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Optimal Selling Mechanisms With Endogenous Seller Outside Offers

Xiaogang Che[1] | Tong Li[2] | Jingfeng Lu[3] | Xiaoyong Zheng[4]

[1]City St George's, University of London, London, UK | [2]Vanderbilt University, Nashville, Tennessee, USA | [3]National University of Singapore, Singapore | [4]North Carolina State University, Raleigh, North Carolina, USA

Correspondence: Tong Li (tong.li@vanderbilt.edu)

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ABSTRACT

We examine a two-stage selling mechanism design problem, where the buyer makes her report and the seller endogenously decides his effort (hidden investment) to generate a possibly better outside offer. The optimal mechanism shows that the seller's effort depends on the reported value of the buyer; a higher value lowers the seller's incentive to invest in the outside offer. After the price of the outside offer is realized, if the buyer's virtual value is less than the price, the seller takes the outside offer, and a termination fee equal to the virtual value is paid to the buyer.

JEL Classification: D44, G33, G34, D82, D86, K22

1 | Introduction

It is common that after a target firm (the seller) and an acquiring firm (the buyer) agree to a merger and acquisition (M&A) deal, the procedure still takes up to a few months to complete (see Bhagwat et al. 2016). During the time, the target firm may not commit to the proposed deal but choose to invest in searching a possibly better outside offer. In this case, the deal is vulnerable to the possibility that the seller will renege on the original transaction if the outside offer turns out to be superior. At the same time, to protect the prospective buyer's interest, the seller is required to promise a certain amount of compensation, called a termination fee,[1] which entails a contingent payment to the buyer when the deal is terminated by the seller. For example, Taqa India Power Ventures paid a \$9 million breakaway fee to Jaiprakash Power to cancel the M&A deal, because "it could get a better deal.…" Another example is that when LinkedIn was acquired by Microsoft, if LinkdIn had accepted an outside offer from Salesforce, it would have paid Microsoft \$725 million. Nowadays, it has become a popular practice to include termination fees in the deal agreements. According to Jeon and Ligon (2011), approximately 73.80% of all M&A announced between January 2001 and December 2007 included termination fee provisions. Moreover, the fees exhibited significant variation in size; on average, the size of a termination fee ranged from 2% to 4% of the deal value.

These practical situations naturally lead to the following research questions. The first question is whether it is optimal for the seller (the target firm) to fully commit to the original transaction or to invest in exploring an outside offer. The former would encourage the current buyer (the acquiring firm) to make a higher payment in the original transaction and the latter gives the seller a chance to potentially obtain a better price. Therefore, it remains unclear which choice is better for the seller. If the seller prefers exploring, the second question concerns how much costly effort he should exert in generating a better outside offer; in particular, such an investment will be contingent on the offer made by the current buyer. Furthermore, if the outside offer turns out to be better and the seller decides to terminate the deal, it naturally leads to the questions of whether a termination fee should be promised to the buyer and what the optimal size of the fee is. Answering these questions helps us understand why sellers agree to include termination fees and have incentives to search and default after the M&A deals.

To shed light on these research questions, this paper studies a two-stage optimal selling mechanism design problem with an endogenously determined ex post seller outside offer. Specifically, we investigate the following class of direct and truthful mechanisms.[2] In the first stage, a buyer with her private value makes a report. Then, the seller, who owns an object and would like to maximize revenue, decides whether to sell. If the seller decides to sell, he does not invest or learn about his outside offer, and the object is allocated to the buyer. If, however, the seller decides not to sell, the game moves to the next stage. In the second stage, the seller first decides how much to invest in generating a better distribution of the price of the outside offer. After that, the price of the outside offer is realized, which is a random draw from the distribution. The seller decides whether to still sell the object to the buyer or to take the outside offer. The related transfers are then implemented. Particularly, when the outside offer is taken, a termination fee may be paid by the seller.

The characterization of the optimal selling mechanism shows that the seller should not fully commit to the original transaction in the first stage, which indicates that exploring the outside offer is better for the seller. In the second stage, the optimal effort invested by the seller depends on the buyer's report in the first stage. The seller has a higher incentive to invest when observing a lower value report from the current buyer. After learning the price of the outside offer, it is optimal to allocate the object to the buyer if the buyer's virtual value is greater than the price of the outside offer. If, however, the value is less than the price, then the seller should take the outside offer. Moreover, a termination fee should be paid to the buyer. The intuition is that although it is costly for the seller, a termination fee has a compensation effect that reinforces the payment by the buyer in the first stage. We further examine the optimal choice of the termination fee. If the termination fee is too high, then the seller will not have any chance to take the outside offer, and that is equivalent to full commitment in the first stage. If the fee is too low, it will discourage the first-stage payment by the buyer. Our analysis shows that the optimal size of the termination fee should be type-dependent, which is equal to the virtual value of the buyer. In addition, we characterize the efficient mechanism in terms of social welfare maximization and then compare it with the optimal selling mechanism. We find that both the allocation and effort rules in the optimal selling mechanism induce detrimental effects on social optimality, generating inefficiency.

Our study contributes to the literature on sequential search in auction design. McAfee and McMillan (1988) examine search mechanisms with N periods, in which the seller in each period needs to incur a search cost to invite a potential buyer, showing that the optimal search mechanism is that in each period the seller chooses to sell if the buyer's reported value is no less than a cutoff value. If no buyer has a value above the cutoff, the seller runs a second-price auction with a reserve price. Crémer et al. (2007), considering a similar setting but with more general assumptions where all the buyers are ex ante asymmetric in the cost and the distribution of their private values, find that the optimal mechanism is equivalent to an optimal search with symmetric information, where the utilities are replaced by virtual utilities. Moreover, the optimal mechanism involves fewer participants, longer search conditional on the same set of participants, and an inefficient sequence of entry, in contrast to the socially efficient mechanism. Crémer et al. (2009) investigate the optimal selling mechanism in which buyers sequentially enter and need to incur costs to learn their private values, showing that the optimal mechanism includes a series of reserve prices in the periods and the seller can still extract full surplus as if he had full control of bidders' information acquisition.

The focus of the literature is on the dynamic trade-off between selling the object in the current period and searching for a second offer in later periods. Our paper contributes to the relevant literature by investigating an endogenous investment decision by the seller in the distribution of the ex post outside offer and its impacts on the design of the revenue-maximizing mechanism. Our results show that the seller's ex post searching effort decreases in the current offer by the buyer, and it is optimal to provide a type-dependent compensation to the buyer when the seller takes the outside offer.

Our study is also closely related to the literature on the role of termination fees in M&A deals. Typically, in M&A deals, a termination fee is an amount that the target firm will pay if the deal fails to be completed, particularly because of the arrival of a better offer for the seller.[3] The literature mainly focuses on questions of whether termination fees should be allowed in the M&A deals and what the optimal size of the termination fees should be.[4] Ayres (1990) examines the role of lockup fees, which are used to protect bidders against competing bids between the signing date and the closing date of a M&A deal. Ayres shows that allocation efficiency is irrelevant to lockup fees in takeover auctions. Differently, other studies view termination fees as a mechanism for target firms to signal and commit to the end of bidding competitions, and that in turn promotes participation by bidders in the initial stage. See Fraidin and Hanson (1994); Rothkopf et al. (2003), and Povel and Singh (2006).

Furthermore, Che and Lewis (2007) examine the impacts of both breakup fees and stock fees on bidder participation in takeover auctions with costly entry, showing that breakup fees are more socially preferred in terms of providing a desirable degree of competition and allocating the objects (the target firms) efficiently, but the target firms would prefer stock fees because of lower costs. Bulow and Klemperer (2009) also consider a bidding competition model with costly entry and suggest that the role of a lockup fee is like a combination of a subsidy to a current entrant and a promise to prohibit subsidies to future bidders, which makes sequential mechanisms to become more efficient and profitable than auctions when selling an object (a firm or an asset).[5] Che et al. (2010) show that when bidders have interdependent valuations on the target firm, using a breakup fee to subsidize entry of a subsequent bidder can overdiscipline the initial bidder's preemption and result in excessive entry by a sequential bidder.[6]

Our study provides an additional rationale for the sellers' (the target firms') adoption of termination fees in the M&A deals, which is one of the fundamental research questions in the literature. In contrast to prior studies, which emphasize costly entry, our analysis focuses on the role of termination fees with investments in the after-market outside offers for the sellers; that is, target firms often choose not to consummate the M&A agreements but walk away for better outside offers.[7] Moreover, the characterization of the optimal size of a termination fee shows that it should be type-dependent and equal to the virtual value of the buyer. This result provides an alternative explanation for why different levels of termination fees are implemented in M&A deals.

The rest of the paper is organized as follows. In Sections 2–3, we present the model setup and analyze the optimal selling mechanism and the socially efficient mechanism, when the seller can endogenously decide how much to invest in the ex post outside offer. Section 4 concludes. All the proofs are relegated to Appendix I. (Online) Appendix II discusses extensions and robustness of our results when some important assumptions are relaxed.

2 | The Model

We here consider a class of direct mechanisms with two stages of buyer value reporting and arrival of a future outside offer for the seller. The seller (He) would like to sell an asset to a buyer (She). The seller's reservation value of the asset is normalized to zero. The buyer's private value, denoted by v, is a random draw from a cumulative distribution function $F(\cdot)$ with density $f(\cdot) > 0$ over the support of $[0, \bar{v}]$, where $\overline{v} > 0$.

In the *first* stage, the buyer is asked to report her value. We use v' to denote the buyer's report. The probability that the object is sold to the buyer who reports v' is denoted by $q_1(v') \in [0, 1]$, and the associated payment to the seller is denoted by $m_1(v') \in \mathbb{R}$. We assume that F is common knowledge among the seller and the buyer, which is *regular* in the sense that the hazard rate $\frac{f(\cdot)}{1-F(\cdot)}$ is increasing.

After the first-stage reporting by the buyer, the seller endogenously decides the effort level θ for the cumulative distribution function of the price of the outside offer p, which takes the following form:[8]

$$H(p, \theta), \text{ with } p \in [0, \bar{v}], \ H_\theta < 0 \text{ and } H_{\theta,\theta} > 0,$$

where H_θ ($H_{\theta,\theta}$) denotes the first (second) derivative of H with respect to θ.[9] We denote the density of $H(p, \theta)$ by $h(p, \theta) = H_p(p, \theta)$. Clearly, a higher θ generates a better distribution of the outside offer from the seller's perspective. We denote the cost of effort θ by $C(\theta)$ with $C(0) = 0$.

The *second* stage of the mechanism can be described as follows: the seller observing the report v' decides the effort level θ which generates $H(p, \theta)$. Then, the outside offer with price p arrives, where p is a random draw from $H(p, \theta)$. The seller makes a report p'. Conditional on the object being unsold in the first stage, the probability of the object being sold in the second stage to the buyer is denoted by $q_2(v', p') \in [0, 1]$ and the associated payment to the seller $m_2(v', p') \in \mathbb{R}$ is implemented. We here assume that the seller commits to the mechanism, and this is run by an ex post budget-balanced escrow agent (i.e., the M&A lawyer team), and the associated payments between the seller and the initial buyer are implemented through the agent.

The design problem consists of $\{q_1(v'), m_1(v'); \theta(v'), q_2(v', p'), m_2(v', p')\}$, where v' and p' are the reports from the buyer and the seller. We here seek a *truthful reporting* incentive-compatible mechanism that maximizes the seller's expected revenue. Specifically, we require that the buyer reports truthfully in the first stage and the seller reports truthfully in the second stage, and the seller's investment is incentive compatible. It is without loss of generality to examine the M&A procedure as the direct mechanism design problem.

Remark 1. The considered class of direct mechanisms includes the following two-stage M&A procedure as a special case: in the first stage, the buyer with private value v makes her payment; the seller in the second stage, after investing effort θ and learning his outside offer p, decides to either complete the original transaction or terminate it by paying a termination fee and then take the outside offer.

We would like to point out that the seller's investment θ is a hidden action in our model. Therefore, the seller does not simply commit to $\theta(v')$. Instead, $\theta(v')$ must be an incentive-compatible choice of the seller based on commitment in the second stage and the buyer's report in the first stage of the mechanism. In the following, we analyze the design problem.[10]

3 | Designs of the Seller-Optimal and Socially Efficient Mechanisms

We first derive the constraints on the allocation rules that make *truthful reporting* incentive compatible for the seller in the second stage of the mechanism (Subsection 3.1) and for the buyer in the first stage of the mechanism (Subsection 3.2). Then, we characterize the allocation and payment rules that maximize the seller's expected revenue subject to these constraints (Subsection 3.3). Finally, we characterize the allocation and payment rules that are socially optimal (Subsection 3.4).

3.1 | Incentive Compatibility in the Second Stage

Given that the object is unsold in the first stage and the seller makes his investment decision θ in the second stage, the seller's revenue by reporting p' in the second stage, denoted by $R_2(p'; p, v)$, can be written as follows:

$$R_2(p'; p, v) = (1 - q_2(v, p'))p + m_2(v, p'). \quad (1)$$

The payment from the buyer in the first stage is already sunk, and therefore, $R_2(p'; p, v)$ can be interpreted as an *additional payoff* from reporting his outside offer in the second stage.

To guarantee the report by the seller to be truthful, incentive compatibility of the seller (**SIC**) requires, for any v,

$$R_2(p; p, v) \ \geq \ R_2(p'; p, v), \forall p, p'.$$

This condition indicates that any untruthful reporting cannot be beneficial for the seller. We then have the following lemma.

Lemma 1. *Given the buyer's first-stage report v and the seller's investment decision θ in the second stage, the second stage of the mechanism is incentive compatible if and only if the following conditions hold:*

i. *$q_2(v, p)$ is nonincreasing in p for any v;*
ii. *$R_2(p; p, v) = R_2(0; 0, v) + \int_0^p (1 - q_2(v, x))\, dx$ for any (v, p).*

Particularly, when $p = 0$, the seller does not take the outside offer for sure, and we therefore have $R_2(0; 0, v) = 0$, bringing no additional gain to the seller.

From the analysis above and together with $H(p, \theta)$, the seller's investment decision problem (before learning the price p of the outside offer) in the second stage can be written as follows:

$$\begin{aligned} & E_{(p|\theta)}\Big\{R_2(p; p, v)\Big\} \ - \ C(\theta) \\ = & \int_0^{\bar{v}} \Big\{R_2(0; 0, v) \ + \ \int_0^p (1 - q_2(v, x))dx\Big\} h(p, \theta) dp \ - \ C(\theta) \quad (2) \\ = & \int_0^{\bar{v}} \Big\{(1 - H(x, \theta))(1 - q_2(v, x))\Big\} dx \ + \ R_2(0; 0, v) \ - \ C(\theta), \end{aligned}$$

where the second equality is obtained as a result of interchanging the order of integration. The seller chooses θ to maximize (2); we call this constraint the investment decision constraint of the seller (**SID**). Note that $R_2(0; 0, v)$ does not depend on the effort level θ.

3.2 | Incentive Compatibility in the First Stage

We then consider the buyer with private value v reporting v' in the first stage. The buyer's interim expected payoff, denoted by π, is given by

$$\begin{aligned} \pi(v'; v) \ = \ q_1(v')v \ - \ m_1(v') \ + \ (1 - q_1(v'))E_{(p|\theta(v'))} \\ \Big\{q_2(v', p)v \ - \ m_2(v', p)\Big\}, \qquad (3) \end{aligned}$$

where the first two terms capture the expected payoff from the first stage, and the third term is the expected payoff from the second stage. To guarantee that the buyer has no incentive to lie about her private value, incentive compatibility of the buyer (**BIC**) requires

$$\pi(v; v) \ \geq \ \pi(v'; v), \forall v, v',$$

indicating that any untruthful reporting cannot be beneficial for the buyer in the mechanism. Individual rationality (**BIR**) requires that the buyer should not be better off by not participating, that is,

$$\pi(v; v) \ \geq \ 0.$$

In particular, the buyer with private value 0 must have $\pi(0; 0) \geq 0$. We say that the first stage is feasible when (**BIC**) and (**BIR**) are both satisfied.

Let us define ξ as follows:

$$\xi(v) \equiv \Big\{q_1(v) \ + \ (1 - q_1(v))E_{(p|\theta(v))}(q_2(v, p))\Big\},$$

which captures the expected probability that the asset is allocated to the buyer with private value v. We then have the following lemma.

Lemma 2. *The first stage of the mechanism is feasible if and only if the following conditions hold:*

i. *$\xi(v)$ is nondecreasing in v;*
ii. *$\pi(v; v) = \pi(0; 0) + \int_0^v \xi(t)dt$;*
iii. *$\pi(0; 0) \geq 0$.*

From Lemma 2, we can further write the ex ante expected payoff of the buyer, denoted by $\mathbb{B}$, as follows:

$$\begin{aligned} \mathbb{B} \ &= \ \int_0^{\bar{v}} \pi(v; v) f(v) dv \\ &= \ \int_0^{\bar{v}} \Big[\pi(0; 0) \ + \ \int_0^v \Big\{q_1(t) \ + \ (1 - q_1(t))E_{(p|\theta(t))}(q_2(t, p))\Big\} dt\Big] f(v) dv \\ &= \ \pi(0; 0) \ + \ \int_0^{\bar{v}} \int_0^v \big(q_1(t) \ + \ (1 - q_1(t))E_{(p|\theta(t))}(q_2(t, p))\big) dt f(v) dv. \end{aligned}$$

By interchanging the order of integration, the expected payoff function can be rewritten as follows:

$$\mathbb{B} \ = \ \pi(0; 0) \ + \ \int_0^{\bar{v}} \big(q_1(v) \ + \ (1 - q_1(v))E_{(p|\theta(v))}(q_2(v, p))\big) \left(\frac{1 - F(v)}{f(v)}\right) f(v) dv. \qquad (4)$$

3.3 | Seller Revenue and the Optimal Mechanism

Now we are ready to characterize features of the allocation rules and the associated payments in the two stages of the mechanism that maximize the seller's expected revenue. We denote the seller's overall expected revenue by $\mathbb{S}$ and the total expected surplus of the seller and the buyer by $\mathbb{T}$. Clearly, $\mathbb{T}$, which depends on the investment level by the seller, can be written as follows:

$$\begin{aligned} \mathbb{T} \ = \ \int_0^{\bar{v}} \{ & q_1(v)v + (1 - q_1(v)) \left[E_{(p|\theta(v))} (q_2(v, p)v \right. \\ & \left. + (1 - q_2(v, p)) \, p) - C(\theta(v))\right]\} f(v) dv, \qquad (5) \end{aligned}$$

where the first term in the curly brackets is the total surplus when the item is sold in the first stage and the second term in the curly brackets is the expected surplus from selling to the buyer or from taking the outside offer in the second stage.[11]

The seller's overall expected revenue is the difference between total surplus and the expected payoff of the buyer, which gives us the following equation:

$$\begin{aligned}\mathbb{S} &= \mathbb{T} - \mathbb{B} \\ &= \int_0^{\bar{v}} \left\{ q_1(v)v + (1-q_1(v))\left[E_{(p|\theta(v))}(q_2(v,p)v + (1-q_2(v,p))p) - C(\theta(v))\right] \right\} f(v)dv \\ &- \int_0^{\bar{v}} (q_1(v) + (1-q_1(v))E_{(p|\theta(v))}(q_2(v,p))) \left(\frac{1-F(v)}{f(v)} \right) f(v)dv - \pi(0;0) \\ &= \int_0^{\bar{v}} \left\{ (1-q_1(v))\left[E_{(p|\theta(v))}\left\{ q_2(v,p)\left(v - \frac{1-F(v)}{f(v)}\right) + (1-q_2(v,p))p \right\} - C(\theta(v)) \right] \right\} f(v)dv \\ &+ \int_0^{\bar{v}} q_1(v)\left(v - \frac{1-F(v)}{f(v)}\right) f(v)dv - \pi(0;0).\end{aligned} \tag{6}$$

The seller's objective is to implement a mechanism that maximizes (6) subject to the constraints of (**BIC**), (**BIR**), (**SIC**), as well as (**SID**). We provide a road map here that describes the steps we will follow to examine the maximization problem. In *Step 1*, we ignore all the above constraints and identify the optimal second- and first-stage allocation rules and the seller's search effort that maximize the seller's revenue in (6). In *Step 2*, we derive the second- and first-stage payment rules, and further show that the identified optimal allocation rules, payment rules, and seller search effort satisfy all the constraints.

Step 1. Let us first consider the second-stage allocation rule. After learning p, the seller's decision on whether to take the outside offer in the second stage is given by the relevant term in the first term of the last line of (6),

$$\begin{aligned}&\left\{ q_2(v,p)\left(v - \frac{1-F(v)}{f(v)}\right) + (1-q_2(v,p))p \right\} \\ &= \left\{ q_2(v,p)\left(v - \frac{1-F(v)}{f(v)} - p\right) + p \right\}.\end{aligned} \tag{7}$$

Therefore, it is best to sell to the buyer in the second stage if and only if the buyer's virtual value is no less than p. To do so, we would maximize (6) at *every* point v. Therefore, we have the following allocation rule (after learning the price of the outside offer) in the second stage:

$$\tilde{q}_2(v,p) = \begin{cases} 1 & \text{if } v - \frac{1-F(v)}{f(v)} \geq p; \\ 0 & \text{if } v - \frac{1-F(v)}{f(v)} < p. \end{cases} \tag{8}$$

Note that $\tilde{q}_2(v,p)$ is nonincreasing in p as required by part (i) of Lemma 1.

We then move to the examination of the optimal effort rule for the seller in determining the distribution of the outsider offer. The following lemma can be easily established.

Lemma 3. *Given the allocation rule (8), the relevant term about the seller's second-stage expected revenue in (6) can be simplified as,*

$$\begin{aligned}\zeta(v;\theta(v)) &= E_{(p|\theta(v))}\left\{ \tilde{q}_2(v,p)\left(v - \frac{1-F(v)}{f(v)}\right) + (1-\tilde{q}_2(v,p))p \right\} - C(\theta(v)) \\ &= \int_{\max\left\{v - \frac{1-F(v)}{f(v)},0\right\}}^{\bar{v}} (1-H(p,\theta(v)))dp + \max\left\{ v - \frac{1-F(v)}{f(v)}, 0 \right\} - C(\theta(v)).\end{aligned} \tag{9}$$

Define $D(v;\theta) \equiv [\int_{\max\left\{v-\frac{1-F(v)}{f(v)},0\right\}}^{\bar{v}} (1-H(p,\theta))\,dp - C(\theta)]$. Let us denote the seller's optimal effort by $\tilde{\theta}(v)$ such that $\tilde{\theta}(v) = \arg\max_{\theta\geq 0} D(v;\theta)$ and we assume that $\tilde{\theta}(v)$ is unique.[12] Clearly, (9) is maximized with $\tilde{\theta}(v)$. Given $H_\theta(p,\theta) < 0$ and $H_{\theta,\theta}(p,\theta) > 0$, for $\theta_1 < \theta_2$ and $v < v'$, we have

$$\begin{aligned}D(v';\theta_2) - D(v;\theta_2) &= -\int_{\max\left\{v-\frac{1-F(v)}{f(v)},0\right\}}^{\max\left\{v'-\frac{1-F(v')}{f(v')},0\right\}} (1-H(p,\theta_2))dp \\ &\leq -\int_{\max\left\{v-\frac{1-F(v)}{f(v)},0\right\}}^{\max\left\{v'-\frac{1-F(v')}{f(v')},0\right\}} (1-H(p,\theta_1))dp \\ &= D(v';\theta_1) - D(v;\theta_1),\end{aligned} \tag{10}$$

indicating that $D(v;\theta)$ is submodular in $(v;\theta)$. By Topkis' Theorem, it immediately implies that $\tilde{\theta}(v)$ is nonincreasing in v. Summarizing the discussion above helps us establish the following lemma.

Lemma 4. *The optimal effort rule $\tilde{\theta}(v)$ is nonincreasing in v.*

This analysis above shows how the seller's effort on generating a better distribution of the outside offer would depend on the reported value of the buyer. Consistent with the intuition, a higher private value lowers the seller's incentives to invest in searching for a better outside offer.

Next, we examine the allocation rule in the first stage. Let $\tilde{\zeta}(v) = \zeta(v;\tilde{\theta}(v))$. We have $\tilde{\zeta}(v) \geq \max\left\{v - \frac{1-F(v)}{f(v)}, 0\right\}$, since $\zeta(v;\theta(v)) \geq \max\left\{v - \frac{1-F(v)}{f(v)}, 0\right\}$ when $\theta(v) = 0$.[13] With optimal second-stage allocation rule $\tilde{q}_2(v,p)$ and optimal seller search effort $\tilde{\theta}(v)$, the seller revenue in (6) can be written as

$$\mathbb{S} = \int_0^{\bar{v}} (1-q_1(v))\tilde{\zeta}(v)f(v)dv + \int_0^{\bar{v}} q_1(v)\left(v - \frac{1-F(v)}{f(v)}\right)f(v)dv - \pi(0;0).$$

To determine the optimal $q_1(v)$, we simply need to compare $\tilde{\zeta}(v)$ and $v - \frac{1-F(v)}{f(v)}$. Let us denote v^M such that $v^M - \frac{1-F(v^M)}{f(v^M)} = 0$, which is Myerson's optimal reserve price. Clearly, the optimal $q_1(v)$ should be zero for $v \leq v^M$ in the first stage, since $v - \frac{1-F(v)}{f(v)} \leq 0$ and $\tilde{\zeta}(v) \geq 0$. We next consider the types with $v > v^M$. Recall $\tilde{\zeta}(v) \geq \max\left\{v - \frac{1-F(v)}{f(v)}, 0\right\} = v - \frac{1-F(v)}{f(v)}$ for these types. This means that the optimal $q_1(v)$ should also be zero for $v > v^M$. Therefore, for all the first-stage types, the allocation rule in the first stage is given by

$$\tilde{q}_1(v) = 0. \quad (11)$$

Step 2. We next establish an associated payment rule $\tilde{m}_2(v, p)$ in the second stage to support the second-stage allocation rule $\tilde{q}_2(v, p)$ using (1) and part (ii) of Lemma 1. We use $\tilde{R}_2(p; p, v)$ to denote the seller's second-stage payoff when the allocation rule is $\tilde{q}_2(v, p)$. Without loss of generality, we set $\tilde{R}_2(0; 0, v) = 0$.[14] Combining (1) and part (ii) of Lemma 1 gives

$$\begin{aligned}\tilde{m}_2(v, p) &= \tilde{R}_2(0; 0, v) + \int_0^p (1 - \tilde{q}_2(v, x))dx - (1 - \tilde{q}_2(v, p))p \\ &= \int_0^p (1 - \tilde{q}_2(v, x))dx - (1 - \tilde{q}_2(v, p))p.\end{aligned}$$

Plugging (8) into the equation above shows: If $v - \frac{1-F(v)}{f(v)} \geq p$, we have $\tilde{q}_2(v, p) = 1$, which gives $\tilde{m}_2(v, p) = 0$. If $v - \frac{1-F(v)}{f(v)} < p$, we have $\tilde{q}_2(v, p) = 0$ and

$$\tilde{m}_2(v, p) = \int_{v-\frac{1-F(v)}{f(v)}}^p 1dx - p = -\left(v - \frac{1-F(v)}{f(v)}\right).$$

Summarizing the discussion gives the associated payment rule as follows:

$$\tilde{m}_2(v, p) = \begin{cases} 0 & \text{if} \quad v - \frac{1-F(v)}{f(v)} \geq p, \\ -\max\left\{v - \frac{1-F(v)}{f(v)}, 0\right\} & \text{if} \quad v - \frac{1-F(v)}{f(v)} < p. \end{cases} \quad (12)$$

In summary, we have three cases. In the first case, where $0 < v - \frac{1-F(v)}{f(v)} < p$, $\tilde{q}_2(v, p) = 0$ and the seller chooses to take the better outside offer. At the same time, a payment, which is given by $\tilde{m}_2(v, p) = v - \frac{1-F(v)}{f(v)}$, is made from the seller to the buyer. In this case, the payment can be interpreted as a *termination fee*, which is the cost of taking the outside offer. Plugging (8) and (12) into (1) gives $\tilde{R}_2(p; p, v) = p - (v - \frac{1-F(v)}{f(v)})$, which is the seller's second-stage revenue in this case. In the second case where $v - \frac{1-F(v)}{f(v)} \leq 0 < p$, we have $\tilde{q}_2(v, p) = \tilde{m}_2(v, p) = 0$. In this case, the seller chooses to take the better outside offer without making a payment to the buyer, and his second-stage revenue is $\tilde{R}_2(p; p, v) = p$. In the third case where $v - \frac{1-F(v)}{f(v)} \geq p$, $\tilde{q}_2(v, p) = 1$. In this case, the seller continues to allocate the object to the buyer and therefore, there is no need to set up a termination fee, that is, $\tilde{m}_2(v, p) = 0$. Plugging (8) and (12) into (1) gives $\tilde{R}_2(p; p, v) = 0$, that is, the seller's second-stage revenue is 0 in this case.

We next examine the associated payment in the first stage of the mechanism, which can be similarly determined following standard procedure by using the first-stage envelope condition and the definition of the buyer's expected payoff function. The characterization is as follows. To save space, we relegate the proof to the Appendix.

Case I. When $v < v^M$ and $v - \frac{1-F(v)}{f(v)} < 0$, we have

$$\tilde{m}_1(v) = 0. \quad (13)$$

Case II. When $v \geq v^M$ and $v - \frac{1-F(v)}{f(v)} \geq 0$, we have

$$\begin{aligned}\tilde{m}_1(v) = vH\left(v - \frac{1-F(v)}{f(v)}, \tilde{\theta}(v)\right) - \int_{v_M}^v H\left(t - \frac{1-F(t)}{f(t)}, \tilde{\theta}(t)\right)dt \\ + \left(v - \frac{1-F(v)}{f(v)}\right)\left[1 - H\left(v - \frac{1-F(v)}{f(v)}, \tilde{\theta}(v)\right)\right],\end{aligned} \quad (14)$$

where the first two terms are the buyer's payment when the object is sold to the buyer, and the third term captures the positive impact of the termination fee on the buyer's payment when the seller takes the outside offer. Clearly, by promising a type-dependent termination fee, a seller can encourage the buyer to pay more aggressively in the first stage.

The associated payment above shows that only the buyer, whose private value is no less than v^M (in other words, her virtual value is no less than zero), will make a positive payment; otherwise, the buyer chooses not to participate (or make a zero payment) in the mechanism. The following lemma can, therefore, be easily established.

Lemma 5. *In the first stage of the mechanism,*

i. *if $v < v^M$, the buyer's payment $\tilde{m}_1(v)$ is zero and $\frac{d\tilde{m}_1(v)}{dv} = 0$;*
ii. *if $v \geq v^M$, the buyer's payment $\tilde{m}_1(v)$ is given by (14), and moreover, $\tilde{m}_1(v = v^M) = 0$ and $\frac{d\tilde{m}_1(v)}{dv} > 0$.*

Let us further check whether the mechanism rules characterized above satisfy the constraints. First, we note that clearly, the allocation rule (8) is nonincreasing in p. In addition, part (ii) of Lemma 1 is used to derive the optimal payment rule $\tilde{m}_2(v, p)$ above. These imply that the optimal second-stage allocation and payment rules (8) and (12) satisfy both conditions in Lemma 1 and hence (**SIC**) is satisfied.

Second, let us check whether the constraint (**SID**) can be satisfied with the optimal effort rule. Plugging the allocation rule (8) into (2) shows that (2) and (9) differ only by the term $\max\left\{v - \frac{1-F(v)}{f(v)}, 0\right\}$, which is not a function of θ. This implies that (**SID**) is satisfied with $\tilde{\theta}$.

Third, we examine whether the allocation rule satisfies the requirements in Lemma 2, and have the following lemma.

Lemma 6. *Constraints (**BIC**) and (**BIR**) are satisfied under allocation rule (11) and by setting $\pi(0, 0) = 0$.*

The characterization above shows that if the buyer accepts the termination fee, which is given by $\max\left\{v - \frac{1-F(v)}{f(v)}, 0\right\}$ in the second stage, then the buyer needs to make a payment of $\tilde{m}_1(v)$ in the first stage. Summarizing the discussion above gives us the main result.

Proposition 1. *The optimal mechanism* $\tilde{\mathbf{M}}$ *in the design problem can be characterized as follows:*

i. *In the first stage, the allocation rule* $\tilde{q}_1(v)$ *is given by (11) and the associated payment* $\tilde{m}_1(v)$ *is given by (13) and (14);*
ii. *In the second stage, the effort rule is given by* $\tilde{\theta}(v) = \arg\max_{\theta \geq 0} D(v;\theta)$*, and the allocation rule* $\tilde{q}_2(v, p)$ *and associated payment (termination fee)* $\tilde{m}_2(v, p)$ *are given by (8) and (12), respectively.*[15]

Although selling the object immediately in the first stage gives the buyer an incentive to increase her expected payment in the first stage, an investment in exploring the ex post outside offer may generate a better price to the seller in the next stage. This is the trade-off the seller faces, and our analysis indicates that the latter effect dominates the former effect; the seller should explore the ex post outside offer rather than sell the object immediately in the first stage. Moreover, the optimal effort level (Lemma 4) is decreasing in the true value of the buyer, indicating that the higher the reported value the buyer has, the lower the incentive the seller has to invest in searching for a better outside offer. Furthermore, it is beneficial to promise a type-dependent termination fee when the seller has a chance to explore the possibility of a better outside offer. The intuition is that when the buyer determines how much to pay, she will consider the possibility that the seller may take a better outside offer in the future, and that, in turn, lowers her willingness to pay. By promising to pay a termination fee, the seller can compensate the buyer who loses the chance of obtaining the object. As a consequence, such a termination fee encourages the buyer to pay more initially.

From the optimal mechanism $\tilde{\mathbf{M}}$, plugging the allocation rules (11) and (8) and the optimal effort into (6) and combining it with (9) and $v \geq v^M$ yields the maximum revenue that the seller can obtain is given by[16]

$$\tilde{\mathbb{S}} = \int_{v^M}^{\bar{v}} \left[\left(v - \frac{1 - F(v)}{f(v)} \right) + \int_{v - \frac{1-F(v)}{f(v)}}^{\bar{v}} (1 - H(p, \tilde{\theta}(v)))dp - C(\tilde{\theta}(v)) \right] f(v)dv, \tag{15}$$

where v^M is given by $v^M - \frac{1-F(v^M)}{f(v^M)} = 0$.

Implementation. Let us consider the following selling procedure to implement the optimal mechanism above. Let $J(v) = v - \frac{1-F(v)}{f(v)}$ denote the virtual value function. Note that under our assumption of increasing hazard rate $\frac{f(\cdot)}{1-F(\cdot)}$, we have that $J(\cdot)$ is increasing. Let $J^{-1}(\cdot)$ be the inverse function of $J(\cdot)$. At stage 1, the seller invites the buyer to choose a termination fee $t \in [0, J(\bar{v})]$. Upon her choosing t, she pays $b(t) = \tilde{m}_1(J^{-1}(t))$ to the seller at stage 1. At stage 2, if the seller decides to terminate the deal, he pays a termination fee of t to the buyer. Otherwise, the transaction is completed and the buyer receives the asset. By Lemma 5, we have $b(t)$ increases with t since both $J(\cdot)$ and $\tilde{m}_1(\cdot)$ are increasing functions. Thus, the buyer pays more for a higher termination fee.

With the above procedure, it is clearly optimal for the buyer to choose $t = J(v)$ and thus pays a first-stage payment $b(t) = \tilde{m}_1(v)$ when her value is v. It follows that the seller's search effort thus must be $\tilde{\theta}(v)$ after the buyer chooses the termination fee of $J(v)$. After the effort by the seller is exerted, an outside offer with a random price p from the corresponding distribution is realized in stage 2, which is the seller's private information. The seller completes the current transaction if the termination fee is no less than the price of the outside offer, that is, $t \geq p$, and the object is allocated to the buyer. Otherwise, the seller takes the outside offer with price p and termination fee t is paid to the buyer.

As a result, the above procedure achieves the same allocation as in the optimal selling mechanism identified in Proposition 1, and the seller collects the same revenue in either stage. Therefore, the above procedure implements the optimal selling mechanism. We formally present the result in the following proposition.

Proposition 2. *At stage 1, the seller invites the buyer to choose a termination fee* $t \in [0, J(\bar{v})]$*. If the buyer chooses* t*, she pays* $b(t) = \tilde{m}_1(J^{-1}(t))$ *to the seller at stage 1. At stage 2, the seller pays a termination fee of* t *to the buyer if he decides to terminate the deal. Otherwise, the transaction is completed and the buyer receives the asset.*

3.4 | Social Optimality and Efficiency

In this subsection, we provide the characterization of the efficient mechanism in terms of social welfare maximization and then discuss the socially optimal allocation rules and the seller's effort level in contrast to the optimal mechanism that maximizes the seller's revenue.

Our characterization shows that the socially preferred allocation rule in the second stage is given by[17]

$$q_2^*(v, p) = \begin{cases} 1 & \text{if } v \geq p; \\ 0 & \text{if } v < p. \end{cases} \tag{16}$$

Let us define $G(v;\theta) \equiv [\int_v^{\bar{v}} (1 - H(p, \theta))\,dp - C(\theta)]$. The socially optimal effort level is given by $\theta^* = \arg\max_{\theta \geq 0} G(v;\theta)$. Moreover, $\theta^*(v)$ is nonincreasing in v. Furthermore, our analysis shows that it is still optimal for the seller not to sell in the first stage, that is, $q_1^*(v) = 0$. The payment rule in the first stage is given by

$$m_2^*(v, p) = \begin{cases} 0 & \text{if } v \geq p; \\ -v & \text{if } v < p. \end{cases} \tag{17}$$

In the second stage, the payment rule is given by

$$m_1^*(v) = \int_0^v (1 - H(t, \theta^*(t)))dt. \tag{18}$$

We denote the socially efficient mechanism by $\mathbf{M}^*$, and we then have the following result.

Proposition 3. *The socially efficient mechanism* $\mathbf{M}^*$ *can be characterized as follows:*

i. *In the first stage, the allocation rule* $q_1^*(v) = 0$ *and the associated payment* $m_1^*(v)$ *is given by (18);*

ii. *In the second stage, the effort rule is given by* $\theta^*(v) = \arg\max_{\theta \geq 0} G(v;\theta)$*, and the allocation rule* $q_2^*(v,p)$ *and the associated payment (termination fee)* $m_2^*(v,p)$ *are given by (16) and (17), respectively.*

$$\mathbb{T}^* = \int_0^{\bar{v}} \left[\int_v^{\bar{v}} (1 - H(p, \theta^*(v)))dp + v - C(\theta^*(v)) \right] f(v)dv. \tag{19}$$

Plugging the allocation rules into the total surplus function yields This gives the maximized total expected surplus under the socially preferred mechanism.

Let us further compare the optimal mechanism and the socially efficient mechanism in terms of allocation rules and the seller's effort. Clearly, we have the following: (a) The object should not be sold in the first stage, that is, $q_1 = 0$, no matter which one is implemented between the optimal mechanism and the socially efficient mechanism. (b) Comparing (8) with (16) shows that given the same value from the buyer, the allocation rule in the optimal mechanism in Proposition 1 has a detrimental effect on efficiency. (c) Furthermore, let us assume the convexity of the cost function $C(\theta)$, which captures the fact that marginal cost (disutility) is increasing when the seller exerts more effort (the search cost rises at an increasing rate as search intensity θ increases). Comparing the first-order conditions of $D(v;\theta)$ and $G(v;\theta)$ shows that given the same value of the buyer, $\theta^*(v) < \tilde{\theta}(v)$; the socially preferred effort level is lower than that of the optimal selling mechanism, indicating that the effort rule in the optimal selling mechanism in Proposition 1 has a detrimental effect on efficiency as well. Summarizing the discussion, we have

Proposition 4. *In contrast to the socially efficient mechanism* $\mathbf{M}^*$*, the optimal mechanism* $\tilde{\mathbf{M}}$ *generates social inefficiency in both the allocation and the seller's investment effort. Specifically, given the same value v from the buyer,*

i. *the second-stage allocation rule in the optimal selling mechanism has a higher cutoff (*$v - \frac{1-F(v)}{f(v)} \geq p$ *instead of* $v \geq p$*) to allocate the object to the buyer;*

ii. $\theta^*(v) < \tilde{\theta}(v)$*; the socially preferred effort level is lower than that of the optimal selling mechanism.*

4 | Conclusion

In this paper, we examined a two-stage selling mechanism with an endogenous search decision by the seller. Specifically, in the first stage, the buyer makes her report; after that, the seller endogenously decides his investment in searching for a better outside offer in the second stage. We first characterize the seller's optimal decision in the investment, showing that how much of effort the seller should invest decreases in the buyer's report; that is, the higher the value the current buyer reports, the lower incentive the seller has to invest. Second, it is optimal for the seller to take the outside offer if the price of the offer is greater than the buyer's virtual value. The characterization of the optimal mechanism further shows that when taking the outside offer, the seller should pay a fee to the buyer to terminate the current transaction, which is equal to the virtual value. Moreover, the presence of the termination fee in turn increases the buyer's payment in the first stage. Our analysis sheds light on sellers' behavior of "after-market" search and costly default. For example, our results explain why it becomes popular for the target firms (sellers) to promise termination fees to the acquiring firms (buyers) in the M&A deals.

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Data Availability Statement

Data sharing not applicable to this article as no data sets were generated or analyzed during the current study.

Endnotes

[1] The termination fee is also called "lockup fee" or "breakup fee" in the literature. In our study, these appellations are interchangeable.

[2] Our setting is closely related to the literature of dynamic mechanism design, in which private information arrives sequentially and the designer commits to the mechanism at the outset; see related studies for the design framework by Crémer et al. (2007);;2009); Krasikov and Lamba (Forthcoming), and Yang (2022). In this study, we assume that the seller commits to the selling mechanism through an escrow agent (see related studies on selling mechanisms with seller commitment by McAfee and McMillan 1987; Myerson 1981; Vartiainen 2013; Skreta 2015; Liu et al. 2019 and buyer commitment by Asker 2000; Zheng 2001;;2009; Krähmer and Strausz 2015; Armstrong and Zhou 2016; Chang 2021; Che et al. 2022).

[3] Differently, if the acquiring firm (the buyer) decides to walk away from the transaction, a fee then needs to be paid to the target firm (the seller). This is called a reverse termination fee. See related studies by Coates et al. (2018); Chen et al. (2021), and Che et al. (2024).

[4] See relevant studies by Fraidin and Hanson (1994); Kahan and Klausner (1996), and Coates and Subramanian (2000).

[5] See related studies by Fishman (1988); Roberts and Sweeting (2013); Gorbenko and Malenko (2014); Gentry and Stroup (2019), and Chen and Wang (2023).

[6] Empirical studies on termination fees have also been conducted in the literature (Coates and Subramanian 2000; Burch 2001). Furthermore, Officer (2003) provides evidence that deals with target termination fees generate significantly higher premiums and success rates than deals without such fees. Bates and Lemmon (2003) find that termination fees truncate a natural bidding process. However, differently, Boone and Mulherin (2007) do not observe such a truncation but instead empirically show that termination fees culminate the takeover process. Jeon and Ligon (2011) empirically examine the optimal size of the

termination fees in merger deals, showing that low- or moderate-size fees serve as efficient contractual devices, while large fees are less beneficial to the target firms' shareholders.

[7] See relevant studies on selling mechanism designs with buyer outside offers by Lewis and Sappington (1989), Jullien (2000); Cherry et al. (2004); Figueroa and Skreta (2007);;2009); Kirchkamp et al. (2009); Lauermann and Virág (2012); Lu and Wang (2021); Che et al. (2022), and Liu and Lu (2022).

[8] In our analysis, the investment in the ex post outside offer can be interpreted as incurring an endogenous search cost to find "an additional buyer" and the price of the outside offer can be viewed as a reduced-form payoff (willingness to pay) from eliciting the private information of the additional buyer. There *may* exist a certain level of correlation between the price and the original buyer's private value, and therefore, we assume that distributions of p and v have the same domain to capture such a correlation. See related studies by Crémer et al. (2007);;2009); Krasikov and Lamba (Forthcoming), and Yang (2022) for the dynamic mechanism design literature. Our study also links to the question of whether a seller should prefer an optimally designed mechanism with N bidders over a mechanism with $N+1$ bidders. See related studies by Bulow and Klemperer (1996) and Kirkegaard (2006).

[9] For example, we can have $H(p,\theta)=\Phi(p)^{\theta+\beta_0}$, where $\beta_0>0$ is a constant, and $\Phi(\cdot)$ is a cumulative distribution function with density $\varphi(\cdot)>0$ over $[0,\bar{v}]$.

[10] There exists another possibility that instead of revealing the buyer's report directly, the escrow agent sends a signal, and the seller, based on the observation of the signal, forms his posterior belief on the buyer's report. In this case, we show that the revenue upper bound is the same as what we have in the current setting. See detailed proof in Appendix II.

[11] In Section 3.4, we discuss the socially preferred allocation rules and seller effort level.

[12] For example, the convexity of cost function ensures the uniqueness of optimal $\tilde{\theta}(v)$ under our assumption $H_{\theta,\theta}>0$.

[13] Note $C(0)=0$.

[14] Actually, any $\tilde{R}_2(0;0,v)\geq 0$ would still work and satisfy the seller's participation constraint. In this paper, we do not explicitly model the seller's participation constraint as we assume the seller commits to participating by offering a contract. As well understood in the literature, setting an arbitrary $\tilde{R}_2(0;0,v)\geq 0$ would not affect the buyer's first-stage IC and IR because we can reduce the buyer's first-stage payment by $\tilde{R}_2(0;0,v)\geq 0$ correspondingly.

[15] Alternatively, the associated transfers in the optimal mechanism can be organized in a slightly different way that in the first stage, the buyer does not make a payment after reporting. When moving to the second stage, the buyer pays $\tilde{m}_1(v)$ if the seller chooses to complete the transaction, while the payment is the difference between $\tilde{m}_1(v)$ and $\tilde{m}_2(v,p)$ if the seller chooses to take the outside offer.

[16] In Appendix I, we also show how the seller's maximum revenue can be derived from the payoffs in both stages.

[17] In Appendix I, we provide details of the characterization of the socially efficient mechanism.

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Supporting Information

Additional supporting information can be found online in the Supporting Information section.

Data S1

Appendix A

Proof of Lemma 1

Given (1), the following equation can be constructed:

$$R_2(p'; p', v) \; = \; R_2(p'; p, v) \; + \; \big(1 - q_2(v, p')\big)(p' - p).$$

Without loss of generality, we focus on the case of $p' < p$. Since $R_2(p'; p, v) \leq R_2(p; p, v)$, we have

$$R_2(p'; p', v) \; \leq \; R_2(p; p, v) \; + \; \big(1 - q_2(v, p')\big)(p' - p),$$

which we can rewrite as follows:

$$R_2(p; p, v) - R_2(p'; p', v) \; \geq \; \big(1 - q_2(v, p')\big)(p - p'). \tag{A.1}$$

Again, given (1), we can construct the following equation:

$$R_2(p; p, v) \; = \; R_2(p; p', v) \; + \; (1 - q_2(v, p))(p - p').$$

Since $R_2(p; p', v) \leq R_2(p'; p', v)$, we have

$$R_2(p; p, v) \; \leq \; R_2(p'; p', v) \; + \; (1 - q_2(v, p))(p - p'),$$

which we can rewrite as follows:

$$R_2(p; p, v) - R_2(p'; p', v) \; \leq \; (1 - q_2(v, p))(p - p'). \tag{A.2}$$

Combining (A.1) and (A.2) gives the following inequality:

$$\big(1 - q_2(v, p')\big)(p - p') \; \leq \; R_2(p; p, v) - R_2(p'; p', v) \; \leq \; (1 - q_2(v, p))(p - p'),$$

which implies part (i), given $p' < p$.

The inequality above can be rewritten for any $\kappa > 0$

$$\big(1 - q_2(v, p')\big)\kappa \; \leq \; R_2(p' + \kappa; p' + \kappa, v) - R_2(p'; p', v) \; \leq \; \big(1 - q_2(v, p' + \kappa)\big)\kappa.$$

Riemann integration implies the following:

$$R_2(p; p, v) \; = \; R_2(0; 0, v) \; + \; \int_0^p (1 - q_2(v, x))dx, \tag{A.3}$$

which gives part (ii).

The above merely shows the necessary condition. We further show the converse—the sufficient condition. It is easy for us to obtain the following cases. If $p \geq p'$, (A.3) gives us

$$\begin{aligned} R_2(p; p, v) \; &= \; R_2(p'; p', v) \; + \; \int_{p'}^{p} (1 - q_2(v, x))dx \; \geq \; R_2(p'; p', v) \; + \; \int_{p'}^{p} \big(1 - q_2(v, p')\big)dx \; = \; R_2(p'; p', v) \; + \; \big(1 - q_2(v, p')\big)(p - p') \\ &= \; R_2(p'; p, v)\,. \end{aligned}$$

Similarly, if $p < p'$, (A.3) gives us

$$\begin{aligned} R_2(p; p, v) \; = \; R_2(p'; p', v) \; - \; \textstyle\int_p^{p'} (1 - q_2(v, x))\,dx \; \geq \; R_2(p'; p', v) \; - \; \textstyle\int_p^{p'} \big(1 - q_2(v, p')\big)\,dx \; &= \; R_2(p'; p', v) \; + \; \big(1 - q_2(v, p')\big)\,(p - p') \\ &= \; R_2(p'; p, v)\,. \end{aligned}$$

So (**SIC**) follows from parts (i) and (ii). We complete the proof of the lemma. □

Proof of Lemma 2

Given (3), we can construct the following equation:

$$\pi(v'; v') \; = \; \pi(v'; v) \; + \; \Big\{ q_1(v')(v' - v) \; + \; (1 - q_1(v'))E_{(p|\theta(v'))}\big(q_2(v', p)(v' - v)\big) \Big\}.$$

Without loss of generality, we focus on the case of $v' < v$. Since $\pi(v';v) \le \pi(v;v)$, we have

$$\pi(v';v') \;\le\; \pi(v;v) \;+\; \Big\{q_1(v')(v'-v) \;+\; \big(1-q_1(v')\big)E_{(p|\theta(v'))}\big(q_2(v',p)(v'-v)\big)\Big\},$$

which we can rewrite as follows:

$$\pi(v;v) \;-\; \pi(v';v') \;\ge\; \Big\{q_1(v')(v-v') \;+\; \big(1-q_1(v')\big)E_{(p|\theta(v'))}\big(q_2(v',p)(v-v')\big)\Big\}. \tag{A.4}$$

Again, given (3), we can construct the following equation:

$$\pi(v;v) \;=\; \pi(v;v') \;+\; \Big\{q_1(v)(v-v') \;+\; (1-q_1(v))E_{(p|\theta(v))}\big(q_2(v,p)(v-v')\big)\Big\}.$$

Since $\pi(v;v') \le \pi(v';v')$, we have

$$\pi(v;v) \;\le\; \pi(v';v') \;+\; \Big\{q_1(v)(v-v') \;+\; (1-q_1(v))E_{(p|\theta(v))}\big(q_2(v,p)(v-v')\big)\Big\},$$

which we can rewrite as follows:

$$\pi(v;v) \;-\; \pi(v';v') \;\le\; \Big\{q_1(v)(v-v') \;+\; (1-q_1(v))E_{(p|\theta(v))}\big(q_2(v,p)(v-v')\big)\Big\}. \tag{A.5}$$

Combining (A.4) and (A.5) gives the following inequality:

$$\begin{aligned}&\Big\{q_1(v')(v-v') \;+\; \big(1-q_1(v')\big)E_{(p|\theta(v'))}\big(q_2(v',p)(v-v')\big)\Big\}\\ &\le\; \pi(v;v) \;-\; \pi(v';v') \;\le\\ &\Big\{q_1(v)(v-v') \;+\; (1-q_1(v))E_{(p|\theta(v))}\big(q_2(v,p)(v-v')\big)\Big\},\end{aligned}$$

which implies part (i), given $v' < v$.

The inequality can be rewritten for any $\delta > 0$

$$\begin{aligned}&\Big\{q_1(v')\delta \;+\; \big(1-q_1(v')\big)E_{(p|\theta(v'))}\big(q_2(v',p)\delta\big)\Big\}\\ &\le\; \pi(v'+\delta;v'+\delta) \;-\; \pi(v';v') \;\le\\ &\Big\{q_1(v'+\delta)\delta \;+\; \big(1-q_1(v'+\delta)\big)E_{(p|\theta(v'+\delta))}\big(q_2(v'+\delta,p)\delta\big)\Big\}.\end{aligned}$$

Riemann integration implies the following:

$$\pi(v;v) \;=\; \pi(0;0) \;+\; \int_0^v \Big\{q_1(t) \;+\; (1-q_1(t))E_{(p|\theta(t))}(q_2(t,p))\Big\}dt, \tag{A.6}$$

which gives part (ii). Furthermore, part (iii) follows directly from $\pi(v;v) \ge 0$.

The above merely shows the necessary condition. We further show the converse—the sufficient condition. First, parts (ii) and (iii) give $\pi(v;v) \ge 0$. Next, it is easy for us to obtain the following cases.

If $v \ge v'$, (A.6) gives us

$$\begin{aligned}\pi(v;v) \;-\; \pi(v';v') \;&=\; \int_{v'}^{v} \Big\{q_1(t) \;+\; (1-q_1(t))E_{(p|\theta(t))}(q_2(t,p))\Big\}dt \;\ge\; \int_{v'}^{v} \Big\{q_1(v') \;+\; \big(1-q_1(v')\big)E_{(p|\theta(v')}\big(q_2(v',p)\big)\Big\}dt\\ &=\; \big[q_1(v') \;+\; \big(1-q_1(v')\big)E_{(p|\theta(v'))}\big(q_2(v',p)\big)\big](v-v'),\end{aligned}$$

which can be rewritten as follows:

$$\pi(v;v) \;\ge\; \pi(v';v') \;+\; \big[q_1(v') \;+\; \big(1-q_1(v')\big)E_{(p|\theta(v')}\big(q_2(v',p)\big)\big](v-v') \;=\; \pi(v';v).$$

Similarly, if $v < v'$, (A.6) gives us

$$\pi(v;v) - \pi(v';v') = -\int_v^{v'} \left\{ q_1(t) + (1-q_1(t))E_{(p|\theta(t))}(q_2(t,p)) \right\} dt \geq -\int_v^{v'} \left\{ q_1(v') + (1-q_1(v'))E_{(p|\theta(v'))}(q_2(v',p)) \right\} dt$$

$$= \left[q_1(v') + (1-q_1(v'))E_{(p|\theta(v'))}(q_2(v',p)) \right](v-v'),$$

which can be rewritten as follows:

$$\pi(v;v) \geq \pi(v';v') + \left[q_1(v') + (1-q_1(v'))E_{(p|\theta(v'))}(q_2(v',p)) \right](v-v')$$
$$= \pi(v';v).$$

So (**BIC**) follows from parts (i) and (ii). We complete the proof of the lemma. □

Proof of Lemma 3

Plugging the allocation rule (8) into the relevant term in (6) yields

$$E_{(p|\theta(v))}\left\{ \tilde{q}_2(v,p)\left(v - \frac{1-F(v)}{f(v)}\right) + (1-\tilde{q}_2(v,p))p \right\} - C(\theta(v))$$

$$= \int_0^{\max\left\{v - \frac{1-F(v)}{f(v)},0\right\}} \left(v - \frac{1-F(v)}{f(v)}\right) h(p,\theta(v))dp + \int_{\max\left\{v - \frac{1-F(v)}{f(v)},0\right\}}^{\bar{v}} ph(p,\theta(v))dp - C(\theta(v)).$$

Furthermore, simplifying the equation shows

$$\begin{aligned}
&\int_0^{\max\left\{v - \frac{1-F(v)}{f(v)},0\right\}} \left(v - \frac{1-F(v)}{f(v)}\right) h(p,\theta(v))dp + \int_{\max\left\{v - \frac{1-F(v)}{f(v)},0\right\}}^{\bar{v}} ph(p,\theta(v))dp - C(\theta(v)) \\
&= \left(v - \frac{1-F(v)}{f(v)}\right) H\left(\max\left\{v - \frac{1-F(v)}{f(v)},0\right\},\theta(v)\right) + pH(p,\theta(v))\Big|_{\max\left\{v - \frac{1-F(v)}{f(v)},0\right\}}^{\bar{v}} \\
&\quad - \int_{\max\left\{v - \frac{1-F(v)}{f(v)},0\right\}}^{\bar{v}} H(p,\theta(v))dp - C(\theta(v)) \\
&= \bar{v} - \int_{\max\left\{v - \frac{1-F(v)}{f(v)},0\right\}}^{\bar{v}} H(p,\theta(v))dp + \max\left\{v - \frac{1-F(v)}{f(v)},0\right\} \\
&\quad - \max\left\{v - \frac{1-F(v)}{f(v)},0\right\} - C(\theta(v)) \\
&= \int_{\max\left\{v - \frac{1-F(v)}{f(v)},0\right\}}^{\bar{v}} (1-H(p,\theta(v)))dp + \max\left\{v - \frac{1-F(v)}{f(v)},0\right\} - C(\theta(v)).
\end{aligned} \tag{A.7}$$

We thus have completed the proof. □

The Buyer's Payment in the First Stage

Setting $\pi(0;0) = 0$, the buyer's interim expected payoff (part (ii) of Lemma 2) under the allocation rules (8) and (11) can be written as follows:

$$\tilde{\pi}(v;v) = \int_0^v \left\{ \tilde{q}_1(t) + (1-\tilde{q}_1(t))E_{(p|\tilde{\theta}(t))}(\tilde{q}_2(t,p)) \right\} dt = \int_0^v \left\{ E_{(p|\tilde{\theta}(t))}(\tilde{q}_2(t,p)) \right\} dt.$$

Since we have

$$E_{(p|\tilde{\theta}(v))}(\tilde{q}_2(v,p)) = H\left(\max\left\{v - \frac{1-F(v)}{f(v)},0\right\},\tilde{\theta}(v)\right),$$

the expected payoff can be rewritten as follows:

$$\tilde{\pi}(v;v) = \int_0^v H\left(\max\left\{t - \frac{1-F(t)}{f(t)},0\right\},\tilde{\theta}(t)\right)dt. \tag{A.8}$$

From (12), the expected termination fee paid by the seller can be written as follows:

$$
\begin{aligned}
E_{(p|\tilde{\theta}(v))}(\tilde{m}_2(v,p)) &= \int_{\max\left\{v-\frac{1-F(v)}{f(v)},0\right\}}^{\bar{v}} -\max\left\{v-\frac{1-F(v)}{f(v)},0\right\} dH(p,\tilde{\theta}(v)) \\
&= -\max\left\{v-\frac{1-F(v)}{f(v)},0\right\}\left[1-H\left(\max\left\{v-\frac{1-F(v)}{f(v)},0\right\},\tilde{\theta}(v)\right)\right].
\end{aligned} \tag{A.9}
$$

Under the allocation rules (8) and (11), plugging (A.9) into the buyer's expected payoff (3) gives

$$
\begin{aligned}
\tilde{\pi}(v;v) &= -\tilde{m}_1(v) + E_{(p|\tilde{\theta}(v))}\left\{\tilde{q}_2(v,p)v - \tilde{m}_2(v,p)\right\} = -\tilde{m}_1(v) + v\left[H\left(\max\left\{v-\frac{1-F(v)}{f(v)},0\right\},\tilde{\theta}(v)\right)\right] \\
&+ \max\left\{v-\frac{1-F(v)}{f(v)},0\right\}\left[1-H\left(\max\left\{v-\frac{1-F(v)}{f(v)},0\right\},\tilde{\theta}(v)\right)\right].
\end{aligned} \tag{A.10}
$$

Combining (A.8) and (A.10) yields

$$
\begin{aligned}
\tilde{m}_1(v) &= E_{(p|\tilde{\theta}(v))}\left\{\tilde{q}_2(v,p)v - \tilde{m}_2(v,p)\right\} - \int_0^v \left\{E_{(p|\tilde{\theta}(t))}(\tilde{q}_2(t,p))\right\}dt = v\left[H\left(\max\left\{v-\frac{1-F(v)}{f(v)},0\right\},\tilde{\theta}(v)\right)\right] \\
&+ \max\left\{v-\frac{1-F(v)}{f(v)},0\right\}\left[1-H\left(\max\left\{v-\frac{1-F(v)}{f(v)},0\right\},\tilde{\theta}(v)\right)\right] - \int_0^v H\left(\max\left\{t-\frac{1-F(t)}{f(t)},0\right\},\tilde{\theta}(t)\right)dt,
\end{aligned} \tag{A.11}
$$

which gives the associated payment from the buyer with private value v in the first stage of the mechanism. Equation (A.11) can further be simplified as follows:

Case I. When $v < v^M$, $v - \frac{1-F(v)}{f(v)} < 0$ and we have

$$
\tilde{m}_1(v) = 0. \tag{A.12}
$$

Case II. When $v \geq v^M$, $v - \frac{1-F(v)}{f(v)} \geq 0$ and we have

$$
\tilde{m}_1(v) = vH\left(v-\frac{1-F(v)}{f(v)},\tilde{\theta}(v)\right) - \int_{v_M}^v H\left(t-\frac{1-F(t)}{f(t)},\tilde{\theta}(t)\right)dt + \left(v-\frac{1-F(v)}{f(v)}\right)\left[1-H\left(v-\frac{1-F(v)}{f(v)},\tilde{\theta}(v)\right)\right]. \tag{A.13}
$$

Cases I and II characterize the buyer's first-stage payment, corresponding to private value v. □

Proof of Lemma 5

Let us separately consider the following two cases:

i. When $v < v^M$, we have

$$
\frac{d\tilde{m}_1(v)}{dv} = 0.
$$

ii. When $v \geq v^M$, we denote $(v - \frac{1-F(v)}{f(v)})$ by $J(v)$ and (14) can be written as follows:

$$
\tilde{m}_1(v) = v\left[H(J(v),\tilde{\theta}(v))\right] - \int_{v_M}^v H\left(J(t),\tilde{\theta}(t)\right)dt + J(v)\left[1-H\left(J(v),\tilde{\theta}(v)\right)\right].
$$

Differentiating with respect to v yields

$$
\begin{aligned}
\frac{d\tilde{m}_1(v)}{dv} &= (v-J(v))h(J(v),\tilde{\theta}(v))\frac{dJ(v)}{dv} + (v-J(v))H_\theta(J(v),\tilde{\theta}(v))\frac{d\tilde{\theta}(v)}{dv} + \left[1-H\left(J(v),\tilde{\theta}(v)\right)\right]\frac{dJ(v)}{dv} \\
&> 0.
\end{aligned}
$$

Therefore, we have $\tilde{m}_1(v)$ is increasing in v. Note that $\tilde{m}_1(v) = 0$ when $v = v^M$. □

Proof of Lemma 6

Let us further check whether the allocation rule in (11) satisfies the requirement in Lemma 2. Plugging $\tilde{q}_1(v)$ and $\tilde{q}_2(v,p)$ into $\xi(v_i)$ yields

$$
\xi(v) = E_{(p|\tilde{\theta}(v))}(\tilde{q}_2(v,p)) = H\left(\max\left\{v-\frac{1-F(v)}{f(v)},0\right\},\tilde{\theta}(v)\right).
$$

$\xi(v)$ can be simplified as follows:

Case I. When $v < v^M$, $v - \frac{1-F(v)}{f(v)} < 0$ and we have

$$\xi(v) = 0. \tag{A.14}$$

Case II. When $v \geq v^M$, $v - \frac{1-F(v)}{f(v)} \geq 0$ and we have

$$\xi(v) = H\left(v - \frac{1-F(v)}{f(v)}, \tilde{\theta}(v)\right). \tag{A.15}$$

We denote $(v - \frac{1-F(v)}{f(v)})$ by $J(v)$. Given the properties of $H(p,\theta)$, $d\tilde{\theta}/dv < 0$, and $dJ/dv > 0$, taking the derivative with respect to v yields

$$\begin{aligned}\frac{d\xi(v)}{dv} &= h\big(J(v),\tilde{\theta}(v)\big)\frac{dJ(v)}{dv} + H_\theta\big(J(v),\tilde{\theta}(v)\big)\frac{d\tilde{\theta}(v)}{dv}\\ &> 0.\end{aligned} \tag{A.16}$$

The analyses in both cases I and II indicate that $\xi(v)$ is nondecreasing in v. Therefore, (**BIC**) is satisfied. Clearly, $\tilde{\pi}(v;v) \geq 0$, indicating (**BIR**) is also satisfied. □

The Seller's Maximum Revenue

The seller's revenue under the optimal mechanism $\tilde{\mathbf{M}}$ includes two parts: the payment from the buyer in the first stage (captured by the first term in the brackets) and the payoff from learning and exercising the outside offer in the second stage (captured by the second and third terms in the brackets), which can be written as follows:

$$\tilde{\mathbb{S}} = \int_0^{\bar{v}} \left[\tilde{m}_1(v) + E_{(p|\tilde{\theta}(v))}\left\{R_2(p;p,v)\right\} - C(\tilde{\theta}(v))\right] f(v)dv.$$

Plugging (A.11) and (1) into the equation above yields

$$\begin{aligned}
&= \int_0^{\bar{v}} \left[E_{(p|\tilde{\theta}(v))}\left\{\tilde{q}_2(v,p)v - \tilde{m}_2(v,p)\right\} - \int_0^{v}\left\{E_{(p|\tilde{\theta}(t))}(\tilde{q}_2(t,p))\right\}dt\right.\\
&\left.+ E_{(p|\tilde{\theta}(v))}\left\{(1-\tilde{q}_2(v,p))p + \tilde{m}_2(v,p)\right\} - C(\tilde{\theta}(v))\right] f(v)dv\\
&= \int_0^{\bar{v}} \left[E_{(p|\tilde{\theta}(v))}\left\{\tilde{q}_2(v,p)v + (1-\tilde{q}_2(v,p))p\right\} - \int_0^{v}\left\{E_{(p|\tilde{\theta}(t))}(\tilde{q}_2(t,p))\right\}dt - C(\tilde{\theta}(v))\right] f(v)dv\\
&= \int_0^{\bar{v}} \left[E_{(p|\tilde{\theta}(v))}\left\{\tilde{q}_2(v,p)v + (1-\tilde{q}_2(v,p))p\right\} - C(\tilde{\theta}(v))\right] f(v)dv - \int_0^{\bar{v}}\left[\int_0^{v}\left\{E_{(p|\tilde{\theta}(t))}(\tilde{q}_2(t,p))\right\}dt\right] f(v)dv.
\end{aligned} \tag{A.17}$$

For the second term in the equation above, interchanging the order of integration gives

$$\int_0^{\bar{v}}\left[\int_0^{v}\left\{E_{(p|\tilde{\theta}(t))}(\tilde{q}_2(t,p))\right\}dt\right] f(v)dv = \int_0^{\bar{v}}\left[\int_t^{\bar{v}} f(v)dv\left\{E_{(p|\tilde{\theta}(t))}(\tilde{q}_2(t,p))\right\}\right]dt = \int_0^{\bar{v}} E_{(p|\tilde{\theta}(v))}(\tilde{q}_2(v,p))\left(\frac{1-F(v)}{f(v)}\right) f(v)dv.$$

We therefore have (A.17) as follows:

$$\tilde{\mathbb{S}} = \int_0^{\bar{v}} \left[E_{(p|\tilde{\theta}(v))}\left\{\tilde{q}_2(v,p)\left(v - \frac{1-F(v)}{f(v)}\right) + (1-\tilde{q}_2(v,p))p\right\} - C(\tilde{\theta}(v))\right] f(v)dv. \tag{A.18}$$

This is equal to (6) with $\tilde{q}_1(v) = 0$. Simplifying the equation as what we had in Step 2 with $v \geq v_M$ gives (15). □

The Characterization of the Socially Efficient Mechanism

Given the total expected surplus function (5), the optimization problem for the socially efficient mechanism can be set up as follows:

$$\begin{aligned}
\mathbb{T} &= \int_0^{\bar{v}} \left\{q_1(v)v + (1-q_1(v))\left[E_{(p|\theta(v))}(q_2(v,p)v + (1-q_2(v,p))p) - C(\theta(v))\right]\right\} f(v)dv\\
&= \int_0^{\bar{v}} \left\{v + (1-q_1(v))\left[E_{(p|\theta(v))}((1-q_2(v,p))(p-v)) - C(\theta(v))\right]\right\} f(v)dv
\end{aligned} \tag{A.19}$$

subject to the constraints of (**BIC**), (**BIR**), (**SIC**) as well as (**SID**). As we did above, we temporarily neglect the constraints to identify the allocation, payment and seller effort rules, and then show that these rules satisfy the constraints.

Step 1. In this step, we identify the socially efficient allocation and seller effort rules. First, the relevant term $(1 - q_2(v,p))(p - v)$ in the equation above indicates that the socially preferred allocation rule in the second stage is given by

$$q_2^*(v,p) \ = \ \begin{cases} 1 & \text{if } v \geq p; \\ 0 & \text{if } v < p. \end{cases} \tag{A.20}$$

This indicates that it is socially optimal to allocate the object to the buyer in the second stage whenever her value is greater than the price of the outside offer; otherwise, the outside offer should be taken.

We then plug the socially preferred allocation rule into the relevant term in the $\mathbb{T}$ function, which leads to,

$$\begin{aligned} \eta(v;\theta(v)) &= E_{(p|\theta(v))}\left\{ q_2^*(v,p)v \ + \ \left(1 - q_2^*(v,p)\right)p \right\} \ - \ C(\theta(v)) \ = \ \int_0^v vh(p,\theta(v))dp \ + \ \int_v^{\bar{v}} ph(p,\theta(v))dp \ - \ C(\theta(v)) \\ &= \int_v^{\bar{v}} (1 - H(p,\theta(v)))dp \ + \ v \ - \ C(\theta(v)). \end{aligned}$$

Given the definition of $G(v;\theta)$ and the properties of $H(p,\theta)$, for $\theta_1 < \theta_2$ and $v < v'$, we have

$$G(v';\theta_2) \ - \ G(v;\theta_2) \ = \ -\int_v^{v'} (1 - H(p,\theta_2))dp \ \leq \ -\int_v^{v'} (1 - H(p,\theta_1))dp \ = \ G(v';\theta_1) - G(v;\theta_1) \tag{A.21}$$

indicating that $G(v;\theta)$ is submodular in $(v;\theta)$. By Topkis' Theorem, it is clear that $\theta^*(v)$ is nonincreasing in v.

Lastly, let us examine the allocation rule in the first stage. Let $\eta^*(v) = \eta(v;\theta^*(v))$. With the second-stage allocation rule $q_2^*(v,p)$ and the search effort $\theta^*(v)$, function $\mathbb{T}$ can be written as follows:

$$\mathbb{T} \ = \ \int_0^{\bar{v}} \left\{ q_1(v)v \ + \ (1 - q_1(v))\eta^*(v) \right\} f(v)dv. \tag{A.22}$$

To determine the first-stage allocation rule $q_1(v)$, we need to compare v and $\eta^*(v)$. Clearly, $\eta^*(v) \geq v$, and therefore, it is optimal to set $q_1^*(v) = 0$. This indicates that when moving to the first stage, both the socially efficient mechanism and the seller optimal mechanism prefer not to allocate the object immediately to the buyer, that is, $\tilde{q}_1(v) = q_1^*(v) = 0$.

Step 2. In this step, we identify the socially efficient payment rules. We first derive the payment rule in the second stage. Given $R_2(0;0,v) = 0$ and $q_2^*(v,p)$, combining (1) and part (ii) of Lemma 1 yields

$$m_2(v,p) \ = \ R_2(0;0,v) \ + \ \int_0^p (1 - q_2(v,x))dx - (1 - q_2(v,p))p \ = \ \int_0^p (1 - q_2(v,x))dx - (1 - q_2(v,p))p,$$

which gives

$$m_2^*(v,p) \ = \ \begin{cases} 0 & \text{if } v \geq p; \\ -v & \text{if } v < p. \end{cases} \tag{A.23}$$

Let us further derive the payment rule in the first stage. Following the same steps we had before, part (II) of Lemma 2 can be rewritten as follows:

$$\pi(v;v) \ = \ \int_0^v \left\{ E_{(p|\theta^*(t))}\left(q_2^*(t,p)\right) \right\} dt \ = \ \int_0^v H(t,\theta^*(t))dt.$$

Combining the equation above with the buyer's expected payoff—(3)—gives

$$\begin{aligned} m_1^*(v) \ &= \ E_{(p|\theta^*(v))}\left\{ q_2^*(v,p)v \ - \ m_2^*(v,p) \right\} - \int_0^v \left\{ E_{(p|\theta^*(t))}\left(q_2^*(t,p)\right) \right\} dt \ = \ vH(v,\theta^*(v)) \ + \ \int_v^{\bar{v}} vh(p,\theta^*(v))dp \ - \ \int_0^v H(t,\theta^*(t))dt \\ &= \ \int_0^v (1 - H(t,\theta^*(t)))dt, \end{aligned} \tag{A.24}$$

which is the payment of the buyer in the first stage.

We further examine whether the constraints are satisfied with the socially efficient allocation, payment, and seller effort rules identified above. Clearly, $q_2^*(v,p)$ is nonincreasing in p, and the characterization of the allocation and the payment rules implies that (**SIC**) is satisfied.

Furthermore, plugging the allocation rule into (2)—the (**SID**) constraint—yields

$$\int_v^{\bar{v}} (1 - H(x,\theta))dx - \ C(\theta),$$

which is the same as $G(v;\theta)$, indicating that (**SID**) is also maximized with θ^*; the constraint is satisfied. Finally, plugging the allocation rules in both stages into the ξ function yields

$$\xi(v) = H(v, \theta^*(v)). \tag{A.25}$$

ξ is nondecreasing in v, indicating that (**BIC**) is satisfied. We set $\pi(0;0) = 0$ so that (**BIR**) is satisfied. □