IOP Conference Series: Materials Science and Engineering

PAPER • OPEN ACCESS

On Rotor Profling of Internally Geared Screw Machines

To cite this article: H Lacevic *et al* 2024 *IOP Conf. Ser.: Mater. Sci. Eng.* **1322** 012006

IOP Conf. Series: Materials Science and Engineering **1322** (2024) 012006 doi:10.1088/1757-899X/1322/1/012006

On Rotor Profiling of Internally Geared Screw Machines

H Lacevic, A Kovacevic, N Stosic and M Read

Department of Engineering, City, University of London, London, UK

E-mail: halil.lacevic@city.ac.uk

Abstract. The internally geared screw machine represents a novel type of positive displacement compressor which consists of an inner and outer rotor. Both rotors rotate in the same direction but are each centered on offset parallel axes. The rotor profiles are designed to create multiple continuous contact points between the rotors, forming several separate working chambers whose volumes vary from minimum to maximum and back to minimum during a single rotation of the outer rotor. For a gas or two-phase working fluid, adjusting the discharge port geometry allows internal compression to occur before discharge. Previous research has focused on using the well-known rotor profiling method, which employs a circular pin to generate the inner and outer rotor profiles. Although the rack method for rotor profile generation has been described and investigated for conventional screw machine rotor profiles, it has never been applied to internally geared screw machine profile generation. This paper provides an initial description and application of the rack method for generating internally geared screw machine profiles. Potential benefits of using the rack method compared to conventional methods for rotor profile generation in internally geared machines are discussed. Additionally, the limitations of using the rack method for internal gearing are presented and illustrated through various examples and applications.

1. Introduction

Conventional twin screw compressors are a type of positive displacement machines used for gas compression and are widely used in industrial and refrigeration applications. Twin screw machines consist of two inter-meshing helical rotors (main and gate rotor) in a fixed casing. A number of separate working chambers with varying volume are formed between rotor lobes and casing as the rotors rotate. On a suction side, when the working chamber is first formed, gas is drawn into the working chamber, trapped, compressed and released through the discharge port on the other side of the machine. These type of compressor machines have presented several advantages, including high efficiency and the ability to handle varying loads and pressures. Geometry of a twin screw machine is very well known and various methods for generating rotor profiles are defined. Apart from the geometry, performance prediction tools including quasi one-dimensional chamber modelling [1] and computational fluid dynamics (CFD)[2] are widely used for twin screw compressors.

Most recent studies on screw compressors have focused on understanding the influence of the various leakages in conventional twin screw machines and improving the geometry of these machines to produce even more efficient configurations. There are not many studies investigating alternatives to the conventional twin screw machines, however a novel design, internally geared screw machine, providing slightly different configuration of rotors has been introduced by Read [3]. Internally geared screw machines are based on a design of gerotor pump which is commonly used in oil pumping applications. Gerotor pumps and internally geared screw machines consist of two internally geared rotors (outer and inner rotor) which

IOP Conf. Series: Materials Science and Engineering **1322** (2024) 012006 doi:10.1088/1757-899X/1322/1/012006

mesh together. To produce separate working chambers, continuous contact between inner and outer rotor is required for both machines, however to reduce power transfer between the rotors in internally geared screw machines, a helical twist is added to both rotors, while gerotor pumps implement straight cut rotors. Other significant difference between gerotor pumps and internally geared screw machines is initial volume of working chamber. To produce efficient compression, a zero initial volume when the working chamber is first formed is required for internally geared screw machines.

Working chambers in internally geared screw machine are formed between outer and inner rotors and gas is drawn in these chambers, carried and released on the other side of the machine. Control of the periods at which the gas is allowed to enter or exit the working chamber allows the compression to occur. The control of these periods is done by providing specifically designed end ports for both suction and discharge side which were initially introduced by Read[3]. An example of end port plates on a model of internally geared screw machine is presented in Figure 1.

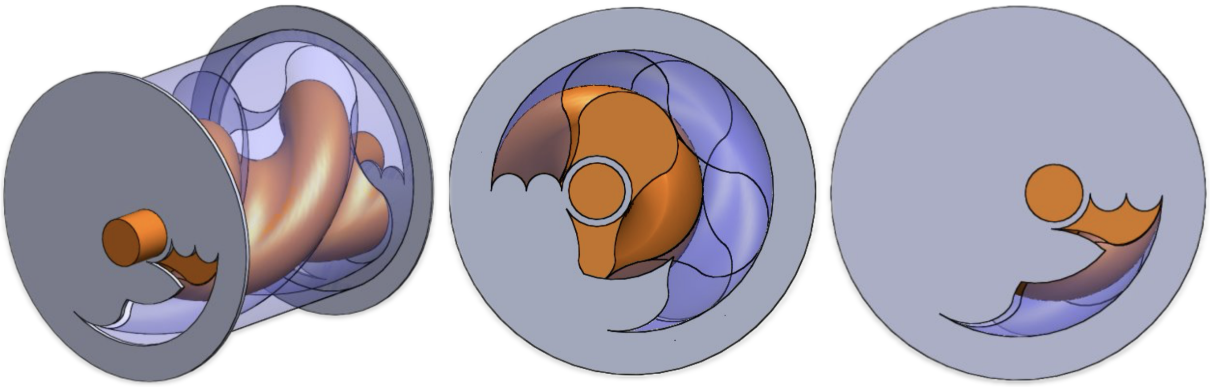


Figure 1. Internally geared screw machine model showing suction and discharge end port plates[3].

Internally geared screw machine has several potential advantages compared to the conventional twin screw machines as mentioned by Read[3]:

- Elimination of rotor-to-casing tip clearances present in conventional screw machines, as the outer rotor encloses the working chamber. The two rotors of the internally geared machine effectively share the same tip leakage areas.
- Elimination of the blow hole area characteristic of conventional machines.
- Reduced viscous drag due to the much lower relative speed at the contact lines between rotors, as compared to the rotor-casing leakage lines in a conventional machine.
- Reduced distortion due to thermal expansion, as the rotation of both rotors will lead to a uniform circumferential temperature in each during operation, with temperature variation limited to the axial direction.

The geometry of internally geared screw machine has been extensively investigated in previous studies [3, 4], which have implemented a variety of rotor profiling methods. In conventional twin screw machines, rotor profiles are commonly generated using the rack method, which offers several advantages compared to other methods, particularly in terms of manufacturing [5]. However, the rack method has not been previously applied to internally geared screw machines. This study focuses on investigating whether the rack method can be utilized for internally geared profiles, which require a zero initial working chamber volume and continuous contact between the rotors. This paper provides initial guidance for rotor profiling in internally geared screw machines and presents work in progress on exploring the potential benefits and limitations of applying the rack method.

IOP Conf. Series: Materials Science and Engineering **1322** (2024) 012006 doi:10.1088/1757-899X/1322/1/012006

2. Internally Geared Screw Machine Rotor Profiles

The design of a compressor machine begins with defining the shapes of rotor profiles, which significantly influence factors such as working chamber volume, leakage paths, and overall machine performance. An internally geared screw machine is consisted of two internally geared rotors (outer and inner) which rotate in the same direction about non-coincident parallel axes. The outer rotor profile forms a conjugate pair with the inner rotor, ensuring continuous points of contact between the rotors during rotation. This continuous contact enables the separation of the volume between the rotors into multiple working chambers. The volume of each working chamber varies with rotor position, depending on the chosen rotor geometry. Another crucial aspect affecting compressor machine performance is the minimum volume of a working chamber, which also depends on the selected geometry. To achieve efficient compression, a zero-minimum volume and continuous contact between the rotors are necessary. Various methods for generating rotor geometry for gerotor machines are described by Colbourne [6], Beard et al. [7], Vecchiato et al. [8], and Hsieh et al. [9].

Previous research concentrated on basic geometries as outlined by Moineau [10], where the internally geared rotor geometry is based on epi & hypocycloid curves. While these profiles offer a convenient method for generating rotor profiles, recent studies have shifted focus towards circular pin-generated rotor profiles, following the method detailed by Vecchiato et al[8]. Circular pin-generated profiles offer a broader range of profile shapes and a relatively straightforward approach to profile generation for internally geared screw machines.

The rack method, as described by Stosic [11], is commonly used for rotor profile generation in conventional twin screw machines. While the rack method has been widely used in conventional twin screw machines, it has not been applied to internally geared screw machines. Further investigation is required to determine whether the rack method can be adapted for internally geared profiles while maintaining a zero-minimum volume of the working chamber and ensuring continuous contact between the rotors. Additionally, it is essential to define the advantages and limitations of the rack method compared to other rotor profiling techniques.

2.1. Circular Pin-Generated Rotor Profiles

Internal pin-generated gearing can be used for generating meshing rotor profiles. The inner rotor must have one more or one less lobe than the outer rotor to ensure continuous contact[4]. In the case of internally geared screw machines, an extended epicycloid profile is formed on the gear with the smaller centrode when the profile of the gear with the larger centrode is partially defined by the circular pin. Circular pins on the outer rotor, consisting of N_1 lobes, are defined in the coordinate system S_1, fixed to the initial position of the outer rotor and centered on the outer rotor pitch circle[4]. Coordinate system S_2 is fixed to the initial position of the inner rotor and centered on the inner rotor pitch circle. Both coordinate systems S_1 and S_2 rotate with the rotors, while coordinate system S_f is fixed to the casing and defines a global coordinate system centered on the outer rotor pitch circle. The angle between coordinate systems S_1 and S_f defines the outer rotor rotational position Φ_1 relative to the global coordinate system.
The gearing ratio between the outer and inner rotors can be calculated as $m_{21} = \frac{N_2}{N_1}$, where $N_2 = N_1 - 1$ is the number of lobes on the inner rotor. In this case, the angle between coordinate systems S_2 and S_f defines the rotational position of the inner rotor $\Phi_2 = \frac{1}{m_{21}}\Phi_1$ relative to the global coordinate system S_f. The centers of coordinate systems S_1 and S_2 are separated by the axis distance between the rotors (E). The transformation of points from coordinate system S_1 to coordinate systems S_2 and S_f can be accomplished using transformation matrices as presented in Equations 1 and 2.

$$M_{12} = \begin{bmatrix} \cos(\Phi_2 - \Phi_1) & \sin(\Phi_2 - \Phi_1) & 0 & -E\sin(\Phi_1) \\ -\sin(\Phi_2 - \Phi_1) & \cos(\Phi_2 - \Phi_1) & 0 & -E\cos(\Phi_1) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1}$$

$$M_{1f} = \begin{bmatrix} \cos(\Phi_1) & -\sin(\Phi_1) & 0 & 0 \\ \sin(\Phi_1) & \cos(\Phi_1) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2}$$

The shapes of rotor profiles are entirely determined and defined by the radius of the circular pin (ρ) and its center distance (a) from the center of the global coordinate system S_f, as illustrated in Figure 2.

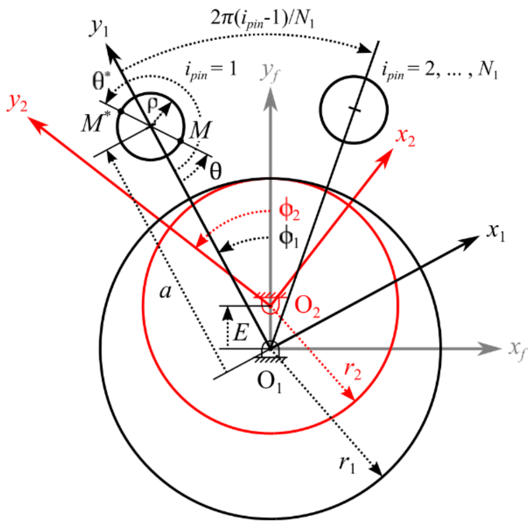
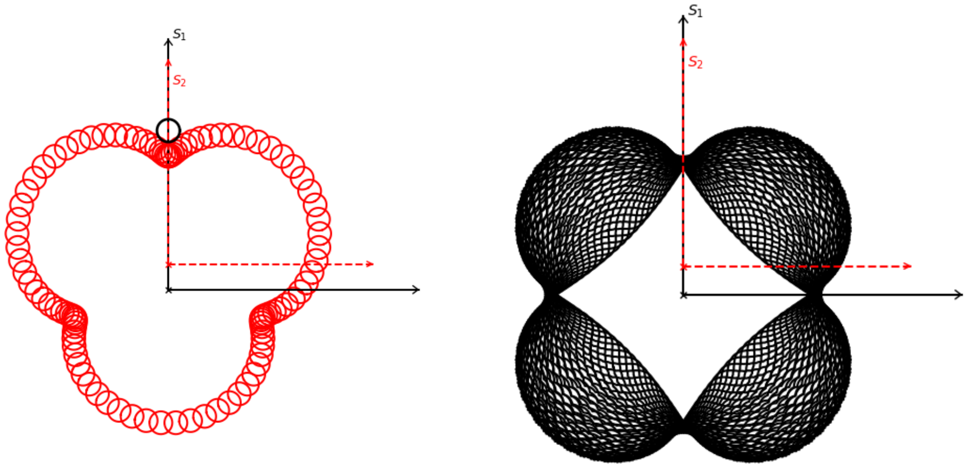


Figure 2. Circular pins and coordinate systems used for internally geared screw machine rotor generation in circular pin-generated profiles[4].

The points of a single circular pin are defined in coordinate system S_1 as a function of θ, where θ ranges from 0 to 2π, and can be expressed in matrix form, as depicted in Equation 3. The points of the inner rotor profile can be determined by rotating a single circular pin in coordinate system S_1 and then transforming it to the second coordinate system S_2, while applying the first equation of meshing as described by Vecchiato[8]. In this configuration, the inner rotor profile is entirely defined by the circular pin and can subsequently be used to generate points for the outer rotor profile. To find the outer rotor profile, the points of the inner rotor profile are transformed from coordinate system S_2 to S_1, while applying the second equation of meshing detailed by Vecchiato[8]. A visual representation of the circular pin-generation method is provided through examples of the inner and outer rotor profile enveloping, as depicted in Figure 3.

$$r_1 = [\rho\cos(\theta) \quad \rho\sin(\theta) \quad 0 \quad 1]^T \tag{3}$$

Figure 4 illustrates examples of profiles achievable through the circular pin-generation method. These profiles are characterized by the lobe number on the outer rotor, N_1, along with the non-dimensional parameters $\lambda = a/r_1$ and $\sigma = \rho/r_1$, and the axis spacing distance $E = r_1/N_1$. However, certain limitations exist on these values to prevent the occurrence of undercuts, which is extensively discussed by Read[4].



(a) Example of circular pin defined in S_1 coordinate system (black) enveloping the inner rotor profile in S_2 (red).

(b) Example of generated inner rotor profile enveloping the outer rotor profile.

Figure 3. Visual representation of circular pin-generation method.

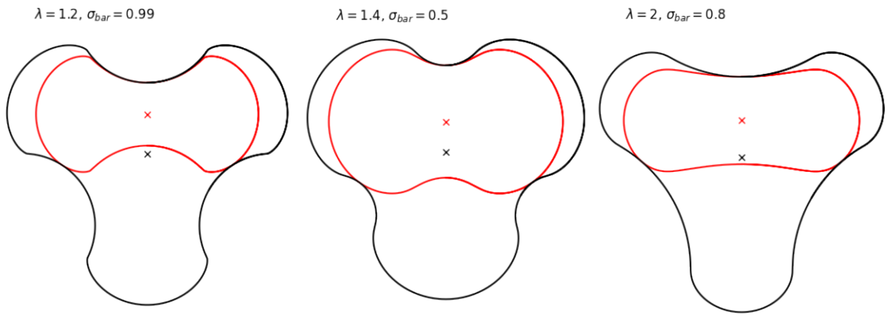


Figure 4: Example of profiles generated using circular pin-generation method with $N_1 = 3$.

2.2. Rack Generated Rotor Profiles

Conventional twin-screw machine rotor profiles are typically generated using the rack method, which leverages the principles of the Envelope Theory formulated by Euler [12] in the field of differential geometry. This theory allows for the mathematical calculation of the equation of a curve that envelopes a family of curves, making it applicable for generating equations of conjugate shapes on paired rotors. Initially introduced by Sakun [13], this method in rotor profiling involves defining a mathematical equation, known as the meshing condition, that must be satisfied by the generated meshing curve on a pair of rotors. The early approach focused on deriving analytical meshing conditions, which provided analytically representable conjugates of selected analytical curves such as circles, lines, and points. An improved approach, described by Stosic [11], significantly simplified the design procedure. This improvement led to much wider set of possible profile shapes and allowed profiles to be defined by various sections on the rack providing the ability to design various type of contact points at specific locations. Using the enveloping method, for any given rotor profile, a corresponding conjugate rotor profile depending on it's pitch circle can be found by applying the meshing condition. The rack rotor represents a special case when the pitch circle has "infinite radius". In other words, rack rotor represents straight rotor that is used to envelope main and gate rotor. The shape of the rack can be defined by segments which makes this method very powerful. An example of externally geared rotor profiles produced using the rack method is shown in Figure 5. In Figure 5 sine wave function is used for generating the rack curve. Rack method for conventional twin screw machines has the following procedure:

- Defining shape of the rack curve.
- Defining pitch circles for both main and gate rotors.
- Solving meshing condition between the rack and main rotor profile.
- Solving meshing condition between the rack and gate rotor profile.

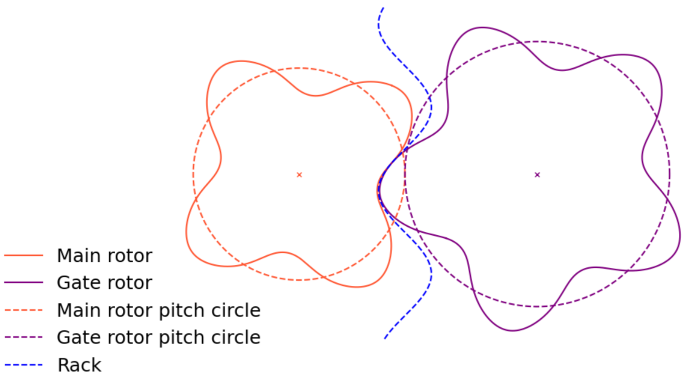


Figure 5. Profiles with external meshing produced using sine wave as a rack.

Although the rack method has been widely used in conventional twin-screw machines, it has never been applied to internally geared screw machines. However, the rack method is commonly used in internal gearing, particularly for generating involute profiles for gears. It is known that the rack method can produce internal gears, but the question remains whether it can produce internal gears that satisfy the necessary conditions for continuous contact between the rotors and a zero-minimum working chamber volume, which is essential for efficient compression in internally geared screw machines.

The rack method for internally geared screw machines begins by defining the rack curve in the global coordinate system. For any given rack curve (x_r, y_r), the coordinates of the inner and outer rotor profiles can be found using Equations 4 and 5, respectively. The pitch circle radii of the inner and outer rotors are denoted as r_{2p} and r_{1p}, while Ψ_i, $i = [1, 2]$ represents the solutions of the meshing condition between the rack and each rotor profile. The solutions to the meshing conditions, as described by Stosic [11], can be determined using Equations 6 and 7.

$$\begin{bmatrix} x_1 \\ y_1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \cos(\Psi_1) & -\sin(\Psi_1) & 0 & r_{1p}\Psi_1 \sin(\Psi_1) \\ \sin(\Psi_1) & \cos(\Psi_1) & 0 & -r_{1p}\Psi_1 \cos(\Psi_1) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r + E \\ y_r \\ 0 \\ 1 \end{bmatrix} \tag{4}$$

$$\begin{bmatrix} x_2 \\ y_2 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \cos(\Psi_2) & -\sin(\Psi_2) & 0 & r_{2p}\Psi_2 \sin(\Psi_2) \\ \sin(\Psi_2) & \cos(\Psi_2) & 0 & -r_{2p}\Psi_2 \cos(\Psi_2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r \\ y_r \\ 0 \\ 1 \end{bmatrix} \tag{5}$$

$$\Psi_1 = (y_r - \frac{\partial x_r}{\partial y_r}(r_{1p} - x_r - E))/r_{1p} \tag{6}$$

IOP Conf. Series: Materials Science and Engineering **1322** (2024) 012006 doi:10.1088/1757-899X/1322/1/012006

$$\Psi_2 = (y_r - \frac{\partial x_r}{\partial y_r}(r_{2p} - x_r))/r_{2p} \tag{7}$$

An example of internally geared profiles generated using the previously described rack method, with a sine wave used as the rack, is shown in Figure 6. From Figure 6, it is evident that the rack method can generate internally geared profiles that mesh around the pitch point, a typical characteristic in gears. However, it is also apparent that using the selected sine wave for the rack curve does not achieve continuous contact between the rotors.

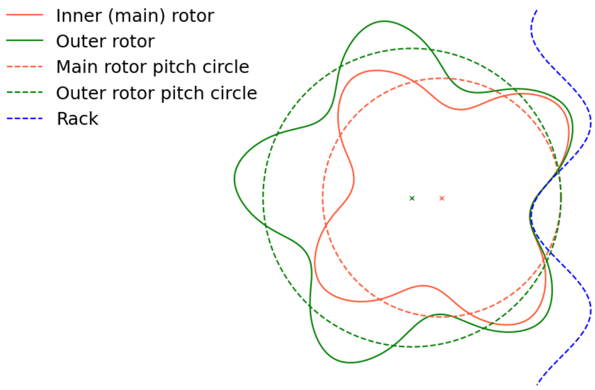


Figure 6. Internally geared profiles generated using the rack method and a sine wave as a rack curve.

As previously mentioned and shown in Figure 6, the rack method can produce profiles with internal meshing. However, whether the rack method can generate profiles that ensure continuous contact between the rotors and a zero-minimum volume of a working chamber has not been investigated. Main focus of this paper is to presents an initial investigation into the use of this method in internally geared screw machines, focusing on the cycloid curve for the rack profile. A cycloid is the curve traced by a point on the circumference of a circular wheel as it rolls along a straight line without slipping, characterized by its repetitive, arch-like shape. The equations for the coordinates of a rack curve based on the cycloid curve, as a function of the angle θ_c, are given in Equations 8 and 9, where $2R_c^{(a)}$ and $2R_c^{(d)}$ define the profile addendum and dedendum, respectively. To ensure that the rack curve defined by the parametric Equations 8 and 9 is continuous, the condition $R_c^{(a)} + R_c^{(d)} = E$ must be met. Examples of rack curves generated using these equations are shown in Figure 7.

$$x_r = \begin{cases} R_c^{(a)}(\theta_c - \sin(\theta_c)) & \text{if } (\theta_c \mod 4\pi) < 2\pi \\ R_c^{(d)}(\theta_c - \sin(\theta_c)) & \text{if } 2\pi \leq (\theta_c \mod 4\pi) < 4\pi \end{cases} \tag{8}$$

$$y_r = \begin{cases} R_c^{(a)}(1 - \cos(\theta_c)) & \text{if } (\theta_c \mod 4\pi) < 2\pi \\ R_c^{(d)}(-1 + \cos(\theta_c)) & \text{if } 2\pi \leq (\theta_c \mod 4\pi) < 4\pi \end{cases} \tag{9}$$

An example of valid rotor profiles with $N_1 = 5$ and $N_2 = 4$ generated using a cycloid-based rack curve with $R_c^{(a)} = [0.5, 0.25]$ is shown in Figure 8. This figure demonstrates that conjugate profiles have been successfully produced using the rack method and that the necessary conditions for achieving continuous contact between the rotors and maintaining a zero-minimum working chamber volume are satisfied.

IOP Conf. Series: Materials Science and Engineering **1322** (2024) 012006 doi:10.1088/1757-899X/1322/1/012006

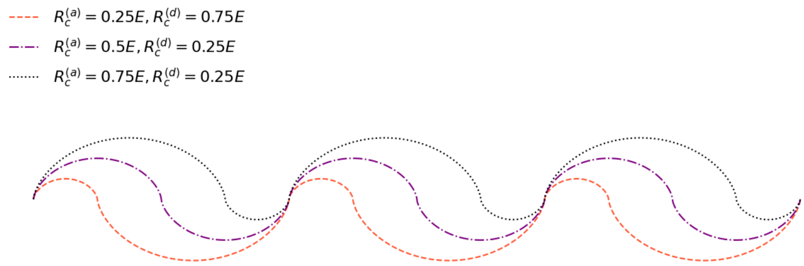
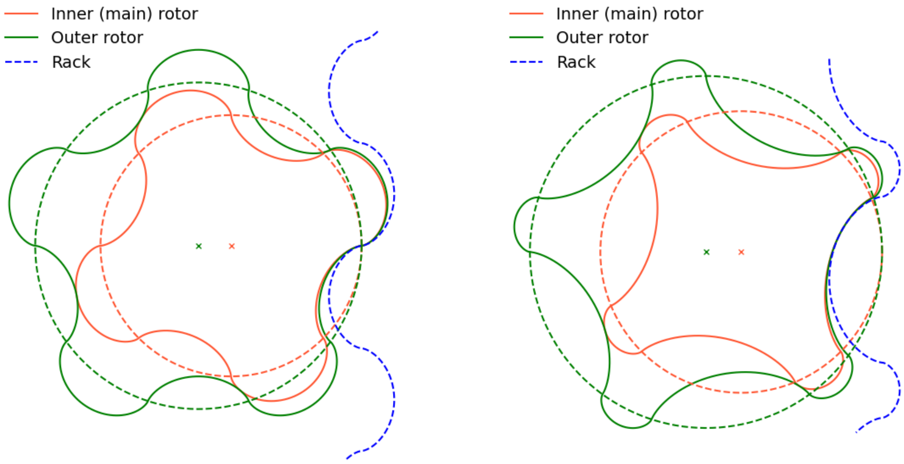


Figure 7. Example of rack curves based on parametric equations given in Equation 8 and 9



(a) Cycloid based rack curve with $R_c^{(a)} = 0.5$. (b) Cycloid based rack curve with $R_c^{(a)} = 0.25$.

Figure 8. Example of valid rotor profiles generated using cycloid based rack curve.

3. Conclusion

Internally geared screw machines represent a novel type of compressor whose efficiency has not been fully investigated. Previous studies on these machines have primarily used circular pin-generated profiles. In contrast, conventional twin screw compressors typically employ the rack method for generating rotor profiles. The rack method offers several advantages, including the ability to produce a wider range of rotor profile shapes and easier manufacturing. Although the rack method has been used to produce internal gears where continuous contact is not required, its potential to generate rotor profiles with continuous contact and a zero-minimum working chamber volume has not been investigated until now.

This study applied the rack method for the first time to generate profiles for internally geared screw machines. The initial investigation introduced a rack curve based on cycloids, demonstrating that the resulting rotor profiles are valid for use in internally geared screw machines. The findings indicate that the rack method can produce rotor profiles that ensure continuous contact and maintain a zero-minimum working chamber volume. However, the limitations of this approach remain unknown.

Future research should focus on determining whether the cycloid-based rack curve is a special case and on generalizing the conditions needed to produce valid rotor profiles using the rack method for internally geared screw machines. Additionally, the limitations and potential benefits of this method compared to other available methods should be thoroughly examined and defined.

References

[1] Hanjalic K and Stosic N 1997 *D*evelopment and optimization of screw machines with a simulation model—PART II: Thermodynamic performance simulation and design optimization *Journal of Fluids Engineering* 119(3) 664–670 doi:10.1115/1.2819296

[2] Kovacevic A, Stosic A and Smith I 2007 *Screw Compressors: Three Dimensional Computational Fluid Dynamics and Solid Fluid Interaction* (Berlin: Springer)

[3] Read M G, Smith I K, and Stosic N 2017 Internally geared screw machines with ported end plates *IOP Conference Series: Materials Science and Engineering* 232 012058 doi:10.1088/1757-899x/232/1/012058

[4] Read M, Smith I, and Stosic N 2020 Influence of rotor geometry on tip leakage and port flow areas in Gerotor-type twin screw compressors *Proceedings of the Institution of Mechanical Engineers, Part E: Journal of Process Mechanical Engineering* 236(1) 94–102 doi:10.1177/0954408920962412

[5] Stosic N, Smith I K, Kovacevic A and Mujic E 2011 Geometry of screw compressor rotors and their tools *Journal of Zhejiang University-SCIENCE A* 12(4) 310–326 doi:10.1631/jzus.a1000393

[6] Colbourne J 1974 The geometry of trochoid envelopes and their application in rotary pumps *Mechanism and Machine Theory* 9(3–4) 421–435 doi:10.1016/0094-114x(74)90025-1

[7] Beard J, Yannitell D and Pennock G 1992 The effects of the generating pin size and placement on the curvature and displacement of epitrochoidal gerotors *Mechanism and Machine Theory* 27(4) 373–389 doi:10.1016/0094-114x(92)90030-l

[8] Vecchiato D, Demenego A, Argyris J and Litvin F L 2001 Geometry of a cycloidal pump *Computer Methods in Applied Mechanics and Engineering* 190(18–19) 2309–2330 doi:10.1016/s0045-7825(00)00236-x

[9] Hsieh C-F and Hwang J-W 2006 Geometric design for a gerotor pump with high area efficiency *Journal of Mechanical Design* 129(12) 1269–1277 doi:10.1115/1.2779887

[10] Moineau R J L 1932 Gear mechanism

[11] Stošić N 1998 On gearing of helical screw compressor rotors *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science* 212(7) 587–594 doi:10.1243/0954406981521556

[12] Guggenheimer H W 1977 *Differential Geometry* (New York: Dover Publications)

[13] Sakun I A 1960 *Screw Compressors* (Moscow: Mashgiz) (in Russian)

Acknowledgement
Funding for this research was received from Carrier Inc., USA and PDM Analysis Ltd., UK.