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Research article

Using Bell violations as an indicator for financial market crisis

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A R T I C L E I N F O

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A B S T R A C T

The failure to identify and measure financial risk carries significant social and economic consequences. This paper introduces a novel framework for analyzing financial stress and crises, based on the Bell inequalities, a foundational framework in causal analysis, originally developed in quantum mechanics. Traditional approaches to crisis analysis do not, in general, adequately represent event-based dependencies and the distribution of tail risks inherent in complex financial systems. The proposed approach is underwritten by a generic causal framework, which we think is suitable for financial analysis: we offer an index for financial stress and we explore its value in detecting extreme market co-movements, which may serve as an early crisis warning signal.

Our analyses employ a rolling-window approach to analyze financial time series data. We utilize S&P 500 and STOXX Europe 600 stocks and consider three historical crises, namely the 2008 financial crisis, the EU debt crisis and the COVID-19 pandemic, which mark some of the largest downturns of financial markets in the last two decades. The findings demonstrate the framework's ability to align the number of observed Bell inequalities violations with observed peaks in market stress. In particular, the framework shows good performance against CDS spreads as a crisis indicator and is less erratic than the traditional Pearson correlation of price returns. It aligns well with implied equity option volatility as measured by VIX. Overall, we think the present causal framework has promising properties and merits further examination.

1. Introduction

Financial crises can be extremely disruptive for society. Part of the problem is that they are hard to detect in early stages. Their quantitative analysis involves the systematic study of financial time series for price levels as well as other economic indicators. Corresponding disruptions often lead to significant loss in value, liquidity shortages, and systemic instability. In the context of stock markets, crisis analysis aims to identify, understand, and ideally forecast periods of intense market stress, such as abrupt price collapses, spikes in volatility, and widespread investor panic. These crises are typically triggered by external shocks, structural imbalances, or cascading sell actions across markets.

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Traditional approaches to crisis analysis are based on various economic and financial indicators, such as stock market volatility indices, credit spreads, and macroeconomic stress measures. Many existing indices focus on isolated metrics or individual markets, not taking into account the intricate web of global financial systems and the cascading effects that can exacerbate instability. Causal models are rarely used and existing statistical analyses based on the correlation of price changes can yield erratic and hard-to-interpret data (Fama, 1965). Despite significant advances in crisis analysis, traditional approaches that model price changes using linear models (Fama et al., 1969) face notable shortcomings. Linear relationships oversimplify the intricate relationships and interactions that characterize crises, particularly during extreme market conditions. For instance, the failure to adequately account for tail dependencies and event-based correlations —representing the relationships and dependencies between specific occurrences across markets — limits the ability of such models to capture the financial risks that arise from extreme co-movements and cascading effects across interconnected markets. Event correlation, as a type of nonlinear correlation, is particularly vital in analyzing the characteristics of a financial crisis (Bluhm and Overbeck, 2006). Although traded volatility, derived from options markets and most commonly embodied in the VIX index, is a valuable crisis indicator, it can have narrow applicability because liquid option prices do not exist for all markets.

These deficiencies highlight the need for a new approach that can complement existing models. We wish to explore a tool with potential to overcome the above challenges and advance the field of crisis analysis. This paper proposes the use of the Bell inequalities as a novel framework. The Bell inequalities can be used outside physics in conjunction with a suitable causal model; we will argue that an approach based on Bell inequalities can potentially offer a novel approach to crisis analysis. Originally developed to detect correlations beyond classical explanations in physics, the Bell inequality provides a mathematical tool to detect inadequacy of a given causal model for describing some observed correlations.

The aim of this paper is to introduce a novel index for crisis analysis, leveraging the mathematical framework of Bell inequalities, with the goal of understanding and predicting financial crises. Building on the work of Gallus et al. (2023), the proposed index analyzes changes in causal relationships between financial variables, offering a fresh perspective on systemic vulnerabilities. Specifically, it detects event-based correlations indicative of increased financial risks. We offer some promising results, concerning validation using historical crisis data through a comparative analysis.

Numerous studies focus on capital requirements for financial market participants to ensure sufficient capitalization in the next crisis. Traditional risk measures, such as Value-at-Risk (VaR) (Jorion, 2006), focus on measuring potential losses under predefined market conditions, while macro-financial stress indices aggregate various indicators to gauge economic stress. In addition, systemic risk models, such as CoVaR (Adrian and Brunnermeier, 2011) and network-based (Shirazi et al., 2017) approaches, explore interdependencies within financial systems, with the aim of identifying critical vulnerabilities. Recent research has also explored advanced statistical techniques, including machine learning (Chatzis et al., 2018) and agent-based modeling (Sornette, 2014). Although these works have great value, e.g, for assessing the adequacy of capital requirements, this is not the main focus. Rather, the present work is about early indicators for financial crises, based on a causal model for the relationship between relevant entities.

This paper is organized as follows. The sections on Bell inequalities in general and Bell inequalities in finance are dedicated to an overview of the Bell inequalities framework. The Methodology Section introduces a violation index for financial crisis detection. The Results Section examines three different financial crises, each with distinct underlying causes, including financial over-indebtedness and exogenous effects. We examine the application of the index to historical datasets and evaluate its predictive precision against stock market indices, that is, the CDS, VIX and VSTOXX volatility indices.

2. Bell inequalities

Bell's (1964) theorem states that certain predictions of quantum mechanics are incompatible with the conjunction of three fundamental principles of classical physics, which are sometimes given the short names 'locality', 'freedom of choice' and 'arrow of time'. These principles are formulated within the standard causal framework (where the latter is typically taken as the 'realism' assumption). Corresponding real world experiments, often referred to as Bell experiments, aim to examine whether there are physical systems where these predictions from quantum mechanics hold true.

Following the theoretical work of Bell (2004) sophisticated experimental tests are possible today. In such experiments, pairs of entangled particles, for example photons or electrons, are created with one particle sent to Alice and the other to Bob. If Alice and Bob are located in different places without the possibility of communication and if it is assumed that Alice can freely choose between two directions ($x = 0$ or $x = 1$) of spin measurement on her particle and that Bob can also choose between two directions of spin measurement ($y = 0$ or $y = 1$, usually rotated by some fixed angle against x) on his particle, then a strong association between the outcomes observed by Alice and Bob is recorded for quantum systems. This experimental fact, described for example in (Aspect et al., 1982; Giustina et al., 2015; Shalm et al., 2015; Hensen et al., 2015), has led to questioning of the fundamental intuitions about how the physical world works.

Both Bell's theorem and corresponding experiments hinge around a set of inequalities constraining observable distributions of measurement outcomes on spatially separated physical systems; these inequalities must hold, if all three 'classical' principles are true. In a nutshell, Bell's inequalities are an empirically verifiable consequence of the idea that the outcome of one measurement on one system cannot depend on which measurement is performed on the other. This idea, called locality is one of the three 'classical' principles. Bell's theorem is formulated in terms of measurement outcomes that are not actually performed, so we have to assume their existence alongside the outcomes of the measurements actually performed: this is the principle of realism, or more precisely, counterfactual definiteness. Finally, we need to assume that we have complete freedom to choose which of several measurements to perform: this is the principle of freedom of choice, also called the no-conspiracy principle or no super determinism (Gill, 2014).

A Bell analysis is underwritten by a causal network, encoding the assumptions of putative sources of influence between the two components of a system. In physics, it is straightforward to provide a corresponding causal network, consistent with the key assumptions of locality and free choice. For example, Robins et al. (2015) and Gill (2014) derived Bell's inequalities using the statistical language of causal interactions, without considering physical experiments. The causal graph (directed, acyclic graph or DAG) of observed and unobserved variables corresponding to a classical physical description of one run of a standard Bell experiment is given in Fig. 1. Note again that, as is standard in the Bell framework, Alice and Bob's settings are binary and the outcome of each measurement is also binary. Observed variables are represented by gray rectangles; the unobserved variable (there is only one, but of course it might be of arbitrarily complex nature) is represented by a white oval. The validity of this causal model places restrictions on the joint distribution of the observed variables.

More formally, two observers, Alice and Bob, perform measurements on a shared system. Each observer chooses a measurement setting, denoted by x for Alice and y for Bob, where $x, y \in \{0, 1\}$, and records an outcome a or b, respectively, where $a, b \in \{-1, 1\}$. The conditional probability distribution $P(a, b|x, y)$ describes the outcomes of these measurements. The expectation values of the measurements for a given choice of settings are defined as follows:

$$\langle ab\rangle_{xy} = \sum_{a,b} ab\, P(a, b|x, y). \tag{1}$$

The Clauser-Horne-Shimony-Holt (CHSH) variant of Bell inequalities introduces four combinations of these expectation values, called S-values:

$$S_1 = \langle ab\rangle_{00} + \langle ab\rangle_{01} + \langle ab\rangle_{10} - \langle ab\rangle_{11}, \tag{2}$$

$$S_2 = \langle ab\rangle_{00} + \langle ab\rangle_{01} - \langle ab\rangle_{10} + \langle ab\rangle_{11}, \tag{3}$$

$$S_3 = \langle ab\rangle_{00} - \langle ab\rangle_{01} + \langle ab\rangle_{10} + \langle ab\rangle_{11}, \tag{4}$$

$$S_4 = -\langle ab\rangle_{00} + \langle ab\rangle_{01} + \langle ab\rangle_{10} + \langle ab\rangle_{11}. \tag{5}$$

For classical local hidden variable models, as shown in Fig. 1, the Bell inequalities are the bound:

$$|S_i| \le 2, \qquad \text{for} \quad i = 1, \ldots, 4. \tag{6}$$

However, in quantum mechanics, entangled systems can violate this bound, up to another bound, called the Tsirelson (Cirel'son, 1980) bound:

$$|S| \le 2\sqrt{2}. \tag{7}$$

Depending on the experimental context and the causal model used, violations of Bell inequalities ($|S| > 2$) can have sharply contrasting interpretations. In realist models for quantum physics, they may be interpreted as violations of free choice or as violations of Bell locality (Gallus et al., 2023), or even as indications of retrocausality (Price, 1996).

3. Bell inequalities in finance

It is possible to apply the Bell framework in any situation where there is a hypothesis about a purported causal model driving the correlation structure. In such applications it is essential to emphasize that the corresponding systems do not have to be quantum physical systems in which case any violations of Bell inequalities would not be interpreted in the way they are in quantum physics (e.g., the idea of physical locality does not apply outside physical applications). When it comes to financial applications, traded prices provide the observed real data. Counterfactual definiteness comes into play once a question like "what would market prices have done

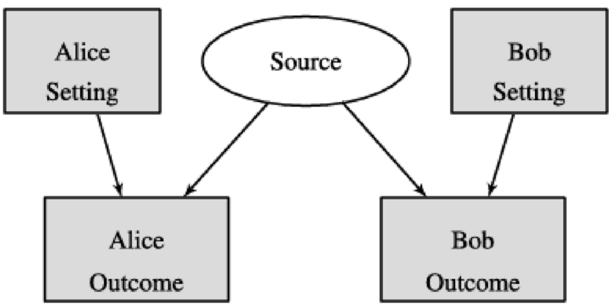


Fig. 1. A classical description of a Bell-CHSH type experiment entails the validity of the graphical model described by this simple causal graph. Rectangles: observed variables; ellipse: unobserved. Settings and outcomes are both binary. Experimental results arguably (via Bell's theorem) show that a description consistent with the above model has to be abandoned (at least for a certain class of quantum experiments). For some physical systems, the probability distribution of experimental data is outside the class of probability distributions allowed by any classical model; see, Gill (2014).

in this scenario ?" is asked. Although this question is impossible to answer in general, it may be well-defined within a specific causal model. Prices are local in the sense that they apply to a specific transaction, but public financial information is distributed fast around the globe, so that, in contrast to Bell experiments in quantum physics, public price information is (more or less) immediately known to all market participants. When translating the Bell causal model shown in Fig. 1 to the analysis of financial markets, the measurement results can be considered as being generated by the simultaneous price changes in two stocks A and B taking the roles of Alice and Bob.

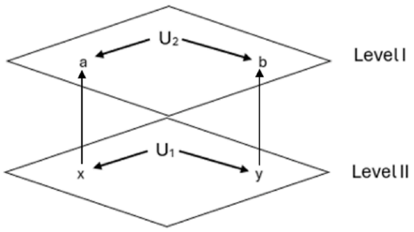
There is no single way to translate the Bell framework to financial analysis of the behavior of stock market indices. In this work, and following Gallus et al. (2023), we assume that the measurement outcomes are defined as $a = 1$ if security A has increased in price over a given period of time and $b = 1$ if security B has increased in price over the same period of time. Similarly, price decreases are denoted by $a = -1$ and $b = -1$, respectively.

In a Bell framework, together with measurement outcomes, we also need to consider measurement settings (recall, two observables are measured for each of Alice and Bob). In the financial application, one way to define different measurement settings is by dividing the financial price data into different regimes, determined by whether the change of the relevant stock index is above or below a particular threshold. This is done in such a way that each day in the joint financial time series of prices is assigned to exactly one financial regime. Specifically, we propose a definition of the S-values using Eqs. (2)–(5) by partitioning the observed financial time series into four different regimes $xy = 00, 01, 10, 11$. For crisis detection, an interesting choice of regimes is based on the distinction between weak (small) and strong (large) price changes. Weak changes are indicated by 1 and strong changes by 0 – a working hypothesis is that strong changes become more prevalent prior to a financial crisis (Gallus et al., 2023).

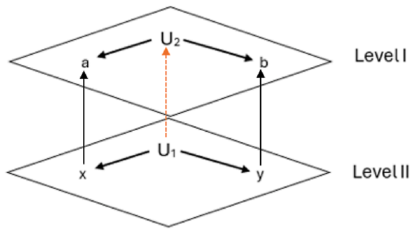
On this basis, we can translate the Bell causal model shown in Fig. 1 to a financial application, but we still have to specify the meaning of the 'source'. In general, the source will be an unknown cause driving an observable price change. We suggest that this unknown cause is composed of two possible (sub) causes, as follows. Fig. 2 shows a two-layer network and can offer a tool to analyze such causal models, by re-structuring a Bell system into interconnected hierarchical layers. The top layer (Level I) represents directional change (e.g. "Up" or "Down"), capturing broad trends, while the bottom layer (Level II) models the magnitude of these movements (e.g. "Large" or "Small") regardless of direction. It may be asked whether such a model would help prediction of future changes. In general, to predict future changes, one would need some insight into the causes of such changes. Even though the proposed approach has been developed in a causal framework, the causes considered are generic and not specific to the particular stocks under study. However, it is common in statistics to employ correlation (and corresponding measures, e.g., regression) for predicting the future, on the basis of the simple principle that the best predictor of the future is the past. Also, arguably, the proposed measure extends correlation-based approaches because it does involve a causal (albeit generic) framework. So, in brief, the approach is considered to have some value for predicting the future, though ideally the causal framework employed would include causes specific to the particular stocks – this is an objective for future works.

In this setup, the black arrows represent dependencies between market observables that are always assumed to exist. Both models (a) and (b) allow for a dependence of the direction of change in a stock on the magnitude of change for that stock. Both models also have a hypothetical unknown cause U_1, which drives the magnitude of price change (i.e., x, y), and a different unknown cause U_2 driving direction (i.e., a, b). U_1 could be interpreted as a market volatility factor that reflects general uncertainty, while U_2 could be regarded as a measure of optimistic versus pessimistic market responses to new information. However, there is no unique way in which even these two simple ideas can be translated into a causal model. The key difference between the simpler model (a) and the more general model (b) is the extent of inter-dependence between magnitude of change and direction of change, for the two systems. In the simpler model, these are assumed to be independent, thereby bounding the S-values to within the classical limit of 2. Under exceptional circumstances (perhaps impending crises), we assume that the more general model applies, magnitude and direction of change between the two systems become intertwined, and the classical limit of 2 for the S-values can be exceeded. More generally, a causal model with more causal links would add accuracy and realism to this approach. However, a technical limitation is that there is no general method for deriving extensions to the basic Bell inequalities for more elaborate causal models.

Compared to the approach in physics, this two-layer approach (Fig. 2) enables a representation of dynamics better suited for financial systems, where Level I and Level II can be linked rather than always be treated independently. Then, by analyzing both intra- and inter-layer correlations, we can better understand the feedback mechanisms and causal dependencies that govern market behavior. This framework facilitates testing the sufficiency of simple versus more complex causal models. Specifically, Bell inequalities allow for a test, via the S-values, of whether model (a) provides an explanation of the observed price action – the simpler model (a) always implies a Bell bound of 2. That is, if we see a violation of this bound, model (a) cannot be used for this pair of stocks. Model (b) will always explain the observed price action as it can generate any probability distribution P (see (Gallus et al., 2023)). On this basis,



(a) Model (a)



(b) Model (b)

Fig. 2. An illustration of the two-layer causal model for the stock price co-movement separating price direction from magnitude of change.

we propose to count the number of pairs of stocks for which model (a) is insufficient in capturing the observed co-dependence. If this number increases in a sliding time window, this may be indicative of stronger links between pairs of stock indices than allowed for in model (a) – our proposal is that this may be indicative of an upcoming crisis.

Risk measures such as the VIX and VSTOXX indices, often referred to as the “fear gauge”, quantify implied volatility in stock markets based on options prices (Brunnermeier, 2009; Fassas and Siriopoulos, 2021). Similarly, Credit Default Swaps (CDS) provide the market-based cost of hedging credit risk by reflecting the cost of insuring against a borrower's default (Weistroffer et al., 2009). CDS spreads are useful for assessing the probabilities of default, but they have been shown to often respond late to financial risks and not fully capture the interconnected nature of markets (Norden and Weber, 2009). These tools have proven value in stock market analysis; they have been studied extensively and their advantages, as well as limitations, are reasonably well-understood. In considering the above framework, there is a question of how it complements (or even exceeds) the existing approaches. In motivating our approach, one consideration is that the partitioning financial regimes into weak and strong price changes as shown in Fig. 2 allows a consideration of market dynamics within a causal framework – causal analysis has proven value in capturing systemic inter dependencies of the systems concerned. Therefore, there is reasonable a priori motivation that the Bell-violation index might offer a useful tool to identify early warning signals of crises and may even outperform traditional volatility and credit risk measures in anticipating financial crises.

4. Methodology

The four S-values can be computed from Eqs. (2)–(5). Of these, the S_1-value is the most interesting to us, because S_1 can be interpreted as correlation when strong changes occur in at least one part of the system. This can be seen directly from Eq. (2), as all correlations with at least one strong change (that is the regimes $xy = 00$, 01 and 10) are added, while the contribution with a weak change in both parts of the system (i.e., $xy = 11$) is subtracted. So, S_1-value can be interpreted as a type of correlation (in the above specific sense), but where contributions that involve a strong change in at least one part are separated from contributions that involve only weak parts (Gallus et al., 2023). On this basis only S_1 is considered in the remainder of this work. Regarding the other three S-values, there is no equivalent interpretation, as expectation values for weak–weak changes are added to expectation values which reflect strong changes in at least one part of the system. So, the separation between ‘strong’ and ‘weak’ changes is not as clean.

4.1. S_1-violation proportion

In this work, the S_1-value is calculated over a rolling time window for pairs of stocks from a particular stock market index, with the idea that exceeding the bound of 2 indicates stronger dependence between the two stocks, than what is allowed for by the simpler model (a) above – as argued, our hypothesis is that this stronger dependence may anticipate increasing market stress. This means that financial crises go hand in hand with periods of high S_1 correlation between stock prices. To be precise, the S_1-violation proportion across a set of stock indices is defined as

$$S_1 - \text{Violation} - \% = \frac{2}{M(M-1)} \sum_{i=1}^{M} \sum_{j=1, j>i}^{M} I_{\{|S_1^{i,j}|>2\}}.$$

In this equation, M represents the number of stocks in a given set, such as a financial index. So, for each pair of stocks (i, j) with $j > i$, we check whether $|S_1^{i,j}| > 2$ and the percentage of such pairs across all pairs of stocks in the set is calculated (details are given in the Appendix).

4.2. Threshold dependence

In this analysis, price change thresholds that distinguish large changes from small ones may be set as fixed numbers (e.g., 0.05) or as multiples of the standard deviation of the specific stock in a pair, as shown in Fig. 3. The question of weak vs. strong price movements is

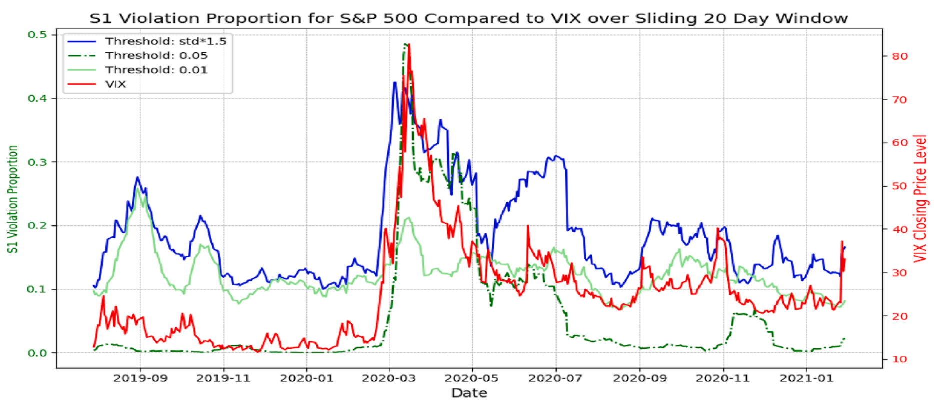


Fig. 3. This figure compares S_1-violations for different threshold specifications, together with the VIX index.

about the tails of the return distribution. A sensitivity analysis using different threshold values is illustrated in Fig. 3. For example, higher thresholds (e.g., $r_i = r_j = 0.05$ for all pairs of stocks) sample the tails of the return distribution, effectively capturing extreme events, while smaller thresholds (e.g., $r_i = r_j = 0.01$ for all pairs of stocks) offer broader but less precise coverage of the distributions. More sophisticated methods could be used, for example setting the threshold to two times standard deviation in case of a Gaussian distribution, or considering confidence intervals from kernel density estimation, but a fixed percentage approach is simplest. In any case, there is a trade-off between using a short time window to derive a measure that is quick to capture market changes, versus using a long time window with better statistical properties for estimation. Note, this is particularly pertinent regarding any threshold based on distributional characteristics, since corresponding estimation would require fairly long windows – currently the windows of 20 points are used, which are brief and so more suitable for capturing rapid changes. Notably, all threshold configurations show trends similar to the VIX index, emphasizing their applicability as crisis indicators. Smaller thresholds exhibit spikes during certain periods, such as September 2019, indicating sensitivity to minor market fluctuations.

The goal of this work is the creation of an index that can signal a major crisis, therefore the thresholds shall be chosen in a way that minor market fluctuations do not produce a signal. In contrast, a higher threshold like 0.05 isolates significant market disruptions and provides the clearest visualization for example in the case of the COVID-19 crisis. This analysis suggests that higher thresholds are better suited is the main purpose in crisis detection, whereas lower thresholds may correspond to less focused signals. This adjustment highlights the flexibility of the S_1 framework in defining thresholds tailored to the scale and rarity of the events being studied. Fig. 4 shows that the most extreme threshold ($r_i = r_j = 0.05$) emphasizes significant market changes, allowing a focus on large market movements, as in, for example, a crisis.

To explore the S_1 methodology, we compared it with key financial indicators, for three financial crises. The Results Section presents a comparison of S_1-violation proportion for stocks in S&P 500 and STOXX Europe 600 with the VIX, VSTOXX and CDS spreads. This analysis reveals the consistency of S_1-violation proportion in identifying three financial crisis periods, such as the 2008 financial crisis, the European debt crisis, and the COVID-19 crisis, even in the absence of forward-looking market instruments. To simplify the methodology, $r_i = r_j = 0.05$ is chosen in the remainder of this work. Data were analyzed using a sliding 20-day window to calculate S_1-violation proportions. For more details on the algorithm, see Appendix.

5. Results

We first consider the extent to which the proportion of S_1-violations overlaps with standard Pearson correlation. Then, we examine with this new index, three major financial crises. Each subsection provides details on the specific crisis, examining it across stocks of both the S&P 500 and STOXX Europe 600 indices. In addition to graphical presentations, some key events for each crisis timeline are described in text. The comparison of interest always concerns the S_1-violation proportion, the regional volatility indices (VIX and VSTOXX), and the regional credit default swap indices for different sectors.

5.1. Proportion of S_1-violations and Pearson correlation

An important empirical question is whether the present approach, using event-based correlation, can produce sharper results than an approach based on standard classical (Pearson) correlation, for the purpose of predicting financial crises. As shown in Fig. 5 for the S&P 500 index, Pearson's correlation exhibits significant variability over time, with peaks occurring even outside of the COVID-19 financial crisis periods. The same pattern can be seen in Fig. 6 for the stocks of the STOXX Europe 600 index, also in the COVID-19 crisis. However, peaks in VIX and the proportion of S_1-violations occur only at actual crisis points, showing their relevance as indicators of systemic risk during periods of market instability. Although VIX is designed to signal market uncertainty, it shows implied volatility from option markets (Brunnermeier, 2009) and is not based on coupled dependencies and hidden causal dependencies

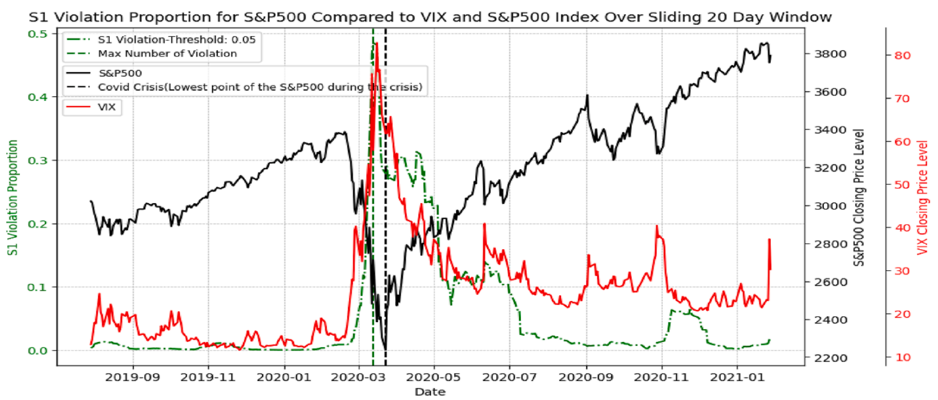


Fig. 4. This figure highlights the 0.05 threshold for S_1-violations. The two vertical lines represent the peak of S_1-violations and the lowest daily closing price of the S&P 500 index during the COVID-19 crisis, respectively. The pattern illustrates the temporal alignment of risk indicators and market downturns.

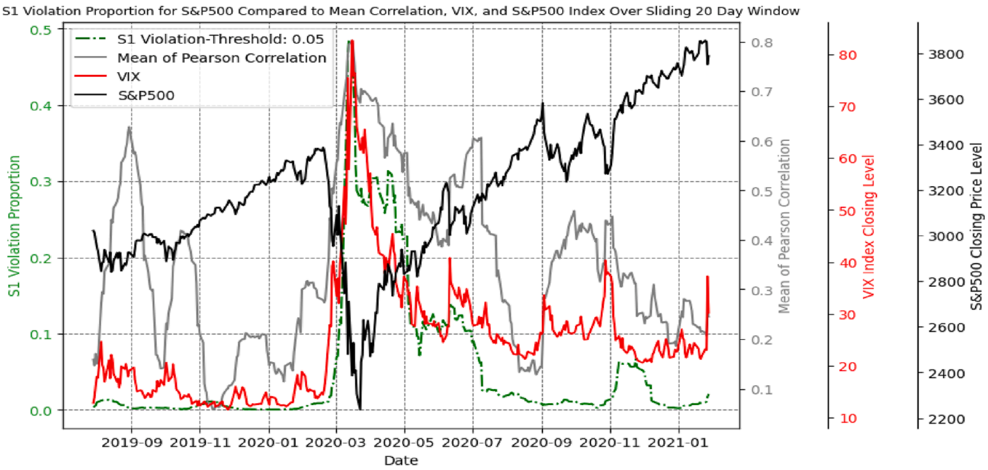


Fig. 5. Average of pairwise Pearson correlations of price returns vs VIX, S_1-violation proportion and daily closing price level of S&P 500 index: COVID-19 Crisis (U.S.).

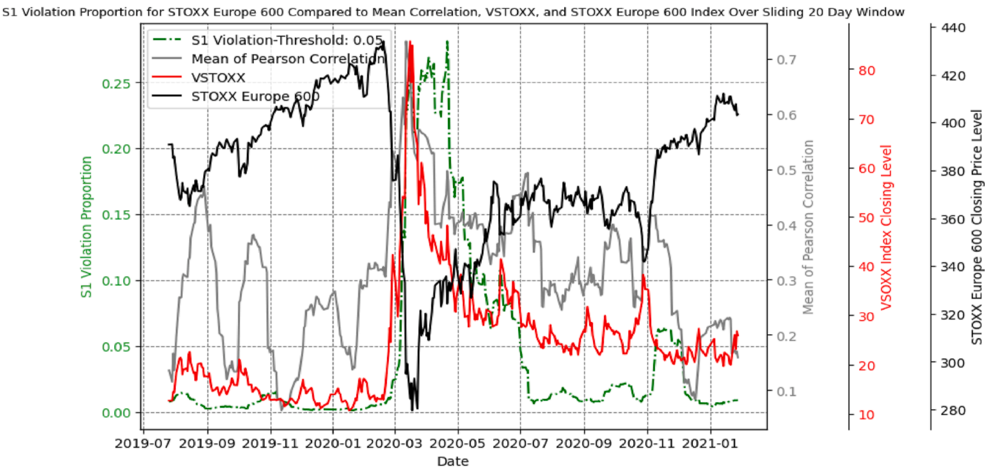


Fig. 6. Average of pairwise Pearson correlations of price returns vs VSTOXX, S_1-violation proportion and daily closing price level of STOXX Europe 600 index: COVID-19 Crisis (EU).

between specific assets in historical price data. We think that the similarity in the green (S_1-violation proportion) and the red lines (VIX) in Fig. 5 around crisis is promising.

5.2. 2008 financial crisis

The 2008 financial crisis originated from the collapse of the U.S. housing market, driven by excessive risk taking, lax regulation, and the proliferation of complex financial instruments such, as mortgage-backed securities (MBS) and collateralized debt obligations (CDOs) (Commission et al., 2011). Subprime mortgages, issued to high-risk borrowers, were repackaged into these securities and sold globally, creating systemic interdependencies. Housing prices peaked in mid-2006 and their subsequent decline triggered a surge in defaults that eroded mortgage-linked assets.

S&P 500 Data and U.S. events: Key points included.

- **15 September 2008**: Bankruptcy of Lehman Brothers, the largest Chapter 11 filing in U.S. history, which triggered a global credit freeze (Paoli and Hill, 2022).
- **16 September 2008**: Bailout of American International Group (AIG) with a $85 billion Federal Reserve loan to prevent systemic collapse (United States Government Accountability Office (GAO), 2011).
- **3 October 2008**: Passage of the Troubled Asset Relief Program (TARP), authorizing $700 billion to stabilize financial institutions (Congressional Budget Office, 2010).

Applying the S_1 framework to the stocks in the S&P 500 index (Fig. 7 upper half), we observe that the red line, representing VIX, exhibited multiple spikes throughout the period, even before the full onset of the crisis. These earlier increases in VIX suggest episodes of market anxiety, but they did not appear to go hand in hand with a systemic breakdown. In contrast, the green dotted line, which represents the S_1-violation proportion, remained relatively stable during these earlier fluctuations and only rose significantly when the financial crisis fully unfolded in mid-September 2008. Unlike VIX, which experienced several sharp declines during the crisis, the S_1-violation proportion remained consistently high throughout the crisis period. This suggests that the S_1-violation proportion serves as a more stable indicator, that responds strongly only when extreme co-movements become persistent and widespread. Peaks of high S_1-violations align reasonably well with historic peaks of VIX. In general, the behavior of the VIX is more erratic than that of the S_1-violation proportion. The peak levels of all CDS indices demonstrate a delayed response to the main drop in the S&P 500, compared to the peaks of VIX and the S_1- violation proportion.

STOXX Europe 600 Data and European events: The crisis spread to Europe through financial contagion, as European banks, heavily exposed to U.S. mortgage-backed securities, faced liquidity shortages, leading to a credit crunch, stock market declines, and government bailouts (Reinhart and Rogoff, 2009). Critical dates include.

- **29 September 2008**: Fortis Bank (Belgium/The Netherlands) was partially nationalized after a liquidity run, followed by a bailout of \$16 billion (The Associated Press, 2008).
- **30 September 2008**: Irish government guarantees all bank liabilities (€440 billion), escalating sovereign risk (The Irish Times, 2008).

The analysis of the stocks in the STOXX Europe 600 index (Fig. 7 lower half) reveals a similar dynamic to those in the S&P 500 index. The red line (VSTOXX) exhibited multiple spikes before the crisis fully emerged, indicating episodes of market stress that ultimately did not escalate into a full-scale financial crisis. Furthermore, we observe that VSTOXX declined before the crisis fully developed, whereas the S_1-violation proportion had already started to increase during this period, suggesting financial stress.

The peak in the S_1-violation proportion on 6 October 2008 coincides with VSTOXX exceeding a closing price level of 80, reaching approximately 0.35. Moreover, during the crisis period, VSTOXX remained highly volatile, experiencing several sharp declines, while the S_1-violation proportion was smoother and remained persistently high. In particular, all European CDS indices, similar to their U.S. counterparts, exhibited lagging behavior, again evidencing a delayed reaction relative to the credit markets.

Our findings concerning the lagging behavior of CDS indices are consistent with Norden and Weber (2009), which also shows that CDS markets tend to lag behind stock markets. Furthermore, our results confirm that the stock market leads a crisis, reinforcing the view that stock prices respond more quickly to information than credit markets.

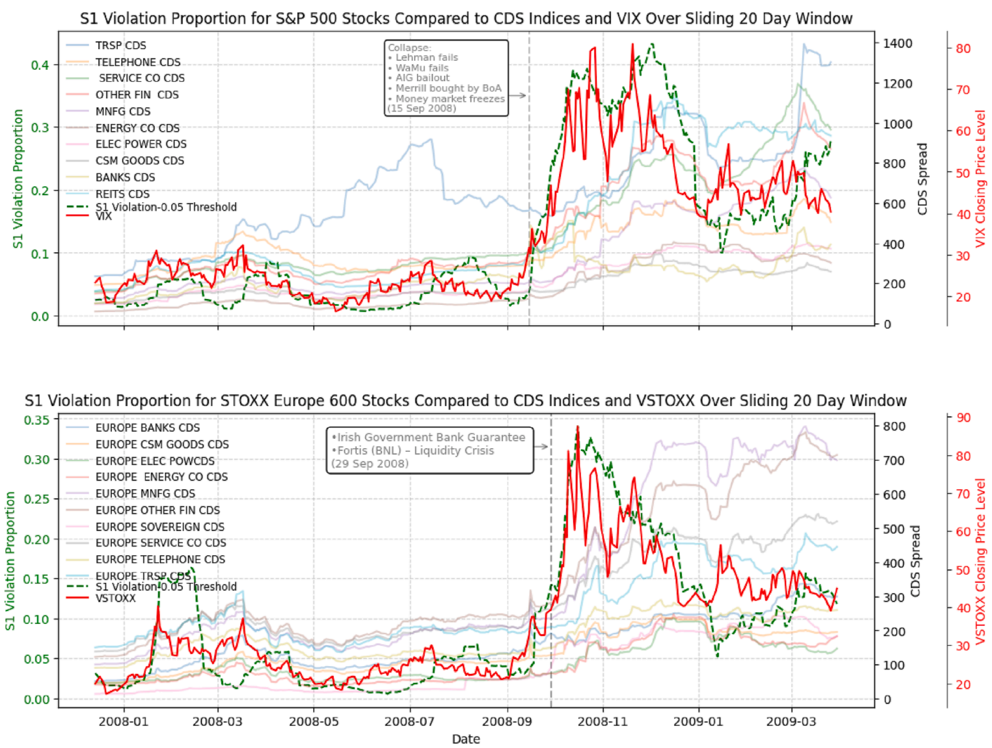


Fig. 7. 2008 financial crisis in the U.S. and EU markets.

5.3. 2010 European Debt Crisis

The European debt crisis, emerged in the aftermath of the 2008 financial crisis. This crisis was triggered primarily by the levels of excessive sovereign debt in several countries in the Eurozone, including Greece, Portugal, Spain, and Italy, raising investor concerns about their ability to meet debt obligations (Lane, 2012). As financial instability spread, volatility in European markets increased, with sovereign bond yields rising sharply and equity markets experiencing significant declines (Arghyrou and Kontonikas, 2012). The crisis demonstrated the transmission of financial distress across borders, particularly through the European banking sector, which suffered substantial losses due to high exposure to risky government bonds (De Grauwe and Ji, 2013).

STOXX Europe 600 Data and European events: During the European debt crisis, particularly around the time of the Greece bailout request on 23 April 2010, the VSTOXX index exhibited significant volatility. It initially spiked sharply, followed by a brief decline, only to surge again shortly thereafter. This surge generally aligns with the increase in the proportion of S_1-violation, as illustrated in Fig. 8, lower half, but the proportion of S_1-violations is lagging. In this crisis, the market reaction is less pronounced, as the problems in the 2010 European Debt Crisis lingered over the markets for a while. The closing level of the VSTOXX index reached a value near 50, while the S_1-violation proportion rose to approximately 0.14. Although the VSTOXX index experienced several sharp declines after the main surge in investors panic and remained highly volatile, the S_1- violation proportion continued to increase throughout the crisis period. The increase in sovereign CDS spreads matches Greece's bailout request and the rise in VSTOXX, while other CDS spreads react later. In general, the reaction of sectorial CDS was much less marked compared to the 2008 financial crisis.

S&P 500 Data: The effects of the European debt crisis were not limited to Europe, but extended to the S&P 500 almost immediately. As uncertainty surrounding the stability of Eurozone economies intensified, global investors sought safer assets, leading to increased demand for U.S. treasuries and a sell-off of riskier assets, such as stocks (Diebold and Yilmaz, 2014). The S&P 500 experienced heightened volatility, with declines during key crisis events, such as the Greece bailout negotiations and credit rating downgrades of European nations. Furthermore, the VIX index increased in response to crisis events, reaching a level of approximately 45, aligned with the peak of the S_1-violation proportion around 0.09, as shown in the upper half of Fig. 8.

In particular, S_1-violations remained elevated throughout the crisis, even when VIX experienced temporary drops. It is also observed that CDS spreads in the U.S. remained largely unchanged despite heightened volatility in the stock market. This aligns with the fact that the crisis originated in Europe, meaning that U.S. credit markets were less directly impacted. These observations further support the idea that credit markets typically lag stock markets in crisis response, with CDS reactions shaped by the crisis's origin and regional financial conditions.

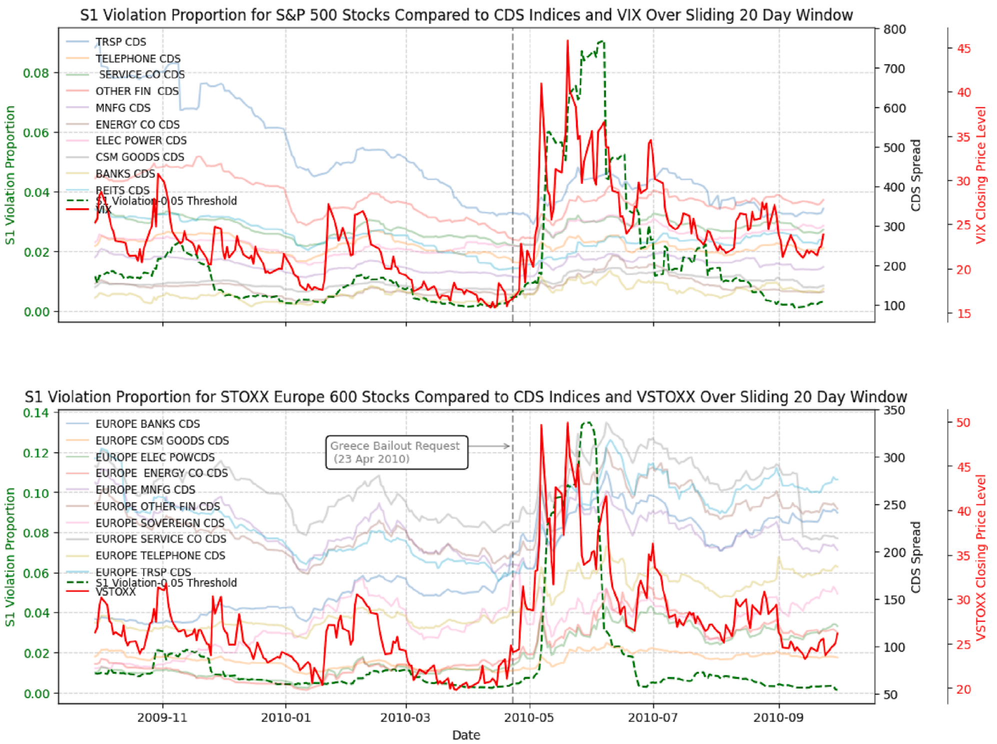


Fig. 8. 2010 European Debt Crisis in the U.S. and EU market.

5.4. 2020 COVID-19 financial crisis

The COVID-19 financial crisis, which began in early 2020, represents one of the most severe shocks to the economy, triggering extreme market volatility. In contrast to the two crises discussed above, this crisis started from an exogenous event affecting daily life and not from a situation of financial over-indebtedness or other mismanagement. The crisis was primarily driven by the rapid spread of the COVID-19 virus and the corresponding response of public authorities, leading to a global lockdown, supply chain disruptions, and dramatic collapse in economic activity. Stock markets around the world experienced large declines, with the S&P 500, STOXX Europe 600, and other major indices dropping sharply, in response to increased uncertainty and economic shutdowns. Central banks and governments implemented massive fiscal and monetary stimulus packages to stabilize financial markets and prevent a prolonged economic downturn (Goodell, 2020).

S&P 500 and STOXX Europe 600 Data: During the COVID-19 financial crisis, particularly in March 2020, there was a sharp increase in the S_1-violation proportion, as illustrated in Fig. 9 in stocks of both S&P 500 and STOXX Europe 600 indices. As market responses were very similar between the U.S. and Europe, a separate analysis is unnecessary. This surge coincided with extreme levels of uncertainty, as reflected in record-high levels of the VSTOXX and VIX indices. Although these volatility indices initially showed sharp spikes followed by declines, responding to evolving news and policy measures, the S_1-violation proportion increased sharply in line with volatility indices. In the U.S., it declined shortly after the peak, whereas it remained high for a longer period of time in Europe. Of course, responses of public authorities were different between the U.S. and Europe. CDS spreads reacted with a short time lag, increasing after the volatility of the stock market surged. However, this time CDS market responses were sector-specific, as certain parts of the real economy were impacted more by the COVID-19 crisis. In Europe, CDS for transport reacted more quickly, reflecting the immediate impact of lockdowns, travel restrictions, and the collapse in demand for transportation services. Meanwhile, in the United States, energy CDS responded faster, likely due to the sharp decline in oil prices and reduced energy consumption as economic activity halted, while U.S. transportation CDS peaked considerably later. Again, there is a difference in the economic structure of transport systems in the U.S. versus Europe, with the U.S relying more on individual forms of transport. So, even when CDS indices are taken into account, implied volatility indices and the S_1-violation proportion give the earliest crisis signal.

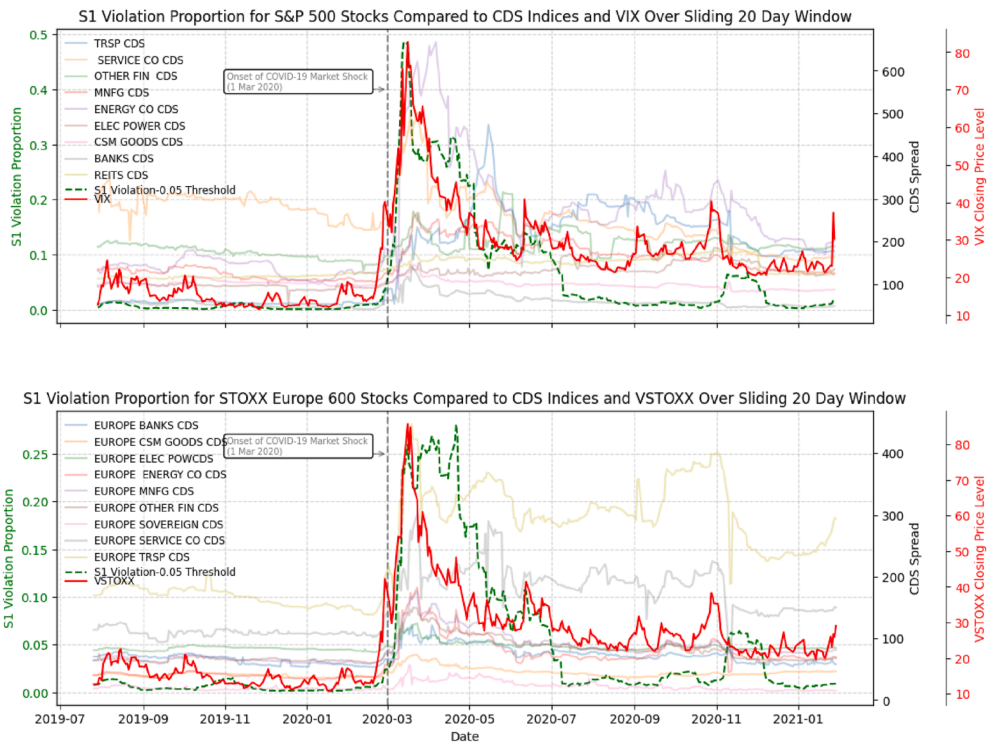


Fig. 9. 2020 COVID-19 financial crisis in the U.S. and EU market.

6. Conclusion

We have argued that the Bell framework in physics can be adapted to provide a measure of event-based causal analysis in the financial domain, which yields useful conclusions when applied over a rolling time window. This adaptation enables the creation of a crisis index for financial systems based on the S_1-violation proportion and using the distinction between strong and weak changes. This index leads to more pronounced signals than average Pearson correlation, which tends to be jumpy and can exhibit steep increases even outside a major crisis, as shown for the COVID-19 crisis. By observing the S_1-violation proportion during three major financial crises for S&P 500 and STOXX Europe 600 stocks, we have shown its usefulness in identifying financial crises early. In the 2008 financial crisis, the S_1-violation proportion increased after the collapse of Lehman Brothers Holdings, coinciding with the increase in the VIX, but remaining high even after the decline of the VIX, highlighting the persistence of financial risk. All CDS indices showed a delayed response to the onset of the crisis. In the 2010 European debt crisis, the S_1-violation proportion increased substantially around key events such as the Greek bailout request, underscoring its ability to detect prolonged market stress. Although VSTOXX showed a temporary decrease at the onset of the crisis, the S_1-violation proportion continued to signal instability. The credit market response was region-specific: U.S.-CDS indices showed little reaction, whereas the European sovereign CDS spreads rose sharply alongside VSTOXX. Other CDS indices responded with a delay relative to the lowest point of the stock marker index. In the 2020 COVID-19 crisis, the S_1-violation proportion captured well the initial financial shock of March 2020. However, it continued to rise even after volatility indices VIX and VSTOXX saw temporary declines, after government interventions. Additionally, sector-specific CDS indices reacted differently across regions, with transport CDS responding faster in Europe and energy CDS leading in the U.S., reflecting the crisis's sectoral impact.

Overall, our results indicate that the S_1-violation proportion is a valuable addition to traditional financial risk measures. It can provide a stable and persistent signal of financial risk, that remains elevated even as implied volatility indices exhibit a temporary jump during the crisis period. Specifically, a comparison of the S_1-violation proportion against volatility and CDS indices allows the following conclusions: First, implied volatility indices (VIX/VSTOXX) react fast but tend to be jumpy and experience multiple ups and downs before and during a crisis. This behavior reflects the demand for equity options but can lead to false alarms and a premature relaxation of caution. Second, while implied volatility indices can only be computed meaningfully when there is a liquid options market that allows the computation of changes in implied volatility, the S_1-violation proportion gives a measure that is always available, as it can be computed from the time series of stock prices. As such, the S_1-violation proportion is readily available for any market or any subsection of a market. Last, CDS indices can only be computed where liquid markets exist for the cost of credit protection. Furthermore, in the three crises analyzed in this work, CDS indices typically lag against the equity volatility indices and the S_1-violation proportion.

7. Directions for future research

Financial crises have repeatedly demonstrated their devastating financial and economic consequences, highlighting the importance of early warning mechanisms to allow timely policy responses. As an example, the 2008 crisis left deep scars on both sides of the Atlantic. In Europe, GDP growth decreased from 0.6 % in 2008 to −4.3 % in 2009, indicating a contraction of 4.9 percentage points during the period (World Bank Group, 2025a), with unemployment doubling in Spain and Ireland, reaching 18 % and 13 %, respectively (World Bank Group, 2025c,b). The crisis exposed structural flaws in European economies, which were later magnified during the 2010–2012 sovereign debt crisis. In the U.S., unemployment peaked at 10 % in October 2009, the highest since 1983 (World Bank Group, 2025d).

The S_1-violation proportion is easily available and can be derived from a collection of economic time series, together with a relevant causal hypothesis. In this work, it was used for the purpose of detecting financial crisis. By identifying warning signals in the form of changes in a causal relationship, as indicated by measures such as the S_1-violation proportion, stakeholders can implement preventive measures, for instance, more effective macroprudential regulations. Ultimately, such foresight could limit the socio-economic fallout of crises, preserving employment, maintaining financial stability, and minimizing long-term structural damage to economies.

Although our research so far has been limited to financial time series data and a specific approach to causality based on magnitude and direction of price change, there are alternative applications worth pursuing in future works. For example the S_1-violation approach could be compared with measures of tail dependence both on a theoretical and on a data-driven basis, though it could be argued that an advantage of the method is that it is based on a particular causal model and so, with elaborations of the causal model, it is possible to see how predictions for the correlation structure are altered. Additionally, a factor complicating a comparison between the present method and measures of tail dependence is that there are many variants of the latter; so, even though such a comparison is important, it is not straightforward. Regarding possible elaborations of the causal model, in future work, we would like to explore the S_1-violation approach using macroeconomic time series, such as such as GDP, unemployment rates, etc. as additional information with which to elaborate the relevant causal models.

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Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix

Mathematical Definition of the S_1-violation Calculation

Let the dataset consist of a time series showing the prices of $M \geq 1$ stocks with N price observations for each stock. These observations are denoted as $P_{i,t}$, where $P_{i,t}$ represents the daily closing price of the stock i at time t. The daily return of stock i at time t, denoted by $R_{i,t}$, is defined as

$$R_{i,t} = \frac{P_{i,t} - P_{i,t-1}}{P_{i,t-1}}.$$

Given a time series and a rolling window size of w, the analysis iteratively moves the window across the dataset. For each endpoint T, the window spans the interval $t \in [T - w + 1, T]$, where.

- T indexes the end of the rolling window and ranges from $T = w$ (the first complete window) to T (the final observation in the dataset).
- t indexes each step (e.g., trading day) within the window.

This process is applied across all stocks $i \in \{1, \ldots, M\}$, for each stock i, a subset of returns $R_{i,t}$ is extracted within the rolling window.

Strong Price Movements

For each stock i in the window.

- Define a threshold $r_i > 0$.
- Compute a binary indicator I to denote returns.

This provides the values for x, y (strong vs. weak). For example, $x = 0$ corresponds to the case when the absolute value of the price change in security A is bigger than the threshold.

- Determine the sign of each return:

$$\mathrm{Sign}(R_{i,t}) = \begin{cases} +1 & \text{if } R_{i,t} \geq 0, \\ -1 & \text{if } R_{i,t} < 0 \end{cases}$$

This provides the values for a, b (direction up or down).

Classification into Regimes

For each pair of stocks $(i, j) = (A, B)$, we classify each time point t in the window into one of four co-movement regimes. These categories can be calculated as the expected values derived from the joint density of returns. For instance, the expectation value for $\langle ab \rangle_{00}$ is given by:

$$\langle ab \rangle_{00} = \frac{\sum_{t \in [T-w+1,T]} \left[\mathrm{sign}(R_{A,t}) \mathrm{sign}(R_{B,t}) I_{\{|R_{A,t}| \geq r_A\}} I_{\{|R_{B,t}| \geq r_B\}} \right]}{\sum_{t \in [T-w+1,T]} I_{\{|R_{A,t}| \geq r_A\}} I_{\{|R_{B,t}| \geq r_B\}}}.$$

Similarly, the expectation values for the other regimes can be computed by substituting the appropriate conditions for each regime.

Counting S_1-violations

The S_1-value, which measures the overall comovement and interdependence dynamics between each pair of stocks such as stocks $i = A$ and $j = B$, is then computed as:

$$S_1^{i,j} = \langle ab \rangle_{00} + \langle ab \rangle_{01} + \langle ab \rangle_{10} - \langle ab \rangle_{11}.$$

Handling of Missing Data

If, for any regime $\langle ab \rangle_{xy}$, there is no valid data within the window (e.g., no observations satisfy the regime's conditions), we set

$\langle ab \rangle_{xy} = 0.$

Using these S_1- values for all pairs of stocks $i, j = 1, \ldots, M, j > i$ we can compute the proportion of violations.

$$S_1 \text{Violation} - \% = \frac{2}{M(M-1)} \sum_{i=1}^{M} \sum_{j=1, j>i}^{M} I_{\{|S_1^{i,j}|>2\}}.$$

So, for each pair of stocks (i, j), it is checked whether $|S_1^{i,j}| > 2$ and the percentage of such pairs across all pairs of stocks for a given index is computed.

Data Description

To establish a robust analytical framework, we utilized financial data sources, based on the availability of the desired data in data source, primarily from Refinitiv (2025). The daily closing prices of the S&P 500 and STOXX Europe 600 stocks were obtained from Refinitiv (2025), ensuring that only stocks consistently included in these indices throughout the study period were considered. This results in approximately $M^2 = 370 \times 370$ pairs of stocks for the 2008 data (note that this excludes, for example, Lehman Brothers Holdings, Fannie Mae, Safeco, etc., as they are no longer part of the index); $M^2 = 420 \times 420$ pairs for the COVID-19 crisis data for the S&P 500 stocks, and approximately $M^2 = 470 \times 470$ pairs for the STOXX Europe 600 stocks. Furthermore, sector-specific 5 years credit default swap indices, as well as the VIX index, were also sourced from Refinitiv (2025). It should be noted that during the COVID- 19 financial crisis, the 5-year telephone CDS in the U.S. market is not plotted due to data inconsistencies and missing values. The VSTOXX index, representing the volatility of the EU market, was obtained directly from its official provider website (STOXX, 2025).

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