The Effect of Mergers on Innovation

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We study the effect of a merger on R&D activity in a dynamic model with uncertainty about the feasibility of innovation. The merger has three effects: it may reduce the number of follow-up innovations (cannibalization effect), increase the probability of the first game-changer innovation (appropriability effect), and bring this innovation forward in time (informational effect). The model suggests mergers are more desirable when R&D outcomes are highly uncertain, but less so when the innovation path is clearer. A surprising policy implication is that the benefit of the merger may be higher if the first and subsequent innovations are closer substitutes.

The idea that more competition leads to more innovation now dominates policy debates. In a 2016 speech at the EDPS-BEUC Conference, EU Competition Commissioner Margrethe Vestager insisted that competition is essential to drive innovation.[1] In this paper, we develop a theoretical model to examine the claim that greater competition drives greater innovation.

A mechanism through which greater competition fosters innovation was clearly articulated in the European Commission's 2016-2017 review of the Dow/DuPont merger.[2] The Commission argued that prior to the merger, each party had in-

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[1] "This is what excites consumers about the new digital economy. Innovative services, which let us do things that simply wouldn't be possible otherwise. To get that kind of service, we need competition." — Commissioner Vestager, "Big Data and Competition," speech at the EDPS-BEUC Conference on Big Data, Brussels, 29 September 2016. https://ec.europa.eu/commission/presscorner/detail/ov/speech_16_5224

[2] The Commission's Dow/DuPont decision can be found at https://ec.europa.eu/competition/mergers/cases/decisions/m7932_13668_3.pdf.

centives to develop new products to capture sales from its competitors. After the merger, however, by introducing new products, instead of capturing sales from its competitors, the merged entity might divert sales from its own products. In other words, an innovation by one merging party now cannibalizes its merger partner's profits. The merger internalizes the reduction of profits that the competition between newly developed products brings. As a result, incentives to innovate are reduced. The described mechanism is commonly referred to as the cannibalization effect[3] and is very much rooted in the traditional horizontal merger analysis.[4]

Although heavily relied on by the Commission, the cannibalization effect does not fully capture all the consequences of a merger for innovation incentives. The existing literature recognizes that innovation incentives also depend on the extent to which an innovator can capture the benefits generated by its innovation efforts. A firm's incentives to innovate are muted if future competition from similar products is expected to rapidly diminish the profit that the innovation generates. A merger may alleviate this concern by giving the merged entity control over the introduction of new products in the future. The increased incentives to innovate due to the improved ability of the merged entity to appropriate the benefits of its innovation efforts are often referred to as the appropriability effect of the merger.[5,6]

Though both the cannibalization and the appropriability effects have been recognized in the existing literature, the trade-off between them is not well understood. We propose a unifying formal framework that encompasses existing verbal arguments (e.g., Shapiro (2012)) and derives novel insights not accessible without a formal model.

We consider two firms engaging in costly R&D activity along a research avenue. The research avenue can be one of two types: good or bad. A good research avenue rewards R&D effort with a product innovation at exponentially distributed time — in the manner of exponential bandits. A bad research avenue never generates an innovation. Initially, the firms do not know the type of the research avenue. Over time, each firm learns about the type of the avenue from its own and its rival's observable research activity and outcomes.[7] There can be, at most, two

[3]Though not explicitly called so, the cannibalization effect was first introduced by Arrow (1962). Tirole (1988) refers to this effect as the replacement effect because, by introducing new products, the merged entity would replace only itself, while absent the merger, each party would replace its competitor.

[4]The very same idea that a merger internalizes the negative externality that one product imposes on the profits from another product is typically used to explain a merged entity's incentives to increase prices. The Commission's Dow/DuPont decision refers to the European Commission's Horizontal Merger Guidelines on numerous occasions.

[5]The appropriability effect was first described in Schumpeter (1942).

[6]In the Dow/DuPont decision, the European Commission equates appropriability to the intellectual property rights protection (paragraph 5, Annex 4 of the decision) and claims that the appropriability is not affected by competition because an innovator can always capture the marginal social value of an innovation (Appendix A to Annex 4 of the decision). This view, however, fails to recognize that the marginal social value of an innovation may decline with the number of similar products in existence.

[7]We assume that a firm's research activity and whether this activity produced an innovation are observable to its rival. This assumption is justified because in many industries, R&D activities are often required to be publicly reported. For example, in the pharmaceutical industry, patented drug develop-

sequential innovations in the market. The innovations are substitutes, and so, the introduction of the second innovation reduces the profit that the first innovation generates.

The goal is to compare the firms' innovation incentives when they compete with each other and when they are merged to form a single entity, referred to as the merged entity. The main difference between the two settings is that, while the competing firms choose their R&D efforts non-cooperatively, the merged entity implements the R&D effort that is jointly optimal for the two merging parties.

In our model, the cannibalization effect arises in relation to the follow-up innovation: once the first innovation has already taken place, the merged entity may block the second innovation to avoid diverting sales from the first innovation. Naturally, this sale diversion is more pronounced and thus is more likely to trigger the cannibalization effect if the first and the follow-up innovations are closer substitutes. Because the first innovation is assumed to be a game-changer (so-called drastic or radical innovation) with no substitutes, the cannibalization effect does not apply to it. In contrast, the appropriability effect arises in relation to the game-changer innovation: for example, when the merged entity blocks the second innovation, the profitability of the first innovation increases, and, hence, the merger increases the probability of the first innovation.

In practice, many mergers occur once the product market is already established, which eliminates uncertainty about the feasibility of future innovations. At this stage, any subsequent innovations typically compete directly with existing products — or, in the language of our model, constitute follow-up innovations that are substitutes to the game-changer innovation. In such mergers, cannibalization concerns may become particularly prominent. For example, regulators reviewing Adobe's proposed 2023 acquisition of Figma feared that the merger would lead the combined firm to shelve development of new products by Figma (a killer acquisition concern) or by Adobe (a reverse killer acquisition concern). The authorities' objections were sufficiently strong that the merging parties ultimately decided to withdraw from the merger.[8]

In other mergers, outcomes of R&D activity are highly uncertain and not clearly linked to existing products, which leads to a trade-off between the cannibalization and appropriability effects. Our model provides new insights for policy makers on how to resolve this trade-off. Perhaps surprisingly, the benefit of the merger, brought by the appropriability effect, may be higher if the first and second in-

ment involves public disclosure of a number of milestone developments, and data set Pharmaprojects tracks all drug projects throughout their whole life-cycle. Similarly, in the early days of generative AI, the big tech companies' major breakthroughs in foundational Large Language Model research were published in public domain. Moreover, in our model, we conjecture that the qualitative results do not change if we assume that only the end results of R&D activities — successful innovations — are observable. We discuss the observability assumption further in Section VI.

[8]Summaries of the regulators' objections to Adobe/Figma merger can be found at https://ec.europa.eu/commission/presscorner/detail/en/ip_23_5778 (concerns of the European Commission in the EU) and https://www.gov.uk/government/news/adobe-figma-deal-could-harm-uk-digital-design-sector (concerns of the Competition and Markets Authority in the UK).

novations are closer demand substitutes.[9] Intuitively, when innovations are close substitutes, expectations of intense competition in the future dampen the competing firms' immediate incentives to innovate, thus reducing the probability of the first innovation. In contrast, the merged entity may avoid competition by not producing the second innovation at all, which implies that an increased substitutability of the innovations does not decrease the merged entity's incentives to produce the first innovation. Thus, within our model, under some conditions, the expectation that the prospective innovations will be close substitutes is an argument for, rather than against, the merger because higher substitutability between the first and second innovations brings about ever higher merger-induced increase in the probability of the first innovation.[10]

In addition, we identify a novel effect of the merger that arises from the dynamic nature of the model. Once a firm launches a newly developed product, its competitor immediately learns that the research avenue is good. Thus, launching a new product generates positive informational externalities for the other firm in the market. The expectation that the rival's costly R&D effort may provide information about the feasibility of the innovation gives rise to the free-riding problem, thereby incentivizing firms to slow down their R&D activities prior to the first innovation. The merged entity internalizes the informational externalities and, thus, has no reason to slow down R&D. We refer to this mechanism as the informational effect. The nature of this effect is familiar from the strategic experimentation literature since Keller, Rady and Cripps (2005), in which the individual incentives to free-ride on others' experimentation efforts lead to an aggregate experimentation level that is lower than socially optimal. Within our model, the informational effect manifests itself through the timing of the first innovation: conditional on the arrival of the first innovation, the merged entity may take less time to generate it.[11,12]

In our model, the slowdown of R&D activities occurs when the competing firms are sufficiently pessimistic that the research avenue is capable of generating a game-changer innovation. In practice, such slowdown was observed in the development of Large Language Models (LLMs). In hindsight, the rise of consumer-facing LLM products might seem inevitable, but in 2017-2018 it was not at all obvious. Driven by such pessimism, despite seemingly having all the

[9]Our conclusion that the benefit of the merger may be higher for closer substitutes may seem surprising because in traditional merger analysis, the price increase following a merger is expected to be stronger when the merging parties' products are closer substitutes. Hence, mergers between close competitors are typically frowned upon.

[10]Our model lends support to the pro-merger arguments put forward by Dow and DuPont (see paragraph 2115 in the Commission's Dow/DuPont decision).

[11]Katz and Shelanski (2007) note that, often, "proponents of the permissive merger policy contend that [...] market consolidation may in fact help to speed innovation by bringing complementary assets together." Although we also conclude that the merger may speed up innovation arrival, the mechanism is different because we assume away any synergy of resources.

[12]Literature on research joint ventures (RJV) also features the free-riding problem in R&D (see Katz (1986) and others). In contrast to our model, RJV models of R&D are static and so cannot provide insights on the timing of innovations; in that literature, free-riding arises because the competitors' R&D effort directly enters each firm's payoff function.

necessary capabilities, Google was delaying the development of consumer-facing LLM products.[13] In late 2022, OpenAI's release of ChatGPT stunned the world — and Google, which released its own LLM (Bard, now Gemini) only months later.

We also examine how the desirability of mergers depends on firms' initial belief that the research avenue is promising. While the cannibalization effect is unaffected by this belief, both the appropriability and informational effects weaken as firms grow more optimistic — that is, as the uncertainty about the feasibility of a game-changer innovation diminishes. Indeed, without such uncertainty, the appropriability and informational effects would not arise. Accordingly, the model suggests that mergers are more desirable when R&D outcomes are highly uncertain, as concentrated market structures allow firms to better appropriate innovation returns and reduce free-riding. By contrast, when the innovation path is clearer and R&D resembles a race rather than a gamble, mergers are less beneficial and may hinder follow-up innovation, making a competitive market structure more favorable.[14]

In the baseline model, we assume that the merger affects neither the cost of R&D nor the profitability of an innovation, keeping the number of innovations in the market fixed. Thus, we implicitly disregard potential resource synergies as well as the price effects associated with increased market power. In Section V, we extend our model to account for the standard price effects of mergers. We show that, while the merged entity's ability to charge higher prices may negatively affect consumer welfare, it also strengthens the incentives to pursue both the first and second innovations — that is, the upside of higher prices is more innovations through stronger appropriability and weaker cannibalization effects. Moreover, the price effects can be so strong that the merged entity always produces the second innovation whenever the competing firms do — that is, the cannibalization effect disappears, making the merger's impact on innovation unambiguously positive.

Our theoretical findings point to a potential bias in the empirical literature, which stems from an asymmetry in detectability: identifying a merger's negative cannibalization effect is empirically easier than detecting its positive appropriability and informational effects. The cannibalization effect shuts down the development of substitute products, while the appropriability and informational effects promote and speed up the development of truly novel products. The absence of novel products is harder to detect than the absence of substitute products. This asymmetry in the ease with which the effects can be identified might bias

[13] In the years leading up to OpenAI's release of ChatGPT in November 2022, Google had developed advanced language models like LaMDA but refrained from launching consumer-facing products due to the uncertainty about the demand for such products (see, for example, https://synthedia.substack.com/p/google-faces-and-innovators-dilemma).

[14] Our recommendation that mergers involving firms on a clear innovation path should be subject to heightened scrutiny is in line with the recommendations in OECD report (2023), "The Role of Innovation in Competition Enforcement," which can be retrieved from www.oecd.org/daf/competition/the-role-of-innovation-in-competition-enforcement-2023.pdf.

the empirical literature towards finding the negative effect of a merger. Indeed, recent empirical papers, such as Ornaghi (2009), Haucap, Rasch and Stiebale (2019) and Cunningham, Ederer and Ma (2021), look at pharmaceutical mergers and document a decline in merged entities' patenting and R&D activity following the merger. Despite the potential bias, there are empirical papers that identify a positive effect of the merger: for example, Guadalupe, Kuzmina and Thomas (2012) find that a target company's innovation increases after an acquisition by a multinational firm.

The relationship between mergers and innovation has been extensively studied in the literature — see Lefouili and Madio (2025) for a comprehensive review.

A number of recent papers analyze the impact of a merger on a one-off innovation in a static setting. For example, Federico, Langus and Valletti (2017, 2018), Denicolò and Polo (2018) and Jullien and Lefouili (2020) work with a simple model of product innovation; Motta and Tarantino (2021) and Mukherjee (2022) consider a model in which firms engage in R&D investments aimed at reducing the cost of production (see Jullien and Lefouili (2018) for comprehensive review). In contrast to this literature, we work in a dynamic setting and our results hinge on the dynamic nature of our model. In particular, the sequential order of stochastic innovations allows us to shed new light on the appropriability/cannibalization trade-off, and the informational effect arises because we model R&D efforts as accruing incrementally over time.

There is a large body of literature on research joint ventures (see, for example, Katz (1986); D'Aspremont and Jacquemin (1988); Kamien, Muller and Zang (1992); Amir, Evstigneev and Wooders (2003)). That literature typically focuses on process innovation, which reduces the cost of production, and compares two settings. In one setting, firms form a joint venture to conduct R&D in a jointly owned lab run; in the other setting, each firm conducts R&D independently in its own lab. In both settings, firms behave non-cooperatively in the product market. In contrast, we focus on product innovation and assume that the merger changes the nature of product market competition. Prior to the merger, firms decide whether to work towards the second innovation in a non-cooperative fashion; after the merger, the merged entity internalizes payoff externality that the second innovation imposes on the first.

We formally model R&D activity as strategic experimentation with exponential bandits and observable actions, as in Keller, Rady and Cripps (2005). In contrast to canonical models that feature only informational externalities, our model features both payoff and informational externalities. Other papers that introduce payoff externalities into a strategic experimentation setting with observable actions include Besanko and Wu (2013), Boyarchenko and Levendorskiĭ (2014), Cripps and Thomas (2019), Thomas (2021), and Das and Klein (2024).[15]

[15] No informational externalities and extreme payoff externalities are featured in dynamic models of R&D races with winner-takes-all payoff structure and irrevocable exit. Within this literature, Malueg and Tsutsui (1997) study the effect of the number of players on R&D incentives.

A growing literature applies variations on the technological ladder model to various R&D and antitrust issues. The technological ladder model was first developed in the context of economic growth (Aghion and Howitt, 1992; Aghion, Harris and Vickers, 1997; Aghion et al., 2001); the current applications consider the impact of firm asymmetries (Cabral, 2018) or of a reduction in the number of firms (Marshall and Parra, 2019) on R&D incentives. More recently, Marshall and Parra (2023) study the impact of a merger that entrails R&D synergies, thus making the merged entity more effective than its competitors at producing a breakthrough. Resorting to numerical methods, Hollenbeck (2020) analyzes a dynamic model in which firms engage in investment, entry, exit, and mergers, finding that horizontal mergers increase long-run innovation. This occurs because the possibility of being acquired encourages new firms to enter the market, leading to more competition and higher overall investment in innovation. The technological ladder literature is complementary to our paper. This literature is concerned with infinite sequences of innovations, whereby today's technology leaders may become technology laggards tomorrow. In contrast, we consider a sequence of, at most, two potential innovations and assume that the leader maintains the first-mover advantage after the follow-up innovation arrives. Moreover, in contrast to our approach, the technological ladder models do not involve learning about the feasibility of innovation.

The rest of the paper is organized as follows. Section I describes the setup. Section II provides a detailed analysis of the model. Section III presents our main result with three effects of the merger on innovation — cannibalization, appropriability and informational effects. Section IV discusses the policy implications of the model. Section V shows that our results are robust to introducing price effects of the merger. Section VI concludes.

I. Model

To identify the effects of a merger, we compare outcomes in two settings: one in which there are two competing firms and one in which these firms are merged into a single entity, referred to as the merged entity. The key metrics for the comparison are the number and the arrival time of innovations. In formulating our policy recommendations, we operate under the implicit assumption that consumers value a greater variety of innovations and their expeditious arrival.[16]

COMPETING FIRMS. — Two firms, indexed by $i \in \{1, 2\}$, undertake R&D to produce an innovation. There can be, at most, two sequential innovations in the market, and each firm can potentially produce both of them. Time is continuous and indexed by $t \in [0, \infty)$. The discount rate is $r > 0$.

[16] In *Comparison* subsections in Online Appendices E.1 and F.3, we use Stackelberg competition model with differentiated products to justify the assumption that consumers always benefit from a greater number of innovations.

Firms undertake research along a research avenue. At each moment t, firm i chooses the research intensity $x_i(t) \in [0,1]$. The flow cost of research is $cx_i(t)$, with $c > 0$.

The research avenue can be one of two types: *good* or *bad*. A bad avenue can never succeed in generating an innovation. A good avenue generates an innovation for firm i according to a Poisson process with intensity equal to firm i's research intensity, $x_i(t)$.

Ex-ante, firms do not know the type of the research avenue; at time $t = 0$, they have a common prior belief $p(0)$ that the avenue is good. Both firms can perfectly observe each other's research intensity allocations and innovation arrivals. Hence, the firms continue to share a common posterior belief $p(t)$ at all t. Since only a good research avenue can generate an innovation, the arrival of the first innovation resolves all uncertainty about the avenue's type: it becomes common knowledge that the avenue is good and therefore the common posterior belief $p(t)$ jumps to 1.

Until a firm innovates, it obtains the flow payoff of 0. If a firm innovates first, as long as it remains the sole innovator, it receives a flow payoff of $\pi > 0$. Once the second innovation arrives, flow payoffs change. If the innovations come from different firms, the leader (which innovated first) gets $\lambda\pi$, and the follower gets $\phi\pi$. If both innovations are developed by the same firm, this firm obtains $(\lambda+\phi)\pi$. Parameters are such that

$$1 \geq \lambda \geq \phi \geq 0. \tag{1}$$

In Appendix B, we use a Stackelberg game with differentiated products to provide microfoundations for the assumptions that competition from the second innovation weakly lowers the payoff from the first innovation ($\lambda\pi \leq \pi$) and that the first innovation's payoff $\lambda\pi$ is weakly higher than the payoff from the second innovation $\phi\pi$.

Merged entity. — The merged entity has two units of research intensity, $X(t) = x_1(t) + x_2(t) \in [0,2]$, to devote to the avenue, with the flow cost of research equal to $cX(t)$. The flow payoff of the merged entity is 0 prior to the first innovation, π after the first innovation, and $(\lambda + \phi)\pi$ after the second innovation. In other words, the merged entity's payoff is simply the sum of the independent firms' payoffs.

Remark on the additive payoff assumption. — The assumption that the merged entity's payoff is simply the sum of the independent firms' payoffs implies that, keeping the dynamics of innovations the same, the merger changes neither the profitability of an innovation nor the cost of R&D. This assumption allows us to focus purely on incentives to innovate, in isolation from the standard unilateral effects of a merger.

In practice, the merged entity's payoff may be higher than the sum of the merging parties' payoffs because the merger may change the competitiveness of the relevant product markets.[17] The merger-induced change in the competitiveness of the product market, studied in Federico, Langus and Valletti (2017), Federico, Langus and Valletti (2018) and Hollenbeck (2020), has an ambiguous effect on consumer welfare. Higher prices hurt consumers after an innovation has been developed. However, the prospect of less intense price competition strengthens the appropriability effect, thus incentivizing R&D.[18] In Section V, we relax the assumption that the merger has no impact on prices.

Potentially, the merger may also bring synergy of resources, thus allowing the merged firm to reduce production or R&D costs.[19] The cost-reduction effect of the merger has been studied, for example, in Atallah (2016), Denicolò and Polo (2021) and Marshall and Parra (2023).

II. Analysis

We begin our analysis with the merged entity's optimization problem (Proposition 2) and then proceed to solve the competing firms' game (Proposition 4). There are two distinct stages in both the competing firms and the merged entity settings: stage 1, when no innovation has arrived yet; and stage 2, which begins after the first innovation has arrived. In each setting, we first analyze stage 2 and then proceed to stage 1.

A. Merged entity

STAGE 2: AFTER THE FIRST INNOVATION. — The merged entity decides to undertake research to get the second innovation only when the second innovation is profitable. The flow payoff with a single innovation is π, while after the second innovation, it is $(\lambda + \phi)\pi$. Thus, the second innovation is profitable if $\lambda + \phi > 1$.

Proposition 1 characterizes the optimal research decision of the merged entity and its payoff at the optimum. It states that, conditional on the second innovation being profitable, the merged entity undertakes research if and only if the flow cost c is sufficiently low.[20]

[17]In the Dow/DuPont decision, according to the EC (see para 2044 in the decision), "*following a merger, the merging parties coordinate the pricing of their products and thus a merger may increase the prices and profits of the merged entity. Less intense competition in the product market can increase the net revenues earned by a product line both when the firms innovate to improve the products in that line and when they do not.*"

[18]In Hollenbeck (2020), the trade-off between the negative effect of the merger on prices and its positive effect on innovation is resolved differently in the short-run and in the long-run. Specifically, mergers may harm consumers in the short run, while leading to higher long-run innovation.

[19]As noted in the Dow/DuPont decision (see para 3296), "efficiencies brought about by a merger may counteract [...] the potential harm to consumers that it might otherwise have."

[20]If the merged entity is indifferent between producing the second innovation and aborting research after the first innovation, we assume it chooses to continue research.

PROPOSITION 1: *If*

$$c \leq \frac{(\lambda + \phi - 1)\pi}{r}, \tag{2}$$

then the merged entity continues research at full intensity until the second innovation arrives; otherwise, it aborts research and the second innovation never arrives. The merged entity's expected payoff from the second stage is

$$V_M = \frac{\pi}{r} + \frac{2}{2+r} \max\left\{\frac{(\lambda + \phi - 1)\pi}{r} - c, 0\right\}. \tag{3}$$

PROOF:

See Appendix A.A1.

STAGE 1: BEFORE THE FIRST INNOVATION. — Prior to any innovation, the merged entity is uncertain about the type of research avenue. In the absence of innovation, the belief of the merged entity that the avenue is good, $p(t)$, evolves according to the law of motion derived from Bayes' rule:

$$p'(t) = -X(t)(1 - p(t))p(t). \tag{4}$$

Formula (4) implies that as the merged entity keeps undertaking research with positive intensity, it becomes progressively more pessimistic that the avenue is good — that is, $p(t)$ decreases.

At the optimum, if the belief drops below a certain threshold — which we refer to as a stopping threshold — the merged entity optimally aborts research and the game ends with no innovation. If the merged entity succeeds before the belief reaches the stopping threshold, $p(t)$ immediately jumps to 1 and the game proceeds to the second stage.

Proposition 2 derives the optimal stopping threshold and uses Proposition 1 to fully characterize the merged entity's uniquely optimal research strategy.

PROPOSITION 2:

1) If

$$\frac{\pi}{r} \leq c, \tag{5}$$

then the merged entity does not undertake research.

2) If

$$\frac{(\lambda + \phi - 1)\pi}{r} < c < \frac{\pi}{r}, \tag{6}$$

then the merged entity undertakes research at full intensity as long as its current belief $p(t)$ is above threshold

$$\check{p} = \frac{cr}{\pi}. \tag{7}$$

Once an innovation arrives or the belief falls to $\check{p}$, the merged entity completely aborts research efforts.

3) If

$$c \leq \frac{(\lambda + \phi - 1)\pi}{r}, \tag{8}$$

then the merged entity undertakes research at full intensity as long as its current belief $p(t)$ is above threshold

$$\check{p} = \frac{cr}{\pi} \frac{2 + r}{2(\lambda + \phi) + r - 2rc/\pi}. \tag{9}$$

Once the belief falls to $\check{p}$, the merged entity aborts research efforts. If an innovation arrives before that, the merged entity undertakes research at full intensity until the second innovation arrives.

PROOF:

See Appendix A.A2.

Given expression (3) for V_M, the stopping threshold, defined in (7) and in (9), is equal to c/V_M. This expression is intuitive because it equalizes the flow cost of research, c, with the expected benefit from the first innovation, pV_M. The difference in definitions (7) and (9) comes from the difference in the expression for V_M, which depends on whether there is research in stage 2.

B. Competing firms

STAGE 2: AFTER THE FIRST INNOVATION. — Both firms have a choice between continuing research and giving up. For the firm that did not innovate — the potential follower — the optimal decision depends on the cumulative payoff that the second innovation brings, $\phi\pi/r$. If it is large relative to the flow cost of research c, then it is optimal to undertake research until the second innovation arrives. For the firm that produced the first innovation — the leader — the optimal decision takes into account not only the cumulative payoff from the leader's second innovation but also the negative payoff externality that this innovation imposes on the first. This externality is equal to $(1 - \lambda)\pi/(1 + r)$, which is smaller than the analogous externality $(1 - \lambda)\pi/r$ for the merged entity. When the follower undertakes research towards to the second innovation, the leader deems its arrival inevitable, but it can be produced either by the leader today or by the follower at some future date. Hence, in contrast to the merged entity, by not introducing the second innovation the leader only delays its arrival. Proposition 3 summarizes the optimal research decision of the firms and their equilibrium payoffs.[21]

PROPOSITION 3:

1) If

$$\frac{\phi\pi}{r} < c, \tag{10}$$

then both firms abort research after the first innovation and the second innovation never arrives. The potential follower's and the leader's expected payoffs are

$$V_F = 0, \quad V_L = \frac{\pi}{r}. \tag{11}$$

2) If

$$\frac{\pi}{r}\left(\phi - \frac{(1-\lambda)r}{1+r}\right) < c \leq \frac{\phi\pi}{r}, \tag{12}$$

then the follower undertakes research at full intensity until the second innovation arrives, while the leader aborts research after producing the first innovation. The follower's and the leader's expected payoffs are

$$V_F = \frac{1}{1+r}\left(\frac{\phi\pi}{r} - c\right), \quad V_L = \frac{\lambda+r}{1+r}\frac{\pi}{r}. \tag{13}$$

3) If

$$c \leq \frac{\pi}{r}\left(\phi - \frac{(1-\lambda)r}{1+r}\right), \tag{14}$$

then both the leader and the potential follower undertake research at full intensity until the second innovation arrives. The potential follower's and the leader's expected payoffs are

(15)

$$V_F = \frac{1}{2+r}\left(\frac{\phi\pi}{r} - c\right), \quad V_L = \frac{\lambda+r}{1+r}\frac{\pi}{r} + \frac{1}{2+r}\left(\frac{\pi}{r}\left(\phi - \frac{(1-\lambda)r}{1+r}\right) - c\right).$$

[21] As for the merged entity, we assume that the firms choose to undertake research whenever indifferent.

PROOF:

See Appendix A.A3.

Comparing Proposition 3 and Proposition 1, we conclude that $V_L + V_F = V_M$ whenever in the second stage the combined research intensity of the competing firms is the same as the research intensity of the merged entity. This is a direct consequence of the additive payoff assumption. However, the conditions under which the aggregate research intensity in the second stage is maximal, (2) and (14), are different. In particular, condition (14) is weaker than condition (2), which means that the merger may block the second innovation. We will come back to this point in Section III.

Stage 1: Before the first innovation. — Prior to any innovation, the common belief that the avenue is good, $p(t)$, evolves according to (4), where $X = x_1 + x_2$.

We are looking for a symmetric Markov perfect equilibrium of the game, in which the firms use identical stationary Markovian strategies with belief p as the state variable. Proposition 4 proves that such an equilibrium is unique and provides the equilibrium characterization. When the potential follower undertakes some research towards the second innovation, the form of the equilibrium closely resembles that in Keller, Rady and Cripps (2005), in which there are no payoff externalities.

In the equilibrium, as in the merged entity setting, if the belief drops below a certain threshold — which we again refer to as a stopping threshold — the firms abort research and the game ends with no innovation.

Before the belief reaches the stopping threshold, the equilibrium research intensity depends on whether there is innovation in the second stage. If there is no research in the second stage, then, in the first stage, the firms undertake research at full intensity. If the potential follower continues research in the second stage, then, in the first stage, after reaching a certain belief threshold $\bar{p}$ above the stopping threshold $\underline{p}$, the firms gradually reduce their equilibrium research intensity from 1 to 0. As a result, their common belief $p(t)$ approaches but never reaches the stopping threshold $\underline{p}$.

PROPOSITION 4:

1) If

$$\frac{\pi}{r} \leq c, \tag{16}$$

then neither firm undertakes research.

2) If

$$\frac{\phi\pi}{r} < c < \frac{\pi}{r}, \tag{17}$$

then the firms undertake research at full intensity as long as their current belief $p(t)$ is above threshold

$$\hat{p} = \frac{cr}{\pi}. \tag{18}$$

Once an innovation arrives or the belief falls to $\hat{p}$, the firms completely abort research efforts.

3) *If*

$$c \leq \frac{\phi\pi}{r}, \tag{19}$$

then the firms undertake research at full intensity as long as their current belief $p(t)$ is above threshold $\bar{p}$. Once the belief falls to $\bar{p}$, the firms undertake research at intensity $x_1(t) = x_2(t) = x^*(p(t))$, defined on $p \in (\underline{p}, \bar{p})$ as

$$x^*(p) = \frac{\dfrac{1}{\underline{p}} - \dfrac{1}{p} - \dfrac{1-p}{p} \ln \dfrac{p(1-\underline{p})}{(1-p)\underline{p}}}{\dfrac{1}{r}\left(\dfrac{1}{p} - \dfrac{1}{\bar{p}}\right) + \dfrac{1}{\underline{p}} - \dfrac{1}{\bar{p}} - \dfrac{1-\bar{p}}{\bar{p}} \ln \dfrac{\bar{p}(1-\underline{p})}{(1-\bar{p})\underline{p}}}. \tag{20}$$

This intensity decreases over time, from 1 at $p(t) = \bar{p}$ to 0 at $t \to +\infty$. In the absence of innovation, the belief approaches threshold $\underline{p}$ at $t \to +\infty$.

a) *If (12) holds, then once an innovation arrives, only the firm that did not innovate undertakes research at full intensity until the second innovation arrives. Threshold $\underline{p}$ is defined as*

$$\underline{p} = \frac{cr}{\pi} \frac{1+r}{\lambda + r}. \tag{21}$$

Threshold $\bar{p} \in [\underline{p}, 1)$ is defined as a unique solution to

$$\frac{1}{\underline{p}} - \frac{1}{\bar{p}} - \frac{1-\bar{p}}{\bar{p}} \frac{r}{1+r} \ln \frac{\bar{p}(1-\underline{p})}{(1-\bar{p})\underline{p}} = \frac{\phi\pi - cr}{cr(1+r)^2}. \tag{22}$$

b) *If (14) holds, then once an innovation arrives, both firms undertake research at full intensity until the second innovation arrives. Threshold $\underline{p}$ is defined as*

$$\underline{p} = \frac{cr}{\pi} \left(\frac{\lambda + r}{1+r} + \frac{1}{2+r} \left(\phi - \frac{(1-\lambda)r}{1+r} - \frac{cr}{\pi} \right) \right)^{-1}. \tag{23}$$

Threshold $\bar{p} \in [\underline{p}, 1)$ *is defined as a unique solution to*

$$\frac{1}{\underline{p}} - \frac{1}{\bar{p}} - \frac{1-\bar{p}}{\bar{p}} \frac{r}{1+r} \ln \frac{\bar{p}(1-\underline{p})}{(1-\bar{p})\underline{p}} = \frac{\phi\pi - cr}{cr(1+r)(2+r)}. \tag{24}$$

PROOF:

See Appendix A.A4.

As in the merged entity setting, the stopping thresholds $\hat{p}$, defined in (18), and $\underline{p}$, defined in (21) and (23), equalize the flow cost of research, c, with the expected benefit from the first innovation, $\hat{p}V_L$ and $\underline{p}V_L$. The difference in the definitions of $\hat{p}$ and $\underline{p}$ comes from the difference in the expression for V_L, which depends on the firms' research decisions in stage 2.

In the region $(\underline{p}, \bar{p})$, where the equilibrium research intensity is strictly between 0 and 1, the firms must be indifferent to undertaking research. Intuitively, the indifference region $(\underline{p}, \bar{p})$ appears because of the free-riding effect: once one firm innovates, the other firm learns that the research avenue is good, thus free-riding on past research efforts of the innovator. Then, during the first stage, in the indifference region, if a competitor undertakes research at full intensity, each firm's best response is to pause its own research efforts and see whether the competitor's research efforts will be successful. Such a free-riding effect was first identified in the Poisson environment by Keller, Rady and Cripps (2005).

In contrast to the competing firms environment in Proposition 4, the optimal research intensity of the merged entity in Proposition 2 is always either 0 or 2 (full intensity) because the merger eliminates the free-riding problem. We will come back to this point in Section III.

III. Main result

By comparing the merged entity and the competing firms' setups, we conclude that the merger has three effects. First, the merged entity in some instances blocks the second innovation that the competing firms would have delivered (*cannibalization effect*). Second, the merger may increase the likelihood of the first innovation (*appropriability effect*), and, third, the merger may bring the first innovation forward in time (*informational effect*).

Cannibalization effect. — The merged entity faces reduced incentives to produce the second innovation because it internalizes negative payoff externalities that the competing firms impose on each other. In particular, in contrast to the competing firms, the merged entity takes into account that the competition from the second innovation reduces, or *cannibalizes*, the profit from the first innovation, π, to $\lambda\pi$. We refer to this as the cannibalization effect.

Appropriability effect. — The flip side of the cannibalization effect is that, for the merged entity, the marginal benefit of undertaking R&D to produce the first innovation is higher. Indeed, due to reduced competition from the second innovation, the first innovation generates higher expected profit, which incentivizes the merged entity to undertake more research to produce the first innovation. Moreover, even if both the competing firms and the merged entity produce the second innovation, the merged entity still has higher incentives to undertake research on the first innovation because, in contrast to the leader of the competing firms, it always reaps the benefit not only from the first innovation, but also from the second innovation. As a result, the merger increases the likelihood that the first innovation will be produced. The positive effect of the merger on this likelihood occurs because the merged entity fully *appropriates* the benefits of its own R&D efforts — that is why we refer to this effect as the appropriability effect.

Informational effect. — The merged entity also internalizes positive informational externalities that the competing firms impose on each other. In particular, an innovation by the leader immediately informs the competitor that the research avenue is good and, thus, is capable of generating another innovation. Hence, before the arrival of the first innovation, the competing firms have incentives to reduce their own research intensity in the hope of free-riding on the opponent's R&D efforts. In contrast, the merged entity cannot fall back on somebody else's efforts to discover whether an innovation is feasible and so undertakes research at higher intensity, which brings the first innovation forward in time. We refer to the positive effect of the merger on the timing of the first innovation as the informational effect because it is connected to *informational* externalities.

Theorems 1, 2 and 3 compare the results of Propositions 2 and 4 in terms of the number and timing of innovations, thus deriving the parameter regions in which the cannibalization, appropriability and informational effects are present. Figure 1 visualizes these regions.

THEOREM 1 (The merger has no impact on innovations): *If*

$$\frac{\phi\pi}{r} < c, \tag{25}$$

then the merger has no impact on the number and timing of innovations.

Theorem 1 corresponds to the blue region in Figure 1 and covers two subregions:

1) $\pi/r \leq c$ (case 1 in Proposition 2 and case 1 in Proposition 4): Neither the competing firms nor the merged entity undertake any research, and so there are no innovations in either setting.

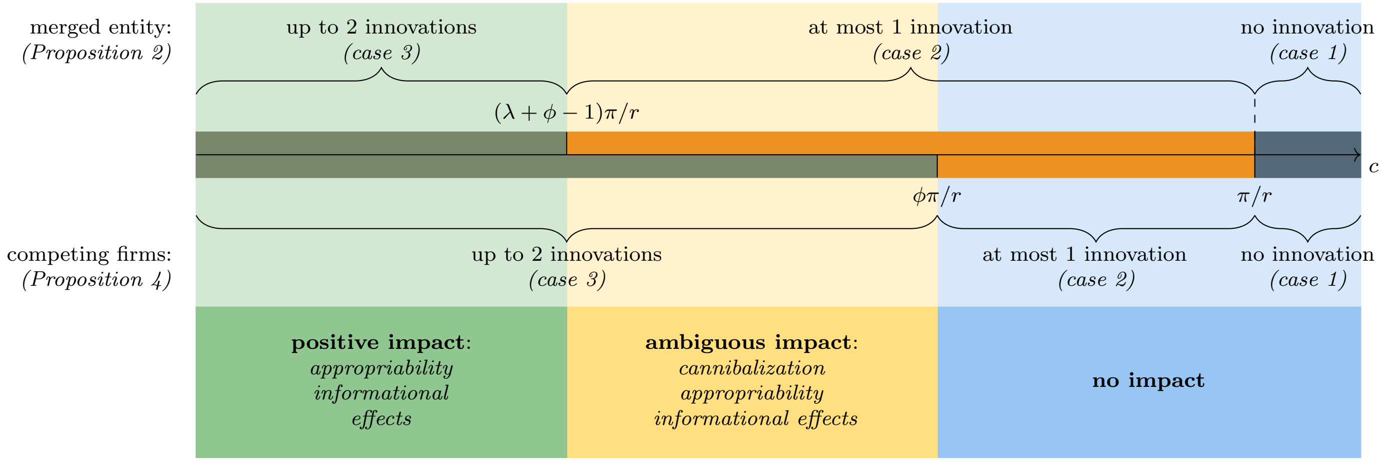


FIGURE 1. THE NUMBER OF INNOVATIONS IN EACH SETTING AND THE EFFECTS OF THE MERGER. THE CASE NUMBERS IN THE BRACKETS CORRESPOND TO CASES IN PROPOSITION 2 FOR THE MERGED ENTITY SETTING AND TO CASES IN PROPOSITION 4 FOR THE COMPETING FIRMS SETTING.

2) $\phi\pi/r < c < \pi/r$ (case 2 in Proposition 2 and case 2 in Proposition 4): In both settings, the cumulative research intensity is 2 until an innovation arrives or until the belief reaches threshold $\hat{p} = \check{p} = cr/\pi$, and there is no research after either of the events.

In sum, if (25) holds, then neither the merged entity nor the competing firms have incentives to work on the second innovation after the first innovation arrives. The merger has no impact because, intuitively, in the absence of the second innovation, there are no payoff externalities, and informational externalities are irrelevant.

THEOREM 2 (The merger has a positive impact on innovations): *If*

$$c < \frac{(\lambda+\phi-1)\pi}{r}, \tag{26}$$

then the merger has an unambiguously positive impact: while it does not block the second innovation, it increases the probability that the first innovation arrives and brings it forward in time.

Theorem 2 corresponds to the green region in Figure 1 and follows from case 3 in Proposition 2 and case 3b in Proposition 4. Case 3a in Proposition 4 is not relevant because, as we discussed after Proposition 3, condition (14) is weaker than condition (26). Both the merged entity and the competing firms have incentives to work on the second innovation after the first innovation arrives. Before the first innovation arrives, the competing firms undertake research at full intensity until

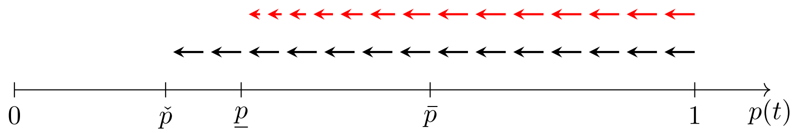


Figure 2. Phase diagram of belief updating (4), assuming $c < \phi\pi/r$. The black arrows indicate the belief trajectory of the merged entity. The red arrows indicate the belief trajectory of the competing firms.

belief threshold $\bar{p}$ and then gradually lower their research intensity, with belief approaching threshold $\underline{p}$. In contrast, the merged entity undertakes research at full intensity until belief threshold $\check{p}$, which by Lemma 1 is lower than $\underline{p}$ (see Figure 2). Indeed, as we discussed in Section II, $\underline{p} = c/V_L$ and $\check{p} = c/V_M$, and so, since $V_M = V_L + V_F$ and $V_F > 0$, $\check{p}$ is less than $\underline{p}$.

LEMMA 1: *Under condition (26), threshold $\check{p}$, defined in (9), is strictly lower than threshold $\underline{p}$, defined in (23).*

PROOF:

See Appendix A.A5.

Thus, the merger positively affects the arrival of the first innovation in two ways.

First, since $\check{p} < \underline{p}$, the competing firms undertake research over a shorter interval of beliefs. By Lemma 2 below, $\check{p} < \underline{p}$ implies that the merger increases the probability of the first innovation — that is, the appropriability effect is at work. Intuitively, a shorter interval of beliefs at which the competing firms undertake some research means that their cumulative investment in research is lower, and so they produce the first innovation with a lower probability.

LEMMA 2: *For both the merged entity and the competing firms, given a prior belief p_0 and a stopping threshold p_S where all research activity ceases, the probability of the first innovation is equal to*

$$1 - \frac{1/p_0 - 1}{1/p_S - 1}, \tag{27}$$

which is decreasing in p_S.

PROOF:

See Appendix A.A6.

Second, the merger increases the research intensity on the belief interval $(\underline{p}, \bar{p})$. Hence, conditional on the first innovation arriving on this belief interval, this innovation takes less time to arrive when the firms are merged — that is, the informational effect is at work. Intuitively, the competing firms produce the second innovation, which triggers free-riding prior to the arrival of the first innovation, thus leading to the informational effect of the merger.

THEOREM 3 (The merger has an ambiguous impact on innovations): *If*

$$\frac{(\lambda+\phi-1)\pi}{r} < c < \frac{\phi\pi}{r}, \tag{28}$$

then the merger blocks the second innovation but increases the probability that the first innovation arrives and brings it forward in time.

Theorem 3 corresponds to the yellow region in Figure 1 and follows from case 2 in Proposition 2 and case 3 in Proposition 4. According to Theorem 3, the merger has three effects.

First, the competing firms have incentives to work on the second innovation after the first innovation arrives, while the merged entity does not. Thus, the merger blocks the second innovation — that is, the cannibalization effect is at work. In practice, the cannibalization effect can manifest itself in both killer and reverse killer acquisitions — phenomena that have attracted considerable attention from industry practitioners. For example, competition authorities raised both such concerns in relation to the Adobe/Figma merger.[22] Our model indicates that these concerns are justified in the yellow region of Figure 1 characterized by condition (28). According to this condition, developing a follow-up innovation — potentially competing with either Adobe's or Figma's existing product — is sufficiently profitable relative to R&D costs if the firms remain independent. At the same time, condition (28) also requires that the follow-up innovation is not attractive for the merged entity to pursue. This is the case, for example, when competition from the follow-up innovation is so intense that developing it is unprofitable for the merged entity even when research is costless — that is, if $\lambda + \phi < 1$.

Second, since the competing firms produce the second innovation while the merged entity does not, the profitability of the first innovation is higher for the merged entity, and, thus, the merged entity produces the first innovation with a higher probability than the competing firms — that is, the appropriability effect is at work. Formally, the merger increases the probability of the first innovation because, as in Theorem 2, by Lemma 3 the stopping threshold of the merged entity is lower than the stopping threshold of the competing firms, $\check{p} < \underline{p}$.

LEMMA 3: *Under condition* (28)*, threshold* $\check{p}$*, defined in* (7)*, is strictly lower than threshold* $\underline{p}$*, defined in* (21) *and* (23)*.*

[22] A killer acquisition concern was that the merger would halt Figma's efforts to develop competing image editing and illustration software; a reverse killer acquisition concern — the merger would lead Adobe to abandon the development of a new product design software that could have competed closely with Figma's existing offering.

PROOF:

See Appendix A.A7.

Finally, the competing firms do not always undertake research at full intensity, as illustrated in Figure 2, and so the merger brings the first innovation forward in time — that is, the informational effect is also at work.

IV. Policy implications

A. The impact of the prior

In this section, we investigate how the desirability of the merger varies with the prior belief that the research avenue is good. While the cannibalization effect is independent of the prior, Theorem 4 states that both the appropriability and the informational effects decrease with the prior. Intuitively, if the initial belief is very high, the first innovation is likely to arrive before the posterior belief reaches threshold $\bar{p}$ (see Figure 2) — that is, before the moment when the difference in the merged entity's and the competing firms' strategies starts to matter. This means that, for high prior, the competing firms and the merged entity are likely to produce the first innovation at the same time. Hence, when firms are more optimistic about the prospect of the game-changer innovation — that is, there is less uncertainty about the feasibility of innovation — the merger becomes less desirable.

THEOREM 4: *Suppose that $c < \phi\pi/r$. Then, the strength of the appropriability effect — that is, the merger-induced increase in the probability of the first innovation — and the strength of the informational effect — that is, the merger-induced decrease in the arrival time of the first innovation*[23] *— both decrease in the prior belief p_0.*

PROOF:

See Appendix A.A9.

Our theoretical results imply that competition authorities should consider the degree of uncertainty surrounding breakthrough innovation when assessing mergers — especially in rapidly evolving sectors such as digital markets. Mergers may be more permissible when innovation is highly speculative and the feasibility of a game-changer product remains unclear. However, once credible innovation paths emerge, mergers become more problematic.

For instance, in the early 2000s, core digital technologies such as mobile operating systems and cloud computing were still emerging, and their commercial potential was far from guaranteed. Google's 2005 acquisition of Android was not seen as anti-competitive at the time, as the future of smartphones remained uncertain. Similarly, the 2008 Google/DoubleClick merger was approved despite

[23] We provide a more precise definition of the strength of the informational effect in Appendix A.A8.

concerns, partly because the digital advertising landscape was still taking shape.[24] Today, by contrast, markets such as AI and productivity tools have matured significantly, and innovation is likely to be more incremental than transformative. In this context, mergers like Adobe/Figma raise greater concerns, as they risk foreclosing potential competition.

To address this, regulators could incorporate an *Innovation Uncertainty Test* into merger review frameworks. This would involve assessing market signals — such as venture capital activity, open-source development, prototype products and market validation — to determine whether innovative potential remains speculative or has entered a commercially credible phase. Mergers involving firms on a clear innovation trajectory should be subject to heightened scrutiny.

B. The impact of innovation substitutability

In this section, we show that the decision of a competition authority to block the merger may be non-monotonic in the degree of substitutability between innovations. This result may seem surprising, as traditional merger analysis typically treats mergers between close competitors with caution. The difference lies in our focus. While traditional analysis emphasizes price effects — the increase in price following a merger is likely to be more pronounced when the merging firms' products are closer substitutes, — we concentrate on the impact on innovation. In Section V, we explore the interaction between the merger's effects on price and innovation.

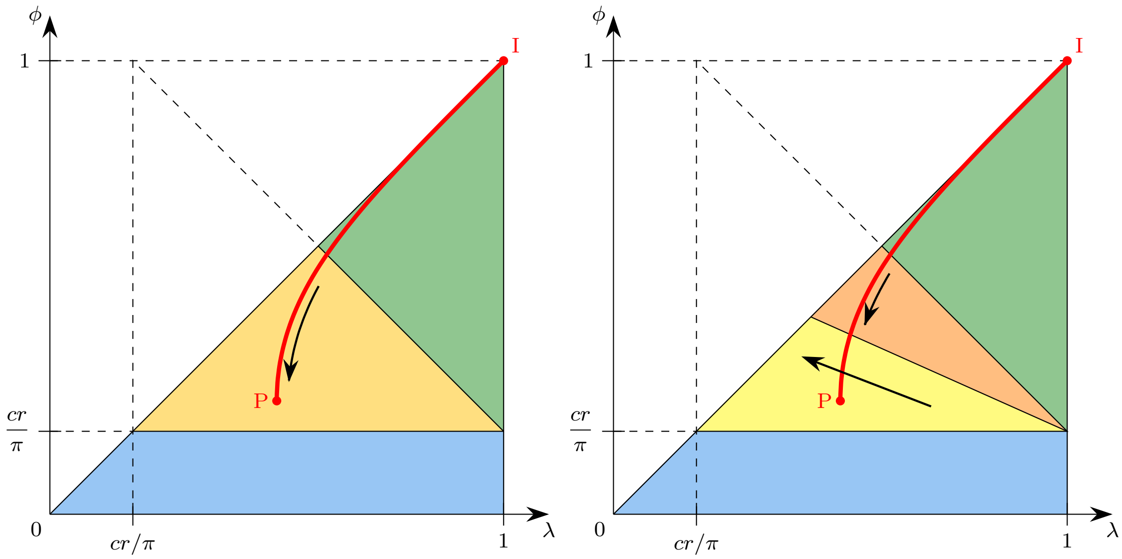
In our model, the degree of substitutability between innovations is captured by λ and ϕ. Intuitively, as innovations become closer substitutes, both ϕ and λ decrease due to more intense competition between innovations. However, due to the first-mover advantage, λ is likely to decrease at a lower rate than ϕ. These two properties shape the red curve in Figure 3, which illustrates one possible parametrization of λ and ϕ through the degree of substitutability between the products and is based on the Stackelberg competition model with differentiated products — see (B.5) in Appendix B.

In addition to the red curve, Figure 3A visualizes on the λ/ϕ plane the regions from Figure 1, in which the merger has positive, ambiguous or no impact on innovation. Since $\lambda > \phi$, the relevant part of the plane is the lower triangle.

Figure 3A allows to trace the impact of substitutability on the desirability of the merger in our model. Starting from the point $\lambda = \phi = 1$, at which consumers view the innovations as independent products, as the degree of demand substitutability increases, the feasible payoffs move down along the red curve, first passing through the green parameter region where the merger is desirable.[25]

[24]For the account of Google's milestone acquisitions, see, for example, https://www.latimes.com/world-nation/story/2020-10-20/how-google-evolved-from-cuddly-startup-to-antitrust-target.

[25]As we show in Online Appendix D, in the green parameter region, as we move down along the red curve, the change in the strength of the appropriability effect is ambiguous — and so it is ambiguous whether the merger becomes more or less desirable as innovations become closer substitutes.



(A) The arrow indicates the direction in which the appropriability effect strengthens.

(B) The arrows indicate the direction in which the informational effect strengthens.

Figure 3. The blue region corresponds to (25). The green region corresponds to (26). The yellow region in panel (A) corresponds to (28). Panel (B) splits this region into two: the bright yellow region corresponds to (29), the orange region corresponds to (30). For payoff parametrization from Appendix B, the red curve traces the feasible payoff pairs $(\lambda(\theta), \phi(\theta))$ as the degree of innovation substitutability θ varies from $\theta = 0$ (independent products, point I) to $\theta = 1$ (perfect substitutes, point P).

As the substitutability of innovations increases further, the payoffs move into the yellow region. At the boundary between the green and the yellow regions, there is a discontinuous drop in the desirability of the merger because in the yellow region, the merger blocks the second innovation. As the substitutability increases further, the payoffs move deeper into the yellow region, where the competition authority needs to trade off the loss of the second innovation, due to the cannibalization effect, against a higher probability and speed of arrival of the first innovation due to the appropriability and informational effects, respectively.

According to Theorem 5 below, in the yellow region, the appropriability effect becomes stronger — that is, the merger brings about an ever higher increase in the probability of the first innovation — as the payoffs move down along the red curve. Consequently, the merger becomes more desirable as substitutability increases. Hence, the decision of the competition authority to block the merger may be non-monotonic in the degree of substitutability between innovations: it may be beneficial to allow the merger when innovations are either very close or very distant substitutes, but to block the merger when innovations are moderate substitutes — that is, when the firms' payoffs lie in the yellow region (where the merger blocks the second innovation) but are close to the boundary with the green region (where the merger does not significantly increase the probability of the first innovation).

The above policy implication relies on the assumption that the competition authority deems whether the first innovation occurs (the appropriability effect) more important than when it occurs (the informational effect). Putting significant weight on the informational effect complicates matters. According to Theorem 5, in the part of the yellow region described in case (A) — that is, the subregion colored bright yellow in Figure 3B — the strength of the informational effect increases in ϕ and decreases in λ. Hence, the change in the strength of the informational effect is ambiguous as the degree of substitutability increases. In particular, as we move down along the red curve, the strength of the informational effect may decrease, counteracting the increase in the appropriability effect, in which case the assessment of whether the merger is more beneficial when innovations are closer substitutes depends on the priorities of the competition authority.

THEOREM 5:

(A) Suppose that

$$\frac{\pi}{r}\left(\phi - \frac{(1-\lambda)r}{1+r}\right) < c \leq \frac{\phi\pi}{r}. \tag{29}$$

Then, the strength of the appropriability effect decreases in λ and does not depend on ϕ. The strength of the informational effect decreases in λ and increases in ϕ.

(B) Suppose that

$$\frac{(\lambda+\phi-1)\pi}{r} < c \leq \frac{\pi}{r}\left(\phi - \frac{(1-\lambda)r}{1+r}\right). \tag{30}$$

Then, the strength of the appropriability effect decreases in λ and in ϕ. The strength of the informational effect decreases in λ and in ϕ.

PROOF:

See Appendix A.A10.

Theorem 5 presents the comparative statics for the appropriability and informational effects with respect to λ and ϕ in the yellow region of Figure 3A. Cases (A) and (B) split this yellow region into two subregions, depicted in Figure 3B in bright yellow (case (A)) and in orange (case (B)). The difference between the cases is due to the leader's behavior after the first innovation. While in case (A), the leader does not attempt to produce the second innovation, in case (B) the leader continues research after producing the first innovation.

Consider case (A). Theorem 5 states that the strength of the appropriability effect does not depend on ϕ and decreases in λ. Intuitively, the incentives of the merged entity are unaffected by ϕ and λ, as the merged entity does not produce the second innovation. In contrast, the competing firms produce the second innovation. For them, the expected payoff from innovating first increases in λ because a higher λ lowers the negative payoff externality that competition from the second innovation imposes on the first innovation. Hence, the increase in the probability of the first innovation, which the merger induces, decreases in λ. At the same time, since the leader does not attempt to produce the second innovation, the expected payoff from innovating first does not depend on ϕ, and so the merger-induced increase in the probability of the first innovation does not depend on ϕ.

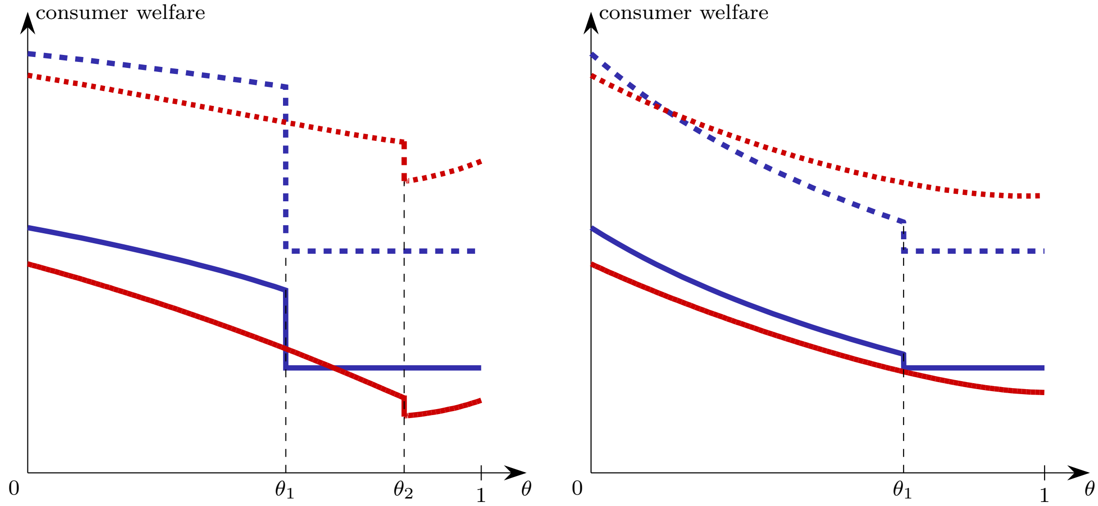
In addition, according to case (A) in Theorem 5, the strength of the informational effect of the merger increases in ϕ and decreases in λ. Intuitively, by increasing the expected payoff from innovating first, a higher λ mitigates the free-riding incentives of the competing firms, thus weakening the informational effect. The impact of ϕ is positive because, by increasing the payoff of the follower, a higher ϕ aggravates the free-riding problem of the competing firms, thus strengthening the informational effect.

When the leader continues research after the first innovation — that is, in case (B) — the comparative statics with respect to ϕ changes. Intuitively, since the second innovation may come from the leader, a higher ϕ increases the expected payoff from innovating first, thus affecting the appropriability and informational effects in the same direction as a higher λ — that is, weakening both effects. At the same time, as in case (A), by increasing the payoff of the follower, a higher ϕ continues to strengthen the informational effect. However, as we show in the proof (see (A.54)), for the informational effect, the former force dominates the latter. Thus, the impact of a higher ϕ on both effects is negative.

C. *Overall consumer surplus: example*

So far, we have examined how each of the appropriability, cannibalization, and informational effects individually influences the number and timing of innovations. Competition authorities, however, would look at these effects in combination and would typically adopt the consumer welfare standard in their assessments. The combined impact of the three effects on consumer welfare is highly sensitive to the specific structure of the utility function and the underlying market environment. In this section, we analyze how these effects interact under the assumptions that a representative consumer has quadratic utility, and that the first and second innovations compete with one another in Stackelberg fashion. We derive consumer welfare under these assumptions in Online Appendix E and illustrate it in Figure 4A.

There are two critical points in Figure 4A. As θ increases to point θ_1, the cannibalization effect becomes active and consumer welfare in the merged entity



(A) Condition (26) holds for all $\theta \in (0, \theta_1)$; condition (30) holds for all $\theta \in (\theta_1, \theta_2)$; condition (29) holds for all $\theta \in (\theta_2, 1)$.

(B) Regions $(0, \theta_1)$ and $(\theta_1, 1)$ corresponds to conditions (26) and (30), respectively, with ϕ replaced by $\Gamma - \lambda$.

Figure 4. Consumer welfare without price effects (panel (A)) and with price effects (panel (B)). The horizontal axis represents the degree of substitutability θ, which affects parameters λ, ϕ and Γ. The red curves correspond to the case where the firms compete, and the blue curves to the case where they are merged. Parameters: $c/\pi = 0.23$, $r = 0.8$; prior belief is $p_0 = 0.3$ for solid curves and $p_0 = 0.45$ for dashed curves.

setting drops abruptly.[26] As θ increases further to point θ_2, the leader no longer undertakes research after producing the first innovation. As a result, aggregate R&D intensity toward the second innovation falls, delaying its arrival and leading to an abrupt decline in consumer welfare in the competing firms setting.

In line with our discussion in Section IV.B, the impact of the merger on consumer welfare is non-monotone in the degree of substitutability θ. In Figure 4A, such non-monotonicity leads to non-monotonicy in the merger approval decision when prior belief is low. Specifically, the merger increases consumer welfare — and so should be approved — for low and high θ (the solid blue curve lies above the solid red curve) but leads to consumer detriment — and so should be blocked — for intermediate θ. This implies that the traditional approach, which presumes that the closer the competitors, the greater the concern about the merger, may be misplaced.

[26]Whenever the cannibalization effect is active — that is, for $\theta > \theta_1$ — its impact on consumer welfare varies with θ. An increase in θ may weaken the impact of the cannibalization effect because the second innovation brings a smaller utility uplift if it is a closer substitute to the existing product. At the same time, an increase in θ may strengthen the impact of the cannibalization effect because when innovations are closer substitutes, consumers may face a sharper decline in prices upon the arrival of the second innovation.

The prior belief has two effects on consumer welfare. First, regardless of the market structure, consumer welfare is higher when the probability that the avenue is good is higher. Furthermore, comparing solid and dashed curves in Figure 4A, we observe that — consistent with our discussion in Section IV.A — the merger is less desirable when prior belief is higher. Moreover, a higher prior may eliminate the non-monotonicity in the merger approval decision — as shown in Figure 4A, when the prior is higher, the merger increases consumer welfare only for low values of θ.

V. Price effects

So far, by assuming that the merged entity's payoff equals the sum of the independent firms' payoffs, we have abstracted from the standard market power effect of a merger. While this simplification helps to isolate innovation incentives from the traditional price effects of a merger, in most real-world markets, price effects matter. As Commissioner Vestager puts it, competition ensures that "consumers get innovative products at the *right* prices." (emphasis added).[27] In this section, we incorporate these effects into our model, bringing it closer to reality and offering insights into the interplay between price effects and R&D incentives.

To capture the price effects of the merger in a tractable way, we introduce a reduced-form specification that reflects the additional market power typically associated with common ownership of multiple products. We assume that once both innovations are introduced, the merged entity earns a flow payoff of $\Gamma\pi$, where

$$\lambda + \phi \leq \Gamma \leq 2. \tag{31}$$

In the competing firms setting, we apply the same logic: if both innovations are developed by the same firm, that firm also receives a flow payoff of $\Gamma\pi$. The payoff multiple Γ exceeds $\lambda + \phi$ when the common ownership of both innovations confers market power, enabling the owner to raise prices for consumers.[28] Our benchmark model corresponds to the special case $\Gamma = \lambda + \phi$. The assumption $\Gamma \leq 2$ states that the payoff from the common ownership of two innovations, $\Gamma\pi$, cannot exceed the payoff from two independent innovations, 2π.

We provide the full analysis of the extended model in Online Appendix F. The main takeaway from our analysis is that the price effects of the merger reinforce the merger's positive impact on the number of innovations via the appropriability and cannibalization effects. Intuitively, the ability to charge higher prices provides the merged entity with additional incentives to pursue both the first and

[27] Commissioner Vestager, "Big Data and Competition," speech at the EDPS-BEUC Conference on Big Data, Brussels, 29 September 2016. https://ec.europa.eu/commission/presscorner/detail/ov/speech_16_5224

[28] The payoff multiple Γ may exceed $\lambda+\phi$ not only due to price effects but also if the common ownership of both innovations creates efficiencies and leads to a reduction in production costs.

the second innovations. Stronger incentives for the first innovation amplify the appropriability effect, while enhanced incentives for the second help mitigate the cannibalization effect. Hence, incorporating price effects may make the merger even more desirable in terms of its impact on the number of innovations.[29]

More formally, if $\Gamma - 1 < \phi$, then all our results remain qualitatively unchanged. In particular, Figure 1, which summarizes the impact of the merger, remains essentially unchanged, except that the sum $\lambda + \phi$ in threshold $(\lambda + \phi - 1)\pi/r$ is replaced with Γ. Hence, since $\Gamma \geq \lambda + \phi$, the cost region in which the merger has a positive impact becomes larger, while the region with the cannibalization effect shrinks — i.e., the cannibalization effect weakens.

To conduct welfare analysis, we impose the same assumptions as made in Section IV.C — specifically, quadratic utility and Stackelberg competition between rival firms. Under these assumptions, condition $\Gamma - 1 < \phi$ holds for any θ. Figure 4B depicts the resulting consumer welfare for specific parameter values and mirrors Figure 4A, which depicts consumer welfare for the same parameter values but without price effects. Figure 4B indicates that incorporating price effects does not overturn our finding that the desirability of the merger may be non-monotone in the degree of substitutability between the innovations. In particular, as the innovations become closer substitutes, the merger may become more desirable, even though the price effects become more pronounced.[30] Furthermore, with price effects, a merger may be desirable for all levels of substitutability when prior belief is sufficiently low.[31]

If $\Gamma - 1 > \phi$, our results change in three respects: the cannibalization effect disappears, the merger may bring the second innovation forward in time and the appropriability effect can now operate alone, without the informational effect.

Condition $\Gamma - 1 > \phi$ means that the second innovation is more profitable to the merged entity than to the potential follower. Thus, now, in contrast to the main model, whenever the potential follower finds it profitable to pursue the second innovation, so does the merged entity. As a result, there is no cannibalization effect and the cost region where the merger has ambiguous impact disappears, as visualized in Figure 5.

[29]The same incentives are also present in the competing firms' setting as the leader of the competing firms may end up producing both innovations. The ability to charge higher prices under common ownership of two innovations makes the leader's role more attractive, thereby straightening the incentives to produce the first innovation. Hence, with price effects, the probability that the competing firms produce the first innovation is higher and the free-riding incentives are weaker. These considerations regarding the competing firms' incentives reduce the desirability of the merger. Nevertheless, whenever the merged entity chooses to develop both innovations, the price effects enhance its incentives to produce the first innovation even more than they do for the competing firms. As a result, with price effects, the appropriability effect of the merger becomes stronger.

[30]The price effects are measured by the difference $\Gamma - (\lambda + \phi)$. As shown in Online Appendix F, this difference is increasing in θ.

[31]When accounting simultaneously for price and innovation effects, Federico, Langus and Valletti (2018) and Bourreau, Jullien and Lefouili (2024) find that mergers are unlikely to benefit consumers absent synergies or spillovers. These are static models where all innovations are simultaneous and there is no learning, so our appropriability and informational effects — which require sequential innovations and learning — are not present.

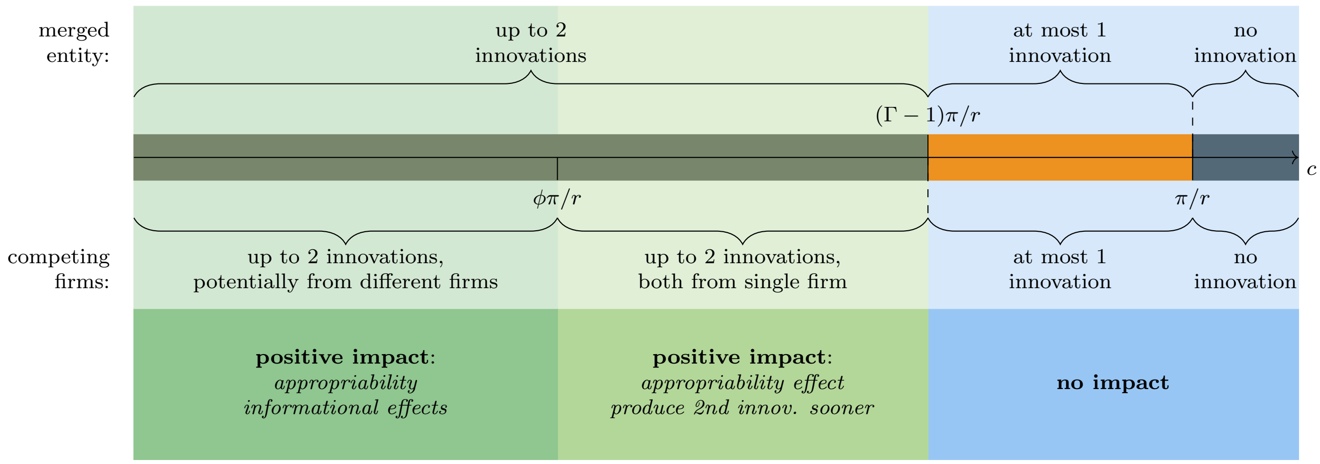


FIGURE 5. THE NUMBER OF INNOVATIONS IN EACH SETTING AND THE EFFECTS OF THE MERGER IF $\Gamma - 1 > \phi$.

Condition $\Gamma - 1 > \phi$ also means that the additional profit from common ownership of both innovations, $\Gamma - \lambda - \phi$, exceeds $1 - \lambda$, the negative externality that the second innovation imposes on the first innovation. Thus, now, in contrast to the main model, upon the arrival of the first innovation, the leader has stronger incentives to pursue the second innovation than its competitor. As a result, there is now a cost region where only the leader and not the follower has incentives to pursue the second innovation:

$$\frac{\phi\pi}{r} < c < \frac{(\Gamma - 1)\pi}{r} \tag{32}$$

In this region, the leader's payoff from the second innovation is the same as that of the merged entity, and so, if the leader finds it optimal to pursue the second innovation, so does the merged entity. However, the merged entity has more resources to devote to R&D — the leader has only one unit of research intensity, but the merged entity has two. Consequently, the competing firms take longer than the merged entity to produce the second innovation — that is, the merger brings the second innovation forward in time.

Finally, in region (32), the discovery that the research avenue is good is of no use to the potential follower, who finds it suboptimal to continue research after the first innovation arrives. Consequently, there is no free-riding effect. At the same time, there is still the appropriability effect. Since the leader has less R&D resources than the merged entity, it obtains a lower expected payoff from the second stage. Hence, in the first stage, the competing firms abort research at higher posterior beliefs than the merged entity, which reduces the probability of the first innovation.

Overall, as Figure 5 illustrates, when condition $\Gamma - 1 > \phi$ holds, the merger either has no impact or unambiguously increases the number of innovations as well as speeds up the arrival of either the first or the second innovation. Furthermore, in region (32), the merger does not harm consumers through higher prices either — upon the arrival of the first innovation, common ownership of the two innovations and the associated monopoly pricing are inevitable, irrespective of the market structure. In other words, in region (32), the firms compete *for* the market (comprising of two innovations), but not *on* the market once it is established. Competition for the market is detrimental for consumers — the merged entity establishes the new market with higher probability. Hence, from ex ante perspective the merger is desirable. However, ex post competition authorities may still want to intervene to promote competition on the market, instead of tolerating perpetual monopoly, but these considerations take us outside of our model.

VI. Conclusion

The Dow/DuPont merger gave rise to a flurry of academic papers that identify various positive and negative effects of mergers on innovation. Despite this heightened interest, the literature failed to come to a consensus regarding a presumed effect of mergers on innovation (see, Jullien and Lefouili (2018)). We contribute to this growing literature by modeling the R&D process in continuous time and allowing for multiple sequential innovations. The dynamic nature of our model allows us to gain new insight into the cannibalization/appropriability trade-off and capture the novel informational effect.

Our model provides clear guidance for competition authorities on how to approach merger review in innovation-driven sectors. First, they should consider the degree of uncertainty surrounding future innovation: in nascent markets, where uncertainty is high, mergers may enhance incentives to pursue breakthrough product innovations and accelerate their arrival. Second, authorities should consider the substitutability between potential innovations — a traditional piece of analysis in any merger review. According to our model, the impact of substitutability on the desirability of a merger is non-monotone — mergers may be more desirable when innovations are closer substitutes. Finally, because the merger is always (weakly) profitable for the firms in our model, a mere notification of the intention to merge does not reveal any information relevant for assessing the merger.

We assume that prior to the merger, there are only two firms, and no other firms undertake R&D along the research avenue. Introducing competitors would complicate the analysis without qualitatively affecting conclusions. With more firms, the effects that we identify in our reduced-form model would be present, though possibly weaker.

In the competing firms setting, our solution relies heavily on the assumption that each firm observes its rival's research intensities. Bonatti and Hörner (2011) considers a model of strategic experimentation in teams in which individual team members' learning intensity is hidden. Marlats and Ménager (2021) allow players to choose whether, at a cost, to observe the other player's experimentation effort and outcomes. Based on the findings in these papers, we conjecture that relaxing the assumption of observable research intensities mitigates the free-riding problem but does not completely eliminate it. Thus, the informational effect of the merger would still be present, albeit weaker.

We assume that the R&D activity never reveals that the research avenue is a dead end, which can never generate a successful innovation. Akcigit and Liu (2016) show that if R&D activity could result in both successful innovations, which are observable by rivals, and dead-end discoveries, which are not observable by rivals, then competition between firms would result in wasteful duplication of dead-end research. In this case, the merger may have an additional positive effect — elimination of wasteful duplicative dead-end research.

Another key assumption in our analysis is that both firms undertake research along the same research avenue. An alternative way to view this assumption is that each firm is endowed with own research avenue, but these avenues are perfectly positively correlated. Reducing the correlation between the firms' avenues weakens the merger's positive impact on the first innovation while strengthening its negative impact on the second innovation. In particular, in Online Appendix G, we argue that if the avenues are uncorrelated or imperfectly negatively correlated, then the appropriability and informational effects are muted. Indeed, with non-positive correlation, the arrival of the first innovation is no longer a good news for the competitor's chances to produce its own innovation. Hence, if the first innovation arrives when the firms are close to abandoning research, the potential follower does not attempt to produce the second innovation. It follows that, at the stopping threshold, (1) leader's continuation payoff coincides with the merged entity's continuation payoff, which means that their incentives to work on the first innovation — that is, their stopping thresholds — are the same (no appropriability effect); and (2) the information that the leader's arm is good is of no use to the follower, which means that there is no free-riding (no informational effect). Furthermore, even when both the merged entity and the competing firms deliver the second innovation, the merger has negative impact. In particular, the merged entity may act as a "lazy" monopolist and take longer to produce the second innovation. By internalizing negative payoff externalities from the second innovation, the merged entity has less incentives than the potential follower to explore the avenue, whose type remains uncertain.

In our analysis, we focus on a single market with symmetric firms — that is, we consider the effects of a horizontal merger. In practice, however, successful commercialization of an innovation often depends on access to complementary resources, products, or customers. Introducing additional markets that interact with the focal market may create asymmetries between firms. For example, an ecosystem or conglomerate — active across multiple markets — may acquire a startup, which operates in a single market. In such cases, vertical relationships matter: the profitability of an innovation in a downstream market may critically depend on the structure and control of upstream markets. Policy discussions increasingly highlight the risk of "kill zones," where dominant firms deter adjacent innovation through exclusionary practices or low-return acquisitions (see, for example, Motta and Peitz (2021) and Kaplow (2024)). These concerns are particularly acute in digital markets and point to an important direction for future research: incorporating vertical relationships, market asymmetries, and barriers to entry when assessing the impact of mergers on innovation.

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Proofs

A1. Proof of Proposition 1

Given the research intensity X, the merged entity's instantaneous payoff is

(A.1)
$$\underbrace{\pi\,\mathrm{dt}}_{\substack{\text{flow payoff}\\ \text{before innovation}}} - \underbrace{cX\,\mathrm{dt}}_{\text{cost of research}} + \overbrace{X\,\mathrm{dt}}^{\text{prob of innovation}} \times \underbrace{(\lambda+\phi)\pi \int_0^{+\infty} e^{-rs}\,\mathrm{ds}}_{\text{payoff after innovation}} = \left(\pi + \frac{(\lambda+\phi)\pi}{r}X - cX\right)\mathrm{dt}\,.$$

Let V_M be the merged entity's expected payoff from the second stage. Then, the merged entity's expected discounted continuation payoff is

$$\text{(A.2)}\qquad \underbrace{(1-X\,\mathrm{dt})}_{\text{prob of no innovation}} \times \underbrace{(1-r\,\mathrm{dt})}_{\text{discount factor}} V_M \overset{\mathrm{dt}\approx 0}{=} (1-X\,\mathrm{dt} - r\,\mathrm{dt})\,V_M.$$

The merged entity's overall payoff is the sum of the instantaneous and expected discounted continuation payoffs:

$$\text{(A.3)}\qquad V_M = \max_{X\in[0,2]}\left\{\pi\,\mathrm{dt} + \left(\frac{(\lambda+\phi)\pi}{r} - c - V_M\right)X\,\mathrm{dt} + (1-r\,\mathrm{dt})V_M\right\},$$

where the maximization is performed over the research intensity at the current moment. Since (A.3) is linear in X, either $X = 0$ or $X = 2$ is optimal. Substituting $X = 0$ into (A.3) gives $V_M = \pi/r$, while substituting $X = 2$ gives $(2+r)V_M = \pi + 2(\lambda+\phi)\pi/r - 2c$. Moreover, (A.3) implies that $X = 2$ is optimal if and only if $(\lambda+\phi)\pi/r - c \geq V_M$. Thus, condition (2) is necessary and sufficient for $X = 2$ to be optimal, and the merged entity's payoff is (3).

A2. Proof of Proposition 2

Consider the merged entity at a given t and let $p(t) = p$ be the current belief that the research avenue is good. Given the research intensity X, the merged entity's instantaneous payoff is

$$\text{(A.4)}\qquad -\underbrace{cX\,\mathrm{dt}}_{\text{cost of research}} + \underbrace{pX\,\mathrm{dt}}_{\text{prob of innovation}} \times V_M,$$

where V_M is its payoff from the second stage. Let $W(p)$ be the merged entity's cumulative payoff, provided that the initial belief is $p(0) = p$. In the absence of innovation, the merged entity updates its belief according to (4). Then, its expected discounted continuation payoff is

(A.5)
$$\underbrace{(1 - pX\,\mathrm{d}t)}_{\text{prob of no innovation}}\overbrace{(1 - r\,\mathrm{d}t)}^{\text{discount factor}}W(p\underbrace{-X(1-p)p\,\mathrm{d}t}_{=\mathrm{d}p}) \overset{\mathrm{d}t\approx 0}{=} (1 - pX\,\mathrm{d}t - r\,\mathrm{d}t)\,W(p) - W'(p)X(1-p)p\,\mathrm{d}t\,.$$

The overall payoff $W(p)$ is the sum of (A.4) and (A.5), maximized over $X \in [0, 2]$. Hence, the Hamilton-Jacobi-Bellman equation is

$$0 = \max_{X\in[0,2]} \left\{ \left(pV_M - c - pW(p) - p(1-p)W'(p)\right) X - rW(p) \right\}. \tag{A.6}$$

The linearity in X of the maximand in (A.6) implies that the belief interval $p \in [0, 1]$ has two regions: (1) a region where $X = 0$ is optimal, (2) a region where $X = 2$ is optimal.

Region with no research. — If $X = 0$, then, by (A.6), $W(p) = 0$. According to (A.6), choosing $X = 0$ is optimal only if $c \geq p\,(V_M - W(p) - (1-p)W'(p))$. Since $W(p) = 0$, this condition can be simplified as

$$c \geq pV_M. \tag{A.7}$$

Region with full intensity. — For the region of beliefs, in which $X = 2$, we can solve the differential equation (A.6):

$$2\left(pV_M - c - pW(p) - p(1-p)W'(p)\right) = rW(p), \tag{A.8}$$

for $W(p)$ explicitly up to a constant of integration:

$$W(p) = \frac{2\,(pV_M - c)}{2+r} - \frac{4c(1-p)}{r(2+r)} + C_M\left(\frac{1-p}{p}\right)^{r/2}(1-p). \tag{A.9}$$

According to (A.6), choosing $X = 2$ is optimal only if $c \leq p\,(V_M - W(p) - (1-p)W'(p))$, and equation (A.8) allows rewriting this condition as

$$W(p) \geq 0. \tag{A.10}$$

Optimum. — To combine the regions into a unique Markovian optimal strategy, first note that $p = 0$ belongs to the region with no research. Indeed, the limit

$p \to 0$ of (A.9) is either $\pm\infty$ or finite and negative, depending on the value of C_M. Negative limit contradicts the optimality condition (A.10). Value $+\infty$ is also non-feasible because the cumulative payoff cannot exceed V_M, the payoff from the discovery net of any research cost.

NO-RESEARCH SOLUTION. — Suppose that the no-research region covers the whole belief interval, $p \in (0, 1)$. Since choosing zero intensity is optimal only if condition (A.7) holds, this is an optimum if and only if $c \geq V_M$, which, given expression (3) for V_M, is equivalent to condition (5). The no-research solution corresponds to case 1 in Proposition 2.

FULL-INTENSITY SOLUTION. — Suppose that there exists $\check{p} \in (0, 1)$ such that there is no research for $p \in [0, \check{p})$ and the beliefs just above $\check{p}$ belong to the full-intensity region. Then the value-matching and smooth-pasting properties, $W(\check{p}) = W'(\check{p}) = 0$, together with (A.9), give the expression for $\check{p}$ and the constant of integration: $\check{p} = c/V_M$ and $C_M = \frac{4c}{r(2+r)} \left(\frac{\check{p}}{1-\check{p}}\right)^{r/2} > 0$. A necessary condition for the full-intensity solution to exist is $c < V_M$ because threshold $\check{p} = c/V_M$ must be below 1.

Denote by $\check{\check{p}} \leq 1$ the highest belief of the full-intensity region. Let us show that condition (A.10) holds (as strict inequality) for all $p \in (\check{p}, \check{\check{p}}]$. Differentiating (A.9) twice yields $W''(p) = C_M \frac{r(2+r)}{4(1-p)p^2} \left(\frac{1-p}{p}\right)^{r/2}$, which is positive because $C_M > 0$. By the value-matching and smooth-pasting properties, $W(\check{p}) = W'(\check{p}) = 0$. Thus, $W(p) > 0$ for all $p \in (\check{p}, \check{\check{p}}]$. Since condition (A.10) holds as strict inequality for all $p \in (\check{p}, \check{\check{p}}]$, at belief $\check{\check{p}}$, this region cannot be adjacent to the no-research region where $W(p) = 0$ because $W(p)$ is continuous. Thus, the full intensity region ends at $\check{\check{p}} = 1$.

The full-research solution corresponds to cases 2 and 3 in Proposition 2.

A3. Proof of Proposition 3

Let V_F be the expected payoff of the potential follower. Given its research intensity x_F and the leader's research intensity x_L, V_F is the sum of the potential follower's instantaneous payoff $(\phi\pi/r - c)\, x_F \,\mathrm{dt}$ and its expected discounted continuation payoff $(1 - x_F \,\mathrm{dt} - x_L \,\mathrm{dt})(1 - r \,\mathrm{dt})V_F$:

$$V_F = \max_{x_F \in [0,1]} \left\{ \left(\frac{\phi\pi}{r} - c - V_F \right) x_F \,\mathrm{dt} + (1 - x_L \,\mathrm{dt} - r \,\mathrm{dt}) V_F \right\}, \tag{A.11}$$

where the maximization is performed over the research intensity at the current moment. Since (A.11) is linear in x_F, either $x_F = 0$ or $x_F = 1$ is optimal. Substituting $x_F = 0$ into (A.11) gives $V_F = 0$, while substituting $x_F = 1$ gives

$(1 + x_L + r)V_F = \phi\pi/r - c$. Moreover, (A.11) implies that $x_F = 1$ is optimal if and only if $\phi\pi/r - c \geq V_F$. Therefore, condition

$$\frac{\phi\pi}{r} \geq c \tag{A.12}$$

is necessary and sufficient for $x_F = 1$ to be optimal, and the potential follower's payoff is

$$V_F = \frac{1}{1 + x_L + r} \max\left\{\frac{\phi\pi}{r} - c, 0\right\}. \tag{A.13}$$

Consider the firm that innovated on the first stage — the leader. Its instantaneous payoff is

$$\underbrace{\pi\,\mathrm{dt}}_{\substack{\text{payoff if}\\ \text{sole innovator}}} \; - \; cx_L\,\mathrm{dt} \; + \; \underbrace{x_F\,\mathrm{dt}}_{\substack{\text{prob of innovation}\\ \text{from follower}}} \times \frac{\lambda\pi}{r} \; + \; \underbrace{x_L\,\mathrm{dt}}_{\substack{\text{prob of innovation}\\ \text{from leader}}} \times \frac{(\lambda + \phi)\,\pi}{r}. \tag{A.14}$$

Thus, the leader's optimization problem is

(A.15)

$$V_L = \max_{x_L \in [0,1]} \left\{ \left(\frac{(\lambda + \phi)\pi}{r} - c - V_L \right) x_L\,\mathrm{dt} + \left(1 + \frac{\lambda x_F}{r}\right) \pi\,\mathrm{dt} + (1 - x_F\,\mathrm{dt} - r\,\mathrm{dt})\, V_L \right\},$$

which implies that continuing research ($x_L = 1$) is optimal if and only if

$$\frac{(\lambda + \phi)\pi}{r} - c \geq V_L. \tag{A.16}$$

If $x_L = 1$ is optimal, then (A.15) implies that

$$0 = \frac{(\lambda + \phi + \lambda x_F + r)\pi}{r} - c - (1 + x_F + r)V_L. \tag{A.17}$$

If $x_L = 0$ is optimal, then (A.15) implies that

$$0 = \frac{(\lambda x_F + r)\pi}{r} - (x_F + r)V_L. \tag{A.18}$$

Substituting V_L from (A.18) into (A.16), we get

$$\frac{(\lambda + \phi)\pi}{r} - c \geq \frac{\lambda x_F + r}{x_F + r}\frac{\pi}{r} \quad \Leftrightarrow \quad c \leq \frac{\pi}{r}\left(\phi - \frac{(1 - \lambda)r}{x_F + r}\right). \tag{A.19}$$

Substituting V_L from (A.17) into (A.16), we also get (A.19). Hence, condition (A.19) is necessary and sufficient for $x_L = 1$ to be optimal. From (A.17) and (A.18), the leader's payoff is

(A.20)
$$V_L = \frac{\lambda x_F + r}{x_F + r}\frac{\pi}{r} + \frac{1}{1 + x_F + r}\left(\frac{\pi}{r}\left(\phi - \frac{(1-\lambda)r}{x_F + r}\right) - c\right) \quad \text{if (A.19) holds,}$$

(A.21)
$$V_L = \frac{\lambda x_F + r}{x_F + r}\frac{\pi}{r} \quad \text{if (A.19) does not hold.}$$

Consider cases 1 and 2, that is, suppose that either (10) or (12) holds. Under either of these conditions, condition (A.19) does not hold irrespective of the value of x_F. Hence, the leader does not undertake research, i.e., $x_L = 0$. In case 1, condition (A.12) does not hold, which means that $x_F = 0$, and so, the payoffs (A.13) and (A.21) become (11). In case 2, condition (A.12) holds, which means that $x_F = 1$, and so, the payoffs (A.13) and (A.21) become (13).

Consider case 3, that is, suppose that (14) holds. Under condition (14), condition (A.12) also holds, which implies $x_F = 1$. Thus, condition (A.19) becomes condition (14), and so, $x_L = 1$ is optimal. Expressions (15) follow from (A.13) and (A.20).

A4. Proof of Proposition 4

Consider firm i at time t. Let $p(t) = p$ be the current belief that the research avenue is good. Fix the instantaneous research intensity of the competitor x_{-i}. Then, by choosing intensity x_i, firm i's instantaneous payoff is

(A.22)
$$-\underbrace{cx_i\,\mathrm{dt}}_{\substack{\text{cost}\\ \text{of research}}} + \underbrace{px_i\,\mathrm{dt}}_{\substack{\text{prob of innovation}\\ \text{from } i}}\times V_L + \underbrace{px_{-i}\,\mathrm{dt}}_{\substack{\text{prob of innovation}\\ \text{from } -i}}\times V_F.$$

Let $V(p)$ be each firm's cumulative payoff in the game, provided that the initial belief is $p(0) = p$. Then, firm i's expected discounted continuation payoff is

(A.23)
$$\underbrace{(1 - p(x_1 + x_2)\,\mathrm{dt})}_{\text{prob of no innovation}}\times\underbrace{(1 - r\,\mathrm{dt})}_{\text{discount factor}}\;V(p + \mathrm{d}p)$$
$$\overset{(4),\,\mathrm{dt}\approx 0}{=} (1 - p(x_1 + x_2)\,\mathrm{dt} - r\,\mathrm{dt})\,V(p) - V'(p)\,(x_1 + x_2)\,(1-p)p\,\mathrm{dt}\,.$$

The overall payoff $V(p)$ is the sum of (A.22) and (A.23), maximized over $x_i \in$

$[0, 1]$. Hence, the Hamilton-Jacobi-Bellman equation is
(A.24)

$$0 = \max_{x_i \in [0,1]} \left\{ -cx_i + px_iV_L + px_{-i}V_F - rV(p) - p(x_1 + x_2)\left(V(p) + (1-p)V'(p)\right) \right\}.$$

Maximization in (A.24) gives firm i's best response correspondence:

$$x_i^* \begin{cases} = 0 & \text{if } c > p\left(V_L - V(p) - (1-p)V'(p)\right), \\ \in [0, 1] & \text{if } c = p\left(V_L - V(p) - (1-p)V'(p)\right), \\ = 1 & \text{if } c < p\left(V_L - V(p) - (1-p)V'(p)\right), \end{cases} \tag{A.25}$$

which, in a symmetric equilibrium, gives rise to three regions in the belief interval $p \in [0, 1]$: (1) a region where both firms do not undertake research, (2) a region where both firms choose the same intermediate intensity (the indifference region), and (3) a region where both firms choose the maximum intensity of research.

Region with no research. — If $x_i = x_{-i} = 0$, then, by (A.24), $V(p) = 0$. According to (A.25), choosing $x_i = 0$ is optimal if and only if $c \geq pV_L$.

The indifference region. — The condition for the middle case in (A.25) gives the differential equation for $V(p)$, which we can solve explicitly up to a constant of integration:

$$V(p) = V_L - c - c(1-p)\ln\frac{p}{1-p} + C_2(1-p). \tag{A.26}$$

Expressing $V'(p)$ through $V(p)$ from the condition for the middle case in (A.25) yields

$$V'(p) = \frac{p\left(V_L - V(p)\right) - c}{p(1-p)}. \tag{A.27}$$

Substituting (A.27) into (A.24) and assigning $x_i = x_{-i} = x^*(p)$ give the expression for the equilibrium intensity:

$$x^*(p) = \frac{rV(p)}{c - p(V_L - V_F)}. \tag{A.28}$$

Since the equilibrium intensity $x^*(p)$ cannot exceed 1, (A.28) implies that function $f(p)$ defined as

$$f(p) = rV(p) - c + p(V_L - V_F) \tag{A.29}$$

is less or equal than 0 for all p in the indifference region.

REGION WITH FULL INTENSITY. — For the region of beliefs, in which $x_i = x_{-i} = 1$, we can solve the differential equation (A.24):

$$p\left(V_L + V_F\right) - c - rV(p) = 2p\left(V(p) + (1-p)V'(p)\right), \tag{A.30}$$

for $V(p)$ explicitly up to a constant of integration:

$$V(p) = \frac{p\left(V_L + V_F\right) - c}{2+r} - \frac{2c(1-p)}{r(2+r)} + C_1\left(\frac{1-p}{p}\right)^{r/2}(1-p). \tag{A.31}$$

According to (A.25), choosing full intensity is optimal if and only if $c \leq p\left(V_L - V(p) - (1-p)V'(p)\right)$, and equation (A.30) allows rewriting this condition as $f(p) \geq 0$, where f is defined in (A.29).

EQUILIBRIUM. — In any equilibrium, each firm's cumulative payoff in the game must be non-negative (otherwise, the firm can deviate to no research) and cannot exceed V_L, the payoff of the first innovator:

$$0 \leq V(p) \leq V_L. \tag{A.32}$$

Condition (A.32) implies that $p = 0$ belongs to the region with no research. Indeed, the limit $p \to 0$ of (A.26) is $+\infty$ and the limit $p \to 0$ of (A.31) is either $\pm\infty$ or finite and negative, depending on the value of C_1. Thus, at $p = 0$, neither (A.26) nor (A.31) is compatible with restriction (A.32), which leaves only the no-research region with $V(p) = 0$.

The remaining construction of the equilibrium relies on the continuity of $V(p)$ and of $V'(p)$ at the regions' borders (the value matching and smooth-pasting properties).

Depending on what kind of region is adjacent to the no-research region on the right, three cases emerge. As we will see, in each case, there exists at most one equilibrium. Moreover, the conditions for equilibrium existence in different cases are non-overlapping, which proves the uniqueness of equilibrium.

NO-RESEARCH EQUILIBRIUM. — Suppose that the no-research region covers the whole belief interval, $p \in (0,1)$. Since choosing zero intensity is optimal only if $c \geq pV_L$, this is an equilibrium if and only if $c \geq V_L$, which, given the expression for V_L provided in Proposition 3, is equivalent to condition (16). The no-research equilibrium corresponds to case 1 in Proposition 4.

Full-intensity equilibrium. — Suppose that there exists $\hat{p} \in (0,1)$ such that there is no research for $p \in [0, \hat{p})$ and the beliefs just above $\hat{p}$ belong to the full-intensity region. Then $V(\hat{p}) = V'(\hat{p}) = 0$, together with (A.31), give the expression for $\hat{p}$ and the constant of integration: $\hat{p} = c/(V_L + V_F)$ and $C_1 = \frac{2c}{r(2+r)} \left(\frac{\hat{p}}{1-\hat{p}}\right)^{r/2} > 0$. A necessary condition for the full-intensity equilibrium to exist is

$$c < V_L + V_F \tag{A.33}$$

because threshold $\hat{p} = c/(V_L+V_F)$ must be below 1. Moreover, $f(\hat{p}) \geq 0$, $V(\hat{p}) = 0$ and $\hat{p} = c/(V_L + V_F)$ imply that $V_F \leq 0$. Hence, another necessary condition for the full-intensity equilibrium to exist is

$$V_F = 0 \tag{A.34}$$

because, by Proposition 3, $V_L > 0$ and $V_F \geq 0$.

Denote by $\hat{\hat{p}} \leq 1$ the highest belief of the full-intensity region. We show that $f(p) > 0$ for all $p \in (\hat{p}, \hat{\hat{p}}]$. The derivative of $f(p)$ is positive at $p = \hat{p}$ because $V'(\hat{p}) = 0$. It is also increasing as differentiating (A.31) twice yields

$$V''(p) = C_1 \frac{r(2+r)}{4(1-p)p^2} \left(\frac{1-p}{p}\right)^{r/2}, \tag{A.35}$$

which is positive because $C_1 > 0$. Consequently, $f(p)$ is increasing for all $p > \hat{p}$ throughout the full-intensity region. At $p = \hat{p}$, $f(p)$ is equal to 0 because $V(\hat{p}) = 0$ and because $V_F = 0$ by (A.34). Thus, $f(p) > 0$ in the full-intensity region $p \in (\hat{p}, \hat{\hat{p}}]$.

Since $f(p) > 0$ for all $p \in (\hat{p}, \hat{\hat{p}}]$, at belief $\hat{\hat{p}}$, the full-intensity region $p \in (\hat{p}, \hat{\hat{p}}]$ cannot be adjacent to the indifference region where $f(p) \leq 0$. Since $V''(p) > 0$ by (A.35) and since $V(\hat{p}) = V'(\hat{p}) = 0$ by the value-matching and smooth-pasting properties, $V(p) > 0$ for all $p \in (\hat{p}, \hat{\hat{p}}]$. Hence, at belief $\hat{\hat{p}}$, the full-intensity region cannot be adjacent to the no-research region where $V(p) = 0$. Thus, the full-intensity region ends at $\hat{\hat{p}} = 1$.

The full-intensity equilibrium corresponds to case 2 and to $c = \phi\pi/r$ in case 3 in Proposition 4. Given the expressions for V_F and V_L in Proposition 3, conditions (A.33) and (A.34) together are equivalent to $\phi\pi/r \leq c < \pi/r$. If $\phi\pi/r < c < \pi/r$, then (10) holds and threshold $\hat{p} = c/(V_L + V_F)$ becomes $\hat{p} = cr/\pi$, as in (18). If $c = \phi\pi/r$, then (12) holds and threshold $\hat{p} = c/(V_L + V_F)$ becomes $\hat{p} = cr/\pi \times (1+r)/(\lambda + r) = \underline{p}$, as in (21). Note that if $c = \phi\pi/r$, then threshold $\bar{p}$, defined in (22), is equal to $\underline{p}$, which means that there is no indifference region in case 3; yet, this case is still different from case 2 in that the potential follower undertakes research in the second stage, which lowers equilibrium $\hat{p} = c/V_L$ from

cr/π to $cr/\pi \times (1+r)/(\lambda+r)$.

Indifference equilibrium. — Suppose there exists $\underline{p} \in (0,1)$ such that there is no research for $p \in [0, \underline{p})$ and the beliefs just above $\underline{p}$ belong to the indifference region. Then $V(\underline{p}) = V'(\underline{p}) = 0$, together with (A.26), give the expression for $\underline{p}$ and the constant of integration: $\underline{p} = c/V_L$ and $C_2 = c \ln \frac{c}{V_L - c} - V_L$. Substituting this expression for C_2 into (A.26) yields

$$V(p) = pV_L - c - c(1-p)\ln\frac{p(V_L - c)}{(1-p)c}. \tag{A.36}$$

From (A.36) it is clear that $V(p) < pV_L - c$ for all $p > \underline{p} = c/V_L$. Hence, by (A.27), the derivative of $V(p)$ is bounded below by $V_L - c/p$, which is positive for all $p > \underline{p} = c/V_L$. Thus, $V(p)$ is increasing in p throughout the indifference region. Since $V(p)$ is equal to 0 at the lower bound of the indifference region, $V(\underline{p}) = 0$, $V(p)$ is positive for all p in the indifference region.

Since $V(p)$ is positive, the numerator in (A.28) is positive for all p in the indifference region. The denominator in (A.28) is positive for beliefs just above $\underline{p} = c/V_L$ if and only if $V_F > 0$, which, given the expression for V_F from Proposition 3, is equivalent to

$$c < \frac{\phi\pi}{r}. \tag{A.37}$$

Since the research intensity $x^*(p)$, defined in (A.28), must be positive throughout the indifference region, condition (A.37) is another necessary condition for the indifference equilibrium to exist.

Note that condition (A.37) implies that $\underline{p} = c/V_L < 1$. Indeed, $V_L \geq \frac{\lambda+r}{1+r}\frac{\pi}{r}$ by Proposition 3, which implies that $c < V_L$ holds if (A.37) holds.

We now argue that $x^*(p)$ is increasing from 0 at $p = \underline{p}$ to 1 at some $p = \bar{p} \in (\underline{p}, 1)$, and so, since the equilibrium research intensity must be less or equal to 1, the highest possible belief in the indifference region is $\bar{p}$. Throughout the indifference region, since $V(p)$ is increasing from $V(\underline{p}) = 0$, the research intensity $x^*(p)$, defined in (A.28), is also increasing from $x^*(\underline{p}) = 0$, as long as the denominator in (A.28) remains positive. If the denominator becomes negative at some belief $\bar{\bar{p}} < 1$, the research intensity $x^*(p)$ becomes unbounded as p approaches $\bar{\bar{p}}$ from below and there is some $\bar{p} \in (\underline{p}, \bar{\bar{p}})$ such that $x^*(\bar{p}) = 1$. If the denominator is still positive at $p = 1$, then $\bar{p} \in (\underline{p}, 1)$ such that $x^*(\bar{p}) = 1$ also exists because $x^*(1) > 1$, which we now show. Substituting $p = 1$ into (A.36) and then into (A.28) yields $x^*(1) = r(V_L - c)/(c - V_L + V_F)$, and so, condition $x^*(1) > 1$ is equivalent to

$$\frac{V_L}{c} - 1 > \frac{V_F}{c(1+r)}. \tag{A.38}$$

Condition (A.38) follows from (A.37), which is a necessary condition for the indifference equilibrium to exist. Indeed, under restriction (A.37), by Proposition 3, (A.38) becomes

(A.39)
$$\frac{\overbrace{\lambda\pi - cr}^{\geq \phi\pi - rc}}{cr(1+r)} + \frac{\overbrace{(\pi/r - c)}^{>0 \text{ if } c<\phi\pi/r}\, r}{c(1+r)} + \frac{1}{c(2+r)} \max\left\{\frac{\pi}{r}\left(\phi - \frac{(1-\lambda)r}{1+r}\right) - c, 0\right\} > \frac{\phi\pi - rc}{cr(1+r)} \overbrace{\frac{1}{1+x_L+r}}^{<1},$$

where $x_L = 0$ if (12) holds and $x_L = 1$ if (14) holds. Inequality (A.39) always holds for $c < \phi\pi/r$. Thus, $x^*(1) > 1$, and so, there always exists $\bar{p} \in (\underline{p}, 1)$ such that $x^*(\bar{p}) = 1$, and $x^*(p)$ is increasing in $p \in (\underline{p}, \bar{p})$ from 0 to 1.

Threshold $\bar{p} \in (\underline{p}, 1)$ is uniquely defined as a solution to $x^*(\bar{p}) = 1$, or equivalently, $f(\bar{p}) = 0$. Substituting $V(\bar{p})$ from (A.36) into equation $f(\bar{p}) = 0$ yields

$$\text{(A.40)} \qquad \frac{V_L}{c} - \frac{1}{\bar{p}} - \frac{1-\bar{p}}{\bar{p}}\frac{r}{1+r}\ln\frac{\bar{p}(V_L - c)}{(1-\bar{p})c} = \frac{V_F}{c(1+r)}.$$

The left-hand side of (A.40) is increasing in $\bar{p}$ for $\bar{p} \in (c/V_L, 1)$, is equal to 0 at $\bar{p} = c/V_L = \underline{p}$, and converges to $V_L/c - 1$ as $\bar{p} \to 1$. Hence, by condition (A.38), equation (A.40) has a unique solution $\bar{p} \in (\underline{p}, 1)$.

Since $V(p)$ is positive for all $p \in (\underline{p}, \bar{p}]$, the indifference region cannot end with the no-research region where $V(p) = 0$; that is, it ends with the full-intensity region. The necessary condition for the full-intensity region, $f(p) \geq 0$, precludes a configuration in which the indifference region ends before $\bar{p}$. Hence, the indifference region ends at $\bar{p}$, at which the full-intensity region starts.

The beliefs just below $p = \bar{p}$ belong to the indifference region, and so, for these beliefs, the relevant expression for $V(p)$ is given in (A.36). The beliefs just above $p = \bar{p}$ belong to the region with full intensity, and so, for these beliefs, the relevant expression for $V(p)$ is given in (A.31). The continuity of $V(p)$ and of $V'(p)$ at $p = \bar{p}$ gives the equation for $\bar{p}$, which coincides with (A.40), and the constant of integration:

$$\text{(A.41)} \qquad C_1 = \frac{2c}{r}\left(1 + \frac{1+r}{2+r}\frac{\bar{p}}{1-\bar{p}}\left(\frac{V_F}{c(1+r)} - \frac{V_L}{c} + 1\right)\right)\left(\frac{\bar{p}}{1-\bar{p}}\right)^{r/2}.$$

Suppose that $V(p)$ is defined by (A.31), with C_1 defined in (A.41), for all $p \in (\bar{p}, 1)$. We show that condition $f(p) > 0$ for all $p \in (\bar{p}, 1)$. The second derivative of $V(p)$, expressed in (A.35), has the same sign for all $p \in (\bar{p}, 1)$. Hence, the derivative of $f(p)$ is monotone. Differentiating (A.31) once and then substituting (A.41) and $p = \bar{p}$ yields

$$V'(\bar{p}) = -\frac{V_L - V_F}{r} + \frac{c}{\bar{p}}\left(\frac{1}{r} + \frac{1+r}{r}\frac{\bar{p}}{1-\bar{p}}\left(\frac{V_L}{c} - \frac{1}{\bar{p}} - \frac{V_F}{c(1+r)}\right)\right) \tag{A.42}$$
$$\overset{(A.40)}{=} -\frac{V_L - V_F}{r} + \frac{c}{\bar{p}}\left(\frac{1}{r} + \ln\frac{\bar{p}(V_L - c)}{(1-\bar{p})c}\right) \overset{\bar{p}>c/V_L}{>} -\frac{V_L - V_F}{r} + \frac{c}{r\bar{p}} > -\frac{V_L - V_F}{r},$$

which implies that the derivative of $f(p)$ is positive at $p = \bar{p}$. Differentiating (A.31) and then taking the limit $p \to 1$ gives $V'(1) = \frac{2c+r(V_L+V_F)}{r(2+r)} > 0$, which implies that $f'(p)$ is positive at the limit $p \to 1$. Hence, $f'(p) > 0$ for all $p \in (\bar{p}, 1)$, and so, $f(p)$ is increasing for all $p \in (\bar{p}, 1)$. Since $f(\bar{p}) = 0$, $f(p) > 0$ for all $p \in (\bar{p}, 1)$.

Denote by $\hat{\hat{p}} \leq 1$ the highest belief of the full-intensity region. Suppose that $\hat{\hat{p}} < 1$. Then, $f(\hat{\hat{p}}) > 0$ because $f(p) > 0$ for all $p \in (\bar{p}, 1)$. Hence, the full-intensity region cannot be adjacent to the indifference region where $f(p) \leq 0$. Moreover, for all $p \in (\bar{p}, 1)$, including $p = \hat{\hat{p}}$, $V(p)$ is positive because

$$rV(p) \overset{f(p)>0}{>} c - V_L + V_F \overset{(A.38)}{>} r(V_L - c) \overset{\underline{p}=c/V_L<1}{>} 0. \tag{A.43}$$

Hence, at belief $\hat{\hat{p}}$, the full intensity region cannot be adjacent to the no-research region where $V(p) = 0$. Thus, the full-intensity region ends at $\hat{\hat{p}} = 1$.

The indifference equilibrium corresponds to $c < \phi\pi/r$ in case 3 in Proposition 4. Given the expressions for V_F and V_L from Proposition 3, threshold $\underline{p} = c/V_L$ becomes (21) and (23), and the equation (A.40) for $\bar{p}$ becomes (22) and (24). The expression (A.28) for the equilibrium intensity becomes (20) after substituting $V(p)$ from (A.36), V_F from (A.40) and $V_L = c/\underline{p}$.

A5. Proof of Lemma 1

$$\underline{p} - \check{p} = \frac{\frac{cr}{\pi}\overbrace{\left(\phi - \frac{cr}{\pi}\right)}^{\geq 1-\lambda>0 \text{ by } (26)}}{\left(\frac{\lambda+r}{1+r} + \frac{1}{2+r}\underbrace{\left(\phi - \frac{(1-\lambda)r}{1+r} - \frac{cr}{\pi}\right)}_{\geq 1-\lambda-\frac{(1-\lambda)r}{1+r}>0 \text{ by } (26)}\right)\underbrace{\left(2(\lambda+\phi) + r - \frac{2rc}{\pi}\right)}_{\geq 2+r>0 \text{ by } (26)}} > 0. \tag{A.44}$$

A6. Proof of Lemma 2

Given a prior belief p_0 and a stopping threshold p_S, the cumulative investment in research is equal to $\int_0^{+\infty} X(t)\,\mathrm{d}t \overset{(4)}{=} \int_{p_S}^{p_0} \frac{\mathrm{d}p}{(1-p)p} = \ln \frac{1/p_S-1}{1/p_0-1}$. Substituting it into the probability of the first innovation $1-\exp\left(-\int_0^{+\infty} X(t)\,\mathrm{d}t\right)$ yields (27).

A7. Proof of Lemma 3

If (12) holds, then $\underline{p}$ is defined in (21) and so $\underline{p}-\check{p}=\dfrac{cr}{\pi}\cdot\dfrac{1-\lambda}{\lambda+r}>0$. If (14) holds, then $\underline{p}$ is defined in (23) and so $\underline{p}-\check{p}=\dfrac{cr}{\pi}\cdot\dfrac{1-\lambda+cr/\pi-\lambda-\phi+1}{2\lambda+r+\phi-cr/\pi}>0$, where the numerator is positive by the left inequality in (28) and the denominator is positive by the right inequality in (28).

A8. Definition of the strength of the informational effect

Our definition of the strength of the informational effect isolates the merger-induced change in the speed of research investment from the appropriability effect — that is, from the change in the cumulative investment in research before it is abandoned. We cap the cumulative R&D investment by exogenously fixing a lower bound $p^\dagger$ on the posterior belief (by (4), the posterior maps one-to-one to the cumulative R&D effort). The strength of the informational effect is then the merger-induced decrease in the arrival time of the first innovation, conditional on event $\mathcal{E}\left(p^\dagger\right)$ that innovation arrives before the posterior reaches $p^\dagger$ in both the competing and merged entity settings.

Let $p^\dagger \in (\underline{p},\bar{p})$. Restriction $p^\dagger > \underline{p}$ ensures that in the absence of innovation, both competing firms and the merged entity do not abandon research before the posterior falls to $p^\dagger$. Restriction $p^\dagger < \bar{p}$ ensures that the arrival time of the first innovation may differ between the two settings.

The merger-induced decrease in the arrival time of the first innovation can be measured as

$$\mathbb{E}\left[T_c - T_m \mid \mathcal{E}\left(p^\dagger\right)\right], \tag{A.45}$$

where T_m and T_c are the arrival times of the first innovation in the merged entity and in the competing firms settings, respectively. A more natural definition from consumer welfare perspective involves exponential discount of the arrival time:

$$\mathbb{E}\left[\exp\left(-rT_m\right)-\exp\left(-rT_c\right) \mid \mathcal{E}\left(p^\dagger\right)\right]. \tag{A.46}$$

We adopt (A.46) as the definition of the strength of the informational effect but in Online Appendix C we show that all our results remain unchanged under definition (A.45).

A9. Proof of Theorem 4

Suppose that $c < \phi\pi/r$, which corresponds to cases 2 and 3 in Proposition 2 — so that the merged entity undertakes research until the belief threshold $\check{p}$ defined either in (7) or in (9) — and case 3 in Proposition 4 — so that the competing firms undertake research at full intensity until the belief threshold $\bar{p}$ and then slow down their research undertaking it at intensity $x^*(p)$ defined in (20) until belief threshold $\underline{p}$, with thresholds $\underline{p}$ and $\bar{p}$ defined either in (21) and (22) or in (23) and (24).

Denote by T_m and T_c the arrival times of the first innovation in the merged entity and in the competing firms setups, respectively.

The strength of the appropiability effect is defined as the merger-induced increase in the probability of the first innovation and is derived in Lemma A.1.

LEMMA A.1: *Given a prior belief $p_0 > \bar{p}$, the strength of the appropiability effect is equal to*

$$\Pr(T_m < +\infty) - \Pr(T_c < +\infty) = \frac{1-p_0}{p_0}\left(\frac{\underline{p}}{1-\underline{p}} - \frac{\check{p}}{1-\check{p}}\right). \tag{A.47}$$

PROOF:

The statement directly follows from Lemma 2.

Since $\underline{p} > \check{p}$ by Lemmas 1 and 3, (A.47) is decreasing in p_0.

Lemma A.2 implies that the strength of the informational effect, defined in (A.46), is decreasing in p_0.

LEMMA A.2: *Given a prior belief $p_0 > \bar{p}$ and threshold $p^\dagger \in (\underline{p}, \bar{p})$, the strength of the informational effect (A.46) is equal to*

$$\frac{\left(\frac{1-p_0}{p_0}\right)^{r/2}}{\frac{p_0}{1-p_0} - \frac{p^\dagger}{1-p^\dagger}} \int_{p^\dagger}^{\bar{p}} \left(\left(\frac{p}{1-p}\right)^{r/2} - \left(\frac{\bar{p}}{1-\bar{p}}\right)^{r/2} \exp\left(-\frac{r}{2}\int_{\underline{p}}^{\bar{p}} \frac{ds}{x^*(s)(1-s)s}\right)\right) \frac{dp}{(1-p)^2}. \tag{A.48}$$

PROOF:

We use notation T for T_m and T_c, that is, the arrival times of the first innovation in either setting. Then, for any upper bound $\hat{T}$ on T, given the overall research intensity $X(t)$, $\mathbb{E}\left[\exp(-rT) \mid T < \hat{T}\right]$ is equal to

(A.49)

$$\int_0^{\hat{T}} \exp(-rT) \frac{X(T)\exp\left(-\int_0^T X(t)\,\mathrm{d}t\right)}{1-\exp\left(-\int_0^{\hat{T}} X(t)\,\mathrm{d}t\right)} \mathrm{d}T \overset{(4)}{=} \int_{\hat{T}}^{0} \exp(-rT) \frac{\frac{p'(T)}{(1-p(T))p(T)} \frac{p(T)(1-p(0))}{p(0)(1-p(T))}}{1-\frac{p(\hat{T})(1-p(0))}{p(0)\left(1-p(\hat{T})\right)}} \mathrm{d}T$$

$$= \left(\frac{p(0)}{1-p(0)} - \frac{p(\hat{T})}{1-p(\hat{T})}\right)^{-1} \int_{p(\hat{T})}^{p(0)} \exp\Bigg(-r \cdot \underbrace{T(p)}_{\overset{(4)}{=} \int_p^{p(0)} \frac{\mathrm{d}s}{X(s)(1-s)s}}\Bigg) \frac{\mathrm{d}p}{(1-p)^2}.$$

For the merged entity, $X(s) = 2$ for all $s > p^{\dagger}$. For the competing firms, $X(s) = 2$ for all $s > \bar{p}$ and $X(s) = 2x^*(s)$ for all $s \in (p^{\dagger}, \bar{p})$. Hence, with $p(0) = p_0$ and $p(\hat{T}) = p^{\dagger}$, (A.49) implies (A.48).

A10. Proof of Theorem 5

CASE (A). — This case corresponds to case 2 in Proposition 2 and case 3a in Proposition 4, in which the leader aborts research after producing the first innovation.

The strength of the appropriability effect derived in Lemma A.1 does not depend on ϕ and decreases in λ. Indeed, for the merged entity, the stopping threshold $\check{p}$ defined in (7) depends on neither λ nor ϕ. For the competing firms, the stopping threshold $\underline{p}$ defined in (21) decreases in λ and is independent of ϕ. Hence, (A.47) decreases in λ and is independent of ϕ.

The strength of the informational effect derived in Lemma A.2 decreases in λ and increases in ϕ. Indeed, by Lemma A.3, the comparative statics of the strength of the informational effect move in the opposite direction to that of the equilibrium intensity $x^*(p)$ on the free-riding region $(\underline{p}, \bar{p})$. By Lemma A.4, $x^*(p)$ in case (A) increases in λ and decreases in ϕ.

LEMMA A.3: *Consider any parameter z of the model (except the discount rate r). If the competing firms' equilibrium research intensity $x^*(p)$ increases in z for all $p \in (\underline{p}, \bar{p})$, then for any prior belief $p_0 > \bar{p}$ and threshold $p^{\dagger} \in (\underline{p}, \bar{p})$, (A.48) decreases in z.*

PROOF:

Parameter z may enter into (A.48) only through $\bar{p}$ and $x^*(p)$. However, the first derivative of (A.48) with respect to $\bar{p}$ is 0: the integrand in (A.48) is 0 at $p = \bar{p}$ and its derivative with respect to $\bar{p}$ is $\dfrac{r\,(1-x^*(\bar{p}))}{2(1-\bar{p})\bar{p}x^*(\bar{p})}\left(\dfrac{\bar{p}}{1-\bar{p}}\right)^{r/2}\exp\left(-\dfrac{r}{2}\displaystyle\int_{\underline{p}}^{\bar{p}}\dfrac{\mathrm{d}s}{x^*(s)(1-s)s}\right)=$ 0 because $x^*(\bar{p})=1$. Then, if $x^*(s)$ increases in z for all s, then (A.48) decreases in z.

LEMMA A.4: *Given thresholds $\underline{p}$ defined in (21) and $\bar{p}$ defined in (22), the equilibrium research intensity $x^*(p)$ defined in (20) increases in λ and decreases in ϕ.*

PROOF:

The equilibrium research intensity $x^*(p)$ defined in (20) decreases in $\bar{p}$: differentiating (20) w.r.t. $\bar{p}$ yields

(A.50)
$$-\frac{\dfrac{1}{\underline{p}}-\dfrac{1}{p}-\dfrac{1-p}{p}\ln\dfrac{p(1-\underline{p})}{(1-p)\underline{p}}}{\left(\dfrac{1}{r}\left(\dfrac{1}{p}-\dfrac{1}{\bar{p}}\right)+\dfrac{1}{\underline{p}}-\dfrac{1}{\bar{p}}-\dfrac{1-\bar{p}}{\bar{p}}\ln\dfrac{\bar{p}(1-\underline{p})}{(1-\bar{p})\underline{p}}\right)^2}\frac{1}{\bar{p}^2r}\left(1+r\ln\frac{\bar{p}(1-\underline{p})}{(1-\bar{p})\underline{p}}\right)<0$$

since $\dfrac{1}{\underline{p}}-\dfrac{1}{p}-\dfrac{1-p}{p}\ln\dfrac{p(1-\underline{p})}{(1-p)\underline{p}}>0$ because it is increasing in $p>\underline{p}$ and equal to 0 at $p=\underline{p}$.

The equilibrium research intensity $x^*(p)$ defined in (20) decreases in $\underline{p}$: differentiating (20) w.r.t. $\underline{p}$ yields

(A.51)
$$-\frac{\dfrac{p-\underline{p}}{r}\left(\dfrac{\bar{p}}{p}-1\right)+\left\{\left(\bar{p}-\underline{p}\right)(1-p)\ln\dfrac{p(1-\underline{p})}{(1-p)\underline{p}}-\left(p-\underline{p}\right)(1-\bar{p})\ln\dfrac{\bar{p}(1-\underline{p})}{(1-\bar{p})\underline{p}}\right\}}{p\bar{p}\left(1-\underline{p}\right)\underline{p}^2\left(\dfrac{1}{r}\left(\dfrac{1}{p}-\dfrac{1}{\bar{p}}\right)+\dfrac{1}{\underline{p}}-\dfrac{1}{\bar{p}}-\dfrac{1-\bar{p}}{\bar{p}}\ln\dfrac{\bar{p}(1-\underline{p})}{(1-\bar{p})\underline{p}}\right)^2}<0.$$

Indeed, the expression in the curly brackets is positive because it is a concave function of p and equal to 0 at $p=\underline{p}$ and at $p=\bar{p}$.

Clearly, threshold $\underline{p}$ defined in (21) decreases in λ and is independent of ϕ.

Threshold $\bar{p}$ defined in (22) decreases in λ because the right-hand side of (22) is independent of both λ and $\bar{p}$, while the left-hand side of (22) increases in $\bar{p}$:

(A.52)
$$\frac{\partial}{\partial\bar{p}}\left(\frac{1}{\underline{p}}-\frac{1}{\bar{p}}-\frac{1-\bar{p}}{\bar{p}}\frac{r}{1+r}\ln\frac{\bar{p}(1-\underline{p})}{(1-\bar{p})\underline{p}}\right)=\frac{1}{\bar{p}^2(1+r)}\left(1+r\ln\frac{\bar{p}(1-\underline{p})}{(1-\bar{p})\underline{p}}\right)>0,$$

and increases in λ: it depends on λ only through $\underline{p}$, which, by (21), decreases in λ, and

$$\text{(A.53)}\qquad \frac{\partial}{\partial\underline{p}}\left(\frac{1}{\underline{p}}-\frac{1}{\bar{p}}-\frac{1-\bar{p}}{\bar{p}}\frac{r}{1+r}\ln\frac{\bar{p}(1-\underline{p})}{(1-\bar{p})\underline{p}}\right)=-\frac{\bar{p}(1-\underline{p})+r(\bar{p}-\underline{p})}{\bar{p}(1+r)(1-\underline{p})\underline{p}^2}<0.$$

Threshold $\bar{p}$ defined in (22) increases in ϕ because the right-hand side of (22) increases in ϕ and is independent of $\bar{p}$, while the left-hand side of (22) increases in $\bar{p}$ by (A.52) and is independent of ϕ.

Since the equilibrium research intensity $x^*(p)$ decreases in $\bar{p}$ and in $\underline{p}$, the comparative statics of $x^*(p)$ follows from the comparative statics of $\bar{p}$ and $\underline{p}$.

CASE (B). — This case corresponds to case 2 in Proposition 2 and case 3b in Proposition 4, in which the leader continues research after producing the first innovation.

By the same logic as in case (A), as λ and ϕ change, the strength of the appropriability effect changes in the same direction as the stopping threshold $\underline{p}$, which, by definition (23), decreases in λ and in ϕ.

The comparative statics of the strength of the informational effect follows from Lemma A.5.

LEMMA A.5: *Given thresholds $\underline{p}$ defined in (23) and $\bar{p}$ defined in (24), the equilibrium research intensity $x^*(p)$ defined in (20) increases in λ and in ϕ.*

PROOF:

As in Lemma A.4, the comparative statics of $x^*(p)$ follows from the comparative statics of $\bar{p}$ and $\underline{p}$. Clearly, threshold $\underline{p}$ defined in (23) decreases in λ and in ϕ. Threshold $\bar{p}$ defined in (24) decreases in λ by exactly the same argument as in the proof of Lemma A.4. It decreases in ϕ — which is different from the result in Lemma A.4 because $\underline{p}$ now depends on ϕ: by the implicit function theorem, since the left-hand side of (24) is increasing in $\bar{p}$ by (A.52), the sign of $\bar{p}'(\phi)$ coincides with the sign of

(A.54)

$$\underbrace{\frac{\partial}{\partial \phi}\left(\frac{\phi\pi - cr}{cr(1+r)(2+r)}\right)}_{>0} - \underbrace{\frac{\partial}{\partial \underline{p}}\left(\frac{1}{\underline{p}} - \frac{1}{\bar{p}} - \frac{1-\bar{p}}{\bar{p}}\frac{r}{1+r}\ln\frac{\bar{p}(1-\underline{p})}{(1-\bar{p})\underline{p}}\right)}_{<0 \text{ by (A.53)}} \underbrace{\underline{p}'(\phi)}_{<0} \overset{(23)}{=}$$

$$\frac{\pi}{cr(1+r)(2+r)} - \frac{\bar{p}(1-\underline{p}) + r(\bar{p}-\underline{p})}{\bar{p}(1+r)(1-\underline{p})\underline{p}^2}\cdot\frac{\pi\underline{p}^2}{cr(2+r)} = -\frac{\pi(\bar{p}-\underline{p})}{c\bar{p}(1+r)(2+r)(1-\underline{p})} < 0.$$

MICROFOUNDATIONS FOR THE PAYOFF STRUCTURE

ONE INNOVATION. — Suppose that only one firm generated an innovation. This innovation allows the firm to produce a product. The inverse demand for the product is $p(q) = Q - q$, where q is the market quantity and $Q > 0$ is a parameter. Assume that the firm's marginal cost is 0. Then, the firm's profit is $qp(q)$, which is maximized at $q = Q/2$. Hence, the sole innovator's profit is

$$\pi = \frac{Q^2}{4}. \tag{B.1}$$

Two innovations. — Suppose that both firms have generated innovations, and, thus, both produce substitute products. Let p_i and q_i be the price and the demand for a product produced by firm i. The inverse demand function for firm i's product is $p_i(q_1, q_2) = Q - q_i - \theta q_{-i}$, where q_{-i} is the demand for the competitor's product. Parameter $\theta \in [0, 1]$ is the degree of substitutability between the products. When $\theta = 0$, the products are independent; when $\theta = 1$, the products are perfect substitutes. Firms' marginal production cost is normalized to 0. Firms compete by choosing quantities. The firm that innovated first has a natural advantage and so sets its quantity first; that is, the game is a Stackelberg competition with differentiated products. We assume that the firms commit to their quantity choices and then forever receive the flow payoff generated by their choices.

Without loss of generality, suppose that firm 1 is the leader and firm 2 is the follower. The follower maximizes $q_2 p_2(q_1, q_2) = q_2 (Q - q_2 - \theta q_1)$ w.r.t. $q_2 \geq 0$, and so, the follower's best response function is

$$q_2(q_1) = \frac{Q - \theta q_1}{2}. \tag{B.2}$$

When choosing its quantity, the leader correctly predicts the follower's best response and maximizing $q_1 p_1(q_1, q_2(q_1)) = q_1 \left(\frac{2-\theta}{2} Q - \frac{2-\theta^2}{2} q_1\right)$ w.r.t. $q_1 \geq 0$ chooses

$$q_1 = \frac{2 - \theta}{2(2 - \theta^2)} Q. \tag{B.3}$$

Hence, the leader's and the follower's payoffs are

$$\lambda\pi = \frac{(2 - \theta)^2}{2(2 - \theta^2)} \frac{Q^2}{4}, \quad \phi\pi = \frac{\left(4 - 2\theta - \theta^2\right)^2}{4(2 - \theta^2)^2} \frac{Q^2}{4}. \tag{B.4}$$

Interpretation of parameters. — Comparing (B.4) with (B.1) yields the expression for λ and ϕ, as functions of θ:

$$\lambda(\theta) = \frac{(2 - \theta)^2}{2(2 - \theta^2)}, \quad \phi(\theta) = \frac{\left(4 - 2\theta - \theta^2\right)^2}{4(2 - \theta^2)^2}. \tag{B.5}$$

To justify restriction (1), note that when products are independent ($\theta = 0$), both firms get the monopoly profit: $\lambda(0) = \phi(0) = 1$. Moreover, for all $\theta \in (0, 1)$, both

$\lambda(\theta)$ and $\phi(\theta)$ are decreasing in θ: $\lambda'(\theta) = -\dfrac{2(2-\theta)(1-\theta)}{(2-\theta^2)^2} < 0$ and $\phi'(\theta) = -\dfrac{8(1-\theta)^2 + \theta(4-2\theta-\theta^3)}{(2-\theta^2)^3} < 0$. This is intuitive because a higher degree of substitutability intensifies competition. Finally, for all $\theta \in (0,1)$, the follower's payoff is lower than the leader's payoff: $\lambda(\theta) - \phi(\theta) = \dfrac{(4-3\theta)\theta^3}{4(2-\theta^2)^2} > 0$, which is a manifestation of the classical Stackelberg first-mover advantage.