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## Energy Commodities and Calendar Spread Options

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*We present a unified framework for pricing calendar spread options on energy commodities under affine models featuring stochastic volatility, jumps, and the Samuelson effect. Expressions for the joint characteristic function of log-futures prices are derived, enabling efficient calibration and valuation. An empirical analysis, across WTI crude oil, HH natural gas, and ULSD heating oil shows that stochastic volatility models consistently outperform others. Jumps enhance short-term fit, while volatility dynamics matter more at longer maturities. The Black model remains competitive for short- and mid-term contracts.*

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**Introduction**

There are different types of spreads in the energy markets, and in this work we focus on calendar spreads on futures contracts of energy commodities. A calendar spread involves the simultaneous purchase and sale of contracts on the same commodity with different maturities. Calendar spread futures and options are used to profit from time decay, price volatility, or neutral price movements of the underlying. In terms of open interest, calendar spread options (CSOs) are the third most traded option type in the energy markets.

In this work we develop a unified pricing framework for CSOs under a broad class of affine models, including those with stochastic volatility, jumps, and the Samuelson effect. While Schneider and Tavin (2018) introduced a futures-based stochastic volatility model and computed the joint characteristic function (JCF) for pricing CSOs, their framework limits its direct applicability to other models. In contrast, we clearly separate these components and generalize the methodology to a wider class of affine models and derive closed-form expressions for the JCF of futures log-prices. This allows for efficient model calibration and pricing. In addition, we propose a novel approach that leverages conditioning arguments to price CSOs under advanced stochastic volatility models, such as those where direct computation of the JCF is numerically demanding. This contributes to a unified efficient framework for model pricing and calibration.

Several authors have proposed approximation formulas for pricing spread options. Kirk (1995) was the first to offer an approximate solution, still widely used in practice, by generalizing the Margrabe (1978) exchange option formula for arbitrary strikes. Numerous extensions of Kirk's approach followed, among which the method presented in Caldana and Fusai (2013) excels. This method proposes a fast and accurate

1-dimensional Fourier inversion method which generalizes the approximation of Bjerksund and Stensland (2014) to any model for which the JCF of the futures log-prices in the spread is available in closed form.

Our empirical analysis focuses on three major energy benchmarks: West Texas Intermediate (WTI) crude oil, Henry Hub (HH) natural gas, and ultra-low sulfur diesel (ULSD) heating oil. Each model is calibrated in two steps, first to plain vanilla options (PVOs) on individual futures, then to CSO quotes to capture the correlation structure. The results show that stochastic volatility models provide superior performance in fitting PVO and CSO prices.

## Calendar Spread Options

We analyze energy benchmarks for crude oil, natural gas, and heating oil. The futures term structure of each commodity reflects a combination of factors specific to its production, usage, and market dynamics: crude oil prices are heavily influenced by production forecasts, natural gas exhibits pronounced seasonal effects, and heating oil is affected by both factors. CSOs are written on the price difference between two futures contracts on the same asset with different maturities and offer a leveraged instrument for hedging or speculating on changes in the shape of the futures term structure.

$F(t,T)$ denotes the futures price observed at time $t$ with maturity at $T$, $f(t,T) \equiv \ln F(t,T)$ is the log-price. $T_1$ and $T_2$ denote the maturities of two futures contracts written on the same asset. The calendar spread $s(t,T_1,T_2)$ at time $t$ is defined as the difference in prices between the shorter and longer maturity contracts: $s(t,T_1,T_2) \equiv F(t,T_1) - F(t,T_2)$ . $T_0$ is the option expiration date, with $0 < T_0 < T_1 < T_2$. The arbitrage-free price at time $t$ of a European CSO with strike $K$ is given by:

$$P_{CSO}(t,T_0,T_1,T_2,K) = P(t,T_0)\mathbb{E}_{T_0}^{Q}[max(\varphi \cdot (s(T_0,T_1,T_2) - K),0)],$$

where $P(t,T_0)$ denotes the $t$-price of a zero-coupon bond maturing at $T_0$, and $\varphi = 1\ (-1)$ corresponds to a call (put).

We consider a pool of seven models from literature, where the underlying asset is quoted in the form of a futures contract. The models are Black (1976), Merton (1976), Heston (1993), Bates (1996), and Schneider and Tavin (2018). Their univariate futures price dynamics are summarized in Table 1 on the next page.

For $j = 1,2$ and in addition to futures price $F(t,T_j)$, $V_t$ denotes the common variance process, $W_t^{F_j}$ and $W_t^V$ the Brownian motions associated with the futures price and the variance, $N_t$ a Poisson process, and $J_F$ the jump size. Key parameters include the futures price volatility $\sigma_F > 0$, $\mathbb{E}\left[dW_t^{F_1}, dW_t^{F_2}\right] = \varphi dt, \varphi \in [-1,1]$, the variance mean-reversion speed $\kappa > 0$, the long-run variance level $\theta > 0$, the variance volatility $\sigma_v > 0$, the jump arrival intensity $\lambda > 0$, and the jump size distribution parameters $\mu_J \in \mathrm{R}$ and $\sigma_J \in R^+$.

**Table 1**
**Univariate Future Price Dynamics**

| Model | Dynamics | Volatility | Jumps |
|---|---|---|---|
| Bla76 | $\frac{dF(t,T)}{F(t,T)} = \sigma_F dW_t^F$ | $\sigma_F$ constant | |
| Mer76 | $\frac{dF(t,T)}{F(t,T)} = -\lambda \mathbb{E}_t^{\mathbb{Q}}\left[e^{J_F}-1\right]dt + \sigma_F dW_t^F + \left(e^{J_F}-1\right)dN_t$ | $\sigma_F$ constant | $J_F \sim \mathcal{N}(\mu_J, \sigma_J^2)$ |
| Hes93 | $\frac{dF(t,T)}{F(t,T)} = \sigma_F \sqrt{V_t} dW_t^F$ | $\sigma_F$ constant | |
| | $dV_t = \kappa\left(\theta - V_t\right)dt + \sigma_V \sqrt{V_t} dW_t^V$ | $\sigma_V$ constant | |
| Bat96 | $\frac{dF(t,T)}{F(t,T)} = -\lambda \mathbb{E}_t^{\mathbb{Q}}\left[e^{J_F}-1\right]dt + \sigma_F \sqrt{V_t} dW_t^F + \left(e^{J_F}-1\right)dN_t$ | $\sigma_F$ constant | $J_F \sim \mathcal{N}(\mu_J, \sigma_J^2)$ |
| | $dV_t = \kappa\left(\theta - V_t\right)dt + \sigma_V \sqrt{V_t} dW_t^V$ | $\sigma_V$ constant | |
| ST18 | $\frac{dF(t,T)}{F(t,T)} = \sum_{i=1}^{n} \sigma_{F_i}(t,T)\sqrt{V_{i,t}} dW_t^{F_i}$ | $\sigma_{F_i}(t,T) = \alpha_i e^{-\gamma_i(T-t)}$ | |
| | $dV_{i,t} = \kappa_i\left(\theta_i - V_{i,t}\right)dt + \sigma_{V,i}\sqrt{V_{i,t}} dW_t^{V_i}$ | $\sigma_{V_i}$ constant | |

Note: Bla76, Mer76, Hes93, Bat96, and ST18 stand for the models in Black (1976), Merton (1976), Heston (1993), Bates (1996), and Schneider and Tavin (2018), respectively.

---

**Group 1** models are uni-factor models (Bla76, Mer76, Hes93, Bat96). In the bivariate case, these models have dynamics given in the general form by:

$$\frac{dF(t,T_j)}{F(t,T_j)} = \sigma_{F_j}\sqrt{V_t}\, dW_t^{F_j} - \lambda_j \mathbb{E}^{\mathbb{Q}}\left[e^{J_F}-1\right]dt + \left(e^{J_F}-1\right)dN_t, \quad dV_t = \kappa(\theta - V_t)dt + \sigma_V\sqrt{V_t}\, dW_t^V,$$

with $\mathbb{E}\left[dW_t^{F_j}, dW_t^V\right] = \varphi_{FV}dt, \varphi_{FV} \in [-1,1]$ for both futures, $J_F \sim \left(\mu_J, \sigma_J^2\right)$, $F(t,T_1) \neq F(t,T_2) > 0$, and $V_0 > 0$. Uni-factor models have i.i.d. increments and multi-factor models incorporate stochastic volatility.

**Group 2** models feature the Samuelson effect (that is, a time-dampening price volatility) (see Trolle and Schwartz, 2009; Schneider and Tavin, 2018; Crosby and Frau, 2022). Our framework can accommodate all Group 2 models. For illustration, we focus on the ST18 model, about which the bivariate dynamics are given by:

$$\frac{dF(t,T_j)}{F(t,T_j)} = \sigma_{F_j}(t,T_j)\sqrt{V_t}\, dW_t^{F_j}, \quad dV_t = \kappa(\theta - V_t)dt + \sigma_V\sqrt{V_t}\, dW_t^V.$$

We define the vectors $\boldsymbol{u} \equiv (u_1, u_2) \in R^2$, $\boldsymbol{T} \equiv (T_1, T_2)$, and $\boldsymbol{F} \equiv \boldsymbol{F}(T_0, \boldsymbol{T}) \equiv \left(F(T_0,T_1), F(T_0,T_2)\right)'$. We consider the transform involving the prices of the two futures forming the spread, which is given by:

$$\Phi_{\mathbf{F}}(\mathbf{u}) \equiv \mathbb{E}_t^{\mathbb{Q}}\left[e^{i\mathbf{u}\cdot\ln \mathbf{F}(T_0,\mathbf{T})}\right] = \mathbb{E}_t^{\mathbb{Q}}\left[e^{iu_1 \ln F(T_0,T_1) + iu_2 \ln F(T_0,T_2)}\right].$$

Caldana and Fusai (2013) provide a robust and computationally tractable alternative to more complex inversion techniques. The lower bound for the $t$ −time price of a CSO call is given by:

$$
\begin{aligned}
LB_{\text{CSO}}(t, T_0, T_1, T_2, K) &= \frac{P(t, T_0)}{\pi}\, e^{-\delta k} \int_0^{\infty} e^{-iuk} \Psi_{\mathbf{F}}(u)\, du, \\
\Psi_{\mathbf{F}}(u) &= \frac{e^{i\eta \ln \Phi_{\mathbf{F}}(0, -i\alpha)}}{i\eta} \left( \Phi_{\mathbf{F}}(\eta - i, -\alpha\eta) - \Phi_{\mathbf{F}}(\eta, -\alpha\eta - i) - K \Phi_{\mathbf{F}}(\eta, -\alpha\eta) \right), \\
k = \ln\left(F(t, T_2) + K\right), &\qquad \eta = u - i\delta, \qquad \alpha = \frac{F(t, T_2)}{F(t, T_2) + K},
\end{aligned}
$$

and $\delta$ is a damping parameter to ensure integrability in the Fourier space. To evaluate the CSO price, one must define the JCF according to the model. For our models, the JCF admits an exponential-affine form. The functions appearing in the exponent are solutions to Riccati ordinary differential equations (ODEs).

**Proposition 1.** *Assuming a general affine process, the JCF of the futures log- prices in spread is given by*:

$$
\mathbb{E}_t^{\mathbb{Q}}\left[ e^{iu_1 f(T_0, T_1) + iu_2 f(T_0, T_2)} \right] = \exp\left\{ iu_1 f(t, T_1) + iu_2 f(t, T_2) + A(T_0 - t; u_1, u_2) + C(T_0 - t; u_1, u_2) V_t \right\}.
$$

Transform techniques relying on JCFs may exhibit slowness and numerical inaccuracy when the evaluation involves special functions (*e.g.*, ST18). For ST18, log-prices are jointly normal conditional on a path of the Brownian motion driving the variance process, which permits the use of conditional Monte Carlo (MC) simulation. We detail the conditioning arguments specific to ST18 in Proposition 2.

**Proposition 2.** *In the ST18 model, $F(T_0, T_j)$ has a conditional normal distribution, with mean*:

$$
\begin{aligned}
& f\left(t, T_j\right) - \frac{\alpha_j \rho_{FV} \kappa \theta}{\sigma_V \gamma_j} e^{-\gamma_j T_j} \left( e^{\gamma_j T_0} - e^{\gamma_j t} \right) + \frac{\alpha_j \rho_{FV}}{\sigma_V} e^{-\gamma_j T_j} \left( e^{\gamma_j T_0} V_{T_0} - e^{\gamma_j t} V_t \right) \\
& - \frac{\alpha_j^2}{2} \int_t^{T_0} \left( e^{-2\gamma_j (T_j - s)} - \frac{2\rho_{FV} \kappa}{\alpha_j \sigma_V} e^{-\gamma_j (T_j - s)} + \frac{2\rho_{FV} \gamma_j}{\alpha_j \sigma_V} e^{-\gamma_j (T_j - s)} \right) V_s\, ds.
\end{aligned}
$$

*The conditional covariance between the log-prices of futures for delivery at times $T_i$ and $T_j$ is given by*:

$$
\alpha_i \alpha_j \rho \left(1 - \rho_{FV}^2\right) \int_t^{T_0} e^{-\gamma_i (T_i - s)} e^{-\gamma_j (T_j - s)} V_s\, ds.
$$

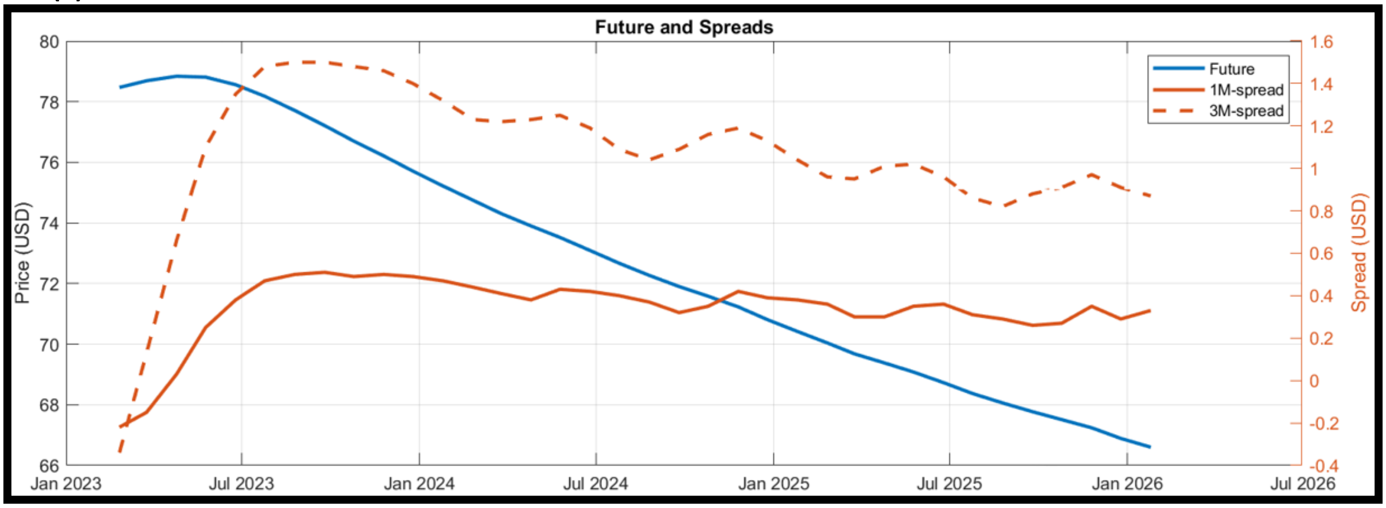
### Empirical Investigation and Model Calibration

Our market dataset consists of futures prices and their corresponding PVOs, spreads on futures and the corresponding CSOs, all written on three energy benchmarks: WTI light sweet crude oil, HH natural gas, and ULSD heating oil. These contracts are listed on NYMEX and quoted in USD. We use official market data from this exchange as of February 8, 2023. In terms of options, we exclude in-the-money (ITM) options.

**WTI**: this dataset consists of ten one-month (1M) futures spreads and the corresponding 1M CSOs; this market is predominantly in backwardation. **HH**: this dataset includes one three-month (3M) spread and the corresponding 3M CSOs; this market is in contango, with a strong seasonal component. **ULSD**: this dataset consists of two 1M spreads and the corresponding 1M CSOs; this market is in backwardation. Figure 1 below represents the term structures of the three benchmarks as of February 8, 2023.

**Figure 1**
**Term-Structures – Futures Prices and Spreads**

**(a) WTI**




**(b) HH**

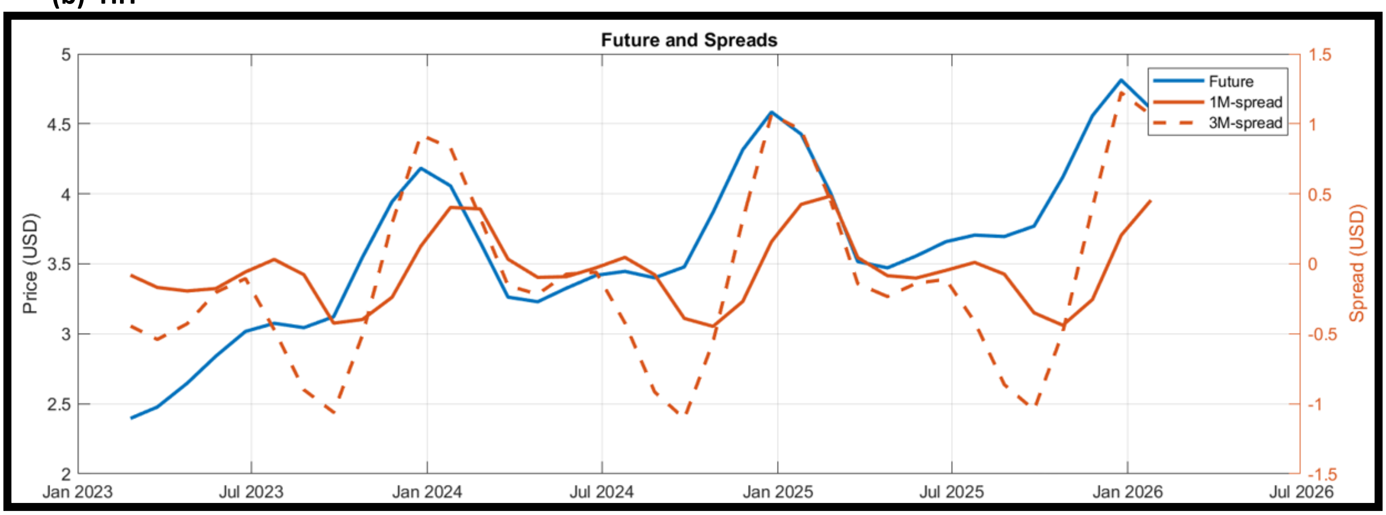
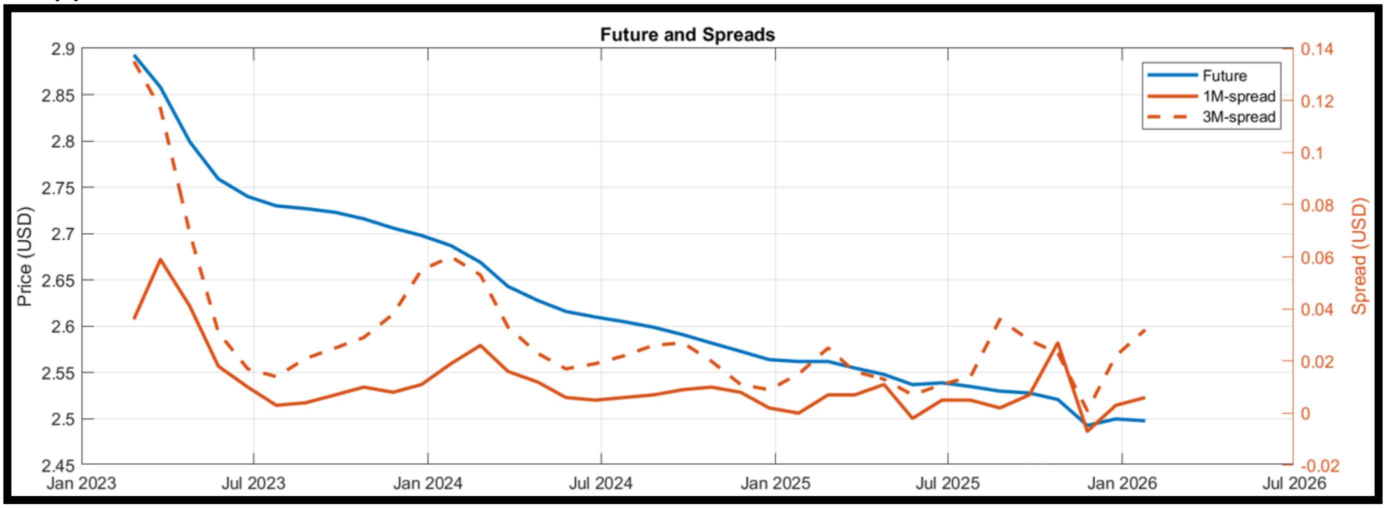


**Figure 1**
**Term-Structures – Futures Prices and Spreads (Continued)**

**(c) ULSD**




We implement a calibration procedure between market and model option prices, in a two-step process. In the **first step**, we calibrate all parameters except for the correlation between the two futures forming the spread. The objective function is constructed using PVOs prices on the futures in the spread; it reads:

$$\min_{\bar{\sigma}_1,\bar{\sigma}_2,\phi}\left\{\sum_{i=1}^{N_1}\left[\frac{p_{\mathrm{PVO}}(K_i,T_1)-\hat{p}_{\mathrm{PVO}}(K_i,T_1;\bar{\sigma}_1,\phi)}{\mathcal{V}^2(K_i,T_1)}\right]^2+\sum_{i=1}^{N_2}\left[\frac{p_{\mathrm{PVO}}(K_i,T_2)-\hat{p}_{\mathrm{PVO}}(K_i,T_2;\bar{\sigma}_2,\phi)}{\mathcal{V}^2(K_i,T_2)}\right]^2\right\},$$

where $p_{PVO}$ and $\hat{p}_{CSO}$ denote market and model option prices, $N_1$ and $N_2$ are the number of strikes for the first and second futures, $\overline{\sigma_1}$ and $\overline{\sigma_2}$ refer to volatility parameters, and $\Phi$ is the vector of remaining parameters. In the **second step**, we calibrate the price correlation ρ, the objective function reads:

$$\min_{\rho}\sum_{i=1}^{N}\left[p_{\mathrm{CSO}}(K_i,T_1,T_2)-\hat{p}_{\mathrm{CSO}}(K_i,T_1,T_2;\bar{\sigma}_1,\bar{\sigma}_2,\phi,\rho)\right]^2,$$

where $N$ is the number of strikes for a given spread. As a proxy for $\hat{p}_{CSO}$ we use the very accurate lower bound $LB_{CSO}$ for Group 1 models, computed using Gauss–Kronrod quadrature. For ST18 we adhere to the conditional MC approach. To assess model fit, we compute the root mean squared error (RMSE).

## Model Results and Analysis

The resulting price correlation $\rho$ estimates are reported in Table 2 on the next page. We observe that $\rho$ from WTI CSO prices is equal to 1 across all contracts in Group 1 models; for HH and ULSD, it slightly differs across models and is at or above 0.90.

### Table 2
**Calibrated Correlation**

| ρ | WTI | | | | | | | | | | HH | ULSD | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Spr.1 | Spr.2 | Spr.3 | Spr.4 | Spr.5 | Spr.6 | Spr.7 | Spr.8 | Spr.9 | Spr.10 | Spr.8 | Spr.1 | Spr.2 |
| **Bla76** | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0.915 | 0.935 | 0.920 |
| **Mer76** | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0.965 | 0.995 |
| **Hes93** | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0.900 | 0.925 | 0.905 |
| **Bat96** | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0.935 | 0.960 |
| **ST18** | 1 | 0.965 | 0.960 | 0.965 | 0.965 | 0.965 | 1 | 1 | 0.900 | 0.0965 | 1 | 0.935 | 0.915 |

Table 3 presents the RMSE for PVOs: stochastic volatility models offer superior performance in fitting options. Table 4 presents the RMSE for CSOs: Heston (1993) consistently delivers the lowest errors across contracts, and Schneider and Tavin (2018) perform comparably.

### Table 3
**PVO Pricing Errors**

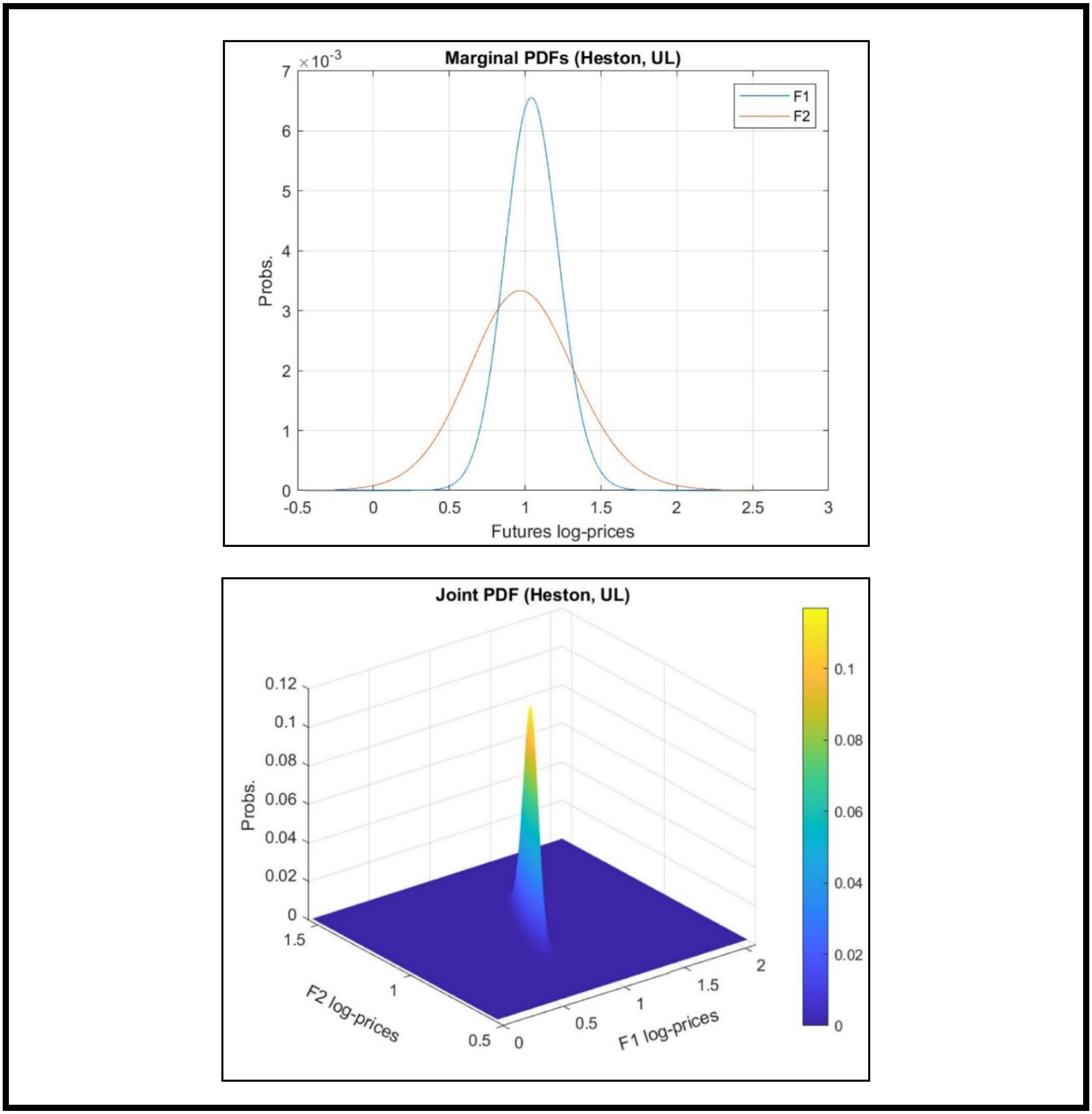
| RMSE | WTI | | | | | | | | | | HH | ULSD | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Spr.1 | Spr.2 | Spr.3 | Spr.4 | Spr.5 | Spr.6 | Spr.7 | Spr.8 | Spr.9 | Spr.10 | Spr.8 | Spr.1 | Spr.2 |
| **Bla76** | 0.0983 | 0.1384 | 0.1546 | 0.1679 | 0.1835 | 0.1963 | 0.2014 | 0.2039 | 0.2232 | 0.2321 | 0.0425 | 0.0012 | 0.0042 |
| **Mer76** | 0.0108 | 0.0120 | 0.0107 | 0.0096 | 0.0100 | 0.0082 | 0.0059 | 0.0076 | 0.0089 | 0.0092 | 0.0020 | 0.0004 | 0.0004 |
| **Hes93** | 0.0077 | 0.0031 | 0.0022 | 0.0027 | 0.0027 | 0.0022 | 0.0024 | 0.0024 | 0.0029 | 0.0056 | 0.0235 | 0.0003 | 0.0004 |
| **Bat96** | 0.0035 | 0.0028 | 0.0023 | 0.0027 | 0.0027 | 0.0022 | 0.0024 | 0.0023 | 0.0028 | 0.0061 | 0.0017 | 0.0007 | 0.0002 |
| **ST18** | 0.0077 | 0.0031 | 0.0022 | 0.0027 | 0.0027 | 0.0022 | 0.0024 | 0.0022 | 0.0029 | 0.0056 | 0.0101 | 0.0003 | 0.0003 |

### Table 4
**CSO Pricing Errors**

| RMSE | WTI | | | | | | | | | | HH | ULSD | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Spr.1 | Spr.2 | Spr.3 | Spr.4 | Spr.5 | Spr.6 | Spr.7 | Spr.8 | Spr.9 | Spr.10 | Spr.8 | Spr.1 | Spr.2 |
| **Bla76** | 0.0757 | 0.0424 | 0.0230 | 0.1379 | 0.0595 | 0.0328 | 0.0516 | 0.0395 | 0.0577 | 0.0449 | 0.0264 | 0.0019 | 0.0006 |
| **Mer76** | 0.1325 | 0.4888 | 0.0230 | 1.1636 | 1.3749 | 1.6885 | 3.4690 | 2.4925 | 3.8310 | 5.1665 | 0.0486 | 0.0046 | 0.0198 |
| **Hes93** | 0.0177 | 0.0283 | 0.0146 | 0.0217 | 0.0451 | 0.0236 | 0.0443 | 0.0368 | 0.0496 | 0.0383 | 0.0166 | 0.0017 | 0.0025 |
| **Bat96** | 0.0248 | 0.0282 | 0.0130 | 0.1008 | 0.1166 | 0.0995 | 0.0681 | 0.2931 | 0.0604 | 0.1204 | 0.0389 | 0.0018 | 0.0041 |
| **ST18** | 0.0002 | 0.0015 | 0.0008 | 0.0045 | 0.0159 | 0.0148 | 0.0353 | 0.0511 | 0.0050 | 0.0106 | 0.0152 | 0.0020 | 0.0022 |

Figure 2 (illustratively for Hes93 on ULSD only) displays the marginal and joint probability densities of the futures in the spreads.

**Figure 2**
**Futures Log-Price Densities Under Hes93 (ULSD)**



## Conclusion

We derive expressions for the JCF of futures log-prices under a broad class of affine models, accommodating stochastic volatility, jumps, and time-dampening features. In addition, we introduce a novel methodology based on conditioning arguments for pricing CSOs under advanced stochastic volatility models such as ST18, where computing the JCF poses significant computational challenges. Our unified approach enables model calibration and provides a tractable, consistent framework for pricing CSOs on energy commodities. We carry out an empirical analysis on three energy benchmarks: WTI crude oil, HH natural gas, and ULSD heating oil. We calibrate each model to PVO prices and to CSO quotes in a two-step procedure. Results show that stochastic volatility models offer superior performance in fitting PVO prices. When evaluating CSO pricing errors, Hes93 and ST18 deliver the best performance.

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## Endnote

Carme Frau presented on this topic at the Finance Forum 2022, Energy Finance Italy EFI8 2023, UIB's DEE research seminar 2023, CEMA annual meeting 2023, II Jornada de Mercados Financieros del Banco de España 2023, Energy Finance Italy EFI9 2024, 31st Global Finance Conference 2024, and QUANT research seminar at Emlyon Business School 2024.

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## Keywords

Energy commodities, bivariate models, joint characteristic function, estimation, calendar spread option.

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