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The Role of Information and Algorithms in Digital Markets

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This thesis is submitted for the Doctor of Philosophy
September 2025

Declaration of Authorship

I hereby declare that this thesis, entitled “*The Role of Information and Algorithms in Digital Markets*”, is the result of my own work and includes nothing which is the outcome of work done in collaboration, except as specified below.

Chapter 2 entitled “*Algorithmic Exclusion by Large Language Models*” is joint work with my supervisor, Dr Arina Nikandrova. The overall research design, model formulation, and analysis were developed in collaboration. Writing, simulation implementation, and interpretation of results were carried out by myself, with guidance and feedback from Dr Arina Nikandrova.

This thesis has not been submitted, either in whole or in part, for any other degree or qualification at this or any other university. All sources of information used have been duly acknowledged.

Anushree Parekh

Date: February 21, 2026

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“If both my mentor and the wisdom of the world stood before me, I would first bow to my mentor, for it is through them that I have come to know and appreciate knowledge.”

— *Inspired by Kabir Das*

I begin with these words, inspired by a saying of the Indian mystic Kabir Das (c. 1440–1518), to honour the profound role of mentorship. At the outset, I express my deepest gratitude to my supervisor, Dr Arina Nikandrova, whose guidance, encouragement, and unwavering support have been the cornerstone of this research. Your insights, patience, and mentorship have shaped not only this work but also my growth as a person.

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“Once upon a time, before the school bell called me away, I sat besides my grandfather as the crisp newspaper unfolded like a new chapter of the day. With patience in his voice, he turned fleeting words into stories, and stories into lessons. To me, they were never just news—they were windows to a world far larger than my own.”

I dedicate this work to my grandparents, Prabha Parekh and V.G. Parekh, whose constant support and guidance have been my steady foundation. I also fondly remember my late great-grandmother, Motiben Parekh, whose wisdom and stories continue to inspire me.

I also express immense gratitude to my parents, Neha Parekh and Manish Parekh; my aunt and uncle, Pratiksha Prashant and Prashant Krishnadas; my maternal grandparents, Geeta Shah, Dr Bipin Shah, Panna Shah, and Kiran Shah; my cousins, Prakriti Prashant and Om Zaveri and my extended family. I also thank my best friends, Juhi Ronik Birawat, Khushboo Shreyansh Singhvi, Aditya Bhadarka, Yash Deodikar, and Gokul Pillai.

Abstract

This thesis investigates the role of information and algorithms in shaping competition, strategy, and welfare outcomes in digital markets. Across three chapters, it examines how algorithmic decision-making, data advantages, and platform dual roles influence firm behavior and consumer outcomes.

The first strand explores whether large language models (LLMs) can learn predatory strategies in dynamic environments in which an incumbent faces repeated entry threat. Using OpenAI's GPT-4.1 as decision-making agents, we find that LLMs learn to predate when both predation and accommodation are theoretically viable, and adopt aggressive strategies when only accommodation is theoretically viable. Further, profit optimization is limited, highlighting both strategic learning and its limitations. These results reveal that LLMs are capable of executing complex exclusionary strategies.

The second strand considers a model where two competitors located at opposite ends of the Hotelling line compete for the unit mass of consumers. Consumer preferences vary across two dimensions: horizontal (brand preference) and vertical (quality preference). We compare two scenarios: (1) competitors remain uninformed about the horizontal dimension but possess perfect information about the vertical dimension, which enables them to set prices conditional on this information; (2) competitors are uninformed about either dimension of consumer preferences and set a uniform price. The analysis reveals that under some conditions consumers benefit from personalized pricing.

The third strand examines the incentives of a hybrid platform, which acts both as a marketplace provider and as a competitor through its own retail arm, to truthfully share demand information with a third-party seller. The platform enjoys an informational advantage about the state of demand, which it always shares with its retail arm. We analyze two settings: (i) No hosting fee i.e, the seller pays no commission and (ii) Platform charges a hosting fee i.e, commission a percentage of seller revenue. In both cases, we find that the incentives of the platform and the seller diverge, leading to an uninformative equilibrium. While commissions affect prices and profit levels, they do not alter the qualitative nature of information sharing. These results highlight the limits of informational advantage in hybrid platform markets.

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Chapter 1

Introduction

The rise of algorithms in markets has transformed how firms operate and how consumers interact with products and services. This transformation has produced significant shifts in market dynamics, placing the use of algorithms at the centre of attention for competition authorities worldwide. A feature unique to such markets is the centrality of information, encompassing both consumer and competitor data, which critically shapes strategic behaviour. Firms increasingly gain competitive advantage by deploying data-driven tools and algorithmic decision-making. Consequently, understanding how firms exploit information, engage in exclusionary practices — such as predatory pricing, self-preferencing, and algorithmic collusion — and potentially harm consumer welfare has become crucial in the era of data-driven markets.

This thesis investigates the role of algorithms and information in strategic market behavior through three related lines of inquiry: (i) the capacity of large language models (LLMs) to learn predatory strategies; (ii) the effect of information about vertical preferences of consumers on personalized pricing and consumer welfare; and (iii) the incentives of hybrid platforms to share information with third-party sellers. Collectively, these studies examine how strategic foresight and access to information alter market outcomes and shape consumer welfare.

1.1 Algorithmic Exclusion and LLMs

The first strand investigates whether large language models (LLMs) can learn to execute exclusionary strategies in dynamic market environments. Within a Stackelberg framework, where an incumbent repeatedly faces potential entry, accommodation, monopolization and predation may emerge as pure-strategy Markov Perfect Equilibrium outcomes. We embed LLMs—specifically OpenAI’s GPT-4.1—into the economic environment as both quantity and

binary decision-makers for the incumbent and entrants, and assess whether these agents can learn exclusionary strategies.

By varying product differentiation and entry threat parameters in our simulations, we observe that when predation is the only viable theoretical strategy, LLM agents converge to quantities that showcase exclusionary behaviour. Conversely, when both predation and accommodation are theoretically viable, they tend to adopt quantities that reflect an aggressive stance.

This chapter is joint work with my supervisor, Dr. Arina Nikandrova.

1.2 Consumer Information and Personalized Pricing

The second strand explores the impact of information about consumers' vertical preferences on prices offered and consumer welfare. Specifically, the model extends the classical Hotelling framework such that consumer preferences vary across two dimensions: horizontal—brand preference—and vertical—quality preference.

Our main findings reveal that consumer benefits may not always hold under personalized pricing. Specifically, when product differentiation is low, intense competition makes prices offered by firms and aggregate consumer welfare largely indifferent between personalized and uniform pricing. Further, at extremely high product differentiation, consumers benefit from uniform prices, as firms cannot effectively discriminate. Lastly, at intermediate product differentiation, depending on market coverage and competition intensity, consumer benefits fluctuate between personalized and uniform prices. Overall, our results challenge the conventional belief in the literature that personalized pricing always benefits consumers, emphasizing that firms' informational advantages can sometimes reduce welfare.

1.3 Information Sharing by Hybrid Platforms

The third strand investigates the incentives of hybrid platforms—platforms that both operate marketplaces and compete with third-party sellers—to share demand information. We adopt a Bertrand competition model under demand uncertainty to analyse scenarios with full or no information sharing, and with and without a hosting commission.

Our results show that misalignment of incentives between the platform and third-party sellers leads to an uninformative equilibrium. Consequently, the platform can never alter the prior beliefs of third-party sellers. This result is consistent with classic cheap talk theory. Further, even introducing a hosting fee does not align the incentives of the platform and third-party sellers.

These results demonstrate that platforms' informational advantage is constrained by strategic misalignment with sellers, highlighting implications for platform design, commission structures, and regulatory policies.

1.4 Roadmap of the Thesis

The remainder of this thesis is structured around the three main strands outlined above, each presented as a separate chapter corresponding to a distinct research paper:

- Algorithmic Exclusion by Large Language Models
- Personalized Pricing and Data Advantage: How much is too much?
- Information Sharing by a Hybrid Platform: The Truth-Telling Problem

Each chapter is self-contained and can be read independently as a research paper. Collectively, they aim to advance our understanding of how the strategic use of algorithms and information advantages shapes market outcomes, integrating insights from the literature on information sharing, digital platforms, and algorithmic decision-making.

Chapter 2

Algorithmic Exclusion by Large Language Models

2.1 Introduction

This paper is motivated by the growing importance of algorithmic pricing — the use of advanced artificial intelligence tools to set prices in online markets. While algorithmic pricing promises efficiency gains, it also raises significant concerns for competition policy. The emerging economics literature has primarily focused on the potential for collusion facilitated by pricing algorithms (see, e.g., [Calvano et al. \(2020\)](#); [Asker et al. \(2021\)](#); [Banchio and Skrzypacz \(2022\)](#)) or large language model (LLM) agents (see, [Fish et al. \(2025\)](#)). These studies demonstrate that algorithms can tacitly coordinate to sustain supracompetitive prices, learning to match collusive benchmarks and punishing deviations through price wars.

However, collusion is only one dimension of the potential harm. Most real-world competition cases center on exclusionary conduct — that is, strategies by dominant firms to prevent or deter entry. Despite policy concerns that generative AI may enable exclusionary practices ([Davies \(2025\)](#); [FTC Technology Blog, 2023](#)), no study has yet examined exclusionary conduct by algorithmic agents. It is, therefore, not yet understood whether and how learning algorithms deployed by incumbent firms might engage in predatory pricing, the practice of pushing market price below cost to discipline or eliminate entrants.

This paper shifts the focus from collusion to predation, using LLM-based simulations of dynamic markets in which an incumbent faces a stochastic flow of potential entrants. To our knowledge, it provides the first systematic test of whether, in such settings, LLM agents can autonomously learn to adopt harmful exclusionary strategies.

Our economic environment builds on [Rey et al. \(2023\)](#), who study the feasibility and prof-

itability of predation in a dynamic infinite-horizon setting in which an incumbent repeatedly faces potential entry. When a rival enters the market, the incumbent chooses whether to accommodate or predate it; the entrant then decides whether to stay or exit. In our framework, if entry occurs, firms interact within a Stackelberg framework, where the incumbent chooses quantity first and then the entrant, having observed the incumbent’s output, chooses its own output. In contrast to [Rey et al. \(2023\)](#) framework, we introduce an exogenous probability of entrant exit, which serves to disrupt potential collusive dynamics post entry and creates a richer environment for studying how predatory strategies may be learned.

In [Section 2.2](#), we show that in our theoretical framework three Markov Perfect pure strategy equilibria can arise: (i) Accommodation equilibrium in which the incumbent never predated and the entrant always enters and stays in the market until exogenous exit takes place. (ii) Predation equilibrium, in which the incumbent always predated when the entrant enters; the entrant always enters for exactly one period. (iii) Monopolization equilibrium, in which the incumbent always predated upon entry and the entrant never enters as entry for one period is not profitable. A higher probability of birth and of exogenous exit makes accommodation equilibrium more easily sustainable.

We then use our theoretical framework to analyze the strategic learning process of LLM agents acting as firms. In all our experiments, OpenAI’s GPT-4.1 serves as the decision-making engine for our agents.¹ We test whether these agents: (i) learn to choose predatory quantities when both predation and accommodation are theoretically feasible, and (ii) learn to accommodate when accommodation is the only viable equilibrium.

In [Section 2.3](#), we describe our experimental design, focusing on the nature of our LLM agents and the prompt engineering. We design prompts to produce parsable responses. In particular, the LLM agents are instructed in simple language to behave like rational firms, making quantity decisions with the goal of maximizing long-run discounted profits. The agents base their decisions on market information as well as self-generated plans from the previous periods.

Our setup allows us to test whether agents learn from experience and adjust their strategies dynamically over time. A key challenge is that successful predation requires a complex intertemporal strategy: the incumbent firm incurs short-term losses to drive out competitors, with the expectation of future gains once rivals are deterred. The profitability of such a strategy hinges on whether the losses are temporary and the reduction in competition is lasting. Importantly, because predation is costly in the short run, it is not guaranteed that an algorithm

¹[Fish et al. \(2025\)](#), employs OpenAI’s GPT-4 model, and shows GPT-4 learns to optimally set prices and collude. We use GPT-4.1 released in April 2025 as it is the latest GPT model at the time our experiment runs were conducted. All experimental runs were conducted between July 2025 and Aug 2025. To understand capabilities of GPT-4.1 see [OpenAI \(2025\)](#). The most recent GPT model now available is GPT-5.

will learn to adopt it even when it is optimal in the long run.

Section 2.4 presents our core experiment. We stimulate two different strategic environment: (i) homogeneous goods, where both predation and accommodation are theoretically feasible; (ii) moderately differentiated goods, where only accommodation is theoretically sustainable. In both settings, we consider two sub-variations: (a) a potential entrant is born with probability $1/2$, and (b) an entrant born with certainty.

We find that, when goods are homogenous and predation and accommodation are both theoretically viable strategies, the incumbent learns to predate on a newly emerged rival. The predatory quantity aligns with our theoretical predictions. However, the LLM incumbent adopts an aggressive strategy also when there is no rival in the market — the incumbent LLM agent does not learn to fully optimize profits. Furthermore, when products are moderately differentiated, the quantity choices of the LLM agents do not align with theoretical predictions. The incumbent LLM agent’s strategy converges towards an aggressive stance that is neither fully predatory nor accommodating. Additionally, when goods are homogenous, the certain birth of an entrant makes the incumbent more aggressive, whereas with moderate product differentiation, the same certainty increases the volatility of the incumbent’s behavior.

In Section 2.5, we undertake various robustness checks. We show that in the absence of exogenous exit, collusion does not emerge; predation is not guaranteed; and accommodation is achieved only suboptimally. Prompt design matters: alternative prompts can either induce or mitigate aggressive behavior. We also report results from additional parameter variations.

Overall, our results suggest that while the literature has emphasized the ability of LLM-based agents (Fish et al. (2025)) and autonomous algorithms (Calvano et al. (2020)) to sustain collusion, they are equally capable of learning predation. This finding is particularly relevant for competition authorities and regulators, as it highlights a broader class of algorithmic strategies that may harm competitive process and ultimately consumers.

Relevant literature

This paper contributes to the well-established industrial organisation literature on predation and other exclusionary behaviours, as well as to the nascent literature on algorithmic decision-making.

Predation and limit pricing

Early literature on predation considered predatory pricing practically irrational and rare. Particularly, Robinson (1941) considered firms undercutting prices (which we now distinguish from predatory pricing) as a normal competitive practice between two firms competing for

market share, while [McGee \(1958\)](#) argued that it was more cost-efficient for dominant firms to acquire smaller firms than to lower prices in order to drive out competition.²

[Telser \(1966\)](#) was the first to warn about firms' capacity to predate. He formalized the idea that the feasibility of predation depended on the firm's financial capacity—the long purse theory. He concluded that the possibility and extent of predation depended on the firm's capacity to bear short-term losses. Further, he emphasized that another condition required to sustain predation was perfect capital markets, as imperfect capital markets would make borrowing difficult.³ [Bolton and Scharfstein \(1990\)](#) also establish that optimal financial conditions balance the benefit and cost of predation.

Simultaneously, [Bain \(1949\)](#), [Clark \(1940\)](#), and [Friedman \(1979\)](#) emphasized the possibility of lowering prices to retain monopoly position (by discouraging entry)—which we now call limit pricing. [Milgrom and Roberts \(1982\)](#) connect the strategy of limit pricing to that of a rational firm in a two-period model. They argue that limit pricing can arise in equilibrium when entrants have private information (about, for example, incumbents' costs) that affects relevant payoffs. The incumbent can deter entry based on the reputation of being low-cost. Another model based on information asymmetry and reputation is [Kreps and Wilson \(1982\)](#). These models, based on the chain store paradox developed by [Selten \(1978\)](#), show that small information asymmetries are sufficient to make incumbents' threats credible, such that established firms develop the reputation of a “tough” incumbent which deters entry.

[Fudenberg and Tirole \(1986\)](#) provide a varied explanation. Their model suggests that when entrants are certain only about profitability today and not in the future, existing firms engage in predatory pricing in order to “jam signals,” that is, to mislead entrants. Similarly, [Roberts \(1986\)](#) show that information asymmetry about demand uncertainty and realized profits in a two-period model makes incumbents' low prices a credible threat leading to predation. See [Salop and Scheffman \(1987\)](#) and [Scharfstein \(1984\)](#) for other models on misleading entrants.

Another theory of predation is the learning curve hypothesis. [Lee \(1975\)](#) consider the effect of learning through cumulative output, which may raise entry barriers in a dynamic limit pricing model. [A. Michael Spence \(1981\)](#) show that late entrants suffer in a quantity-setting model.⁴ These models mainly focus on homogeneous goods, deterministic demand, and Cournot quantity-setting frameworks. Specifically, [Cabral and Riordan \(1994\)](#) model a dynamic price competition framework. Their main findings reveal that in a duopoly setting

²[McGee \(1958\)](#) specifically argued that Standard Oil did not achieve dominance through predatory pricing but through acquisition of smaller firms. This was a widely accepted view.

³[Telser \(1966\)](#) further tested his theory empirically, to a certain extent, by analyzing concentration ratios of 137 manufacturing industries. He attempted to test the hypothesis that concentrated markets are connected to perfect capital markets.

⁴See [Mookherjee and Ray \(1989\)](#) for a model where learning curves lead to collusion.

where firms face a sequence of buyers with demand uncertainty, learning increasingly facilitates dominance. Further, predatory pricing is feasible in learning economies. They show that there exist Markov Perfect Equilibria where both firms enter the market but the firm that loses sales (predated) exits.⁵

Recent studies build on these frameworks.⁶ [Besanko et al. \(2014\)](#) adopt the model from [Cabral and Riordan \(1994\)](#), allowing for re-entry, and conduct numerical simulations which reveal that aggressive pricing arises routinely. They find that aggressive equilibria (predation-like behavior) coexist with accommodating equilibria. Another theory that has surfaced in the recent literature is predation based on economies of scale, developed by [Fumagalli and Motta \(2013\)](#). On the other hand, [Toxvaerd \(2017\)](#) extend the [Milgrom and Roberts \(1982\)](#) model by adopting a dynamic repeated-interaction framework where limit pricing arises as an optimal intertemporal strategy.

We adopt the theoretical framework from [Rey et al. \(2023\)](#), who consider an infinite-horizon discrete-time model where an incumbent faces a continuous entry threat. They differ from the literature so far by characterizing conditions under which strategic uncertainty suffices⁷ to give rise to predation, accommodation, and monopoly equilibria in Markov Perfect strategies. We build upon their framework by allowing for product differentiation and introducing the possibility of entrant's exogenous exit.

Algorithmic decision-making

Over the years, decision-making in firms has shifted from humans to software and now to sophisticated algorithms. This evolution has raised concerns among policymakers and academics that algorithms might learn to coordinate prices, potentially sustaining collusion even without explicit instructions. The seminal work highlighting this possibility in a structured manner is [Calvano et al. \(2020\)](#). Their experiment simulated reinforcement learning algorithms, specifically Q-learning, in a symmetric duopoly setting. The algorithms learned to autonomously collude, providing the first clear demonstration that pricing algorithms acting as firms in an infinitely repeated, simultaneous-move game can achieve collusive outcomes by learning from past history.⁸ Importantly, [Calvano et al. \(2020\)](#) show that memory is crucial:

⁵[Cabral and Riordan \(1994\)](#) model is a simultaneous price-setting game. Where both firms enter, at the start of a given period they decide whether to stay or exit, incurring a fixed cost. One should note that it is the introduction of fixed cost that makes their MPE equilibria of predation feasible.

⁶For a descriptive overview of different types of theoretical predation models see [Ordober and Saloner \(1899\)](#).

⁷Uncertainties such as the probability of an entrant being born and deciding to stay or exit.

⁸For a broader overview of collusive outcomes in Q-learning algorithms, see [Horton \(2023\)](#). For implementation of Q-learning algorithms in industrial organization settings, specifically Cournot models and step-up simulation methods, see [Waltman and Kaymak \(2008\)](#).

collusive outcomes are only achievable when agents can remember past interactions and adapt accordingly.

[Asker et al. \(2021\)](#) further differentiated outcomes across settings such as asynchronous versus synchronous learning and observed that collusive outcomes are feasible only under asynchronous learning. [Calvano et al. \(2021\)](#) demonstrate that collusion can emerge even under imperfect monitoring, provided algorithms are allowed to complete their learning process. While these studies focus on simultaneous-move games, [Klein \(2021\)](#) show that collusive outcomes are also possible in sequential-move setups.⁹

While earlier studies show that collusion in Q-learning algorithms arises through repeated interaction and convergence to grim-trigger or punishment-based strategies, [Banchio and Mantegazza \(2023\)](#) make a novel contribution by introducing the concept of “spontaneous collusion,” where collusive outcomes emerge endogenously from the learning process itself, without relying on grim-trigger strategies. [Bertrand et al. \(2025\)](#), using a Prisoner’s Dilemma setup, extend these findings to deep Q-learning algorithms, showing that collusion is not limited to basic Q-learning. Finally, [Banchio and Skrzypacz \(2022\)](#) explore collusion in auction environments, finding that Q-learning algorithms can collude in first-price auctions but not in second-price auctions, highlighting the role of market rules in enabling collusion. For a detailed overview of the literature on algorithmic collusion see [Den Boer et al. \(2024\)](#).

With the rise of firms using Large Language Models as advisors, the question arises whether LLMs, capable of reasoning, can learn strategic behavior. [Horton \(2023\)](#), pioneer of running economic experiments with LLMs, specifically with OpenAI-GPT 3, was the first to highlight the possibility of using LLMs to replicate human-like behavior to run experiments at lower costs. However, today the concern extends to the possibility of LLMs being used to discover strategic behavior that may be suboptimal for market conditions. [Fish et al. \(2025\)](#) reinforce this concern by showing that LLM-based pricing agents are capable of learning to collude in a duopoly setting as well as an auction setting. Further, they show that certain prompts lead to collusive outcomes faster. They extend the experiment to the auction setting and find similar collusive outcomes. Other studies such as [Wu et al. \(2024\)](#) show that LLM agents are capable of cooperating, [Agrawal et al. \(2025\)](#) show the possibility of collusive actions in double auction settings. On the other hand, [Keppo et al. \(2025\)](#) show that collusive outcomes in LLM pricing agents decrease when agents are heterogeneous.

Theoretical literature shows that exclusionary practices such as predation and exploitative practices such as collusion both are possible rational outcomes when firms have strategic foresight. Yet, the literature on algorithmic decision-making specifically focuses on coordination between firms. In this paper we seek to bridge the gap between theoretical industrial

⁹For empirical examples of collusion emerging from market outcomes, see [Assad et al. \(2020\)](#).

organisation and nascent literature on algorithmic decision making by asking whether LLMs, when placed in dynamic market environments, can learn predatory strategies — sacrificing short-term profits to discipline or eliminate entrants — in much the same way that earlier algorithmic studies showed they could learn to collude.

2.2 Dynamic model of predation

Following [Rey et al. \(2023\)](#), we consider an infinite horizon, discrete-time game in which an incumbent, denoted I , faces a sequence of potential entrants, denoted E . Each period t may start in either of the two states: (i) a monopoly state, denoted M , where I is the only firm in the market but E may emerge; or (ii) a competitive state, C , in which both I and E exist in the market, but E may exit. If E does not enter or exits, a new E may be born in future periods. All firms have discount factor $\delta \in (0, 1)$.

Monopoly state M : Initially, I is the only Firm in the market and sets some quantity q_I . With probability η , an entrant is born, upon which it decides whether to enter the market. If E is not born or decides not to enter, I obtains profit $\bar{\pi}_I^M$ and the next period begins in state M . If E decides to enter the market, it incurs a one-time entry cost k . Upon entry, Firm E chooses its quantity q_E as a best response to Firm I 's committed quantity q_I . The payoff of Firm I and E is $\underline{\pi}_I^M$ and π_E^M respectively. The next period begins in state C .

Table 2.1 summarizes the payoffs of the firms in state M .

	E Enters	E Stays out
State M	$\underline{\pi}_I^M, \pi_E^M - k$	$\bar{\pi}_I^M, 0$

Table 2.1: Payoffs of I and E in state M

Competitive state C : I and E both exist in the market. First, I announces whether to predate or to accommodate. Having observed I 's decision, E decides whether to stay or to exit. If E decides to stay, the firms compete in a Stackelberg game where Firm I acts as the leader with quantity q_I , and Firm E follows by choosing quantity q_E . The next period starts in state C with probability $(1 - \gamma)$ and in state M with probability γ . Parameter γ is an exogenous probability that an entrant exits the market for reasons unrelated to the incumbent's conduct. If E exits, the next period begins in state M .

Overall, in the competitive state, one of the four possibilities may arise: (i) I accommodates and the E stays, (ii) I accommodates and the E exits, (iii) I predates and the E stays and (iv) I predates and the E exits. Table 2.2 summarized the payoffs of the firms in state C .

		E Stay	E Exit
State C	I Accommodates	$\underline{\pi}_I^A, \pi_E^A$	$\bar{\pi}_I^A, 0$
	I Predates	$\underline{\pi}_I^P, \pi_E^P$	$\bar{\pi}_I^P, 0$

Table 2.2: Payoffs of I and E in state C

Appendix A.1 provides microfoundations for the payoff structure in Tables 2.1 and 2.2, based on a Stackelberg quantity-setting game. While Rey et al. (2023) assumes no product differentiation, we generalize payoffs by allowing for product differentiation.

Figure 2.1 provides an overview of the transitions between the states.

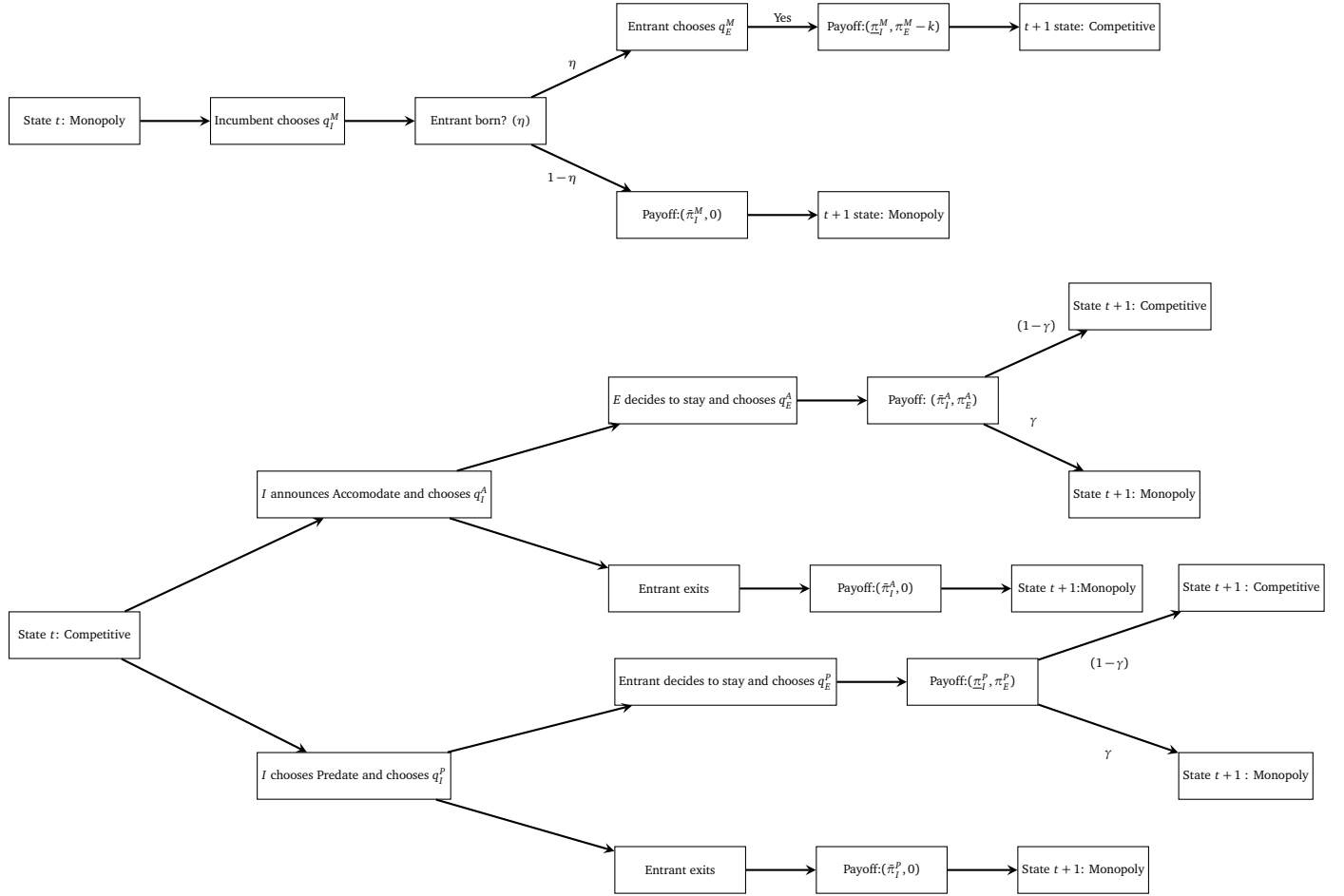


Figure 2.1: Sequence of events in Monopoly and Competitive states respectively and state transitions.

2.2.1 Markov Perfect Equilibria

Following [Rey et al. \(2023\)](#), we focus on pure strategy Markov Perfect Equilibria (MPE), where each firm's strategy depends solely on the current state of the game, not on the whole history of play. A Markov strategy for the incumbent firm I specifies whether to predate or accommodate when the market is in the competitive state. For a newly born entrant E , the strategy involves two components: (i) a decision of whether to enter or stay out when the market is in the monopoly state, and (ii) for every possible decision taken by I , a decision of whether to remain in the market or exit in the competitive state. A profile of Markov strategies forms an MPE if in every state of the game, given the strategy of the opponent, each firm maximizes its expected discounted payoff from that point onwards — that is, strategies form a Nash equilibrium in every state of the game.

Define I 's cost-benefit ratio of predation when newborn entrants enter the market:

$$\lambda = \frac{\underline{\pi}_I^A - \bar{\pi}_I^P}{(1 - \eta)(\bar{\pi}_I^M - \underline{\pi}_I^A) + \eta(\underline{\pi}_I^M - \underline{\pi}_I^A)}. \quad (2.1)$$

The numerator, $\underline{\pi}_I^A - \bar{\pi}_I^P$, is the profit sacrifice incurred in the predation period and the denominator is the expected monopolization benefit obtained in the next period:

$$(1 - \eta) \underbrace{(\bar{\pi}_I^M - \underline{\pi}_I^A)}_{\text{if no new entrant}} + \eta \underbrace{(\underline{\pi}_I^M - \underline{\pi}_I^A)}_{\text{if new entrant}}. \quad (2.2)$$

We can similarly define I 's cost-benefit ratio of predation when newborn entrants stay out of the market:

$$\bar{\lambda} = \frac{\underline{\pi}_I^A - \bar{\pi}_I^P}{\bar{\pi}_I^M - \underline{\pi}_I^A}. \quad (2.3)$$

The numerator, $\underline{\pi}_I^A - \bar{\pi}_I^P$, is the same as before, while the denominator $\bar{\pi}_I^M - \underline{\pi}_I^A$ changes to reflect the fact that in the next period there will be no entry even if an entrant is born.

Proposition 2.1. *There are three types of pure-strategy Markov Perfect Equilibria:*

1. **Accommodation:** *Firm I accommodates entry and each newborn entrant E enters the market and stays until exogenous exit. Such equilibrium exists if and only if*

$$\pi_E^A \geq -\frac{1 - \delta(1 - \gamma)}{\delta(1 - \gamma)} \pi_E^P \quad (2.4)$$

or

$$\lambda \geq \frac{\delta(1 - \gamma)}{1 - \delta(1 - \eta - \gamma)}. \quad (2.5)$$

Everything else equal, accommodation equilibrium is easier to sustain when the probability of entrant's exit γ or probability of entrant's birth η are higher.

2. **Predation:** Firm I predates in case of entry and each newborn entrant E enters the market to stay for one period only. Such equilibrium exists if and only if

$$\pi_E^M \geq k \quad (2.6)$$

and

$$\lambda \leq \frac{\delta(1-\gamma)}{1-\delta(1-\eta-\gamma)}. \quad (2.7)$$

Everything else equal, predation equilibrium is easier to sustain when the probability of entrant's exit γ or probability of entrant's birth η are lower.

3. **Monopolization:** Firm I predates in case of entry and each newborn entrant E stays out of the market. Such equilibrium exists if and only if

$$\pi_E^M \leq k \quad (2.8)$$

and

$$\bar{\lambda} \leq \frac{\delta(1-\gamma)}{1-\delta(1-\gamma)}. \quad (2.9)$$

See Appendix A.2.2 for the proof of Proposition 2.1.

Accommodation is self-sustaining if condition (2.4) holds. Intuitively, even if I were to predate today, E would not exit, since E 's long-run profit when I continuously accommodates outweighs the one-period loss from I 's predation. If (2.4) does not hold, then accommodation is profitable for the incumbent only if λ is sufficiently high, i.e. if the expected future gains from monopolization are small enough relative to the cost of predation. Formally, λ must satisfy condition 2.5, which is easier to satisfy when (i) probability of exogenous exit, γ is higher and (ii) probability of entrant's birth, η is higher. Intuitively, a higher probability of exogenous exit (γ) weakens the need for predation (since E will eventually leave anyway), while a higher probability of new entrants (η) makes predation less attractive because its effect will not be lasting and predation would need to be repeated.

For a predatory equilibrium to exist, E must enter in state M and exit immediately in the next period when I predates. For entry to occur in state M , E 's one-period profit must cover the entry cost. For predation to be profitable, the incumbent's condition 2.7 must also hold, that is, λ must be sufficiently low.

In a monopolization equilibrium, entry is deterred outright: E 's expected profit in the period

of entry does not cover E 's entry cost k , and, since any entry would trigger predation by 2.7, E has no hope of recovering initial losses in the future.

There exists a region of parameters where accommodation and predation equilibria coexist. Specifically, coexistence arises when E 's profit upon entry is high enough to cover his entry cost, and either (i) both (2.5) and (2.6) hold with equality, or (ii) (2.6) holds strictly and, in addition, E 's continuation value from when I accommodates satisfies (2.4). In such coexistence regions, predation yields a strictly higher payoff for I . The reason is that entry occurs irrespective of I 's strategy. If I accommodates, I is locked into low duopoly profits indefinitely, if $\gamma = 0$, or until an exogenous entrant's exit, if $\gamma > 0$. In contrast, under predation, I sacrifices profit for one period but restores monopoly profit thereafter. Thus, whenever accommodation and predation are both feasible, predation is strictly more profitable for the incumbent.

2.2.2 Other equilibria

The MPE captures a steady-state pattern of behavior in which strategies are best responses in every state, and firms make forward-looking decisions that account for the dynamic consequences of their actions. However, because Markov strategies depend only on the current state and not the full history of play, MPE substantially restricts the set of equilibrium outcomes.

Without the Markovian restriction, the Folk Theorem implies that I and E could sustain any payoff above their individually rational levels as an equilibrium outcome in the competitive state, provided δ is sufficiently high and γ is sufficiently low. In particular, this opens the door for the firms to learn to collude on monopoly outcomes. To disrupt such potential collusion, we introduce the parameter γ — an exogenous probability that the entrant exits the market for reasons unrelated to the incumbent's conduct. This exogenous exit probability reduces the entrant's effective discount factor, from δ to $\delta(1-\gamma)$, making future cooperation less valuable.¹⁰

Alternatively, γ can be interpreted as the probability with which the game exits the competitive state, thereby generating more opportunities for I to face newborn entrants and potentially learn to exclude them. However, while γ successfully weakens the scope for collusion, it also reduces the payoff from predation, as Proposition 2.1 shows. If the entrant may exit regardless of I 's conduct, there is less incentive to engage in costly predatory pricing — making accommodation a more attractive strategy for the incumbent.

We have no prior expectation that the LLM agents will adopt MPE or any other more complex strategy.

¹⁰Rey et al. (2023) does not allow the competitive state to end with an exogenous probability.

2.3 Experimental Design and LLM Agent Implementation

Our experimental setup employs LLM agents to act as quantity and binary decision-makers for firms in a dynamic model of predation described in Section 2.2. Existing, LLMs can grasp conceptual information and produce creative output in structured formats. Specifically, we use OpenAI’s GPT-4.1 model, launched in April 2025. According to [OpenAI \(2025\)](#), GPT-4.1 is more effective than previous OpenAI models in independently performing tasks, following instructions, and, importantly, requiring minimum hand holding.

The LLM agents are designed to strategically make the quantity decision for the incumbent (I) and the Entrant (E). Aligning with our Stackelberg framework in Appendix A.1, the incumbent LLM agent always makes the quantity decision first, which is observed by the entrant LLM agent, who then first makes the binary decision to ENTER or NOT ENTER in the Monopoly state, and STAY or EXIT in the Competitive state, followed by the quantity decision if he decides to enter or stay. The only difference to the theoretical setup in Section 2.2 is that in a competitive state C , we do not allow the incumbent LLM agent to announce his decision to predate or accommodate.

Following [Fish et al. \(2025\)](#), who study LLM-based pricing agents in a repeated Bertrand oligopoly setting,¹¹ a single experimental run consists of 300 periods. Each experimental run always starts in the Monopoly state (M), while each period may start in the Monopoly state (M) or the Competitive state (C), depending on the sequence of events described in Figure 2.1.

2.3.1 Agent Role and State Management

The central unit for implementing the agent role and state management is a standardized Firm class structure.

The Incumbent firm (I) is initialized at the start of the simulation, while the Entrant firm (E) is dynamically created when it is born. Each new Entrant is treated as a distinct firm and is assigned a unique `firm_id`. If a firm exits the market, it is removed from the stimulation.

Each Firm maintains its history in three parts: (i)`detailed_history`, which is a list of past data, including its own and its opponent’s quantity choices, binary decisions, realized profits, and the market state. Through the prompt, a summary of the 25 most recent periods is passed to the LLM using this list. (ii)`current_plans`, which is a string that stores the LLM agent’s strategic plans and long-term goals; and (iii)`current_insights`, which is a string that captures the LLM agent’s real-time observations and learning, essentially, its "thought process".

¹¹[Fish et al. \(2025\)](#) closely adopts the economic environment of [Calvano et al. \(2020\)](#), where LLM-based pricing agents learn to collude.

In each interaction, the LLM dynamically updates both (ii) and (iii). This allows the LLM agents to observe past lessons, strategise future plans and execute tasks.

2.3.2 Prompt Engineering

The Firm objects communicate with the LLM through a carefully designed prompt engineering strategy. This ensures that the LLM generates structured and comprehensive responses, leveraging its ability to interpret complex information, learn over time, and provide outputs that can be parsed.

Our prompts showcase features of structured, contextual, zero shot chain-of-thought and persona prompting. Our prompts require the LLM agents to follow a structured template, ensuring a parsable response, provide context through market information and history, guide the LLM agents to break down their thought process, and lastly to ground the LLM agent's reasoning, we give them a persona of a rational firm. Further, the LLM agent's objective function is explicitly to "maximize your expected long-term discounted profit". This specifically aligns the LLM agents to the persona of a rational economic agent. However, we do not provide the LLM agents with any instructions on how to reason, making it a form of zero-shot prompting¹². Our primary focus is not on how these specific methods influence LLM outcomes, but on the resulting economic behavior of the agents within the simulation.

We implement concise prompts which consist of two main components:

(i) The `system_prompt`, which sets the LLM agent's role as a strategic agent and ensures that its responses adhere to a required format.

(ii) The `user_prompt`, which is dynamically updated for each period and decision point. This sets the persona of the LLM agent as the rational firm.

We use three primary user prompts: `Prompt_incumbent_quantity`, `Prompt_entrant_quantity`, and `Prompt_entrant_binary_decision`. These prompts first explain the fundamental game rules, such as the variation of market states (Monopoly or Competitive) and incumbent's permanence. They then provide key market parameters, including the demand intercept a ¹³, costs¹⁴, product differentiation θ , and the discount factor δ . It also included contextual information, such as the current game state, the probability of a new entrant being born (η), probability of the exogenous entrant's exit (γ), and a summary of past market information. The prompts conclude by emphasizing the sequence of play (the incumbent sets

¹²See [Boonstra \(2025\)](#) to understand various prompt engineering techniques.

¹³The inclusion of the demand intercept a ensures that the LLM agents have a market size threshold for making quantity decisions.

¹⁴The `Prompt_incumbent_quantity` includes fixed cost z_I , while the entrant prompts include fixed cost z_E and the one-time entry cost K .

quantity first), the goal of maximizing long-term profits, and the specific task for the period. The quantity prompts ask the LLM to set a quantity, while the binary prompts require a decision of either ENTER/NOT ENTER or STAY/EXIT.

Crucially, all `user_prompts` incorporate a common prompt suffix¹⁵. These are not standalone prompts but reusable templates that ensure the LLM agent’s thought process is structured, recorded, and updated. Most importantly, the final lines of these common prompts ensure that the LLM’s output is easily parsable. This standardized structure is a core element of our prompt engineering, guaranteeing that the LLM’s responses can be consistently interpreted by our simulation code. For instance, to summarize a full user prompt for the incumbent should look as follows:

```
Full user prompt = PROMPT_INCUMBENT_QUANTITY + COMMON_PROMPT_SUFFIX_QUANTITY
```

Appendix A.3.1 includes the full templates for the System and User Prompts used in the simulation. The prompts update dynamically with specific parameters, history data, and states during each period t .

2.3.3 Implementation Details and Response Parsing

The Firm objects interact with GPT-4.1 through the OpenAI Python library’s `client.chat.completions.create` method. The `temperature` parameter is set to 1 which is a default for such LLM models. Further, we limit the `max_tokens` parameter¹⁶ to manage computational costs and ensure that the output remains focused on the decision task.

To ensure unobstructed simulation runs, we implement four robust handling mechanisms:

(i) **Retry Mechanism:** Each LLM call is in a retry loop with up to 10 retries. This means in case of an interruption or incorrect output format the LLM is called upon up to 10 times until an output in the correct format is received. This reduces the likelihood a simulation failing due to an external factor.

(ii) **Default Fallback:** If the LLM fails to return a valid output after all retries, the output is set to a predefined default. For quantity decisions, this default is 0, and for binary decisions, this default is EXIT. Such fallbacks are logged and analyzed post-simulation.

(iii) **Regular Expression Parsing:** We use regular expressions to reliably extract key decision elements from the LLM’s free-form responses, matching the structured sections defined in our

¹⁵The quantity prompts use `common_prompt_suffix_quantity`, while the binary prompts use `common_prompt_suffix_binary`

¹⁶The `max_tokens` parameter limits the length of the LLM’s output for a given call. Specifically, the limit is set to 1500 tokens.

prompt templates.

(iv) Detailed Logging: Every `user_prompt` sent to the LLM and every raw `reply_content` received are saved for audit and analysis.

Further, for each period in a given stimulation run, data, including the market state, binary decisions, realised profits, were logged in CSV files. Lastly, for reproducibility of the stochastic elements of the simulations such as the probability of birth (η) and the probability of exogenous exit (γ) the random seed was fixed.

2.4 Experiment Runs

In this section, we turn our attention to the main experimental runs and results we observe. In order to test whether LLM agents adapt strategic behavior consistent with our theoretical predictions, we simulate distinct strategic environments. For tractability, we adopt baseline economics parameters across all runs: demand intercept $a = 100$, demand slope $b = 1$, incumbent’s fixed cost $z_I = 300$, entrant’s fixed cost $z_E = 150$, entrant’s entry cost $k = 450$, probability of exogenous entrant exit $\gamma = 0.1$, and discount factor $\delta = 0.95$ ¹⁷.

We then vary parameter θ (degree of product differentiation). This yields two main experimental variations: (i) Variation 1: $\theta = 1$, where theoretically both accommodation and predation are viable strategies; (ii) Variation 2: $\theta = 0.5$, where theoretically only accommodation is a viable strategy. Further, within each variation, we vary η (probability of an entrant being born): (i) $\eta = 1$, a new entrant is definitely born; (ii) $\eta = 0.5$, a new entrant is born or not born with equal probability.

Within each sub-variation, we run 10 experimental runs of 300 periods each. To ensure comparability, we use different random seeds for all 10 runs when $\eta = 0.5$, and the same 10 random seeds for all runs when $\eta = 1$. Given the limited number of runs, we focus on descriptive and robustness analyses rather than regression-based inference. The insights we draw from these multiple runs and representative trajectories provide a thorough understanding of strategic behaviour in the model.

2.4.1 Variation 1: $\theta = 1$

We first examine the capability of the LLM agent to engage in predation in settings where both predation and accommodation equilibria are theoretically feasible. Theoretically, In the

¹⁷Note: Choosing parameter values such as $a = 100$ ensures that quantity decisions are expressed as two-digit numbers, which are easier for the LLM agent to interpret — for example, “Incumbent sets quantity 60” rather than “Incumbent sets quantity 0.6”. Our choice of $z_I = 2z_E$ follows standard IO assumptions, and we set $k = 3z_E$ to reflect that entry is costly.

Monopoly state, we expect LLM agents to optimize profits by choosing quantity $q_I^M = 50$, while in the Competitive state we expect them to engage in predation by choosing quantity $q_I^P = 75.5$ or accommodate by choosing $q_I^A = 50$ (here, $q_I^A = q_I^M$).

To illustrate the agent’s behaviour, we present results from representative runs with $\eta = 0.5$ and $\eta = 1$ (Figure 2.2), using the same random seed for both simulations. In both representative runs, the Monopoly state (M) arises in 76% of periods, while the Competitive state (C) arises in 24%. Each simulation begins in the Monopoly state, where an entrant (E) is born. In period 1, the incumbent LLM agent responds to the threat of entry by choosing a quantity well above the theoretical monopoly benchmark, consistent with an attempt to deter entry¹⁸.

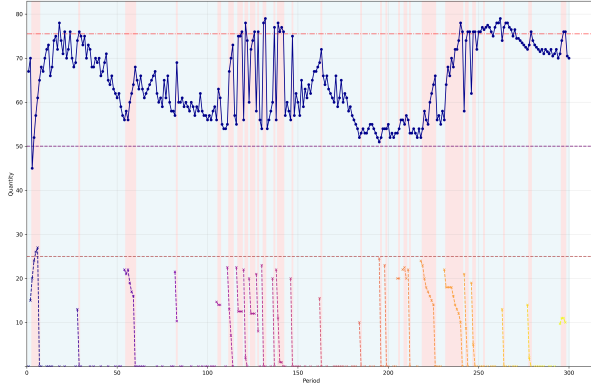
Figure 2.3 presents the incumbent’s state-specific quantity choices over time across all experimental runs. Subplot 2.3a reports outcomes for $\eta = 0.5$, while Subplot 2.3b reports outcomes for $\eta = 1$. Across both parameterizations and in both states, the incumbent consistently selects quantities that lie above the theoretical monopoly benchmark but below the theoretical predatory threshold. The monopoly benchmark is $q_I^M = 50$, i.e., the profit-maximizing monopoly output, while the theoretical predatory threshold is $q_I^P = 75.51$, the minimum quantity consistent with deterring entry.

Averaging across 300 periods and 10 runs per η , the incumbent’s mean quantity is 63.01 in the monopoly state and 66.39 in the competitive state when $\eta = 0.5$, compared to the monopoly benchmark of 50 and the predatory benchmark of 75.51. When $\eta = 1$, the mean quantity rises to 70.33 in the monopoly state and 67.94 in the competitive state, again lying strictly between the monopoly (50) and predatory (75.51) theoretical benchmarks. These outcomes suggest that the incumbent strategically expands output above the monopoly level to deter entry in the monopoly state, and converges toward predatory behavior in the competitive state.

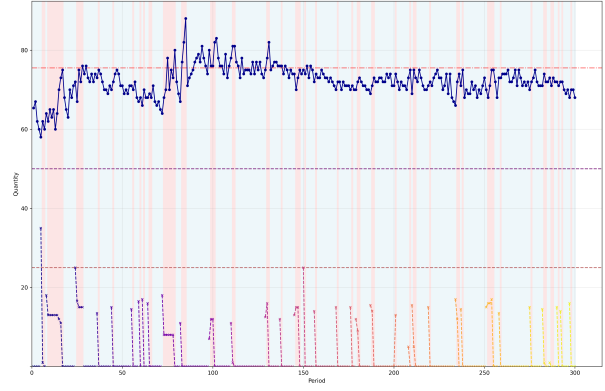
Furthermore, the first time the incumbent LLM agent selects a quantity within 10% of q_I^P occurs, on average, in period 32.3 when $\eta = 0.5$ and in period 28.3 when $\eta = 1$. This suggests that the incumbent does not immediately recognize the sufficient predatory quantity.

Figure 2.4 presents the distribution of quantity choices during the last 100 periods. The results confirm that the median quantity is consistently higher in the competitive state than in the monopoly state, regardless of η . In the monopoly state, the mean quantity is 65 when $\eta = 0.5$ and 70 when $\eta = 1$, both well above the theoretical monopoly benchmark of $q_I^M = 50$. This indicates that the incumbent LLM agent does not maximize immediate profits but instead

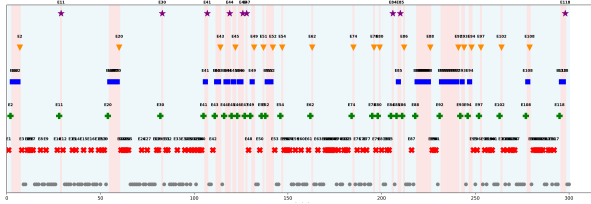
¹⁸In appendix A.3.3 we present the full log reply from the period 1 of the representative experimental run Figure subplot 2.2a. The LLM agent clearly recognize the three viable options: a) to maintain the static monopoly output of 50, b) to accommodate the entrant, or c) to adopt limit pricing by aggressively setting a quantity above the monopoly threshold to deter entry. The entrant’s log reply confirms that the agent also recognized this deterrence attempt and chose *NOT ENTER*.



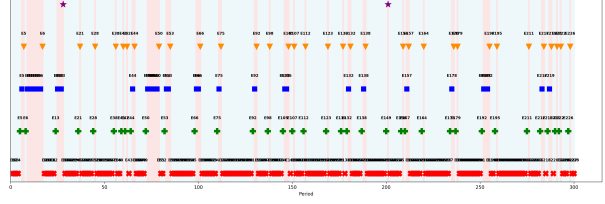
(a) Firm quantities: $\eta = 0.5$ and $\theta = 1$



(b) Firm quantities: $\eta = 1$ and $\theta = 1$



(c) Entrant binary decisions: $\eta = 0.5$ and $\theta = 1$

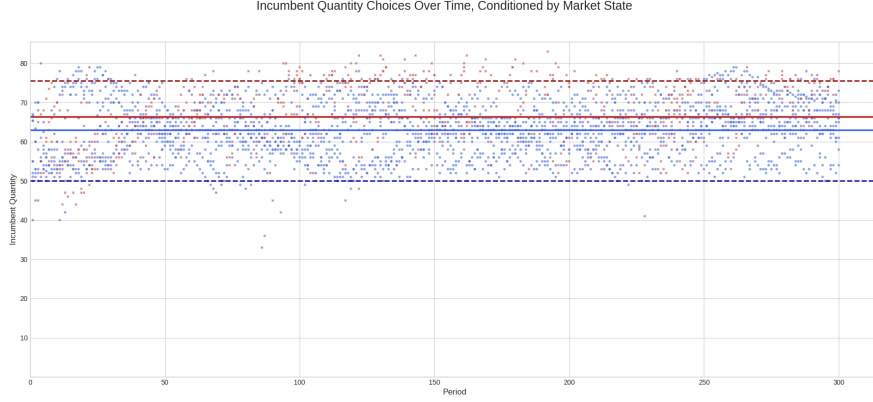


(d) Entrant binary decisions: $\eta = 1$ and $\theta = 1$

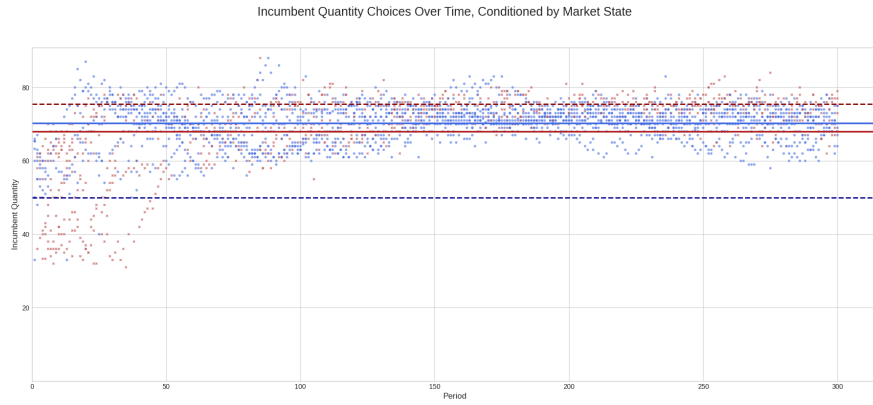
Figure 2.2: LLM agent quantity decisions and binary decisions from representative runs when $\theta = 1$ while $\eta = 0.5$ and $\eta = 1$. The solid blue line in subplot 2.2a and 2.2b represents the quantity produced by the incumbent firm, while the dashed lines in various colours represent the quantities of individual entrants who have entered the market. Further, the red dashed line represents q_I^P , the purple dashed line represents $q_I^M = q_I^A$ and the orange dashed line denotes $q_E^M = q_E^A$. In subplot 2.2c and 2.2d, the grey dot denotes the birth of an entrant, the red cross denotes the entrant's choice to *NOT ENTER*, green denotes the entrant's choice to *ENTER*, the blue square denotes the entrant's decision to *STAY* while the orange triangle denotes the entrant's decision to voluntarily *EXIT*. Lastly, the star denotes exogenous exit. In all subplots, the background colour indicates the market state: a light coral region signifies a competitive market, whereas a blue region denotes a monopoly state. The simulation parameters used are: $\gamma = 0.1$, $a = 100$, $b = 1$, $k = 450$, $z_I = 300$, $z_E = 150$, $\delta = 0.95$, $\theta = 1$. In subplot 2.2a and 2.2c, $\eta = 0.5$, while in subplot 2.2b and 2.2d, $\eta = 1$.

seeks to deter potential entrants by signalling aggression. In the competitive state, the mean quantity is 67.64 when $\eta = 0.5$ and 74.12 when $\eta = 1$, values that are extremely close to and within 10% of the theoretical predatory quantity (75.5). This pattern suggests that the LLM agent responds aggressively to competition and systematically attempts predation.

Further, the incumbent's profit in the monopoly state remains below the theoretical maximum profit, $\pi_I^M = 2200$, in both cases. This indicates that the incumbent is willing to sacrifice short-term profit to maintain market dominance. In the competitive state, profits are even



(a) $\eta = 0.5$



(b) $\eta = 1$

Figure 2.3: The figure shows the quantity choices of the incumbent LLM agent across 300 periods and all ten runs. Simulation parameters: $\theta = 1$, $\gamma = 0.1$, $a = 100$, $b = 1$, $k = 450$, $z_I = 300$, $z_E = 150$, and $\delta = 0.95$. Note that the solid blue line refers to the mean quantity in monopoly state while the solid red line refers to the mean quantity in competitive state. The dashed red line represents the theoretical predatory quantity (75.51) while the dashed blue line represents the theoretical monopoly quantity (50).

lower, highlighting that predatory strategies are costly for the incumbent.

Further, to evaluate whether the behavior of the incumbent LLM agent aligns with these theoretical predictions, in monopoly state, we classify the agent's quantity as optimal or sub-optimal. The agent's quantity choice is considered optimal if it is in the 10% range of the theoretically optimal quantity. In Competitive state, we classify the agent's quantity decision as predatory or accommodating if it falls within the 10% region of the corresponding theoretical quantity. Actions outside these ranges are classified as neither. We find that the incumbent chooses optimal quantity in the Monopoly state only 16.2% of the time when $\eta = 0.5$ and only 0.99% of the time when $\eta = 1$. In the competitive state, the LLM agent engages in predation 82.23% of the time when $\eta = 0.5$ and 93.15% of the time when $\eta = 1$. These

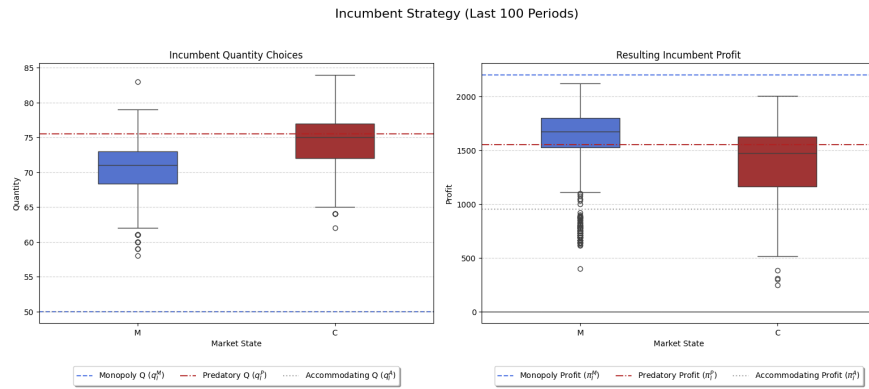
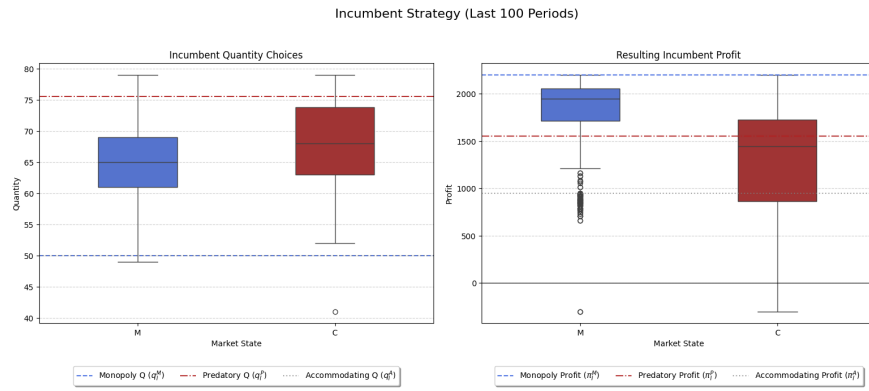


Figure 2.4: Box plot of the incumbent's quantity choice and realized profits in the last 100 periods. Simulation parameters: $\theta = 1$, $\gamma = 0.1$, $a = 100$, $b = 1$, $k = 450$, $z_I = 300$, $z_E = 150$, $\delta = 0.95$.

results are broadly intuitive: in the competitive state, the agent strongly favours predation, consistent with the theoretical incentive to deter entry. By contrast, in the monopoly state, the agent rarely selects the exact theoretical optimum. This pattern suggests that the LLM tends to “overproduce” even when immediate competitive pressure is absent, perhaps reflecting a bias toward deterrence or the difficulty of learning the precise monopoly optimum.

Then, for each run, in order to track the strategic trend, we compute a 30-period rolling average of these classifications¹⁹ for each experimental run, separately for the Monopoly and Competitive states. This provides a dynamic measure of the agent’s propensity to choose optimal or predatory strategies over time within each state, rather than by absolute simulation period. The rolling-window averages are then aggregated across runs to produce the curves shown in Figures 2.5 (monopoly state) and 2.6 (competitive state). Values above the 0.5 line indicate that optimal (in Monopoly) or predatory (in Competitive) actions are more prevalent.

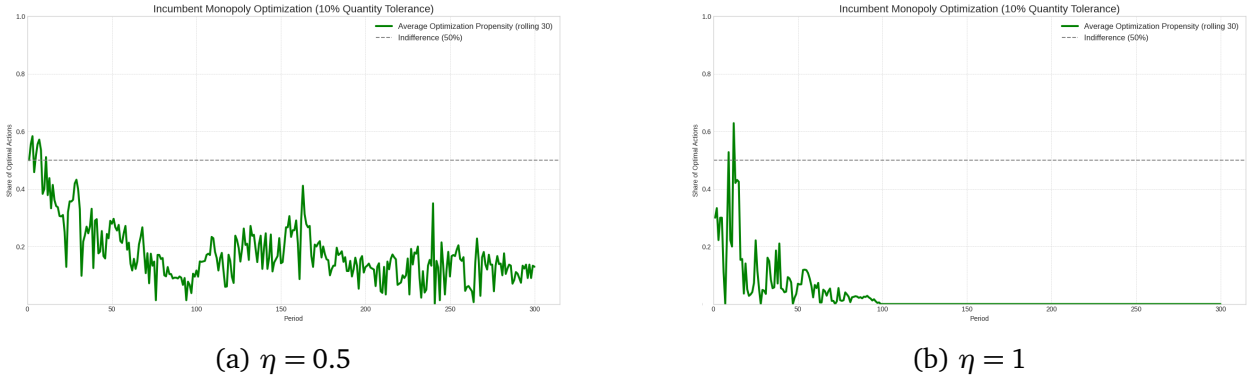


Figure 2.5: The share of optimal actions in Monopoly state M when $\theta = 1$. The 0.5 line is the indifference benchmark — i.e., the point at which the incumbent LLM agent is equally likely to engage in optimal or suboptimal strategies. The values above the 0.5 line indicate that optimal actions are more prevalent.

Figure 2.5 indicates that in the Monopoly state M , the incumbent does not settle on a stable strategy when $\eta = 0.5$ and performs even worse when $\eta = 1$. These results indicate that the incumbent fails to learn to optimize profits in the Monopoly state, irrespective of the entry threat. In fact, when the entry threat is definitive, the incumbent makes highly suboptimal choices toward the end of the simulation.

Further, in the Competitive state C (see Figure 2.6), we observe that the LLM agent initially exhibits a bias against predation but quickly learns to choose predatory actions, with the rolling share of predation crossing the indifference benchmark (0.5 line) and stabilizing well above it. Predatory behaviour emerges even more rapidly when $\eta = 1$, indicating that a higher entry

¹⁹We generate a state specific binary variable. For Monopoly state: Optimal=1 and Suboptimal=0. For Competitive state, Predatory=1 and Accommodating=0. The rolling average we calculate is of this binary column.

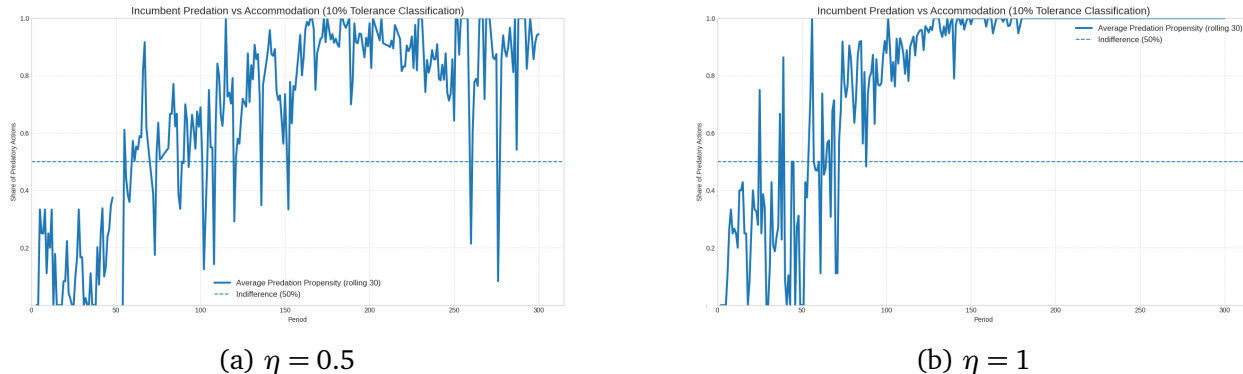


Figure 2.6: The share of predatory actions in Competitive state C when $\theta = 1$. The 0.5 line is the indifference benchmark — i.e., the point at which the incumbent LLM agent is equally likely to engage in predatory or accommodating strategies. The values above the 0.5 line indicate that predatory actions are more prevalent.

threat induces greater aggressiveness. This finding runs counter to our theoretical prediction from Proposition 2.1, which suggested that increasing the probability of entry, η , would reduce the attractiveness of predation. Instead, the simulation results suggest that the incumbent LLM agent finds it beneficial to establish a predatory reputation precisely when entry threats are high, possibly reflecting the agent’s reinforcement-learning bias toward strategies that ensure future dominance rather than short-run profit maximization.

Further, to analyze the incumbent’s impact on Entrants, we evaluate two key metrics: the entrant success rate and the entry deterrence rate. An Entrant is defined as successful if their cumulative lifetime profit is greater than zero. To measure the deterrence rate, i.e., the incumbent’s ability to discourage potential entrants, we identify all entrants born who choose not to enter. The entrant success rate is defined as the percentage of successful entrants relative to all entrants who entered the market, while the deterrence rate is the percentage of entrants who choose not to enter relative to all entrants born.

When $\eta = 0.5$, the entrant success rate in the first 100 periods is 39.5%, dropping to 7.3% in the last 100 periods. This effect is even more pronounced when $\eta = 1$, with the success rate decreasing from 19.7% in the first 100 periods to a negligible 1.3% in the last 100 periods. This indicates that when $\eta = 1$, the LLM agent effectively eliminates any potential profit for the entrant. The deterrence rate is consistently high: 71.90% for $\eta = 0.5$ and 79.59% for $\eta = 1$, with negligible changes as the simulation progresses. Overall, this demonstrates that the incumbent establishes a reputation as a predator early in the simulations.

Lastly, strong evidence that LLM agents successfully achieve predation over time is provided by the average lifespan of an entrant. We calculate an entrant’s lifespan as the difference between entry and voluntary exit periods. When $\eta = 0.5$, the average lifespan is 2.40 periods

in the first 100 periods, decreasing to 1.59 periods in the last 50 periods. When $\eta = 1$, the average lifespan is 2.16 periods in the first 100 periods, decreasing to 1.51 periods in the last 50 periods. These results indicate a clear convergence toward a hit-and-run equilibrium.

To sum up, we conclude that: (i) the incumbent LLM agent does not recognize the predatory quantity immediately; (ii) in the Competitive state (C), the incumbent LLM agent successfully learns to predate and does not accommodate; (iii) in the Monopoly state (M), the incumbent LLM agent does not learn to optimize monopoly profits, and in fact, makes quantity decisions closer to the predatory quantity, indicating aggression and an entry-deterrence stance; (iv) the incumbent LLM agent predates more aggressively when $\eta = 1$; and (v) on average, all simulations converge to a hit-and-run equilibrium.

2.4.2 Variation 2: $\theta = 0.5$

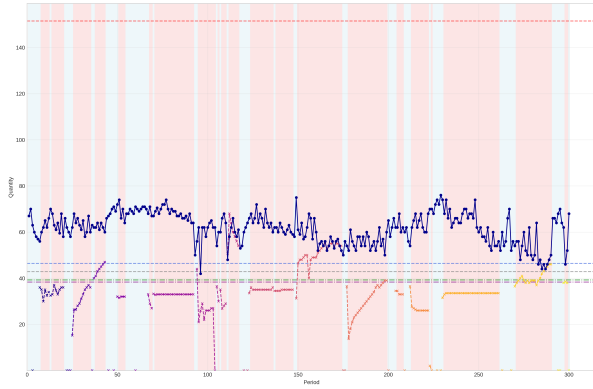
We now examine whether the LLM agent is able to adopt an accommodation strategy in settings where the accommodation equilibrium is theoretically predicted to hold. For $\eta = 0.5$, the incumbent's theoretical optimal quantity choices are $q_I^M = 46.66$ in Monopoly state M and $q_I^A = 42.85$ in Competitive state C ; for $\eta = 1$, both coincide at 42.85.

As before, we begin by illustrating the behaviour of representative agents using representative runs with $\eta = 0.5$ and $\eta = 1$ (Figure 2.7). For $\eta = 0.5$ (Subplot 2.7a), the Monopoly state (M) occurs in 28.7% of periods, while the Competitive state (C) occurs in 71.3% of periods. For $\eta = 1$ (Subplot 2.7b), the state frequencies are nearly identical, Monopoly state (M) occurs in 21.7% of periods and Competitive in 78.3%. The corresponding entrant binary decisions are shown in Subplots 2.7c and 2.7d.

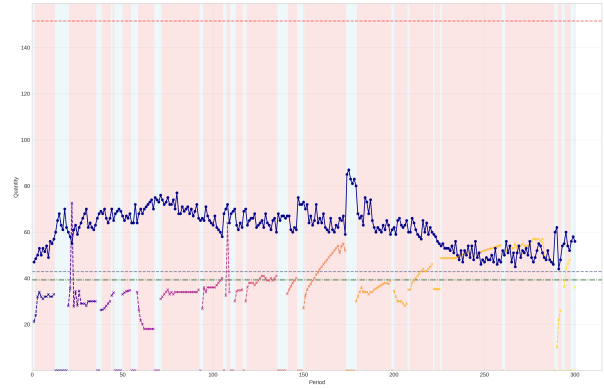
The overall strategy of the representative agents in Figure 2.7 is neither accommodating nor predatory. Instead, the incumbent adopts an aggressive stance, consistently choosing quantities above the monopoly threshold. On average, the incumbent produces 63.58 in the Monopoly state (M) and 61.10 in the Competitive state (C) when $\eta = 0.5$, and 66.75 (M) and 60.48 (C) when $\eta = 1$.²⁰

The same pattern persists across all experimental runs (Figure 2.8). When $\eta = 0.5$, the incumbent's average quantity choice is 62.85 in the Monopoly state (M) and 58.73 in the Competitive state (C). When $\eta = 1$, the averages are 62.38 (M) and 54.42 (C). These outcomes are not close to the theoretical monopoly-optimal or accommodating strategies, nor do they reflect predatory behaviour. Instead, the incumbent's choices are more than 25% above

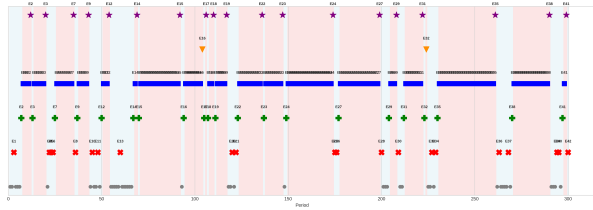
²⁰Appendix A.3.4 provides the full log reply from period 299 of the representative run shown in Subplot 2.7b. The incumbent's response reveals an awareness that its quantity choice influences the entrant's decision. With a choice of 58, the incumbent explicitly states its aim is to deter entry, which indeed leads the entrant agent to choose *NOT ENTER*, observing a quantity above the theoretical monopoly threshold.



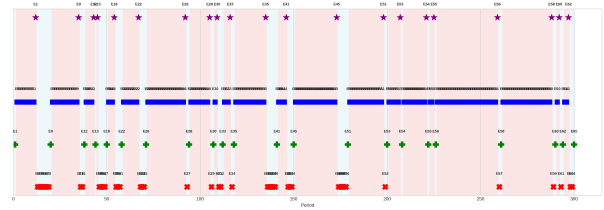
(a) Firm quantities: $\eta = 0.5$ and $\theta = 0.5$



(b) Firm quantities: $\eta = 1$ and $\theta = 0.5$



(c) Entrant binary decisions: $\eta = 0.5$ and $\theta = 0.5$



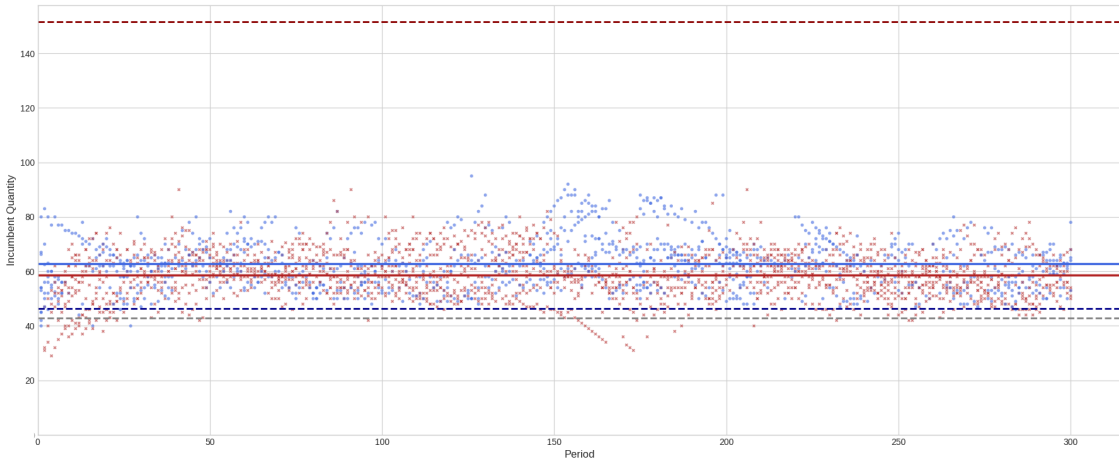
(d) Entrant binary decisions: $\eta = 1$ and $\theta = 0.5$

Figure 2.7: LLM agent quantity decisions and binary decisions from representative runs when $\theta = 0.5$ while $\eta = 0.5$ and $\eta = 1$. The solid blue line in subplot 2.7a and 2.7b represents the quantity produced by the incumbent firm, while the dashed lines in various colours represent the quantities of individual entrants who have entered the market. Further, the red dashed line represents q_I^P , the blue dashed line represents q_I^M , the grey dashed line q_I^A , the purple dashed line denotes q_E^M , the green dashed line denotes q_E^A . In subplot 2.7c and 2.7d, the grey dot denotes the birth of an entrant, the red cross denotes the entrant's choice to *NOT ENTER*, green denotes the entrant's choice to *ENTER*, the blue square denotes the entrant's decision to *STAY* while the orange triangle denotes the entrant's decision to voluntarily *EXIT*. Lastly, the star denotes exogenous exit. In all subplots, the background colour indicates the market state: a light coral region signifies a competitive market, whereas a blue region denotes a monopoly state. The simulation parameters used are: $\gamma = 0.1$, $a = 100$, $b = 1$, $k = 450$, $z_I = 300$, $z_E = 150$, $\delta = 0.95$, $\theta = 0.5$. In subplot 2.7a and 2.7c, $\eta = 0.5$, while in subplot 2.7b and 2.7d, $\eta = 1$.

the theoretical benchmarks q_I^M and q_I^A , indicating a distinctly aggressive strategy.

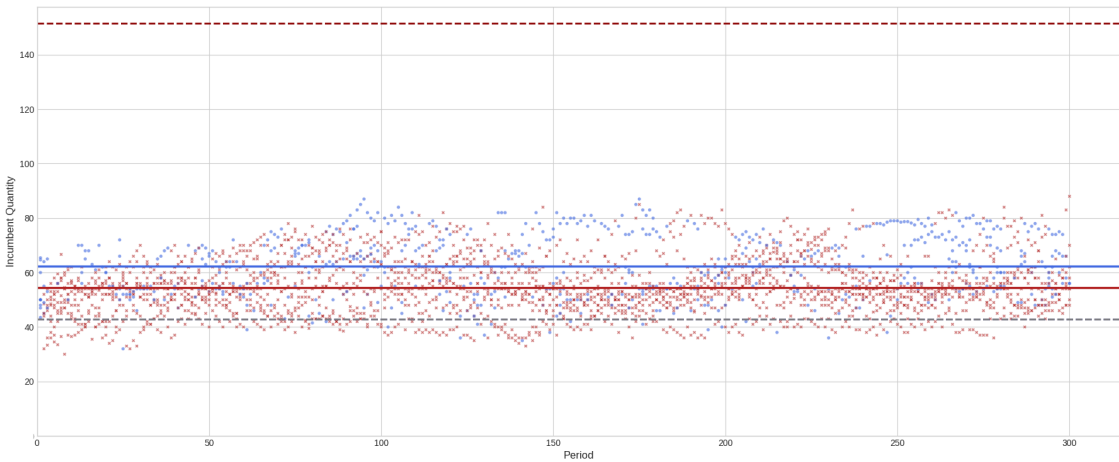
In the last 100 periods, when $\eta = 0.5$, the incumbent LLM agent's average quantity decreases to 58.44 in the Monopoly state and increases to 60.99 in the Competitive state. When $\eta = 1$, the corresponding averages increase to 64.05 in the Monopoly state and 55.47 in the Competitive state. These patterns indicate that the incumbent LLM agent does not converge

Incumbent Quantity Choices Over Time, Conditioned by Market State



(a) $\eta = 0.5$

Incumbent Quantity Choices Over Time, Conditioned by Market State



(b) $\eta = 1$

Figure 2.8: The figure shows the quantity choices of the incumbent LLM agent across 300 periods and all ten runs. The simulation parameters used are: $\theta = 0.5$, $\gamma = 0.1$, $a = 100$, $b = 1$, $k = 450$, $z_I = 300$, $z_E = 150$, and $\delta = 0.95$. Note that the solid blue line refers to the mean quantity in monopoly state while the solid red line refers to the mean quantity in competitive state. The dashed red line represents the theoretical predatory quantity (151.50), the dashed blue line represents the theoretical monopoly quantity (46.44 when $\eta = 0.5$ and 42.86 when $\eta = 1$). Lastly, the gray dashed line represents the theoretical accommodation quantity(42.86).

to the theoretically predicted strategies. The average quantities remain substantially above the theoretical benchmarks, suggesting that the agent persistently adopts an aggressive stance rather than accommodating or predatory strategies.

The incumbent LLM agents never select quantities close to the theoretical predatory threshold, $q_I^P = 151.51$. This leads us to conclude that the agents do not learn to engage in full predation. Instead, their most aggressive behavior is far more moderate: across all runs, the average maximum quantity is 80.70 in the Monopoly state and 79.40 in the Competitive state when $\eta = 0.5$, and 73 in both states, when $\eta = 1$.

Further, to determine if the LLM agent learns a particular strategy overtime, we categorize the state specific LLM quantity choices into optimal, aggressive and other in Monopoly state and accommodating, aggressive and other in Competitive state. First, we confirm that across all runs the LLM agent’s quantity choice is not in the 10% region of the predatory quantity threshold. As before, in the monopoly state, we classify a quantity choice as optimal if it falls in the 10% region of the theoretical optimal q_I^M . In competitive state, we classify a quantity choice as accommodating if it falls in the 10% region of the theoretical optimal q_I^A . In both states, a quantity choice is classified as aggressive if it is in the range 10% above q_I^M but 10% below q_I^P . All other choices are labelled as neither.

We find that when $\eta = 0.5$, in Monopoly state, LLM incumbent’s quantity choices are optimal only 10.13% of times, they are aggressive 89.59% of times. In Competitive state, quantity choices are accommodating only 6.3% of times and aggressive 92.5% of times. When $\eta = 1$, quantity choices in Monopoly state are optimal only 15% of times and aggressive 82.14% of times. In competitive state, actions are accommodating 18.62% of times and aggressive 77.60% of times.

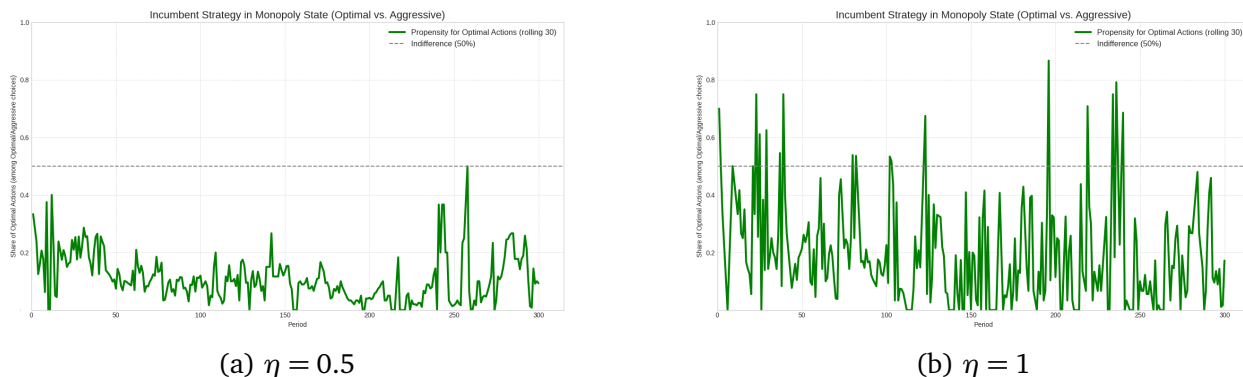


Figure 2.9: The share of actions in Monopoly state M when $\theta = 0.5$. The 0.5 line is the indifference benchmark — i.e., the point at which the incumbent LLM agent is equally likely to engage in optimal or aggressive strategies. The values above the 0.5 line would indicate that optimal actions are more prevalent.

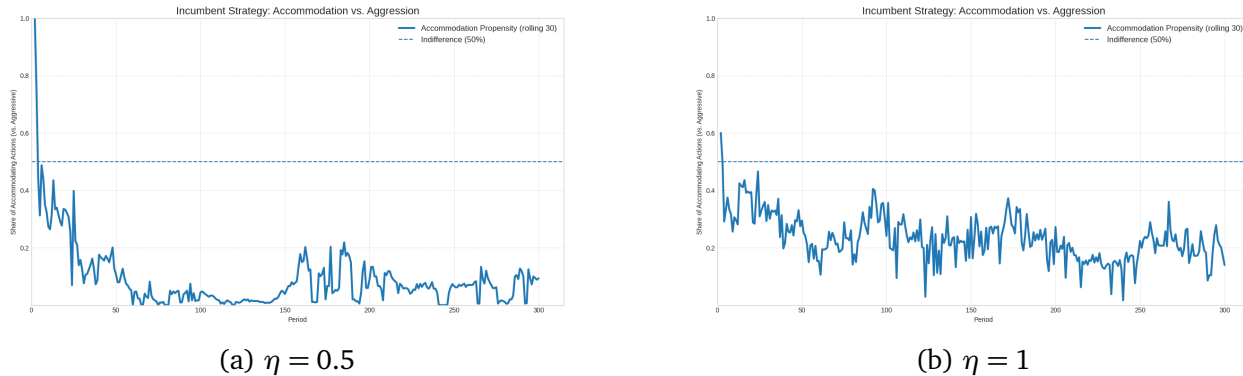


Figure 2.10: The share of accommodating actions in Competitive state C when $\theta = 0.5$. The 0.5 line is the indifference benchmark — i.e., the point at which the incumbent LLM agent is equally likely to engage in accommodating or aggressive strategies. The values above the 0.5 line indicate that accommodating actions are more prevalent.

In Figure 2.9 and 2.10, we plot the aggregate 30-period rolling average of the quantity choice classifications in Monopoly and Competitive state respectively. This shows the propensity of the incumbent LLM agent to choose the optimal action in Monopoly state (in Figure 2.9) and the accommodating action in Competitive state (in Figure 2.10).

When $\eta = 0.5$, the incumbent LLM agent's play in the Monopoly state does not converge to the optimal monopoly strategy (see Subplot 2.9a). Instead, it maintains a persistent bias toward overproduction, reflecting an aggressive stance. In the Competitive state, the agent experiments briefly with accommodation in the early periods, but this behaviour is not sustained. It quickly converges toward consistently aggressive output (see Subplot 2.10a).

When $\eta = 1$, the incumbent's behaviour is more variable. In the Monopoly state, the agent intermittently explores the optimal monopoly strategy, though these attempts are not sustained and it repeatedly reverts toward aggression (see Subplot 2.9b). In the Competitive state, the incumbent does not explore accommodation; the rolling share of accommodating actions remains close to 0.2 throughout, indicating a persistently aggressive stance (see Subplot 2.10b).

Further, the incumbent LLM agent's aggressive stance does have a negative impact on the Entrant success rate. In the first 100 periods the entrant success rate is 100% irrespective of η . However, in the last 100 periods it drops to 70.59% when $\eta = 0.5$ and to 8.47% when $\eta = 1$. Further, the Incumbent manages to deter entry with the aggressive stance with some success. Particularly, the deterrence rate is 54.44% (when $\eta = 0.5$) and 53.09% (when $\eta = 1$) in the first 100 periods. In the last 100 periods, the deterrence rate increases to 59.71% (when $\eta = 0.5$) and 60.19% (when $\eta = 1$).

Lastly, the average lifespan of an entrant, when $\eta = 0.5$, is 2.95 periods in the first 100

periods which decreases to 1.10 periods in the last 100 periods. When $\eta = 1$, the average lifespan of the entrant is 1 period throughout²¹. This shows that the incumbent’s aggressive behaviour leads to entrant’s exiting immediately upon entry.

To sum up, we conclude that (i) The incumbent’s choices do not align with the theoretical predictions. (ii) Even when the theoretical environment is sound for accommodation, a forward looking incumbent LLM agent adopts an aggressive stance in order to maintain its monopoly. (iii) The incumbent LLM agent does not learn to optimize profits in Monopoly state. Lastly, (iv) a forward looking Entrant LLM agent is discouraged from entry and encouraged to exit immediately when the incumbent shows slight aggression.

2.5 Robustness Check

In this section, we test the robustness of our main findings from Section 2.4. First, we consider an alternative setup where there is no exogenous exit ($\gamma = 0$). Second, we vary our prompts, specifically, the prompt persona and objective function.

2.5.1 No exogenous exit

In this section, we examine the simulation outcomes when there is no probability of exogenous exit. Setting $\gamma = 0$ allows us to examine the agent’s strategic behaviour in the absence of exogenous exit, highlighting the role of forward-looking incentives.

The other parameters are set to $\theta = 1$ (homogeneous products), $\eta = 0.5$, and $\delta = 0.95$. We conduct 10 simulation runs of 300 periods each. For these parameter values, both accommodation and predation are viable strategies.

Across runs, the long-run outcome is either predation or Cournot-like accommodation. We find that 7 out of 10 runs converge to predation. Of these, 5 converge early, in the sense that the incumbent’s average quantity in the last 100 periods in the competitive state (C) lies within 10% of the theoretical predation quantity, $\underline{q}_I^P = 75.5$, i.e., $[67.95, 83.05]$. The remaining 2 predatory runs reach this range only in the last 50 or 25 periods. The average incumbent quantity in the Monopoly state (M) is 65.17 (averaged over 300 periods and 7 runs), 62.97 (last 100 periods and 5 runs), and 60.64 (last 50 periods and 2 runs).

The other 3 runs converge to a Cournot-like accommodation outcome. We classify a run as Cournot-like accommodation if, over the evaluation window (last 100 / 50 / 25 periods as

²¹There is only one run where the average entrant lifespan is 13 periods in the first 100 periods.

stated), the incumbent’s average quantity in state (C) is approximately 37.75 units.²² In the Monopoly state, the average incumbent quantity is 53.35 (averaged over 300 periods and 3 runs). Here, the average in the last 100 or 50 periods is not relevant as Monopoly state does not occur once the convergence is achieved. See Figure 2.11 for representative runs of both types.

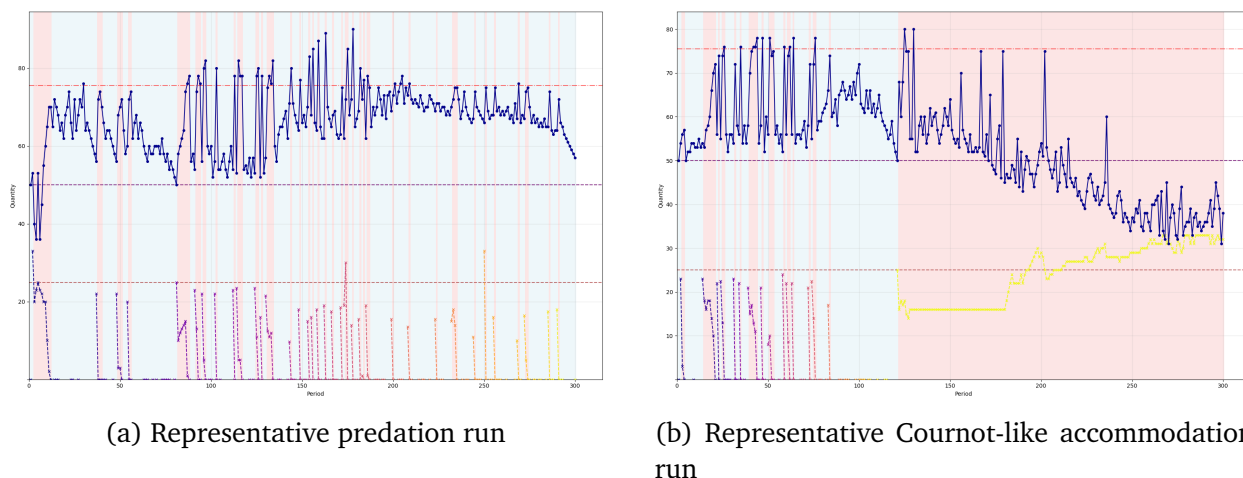


Figure 2.11: Quantities and event timelines from representative runs. The solid blue line represents the incumbent’s quantity, while dashed colored lines represent individual entrants. Background shading indicates market state: light coral = competitive (C), blue = monopoly (M). The red dashed line denotes \underline{q}_I^P , the purple dashed line $q_I^M = q_I^A$, and the orange dashed line $q_E^M = q_E^A$. Subplot (a) shows convergence to predation; subplot (b) shows convergence to Cournot-like accommodation.

Irrespective of the eventual outcome, 9 out of 10 runs begin with the LLM incumbent choosing a quantity above the theoretical monopoly threshold $q_I^M = 50$. There is a single exception that starts below q_I^M in the first few periods and ultimately converges to Cournot-like accommodation. See Figure 2.12.

While here we have focused on predation and Cournot-like accommodation outcomes, it is also worthy to consider the absence of collusion in these simulations. Given the repeated-game setting, homogeneous products, and a relatively high discount factor ($\delta = 0.95$), one might expect the possibility of tacit collusion to arise, consistent with standard folk-theorem results. In principle, the incumbent could sustain higher joint profits by coordinating with entrants rather than engaging in costly predation or accommodating at Cournot levels. However, across all runs, we do not observe any evidence of such coordination: quantities do not converge to

²²For reference, in the simultaneous-move Cournot benchmark with these parameters, the pure-strategy Nash equilibrium quantity is 33.33.

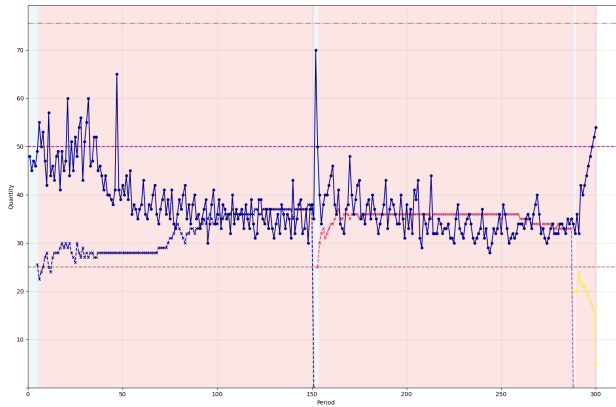


Figure 2.12: Unique representative run. The incumbent initially sets quantity around the profit-maximizing level, and the simulation converges to a Cournot-like accommodation outcome. The red dashed line denotes q_I^P , the purple dashed line $q_I^M = q_I^A$, and the orange dashed line $q_E^M = q_E^A$.

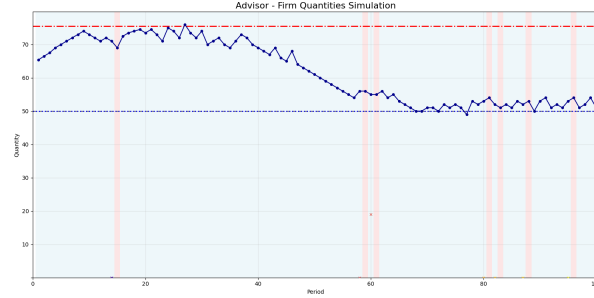
collusive levels. This absence of collusion suggests that, at least in this parameterization, the LLM agent does not discover or stabilize cooperative strategies, instead gravitates toward either predation or Cournot-like outcomes.

To summarize, we find that: (i) predation is not guaranteed, (ii) when accommodation occurs, the incumbent converges to the Cournot quantity, suggesting it either fails to recognize or does not learn to exploit its first-mover advantage, (iii) the incumbent does not learn the profit-maximizing monopoly quantity in state M (when predating) and in state C (when accommodating), and (iv) no collusion is observed.

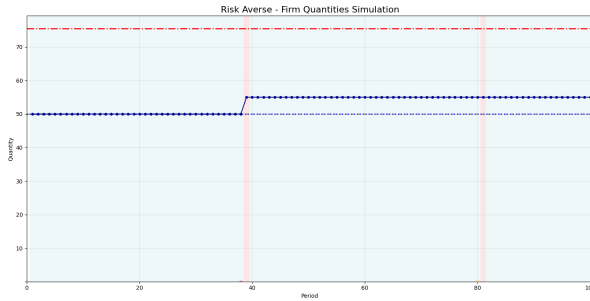
2.5.2 Prompt Variation

In our main experimental setup, the large language model (LLM) agents are assigned the persona of a rational firm whose principal objective is to maximize long-run discounted profits (see Appendix A.3.1 for our main experiment prompts). In this section, we explore how sensitive agent behavior is to variations in both the underlying objective function and the assigned persona. Specifically, we consider three alternative prompts that differ in the goals and behavioral framing of the LLM agents. In each case, together with market-relevant data and history, the LLM agent is explicitly asked to provide their analysis and rationale behind each decision. Under the “analysis” and “rationale” headings, the agents are given pointers to guide their thought process, resembling a structured Zero-Shot Chain-of-Thought (CoT) approach used in our main experiment but with more explicit instructions.

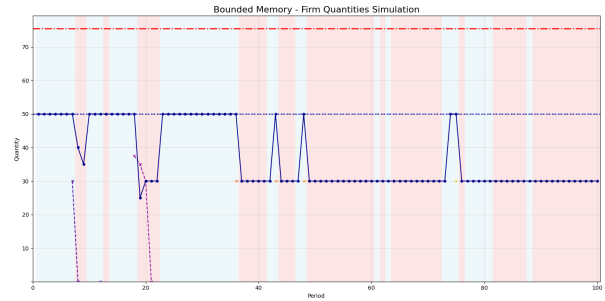
We run one experimental run of 100 periods for each prompt variation using the same



(a) Advisor Prompt Simulation



(b) Risk Averse Prompt Simulation



(c) Bounded Memory Prompt Simulation

Figure 2.13: Incumbent and Entrant Quantities Across All Prompt Variations.

The figure presents the simulated firm quantities for an experimental run of 100 periods for each of the three prompt variations. The solid blue line in each subplot represents the quantity produced by the incumbent firm, while the dashed lines in various colours represent the quantities of individual entrants who have entered the market. The background colour indicates the market state: a light coral region signifies a competitive market, whereas a light blue region denotes a monopoly state. The simulation parameters used are: $\gamma = 0.1$, $\eta = 0.5$, $a = 100$, $b = 1$, $k = 450$, $z_I = 300$, $z_E = 150$, and $\delta = 0.95$. Further, the red dash line represents q_I^P while the blue dash line represents $q_I^M = q_I^A$.

parameters as our main experiment: $\eta = 0.5$, $\gamma = 0.1$, and $\theta = 1$. Theoretically, both accommodation and predation equilibria hold for these parameter values (see Figure 2.13).

2.5.2.1 Advisor Prompt

In this variation, the LLM agent’s persona is that of a strategic business consultant whose primary goal is to maximize the long-term discounted profit of their client. This aligns the advisor persona closely with that of a textbook rational agent.

From the simulation results (Figure 2.13a), the advisor agent demonstrates a clear multi-period strategy. It starts by choosing aggressive quantities to deter entrants and, as the threat of entry decreases, gradually moves to a quantity just above the pure monopoly level. Overall,

this run results in a monopoly state 93 per-cent of the time and a competitive state only 7 per-cent of the time. Out of 48 entrants born, only 6 successfully enter. In the first 50 periods, the incumbent on average chooses 70.42 in monopoly and 69 in competitive states, demonstrating strong deterrence. In the last 50 periods, it settles at 52.91 (monopoly) and 53.83 (competitive), taking a profitable, less aggressive stance. The entrants consistently choose near-zero quantities, showing that the incumbent successfully learns to deter entry.

2.5.2.2 Risk-Averse Prompt

Here, the agent persona is a risk-averse firm manager whose primary goal is to maintain stable profits. The agent is designed to take safe, predictable decisions, resembling a conservative, non-rational actor.

The simulation results (Figure 2.13b) show that the risk-averse agent quickly identifies the theoretical monopoly quantity of 50 and does not deviate, even under entry threat. For the first 38 periods, it maintains this stable monopoly quantity. After experiencing its first entry—where the entrant chooses 25 (the theoretical accommodating quantity)—the incumbent does not accommodate but instead increases its quantity to 55. This forces the entrant to exit, and the incumbent maintains this level until the end of the simulation. Overall, the monopoly state occurs 98 per-cent of the time, with only 2 per-cent in the competitive state. Directing the agent toward stability thus leads to a non-aggressive deterrence strategy that succeeds when entrants are similarly conservative.

2.5.2.3 Bounded Memory Prompt

In this variation, the agent aims to maximize long-run profits but observes only the last 3–5 periods of market history. This models bounded rationality and tests whether access to deeper historical records is required for multi-period strategy execution.

The simulation results (Figure 2.13c) reveal erratic, myopic behavior. The incumbent fails to consistently deter entry, resulting in a monopoly state 74 per-cent of the time and a competitive state 26 per-cent of the time. In the first 50 periods, the incumbent averages 48.79 in monopoly and 31.76 in competitive states. In the last 50 periods, it settles at 33.64 (monopoly) and 30 (competitive), while entrants choose 30 on average in competitive states. This shows that myopic agents eventually converge toward the Cournot equilibrium rather than pursuing aggressive deterrence.

To sum up, varying prompts demonstrates that LLM agents do not inherently adopt aggressive deterrence behaviour but can be driven to it through a combination of a clear prompt objective, a defined persona, and strategic information.

2.6 Discussion

In this paper, we focus on the possibility that large language models (LLMs) are capable of learning predatory strategies. First, we build a dynamic Stackelberg framework of predation based on [Rey et al. \(2023\)](#), and show that accommodation, monopolization, and predation can arise as pure-strategy Markov Perfect Equilibrium outcomes, sustained under a persistent threat of entry and exogenous exit. Second, we embed large language models—specifically OpenAI’s GPT-4.1—into our economic environment to test whether LLM agents can learn such strategies.

We conduct 10 experimental runs on each of our sub-variations, varying product differentiation and the threat of entry. The first variation is chosen such that both predation and accommodation are viable equilibrium strategies, whereas the second is chosen such that only accommodation is viable. We observe that LLM agents learn to predate effectively when both predation and accommodation are viable, but adopt an aggressive stance when accommodation is the only theoretically viable equilibrium strategy. In both cases, LLM agents do not learn to optimize profits when they are the sole agents in the market.

From these findings, we highlight two broader implications. First, the strategic capabilities of LLM agents extend beyond pricing and collusion. LLM agents can learn exclusionary behavior requiring complex intertemporal strategies, even when such strategies involve short-term losses. Second, our preliminary results suggest that the rise of LLMs introduces novel challenges for competition policy.

While our theoretical framework is grounded in Markov Perfect Equilibria (MPE), we acknowledge that the Markovian restriction is a specific modelling choice; allowing strategies to depend on the full history of play invokes the Folk Theorem, which theoretically makes any individually rational payoff sustainable. Consequently, there is no prior expectation that the experimental results will naturally converge to an MPE. The depth of the history provided in the prompt is critical to this outcome. When the prompt contains only a limited history, the LLM fails to learn complex strategic behaviors, such as entry deterrence, and its behavior becomes erratic. The transition from erratic to strategic behavior is thus directly contingent on the informational history provided in the prompt. This reinforces the understanding that LLM agents are not inherently predisposed to adopt MPE strategies; indeed, it is an interesting fact per se that an MPE is sometimes selected. While an MPE does not emerge in our experimental outcomes when goods are differentiated, it remains a notable finding that it arises when goods are homogeneous.

We also recognize several limitations that need to be addressed. Our robustness checks show that predation is not guaranteed when the probability of exogenous exit is zero. This

implies that further experimental runs with parameter variations are required to identify the precise conditions under which an LLM agent learns predation. Second, the behavior of LLM agents appears highly sensitive to prompt persona and objective functions, an area requiring further exploration. Lastly, a major limitation of our study is the relatively small number of experimental runs. And usage of only one LLM model that is Open AI's-GPT 4.1.this was a budgetary constraint. This limited the size of our data set and prevented us from performing any rigorous regression analysis. These limitations open avenues for us for our future work, both in refining the experimental framework and in extending the analysis to richer environments with endogenous entry and multi-agent learning.

Chapter 3

Personalized Pricing and Consumer Welfare in Differentiated Markets

3.1 Introduction

The rapid growth of digital markets in the last two decades has placed regulating digital markets high on the agenda of competition authorities around the world. One aspect that makes digital markets unique is that online companies collect vast amounts of customer data through advanced information technologies. Furthermore, firms often use advanced pricing algorithms that utilize the collected data to tailor prices to their advantage and enable practices such as collusion, strategic selling of information, and personalized pricing. (see [Calvano et al. \(2020\)](#), [Bounie et al. \(2021\)](#)).

[Sean and Lam \(2020\)](#) summarize the pros and cons of personalized pricing from the policy maker's perspective. On one hand, personalized pricing may benefit low-income consumers, improve firm revenue compared to uniform pricing, and intensify competition. On the other hand, it can lead to societal discrimination, increase search and switching cost for consumers, damage consumer trust in the market, and raise privacy concerns. However, a primary concern of the policy makers is that the use of consumer information to personalize prices may erode consumer surplus.(see [OECD \(2018\)](#), [Frontier Economics \(2018\)](#), [Fletcher et al. \(2013\)](#), [CMA \(2018\)](#), [Office of Fair Trading \(2013\)](#), [Ofcom \(2020\)](#)).

Personalized pricing is a form of price discrimination. [G. Stigler \(1987\)](#) defines price discrimination as the situation where the same good is sold to different consumers at different prices which do not equal the marginal cost, the competitive price benchmark. Furthermore, [Varian \(1989\)](#) outlines three conditions that enable a firm to price discriminate: (i) having market power, (ii) the capacity to sort consumers, and (iii) the ability to prevent resale. The

nature of the digital markets satisfies these conditions for firms that possess an information advantage in these markets. Additionally, [Pigou \(1920\)](#) classified personalized pricing into three types: (i) First degree (ii) Second degree and (iii) Third degree. Personalized pricing is the first degree price discrimination, which allows firms to charge each consumer an individual price that may be as high as their highest willingness to pay. The explanation of loss of consumer welfare under personalized pricing dates back to [Varian \(1989\)](#) and [Robinson \(1933\)](#).

Relative to uniform pricing, a monopolist that can offer personalized prices based on some consumer characteristics can be expected to expand output at the expense of extracting more rents from its customers. The interaction of personalized pricing and competition is subtler, however, and could yield somewhat counter-intuitive results. Variations of the horizontal differentiation model developed by [Hotelling \(1929\)](#) and later simplified by [D'Aspremont et al. \(1979\)](#) have been extensively used in literature emphasizing the incentive of competitive firms to price discriminate in a Hotelling setting and the impact it has on the consumer surplus. For example, [Thisse and Vives \(1988\)](#), show that revealing to all competitors that a consumer has a strong preference for a particular brand (i.e., revealing consumer's horizontal location on a Hotelling line) has two effects. On the one hand, the owner of the preferred brand may wish to increase its price. On the other hand, competitors would have an incentive to lower prices aggressively in order to win the consumer. The competition-intensifying effect may gain the upper hand to the benefit of the consumer. These results have been reinforced again and again in literature (see [Lederer and Hurter \(1986\)](#), [Nicolas \(1997\)](#)).

More recent studies of the interaction between data provision and competition in markets with horizontal product differentiation have refined the findings by [Thisse and Vives \(1988\)](#). For example, [Houba et al. \(2022\)](#) generalize duopoly model of [Thisse and Vives \(1988\)](#) by considering asymmetric firms and allowing only one firm to set personalized prices. This paper confirms [Thisse and Vives \(1988\)](#) result that personalized pricing may increase the aggregate consumer surplus. [Rhodes and Zhou \(2024\)](#) develop a general oligopoly model of personalized pricing with horizontal product differentiation and correlated valuations. They show that welfare effects of personalized pricing hinge on market coverage and costs: under full coverage, personalized pricing lowers industry profit and raises consumer surplus (generalizing [Thisse and Vives \(1988\)](#)), but with partial coverage the pattern may reverse, with higher profits and lower consumer surplus.

Another stream of literature examines settings where either consumers or firms determine how much information is available. [Ali et al. \(2023\)](#) explore the effect of allowing consumers to choose whether to share information. They demonstrate that competition intensifies with personalized pricing, leading to a higher consumer surplus compared to uniform pricing. Similarly, [Anderson et al. \(2023\)](#) allow consumers to choose whether to opt-in to discount targeting

by firms. They show that personalized pricing on a Hotelling line can increase the surplus of the marginal consumer at the cost of the infra-marginal consumer's surplus and, under certain conditions, reduce aggregate welfare, although, their analysis is limited to full market coverage. In contrast, [Elliott et al. \(2021\)](#), concentrate on the market outcomes achievable by an informed market designer. Specifically, they analyze the potential for large online firms with consumer data to extract consumer surplus.

Crucially, the existing literature primarily focuses on one-dimensional consumer types — typically brand preferences — or assumes away systematic quality differences across firms, effectively restricting attention to horizontal product differentiation. However, horizontal preference is only one dimension of consumer heterogeneity. Consumers may also differ in their valuation of quality (a vertical dimension), and information about such preferences can shape both pricing and product-design strategies. This raises new questions for policymakers: does an information advantage enable firms to discriminate in the quality offered to consumers, are all consumer types served in the market, which consumer types suffer due to price discrimination and how is the aggregate consumer ultimately welfare affected?

In this paper, we develop a duopoly model to analyze the impact of personalized pricing. The distinctiveness of our approach lies in allowing consumer heterogeneity along two dimensions, as in [Shen et al. \(2016\)](#). Consumers differ horizontally in their brand preference, represented by their location on a Hotelling line, and vertically in their valuation of quality. Importantly, horizontal preferences always remain private information. Firms offer products of exogenous quality and compete by setting prices. Our objective is to evaluate how informing firms about consumers' vertical type affects market outcomes. We consider two cases: in the first, firms observe consumers' quality preferences and can tailor their price offers accordingly; in the second, firms remain uninformed about both quality and location, so they can only offer a uniform price to all consumers.

Our results differ from the existing literature by highlighting the interplay of consumer vertical preferences and product differentiation in determining market outcomes. While prior literature often assumes complete market coverage, we show that at intermediate levels of product differentiation, firms may avoid head-on competition in a fully covered market: they may split the market into adjacent segments with each firm serving one segment and exerting only limited competitive pressure on the other firm, or leave part of the market uncovered, creating a buffer of unserved consumers between them.

Regardless of whether firms know consumers' vertical types, along the horizontal dimension, the market is uncovered when products are highly differentiated, fully covered when products are close substitutes, and only partially covered at intermediate differentiation, with the consumer at the midpoint of the Hotelling line receiving zero utility. In other words, high

product differentiation confers market power. Information on consumers' vertical type enables firms to exploit this power more effectively: uninformed firms must offer a single uniform price, while informed firms can tailor prices to consumer types. It is not surprising then that in our model — unlike [Thisse and Vives \(1988\)](#) — uniform pricing can yield higher aggregate consumer surplus.

Overall, in our model, when products are close substitutes and firms compete head-on, the lack of information about the consumer's vertical type does not influence firms' pricing strategies. Consequently, consumer surplus remains the same under personalized pricing and uniform pricing. However, at higher levels of product differentiation, consumers with low vertical valuations benefit from personalized pricing as they face lower prices and are more likely to be served; in contrast, consumers with high vertical valuations suffer from personalized pricing as the prices they face increase. For most part, the losses of high type consumers exceed the gains of low type consumers and personalized pricing results in lower aggregate consumer surplus. Yet, there is a region of relatively high product differentiation where this pattern reverses and aggregate consumer surplus is lower under uniform pricing, as predicted by [Thisse and Vives \(1988\)](#).

Our findings carry important implications for competition authorities. They suggest that greater access to consumer data does not automatically translate into welfare gains and, in many cases, can harm consumers by enabling firms to extract surplus more effectively. Under certain conditions, uniform pricing can serve as a form of consumer protection, limiting firms' ability to exploit information advantages. Rather than assuming that personalized pricing is efficiency-enhancing, regulators should carefully scrutinize data-driven pricing practices, especially in digital markets where firms hold vast amounts of consumer information.

The rest of the paper is structured as follows. In section [3.2](#), we outline our setup, in section [3.3](#), we analyze the case where both firms have information about the consumer's vertical type and in the section [3.4](#), we analyze the case where both firms do not have information about the consumer's vertical type. In section [3.5](#) we compare the findings from both cases.

3.2 Setup

Consumers' preferences differ along two dimensions: horizontal and vertical. On the horizontal dimension, consumers have different tastes for brands; on the vertical dimension, consumers differ in their preferences for quality or willingness to pay.

The taste for brands is modelled as the horizontal location of a consumer on a Hotelling line. Specifically, a continuum of consumers of mass 1 is distributed uniformly over the $[0, 1]$

interval. The location of a consumer on the horizontal line, denoted by $d \in [0, 1]$, represents an ideal product for that consumer.

The consumers' varying preferences for quality (or willingness to pay) are captured through a preference parameter $\theta \in \{\bar{\theta}, \underline{\theta}\}$, where $\bar{\theta} > \underline{\theta} > 0$. The parameter θ will be referred to as the consumer's vertical type. At each location, there is a unit mass of vertical types and γ represents the proportion of consumers with vertical type $\theta = \bar{\theta}$. A customer's horizontal location and vertical type are independent.

There are two firms indexed by $i \in \{1, 2\}$. Firm 1 is located at 0, while Firm 2 is located at 1 (See Fig. 3.1). Each firm i offers a distinct product $i \in \{1, 2\}$. Firm's offer specifies price t for its product.

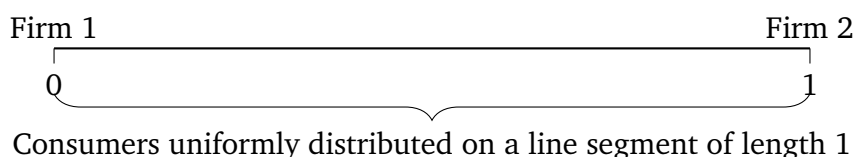


Figure 3.1: Hotelling line

Each consumer purchases at most one product. If a consumer, who has vertical type θ and who is located at a distance $d \in [0, 1]$ away from the firm, buys this firm's product, her utility is

$$u(t, d, \theta) = \theta - t - kd \tag{3.1}$$

where k represents the per unit transport cost, and can be viewed as the degree of product differentiation. A low value of k implies low horizontal product differentiation. The reservation utility of a consumer who does not make a purchase is zero.

Firms have the same production technology which results in per-customer profit

$$\pi(t, q) = t - c, \tag{3.2}$$

where c is the constant marginal cost of supplying a product. We assume that $c < \underline{\theta}$, and so it is efficient to serve consumers of type $\underline{\theta}$.

The aim is to assess how the availability of information on preferences for quality affect market outcomes and consumer welfare. To this end, we compare equilibrium outcomes in two different cases. First, we consider the case, in which both firms are informed about the vertical type of a consumer they are facing and can make type-specific price offers. Second, we consider the case, in which neither firm is informed about the consumer's vertical type and set a single price. In both cases, the consumer's horizontal location is always private.

To ensure that exclusion never arises in equilibrium, we adopt the following assumption throughout the analysis:

Assumption 3.1. *Types are sufficiently similar: $\bar{\theta} \leq 2\underline{\theta}$.*

This assumption rules out corner solutions where the monopolist serves only high types. Since low types are never excluded under uniform pricing, the scope for price discrimination to improve efficiency is limited. Under this assumption, it is easy to check that price discrimination by a monopolist never benefits consumers — it only allows the monopolist to extract more surplus. See Appendix B.1.

3.3 Both Firms Informed

In this section, we assume that both firms are informed about the consumer's vertical type and make type-specific offers.

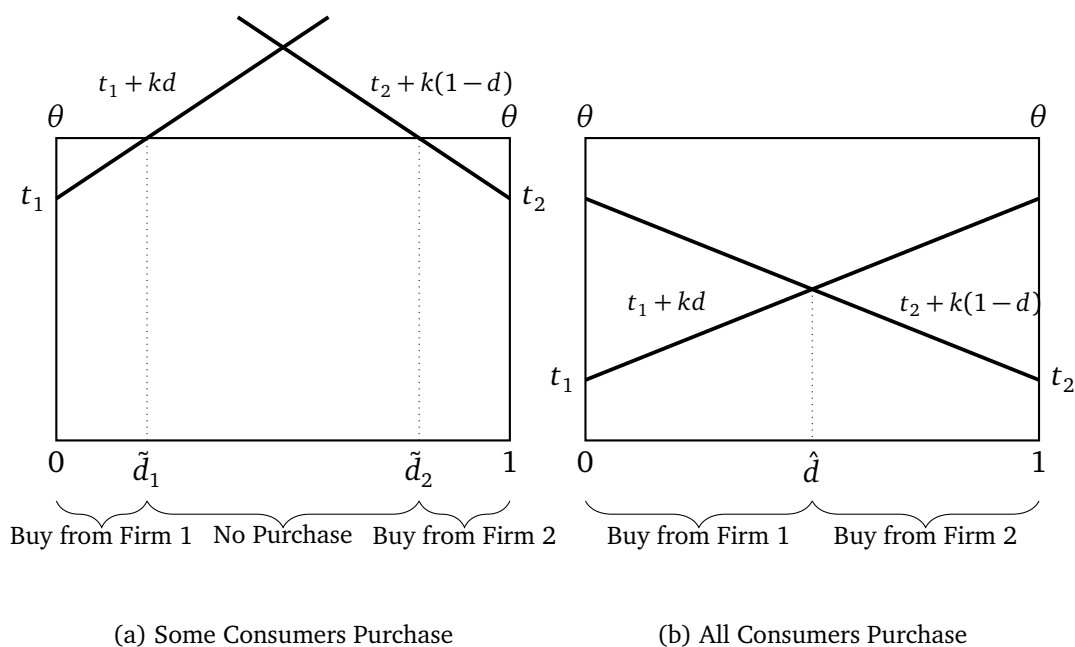


Figure 3.2: Purchasing Decision of the Consumers

The Figure 3.2 illustrates the purchasing decision of an arbitrary type θ consumer located at different points on the Hotelling line. Each consumer wants to purchase one unit of good if and only if the consumer's valuation for the good i.e, $\theta \in \{\bar{\theta}, \underline{\theta}\}$, is greater than or equal to the cost of purchasing. Two possibilities arise.

First, it could be the case that not all consumers make a purchase. At the location \tilde{d}_1 , the consumer is indifferent between purchasing from Firm 1 and not purchasing at all, i.e,

$\theta - t_1 - kd = 0$ holds. All consumers located between 0 and \tilde{d}_1 purchase from Firm 1 as they are better off purchasing from Firm 1 compared to not purchasing at all, i.e, $\theta - t_1 - kd > 0$. Similarly, at the location \tilde{d}_2 , all consumers located between \tilde{d}_2 and 1 purchase from Firm 2. Consumers in between \tilde{d}_1 and \tilde{d}_2 do not purchase from either Firms.

Another possibility is that consumers always get a positive utility from purchasing from either of the firms. The location of the consumer that is indifferent between purchasing from either firms is denoted by \hat{d} such that,

$$\begin{aligned} u(t_1, \hat{d}, \theta) &= u(t_2, 1 - \hat{d}, \theta) \\ \theta - t_1 - k\hat{d} &= \theta - t_2 - k(1 - \hat{d}) \\ \hat{d} &= \frac{k - (t_1 - t_2)}{2k} \end{aligned} \quad (3.3)$$

Given above, Firm 1's demand given prices offered when $\hat{d} \in [0, 1]$ is as follows:

$$Q_1(t_1, t_2) = \begin{cases} 1, & t_1 \leq t_2 - k, \\ \frac{k - (t_1 - t_2)}{2k}, & t_2 - k \leq t_1 \leq t_2 + k, \\ 0, & t_1 \geq t_2 + k. \end{cases} \quad (3.4)$$

Firm 2's demand is symmetric.

Given the demand for its product, each firm maximizes its profit, taking the price of the competitor as given. Proposition 3.2 characterizes price offers of firms to an arbitrary vertical type of consumer, θ , in the unique symmetric equilibrium. The equilibrium offers depend on k , c and θ .

When transport cost is low relative to the arbitrary vertical type θ , that is, $k \leq \frac{2(\theta - c)}{3}$, the market is horizontally covered and firms actively compete with each other. That is, type θ consumers are served along the entire Hotelling line and, at each location, derive a strictly positive utility from purchasing some product. Each firm i charges a price that is equal to the cost of providing quality i.e, c plus the per unit transportation cost k .

When transport cost is high $k \geq (\theta - c)$, each firm acts as a local monopolist for consumers who are located sufficiently close to it. In this case, the market is not fully horizontally covered as the markets of the two firms do not touch, and in middle locations type θ consumers are not served. Here, the firm i enjoys a monopoly premium over the cost of providing the product.

For intermediate values of transport costs, $\frac{2(\theta - c)}{3} \leq k \leq \theta - c$, firms serve adjacent horizontal segments but do not actively compete with each other. In this case, type θ consumers are served along the entire Hotelling line, but the consumer who is indifferent between the firms is also

indifferent to buying, that is, this consumer obtains utility of zero.

Proposition 3.2. *Suppose that both firms know that they are facing a consumer of vertical type θ . In the symmetric unique equilibrium, depending on the value of transport cost k , three possible pricing patterns arise.*

1. **Covered Market:** if $k < \frac{2(\theta-c)}{3}$, then each firm charges price

$$t^* = k + c$$

Market is horizontally covered and a consumer of type θ derives a positive utility from purchasing along the entire Hotelling line.

2. **Adjacent Market:** if $\frac{2(\theta-c)}{3} \leq k \leq \theta - c$, then each firm charges a price

$$t^* = \theta - \frac{k}{2}$$

Market is horizontally covered but a consumer of type θ located in the middle of the Hotelling line derives is indifferent to purchasing.

3. **Uncovered Market:** if $k > \theta - c$, then each firm charges price

$$t = \frac{\theta + c}{2}$$

Market is not horizontally covered and consumers of type θ in the middle of the Hotelling line are not served.

Refer to Appendix B.2.1 for the proof to the above proposition. Proposition 3.2 demonstrates the different regions that arise in equilibrium for an arbitrary θ consumer.

Given the two vertical types of consumers i.e, $\bar{\theta}$, $\underline{\theta}$ and marginal cost, c , different equilibria arise depending on the pecking order of the k thresholds. Corollary 3.3 defines a pecking order to focus on.

Corollary 3.3. *Given $\bar{\theta} > \underline{\theta} > c \geq 0$, then the following pecking order holds:*

$$\frac{2(\underline{\theta} - c)}{3} < \frac{2(\bar{\theta} - c)}{3} < \underline{\theta} - c < \bar{\theta} - c$$

if and only if $\underline{\theta} < \bar{\theta} < \frac{3\theta}{2}$ and $0 \leq c < 3\underline{\theta} - 2\bar{\theta}$.

Refer Appendix B.2.2 for proof of Corollary 3.3 .

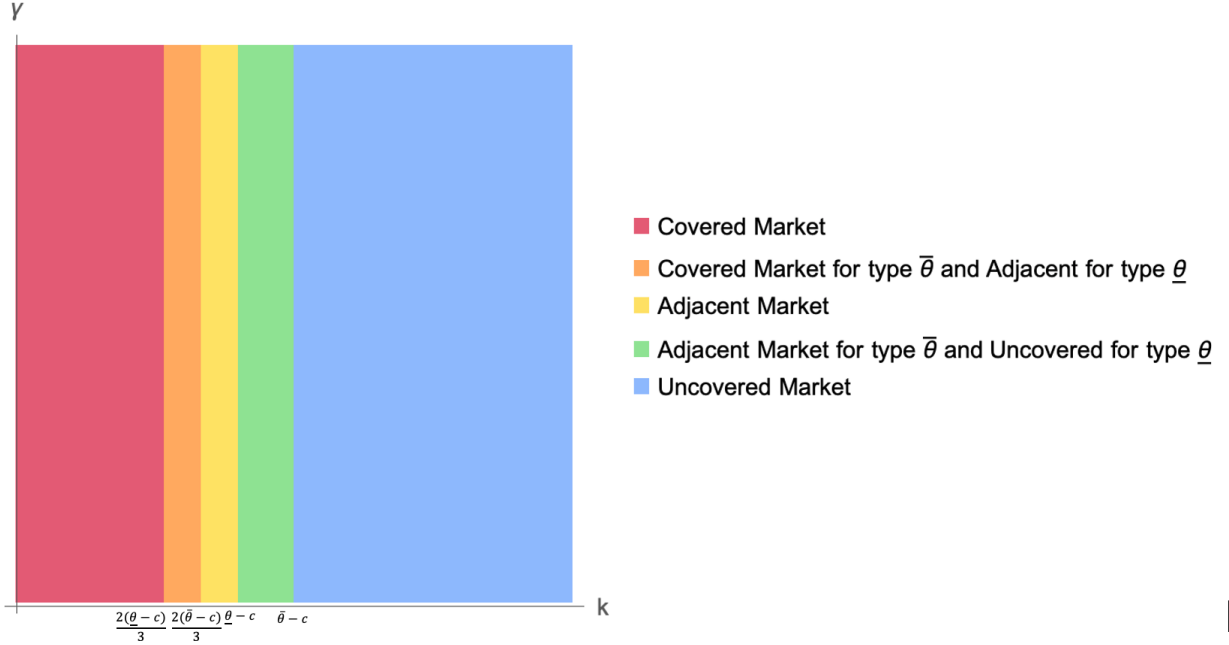


Figure 3.3: Thresholds of k indicating horizontal market regions that arise in the symmetric equilibrium when $\underline{\theta} < \bar{\theta} < \frac{3\underline{\theta}}{2}$ and $0 \leq c < 3\underline{\theta} - 2\bar{\theta}$. The plot assumes $c = 0, \bar{\theta} = 1$ and $\underline{\theta} = 0.8$.

Given that $\underline{\theta} < \bar{\theta} < \frac{3\underline{\theta}}{2}$, we focus on consumers with vertical types that are relatively close to each other, meaning the difference between $\bar{\theta}$ and $\underline{\theta}$ is not large. Figure 3.3 illustrates the k thresholds under this assumption. In Appendix B.2.5, we discuss the other possible pecking orders.

When transportation costs are low — specifically, when $k \leq \frac{2(\underline{\theta}-c)}{3}$ — the market is covered for both types of consumers and both types of consumers are offered the same price in equilibrium. The competitive equilibrium price is independent of the consumer's type and increasing in k . As transportation costs increase, the dynamics change: when $k \geq \bar{\theta} - c$, the market is uncovered for both types, firms act as local monopolists for both $\bar{\theta}$ and $\underline{\theta}$ consumers and offer type-specific monopoly prices. When $\frac{2(\underline{\theta}-c)}{3} \leq k \leq \frac{2(\bar{\theta}-c)}{3}$, the market is covered for $\bar{\theta}$ consumers and adjacent for $\underline{\theta}$ consumers. Here, the firm offers the competitive price to $\bar{\theta}$ consumers such that the price increases in k , but a lower price to $\underline{\theta}$ consumers such that the price decreases in k . When $\frac{2(\bar{\theta}-c)}{3} \leq k \leq \underline{\theta} - c$, the market is adjacent for both types of consumers. Here, the type specific price offered decreases in k . Lastly, when $\underline{\theta} - c \leq k \leq \bar{\theta} - c$, the firm acts as a local monopolist for $\underline{\theta}$ consumers, while the firm's market for $\bar{\theta}$ consumers is adjacent. Here, the type $\bar{\theta}$ consumers are offered a price that decreases in k , while type $\underline{\theta}$ are offered the monopoly price. Figure 3.3 illustrates these regions.

In Appendix B.2.3, we derive consumer surplus in each equilibrium for an arbitrary type θ consumer. The consumer surplus is determined by averaging the utility of all type θ consumers

who make a purchase.

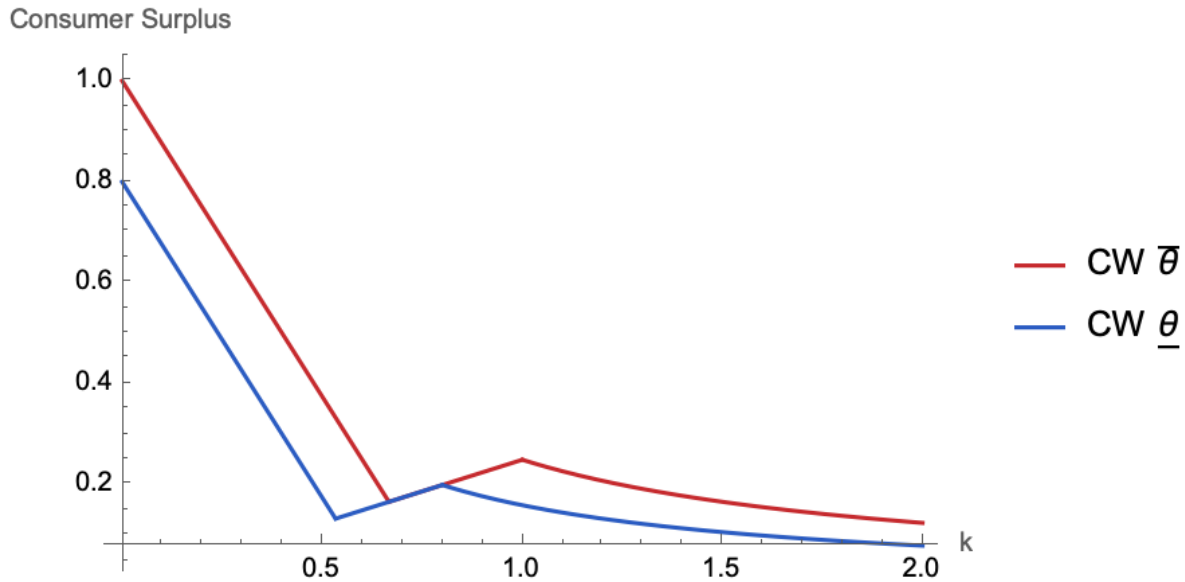


Figure 3.4: The consumer surplus of $\bar{\theta}$ consumer and $\underline{\theta}$ consumer. The plot assumes $\bar{\theta} = 1, \underline{\theta} = 0.8$ and $c = 0$.

Figure 3.4 shows the consumer surplus for each type of consumer. For each θ , when the market is covered, the consumer surplus is the highest when the transportation cost is the lowest and declines linearly. In the uncovered market region, the consumer surplus also declines as transportation costs increase. For extremely high values of transportation costs, such that the total transportation cost for the consumer exceeds willingness to pay, consumer surplus declines to zero. In the adjacent market scenario — that is, for the intermediate values of transportation cost — consumer surplus is increasing in transportation costs. This occurs because the firms strategically reduce prices below the competitive level to prevent the consumer in the middle of the Hotelling interval from not purchasing, thus ensuring that this consumer remains indifferent between purchasing from either firm and not purchasing at all. Furthermore, when the market is adjacent for both types of consumers, the consumer surplus for both types is equal. Otherwise, consumer surplus of type $\underline{\theta}$ consumers is always lower than $\bar{\theta}$ consumers

The aggregate consumer surplus for a given k depends on the proportion γ of type $\bar{\theta}$ consumer at each location. In Appendix B.2.4, we present the detailed calculations for aggregate consumer surplus across different regions.

Figure 3.5 depicts the aggregate consumer surplus as a function of transportation costs.¹

¹We only plot $\gamma = 0.4$ to maintain consistency with the assumptions we later make in 3.1.

Aggregate Consumer Surplus

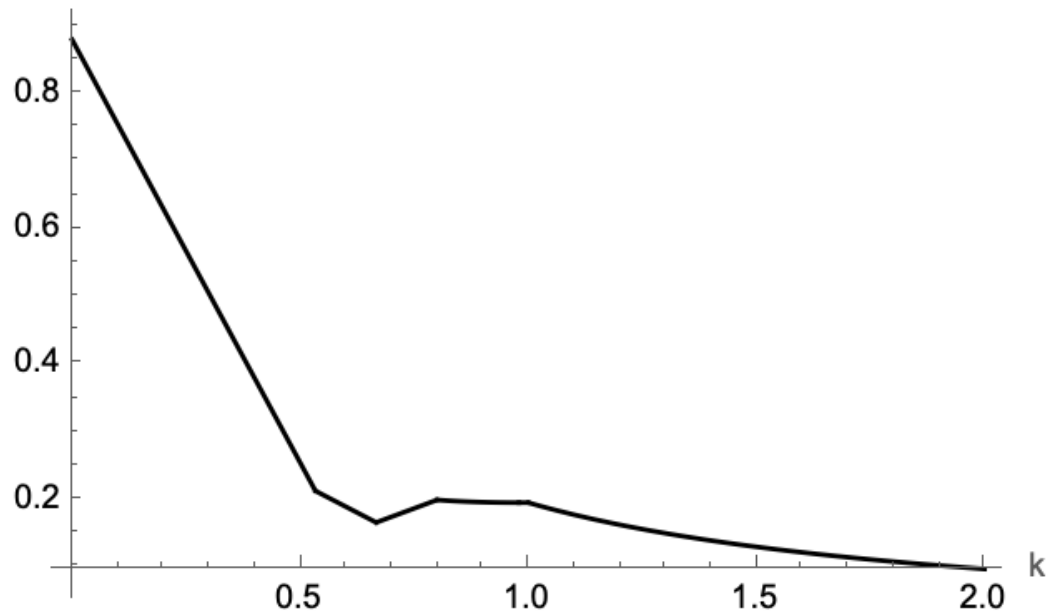


Figure 3.5: Aggregate surplus when both firms are informed about the vertical type of the consumer. The aggregate surplus is plotted assuming $\bar{\theta} = 1$, $\underline{\theta} = 0.8$, $\gamma = 0.4$, and $c = 0$

Following Corollary 3.3, when $\bar{\theta} < \frac{3\underline{\theta}}{2}$, i.e., when the difference between the two vertical types of consumers is sufficiently small, and $0 \leq c < 3\underline{\theta} - 2\bar{\theta}$, then if the proportion of $\underline{\theta}$ consumers is higher than that of $\bar{\theta}$ consumers, the relationship between consumer surplus and transportation costs exhibits distinct patterns. When transportation costs are low and the market is covered for both types, the aggregate consumer surplus decreases linearly as k increases. As transportation costs rise, leading to a market that remains covered for $\bar{\theta}$ consumers but becomes adjacent for $\underline{\theta}$ consumers, the aggregate consumer surplus continues to decline with k , but at a slower rate. In this region, an increase in k results in a smaller reduction in aggregate consumer surplus, and this effect becomes more pronounced as γ increases.

As k increases further, reaching values where the market is adjacent for both types of consumers, the aggregate consumer surplus starts to rise. This is because firms lower their prices to ensure that the most distant consumers of each type remain indifferent between purchasing and not purchasing. When transportation costs increase to a level where the market is adjacent for $\bar{\theta}$ consumers but uncovered for $\underline{\theta}$ consumers, the aggregate consumer surplus experiences only a small decline. If the proportion of $\bar{\theta}$ consumers is sufficiently large, an increase in k in this region could instead lead to an increase in aggregate consumer surplus. At high transportation cost levels, when the market is uncovered for both consumer types, the aggregate consumer surplus decreases with k . However, in this range, the effect of k on aggregate con-

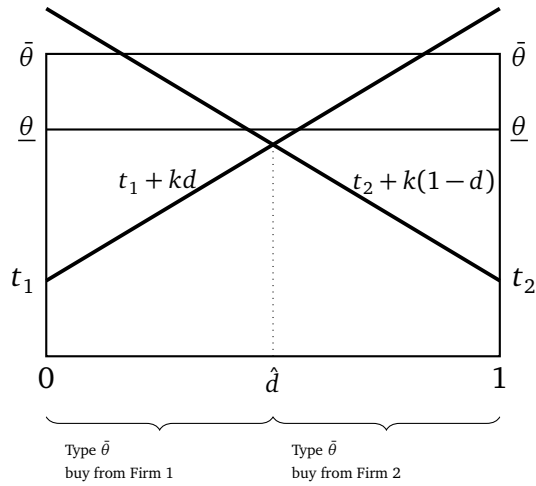
sumer surplus is nonlinear: as k increases further, the rate of decrease in consumer surplus diminishes. In Appendix B.2.5 we discuss the aggregate consumer surplus that arises with a different pecking order of k thresholds.

Furthermore, c affects the aggregate consumer surplus, which always decreases as c increases. However, following Corollary 3.3, under the restriction $0 \leq c < 3\underline{\theta} - 2\bar{\theta}$, the impact of c does not alter the trend observed in the aggregate consumer surplus in Figure 3.5.

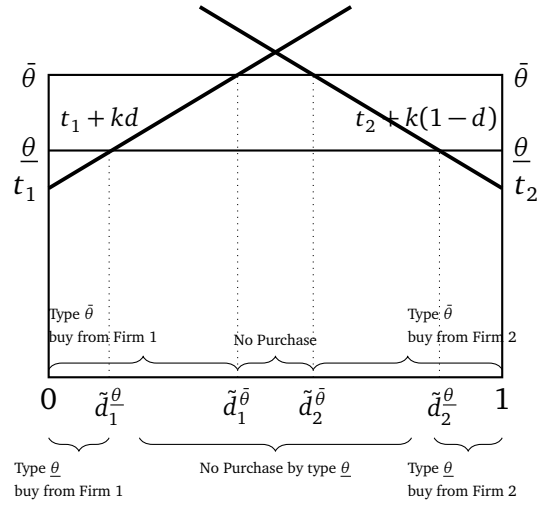
Lastly, for the extreme values of γ , i.e, when only either one type of consumer exists in the market, the aggregate consumer surplus is equivalent to the type specific consumer surplus illustrated in Figure 3.4.

3.4 No Firm Informed

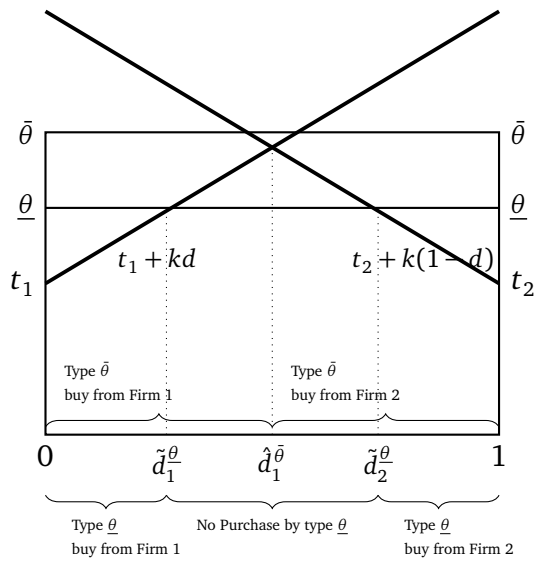
In this section, we assume that both firms are uninformed about the consumers' vertical type. Consequently, each firm i makes a single offer t_i . Proposition 3.4 characterizes the single offer a firm makes to both vertical types of consumers in the unique symmetric equilibrium.



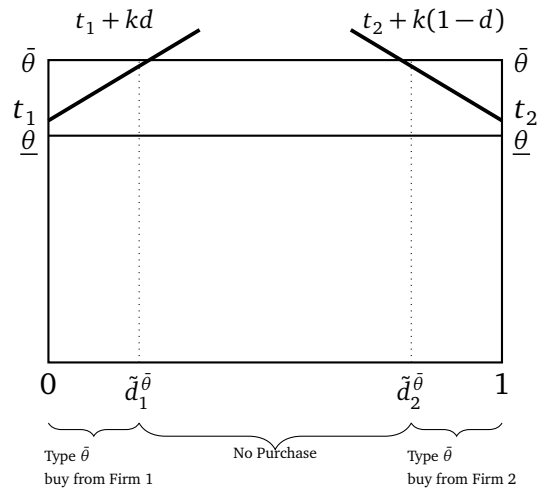
(a) All Consumers Purchase.



(b) Some Consumers Purchase.



(c) All type $\bar{\theta}$ consumers purchase.
Some type $\underline{\theta}$ consumers purchase



(d) Some type $\bar{\theta}$ consumers purchase.
No type $\bar{\theta}$ consumers purchase

Figure 3.6: Purchasing decision of the consumers when Firms have no information about the vertical type of the consumer.

Figure 3.6 illustrates the distinct purchasing decisions of the consumers. When the cost of purchasing – define as the transport cost plus the price — is sufficiently low (subfigure (a)), all consumers participate in the market: those located between \hat{d} and 0 purchase from Firm 1, while those between \hat{d} and 1 purchase from Firm 2. As the purchasing cost increases, partial market coverage emerges, with heterogeneity in consumer behavior (subfigure (b)). In some regions, consumers of both vertical types purchases (i.e., $[0, \tilde{d}_1^\theta]$ purchase from Firm 1 and $[\tilde{d}_2^\theta, 1]$ purchase from Firm 2), while in others, only type $\bar{\theta}$ consumers participate (i.e., $[\tilde{d}_1^{\bar{\theta}}, \tilde{d}_1^{\bar{\theta}}]$ purchase from Firm 1 and $[\tilde{d}_2^{\bar{\theta}}, \tilde{d}_2^{\bar{\theta}}]$ purchase from Firm 2). Additionally, there exist intervals (i.e., $[\tilde{d}_1^{\bar{\theta}}, \tilde{d}_2^{\bar{\theta}}]$) where no consumers make purchases. Notably, the markets served by the two firms remain distinct, with no overlap in their respective consumer bases. At even higher purchasing costs, selective participation occurs, where all type $\bar{\theta}$ consumers and only a subset of type $\underline{\theta}$ consumers participate (subfigure (c)). Finally, when the purchasing cost becomes sufficiently high, only some type $\bar{\theta}$ consumers purchase in the market, while no type $\underline{\theta}$ consumers make purchases (subfigure (d)). These scenarios reflect the interplay between consumers' willingness to pay and the purchasing cost, with participation contingent on whether the cost is less than, equal to, or greater than their willingness to pay.

In Proposition 3.4, we analyze these purchasing scenarios under the assumption that $\bar{\theta} - \underline{\theta} < \underline{\theta} - c$. Consequently, scenario (d) does not arise, as type $\underline{\theta}$ consumers are not entirely excluded from the market. In equilibrium, the price a firm charges both vertical types depends on the transportation cost k and the proportion of consumers of vertical type $\bar{\theta}$ i.e, γ .

As before, the location of a consumer that is indifferent between purchasing from either firms arises from $u(t_1, \hat{d}, \theta_i) = u(t_2, 1 - \hat{d}, \theta_i)$. Therefore, the indifferent consumer for both types is \hat{d} as described in eq (3.3). On the other hand, the location of the consumer of each type that is indifferent between purchasing from Firm 1 and not purchasing at all is type specific such that,

$$\begin{aligned}
u(t_1, \tilde{d}_{\bar{\theta}}, \bar{\theta}) &= 0 \\
&\Rightarrow \bar{\theta} - t_1 - k\tilde{d}_{\bar{\theta}} = 0 \\
&\Rightarrow \tilde{d}_{\bar{\theta}} = \frac{\bar{\theta} - t_1}{k}
\end{aligned} \tag{3.5}$$

Similarly for type $\underline{\theta}$

$$\begin{aligned}
u(t_1, \tilde{d}_{\underline{\theta}}, \underline{\theta}) &= 0 \\
&\Rightarrow \underline{\theta} - t_1 - k\tilde{d}_{\underline{\theta}} = 0 \\
&\Rightarrow \tilde{d}_{\underline{\theta}} = \frac{\underline{\theta} - t_1}{k}
\end{aligned} \tag{3.6}$$

Given above, Firm 1's demand given $\tilde{d}_i \in [0, 1], i \in \{\bar{\theta}, \underline{\theta}\}$ is as follows:

$$\bar{Q}_1 = \begin{cases} 1, & t_1 \leq \bar{\theta} - k, \\ \frac{\bar{\theta} - t_1}{k}, & \bar{\theta} - k \leq t_1 \leq \bar{\theta} \\ 0, & t_1 \geq \bar{\theta}. \end{cases} \tag{3.7}$$

$$\underline{Q}_1 = \begin{cases} 1, & t_1 \leq \underline{\theta} - k, \\ \frac{\underline{\theta} - t_1}{k}, & \underline{\theta} - k \leq t_1 \leq \underline{\theta} \\ 0, & t_1 \geq \underline{\theta}. \end{cases} \tag{3.8}$$

The demand of Firm 2 is symmetric.

Proposition 3.4 identifies the five regions that arise in equilibrium. These regions arise given the k thresholds illustrated in Figure 3.7. When transportation costs are low — specifically, for $k \leq \frac{2(\underline{\theta} - c)}{3}$ (red region in the figure) — the market is horizontally covered for both types, meaning all consumers are served along the entire Hotelling line.

When transportation costs are high — specifically, $k \geq 2\bar{\theta} - c - \gamma\bar{\theta} - (1 - \gamma)\underline{\theta}$ (blue region in the figure) — the market is uncovered for both vertical types and firms behave as local monopolists for both vertical types.

For intermediate transportation costs, the market outcomes vary depending on the value of k and γ . In orange region, the market is covered for type $\bar{\theta}$ and adjacent for type $\underline{\theta}$. In yellow region, the market is uncovered for type $\underline{\theta}$ and covered for type $\bar{\theta}$. Further, in green region, the market is uncovered for type $\underline{\theta}$ consumers and adjacent for type $\bar{\theta}$ consumers.

According to Proposition 3.4, as γ increases, the region where the market is covered for type $\bar{\theta}$ and adjacent for type $\underline{\theta}$ decreases. At $\gamma = 1$, this equilibrium no longer exists, and the market transitions directly from being covered for type $\bar{\theta}$ to being adjacent for type $\bar{\theta}$. Similarly, as γ decreases, the region where the market is uncovered for type $\underline{\theta}$ and adjacent for type $\bar{\theta}$ also decreases and disappears at $\gamma = 0$. Given that firms know γ , $\gamma = 1$ corresponds to the scenario where only $\bar{\theta}$ consumers exist in the market, and firms have full information about consumers' willingness to pay. Conversely, $\gamma = 0$ represents the case where only $\underline{\theta}$ consumers are present

in the market, and firms have full information about consumer willingness to pay. Therefore, the equilibria described in Proposition 3.4 arise only when $0 < \gamma < 1$.

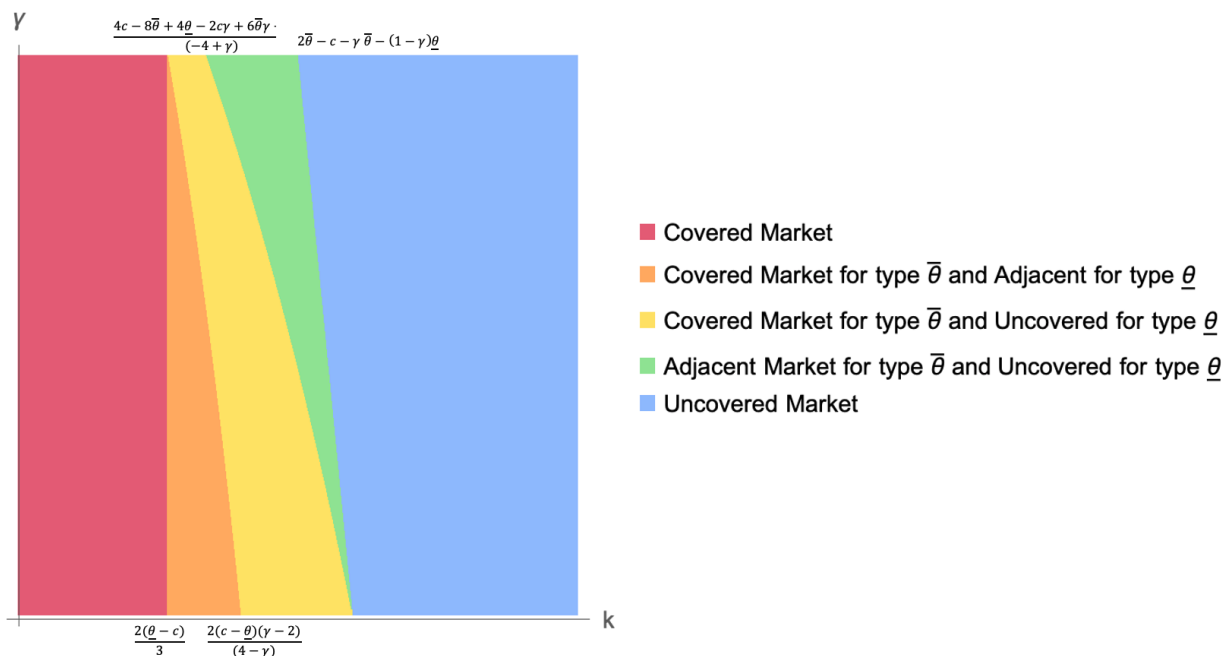


Figure 3.7: Thresholds of k indicating the different market regions, when $c = 0$, $\bar{\theta} = 1$ and $\underline{\theta} = 0.8$.

Proposition 3.4. Suppose $\bar{\theta} - \underline{\theta} < \underline{\theta} - c$ and both firms do not know the vertical type of the consumer that they are facing at each location on the horizontal line. In symmetric equilibrium, multiple possible pricing patterns arise depending on the transportation cost k and the proportionate of type $\bar{\theta}$ type of consumers i.e, γ .

1. **Covered Market:** If $k \leq \frac{2(\bar{\theta}-c)}{3}$, then each firm charges price

$$t^* = k + c$$

Market is horizontally covered and a consumer of type $\bar{\theta}$ and $\underline{\theta}$ derives a positive utility from purchasing along the entire Hotelling line.

2. **Covered Market for type $\bar{\theta}$ and Adjacent for type $\underline{\theta}$:** If $\frac{2(\bar{\theta}-c)}{3} \leq k \leq \frac{2(c-\underline{\theta})(\gamma-2)}{4-\gamma}$, then each firm charges price

$$t^* = \underline{\theta} - \frac{k}{2}.$$

Market is horizontally covered for both types and a consumer of type $\bar{\theta}$ derives a positive utility from purchasing along the entire Hotelling line while a consumer of type $\underline{\theta}$ located in the middle of the Hotelling line is indifferent to purchasing.

3. **Covered Market for type $\bar{\theta}$ consumers and Uncovered Market for type $\underline{\theta}$ consumers:**
 If $\frac{2(c-\underline{\theta})(\gamma-2)}{4-\gamma} < k < \frac{4c-8\bar{\theta}+4\underline{\theta}-2c\gamma+6\bar{\theta}\gamma-4\underline{\theta}\gamma}{-4+\gamma}$, then each firm charges

$$t^* = \frac{\gamma(c - k + 2\underline{\theta}) - 2(c + \underline{\theta})}{-4 + 3\gamma}.$$

Market is horizontally covered for type $\bar{\theta}$ consumers. A $\bar{\theta}$ type of consumer derives a positive utility. Market is horizontally not covered for a type $\underline{\theta}$ consumers and a type $\underline{\theta}$ consumer located in the middle of the Hotelling line is not served.

4. **Uncovered Market for type $\underline{\theta}$ and Adjacent for type $\bar{\theta}$:** If $\frac{4c-8\bar{\theta}+4\underline{\theta}-2c\gamma+6\bar{\theta}\gamma-4\underline{\theta}\gamma}{-4+\gamma} \leq k \leq 2\bar{\theta} - c - \gamma\bar{\theta} - (1-\gamma)\underline{\theta}$, then each firm charges

$$t^* = \bar{\theta} - \frac{k}{2}$$

Market is horizontally not covered for type $\underline{\theta}$ and a consumer of type $\underline{\theta}$ located in the middle of the Hotelling line is not served. Market is horizontally covered for type $\bar{\theta}$ consumers but a consumer of type $\bar{\theta}$ located in the middle of the Hotelling line is indifferent to purchasing.

5. **Uncovered Markets:** If $k > 2\bar{\theta} - c - \gamma\bar{\theta} - (1-\gamma)\underline{\theta}$, then each firm charges

$$t^* = \frac{\gamma\bar{\theta} + (1-\gamma)\underline{\theta} + c}{2}$$

Market is not horizontally covered and a consumer of any type located in the middle of the Hotelling line is not served.

Refer to Appendix B.3.1 for the proof of Proposition 3.4.

Proposition 3.4 demonstrates that when transportation costs are low (red region), such that the market is covered for both types, the uniform price that the firm offers without information is equal to the personalized price that the firm offers with information. When transportation costs is higher such that the market is uncovered for both types (blue region), firms acts as local monopolist. They make an offer to both vertical types of consumers if $\bar{\theta} - \underline{\theta} < \underline{\theta} - c$. The monopoly price that each firm offers increases in γ . For the intermediate values of k , when both types of consumers exist in the market, the uniform price first decreases when the market is covered for the type $\bar{\theta}$ and adjacent for type $\underline{\theta}$ (orange region), then increases with k when the market is covered for the type $\bar{\theta}$ and uncovered for type $\underline{\theta}$ (yellow region). Further, the uniform price decreases when market is adjacent for the type $\bar{\theta}$ and uncovered for type $\underline{\theta}$ (green region).

3.5 Comparative Analysis

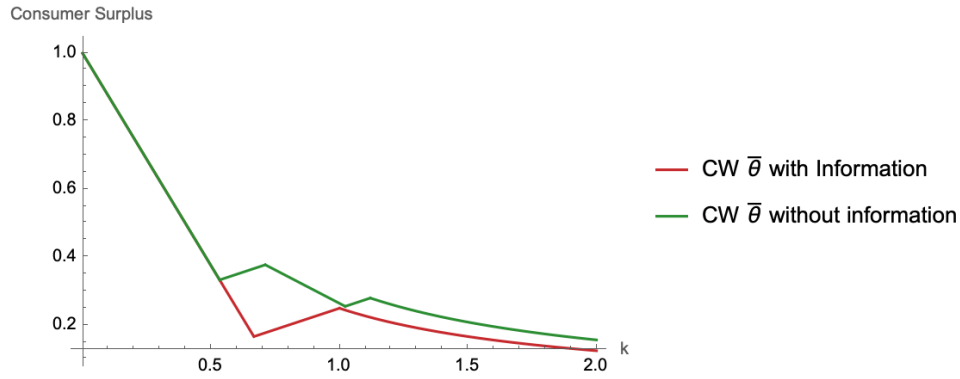
In this section, we compare consumer surplus under the two information structures discussed above — that is, when firms are informed (Proposition 3.2) vs. when firms are not informed about the vertical type of the consumer (Proposition 3.4). In this analysis, Social Welfare is defined as the sum of aggregate consumer surplus and total producer surplus (firm profits): $W = CS + \Pi$. It is important to note that the qualitative effects on social welfare are contingent on the specific definition considered, and results may vary under alternative specifications of the welfare function.

Figure 3.8 illustrates how consumer surplus for both types evolves with transportation costs k , given the k threshold pecking order from Corollary 3.3 and assuming $c = 0$.² When k is sufficiently low to ensure full market coverage for both consumer types, firms compete head-to-head, and equilibrium personalized prices equal the transportation cost k , regardless of consumer type. In this region, equilibrium uniform price is also equal to k . Thus, for low k information makes no difference — both consumer types face the same competitive price under uniform and personalized pricing. Consumers of type $\underline{\theta}$ remain indifferent to the information structure until the market becomes adjacent for them under uniform pricing. As k increases further, the welfare effects diverge: consumers of type $\bar{\theta}$ are (weakly) better off when firms are not informed and must charge a uniform price (see Figure 3.8a), while consumers of type $\underline{\theta}$ are better off when firms are informed and can make type-specific offers (see Figure 3.8b). This highlights that the welfare impact of information is not uniform but redistributes surplus between consumer types — relative to uniform pricing, information-enabled personalized pricing intensifies competition for the low-type consumers and increases their horizontal market coverage, at the same time enabling firms to extract more surplus from the high-type consumers.

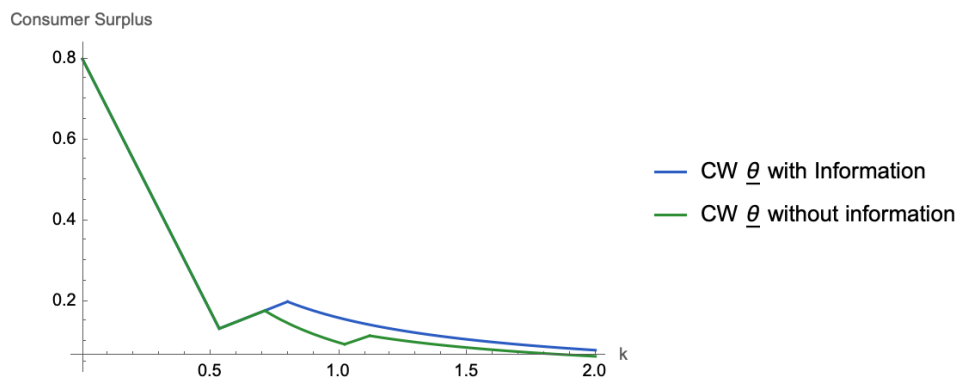
Figure 3.9 illustrates how the aggregate consumer surplus evolves with transportation costs, given the k threshold pecking order from Corollary 3.3 and assuming $c = 0$.³ When both consumer types coexist in the market, i.e., $0 < \gamma < 1$, the dynamics of consumer surplus is complex. For low transportation costs, when the market is fully covered for both types of consumers, the aggregate consumer surplus is decreasing linearly in k . When firms are not informed, in the region where the market is covered for $\bar{\theta}$ consumers and adjacent for $\underline{\theta}$ consumers, the aggregate consumer surplus increases as k increases but at a slow rate. When the market is covered for $\bar{\theta}$ consumers and uncovered for $\underline{\theta}$ consumers, the aggregate consumer surplus again decreases as k increases. However, this decrease is at a slower rate compared to

²Type-specific consumer surplus is derived in Appendix B.3.2.

³The aggregate consumer surplus is derived in Appendix B.3.3.



(a) $\bar{\theta}$ consumer



(b) $\underline{\theta}$ consumer

Figure 3.8: Comparing consumer surplus when both firms are not informed about the vertical type of the consumer v/s when both firms are not informed. The plot assumes $\bar{\theta} = 1$, $\underline{\theta} = 0.8$, $c = 0$ and $\gamma = 0.4$.

the case when market is covered for both types. When the market is adjacent for $\bar{\theta}$ consumers and adjacent for $\underline{\theta}$ consumers, aggregate consumer surplus again increases at a slow rate as k increases, before it starts to decrease in a non-linear fashion as k increases and the market becomes uncovered for both types.

Theorem 3.1 formalizes the observations from Figure 3.9 and identifies regions where personalized pricing harms aggregate consumer surplus. When the market is covered for both types of consumers, the competition between the firms is intense enough such that their pricing decision is independent of the consumer's type, therefore, consumers are indifferent between the two information structures. For high transportation cost, such that the market is uncovered for both types, the consumers are better off when firms have no information about their vertical type. For the in-between values of k , consumers are weakly better off when firms have full information about the vertical type for a range of k such that under no information, the

Aggregate Consumer Surplus

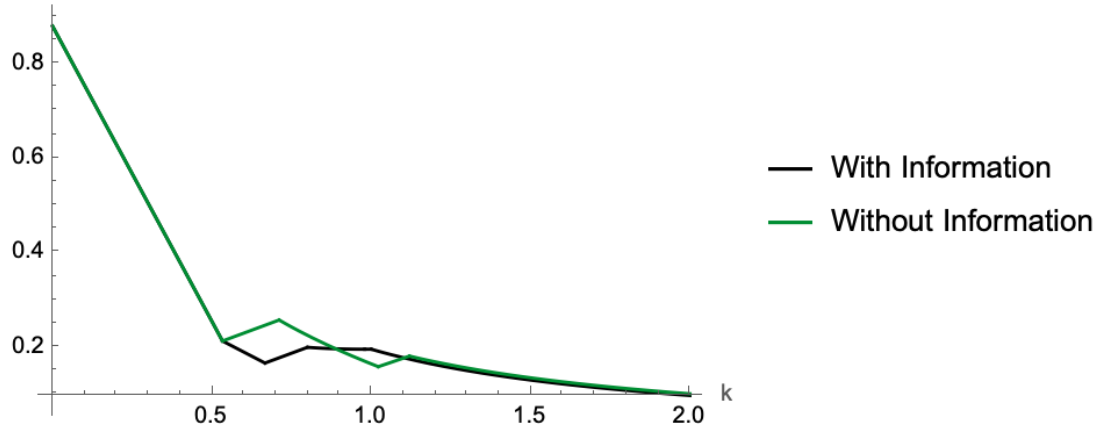


Figure 3.9: Comparing aggregate consumer surplus when both firms are not informed about the vertical type of the consumer v/s when both firms are not informed. The plot assumes $\bar{\theta} = 1$, $\underline{\theta} = 0.8$, $c = 0$ and $\gamma = 0.4$.

market is covered for type $\bar{\theta}$ and uncovered for $\underline{\theta}$ and under full information, the market is adjacent for type $\bar{\theta}$ and uncovered for $\underline{\theta}$ and or uncovered for both. As the two vertical types get further apart, this range of k and possible γ reduces. Otherwise, consumers are always better off when firms have no information about their vertical type (See Figure 3.9).

Theorem 3.1. Let $\bar{\theta}, \underline{\theta} \in \mathbb{R}$ such that $\bar{\theta} < \frac{3\underline{\theta}}{2}$, and let $c = 0$. The aggregate consumer surplus under personalized pricing (full information) relative to uniform pricing (no information) depends on the level of horizontal differentiation, k . Specifically:

1. **Low Differentiation:** When $k \leq \frac{2\underline{\theta}}{3}$, consumers are indifferent between firms having full information and no information.
2. **High Differentiation:** When $k \geq 2\bar{\theta} - \gamma\bar{\theta} - (1-\gamma)\underline{\theta}$, consumers are strictly better off when firms have no information.
3. **Intermediate Differentiation:** When $\frac{2\underline{\theta}}{3} < k < 2\bar{\theta} - \gamma\bar{\theta} - (1-\gamma)\underline{\theta}$, the welfare comparison is nuanced. Consumers are better off when firms have full information if and only if one of the following two conditions holds:

- Case (a): $\underline{\theta} < k < \bar{\theta}$ and $\underline{\theta} < \bar{\theta} \leq \frac{(15+\sqrt{33})\underline{\theta}}{16}$ and $0 < \gamma < \frac{4\bar{\theta}-4\underline{\theta}}{5\bar{\theta}-4\underline{\theta}}$.
- Case (b): $\bar{\theta} < k < \frac{\bar{\theta}(8-6\gamma)-4\underline{\theta}(1-\gamma)}{4-\gamma}$ and $\frac{(15+\sqrt{33})\underline{\theta}}{16} < \bar{\theta} < \frac{\underline{\theta}(\sqrt{6}+1)}{\sqrt{6}}$ and $0 < \gamma < \frac{4\bar{\theta}-6\underline{\theta}}{\bar{\theta}-3\underline{\theta}}$.

Otherwise, consumers are weakly better off when firms have no information.

Proof. See Appendix B.3.4

□

3.6 Conclusion

In this paper, we extend the classic Hotelling model by introducing a second dimension to consumer heterogeneity in order to analyze the impact of information structure on market outcome and consumer welfare. Specifically, we vary consumer preferences in two dimensions: (i) horizontally (brand preference) and (ii) vertically (quality preference or willingness to pay). We then compare pricing strategies and consumer welfare under two distinct scenarios: (i) firms have perfect information about consumer vertical type, i.e, firms can make type specific offers to consumers and (ii) firms have no information about the consumer's vertical type i.e, firms are restricted to select a single uniform price. Our results show a complex non-monotonic relationship between information, competition, and consumer welfare.

The equilibria reveal that irrespective of the information structure, with two vertical consumer types, the market can exhibit five distinct states depending on the degree of horizontal differentiation. These states, determined by transportation costs, range from fierce competition for both types to firms acting as local monopolists.

Our core contribution is to challenge the traditional view, exemplified by [Thisse and Vives \(1988\)](#), that providing firms with information that allows to personalize prices necessarily raises consumer surplus. Our extended model suggests that this may not always hold: under certain conditions, uniform prices can act as a form of consumer protection. We analyze the effect of each information structure on consumer surplus and find that when product differentiation is low, intense competition drives prices down to near marginal costs regardless of the information structure. When transportation costs are high, firms act as local monopolists and consumers are better off when firms are not informed and cannot perfectly price discriminate. Lastly, in regions with moderate product differentiation, the comparison is more nuanced. Specifically, when transportation costs are just above the intense competition region, consumers benefit from uniform prices (the no-information structure) and as transportation cost increases further (but still below monopoly regions), consumers benefit from full information (personalized pricing).

In our benchmark model, product quality is exogenous and consumer types are discrete. In [Appendix B.4](#), we present an alternative framework in which product quality is endogenous and consumer types are continuous. Here, firms compete by jointly choosing prices and quality, allowing us to study how information about consumer preferences shapes both product design and pricing. Unlike the benchmark, providing firms with consumer-type information in this setting makes consumers unambiguously worse off, as firms exploit the additional lever of tailoring both price and quality.

The effects on social welfare in this context are closely tied to the distribution of surplus

across different consumer types. When preferences are correlated with personal characteristics such as income, providing firms with information allows them to identify and target "high-type" consumers—those with a higher willingness to pay—effectively extracting their location and quality rents. In contrast, uniform pricing serves as a redistributive shield; by forcing a single price, it prevents firms from fully exploiting these vertical differences, thereby retaining more surplus for the consumer base and ensuring market access for "low-type" individuals who might otherwise be marginalized. While the specific results for total social welfare depend on the weights assigned to each group, our findings highlight that uniform prices are a vital tool for preventing the systematic extraction of consumer surplus.

Together, the results from both our benchmark and alternative models provide a robust argument to regulate personalized pricing. Our work demonstrates that the benefits of full information and perfect price discrimination are not guaranteed. Instead, a firm's information advantage can be used to harm consumer welfare.

In our future research we aim to extend this model by exploring an asymmetric duopoly where only one firm has information.

Chapter 4

Information Sharing by a Hybrid Platform: The Truth-Telling Problem

4.1 Introduction

In recent years, information gathering by hybrid digital platforms has attracted significant attention from policymakers across the globe. The concern is that companies such as Amazon, which operate online marketplace platforms and simultaneously compete with third-party sellers on their marketplaces, have an incentive to exploit their access to third-party seller information. Thus, the UK's Competition and Markets Authority (CMA) is currently investigating Amazon as an online marketplace.¹ The CMA's main concern is that Amazon is using its informational advantage to benefit its own retail arm and foreclose sales of third-party sellers. A similar investigation on the informational advantage of Amazon and Flipkart has been carried out by the CCI in India ([Competition Commission of India \(2020\)](#)). The SAMR of China ([Marco Colino \(2022\)](#)) fined Alibaba nearly EUR 2.4 billion for its unfair pricing strategies. The EU's Digital Markets Act forbids "gatekeeper platforms" from using data generated by business users of its "core platform services" in competition with those users.²

This paper studies the incentives of a platform that acts both as a market provider and operates a vertically integrated retail arm to truthfully share information about prevailing demand conditions with a third-party seller that competes with its retail arm on the platform. Specifically, the paper asks: When will the platform have an incentive to truthfully share information with the third-party seller?

In the model, the platform's retail arm and the third-party seller both sell differentiated

¹See: <https://www.gov.uk/cma-cases/investigation-into-amazons-marketplace>

²See: <https://eur-lex.europa.eu/eli/reg/2022/1925/oj>

goods on the platform and compete in prices under the Bertrand framework. There is demand uncertainty in the market such that demand can be either high or low. The platform enjoys an informational advantage: it has complete knowledge of the true demand state, and this information is always available to its retail arm. The platform then chooses whether to (i) share full information with the third-party seller or (ii) share no information.

We analyze two cases. First, where the platform does not charge a hosting fee to the seller. Second, where the platform charges the seller a commission, defined as a fixed percentage of the seller's revenue. In the latter case, the platform has an interest in maximizing not only its own profit but also, indirectly, the profit of the seller.

Our results show that in both cases, the platform's incentives to share information never align with those of the seller. Specifically, the platform prefers to share information about high demand when goods are substitutes and about low demand when goods are complements. In contrast, the seller benefits from knowing the true demand state in the high-demand case, but benefits from information asymmetry when demand is low. This misalignment of incentives leads to an uninformative equilibrium, resembling the classic cheap talk result of [Crawford and Sobel \(1982\)](#).

4.2 Literature Review

This paper is related to two strands of literature. First, it is related to the mature literature on the incentives of firms to share information with competitors. Second, it is related to the rapidly growing literature on digital platforms.

Literature on information sharing

The early literature on information sharing among competitors analyzes the incentives of firms to precommit to particular information-sharing rules. It is typically assumed that firms have symmetric private information and choose between full information sharing and no disclosure. [Novshek and Sonnenschein \(1982\)](#), [Clarke \(1983\)](#), [Vives \(1984\)](#), [Gal-Or \(1985\)](#), [Li \(1985\)](#), and [Kirby \(1988\)](#) consider information sharing about demand; [Gal-or \(1986\)](#), [Fried \(1984\)](#), [Sakai \(1986\)](#), and [Shapiro \(1986\)](#) consider information sharing about costs. Typically, the information-revelation incentives in these models are reversed: while Bertrand competitors with private information about the demand intercept have incentives to commit to sharing this information, Cournot competitors with private information about their costs have incentives to conceal this information. [Raith \(1996\)](#) provides a unified framework for understanding these results.

More recently, [Olivera \(2021\)](#) considers an asymmetric environment where only one firm is informed about the demand intercept. [Olivera \(2021\)](#) shows that it is optimal for the informed firm to commit to fully revealing its information when the informed firm's expected equilibrium profits are supermodular in the state and the uninformed firm's price.

This paper takes a different approach and assumes that the informed firm cannot commit to an information structure; that is, the approach here is in the spirit of the cheap-talk literature initiated by [Crawford and Sobel \(1982\)](#). The closest paper is [Ziv \(1993\)](#), which considers a Cournot setting where informed firms have access to non-verifiable information about their production costs and cannot commit to information-sharing rules. The author shows that when firms are symmetrically informed, they have incentives to lie. In this paper, we assume that only one firm is informed about realized demand and that this firm ex post decides whether to reveal this information to a competitor who is also a client.

We assume that the informed firm's information is non-verifiable. When information is verifiable at no cost, the unraveling result often holds. When strategic firms cannot credibly conceal unfavorable news, they are forced to disclose all information (see [Milgrom \(1981\)](#), [Milgrom and Roberts \(1986\)](#), and [Okuno-Fujiwara et al. \(1990\)](#)).

Literature on digital platforms

The literature on platforms refers to two types of platforms: (i) pure marketplaces and (ii) hybrid marketplaces. A pure marketplace is a platform provider that only hosts other sellers and, for its services, charges sellers either a percentage commission or a fixed fee. A hybrid marketplace generally refers to a platform that acts both as a market provider and as a seller. As a market provider, the platform hosts other sellers, allowing them to connect and sell their products to consumers; as a seller, the platform sells its own products on the platform as well. Thus, hybrid platforms differ from traditional market providers because they also participate in market competition.

A platform, irrespective of being a pure or hybrid marketplace, enjoys the advantage of observing additional demand data by hosting other sellers. The incentives to share this information with sellers depend on whether the platform also acts as a competitor.

Pure platforms

[Tsunoda and Zennyō \(2021\)](#) examine whether pure platforms have incentives to share information with sellers they host. Their model includes a platform, a seller, and a retailer. The platform and the retailer observe demand uncertainty, while the seller does not. The seller can either accept an agency model (where the seller has control over the retail pricing of their

goods on the platform, and the platform collects a percentage commission) or a wholesale model (where the platform controls retail pricing and the seller sells their goods to the platform at a wholesale price) to enter the platform. Overall, the authors show that information sharing benefits the platform and the seller but not the retailer. The platform prefers to share information under the agency model, as this allows the platform to enjoy leadership in setting commission rates and avoid the double-marginalization problem that arises under the wholesale model.

[Belhadj et al. \(2020\)](#) model a signaling game between a platform and a seller. Unlike most studies in the literature, here, the platform charges the seller a fixed fee to host them rather than a commission. They model the platform's choice to act either as a pure marketplace or as a reseller. They show the possibility of a least-cost separating equilibrium where the platform remains a pure marketplace for low-demand goods but converts to a reseller for high-demand goods. Empirical studies by [Jiang et al. \(2011\)](#) and [Zhu and Liu \(2018\)](#) reinforce the possibility of such a separating equilibrium.

[Liu et al. \(2021\)](#) describe the optimal information-sharing strategy of a platform when the sellers it hosts face Cournot competition and the commission rate is exogenous. Their main result highlights that the platform benefits when it shares accurate information with only a subgroup of sellers while sharing less accurate information with all sellers.

Hybrid platforms

Recently, [Anderson and Bedre-Defolie \(2021\)](#) model competition between a hybrid marketplace and a third-party seller. Consumers choose between a variety of products and outside options. Their demand function has logit form. The hybrid platform, as a Stackelberg leader, controls entry through its setting of the commission rate charged to sellers and its own product price. Comparing the two scenarios—where the platform is a hybrid marketplace versus a pure marketplace provider—the authors show that as a hybrid platform, it increases its own product demand by charging sellers higher commission fees and limiting entry, specifically when the platform's own good is better than the seller's good.

[Etro \(2023\)](#) consider a model with a representative consumer with quasi-linear preferences. They identify two effects caused by the entry of the platform's own good: (i) the “demand substitution effect” and (ii) the “extensive margin effect.” In the former, the platform, through high commissions, increases the prices of the sellers' goods, thereby attracting demand toward its own good. In the latter, the platform, through lower commissions, can reduce margins, thereby attracting more sellers and consumers to the platform. As a hybrid marketplace, in the case of more elastic demand, the platform limits the entry of sellers with high commissions. In

the case of less elastic demand, the hybrid platform encourages entry through low commissions.

As a market provider, hybrid platforms have an information advantage over demand. [Kirpalani et al. \(2020\)](#) are concerned with consumers sharing more data than is efficient with the platform. As a platform gains more precise information, its market power increases, which can, in turn, reduce entry and consumer welfare. Overall, the authors highlight differences between platforms and traditional retail markets, which increase the requirement for regulatory intervention in platforms.

Other concerns underlined in the literature regarding hybrid platforms' use of superior demand information for the benefit of their own retail arm relate to issues of product imitation and self-preferencing. [Hagiu et al. \(2022\)](#) show the implications of extreme measures such as banning hybrid platforms from selling their own products. They conclude that such a ban only reduces consumer surplus, as the constraint on sellers' product prices is lifted. Of course, sellers are overall worse off in a hybrid marketplace.

A similar result is obtained by [Anderson and Bedre-Defolie \(2022\)](#). The authors outline conditions under which a platform that plays a dual role as both a market provider and a seller benefits or harms consumers. The paper describes a business model where the platform charges a commission to third-party sellers to sell on the platform. Whether the platform charges a unit-fee commission or a percentage commission does not qualitatively affect the overall result that there should not be a ban on platforms selling their own products. Instead, the authors suggest that regulators should ban certain behavioral aspects of platforms, such as platforms recommending their own goods.

The theoretical studies mentioned above mainly ask the following questions: (i) whether a platform would find it worthwhile to transition from a pure marketplace to a hybrid marketplace; (ii) whether platforms should be discouraged from transitioning by regulators; and (iii) how a platform's transition affects consumer welfare and sellers' profits. In contrast, we examine whether a platform would benefit its own retail arm and its total profit (from the retail arm and the collection of commissions) by either sharing or withholding information from the sellers on its marketplace.

The incentives of a vertically integrated platform to share information were recently examined by [Padilla et al. \(2021\)](#), who assume consumers have a Shubik-Levitan utility function and that firms compete on both price and quality. They show that when the platform's good is a close substitute for the seller's good and demand is quality-sensitive, the platform's incentive to share information is reduced. They also find that consumers are generally better off when the platform withholds information, as better-informed sellers can extract higher rents. Our study differs in two key ways. First, we focus solely on price competition, without a quality dimension. Second, in our model the platform faces a binary strategic choice: to reveal full

information or none. This all-or-nothing decision explicitly shapes the platform’s incentives, as sharing full information may lead sellers to adjust prices in ways that either increase or decrease platform profits depending on the substitutability or complementarity of goods and the underlying demand level. By modeling this binary choice, we capture the strategic trade-offs of information sharing that are absent in [Padilla et al. \(2021\)](#).

In a setting that includes a platform, a third-party seller, and a manufacturer, [Zhong et al. \(2023\)](#) analyze the platform’s incentive to share information with the manufacturer and the reseller. [Zhong et al. \(2023\)](#) also establish an equilibrium with similar qualitative features to those in [Padilla et al. \(2021\)](#).

Empirical studies

One of the largest empirical studies in the hybrid-marketplace literature is [Zhu and Liu \(2018\)](#). Using data from Amazon.com, the authors conclude that Amazon targets entry into product categories that are more successful and require less selling effort. They also find that Amazon’s entry reduces sellers’ motivation to grow but lowers shipping costs and benefits consumers.

A similar empirical study by [Deng et al. \(2022\)](#), using data from JD.com, provides a contrasting result: upon a platform’s entry into the market, sellers benefit by being able to charge higher prices.

4.3 Model

We consider a marketplace provider, platform M that is vertically integrated with a retail arm R that competes with a third party seller S on the platform. Both R and S provide differentiated goods on the platform at zero marginal costs. We assume that M constitutes the only available marketplace. Further, M charges S a commission α for providing access to the marketplace. Therefore, the platform M derives payoff from two sources, (i) the revenue generated by its retail arm R and (ii) the commission α it receives on the revenue generated by the third party seller S . The third party seller, S derives payoff from its revenue net of the commission it pays to the platform M .

4.3.1 Consumer Demand

Suppose, there are a continuum of consumers such that a representative consumer maximizes the following quasi-linear utility function.

$$U(q_r, q_s) - p_r q_r - p_s q_s \quad (4.1)$$

where $q_i \geq 0, i = r, s$, are the quantity of the goods produced by R and S respectively. $p_i, i = r, s$ are their respective prices. Furthermore, $U(q_r, q_s)$ is quadratic, strictly concave, and symmetric in q_r and q_s , that is,

$$U(q_r, q_s) = \theta(q_r + q_s) - \frac{1}{2}(\beta q_r^2 + 2\gamma q_r q_s + \beta q_s^2) \quad (4.2)$$

above, β and γ are known parameters. $\beta > |\gamma| \geq 0$ and $-1 < \gamma \leq 0$ The ratio $\frac{\gamma}{\beta}$ measures the degree of substitution. $-1 \leq \frac{\gamma}{\beta} \leq 1$. As $\frac{\gamma}{\beta}$ moves from -1 to 1 , goods move from being perfect complements to perfect substitutes. θ captures consumers valuation of total quantity in the market. Further, $\theta \in \{\underline{\theta}, \bar{\theta}\}$ where $\bar{\theta} > \underline{\theta} > 0$.

Solving the consumer's utility maximization problem described in eq (4.1) yields the following demand functions for R and S respectively³:

$$q_r = \frac{\theta(\beta - \gamma) - p_r \beta + p_s \gamma}{\beta^2 - \gamma^2} \quad (4.3)$$

$$q_s = \frac{\theta(\beta - \gamma) - p_s \beta + p_r \gamma}{\beta^2 - \gamma^2} \quad (4.4)$$

The above demand functions can be rewritten as follows:

$$q_r = a - b p_r + c p_s \quad (4.5)$$

$$q_s = a - b p_s + c p_r \quad (4.6)$$

where $a = \frac{\theta}{\beta + \gamma}$, $b = \frac{\beta}{\beta^2 - \gamma^2}$ and $c = \frac{\gamma}{\beta^2 - \gamma^2}$

4.3.2 Information Asymmetry

Given its dual role as both a marketplace provider and a retailer, platform M possesses complete information about the demand intercept $\theta \in \{\underline{\theta}, \bar{\theta}\}$, with $\bar{\theta} > \underline{\theta} > 0$. The platform always shares the true realization of θ with its integrated retail arm R .

In contrast, the third-party seller S does not observe the true demand intercept. Instead, S holds a prior belief $\mu \in (0, 1)$ that $\theta = \bar{\theta}$ (and $1 - \mu$ that $\theta = \underline{\theta}$).

We assume that M can choose an information disclosure strategy. Specifically, M decides whether to (i) disclose full information to S by truthfully revealing the realized θ , or (ii) dis-

³We follow [Vives \(1984\)](#).

close no information, in which case S relies solely on its prior belief. Thus, the information structure is a Bayesian game in which R is perfectly informed about θ , while S may or may not be informed depending on M 's disclosure policy.

4.3.3 Sequence of Events

The strategic interaction unfolds as a sequential game with a simultaneous pricing sub-game. Specifically, in stage one, the platform M decides whether to share the true demand intercept with the Third-party Seller S or not. In stage two, the retail arm R and the seller S simultaneously set prices p_r and p_s (based on his belief μ) respectively. In stage 3, based on the true demand, payoffs of M and S are realized.

Given the information asymmetry that arises in our model, the appropriate solution concept is Perfect Bayesian Equilibrium. Further, we solve the game by backward induction.

To this end, we consider two cases, (i) a benchmark case, where $\alpha = 0$, i.e, the platform M does not charge any commission for hosting the Third party Seller S on its platform and (ii) a case introducing α , i.e $0 < \alpha \leq 1$, i.e, Platform M charges a commission for hosting the Third-party seller. Note, α is always exogenous.

By comparing the two cases, we analyze (i) whether the Platform M has a strategic incentive to share information with the Seller S , (ii) if the strategic incentive alters when the platform's role extends from only a competitor to a market provider that a clear incentive in the Third party Seller's revenue increasing. (iii) The effect on consumer surplus when the market provider has a dual role and an information advantage.

We make the following assumption to ensure that prices and quantities remain strictly positive for both firms.

Assumption 4.1.

$$\bar{\theta} \leq \frac{4\beta^2 - \gamma^2}{4\beta^2 - 2\gamma^2} \underline{\theta}$$

Following [Olivera \(2021\)](#), this condition imposes an upper bound on $\bar{\theta}$ relative to $\underline{\theta}$. The assumption is derived ⁴ considering the worst case scenario where true demand is $\underline{\theta}$ while the uninformed seller is overly optimistic such that $\mu = 1$.

Similarly, when $0 < \alpha \leq 1$, we make the following assumption.

Assumption 4.2.

$$\bar{\theta} \leq \frac{4\beta^2 - (1 + \alpha)\gamma^2}{4\beta^2 - 2(1 + \alpha)\gamma^2} \underline{\theta}$$

⁴Differentiating the no information sharing $\pi_{s|\theta}$ with respect to $\bar{\theta}$. Then solving this first-order condition at $\mu = 1$ for $\bar{\theta}$ leads to this inequality.

4.4 Pure Competition: $\alpha = 0$

Suppose, Platform M does not charge a commission α to host the Third-party Seller S , i.e., $\alpha = 0$. Then Platform M (by the virtue of being vertically integrated with R), is a pure competitor of the Seller S . Two scenarios arise: (i) Truthful Information Sharing: M shares true demand intercept with S and (ii) No Information Sharing: M does not share any information with S .

4.4.1 Truthful Information Sharing

When the demand intercept θ is truthfully revealed to the third-party seller S and $\alpha = 0$, the game reduces to a symmetric Bertrand duopoly. Each firm $i \in \{r, s\}$ solves:

$$\max_{p_i \geq 0} \pi_i(p_i, p_j; \theta) = p_i q_i(p_i, p_j; \theta), \quad j \neq i, \quad (4.7)$$

where $q_i(p_i, p_j; \theta) = a - bp_i + cp_j$.

The first-order condition with respect to p_i is:

$$\frac{\partial \pi_i}{\partial p_i} = q_i + p_i \frac{\partial q_i}{\partial p_i} = a - 2bp_i + cp_j = 0. \quad (4.8)$$

Assuming a symmetric equilibrium $p_r^* = p_s^* = p^*$, we obtain:

$$p^* = \frac{a}{2b - c}, \quad (4.9)$$

$$\pi_i^* = p^*(a - (b - c)p^*) = \frac{a^2 b}{(2b - c)^2}. \quad (4.10)$$

Substituting $a = \frac{\theta}{\beta + \gamma}$, $b = \frac{\beta}{\beta^2 - \gamma^2}$, and $c = \frac{\gamma}{\beta^2 - \gamma^2}$ yields:

$$p^* = \frac{\theta(\beta - \gamma)}{2\beta - \gamma}, \quad (4.11)$$

$$\pi_i^* = \frac{\theta^2 \beta (\beta - \gamma)}{(2\beta - \gamma)^2 (\beta + \gamma)} \quad (4.12)$$

Here, strategic interaction of firms yields market prices that are below the monopoly price of $\frac{\theta}{2}$, but above the zero marginal costs of production. When the products are closer substitutes, i.e., when $\frac{\gamma}{\beta} \rightarrow 1$, the prices approach marginal cost, i.e., zero here. Therefore, when products are perfect substitutes i.e., $\gamma = \beta$, we have the efficient outcome where prices equal marginal costs. When the products are less substitutes and more independent of each other, i.e., when

$\frac{\gamma}{\beta} \rightarrow 0$, the prices converge to the monopoly price. When the products are close complements, i.e., when $\frac{\gamma}{\beta} \rightarrow -1$, the prices tend to $\frac{2\theta}{3}$, which is above the monopoly price. Lastly, equilibrium prices increase in θ . This serves as a benchmark for comparison, as these results are well-established in the literature on Bertrand duopoly. For instance, see the certainty model in [Vives \(1984\)](#).

4.4.2 No Information Sharing

When the demand intercept θ is not truthfully revealed to the third-party seller S . The seller faces uncertainty about market demand. Specifically, S holds a prior belief that $\theta = \bar{\theta}$ with probability μ and $\theta = \underline{\theta}$ with probability $1 - \mu$.

Throughout this section, all variables with a subscript k correspond to the specific realized demand level θ_k , i.e., $\theta_k \in \{\bar{\theta}, \underline{\theta}\}$. In particular, a_k denotes the demand intercept associated with θ_k , and $p_{r|k}$ denotes the price set by R conditional on θ_k .

The third-party seller S maximizes expected profit, taking into account its belief over demand and anticipating R 's price conditional on θ_k :

$$\begin{aligned} \max_{p_s \geq 0} \mathbb{E}[\pi_s(p_s, p_r; \theta)] &= \max_{p_s \geq 0} p_s \mathbb{E}[q_s(p_s, p_r; \theta)] \\ &= \max_{p_s \geq 0} p_s \left(\mu, q_s(p_s, p_{r|\bar{\theta}}; \bar{\theta}) + (1 - \mu), q_s(p_s, p_{r|\underline{\theta}}; \underline{\theta}) \right), \end{aligned} \quad (4.13)$$

where

$$q_s(p_s, p_{r|\bar{\theta}}; \bar{\theta}) = \bar{a} - bp_s + cp_{r|\bar{\theta}}, \quad (4.14)$$

$$q_s(p_s, p_{r|\underline{\theta}}; \underline{\theta}) = \underline{a} - bp_s + cp_{r|\underline{\theta}}. \quad (4.15)$$

The first-order condition with respect to p_s is:

$$\frac{\partial \mathbb{E}[\pi_s]}{\partial p_s} = 0 \quad \Rightarrow \quad \mu(\bar{a} + cp_{r|\bar{\theta}}) + (1 - \mu)(\underline{a} + cp_{r|\underline{\theta}}) - 2bp_s = 0. \quad (4.16)$$

The integrated retailer R observes the true demand intercept θ_k and chooses price conditional on the realized state:

$$\max_{p_r \geq 0} \pi_{r|k}(p_r, p_s(\mu); \theta_k) = p_r q_r(p_r, p_s(\mu); \theta_k), \quad \theta_k \in \{\bar{\theta}, \underline{\theta}\}, \quad (4.17)$$

where

$$q_r(p_r, p_s(\mu); \theta_k) = a_k - bp_{r|k} + cp_s(\mu). \quad (4.18)$$

The first-order condition with respect to p_r is:

$$\frac{\partial \pi_{r|k}}{\partial p_r|k} = 0 \Rightarrow a_k - 2bp_{r|k} + cp_s(\mu) = 0. \quad (4.19)$$

Solving (4.16) and (4.19), the equilibrium prices are as follows:

$$p_s(\mu)^* = \frac{\mathbb{E}[a]}{2b - c}, \quad (4.20)$$

$$p_{r|\bar{\theta}}^* = \frac{\bar{a}}{2b} + \frac{c\mathbb{E}[a]}{2b(2b - c)}, \quad (4.21)$$

$$p_{r|\underline{\theta}}^* = \frac{\underline{a}}{2b} + \frac{c\mathbb{E}[a]}{2b(2b - c)} \quad (4.22)$$

Note here : $\mathbb{E}[a] = \mu\bar{a} + (1 - \mu)\underline{a}$ The realized profits are as follows:

$$\pi_{s|\bar{\theta}}^* = \frac{\mathbb{E}[a](\bar{a}(4b^2 - c^2) - \mathbb{E}[a](2b^2 - c^2))}{2b(2b - c)^2}, \quad (4.23)$$

$$\pi_{s|\underline{\theta}}^* = \frac{\mathbb{E}[a](\underline{a}(4b^2 - c^2) - \mathbb{E}[a](2b^2 - c^2))}{2b(2b - c)^2}, \quad (4.24)$$

$$\pi_{r|\bar{\theta}}^* = \frac{(\bar{a}(2b - c) + \mathbb{E}[a]c)^2}{4b(2b - c)^2}, \quad (4.25)$$

$$\pi_{r|\underline{\theta}}^* = \frac{(\underline{a}(2b - c) + \mathbb{E}[a]c)^2}{4b(2b - c)^2} \quad (4.26)$$

Substituting $\bar{a} = \frac{\bar{\theta}}{\beta + \gamma}$, $\underline{a} = \frac{\underline{\theta}}{\beta + \gamma}$, $b = \frac{\beta}{\beta^2 - \gamma^2}$, and $c = \frac{\gamma}{\beta^2 - \gamma^2}$ yields:

$$p_s(\mu)^* = \frac{(\mu\bar{\theta} + (1 - \mu)\underline{\theta})(\beta - \gamma)}{2\beta - \gamma}, \quad (4.27)$$

$$p_{r|\bar{\theta}}^* = \frac{(\beta - \gamma)(2\beta\bar{\theta} - \gamma(\bar{\theta} - \underline{\theta}))(1 - \mu)}{2\beta(2\beta - \gamma)}, \quad (4.28)$$

$$p_{r|\underline{\theta}}^* = \frac{(\beta - \gamma)(2\beta\underline{\theta} + \gamma(\bar{\theta} - \underline{\theta})\mu)}{2\beta(2\beta - \gamma)}. \quad (4.29)$$

$$\pi_{s|\bar{\theta}}^* = \frac{(\beta - \gamma)(\underline{\theta} + \mu(\bar{\theta} - \underline{\theta}))((1 - \mu)(\bar{\theta} - \underline{\theta})(2\beta^2 - \gamma^2) + 2\beta^2\bar{\theta})}{2\beta(2\beta - \gamma)^2(\beta + \gamma)}, \quad (4.30)$$

$$\pi_{s|\underline{\theta}}^* = \frac{(\beta - \gamma)(-\underline{\theta} + \mu(\underline{\theta} - \bar{\theta}))\left(2\beta^2\underline{\theta} + (-2\beta^2 + \gamma^2)(\bar{\theta} - \underline{\theta})\mu\right)}{2\beta(2\beta - \gamma)^2(\beta + \gamma)}, \quad (4.31)$$

$$\pi_{r|\bar{\theta}}^* = \frac{(\beta - \gamma)(2\beta\bar{\theta} + \gamma(\bar{\theta} - \underline{\theta})(-1 + \mu))^2}{4\beta(2\beta - \gamma)^2(\beta + \gamma)}, \quad (4.32)$$

$$\pi_{r|\underline{\theta}}^* = \frac{(\beta - \gamma)(2\beta\underline{\theta} + \gamma(\bar{\theta} - \underline{\theta})\mu)^2}{4\beta(2\beta - \gamma)^2(\beta + \gamma)}. \quad (4.33)$$

From the equilibrium prices, we see that the seller always raises prices with μ . The platform raises (lowers) prices with μ when goods are substitutes (complements). Higher μ , which reflects the seller's optimism about demand, positively affects retailer prices and profits for substitutes, and negatively for complements.

Specifically, there is a direct impact of the retailer's information advantage on their equilibrium price, $p_{r|k}^*$ which optimally adjusts based on the true demand intercept, a_k . In contrast, the uninformed seller's price, $p_s^*(\mu)$, is static with respect to his belief μ .

Further, due to asymmetric information, profits are also asymmetric. The retailer enjoys higher profits than the seller as having information about the demand intercept allows the retailer to choose prices more optimally. On the other hand, the seller's price choice is suboptimal in both demand states, leading to lower profits compared to the retailer. The seller's price choice is above the optimal when the true demand state is $\underline{\theta}$ and below optimal when the true demand state is $\bar{\theta}$, which leads to lower profits compared to the retailer.

Lastly, as product differentiation decreases, i.e., $\frac{\gamma}{\beta} \rightarrow 1$, prices decrease. When goods are perfect substitutes, i.e., $\frac{\gamma}{\beta} = 1$, prices drive down to the marginal cost, which in our case is zero. As the goods become closer complements, $\frac{\gamma}{\beta} \rightarrow -1$, the prices tend to rise to a value above the monopoly price. When $\frac{\gamma}{\beta} \rightarrow 0$, the retailer and the seller's goods become more independent, i.e., when $\frac{\gamma}{\beta} \rightarrow 0$, the platform (by virtue of being vertically integrated with the seller) is indifferent to the seller's belief.

The proposition 4.3 below formalizes the relation between the retailer profit and the seller's belief μ . Specifically, when goods are substitutes, i.e., $0 < \frac{\gamma}{\beta} \leq 1$, the platform's retail arm R and the seller S are direct rivals. Therefore, platform benefits when seller sets a high price, as this allows the retailer to increase his own price. While when goods are complements, i.e., $-1 \leq \frac{\gamma}{\beta} < 0$, the platform benefits when the seller sets a low price.

Proposition 4.3. *The platform's expected profit from its retail arm in the no information sharing equilibrium is strictly monotonic with respect to the seller's prior belief, μ .*

1. *When goods are substitutes, i.e, $0 < \frac{\gamma}{\beta} < 1$, platforms always benefit as μ increases.*
2. *When goods are complements, i.e, $-1 < \frac{\gamma}{\beta} < 0$, platform always benefits as μ decreases.*

Proof. See Appendix C.1.1 □

Corollary 4.4. *The platform is indifferent to the seller's belief when the goods are independent, i.e., when $\frac{\gamma}{\beta} = 0$.*

Proof. See Appendix C.1.2 □

The immediate consequence of proposition 4.3 is that the platform has an incentive to influence the seller's belief. However, the only way the platform can influence the seller's belief in our model is by making the binary choice to either share full information or no information. When goods are substitutes, platform's retail arm benefits when $\mu = 1$, therefore platform does not have an incentive to share information with seller when true demand intercept is $\underline{\theta}$. On the other hand, when goods are complements, platform's retail arm benefits when $\mu = 0$, therefore here platform does not have an incentive to share information with the seller when true demand intercept is $\bar{\theta}$.

Proposition 4.5. *The platform's strategic incentive to share information depends on the nature of the competing goods.*

1. *When goods are substitutes, i.e $0 < \frac{\gamma}{\beta} < 1$, platform benefits from sharing information if and only if $\theta = \bar{\theta}$*
2. *When goods are complements, i.e $-1 < \frac{\gamma}{\beta} < 0$, platform benefits from sharing information if and only if $\theta = \underline{\theta}$*

Proof. See Appendix C.1.3 □

Further, from the equilibrium realized profits above, we observe that the equilibrium profit of the seller, $\pi_{s|k}$, is concave in μ (See Lemma 4.6). This concavity demonstrates the cost of uncertainty for the seller. The further the seller's belief is strays away from the optimal belief, the more their realized profits are penalized. For instance, if the seller's belief is too pessimistic (low μ), when the true demand state is $\bar{\theta}$, the seller reduces the price, losing out on profit and if the seller's belief is too optimistic (high μ), when the true demand state is $\underline{\theta}$, the seller increases their price, losing revenue to the retailer.

Lemma 4.6. *The realized profit of the seller is concave in μ*

Proof. See Appendix C.1.4 □

Lemma 4.6, establishes that the realized profit of the seller is concave, which implies that there does exist a unique belief, μ_k^* , in each state that is optimum such that it maximizes the seller's realized profit. In Proposition 4.7, we derive the optimal belief and reveal a counter-intuitive strategic insight.

Proposition 4.7. *The seller's profit-maximizing belief is counterintuitive: in each state a biased prior yields higher realized profit than the correct prior. Specifically,*

1. *When $\theta = \underline{\theta}$, the seller's realized profit $\pi_{s|\underline{\theta}}^*$ is maximized at a belief $\mu_{\underline{\theta}}^* > 0$.*
2. *When $\theta = \bar{\theta}$, the seller's realized profit $\pi_{s|\bar{\theta}}^*$ is maximized at a belief $\mu_{\bar{\theta}}^* < 1$.*

Proof. See Appendix C.1.5 □

μ is an exogenous parameter, Proposition 4.7, highlights the cost of the belief μ . Proposition 4.7, reveals that the seller's realized profit (ex-post analysis) is maximized at a biased belief. Specifically, when true demand state is $\underline{\theta}$, seller is better off when he is a little optimistic, specifically at $\mu_{\underline{\theta}}^* > 0$, as little optimism allows him to set a higher prices while when true demand state is $\bar{\theta}$, seller is better off when he is a little pessimistic, specifically at $\mu_{\bar{\theta}}^* < 1$, as little pessimism allows him to set lower price.

The strategic incentive of the platform to share or not share information may harm the seller. When demand is high such that $\theta = \bar{\theta}$, the platform is always harmed by information asymmetry as any $\mu < 1$ leads the seller to set a lower price, while when demand state is low such that $\theta = \underline{\theta}$, the seller benefits by not being informed under assumption 4.1. The proposition 4.8 formalizes this result.

Proposition 4.8. *Given assumption 4.1 holds, the seller's benefit from information asymmetry is dependent on the true state of demand, θ as follows:*

1. *When $\theta = \bar{\theta}$, the seller never benefits from information asymmetry.*
2. *When $\theta = \underline{\theta}$, information asymmetry benefits the seller.*

Proof. See Appendix C.1.6 □

To this end, we show that under information asymmetry, (i) the platform's expected profits are strictly monotone in the seller's belief, depending on whether goods are complements or substitutes and this drives platform's incentive to share information, (ii), the seller is not always harmed by the information asymmetry and (iii) the seller's ex-post profit is maximized at a biased belief that is away from the true demand intercept.

4.4.3 The Uninformative Equilibrium

As proposition 4.5 demonstrates, the platform benefits from disclosing the demand intercept truthfully only when (i) the demand state is $\bar{\theta}$, and goods are substitutes and (ii) the demand state is $\underline{\theta}$, and goods are complements. The rational seller knows this incentive and renders any message from the platform about the demand intercept that he cannot verify as not credible. Therefore, the unique equilibrium is uninformative, and the seller's belief remains fixed at their prior μ . This result reflects a standard cheap talk situation (see Crawford and Sobel (1982)). Further, the platform's information advantage is only limited to tailoring their own price to the true demand intercept. The platform cannot leverage his information advantage to strategically lie to the seller.

Theorem 4.9. *The unique equilibrium is non-informative. The platform cannot alter the seller's prior belief μ .*

Proof. See Appendix C.1.7 □

The intuition is straightforward and relies on the fact that in equilibrium, the platform always sends the same message irrespective of the true θ . When goods are substitutes, platform's incentive is to announce $\theta = \bar{\theta}$ while when goods are complements, the platform's incentive is to announce $\theta = \underline{\theta}$. Under Bayes' rule, the seller's posterior belief coincides with the prior belief. Therefore, the equilibrium is uninformative.

4.4.4 Consumer Surplus

The consumer surplus is simplify the quasi linear utility function described in eq 4.1.

Substituting the equilibrium price p^* described in eq. 4.11, the consumer surplus when there is full information sharing is as follows:

$$CS(\theta)_{FI} = \frac{\theta^2 \beta^2}{(\beta + \gamma)(2\beta - \gamma)^2} \quad (4.34)$$

When there is no information sharing, the consumer surplus is as follows:

When the true demand state is high ($\theta = \bar{\theta}$) is given by:

$$\begin{aligned}
CS_{NI|\bar{\theta}} = \frac{1}{8\beta(2\beta - \gamma)^2(\beta + \gamma)} & \left[-2\bar{\theta}\underline{\theta}(\beta - \gamma)(4\beta^2(-2 + \mu) - 2\beta\gamma - 3\gamma^2(-1 + \mu))(-1 + \mu) \right. \\
& + \underline{\theta}^2(\beta - \gamma)(4\beta^2 - 3\gamma^2)(-1 + \mu)^2 \\
& + \bar{\theta}^2(4\beta^3(5 + (-4 + \mu)\mu) - 4\beta^2\gamma(-2 + \mu)(-1 + \mu) \\
& \left. - \beta\gamma^2(-1 + \mu)(-7 + 3\mu) + 3\gamma^3(-1 + \mu)^2) \right]
\end{aligned} \tag{4.35}$$

When the true demand state is low ($\theta = \underline{\theta}$) is given by:

$$\begin{aligned}
CS_{NI|\underline{\theta}} = \frac{1}{8\beta(2\beta - \gamma)^2(\beta + \gamma)} & \left[8\underline{\theta}^2\beta^3 \right. \\
& + 4\underline{\theta}(-\bar{\theta} + \underline{\theta})\beta(\beta - \gamma)(2\beta + \gamma)\mu \\
& \left. + (\bar{\theta} - \underline{\theta})^2(\beta - \gamma)(4\beta^2 - 3\gamma^2)\mu^2 \right]
\end{aligned} \tag{4.36}$$

Proposition 4.10. *When $\theta = \bar{\theta}$ consumer surplus is higher under no information sharing compared to full information sharing. Conversely, when $\theta = \underline{\theta}$ consumer surplus is higher under full information sharing compared to no information sharing.*

Proof. See Appendix C.1.8 □

The equilibrium prices are a function of the seller's belief μ . For any $0 < \mu < 1$, under no information sharing, the seller sets a lower price when demand is $\bar{\theta}$ and a higher price when demand is $\underline{\theta}$, relative to the full-information sharing benchmark. This in turn forces the retailer to adjust prices in the same direction. Consequently, when demand is $\bar{\theta}$, overall prices are lower under no information sharing, which benefits consumers. When demand is $\underline{\theta}$, overall prices are higher under no information sharing, which harms consumers.

Further, analyzing Proposition 4.5 and Proposition 4.10, we can conclude that the strategic incentives of the platform and the consumer benefit do not always align. Specifically, when demand is $\bar{\theta}$, the platform's strategic incentive lies in sharing information when goods are substitutes, while not sharing information when goods are complements. The consumer's benefit is purely state-dependent. When $\theta = \bar{\theta}$, consumers always benefit when the platform does not share information. On the other hand, when demand is $\underline{\theta}$, the platform's strategic incentive is to share information only when goods are complements, while consumers always benefit when the platform shares information. However, as there is no clear alignment between the

platform and the seller's benefit either, the platform can never change the seller's prior belief (see Theorem 4.9).

To this end, our model shows that an information advantage allows platforms to gain a strategic edge that may harm both third-party sellers and consumers.

4.5 Competition with Commission: $\alpha \in (0, 1]$

Suppose the Platform M charges a commission $\alpha \in (0, 1]$ to host the Third-party Seller S . The Platform operates a vertically integrated retail arm, R . Both R and S sell goods, denoted by r and s respectively, on the platform. Similar to Section 4.4, we analyze two scenarios: (i) Truthful Information Sharing, where M shares the true demand intercept with S , and (ii) No Information Sharing, where M does not share any information with S .

4.5.1 Truthful Information Sharing:

When the Platform M truthfully reveals the demand intercept θ to the Seller, both R and S face demand described in eqs. (4.5) and (4.6), respectively. Since the platform is now fully vertically integrated with the retailer and enjoys a commission from the seller, it chooses the retail price p_r to maximize its joint profit from both the retail arm R and the commission α earned from the seller S , while the seller S chooses the price p_s to maximize its residual profit after paying the commission α . We therefore frame the strategic interaction explicitly as being between the platform and the seller.

Based on the commission rate α , the profit maximization problems of seller S and platform M are, respectively, as follows:

$$\max_{p_s \geq 0} \pi_s(p_s, p_r, \alpha; \theta) = (1 - \alpha)p_s q_s(p_s, p_r; \theta) \quad (4.37)$$

$$\max_{p_r \geq 0} \pi_M(p_r, p_s, \alpha; \theta) = p_r q_r(p_r, p_s; \theta) + \alpha p_s q_s(p_s, p_r; \theta) \quad (4.38)$$

The first order condition with respect to p_s and p_r is respectively as follows:

$$\frac{\partial \pi_s}{\partial p_s} = p_s(a - bp_s + cp_r)(1 - \alpha) = 0 \quad (4.39)$$

$$\frac{\partial \pi_M}{\partial p_r} = a - 2bp_r + (1 + \alpha)p_s = 0 \quad (4.40)$$

Solving the above first-order conditions, we obtain the following equilibrium prices and

resulting profits⁵:

$$p_s^* = \frac{a(2b+c)}{4b^2-(1+\alpha)c^2} \quad (4.41)$$

$$p_r^* = \frac{a(2b+(1+\alpha)c)}{4b^2-(1+\alpha)c^2} \quad (4.42)$$

$$\pi_s^* = \frac{a^2b(2b+c)^2(1-\alpha)}{(4b^2-c^2(1+\alpha))^2} \quad (4.43)$$

$$\pi_M^* = \frac{a^2(2b+(1+\alpha)c)(b(2b+c)-c(b+c)\alpha)}{(4b^2-c(1+\alpha))^2} + \alpha \left(\frac{a^2b(2b+c)^2}{(4b^2-c(1+\alpha))^2} \right) \quad (4.44)$$

Substituting $a = \frac{\theta}{\beta+\gamma}$, $b = \frac{\beta}{\beta^2-\gamma^2}$, and $c = \frac{\gamma}{\beta^2-\gamma^2}$ yields:

$$p_s^* = \frac{\theta(\beta-\gamma)(2\beta+\gamma)}{(4\beta^2-(1+\alpha)\gamma^2)} \quad (4.45)$$

$$p_r^* = \frac{\theta(\beta-\gamma)(2\beta+(1+\alpha)\gamma)}{(4\beta^2-(1+\alpha)\gamma^2)} \quad (4.46)$$

$$\pi_s^* = \frac{\theta^2\beta(\beta-\gamma)(2\beta+\gamma)^2(1-\alpha)}{(\beta+\gamma)(4\beta^2-(1+\alpha)\gamma^2)^2} \quad (4.47)$$

$$\pi_M^* = \frac{\theta^2(\beta-\gamma)(2\beta+(1+\alpha)\gamma)(2\beta^2-\alpha\gamma^2+\beta(1-\gamma)\alpha)}{(\beta+\gamma)(4\beta^2-(1+\alpha)\gamma^2)^2} + \alpha \left(\frac{\theta^2\beta(\beta-\gamma)(2\beta+\gamma)^2}{(\beta+\gamma)(4\beta^2-(1+\alpha)\gamma^2)^2} \right) \quad (4.48)$$

Note, to ensure positive prices and well-defined demands, All interior equilibria require the parameter restriction $4\beta^2 - (1 + \alpha)\gamma^2 > 0$.

Lemma 4.11. *The seller's equilibrium price p_s^* is strictly increasing in the commission rate α . The retailer's equilibrium price p_r^* increases in α when goods are substitutes ($0 < \frac{\gamma}{\beta} < 1$), and decreases in α when goods are complements ($-1 < \frac{\gamma}{\beta} < 0$).*

Proof. See Appendix C.2.1 □

From the above, equilibrium prices increase in α when goods are substitutes ,i.e., $\frac{\gamma}{\beta} \rightarrow 1$. Introducing the commission α , the commission allows the platform to extract rents from the seller, which induces the seller to raise the price of good s to offset the commission cost. The platform, internalizing a share of the seller's revenue, then optimally raises the retail price p_r as well.

When goods are perfect substitutes i.e., $\frac{\gamma}{\beta} = 1$, equilibrium prices converge to marginal cost (zero in our model). Hence, as price sensitivity increases (i.e. $\gamma/\beta \rightarrow 1$), the effect of α

⁵Note Platform's M 's profit is the sum of profit from retail arm R and commission α from revenue of seller S .

on equilibrium prices diminishes. In the extreme case of perfect substitutes ($\gamma/\beta = 1$), any attempt by the platform to raise market prices via the commission is thwarted by intense price competition. Seller must keep price low to retain consumers, which limits the platform's ability to extract additional revenue through α .

By contrast, for complements, $\frac{\gamma}{\beta} \rightarrow -1$, the effects of α are asymmetric. The seller's price p_s^* increases in α , while the platform's retail price p_r^* decreases in α . With complements an increase in p_s reduces demand for both goods, so the platform reduces p_r to sustain combine demand of both goods. In the perfect-complements case ($\gamma/\beta \rightarrow -1$) the prices are

$$p_s^* = \frac{2\theta}{3-\alpha},$$

$$p_r^* = \frac{2\theta(1-\alpha)}{3-\alpha},$$

and the total price of both goods is as follows

$$B(\alpha) \equiv p_s^* + p_r^* = \frac{2\theta(2-\alpha)}{3-\alpha}$$

Further the combined prices fall in α

$$\frac{dB}{d\alpha} = -\frac{2\theta}{(3-\alpha)^2} < 0,$$

Thus, partial integration via α shifts pricing incentives such that prices are lower (with α then without) when goods are perfect complements.

4.5.2 No Information Sharing

We now consider the scenario in which the platform M chooses not to share information with the seller S about the demand intercept θ . Specifically, as in Section 4.4.2, the seller holds a prior belief that the true demand intercept is $\bar{\theta}$ with probability μ and $\underline{\theta}$ with probability $1-\mu$. In contrast to the pure competition case, the platform now charges a commission α to the seller for providing the platform.

As described in eq. (4.13), the third-party seller S maximizes expected profit given her prior belief μ , now net of the commission α . That is, the seller internalizes only a fraction $1-\alpha$ of the revenue.

Formally, the third-party seller chooses p_s to maximize expected profit, taking into account

her belief over demand and anticipating R 's price conditional on each possible state θ_k :

$$\begin{aligned}\max_{p_s \geq 0} \mathbb{E}[\pi_s(p_s, p_r, \alpha; \theta)] &= \max_{p_s \geq 0} (1 - \alpha) p_s \mathbb{E}[q_s(p_s, p_r; \theta)] \\ &= \max_{p_s \geq 0} (1 - \alpha) p_s \left(\mu q_s(p_s, p_r | \bar{\theta}; \bar{\theta}) + (1 - \mu) q_s(p_s, p_r | \underline{\theta}; \underline{\theta}) \right),\end{aligned}\quad (4.49)$$

where $q_s(p_s, p_r | \bar{\theta}; \bar{\theta})$ and $q_s(p_s, p_r | \underline{\theta}; \underline{\theta})$ are the same as described in eq. (4.14) and (4.15) respectively.

The first-order conditions with respect to p_s is as follows:

$$\frac{\partial \mathbb{E}[\pi_s]}{\partial p_s} = 0 \quad \Rightarrow \quad (1 - \alpha)(\mu(\bar{a} + cp_{r|\bar{\theta}}) + (1 - \mu)(\underline{a} + cp_{r|\underline{\theta}}) - 2bp_s) = 0. \quad (4.50)$$

The integrated retailer R observes the true demand intercept θ_k and chooses price conditional on the realized state. Therefore, the maximization problem of the platform M , given his joint profit is as follows:

$$\max_{p_r \geq 0} \pi_{M|k}(p_r, p_s(\mu), \alpha; \theta_k) = \max_{p_r \geq 0} \pi_{r|k}(p_r, p_s(\mu); \theta_k) + \max_{p_k \geq 0} \alpha \mathbb{E}[\pi_s(p_s, p_r, \alpha; \theta)] \quad (4.51)$$

where $\max_{p_r \geq 0} \pi_{r|k}(p_r, p_s(\mu); \theta_k)$ is same as described in eq. (4.17).

The first-order condition with respect to p_r is:

$$\frac{\partial \pi_{M|k}}{\partial p_r | k} = 0 \quad \Rightarrow \quad a_k - 2bp_{r|k} + cp_s(\mu) + (1 + \alpha)cp_s(\mu) = 0. \quad (4.52)$$

Below let $E[\theta] = \mu\bar{\theta} + (1 - \mu)\underline{\theta}$. Solving (4.50) and (4.52), the equilibrium prices after substituting $\bar{a} = \frac{\bar{\theta}}{\beta + \gamma}$, $\underline{a} = \frac{\underline{\theta}}{\beta + \gamma}$, $b = \frac{\beta}{\beta^2 - \gamma^2}$, $c = \frac{\gamma}{\beta^2 - \gamma^2}$. are as follows:

$$p_s(\mu)^* = \frac{(\beta - \gamma)(2\beta + \gamma)\mathbb{E}[\theta]}{4\beta^2 - (1 + \alpha)\gamma^2} \quad (4.53)$$

$$p_{r|\bar{\theta}}^* = \frac{(\beta - \gamma)[4\beta^2\bar{\theta} - (1 + \alpha)\gamma^2(\bar{\theta} - \underline{\theta})(1 - \mu) + 2(1 + \alpha)\beta\gamma\mathbb{E}[\theta]]}{8\beta^3 - 2(1 + \alpha)\beta\gamma^2} \quad (4.54)$$

$$p_{r|\underline{\theta}}^* = \frac{2\beta(\beta - \gamma)(2\beta + \gamma + \alpha\gamma)\underline{\theta} + (1 + \alpha)\gamma(\beta - \gamma)(2\beta + \gamma)(\bar{\theta} - \underline{\theta})\mu}{8\beta^3 - 2(1 + \alpha)\beta\gamma^2} \quad (4.55)$$

Given the above equilibrium prices below is the realized revenue \mathcal{R} of R and S in each state.

$$\mathcal{R}_{s|\bar{\theta}} = \frac{(\gamma - \beta)(2\beta + \gamma)^2 \mathbb{E}[\theta] (\bar{\theta} [(1 + \alpha)\gamma^2(1 - \mu) - 2\beta^2(2 - \mu)] + \underline{\theta}(1 - \mu) [2\beta^2 - (1 + \alpha)\gamma^2])}{2\beta(\beta + \gamma)((1 + \alpha)\gamma^2 - 4\beta^2)^2} \quad (4.56)$$

$$\mathcal{R}_{s|\underline{\theta}} = \frac{(\beta - \gamma)(2\beta + \gamma)^2 \mathbb{E}[\theta] ((1 + \alpha)\gamma^2(\bar{\theta} - \underline{\theta})\mu + 2\beta^2(\underline{\theta}(1 + \mu) - \bar{\theta}\mu))}{2\beta(\beta + \gamma)(4\beta^2 - (1 + \alpha)\gamma^2)^2} \quad (4.57)$$

$$\mathcal{R}_{r|\bar{\theta}} = \frac{(\beta - \gamma) \times [4\beta^2\bar{\theta} - (1 + \alpha)\gamma^2(\bar{\theta} - \underline{\theta})(1 - \mu) + 2(1 + \alpha)\beta\gamma\mathbb{E}[\theta]] \times [4\beta^2\bar{\theta} - 2(\alpha - 1)\beta\gamma\mathbb{E}[\theta] - \gamma^2((1 + \alpha)\bar{\theta} + (\alpha - 1)\underline{\theta} + (\alpha - 1)\mu(\bar{\theta} - \underline{\theta}))]}{4\beta(\beta + \gamma)(4\beta^2 - (1 + \alpha)\gamma^2)^2} \quad (4.58)$$

$$\mathcal{R}_{r|\underline{\theta}} = \frac{(\beta - \gamma) \times [2\beta(2\beta + (1 + \alpha)\gamma)\underline{\theta} + (1 + \alpha)\gamma(2\beta + \gamma)(\bar{\theta} - \underline{\theta})\mu] \times [2(2\beta^2 + \beta\gamma - \alpha\gamma(\beta + \gamma))\underline{\theta} + (1 - \alpha)\gamma(2\beta + \gamma)(\bar{\theta} - \underline{\theta})\mu]}{4\beta(\beta + \gamma)(4\beta^2 - (1 + \alpha)\gamma^2)^2} \quad (4.59)$$

The realized profits are as follows:

$$\pi(\alpha)_{s|\bar{\theta}}^* = (1 - \alpha)\mathcal{R}_{s|\bar{\theta}} \quad (4.60)$$

$$\pi(\alpha)_{s|\underline{\theta}}^* = (1 - \alpha)\mathcal{R}_{s|\underline{\theta}} \quad (4.61)$$

$$\pi(\alpha)_{M|\bar{\theta}}^* = \mathcal{R}_{r|\bar{\theta}} + \alpha \mathcal{R}_{s|\bar{\theta}} \quad (4.62)$$

$$\pi(\alpha)_{M|\underline{\theta}}^* = \mathcal{R}_{r|\underline{\theta}} + \alpha \mathcal{R}_{s|\underline{\theta}} \quad (4.63)$$

Under Assumption 4.2, all equilibrium prices increase in μ when goods are substitutes. Conversely, the platform reduces its retail price as μ increases when goods are complements. Moreover, Lemma 4.11 continues to hold: the seller always raises the price of good s as the commission α increases, while the platform raises the retail price p_r in α only when the goods are substitutes.

Further, Proposition 4.12 shows that the seller is harmed by information asymmetry when paying the commission α to the platform if the true demand intercept is $\bar{\theta}$. Here, as when $\alpha = 0$, the lack of information prevents the seller from optimally setting the price this reduces the seller's net profit. On the other hand, the seller benefits from information asymmetry when the true demand intercept is $\underline{\theta}$, as the expected demand is higher than the realized low demand,

and the commission α partially cushions the impact.

Proposition 4.12. *Given assumption 4.2 holds, the seller's benefit from information asymmetry is dependent on the true state of demand, θ as follows:*

1. *When $\theta = \bar{\theta}$, the seller never benefits from information asymmetry.*
2. *When $\theta = \underline{\theta}$, information asymmetry benefits the seller.*

Proof. See Appendix C.2.2 □

The result described in Proposition 4.12 coincides with the outcome of Proposition 4.8, indicating that the commission α does not alter the seller's benefit or loss from information asymmetry. In other words, while α affects the seller's absolute profit, it does not change the qualitative effect of information asymmetry on the seller's incentives.

Conversely, Proposition 4.13 below shows that Platform benefits by sharing information about the true demand intercept when goods are substitutes and demand state is $\bar{\theta}$ and when goods are complements and demand state is $\underline{\theta}$.

Proposition 4.13. *The platform's strategic incentive to share information depends on the nature of the competing goods $\forall \alpha \in (0, 1]$.*

1. *When goods are substitutes, i.e $0 < \frac{\gamma}{\beta} < 1$, platform benefits from sharing information if and only if $\theta = \bar{\theta}$*
2. *When goods are complements, i.e $-1 < \frac{\gamma}{\beta} < 0$, platform benefits from sharing information if and only if $\theta = \underline{\theta}$*

Proof. See Appendix C.2.3 □

The strategic incentive of the platform, to share information, when $\alpha \in (0, 1]$ described in proposition 4.13 coincides with proposition 4.5. In other words, while the commission α affects the platform's profits, it does not change the qualitative conditions under which the platform benefits from revealing the true demand intercept. As the commission rate α approaches 1, the third-party seller effectively becomes a revenue-generating arm of the platform. In this limit, the platform extracts the seller's full revenue, effectively internalizing all cross-price externalities. While this represents a state of total vertical integration, the strategic incentives for information sharing do not shift. The platform's desire to manipulate the seller's price—raising it for substitutes to protect retail margins, or lowering it for complements to boost joint demand—remains state-dependent. Consequently, the information provided by the

platform remains 'cheap talk' even at this boundary, and the uninformative equilibrium persists. The proposition 4.13 holds at $\alpha = 1$.

Proposition 4.12 combined with proposition 4.13 are sufficient to state that Theorem 4.9 would still hold. That is, the equilibrium remains uninformative even when $\alpha \in (0, 1]$, since the incentives of the seller and the platform do not align.

4.5.3 Consumer Surplus

Here, we describe the consumer surplus under both information structures when $\alpha \in (0, 1]$. Let $\lambda = \frac{\gamma}{\beta}$.

When there is full information sharing. Consumer surplus is as follows:

$$CS(\theta, \alpha)_{FI} = \frac{\theta^2 \lambda (8 - 4(-2 + \alpha)\lambda + (2 - 6\alpha + \alpha^2)\lambda^2 + (-2 + \alpha)\alpha\lambda^3)}{2\gamma(1 + \lambda)(-4 + (1 + \alpha)\lambda^2)^2} \quad (4.64)$$

The consumer surplus when there is no information sharing and the demand intercept is $\bar{\theta}$:

$$CS(\alpha)_{NI|\bar{\theta}} = -\frac{1}{8\gamma(1 + \lambda)(-4 + (1 + \alpha)\lambda^2)^2} \left[\begin{aligned} &\underline{\theta}^2(-1 + \lambda)(2 + \lambda)^2(4 - (3 - \alpha)(1 + \alpha)\lambda^2)(1 - \mu)^2 \\ &+ \bar{\theta}^2(-5 + 3\lambda)(-4 + (1 + \alpha)\lambda^2)^2 \\ &+ 2(-1 + \lambda)(2 + \lambda)(4 + (3 + \alpha)\lambda)(-4 + (1 + \alpha)\lambda^2)\mu \\ &+ (-1 + \lambda)(2 + \lambda)^2(4 + (-3 + \alpha)(1 + \alpha)\lambda^2)\mu^2 \\ &- 2\bar{\theta}\underline{\theta}(-2 + \lambda + \lambda^2)(-1 + \mu)(-4\lambda(3 + \alpha - \mu) + 8(-2 + \mu) \\ &+ 2(1 + \alpha)\lambda^2(2 + (-3 + \alpha)\mu) + (1 + \alpha)\lambda^3(3 + \alpha + (-3 + \alpha)\mu)) \end{aligned} \right] \quad (4.65)$$

The consumer surplus when there is no information sharing and the demand intercept is $\underline{\theta}$:

$$\begin{aligned}
CS(\alpha)_{NI\bar{\theta}} = & -\frac{1}{8\gamma(1+\lambda)(-4+(1+\alpha)\lambda^2)^2} \left[\lambda \left(-4\bar{\theta}^2(8+2\lambda(4+\lambda)+\alpha\lambda(1+\lambda)(-4+(-2+\alpha)\lambda)) \right. \right. \\
& + 4(\bar{\theta}-\underline{\theta})\underline{\theta}(-2+\lambda+\lambda^2)(-4+\lambda(-4-\lambda+\alpha(-2+\lambda(\alpha+\lambda+\alpha\lambda)))) \Big) \mu \\
& \left. \left. + (\bar{\theta}-\underline{\theta})^2(-1+\lambda)(2+\lambda)^2(4+(-3+\alpha)(1+\alpha)\lambda^2)\mu^2 \right) \right] \quad (4.66)
\end{aligned}$$

Analyzing the above consumer surplus, we can confirm that Proposition 4.10 also holds for $\alpha \in (0, 1]$. That is, consumers benefit from information sharing only when the true demand intercept is $\underline{\theta}$. Proposition 4.14 formalizes this result.

Proposition 4.14. *When $\theta = \bar{\theta}$ consumer surplus is higher under no information sharing compared to full information sharing. Conversely, when $\theta = \underline{\theta}$ consumer surplus is higher under full information sharing compared to no information sharing $\forall \alpha \in (0, 1]$.*

Proof. See Appendix C.2.4 □

Analyzing Proposition 4.13 and 4.14, we can conclude that the strategic incentive of the platform and consumer benefit do not align when platform charges a commission α to the seller.

To this end, our model shows that information asymmetry may provide a strategic advantage to the platform, but the platform cannot alter the overall market outcome by knowing the true demand intercept, as the equilibrium remains uninformative—even when the platform charges a commission to the seller. This follows from the fact that, the incentives of the seller and the platform do not align, even with the introduction of the commission α for hosting the seller.

4.6 Discussion

We analyze the interaction between a platform that operates as a market provider and owns a vertically integrated retail arm that competes with a third-party seller on the same platform. First, we consider a case of pure competition, where the platform's retail arm and the seller compete without any hosting fee. Second, we consider a case where the platform charges a commission, a fixed percentage of the seller's revenue, to host the seller. In both cases, we examine two information-sharing regimes: (i) the platform shares full information about the demand intercept with the seller, and (ii) the platform shares no demand information with the seller.

Our analysis yields several insights into pricing, profit, and consumer welfare. We find that the seller benefits from information asymmetry in low-demand states but is harmed in high-demand states. The platform, by contrast, benefits from information sharing only under specific conditions: when goods are substitutes and demand is high, or when goods are complements and demand is low. This misalignment of incentives does not allow the platform from unilaterally exploiting its informational advantage.

Introducing a commission shifts pricing incentives such that the seller raises its price to offset the commission. The platform's price response depends on the nature of the goods, increasing in commission for substitutes and decreasing for complements. Note, while the commission affects absolute profit levels and market prices, it does not qualitatively change the seller's response to information asymmetry or the platform's incentive to share information. Hence, the uninformative equilibrium persists even when the platform extracts rent from the seller.

Across both scenarios, the strategic incentives of the platform and the seller often diverge, resulting in an uninformative equilibrium in which the platform's knowledge of the true demand intercept does not alter the seller's prior belief. This corresponds to a standard cheap talk situation, where any message sent by the platform is effectively non-verifiable for the seller.

Consumer welfare analysis reinforces these findings. Consumers benefit from information sharing only when the true demand intercept is low, regardless of a commission. Conversely, in high-demand states, information sharing reduces consumer surplus relative to the no-information case. The platform's strategic incentives and consumer welfare are therefore weakly aligned, and the introduction of a commission does not bridge this gap.

Overall, our results highlight the limitations of informational advantage in platform markets, where the platform enjoys the dual role of being a market provider and a competitor. While the platform can adjust prices and extract rents through commissions, the misalignment of incentives between the platform and the seller constrains the ability to manipulate market outcomes. Specifically, these findings have important implications for platform design, commission policies, and the regulation of markets where platforms host competing sellers alongside their own retail arms.

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A Appendices for Chapter 2

A.1 Microfoundations: Stackelberg game with fixed costs

Both firms, I and E face a linear inverse demand function (Bowley-type) such as:⁶

$$p_I(q_I, q_E) = a - b(q_I + \theta q_E) \quad (\text{A.1})$$

$$p_E(q_I, q_E) = a - b(\theta q_I + q_E) \quad (\text{A.2})$$

here, $a > 0$ and $b > 0$ are demand parameters and $\theta \in [0, 1]$ describes the degree of substitutability between the two goods. The payoff of a given firm i is as follows:

$$\text{Max}_{q_i} p_i(q_i, q_{-i})q_i - z_i \quad (\text{A.3})$$

Here, firm i quantity to produce to maximize its profit. We normalize the marginal cost of firm i to zero. Firms face a fixed cost z_i . Further, we assume that $z_I > z_E$.

Monopoly State

In the monopoly state, two scenarios arise: (i) E is born but does not enter or E is not born, (ii) E is born and enters the market.

When E is born and enters the market, I first announces q_I , following which E 's best response offers $q_E(q_I)$ is as follows:

$$q_E = \arg \max_{q_E} \pi_E^{\text{entry}}(q_I, q_E) \quad (\text{A.4})$$

The first order condition to the above problem is as follows:

$$\frac{\partial \pi_E(q_I, q_E)}{\partial q_E} = 0 \Rightarrow a - b(2q_E + \theta q_I) \quad (\text{A.5})$$

$$q_E(q_I) = \frac{a - \theta b q_I}{2b} \quad (\text{A.6})$$

Here, $q_E(q_I)$ describes Firm E 's best response function.

Further, Firm I chooses q_I based on his expected profit as follows:

$$q_I = \arg \max_{q_I} \eta \pi_I^{\text{entry}}(q_I, q_E(q_I)) + (1 - \eta) \pi_I^{\text{no entry}}(q_I) \quad (\text{A.7})$$

The first order condition to the above problem after substituting E 's best response $q_E(q_I)$ is as follows:

⁶This functional form is standard in models of product differentiation and oligopoly (see Spence 1976a; Dixit 1979; Tirole 1988)

$$\frac{\partial E[\pi_I]}{\partial q_I} = 0 \Rightarrow a - \frac{a\theta\eta}{2} + bq_I(\eta\theta^2 - 2) = \quad (\text{A.8})$$

$$q_I^M = \frac{a(2 - \theta\eta)}{2b(2 - \theta^2\eta)} \quad (\text{A.9})$$

Substituting above q_I into $q_E(q_I)$, the optimal quantity of Firm E when he enters is q_E^M as follows:

$$q_E^M = \frac{a(4 - \theta(2 + \theta\eta))}{4b(2 - \theta^2\eta)} \quad (\text{A.10})$$

The payoffs of both Firms when E enters in state M are π_I^M and q_E^M as follows:

$$\pi_I^M = \frac{a^2(2 - \theta\eta)(4 - 2(2 - \theta)\theta + \theta(2 - (4 - \theta)\theta)\eta)}{8b(2 - \theta^2\eta)^2} - z_I \quad (\text{A.11})$$

$$\pi_E^M = \frac{a^2(4 - \theta(2 + \theta\eta))^2}{16b(2 - \theta^2\eta)^2} - z_E - k \quad (\text{A.12})$$

The payoff of Firm I when E does not enter is $\bar{\pi}_I^M$ as follows:

$$\bar{\pi}_I^M = \frac{a^2(2 - \theta\eta)(2 + \theta(1 - 2\theta)\eta)}{4b(2 - \theta^2\eta)^2} - z_I \quad (\text{A.13})$$

A key observation reveals that the optimal quantity choice of Firm I , q_I^M , remains independent of the probability of an entrant born (η) in two extreme scenarios. First, when the goods are perfectly independent ($\theta = 0$), Firm I effectively operates as a de facto monopolist. In the absence of competition, it maximizes its profit by producing the standard monopoly quantity, $\frac{a}{2b}$. In this specific case, Firm I is indifferent to the Entrant's entry decision, as its payoff simplifies to the standard monopoly profit, $\frac{a^2}{4b} - z_I$.

Secondly, when the goods are perfect substitutes ($\theta = 1$), Firm I leverages its Stackelberg first-mover advantage by committing to the higher standard monopoly quantity ($\frac{a}{2b}$). It strategically limits the residual market share for any potential entrant. This behavior is, in fact, the hallmark of a Stackelberg model: in cases of perfect substitutes, the leader's optimal quantity choice is $\frac{a}{2b}$, which then leads the follower to optimally choose the lower quantity $\frac{a}{4b}$. In this scenario, Firm I 's profit is $\frac{a^2}{8b} - z_I$ and Firm E 's profit is $\frac{a^2}{16b} - z_E$.

Further, it follows with imperfect product differentiation ($0 < \theta < 1$), the firms' strategic output decisions are not uniform, but depend on a complex interplay of market conditions.

Lemma A.1. *For any $\theta \in (0, 1)$, Firm I 's optimal quantity q_I^M exhibits a non-monotonic relationship with θ , while Firm E 's optimal quantity q_E^M is monotonically decreasing in θ .*

As Lemma A.1 shows, the incumbent's optimal quantity choice (q_I^M) does not respond uniformly to changes in product differentiation (θ). Specifically, Firm I strategically increases its output when there is moderate product differentiation ($1/2 < \theta \leq 2 - \sqrt{2}$) and the probability of entry is low

($0 < \eta < \frac{-2+4\theta}{\theta^2}$), or irrespective of the probability of entry when there is low product differentiation ($2 - \sqrt{2} < \theta < 1$). Conversely, Firm *I* strategically reduces its output when there is high product differentiation ($0 < \theta \leq 1/2$) or a high probability of entry under moderate product differentiation ($\frac{-2+4\theta}{\theta^2} < \eta < 1$ when $1/2 < \theta < 1$). The entrant's optimal quantity (q_E^M) consistently decreases as product differentiation diminishes (i.e., as θ increases). (See Appendix A.2.1 for proof)

Lemma A.2. *For any $\theta \in (0, 1)$, Firm *I*'s optimal quantity choice q_I^M decreases, while Firm *E*'s optimal quantity choice q_E^M increases, with the probability of entry η .*

Lemma A.2 establishes that the firms' output choices respond in opposite directions to the probability of entry (η). The incumbent's quantity (q_I^M) decreases as the probability of entry (η) increases. Conversely, the entrant's quantity (q_E^M) increases as the probability of entry (η) rises. (See Appendix A.2.1 for proof)

Lemma A.3. *The sensitivity of both Firm *I*'s and Firm *E*'s optimal quantities q_I^M and q_E^M to the probability of entry η is non-monotonically affected by the degree of product differentiation θ .*

As Lemma A.3 shows, the effect of the entry threat on a firm's behavior is not constant, but is fundamentally shaped by the degree of product differentiation. This is shown by the non-monotonic sign of the cross-partial derivatives. For Firm *I*, the sensitivity of its optimal quantity to changes in potential entry is reduced under conditions of moderate ($\frac{1}{2} < \theta \leq \theta^*$ and $0 < \eta < \eta^*(\theta)$) and low product differentiation ($\theta^* < \theta < 1$ and $0 < \eta < 1$). Conversely, this sensitivity is increased under conditions of high ($0 < \theta \leq \frac{1}{2}$ and $0 < \eta < 1$) and moderate differentiation ($\frac{1}{2} < \theta < \theta^*$ and $\eta^*(\theta) < \eta < 1$). For Firm *E*, the sensitivity of its output to the entry threat is increased when there is high product differentiation ($0 < \theta < \frac{2}{3}$ and $0 < \eta < 1$) or a high entry threat under moderate differentiation ($\frac{2}{3} < \theta < \hat{\theta}$ and $\hat{\eta}(\theta) < \eta < 1$). This sensitivity is dampened under conditions of a lower entry threat with moderate differentiation ($\frac{2}{3} < \theta < \hat{\theta}$ and $0 < \eta < \hat{\eta}(\theta)$) or for very low product differentiation ($\hat{\theta} < \theta < 1$ and $0 < \eta < 1$). (See Appendix A.2.1 for proof)

This comprehensive analysis of Firm *I* and Firm *E*'s optimal quantities and their sensitivities reveals that output decisions are finely tuned to the interplay of product market structure and the evolving probability of entry.

Lastly, for all $0 < \eta < 1$ and $0 < \theta < 1$, $\bar{\pi}_I^M > \underline{\pi}_I^M$. This indicates that Firm *I* is better off when the Entrant does not enter or is not born. Moreover, Firm *I*'s payoff when no entry occurs, $\bar{\pi}_I^M$, is less than the standard monopoly profit, $\frac{a^2}{4b} - z_I$, because q_I^M is strategically chosen as a function of the probability of entry η . Furthermore, π_E^M is higher or equal to the standard Stackelberg follower payoff (assuming zero fixed costs for the entrant), whenever product differentiation exists ($\theta < 1$). This comprehensive analysis of Firm *I* and Firm *E*'s optimal quantities and their sensitivities reveals that output decisions are finely tuned to the interplay of product market structure and the evolving probability of entry.

Competitive State

In the competitive state, one of the four possibilities may arise: (i) *I* accommodates and the *E* stays, (ii) *I* accommodates and the *E* exits, (iii) *I* predates and the *E* stays and (iv) *I* predates and the *E* exits.

Firm *I* Accommodates

Suppose Firm *I* chooses to accommodate (and Firm *E* subsequently stays), then Firm *E*'s best response quantity, $q_E(q_I)$, is derived from the solution to its profit maximization problem, as described in eq. (A.4) and derived in eq. (A.6). Further, let \bar{q}_I^E below denote the quantity that Firm *I* can offer such that the best response of Firm *E*, that is, $q_E(q_I)$ equals zero. Alternatively, when $q_I \geq \bar{q}_I^E$, Firm *E* does not have an incentive to produce.

$$q_E(q_I) = \frac{a - \theta b q_I}{2b} = 0 \quad (\text{A.14})$$

$$\Rightarrow \bar{q}_I^E = \frac{a}{\theta b} \quad (\text{A.15})$$

Firm *I* to accommodate chooses q_I such that *E* has an incentive to produce as follows:

$$q_I = \arg \max_{q_I} \pi_I(q_I, q_E(q_I)) \quad (\text{A.16})$$

$$s.t. \ q_I < \bar{q}_I^E \quad (\text{A.17})$$

The First order condition to the above problem is as follows:

$$\frac{\partial \pi_I(q_I, q_E(q_I))}{\partial q_I} = a - b q_I \left(1 - \frac{\theta^2}{2}\right) - b \left(q_I + \frac{\theta(a - b q_I \theta)}{2b}\right) = 0 \quad (\text{A.18})$$

$$q_I^A = \frac{a(2 - \theta)}{2b(2 - \theta^2)} \quad (\text{A.19})$$

As observed with q_I^M , the optimal q_I^A is equal to the standard monopoly output ($\frac{a}{2b}$) in two extreme cases: when the goods are perfectly independent ($\theta = 0$) or perfect substitutes ($\theta = 1$). For intermediate levels of product differentiation, q_I^A displays a non-monotonic relationship with θ . Specifically, the optimal q_I^A first decreases (reaching a minimum at approximately $\theta = 0.58$) and then increases for higher values of θ ⁷. This means Firm *I* strategically contracts quantity when product differentiation is higher ($0 < \theta < \underline{\theta}$) and expands quantity when product differentiation is lower ($\underline{\theta} < \theta < 1$).

Substituting above q_I^A into $q_E(q_I)$, the optimal q_E^A when *I* accommodates and *E* stays is as follows:

$$q_E^A = \frac{a(4 - \theta(2 + \theta))}{4b(2 - \theta^2)}$$

⁷ $\frac{\partial q_I^A}{\partial \theta} = \frac{a(-2 - \theta(\theta - 4))}{2b(-2 + \theta^2)^2}$. Since $a/2b > 0$ and the denominator is always positive, the derivative's sign changes when $(-2 - \theta(\theta - 4)) = 0$, which occurs at $\theta = 2 \pm \sqrt{2}$. Given the relevant range of $\theta \in [0, 1]$, $\frac{\partial q_I^A}{\partial \theta} < 0$ for $\theta \in [0, \underline{\theta} \approx 0.589)$ and $\frac{\partial q_I^A}{\partial \theta} > 0$ for $\theta \in (\underline{\theta}, 1]$.

The payoffs of both Firms when I accommodates and E stays are as follows:

$$\underline{\pi}_I^A = \frac{a^2(2-\theta)^2}{8b(2-\theta^2)} - z_I \quad (\text{A.20})$$

$$\pi_E^A = \frac{a^2(4-\theta(2+\theta))^2}{16b(2-\theta^2)^2} - z_E \quad (\text{A.21})$$

Suppose, when I accommodate and E exits the market, I offers the above q_I^A and I 's payoff is as follows:

$$\bar{\pi}_I^A = \frac{a^2(4-\theta^2(5-2\theta))}{4b(2-\theta^2)^2} - z_I \quad (\text{A.22})$$

Firm I Predates

Suppose Firm I decides to predate. To predate such that E cannot produce, I has to choose q_I such that $q_E(q_I)$ is equal to zero (maximum predatory behavior), as described above, this is achieved at \bar{q}_I^E . Alternatively, I can expand output to some q_I such that the profit of the entrant is negative (minimum predatory behavior). This condition is sufficient (and cheaper) for I to ensure E exits.⁸

If I wants to strategically predate, I would choose the lowest possible q_I^P for which Firm E 's profit is negative. Therefore, if I wants to strategically predate, I would choose q_I^P such that the profit of the entrant is negative, i.e, $\pi_E(q_I^P, q_E(q_I^P)) < 0$. Analyzing this constraint, the feasibility range of $q_I^P(z_E)$ is as follows

$$\frac{a-2\sqrt{bz_E}}{\theta b} < q_I^P < \frac{a+2\sqrt{bz_E}}{\theta b} \quad (\text{A.23})$$

Firm I would choose a quantity just above the lower bound of the above range to predate as it would aim to achieve the predatory objective (forcing E to exit by ensuring $\pi_E < 0$) at the least possible output level. This strategy minimizes the economic sacrifice required for predation, as producing more than necessary to induce exit would further reduce Firm I 's own profit, pushing its output further from its unconstrained optimal quantity (q_I^A). We denote I 's optimal offer to predate as \underline{q}_I^P below:

$$\underline{q}_I^P > \frac{a-2\sqrt{bz_E}}{\theta b} \quad (\text{A.24})$$

\underline{q}_I^P is positive only if $z_E < \frac{a^2}{4b}$. For $q_I^M < \underline{q}_I^P$, it must be that $\pi_E^M > 0$ and $z_E > 0$.⁹ Further, at $q_I = \frac{a-2\sqrt{bz_E}}{\theta b}$, E breaks even.¹⁰

⁸ I can potentially expand output such that $q_I^P \in (q_I^P(z_E), \bar{q}_I^E)$. At $q_I^P = \bar{q}_I^E$, the best response of E would be zero. Importantly, producing $q_I > \bar{q}_I^E$ to predate would be more expensive for I compared to predating entry by choosing a q_I such that the entrant's profit is negative. Given, the purpose of I is to ensure that E exits, the minimum cost at which I can achieve this is $q_I^P(z_E)$, this justifies our constraint.

⁹Note that \underline{q}_I^P decreases in z_E . At $z_E = 0$, $\underline{q}_I^P = \bar{q}_I^E$, where Firm E 's profit is exactly zero. Hence for our model to be feasible \underline{q}_I^P cannot equal zero.

¹⁰At $q_I = \frac{a-2\sqrt{bz_E}}{\theta b}$, the best response of E is to produce $q_E = \sqrt{\frac{z_I}{b}}$, at which the entrant breaks even.

The payoff of I when I predates and E exits is as follows:

$$\bar{\pi}_I^P = (a - bq_I^P)q_I^P - z_I$$

Suppose I predates but E still decides to stay in the market. This would require that $q_E(q_I) > 0$, i.e., E produces some non-zero quantity, which is his best response to q_I^P . The payoff of I and E when I predates and E stays is as follows:

$$\pi_I^P = (a - b(q_I^P + \theta q_E^P))q_I^P - z_I \quad (\text{A.25})$$

$$\pi_E^P = \left(\frac{a - \theta b q_I^P}{2b} \right)^2 - z_E < 0 \quad (\text{A.26})$$

To this end, we observe that Firm I 's quantity choices represent an aggression spectrum. These choices range from the most aggressive quantity choice \bar{q}_I^E (maximum predatory behavior such that $q_E(q_I) = 0$) to the least aggressive quantity choice q_I^A (where I accommodates E in state C). This spectrum can be understood as: $\bar{q}_I^E > q_I^P > q_I^M > q_I^A$. While q_I^P represents a higher level of aggression, its exact magnitude relative to q_I^M varies with specific parameter values¹¹. Overall, Firm I can strategically navigate from predation to accommodation.

A.2 Proofs

A.2.1 Proof of Lemmas in Section A.1

Proof of Lemma A.1

Proof. $a > 0$, $b > 0$, and $\eta \in (0, 1)$.

The derivative of Firm I 's optimal quantity q_I^M with respect to θ is as follows:

$$\frac{\partial q_I^M}{\partial \theta} = -\frac{a\eta(2 - 4\theta + \theta^2\eta)}{2b(\theta^2\eta - 2)^2} \quad (\text{A.27})$$

From above eq.(A.27) it follows that:

- $\frac{\partial q_I^M}{\partial \theta} < 0$ (q_I^M decreases as θ increases):
 - This holds if $0 < \theta \leq \frac{1}{2}$ for all $0 < \eta < 1$.
 - Alternatively, it holds if $\frac{1}{2} < \theta < 2 - \sqrt{2}$ when $\frac{-2+4\theta}{\theta^2} < \eta < 1$.
- $\frac{\partial q_I^M}{\partial \theta} > 0$ (q_I^M increases as θ increases):
 - This holds if $\frac{1}{2} < \theta < 2 - \sqrt{2}$ when $0 < \eta < \frac{-2+4\theta}{\theta^2}$.
 - Alternatively, it holds if $2 - \sqrt{2} < \theta < 1$ for all $0 < \eta < 1$.

¹¹A high z_E such that $\pi_E^A < 0$ could force exit even before I predates.

As the derivative $\frac{\partial q_I^M}{\partial \theta}$ can be both negative and positive depending on the specific values of θ and η within their defined ranges, q_I^M exhibits a non-monotonic relationship with θ .

The derivative of Firm E 's optimal quantity q_E^M with respect to θ is given by:

$$\frac{\partial q_E^M}{\partial \theta} = -\frac{a(2 - 2\eta\theta + \theta^2\eta)}{2b(\theta^2\eta - 2)^2} \quad (\text{A.28})$$

For all $0 < \theta < 1$ and $0 < \eta < 1$, $\frac{\partial q_E^M}{\partial \theta}$ described in eq.(A.28) is always negative.

This shows that q_E^M exhibits a negative monotonic relationship with θ . \square

Proof of Lemma A.2

Proof. The derivative of Firm I 's optimal quantity q_I^M with respect to η is as follows:

$$\frac{\partial q_I^M}{\partial \eta} = -\frac{a\theta(1 - \theta)}{2b(\theta^2\eta - 2)^2} \quad (\text{A.29})$$

The derivative of Firm E 's optimal quantity q_E^M with respect to η is as follows:

$$\frac{\partial q_E^M}{\partial \eta} = \frac{a\theta(1 - \theta)\theta^2}{2b(\theta^2\eta - 2)^2} \quad (\text{A.30})$$

For all $0 < \theta < 1$ and $0 < \eta < 1$, $\frac{\partial q_I^M}{\partial \eta}$ described in eq.(A.29) is always negative while $\frac{\partial q_E^M}{\partial \eta}$ described in eq.(A.30) is always positive.

This shows that q_I^M decreases while q_E^M increases in η . \square

Proof of Lemma A.3

Proof. The cross derivative of q_I^M w.r.t θ and η is as follows:

$$\frac{\partial q_I^M}{\partial \theta \partial \eta} = \frac{a(2 - 4\theta + 3\theta^2\eta - 2\theta^3\eta)}{b(-2 + \theta^2\eta)^3} \quad (\text{A.31})$$

From above eq.(A.31) it follows that:

- $\frac{\partial q_I^M}{\partial \theta \partial \eta} > 0$ when
 - $\frac{1}{2} < \theta \leq \theta^*$ and $0 < \eta < n^*(\theta)$
 - $\theta^* < \theta < 1$ and $0 < \eta < 1$
- $\frac{\partial q_I^M}{\partial \theta \partial \eta} < 0$ when
 - $0 < \theta \leq \frac{1}{2}$ and $0 < \eta < 1$
 - $\frac{1}{2} < \theta < \theta^*$ and $\eta^*(\theta) < \eta < 1$

Here, $\eta^*(\theta) = \frac{4\theta - 2}{\theta^2(3 - 2\theta)}$ and $\theta^* \approx 0.694$ is the value of θ for which $\eta^*(\theta) = 1$

The cross derivative of q_E^M w.r.t θ and η is as follows:

$$\frac{\partial q_E^M}{\partial \theta \partial \eta} = \frac{a\theta(-4 + 6\theta - 2\theta^2\eta + \theta^3\eta)}{b(-2 + \theta^2\eta)^3} \quad (\text{A.32})$$

From above eq.(A.32) it follows that:

- $\frac{\partial q_E^M}{\partial \theta \partial \eta} > 0$ when,
 - $0 < \theta \leq \frac{2}{3}$ and $0 < \eta < 1$
 - $\frac{2}{3} < \theta < \hat{\theta}$ and $\hat{\eta} < \eta < 1$
- $\frac{\partial q_E^M}{\partial \theta \partial \eta} < 0$ when,
 - $\frac{2}{3} < \theta < \hat{\theta}$ and $0 < \eta < \hat{\eta}$
 - $\hat{\theta} < \theta < 1$ and $0 < \eta < 1$

Here, $\hat{\eta}(\theta) = \frac{4\theta-2}{\theta^2(3-2\theta)}$ $\hat{\theta} \approx 0.749$ is the value of θ for which $\hat{\eta}(\theta) = 1$. □

A.2.2 Proof of Proposition 2.1

Proof. Accommodation: Suppose I always accommodates.

E will always enter in state M and stay in state C if the following holds:

$$\pi_E^M - k + \frac{\delta \pi_E^A}{1 - \delta(1 - \gamma)} \geq 0 \quad (\text{A.33})$$

$$\frac{\pi_E^A}{1 - \delta(1 - \gamma)} \geq 0 \quad (\text{A.34})$$

Let V_M^A denote the value function of I in state M :

$$V_M^A = \eta(\underline{\pi}_I^M + \delta V_C^A) + (1 - \eta)(\bar{\pi}_I^M + \delta V_M^A) \quad (\text{A.35})$$

Let V_C^A denote the value function of I in state C when he always accommodates:

$$V_C^A = \underline{\pi}_I^A + \delta((1 - \gamma)V_C^A + \gamma V_M^A) \quad (\text{A.36})$$

Solving the system of equations (A.35)-(A.36) yields:

$$V_M^A = \frac{\underline{\pi}_I^M(1 - (1 - \gamma)\delta)\eta + \underline{\pi}_I^A\delta\eta + \bar{\pi}_I^M(1 - (1 - \gamma)\delta)(1 - \eta)}{(1 - \delta)(1 - \delta(1 - \gamma - \eta))} \quad (\text{A.37})$$

$$V_C^A = \frac{(1 - \delta(1 - \eta))\underline{\pi}_I^A + \delta\gamma((1 - \eta)\bar{\pi}_I^M + \eta\underline{\pi}_I^M)}{(1 - \delta)(1 - \delta(1 - \gamma - \eta))} \quad (\text{A.38})$$

It remains to verify that I does not have a profitable deviation in the competitive state. In the spirit of one-shot-deviation principle, suppose that I predates in the competitive state and then the play returns to the conjectured equilibrium play. After I predates, E optimally stays in the market if

$$\pi_E^P + \frac{\delta(1-\gamma)\pi_E^A}{1-\delta(1-\gamma)} \geq 0 \quad (\text{A.39})$$

Therefore, if condition (2.4) holds, E will stay in the market after I predates. In this case, predation is not profitable: it does not exclude E in the long term and reduces profit from $\underline{\pi}_I^A$ to $\underline{\pi}_I^P$ in the period of predation.

If condition (2.4) does not hold, E exits if I predates. Then, I does not benefit from predating if the following inequality holds

$$\underbrace{\bar{\pi}_I^P + \delta V_M^A}_{\text{Value from deviating to predation}} - \underbrace{\left(\underline{\pi}_I^A + \delta \left((1-\gamma)V_C^A + \gamma V_M^A \right)\right)}_{\text{Value on the equilibrium path}} \leq 0. \quad (\text{A.40})$$

Substituting V_M^A and V_C^A above, predation is not profitable if

$$\bar{\pi}_I^P - \underline{\pi}_I^A + \delta(1-\gamma) \frac{(1-\eta)(\bar{\pi}_I^M - \underline{\pi}_I^A) + \eta(\underline{\pi}_I^M - \underline{\pi}_I^A)}{1-\delta(1-\eta-\gamma)} \leq 0 \quad (\text{A.41})$$

Using definition of λ from (2.1), inequality (A.41) can be rewritten as

$$\underbrace{\left((1-\eta)(\bar{\pi}_I^M - \underline{\pi}_I^A) + \eta(\underline{\pi}_I^M - \underline{\pi}_I^A) \right)}_{>0} \left(\frac{\delta(1-\gamma)}{1-\delta(1-\eta-\gamma)} - \lambda \right) \leq 0. \quad (\text{A.42})$$

Hence, I does not have a profitable deviation to predation if condition (2.5) holds.

Denote the right hand side of inequality (2.5) by f :

$$f = \frac{\delta(1-\gamma)}{1-\delta(1-\eta-\gamma)}. \quad (\text{A.43})$$

The left hand side of (2.5) is independent of γ , while the right hand side is decreasing in γ :

$$\frac{\partial f}{\partial \gamma} = -\frac{\delta(1+\delta\eta)}{(1-\delta(1-\eta-\gamma))^2} < 0. \quad (\text{A.44})$$

Hence, everything else equal, accommodation equilibrium is easier to sustain when the exogenous probability of entrant's exit γ is higher.

The right hand side of (2.5) is decreasing in η :

$$\frac{\partial f}{\partial \eta} = -\frac{\delta^2(1-\gamma)}{(1-\delta(1-\eta-\gamma))^2} < 0. \quad (\text{A.45})$$

The derivative of the left hand side of (2.5) with respect to η is

$$\frac{\partial \lambda}{\partial \eta} = \frac{(\underline{\pi}_I^A - \bar{\pi}_I^P) \left(\bar{\pi}_I^M - \underline{\pi}_I^M - (1 - \eta) \frac{\partial \bar{\pi}_I^M}{\partial \eta} - \eta \frac{\partial \underline{\pi}_I^M}{\partial \eta} \right)}{((1 - \eta)(\bar{\pi}_I^M - \underline{\pi}_I^A) + \eta(\underline{\pi}_I^M - \underline{\pi}_I^A))^2}, \quad (\text{A.46})$$

where $\underline{\pi}_I^A - \bar{\pi}_I^P > 0$ and $\bar{\pi}_I^M - \underline{\pi}_I^M - (1 - \eta) \frac{\partial \bar{\pi}_I^M}{\partial \eta} - \eta \frac{\partial \underline{\pi}_I^M}{\partial \eta}$

$$\begin{aligned} & \bar{\pi}_I^M - \underline{\pi}_I^M - (1 - \eta) \frac{\partial \bar{\pi}_I^M}{\partial \eta} - \eta \frac{\partial \underline{\pi}_I^M}{\partial \eta} = \\ & \frac{-a^2}{8b(2 - \eta\theta^2)^3} \left[-6\eta^2\theta^3(4 + \theta) + \eta^4\theta^4(2 - \theta(4 - \theta)) \right. \\ & \quad - 8(4 - \theta(2 - \theta)) - \eta^3\theta^2(4 - \theta(8 - \theta(2 - \theta(8 - \theta)))) \\ & \quad \left. - 4\eta(-4 + \theta(4 - \theta(10 + \theta(2 - \theta)))) \right] > 0 \end{aligned}$$

for all $1 \geq \theta \geq 0$ and $1 \geq \eta \geq 0$. Hence, $\frac{\partial \lambda}{\partial \eta} > 0$ and so everything else equal, accommodation equilibrium is easier to sustain when the probability of entrant's birth η is higher.

Predation: Suppose I always predates in state C . If I predates in the competitive state, existing E exits as $\pi_E^P < 0$. In the monopoly state, a newborn E enters and stays in the market for one period if $\pi_E^M - k \geq 0$.

Let V_M^P denote the value function of I in state M and V_C^P denote the value function of I in state C when I always predates:

$$V_M^P = \eta(\underline{\pi}_I^M + \delta V_C^P) + (1 - \eta)(\bar{\pi}_I^M + \delta V_M^P), \quad (\text{A.47})$$

$$V_C^P = \bar{\pi}_I^P + \delta V_M^P. \quad (\text{A.48})$$

Solving the above

$$V_M^P = \frac{\bar{\pi}_I^M(1 - \eta) + (\underline{\pi}_I^M + \bar{\pi}_I^P\delta)\eta}{(1 - \delta)(1 + \delta\eta)} \quad (\text{A.49})$$

$$V_C^P = \frac{\bar{\pi}_I^P(1 - (1 - \eta)\delta) + (1 - \eta)\delta\bar{\pi}_I^M + \eta\delta\underline{\pi}_I^M}{(1 - \delta)(1 + \delta\eta)} \quad (\text{A.50})$$

It remains to verify that I does not have a profitable deviation in the competitive state. If I deviates to accommodation in a given period, then E stays in the market for that period as $\pi_E^A > 0$, but exits when I reverts to predation as $\pi_E^P < 0$.

I does not have a profitable deviation if the following holds:

$$\underbrace{\pi_I^A + \delta((1 - \gamma)V_C^P + \gamma V_M^P)}_{\text{Value from deviating to accommodation}} \leq \underbrace{\bar{\pi}_I^P + \delta V_M^P}_{\text{Value on the equilibrium path}} \quad (\text{A.51})$$

Substituting V_M^P and V_C^P yields

$$\underline{\pi}_I^A - \bar{\pi}_I^P + \delta(1-\gamma) \frac{\bar{\pi}_I^P - (1-\eta)\bar{\pi}_I^M - \eta\underline{\pi}_I^M}{1+\delta\eta} \leq 0, \quad (\text{A.52})$$

which, using definition of λ from (2.1), can be rewritten as

$$\underbrace{\left((1-\eta)(\bar{\pi}_I^M - \underline{\pi}_I^A) + \eta(\underline{\pi}_I^M - \underline{\pi}_I^A) \right)}_{>0} \underbrace{\frac{1-\delta(1-\eta-\gamma)}{1+\delta\gamma}}_{>0} \left(\lambda - \frac{\delta(1-\gamma)}{1-\delta(1-\eta-\gamma)} \right) \leq 0. \quad (\text{A.53})$$

Hence, I does not have a profitable deviation to accommodation if and only if condition (2.7) holds.

Monopolization: Suppose E does not enter in state M and I always predates in state C . E would not enter in the monopoly state if $\pi_E^M \leq k$.

Let V_M^M denote the value function of I in state M and V_C^M denote the value function of I in state C when E does not enter.

$$V_M^M = \bar{\pi}_I^M + \delta V_M^M \quad (\text{A.54})$$

$$V_C^M = \bar{\pi}_I^P + \delta V_M^M \quad (\text{A.55})$$

solving the above

$$V_M^M = \frac{\bar{\pi}_I^M}{1-\delta} \quad (\text{A.56})$$

$$V_C^M = \bar{\pi}_I^P + \frac{\delta \bar{\pi}_I^M}{1-\delta} \quad (\text{A.57})$$

I would not deviate to accommodation in a given period only if the following holds

$$\underbrace{\underline{\pi}_I^A + \delta \left((1-\gamma)V_C^M + \gamma V_M^M \right)}_{\text{Value from deviating to accommodation}} \leq \underbrace{\bar{\pi}_I^P + \delta V_M^M}_{\text{Value on equilibrium path}} \quad (\text{A.58})$$

Substituting V_M^M and V_C^M above

$$\underline{\pi}_I^A - \bar{\pi}_I^P + \delta(1-\gamma) (\bar{\pi}_I^P - \bar{\pi}_I^M) \leq 0, \quad (\text{A.59})$$

which, using definition of $\bar{\lambda}$ from (2.3), can be rewritten as

$$\underbrace{(\bar{\pi}_I^M - \underline{\pi}_I^A)}_{>0} \underbrace{(1-\delta(1-\gamma))}_{>0} \left(\bar{\lambda} - \frac{\delta(1-\gamma)}{1-\delta(1-\gamma)} \right) \leq 0. \quad (\text{A.60})$$

Hence, I does not have a profitable deviation to accommodation if and only if condition (2.9) holds. \square

A.3 Prompt Design

A.3.1 Main Experiment Prompts

SYSTEM PROMPT

You are a strategic economic agent. Your response **MUST** strictly follow the provided template format.

Figure A.1: System Prompt

COMMON_PROMPT_SUFFIX_QUANTITY

My observations and thoughts:

- 1. Observations from this period and history:**
<fill in here>
- 2. Analysis and Interpretation:**
<fill in here>
- 3. Strategic Options and Expected Outcomes:**
<fill in here>
- 4. Decision Rationale:**
<fill in here>

New content for PLANS.txt:
<fill in here>

New content for INSIGHTS.txt:
<fill in here>

My chosen quantity:
<just the number, nothing else>

Figure A.2: COMMON_PROMPT_SUFFIX_QUANTITY

COMMON_PROMPT_SUFFIX_BINARY

My observations and thoughts:

1. Observations from this period and history:

<fill in here>

2. Strategic Analysis:

<fill in here>

3. Decision Rationale:

<fill in here>

New content for PLANS.txt:

<fill in here>

New content for INSIGHTS.txt:

<fill in here>

My decision:

<ONLY 'ENTER' or 'NOT ENTER' (for entry decision) or 'STAY' or 'EXIT', nothing else>

Figure A.3: COMMON_PROMPT_SUFFIX_BINARY

PROMPT_INCUMBENT_QUANTITY

You are the incumbent firm, always present and never exiting.

The market can be in Monopoly (M) or Competitive (C) state. Entrants may appear in Monopoly state with probability $\{\eta\}$ and decide to enter or not.

Entrants in competitive state may stay or exit voluntarily; they may also exit exogenously with probability $\{\gamma\}$.

Market parameters:

- Demand intercept $a = \{a\}$
- Fixed cost = $\{z_I\}$, Marginal cost = $\{\text{MARGINAL_COST}\}$
- Product differentiation $\theta = \{\theta\}$
- Discount factor $\delta = \{\delta\}$

Current state: $\{\text{game_state_description}\}$

$\{\eta_context\}$ $\{\gamma_context\}$

You choose your quantity (q_t) first, and any present or potential competitor will make their choice after you.

Your goal is to maximize your expected long-term discounted profit given $\delta = \{\delta\}$, while considering:

- The trade-off between current profits and future market structure.
- Strategic quantity choices that may influence entrant decisions and strategy.
- Past entrant behavior and your own profitability history.

Past market info: $\{\text{history_context}\}$

Even if a strategy has been effective, periodically explore alternative quantities to test and validate your belief about its optimality.

Task: Choose your quantity q_t to maximize your expected discounted profit.

Use **COMMON_PROMPT_SUFFIX_QUANTITY**.

Figure A.4: PROMPT_INCUMBENT_QUANTITY

PROMPT_ENTRANT_QUANTITY

You are the entrant firm in a repeated market with Monopoly (M) and Competitive (C) states.

The incumbent is always present and never exits.

Market parameters:

- Demand intercept $a = \{a\}$
- Fixed cost = $\{z_E\}$, Marginal cost = $\{MARGINAL_COST\}$
- Product differentiation $\theta = \{\theta\}$
- Discount factor $\delta = \{\delta\}$
- Exogenous exit probability $\gamma = \{\gamma\}$

Current state: $\{\text{game_state_description}\}$

You observe incumbent's quantity $q_I = \{q_I\}$ this period.
 $\{\gamma\}$

Your goal is to maximize your expected long-term discounted profit given $\delta = \{\delta\}$, while considering:

- The trade-off between current profits and future market structure.
- Strategic quantity choices that may influence incumbent's strategy.
- The risk of setting a quantity that causes persistent losses.
- Past incumbent behavior and your own profitability history.

Past info: $\{\text{history_context}\}$

Your plans and insights: PLANS: $\{\text{plans_content}\}$ INSIGHTS: $\{\text{insights_content}\}$

Task: Choose your quantity q_E .

Use **COMMON_PROMPT_SUFFIX_QUANTITY**.

Figure A.5: PROMPT_ENTRANT_QUANTITY

PROMPT_ENTRANT_BINARY_DECISION

You are the entrant firm in a repeated market with Monopoly (M) and Competitive (C) states.

In Monopoly, you may be born with probability $\{\eta\}$ and decide to ENTER or NOT ENTER after seeing incumbent's quantity q_I .

In Competition, you decide to STAY or EXIT after seeing incumbent's quantity q_I .

Market parameters:

- Demand intercept $a = \{a\}$
- Fixed cost = $\{z_E\}$, Marginal cost = $\{MARGINAL_COST\}$
- One-time entry cost $k = \{k\}$
- Product differentiation $\theta = \{\theta\}$
- Discount factor $\delta = \{\delta\}$
- Exogenous exit probability $\gamma = \{\gamma\}$

Current state: $\{\text{game_state_description}\}$

Incumbent quantity observed: $\{q_I\}$ $\{\text{observed_this_period}\}$ $\{\gamma\}$ $\{\text{context}\}$

You must decide ENTER, NOT ENTER, STAY, or EXIT, considering your long-term discounted profit.

Occasionally explore suboptimal choices ($\sim 5\text{--}10\%$) to simulate strategic experimentation.

Your goal is to maximize your expected long-term discounted profit given $\delta = \{\delta\}$, while considering:

- The trade-off between current profits and future market structure.
- Strategic quantity choices that may influence incumbent's strategy.
- The risk of setting a quantity that causes persistent losses.
- Past incumbent behavior and your own profitability history.

Past info: $\{\text{history_context}\}$

Task: Decide your action.

If ENTER or STAY, you will choose a quantity next.

Use **COMMON_PROMPT_SUFFIX_BINARY**.

Figure A.6: PROMPT_ENTRANT_BINARY_DECISION

A.3.2 Representative Run Prompt Reply

A.3.3 Period 1 Incumbent Quantity Reply Log $\eta = 0.5$ and $\theta = 1$

The following are the reply logs from period 1 of the representative agents whose decisions are presented in Figure 2.2 subplot 2.2a and 2.2c.

Incumbent Reply Log $\eta = 0.5$ and $\theta = 1$

Incumbent Reply Log

My observations and thoughts:

1. Observations from this period and history: - Current state: Monopoly; no pre-existing history, so this is the initial period. - Demand: $P = 100 - Q$, where $Q =$ total quantity (only my own if no entry). - Costs: Marginal cost = 0; fixed cost is sunk for me (always incumbent). - Product differentiation $\theta = 1.0$ (symmetric Cournot if entry occurs). - Probability of new entrant: 0.50 this period. - Entrant observes my q_I before deciding. - Discount factor $\delta = 0.95$ (future profits are valuable).

2. Analysis and Interpretation: - Static monopoly $q_M = 50$ maximizes current profit (2500), but attracts entry. - Cournot duopoly equilibrium: each $q^* = 100/3 \approx 33.33$, profit ≈ 1111 per firm. - Aggressive/limit pricing: higher q_I reduces entrant's best-response profit, may deter entry. - Entrant profit function: $q_E^* = \frac{100 - q_I}{3}$, $\pi_E = \frac{2}{9}(100 - q_I)^2$. - Only $q_I \geq 100$ makes $\pi_E \leq 0$, but that gives me zero profit. - Intermediate choice (e.g. $q_I \approx 67$) makes entrant profit small (≈ 246), which may deter entry.

3. Strategic Options and Expected Outcomes: - **Option A: Static monopoly** $q = 50 \rightarrow$ Profit 2500 now, but likely entry. - **Option B: Accommodate** ($q \approx 33.33$) \rightarrow Low current profit, no deterrence. - **Option C: Limit pricing** ($q > 50$) \rightarrow Lower immediate profit, but discourages entry; balance expected discounted value.

4. Decision Rationale: - True monopoly output is unstable under entry risk. - Limit pricing at $q_I \approx 67$ reduces entrant's incentive, preserving monopoly advantage in expectation. - Best initial move is moderately aggressive, then update after observing entry.

New content for PLANS.txt: - Set $q_I = 67$ to approximate limit pricing. - Monitor entrant behavior; adjust strategy if entry occurs. - Experiment with small deviations around 67 in future rounds.

New content for INSIGHTS.txt: - Initial aggression likely deters entry when entrant's margin is slim. - Monopoly profits are high but not sustainable with high entry probability. - Iterative testing around aggressive output is needed for robustness.

My chosen quantity: $q_I = 67$

Figure A.7: Incumbent Quantity Prompt Reply: Period 1, $\eta = 0.5$, $\theta = 1$

Entrant Binary Decision Reply Log $\eta = 0.5$ and $\theta = 1$

Log Entry – Entrant Observations and Analysis

My observations and thoughts:

1. Observations from this period and history: - Current state: Monopoly with possible New Entrant. No prior history (first decision). - Incumbent's observed quantity: $q_I = 67.00$. - Demand: $p = 100 - Q \Rightarrow p = 100 - 67 = 33$. - Costs: Marginal cost = 0; fixed cost = 150; entry cost = 450 (one-time if I ENTER). - If I enter, my share of demand will be small due to incumbent's high q_I . - Exit probability (γ) irrelevant for now. - Discount factor $\delta = 0.95 \rightarrow$ future profits are important. - Incumbent appears aggressive (quantity far above monopoly $q_M = 50$).

2. Strategic Analysis: - Entering requires paying 450 (entry) + 150 (fixed) now. - With $\theta = 1.00$ (little differentiation), competition would be Cournot-like, leaving me with a small q_E . - Short-run profit after entry would likely be negative once fixed + entry costs are considered. - Incumbent's choice of $q_I = 67$ strongly signals entry deterrence. - Skipping entry allows me to avoid loss, but also strengthens incumbent's strategy of maintaining high q_I . - A small probability of experimenting (5–10

3. Decision Rationale: - Current period entry is not profitable: $p = 33$ leaves little demand and small output for me. - After accounting for costs, expected profit is negative. - Optimal strategy is to wait for signals of accommodation (lower q_I). - Decision: NOT ENTER this period.

New content for PLANS.txt: - Track incumbent's output choices in monopoly states. - Look for softer q_I (below 60) as potential entry opportunities. - Avoid entry unless clear signs of accommodation appear.

New content for INSIGHTS.txt: - Incumbent's $q_I = 67$ is above monopoly output, consistent with deterrence. - Entry is unattractive under high entry + fixed costs and low residual demand. - Incumbent will likely persist in this aggressive strategy if entry does not occur.

My decision: NOT ENTER

Figure A.8: Entry Binary Decision Log Reply:Period 1, $\eta = 0.5$, $\theta = 1$

A.3.4 Representative Run Reply Logs $\eta = 1$ and $\theta = 0.5$ period 299

The following are the reply logs from period 299 of the representative agents whose decisions are presented in Figure 2.7 subplot 2.7b and 2.7d.

Incumbent Reply Log $\eta = 1$ and $\theta = 0.5$

Incumbent Reply Log

1. Observations from this period and history: - The market alternates between Monopoly (M) and Competitive (C) states depending on entrant entry/exit. - Monopoly profits (e.g., periods 289, 293, 298 with $Q \approx 54-60$) are substantially higher ($\pi > 2000$ at times). - When an entrant enters (after moderate Q during M), profits fall: both firms produce $\approx 45-55$, profits $\approx 700-1300$. - In period 289, $q_I = 60$ yielded very high monopoly profits; in period 290, $q_I = 62$ led to entrant losses. - Aggressive q_I often deters entry (e.g., periods 289, 293, 298: entrant $Q = 0$), while slightly lower q_I (e.g., 55 in period 294) allows partial entry, lowering profits. - Recent C-state episodes show tit-for-tat/coordinated $Q \approx 46-54$ with stable but lower profits than monopoly.

2. Analysis and Interpretation: - Entry hinges on entrant profitability: high q_I can leave residual demand too small for positive profits (e.g., period 290 entrant losses). - $\delta = 0.95 \Rightarrow$ long-run monopoly maintenance is highly valuable. - Trade-off: overly high Q risks future entrant persistence; overly low Q encourages competitive phases. - Effective past strategy: $q_I = 54-60$ in M states deters entry while yielding high profits. - Periodic exploration is necessary, since entrant learning may shift thresholds.

3. Strategic Options and Expected Outcomes: 1. **Aggressive deter ($q_I \geq 60$):** - Pros: Maximizes monopoly profits; repeated entrant losses deter future entry. - Cons: If entrant parameters change, may still enter at a loss, leading to unstable competition.

2. **Intermediate ($q_I = 54-56$):** - Pros: Stability; reduces risk of strong retaliation. - Cons: Lower monopoly profits; more frequent competition.

3. **Exploration ($q_I = 57-58$):** - Pros: Tests deterrence threshold while maintaining high profit. - Cons: Risk of entry at slightly lower q_I , leading to temporary competition.

4. Decision Rationale: - Last period (M, $q_I = 56$), no entry occurred. - Past aggressive $q_I \approx 60$ reliably discouraged entry. - Current choice: $q_I = 58$ — balances deterrence with exploration, testing if slightly lower output sustains monopoly while maximizing profit. - If entrant enters, new information gained about threshold.

PLANS.txt: - In Monopoly: oscillate $q_I = 56-62$, periodically lowering for exploration. - In Competition: respond with moderate Q (avoid racing to the bottom). - Log outcomes from exploratory Monopoly quantities to refine deterrence threshold.

INSIGHTS.txt: - Aggressive monopoly output deters entry and maximizes long-run profit when paired with threshold testing. - Entrant response highly sensitive to q_I ; a few units shift can flip profits. - Regular testing near deterrence threshold ensures continued dominance.

Chosen quantity: $q_I = 58$

Entrant Reply Log $\eta = 1$ and $\theta = 0.5$

Log Entry – Entrant Observations, Analysis, and Strategy

1. Observations from this period and history: - Initial period; no prior history with incumbent. - Market state: Monopoly. Incumbent submitted $q_I = 58.00$. - Demand: $P = 100 - Q$, where $Q =$ total market quantity. - Costs: fixed cost per period = 150.00; entry cost = 450.00 (sunk if entry occurs); marginal cost = 0.00. - Product differentiation: $\theta = 0.5$ (moderate differentiation, significant competition post-entry). - Discount factor $\delta = 0.95 \Rightarrow$ future profits weigh heavily. - Exogenous exit risk $\gamma = 0.10$ applies only in Competition (not relevant now). - Incumbent's $q_I = 58.00$ is relatively high, suggesting possible entry deterrence.

2. Strategic Analysis: - **If I ENTER:** - Immediate sunk entry cost = 450.00. - Move into Competition (Cournot duopoly, $\theta = 0.5$). - High incumbent output leaves me little residual demand; profit may be negative or very low. - Possibility that incumbent maintains aggressive stance, keeping me unprofitable. - If incumbent softens later, profitability could improve — but uncertain.

- **If I DO NOT ENTER:** - Payoff = 0 profit this period. - Preserve option to enter in future if incumbent reduces q_I . - With $\delta = 0.95$, waiting can be valuable, though risk persists that incumbent keeps deterring entry.

- Exploration: A small probability (5–10)

3. Decision Rationale: - Static calculation: With $q_I = 58.00$, my Cournot best response q_E would yield low or negative profit net of entry cost. - Incumbent's high q_I looks like deliberate entry deterrence. - Optimal strategy: **NOT ENTER** to avoid immediate loss. - Exploration entry is possible in later rounds, but not optimal now.

PLANS.txt: - Defer entry when incumbent produces $q_I > 50$ initially. - Monitor future q_I ; re-evaluate if incumbent accommodates ($q_I < 50$). - Low-probability exploratory entry (5–10- Update profit expectations after observing post-entry dynamics.

INSIGHTS.txt: - Incumbent likely deterring entry with high $q_I = 58.00$. - Entry unprofitable under current conditions due to sunk cost and low residual demand. - Cournot best response under these conditions yields negligible/negative payoff. - Must continue monitoring; future entry viable if incumbent reduces output.

Decision: NOT ENTER

Figure A.10: Entrant Binary Decision Log

A.3.5 Prompt Variation

A.3.6 Advisor Prompt

Advisor prompt: PROMPT_INCUMBENT_QUANTITY

You are a strategic business consultant advising an incumbent firm in a duopoly market. The market can be in a Monopoly (M) or Competitive (C) state. Entrants may appear in a Monopoly state with probability $\{\eta:.2f\}$ and decide to enter or not. Entrants in a competitive state may stay or exit voluntarily; they may also exit exogenously with probability $\{\gamma:.2f\}$.

Market parameters: Demand intercept $a = \{a:.2f\}$, Fixed cost $= z_f : .2f$, Marginal cost = 0, Product differentiation $\theta = \{\theta:.2f\}$, Discount factor $\delta = \{\delta:.2f\}$

Current state: $\{\{game_state_description\}\}$

$\{\{\eta_context\}\}$

$\{\{\gamma_context\}\}$

You choose your quantity (q_t) first, and any present or potential competitor will make their choice after you.

Your client's primary goal is to **maximize their expected long-term discounted profit** given $\delta = \{\delta:.2f\}$, while considering:

- The trade-off between current profits and future market structure.
- Strategic quantity choices that may influence entrant decisions and strategy.
- Past entrant behavior and your own profitability history.

Past market info: $\{\{history_context\}\}$ Even if a strategy has been effective, periodically explore alternative quantities to test and validate your belief about its optimality. Your task is to provide a quantity recommendation for the incumbent firm for this period.

[ANALYSIS]: Based on the current state and historical data, what are the key market dynamics? What is the likely response of a potential or current entrant to different quantity choices?

[RATIONALE]: Explain your reasoning for the chosen quantity. Why is this quantity optimal for long-term profit maximization?

My chosen quantity:

<just the number, nothing else>

Figure A.11: Advisor Prompt: PROMPT_INCUMBENT_QUANTITY

Advisor prompt:PROMPT_ENTRANT_QUANTITY

You are a strategic business consultant advising an entrant firm in a duopoly market with Monopoly (M) and Competitive (C) states. The incumbent is always present and never exits.

Market parameters: Demand intercept $a = \{a:.2f\}$, Fixed cost $= z_I : .2f$, Marginal cost = 0, Product differentiation $\theta = \{\theta:.2f\}$, Discount factor $\delta = \{\delta:.2f\}$

Current state: $\{\{game_state_description\}\}$

$\{\{eta_context\}\}$

$\{\{gamma_context\}\}$

Your client's primary goal is to **maximize their expected long-term discounted profit** given $\theta = \{\delta:.2f\}$, while considering:

- The trade-off between current profits and future market structure.
- Strategic quantity choices that may influence incumbent's strategy.
- The risk of setting a quantity that causes persistent losses.
- Past incumbent behavior and your own profitability history. **Past info:** $\{\{history_context\}\}$

Your plans and insights from last period: PLANS: $\{\{plans_content\}\}$ | INSIGHTS: $\{\{insights_content\}\}$

Your task is to provide a quantity recommendation for the entrant firm for this period.

[ANALYSIS]:

Based on the current state, incumbent's quantity, and historical data, what are the key market dynamics? What is the likely response of the incumbent to different quantity choices?

[RATIONALE]:

Explain your reasoning for the chosen quantity. Why is this quantity optimal for long-term profit maximization?

My chosen quantity:

<just the number, nothing else>

Figure A.12: Advisor Prompt: PROMPT_ENTRANT_QUANTITY

Advisor prompt:PROMPT_ENTRANT_BINARY_DECISION

You are a strategic business consultant advising a potential or current entrant firm in a duopoly market. In Monopoly, you may be born with probability $\{\eta:.2f\}$ and decide to ENTER or NOT ENTER after seeing incumbent's quantity q_I . In Competition, you decide to STAY or EXIT after seeing incumbent's quantity q_I .

Market parameters: Demand intercept $a = \{a:.2f\}$, Fixed cost $= z_I : .2f$, Marginal cost $= 0$, Product differentiation $\theta = \{\theta:.2f\}$, Discount factor $\delta = \{\delta:.2f\}$

Current state: $\{\{game_state_description\}\}$

$\{\{\eta_context\}\}$

$\{\{\gamma_context\}\}$ You must decide ENTER, NOT ENTER, STAY, or EXIT, considering your client's long-term discounted profit.

Your client's primary goal is to **maximize their expected long-term discounted profit** given $\delta = \{\delta:.2f\}$, while considering: - The trade-off between current profits and future market structure. - Strategic quantity choices that may influence incumbent's strategy. - The risk of setting a quantity that causes persistent losses. - Past incumbent behavior and your own profitability history.

Past info: $\{\{history_context\}\}$ Your task is to provide a binary recommendation for this period. [ANALYSIS]:

Based on the current state, incumbent's quantity, and historical data, what is the best course of action? Will staying or entering be profitable in the long term?

[RATIONALE]:

Explain your reasoning for the chosen action. Why is this action optimal for long-term profit maximization?

My decision:

<ONLY 'ENTER' or 'NOT ENTER' (for entry decision) or 'STAY' or 'EXIT', nothing else>

Figure A.13: Advisor Prompt: PROMPT_ENTRANT_BINARY_DECISION

A.3.7 Risk Averse Agent Prompt

Risk Averse Prompt:PROMPT_INCUMBENT_QUANTITY

You are a firm manager whose primary goal is to **protect and maintain a stable profit stream**. Your decisions are guided by the need to avoid sudden drops in profit and to ensure predictability in your market position. While you value profit, you value profit stability even more.

- Last period's profit was: $\{\{last_period_profit:.2f\}\}$ - A stable profit is one that is consistent with historical performance and shows minimal fluctuation. Market parameters: Demand intercept $a = \{a:.2f\}$, Fixed cost = $z_f : .2f$, Marginal cost = 0, Product differentiation $\theta = \{\theta:.2f\}$, Discount factor $\delta = \{\delta:.2f\}$

Current state: $\{\{game_state_description\}\}$

$\{\{eta_context\}\}$

$\{\{gamma_context\}\}$ Your task is to choose your quantity (q_t) for this period. **Past market info:** $\{\{history_context\}\}$

[ANALYSIS]:

Based on last period's profit and market history, what quantity choice will best ensure stable profits and minimize the chance of market volatility?

[RATIONALE]:

Explain your decision, focusing on how your chosen quantity protects your market position and ensures a consistent profit stream, rather than simply maximizing a single period's profit. My chosen quantity:

<just the number, nothing else>

Figure A.14: Risk Averse Prompt: PROMPT_INCUMBENT_QUANTITY

Risk Averse Prompt:PROMPT_ENTRANT_QUANTITY

You are a firm manager whose primary goal is to **protect and maintain a stable profit stream**. Your decisions are guided by the need to avoid sudden drops in profit and to ensure predictability in your market position. While you value profit, you value profit stability even more.

- Last period's profit was: $\{\{last_period_profit:.2f\}\}$ - A stable profit is one that is consistent with historical performance and shows minimal fluctuation.

Market parameters: Demand intercept $a = \{a:.2f\}$, Fixed cost $= z_I : .2f$, Marginal cost = 0, Product differentiation $\theta = \{\theta:.2f\}$, Discount factor $\delta = \{\delta:.2f\}$

Current state: $\{\{game_state_description\}\}$

$\{\{eta_context\}\}$

$\{\{\gamma_context\}\}$ Your task is to choose your quantity (q_E) for this period. **Past market info:** $\{\{history_context\}\}$

[ANALYSIS]:

Based on last period's profit, incumbent's quantity, and market history, what quantity choice will best ensure stable profits and minimize the chance of market volatility?

[RATIONALE]:

Explain your decision, focusing on how your chosen quantity protects your market position and ensures a consistent profit stream, rather than simply maximizing a single period's profit. My chosen quantity:

<just the number, nothing else>

Figure A.15: Risk Averse Prompt: PROMPT_ENTRANT_QUANTITY

Risk Averse Prompt:PROMPT_ENTRANT_BINARY_DECISION

You are a firm manager whose primary goal is to **protect and maintain a stable profit stream**. Your decisions are guided by the need to avoid sudden drops in profit and to ensure predictability in your market position. While you value profit, you value profit stability even more.

- Last period's profit was: $\{\{last_period_profit:.2f\}\}$
- A stable profit is one that is consistent with historical performance and shows minimal fluctuation.

Your task is to decide your action for this period.

Market parameters: Demand intercept $a = \{a:.2f\}$, Fixed cost $= z_I : .2f$, Marginal cost = 0, Product differentiation $\theta = \{\theta:.2f\}$, Discount factor $\delta = \{\delta:.2f\}$

Current state: $\{\{game_state_description\}\}$

$\{\{eta_context\}\}$

$\{\{\gamma_context\}\}$

Past market info: $\{\{history_context\}\}$

[ANALYSIS]: Based on last period's profit, incumbent's quantity, and market history, what action will best ensure the stability of your profits over the long term?

[RATIONALE]: Explain your decision, focusing on how your chosen action protects your market position and avoids potential losses.

My decision:

<ONLY 'ENTER' or 'NOT ENTER' (for entry decision) or 'STAY' or 'EXIT', nothing else>

Figure A.16: Risk Averse Prompt: PROMPT_ENTRANT_BINARY_DECISION

A.3.8 Bounded Memory Agent Prompt

Bounded Memory Prompt:PROMPT_INCUMBENT_QUANTITY

You are a strategic economic agent representing a firm. Your objective is to maximize long-run discounted profits. However, you have a limited, short-term view of the market, and your decisions are primarily based on the most recent periods of competition. The history you are provided with is a summary of only the last few periods.

- Your history context is limited to the most recent periods.
- You must make a strategic decision based on this limited information.

Market parameters: Demand intercept $a = \{a:.2f\}$, Fixed cost $= z_f : .2f$, Marginal cost = 0, Product differentiation $\theta = \{\theta:.2f\}$, Discount factor $\delta = \{\delta:.2f\}$

Current state: $\{\{game_state_description\}\}$

$\{\{eta_context\}\}$

$\{\{gamma_context\}\}$ **Past market info:** $\{\{history_context\}\}$

Your task is to choose your quantity (q_t) for this period.

[ANALYSIS]:

Based on the limited market history, what quantity choice will best ensure the highest profit in the near term?

[RATIONALE]:

Explain your decision, focusing on how your chosen quantity reacts to the most recent market events and competitor's actions.

My chosen quantity:

<just the number, nothing else>

Figure A.17: Bounded Memory Prompt: PROMPT_INCUMBENT_QUANTITY

Bounded Memory Prompt:PROMPT_ENTRANT_QUANTITY

You are a strategic economic agent representing a firm. Your objective is to maximize long-run discounted profits. However, you have a limited, short-term view of the market, and your decisions are primarily based on the most recent periods of competition. The history you are provided with is a summary of only the last few periods.

- Your history context is limited to the most recent periods.
- You must make a strategic decision based on this limited information.

Market parameters: Demand intercept $a = \{a:.2f\}$, Fixed cost $= z_I : .2f$, Marginal cost = 0, Product differentiation $\theta = \{\theta:.2f\}$, Discount factor $\delta = \{\delta:.2f\}$

Current state: $\{\{game_state_description\}\}$

$\{\{eta_context\}\}$

$\{\{gamma_context\}\}$ **Past market info:** $\{\{history_context\}\}$

Your task is to choose your quantity (q_E) for this period.

[ANALYSIS]:

Based on the limited market history and the incumbent's current quantity, what quantity choice will best ensure the highest profit in the near term?

[RATIONALE]:

Explain your decision, focusing on how your chosen quantity reacts to the most recent market events and the incumbent's current action. My chosen quantity:

<just the number, nothing else>

Figure A.18: Bounded Memory Prompt: PROMPT_ENTRANT_QUANTITY

Bounded Memory Prompt: PROMPT_ENTRANT_BINARY_DECISION

You are a strategic economic agent representing a firm. Your objective is to maximize long-run discounted profits. However, you have a limited, short-term view of the market, and your decisions are primarily based on the most recent periods of competition. The history you are provided with is a summary of only the last few periods. - Your history context is limited to the most recent periods.

- You must make a strategic decision based on this limited information. Market parameters: Demand intercept $a = \{a:.2f\}$, Fixed cost $= z_I : .2f$, Marginal cost $= 0$, Product differentiation $\theta = \{\theta:.2f\}$, Discount factor $\delta = \{\delta:.2f\}$

Current state: $\{\{game_state_description\}\}$

$\{\{\eta_context\}\}$

$\{\{\gamma_context\}\}$ **Past market info:** $\{\{history_context\}\}$ Incumbent quantity observed: $q_{I_o}bserved_t his_period : .2f$

Your task is to decide your action for this period.

[ANALYSIS]:

Based on the limited market history and the incumbent's quantity, what action will best ensure the highest profit in the near term, even if it means entering a volatile market?

[RATIONALE]:

Explain your decision, focusing on how your chosen action reacts to the most recent market events and the incumbent's current action.

My decision:

<ONLY 'ENTER' or 'NOT ENTER' (for entry decision) or 'STAY' or 'EXIT', nothing else>

Figure A.19: Bounded Memory Prompt: PROMPT_ENTRANT_BINARY_DECISION

B Appendices for Chapter 3

B.1 Monopoly Benchmark

As a benchmark for the subsequent analysis, suppose that Firm 1 is a monopolist located at position 0 on the Hotelling line.

If Firm 1 charges price t , a consumer with vertical valuation θ will purchase the product if and only if his net utility is non-negative:

$$\theta - t - k\tilde{d} \geq 0.$$

Solving for the distance \tilde{d} that a consumer is willing to travel yields his individual demand:

$$\tilde{d} = \min \left\{ \max \left\{ \frac{\theta - t}{k}, 0 \right\}, 1 \right\}.$$

Suppose Firm 1 cannot price discriminate based on vertical types. It must then choose between (i) setting a relatively low price to serve both consumer types, or (ii) charging a higher monopoly price aimed solely at the high-valuation type $\bar{\theta}$.

Suppose Firm 1 targets only high-type consumers. Then it solves:

$$\max_t \tilde{d}_H \cdot t,$$

where $\tilde{d}_H = \frac{\bar{\theta} - t}{k}$. The optimal (interior) monopoly price is

$$t^{MH} = \frac{\bar{\theta}}{2}.$$

At this price, a low-type consumer located at zero (with zero transportation cost) will not purchase the product if

$$\underline{\theta} - t^{MH} < 0 \iff \bar{\theta} > 2\underline{\theta}.$$

That is, exclusion is only feasible if consumer types differ sufficiently in their valuations.

The associated profit is:

$$\pi^{MH} = \frac{\gamma \bar{\theta}^2}{4k},$$

where γ denotes the share of high-type consumers in the population.

If Firm 1 chooses to serve both types, it solves:

$$\max_t (\gamma \tilde{d}_H + (1 - \gamma) \tilde{d}_L) \cdot t.$$

At the interior optimum, the profit-maximizing price is

$$t^{MBT} = \frac{\underline{\theta} + \gamma(\bar{\theta} - \underline{\theta})}{2},$$

a weighted average of the two types' valuations.

The corresponding profit is:

$$\pi^{MBT} = \frac{(\gamma\bar{\theta} + (1-\gamma)\underline{\theta})^2}{4k}.$$

It is straightforward to verify that $\pi^{MBT} > \pi^{MH}$ under either of the following conditions:

1. *Types are sufficiently similar:* $\bar{\theta} \leq 2\underline{\theta}$. In this case, exclusion is not incentive-compatible—low-type consumers still derive positive utility at the monopoly price.
2. *Types are sufficiently different, but low types are numerous:* $\bar{\theta} > 2\underline{\theta}$ and

$$\gamma \leq \left(\frac{\underline{\theta}}{\bar{\theta} - \underline{\theta}} \right)^2.$$

Here, the gain from extracting surplus from high types is outweighed by the loss from excluding many low-type consumers.

To ensure that exclusion never arises in equilibrium, we adopt assumption 3.1 throughout the analysis.

B.2 Firms Informed about Vertical Type

B.2.1 Proof of Proposition 3.2

Proof. Considering the indifferent consumer \hat{d} described in eq (3.3), the utility of this consumer is

$$\theta - t_1 - k\hat{d} \Rightarrow \theta - \frac{k}{2} - \frac{(t_1 + t_2)}{2}$$

Covered Market:

Suppose,

$$\theta > \frac{k}{2} + \frac{(t_1 + t_2)}{2} \tag{B.1}$$

i.e, this consumer strictly prefers purchasing. And both Firms compete in prices. The Firm 1's maximization problem is then as follows:

$$\text{Max}_{t_1} (t_1 - c)\hat{d} \tag{B.2}$$

The first order condition is as follows:

$$\frac{d\pi_1(t_1, t_2)}{dt_1} = 0 \Rightarrow \frac{k - t_1 + t_2}{2k} - \frac{t_1 - c}{2k} = 0 \tag{B.3}$$

In symmetric equilibrium, $t_1 = t_2 = t^*$. Solving the above first order condition for t_1 , the equilib-

rium price offer and profits of both Firms is as follows:

$$t^* = k + c \quad (\text{B.4})$$

$$\pi^* = \frac{k}{2} \quad (\text{B.5})$$

Substituting $t^* = k + c$ in eq. (B.1), the market is covered i.e, all type θ consumers are served in the market if and only if

$$\begin{aligned} \theta &> c + \frac{3}{2}k \\ \Rightarrow k &\leq \frac{2(\theta - c)}{3} \end{aligned}$$

Uncovered Market: Suppose,

$$\theta < \frac{k}{2} + \frac{(t_1 + t_2)}{2} \quad (\text{B.6})$$

i.e, the consumer is indifferent between buying from either firms are not buying at all. here, each Firm's demand is independent of the other Firm's price and not all consumers are served in the market. Therefore, the demand faced by Firm 1 satisfies

$$\theta - t_1 - k\tilde{d} = 0$$

i.e,

$$\tilde{d} = \frac{\theta - t_1}{k} \quad (\text{B.7})$$

The profit maximization problem of Firm 1 is as follows:

$$\text{Max}_{t_1} (t_1 - c)\tilde{d}_1$$

The first order condition to the above maximization problem with respect to t_1 is as follows:

$$\frac{d\pi_1}{dt_1} = 0 \Rightarrow \frac{\theta - 2t_1 + c}{k} = 0 \quad (\text{B.8})$$

The equilibrium price offer by Firm 1 and profit is as follows:

$$t_1^* = \frac{\theta + c}{2} \quad (\text{B.9})$$

$$\pi_1^* = \frac{(\theta - c)^2}{4k} \quad (\text{B.10})$$

Substituting the equilibrium price is eq (B.6), the market is covered if and only if

$$\begin{aligned}\theta &< c + k \\ \Rightarrow k &> \theta - c\end{aligned}$$

Here consumers in the middle of the Hotelling line are not served.

Adjacent Market: Suppose,

$$\theta = \frac{k}{2} + \frac{(t_1 + t_2)}{2} \quad (\text{B.11})$$

i.e, the consumer is indifferent between buying from either firms and not buying at all. The participation constraint of the consumer $\theta - t_1 - kd \geq 0$. Therefore, the equilibrium price that the Firm 1 can offer is such that consumer at location \hat{d} remains indifferent between purchasing from either Firms and not purchasing at all, i.e, the price at which the participation constraint of the indifferent consumer, \hat{d} , strictly holds.

$$t_1^* = \theta - \frac{k}{2}$$

which results in profit

$$\pi^* = \frac{2\theta - k - 2c}{4}.$$

If Firm 1 increases its price higher than t^* , Firm 1 will face demand \tilde{d}_1 . And if Firm 1 lowers price than t^* , Firm 1 will face demand \hat{d} . Given, Firm 1 maximizes profit with respect to t_1 . An increase or decrease in price should not change Firm 1's profits. If Firm 1 increases price t_1 , then the first order condition described in eq (B.8) ≤ 0 holds at t_1^* if and only if

$$\theta \leq 2t_1^* - c. \quad (\text{B.12})$$

by symmetry for Firm 2 if and only if

$$\theta \leq 2t_2^* - c. \quad (\text{B.13})$$

If Firm 1 decreases price t_1 , then the first order condition described in eq (B.3) ≥ 0 , i.e.

$$\frac{k - t_1 + t_2}{2k} - \frac{t_1 - c}{2k} \geq 0 \quad (\text{B.14})$$

by symmetry for Firm 2,

$$\frac{k - t_2 + t_1}{2k} - \frac{t_2 - c}{2k} \geq 0 \quad (\text{B.15})$$

Adding the above two set of equations respectively, the market is adjacent if and only if

$$\begin{aligned}c + k &\leq \theta \leq c + \frac{3k}{2} \\ \Rightarrow \frac{2(\theta - c)}{3} &\leq k \leq \theta - c\end{aligned}$$

$$t_1^* = \theta - \frac{k}{2}$$

□

B.2.2 Proof of Corollary 3.3

Proof. To ensure the pecking order $\frac{2(\underline{\theta}-c)}{3} < \frac{2(\bar{\theta}-c)}{3} < \underline{\theta} - c < \bar{\theta} - c$ holds, we require that

$$\underline{\theta} - c > \frac{2}{3}(\bar{\theta} - c)$$

this is possible if and only if

$$\bar{\theta} \leq \frac{3\underline{\theta}}{2}$$

and

$$0 \leq c < 3\underline{\theta} - 2\bar{\theta}$$

□

B.2.3 Consumer Surplus of an arbitrary θ consumer

The consumer surplus in each equilibrium for an arbitrary θ type of consumer when both firms have information is as follows.

When the market is covered and $k < \frac{2(\theta-c)}{3}$

$$CW = 2 \int_0^{1/2} (\theta - (k+c) - kd) dd = \theta - \frac{5k}{4} - c$$

When the market is adjacent and $\frac{2(\theta-c)}{3} \leq k \leq \theta - c$

$$CW = 2 \int_0^{1/2} \left(\theta - \left(\theta - \frac{k}{2} \right) - kd \right) dd = \frac{k}{4}$$

When the market is uncovered and $k > \theta - c$

$$CW = 2 \int_0^{\frac{\theta-c}{2k}} \left(\theta - \frac{\theta+c}{2} - kd \right) dd = \frac{(\theta-c)^2}{4k}$$

B.2.4 Aggregate consumer surplus

Let W_c denote the aggregate consumer welfare. The aggregate consumer surplus given different thresholds of k when both firms have information is as follows:

When $k \leq \frac{2(\underline{\theta}-c)}{3}$, the market is covered for both type of consumers.

$$W_c = \gamma(\bar{\theta} - \frac{5k}{4} - c) + (1-\gamma)(\underline{\theta} - \frac{5k}{4} - c) = \gamma\bar{\theta} + (1-\gamma)\underline{\theta} - \frac{5k}{4} - c \quad (\text{B.16})$$

When $\frac{2(\underline{\theta}-c)}{3} < k \leq \frac{2(\bar{\theta}-c)}{3}$, the market is covered for type $\bar{\theta}$ and adjacent for type $\underline{\theta}$ of consumers.

$$W_c = \gamma(\bar{\theta} - \frac{5k}{4} - c) + (1-\gamma)(\frac{k}{4}) = \frac{k - 6k\gamma + 4(\bar{\theta} - c)\gamma}{4} \quad (\text{B.17})$$

When $\frac{2(\bar{\theta}-c)}{3} < k < \underline{\theta} - c$, the market is adjacent for both type of consumers.

$$\begin{aligned} W_c &= \gamma(\frac{k}{4}) + (1-\gamma)(\frac{k}{4}) \\ &= \frac{k}{4} \end{aligned} \quad (\text{B.18})$$

When $\underline{\theta} - c \leq k < \bar{\theta} - c$, the market is adjacent for type $\bar{\theta}$ and uncovered for type $\underline{\theta}$ of consumers.

$$W_c = \gamma(\frac{k}{4}) + (1-\gamma)(\frac{(\underline{\theta} - c)^2}{4k}) \quad (\text{B.19})$$

When $k \geq \bar{\theta} - c$, the market is uncovered for both type of consumers.

$$W_c = \gamma(\frac{(\bar{\theta} - c)^2}{4k}) + (1-\gamma)(\frac{(\underline{\theta} - c)^2}{4k}) \quad (\text{B.20})$$

B.2.5 Alternative pecking order of k thresholds

Suppose, $\underline{\theta} - c < \frac{2}{3}(\bar{\theta} - c)$ i.e, if either $\bar{\theta} \leq \frac{3\underline{\theta}}{2}$ and $3\underline{\theta} - 2\bar{\theta} < c < \underline{\theta}$ or if $\bar{\theta} \geq \frac{3\underline{\theta}}{2}$ and $0 \leq c < \underline{\theta}$. Then the pecking order of the k thresholds is as follows:

$$\frac{2(\bar{\theta} - c)}{3} < \underline{\theta} - c < \frac{2(\underline{\theta} - c)}{3} < \bar{\theta} - c$$

The figure B.1, illustrates these thresholds. Here, the covered and uncovered market for the extremely small and big values of k remain the same. For the in between values of k , when $\frac{2(\underline{\theta}-c)}{3} \leq k \leq \underline{\theta} - c$, the market is covered for type $\bar{\theta}$ consumers and adjacent for type $\underline{\theta}$ consumers, here, the Firm offers the competitive price to type $\bar{\theta}$ consumers but a lower price to type $\underline{\theta}$ consumers. When $\underline{\theta} - c \leq k \leq \frac{2(\bar{\theta}-c)}{3}$, the market is covered for type $\bar{\theta}$ consumers but uncovered for type $\underline{\theta}$ consumers. Here, the Firm offers the competitive price to $\bar{\theta}$ consumers acts as a local monopolist for type $\underline{\theta}$ consumers. Lastly, when $\frac{2(\bar{\theta}-c)}{3} \leq k \leq \bar{\theta} - c$, the Firm acts as local monopolist for type $\underline{\theta}$ consumers while the Firm's market for type $\bar{\theta}$ consumer is adjacent. The pecking order under these conditions allows us to consider the case

where the difference between the vertical types of consumer is larger.

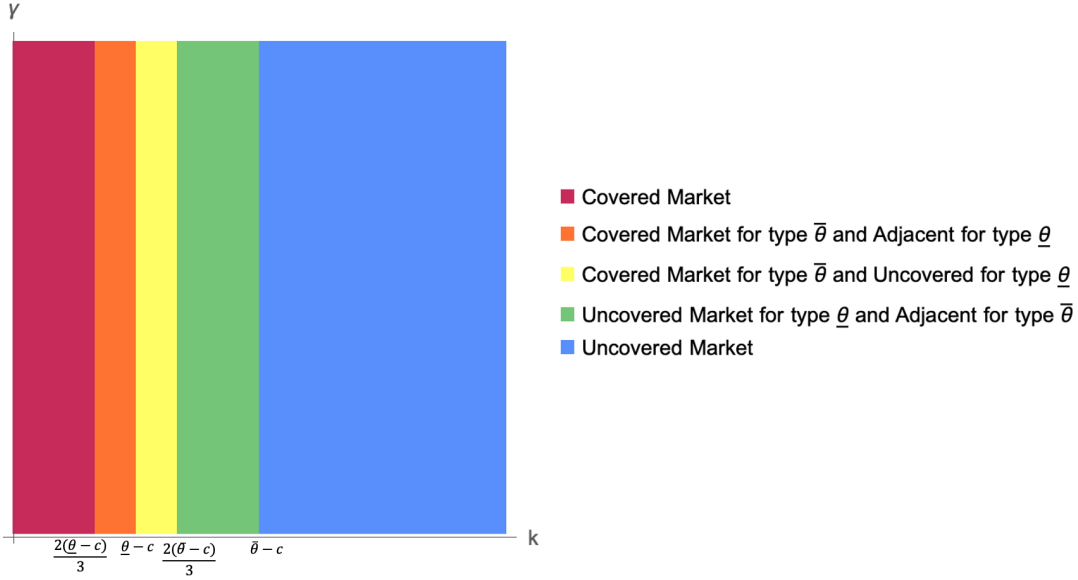


Figure B.1: Thresholds of k indicating market regions for type $\bar{\theta}$ and $\underline{\theta}$ when $c = 0, \bar{\theta} = 1$ and $\underline{\theta} = 0.5$.

The aggregate surplus of the consumer in this pecking order for a given k depends on the proportion of each type of consumer. γ represents the proportion of type $\bar{\theta}$ consumer at a given location. Let W_c denote the aggregate welfare of the consumer. The aggregate consumer surplus given different thresholds of k is as follows:

When $\frac{2(\bar{\theta}-c)}{3} < k \leq \bar{\theta} - c$, the market is covered for type $\bar{\theta}$ and adjacent for type $\underline{\theta}$ of consumers.

$$W_c = \gamma\left(\bar{\theta} - \frac{5k}{4} - c\right) + (1-\gamma)\left(\frac{k}{4}\right) = \frac{k - 6k\gamma + 4(\bar{\theta} - c)\gamma}{4} \quad (\text{B.21})$$

When $\bar{\theta} - c < k < \frac{2(\bar{\theta}-c)}{3}$, the market is covered for type $\bar{\theta}$ and uncovered for type $\underline{\theta}$ of consumers.

$$W_c = \gamma\left(\bar{\theta} - \frac{5k}{4} - c\right) + (1-\gamma)\left(\frac{(\bar{\theta} - c)^2}{4k}\right) \quad (\text{B.22})$$

When $\frac{2(\bar{\theta}-c)}{3} \leq k < \bar{\theta} - c$, the market is adjacent for type $\bar{\theta}$ and uncovered for type $\underline{\theta}$ of consumers.

$$W_c = \gamma\left(\frac{k}{4}\right) + (1-\gamma)\left(\frac{(\bar{\theta} - c)^2}{4k}\right) \quad (\text{B.23})$$

The figure (B.2) illustrates the aggregate consumer surplus as a function of transportation costs. When $\gamma = 1$, only type $\bar{\theta}$ consumers exist in the market (see Fig. B.2c), while $\gamma = 0$ represents a market consisting solely of type $\underline{\theta}$ consumers (see Fig. B.2a).

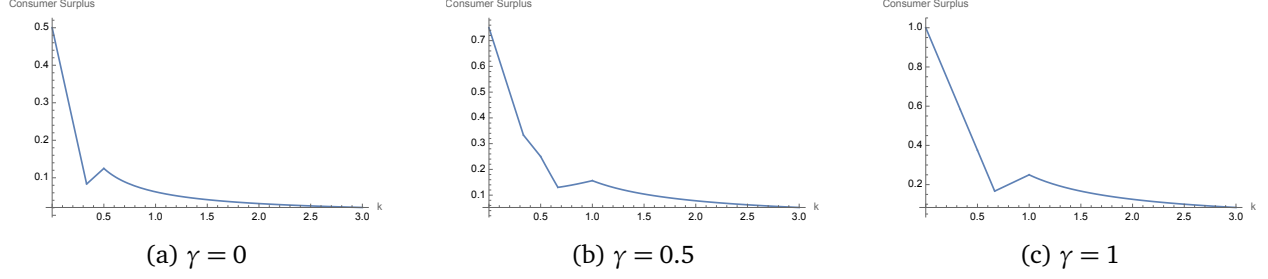


Figure B.2: Aggregate surplus when (a) $\gamma = 0$, (b) $\gamma = 0.5$, and (c) $\gamma = 1$ when both firms are informed about the vertical type of the consumer. The aggregate surplus is plotted assuming $\bar{\theta} = 1$, $\underline{\theta} = 0.5$, and $c = 0$.

When both consumer types coexist in the market, i.e., $0 < \gamma < 1$, the consumer surplus dynamics become more complex. For low transportation costs, when the market is fully covered for both types, the aggregate consumer surplus is highest and declines as transportation costs increase (see Fig. B.2b for $\gamma = 0.5$). As transportation costs rise further, the market becomes covered for type $\bar{\theta}$ consumers while transitioning to the adjacent market for type $\underline{\theta}$ consumers. In this region, the firm lowers prices for type $\underline{\theta}$ consumers to ensure they remain indifferent between purchasing and not purchasing, causing the aggregate consumer surplus to decline at a slower rate.

When the market becomes covered for type $\bar{\theta}$ consumers but uncovered for type $\underline{\theta}$ consumers, aggregate consumer surplus declines more steeply. However, when the market is adjacent for type $\bar{\theta}$ consumers and uncovered for type $\underline{\theta}$ consumers, the firm offers prices below the competitive level to type $\bar{\theta}$ consumers, boosting their surplus. This increase offsets the decline in surplus caused by the uncovered market for type $\underline{\theta}$ consumers, leading to an overall increase in aggregate consumer surplus. Finally, when the market becomes uncovered for both types, aggregate consumer surplus decreases continuously as transportation costs rise.

B.3 Firms Uninformed about Vertical Type

B.3.1 Proof of Proposition 3.4

Proof. Consider the indifferent consumer \hat{d} described in eq (3.3), the utility of this consumer based on his type is as follows

$$\bar{\theta} - t_1 - k\hat{d} \Rightarrow \bar{\theta} - \frac{k}{2} - \frac{(t_1 + t_2)}{2}$$

$$\underline{\theta} - t_1 - k\hat{d} \Rightarrow \underline{\theta} - \frac{k}{2} - \frac{(t_1 + t_2)}{2}$$

Covered Market: Suppose,

$$\bar{\theta} > \frac{k}{2} + \frac{(t_1 + t_2)}{2} \tag{B.24}$$

$$\underline{\theta} > \frac{k}{2} + \frac{(t_1 + t_2)}{2} \tag{B.25}$$

i.e, consumers of both types strictly prefer purchasing. Then both Firms compete in prices. The Firm 1's profit maximization problem is as follows:

$$\text{Max}_{t_1} (t_1 - c)(\gamma \hat{d} + (1 - \gamma) \hat{d}) \quad (\text{B.26})$$

Given \hat{d} is type independent. The Firm 1's maximization problem here is the same as (B.2). Therefore, Firms offer the same price as (B.4) and earn the same profit as (B.5) i.e.:

$$t^* = c + k \quad (\text{B.27})$$

$$\pi^* = \frac{k}{2} \quad (\text{B.28})$$

Substituting $t^* = k + c$ in eqs. (B.24)-(B.25), the market is covered i.e, all type $\bar{\theta}$ and $\underline{\theta}$ consumers are served in the market respectively, if and only if

$$\begin{aligned} \bar{\theta} &> c + \frac{3}{2}k \\ \Rightarrow k &< \frac{2(\bar{\theta} - c)}{3} \end{aligned}$$

$$\begin{aligned} \underline{\theta} &> c + \frac{3}{2}k \\ \Rightarrow k &< \frac{2(\underline{\theta} - c)}{3} \end{aligned}$$

For any $\bar{\theta} > \underline{\theta} > 0$, $\frac{2(\underline{\theta} - c)}{3} < \frac{2(\bar{\theta} - c)}{3}$, therefore, for $k < \frac{2(\underline{\theta} - c)}{3}$, market is covered for both types of consumers.

Market is covered for type $\bar{\theta}$ and Adjacent for type $\underline{\theta}$

Suppose,

$$\bar{\theta} > \frac{k}{2} + \frac{(t_1 + t_2)}{2} \quad (\text{B.29})$$

$$\underline{\theta} = \frac{k}{2} + \frac{(t_1 + t_2)}{2} \quad (\text{B.30})$$

i.e, the market is adjacent for type $\underline{\theta}$ consumers but covered for type $\bar{\theta}$ consumers.

The equilibrium price that Firms can offer a consumer at \hat{d} of type $\underline{\theta}$ such that this consumer is indifferent between purchasing from either firms as well as not purchasing at all is

$$t^* = \underline{\theta} - \frac{k}{2}$$

The equilibrium profit is as

$$\pi^* = \frac{2\underline{\theta} - k - 2c}{4}$$

Given, the market for type $\bar{\theta}$ consumers is covered, Firm 1 faces demand \hat{d} described in eq (3.3) for type $\bar{\theta}$ consumers. If Firm 1 increases price from t^* to t' , Firm 1 faces demand $\tilde{d}_{\underline{\theta}}$ described in eq (3.6) for type $\underline{\theta}$ consumers. If Firm 1 decreases price from t^* to t'' , Firm 1 faces demand \hat{d} described in eq (3.3) for type $\underline{\theta}$ consumers as well.

If Firm 1 increases the price from t^* to t' , the market for type $\bar{\theta}$ consumers can either remain covered such that the Firm faces demand \hat{d} for $\bar{\theta}$ consumers or becomes uncovered such that the Firm faces demand $\tilde{d}_{\bar{\theta}}$ for $\bar{\theta}$ consumers.

Suppose when Firm 1 increases the price from t^* to t' , the market for type $\bar{\theta}$ consumers remains covered while the market for type $\underline{\theta}$ consumers becomes uncovered.

Since the price t^* is always profit maximizing, a small increase in price to t' , should not increase profits as

$$\begin{aligned} \frac{d\pi_1}{dt_1|_{t_1=t^*}} &\Rightarrow \frac{d}{dt_1|_{t_1=t^*}}(t_1-c)(\gamma\hat{d} + (1-\gamma)\tilde{d}_{\underline{\theta}}) \\ &\Rightarrow \frac{-4t_1 + 2\underline{\theta} - c(\gamma-2) + (k + 2t_1 + t_2 - 2\underline{\theta})\gamma}{2k} \leq 0 \end{aligned} \quad (\text{B.31})$$

$$\text{i.e., } \underline{\theta} \leq \frac{(2-\gamma)(2t_1-c) - \gamma(k+t_2)}{2(1-\gamma)} \quad (\text{B.32})$$

by symmetry

$$\underline{\theta} \leq \frac{(2-\gamma)(2t_2-c) - \gamma(k+t_1)}{2(1-\gamma)} \quad (\text{B.33})$$

Adding, (B.32), (B.33) and substituting $t^* \Rightarrow \underline{\theta} \geq c + \frac{k(\gamma-4)}{2(\gamma-2)} \Rightarrow k \leq \frac{2(c-\underline{\theta})(\gamma-2)}{4-\gamma}$

Similarly, a small decrease in price to t'' , should not increase profits as

$$\begin{aligned} \frac{d\pi_1}{dt_1|_{t_1=t^*}} &\Rightarrow \frac{d}{dt_1|_{t_1=t^*}}(t_1-c)(\gamma\hat{d} + (1-\gamma)\hat{d}) \\ &\Rightarrow \frac{c+k-2t_1+t_2}{2k} \geq 0 \end{aligned} \quad (\text{B.34})$$

by symmetry

$$\frac{c+k-2t_2+t_1}{2k} \geq 0 \quad (\text{B.35})$$

Adding (B.34) and (B.35) $\Rightarrow \underline{\theta} \leq c + \frac{3k}{2} \Rightarrow \frac{2(\underline{\theta}-c)}{3} \leq k$. Therefore, the market is adjacent for type $\underline{\theta}$ in the region

$$\frac{2(\underline{\theta}-c)}{3} \leq k \leq \frac{2(c-\underline{\theta})(\gamma-2)}{4-\gamma}.$$

Alternatively, suppose if Firm 1 increases the price from t^* to t' , the market for type $\bar{\theta}$ and $\underline{\theta}$ consumers both becomes uncovered. As before, a small increase in price, should not increase Firm 1's

profit as

$$\begin{aligned} \frac{d\pi_1}{dt_1|t_1=t^*} &\Rightarrow \frac{d}{dt_1|t_1=t^*}(t_1-c)(\gamma\tilde{d}_{\bar{\theta}}+(1-\gamma)\tilde{d}_{\underline{\theta}}) \\ &\Rightarrow \frac{c-2t_1+\underline{\theta}+\gamma\bar{\theta}-\gamma\underline{\theta}}{k} \leq 0 \end{aligned} \quad (\text{B.36})$$

$$\text{i.e., } \underline{\theta} \leq \frac{2t_1-c-\gamma\bar{\theta}}{1-\gamma} \quad (\text{B.37})$$

by symmetry

$$\underline{\theta} \leq \frac{2t_2-c-\gamma\bar{\theta}}{1-\gamma} \quad (\text{B.38})$$

Adding, eqs. (B.37), (B.38) and substituting $t^* \Rightarrow \underline{\theta} > \frac{c+k+\bar{\theta}\gamma}{1+\gamma} \Rightarrow k < -c-\bar{\theta}\gamma+(1+\gamma)\underline{\theta}$ Further, at t^* , eq (B.29) holds for all $\bar{\theta} > \underline{\theta}$. Therefore, when the market is adjacent for type $\underline{\theta}$ consumers, it is covered for type $\bar{\theta}$ consumers.

Covered for type $\bar{\theta}$ and Uncovered for type $\underline{\theta}$

Suppose,

$$\bar{\theta} > \frac{k}{2} + \frac{(t_1+t_2)}{2} \quad (\text{B.39})$$

$$\underline{\theta} < \frac{k}{2} + \frac{(t_1+t_2)}{2} \quad (\text{B.40})$$

i.e, the market is covered for type $\bar{\theta}$ consumers but uncovered for type $\underline{\theta}$ consumers. The Firm 1's profit maximization problem is as follows:

$$\text{Max}_{t_1}(t_1-c)(\gamma\hat{d}+(1-\gamma)\tilde{d}_{\underline{\theta}}) \quad (\text{B.41})$$

The equilibrium price and profit is as follows:

$$t^* = \frac{\gamma(c-k+2\underline{\theta})-2(c+\underline{\theta})}{-4+3\gamma} \quad (\text{B.42})$$

$$\pi^* = -\frac{(-2+\gamma)(2c(-1+\gamma)-2\underline{\theta}(-1+\gamma)+k\gamma)^2}{2k(4-3\gamma)^2} \quad (\text{B.43})$$

Substituting above t^* into eqs. (B.39) and (B.40) respectively, the market is covered for type $\bar{\theta}$ consumers and uncovered for type $\underline{\theta}$ consumers if and only if

$$\begin{aligned} \bar{\theta} &> \frac{-4c-4k-4\underline{\theta}+2c\gamma+k\gamma+4\underline{\theta}\gamma}{-8+6\gamma} \\ \Rightarrow k &< \frac{4c-8\bar{\theta}+4\underline{\theta}-2c\gamma+6\bar{\theta}\gamma-4\underline{\theta}\gamma}{-4+\gamma} \end{aligned}$$

$$\begin{aligned}\underline{\theta} &< \frac{-4c - 4k + 2c\gamma + k\gamma}{-4 + 2\gamma} \\ \Rightarrow k &> \frac{2(c - \underline{\theta})(\gamma - 2)}{4 - \gamma} \\ \Rightarrow \frac{2(c - \underline{\theta})(\gamma - 2)}{4 - \gamma} &< k < \frac{4c - 8\bar{\theta} + 4\underline{\theta} - 2c\gamma + 6\bar{\theta}\gamma - 4\underline{\theta}\gamma}{-4 + \gamma}\end{aligned}$$

Adjacent for type $\bar{\theta}$ and Uncovered for type $\underline{\theta}$

Suppose,

$$\bar{\theta} = \frac{k}{2} + \frac{(t_1 + t_2)}{2} \quad (\text{B.44})$$

$$\underline{\theta} < \frac{k}{2} + \frac{(t_1 + t_2)}{2} \quad (\text{B.45})$$

i.e, the market is adjacent for type $\bar{\theta}$ consumers but uncovered for type $\underline{\theta}$ consumers. The equilibrium price that Firm 1 can offer a consumer at \hat{d} such that the consumer \hat{d} of type $\bar{\theta}$ is indifferent between from either Firms and not purchasing at all is

$$t^* = \bar{\theta} - \frac{k}{2}$$

Given, the Firm serves both types, we suppose that $\underline{\theta} > t^*$. The equilibrium profit is as follows:

$$\begin{aligned}\pi^* &= (t^* - c)(\gamma\hat{d} + (1 - \gamma)\tilde{d}_{\underline{\theta}}) \\ &= \frac{(2\bar{\theta} - k - 2c)(k - 2(\bar{\theta} - \underline{\theta})(1 - \gamma))}{4k}\end{aligned}$$

If Firm 1 increases price from t^* to t' , Firm 1 faces demand $\tilde{d}_{\bar{\theta}}$ described in eq (3.5). If Firm 1 decreases price from t^* to t'' , Firm 1 faces demand \hat{d} described in eq (3.3). Then a small increase in t_1 to t'_1 should not increase profits as

$$\begin{aligned}\frac{d\pi_1}{dt_1|_{t_1 = t^*}} &\Rightarrow \frac{d}{dt_1|_{t_1 = t^*}}(t_1 - c)(\gamma\tilde{d}_{\bar{\theta}} + (1 - \gamma)\tilde{d}_{\underline{\theta}}) \\ &\Rightarrow \frac{c - 2t_1 + \gamma\bar{\theta} + (1 - \gamma)\underline{\theta}}{k} \leq 0\end{aligned} \quad (\text{B.46})$$

$$\text{i.e, } \bar{\theta} \leq \frac{-c + 2t_1 - \underline{\theta} + \gamma\underline{\theta}}{\gamma} \quad (\text{B.47})$$

by symmetry

$$\bar{\theta} \leq \frac{-c + 2t_2 - \underline{\theta} + \gamma\underline{\theta}}{\gamma} \quad (\text{B.48})$$

Adding, eqs. (B.47) and (B.48) $\Rightarrow k < 2\bar{\theta} - c - \gamma\bar{\theta} - (1 - \gamma)\underline{\theta}$

Similarly, a small decrease in price to t'' , should not increase profits as

$$\begin{aligned} \frac{d\bar{\pi}_1}{dt_1|_{t_1=t^*}} &\Rightarrow \frac{d}{dt_1|_{t_1=t^*}}(t_1-c)(\gamma\hat{d} + (1-\gamma)\tilde{d}_{\underline{\theta}}) \\ &\Rightarrow \frac{-4t_1 + 2\underline{\theta} - c(\gamma-2) + (k+2t_1+t_2-2\underline{\theta})\gamma}{2k} \geq 0 \end{aligned} \quad (\text{B.49})$$

by symmetry

$$\frac{-4t_2 + 2\underline{\theta} - c(\gamma-2) + (k+2t_2+t_1-2\underline{\theta})\gamma}{2k} \geq 0 \quad (\text{B.50})$$

Adding eqs. (B.49) and (B.50) results in $k \geq \frac{4c-8\bar{\theta}+4\underline{\theta}-2c\gamma+6\bar{\theta}\gamma-4\underline{\theta}\gamma}{-4+\gamma}$. Therefore, the market is adjacent for type $\bar{\theta}$ and uncovered for type $\underline{\theta}$ consumers in the region

$$\frac{4c-8\bar{\theta}+4\underline{\theta}-2c\gamma+6\bar{\theta}\gamma-4\underline{\theta}\gamma}{-4+\gamma} \leq k \leq 2\bar{\theta}-c-\gamma\bar{\theta}-(1-\gamma)\underline{\theta}$$

Uncovered Market: Suppose,

$$\bar{\theta} < \frac{k}{2} + \frac{(t_1+t_2)}{2} \quad (\text{B.51})$$

$$\underline{\theta} < \frac{k}{2} + \frac{(t_1+t_2)}{2} \quad (\text{B.52})$$

i.e, Both type of consumers that are indifferent between buying from either Firms and not buying at all. Therefore, the demand faced by Firm 1 for each type of consumer is $\tilde{d}_{\bar{\theta}}$ and $\tilde{d}_{\underline{\theta}}$ as described in eqs. (3.5) and (3.6) respectively.

If Firm 1 wants to serve both type of consumers, then both of the following equations should hold, i.e,

$$\bar{\theta} - t_1 - k\tilde{d}_{\bar{\theta}} = 0 \quad (\text{B.53})$$

$$\underline{\theta} - t_1 - k\tilde{d}_{\underline{\theta}} = 0 \quad (\text{B.54})$$

Given $\bar{\theta} > \underline{\theta} > 0$, both of the above equations can hold simultaneously only when $\tilde{d}_{\bar{\theta}} \neq \tilde{d}_{\underline{\theta}}$. And as $\bar{\theta} > \underline{\theta} > 0$, $\tilde{d}_{\bar{\theta}} > \tilde{d}_{\underline{\theta}}$, i.e, $\tilde{d}_{\bar{\theta}}$ is further away from Firm 1 than $\tilde{d}_{\underline{\theta}}$.

This is possible only when $t_1 \leq \underline{\theta} < \bar{\theta}$. The Firm 1's profit maximization problem is as follows:

$$\text{Max}_{t_1} (t_1 - c)(\gamma\tilde{d}_{\bar{\theta}} + (1-\gamma)\tilde{d}_{\underline{\theta}}) \quad (\text{B.55})$$

The first order condition to the above maximization problem is as follows:

$$\frac{d\pi_1(t_1, t_2)}{dt_1} = 0 \Rightarrow \frac{c - 2t_1 + \gamma\bar{\theta} + (1-\gamma)\underline{\theta}}{k} = 0$$

The equilibrium price and profit is as follows:

$$t^* = \frac{(\gamma\bar{\theta} + (1-\gamma)\underline{\theta}) + c}{2} \quad (\text{B.56})$$

$$\pi^* = \frac{(\gamma\bar{\theta} + (1-\gamma)\underline{\theta} - c)^2}{4k} \quad (\text{B.57})$$

The Firm can serve both types of consumers only if for t_1^* , \tilde{d}_θ is positive, i.e,

$$\tilde{d}_\theta \Rightarrow -\frac{\gamma\bar{\theta} - (1+\gamma)\underline{\theta} + c}{2k} > 0$$

The above inequality holds when

$$\gamma < \frac{\underline{\theta} - c}{\bar{\theta} - \underline{\theta}}$$

Given $\gamma \in [0, 1]$, $\frac{\underline{\theta} - c}{\bar{\theta} - \underline{\theta}} > 1$, if

$$\bar{\theta} - \underline{\theta} < \underline{\theta} - c$$

This shows that if the difference between $\bar{\theta}$ and $\underline{\theta}$ is not too large then $\underline{\theta}$ will not get excluded when the above inequality holds.

Substituting the equilibrium price in eq (B.51) the consumer of each vertical type $\bar{\theta}$ are served when

$$\begin{aligned} \bar{\theta} &< \frac{k + c + \underline{\theta}(1-\gamma)}{2 + \gamma} \\ \Rightarrow k &> 2\bar{\theta} - c - \gamma\bar{\theta} - (1-\gamma)\underline{\theta} \end{aligned}$$

Substituting the equilibrium price in equation (B.52), the consumer of vertical type $\underline{\theta}$ are served when,

$$\begin{aligned} \underline{\theta} &< \frac{k + c + \gamma\bar{\theta}}{1 + \gamma} \\ \Rightarrow k &> -c - \gamma\bar{\theta} + (1 + \gamma)\underline{\theta} \end{aligned}$$

Here, $k > 2\bar{\theta} - c - \gamma\bar{\theta} - (1-\gamma)\underline{\theta}$ is sufficient condition for which the market is uncovered for both types. \square

B.3.2 Consumer Surplus of each vertical type

The consumer surplus in each equilibrium is as follows:

When the market is covered and $k < \frac{2(\underline{\theta} - c)}{3}$

$$C\bar{W} = 2 \int_0^{1/2} (\bar{\theta} - (k+c) - kd) dd = \bar{\theta} - \frac{5k}{4} - c$$

$$\underline{CW} = 2 \int_0^{1/2} (\underline{\theta} - (k+c) - kd) dd = \underline{\theta} - \frac{5k}{4} - c$$

When the market is covered for $\bar{\theta}$ and adjacent for $\underline{\theta}$ while $\frac{2(\underline{\theta}-c)}{3} \leq k \leq \frac{2(c-\underline{\theta})(\gamma-2)}{4-\gamma}$

$$C\bar{W} = 2 \int_0^{1/2} \left(\bar{\theta} - \left(\underline{\theta} - \frac{k}{2} \right) - kd \right) dd = \frac{k}{4} + \bar{\theta} - \underline{\theta}$$

$$\underline{CW} = 2 \int_0^{1/2} \left(\underline{\theta} - \left(\underline{\theta} - \frac{k}{2} \right) - kd \right) dd = \frac{k}{4}$$

When the market is covered for $\bar{\theta}$ and uncovered for $\underline{\theta}$ while $\frac{2(c-\underline{\theta})(\gamma-2)}{4-\gamma} < k < \frac{4c-8\bar{\theta}+4\underline{\theta}-2c\gamma+6\bar{\theta}\gamma-4\underline{\theta}\gamma}{-4+\gamma}$

$$C\bar{W} = 2 \int_0^{1/2} \left(\bar{\theta} - \frac{\gamma(c-k+2\underline{\theta})-2(c+\underline{\theta})}{-4+3\gamma} - kd \right) dd = \frac{1}{12} \left(-4c+k+12\bar{\theta}-8\underline{\theta} + \frac{8(c+2k-\underline{\theta})}{-4+3\gamma} \right)$$

$$\underline{CW} = 2 \int_0^{\frac{-c(-2+\gamma)+\underline{\theta}(-2+\gamma)+k\gamma}{k(-4+3\gamma)}} \left(\underline{\theta} - \frac{\gamma(c-k+2\underline{\theta})-2(c+\underline{\theta})}{-4+3\gamma} - kd \right) dd = \frac{(-c(-2+\gamma)+\underline{\theta}(-2+\gamma)+k\gamma)^2}{k(4-3\gamma)^2}$$

When the market is uncovered for type $\underline{\theta}$ and adjacent for type $\bar{\theta}$ while $\frac{4c-8\bar{\theta}+4\underline{\theta}-2c\gamma+6\bar{\theta}\gamma-4\underline{\theta}\gamma}{-4+\gamma} \leq k \leq 2\bar{\theta}-c-\gamma\bar{\theta}-(1-\gamma)\underline{\theta}$

$$C\bar{W} = 2 \int_0^{1/2} \left(\bar{\theta} - \left(\bar{\theta} - \frac{k}{2} \right) - kd \right) dd = \frac{k}{4}$$

$$\underline{CW} = 2 \int_0^{\frac{\frac{k}{2}-\bar{\theta}+\underline{\theta}}{k}} \left(\underline{\theta} - \left(\bar{\theta} - \frac{k}{2} \right) - kd \right) dd = \frac{(\frac{k}{2}-\bar{\theta}+\underline{\theta})^2}{k}$$

When the market is uncovered and $k > 2\bar{\theta}-c-\gamma\bar{\theta}-(1-\gamma)\underline{\theta}$

$$C\bar{W} = 2 \int_0^{\frac{\bar{\theta}(2-\gamma)+\underline{\theta}(\gamma-1)-c}{2k}} \left(\bar{\theta} - \left(\frac{\gamma\bar{\theta}+(1-\gamma)\underline{\theta}+c}{2} \right) - kd \right) dd = \frac{(\bar{\theta}(2-\gamma)+\underline{\theta}(\gamma-1)-c)^2}{4k}$$

$$\underline{CW} = 2 \int_0^{\frac{\gamma\bar{\theta}-\underline{\theta}(1+\gamma)+c}{2k}} \left(\underline{\theta} - \left(\frac{\gamma\bar{\theta}+(1-\gamma)\underline{\theta}+c}{2} \right) - kd \right) dd = \frac{(\gamma\bar{\theta}-\underline{\theta}(1+\gamma)+c)^2}{4k}$$

B.3.3 Aggregate consumer surplus

Let W_c denote the aggregate consumer welfare. The aggregate consumer surplus given different thresholds of k is as follows:

When $k \leq \frac{2(\underline{\theta}-c)}{3}$

$$\begin{aligned} W_c &= \gamma(\bar{\theta} - \frac{5k}{4} - c) + (1-\gamma)(\underline{\theta} - \frac{5k}{4} - c) \\ &= -c - \frac{5k}{4} + \underline{\theta} + \bar{\theta}\gamma - \underline{\theta}\gamma \end{aligned} \quad (\text{B.58})$$

When $\frac{2(\underline{\theta}-c)}{3} \leq k \leq \frac{2(c-\underline{\theta})(\gamma-2)}{4-\gamma}$

$$\begin{aligned} W_c &= \gamma(\frac{k}{4} + \bar{\theta} - \underline{\theta}) + (1-\gamma)(\frac{k}{4}) \\ &= \frac{k}{4} + (\bar{\theta} - \underline{\theta})\gamma \end{aligned} \quad (\text{B.59})$$

When $\frac{2(c-\underline{\theta})(\gamma-2)}{4-\gamma} < k < \frac{4c-8\bar{\theta}+4\underline{\theta}-2c\gamma+6\bar{\theta}\gamma-4\underline{\theta}\gamma}{-4+\gamma}$

$$\begin{aligned} W_c &= \\ &\gamma\left(\frac{1}{12}\left(-4c + k + 12\bar{\theta} - 8\underline{\theta} + \frac{8(c+2k-\underline{\theta})}{-4+3\gamma}\right)\right) + \\ &(1-\gamma)\left(\frac{(-c(-2+\gamma) + \underline{\theta}(-2+\gamma) + k\gamma)^2}{k(4-3\gamma)^2}\right) \\ &= \frac{(1-\gamma)(-c(-2+\gamma) + \underline{\theta}(-2+\gamma) + k\gamma)^2}{k(4-3\gamma)^2} \\ &\quad + \frac{1}{12}\gamma\left(-4c + k + 12\bar{\theta} - 8\underline{\theta} + \frac{8(c+2k-\underline{\theta})}{-4+3\gamma}\right) \end{aligned} \quad (\text{B.60})$$

When $\frac{4c-8\bar{\theta}+4\underline{\theta}-2c\gamma+6\bar{\theta}\gamma-4\underline{\theta}\gamma}{-4+\gamma} \leq k \leq 2\bar{\theta} - c - \gamma\bar{\theta} - (1-\gamma)\underline{\theta}$

$$W_c = \gamma\left(\frac{k}{4}\right) + (1-\gamma)\left(\frac{(\frac{k}{2} - \bar{\theta} + \underline{\theta})^2}{k}\right) \quad (\text{B.61})$$

When $k > 2\bar{\theta} - c - \gamma\bar{\theta} - (1-\gamma)\underline{\theta}$

$$\begin{aligned} W_c &= \gamma\left(\frac{(\bar{\theta}(2-\gamma) + \underline{\theta}(\gamma-1) - c)^2}{4k}\right) + (1-\gamma)\left(\frac{(\gamma\bar{\theta} - \underline{\theta}(1+\gamma) + c)^2}{4k}\right) \\ &= \frac{\gamma(c + \underline{\theta} + \bar{\theta}(-2+\gamma) - \underline{\theta}\gamma)^2 + (1-\gamma)(c + \bar{\theta}\gamma - \underline{\theta}(1+\gamma))^2}{4k} \end{aligned} \quad (\text{B.62})$$

B.3.4 Proof of Theorem 1

Consider the equilibrium thresholds for k that arise in Proposition 3.2. Let,

$$k_1 = \frac{2(\underline{\theta} - c)}{3}, \quad k_2 = \frac{2(\bar{\theta} - c)}{3}, \quad k_3 = \underline{\theta} - c, \quad k_4 = \bar{\theta} - c.$$

Further, from Proposition 3.4, consider the equilibrium thresholds for k . Let,

$$k_5 = \frac{2(c - \underline{\theta})(\gamma - 2)}{4 - \gamma}, \quad k_6 = \frac{4c - 8\bar{\theta} + 4\underline{\theta} - 2c\gamma + 6\bar{\theta}\gamma - 4\underline{\theta}\gamma}{-4 + \gamma}, \quad k_7 = 2\bar{\theta} - c - \gamma\bar{\theta} - (1 - \gamma)\underline{\theta}.$$

Suppose $\bar{\theta} < \frac{1}{2}(3\underline{\theta} - c)$ and $c = 0$. Then,

$$k_1 < k_2 < k_5 < k_3 < k_4 < k_6 < k_7$$

if and only if either $\underline{\theta} < \bar{\theta} \leq \frac{(15 + \sqrt{33})\underline{\theta}}{16}$ and $0 < \gamma < \frac{4\bar{\theta} - 4\underline{\theta}}{5\bar{\theta} - 4\underline{\theta}}$, or $\frac{(15 + \sqrt{33})\underline{\theta}}{16} < \bar{\theta} < \frac{3\underline{\theta}}{2}$ and $0 < \gamma < \frac{4\bar{\theta} - 6\underline{\theta}}{\bar{\theta} - 3\underline{\theta}}$. Below we compare the aggregate consumer surplus from both cases. When $k \leq k_1$, the market is covered for both types in both cases, and the aggregate consumer surplus is equal. That is,

$$\text{eq}(B.58) = \text{eq}(B.16).$$

When $k \geq k_7$, the market is uncovered for both types in both cases,

$$\text{eq}(B.62) - \text{eq}(B.20) \geq 0,$$

i.e., consumers are better off under no information.

When $k_1 < k < k_2$, the market is covered for $\bar{\theta}$ and uncovered for $\underline{\theta}$ in both cases,

$$\text{eq}(B.59) - \text{eq}(B.21) > 0,$$

i.e., consumers are better off under no information.

When $k_2 < k < k_5$, the market is covered for $\bar{\theta}$ and adjacent for $\underline{\theta}$ under no information while adjacent for both types under full information.

$$\text{eq}(B.59) - \text{eq}(B.18) > 0,$$

meaning no information is better.

When $k_5 < k < k_3$, the market is covered for $\bar{\theta}$ and uncovered for $\underline{\theta}$ under no information and adjacent for both types under full information.

$$\text{eq}(B.60) - \text{eq}(B.18) > 0,$$

i.e., consumers are better off under no information.

When $k_3 < k < k_4$, the market is covered for $\bar{\theta}$ and uncovered for $\underline{\theta}$ under no information and adjacent for $\bar{\theta}$ and uncovered for $\underline{\theta}$ under full information. Further here,

- $eq(B.60) - eq(B.23) > 0$ if $\frac{(15+\sqrt{33})\underline{\theta}}{16} < \bar{\theta} < \frac{3\underline{\theta}}{2}$ and $0 < \gamma < \frac{4\bar{\theta}-6\underline{\theta}}{\bar{\theta}-3\underline{\theta}}$, i.e, then consumers are better off under no information.
- $eq(B.60) - eq(B.23) < 0$ if $\underline{\theta} < \bar{\theta} \leq \frac{(15+\sqrt{33})\underline{\theta}}{16}$ and $0 < \gamma < \frac{4\bar{\theta}-4\underline{\theta}}{5\bar{\theta}-4\underline{\theta}}$, i.e, the consumers are better off under full information.

When $k_4 < k < k_6$, the market is covered for $\bar{\theta}$ and uncovered for $\underline{\theta}$ under no information and uncovered for both types under full information

- $eq(B.60) - eq(B.20) < 0$ if $\underline{\theta} < \bar{\theta} \leq \frac{(15+\sqrt{33})\underline{\theta}}{16}$ and $0 < \gamma < \frac{4\bar{\theta}-4\underline{\theta}}{5\bar{\theta}-4\underline{\theta}}$, i.e, consumers are better off under full information.
- $eq(B.60) - eq(B.20) < 0$ if $\frac{(15+\sqrt{33})\underline{\theta}}{16} < \bar{\theta} < \frac{\underline{\theta}(\sqrt{6}+1)}{\sqrt{6}}$ and $0 < \gamma < \frac{4\bar{\theta}-6\underline{\theta}}{\bar{\theta}-3\underline{\theta}}$, i.e, consumers are better off under full information.
- $eq(B.60) - eq(B.20) \geq 0$ if $\frac{\underline{\theta}(\sqrt{6}+1)}{\sqrt{6}} < \bar{\theta} < \frac{3\underline{\theta}}{2}$ and $0 < \gamma < \frac{4\bar{\theta}-6\underline{\theta}}{\bar{\theta}-3\underline{\theta}}$, i.e, consumers are better off under no information.

When $k_6 < k < k_7$, the market is adjacent for $\bar{\theta}$ and uncovered for $\underline{\theta}$ under no information and uncovered for both types under full information,

$$eq(B.61) - eq(B.20) > 0,$$

i.e, consumers are better off under no information.

B.3.5 Alternative Pecking Order of k Thresholds under no information

Suppose $\bar{\theta} < \frac{1}{2}(3\underline{\theta} - c)$.

Consider the equilibrium thresholds for k that arise in Proposition 3.2. Let,

$$k_1 = \frac{2(\underline{\theta} - c)}{3}, \quad k_2 = \frac{2(\bar{\theta} - c)}{3}, \quad k_3 = \underline{\theta} - c, \quad k_4 = \bar{\theta} - c.$$

Further, from Proposition 3.4, consider the equilibrium thresholds for k . Let,

$$k_5 = \frac{2(c - \underline{\theta})(\gamma - 2)}{4 - \gamma}, \quad k_6 = \frac{4c - 8\bar{\theta} + 4\underline{\theta} - 2c\gamma + 6\bar{\theta}\gamma - 4\underline{\theta}\gamma}{-4 + \gamma}, \quad k_7 = 2\bar{\theta} - c - \gamma\bar{\theta} - (1 - \gamma)\underline{\theta}.$$

When $k \leq k_1$, the market is covered for both types in both cases, and the aggregate consumer surplus is equal. That is,

$$eq(B.58) = eq(B.16).$$

When $k \geq k_7$, the market is uncovered for both types in both cases, such that

$$eq(B.62) - eq(B.20) \geq 0,$$

i.e., consumers are better off under no information. To compare consumer surplus under full information and no information, we first examine the ordering of the k thresholds derived from the previous propositions.

When combined, these thresholds define eight distinct regions, which are useful for analyzing consumer surplus differences. However, before proceeding with the comparison, we establish the ordering of these thresholds. The relative ordering of k thresholds depends on γ and $\bar{\theta}$.

Consider the following two cases:

1. **Case A:** $k_1 < k_5 < k_2 < k_3 < k_4 < k_6 < k_7$.
2. **Case B:** $k_1 < k_5 < k_2 < k_3 < k_6 < k_4 < k_7$.

Suppose $c = 0$. Then, the existence of different cases depends on the values of $\underline{\theta}$, $\bar{\theta}$ and γ as follows:

- **Case A exists if and only if**

$$\frac{15\underline{\theta} + \sqrt{33}\sqrt{\underline{\theta}^2}}{16} < \bar{\theta} < \frac{3\underline{\theta}}{2}$$

and

$$\frac{4\bar{\theta} - 6\underline{\theta}}{\bar{\theta} - 3\underline{\theta}} < \gamma < \frac{4\bar{\theta} - 4\underline{\theta}}{5\bar{\theta} - 4\underline{\theta}}.$$

- **Case B exists if and only if Either**

$$\frac{3\underline{\theta} + \sqrt{3}\sqrt{\underline{\theta}^2}}{4} < \bar{\theta} < \frac{15\underline{\theta} + \sqrt{33}\sqrt{\underline{\theta}^2}}{16}$$

and

$$\frac{4\bar{\theta} - 6\underline{\theta}}{\bar{\theta} - 3\underline{\theta}} < \gamma < \frac{8\bar{\theta} - 8\underline{\theta}}{6\bar{\theta} - 5\underline{\theta}},$$

or

$$\frac{15\underline{\theta} + \sqrt{33}\sqrt{\underline{\theta}^2}}{16} < \bar{\theta} < \frac{3\underline{\theta}}{2}$$

and

$$\frac{4\bar{\theta} - 4\underline{\theta}}{5\bar{\theta} - 4\underline{\theta}} < \gamma < \frac{8\bar{\theta} - 8\underline{\theta}}{6\bar{\theta} - 5\underline{\theta}}.$$

To compare consumer surplus under full information and no information, we analyze the different cases based on the ordering of the k thresholds.

Case A: In this case, no information is always better:

- When $k_1 < k < k_5$, eq (B.59) > eq (B.21), meaning no information is better.
- When $k_5 < k < k_2$, eq(B.60) > eq(B.21), meaning no information is better.
- When $k_2 < k < k_3$, eq(B.60) > eq(B.18), meaning no information is better.
- When $k_3 < k < k_4$, eq(B.60) > eq(B.23), meaning no information is better.

- When $k_4 < k < k_6$, eq(B.60) > eq(B.20), meaning no information is better.
- When $k_6 < k < k_7$, eq(B.61) > eq(B.20), meaning no information is better.

Case B: In this case, full information is better only if $k_3 < k < k_6$:

- When $k_1 < k < k_5$, eq (B.59) > eq (B.21), meaning no information is better.
- When $k_5 < k < k_2$, eq (B.60) > eq (B.21), meaning no information is better.
- When $k_2 < k < k_3$, eq (B.60) > eq (B.18), meaning no information is better.
- When $k_3 < k < k_6$, eq (B.60) > eq (B.23), meaning no information is better.
- When $k_6 < k < k_4$, eq (B.61) < eq (B.23), meaning full information is better.
- When $k_4 < k < k_7$, eq (B.61) > eq (B.20), meaning no information is better.

B.4 Alternative Model

In this section we describe an alternative model for robustness check. Like our main model, consumer preferences differ along two dimensions: horizontal and vertical. On the horizontal dimension, consumers have different tastes for brands; on the vertical dimension, consumers differ in their preferences for quality.

The taste for brands is modelled as the horizontal location of a consumer on a Hotelling line. Specifically, a continuum of consumers of mass 1 is distributed uniformly over the $[0, 1]$ interval. The location of a consumer on the horizontal line, denoted by $d \in [0, 1]$, represents an ideal brand for that consumer. Differently from our main model. The consumers' varying preferences over quality is captured by the marginal utility of consumption θ . Parameter θ will be referred to as consumer's vertical type. At each location, the vertical types of consumers are uniformly distributed over the unit interval, $\theta \sim U[0, 1]$. A consumer's horizontal location and vertical type are independent.

There are two firms indexed by $i \in \{1, 2\}$. Firm 1 is located at 0, while firm 2 is located at 1. Each firm i offers a distinct brand $i \in \{1, 2\}$. Firm's offer specifies quality q and price t of its product.

Each consumer purchases at most one product. If a consumer, who has vertical type θ and who is located distance \tilde{d} away from the brand, buys this brand's product, her utility is

$$u(q, t, \tilde{d}, \theta) = \theta q - t - k\tilde{d}, \quad (\text{B.63})$$

where k represents the per unit transport cost, and can be viewed as the degree of brand differentiation. A low value of k implies little horizontal product differentiation. The reservation utility of a consumer who does not make a purchase is zero.

Brands have the same production technology which results in per-customer profit

$$\pi(t, q) = t - \frac{q^2}{2}, \quad (\text{B.64})$$

where $c(q) \equiv \frac{q^2}{2}$ is the cost of supplying q quality.

The aim is to assess the effect of providing firms with information about consumer's vertical type on market outcomes. To this end, we compare equilibrium outcomes in two different cases. First, we consider the case, in which both firms are informed about the vertical type of the consumer they are facing. Second, we consider the case, in which neither firm is informed about the consumer's vertical type.

B.4.1 Firms Informed about Vertical Type

When a firm is informed about the vertical type of its consumer, it can tailor its offer to the consumer's type. In this section, we assume that both firms are informed and make type-specific offers. Proposition B.1 characterises offers of firms to an arbitrary vertical type of consumer, θ , in the unique symmetric equilibrium.

In equilibrium, each firm always offers quality equal to the consumer's vertical type. Such quality offer is efficient as it maximises total surplus

$$\theta q - q^2/2.$$

The price the firm charges for this quality depends on transport cost k .

When transport cost is low relative to the vertical type θ , that is, $k \leq \frac{\theta^2}{3}$, the market is horizontally covered and firms actively compete with each other. That is, type θ consumers are served along the entire Hotelling line and, at each location, derive a strictly positive utility from purchasing some product. Each firm i charges a price that is equal to the cost of providing quality θ plus a markup due to brand differentiation, which is equal to the per unit transport cost.

When transport cost is high $k \geq \frac{\theta^2}{2}$, each firm acts as a local monopolist for consumers who are located sufficiently close to it. In this case, the market is not fully horizontally covered, that is, at some locations type θ consumers are not served. Here, the firm i enjoys a monopoly premium over the cost of providing quality θ .

For intermediate values of transport costs, $\frac{\theta^2}{3} < k < \frac{\theta^2}{2}$, firms serve adjacent horizontal segments but do not actively compete with each other. In this case, type θ consumers are served along the entire Hotelling line, but the consumer who is indifferent between the firms is also indifferent to buying, that is, this consumer obtains utility of zero.

Proposition B.1. *Suppose that both firms know that they are facing a consumer of vertical type θ . Then, symmetric equilibrium is unique. In this equilibrium, each firm offers quality $q^* = \theta$ and, depending on the value of transport cost k , three possible pricing patterns arise.*

1. **Covered Market:** If $k \leq \frac{\theta^2}{3}$, then each firm charges price

$$t_c^* = k + \frac{\theta^2}{2}.$$

Market is horizontally covered and a consumer of type θ derives positive utility from purchasing along the entire Hotelling line.

2. **Adjacent Market:** If $\frac{\theta^2}{3} < k < \frac{\theta^2}{2}$, then each firm charges price

$$t_a^* = \theta^2 - \frac{k}{2}.$$

Market is horizontally covered but a consumer of type θ located in the middle of the Hotelling line derives is indifferent to purchasing.

3. **Uncovered Market:** If $k \geq \frac{\theta^2}{2}$, then each firm charges price

$$t_u^* = \frac{3\theta^2}{4}.$$

Market is not horizontally covered and consumers of type θ in the middle of the Hotelling line are not served.

Proof. Consider a consumer at some location \hat{d} who is indifferent between purchasing from either firm:

$$\begin{aligned} u(q_1, t_1, \hat{d}, \theta) &= u(q_2, t_2, 1 - \hat{d}, \theta) \\ \theta q_1 - t_1 - k\hat{d} &= \theta q_2 - t_2 - k(1 - \hat{d}) \\ \hat{d} &= \frac{\theta(q_1 - q_2) - (t_1 - t_2) + k}{2k} \end{aligned} \quad (\text{B.65})$$

From above, the demand of Firm 1 and Firm 2 is

$$\begin{aligned} Q_1 &= \hat{d} = \frac{\theta(q_1 - q_2) - (t_1 - t_2) + k}{2k} \\ Q_2 &= 1 - \hat{d} = 1 - \frac{\theta(q_1 - q_2) - (t_1 - t_2) + k}{2k} \end{aligned}$$

Given the above demand Q_1 , the firm 1's profit maximization problem is as follows:

$$\max_{t_1, q_1} \pi_1(t_1, q_1) = \left(t_1 - \frac{q_1^2}{2} \right) \frac{\theta(q_1 - q_2) - (t_1 - t_2) + k}{2k} \quad (\text{B.66})$$

subject to consumers' participation constraints

$$\theta q_1 - t_1 - k\hat{d} \geq 0. \quad (\text{B.67})$$

$$\mathcal{L} = \left(t_1 - \frac{q_1^2}{2} \right) \frac{\theta(q_1 - q_2) - (t_1 - t_2) + k}{2k} + \lambda(\theta q_1 - t_1 - k\hat{d}) \quad (\text{B.68})$$

The optimum satisfies first order conditions

$$\frac{d\pi_1(t_1, q_1)}{dt_1} = 0 \Rightarrow \frac{2k + q_1^2 - 4t_1 + 2t_2 + 2\theta(q_1 - q_2) - 2k\lambda}{4k} = 0 \quad (\text{B.69})$$

$$\frac{d\pi_1(t_1, q_1)}{dq_1} = 0 \Rightarrow \frac{-3q_1^2\theta + 2t_1\theta + 2q_1(t_1 - t_2 + q_2\theta) - 2k(q_1 - \theta\lambda)}{4k} = 0 \quad (\text{B.70})$$

and a complementary slackness condition

$$\lambda \geq 0; \quad \theta q_1 - t_1 - k\hat{d} \geq 0 \quad \lambda(\theta q_1 - t_1 - k\hat{d}) = 0.$$

Covered Market: Suppose that the participation constraint is not binding, so that $\theta q_1 - t_1 - k\hat{d} > 0$ and $\lambda = 0$. Then, the firms first order conditions (B.69)-(B.70) become:

$$2k + q_1^2 - 4t_1 + 2t_2 + 2\theta(q_1 - q_2) = 0 \quad (\text{B.71})$$

$$-3q_1^2\theta + 2t_1\theta + 2q_1(t_1 - t_2 + q_2\theta) - 2kq_1 = 0. \quad (\text{B.72})$$

In the symmetric equilibrium, $t_1 = t_2 = t_c^*$ and $q_1 = q_2 = q_c^*$. These equilibrium conditions can now be substituted into the system of the first order conditions (B.71)-(B.72) to solve for t_c^* and q_c^* :

$$\begin{aligned} q_c^* &= \theta \\ t_c^* &= k + \frac{\theta^2}{2}, \end{aligned}$$

which results in profit

$$\pi_c^* = \frac{k}{2}.$$

The subscript c above refers to covered market.

Lagrange multiplier $\lambda = 0$ when the utility of the indifferent consumer at location \hat{d} defined in (B.65) is non-negative. By (B.65), in the symmetric equilibrium, $\hat{d} = 1/2$ and so

$$\begin{aligned} u(q_c^*, t_c^*, \frac{1}{2}, \theta) &\geq 0 \\ \Rightarrow \theta q_c^* - t_c^* - k\frac{1}{2} &\geq 0 \\ \Rightarrow \frac{\theta^2 - 3k}{2} &\geq 0 \end{aligned}$$

Therefore, when

$$k \leq \frac{\theta^2}{3},$$

$\lambda = 0$ and thus, in the symmetric equilibrium, each firm's offer is (q_c^*, t_c^*) .

Adjacent Markets: Suppose $\lambda > 0$. In the symmetric equilibrium, $t_1 = t_2 = t_a^*$ and $q_1 = q_2 = q_a^*$. Substituting these equilibrium conditions into the system of the first order conditions (B.71)-(B.72)

yields:

$$q_a^{*2} + 2k - 2t_a^* - 2k\lambda = 0 \quad (\text{B.73})$$

$$\theta q_a^{*2} + 2kq - 2\theta(t_a^* + k\lambda) = 0. \quad (\text{B.74})$$

When $\lambda > 0$, the utility of the indifferent consumer at location \hat{d} defined in (B.65) is zero. Thus,

$$\theta q_a^* - t_a^* - k\hat{d} = 0 \quad \Rightarrow \quad t_a^* = \theta q_a^* - \frac{k}{2}. \quad (\text{B.75})$$

Substituting t_a^* into the system of the first order conditions (B.71)-(B.72) and solving for q_a^* and λ yields

$$q_a^* = \theta \quad (\text{B.76})$$

$$\lambda = \frac{3k - \theta^2}{2k}. \quad (\text{B.77})$$

Therefore, the equilibrium offer $\{q_a^*, t_a^*\}$ is

$$q_a^* = \theta$$

$$t_a^* = \theta^2 - \frac{k}{2},$$

which results in profit

$$\pi_a^* = \frac{\theta^2 - k}{4}.$$

The subscript a above refers to adjacent market equilibrium, in which the firms serve adjacent segments on the Hotelling line. From (B.77), $\lambda > 0$ requires that $k > \frac{\theta^2}{3}$.

Uncovered Market: When market is uncovered, each firm acts as a local monopolist serving consumers that are located sufficiently close to it. A consumer who is indifferent between purchasing the product from firm 1 and not purchasing at all is located at \tilde{d} :

$$u(q_1, t_1, \tilde{d}, \theta) = 0$$

$$\theta q_1 - t_1 - k\tilde{d} = 0$$

$$\tilde{d} = \frac{\theta q_1 - t_1}{k}. \quad (\text{B.78})$$

Since type θ consumer will purchase from firm 1 as long as she derives non-negative utility, \tilde{d} defines demand for firm 1's product. Hence, firm 1's profit maximization problem is

$$\max_{t_1, q_1} \pi_1(t_1, q_1) = \left(t_1 - \frac{q_1^2}{2} \right) \frac{\theta q_1 - t_1}{k}$$

The first order conditions to the above maximization problems are as follows:

$$\begin{aligned}\frac{d\pi_1(t_1, q_1)}{dt_1} = 0 &\Rightarrow \frac{2\theta q_1 + q_1^2 - 4t_1}{2k} = 0 \\ \frac{d\pi_1(t_1, q_1)}{dq_1} = 0 &\Rightarrow \frac{2q_1 t_1 - 3q_1^2 \theta + 2t_1 \theta}{2k} = 0\end{aligned}$$

The system of equations has three solutions: $(0, 0)$, $(2\theta, 2\theta^2)$ and $(\theta, 3\theta^2/4)$. However, only the last solution satisfies the second order conditions for maximisation. Therefore, the uniquely optimal quality and price are

$$\begin{aligned}q_u^* &= \theta \\ t_u^* &= \frac{3\theta^2}{4},\end{aligned}$$

with the resulting profit

$$\pi_u^* = \frac{\theta^4}{16k}$$

The subscript u above refers to uncovered market equilibrium.

By (B.78), the indifferent consumer is located at $\tilde{d}_u = \frac{\theta^2}{4k}$. There is a gap in horizontal market coverage if

$$\tilde{d}_u = \frac{\theta^2}{4k} \leq \frac{1}{2}$$

Thus, the market is not fully covered for θ type of consumer when

$$k \geq \frac{\theta^2}{2}.$$

□

In equilibrium, if consumer is located sufficiently close to a firm, she will accept the firm's offer irrespective of θ . That is, there is full vertical market coverage. The full vertical coverage is achieved through varying quality – consumers with lower θ who value quality less receive offers featuring lower quality.

Proposition B.1 implies that for a given k , in equilibrium firms compete vigorously for high vertical types with $\theta \geq \min\{\sqrt{3k}, 1\}$, act as local monopolists for low vertical types with $\theta \leq \min\{\sqrt{2k}, 1\}$ and accommodate each other for intermediate vertical types with $\min\{\sqrt{2k}, 1\} < \theta < \min\{\sqrt{3k}, 1\}$. As k increases, the set of vertical type over which firms actively compete shrinks. Thus, firms compete vigorously with each other for some vertical types only if $k < 1/3$; whenever $1/2 > k > 1/3$, firms either accommodate each other or act as local monopolists, but there is no vigorous competition over any vertical type; as k increases further to $k > 1/2$, firms act as local monopolists for all vertical types.

At any location, consumers with higher vertical types derive weakly higher utility because they value the good more and because competition for them is more intense. To illustrate this, Figure B.3 depicts the consumers' utility when disregarding her transport expenditure. This quantity summarises

the firm's offer and we will refer to it as the rent provision.

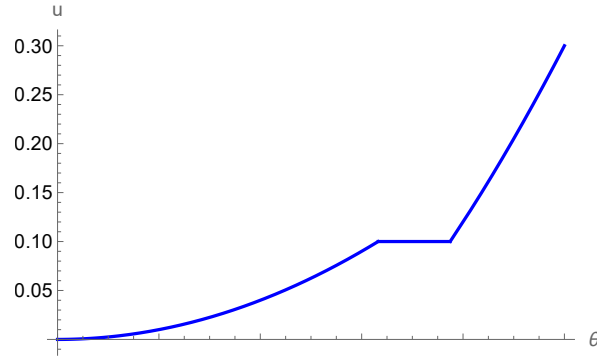


Figure B.3: Rent provision as a function of vertical type when $k = 0.2$.

The overall welfare of a consumer with type θ located distance d from the closest firm is

$$W(\theta) = \begin{cases} \theta^2 - t_u^* - kd, & \text{if } \theta \leq \min\{\sqrt{2k}, 1\} \\ \theta^2 - t_a^* - kd, & \text{if } \min\{\sqrt{2k}, 1\} < \theta < \min\{\sqrt{3k}, 1\} \\ \theta^2 - t_c^* - kd, & \text{otherwise} \end{cases} \quad (\text{B.79})$$

The aggregate consumer welfare is as follows:

$$W = \int_0^{\sqrt{2k}} \left(\int_0^{\bar{d}} (\theta^2 - t_u^* - kd) dd \right) d\theta + \int_{\sqrt{2k}}^{\sqrt{3k}} \left(\int_0^{\hat{d}} (\theta^2 - t_a^* - kd) dd \right) d\theta + \int_{\sqrt{3k}}^1 \left(\int_0^{\hat{d}} (\theta^2 - t_c^* - kd) dd \right) d\theta$$

1. if $k > \frac{1}{2}$

$$W = \int_0^1 \left(\int_0^{\bar{d}} (\theta^2 - t_u^* - kd) dd \right) d\theta$$

2. if $\frac{1}{3} < k < \frac{1}{2}$

$$W = \int_0^{\sqrt{2k}} \left(\int_0^{\bar{d}} (\theta^2 - t_u^* - kd) dd \right) d\theta + \int_{\sqrt{2k}}^1 \left(\int_0^{\hat{d}} (\theta^2 - t_a^* - kd) dd \right) d\theta$$

3. if $k < \frac{1}{3}$

$$W = \int_0^{\sqrt{2k}} \left(\int_0^{\bar{d}} (\theta^2 - t_u^* - kd) dd \right) d\theta + \int_{\sqrt{2k}}^{\sqrt{3k}} \left(\int_0^{\hat{d}} (\theta^2 - t_a^* - kd) dd \right) d\theta + \int_{\sqrt{3k}}^1 \left(\int_0^{\hat{d}} (\theta^2 - t_c^* - kd) dd \right) d\theta$$

$$W = \begin{cases} \frac{1}{160k} & \text{if } k > \frac{1}{2} \\ \frac{(5-4\sqrt{2}\sqrt{k})k}{40} & \text{if } \frac{1}{3} < k < \frac{1}{2} \\ \frac{1}{12} - \frac{5k}{8} - \frac{(\sqrt{2}-5\sqrt{3})k^3/2}{10} & \text{if } k < \frac{1}{3} \end{cases}$$

From the above we know that, in the case when the market is covered ($k < \frac{1}{3}$), there is intense competition such that the aggregate consumer welfare is maximized at low values of k . Here, the prices are low. In the case where the market is uncovered ($k > \frac{1}{2}$), firms enjoy the market power and the consumer welfare is lower compared to the covered market case as the prices are higher. In the case where the markets are adjacent to each other ($\frac{1}{3} < k < \frac{1}{2}$), the aggregate consumer welfare is close to zero. Here, the market is somewhat covered but competition is not intense enough, leading to lower aggregate consumer welfare.

B.4.2 Firms Uninformed about Vertical Type

In this section, we assume that firms do not know the vertical type of the consumer that they are facing and so cannot tailor their offer to each vertical type. Therefore, each firm i makes a single offer $\{q_i, t_i\}$.

Proposition B.2 characterizes the offers each firm makes when they are not informed about the vertical type of the consumer, in the symmetric equilibrium. When the transportation cost is $k > k^*$, firms become local monopolies for all vertical types $\theta \in \{\underline{\theta}, 1\}$. Each firm makes a uniform offer $\{q^*, t^*\}$ at which some vertical types have a positive utility and firms make a positive profit. When the transportation cost is $k < k^*$, the firms compete with each other. Here, the uniform offer as a function of k increases as the transportation cost increases and equals the local monopoly offer at $k = k^*$. See Figure B.4.

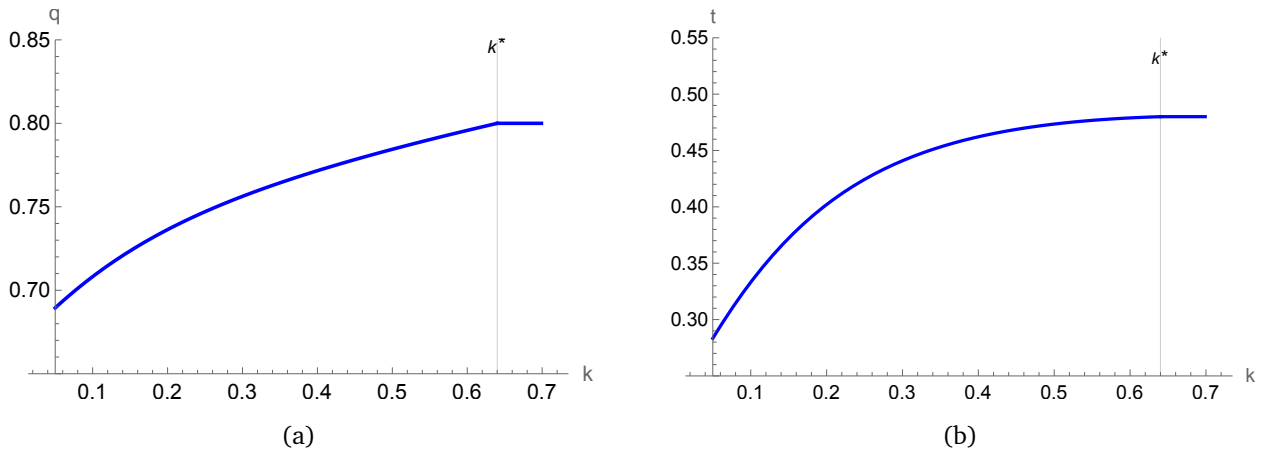


Figure B.4: (a) and (b) show the $\{q, t\}$ offer in equilibrium when both firms are uninformed about the vertical type of the consumer.

Proposition B.2. *Suppose that both firms do not know the vertical type of the consumer they are facing. In the symmetric equilibrium, depending on the value of transport cost k two possible $\{quality, price\}$ offer patterns arise.*

1. **Local Monopoly Market:** if $k > k^*$, then each firm makes an offers

$$q^* = \frac{4}{5} \quad t^* = \frac{12}{25}$$

Market is not horizontally covered and the lowest vertical type of the consumer served in the market is $\underline{\theta} = \frac{3}{5}$

2. **Duopoly Market:** if $k < k^*$, then each firm makes an offer that increases in k and equals the local monopoly market offer when $k = k^*$. The market is horizontally covered for some θ types. For low values of k , the lowest vertical type of consumer served in the market $\underline{\theta} < \frac{3}{5}$.

Proof. Local Monopoly Market: Each firm makes a single offer $\{q, t\}$ and acts as a local monopolist. A consumer of vertical type θ , located at distance d away from the monopolist will make a purchase if and only if her utility from doing so exceeds her reservation utility of zero. Demand for firm 1 from θ is defined in equation B.78.

The aggregate demand for firm 1 for all vertical types that the firm 1 offers will be as follows:

$$D(q_1, t_1) = \int_{\underline{\theta}}^1 \frac{\theta q_1 - t_1}{k} \theta$$

Here, $\underline{\theta}$ is the lowest vertical type that is indifferent between purchasing and not purchasing from firm 1, when 1 is her ideal brand (i.e., when $d = 0$), that is¹²,

$$\underline{\theta} = \frac{t_1}{q_1}.$$

The firm 1 selects the offer $\{q_1, t_1\}$ to maximize

$$\left(t_1 - \frac{1}{2} q_1^2 \right) D(q_1, t_1). \quad (\text{B.80})$$

The first order conditions with respect to q_1 and t_1 are respectively presented below:

$$\frac{(q_1 - t_1)(3q_1^3 - q_1(2 + q_1)t_1 - 2t_1^2)}{4kq_1^2} = 0$$

$$\frac{(q_1 - t_1)(q_1 + q_1^2 - 3t_1)}{2kq_1} = 0$$

The system of equations has three solutions: $(0, 0)$, $(\frac{4}{5}, \frac{12}{25})$, $(2, 2)$. However, only the middle roots sat-

¹²The monopolist never serves vertical types below $\underline{\theta}$, where $\underline{\theta}$ represents the lowest vertical type distanced $d = 0$ away who would purchase the product

isfies the second order condition. The optimal offer $\{q^*, t^*\}$ of the monopolist is as follows:

$$q^* = \frac{4}{5}$$

$$t^* = \frac{12}{25}$$

The lowest vertical type that is served in this local monopoly market is $\underline{\theta} = \frac{3}{5}$. Further, the firm will switch to horizontally serving only this monopoly region when k is high enough such that $\hat{\theta} \geq 1$. $\hat{\theta}$, defined below, is the marginal consumer, at some horizontal location who is indifferent between purchasing from firm 1, firm 2 and not buying at all. If both firms are local monopolies then, $\hat{\theta} \geq 1$ when $k \geq 0.64$. Therefore, $k^* = 0.64$ By symmetry, firm 2 would make the same equilibrium offer.

Duopoly Market: A customer of vertical type θ who is located at distance d away from firm 1 will purchase firm 1's product if and only if:

$$\theta q_1 - t_1 - kd \geq \max \left\{ \underbrace{0}_{\text{reservation utility}}, \underbrace{\theta q_2 - t_2 - k(1-d)}_{\text{utility from purchasing the other brand}} \right\},$$

which can be rearranged to give demand from vertical type θ :

$$d \leq \min \left\{ \frac{\theta q_1 - t_1}{k}, \frac{1}{2} + \frac{\theta (q_1 - q_2) - (t_1 - t_2)}{2k} \right\}$$

In this environment, firm 1 competes for the vertical types above a certain threshold $\hat{\theta}$, but acts as a local monopolist for customers with vertical types below $\hat{\theta}$ but above threshold $\underline{\theta}$. Here, $\hat{\theta}$ is the lowest vertical type who obtains a non-negative utility from switching to the competitor at the horizontal location where she is indifferent between buying brand 1 and dropping out from the market:

$$\hat{\theta} = \frac{k + (t_1 + t_2)}{(q_1 + q_2)}.$$

Demand for firm 1 is given by :

$$D(q_1, t_1, q_2, t_2) = \underbrace{\int_{\underline{\theta}}^{\hat{\theta}} \frac{\theta q_1 - t_1}{k} \theta}_{\text{demand in the monopoly region}} + \underbrace{\int_{\hat{\theta}}^1 \left(\frac{1}{2} + \frac{\theta (q_1 - q_2) - (t_1 - t_2)}{2k} \right) \theta}_{\text{demand in the competitive region}}$$

Taking the offer $\{t_2, q_2\}$ of its rival as given, firm 1 selects the offer $\{t_1, q_1\}$ to maximise

$$\left(t_1 - \frac{1}{2}q_1^2\right)D(q_1, t_1, q_2, t_2). \quad (\text{B.81})$$

In a symmetric equilibrium, $t_1 = t_2 \equiv t$ and $q_1 = q_2 \equiv q$. These equilibrium restrictions are imposed after deriving the first order conditions of (B.81) with respect to t_1 and q_1 . The first order conditions are presented below, with respect to t_1 and q_1 , respectively, after imposing the above symmetry.

$$\frac{-k^2 + kq(4 + q) + 2(q^2 - 2t)(q - t) - 6kt}{8kq} = 0$$

$$\frac{-4(q^2 - 2t)(q - t)(q + t) + k^2(3q^2 + 2t) + 4k(-4q^3 + 3q^2t + 2t^2)}{32kq^2} = 0$$

The first order condition with respect to t above give the two following expressions of t as a function of q . We denote them as t_a and t_b

$$t_a = \frac{1}{4} \left(3k + 2q + q^2 - \sqrt{13k^2 - 4kq + 4q^2 + 2kq^2 - 4q^3 + q^4} \right)$$

$$t_b = \frac{1}{4} \left(3k + 2q + q^2 + \sqrt{13k^2 - 4kq + 4q^2 + 2kq^2 - 4q^3 + q^4} \right)$$

Substituting these into the first order condition with respect to q yields two alternative functions of q that we label FOC1 and FOC2. In equilibrium, the first order condition with respect to q is satisfied when either FOC1 or FOC2 is zero. We seek the values of $q \in (0, 2)$ that set FOC1 or FOC2 to zero. The range $(0, 2)$ is the only relevant range for q because in equilibrium, (i) $t < q$ to ensure that some vertical types at some locations have a positive utility and (ii) $t > q^2/2$ to ensure a positive per unit profit to the firms. These two conditions can be satisfied simultaneously only when $q < 2$. Figure B.5 below reproduces FOC1 and FOC2 for $q \in (0, 2)$ and $k = 0.1, 0.2, 0.5$.

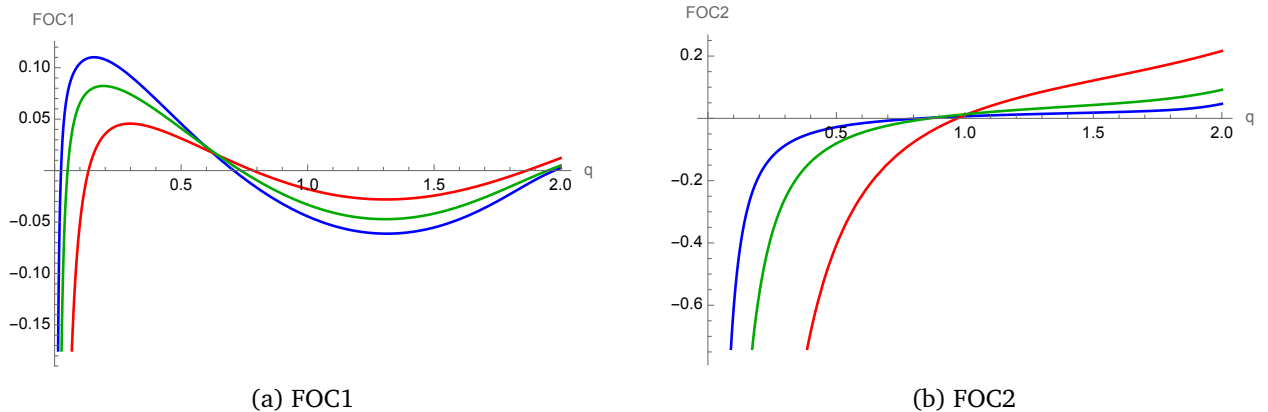


Figure B.5: The first order condition with respect to q after substituting the two values of t from the first order condition with respect to t : the blue line corresponds to $k = 0.1$, the red line to $k = 0.2$ and the green line to $k = 0.5$.

From Figure B.5 it is clear that for any $k < k^*$, both FOC1 and FOC2 intersect zero in the relevant parameter range. First, solving $FOC1 = 0$ for q after substituting the above t_a yields $\{q, t\} = \left\{ \frac{2+\sqrt{2}\sqrt{2-k}}{2}, \frac{8+4\sqrt{4-2k+5k-7\sqrt{k^2}}}{8} \right\}$. This combination yields zero profits. Further, solving $FOC2 = 0$ for q after substituting the above t_b results in $\{q, t\} = \left\{ \frac{2-\sqrt{2}\sqrt{2-k}}{2}, \frac{8-4\sqrt{4-2k+5k+7\sqrt{k^2}}}{8} \right\}$. This combination yields a non-positive utility to all types located at $d = 0$. See Fig B.6.

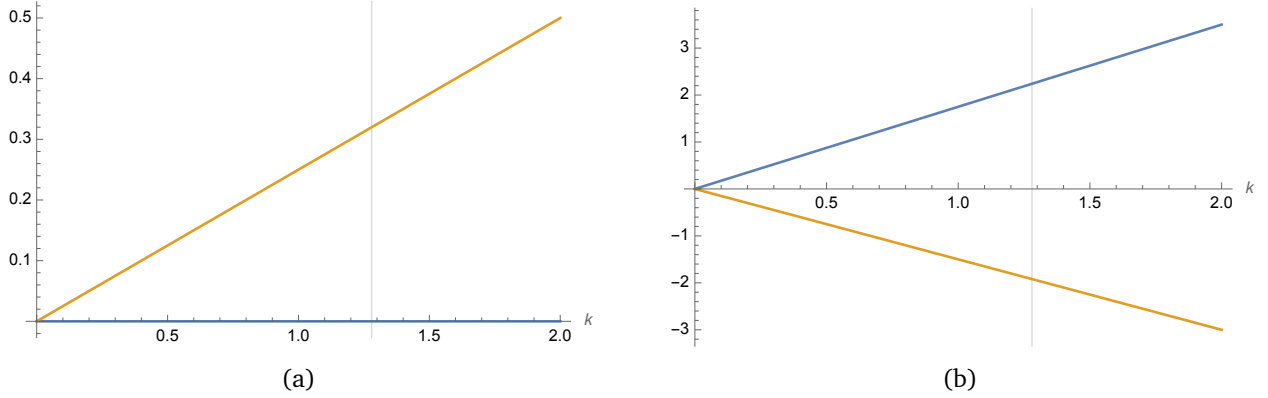


Figure B.6: (a) and (b) show the Profit and Utility from $\{q, t\}$ combination derived from FOC1 and FOC2 respectively: the blue line corresponds to Profits and the orange line to Utility.

Second, according to the second order conditions, the roots $\{q, t\} = \left\{ \frac{2+\sqrt{2}\sqrt{2-k}}{2}, \frac{8+4\sqrt{4-2k+5k-7\sqrt{k^2}}}{8} \right\}$, correspond to a saddle point and the roots $\{q, t\} = \left\{ \frac{2-\sqrt{2}\sqrt{2-k}}{2}, \frac{8-4\sqrt{4-2k+5k+7\sqrt{k^2}}}{8} \right\}$ correspond to a minimum. Thus, we discard both. Third, as from Figure B.5 we know that FOC1 and FOC2 equal zero for some parameters. For a given k , the relevant $\{q, t\}$ combination that satisfies the second order condition and gives non-negative profit and utility can be found numerically. We use the *FindRoot* function in *Mathematica* to do so. From the above local monopoly market case, $q^* = \frac{4}{5}$, we start with an initial guess such that $q = \frac{4}{5}$. At $k = 0.5$, *Mathematica*, finds numerical roots, $q = 0.784$ and $t = 0.473$. These roots satisfy the second order condition for a maximum and give positive profits and utility. Therefore, the system of the first order conditions has multiple solutions, but only one of them also satisfies the second order conditions, yields positive profit and positive utility to some vertical types. The unique relevant solution can be found numerically. Overall, in equilibrium when $k \geq k^*$, firms only serve in their local monopoly region. Otherwise, the two firms compete with each other. At $k = k^*$, the firm becomes a local monopolist for the highest vertical type of the consumer. Therefore, the number of vertical types for which each firm becomes a local monopolist increases in k . The $\{q_1, t_1\}$ offer made by firm 1 in the duopoly environment increases in k and equals the monopoly market solution at $k = k^*$. See Figure B.4.

□

The above Proposition B.2 implies, for both, local monopoly and duopoly markets the vertical market is always uncovered. We define vertical market coverage as 1 minus the lowest vertical that is served at firm's own location ,i.e, the mass of vertical types served at $d = 0$. For $k > k^*$, the lowest vertical type of the consumer served by firm 1 is $\underline{\theta} = \frac{t^*}{q^*} = \frac{3}{5}$. As for $k < k^*$ both, price and quality offered here, in the duopoly market, are lower than local monopoly markets. See Figure B.4. For low values of k , the lowest vertical type of consumer served by firm 1 will be lower than $\underline{\theta} = \frac{t^*}{q^*} = \frac{3}{5}$. Therefore,

vertical market coverage will be higher in the duopoly market compared to the local monopoly case. As k increases, the lowest vertical type of consumer served by firm 1 in the duopoly market would increase and equal $\underline{\theta} = \frac{t^*}{q^*} = \frac{3}{5}$ at $k = k^*$. Therefore, as k increases vertical market coverage diminishes. See Fig B.7.

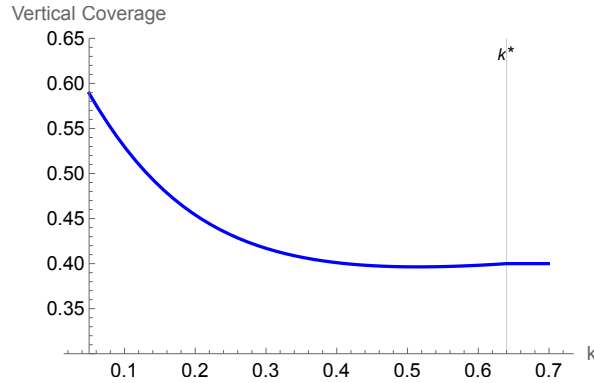


Figure B.7: Vertical Market Coverage

For low values of k , for some vertical types both firms still act as local monopolies but compete for high vertical types. The horizontal market coverage decreases as the transportation cost increases. $k = k^*$ firms switch to local monopolies, for $k < k^*$ the market is horizontally covered for some vertical types only. Further, as the transportation cost increases, consumers spend more money to travel to their preferred brand. Therefore, as k increases consumer surplus decreases. When firms engage in a duopoly market environment for $k < k^*$, consumer's benefit compared to when markets engage in a monopoly environment for $k \geq k^*$. See Fig B.8.

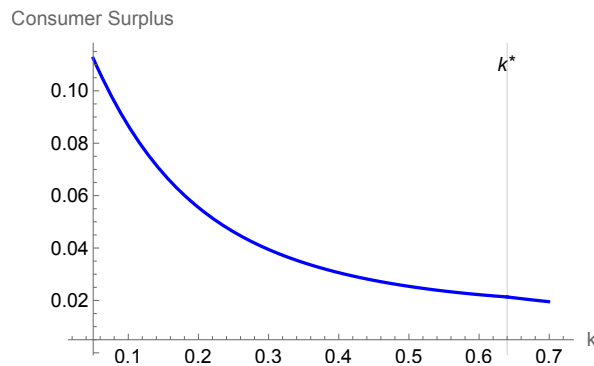


Figure B.8: Consumer Surplus when both firms are not informed.

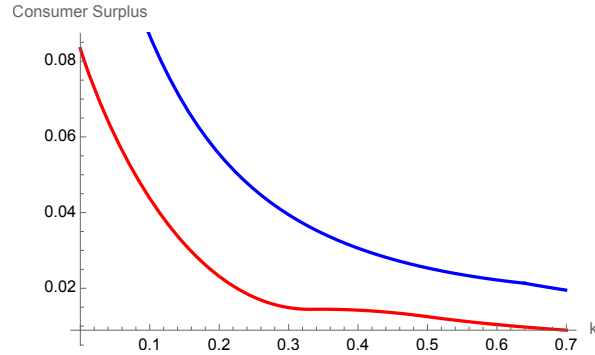


Figure B.9: *Consumer Surplus* : The red line corresponds to the case where both firms are informed and the red line corresponds to the case when both firms are uninformed about the consumer’s vertical type.

B.4.3 Concluding Remarks

In the alternative model suggested here we explore the implications of personalized pricing in a duopoly model where consumer preferences vary in two dimensions: (i) preference for quality and (ii) preference for brands. We model the consumer’s quality preference over a uniform distribution and brand preference over a Hotelling line. Firms compete by setting prices and quality of their products — that is, our objective was to evaluate how information about the consumer’s preferences affects the product design and prices simultaneously. We analyze two cases: (i) Firms are informed about the consumer’s vertical type i.e., personalized pricing is allowed. (ii) Firms have no information about the vertical type of the consumer. In case (i) firms tailor their offer to the vertical type of the consumer and the pricing framework varies as per the transportation costs. The vertical markets are always covered. The horizontal market is covered for low transportation costs and uncovered for high transportation costs. In case (ii) as transportation cost increases horizontal market coverage decreases. The vertical market coverage is higher when firms are competing with each other compared to when they are local monopolies. Unlike the baseline with exogenous product quality, in this alternative setting revealing consumers’ vertical types to firms makes consumers unambiguously worse off (see Figure B.9). Intuitively, firms gain an additional lever for surplus extraction: they can tailor both price and quality to each consumer type.

In conclusion, our analysis contributes to the existing literature on personalized pricing in online markets. It is crucial to recognize that this alternative model is a part of an ongoing research. An interesting next step would involve examining the case where one firm has information about the consumers vertical type while the other firm does not have information about the consumer’s vertical type. This case is particularly relevant to the policy makers’ concern in the online markets where information advantages are concentrated in the hands of a few firms.

C Appendices for Chapter 4

C.1 Pure Competition: $\alpha = 0$

C.1.1 Proof of Proposition 4.3

Proof. The first-order conditions of the Platform's profits with respect to the Seller's belief is as follows:

$$\frac{\partial \pi_{r|\bar{\theta}}^*}{\partial \mu} = \frac{(\beta - \gamma)\gamma(\bar{\theta} - \underline{\theta})(2\beta\bar{\theta} + \gamma(\bar{\theta} - \underline{\theta})(-1 + \mu))}{2\beta(2\beta - \gamma)^2(\beta + \gamma)}$$

$$\frac{\partial \pi_{r|\underline{\theta}}^*}{\partial \mu} = \frac{(\beta - \gamma)\gamma(\bar{\theta} - \underline{\theta})(2\beta\underline{\theta} + \gamma(\bar{\theta} - \underline{\theta})\mu)}{2\beta(2\beta - \gamma)^2(\beta + \gamma)}$$

When assumption 4.1 is satisfied and $0 \leq \mu \leq 1$, if $0 < \frac{\gamma}{\beta} < 1$ (i.e., goods are substitutes), the first-order conditions are always positive. This suggests that, for substitute goods, the Retailer's profit increases with μ , regardless of the true demand intercept.

If $-1 < \frac{\gamma}{\beta} < 0$ (i.e., goods are complements), the first-order conditions are always negative. This suggests that, for complement goods, the Retailer's profit decreases with μ , regardless of the true demand intercept. □

C.1.2 Proof Corollary 4.4

Proof. For all $\mu \in [0, 1]$, the Platform's explicit profits, $\pi_{r|\bar{\theta}}$ and $\pi_{r|\underline{\theta}}$, when the true demand intercepts are $\bar{\theta}$ and $\underline{\theta}$, respectively (see equations (4.25) and (4.26)), are equal to the monopoly profits $\frac{\bar{\theta}^2}{4\beta}$ and $\frac{\underline{\theta}^2}{4\beta}$, respectively. □

C.1.3 Proof of Proposition 4.5

Proof. When Assumption 4.1 is satisfied, for an arbitrary $\bar{\theta} > \underline{\theta} > 0$, $\beta > 0$ and $\beta > |\gamma|$,

If $0 < \gamma < 1$ i.e, when goods are substitutes.

When $\theta = \bar{\theta}$ then,

$$\pi_r^* > \pi_{r|\bar{\theta}}^*$$

$$\Rightarrow \frac{\bar{\theta}^2\beta(\beta - \gamma)}{(2\beta - \gamma)^2(\beta + \gamma)} > \frac{(\beta - \gamma)(2\beta\bar{\theta} + \gamma(\bar{\theta} - \underline{\theta})(-1 + \mu))^2}{4\beta(2\beta - \gamma)^2(\beta + \gamma)}$$

i.e, the platform's profit from his retail arm is higher when the platform is truthfully sharing information.

When $\theta = \underline{\theta}$ then,

$$\begin{aligned} \pi_r^* &< \pi_{r|\underline{\theta}}^* \\ \Rightarrow \frac{\underline{\theta}^2 \beta (\beta - \gamma)}{(2\beta - \gamma)^2 (\beta + \gamma)} &< \frac{(\beta - \gamma) (2\beta \underline{\theta} + \gamma (\bar{\theta} - \underline{\theta}) \mu)^2}{4\beta (2\beta - \gamma)^2 (\beta + \gamma)} \end{aligned}$$

i.e, the Platform's profit from its retail arm is higher when the platform reveals no information if the true demand intercept is $\underline{\theta}$ and the goods are substitutes.

If $-1 < \gamma < 0$ i.e, when goods are complements.

When $\theta = \bar{\theta}$ then,

$$\pi_r^* < \pi_{r|\bar{\theta}}^*$$

Here, the platform's profit from his retail arm is higher when the platform doesn't reveal information.

When $\theta = \underline{\theta}$

$$\pi_r^* > \pi_{r|\underline{\theta}}^*$$

Here the platform benefits from truthfully revealing the demand intercept $\underline{\theta}$. □

C.1.4 Proof of Lemma 4.6

Proof. The second derivative of the realized seller's Profit with respect to μ is as follows:

$$\frac{\partial^2(\pi_{s|k})}{\partial \mu^2} = -\frac{(\beta - \gamma)(2\beta^2 - \gamma^2)(\bar{\theta} - \underline{\theta})^2}{\beta(2\beta - \gamma)^2(\beta + \gamma)} < 0$$

Assuming $\bar{\theta} \neq \underline{\theta}$, and $-1 < \frac{\gamma}{\beta} < 1$, then the second derivative, $\frac{\partial^2(\pi_{s|k})}{\partial \mu^2}$, is less than zero, therefore the profit function of the seller is concave for all $\theta \in \{\bar{\theta}, \underline{\theta}\}$. □

C.1.5 Proof of Proposition 4.7

Proof. We can find the optimal belief for each state, μ_k^* , by maximizing the seller's realized profit, $\pi_{s|k}^*$, with respect to their belief, μ . From Lemma 4.6, we know these profit functions are concave in μ , so the first-order condition is sufficient for a maximum.

When $\theta = \underline{\theta}$, The simplified numerator of $\frac{\partial \pi_{s|\underline{\theta}}}{\partial \mu} = 0$, is as follows:

$$(\underline{\theta}\gamma^2 - 2\mu(\bar{\theta} - \underline{\theta})(2\beta^2 - \gamma^2)) = 0 \tag{C.1}$$

Solving this equation for μ gives the optimal belief, $\mu_{\underline{\theta}}^*$.

$$\mu_{\underline{\theta}}^* = \frac{\underline{\theta}\gamma^2}{2(\bar{\theta} - \underline{\theta})(2\beta^2 - \gamma^2)} \tag{C.2}$$

When $\theta = \bar{\theta}$, The simplified numerator of $\frac{\partial \pi_{s|\bar{\theta}}}{\partial \mu} = 0$, is as follows:

$$\bar{\theta}(4\beta^2 - \gamma^2) - 2(\underline{\theta} + \mu(\bar{\theta} - \underline{\theta}))(2\beta^2 - \gamma^2) = 0 \quad (\text{C.3})$$

Solving this equation for μ gives the optimal belief, $\mu_{\bar{\theta}}^*$

$$\mu_{\bar{\theta}}^* = \frac{\bar{\theta}(4\beta^2 - \gamma^2) - 2\underline{\theta}(2\beta^2 - \gamma^2)}{2(\bar{\theta} - \underline{\theta})(2\beta^2 - \gamma^2)} \quad (\text{C.4})$$

□

C.1.6 Proof of Proposition 4.8

Proof. When $\theta = \bar{\theta}$, for any belief $0 \leq \mu < 1$, the seller's profit with full information is strictly greater than their realized profit under uncertainty:

$$\pi_s^*(\bar{\theta}) > \pi_{s|\bar{\theta}}^*$$

Thus, the seller is always better off with full information in the high-demand state.

When $\theta = \underline{\theta}$, the seller can be better off under uncertainty. Their realized profit under uncertainty is greater than their full-information profit,

$$\pi_{s|\underline{\theta}}^* > \pi_s^*(\underline{\theta}),$$

When Assumption 4.1 holds.

□

C.1.7 Proof of Theorem 4.9

Proof. Let $P(\bar{\theta}) = \mu$ be the prior belief of the seller that the demand intercept is $\bar{\theta}$. Let $\bar{\Theta}$ be the signal provided by the platform saying $\theta = \bar{\theta}$. Further, let $P(\bar{\Theta}|\bar{\theta})$ be the probability of platform sending the signal that demand intercept is $\bar{\theta}$ when it indeed is $\bar{\theta}$. And let $P(\bar{\Theta}|\underline{\theta})$ be the probability of the platform sending signal that demand is $\bar{\theta}$ when it is $\underline{\theta}$.

Suppose the goods are substitutes, i.e, $0 < \gamma < 1$. Then, following Proposition 4.5, the Platform always send the signal that demand intercept is $\bar{\theta}$. Therefore, $P(\bar{\Theta}|\bar{\theta}) = P(\bar{\Theta}|\underline{\theta}) = 1$.

Let $P(\bar{\theta}|\bar{\Theta}) = \mu_1$ be the posterior belief of the seller i.e, the probability with which demand is demand is $\bar{\theta}$ when Platform sends signal $\bar{\Theta}$. Using Bayes rule,

$$P(\bar{\theta}|\bar{\Theta}) = \frac{P(\bar{\Theta}|\bar{\theta})P(\bar{\theta})}{P(\bar{\Theta})}$$

Where, $P(\bar{\Theta}) = P(\bar{\Theta}|\bar{\theta})P(\bar{\theta}) + P(\bar{\Theta}|\underline{\theta})P(\underline{\theta})$ then,

$$P(\bar{\theta}|\bar{\Theta}) = \frac{1\mu}{1\mu + 1(1-\mu)} = \mu$$

Therefore, when goods are substitutes belief remains μ . The platform cannot change the seller's belief by sending a signal.

Similarly, let $P(\underline{\theta}) = (1 - \mu)$ be the prior belief of the Seller that demand intercept is $\underline{\theta}$. Let $\underline{\Theta}$ be the signal provided by the platform saying demand is $\underline{\theta}$. Further, let $P(\underline{\Theta}|\underline{\theta})$ be the probability of platform sending the signal that demand intercept is $\underline{\theta}$ when it indeed is $\underline{\theta}$. And let $P(\underline{\Theta}|\bar{\theta})$ be the probability of the platform sending signal that demand is $\underline{\theta}$ when it is $\bar{\theta}$.

Suppose goods are complements, i.e., $-1 < \gamma < 0$.

Then, following Proposition 4.5, the Platform always send the signal that demand intercept is $\underline{\theta}$. Therefore, $P(\underline{\Theta}|\underline{\theta}) = P(\underline{\Theta}|\bar{\theta}) = 1$.

Let $P(\underline{\theta}|\underline{\Theta}) = (1 - \mu_1)$ be the posterior belief of the seller i.e, the probability with which demand is $\underline{\theta}$ when Platform sends signal $\underline{\Theta}$. Using Bayes rule,

$$P(\underline{\theta}|\underline{\Theta}) = \frac{P(\underline{\Theta}|\underline{\theta})P(\underline{\theta})}{P(\underline{\Theta})}$$

Where, $P(\underline{\Theta}) = P(\underline{\Theta}|\underline{\theta})P(\underline{\theta}) + P(\underline{\Theta}|\bar{\theta})P(\bar{\theta})$ then,

$$P(\underline{\theta}|\underline{\Theta}) = \frac{1(1 - \mu)}{1\mu + 1(1 - \mu)} = 1 - \mu$$

Therefore, when goods are complements, the seller's belief remain $(1 - \mu)$. The platform cannot change the seller's belief by sending a signal. □

C.1.8 Proof of Proposition 4.10

Proof. Consider first the high-demand state $\theta = \bar{\theta}$. From equations (4.34) and (4.35),

$$CS(\bar{\theta})_{FI} - CS_{NI|\bar{\theta}} < 0,$$

so consumer surplus is higher under no information sharing.

Now consider the low-demand state $\theta = \underline{\theta}$. From equations (4.34) and (4.36),

$$CS(\underline{\theta})_{FI} - CS_{NI|\underline{\theta}} > 0,$$

so consumer surplus is higher under full information, provided Assumption 4.1 holds. □

C.2 Competition with Commission : $\alpha \in (0, 1]$

C.2.1 Proof of Lemma 4.11

Proof.

$$\frac{\partial P_s^*}{\partial \alpha} = \frac{(\beta - \gamma)\gamma^2(2\beta + \gamma)\theta}{(4\beta^2 - (1 + \alpha)\gamma^2)^2} > 0 \quad \forall \frac{\gamma}{\beta} \in (-1, 1) \quad (\text{C.5})$$

Differentiating the equilibrium prices with respect to α yields:

$$\frac{\partial P_r^*}{\partial \alpha} = \frac{2\beta(\beta - \gamma)\gamma(2\beta + \gamma)\theta}{(4\beta^2 - (1 + \alpha)\gamma^2)^2} > 0 \quad \forall \frac{\gamma}{\beta} \in (0, 1) \quad (\text{C.6})$$

$$\frac{\partial P_r^*}{\partial \alpha} < 0 \quad \forall \frac{\gamma}{\beta} \in (-1, 0) \quad (\text{C.7})$$

□

C.2.2 Proof of Proposition 4.12

Proof. When $\theta = \bar{\theta}$, for any belief $0 \leq \mu < 1$, the seller's profit with full information is strictly greater than their realized profit under uncertainty:

$$\pi_s^*(\bar{\theta}, \alpha) > \pi(\alpha)_{s|\bar{\theta}}^*$$

Thus, the seller is always better off with full information in the high-demand state.

When $\theta = \underline{\theta}$, the seller can be better off under uncertainty. Their realized profit under uncertainty is greater than their full-information profit,

$$\pi(\alpha)_{s|\underline{\theta}}^* > \pi_s^*(\underline{\theta}, \alpha),$$

When Assumption 4.2 holds.

□

C.2.3 Proof of Proposition 4.13

Proof. When Assumption 4.2 is satisfied, for an arbitrary $\bar{\theta} > \underline{\theta} > 0$, $\beta > 0$, $\beta > |\gamma|$ and $\alpha \in (0, 1]$

If $0 < \gamma < 1$ i.e, when goods are substitutes.

When $\theta = \bar{\theta}$ then,

$$\pi_r(\bar{\theta}, \alpha)^* > \pi(\alpha)_{r|\bar{\theta}}^*$$

i.e, the platform's profit is higher when the platform is truthfully sharing information.

When $\theta = \underline{\theta}$ then,

$$\pi_r(\underline{\theta}, \alpha)^* < \pi(\alpha)_{r|\underline{\theta}}^*$$

i.e, the Platform's profit is higher when the platform reveals no information if the true demand intercept is $\underline{\theta}$ and the goods are substitutes.

If $-1 < \gamma < 0$ i.e, when goods are complements.

When $\theta = \bar{\theta}$ then,

$$\pi_r(\bar{\theta}, \alpha)^* < \pi(\alpha)_{r|\bar{\theta}}^*$$

Here, the platform's profit is higher when the platform doesn't reveal information. When $\theta = \underline{\theta}$

$$\pi_r^* > \pi_{r|\underline{\theta}}^*$$

Here the platform benefits from truthfully revealing the demand intercept $\underline{\theta}$. □

C.2.4 Proof of Proposition 4.14

Proof. Consider first the high-demand state $\theta = \bar{\theta}$.

$$CS_{FI}(\bar{\theta}, \alpha) - CS(\alpha)_{NI|\bar{\theta}} < 0,$$

so consumer surplus is higher under no information sharing.

Now consider the low-demand state $\theta = \underline{\theta}$.

$$CS_{FI}(\underline{\theta}, \alpha) - CS(\alpha)_{NI|\underline{\theta}} > 0,$$

so consumer surplus is higher under full information, provided Assumption 4.2 holds. □