.

Chapter 10. A new paradigm: Emergence of a dual microtonal compositional practice

Reflecting on all my compositions up to *Seeking Gnosis* led me to conclude that, despite my aims of making music which pragmatically approximates alternative EDOs, such practice could be streamlined into two parallel ones. One practice would correspond to conceptually intricate microtonal approximations which employ notational systems beyond 12 and 24NPO and tuneable intervals and would be more appropriate for performers specialised in microtonality, or non-specialists who would have sufficient rehearsal time with a specialist to guide them into immersing themselves into these sonic worlds. The other new practice would involve more extreme levels of compromise - particularly conscientious employments of quarter tones - and would be more appropriate for non-specialist performers, or for performances involving limited rehearsal time.

In retrospect, I consider that most of my output so far would tend towards the first category (despite how much I would wish it to be otherwise). The exception to this would perhaps be *The Fable of the Pilgrim*, given how pragmatic its microtonal explorations are for performance through scordatura. Based on this reflection I decided that the following (and last) piece should explore the second practice.

Chapter 11. *Convergence*

This chapter addresses the composition process of *Convergence* for sinfonietta. Subsequent subchapters focus on the creative crossover between the religious concepts which influenced the multi-movement structure of this piece and the microtonal systems explored throughout it (11.1), the adaptation of 'Piece Z'[142] into the first movement of this piece (11.2) and a commentary of some of the techniques explored in the various movements (11.3).

[142] See Chapter 8 for a commentary on the composition process of 'Piece Z'.

11.1. Extramusical influences and early considerations

Before working on this piece, I had been reading a variety of books on Abrahamic religions, one of them being Eileen Maddocks's book *1844: Convergence in Prophecy for Judaism, Christianity, Islam and the Baha'i Faith*. Similarly to how the title of this book suggests a convergence between various religions, I wanted to conceptualize this piece as a convergence of many of the ideas that I have been exploring in my research (as its culmination) and of certain religious themes that I came across and which I found conceptually compelling. I therefore named this piece after this book and designed it to span multiple movements, each representing one of these themes[143] and focused on different approximation strategies for different instrument families.

A piece for brass ensemble was originally planned to be written before this piece. Due to time constraints, I decided that it would not be written. Instead, *Convergence* would be longer, but include an expanded brass section and feature at least one movement (the fifth) focused on it. 'Piece Z' would be adapted for this ensemble and completed to be *Convergence*'s first movement. Alongside these two movements, I began to sketch what would become the second, third and fourth movements.

Because of my daily usage of Bosanquet-Wilson mappings of a variety of EDOs on the Lumatone, I became interested in the different varieties of diatonic scales (or otherwise) which these mappings highlight. Unsurprisingly, I noticed that in each movement I was focusing on different EDOs associated with the different categories from my hybrid model at the end of Chapter 2.5.[144]

11.2. Adaptation of 'Piece Z' into the first movement of *Convergence*

Given the considerations from Chapter 10, the microtonality of 'Piece Z', including the excerpts discussed in Chapter 8.2, was simplified to rely solely on quarter tones (24NPO). The electric guitar part was carefully rearranged into the string section by relying as much as possible on fixed left-hand positions.

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[143] See the performance notes of this piece for a detailed description of the symbolism behind each movement.

[144] See the performance notes for a complete exposition of the various systems that were used.

This also led to the guitar riff from Chapter 8.2.1 becoming an extended ‘neutral chord’ on Ed. In ‘Piece Z’, this passage was perhaps too harmonically repetitive. By improvising with these materials on the Lumatone, a chord progression was sketched by moving the two fifths between the Ed, Bd and F≠ of this chord up and down by quarter tones, while leaving the two fifths between G, D and A mostly unmoved. This would create a variation between the Ed neutral 9th chord (with a ≠11th) with Eb major 9th chord (with #11th) and E minor 11th chord. Other chords were also added: C minor 13th, F minor 11th, C neutral 13th and Eb supermajor 9th (with #11th).[145] These chord progressions are explored from rehearsal mark H onwards.

The strategies of mapping semitone and quarter tone based diatonic/pentatonic scales on different instruments to approximate 17EDO[146] were only thoroughly used in this (completed) movement. I wondered if there would be much differentiation from approximating other EDOs using these techniques, because 12EDO/NPO ends up being so prevalent. Nonetheless, I found the results convincing both when making MIDI mockups and in the recorded rehearsal (even with unstable intonation throughout the clarinet’s register).[147] When I played these passages on the Lumatone in 17EDO, I found the approximated results convincing (although I can still recognize that they are approximations).

Even though the oboe’s tendency towards 12EDO would lead its diatonic scales to be categorized as Meantone (although still being close to Pythagorean Tuning),[148] 17EDO is still a Superpythagorean system. However, Meantone and Superpythagorean diatonic scales are, nonetheless, diatonic. To my perception, diatonicism is an intonation-agnostic musical reference. Whether it is rendered in 19, 17 or 12EDO, I recognize a diatonic scale before I notice the proportion between its whole tones and semitones. Maybe it is that reference that aids tricking my ear when earing a 12NPO melody on top of a 24NPO rounding of 17EDO harmony.

Given the limited amount of rehearsal time allotted to this movement, it was recorded by workshopping shorter sections of it and then joining them together when editing the final

[145] The terminology being used to identify these extended chords is mostly based on jazz theory. In the score, these chords are indicated, but simplified, as if they were triads.

[146] As discussed in Chapter 8.2.2.

[147] See Chapter 11.3.3.

[148] See the ‘EDO Diatonicism’ spreadsheet in Global Properties of EDOs to compare the intervallic statistics between 12EDO and 53EDO (and ED which is almost indistinguishable from Pythagorean Tuning).

recording. I find the movement mostly successful in terms of the quality of the recording, and the performability of its microtonality (as notated in the score).

However, the adaptation of the electric guitar part of 'Piece Z' into the string section was only partly successful. The intonational precision that was achieved was, to me, not sufficient. More rehearsals could have been enough to reach more precise intonation. However, I suspect the problem lies on the reliance of fast unusual microtonal sonorities (with harmonies based on extensions of triadic chords) which do not allow for a relational approach to intonation. This would be a similar phenomenon to my experiences of writing in 48NPO for strings before the start of this research, but instead within a 24NPO context.

11.3. Commentary on some technical aspects

11.3.1. Subdivisions of 12NPO intervals in the strings

Given the scales used in *Odd [s]Paces* repeat at the 3/2 ratio, I wondered if the core idea behind them could be simplified to be applied in this piece. I concluded that the subdivision of the 12NPO intervals,[149] either into two or three parts, would be a logical path to follow.

Theoretically, the precise cent sizes of the intervals of these scales cannot be predetermined, since 12NPO is an intonation-agnostic generalization. However, by rooting this approach on the intervallic references of performers (whose practice arguably could be classified as 12NPO), my deduction was that such an approach would be easily absorbed by the performers, particularly melodically.

Having used this approach in the string section, particularly in the third and fourth movements, I concluded that such it is viable. Since it was used to subdivide the perfect fifth and fourth and major and minor thirds (intervals that, in string practice, tend to be simpler to intone than semitones),[150] therefore I hypothesized that their subdivisions would also be easier to calculate than those of semitones. It did not take long for the performers to grasp this concept and successfully applying it.

A passage in which I found such an approach to be particularly successful is represented in Figure 80, particularly in bars 50-52 of the third movement, showing a 'crossfade' from

[149] By this it is meant the subdivision of the intervals commonly associated with the 12-tone chromatic scale without imposing any fixed intonation approach for these intervals.

[150] See Chapter 6.3.1 for my reflections on using semitones as unreliable intervallic references for microtonal exploration in string players.

9NPO to 6NPO (which explores a modulation from one subdivision of the major third, from three parts to two)[151] rooted on C. Bars 53-55 show another cadence/modulation from 8NPO (minor thirds divided in two) to 6NPO, also rooted on C. Through these modulations, harmonic movement would be introduced to these harmonically static and limited tuning/scales. This last passage, however, proved to be more problematic.

Figure 80. Bars 50-55 of the third movement of Convergence.

Arguably, bars 50-52 show a melodic driven application of this approach of subdividing 12NPO intervals. This means that performers would (and did) give primacy to melodic direction (and leave vertical intonation somewhat up to chance). Bars 53-55 would present a more harmonic driven application. However, given that the 8NPO quarter tones are not played relationally (instead being calculated individually by each performer, similarly to some passages in *Odd [s]Paces*), a retuned keyboard had to be used as reference (whereas in bars 50-52 it was not needed). Nonetheless, the successful interchangeability from one subdivision of the major third to another in bars 50-52 is another example of the viability of exploring subdivisions of 12NPO intervals beyond the semitone in string instruments. However, it could

[151] The microtones of 9NPO have been notated with quarter tone accidentals only for convenience since, theoretically, such microtones would correspond to sixth tone deviations.

be argued that, for melodic purposes, the three premises of the 'Benjamin test'[152] were met on bars 50-52 (hence its success). On bars 53-55, one of its premises (an acoustical reference against which to tune by ear) was not met, and therefore this would justify why this passage was not as successful. It can be concluded that the viability of this approach of subdividing 12NPO intervals in string instruments is dependent on the premises of the 'Benjamin test' being met.

One curious situation, however, happened in the second violin solo in the beginning of the sixth movement (Figure 81). Given how slow it is, the 9NPO melodic line could be conceptualized melodically, by visually calculating in the violin the subdivision of the major third in three parts within a single string, and harmonically, by identifying whenever a 7/6 'subminor third' (and 'easy' tuneable interval approximated in 9NPO by the interval 2\9NPO) would be used. Nonetheless, the performer (who is also a composer with some experience with microtonality) asked me to write in their part the cent deviations from 12EDO for each microtonal pitch in their part. Given my reflections from Chapter 6.3.1, I was surprised to note that the performer's reliance on cent deviations (which have the 12EDO semitones as reference, converging with Spohr and Habeneck's approach to intonation in Chapter 2.6) led to a performance which I perceive to sound intonationally identical to the MIDI mock-up that was made in Dorico.[153] This was likely due to how acquainted this performer was with 20th century microtonal music and his own training as a composer.

[152] See Chapter 3.3.4.

[153] If I were to do a spectral analysis, their intonation would certainly not be the same as the mock-up, but its deviations would be imperceivable to my ear.

Figure 81. Bars 1-7 of the sixth movement of Convergence.

Both the violist and the cellist who performed in this piece were the same performers from *Seeking Gnosis*, and my comments on how they approach their parts (in Chapter 9.3.2) are the same for this piece.

11.3.2. Brass writing and performance of the fifth and second movements

During the rehearsal and recording of the fifth movement for the brass section, I found that the intonation of the players which played this piece tends to converge towards Just Intonation consonances more than I had originally assumed. My overly detailed notational approach in their parts seemed unnecessary in bars 18-20 and 27-29.[154] Some of the chords that were not supposed to sound as consonant, ended up sounding as such to me, likely because of this.

The trumpet and (especially) the horn, in certain ranges, have some limitations regarding microtonal fingerings. Some pitches were unsuccessful, either because the instruments simply could not play them (even though, after studying fingering charts, I was convinced that they

[154] See figure 112 in Appendix 10.

could), or because of other unknown reasons. One example is the concert pitch A≠ on the horn part in bars 10 and 13 (Figure 82). The alternative fingering required to play this note needed to be compensated with embouchure adjustments. This led to an extremely unstable pitch.[155] I conclude that I was overly optimistic regarding microtonal fingerings in the horn (similarly to my clarinet writing, generally).

Figure 82. Bars 10-13 of the fifth movement of Convergence.

One curious combination of situations during the recorded rehearsal was that of the performance of bars 9-12 from the second movement and bars 6-9 from the fifth movement, which share exactly the same pitches in the trumpet parts (see Figure 83).

[155] The performer, however, told me that horn players generally will need to do different intonational adjustments during performance with different instruments. The way horns are built leads to the intonation of the natural harmonics not always being at the same pitch height between instruments. This means that, between different horns, the pitch height of natural harmonics will diverge differently from the harmonic series (as theorized) and converge also differently with 12EDO. With another horn, that fingering used to play an A≠ could potentially lead to a pitch closer to the desired one and not require embouchure adjustments, or nearly as much.

a)

b)

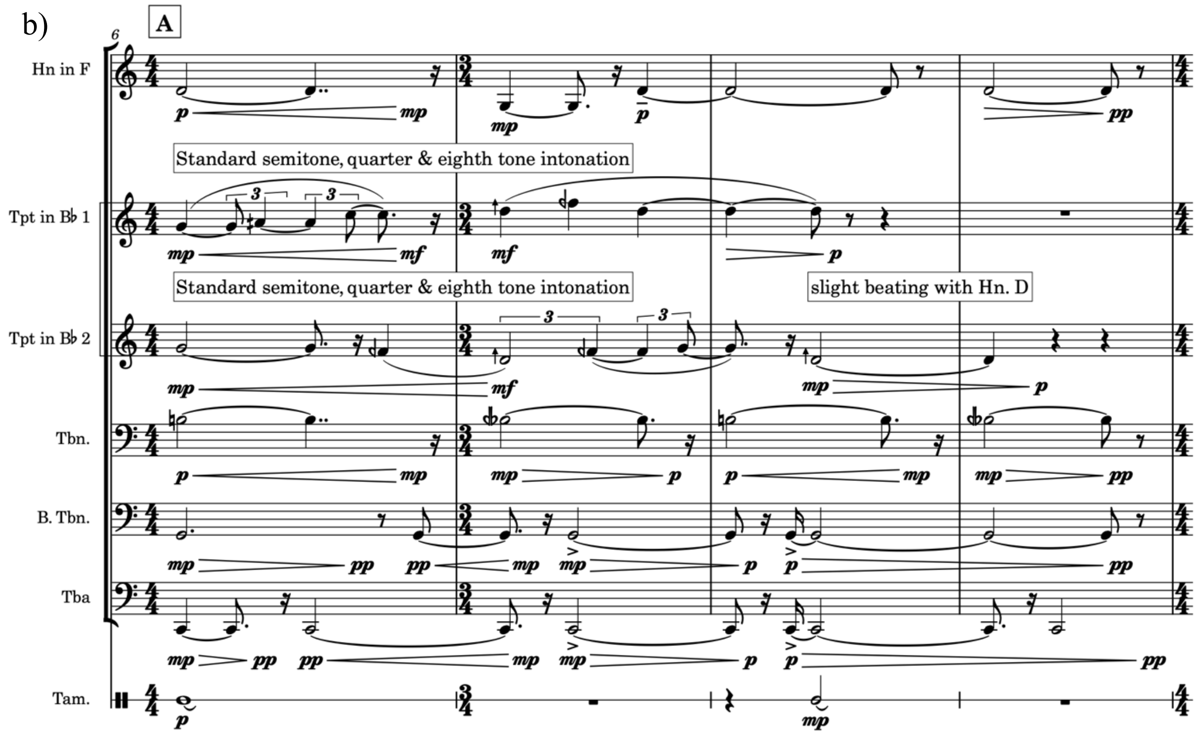

Figure 83. Bars 9-12 of the second movement of Convergence *(a) and bars 6-9 of the fifth movement (b).*

Due to unforeseen circumstances, bars 9-12 of the second movement were recorded without my presence. However, after they were recorded, it was decided that this passage would not be replayed due to time constraints. Later, when bars 6-9 from the fifth movement were being workshopped, I spent some time working with the trumpet players to help them achieve the quarter tones in the score - which vertically approximate tuneable intervals and, therefore, can be conceptualized as consonances. I gave up on trying to have the eighth tones intoned because of how much time it would take to help the performers fine tune those musically-not-so-relevant dissonances, when these performers are especially good at tuning consonances. Having those quarter tones intoned would lead to much more relevant sonorities than the more subtle eighth tone dissonances. Nonetheless, at certain times, the second trumpet did reach the desired 'beating effect' against other instruments that can be reached with eighth tones.

After the recorded rehearsal, but before I had listened to the recordings of both of these passages, the conductor (who is also a composer) told me that, even though he is experienced with microtonality in a spectralist context, my presence was essential for the workshopping of this piece and, had I not been present in the recorded rehearsal, the performance would have sounded very different. These two passages illustrate just that. This piece is filled with 'niche' sonorities that are not familiar to the performers with whom I collaborated. Therefore, I conclude that the rehearsing of this kind of music – even with these more simplified microtonal approaches, compared to my previous pieces – should still be supervised by a specialist.

11.3.3. Reflections on the clarinet part

The rehearsal process of *Convergence* reveals a weakness of my choice to use a clarinet retuned a quarter tone lower: not to expect consistent intonation throughout the register of the clarinet (and potentially other retuned woodwinds).

For scheduling reasons, two different clarinettists played different passages of *Convergence*. One performer had to spend time retuning their instrument multiple times, depending on which register of the clarinet would be used the most in which passage. The other performer, however, only had to retune their clarinet a quarter tone lower once, only needing to do slight adjustments as often as the other performers. In both cases, the intonation of the clarinets proved to be irregular throughout their registers.

This experience, however, presented an opportunity to reflect on how much intonational deviation my ear is willing to accept within a quarter tonal context. While the clarinet part only includes quarter tones, they are often approximations of pitches from systems unrelated to

24EDO. This, alongside the elasticity of my pitch perception,[156] may justify why many times throughout the recorded rehearsal, I found the irregular intonation of the clarinets not to be as problematic as it would otherwise seem. Not only that, but often I enjoyed the unexpected results.

However, in Interlude III, I found this irregularity to be problematic, since it relies on the illusion that one is hearing a group of ascendingly played EDOs played by the cor anglais and the clarinet. For this effect to happen, there cannot be a huge intonational discrepancy between the intervals of consecutive steps. However, that is what happens with the irregular intonation of the retuned clarinet.

[156] See Chapter 2.2 for my systematization of how pitch perception, notation and intonation relate to each other.

Part III. Concluding remarks and further research avenues

Whilst the first three research questions of this project have led to concrete theorisation and rewarding learning, responses to the fourth question – concerning creative practices – have been necessarily subjective and based on my own work, summarized here.

Throughout this research, my approach for approximating equal tunings, when writing for performers on Western concert instruments, has gradually evolved. On the one hand, I gradually moved away from rounding as I realised that this method did not always achieve the goals for which it was designed. These goals include the distortion of musical materials through their approximation into tuning systems different from the one in which they were originally conceived. As will be revealed, this was always done with a view to retaining at least some of their acoustical properties whilst improving their performability for performers of Western concert music instruments. Rounding, nonetheless, proved to be a useful tool particularly with electronic media and instruments whose intonation needs to be fixed prior to performance (such as guitars and keyboards). *Dreams from an Old Memory*[157] and *The Fable of the Pilgrim*[158] illustrate this.

On the other hand, I became more familiar with intonation frameworks beyond the guitar and the modern piano, particularly of string players and of renaissance/baroque keyboard temperaments. Generalizing their features and mapping them into the Lumatone (and playing them), and incorporating them into interactive software (Equal Tuning Lab) and data spreadsheets (Global Properties of EDOs) that I have developed concurrently with my compositional practice, has helped me developing a more global and embodied understanding of different intonation standards and how they relate to each other.[159] Intonation frameworks are also culturally linked with notation,[160] and different performers (and groups of performers)

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[157] See Chapter 4.1.

[158] See Chapters 7.1 and 7.2.

[159] See Chapters 2.5 and 2.6.

[160] See the following writings for a review of the reciprocity between intonation, tuning systems and notation. Patrizio Barbieri and Sandra Mangsen, 'Violin Intonation: A Historical Survey', *Early Music*, 19.1, 1991, pp. 69–88., and Mieko Kanno, 'Thoughts on How to Play in Tune: Pitch and Intonation', *Contemporary Music Review*, 22.1–2, 2003, pp. 35–52 <https://doi.org/10.1080/0749446032000134733>. Also see Duffin, *How Equal Temperament Ruined Harmony*. Although Duffin does not seem to problematize the Western Chromatic Notation

have both shared and idiosyncratic ways of dealing with them. Therefore, some of these generalizations may be broadly applicable amongst a variety of performers of different instruments, such as 12NPO – the conceptual 12-tone chromatic scale unattached to any particular intonation standard. Other generalizations may only be appropriate for a more contained set of individuals (e.g. tuneable intervals for string players whose technique converges more with Tartini's approach, or Pythagorean Tuning/Schismatic systems for those who converge more with Campagnoli's).[161] In this research I have argued that notation is the bridge that connects intonation (the pitch choices within the performers' agency) with microtonality (the pitch choices within the composers' agency), and that there is reciprocity between both. They share a 'feedback loop'.

This showed me that rounding, as a microtonal approximation strategy, while being useful in many aspects, does not relate to many historical and current intonation practices of performers of many of these instruments. Its weaknesses in performability are then due to this. *Rounding* is not necessarily informed by this feedback loop.

This led me towards hybridizing these two fields (rounding and my knowledge of intonation techniques) in different stages of the research. This was done firstly by incorporating approximations of Sabat and Hayward's Just Intonation tuneable intervals in string instruments, mostly written in 72NPO notation (therefore mixing these two modalities by using one to represent the other), alongside rounded material in *Adam*[162] and *Odd [s]Paces*.[163] In later stages, other approximation approaches were explored, some partially incorporating instances of rounding (such as in the incomplete 'Piece Z', mixing rounding with diatonicism in wind players, by somewhat detaching harmony from melody),[164] and others that do not (such as free approximations into 48EDO in the electric guitar part at the end of *Seeking Gnosis*).[165]

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system, his whole argument about different intonation approaches and temperaments is premised on attaching different intonation standards to it (similarly to how Circle of Fifths Notation functions).

161 See Chapter 2.6.

162 See Chapter 3.2.

163 See Chapter 6.2.

164 See Chapter 8.2.2. Since this strategy was only explored once in the unfinished 'Piece Z' (later adapted and completed as the first movement of *Convergence*), it is not yet certain to me that using it again to approximate other equal tunings would lead to significantly distinct results. Therefore, this approach should be further explored in future compositions.

165 See Chapter 9.4.

Simultaneously, 12NPO gradually became the ‘lowest common denominator’ when writing for any pitched instrument, usually through approximations (except for *Dreams from an Old Memory*, since its intonation is manipulated electronically). This happened because, while there are diverging intonational techniques between different instruments and performers, there are still certain commonalities. 12NPO would then be complemented with other approaches seemingly more suitable for each context, informed by my knowledge of intonation practices associated with performers of different instruments. One example of this is the use of scales based on subdividing intervals of 12NPO which anchor to it every few notes (e.g. 8NPO, which divides the minor third in two parts, and 9NPO, which divides de major third in three parts) mostly in string instruments (but not only). This allows for some level of relationality between technique and visual and muscular calculations in performance. Such scales were explored in *Convergence*.[166]

However, my experiences with wind players (particularly clarinettists) have led to more inconsistent performative results in my compositions than with string and guitar players. This suggests that I am less familiar with the intonation techniques of performers of these instruments. Therefore, further investigation on wind intonation techniques is still a necessary avenue to pursue to achieve a more idiomatic writing for these instruments. This in turn may lead to new ways of conceptualizing pitch and new compositional approaches informed by these techniques, similarly to what happened with string instruments.

The music that I have written with these systems arguably has neo-modal tendencies, as I have materialized many of these systems to resemble the structure of scales or used equal tunings that already resemble scales (such as 5 and 7EDO). It also tends to be not too overtly xenharmonic (to my perception). This is likely due to the types of instruments for which this research was designed, but also personal taste. My rhythmical language developed particularly from *Odd [s]Paces* onwards, and I gradually found ways of interlinking rhythm and pitch with some conceptual consistency (many times through repetitive variations), while still leaving enough space for spontaneous creativity. I find such an interlink to be an appealing way to compose given my interest in numerical schema, even though its existence is not strictly necessary. However, I do not consider having made significant advances in my use of form, which tends to result in either short movements grouped together and/or longer forms based on

[166] See Chapter 11.3.1.

intercalated blocks of materials. Form, and how it can relate to the pitch explorations of this project, will have to be a particular focus of future pieces.

This research was originally designed to have roundings into subdivisions of 12NPO as its central approach, but the gradual decline of its use leads to reconsidering the framework of this research, particularly in notation. Given that rounding has always been intrinsically linked with 12NPO and its subdivisions, a notational approach based on these systems (which I have been referring as 12, 24, 48 and 72NPO notation)[167] always seemed appropriate. However, if rounding is no longer the main feature or, perhaps, even a feature of future compositions, then the appropriateness of this research's notational framework is no longer guaranteed.

Moreover, the employment of different notational approaches which relate to substantially different intonation frameworks (such as HEJI, Circle of Fifths, or even Sagittal notation) could be the next epistemic impulse of this project. This would set the next stage of this research and rebalance it, particularly the weight that each of the four research questions poses. A notation system would no longer be a consequence of the approximation approaches being explored in each piece, but instead it would drive the search for new approximation approaches compatible with it. This could involve composing pieces that approximate certain equal tunings only through Just Intonation (such as tuneable intervals above a drone, or movable drone) by having them notated with HEJI or Sagittal notation.

Another consequence of this shift may also involve including in future compositions special instruments that have been avoided so far, such as guitars with alternative fretboards or even the Lumatone. For example, a piece for cello and Lumatone notated in either Circle of Fifths or Sagittal notation could involve retuning the fifths of the open strings to converge with the fifths of an EDO mapped on the Lumatone (which would strategically relate to the string intonation standards from Chapter 2.6 in some form). This mapped EDO on the Lumatone, the new notational approach and the retuning of the strings would then offer a new intonation standard to be followed by the cellist. The first example after the first paragraph of Chapter 6.3.1 could be simulated in such a piece, which would shift the intonation standard of the cellist away from 12NPO and its subdivisions and instead towards 17NPO and its subdivisions (which can be seen as an exaggeration of Campagnoli's approach).

[167] See Chapter 3.2 for a description of how this notation standard was conceptualized to be employed in *Adam*.

Throughout this portfolio, I have moved away from a universal pre-established formula, to be applied for all instruments (roundings of equal tunings), towards a multimodal methodology dependent on context which allows for the application of different strategies in different circumstances, while still relating to a unifying principle – exploring materials that resemble equal tunings in different ways. In other words: beginning with an approach and adapting accordingly.

Appendix 1. Glossary of regularly used advanced terminology

Bosanquet-Wilson mapping – a mapping of pitches used in hexagonal generic keyboards (such as the **Lumatone**) which maps **EDOs** into these keyboards and, particularly, highlights the heptatonic (possibly **Diatonic**) scales derived by multiple six iterations of a **generator** interval always with the same shape (which, to a certain extent, resembles, but expands, the layout of a piano).

Cent – a unit used to measure the size of intervals. 1 cent (or 1¢) is one hundredth of the 12EDO semitone (which, therefore, has a size of 100¢).

Circle of Fifths Notation – a notational approach used to notate a variety of **tuning systems** by naming pitches based on their location in a circle of fifths. Visually, this notation system is indistinguishable from the **Western Chromatic Notation** system, since its system of accidentals is based on adding sharps and flats to the C, D, E, F, G, A, B note names.

Comma – a small **Just Intonation** interval that is explored in the context of temperament theory. In this research (except for Appendix 10), the only comma that is referenced is the Syntonic Comma (81/80) which is the difference between the **Pythagorean** major third (81/64) and the 5/4 major third, and which is used as the starting point to create **Meantone Temperaments** – by subtracting the **generator** perfect fifth of **Pythagorean Tuning** by a part of this comma: 1/3 of it, in the case of 1/3 Comma Meantone Temperament, and 1/4, in the case of 1/4 Comma Meantone Temperament.

Diatonic – a heptatonic scale which is derived by a circle of fifths (whose size is greater than 4\7EDO, at about 686¢, and 3\5EDO, at 720¢) and has an intervallic structure which converges with the major and minor scale and the seven gregorian modes. This term can also be used as an adjective to musical structures which relate to diatonic scales. (This strict terminology is only employed for the purposes of this research, as this term is commonly used to have a broader meaning.)

Double rounding (see **Rounding**) – the act of **rounding** intervals from a system into another system and then **rounding** the results into yet another system.

Equal tuning – a **tuning system** whose steps are logarithmically equally distanced in frequency. In this research, an individual equal tuning is always referred in the format 'xEDi' (or ‘x Equal Divisions of *interval*’), since all equal tunings used in this research repeat at a certain interval. The most well-known equal tuning in Western music is 12EDO.

EDO – Equal Divisions of the Octave. A term used to refer to **equal tunings** which repeat at the octave.

ED3/2 – Equal Divisions of the 3/2 ratio. A term used to refer to **equal tunings** which repeat at the 3/2 ratio.

Eighth tone – an interval with the size of roughly a quarter of a **semitone** (and an eighth of a whole tone). This is the smallest interval found in 48EDO/NPO- It is commonly considered to be a **microtone**.

Enharmonic (pitch equivalencies) – the relationship between two different pitches with different note names, which are derived from a circle of fifths, and which share the same frequency (e.g. C# and Db in 12EDO). Each **EDO** has its own collection of enharmonic equivalencies.

Enharmonic (genus) – one of the three ancient Greek genera of musical scales, which has intervals that could be considered **quarter tones**. When compared to the other two genera (diatonic and chromatic) this genus could be considered the most **xenharmonic**.

Expressive intonation – the intonation standard of the 20th century Spanish cellist Pablo Casals. It shares similarities with early romantic period violinist Bartolomeo Campagnoli’s intonation approach.

Fake octave – a musical interval close to the 2/1 perfect octave but distanced to it by a **microtone**.

Generator – the musical interval used to create a variety of **tuning systems** (such as **EDOs**, **Pythagorean Tuning** and **Meantone Temperaments**) and scales (such as the heptatonic **Diatonic** scales), by stacking multiple iterations of it. In the context of this research, generator intervals are conceptualized as perfect fifths.

Harmonic series – an intervallic structure found in nature and is the basis for the acoustic spectra of many sounds with a defined pitch. It is organized in a series of infinite partials equally distanced in frequency on a linear scale. The nth partial of the harmonic series is calculated by multiplying n with the frequency of the fundamental. This structure is the basis for **Just Intonation**.

Inharmonic – an intervallic structure which does not converge with the **harmonic series**. Inharmonic intervals are intervals not found in the **harmonic series** (such as the ones found in **equal tunings**). Inharmonic chords have structures that do not conform with the **harmonic series** (even if they are comprised of intervals found in it, but whose pitch distribution does not correspond to any subset of it).

Interval class – in the context of this research, this classification system is used to conceptualize as being equivalent intervals that are octave duplications of each other (such as a major second and a major ninth) and is particularly used in the context of **Just Intonation** ratios. The interval class of a musical interval corresponds to its most octave-reduced version (e.g. the interval class of both the 9/8 and 9/4 ratios is the 9/8 interval class, since 9/8 cannot be reduced by an octave, and 9/4 is a 9/8 ratio plus an octave).

Interval Class Rank – a tentative classification standard for how problematic in a performance context an **interval class** found in 72NPO might be, by having it classified in a rank from A to J. The rank of an interval class is based on the amount and the level of difficulty of the **tuneable intervals** in the vicinity of the intervals that are classified with that **interval class** (in other words; the level of difficulty of its associated **quasi-tuneable** intervals, if there are any).

Intonation – 'the art of selecting pitches, or (more accurately) pitch-"regions" along the glissando-continuum of pitch-height […]. The "tolerance" or exactitude of such regions varies based on the instrument and musical style'.[168] A variety of fixed tuning systems and temperaments are thus included in this definition.

Isobematic – a term, coined by Christopher Bochmann, used to describe a type of post-serial atonal music in which the different musical parameters can be conceptualized and/or measured by equal steps. According to this definition, isobematic music could be composed with **equal tunings** and **NPOs**.

Just Intonation – an intonation standard in which all musical intervals are derived by ratios of frequencies (with natural numbers, such as 3/2, 5/4, etc.) found in the **harmonic series**.

Lumatone – a generic and isomorphic keyboard with hexagonal keys which can be programmed to map a variety of **tuning systems** (particularly **EDOs**, with **Bosanquet-Wilson**

[168] Sabat, 'Intonation and Microtonality' (para. 2 of 19).

mappings and other layouts). Its keys can also be programmed by customising their colours according to the user's preferences.

Macrotone - a musical interval, conceptualized within the context of **microtonality**, which is larger than a **semitone** (usually the 100¢ wide 1\12EDO is used as the reference). **Tuning systems** can be considered macrotonal (as opposed to 'microtonal') if their smallest steps are bigger than a **semitone** (such as **EDOs** with less steps than 12, or **ED3/2s** with less steps than 7).

Meantone – the classification of a **tuning system**/**temperament** derived by a circle of fifths with a **generator** size between 4\7EDO (about 686¢) and the 3/2 ratio (about 702¢). **EDOs** can be classified as 'Meantone' if they can be created with a **generator** perfect fifth with a compatible size (such as 19 and 31EDO). This term can also be used to refer to the type of **Diatonic** scale present in a Meantone **temperament**/**EDO**. Many historical **temperaments** (derived by contracting the size of their **generator** 3/2 perfect fifths by a fraction on the 81/80 Syntonic **Comma**) are Meantone **temperaments** (such as 1/4 and 1/3-Comma Meantone).

Microtonality (see **Intonation**) – 'an approach to pitch which acknowledges the musical possibility of this entire glissando-continuum and is not limited to the conventional twelve equal tempered pitch-classes'.[169]

Microtone – a musical interval, conceptualized within the context of **microtonality**, which is smaller than a **semitone** (usually the 100¢ wide 1\12EDO is used as the reference). **Tuning systems** can be considered microtonal (as opposed to 'macrotonal') if their smallest steps are smaller than a **semitone** (such as **EDOs** with more steps than 12, or **ED3/2s** with more steps than 7).

Neutral (interval) – an interval which can be classified as being in between major and minor (e.g. the 350¢ 'neutral third'). This intervallic classification is particularly common in even **subdivisions** of 12EDO/NPO, such as 24EDO/NPO.

Neutral (chord) – a triadic chord formed by two neutral thirds. In 24EDO, given the size of its thirds, neutral chords also have perfect fifths (because 350¢+350¢=700¢). Neutral chords are then notated with a mix of chromatic accidentals (whenever appropriate) and quarter tone

[169] Sabat, 'Intonation and Microtonality', (para. 3 of 19).

accidentals. An extended neutral chord on C would then include the pitches C, Ed, G, Bd, D, F≠, A, etc.

Neutral (tuning system) – a **tuning system** or **temperament** with a perfect fifth size of 700¢ (12EDO and all its **subdivisions**).

NPO – Notes per octave. A term used to refer both to notation systems with 'x' notes per octave, as well as a generic term for **tuning systems** and **intonation** standards whose conceptualization is based on the same number of pitches per octave. This term is intonation-agnostic and is used mostly to refer to situations in which pitches are, in practice, not equally spaced, such as in performances of non-fixed pitch instruments with scores written in 12, 24, 48 or even 72NPO, or in historical **tuning systems** and **temperaments** of unevenly spaced 12 pitches per octave (such as **Pythagorean Tuning**, **Meantone** and Well **Temperaments**).

Orgone[7] – a non-**diatonic** heptatonic and octave-repeating **xenharmonic** and **microtonal** scale related both to Regular Temperament and Moments of Symmetry (MOS) theory which, in 11EDO, as a structure of LsLsLsL (in which 'L' corresponds to two steps of 11EDO, or 2\11EDO, and 's' to one step, or 1\11EDO). It can also be created by rounding 7EDO into 11EDO. In this research it is also referred as the '**tense dorian**' scale.

Pitch drifting – a phenomenon associated with **Just Intonation** harmony which results in the pitch centre of a musical passage moving (or 'drifting') upwards or downwards in frequency when, in a musical score written in the **Western Chromatic Notation** system, a pitch modulation would not be expected. This phenomenon has historically been eliminated in keyboard music through **temperament**.

Pythagorean Tuning – a **tuning system** derived by a circle of fifths with a **generator** size of 3/2 (about 702¢). Because of this, all its intervals are in **Just Intonation**. For the purposes of this research, 53EDO (and its **subdivisions**) are classified as 'Pythagorean' because their **generator** fifth size is very close to the 3/2 ratio (at a difference of about 0.1¢). This term can also be used to refer to the **Diatonic** scale present in Pythagorean Tuning, 53EDO and its **subdivisions**.

Quarter tone – an interval with the size of roughly half of a **semitone** (and a quarter of a whole tone). This is the smallest interval found in 24EDO/NPO. It is commonly considered to be a **microtone**.

Quasi-tuneable intervals – **inharmonic** intervals which are close to **tuneable intervals**. In this research, this term is usually used in the context of **equal tunings** and **NPOs**.

Rounding – the act of systematically approximating intervals from one system into the closest possible interval in another system. This term is mostly used to refer to this kind of approximation for structures present in an **equal tuning** into another one or into a **NPO** notation system (such as 12, 24, 48 and 72NPO).

Semitone – an interval with the size of roughly half of a whole tone. This is the smallest interval found in 12EDO/NPO. Semitones are also a group of musical intervals which correspond to the smallest interval in **diatonic** and chromatic contexts. Moving by a **diatonic** semitone has the implication of moving from one note to another one with a different name, but the same number of accidentals (e.g. B-C, E#-F#, Cb-Bb), whereas moving by a chromatic semitone results in the note name remaining the same, but an accidental having to be either added or subtracted (e.g. C-C#, Eb-E). 12EDO is the only **tuning system** in which its **diatonic** and chromatic semitones are the same size (100¢).

Sixth tone – an interval with the size of roughly a third of a **semitone** (and a sixth of a whole tone). This is the smallest interval found in 36EDO/NPO, and the second smallest found in 72EDO/NPO. It is commonly considered to be a **microtone**.

Subdivision – a term usually used to refer to **tuning systems** and **NPO** notation systems whose number of pitches are a multiple of other ones (because the intervals of those initial systems are 'subdivided). Most of the time, it is used to refer to 24, 48 and 72EDO/NPO, which are subdivisions of 12EDO/NPO.

Subminor (interval) – an interval which, in 24EDO, would be about a **quarter tone** narrower than a minor interval (e.g. a subminor third would be 250¢ wide).

Supermajor (interval) – an interval which, in 24EDO, would be about a **quarter tone** wider than a major interval (e.g. a supermajor third would be 450¢ wide).

Supermajor (chord) – a triadic chord formed by a supermajor and a **subminor** third. In 24EDO, given the size of its thirds, supermajor chords also have perfect fifths (because 450¢+250¢=700¢).

Superpythagorean – the classification of a **tuning system** derived by a circle of fifths with a **generator** size between the 3/2 ratio (about 702¢) and 3\5EDO (720¢). **EDOs** can be classified as ‘Superpythagorean’ if they can be created with **generator** perfect fifth with a compatible size (such as 17 and 22EDO). This term can also be used to refer to the type of **Diatonic** scale present in a Superpythagorean **EDO**.

Tense dorian – another name used for the **orgone[7]** scale. Except for its fourth (of about 545¢) and fifth (approximately 655¢), this scale has some intervallic resemblances with the dorian mode but sounds more **xenharmonic** and tense.

Tuning system – a system of fixed fine-tuned pitches used, amongst other mediums, in fixed-pitch instruments (such as keyboards) and which is not used to temper **Just Intonation** ratios. In this category are included both historical and modern systems, such as collections of untempered **Just Intonation** ratios (such as **Pythagorean Tuning),** and **Equal Tunings**.

Temperament – a system of fixed fine-tuned pitches used, amongst other mediums, in fixed-pitch instruments (such as keyboards) to approximate **Just Intonation** intervals through a variety of intervallic criteria. In this category are included both historical and modern systems although in this research only historical temperaments are referred (such as **Meantone** Temperaments).

Tuneable Intervals – **Just Intonation** intervals which Marc Sabat and Robin Hayward have considered to be tuneable by ear by playing one pitch over another and being able to tune by ear their desired resonance. These intervals are classified as ‘easy’, ‘more difficult’ and ‘hard’.

Twelfth tone – an interval with the size of roughly a sixth of a **semitone** (and a twelfth of a whole tone). This is the smallest interval found in 72EDO/NPO. It is commonly considered to be a **microtone**.

Western Chromatic Notation – the most common notation standard in Western music. Alongside other parameters, it is compatible with the convention of 12 different pitch classes (including enharmonic equivalents) per octave and with multiple **tunings systems**, **temperaments** and **intonation** practices. Its system of accidentals is based on adding sharps and flats to the C, D, E, F, G, A, B note names.

Whole tone pentatonic – a **xenharmonic** and microtonal pentatonic scale which can be created by rounding 5EDO into 11EDO. This scale has intervallic characteristics which make it somewhat resemble a subset of the hexatonic whole tone scale.

Xenharmonic – coined by Ivor Darreg, it is a description referring to musical structures (scales, harmonies, melodies) outside of the convention of **tuning systems**, **temperaments** and **intonation** practices which relate to 12EDO. Although not part of Darreg's definition, a continuum of **microtonal** systems and **intonation** practices spanning from 'non-xenharmonic' to 'xenharmonic' could be used to describe their intervallic qualities.

Appendix 2. Bosanquet-Wilson mappings of non-Diatonic EDOs on the Lumatone[170]

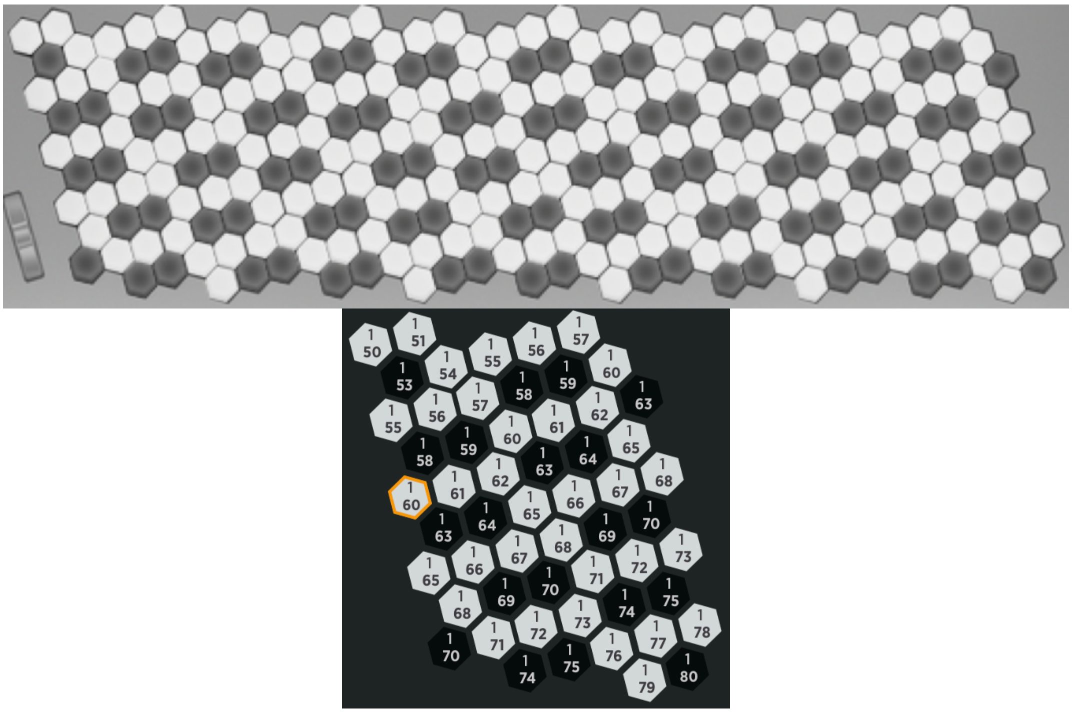


Figure 84. Bosanquet-Wilson mapping of 11EDO (with a chosen fifth size 6\11EDO), a Superflat rendering of this EDO according to Giedraitis.

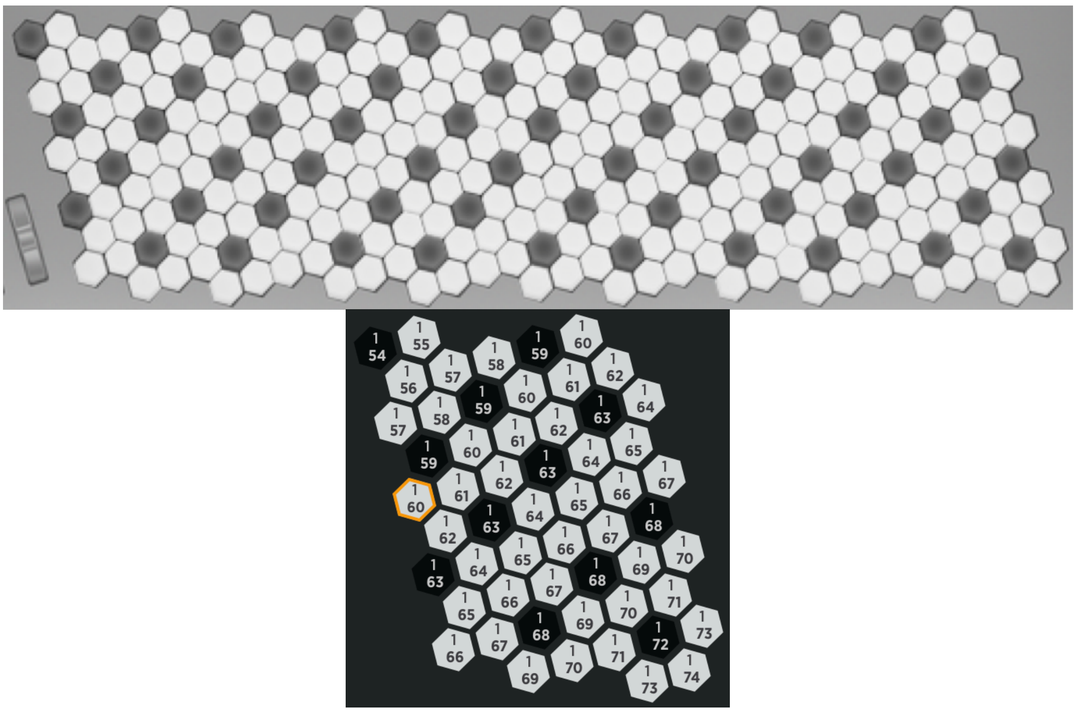


Figure 85. Bosanquet-Wilson mapping of 9EDO, a Superflat EDO according to Giedraitis.

[170] Trivial EDOs have not been added, since they are included within 12EDO.

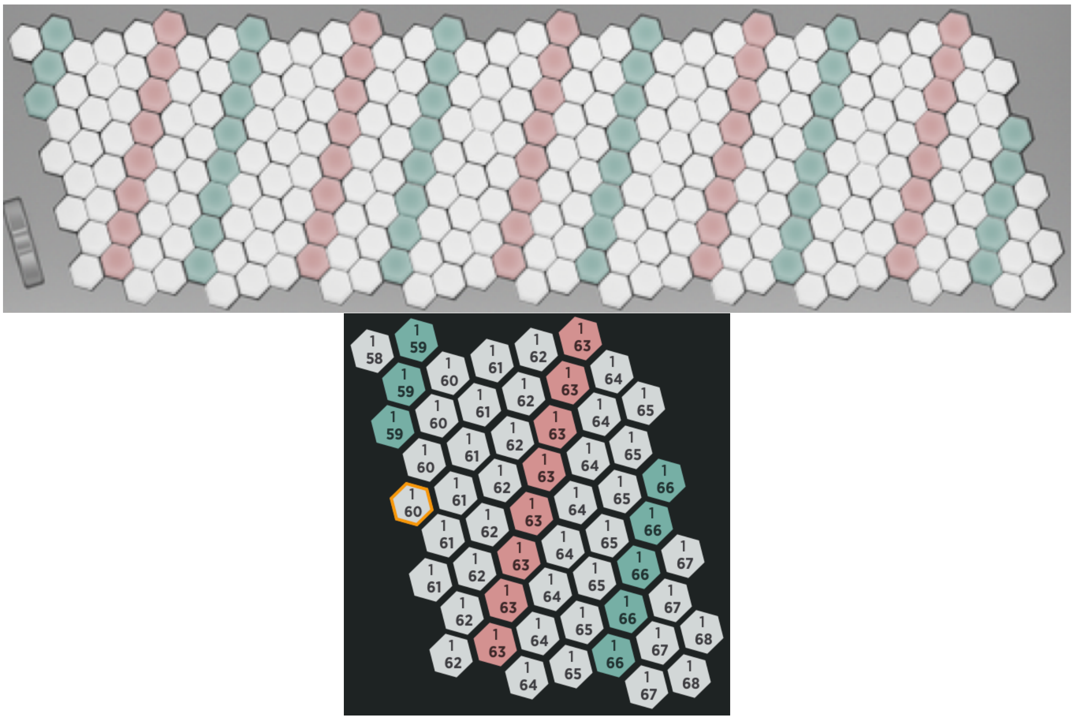


Figure 86. Bosanquet-Wilson mapping of 7EDO, a Perfect EDO according to Giedraitis.

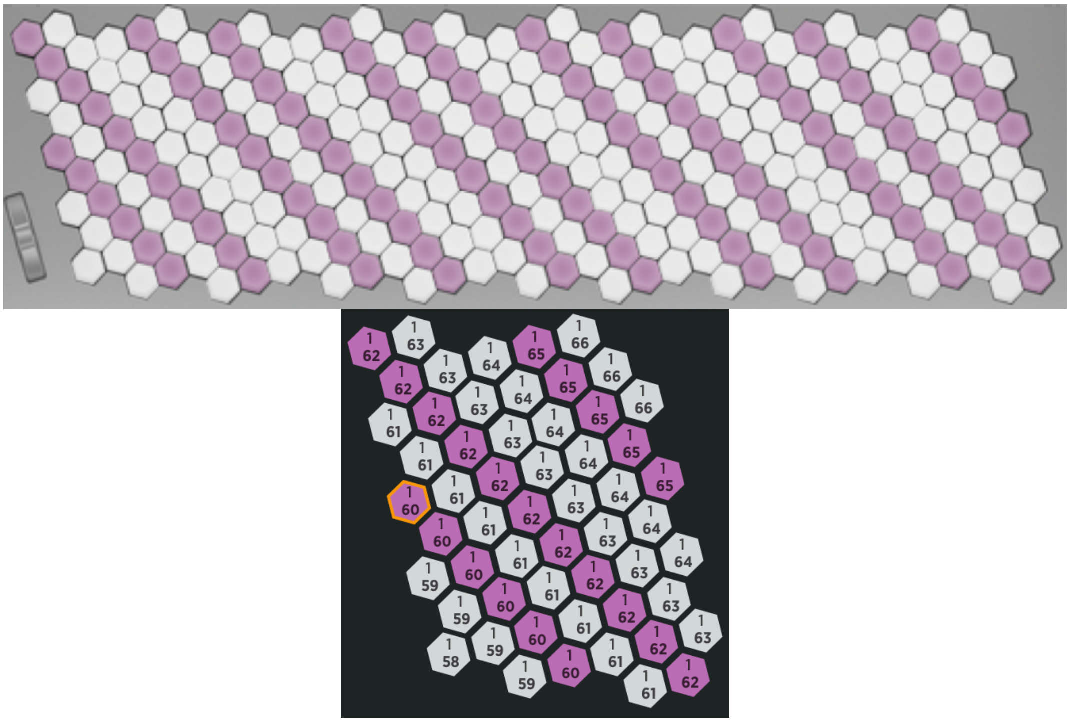


Figure 87. Bosanquet-Wilson mapping of 5EDO, a Pentatonic EDO according to Giedraitis.

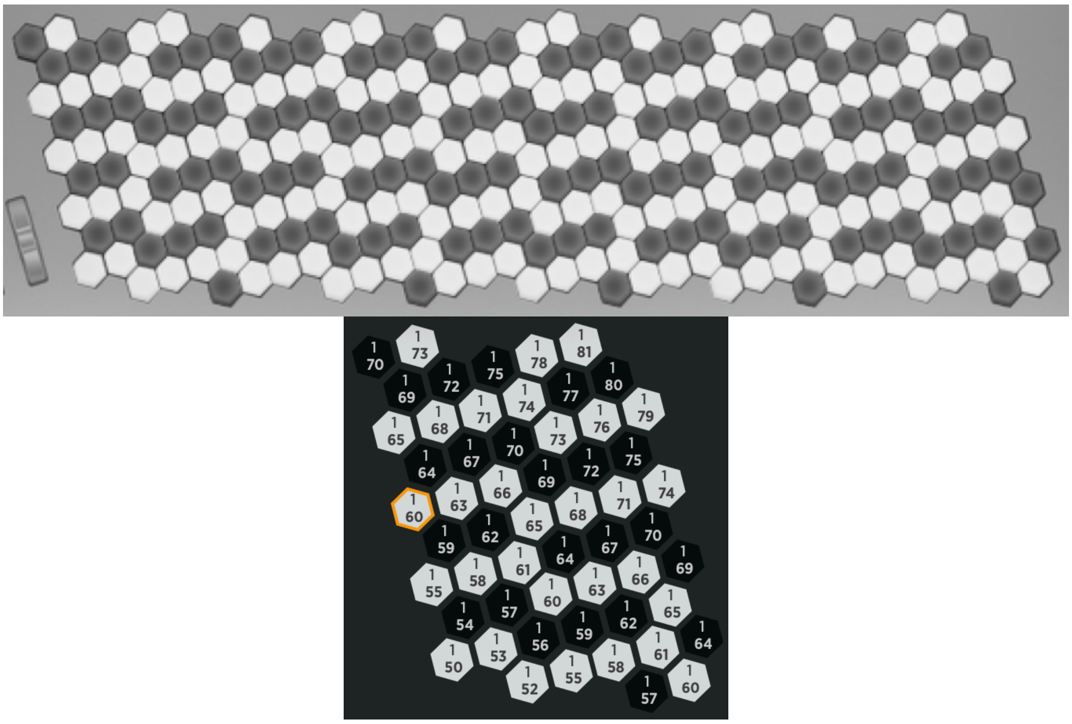


Figure 88. Bosanquet-Wilson mapping of 13EDO (with a chosen fifth size of 8\13EDO), a Supersharp rendering of this EDO according to Giedraitis.

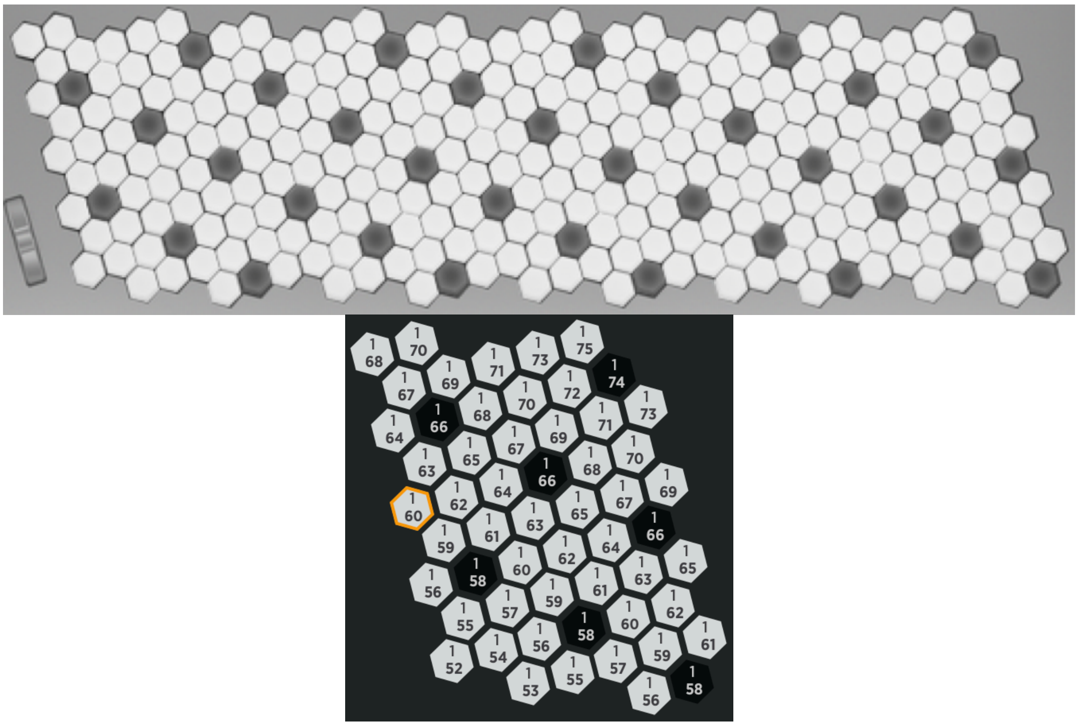


Figure 89. Bosanquet-Wilson mapping of 8EDO, a Supersharp EDO according to Giedraitis.

Appendix 3. List of 12, 24, 48 and 72NPO pitches and enharmonic equivalencies


1
100¢
2
200¢
12NPO
#
b
2
100¢
3
150¢
4
200¢
24NPO
#
b
4
100¢
5
125¢
6
150¢
7
175¢
8
200¢
48NPO
^#
v#
^b
vb
6
100¢
7
116.7¢
8
133.3¢
9
150¢
10
166.7¢
11
183.3¢
12
200¢
72NPO
^#
v#
^b
vb


12NPO
#
b
2
200¢
3
300¢
24NPO
#
b
4
200¢
5
250¢
6
300¢
48NPO
^#
v#
^b
vb
8
200¢
9
225¢
10
250¢
11
275¢
12
300¢
72NPO
^#
v#
^b
vb
12
200¢
13
216.7¢
14
233.3¢
15
250¢
16
266.7¢
17
283.3¢
18
300¢


3
300¢
4
400¢
12NPO
#
b
6
300¢
7
350¢
8
400¢
24NPO
#
b
12
300¢
13
325¢
14
350¢
15
375¢
16
400¢
48NPO
^#
v#
^b
vb
18
300¢
19
316.7¢
20
333.3¢
21
350¢
22
366.7¢
23
383.3¢
24
400¢
72NPO
^#
v#
^b
vb


4
400¢
5
500¢
12NPO
#
b
8
400¢
9
450¢
10
500¢
24NPO
#
b
16
400¢
17
425¢
18
450¢
19
475¢
20
500¢
48NPO
^#
v#
^b
vb
24
400¢
25
416.7¢
26
433.3¢
27
450¢
28
466.7¢
29
483.3¢
30
500¢
72NPO
^#
v#
^b
vb


5
500¢
6
600¢
12NPO
#
b
10
500¢
11
550¢
12
600¢
24NPO
#
b
20
500¢
21
525¢
22
550¢
23
575¢
24
600¢
48NPO
^#
v#
^b
vb
30
500¢
31
516.7¢
32
533.3¢
33
550¢
34
566.7¢
35
583.3¢
36
600¢
72NPO
^#
v#
^b
vb

6
600¢
7
700¢
12NPO
#
b
12
600¢
13
650¢
14
700¢
24NPO
#
b
24
600¢
25
625¢
26
650¢
27
675¢
28
700¢
48NPO
^#
v#
^b
vb
36
600¢
37
616.7¢
38
633.3¢
39
650¢
40
666.7¢
41
683.3¢
42
700¢
72NPO
^#
v#
^b
vb


7
700¢
8
800¢
12NPO
#
b
14
700¢
15
750¢
16
800¢
24NPO
#
b
28
700¢
29
725¢
30
750¢
31
775¢
32
800¢
48NPO
^#
v#
^b
vb
42
700¢
43
716.7¢
44
733.3¢
45
750¢
46
766.7¢
47
783.3¢
48
800¢
72NPO
^#
v#
^b
vb


8
800¢
9
900¢
12NPO
#
b
16
800¢
17
850¢
18
900¢
24NPO
#
b
32
800¢
33
825¢
34
850¢
35
875¢
36
900¢
48NPO
^#
v#
^b
vb
48
800¢
49
816.7¢
50
833.3¢
51
850¢
52
866.7¢
53
883.3¢
54
900¢
72NPO
^#
v#
^b
vb

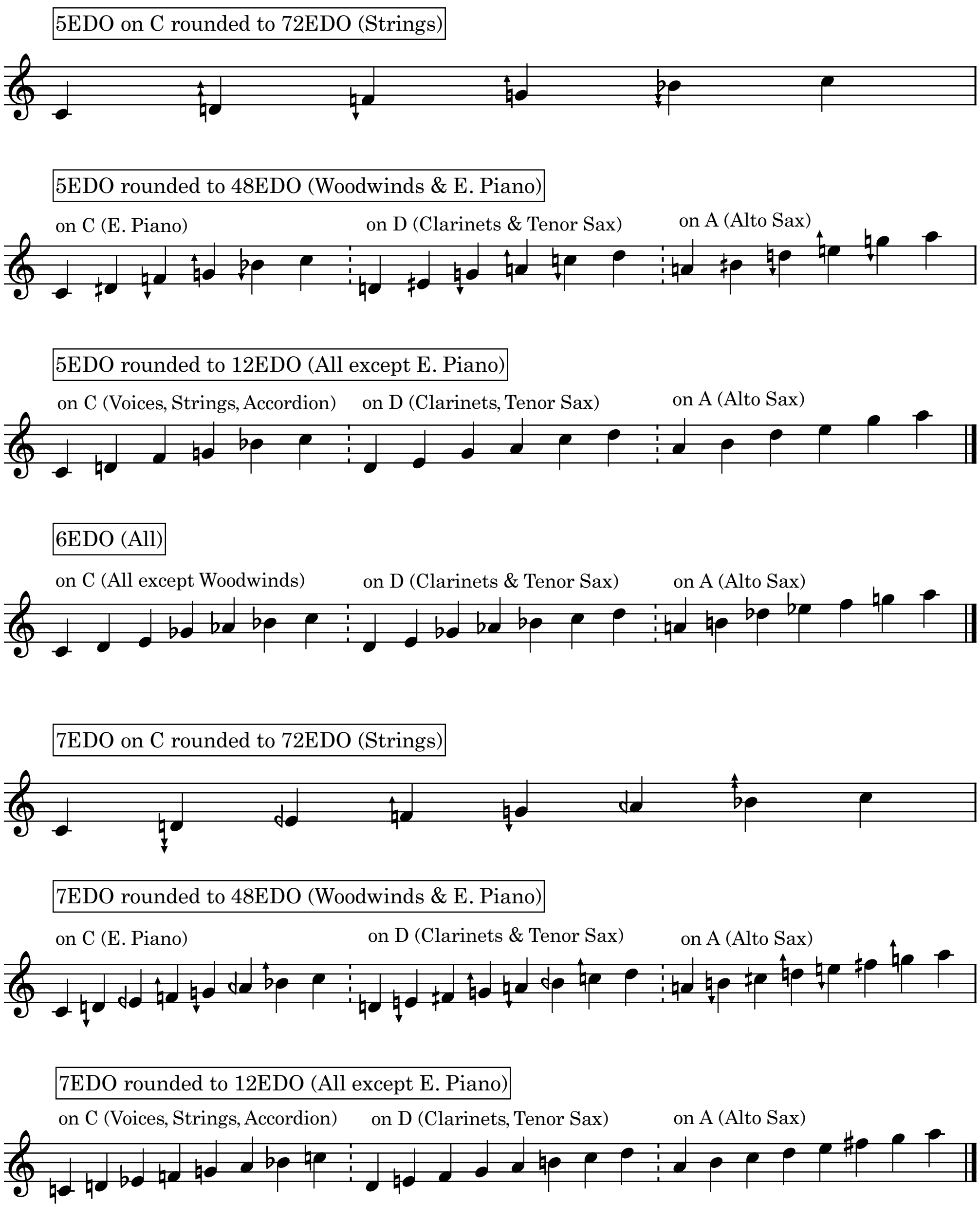

9
900¢
10
1000¢
12NPO
#
b
18
900¢
19
950¢
20
1000¢
24NPO
#
b
36
900¢
37
925¢
38
950¢
39
975¢
40
1000¢
48NPO
^#
v#
^b
vb
54
900¢
55
916.7¢
56
933.3¢
57
950¢
58
966.7¢
59
983.3¢
60
1000¢
72NPO
^#
v#
^b
vb


10
1000¢
11
1100¢
12NPO
#
b
20
1000¢
21
1050¢
22
1100¢
24NPO
#
b
40
1000¢
41
1025¢
42
1050¢
43
1075¢
44
1100¢
48NPO
^#
v#
^b
vb
60
1000¢
61
1016.7¢
62
1033.3¢
63
1050¢
64
1066.7¢
65
1083.3¢
66
1100¢
72NPO
^#
v#
^b
vb

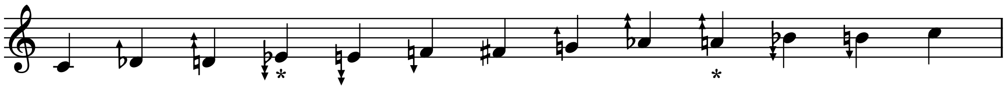

11
1100¢
12
1200¢
12NPO
#
b
22
1100¢
23
1150¢
24
1200¢
24NPO
#
b
44
1100¢
45
1125¢
46
1150¢
47
1175¢
48
1200¢
48NPO
^#
v#
^b
vb
66
1100¢
67
1116.7¢
68
1133.3¢
69
1150¢
70
1166.7¢
71
1183.3¢
72
1200¢
72NPO
^#
v#
^b
vb

Appendix 4. Rounding and Notation of Tuning Systems in *Adam*

Rounding and Notation of Tuning Systems in *Adam*

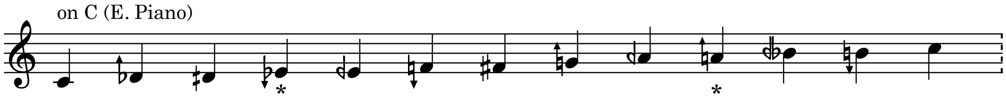


10EDO and two notes of 9EDO* on C rounded to 72EDO (Strings)



10EDO and two notes of 9EDO* rounded to 48EDO (Woodwinds & E. Piano)

on C (E. Piano)

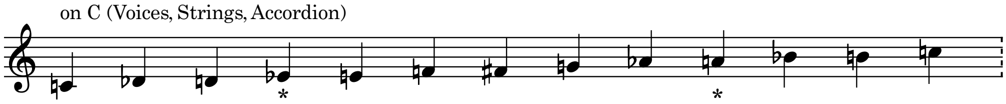


on D (Clarinets & Tenor Sax)

on A (Alto Sax)

10EDO and two notes of 9EDO* rounded to 12EDO (All except E. Piano)

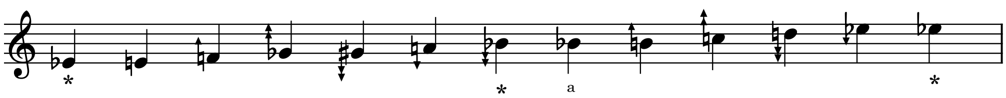
on C (Voices, Strings, Accordion)



on D (Clarinets & Tenor Sax)

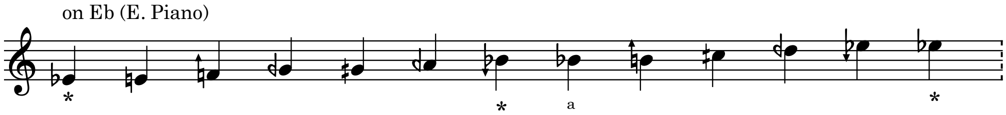
on A (Alto Sax)

Mode of 10EDO and two notes of 9EDO* re-anchored to Bb[a] on Eb & rounded to 72EDO (Strings)



Mode of 10EDO and two notes of 9EDO* re-anchored to Bb[a] & rounded to 48EDO (Woodwinds & E. Piano)

on Eb (E. Piano)

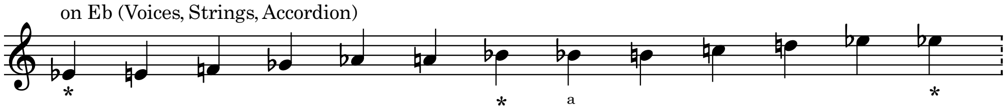


on F (Clarinets & Tenor Sax)

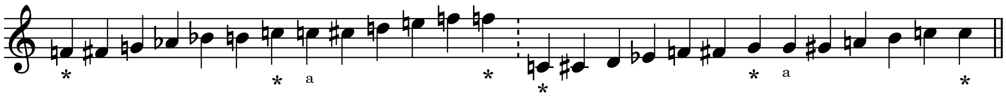
on C (Alto Sax)

Mode of 10EDO and two notes of 9EDO* re-anchoredto Bb[a] & rounded to 12EDO (All except E. Piano)

on Eb (Voices, Strings, Accordion)



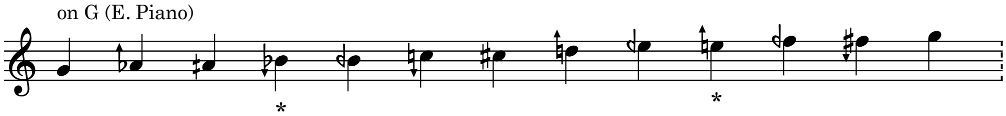
on F (Clarinets & Tenor Sax) on C (Alto Sax)



10EDO and two notes of 9EDO* in G rounded to 72EDO (Strings)

10EDO and two notes of 9EDO* rounded to 48EDO (Woodwinds & E. Piano)

on G (E. Piano)

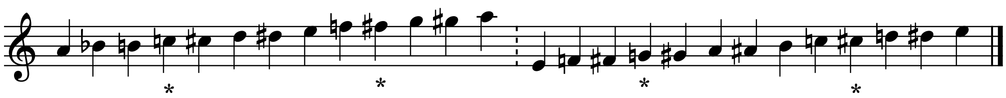


on A (Clarinets & Tenor Sax) on E (Alto Sax)

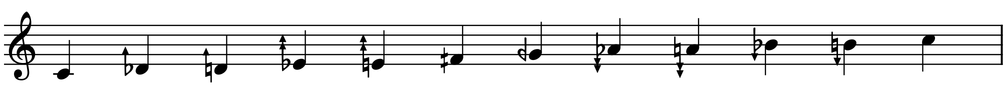
10EDO and two notes of 9EDO* rounded to 12EDO (All except E. Piano)

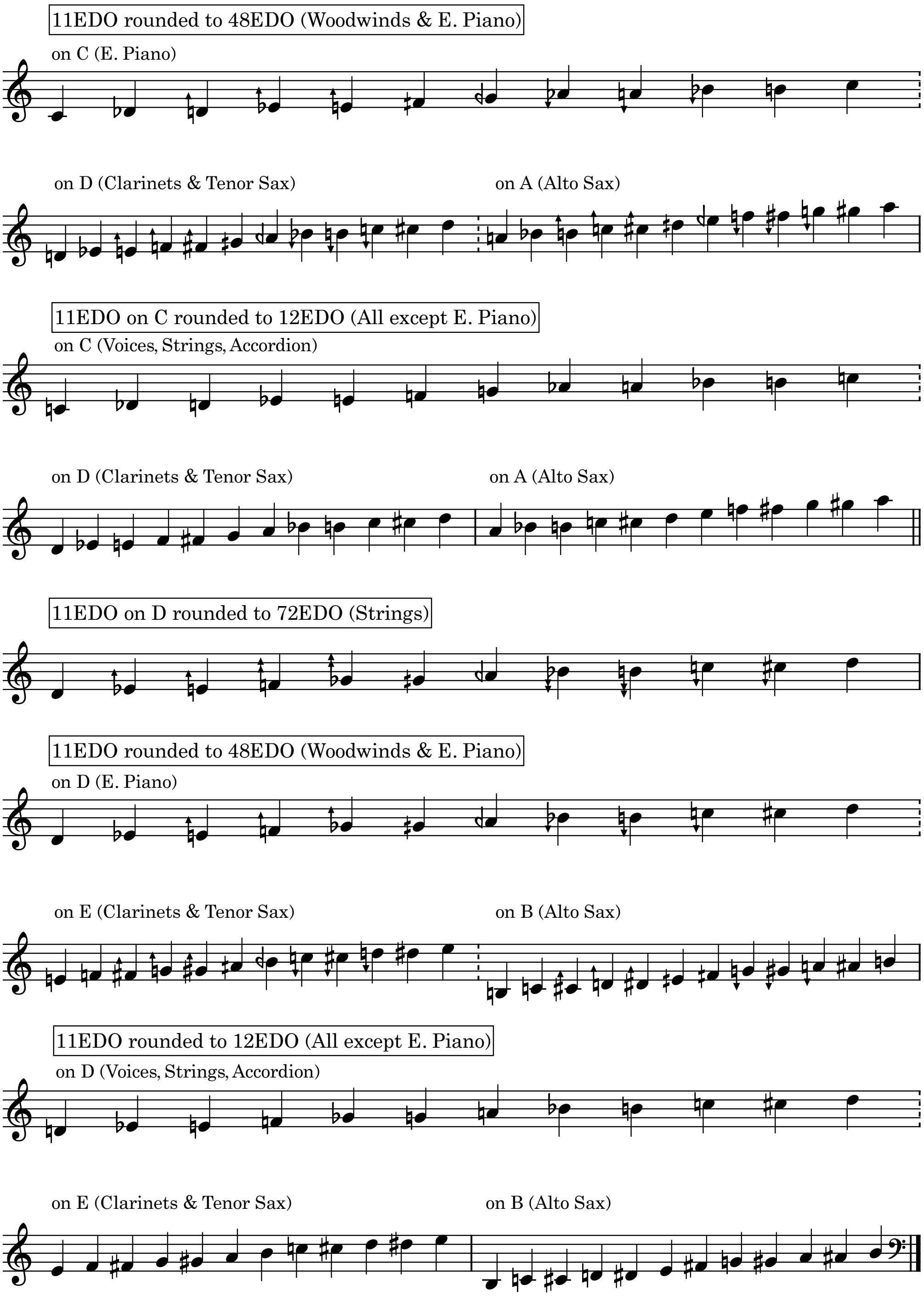
on G (Voices, Strings, Accordion)

on A (Clarinets & Tenor Sax) on E (Alto Sax)

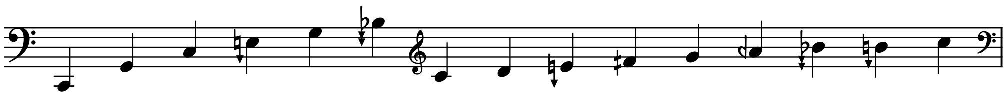


11EDO on C rounded to 72EDO (Strings)



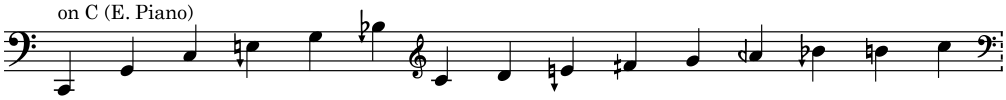

11EDO rounded to 48EDO (Woodwinds & E. Piano)
on C (E. Piano)
on D (Clarinets & Tenor Sax)
on A (Alto Sax)
11EDO on C rounded to 12EDO (All except E. Piano)
on C (Voices, Strings, Accordion)
on D (Clarinets & Tenor Sax)
on A (Alto Sax)
11EDO on D rounded to 72EDO (Strings)
11EDO rounded to 48EDO (Woodwinds & E. Piano)
on D (E. Piano)
on E (Clarinets & Tenor Sax)
on B (Alto Sax)
11EDO rounded to 12EDO (All except E. Piano)
on D (Voices, Strings, Accordion)
on E (Clarinets & Tenor Sax)
on B (Alto Sax)

Harmonic Series on C (Partials 1-16) rounded to 72EDO (Strings)

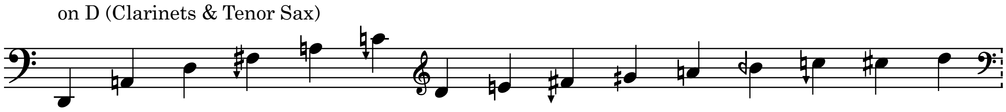


Harmonic Series (Partials 1-16) rounded to 48EDO (Woodwinds & E. Piano)

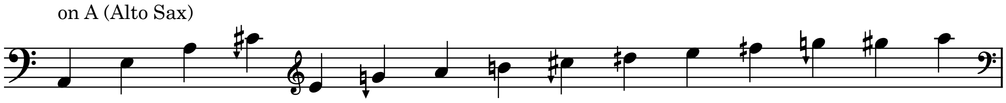
on C (E. Piano)



on D (Clarinets & Tenor Sax)



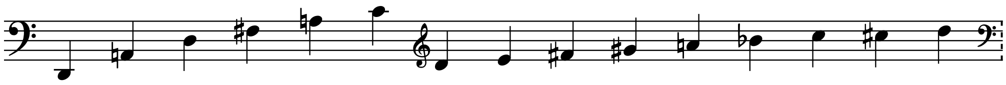
on A (Alto Sax)



Harmonic Series (Partials 1-16) rounded to 12EDO (All except E. Piano)

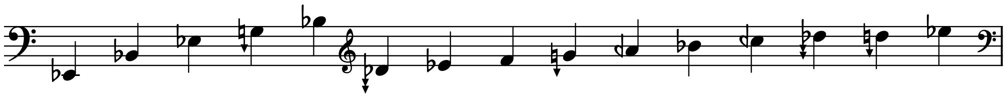
on C (Voices, Strings, Accordion)

on D (Clarinets, Tenor Sax)



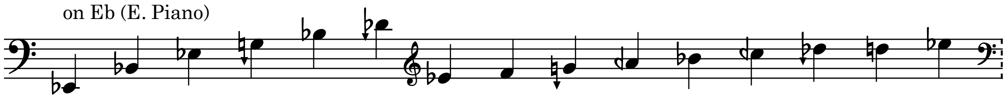
on A (Alto Sax)

Harmonic Series on Eb (Partials 1-16) rounded to 72EDO (Strings)

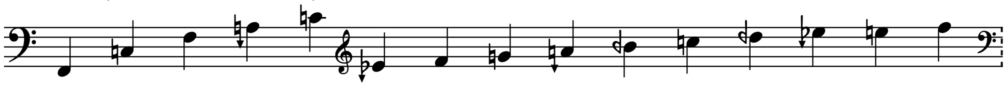


Harmonic Series (Partials 1-16) rounded to 48EDO (Woodwinds & E. Piano)

on Eb (E. Piano)



on F (Clarinets & Tenor Sax)

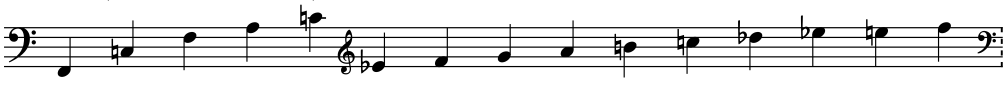


on C (Alto Sax)

Harmonic Series (Partials 1-16) rounded to 12EDO (All except E. Piano)

on Eb (Voices, Strings, Accordion)

on F (Clarinets & Tenor Sax)



on C (Alto Sax)

Appendix 5. Interval Class Ranking System for 72NPO

Based on the discrepancy between tuneable intervals and their associated interval classes regarding tuneability (as mentioned in Chapter 2.4), the variable levels of difficulty associated to tuneable intervals, and the appropriateness of 72NPO to notate tuneable and quasi-tuneable intervals as being equivalent, I have been developing a ranking system for 72NPO intervals. Building upon Sabat and Hayward's research into the tuneability of a variety of Just Intonation intervals,[171] this system classifies from 'A' to 'J' how problematic, in terms of tuneability, an interval class (represented by an interval of any size within the range of a unison and an octave) may be in its performative application.

When stretching an interval within a one-octave span (which represents an interval class) by octave increments up to a three-octave range, if any of the two new resulting intervals (plus the original one), when approximated to their closest intervals in 72EDO, correspond to any of 72EDO's quasi-tuneable intervals, then the different levels of difficulty of these tuneable intervals are used to classify the tested interval. For the purposes of this classification system, the quasi-tuneable intervals of 72EDO are the intervals of this system which are the closest to the tuneable intervals that were identified by Sabat and Hayward.

To the three intervals associated to each interval class (the original one and the two new ones that were generated), a set number of points is attributed to each according to their identified level of difficulty: 3 for 'easy', 2 for 'more difficult', 1 for 'hard' and 0 for 'impossible' (similarly to Sabat and Hayward's classification). By adding all points attributed to each of the three intervals associated to the same interval class, the sum shall indicate the rank of that interval class: 'A' for 9 points (all 3 transpositions are tuneable and easy to perform) up to 'J' for 0 points (all 3 transpositions are impossible to perform). See Figures 90 to 98 for visual illustrations and notation of these interval classes per rank.

In this research, mostly Rank A, B and C tuneable intervals have been employed, particularly in string and brass instruments. Even though this classification system needs to be further explored to test its usefulness as a compositional and performative framework, it was incorporated into Global Properties of EDOs, in its list of 'reference intervals'.

[171] See Sabat and Hayward, pp. 6-10.

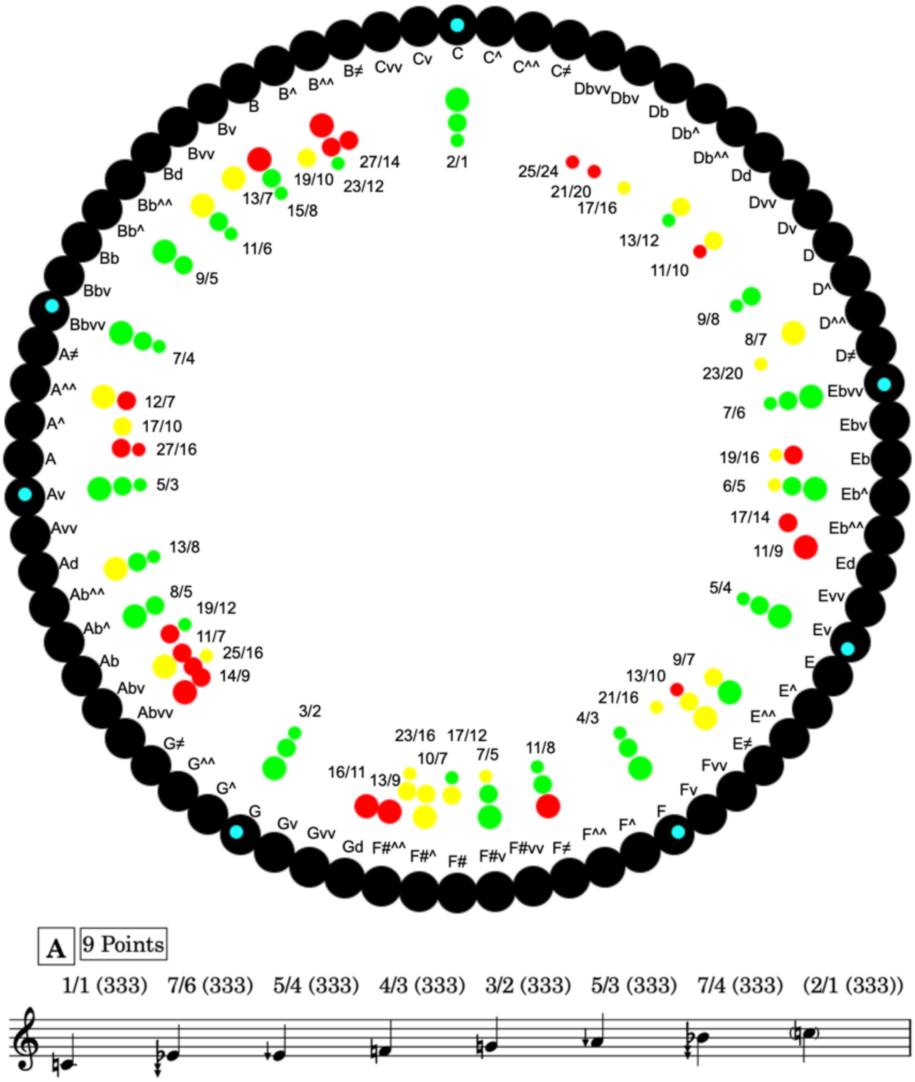


Figure 90. Rank A intervals (cyan dots) above C and their 72NPO musical notation. In the score, above each note, the equivalent ratio that is formed between each note over a middle C is given alongside the amount of points (3 for 'easy/green', 2 for 'more difficult/yellow', 1 for 'hard/red' and 0 for 'impossible/no dot') attributed to each of the three versions of these intervals that are achieved through octave incrementations (original interval, original interval plus one octave, and original interval plus two octaves) in within a three octave range. In Rank A intervals, the sum of these points equals 9; in Rank B, 8 points; in Rank C, 7 points; until, in Rank I, just 1 point. Intervals with 0 points are classified as Rank J.

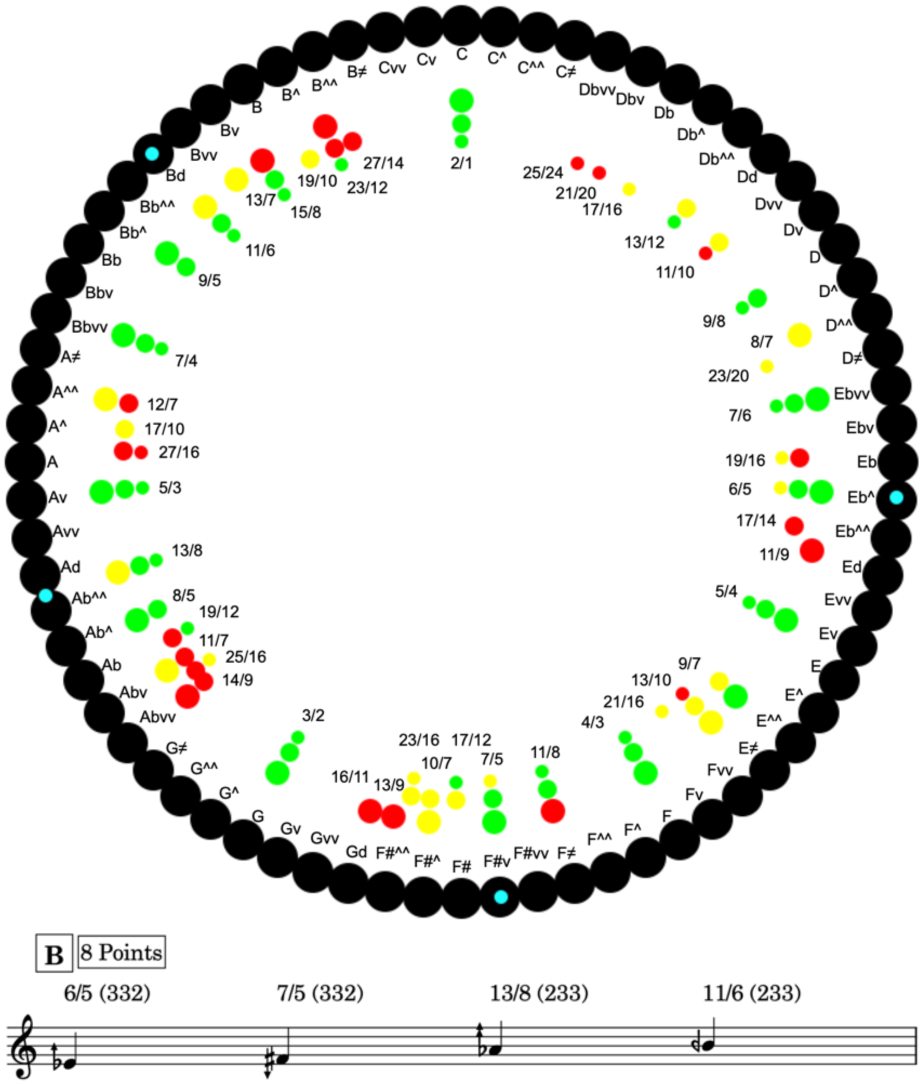


Figure 91. Rank B intervals (cyan dots) above C and their 72NPO musical notation.

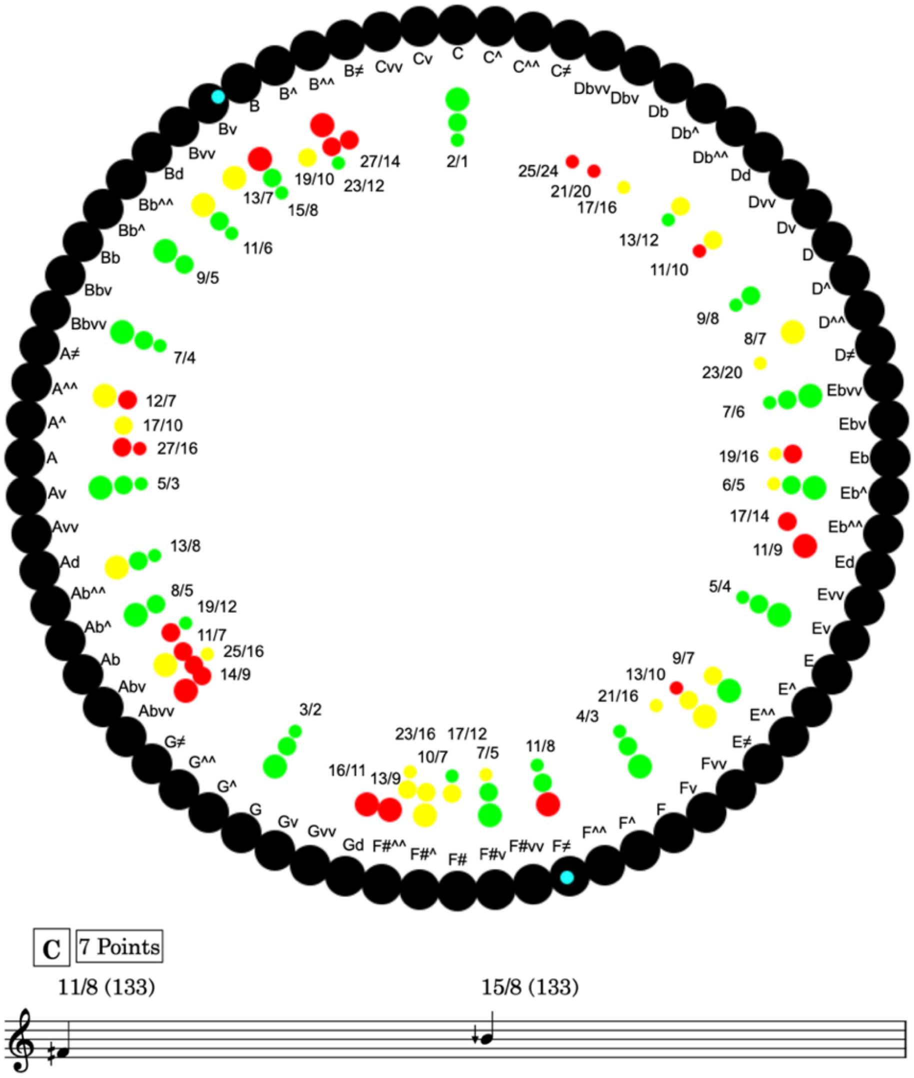


Figure 92. Rank C intervals (cyan dots) above C and their 72NPO musical notation.

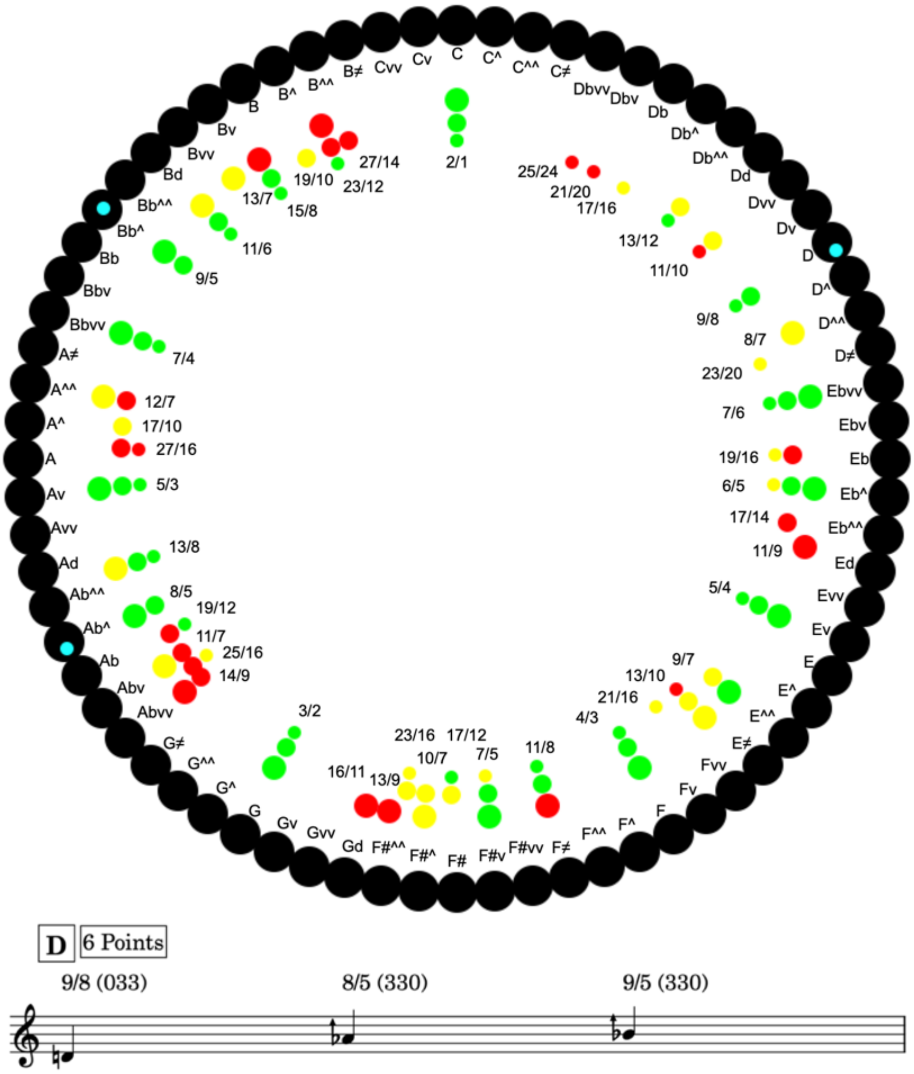


Figure 93. Rank D intervals (cyan dots) above C and their 72NPO musical notation.

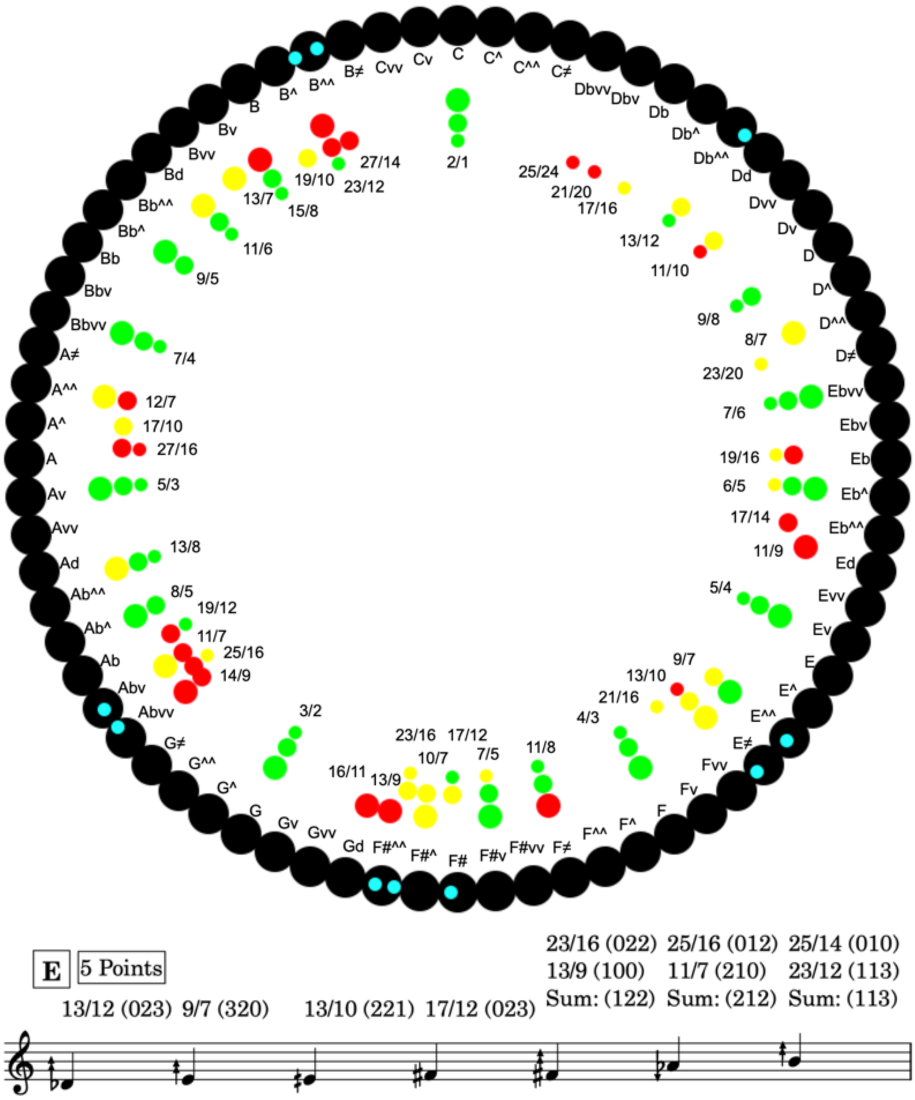


Figure 94. Rank E intervals (cyan dots) above C and their 72NPO musical notation. Two different 'interval classes' sharing the same notation in 72NPO are regarded as just one pitch class. Its resulting points equals the sum of the points of the transpositions of the original interval classes with the highest score. E.g. in its first transposition, F#^^ (13/9 and 23/16) has 1 point and its second and third transpositions each have 2 points, making a total of 5 points. In 72NPO, the interval between C and F#^^ is then a Rank E Interval.

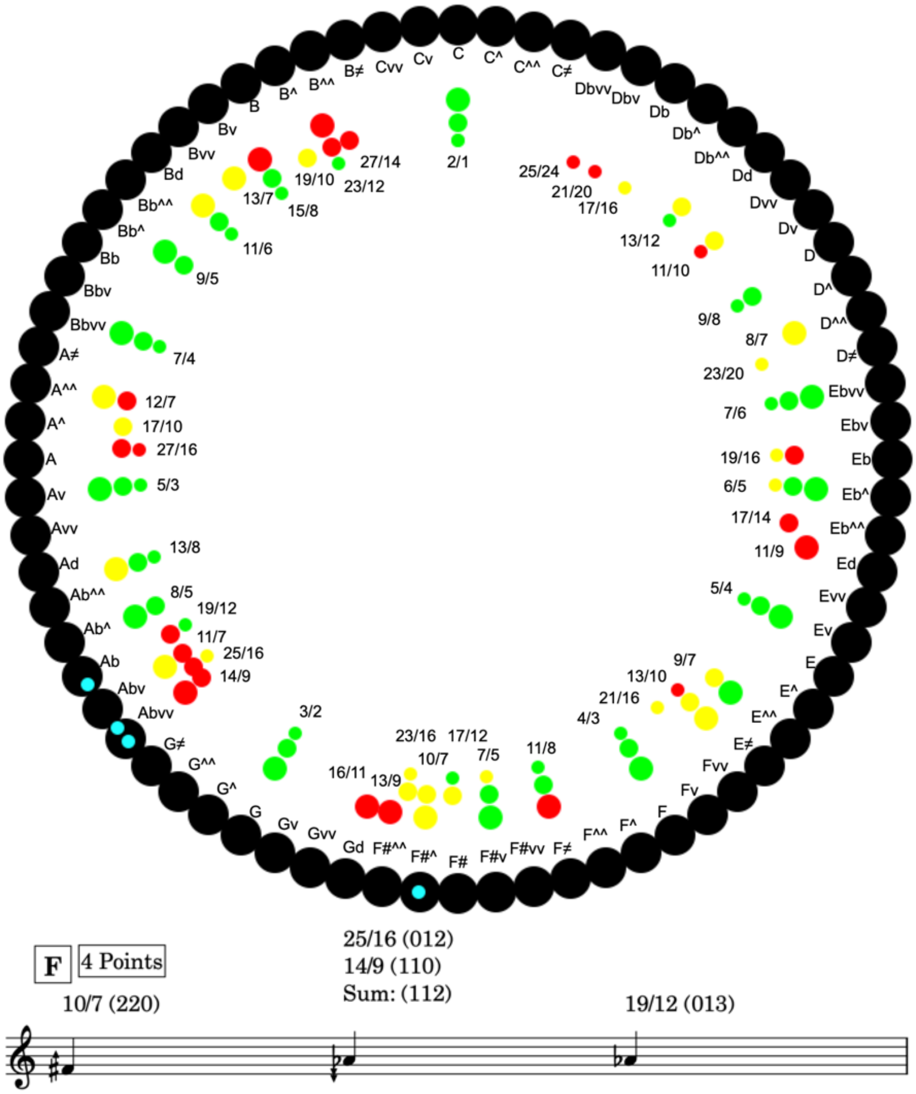


Figure 95. Rank F intervals (cyan dots) above C and their 72NPO musical notation.

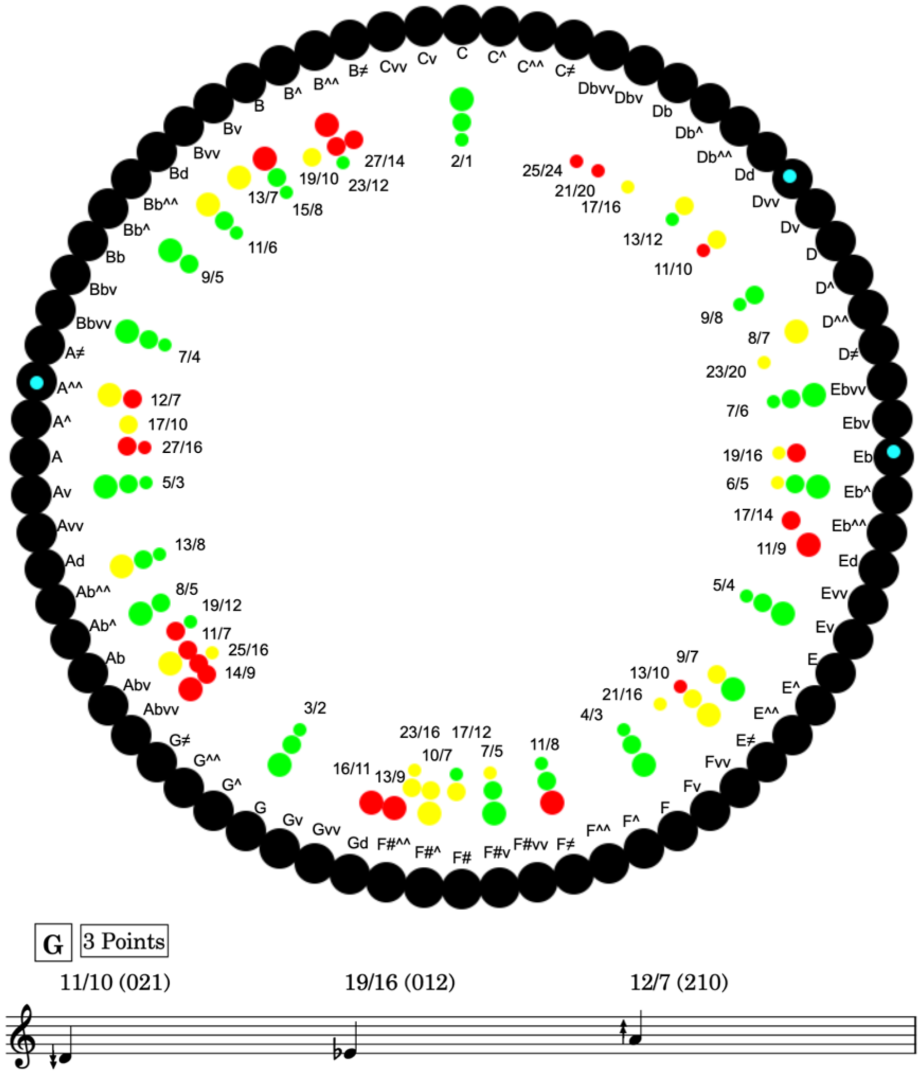


Figure 96. Rank G intervals (cyan dots) above C and their 72NPO musical notation.

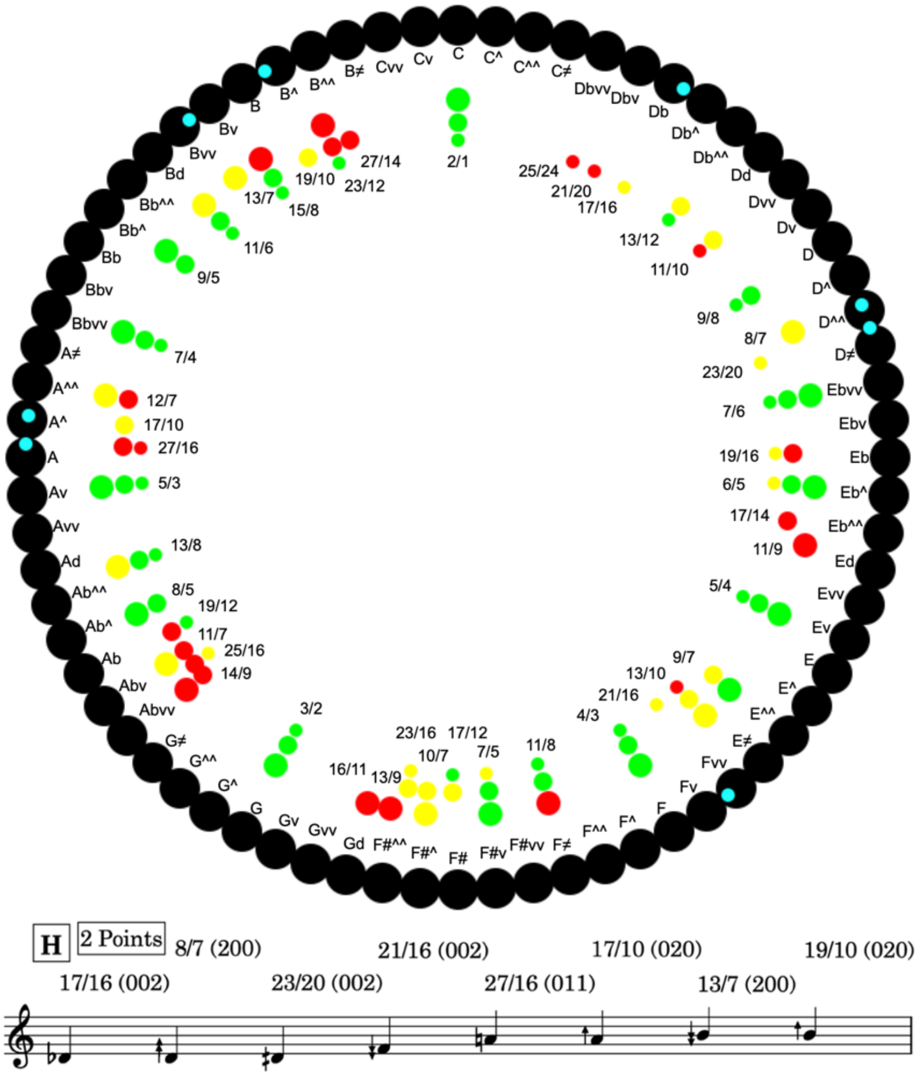


Figure 97. Rank H intervals (cyan dots) above C and their 72NPO musical notation.

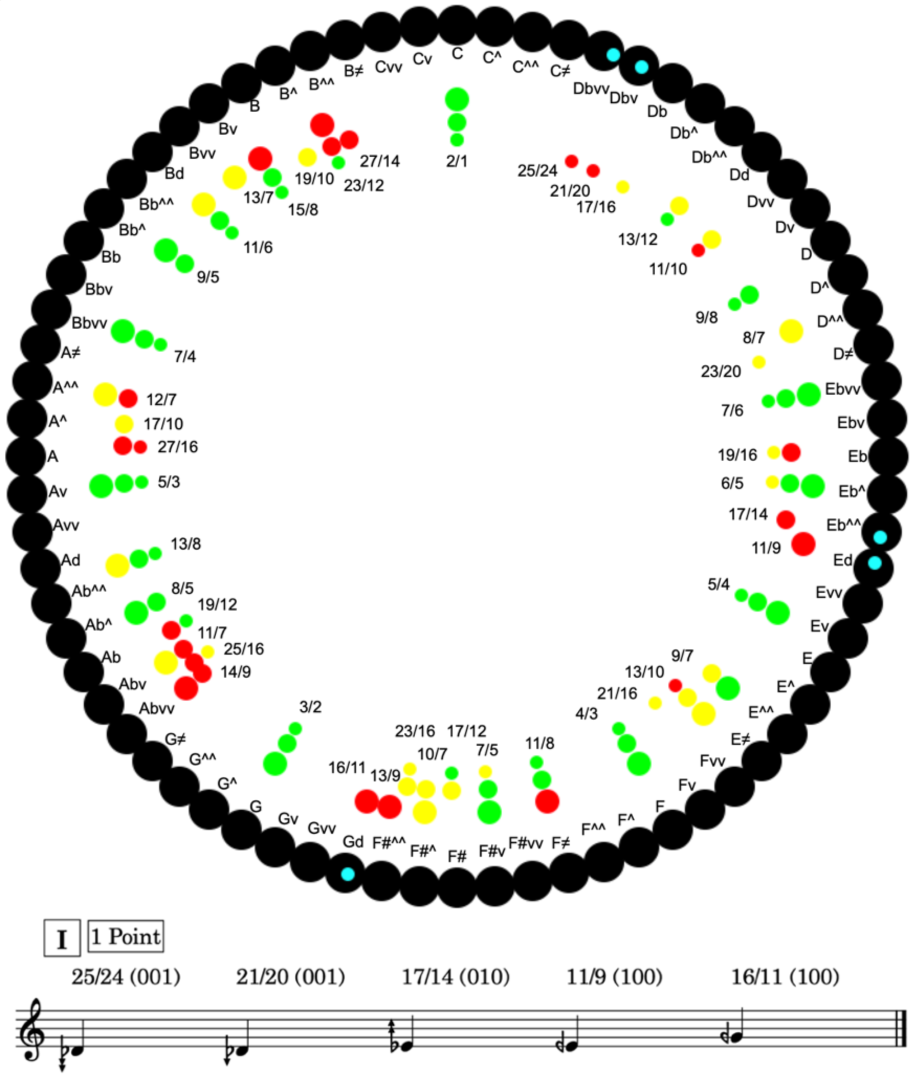


Figure 98. Rank I intervals (cyan dots) above C and their 72NPO musical notation.

Appendix 6. How I perceive intervals and how I would notate them

This document was created during the composition process of *Odd [s]Paces* as an exercise for me to understand how I hear and identify intervallic regions, using 72EDO intervals and 72NPO notation, and identify which intervals I felt were redundant (acoustically and in terms of notation). I would play these intervals on my keyboard with bitKlavier using long notes with a pipe organ sample, listening to them (sometimes isolated, sometimes switching between intervals in their vicinity) and take note of how I perceived them.

Through this exercise I understood that I tend to be more sensitive towards intervals larger than a major second, 24NPO pitch classes (quarter tone-based intervals, but not necessarily their strict 24EDO renderings), and that my perception of consonance is broadly in line with Marc Sabat and Robin Hayward's definition of tuneable intervals.

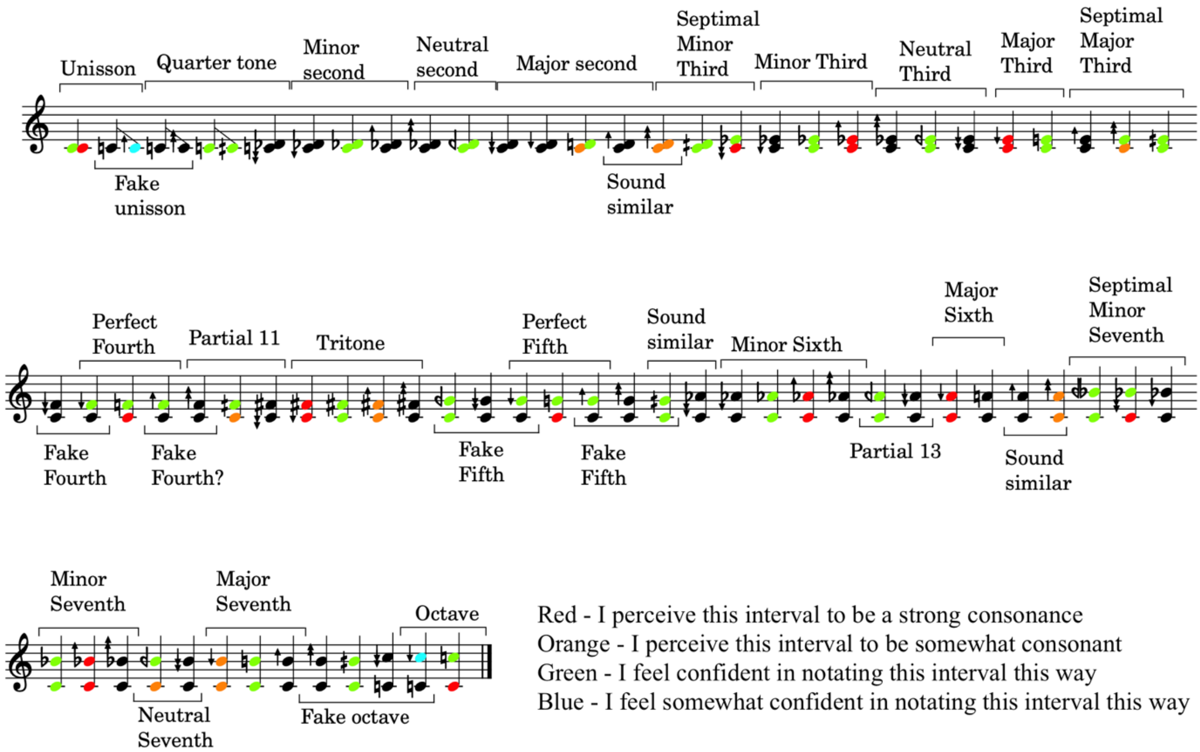


Figure 99. Representation of my perception of 72NPO intervals.

Appendix 7. A scalar rounding model for 5, 7, 11 and 12EDO

Throughout the composition process of *Adam*, *Dreams from an Old Memory* and *The Fable of the Pilgrim*, I found myself gravitating towards mixing 5EDO, 7EDO, 11EDO and the 12EDO second pentatonic mode and the dorian mode. Additionally, the intermixing of these materials became gradually more systematic, with rounding being the central criteria for harmonic and melodic intonational variation within these tunings/scales, linking them conceptually.[172] This led to the development of a rounding model (Figure 100).

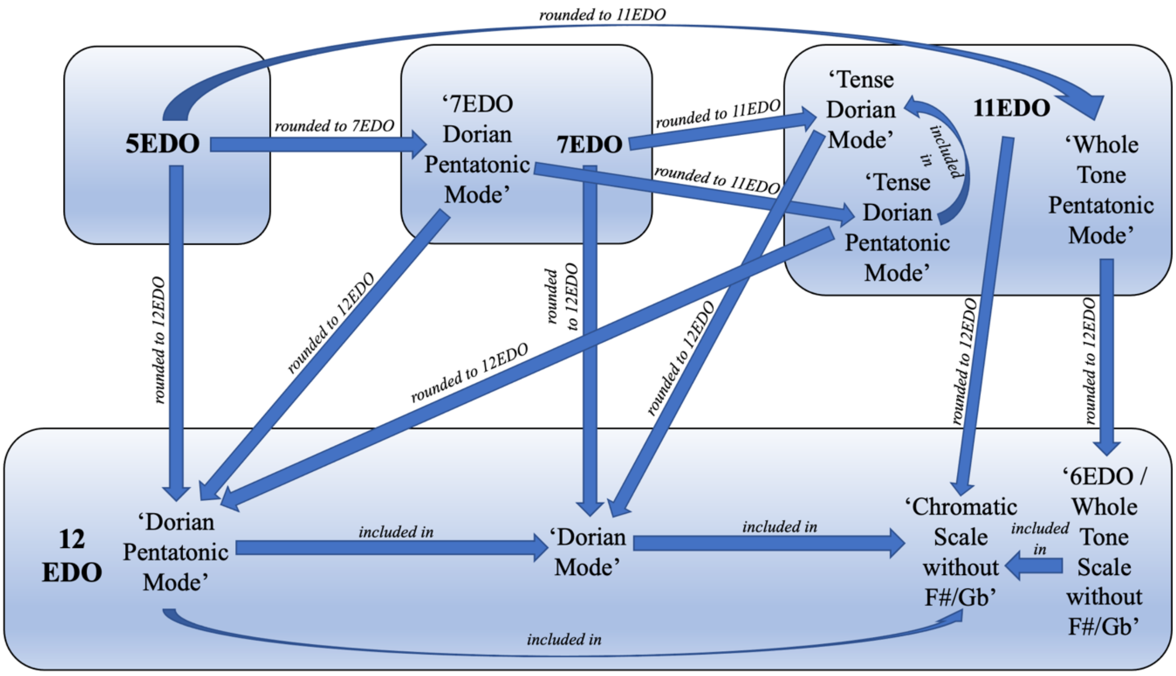


Figure 100. The 5, (6,) 7, 11 & 12EDO rounding model.

In this model, 5 and 7EDO are used as the 'source scales', from which all other microtonal speculations unfold, leading to a variety of pentatonic and heptatonic scales. 5 and 7EDO could be directly rounded to 12EDO, leading to the second pentatonic mode and the dorian mode (Figures 101 and 102, respectively), or could be rounded into 11EDO, leading to the 'whole tone pentatonic' scale (Figure 103) and the 'tense dorian/orgone[7]' scale (Figure 104). These scales can then be rounded into 12EDO, leading to an incomplete whole tone scale (Figure 105), and the dorian mode (Figure 106).

[172] See Chapter 8 for how, in 'Piece Z' (later adapted as the first movement of *Convergence*), I started to explore different approximation approaches for harmony and melody. Nonetheless, this model is relevant for most pieces written during this research.

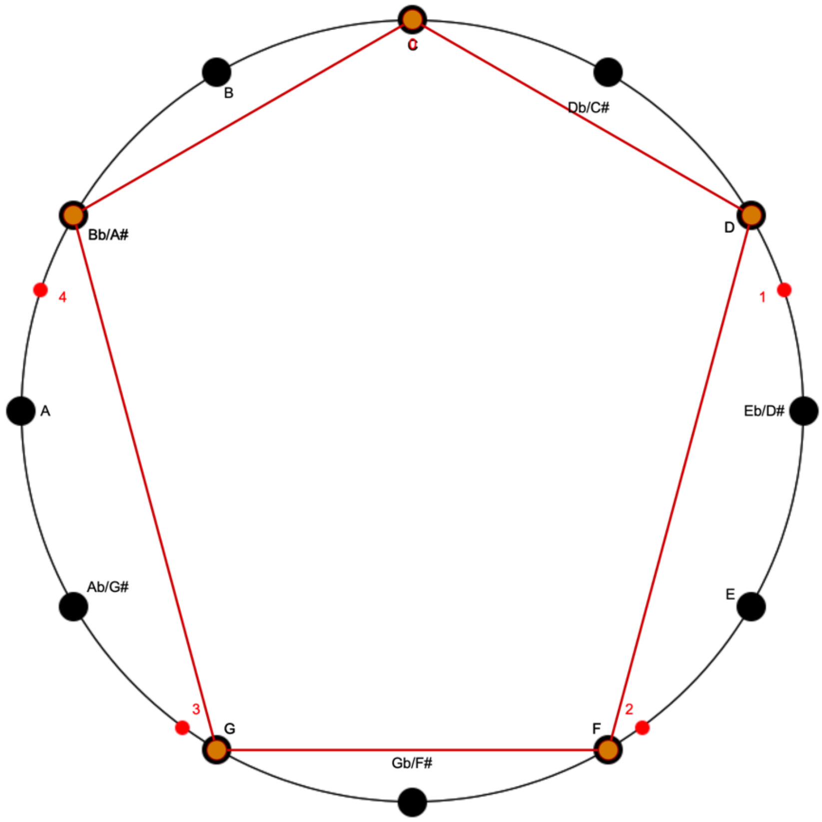


Figure 101. Rounding (polygon and orange dots) of 5EDO (red dots and numbers) into 12EDO (black dots and notation).

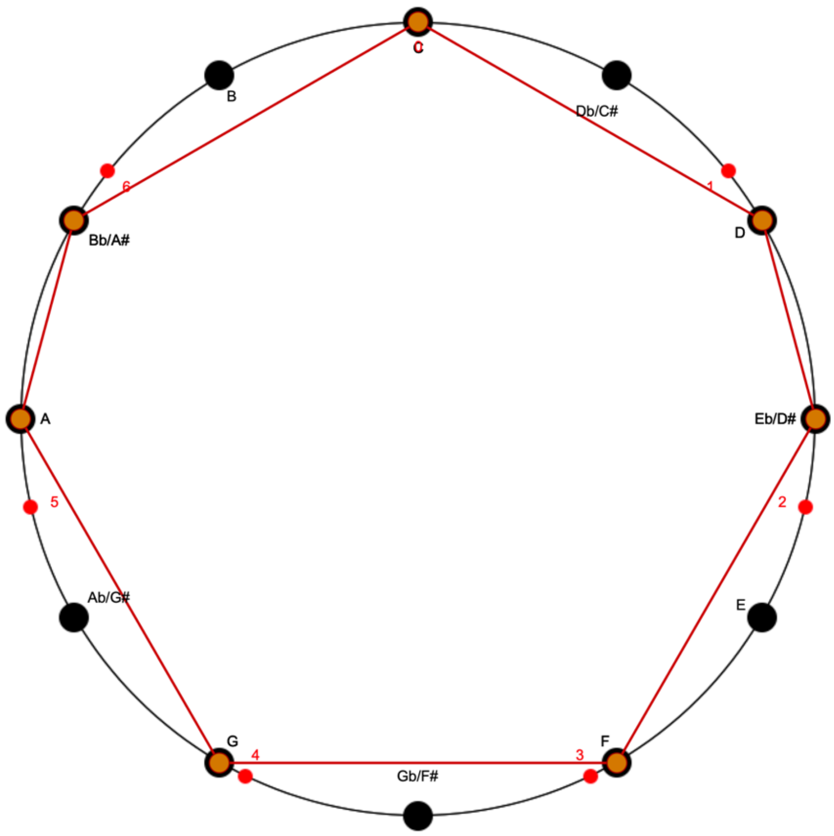


Figure 102. Rounding (polygon and orange dots) of 7EDO (red dots and numbers) into 12EDO (black dots and notation).

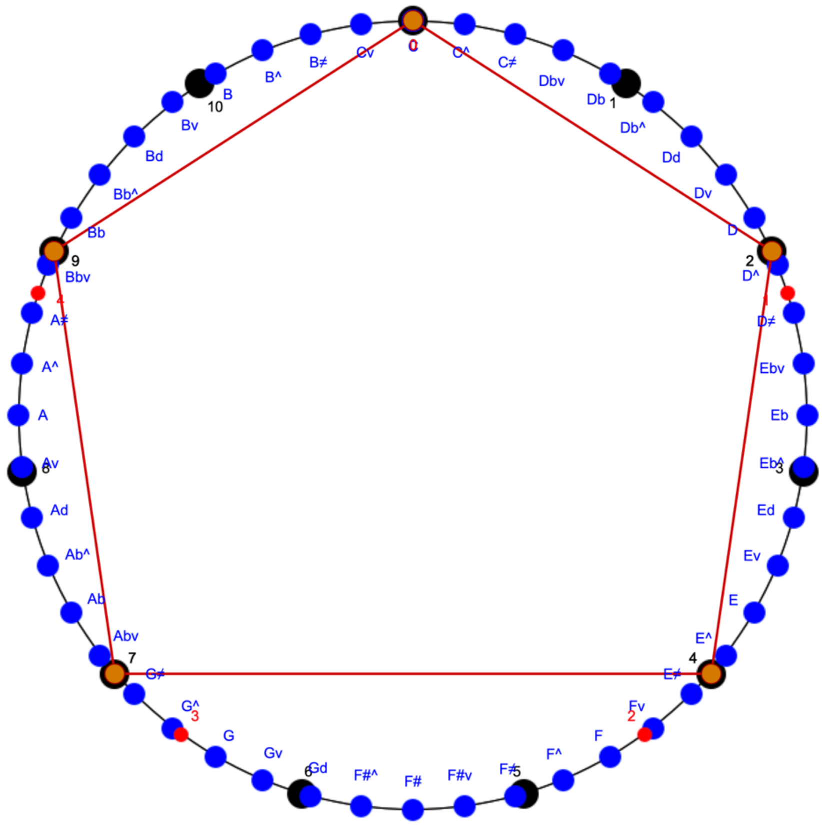


Figure 103. Rounding (polygon and orange dots) of 5EDO (red dots and numbers) into 11EDO (black dots and numbers), against 48EDO (blue dots and notation).

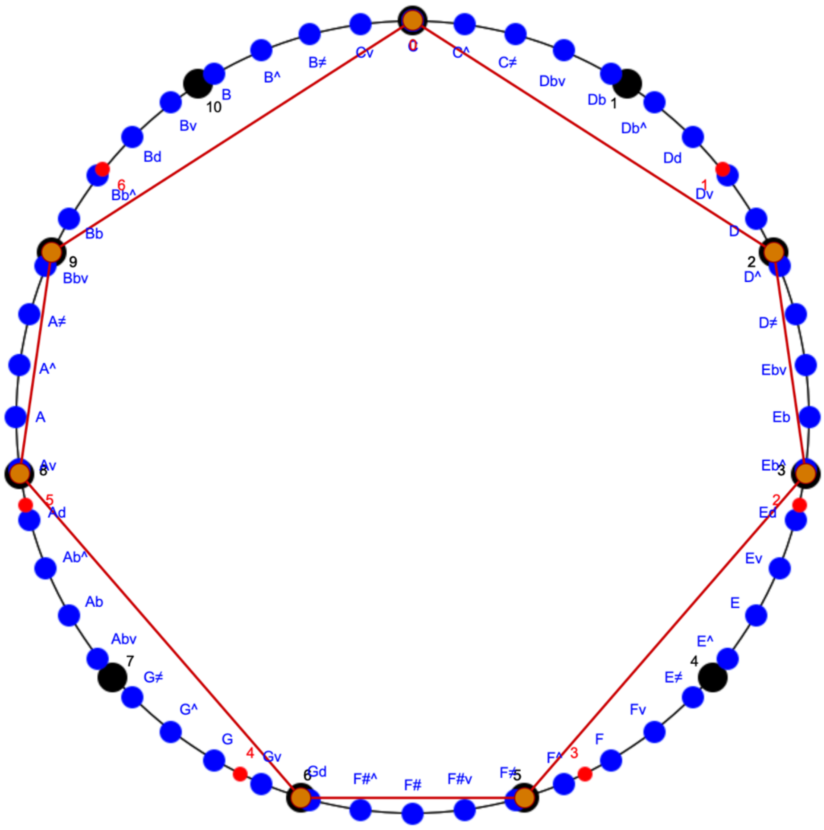


Figure 104. Rounding (polygon and orange dots) of 7EDO (red dots and numbers) into 11EDO (black dots and numbers), against 48EDO (blue dots and notation).

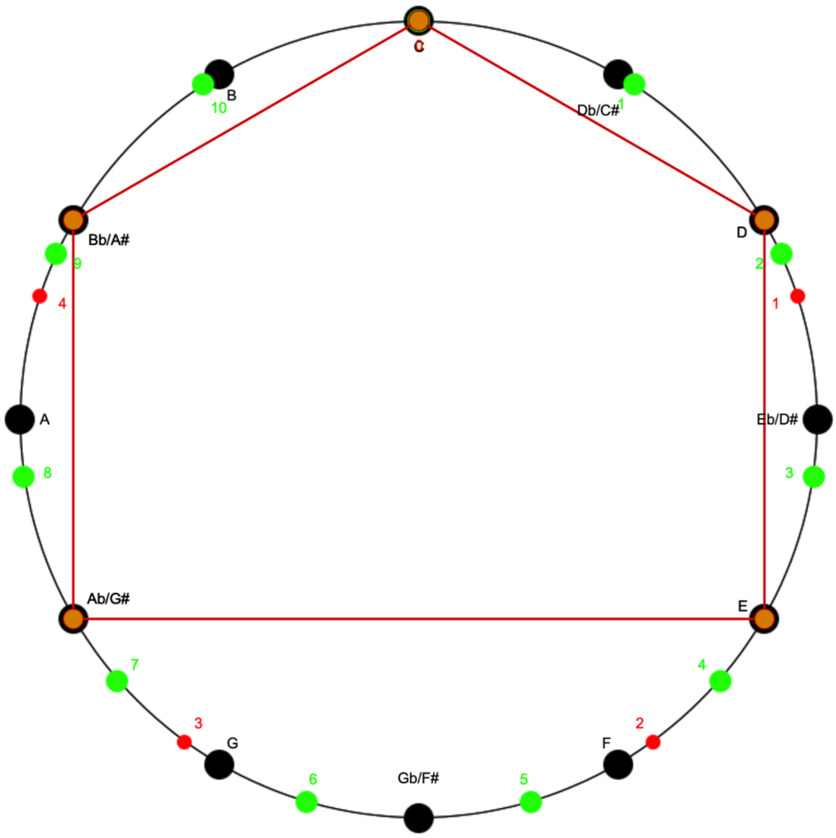


Figure 105. Double rounding (polygon and orange dots) of 5EDO (red dots and numbers) into 11EDO (green dots and numbers) into 12EDO (black dots and notation).

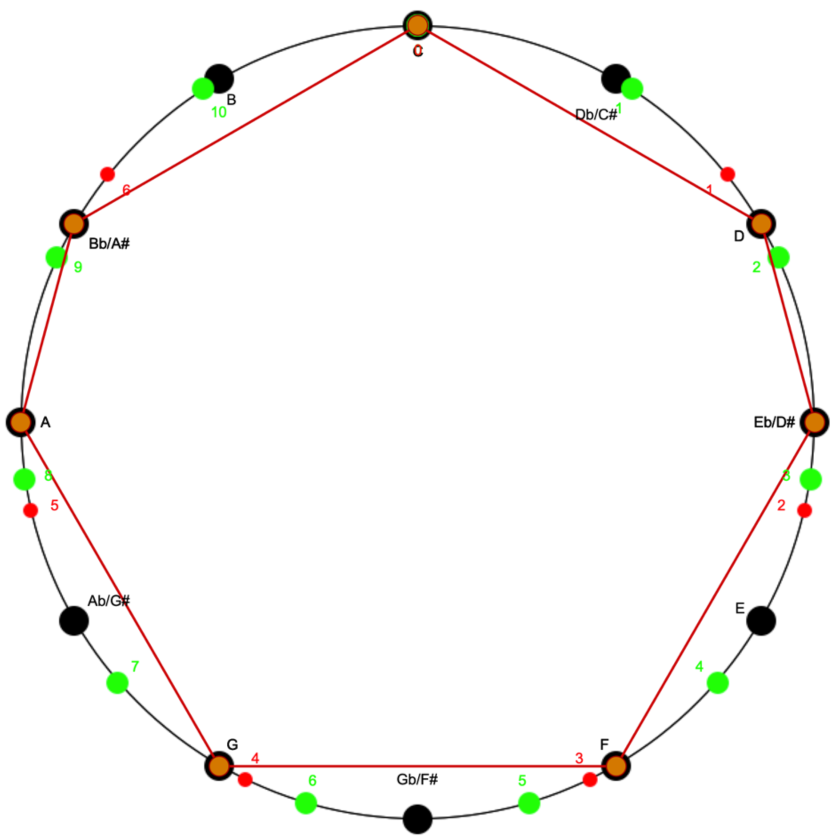


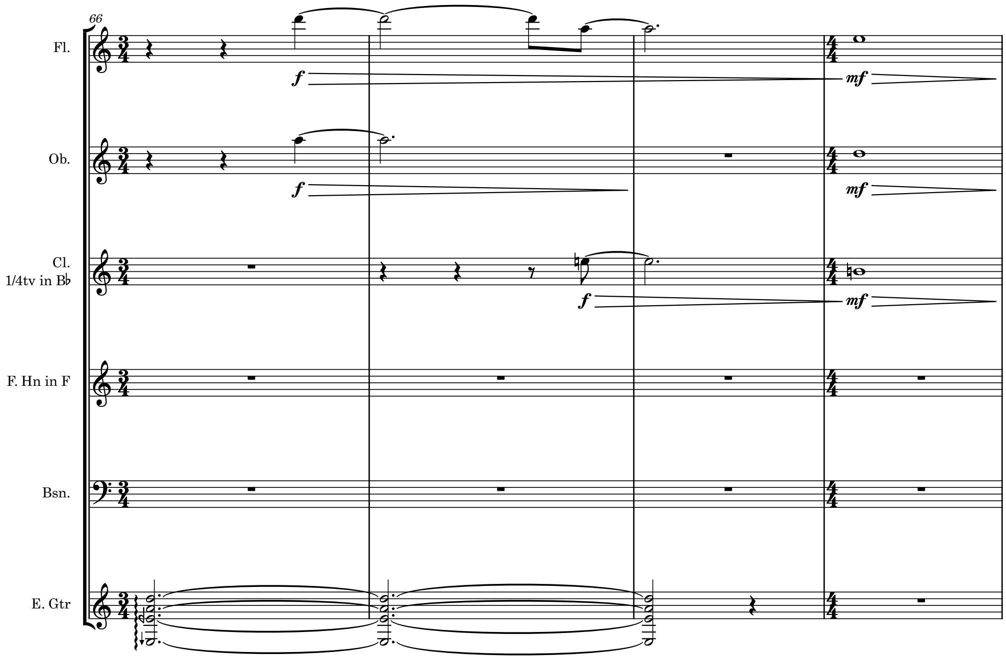
Figure 106. Double rounding (polygon and orange dots) of 7EDO (red dots and numbers) into 11EDO (green dots and numbers) into 12EDO (black dots and notation).

The model of Figure 100 reflects my increasing interest in prime numbers (5, 7 and 11EDO are all prime numbered EDOs), and could be expanded to include other prime numbered EDOs (or even other non-prime numbered ones). Such future explorations, given the increasing number of notes per octave beyond 12, may require this expanded model to include subdivisions of 12EDO. In practice, 24, 48 and 72EDO used in this research to explore approximations of the materials included in the model of Figure 100, but only to make them more performable on acoustic instruments. They were not conceptualized as being part of this model (as 12EDO has). Therefore, further explorations which would include these subdivisions of 12EDO as part of this model could be of musical/research value.

Appendix 8. Sketch of 'Piece Z'


23
B a tempo
Fl.
Ob.
Cl.
1/4tv in B♭
F. Hn in F
Bsn.
mf
mf
mf
senza vib.
mf
30
C
molto rit.
Adagietto ♩ = 72
senza vib.
fp
senza vib.
fp
senza vib.
fp
38
senza vib.
p
fp
fp
fp
senza vib.
p
mf
mf
mf
p
mf

46
♩= 80
Fl.
Ob.
Cl.
1/4tv in B♭
F. Hn in F
Bsn.
p
mf
5th harmonic
53
59
adjust manually to sound high?
is there a way to intonate this lower?


66
Fl.
Ob.
Cl.
1/4tv in B♭
F. Hn in F
Bsn.
E. Gtr


70
Develop crescendo more
Fl.
Ob.
Cl.
1/4tv in B♭
adjust manually to sound high?
F. Hn in F
Bsn.
E. Gtr


D
75
Fl.
Ob.
Cl.
1/4tv in B♭
F. Hn in F
Bsn.
E. Gtr
intonate lower?
intonate lower?
intonate lower?
80
Fl.
Ob.
Cl.
1/4tv in B♭
F. Hn in F
Bsn.
E. Gtr
intonate lower?


E
85
Fl.
Ob.
Cl.
1/4tv in B♭
F. Hn in F
Bsn.
E. Gtr
mf
l.v. (as much as possible)
89
Fix this pitch. Ed ideally
93


97
Fl.
Ob.
Cl.
1/4tv in B♭
F. Hn in F
Bsn.
E. Gtr
p
mf
If possible, intonate higher than quarter sharp to produce a fake octave with Horn
101
106


110
Random wind sounds
Fl.
Random wind sounds
Ob.
Random wind sounds
Cl.
1/4tv in B♭
Random wind sounds
F. Hn in F
Random wind sounds
Bsn.
E. Gtr
mf
114
119


124
Fl.
Ob.
Cl.
1/4tv in B♭
F. Hn in F
Bsn.
E. Gtr
127
mf
F
♩ = 107
131
pp
f

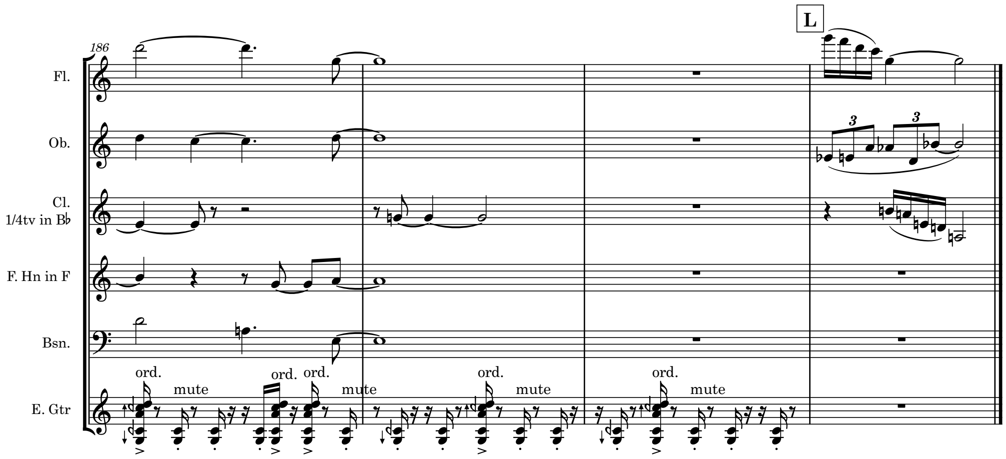

136
Fl.
Ob.
Cl.
1/4tv in B♭
F. Hn in F
Bsn.
E. Gtr


142
G
Fl.
Ob.
Cl.
1/4tv in B♭
F. Hn in F
Fix Basson part
Bsn.
E. Gtr


148
Fl.
Ob.
Cl.
1/4tv in B♭
F. Hn in F
Bsn.
E. Gtr
mf
154
160
H
p
f


164
I
Fl.
Ob.
Cl.
1/4tv in B♭
F. Hn in F
Bsn.
E. Gtr
mute
169
ord.
mute
173


176
J
180
K
183
Fl.
Ob.
Cl.
1/4tv in B♭
F. Hn in F
Bsn.
E. Gtr
ord.
mute


186
L
Fl.
Ob.
Cl.
1/4tv in B♭
F. Hn in F
Bsn.
E. Gtr
3
3
ord.
mute
ord.
ord.
mute
ord.
mute
ord.
mute

Appendix 9. Mathematical considerations about the 'reverse-temperament' of equal tunings

While experimenting with the concept of using Just Intonation ratios to approximate intervals of equal tunings (the reverse action of tempering ratios into temperaments) some mathematical considerations have been taken.

An equal tuning is formed by logarithmically equally dividing an interval serving as a *functional octave*, 'ø', into a modulo of 'm' parts. A musical interval formed of 's' steps of an equal tuning is then represented as $ø^{\frac{s}{m}}$.

A Just Intonation ratio of frequencies 'r' can be used to approximate an interval $ø^{\frac{s}{m}}$ (or 'reverse-temper' it). Similarly, any number of 'i' iterations of a ratio r, 'r^i', can be used to approximate the same amount of i iterations of an interval $ø^{\frac{s}{m}}$, '$ø^{\frac{si}{m}}$' (where $\frac{si}{m}$ is always written in its most simplified form).

By doing this, a comma, 'c' (which is also represented as a ratio of frequencies), is generated (in opposition to having it 'tempered out' when tempering Just Intonation with an equal tuning). Also, $c=\frac{ø^s}{r^m}$, because $r^i \times c^{\frac{i}{m}}=ø^{\frac{si}{m}}$. This means that '$c^{\frac{i}{m}}$' is the interval between the pitch generated by doing i iterations of ratio r (r^i) above a root and the pitch generated by doing the same amount of i iterations of interval $ø^{\frac{s}{m}}$ ($ø^{\frac{si}{m}}$) above the same root, in relation to that comma.

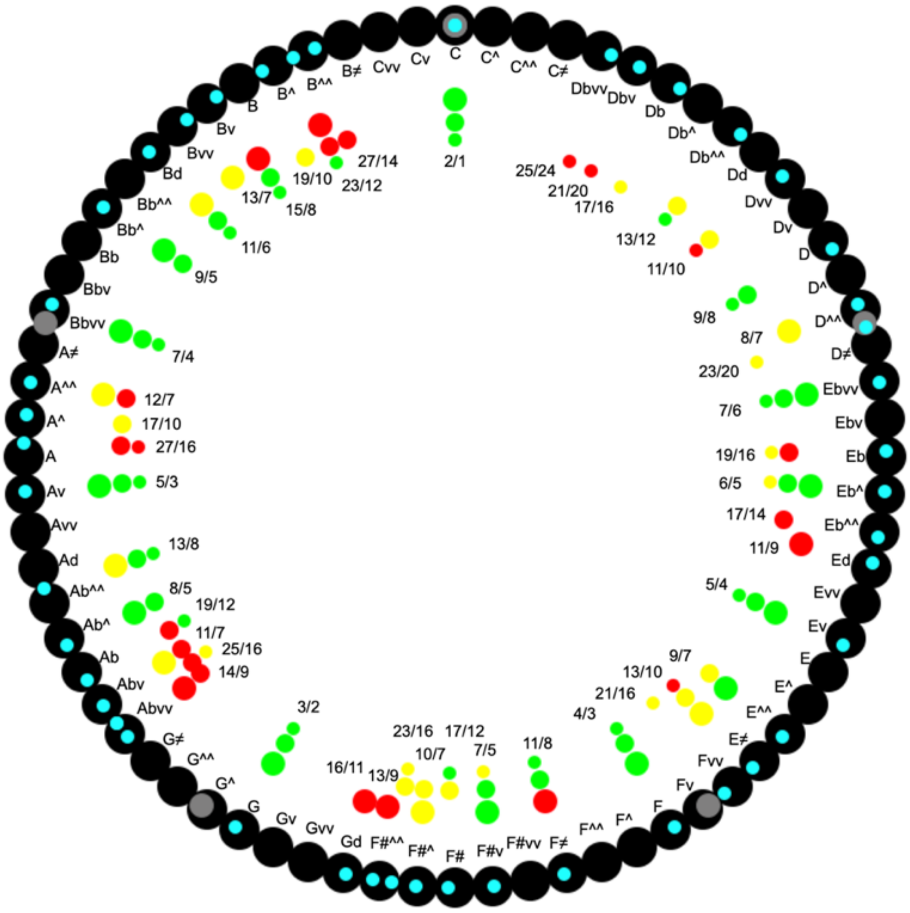


Figure 107. 5EDO (grey dots) against Sabat and Hayward's tuneable intervals.

If one wishes to approximate the interval 4\5EDO (i.e. the interval between the pitches of 5EDO represented by the grey dots that are closest to C and Bbvv in Figure 107, henceforth referred as $2^{\frac{4}{5}}$), with the 7/4 ratio (henceforth referred as $\frac{7}{4}$, alongside all other ratios) then the comma c, generated by the ratio $\frac{7}{4}$ and the interval $2^{\frac{4}{5}}$ above C, would be calculated this way:

$$c=\frac{\o^s}{r^m}, \text{ therefore } c=\frac{2^4}{(\frac{7}{4})^5}=\frac{2^4}{\frac{7^5}{4^5}}=\frac{16}{\frac{16807}{1024}}=\frac{16}{1}\times\frac{1024}{16807}=\frac{16384}{16807}.$$

Knowing that the comma generated by $\frac{7}{4}$ and $2^{\frac{4}{5}}$ above C is $\frac{16384}{16807}$, then $2^{\frac{4}{5}}$ can also be re-written in relation to the $\frac{7}{4}$ ratio and the $\frac{16384}{16807}$ comma.

$r^i\times c^{\frac{i}{m}}=\o^{\frac{si}{m}}$. In this case, $\o^{\frac{si}{m}}$ is $2^{\frac{4}{5}}$, therefore $2^{\frac{4}{5}}=(\frac{7}{4})^1\times(\frac{16384}{16807})^{\frac{1}{5}}=\frac{7}{4}\times(\frac{16384}{16807})^{\frac{1}{5}}=\frac{7}{4}(\frac{16384}{16807})^{\frac{1}{5}}$.

Moreover, $(\frac{16384}{16807})^{\frac{1}{5}}$ is the interval between the pitch a $\frac{7}{4}$ above C and the pitch a $2^{\frac{4}{5}}$ also above C. Because the numerator of the comma is smaller than its denominator, then the interval between these two pitches is a descending interval from $\frac{7}{4}$ above C to $2^{\frac{4}{5}}$ above C.

Similarly, if one wishes to approximate $2^{\frac{3}{5}}$ with $\frac{3}{2}$, then:

$c=\frac{ø^s}{r^m}$, therefore $c=\frac{2^3}{(\frac{3}{2})^5}=\frac{2^3}{\frac{3^5}{2^5}}=\frac{8}{\frac{243}{32}}=\frac{8}{1}\times\frac{32}{243}=\frac{256}{243}$.

Knowing that the comma generated by $\frac{3}{2}$ and $2^{\frac{3}{5}}$ above C is $\frac{256}{243}$, then $2^{\frac{3}{5}}$ can also be re-written in relation to the $\frac{3}{2}$ ratio and the $\frac{256}{243}$ comma.

$r^i\times c^{\frac{i}{m}}=ø^{\frac{si}{m}}$. In this case, $ø^{\frac{si}{m}}$ is $2^{\frac{3}{5}}$, therefore $2^{\frac{3}{5}}=(\frac{3}{2})^1\times(\frac{256}{243})^{\frac{1}{5}}=\frac{3}{2}\times(\frac{256}{243})^{\frac{1}{5}}=\frac{3}{2}(\frac{256}{243})^{\frac{1}{5}}$.

Moreover, $(\frac{256}{243})^{\frac{1}{5}}$ is the interval between the pitch $\frac{3}{2}$ above C and the pitch a $2^{\frac{3}{5}}$ also above C. Because the numerator of this comma is higher than its denominator, then interval between these two pitches is an ascending interval from $\frac{3}{2}$ above C to $2^{\frac{3}{5}}$ above C.

These two examples correspond to approximations into tuneable intervals which were used in the string parts of *Adam*, in the 3rd scene.[173] The intervals $(\frac{16384}{16807})^{\frac{1}{5}}$ and $(\frac{256}{243})^{\frac{1}{5}}$ were, in theory (unless intonated otherwise), present in the performance whenever the strings were performing both $\frac{7}{4}$ and $\frac{3}{2}$ approximations alongside the electric piano's un-approximated $2^{\frac{4}{5}}$ and $2^{\frac{3}{5}}$ intervals, respectively.

While other further potential applications for this mathematical model are yet unknown to me, these mathematical considerations could potentially be useful as a starting point for a compositional and theoretical approach following the reverse paradigm of temperament practice: Just Intonation 'reverse-tempering' equal tunings. Conceptualizing irrational intervals from equal tunings such as $2^{\frac{4}{5}}$ and $2^{\frac{3}{5}}$ in a format visually closer to the notation of just ratios, $\frac{7}{4}(\frac{16384}{16807})^{\frac{1}{5}}$ and $\frac{3}{2}(\frac{256}{243})^{\frac{1}{5}}$, respectively, may also be creatively enticing and convenient for musicians and theorists. Such format, while also visually displaying the irrational nature of these intervals, presents them as the result of an operation between two ratios (the one which approximates them, and a resulting comma). Even though $\frac{7}{4}(\frac{16384}{16807})^{\frac{1}{5}}=2^{\frac{4}{5}}$ and $\frac{3}{2}(\frac{256}{243})^{\frac{1}{5}}=2^{\frac{3}{5}}$, these two different notational approaches imply two different strategies of conceptualizing the same interval sizes.

[173] See Table 1 in Chapter 3.1 and the considerations about the string parts in Chapter 3.3.2

Appendix 10. Excerpts of individual parts

Figure 108. Excerpt of the cello part of Adam.

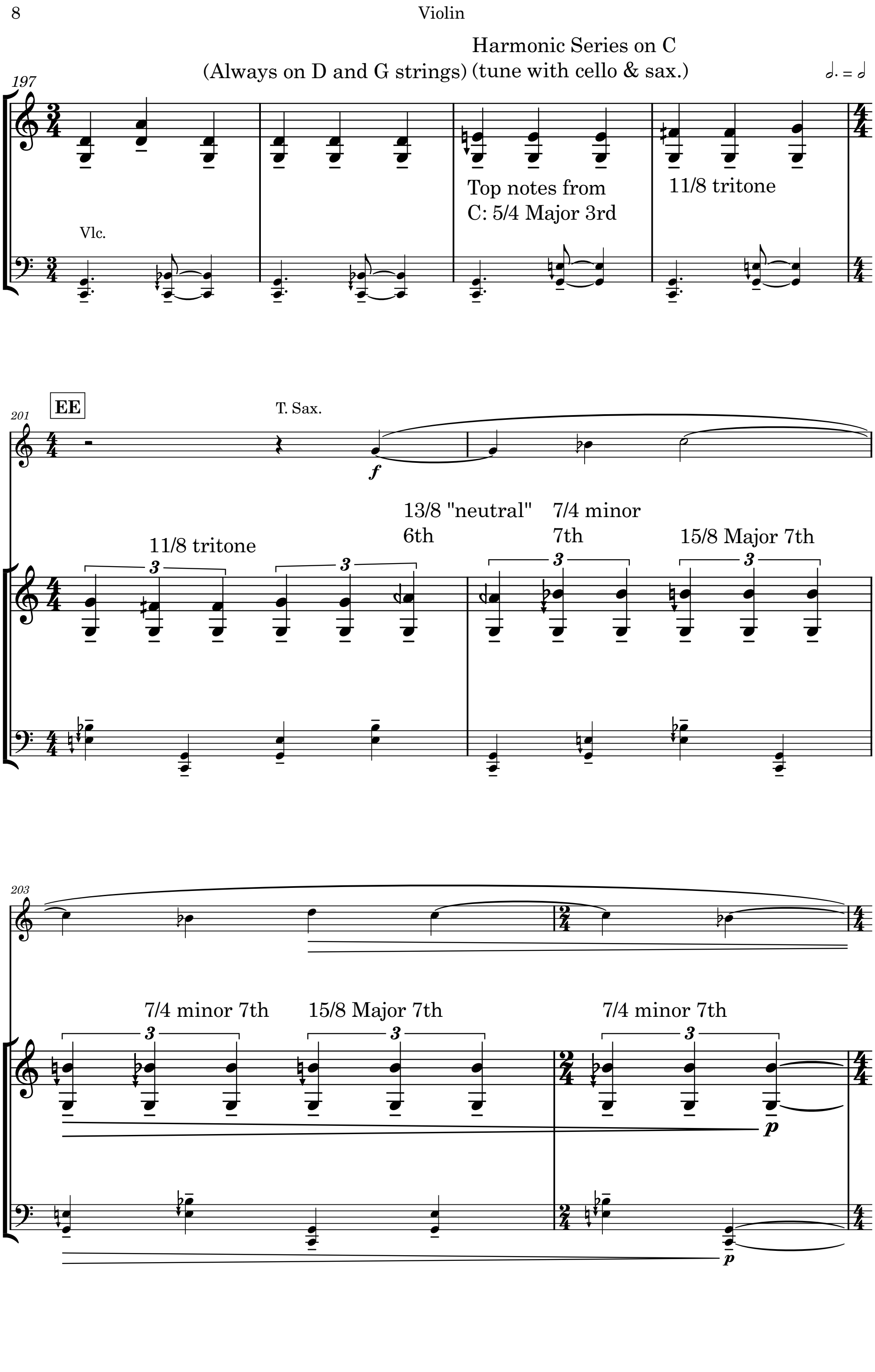


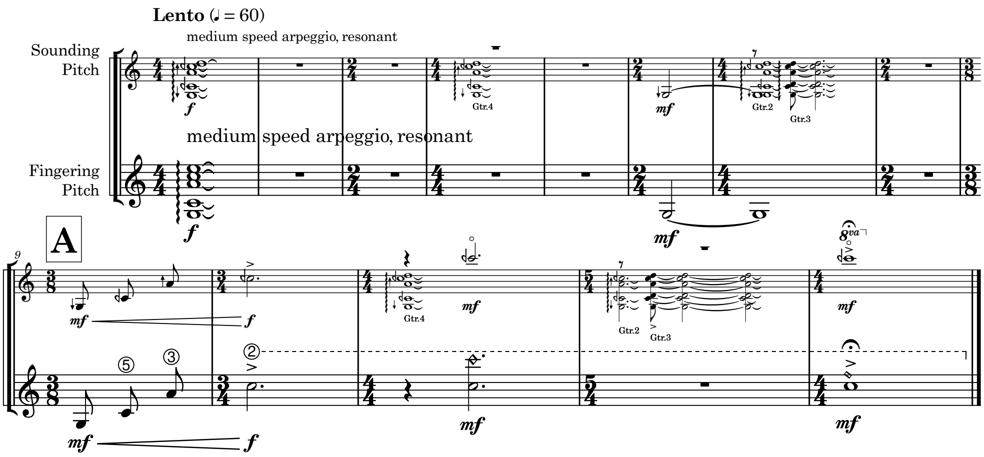
Figure 109. Excerpt of the violin part of Adam*.*

Guitar 1

The Fable of the Pilgrim

Pedro Laranjeira Finisterra

n. Incipit



I. The Mystery

Figure 110. First page of the 'dual notation' version of the first guitar's part of The Fable of the Pilgrim.

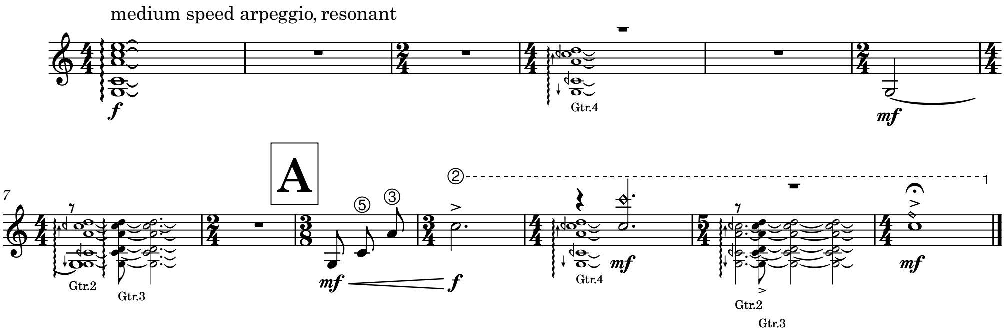
Guitar 1

The Fable of the Pilgrim

Pedro Laranjeira Finisterra

n. Incipit

Lento (♩ = 60)



I. The Mystery

Allegro moderato (♩ = 120)

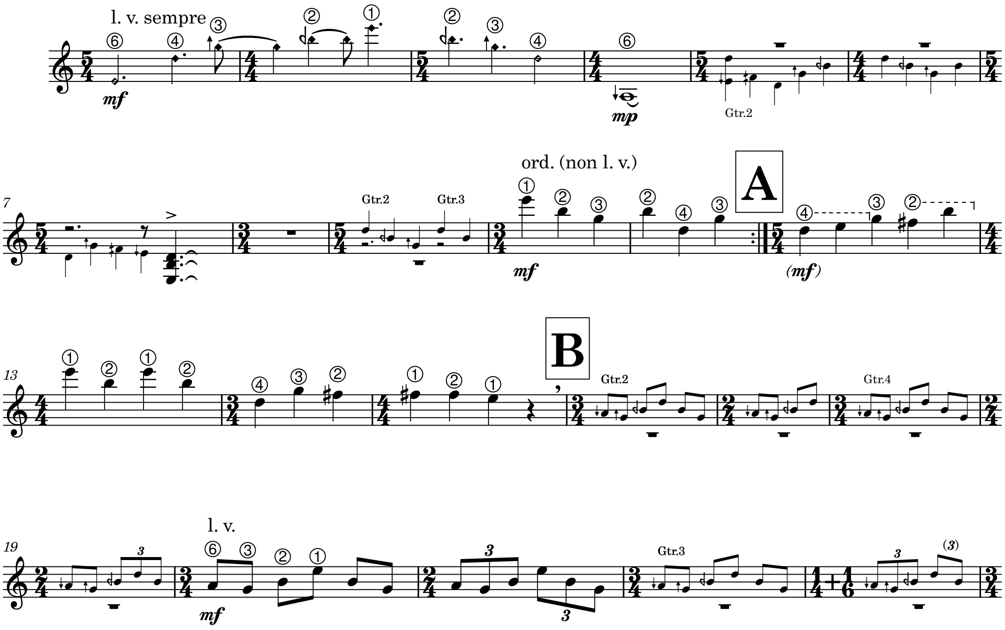


Figure 111. First page of the 'scordatura notation' version of the first guitar's part of The Fable of the Pilgrim.

V. Fullness


Sublime ♩ = 60
Tam.
Tbn.
gliss.
A
Standard semitone, quarter & eighth tone intonation
(with Trumpet 2)
mute off
6
mp
mf
p
B
10
f
mf
14
as if an 11th partial over B. Tbn. D
P5 above Tpt.2 C
Natural M6 +Octave over B. Tbn. D
Natural M3+Octave over Tbn. A or P5 above Hn. E
18
Tpt 2
Hn
Tbn.
B. Tbn.

Figure 112. First trumpet part of the fifth movement of Convergence.

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